D-jump in rough sloping channels at low Froude numbers

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Abstract: Hydraulic jump is a phenomenon which usually occurs in rivers and in correspondence with hydraulic structures. It is characterized by a rapid transition from a super- to sub-critical flow conditions, involving a huge energy dissipation. For its characteristics and its relevance in hydraulics, it has been studied since the early years of the last century. Nevertheless, many features still require a further investigation, e.g., scale effects, air entrainment process, effect of bed roughness on conjugate depths, etc. In particular, in the last few decades, the analysis of the main geometric parameters has been further developed. Nevertheless, no studies dealing with D-jump occurring on a rough sloping channel are present in literature. Therefore, the present paper reports the results of experimental investigations, which allowed to develop a semi-theoretical approach in order to evaluate the D-jump conjugate depth in a wide range of channel bed geometric and boundary configurations.

Keywords: Bed roughness; Friction factor; Hydraulic jumps; Hydraulic models; Sloping channels

1. Introduction:

Hydraulic jump is a phenomenon which generally occurs in correspondence with hydraulic structures, thus its characteristics should be carefully predicted in order to avoid structural damages. It is an efficient energy dissipation system, which can be helpful in river restoration projects, if opportunely controlled (Pagliara and Palermo, 2008a). Therefore, the interest of hydraulic engineers in understanding its dynamics and predicting its main geometric parameters has been and still are relevant. Earlier studies mainly focused on the evaluation of the conjugate depth ratio and on both jump and roller lengths (Peterka, 1983). In particular, the first theoretical approach was proposed by Bélanger, resulting in the well-known Bélanger's equation. Nevertheless, Bélanger's equation is valid in a quite limited range, as it was derived applying the momentum principles and assuming a rectangular horizontal smooth-walled channel configuration. In addition, Bélanger neglected the boundary flow

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resistance and considered a uniform flow velocity and hydrostatic distribution in both upstream and downstream sections of the hydraulic jump. These assumptions are evidently not confirmed in most of the cases occurring in practical applications. Therefore, the analysis of the phenomenon was further developed by other authors, including the effects of both channel geometry and bed roughness.

In particular, the effect of bed flow resistance on conjugate depth is significant. Several authors proposed different approaches to estimate the main characteristics of the hydraulic jump for horizontal channel bed, concluding that there is a significant reduction of the sequent depth mostly due to the increase of bed shear stresses and an intense decrease of the velocity field above the rough bed (Ead and Rajaratnam, 2002).

Namely, different horizontal artificially roughened beds were tested by several authors in order to evaluate the main hydraulic jump characteristics. For example, Leutheusser and Schiller (1975) studied hydraulic jump features on roughened horizontal beds using both spheres and strips and Hughes and Flack (1984) used both strips and gravels of several selected granulometries. Similar analyses were conducted by Ead and Rajaratnam (2002) on a corrugated bed and by Carollo et *al*. (2007) and Pagliara et *al*. (2008) on rough gravel beds. These last two studies systematically analysed the effect of different flow bed resistance induced by gravel beds on sequent depth ratio. The approaches adopted by the authors were different but both the studies showed that an essential role is played by the approach flow Froude number and the relative roughness. In particular, Pagliara et *al*. (2008) extended their analysis to the case in which boulders were located on a rough bed, showing that they contribute to further reduce the conjugate depth.

Other studies focused on the hydraulic jump properties on horizontal rough bed, adopting a modified approach flow Froude number to derive predicting relationships for hydraulic jump lengths (Bhuiyan et *al*., 2011; Afzal et *al*., 2011). Nevertheless, only very recently, Pagliara and Palermo (2015) proposed a semi-theoretical analysis of the hydraulic jump properties on rough beds involving also the air concentration effect on the conjugate depths. In particular, they proposed a comprehensive analysis involving both the effects of flow bed resistance and channel bed configuration. They proposed an aggregate approach to evaluate the sequent depth value valid for a wide range of relative roughness and for both horizontal and adverse-sloped beds up to -15%. They showed that the effect of both the channel configuration (adverse-sloped) and bed roughness is extremely significant. In addition, they validated their relationships including the experimental data of many other authors who analysed the hydraulic jump properties on both adverse-sloped smooth beds (Stevens, 1942; Okada and Aki, 1955; McCorquodale and Mohamed, 1994; Pagliara and Peruginelli, 2000) and on horizontal rough beds (Hughes and Flack, 1984; Carollo et *al*., 2007; Pagliara et *al*., 2008).

Despite the conspicuous literature on the topic, there is still a lack of knowledge relative to D-jump properties on rough beds. Namely, earlier studies focused on hydraulic jump characteristics in smooth sloping channels and for relatively high approach flow Froude numbers. Kindsvarter (1944) classified jumps according to their toe position relative to the bottom kink. He distinguished four jump typologies: A-jump for which the toe is at the kink; B-jump is intermediate to A- and C-jumps; C-jump for which the end of the roller is above the kink; and D-jump when the entire jump is occurring on the sloping channel.

Significant studies on hydraulic jump in smooth sloping channels were conducted, among others, by Bakhmeteff and Matzke (1938), Chow (1959), Rajaratnam (1966), Rajaratnam (1967), Kawagoshi and Hager (1990), and Ohtsu and Yasuda (1991). In particular, these studies proposed semi-empirical equations by which it is possible to evaluate the sequent depth in a wide range of channel slopes. More recently, Carollo et *al*. (2013) analysed the B-jump on rough beds and evaluated the main hydraulic jump characteristics.

But, to the authors' knowledge, there are no studies dealing with D-jump on rough sloping beds, especially for low approach flow Froude numbers. In fact, D-jump frequently occurs in correspondence with low-head river restoration structures, such as block ramps (Pagliara and Palermo, 2008b, 2010, 2011, 2012; Pagliara et *al*., 2009, 2011, 2012, 2016) and rock-grade control structures (Pagliara and Palermo, 2013; Pagliara and Mahmoudi Kurdistani, 2015). Therefore, it is fundamental to understand its characteristics in order to prevent structural failures and damages. The present paper aims to furnish a general predicting relationship for D-jump characteristics, valid in a wide range of channel geometric and bed roughness conditions. Based on the semi-theoretical approach proposed by Pagliara and Palermo (2015), a new relationship to evaluate the sequent depth for D-jumps on rough sloping beds was derived and then it was validated using ad hoc experimental tests. Along with the equation furnished by Pagliara and Palermo (2015), the analysis proposed in the present paper furnishes a valid analytical tool to estimate the sequent depth ratio of hydraulic jumps occurring in rough sloping channels (either positive or adverse).

2. Experimental Facilities:

Experimental tests were conducted in a rectangular channel whose geometric characteristics are: 0.345 m wide, 6.0 m long and 0.5 m high.

Two base materials were glued on steel sheets in order to simulate different bed roughness conditions. Namely, the tested materials were $E2$ and $E4$ (Pagliara *et al.*, 2008; Pagliara and Palermo, 2015). The granulometric characteristics of the material $E2$ are d_{50}=6.26 mm, d_{90}=7.48 mm and non-uniformity coefficient $\sigma=(d_{84}/d_{16})^{0.5}$=1.18; whereas for material $E4$,

d_{50}=30.62 mm, d_{90}=33.30 mm and σ=1.08. Note that d_{xx} is the size of the base material for which *xx*% is finer.

95 tests were conducted in the experimental flume. The downstream tailwater level was regulated in such a way that the hydraulic jump occurred entirely on the sloping bed (D-jump). Several hydraulic conditions were simulated. Namely, the approach flow Froude number F_1, ranged between 2 and 7; the approach flow velocity V_1 between 1.2 and 3.0 m/s; the upstream conjugate depth y_1 between 0.018 and 0.056 m; the downstream conjugate depth y_2 between 0.083 and 0.279 m; and the water discharge Q between 0.01 and 0.03 m^3/s. Furthermore, the channel bed configuration was varied in order to test three different bed slopes i (i.e., i=0.05, 0.10 and 0.15); and the relative roughness varied in the range 0.07< d_{50}/k<0.58, where k is the critical depth.

In order to limit both the flow disturbance and the air entrainment, tests were conducted in such a way that rooster tail formation occurred in rare cases. In the presence of rough bed, a particular importance has to be given to the location of the virtual bed level, i.e., the reference bed level. Namely, according to Hughes and Flack (1984), two different levels were distinguished: the effective top *ET* and the physical top *PT*. The physical top level *PT* is the average level of bed material tops, whereas *ET* is set at $0.2d_{65}$ below the *PT*. According to Hughes and Flack (1984), the virtual bed level coincides with the *ET* level and it is assumed as the reference plane from which the water depths were measured. This methodology is widely accepted in literature and it is commonly adopted in order to locate the effective bed in the presence of rough materials. This is mostly due to the fact that, especially for hydraulic jumps, Hughes and Flack (1984) showed that the error introduced in the results due to the estimation of the *ET* is negligible, as a variation of ±20% in the *ET* level results in a ±5% variation of the conjugate depth ratio.

Both the channel bed and the water surface were carefully measured using a point gauge ±0.1 mm precise. The data obtained were used to determine both the *PT* and *ET* levels and the conjugate depths y_1 and y_2. Note that y_1 and y_2 are the water depths (measured orthogonally to the bed) at the upstream and downstream sections of the hydraulic jump, respectively. Figure 1 shows a diagram sketch of a hydraulic jump occurring on a sloping channel. The main hydraulic and geometric parameters are reported in Figure 1, including the hydraulic jump length L_j. Figure 2 shows a picture of an experimental test conducted on a rough sloping channel, with base material *E*2.

3. Theoretical background:

Based on the approach proposed by Pagliara and Palermo (2015), the conjugate depth ratio $Y=y_2/y_1$ can be derived by applying the momentum equation to a unit width in a rectangular sloping channel in the presence of a rough bed (Figure 1). Namely, the momentum equation can be written as follows:

$$P_1+M_1+W\sin\alpha=P_2+M_2+F_\tau \tag{1}$$

with α as the angle of the bed slope with respect to horizontal (negative for adverse-sloped bed and positive for sloping channels). In Eq. (1), P_j represents the hydrostatic force, M_j the momentum flux, W the weight of the water in the control volume and F_τ the integrated shear stress per unit width. Note that the subscript j is either equal to 1 (upstream section of the hydraulic jump) or 2 (downstream section of the hydraulic jump). Therefore, the components of the Eq. (1) can be explicitly written as follows:

$$P_j = 0.5\gamma y_j^2\cos\alpha \tag{2a}$$

$$M_j = \rho y_j U_j^2 \tag{2b}$$

$$W = 0.5\gamma L_j K(y_1+y_2) \tag{2c}$$

with U_j average velocity, ρ water density, γ water specific weight, L_j hydraulic jump length and K a correction coefficient for control volume weight.

In addition, according to Carollo et *al*. (2007), F_τ can be evaluated as:

$$F_\tau=\beta(M_1-M_2) \tag{2d}$$

with $0\leq\beta<1$, i.e., the momentum deficit parameter.

From previous equations, the following general relationship (valid for both positive sloping channels and adverse-sloped channels, i.e., $\alpha>0$ and $\alpha<0$, respectively) can be easily derived (see also Pagliara and Palermo, 2015):

$$Y=\frac{-1+\sqrt{1+8\mathsf{G}_1^2(1-\beta)}}{2} \tag{3}$$

According to Pagliara and Peruginelli (2000) and Pagliara and Palermo (2015), G_1 can be expressed as follows:

$$\mathsf{G}_1^2=\frac{\mathsf{F}_1^2}{\cos\alpha-\dfrac{\lambda K\sin\alpha}{Y-1}} \tag{4}$$

where $\lambda=L_j/y_1$ is the non-dimensional jump length, $\mathsf{F}_1=U_1/(gy_1)^{0.5}$ is the Froude number in the upstream section of the hydraulic jump and g is the acceleration due to gravity. For hydraulic jumps occurring on adverse-sloped channels, Pagliara and Palermo (2015) showed that the expression for G_1 proposed by Pagliara and Peruginelli (2000) for smooth beds is still valid for rough beds. Namely, for both smooth and rough adverse-sloped beds G_1 can be calculated as follows:

$$G_1=a^{bi}F_1 \qquad (5)$$

with a=3.32 and b=1.52 for $-0.25\leq i\leq 0$.

In addition, for adverse-sloped beds, McCorquodale and Mohamed (1994) and Pagliara and Peruginelli (2000) assumed that the roller length $L_r\approx L_j$ and the correction factor $K\approx 1$. For rough adverse-sloped bed, Pagliara and Palermo (2015) developed a novel methodology to evaluate the momentum deficit parameter β. Namely, based on the studies of Leuthesser and Kartha (1972), Hughes and Flack (1984) and Habibzadeh and Omid (2009), they derived the following expression for β:

$$\beta=-0.14\left(1-e^{2.38(d_{50}/k)}\right) \qquad (6)$$

It has to be noted that β accounts for the effect of bed roughness and increases with the relative roughness d_{50}/k. For smooth bed β=0, as d_{50}=0.

In conclusion, for both rough and smooth adverse-sloped bed, combining Eq. (3), (5) and (6), Pagliara and Palermo (2015) proposed the following expression of the sequent depth ratio:

$$Y=\frac{-1+\sqrt{1+8\left(3.32^{1.52i}F_1\right)^2\left(1+0.14\left(1-e^{2.38(d_{50}/k)}\right)\right)}}{2} \qquad (7)$$

which coincides with Bélanger's equation for smooth (d_{50}=0) horizontal beds (i=0) and with Pagliara and Peruginelli (2000) equation for smooth adverse-sloped beds (i<0 and d_{50}=0).

In the case of (positive) smooth sloping channels, a similar analysis was conducted by different authors (among others Chow, 1959; Rajaratnam, 1966; Ohtsu and Yasuda, 1991). In particular, based on the experimental data of Kindsvater (1944) and Bakhmeteff and Matzke (1938), Chow (1959) proposed a chart by which it is possible to estimate the sequent depth Y for D-jump on smooth beds with $i\leq 0.3$. Similarly, both Rajaratnam (1966) and Ohtsu and Yasuda (1991) proposed two different equations in order to calculate Y, for a wider range of (positive) channel bed slopes.

4. Results and discussion:

As stated above, for sloping channels (Figure 1), the governing equation is still represented by Eq. (3). Nevertheless, in this last case, according to Ohtsu and Yasuda (1991), the correction coefficient K cannot be assumed equal to 1, as it depends on the channel slope ($\tan\alpha=i$). In particular, they proposed the following equation to estimate the coefficient K:

$$K=1+10^{-(2.8\tan\alpha+0.74)} \qquad (8)$$

Furthermore, for D-jumps, the jump length L_j differs from roller length L_r, whereas in the case of adverse-sloped bed $L_r \approx L_j$, as shown by McCorquodale and Mohamed (1994) and Pagliara and Peruginelli (2000). From these two last observations, it is evident that Eq. (5) cannot be extended to $i>0$. Therefore, a new relationship should be proposed.

Regarding the momentum deficit parameter β, a reasonable hypothesis can be done. Equation (6), developed for the adverse-sloped and horizontal cases, is assumed to be still valid also for (positive) sloping configuration. In fact, Pagliara and Palermo (2015) derived Eq. (6) assuming that the mean value of the boundary shear stress ($\tau_{0\text{mean}}$) can be expressed as $\tau_{0\text{mean}}=(\tau_{01}\tau_{02})^{0.5}$, where τ_{01} and τ_{02} are the shear stresses at the downstream and upstream ends of the jump (see also Hughes and Flack, 1984; Habibzadeh and Homid, 2009). Being the channel slope relatively small ($i<0.15$) and the relative submergences (y_1/d_{50} and y_2/d_{50}) in the same ranges tested by Pagliara and Palermo (2015), it seems reasonable to extend the validity of Eq. (6) to D-jumps. Nevertheless, the validity of this simplifying assumption will be experimentally shown in the following.

The data elaboration and general equation validation was conducted in steps. The first step of data elaboration aimed to find a novel predicting equation for G_1. Namely, substituting the measured data of the parameters appearing in Eq. (4) and assuming that K can be expressed as per Eq. (8), it was possible to derive the values of the variable G_1. As stated by Chow (1959), also for (positive) sloping channels, G_1 depends only on the variables F_1 and i. In order to preserve the analytical expression of the global equation proposed by Pagliara and Palermo (2015), the functional relationship reported in Eq. (5) was adopted to interpolate the experimental data. It was experimentally shown that, if a=3.32 and b=2.7, a very good data fit is obtained (R^2=0.95). Therefore, for $0 \leq i \leq 0.15$, Eq. (5) can be re-written as follows:

$$\mathsf{G}_1=3.32^{2.7i}\mathsf{F}_1 \qquad (9)$$

Note that $\mathsf{G}_1=\mathsf{F}_1$ for i=0. Therefore, for horizontal beds, both Eq. (5) and (9), valid for adverse-sloped beds and sloping channels, respectively, furnish exactly the same analytical solution, i.e., no analytical discontinuity occurs.

The second step of the data analysis consisted in the validation of Eq. (3) for smooth (positive) sloping channels, i.e., when d_{50}=0 (and β=0) and G_1 can be expressed by Eq. (9). Therefore, Eq. (3) can be re-written as follows:

$$Y=\frac{-1+\sqrt{1+8\left(3.32^{2.7i}\mathsf{F}_1\right)^2}}{2} \qquad (10)$$

valid for $0 \leq i \leq 0.15$ and smooth channels.

Equation (10) was therefore plotted in graphs $Y(\mathsf{F}_1)$ and compared with other authors' equations, i.e., with the equations proposed by Chow (1959), Rajaratnam (1966), and Ohtsu and Yasuda (1991), respectively. Figure 3a-c shows that there is a reasonably good agreement between all the mentioned equations, including Eq. (10), especially within the tested range of approach flow Froude numbers ($2<\mathsf{F}_1<7$), for all the tested slopes.

Figure 3 confirms that Eq. (10) furnishes a good estimation of the sequent depth Y also outside of its experimental range of validity. In addition, it is also evident that G_1 is a monothonic increasing function of the variable i. It is worth noting that, comparing the two expressions of G_1 (Eq. 5 valid for adverse-sloped beds and Eq. 9 valid for positive sloping channels), the only variation is in the value of the exponent b, i.e., b=1.52 for $i<0$ and b=2.7 for $i>0$. This variation is mostly due to the fact that $L_r \approx L_j$ for $i<0$ (Pagliara and Peruginelli, 2000; McCorquodale and Mohamed, 1994), whereas for $i>0$ $L_r < L_j$ (Ohtsu and Yasuda, 1991).

The third step of data analysis consisted in the validation of the hypothesis that the momentum deficit parameter β can be estimated using Eq. (6) also for (positive) sloping channels. In other words, for $0<i\leq 0.15$ and $0\leq d_{50}/k<0.60$, the governing equation has the following general expression:

$$Y=\frac{-1+\sqrt{1+8\left(3.32^{2.7i}\mathsf{F}_1\right)^2\left(1+0.14\left(1-e^{2.38(d_{50}/k)}\right)\right)}}{2} \qquad (11)$$

The validation of the previous equation was conducted using all the experimental data. Namely, for each tested channel slope, available data were grouped for different relative roughness ranges, i.e., $0<d_{50}/k<0.10$, $0.10<d_{50}/k<0.20$, $0.20<d_{50}/k<0.30$, $0.30<d_{50}/k<0.40$, $0.40<d_{50}/k<0.50$, and $0.50<d_{50}/k<0.60$. Experimental data relative to different d_{50}/k ranges were plotted in graphs $Y(\mathsf{F}_1)$, along with Eq. (11). In Eq. (11), d_{50}/k was assumed as the mean value of the considered range (e.g., d_{50}/k=0.35 for $0.30<d_{50}/k<0.40$). Figure 4a-d shows the mentioned graphs for i=0.05 and $0<d_{50}/k<0.10$, $0.30<d_{50}/k<0.40$, $0.40<d_{50}/k<0.50$, and $0.50<d_{50}/k<0.60$, respectively. Figure 5a-c shows the same for i=0.10 and $0<d_{50}/k<0.10$, $0.30<d_{50}/k<0.40$, and $0.40<d_{50}/k<0.50$, respectively. Finally, Figure 6a-b shows the same for i=0.15 and $0.30<d_{50}/k<0.40$, and $0.40<d_{50}/k<0.50$, respectively.

From the previous figures, it appears evident that Eq. (11) furnishes a reasonably good estimation of all the experimental data, therefore, the hypothesis assumed above relative to the validity of Eq. (6) also for the (positive) sloping channel case is experimentally confirmed. It is worth noting that in the case of the (positive) sloping channels, the sequent depth is a monotonic increasing function of the channel slope i, thus confirming the findings of other authors obtained for smooth sloping channels. There is a substantial similitude with both the hydraulic jump occurring either on a horizontal or adverse-sloped rough bed: the effect of bed roughness is to reduce the sequent depth, being constant both the channel slope and the inflow conditions (i.e., the approach flow Froude number). The reason for which on rough beds a substantial reduction of the hydraulic jump sequent depth occurs for all the tested geometric configurations (positive, horizontal and negative channel slope) was furnished by Ead and Rajaratnam (2002). They observed that the bed roughness induces an intense mixing of the flow resulting in significant Reynolds shear stresses. In addition, they conducted velocity measurements within the hydraulic jump, showing that a significant reduction of the velocity field occurs in the presence of rough beds. In conclusion, the increase of bed shear stresses can cause a prominent reduction of the sequent depth.

Nevertheless, it is worth noting that the effect of the roughness on the sequent depth ratio mainly depends on the relative roughness and it is not depending on the channel slope, if it is relatively small ($-0.15 \leq i \leq 0.15$). Therefore, the momentum deficit parameter can be expressed by the same equation in the aforementioned range of bed channel slopes. Just for example, for both $i=-0.1$ and $i=0.1$, the conjugate depth ratio can vary from 1% (for $d_{50}/k=0.05$, and for both $\mathsf{F}_1=2$ and 6) up to 17% (for $d_{50}/k=0.45$ and for both $\mathsf{F}_1=2$ and 6) respect to the corresponding value of Y relative to smooth bed configuration. Therefore, especially for high d_{50}/k values, the equations valid for smooth bed can significantly over-estimate the conjugate depth ratio. Whereas, the proposed equations (which take into account the effect of roughness) furnish a good estimation of the experimental values.

It can be noted that both the Eq. (7) and Eq. (11) proposed by the authors have essentially the same analytical structure. They both converge to Bélanger equation for $\beta=0$ and $i=0$, i.e., they were derived in such a way that no analytical discontinuity occurs for classical hydraulic jump on smooth bed. In addition, the combination of these two equations constitutes the general solution for the evaluation of the sequent depth ratio of a hydraulic jump entirely occurring on either positive or adverse (negative) sloping channel. Their validity was also successfully tested with other authors data, showing a good agreement with the existing literature on the topic. Figure 7 shows the comparison between measured (Y_{meas}) and calculated with Eq. (11) (Y_{calc}) values of the sequent depth Y for experimental tests conducted on rough sloping channel.

Finally, it is worth highlighting how potential scale effects can affect the proposed relationships. Namely, the adoption of Froude similitude in hydraulic jump studies results in smaller model Reynolds numbers than in prototypes (Chanson, 2009). Thus, two-phase flow properties can be affected. Nevertheless, Chanson (2009) and Heller (2011) conducted a series of experimental tests showing that air-water flow properties are not significantly affected by scale effects when $B/y_1>10$, where B is the channel width. Furthermore, Chanson & Murzyn (2008) showed that a self-similarity of the void fraction profiles in the developing shear layer occurs for Reynolds numbers Re>40000, where $\mathrm{Re}=\rho U_1 y_1/\mu$, in which μ is the dynamic viscosity of water. Thus, it can be reasonably concluded that scale effects do not significantly affect the results of the present study (see also Pagliara and Palermo, 2015), as most of the experimental tests are characterized by $B/y_1>10$ and Re much larger than 40000.

5. Conclusion:

A general equation for the D-jump sequent depth estimation on rough (positive) sloping channels for low approach flow Froude numbers was derived using a semi-theoretical approach. The results of data analysis showed that the proposed methodology is in good agreement with all the existing literature on the topic for smooth sloping channels. In addition, it was experimentally shown that for relatively low channel slopes (either positive or negative), the effect of channel bed roughness on the sequent depth ratio is mainly depending on the relative roughness. Nevertheless, the increase of Reynolds shear stresses occurring on rough beds causes a reduction of the sequent depth ratio, which becomes more prominent by increasing the relative roughness. The combined use of both the equations proposed by the authors (for negative and positive sloping channels) constitutes a valid tool for the estimation of the sequent depth in a wide range of channel bed geometric configurations and relative roughness conditions. The results of this paper can be particularly useful in designing low-head structures for river restoration (e.g., block ramps and rock-grade control structures), as for this structural typology, generally, the hydraulic conditions are within the tested range of parameters.

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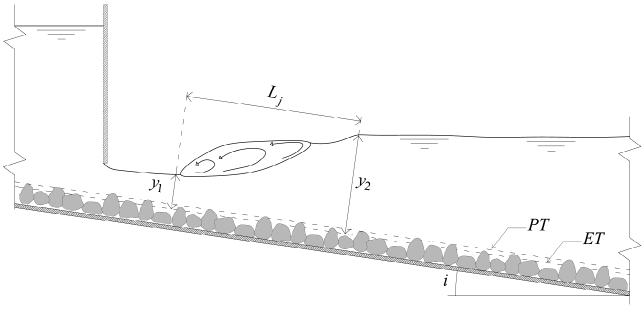

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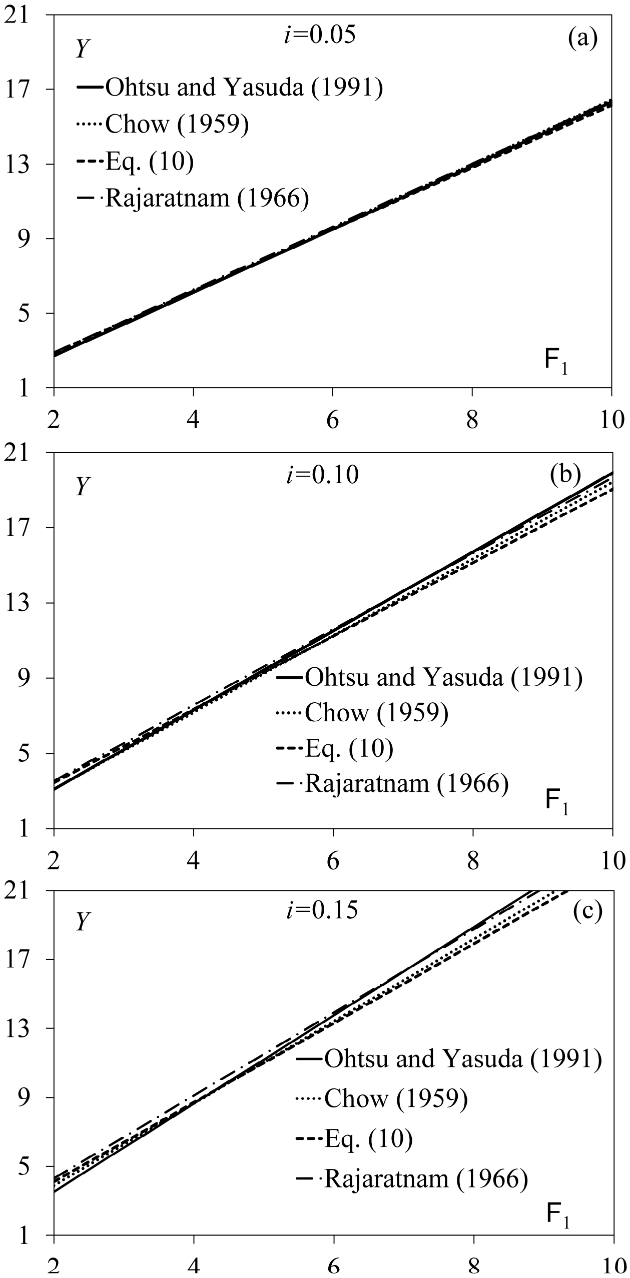

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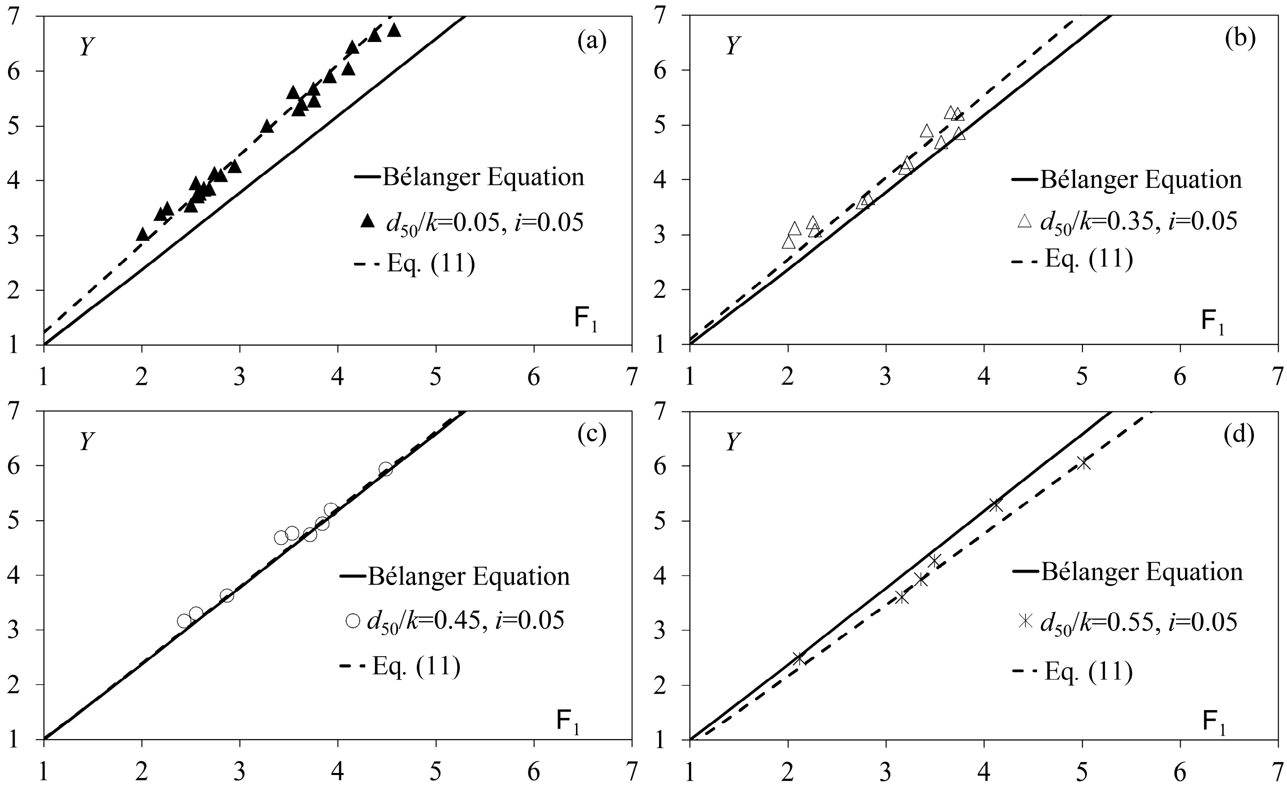

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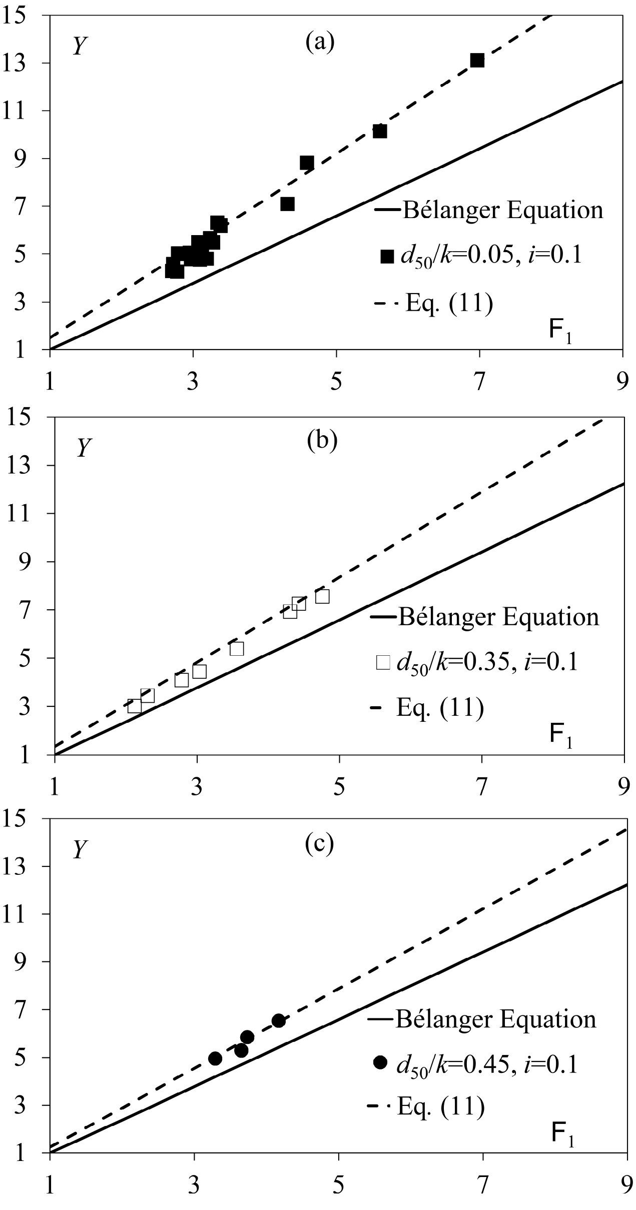

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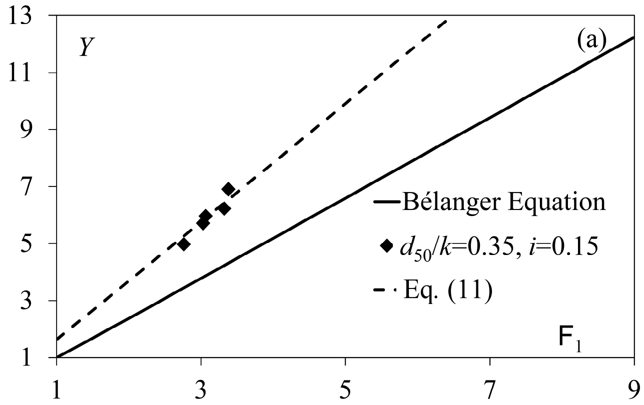

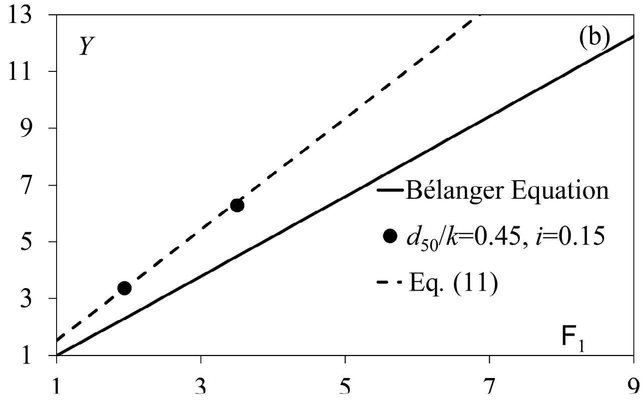

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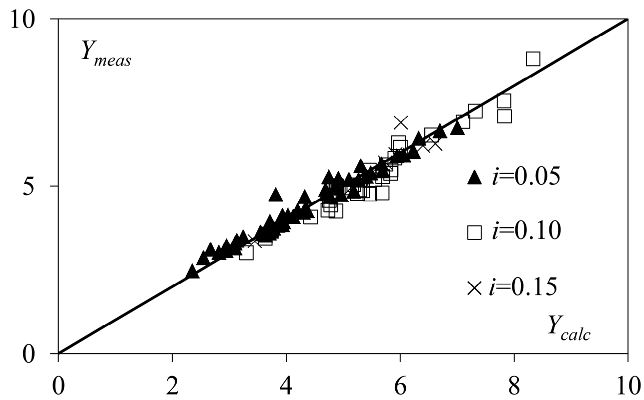

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