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A REVIEW OF HYDRAULIC JUMP PROPERTIES ON BOTH SMOOTH AND ROUGH BEDS IN SLOPING AND ADVERSE CHANNELS

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Abstract. Hydraulic jump is a phenomenon which has received significant attention in recent years and it is still studied because of its capacity to dissipate a considerable amount of the flow energy. Nevertheless, the importance of the topic still requires significant efforts from the scientific community. Namely, the prediction of the main lengths of the hydraulic jump are still an open question, as the actual knowledge on the topic does not cover all the possible configurations and boundary conditions which can usually be found in practical applications. In particular, the effects of bed roughness, bed slope, channel geometry, and air concentration on the conjugate depths ratio are still not fully understood. The present paper aims to furnish a synthetic picture of the state of art regarding the hydraulic jump properties in a wide range of both boundary conditions and geometric configurations. In particular, the analysis will be focused on the effect of both relative roughness and bed slope on the conjugate depth ratio, including the effect of air entrainment on the estimation of the effective depth. Furthermore, some predicting relationships proposed by different authors will be compared and discussed.

Keywords: Aerated flows, bed roughness, hydraulic jump, sloping channel, stilling basin.

INTRODUCTION

Hydraulic jump is a fascinating topic for hydraulic engineers. The analysis of this hydraulic phenomenon has been conducted by many researchers during the last century, but there are still many aspects which should be deepened. The first systematic study was conducted by Bélanger [1828], who analyzed the hydraulic jump occurring on a smooth horizontal rectangular channel. He derived the well-known Bélanger's equation by which it is possible to estimate the conjugate depth ratio. Nevertheless, the simplifying assump-

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tions made by Bélanger do not allow for a correct estimation of the main hydraulic jump characteristics when either the channel bed is not smooth or the channel geometry is not prismatic. In addition, the bed slope is another key factor which should be carefully taken into consideration, as even in the case of rectangular prismatic channels it can deeply modify hydraulic jump characteristics.

Considering the relevant limitations of Bélanger's theory, many researchers conducted experimental tests in order to understand the complexity of the phenomenon in a wider range of hydraulic and geometric configurations. Namely, several studies were conducted in order to analyze the hydraulic jump properties on smooth sloping beds [e.g., Kindsvarter 1944, Bakhmeteff and Matzke 1938, Chow 1959, Rajaratnam 1966, Rajaratnam 1967, Kawagoshi and Hager 1990, and Ohtsu and Yasuda 1991]. For sloping channels, four jump typologies were distinguished [Kindsvarter, 1944]: A-jump, when the toe is at the kink; B-jump is intermediate to A- and C-jumps; C-jump, when the end of the roller is above the kink; and D-jump when the entire jump is occurring on the sloping channel. It was shown that both in the cases of B- and D-jumps, the sequent depth ratios increases with the channel slope, whereas a sensible reduction of the hydraulic jump length takes place. Figure 1 shows diagram sketches of different hydraulic jump types. More recently the analysis was also extended to the case of adverse-sloped smooth channels [McCorquodale and Mohamed 1994, Pagliara and Palermo 2015]. In this case, the experimental evidences confirmed that the hydraulic jump behaviour is significantly different from that previously mentioned. In fact, the sequent depth ratio decreases with the absolute value of the channel slope and the hydraulic jump is much more unstable than that occurring on sloping channels for the same approaching flow conditions.

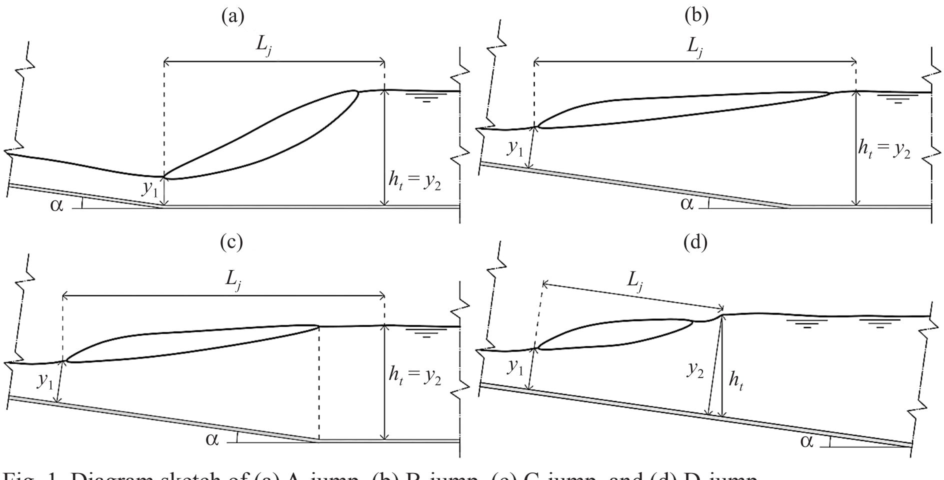

Fig. 1. Diagram sketch of (a) A-jump, (b) B-jump, (c) C-jump, and (d) D-jump

But in practical applications generally the channel bed is not smooth. Just for example, the presence of protection by riprap or cobbles in the stilling basin significantly modify the bed roughness, resulting in a large effect on both velocity profiles and shear stresses [Ead and Rajaratnam 2002]. In addition, the bed roughness deeply modifies the air entrainment process [Pagliara et al. 2008a, Pagliara and Palermo 2015], thus contributing to varying jump properties. Several studies were conducted on this particular topic. Namely, experimental tests showed that the hydraulic jump properties on a rough horizontal bed are significantly influenced by two main parameters: the approaching flow conditions and the relative roughness [Hughes and Flack 1984, Afzal et al. 2011, Bhuiyan et al. 2011, Carollo et al. 2013]. Furthermore, experimental tests were also conducted in the presence of both sloping channels and adverse-sloping channels [Pagliara and Palermo 2015]. It was experimentally proven that for adverse-sloping channels, the sequent depth ratio decreases with relative roughness, if both the channel slope and the approaching flow conditions are held constant. Furthermore, the relative roughness plays a fundamental role as it increases air entrainment, thus modifying the effective water depths. For sloping channels, just a few studies were found dealing with effect of roughness on sequent depth ratio and they are all limited to the B-jump configuration [e.g., Carollo et al. 2013].

The aim of this paper is to compile the main results present in literature for all the aforementioned configurations. In addition, a critical comparison of the different methodologies will be presented in order to highlight both similarities and differences.

HYDRAULIC JUMP ON HORIZONTAL BEDS

The theoretical analysis of the hydraulic jump was conducted by Bélanger, who furnished the well-known Eq. (1) to estimate the conjugate depth ratio $Y = y_2/y_1$, where F_1 is the Froude number at the upstream section of the hydraulic jump and $\lambda = 8$:

$$Y = 0.5\left[-1 + \sqrt{1 + \lambda F_1^2}\right] \tag{1}$$

The previous equation is valid under the following hypotheses: horizontal rectangular channel; uniform velocity distribution; hydrostatic pressure distribution; and negligence of boundary flow resistance. Figure 2a shows a diagram sketch of a hydraulic jump on a smooth horizontal bed. It is evident that such conditions are very unusual in practical applications. Furthermore, successive studies showed that some corrections to the Bélanger's equations were necessary to predict Y, especially for high F_1, for which Eq. (1) systematically over-estimates the sequent depth ratio. In particular, Govinda Rao and Ramaprasad [1966] proved that λ depends on the velocity distribution and, in general, $\lambda < 8$.

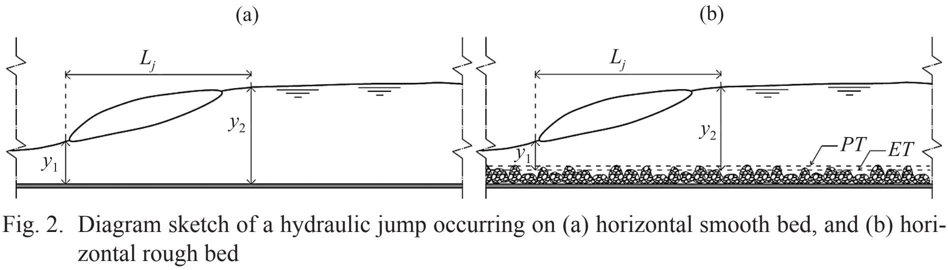

Fig. 2. Diagram sketch of a hydraulic jump occurring on (a) horizontal smooth bed, and (b) horizontal rough bed

Successive studies involved bed roughness effect (fig. 2b). The definition of the effective bed level (ET), from which the water depths are measured, is of relevant importance. According to Hughes and Flack [1984], ET can be set at $0.2d_{65}$ below the average level of the physical tops of the bed material (PT), in the case of spatial uniformity of bed roughness. Rajaratnam [1966] and Ead and Rajaratnam [2002] showed that the bed roughness represents one of the most influencing parameters, deeply modifying the dissipative mechanism and the characteristic lengths of the hydraulic jump. In other words, the presence of corrugated/rough bed increases shear stresses resulting in a significant modification of velocity profiles and in a reduction of the variable Y. Therefore, Rajaratnam [1966] analyzed the hydraulic jump occurring on horizontal rough beds, applying the general expression of the momentum equation:

$$P_1 + M_1 + W\sin\alpha = P_2 + M_2 + F_\tau \tag{2}$$

where:

P_1 and P_2 – the hydrostatic forces,
M_1 and M_2 – momentum fluxes,
in which the subscripts 1 and 2 represent the upstream and downstream hydraulic jump sections, respectively,
F_τ – the integrated shear stress per unit width,
W – the weight of the control volume,
α – the angle of the bed slope respect to horizontal (i.e., $\alpha = 0$ for horizontal beds, $\alpha < 0$ for adverse-sloped beds and $\alpha > 0$ for sloping beds).

He proposed that the integrated shear stress F_τ can be assumed equal to εP_1, where ε is the non-dimensional shear force coefficient. Following a similar approach, Ead and Rajaratnam [2002] showed that F_τ can be also expressed as function of M_1, resulting in $F_\tau = \varepsilon_1 M_1$. In both cases, the effect of shear stresses only depends on the flow conditions at the upstream section of the hydraulic jump. This assumption allowed derivation of a simple analytical expression for the sequent depth Y, as follows [Ead and Rajaratnam 2002]:

$$Y^3 - \left(1 + 2\mathrm{F}_1^2 - 2\varepsilon_1\mathrm{F}_1^2\right)Y + 2\mathrm{F}_1^2 = 0 \tag{3a}$$

Where, for both smooth and rough beds,

$$\varepsilon = 0.16\mathrm{F}_1^2 - 0.80\mathrm{F}_1 + 1 \tag{3b}$$

and $\varepsilon = 2\varepsilon_1\mathrm{F}_1^2$. It is worth noting that Ead and Rajaratnam [2002] estimated the integrated bed shear stress for both smooth and rough beds, showing that, in the tested range of parameters, the local friction coefficient c_f was either equal to 0.011 or 0.069, respectively. The large difference of c_f values coupled with the experimental validation of Eq. (3a) for both smooth and rough beds states that much larger Reynolds shear stresses should occur in the presence of rough beds, involving a significant reduction of the sequent depth ratio for a constant inflow Froude number (F_1). The same results were also obtained by other researchers, who followed different approaches, including the estimation of the average boundary shear stress over the wetted perimeter of the jump. Namely, Leutheusser and Kartha [1972] proposed the following general equation to estimate Y:

$$Y = 0.5\left[-1+\sqrt{1+8(1+\delta)\mathrm{F}_1^2}\right] \tag{4}$$

where δ depends on the vertical velocity distribution, the longitudinal flux of turbulent momentum, pressure distribution and the channel bed boundary conditions. Nevertheless, Eq. (4), as theoretically derived by Leutheusser and Kartha [1972], presents some difficulties in practical applications, as Hughes and Flack [1984] noted that a quantitative assessment of δ is not so easy to be done in most of the cases. Therefore, Hughes and Flack [1984] and Pagliara et al. [2008a] proposed a simplification of the mentioned approach. In particular, Pagliara et al. [2008a] analysed the conjugate depth ratio reduction on rough beds made of both uniform and non-uniform materials. They proposed the following equation to estimate the correction coefficient δ:

$$\delta = -0.05t\left(0.256Y^2 - 1.256Y + 1\right) - 1.1/\psi_{nu}^{4.5} \tag{5a}$$

$$\psi_{nu} = \frac{\sigma^2 d_{90}}{\left(\sigma^2 - 1\right)d_{84}} \tag{5b}$$

where

ψ_{nu} – termed as non-uniformity number,

$\sigma = (d_{84}/d_{16})^{0.5}$ – the non-uniformity coefficient,

d_{xx} – the diameter of the bed material for which xx% is finer and $t = d_{65}/y_1$ is the relative roughness.

Note that if the bed material is uniform ψ_{nu} tends to infinity. Therefore, the correction coefficient δ for uniform bed material can be easily derived from Eq. (5a). Furthermore, for smooth beds $t = 0$ and $\delta = 0$, i.e. Eq. (4) becomes Eq. (1).

Nevertheless, both Hughes and Flack [1984] and Pagliara et al. [2008a], adopted the variable t to define the relative roughness. Other researchers [Carollo et al. 2009, Pagliara and Palermo 2015] introduced a different definition of the relative roughness, in order to simplify the application of the derived relationships. Namely, they defined the relative roughness as d_{50}/k, where k is the critical depth. Both Carollo et al. [2009] and Pagliara and Palermo [2015] assumed $F_\tau = \beta(M_1 - M_2)$, where β is a coefficient whose values are less than 1. Based on this assumption, Carollo et al. [2009] proposed the following relationship to evaluate Y.

$$Y = 1 + \sqrt{2}\,\exp\left(-\frac{d_{50}}{k}\right)\left(\mathrm{F}_1 - 1\right)^{0.963} \tag{6}$$

From the previous Eq. (6), it appears evident that the effect of bed roughness on Y is significant resulting in a prominent reduction of the dependent variable with the relative roughness, thus confirming the previous findings obtained with different approaches [e.g., Leutheusser and Kartha 1972]. Nevertheless, all the mentioned studies did not take into account the effect of air entrainment. Namely, the negligence of air concentration introduces errors in the estimation of effective water depths, whose correct assessment is fundamental for the definition of the variables of the general momentum equation

(Eq. 2). Therefore, Pagliara and Palermo [2015] developed a semi-theoretical methodology involving the measurement of the air concentration profiles at the upstream and downstream sections of the hydraulic jump. Figure 3a–b shows a picture of a hydraulic jump occurring on an adverse-sloped rough bed and the probe adopted to measure air concentration inside hydraulic jump. The air concentration measurements allowed for a correct estimation of the effective conjugate depths. Thus, the following equation was derived to estimate Y. Figure 4 illustrates the effect of d_{50}/k on Y.

$$Y = 0.5\left[-1+\sqrt{1+8F_1^2\left[1+0.14\left(1-e^{2.38\frac{d_{50}}{k}}\right)\right]}\right] \tag{7}$$

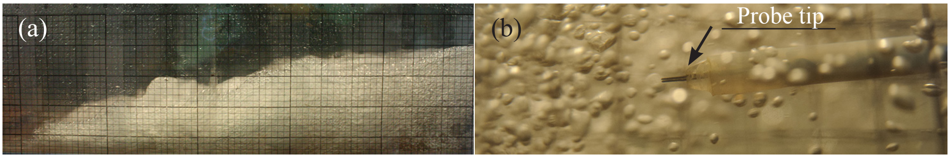

Fig. 3. Picture showing a hydraulic jump on a horizontal rough bed (a); particular of the air-concentration probe adopted to evaluate the effective hydraulic jump depths (b)

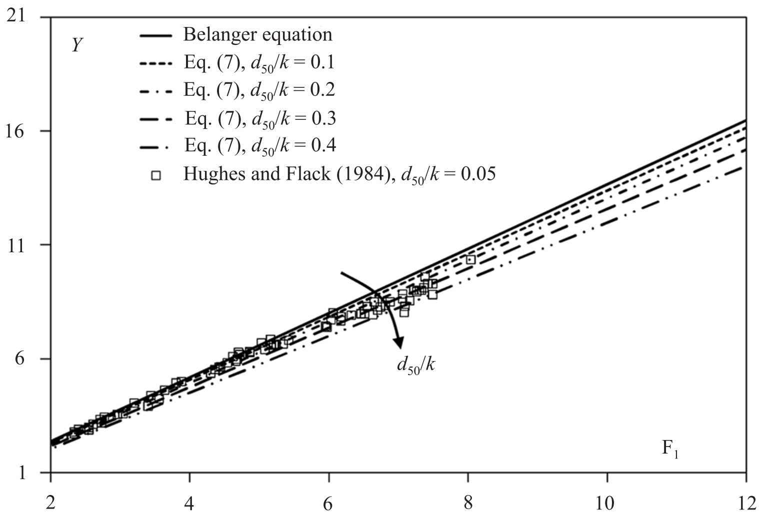

Fig. 4. Plots of Eq. (7) for different d_{50}/k, including data derived from Hughes and Flack (1984) for $0 < d_{50}/k \leq 0.1$

It has to be noted that Eq. (7) reduces to Bélanger's equation for smooth bed, i.e., when $d_{50}/k = 0$. Nevertheless, it represents a generalization of all the previous proposed equations, as it was tested using a database derived by several authors. As mentioned above, Eq. (7) also considers the effect of air concentration on the sequent depth ratio Y. In particular, sequent depth ratio results slightly larger than that estimated in previous studies for $F_1 > 6$. This is mostly due to the fact that air concentration increases with F_1, especially at the upstream section of the hydraulic jump, resulting in a slight increase of the variable Y respect to the case in which the air concentration is not taken into account. This occurrence is evident by observing experimental data derived from Hughes and Flack [1984] and reported in Figure 4. For $F_1 < 6$, the trend of experimental data well follows the semi-theoretical equation valid for $d_{50}/k = 0.1$. Whereas, the proposed equation slightly (≈10%) overestimates it for $F_1 > 6$.

HYDRAULIC JUMP ON ADVERSE-SLOPED BEDS

In the presence of an adverse-sloped bed, the hydraulic jump properties are subjected to significant modification. In particular, the weight of the control volume (W in Eq. 2) plays a relevant role. Furthermore, the hydraulic jump occurring on smooth adverse-sloped beds becomes quite unstable (particularly for $F_1 < 4$); therefore, generally sills are used to stabilize jump. This jump configuration on smooth beds was analyzed by several authors [among others, Okada and Aki 1955, McCorquodale and Mohamed 1994, Pagliara and Peruginelli 2000]. All the mentioned studies concluded that the bed slope $i = \tan\alpha$ (negative for adverse sloped bed) contributes to significantly modify the sequent depth ratio Y. Namely, Y is a monotonic increasing function of i, i.e., for higher bed slopes the sequent depth reduction is more prominent. Both McCorquodale and Mohamed [1994] and Pagliara and Peruginelli [2000], derived a semi-theoretical equation to evaluate Y, introducing the adverse jump parameter $G_1 = f(i, F_1)$, for which they assumed that the roller length can be considered as the jump length and the control volume per unit width can be calculated as $0.5K(y_1 + y_2)L_j$, where K was assumed equal to 1.08 and 1 by McCorquodale and Mohamed [1994] and Pagliara and Peruginelli [2000], respectively, and it represents the coefficient for the determination of the weight of control volume. Figure 5 shows diagram sketches of hydraulic jumps on adverse-sloped smooth bed (Fig. 5a), and adverse-sloped rough bed (Fig. 5b).

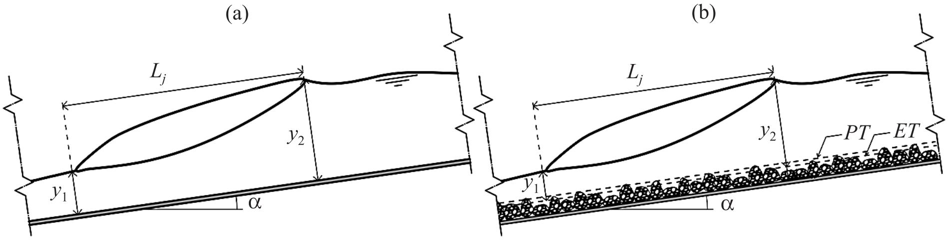

Fig. 5. Diagram sketch of a hydraulic jump occurring on (a) adverse-sloped smooth bed, and (b) adverse-sloped rough bed

In particular, Pagliara and Peruginelli [2000] proposed the following Eq. (8) to estimate Y. Figure 6a reports the plots of Eq. (8) in a graph $Y(F_1)$ in which the effect of the bed slope i is clearly illustrated. The proposed relationship was compared and tested with both equations and experimental data proposed by Okada and Aki [1955] and

McCorquodale and Mohamed [1994], resulting in a very good agreement (±10%) with the previous literature.

$$Y = 0.5\left[-1 + \sqrt{1 + 8\left(3.32^{1.52i}\,\mathrm{F}_1\right)^2}\right] \tag{8}$$

More recently, further studies were conducted by Pagliara and Palermo [2015]. They analyzed the hydraulic jump characteristics on both smooth and rough adverse-sloped bed, including air concentration measurements in order to estimate the effective water depths. This study represents the first attempt to furnish a comprehensive analysis of the hydraulic jump properties on both horizontal and adverse-sloped bed in which the effect of air concentration is also taken into consideration. Based on experimental data, Pagliara and Palermo [2015] proposed a general equation valid for both horizontal and averse-sloped beds and for both smooth and rough bed configurations. Namely, applying the momentum equation (Eq. 2), they derived the following general relationship, valid for a wide range of relative roughness ($d_{50}/k < 0.5$) and bed slopes ($-0.25 < i < 0$):

$$Y = 0.5\left[-1 + \sqrt{1 + 8\left(3.32^{1.52i}\,\mathrm{F}_1\right)^2\left[1 + 0.14\left(1 - e^{2.38\frac{d_{50}}{k}}\right)\right]}\right] \tag{9}$$

It has to be noted that Eq. (9) coincides with Eq. (8) for smooth beds (as $d_{50}/k = 0$) and with Eq. (7) for horizontal beds, i.e., it represents a generalization of the previous relationships present in literature. Furthermore, it gives an effective tool to estimate the effect of bed roughness on the jump properties also for adverse-sloped beds. In particular, Figure 6b illustrates the effect of relative bed roughness for different bed slopes. It is easy to observe that also in the case of adverse-sloped beds, the relative roughness contributes to reduce the sequent depth ratio Y, i.e., Y is a monotonic decreasing function of d_{50}/k, thus confirming the findings valid for horizontal rough beds. For practical applications, the proposed Eq. (9) appears very useful, as it furnishes one unique and comprehensive tool to evaluate the hydraulic jump properties in a wide range of both hydraulic parameters and bed configurations. Furthermore, it paves the way for further research on the topic, as Pagliara and Palermo [2015] showed that all the hydraulic jump lengths (i.e., Y and L_j) decreases with both bed slope and relative roughness. This occurrence is particularly important as it implies that the length of the stilling basin downstream of a hydraulic structure can be significantly shorter if opportunely design, thus reducing the construction costs. It is worth noting that potential scale effects on air–water flow properties are not significant if $B/y_1 > 10$, where B is the channel width, and Reynolds number $Re > 40000$ [Chanson 2009, Heller 2011]. The experimental validation of Eq. (9) was conducted for B/y_1 and Re values larger than those previously mentioned; therefore, potential scale effects do not significantly affect the estimation of Y. Figure 7 shows an example of a hydraulic jump occurring on an adverse-sloped rough bed.

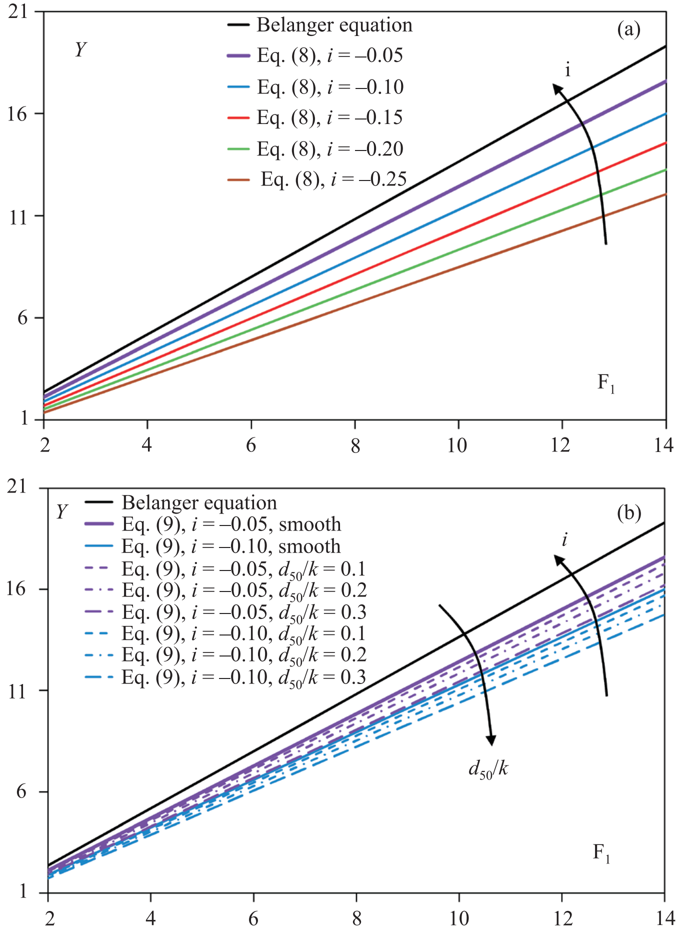

Fig. 6. Hydraulic jump on adverse-sloped beds: effect of bed slope i on $Y(F_1)$ for (a) smooth beds and (b) rough beds

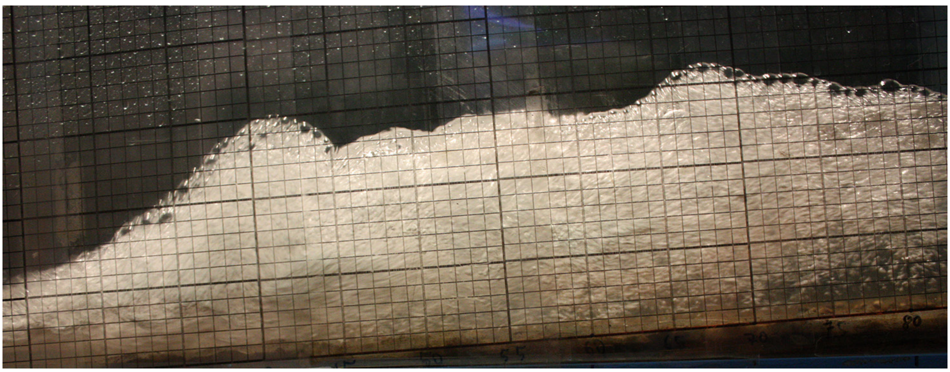

Fig. 7. Picture showing a hydraulic jump occurring on an adverse-sloped rough bed

HYDRAULIC JUMP ON SLOPING BEDS

Other hydraulic jump configurations which frequently occur in practical application are illustrated in Figure 8a–c. Namely, Figure 8a shows a D-jump occurring on a smooth sloping channel, whereas Figure 8b, c shows a B-jump occurring on a smooth and rough bed, respectively. The smooth sloping channel configuration was analyzed by several authors [among others Chow 1959, Rajaratnam 1966, Kawagoshi and Hager 1990, and Ohtsu and Yasuda 1991], who proposed different approaches and relationships to estimate the sequent depth ratio Y. Namely, for D-jump on smooth beds, Rajaratnam [1966] proposed the following Eq. (10) (with α in rad), whereas Ohtsu and Yasuda [1991], applying momentum equation, derived Eq. (11), valid for $0° \leq \alpha \leq 19°$ and $4 \leq F_1 \leq 14$, in which $Y_D = h_t/y_1$, where h_t is the vertical water depth measured at the downstream section of the hydraulic jump (Fig. 8a):

$$Y = \sqrt{2} \cdot 10^{0.027\alpha} F_1 - 0.5 \tag{10}$$

$$Y_D = \left(0.077\alpha^{1.27} + 1.41\right)\left(F_1 - 1\right) + 1 \tag{11}$$

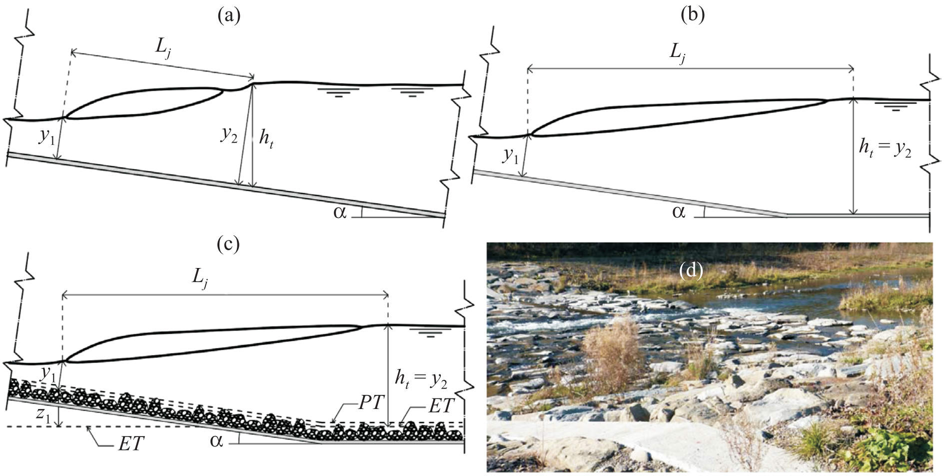

Fig. 8. Diagram sketch of a hydraulic jump occurring on a sloping bed: (a) D-Jump on a smooth bed, (b) B-jump on a smooth bed, and (c) B-jump on a rough bed; example of a block ramp with the downstream protected stilling basin (flow from left to right)

Figure 9 reports plots of the mentioned equations valid for D-jump on smooth beds, along with the curves proposed by Chow [1959], in a graph $Y(F_1)$ for a large range of bed slopes ($0 < i < 0.15$). It can be observed that all the proposed equations are in very good agreement. Nevertheless, by increasing the bed slope, some slight differences can be detected. In particular, for $i = 0.15$, the curve proposed by Chow [1959] seems to underestimate the values of the variable Y (which is almost equal to Y_D in the tested ranges of bed slopes), especially for high F_1. This occurrence is probably due to the fact that Ohtsu and

Yasuda [1991] conducted experimental tests in a wider range of inflow Froude numbers and derived their relationship by applying the momentum equation for which the estimation of the jump length is required. This last parameter is not easily estimated, as the hydraulic jump is a "dynamic" phenomenon. Furthermore, another important parameter which requires particular attention is the weight correction coefficient K, accounting for the difference between the actual weight of the control volume and the weight of the idealized control volume (straight line surface-profile between upstream and downstream sections of the jump). Ohtsu and Yasuda [1991] furnished an expression for K, stating that it decreases with i, tending to 1 for high bed slopes. Nevertheless, despite the complexity of the phenomenon and difficulties in lengths estimation, all the proposed relationships are in a good agreement, showing that the sequent depth ratio Y strongly depends on i, i.e., it is an increasing monotonic function of i. By comparing Figures 6a and 9, it is worth noting that the effect of the slope i is more prominent in the case of sloping channels than in the case of adverse-sloped channels.

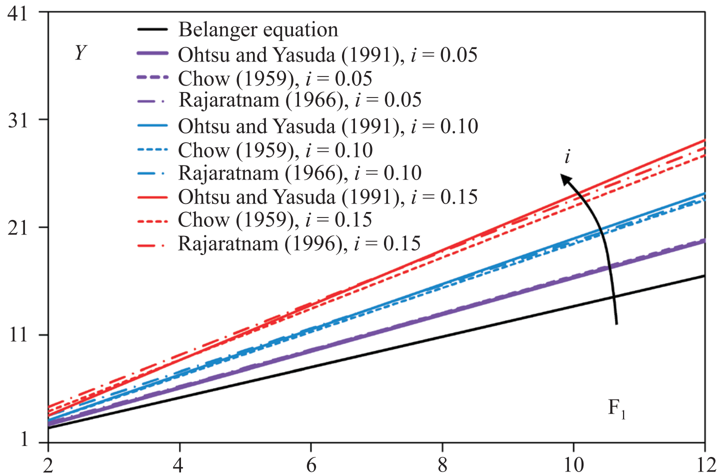

Fig. 9. Comparison of selected predicting equations to estimate the sequent depth ratio Y for different bed slope configurations

According to Kindsvarter [1944], B-jump can also occur on sloping channels. This last jump configuration is particularly frequent in practical applications, especially downstream of low head hydraulic structures. In particular, Pagliara et al. [2008b] analyzed this jump configuration in terms of energy dissipation on both fixed and movable beds. Furthermore, the analysis of hydraulic jump occurring in correspondence with a block ramp (Figure 8d) was further developed by Pagliara and Palermo [2008a] and Pagliara and Palermo [2008b], both in the presence and absence of a protection rock layer downstream of the ramp. They showed that the erosive phenomenon mostly depends on the hydraulic jump properties. In fact, by comparing the erosive process occurring in the presence either of a smooth or a rough sloping bed, it was possible to conclude that the erosive action of the hydraulic jump on the downstream is more prominent for smooth beds as a less energy dissipation occurs. This evidence is valid for both clear-water and live-bed conditions as shown by Pagliara et al. [2011] and Pagliara et al. [2012] and can be extended also to other structure typologies, such as rock grade control structures [Pagliara and Palermo 2013]. Nevertheless, in the case of movable stilling basin, the hydraulic jump properties strongly depend on the stilling basin geometry and configurations [Pagliara and Palermo 2011, Pagliara and Palermo 2012]. Namely, the hydraulic jump can be either 2D or 3D, according to the geometry of the downstream stilling basin (i.e., prismatic channel for which stilling basin has the same width of the upstream sloping channel or enlarged rectangular basin), resulting in different energy dissipation mechanisms [Nashta and Garde 1988, Bremen and Hager 1993, Pagliara et al 2009]. In particular, Bremen and Hager [1993] and Pagliara et al. [2009] showed that for channel configurations illustrated in Figure 8c-d, according to the stilling basin geometry several jump typologies can be distinguished. In synthesis, depending on both bed slope and downstream tailwater, the stilling basin geometry can strongly modify the flow pattern inside the jump (and consequently the dissipative process), resulting in an increase of energy dissipation in the presence of a 3D jump occurring in an enlarged movable stilling basin. Nevertheless, the physics of the phenomenon in the presence of a movable bed still requires significant efforts to be fully understood.

But, in the case of either smooth or rough fixed beds and prismatic channels (Figure 8b–c), hydraulic jump properties can be estimated following the approaches adopted by Ohtsu and Yasuda [1991] for smooth beds and Carollo et al. [2013] for rough beds. Namely, Ohtsu and Yasuda [1991] analysed the B-jump on smooth sloping channel (Figure 8b) and proposed an analytical expression to estimate y_2/y_1. They showed that, for the same inflow conditions (i.e., same F_1), the sequent depth ratio of a classical hydraulic jump is larger than that occurring in the case of a B-jump for $\alpha > 19°$. They proved that if B-jump is formed, the surface roller deeply influences the maximum cross sectional velocity and its decay becomes larger than classical jump. Furthermore, when $\alpha > 23°$, the curvature of the streamlines in correspondence with the slope transition becomes larger, resulting in the formation of an acceleration zone just downstream of the sloping part of the channel. These evidences were also noted by Carollo et al. [2013] who analysed the B-jump on rough sloping channel (Figure 8c). The authors generalized their Eq. (6), valid for horizontal rough beds, including the effect of the upstream channel slope i and the location of the jump toe position, expressed by the parameter $E = (y_2 - z_1)/y_2$, where z_1 is the bottom elevation at jump toe section. Namely, they proposed the following Eq. (12):

$$Y = \sqrt{2}\,\exp\left(-\frac{d_{50}}{k}\right)\exp\left(-\frac{\tan\alpha}{11.01}\right)E^{-\frac{0.202}{(\tan\alpha)^{0.644}}}\left(F_1 - 1\right)^{0.963} + \frac{1}{E} \tag{12}$$

Note that $E = 1$ for horizontal beds, thus Eq. (12) is analytically consistent with Eq. (6). Also in this case, the authors identified the effect of the bed roughness on the sequent depth. In particular, by comparing the sequent depth ratio in the same hydraulic conditions and channel bed geometry, they experimentally proved that Y decreases with relative roughness also in the case of B-jump. The reason for such occurrence is essentially the same identified by Ead and Rajaratnam [2002], i.e., the presence of roughness induces much larger Reynolds shear stresses.

CONCLUSIONS

Despite of the significant number of studies focusing on the hydraulic jump, the actual knowledge on such fascinating hydraulic phenomenon is still limited. Namely, hydraulic jump occurs in many practical applications and the large range of both hydraulic conditions and configurations do not allow for full comprehension of all the physical dynamics involved in a such complex phenomenon. Nevertheless, the efforts of the scientific community have substantially increased the understanding of the main properties of this phenomenon. This paper aimed to synthetize and discuss the main characteristics of the hydraulic jump occurring on different bed configurations and under several boundary conditions. It appeared clear that the bed slope, the relative roughness and air entrainment play fundamental roles in modifying the jump properties. Namely, despite of the channel bed configuration, the sequent depth ratio decreases with the relative roughness.

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PRZEGLĄD CECH ODSKOKU HYDRAULICZNEGO W KORYTACH NACHYLONYCH W DÓŁ LUB W GÓRĘ (TZW. ODWRÓCONYCH) Z GŁADKIM LUB SZORSTKIM DNEM

Streszczenie. Odskok hydrauliczny jest zjawiskiem, które w ostatnich latach zwraca szczególną uwagę i stanowi istotny przedmiot badań ze względy na swoją możliwość rozpraszania znacznych ilości energii przepływowej. Waga tematu zobowiązuje środowisko naukowe do podejmowania wciąż nowych wysiłków badawczych. Prognoza głównych długości odskoku hydraulicznego pozostaje bowiem nadal kwestią otwartą, gdyż bieżąca wiedza na ten temat nie dotyczy wszystkich możliwych konfiguracji oraz warunków brzegowych, które zazwyczaj występują w praktycznych zastosowaniach. Szczególnie oddziaływania szorstkości dna, jego spadku, geometrii koryta i koncentracji powietrza na współczynnik głębokości sprzężonych nie są dotąd w pełni poznane. Celem pracy jest przedstawienie syntetycznego obrazu stanu wiedzy dotyczącej właściwości odskoku hydraulicznego w szerokim zakresie zarówno warunków brzegowych, jak i konfiguracji geometrycznych. Analiza będzie się w szczególności skupiać na tym, jaki wpływ na współczynnik głębokości sprzężonych mają zarówno względna szorstkość koryta, jak i jego spadek, a także na tym, jaki jest wpływ deflacji powietrza na ocenę rzeczywistej głębokości. Ponadto porównane zostaną i poddane dyskusji niektóre przewidywane przez różnych autorów zależności.

Słowa kluczowe: napowietrzane przepływy, szorstkość dna, odskok hydrauliczny, pochyłe koryto, niecka wypadowa

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