1. INTRODUCTION

Cyclotron waves are an important constituent of plasmas in solar corona, solar wind and planetary magnetospheres. As it is known, energetic particles with anisotropic temperature can excite a wide class of cyclotron wave instabilities. Kinetic theory of such waves in the straight magnetic field plasmas is developed very well, see e.g. Refs. [1-6]. However, plasma models in the straight magnetic field are quite rough for planetary magnetospheres which are three-dimensional in the general case. As more suitable, the Earth’s magnetosphere can be considered as a two-dimensional (2D) dipole magnetic field configuration. Another interesting 2D magnetospheric plasma model is a configuration with circular magnetic field lines, which is artificial but simpler and helpful to describe the principal wave processes in the Earth’s magnetosphere. The dispersion equations for cyclotron waves in magnetospheric plasmas with dipole and circular magnetic field lines were derived in Ref. [7]. In this paper we analyze the dispersion characteristics of the electromagnetic ion-cyclotron (EMIC) waves in the hydrogen plasma confined in the last plasma model including the energetic protons with the bi-Maxwellian distribution function.

2. DISPERSION EQUATION FOR FIELD-ALIGNED CYCLOTRON WAVES

According to Ref. [7], the dispersion equations for field aligned cyclotron waves in magnetospheric plasmas with circular magnetic field lines can be rewritten by analogy with the straight magnetic field case in the form:

\[ Y^+_{\nu_j}(u, \nu) = \frac{n \pi c}{\omega R_0 L_{\parallel 0}} = 1 + 2 \sum_{\nu_j} e^{i \omega_{\nu_j}}(L), \]

where \( \sigma \) denotes the particle species (electron, proton, heavy ions); \( n \) is the mode number along the geomagnetic field; \( L = R L_{\parallel 0} \cos \theta \) is the non-dimensional L-shell parameter; \( \theta \) is the geographical latitude; and the transverse permittivity elements are

\[ \varepsilon^{\perp \perp}_{\nu_j} = \frac{\omega_{\nu_j}^2 L_{\parallel 0}}{2 \pi \tan \theta_0 T_{\perp}}, \quad \sum_{\nu_j} \int_{-1}^{1} \left( 1 - 2 \nu \right) d\nu \left( 1 - 1 + T_{\perp} T_{\parallel 0} \right) \times \left[ \frac{b(\theta) - 1 + T_{\perp} / T_{\parallel 0}}{1 - (1 - 2 \nu)^2 b(\theta)} + \frac{\pi \nu \nu_{\parallel 0}}{\omega R_0 L_{\parallel 0}} \right] \left( 1 - T_{\perp} / T_{\parallel 0} \right) \times \left\{ \cos \left[ \frac{n \pi}{\theta_0} - p \frac{2 \pi}{\tau_{\nu_j}} + \frac{IR_{\nu_j} 0}{L_{\nu_j}} C(\theta) \right] + \left(1 - \nu \right)^2 \left[ p \frac{2 \pi}{\tau_{\nu_j}} - \frac{IR_{\nu_j} 0}{L_{\nu_j}} C(\theta) \right] \right\} d\theta, \]

\[ \lambda_{\nu_j}(u, \nu) = \frac{\omega_{\nu_j}^2 L_{\parallel 0}}{2 \pi \tan \theta_0 T_{\perp}}, \quad \sum_{\nu_j} \int_{-1}^{1} \left( 1 - 2 \nu \right) d\nu \left( 1 - 1 + T_{\perp} T_{\parallel 0} \right) \times \left[ \frac{b(\theta) - 1 + T_{\perp} / T_{\parallel 0}}{1 - (1 - 2 \nu)^2 b(\theta)} + \frac{\pi \nu \nu_{\parallel 0}}{\omega R_0 L_{\parallel 0}} \right] \left( 1 - T_{\perp} / T_{\parallel 0} \right) \times \left\{ \cos \left[ \frac{n \pi}{\theta_0} - p \frac{2 \pi}{\tau_{\nu_j}} + \frac{IR_{\nu_j} 0}{L_{\nu_j}} C(\theta) \right] + \left(1 - \nu \right)^2 \left[ p \frac{2 \pi}{\tau_{\nu_j}} - \frac{IR_{\nu_j} 0}{L_{\nu_j}} C(\theta) \right] \right\} d\theta, \]

Here we have used the following definitions:

\[ Y^+_{\nu_j}(u, \nu) = \frac{\omega_{\nu_j}^2 L_{\parallel 0}}{2 \pi \tan \theta_0 T_{\perp}}, \quad \sum_{\nu_j} \int_{-1}^{1} \left( 1 - 2 \nu \right) d\nu \left( 1 - 1 + T_{\perp} T_{\parallel 0} \right) \times \left[ \frac{b(\theta) - 1 + T_{\perp} / T_{\parallel 0}}{1 - (1 - 2 \nu)^2 b(\theta)} + \frac{\pi \nu \nu_{\parallel 0}}{\omega R_0 L_{\parallel 0}} \right] \left( 1 - T_{\perp} / T_{\parallel 0} \right) \times \left\{ \cos \left[ \frac{n \pi}{\theta_0} - p \frac{2 \pi}{\tau_{\nu_j}} + \frac{IR_{\nu_j} 0}{L_{\nu_j}} C(\theta) \right] + \left(1 - \nu \right)^2 \left[ p \frac{2 \pi}{\tau_{\nu_j}} - \frac{IR_{\nu_j} 0}{L_{\nu_j}} C(\theta) \right] \right\} d\theta, \]

\[ \lambda_{\nu_j}(u, \nu) = \frac{\omega_{\nu_j}^2 L_{\parallel 0}}{2 \pi \tan \theta_0 T_{\perp}}, \quad \sum_{\nu_j} \int_{-1}^{1} \left( 1 - 2 \nu \right) d\nu \left( 1 - 1 + T_{\perp} T_{\parallel 0} \right) \times \left[ \frac{b(\theta) - 1 + T_{\perp} / T_{\parallel 0}}{1 - (1 - 2 \nu)^2 b(\theta)} + \frac{\pi \nu \nu_{\parallel 0}}{\omega R_0 L_{\parallel 0}} \right] \left( 1 - T_{\perp} / T_{\parallel 0} \right) \times \left\{ \cos \left[ \frac{n \pi}{\theta_0} - p \frac{2 \pi}{\tau_{\nu_j}} + \frac{IR_{\nu_j} 0}{L_{\nu_j}} C(\theta) \right] + \left(1 - \nu \right)^2 \left[ p \frac{2 \pi}{\tau_{\nu_j}} - \frac{IR_{\nu_j} 0}{L_{\nu_j}} C(\theta) \right] \right\} d\theta, \]
Our dispersion equation is suitable to analyze the instabilities of both the right-hand (if \( l=1 \)) and left-hand (if \( l=-1 \)) polarized waves. Further, Eq. (1) should be resolved numerically for the real and imaginary parts of the wave frequency, \( \omega = \text{Re} \omega + i \text{Im} \omega \), to define the instability conditions. As it is well known, in the straight magnetic field case, the squared refractive index of the EMIC waves (\( l=-1 \)) in the hydrogen plasma is defined by the expression

\[
\left( \frac{k_i c}{\text{Re} \omega} \right)^2 = \frac{\Omega_{\gamma}^2}{\Omega_{c,\varphi}^2 - \text{Re} \omega}, \tag{3}
\]

where \( \Omega_{\gamma}^2 = 4\pi N_p e^2 / M_p \) is the squared proton plasma frequency, \( N_p = N_{p+} + N_{p-} \), \( N_{p+} \) and \( N_{p-} \) are the densities of the cold and hot protons, respectively;

\[
\Omega_{c,\varphi} = \frac{e B(L,0)}{M_p c} = \frac{\omega_{c,\varphi}}{L_i^3}
\]

is the cyclotron frequency of the L-shell protons at the equatorial plane. Since the parallel wavenumber \( k_i \) is connected with the mode numbers \( n \) as \( k_i = n \pi / (R_p L_\theta ) \), for plasmasphere with circular magnetic field lines, so that the mode numbers can be estimated as

\[
n = \frac{R_p L_\theta \gamma_{p\varphi} \text{Re} \omega}{\pi \sqrt{\Omega_{\gamma}^2(\Omega_{c,\varphi} - \text{Re} \omega)}}. \tag{4}
\]

Where as the increment (decrement) of the EMIC waves in a hydrogen plasma \( \gamma \), if \( \gamma = \text{Im} \omega < \text{Re} \omega \), is defined by the expression

\[
\gamma = -2 \frac{\text{Re} \omega}{\Omega_{c,\varphi}^2} \left( \frac{\Omega_{c,\varphi} - \text{Re} \omega}{\Omega_{\gamma}^2(\Omega_{c,\varphi} - \text{Re} \omega)} \right) \text{Im} \varepsilon_{-1,h}, \tag{5}
\]

where

\[
\text{Im} \varepsilon_{-1,h} = \frac{\Omega_{\gamma}^2}{2(\text{Re} \omega)^2 k_i^2} \left[ \frac{\text{Re} \omega}{\Omega_{c,\varphi}} \left( 1 - \frac{\text{Re} \omega}{\Omega_{c,\varphi}} \right) \left( \frac{T_{sh}}{T_{ph}} \right)^{-1} \right] \times \exp \left[ -\frac{(\text{Re} \omega - \Omega_{c,\varphi})}{k_i^2 \nu_{pph}} \right].
\]

By index ‘\( h \)’ we denote the plasma parameters for the resonant hot protons. As follows from Eqs. (3), (4), the proton cyclotron instability (PCI) of EMIC waves, \( \gamma > 0 \), is possible if \( \text{Im} \varepsilon_{-1,h} < 0 \), i.e., if \( T_{sh} > T_{ph} \).

3. NUMERICAL RESULTS

Now, let us compare the PCI growth rates in the plasmas confined in the straight magnetic field \( \gamma_c \), and in the 2D magnetosphere with circular magnetic field lines \( \gamma_c \). For simplicity, there are considered the hydrogen plasmas at the geostationary orbit, \( L=6.6 \), including the cold electrons with \( N_e=11 \text{ cm}^{-3} \), cold protons with \( N_{p}=10 \text{ cm}^{-3} \), and energetic protons with \( N_{p}=1 \text{ cm}^{-3} \). The parallel and transverse temperatures of the energetic protons are equal to \( T_{ph}=10 \text{ keV} \) and \( T_{sh}=30 \text{ keV} \), whereas the temperature of the cold particles is small and isotropic. In this case, the mode numbers \( n \) of the field aligned EMIC waves can be defined by Eqs. (3), (4); the corresponding dependence \( n(\omega) \) is plotted in Fig. 1.

![Fig. 1. Dependence of the mode numbers on the wave frequency for EMIC waves in a hydrogen plasma](image1)

The PCI growth rate \( \gamma_c \) for EMIC waves in the straight magnetic field plasma we estimate, as usually, by Eq. (5). As for \( \gamma_c \), for EMIC waves in the magnetospheric-like plasma with circular magnetic field lines, we use the similar expression:

\[
\gamma_c = -2 \frac{\text{Re} \omega}{\Omega_{c,\varphi}^2} \left( \frac{\Omega_{c,\varphi} - \text{Re} \omega}{\Omega_{\gamma}^2(\Omega_{c,\varphi} - \text{Re} \omega)} \right) \text{Im} \varepsilon_{c,h}, \tag{6}
\]

where

\[
\text{Im} \varepsilon_{c,h} = \sum_{\nu=1}^{\infty} \frac{\Omega_{\gamma}^2}{2 \pi \nu \theta_{p} T_{\nu h}^2} \left( 1 - 2 \nu \right)^{2} \left( \frac{Z_{\nu h}}{p} \right) \times \int Y_{\nu h}^{2} \left( \frac{Z_{\nu h}}{p} \right) Z_{\nu h}^{2} \exp \left[ -\frac{Z_{\nu h}^{2}}{p} \left( 1 - (1 - 2 \nu)^{2} \left( 1 - \frac{T_{h}}{T_{c}} \right) \right) \right] d\nu.
\]

![Fig. 2. The PCI growth rates versus \( \omega \) for EMIC waves in the hydrogen plasmas confined in the straight uniform magnetic field (a) and in the 2D magnetosphere (b)](image2)

The PCI growth rates versus \( \omega \) are presented in Fig. 2a for EMIC waves in the straight magnetic field plasma by Eq. (5), and in Fig. 2b for EMIC waves in the 2D magnetosphere-like plasma with circular magnetic field lines. The computations of \( \gamma_c \) are carried out in the interval \( 2 \text{Hz} \leq \omega \leq 7 \text{Hz} \), whereas the minimal gyrofrequency of the protons at \( L=6.6 \) is close to \( \Omega_{c,\varphi} \approx 1 \text{Hz} \). As shown in Fig. 2a and Fig. 2b, the instability of EMIC waves is possible for both plasma models in the frequency range \( \omega < \Omega_{c,\varphi} \). It should be noted that the proton-cyclotron instability is impossible for EMIC waves in the frequency range \( \Omega_{c,\varphi} < \Omega_{c,\varphi} h(\theta_a) \), where \( \Omega_{c,\varphi} h(\theta_a) \) is the maximal gyrofrequency of the protons at the given L-shell magnetic field line.
As one can see, the dependence $\gamma_\parallel$($\omega$) and $\gamma_\perp$($\omega$) on the wave frequency $\omega$ are similar; however, $\gamma_\parallel$($\omega$)$<\gamma_\perp$($\omega$) under the same bulk parameters. The ratio $\gamma_\parallel$ / $\gamma_\perp$ $\propto$ 4 + 10 versus $\omega$ for considered magnetospheric-like plasmas is presented in Fig. 3. This dependence is not linear; the difference is very large (by factor 10) for EMIC waves in the range of $\omega$ ~ 2Hz and is smaller (by factor 4) in the range of high frequencies $\omega$ ~ 7Hz.

The large difference between $\gamma_\parallel$ and $\gamma_\perp$ is connected with the fact that the wave-particle interaction in the straight magnetic field plasma is more effective since the resonant particles move along the uniform magnetic field line with the constant parallel velocity and interact permanently (in time) with the wave according to the well known resonance condition $\omega - \Omega_{ci} = k_\parallel v_\parallel$. As for 2D magnetospheric plasmas, since $v_\parallel$ $\neq$ const for the trapped particles, there is another wave-particle resonance condition involving the particle energy, pitch angle, cyclotron and bounce frequencies. As a result, the trapped particle bouncing between the reflection points only part of the bounce-time can interact effectively with the wave.

**CONCLUSIONS**

Dispersion equation is analyzed for EMIC waves in a hydrogen magnetospheric plasma with circular B-field lines. As in the straight B-field plasmas, the growth rate of PCI is defined by the contribution of the resonant particles to the imaginary part of the transverse permittivity elements. The comparison of the growth rates is carried out for EMIC waves in the hydrogen plasmas with the straight and circular B-field lines under the same macroscopic bulk parameters at the geostationary orbit $L$=6.6. It is shown that the PCI growth rate in the 2D magnetosphere is much less than it is in the straight uniform B-field case. Of course, the similar approach can be used to analyze the dispersion characteristics of the EMIC waves in 2D magnetospheric multi-ions plasmas with dipole and circular magnetic field lines including the protons and heavy ions (He++, O+) with the temperature anisotropy.

**REFERENCES**