SURPRISE-DRIVEN ABDUCTIONS IN DGEs

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Abductive inferences, which are the only types of inference that produce new ideas, are important in mathematical problem solving. Such inferences, according to Peirce, arise from surprising or unexpected situations. Therefore, one way to improve student problem solving may be to provide them with environments that are designed to evoke surprise. In this paper, we examine the potential of dynamic geometry environments (DGEs)to foster surprise. We conjecture that the ease with which students can explore configurations, along with the immediate feedback, may lead them to encounter surprising situations. We analyse three different examples of student problem solving featuring surprised-provoked abduction, and identify the specific role that the DGE played.

Keywords: abduction, dynamic geometry, feedback, surprise

ABDUCTION AND SURPRISE

Peirce introduced the notion of abduction as a type of inference distinct from both deduction and induction. In thispaper we will refer strictly to Pierce's definition of abduction of the general form, thatis: (fact) a fact A is observed; (rule) if C were true, then A would certainly be true;(hypothesis) so, it is reasonable to assume C is true.Peirce saw abduction as the only inference capable of producing new ideas. Both induction and deduction constitute a closed system in that all propositions must come from within the system. As Mason (1995) writes "The pedestrian can argue inductively from a series of instances; the pedant can argue deductively from what is known. But both have to know where they are going" (p. 5). In contrast, abduction is capable of reaching outside the system to introduce new propositions. The inference is based on one premise and involves the generation of the abduction through the positing of the rule. However, frequently, only the result is stated, while the rule is verbally suppressed. This can make it difficult for an outside observer to identify instances of abduction; as Mason writes "The tricky part about abduction is locating at the same time the appropriate rule and the conjectured case" (p. 5).

Peirce also emphasized an aspect of abduction related to its initiation.One feels the need to explain a situation because it strikes one as surprising or unusual.The role that surprise plays in giving rise to abduction strikes us as particularly interesting, especially since neither Reid (2002), Arzarello et al. (1998), nor Cifarelli (1997) acknowledge it in their analyses of student abductions. Is it possible that students experience a somewhat different emotional response than that of surprise, perhaps more along the lines of confusion or helplessness? Thagard (2007) claims that the initiation of abduction is inherently emotional, spurred on by feelings of astonishment, surprise, or even less extreme reactions such as puzzlement (as Peirce also had noticed). While he has little empirical evidence to support this claim, the work of Damasio (1994) on the fundamental role of emotion in decision-making would seem to corroborate Thagard's claim. We thus hypothesise that student abductions will also possess some kind of emotional component, if not surprise, and that this emotional component will be important both in identifying instances of abduction and in understanding the cognitive dimension of the abductive inference in the problem-solving process.

ABDUCTION AND DGES

In mathematics education there has been renewed interest in the concept of abduction inthe context of conjecturing and problem solving in DGEs (e.g., Arzarello et al., 2002; Antonini&Mariotti, 2010; Baccaglini-Frank, 2010b; Baccaglini-Frank &Mariotti, 2011; Samper et al., 2012).A feature offered by a DGE is the dragging tool, that can be exploited in various ways by the solver, and that can support conjecture-generation. Research carried out by Arzarello, Olivero, Paola, and Robutti (2002) led to thedescription of a set of dragging modalities, classified through an a posteriori analysis of solvers' work, that can beobserved while a solver is producing a conjecture in a DGE. A key moment of the process of conjecture-generation isdescribed in Arzarello et al.'s model as an abduction, which seems to be related to the use of dummy locus dragging, later referred to here as maintaining dragging (MD) (Baccaglini-Frank, 2010a; Baccaglini-Frank &Mariotti, 2010). However, various aspects of the relationship betweendragging and abduction needed further clarification.

Starting from the results of these studies Baccaglini-Frank and Mariotti (2010; 2011) carried out a new study to further clarify the various aspects of the relationship between dragging and abduction, finding that when a particular dragging modality, maintaining dragging, is used successfully, the abduction leading to the discovery of a new geometric property is of a different nature than those described previously. This abduction makes use of a rule that has to do with the behaviour of the instrument "maintaining dragging" and it leads to feedback from the software, that can be interpreted by the solver as a new property, "causing" the one intentionally maintained. So in this case, the abduction (referred to as "instrumented") is supported by the software, which also keeps hidden (to the solver) other geometrical properties and theorems that connect the new property with the one induced intentionally through dragging. However, in a case in which physical use of maintaining dragging failed, the students acted "as if" they were using maintaining dragging, but left the figure static and reasoned abductively about how the dragged point would have to move in order for their desired property to be maintained (Baccaglini-Frank, 2010b). Indeed, later analyses of the data led Baccaglini-Frank and Antonini (2016) to a new hypothesis, that is, that different types of maintaining dragging – either as a physical tool or as a psychological tool for generating conjectures – can influence students' subsequent processes of proving. In particular it seems that using maintaining dragging as a physical tool can foster continuity between the conjecturing phase and the proving phase, allowing the students to discover geometrical properties that can bridge the logical gap between the premise and the conclusion of the conjecture, and be successfully pieced together in a proof.

LINKING ABDUCTION, SURPRISE AND DGEs

In her exposition of the different sources of surprise in the teaching and learning of mathematics Movshovitz-Hadar (1988) identifies several that seem a priori relevant to DGEs. For example: "A common property in a random collection of objects," "Unexpected existence, and non-existence of the expected," and "Refutation of a conjecture obtained inductively" (p. 35). Indeed, although not analysed in terms of surprise, it is evident in the examples of abduction described in Baccaglini-Frank and Antonini (2016) that the abductions arose in situations in which the students saw something they had not expected—which corresponds to one of Movshovitz-Hadar's sources of surprise. Given the ease with which students can experiment with many different cases of a configuration, it is also possible that they will be able to refute a conjecture that they had obtained inductively. For example, they might drag the vertex of a triangle in a certain manner and observe that the intersection of the altitudes can fall outside of the triangle. The exiting of the triangle centre could produce surprise.

Ruthven (2017) has argued that since students are often not surprised by the fact that the three altitudes or perpendicular bisectors of a triangle meet at a point, it is important to design tasks in which the surprising aspect of that fact can be appreciated. He provides the example of constructing three circles centred at an arbitrary point P in the triangle, passing through each one of the vertices. By dragging the point P, students will see that it is possible to have the three circles coincide exactly, a phenomenon that might elicit more surprise because it arises from a more complex and messy configuration. Although Ruthven does not link the eliciting of surprise to abduction, his argument is important because it highlights the fact that DGEs on their own do not necessarily elicit surprise; rather, it depends on the task that is offered to students.

Since the study of Baccaglini-Frank and Antonini focused on conjecturing, and raised issues about the problematic use of maintaining dragging in relation to abduction and the proving process, we decided to investigate tasks involving problem solving, where constructions played an important role. We assumed, based on Peirce's point of view, that when abductions occurred in these situations, they would also arise from surprise. From this, we were interested in understanding what specific role the DGE played in promoting experiences of surprise. Our hypothesis is that there may be particular aspects of DGE use that are more conducive to eliciting surprise.

THREE EXAMPLES

We present three examples, two of which were reported in prior research conducted by the first author and a third that comes from research conducted by the second author. They were each selected because they had previously been identified as featuring abduction. We will describe that abduction, show how it arises from some form of surprise, and then examine the role played by the DGE in eliciting that surprise.

Impossible construction.

The first case was reported in Baccaglini-Frank,Antonini, Leung, &Mariotti (2013). The task, given to high school students in Italy, was as follows: Is it possible to construct a triangle with two perpendicular internal angle bisectors? If so, provide steps for a construction. If not, explain why. Giulio and Stefano immediately advanced the hypothesis that the construction is not possible, but Giulio also started to construct a figure in *Cabri*—two perpendicular lines (Fig. 1a)—and referred to them as the bisectors of the triangle. In the transcripts below "Int" refers to the interviewer, and the bold letters refer to the solver who is holding the mouse.

Int:	Can you show me the triangle?
Giu:	No, it's this that moves [*drags one of the lines to turn the configuration around, as in Fig. 1b*]…I'm stupid. Aaaaaaaah[*sighs*].
Ste:	<u>No, the only way is to have 90 degree angles</u>.
Giu:	That for a triangle is a bit difficult! [*giggling*]So…they have to be… [*he then constructs two points on the bisectors, indicating them as endpoints of the triangle's side*]
Ste:	If triangles have four angles…[1]

Once the perpendicular lines have been turned, the students seem to see a triangle with a side that is horizontal, joining both of the lines below the intersection point. In that configuration, Stefano makes an abduction (underlined), whose structure is as follows:

- fact: <u>The only way (to make the triangle with the two lines as perpendicular bisector) is to have 90 degree angles</u>
- rule: If the sides of the triangle are vertical, the trianglewill have 90 degree angles
- hypothesis: The sides of the triangle are vertical (perpendicular to the imagined based of the triangle)

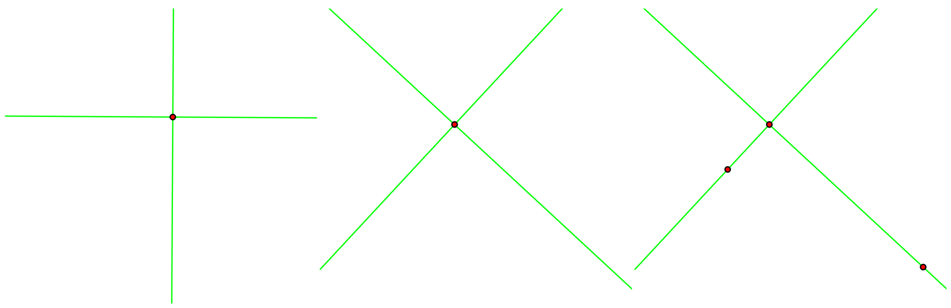

Figure 1. (a) the initial construction. (b) turning the configuration around. (c)the endpoints of the triangle's side.

The abduction starts with a fact that is not true necessarily: it is not true that *the only way* is to have 90 degree angles (what is true is that the only way for a triangle to have perpendicular internal angle bisectors is for the sum of the two angles containing the imagined horizontal side to be 180 degrees, but not necessarily 90 degrees each), however the same (correct) abduction could have been made starting from the statement: "It has 90 degree angles". Moreover, it is restrictive to think of the side common to the two angles of which the bisectors are drawn to be horizontal (in the configuration of Fig. 1b). Indeed Giu seems to realize this and when he constructs the endpoints of the triangle's side (Fig 1c) he moves them to an evidently non-horizontal configuration, then proceeds to construct the other two sides ofthe triangle. Before the abduction takes place, there seems to be surprise arising from an assumed impossibility of the construction, which the students state even though they have not yet determined how to explain it, and are perhaps not entirely convinced of. Possibly, the abduction takes place in an attempt to explain the non-existence of the expected.The fact that they can easily and precisely construct the two perpendicular lines and then rotate that configuration, enables them to imagine the triangle whose base is horizontal, below the point of intersection, and whose sides are vertical. Without *Cabri*, they could have certainly drawn two perpendicular lines on a piece of paper and turned that paper around. However, it is the robustness of the construction that they are making in *Cabri* that first entices them to attempt the construction, which would have been impossible to realize to such a degree of precision on a piece of paper. Moreover, the software will guide the refutation of the statement: “No, the only way is to have 90 degree angles” and of the abductive hypothesis reached, that the other two sides of the triangle are vertical. Through the visual and dynamic feedback provided by the software the students will formally explain the impossibility of the construction through a generalization of their ideas (parallel sides or triangles with 4 angles), explaining why the construction is impossible.

Unexpected breach

Two 15-year-old students in the second year of an Italian high school, Francesco andGianni, are working on the problem in the example described above. Initially, in orderto obtain the desired property (that we will indicate with P_d) “ABCD parallelogram”the students have chosen diagonals intersecting at their midpoints (P_1) as the propertyto induce intentionally through maintaining dragging (the student holding the mouse isin bold). However their attempt fails.

Gui: If you do like, maintaining dragging……….leaving them to remain more or less the same [*referring to PB and PD*]

Fab: exactly[…]They go through the types of quadrilaterals that they think are possible[…]

Gui: Uh, then we had done? parallelogram.

Fab: For the parallelogram, uh, let’s try to do it with the trace to see if some kind of form emerges[*they turn on trace on D*]

Gui: Uh, and now what are we doing? Oh, for the parallelogram?

Fab: Yes, it’s a good idea to see when it remains a…. parallelogram.

Fab: more or less

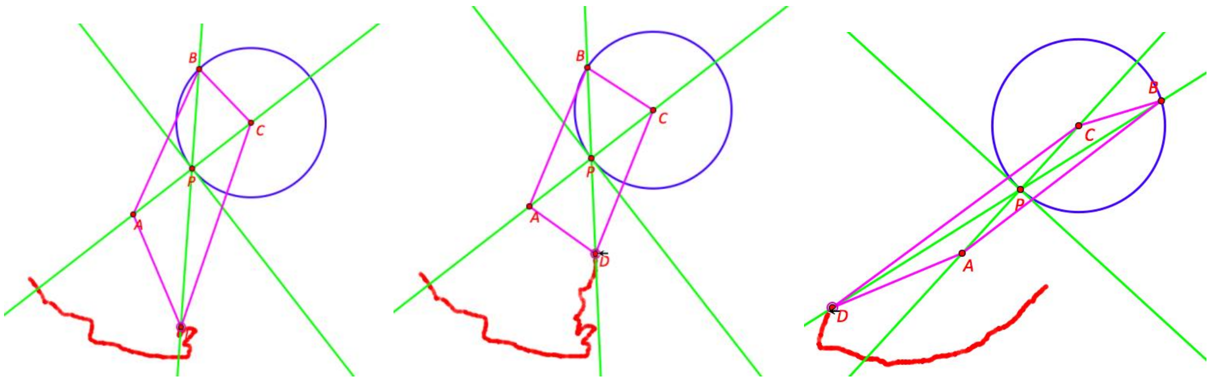

Figure 2. (a). First attempt at maintaining dragging, expectation on shape of trace mark is made explicit; (b). The expectation is not met; (c) Second attempt at maintaining dragging; the expectation on the trace mark is not met.

Gui: Yes, ok, the usual circle comes out. (see Fig. 2a)

Fab: Wait wait….what… oh no!

Fab:	Oh dear, where is it going? [*They erase the trace and try again, now Gui is dragging; se Fig. 2b*]
Fab:	But maybe it’s not necessarily the same circle. Try to make it go around… go, go.
Fab:	Because, like, at some point, I don’t know… go…
Fab:	by tomorrow….keep going[*Gui is dragging slowly*]
Fab:	Careful, you are making it too long!
Fab:	But you see? This one is always longer than this one… it’s too long. If you go along this circle, let’s say here, this one is too long [*referring to PD; Fig 2c*]
Fab:	So maybe it’s not necessarily the case that D is on a circle so that ABCD is a parallelogram. [*and he takes the mouse back*]
Gui:	Put it back decently, so we can understand something.[…]
Gui:	We discovered when?
Fab:	Let’s also try without trace.
Gui:	Let’s try to think about it without, like...becauseif when you move this [*point D*] while always maintaining the same distance [*DP=PB*]… look.
Gui:	it’s a chord, no? We have to… it means that this must be a chord equal to another circle, which I think has centre A.
Gui:	Because I think that if, let’s say, you do a circle of the centre
Fab:	A, you mean…
Gui:	symmetric with respect to that, you have to do it from the centre A.
Fab:	uhm.
Gui:	Do it.
Fab:	From centre A and radius AP?
Gui:	From centre A and radius AP. Yes, I think so.

Gui observes that PB is a chord of the circle, which initiates an abductive process:

- Fact: chord PB of a circle. PB=PD
- Rule: if PD is a chord of a circle symmetrical to the observed one, then PD=PB.
- Hypothesis: PD is a chord of the symmetrical circle.

In this case, the abduction arises from the unexpected fact that the regularity that can be inferred from the trace is not the belonging of its point to the circle the students have in mind (possibly a “big” circle to which the one on the screen is tangent internally, which is part of a conjecture obtained inductively), which surprises the two boys and prompts a need to explain why. For this situation to have been surprising, it was helpful that the students had previously made a conjecture in an earlier problem in which the regularity of the trace had indeed been a circle. Compared to the previous example, the DGE plays a more significant role in eliciting the surprise through the use of the tracing tool and the maintaining dragging. The trace provides a compelling and persuasive counter-example to the inductively generated conjecture that the students had made when they first started dragging. The refutation of the initial (partially implicit) conjecture obtained inductively, thanks to the feedback given by the DGE during students’ use of maintaining dragging, is what prompts the abduction, that is then made without moving the figure. The hypothesis reached through the abduction is then tested by the students who construct the geometrical element in this hypothesis (a symmetrical circle with PD as a chord) and try dragging D along it, this time receiving expected feedback from the DGE,

Constructing a parabola.

The third example occurred in an undergraduate course at a Midwestern university. The teacher (T) is working with two students Lucie and Brendan, who are trying to solve the problem of geometrically constructing a parabola in *Sketchpad* (using the circle and line tools) given a focus point *P* and a directrix line *j*. Such a curve consists of all points equidistant to *P* and *j*. The students have already constructed the envelope of the parabola by tracing the perpendicular bisector of *PB* where *B* is a point on *j* that they can drag back and forth along the line (Figure 3 shows this trace). After creating the envelope Brendan points out that they must try to find a way to construct the curve itself. So the students begin looking for ways to construct a point that depends on B somehow, so that, as they move B along *j*, it might trace out the parabola. At first, they place a point on *PB* right where the segment first touches the envelope edge. When Lucie drags B, they realize that this point doesnot always lie on the curve, so they delete the point. Brendan notices that if the solution point is on *PB,* then it could reach the upper parts of the parabola. Lucie then comes up with the hypothesis (underlined below) that this point will have to lie on a line passing through *P* and perpendicular to *j*.

B: So we gotta figure out what line goes right there [*traces the curve formed by the envelope with his finger*].

L: Laughs. Ummmmmm [*drags B back and forth alongj*]

B: We can't have [..] Well like [….] like see that point has to be able to get up here right? [*points to j with his pen and then points to the top left of the curve with his pen and then his finger*.]

L: uhuh

B: which means it can't touch the line. Yep. So then… let's say…. [*constructs the line though P perpendicular to j*]. Maybe that's the line … cuz um… the distance from like… here to here [*points to the distance between the curve on the left and her new line*] would be the same as that one? But I don't know if that's right.[*Points to her new line and the curve on the right.*]

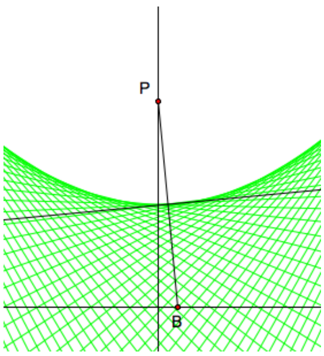

Figure 3.The envelope of the parabola.

Lucie's hypothesis, "Maybe that's the line" follows the abductive form. The fact comes from Brendan's observation that if the point is going to trace out a parabola, it has to be able to "get up there". It seems that Brendan and Lucie see the problem of getting the point to go up far enough provides the sense of puzzlement that motivates the abduction. Lucie and Brendan show several signs of puzzlement as they begin working on the problem, with Brendan stating the problem twice—perhaps surprised that it's possible to construct the envelope, but not the curve—and both being unsure of how to proceed.

- Fact: The (solution) point has to go up

- Rule: If it’s on that line, it would go up
- Hypothesis: The point is on that line

Prior to the inference, there are several pauses, one right before she begins constructing the line, and one while she is constructing the line. In effect, her verbal statement (Maybe that’s the line) is preceded by an action (constructing the line), an action that communicates her result even before she states it. Lucie’s abduction came as a result of Brendan’s deduction (that the point couldn’t possibly be on the segment *PB*). And her abduction then led right to an inductive inference, as she tries to provide a rationale for her abduction: she infers that her new line will be equidistant to each side of the parabola.

In this case, the ease with which the students were able to create the envelope, and for all intents and purposes, see the curve, was significant to the abduction. The possibility of dragging and receiving visual feedback from the DGE (in the form of a very large number of tangents to the parabola seen simultaneously thanks to the trace functionality)enabled them to attend to the far reaches of the curve. They were stuck at the need to move from a trace to a locus, that is, to give a geometrical description (in function of the constructed elements) of the traced points of the parabola. However it seems to be thanks to the trace and what it showed them that they were convinced that it was possible to construct that parabola.

DISCUSSION

In the three examples we analysed above, we identified three different sources of surprise. The first was related to expectations on reasons for theimpossibility of the construction. The second involved refutation of an inductively generated conjecture. The third seemed to also be related to expectations on (im)possibility in that the students wondered how the point was going to be able to reach the top part of the parabola. But it was perhaps also about the juxtaposition of a simple construction (the trace) with a more difficult one (the locus). In both the first and third cases, the fact of being able to construct something precise and transformable, through which implications of the constructed properties could be guessed at, seemed to enable the students to experience surprise, or at least curiosity, and subsequently feel the need to find an explanation; and this process, in both cases involved abduction. We thus propose two types of construction tasks as being potentially fruitful in terms of eliciting abductions: impossible constructions and locus constructions. In the second example, the abduction arose from a combination of using maintaining dragging with a trace and encountering a trace that was unexpected (because it seemed to be conflicting with the pattern they had in mind). When the abduction took place the students had decided to stop using the dragging tool and “think about it”. Although maintaining dragging was not being used at the moment of the abduction, be argue that it is because of interiorization if this dragging scheme that this abduction was possible. Indeed the students seem to mentally manipulate the figure as if they were using maintaining dragging; and in order to control such manipulation of the figure, through abduction they come up with a geometric property which indeed satisfies the constraints they are imagining on the movement of the figure. This might provide the basis for designing a sequence of tasks involving maintaining dragging in which the shape of the trace eventually changes and students, who have learned to use maintaining dragging are encouraged to predict the dynamic behaviour of the figures even before actually manipulating them.

Notes

1. For the sake of completeness, this is also an abduction.Fact the symmetric images of the endpoints across the two bisectors give two other points on the sides of the triangle. Rule: if a triangle has four angles, I would have constructed my triangle with robust perpendicular angle bisectors. Abductive hypothesis: a triangle has four angles.

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