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Nonlinear excitation of zonal flows by Rossby wave turbulence

J T Mendonça[1,2,6], **P K Shukla**[2,3,4,5] **and R Bingham**[2,5]

[1] IPFN, Instituto Superior Técnico, Av. Rovisco Pais 1, 1049-001 Lisboa, Portugal
[2] Centre for Fundamental Physics (CfFP), STFC Rutherford Appleton Laboratory, Chilton, Didcot, UK
[3] Institut für Theoretische Physik IV, Ruhr-Universität Bochum, D-44780 Bochum, Germany
[4] Department of Physics, Umeå University, SE-90187 Umeå, Sweden
[5] SUPA Department of Physics, University of Strathclyde, Glasgow G4 0NG, UK
E-mail: titomend@ist.utl.pt

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Abstract. We apply the wave-kinetic approach to study nonlinearly coupled Rossby wave-zonal flow fluid turbulence in a two-dimensional rotating fluid. Specifically, we consider for the first time nonlinear excitations of zonal flows by a broad spectrum of Rossby wave turbulence. Short-wavelength Rossby waves are described here as a fluid of quasi-particles, and are referred to as the 'Rossbyons'. It is shown that Reynolds stresses of Rossbyons can generate large-scale zonal flows. The result should be useful in understanding the origin of large-scale planetary and near-Earth atmospheric circulations. It also provides an example of a turbulent wave background driving a coherent structure.

[6] Author to whom any correspondence should be addressed.

Contents

1. Introduction

Although studies of fluid turbulence have been carried out over many decades, it is still one of the most important and challenging problems in physics. It has stimulated a wide range of new theoretical concepts [1, 2]. Its standard description involves the concept of an energy cascade through the fluid perturbation scale length, flowing from the unstable region where turbulence is generated to the dissipative region where it is absorbed. Direct and inverse cascades, from lower to larger scale lengths, and the inverse, are usually invoked.

In this paper, we propose a new view on fluid turbulence, where a direct energy exchange between large- and short-scale length perturbations is considered, without the need for any of the usual energy cascades. This is directly inspired by our model for the plasma wave turbulence [3], where short-scale oscillations of the medium provide an arbitrary turbulent spectrum as the background for the excitation and absorption of large-scale turbulence. In the present model, we describe the short-scale turbulence as a fluid of quasi-particles, described by an appropriate wave kinetic equation. Exact and approximate wave kinetic equations have been derived [4] and applied to many specific plasma physics situations. They include both electromagnetic [5]–[7], and electrostatic plasma turbulence [8]–[10]. Of particular relevance is the case of zonal flows in the presence of drift wave (or drifton) turbulence as explored recently by us [11], which has been successfully applied to explain satellite observations in the magnetosphere [12].

In the following, we apply the wave kinetic description to the case of a rotating two-dimensional fluid turbulence, where both large-scale zonal flows and short-scale Rossby waves can coexist. They can both be described by the Charney equation [13], which also admits vortex solutions [14, 15]. Nonlinear couplings between zonal flows and Rossby waves have already been discussed in [16]. Our interest here is not to address the planetary atmospheric turbulence in all its complexity, or to discuss the interesting relations between the well-known beta-plane and rotating shallow water models [17, 18], but to propose a different approach to fluid turbulence, and to illustrate it by a specific but physically relevant example. It should also be noticed that successful two-dimensional models for plasma turbulence very similar to Charney type equations for fluid turbulence have been established in the past [19, 20]. But here we approach the two-dimensional fluid problem from the point of view of the wave kinetic theory, which describes the short-scale Rossby waves as a fluid of quasi-particles (we refer to them as the *Rossbyons*), and will allow us to consider an arbitrary Rossby wave turbulent spectrum and to establish a clear and solid basis for comparison between the plasma and fluid turbulences.

We first show that a wave kinetic equation can be derived from the Charney equation. We then derive a nonlinear dispersion relation for zonal flows, allowing us to describe the excitation of zonal flows in a rotating fluid, due to the presence of a broad spectrum of Rossby waves (or equivalently, an arbitrary distribution of Rossbyons). The particular example of a mono-kinetic distribution, or a Rossbyon beam, is used to show that zonal flows can become unstable and to derive the corresponding nonlinear growth rate. A Landau resonance is shown to exist in the kinetic dispersion relation for the zonal flows. The role of such a resonance and the main differences with respect to plasma turbulence will be discussed.

2. Basic equations

We start from the generalized Charney equation, which can describe the existence of both Rossby waves and zonal flows in a two-dimensional fluid model of a rotating fluid. We have [13, 15, 16]

$$\left(1 - r_0^2 \nabla_\perp^2\right) \frac{\partial h}{\partial t} - v_0 \frac{\partial h}{\partial y} - U(h) = 0, \tag{1}$$

where r_0 and v_0 are the Rossby radius and the Rossby velocity, respectively, as defined by

$$r_0 = \frac{1}{\Omega_c}\sqrt{g H_0}, \quad v_0 = g \frac{\partial}{\partial x}\left(\frac{H_0}{\Omega_c}\right). \tag{2}$$

The total depth of the fluid is determined by $H = H_0 + h$, where H_0 is its equilibrium or unperturbed value and h the wave amplitude perturbation. We have also used g, the acceleration of gravity, and Ω_c, the Coriolis frequency. In equation (1), we have introduced the potential

$$U(h) = r_0^2 \mu \nabla_\perp^4 h - \epsilon \left(\hat{z} \times \nabla h \cdot \nabla\right) \nabla_\perp^2 h, \tag{3}$$

where μ is a kinematic fluid viscosity, $\hat{z}$ the vertical unit vector, $\nabla_\perp$ the nabla operator in the perpendicular plane and $\epsilon = g^2 H_0 / \Omega_c^3$ the nonlinear coupling coefficient.

Let us now write $h = \tilde{h} + a$, where $\tilde{h}$ represents the short-scale Rossby wave turbulence, and a the large-scale zonal flows. From the Charney equation (1) we can then obtain a pair of coupled equations, one describing the Rossby waves

$$\left(1 - r_0^2 \nabla_\perp^2\right) \frac{\partial \tilde{h}}{\partial t} - v_0 \frac{\partial \tilde{h}}{\partial y} + r_0^2 \mu \nabla_\perp^4 \tilde{h} = J(\tilde{h}, a) + J(a, \tilde{h}) \tag{4}$$

and another for zonal flows

$$\left(1 - r_0^2 \nabla_\perp^2\right) \frac{\partial a}{\partial t} + r_0^2 \mu \nabla_\perp^4 a = J(\tilde{h}, \tilde{h}), \tag{5}$$

where we have defined the nonlinear terms generically as

$$J(a, b) = \epsilon \left(\hat{z} \times \nabla a \cdot \nabla\right) \nabla_\perp^2 b, \tag{6}$$

for arbitrary functions a and b. For Rossby waves of the form $\exp(\mathrm{i}\vec{k} \cdot \vec{r} - \mathrm{i}\omega t)$, and using the linear approximation $\epsilon = 0$, we can easily derive from equation (4), the dispersion relation

$$\omega = -\frac{1}{1 + k_\perp^2 r_0^2}\left(k_y v_0 + \mathrm{i}\mu k_\perp^4 r_0^2\right), \tag{7}$$

which exhibits the existence of wave damping, due to the fluid viscosity μ. Similarly, for zonal flows oscillating in space and time as $\exp(\mathrm{i}\vec{q}\cdot\vec{r}-\mathrm{i}\Omega t)$, we obtain from the linearized version of equation (5), the dispersion relation

$$\Omega \equiv -\mathrm{i}\Gamma_0, \quad \Gamma_0 = \mu \frac{q_\perp^4 r_0^2}{1+q_\perp^2 r_0^2}, \tag{8}$$

which is a purely damped mode, with the damping rate also proportional to the kinematic fluid viscosity.

The nonlinear properties of equations (4) and (5) will be discussed next.

3. The Rossbyon fluid

Let us consider a broad spectrum of short-wavelength Rossby wave turbulence, generically described by the Fourier integral

$$\tilde{h}(\vec{r},t) = \int \tilde{h}(\vec{k}) \exp\left(\mathrm{i}\vec{k}\cdot\vec{r}-\mathrm{i}\omega_k t\right) \frac{\mathrm{d}\vec{k}}{(2\pi)^3}, \tag{9}$$

where $\omega_k \equiv \omega(\vec{k})$ is determined by the dispersion relation (7), and $\tilde{h}(\vec{k})$ are slowly varying spectral amplitudes. Replacing this in equation (4), and assuming that the low-frequency fluid perturbations appearing in the nonlinear terms are simply described by $a(\vec{r},t) = a_0 \exp(\mathrm{i}\vec{q}\cdot\vec{r}-\mathrm{i}\Omega_k t)$, we can derive the following evolution equation for the spectral amplitudes:

$$\frac{\partial h}{\partial \tau} - 2\mathrm{i}\mu r_0^2 k_\perp^2 \left(\vec{k}_\perp \cdot \nabla_\perp\right) h = \alpha_- a_0 h_- - \alpha_+ a_0^* h_+, \tag{10}$$

where we have used $h \equiv \tilde{h}(\vec{k})$, to simplify, and defined the differential operator

$$\frac{\partial}{\partial \tau} = \left(1+r_0^2 k_\perp^2\right)\frac{\partial}{\partial t} - v_0 \frac{\partial}{\partial y}. \tag{11}$$

We have also defined $h_\pm \equiv \tilde{h}(\vec{k}_\pm)$, with $\vec{k}_\pm = \vec{k} \pm \vec{q}$, and the new nonlinear coefficients are determined by

$$a_\pm = \epsilon \left[\left(\hat{z}\times\vec{q}\cdot\vec{k}_\pm\right) k_{\pm\perp}^2 + \left(\hat{z}\times\vec{k}_\pm\cdot\vec{q}\right) q_\perp^2\right]. \tag{12}$$

In the extreme case of infinitesimal wavelengths, such that $|k| \gg |q|$, we can use the approximation

$$h_\pm = h \pm \vec{q}\cdot\frac{\partial h}{\partial \vec{k}}, \tag{13}$$

which corresponds to taking the geometric optics approximation for short-wavelength (in comparison with the Rossby radius) Rossby waves. Equation (11) can then be reduced to a much simpler form as

$$\frac{\partial h}{\partial \tau} - \mathrm{i}\vec{\beta}\cdot\nabla_\perp h = Ah - B\vec{q}\cdot\frac{\partial h}{\partial \vec{k}}, \tag{14}$$

where we have assumed $\vec{\beta} = 2\mu r_0^2 k_\perp^2 \vec{k}_\perp$, and defined new coefficients as

$$A = \alpha_- a_0 - \alpha_+ a_0^*, \quad B = \alpha_- a_0 + \alpha_+ a_0^*. \tag{15}$$

Let us now define the Rossbyon occupation number, as $N = |h|^2$, or more explicitly $N(\vec{k}) = |\tilde{h}(\vec{k})|^2$. The evolution equation for this new quantity can be derived from equation (14), and takes the form

$$\frac{\partial N}{\partial \tau} - \mathrm{i}\vec{\beta} \cdot \left[h^* \nabla_\perp h - h \nabla_\perp h^*\right] = (A + A^*)N - B\vec{q} \cdot \frac{\partial N}{\partial \vec{k}}, \tag{16}$$

where we have

$$(A + A^*) = -2\epsilon(\hat{z} \times \vec{q} \cdot \vec{k})\vec{k}_\perp \cdot \vec{q}_\perp (a_0 + a_0^*). \tag{17}$$

Equation (16) can be rewritten in a more compact and physically more appealing form by noting that the term $(A + A^*)N$ can be absorbed into the dispersion relation for the Rossbyons, as a small correction term, due to the presence of zonal flows, in such a way that the real part of the Rossbyon frequency becomes

$$\Re(\omega) = -\frac{k_y v_0 - (A + A^*)}{1 + k_\perp^2 r_0^2}. \tag{18}$$

Of course, for infinitesimal values of the zonal flow amplitude a_0, such a correction is negligible and equation (7) can be used. On the other hand, it can be shown by using energy conservation arguments (strictly valid in the absence of the nonlinear coupling ϵ) that the dissipative term associated with $\vec{\beta}$ leads to the decay of the occupation number as $\exp(-2\gamma_k t)$, where $\gamma_k = \Im(\omega)$ as determined by (7). We are then left with a kinetic equation for the Rossbyon occupation number, which can be written in terms of the space and time variables y and t as

$$\left(\frac{\partial}{\partial t} - v_y \frac{\partial}{\partial y} + B\vec{q} \cdot \frac{\partial}{\partial \vec{k}}\right) N(\vec{k}) = -2\gamma_k N(\vec{k}). \tag{19}$$

This will be our basic equation for the kinetic description of the Rossby wave turbulence. In such an equation, it is obvious that $B\vec{q}$ describes a force acting on the Rossbyon quasi-particles, due to the presence of zonal flows.

4. Dispersion relation

We now turn to the nonlinear evolution equation for zonal flows. Introducing the Rossbyon occupation number in the nonlinear terms of equation (5), and using the quasi-optical approximation for these quasi-particles (valid for $|k| \gg |q|$), we can write

$$\left(1 - r_0^2 \nabla_\perp^2\right) \frac{\partial a}{\partial t} + r_0^2 \mu \nabla_\perp^4 a = \int \epsilon_k N(\vec{k}) \frac{\mathrm{d}\vec{k}}{(2\pi)^3}, \tag{20}$$

with $\epsilon_k = \epsilon[(\hat{z} \times \vec{q} \cdot \vec{k})\vec{k}_\perp^2]$. For perturbations of the form $a = a_0 \exp(\mathrm{i}\vec{q} \cdot \vec{r} - \mathrm{i}\Omega t)$ and $N(\vec{k}) = N_{k0} + \tilde{N}_k \exp(\mathrm{i}\vec{q} \cdot \vec{r} - \mathrm{i}\Omega t)$, where N_{k0} is the unperturbed value, we obtain

$$a_0 = \frac{\mathrm{i}}{\Omega(1 + r_0^2 q_\perp^2) + \mathrm{i} r_0^2 \mu q_\perp^4} \int \epsilon_k \tilde{N}(\vec{k}) \frac{\mathrm{d}\vec{k}}{(2\pi)^3}. \tag{21}$$

On the other hand, the linearized form of the kinetic equation (19) allows us to write

$$\tilde{N}_k = -\mathrm{i}\epsilon_k a_0 \frac{\vec{q} \cdot \partial N_{k0}/\partial \vec{k}}{(\Omega + v_y q_y) + 2\mathrm{i}\gamma_k}. \tag{22}$$

From these two equations, we can then derive the nonlinear dispersion relation for the zonal flows in the presence of Rossbyon turbulence, in the following form:

$$\Omega\left(1+r_0^2q_\perp^2\right)+\mathrm{i}r_0^2\mu q_\perp^4-\int f(\vec{k})\,\vec{q}\cdot\frac{\partial N_{k0}}{\partial\vec{k}}\frac{\mathrm{d}\vec{k}}{(2\pi)^3}=0, \tag{23}$$

with

$$f(\vec{k})=\frac{\epsilon_k^2}{(\Omega+v_yq_y)+2\mathrm{i}\gamma_k}. \tag{24}$$

In the absence of the Rossby wave turbulence, this will of course reduce to the linear dispersion relation (7). But, in the presence of the Rossby turbulence, this shows the occurrence of a Landau resonance, which will be discussed below. Such a resonance is clearly recognized when we neglect the wave damping term $2\mathrm{i}\gamma_k$. The singularity in the expression of $f(\vec{k})$, is then determined by the equality

$$v_y\equiv-\frac{v_0}{(1+r_0^2k_\perp^2)}=-\frac{\Omega}{q_y}, \tag{25}$$

which corresponds to a situation where the velocity of the Rossbyon quasi-particle along y, exactly matches the phase velocity of the zonal flow $(-\Omega/q_y)$ along the same direction.

A completely new picture of the fluid turbulence emerges from here where, instead of the occurrence of a (direct or indirect) energy cascade, we have a direct link between short- and large-scale fluid perturbations, eventually mediated by a Landau resonance.

5. Beam-like instability

In order to understand the physical relevance of the above nonlinear dispersion relation for the zonal flows, let us consider a simple but illustrative case. We assume an initial turbulent distribution such that $N_{k0}=(2\pi)^3N_0\delta(\vec{k}-\vec{k}_0)$. Equations (23) and (24) can then be reduced to

$$\Omega\left(1+r_0^2q_\perp^2\right)+\mathrm{i}r_0^2\mu q_\perp^4+\frac{N_0\vec{q}\cdot\vec{g}(\vec{k}_0)}{(\Omega+v_yq_y)+2\mathrm{i}\gamma_k}=0, \tag{26}$$

where

$$\vec{g}(\vec{k})=\frac{\partial}{\partial\vec{k}}\frac{\epsilon_k^2}{(1+r_0^2k_\perp^2)}. \tag{27}$$

The contribution from the nonlinear term is maximum for the case $q_y=0$ and $\Omega=\mathrm{i}\Gamma$. This leads to

$$\Gamma=\Gamma_0+\frac{N_0\vec{q}\cdot\vec{g}(\vec{k}_0)}{(\Gamma+2\gamma_k)(1+r_0^2q_\perp^2)}, \tag{28}$$

where the first term corresponds to the linear damping rate of the zonal flow, as determined by equation (8). Solving for Γ, we obtain

$$\Gamma=-\frac{1}{2}(\Gamma_0+2\gamma_k)\left[1\pm\sqrt{1+N_0\frac{\vec{q}\cdot\vec{g}(\vec{k}_0)}{(\Gamma_0+2\gamma_k)^2(1+r_0^2q_\perp^2)}}\right]. \tag{29}$$

We see that the zonal flows can be destabilized by the short-scale Rossby wave turbulence, as long as $\vec{q}\cdot\vec{g}(\vec{k}_0) > 0$, therefore $\Gamma > 0$. In this case, and assuming that the linear terms are still the dominant ones, we obtain an approximate expression for the growth rate

$$\Gamma \simeq \frac{N_0}{4}\frac{[\vec{q}\cdot\vec{g}(\vec{k}_0)]}{(\Gamma_0+2\gamma_k)^2(1+r_0^2q_\perp^2)}. \tag{30}$$

From equation (27) we clearly see that such a situation can always occur, for a given range of $\vec{q}$, by noting that

$$\vec{q}\cdot\vec{g}(\vec{k}) = \frac{\vec{q}}{(1+r_0^2k_\perp^2)}\cdot\left[\frac{\partial\epsilon_k^2}{\partial\vec{k}} - \frac{2r_0^2\vec{k}_\perp}{(1+r_0^2k_\perp^2)}\right]. \tag{31}$$

If the second term is dominant, we only have to consider an adequate direction of $\vec{q}$ in order to make the quantity $(-\vec{q}\cdot\vec{k}_\perp)$ negative. Similarly, when the first term is dominant, we can always find a spectral region of $\vec{q}$ such that $\vec{q}\cdot(\partial\epsilon_k^2/\partial\vec{k})$ is positive. In conclusion, we can say that the zonal flows are always unstable in the presence of a beam of Rossbyons.

Finally, let us consider the possible occurrence of a Landau damping. We go back to the nonlinear dispersion relation (23) and (24), and consider the case $\Omega = \mathrm{i}\Gamma$ and $q_y = 0$ as before, but we now retain an arbitrary Rossbyon distribution N_{k0}. Accordingly, we obtain

$$\Gamma\left(1+r_0^2q_\perp^2\right) + r_0^2\mu q_\perp^4 + \vec{q}\cdot\int\frac{\epsilon_k^2}{(1+r_0^2k_\perp^2)}\frac{\partial N_{k0}/\partial\vec{k}}{\Gamma+2\gamma_k}\frac{\mathrm{d}\vec{k}}{(2\pi)^3} = 0. \tag{32}$$

As we see, the integral in the last term of the above equation can be split into its principal part and the contribution from the pole, which corresponds to a given value of $\vec{k}$ such that $\gamma_k = -\Gamma/2$. The principal part of the integral is real, and leads to the occurrence of a nonlinear contribution to the zonal flow damping rate Γ, as already shown. On the other hand, the contribution from the pole is imaginary, and leads to a very small correction to the real part of the frequency Ω.

We can therefore conclude that no Landau damping exists in this wave kinetic model of a rotating viscous fluid. A Landau resonance still exists, but it only contributes to the real part of the frequency. This results from the fact that the zonal flow is essentially a purely growing mode, dominated by viscosity. This is in clear contrast with the plasma turbulence, where the kinetic damping resulting from the Landau resonance can become the dominant process.

6. Discussion and conclusions

A new picture of fluid turbulence was explored here, where large-scale fluid perturbations are considered in a background of short-scale turbulence. In our description, zonal flows were studied in the presence of an arbitrary spectrum of Rossby waves, described as quasi-particles. This means that we have assimilated this spectrum to a kinetic distribution of Rossbyons. A kinetic equation for Rossbyons, valid in the limit of geometric optics, was derived. Nonlinear coupling between the Rossbyons and the zonal flows (driven by Rossbyon fluid stresses) was considered. A new dispersion relation was derived, which establishes a direct link between the short- and large-scale fluid perturbations. No cascading process is involved. Using the particular but physically relevant case of a mono-kinetic spectrum of Rossbyons, we have shown that zonal flows can become unstable by a direct nonlinear coupling with Rossbyons, and approximate expressions for the growth rates were derived.

We have also examined the importance of Landau resonances. We have shown that, contrary to plasma wave turbulence, Landau damping for nonlinearly coupled Rossbyons and zonal flows cannot take place. A Landau resonance only leads to a small nonlinear contribution to the zonal flow frequency Ω, which is purely imaginary in the linear case. Such a qualitative difference between neutral fluid turbulence and plasma turbulence is due to the fact that in the plasma case collisional damping and viscosity effects can be neglected in many important physical situations, in contrast with the fluid turbulence, where a kinematic fluid viscosity is always dominant. However, the present work will not allow us to conclude that Landau damping is completely absent from fluid turbulence, as will be shown in a future work. We have therefore been able to establish a clear theoretical link between the fluid and plasma turbulences, where similar concepts, such as quasi-particles can be used, and where similarities and main qualitative differences can be explicitly stated and the dominant physical processes can be understood. The present results should be useful in understanding the origin of large-scale planetary and near-Earth atmospheric circulations.

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