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LOOK ANGLES FOR A CELESTIAL BODY

by

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SUMMARY

This report presents and solves the problem of computing look angles for a celestial body. The astronomical triangle and its solution is presented and applied to look-angle calculations. Specific examples of computing the position of the sun, the moon, and Polaris by means of trigonometric formulae, tabulated values, and a computer program are given. The accuracy of the computer output is shown to exceed greatly the pointing accuracy required for pencil-beam microwave antennas. Explanations of horizontal parallax and the Equation of Time are also given. Finally, practical applications of aligning and dynamic checking of tracking accuracy of a microwave antenna are also presented.

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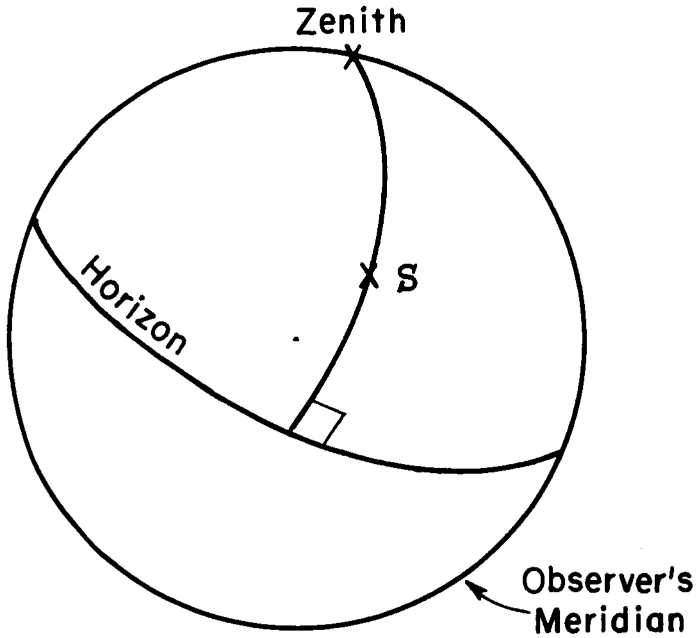
LOOK ANGLES FOR A CELESTIAL BODY

I. INTRODUCTION

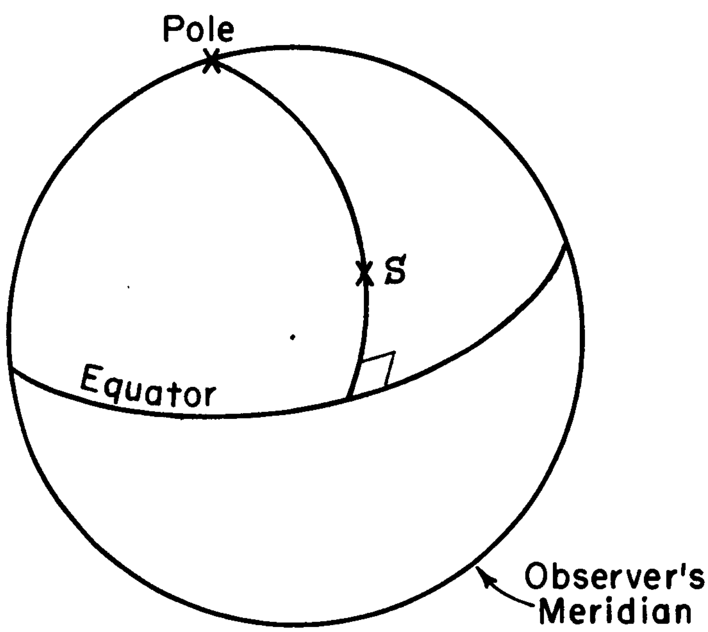
In the course of installation and operation of the four 30-foot antennas of The Ohio State University Satellite Communications Center the problem of precise alignment and pointing of the individual elements and the entire array had to be solved. Also, some ready means for periodic checking of the pointing accuracy had to be provided. The initial alignment was done by sighting on Polaris and periodic checking was done by tracking the sun and the moon. The purpose of this report is to present a simple method of computing look angles for a celestial body, that is to say, to present the necessary angular relationship enabling one to point the antenna, whose geographic location is known, toward an object in space whose location on the celestial sphere is known at a given instant of time.

II. THE ASTRONOMICAL TRIANGLE

The problem of pointing an antenna toward a celestial object is essentially the reverse of the sailor's problem. In the present case, it is assumed that the geographical location of the antenna site is well known and a particular pointing direction is sought, whereas in the case of the navigator, the pointing is read from the scales of the sextant and the geographical location at the time of observation is sought. The basis of all calculations of this type is a solution to the astronomical triangle which is formed by arcs of great circles passing through the pole, the zenith, and the celestial object (Fig. 1c). The point directly above the point of observation is zenith; by drawing a great circle through the pole and the zenith one obtains the great circle labeled observer's meridian in Fig. 1a. On it the latitude of the observer is measured. Next a great circle is drawn through the observer's zenith and the celestial object; the elevation of the object is measured on it. Perpendicular to the latter and to the observer's meridian is the observer's horizon circle. For clarity these three great circles, or arcs of them are shown in Fig. 1a. Figure 1b shows the observer's meridians again; a portion of the great circle through the pole and the object S on which the object's declination is measured; and the equatorial great circle, perpendicular to the pole-object great circle, on which the right ascension

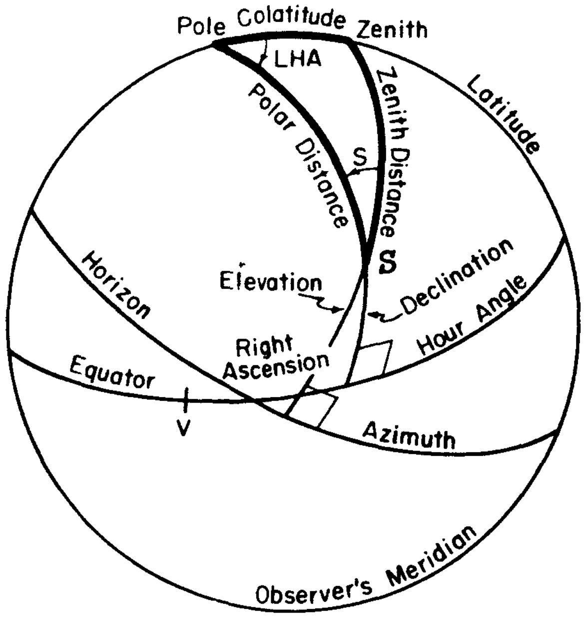


(a)



(b)

Figs. 1a,b. Components of the Astronomical Triangle.



(c)

Fig. 1c. The Astronomical Triangle.

and hour angle are measured. By superimposing Fig. 1a on Fig. 1b one obtains the astronomical triangle shown in heavy lines in Fig. 1c. Azimuth is the arc measured on the horizon circle, in the clockwise sense, from the point closest to the pole where the horizon circle and the observer's meridian intersect to the intersection of the horizon circle and the zenith-object great circle. Elevation is the arc measured from this last point along the zenith-object great circle toward zenith and up to the location of the object S. Hour angle is the arc measured on the equator in the clockwise sense from the observer's meridian to the meridian of the object (the great circle through the pole and the object). Hour angle can be expressed in time measure (e. g., rising sun: + 18 hours; setting sun: + 6 hours) or in angular measure (90° and 270°, respectively). Declination is the arc measured from this last point along the meridian of the object toward the pole and up to the location of the object S. Right ascension is the arc measured on the equatorial circle in a counter-clockwise sense from a point called the Vernal Equinox, V, which is a reference on the celestial sphere just like the meridian passing through Greenwich is a reference on the globe. Latitude is the arc measured on the observer's meridian from the intersection of this with the equatorial circle toward the pole and up to the observer's zenith. Complementary arc segments, e.g., co-latitude, are self-explanatory.

From spherical trigonometry one obtains the following basic formula:

(1) $$\cos \alpha = \cos \beta \cos \gamma + \sin \beta \sin \gamma \cos a.$$

Let a = Local Hour Angle, LHA; $\alpha = 90^\circ - EL$ (EL = elevation of object); $\beta = 90^\circ - L$ (L = latitude of observer); $\gamma = 90^\circ - D$ (D = declination of object S), and substitute these into Eq. (1) obtain

(2) $$\cos(90^\circ - EL) = \cos(90^\circ - L) \cos(90^\circ - D) + \sin(90^\circ - L) \sin(90^\circ - D) \cos LHA,$$

which reduces to

(3) $$\sin EL = \sin L \sin D + \cos L \cos D \cos LHA.$$

Equation (3) gives the elevation of the object, as seen from a given geographical location, in terms of the observer's latitude and the declination and Local Hour Angle of the object:

(4) $$\text{Elevation} = \text{Arc sine}[\sin L \sin D + \cos L \cos D \cos LHA].$$

Now let a = Azimuth, AZ; $\alpha = 90^\circ - D$; $\beta = 90^\circ - L$; $\gamma = 90^\circ - EL$; and substitute into Eq. (1) to obtain

(5) $$\cos(90^\circ - D) = \cos(90^\circ - L) \cos(90^\circ - EL) + \sin(90^\circ - L) \sin(90^\circ - EL) \cos AZ,$$

which reduces to

(6) $$\cos AZ = \frac{\sin D}{\cos L \cos EL} - \tan L \tan EL .$$

Equation (6) yields the aximuth of the object, as seen from a given geographical location, in terms of the observer's latitude and the elevation and declination of the object:

$$\text{(7)} \qquad \text{Azimuth} = \text{Arc cosine}\left[\frac{\sin D}{\cos L \cos EL} - \tan L \tan EL\right].$$

Equations (4) and (7) are the ones used for obtaining look angles for a celestial body.

III. LOOK ANGLES FOR A CELESTIAL BODY

A. Sun

By using Eqs. (4) and (7) it is a simple matter to find the position of the sun or of any distant star. It is necessary to know the latitude and longitude of the point of observation; the accurate time and the right ascension, RA; and the declination of the star. The accurate location of the point of observation can be obtained from well-made maps, such as the ones made by the U.S. Coast and Geodetic Survey. Accurate time can be obtained by tuning a receiver to WWV or CHU, or by use of a chronometer. Information about the positions of celestial objects can be obtained from the American Ephemeris and Nautical Almanac published by the U.S. Government Printing Office, or from the shorter and occasionally more practical version of this, The Nautical Almanac, published by the same office. As an example, suppose the position of the sun must be known accurately on August 15, 1964, at 12 hours 38 minutes 22 seconds EST at the location of The Ohio State University Satellite Communications Center (40^{o} 00' 10"N, 083^{o} 02' 30"W).

Step one is to find the Local Apparent Sidereal Time (LAST) when the observation will be made. Eastern Standard Time (EST), is used in North America in a region approximately 15^{o} wide in longitude centered on longitude 075^{o}W. Since the location is at 083^{o}W, the Local Mean Solar Time (LMST) is (83^{o} 02' 30" - 75^{o}) x 1 hour/15^{o} = 32 minutes 10 seconds behind EST. A sidereal day is equivalent to 23 hours 56 minutes 04.1 seconds of mean solar time, or a mean solar day is 24 hours 03 minutes 56.6 seconds of mean sidereal time; hence, to find sidereal time, one adds to solar time 3 minutes 56.6 seconds x LMST/24 hr. Since the entries in the Ephemeris are for longitude of

Greenwich and the longitude of observation is west of Greenwich, one further adds to solar time 3 minutes 5.66 seconds x $83^{\circ}\ 02'\ 30''/360^{\circ}$ = 55 seconds. From the "Universal and Sidereal Times, 1964" Tables in the Ephemeris the Greewich Mean Sidereal Time (GMST, Hour Angle of the First Point in the Constellation Aries) can be found. By adding LMST, GMST, and the above correction and substracting the "Equation of Equinoxes" (Tabulated in the Ephemeris alongside GMST) the Local Apparent Sidereal Time can be found for the above chosen example at 12 hours 38 minutes 22 seconds EST:

LMST =	+	12h	06m	12s
Conversion to Sidereal =	+		01	59
Longitude Reduction =	+			55
GMST =	+	20	38	32
Equation of Equinoxes =	-			01
		32h	45m	158s .

The local apparent sidereal time at the location and time given above is 08 hours 47 minutes 38 seconds.

The next step is to find the Apparent Right Ascension and the declination. The Ephemeris publishes one value of these at zero hour Ephemeris Time (Ephemeris Time = Universal Time + 35 seconds for 1964; here this small correction is neglected) for the sun for each day of the year, along with tabular differences. The starting time was 12 hours 38 minutes 22 seconds EST. This corresponds to 12.64 + 5.0 = 17.64 hours Ephermeris Time (EST time zone is 5 hours to the West of Greenwich), hence the daily tabular differences must be multiplied by 17.64/24 and the results added to the tabulated values. Thus:

Tabulated Apparent Right Ascension of Sun:	8 hr 44 min 46sec
Correction: $233 \times 17.64/24$	172sec
Extrapolated Apparent RA:	8 hr 47 min 38sec .

The Apparent Right Ascension at the time of observation is 8 hours 47 minutes 38 seconds.

The Local Hour Angle, Right Ascension, and Sidereal time are related by the following formula:

(8) $$LHA = LAST - RA.$$

Substituting the values found above into Eq. 8, one finds that, within the accuracy of the calculations, the LHA of the sun at Columbus, Ohio, at the indicated time is

LHA = 8 hr 47 min 38 sec - 8 hr 47 min 38 sec = 0h 0m 0s .

The apparent declination is found by extrapolating the tabulated value to the time of observation in the same manner as done above with the Right Ascension.

Tabulated Apparent Declination:	$+ 18^\circ\ 04'\ 21''$
Correction: $-894'' \times 17.64/24 =$	$-\ 10'\ 57''$
Extrapolated Apparent Declination	$+ 17^\circ\ 53'\ 24''$.

The apparent declination is then $17^\circ\ 53'\ 24''$.

It is worth mentioning at this point that Right Ascension is always positive; its tabular differences are also positive, that is to say, RA is always increasing; and its value is between 0 and 24 hours. Declination can be positive (sun is above the Equator from March to September); or negative (sun is between the Equator and the Tropic of Capicorn from September to March); it can be increasing (from December to June); or decreasing (from June to December). Hence, the sign of tabulated declination and the sign of the tabular differences must be carefully observed.

Using Eq. (4) one finds the elevation:

$$\text{Elevation} = \text{Arcsine}\left[\sin(40^\circ\ 00'\ 10'')\sin(17^\circ\ 53'\ 24'') + \cos(40^\circ\ 00'\ 10'')\cos(17^\circ\ 53'\ 24'')\cos(0^\circ)\right]$$

$$= \text{Arcsine}\ [0.92644) = 67^\circ\ 53'\ 10''\ .$$

By using Eq. (7) and the value of elevation found above, one finds the azimuth:

$$\text{Azimuth} = \text{Arccosine}\left[\frac{\sin(17^\circ\ 53'\ 24'')}{\cos(40^\circ\ 00'\ 10'')\cos(67^\circ\ 53'\ 10'')} - \tan(40^\circ\ 00'\ 10'')\tan(67^\circ\ 53'\ 10'')\right]$$

$$= \text{Arccosine}\ (-0.9999^{+}) = 180^\circ\ .$$

These results show that the sun was on the local meridian of the OSU Satellite Communications Center on August 1, 1964, at 12h 38m 22s; the local solar noon occurred at 38m 22s past 12 O'clock EST. The accuracy of the above calculations depends, of course, on the accuracy of the trigonometric tables used in the computation. Tables accurate to five places or more are recommended.

As a check on the above calculations for azimuth, and also as an important point of interest, the occurrence of the local solar noon (the culmination, or meridian passage of the sun) can be readily calculated by using the Equation of Time. The Equation of Time is the difference between the mean time and the apparent time, that is, the difference in position between the real sun and the mean sun. This difference can be - 14 and + 16 minutes and the exact value is tabulated in the Ephemeris in the same table (Sun, for 0 h Ephemeris Time) as the RA and the declination. There are several reasons for the existence of the Equation of Time, the major ones being the fact that the angular motion of the Earth is not always the same and that the motion of the real sun is measured in a different plane than the one in which the motion of the mean sun is reckoned (the plane of the eliptic and the plane of the equator respectively).

For the example chosen for illustration, one finds the Equation of Time to be -6 m 15.2 s. The tabular difference is +3.8 s, hence, the corrected Equation of Time is -6m 12s. The location is 32m 10s from the center of Eastern Standard Time Zone, hence the accurate time of the local solar noon (maximum elevation of sun for the day, azimuth 180°) is 12h + 32m 10s -(6m -12s) = 12h 38m 22s, as calculated above. To check the calculations for elevation one makes use of the computed declination and the co-latitude of the observer. The co-latitude is 90° - Latitude = 90° 00' 00" - 40° 00' 00" = 49° 59' 50"; the computed declination was $+17^{\circ}$ 53' 24", hence the colmination of the sun must have been 49° 59' 50" + 17° 53' 24" = 67° 53' 14", which is exactly the elevation angle computed above for the local solar noon.

A considerable amount of time can be saved by using the Nautical Almanac when computing LHA. Among other useful data the Almanac tabulates the Greenwich Hour Angle, GHA, and declination of the sun. The relationship between LHA and GHA is given as

(9) $$LHA = GHA \pm \text{Longitude}.$$

The minus sign in Eq. (9) goes with westerly and the plus sign with easterly longitudes. Looking on the page for the date of observation (August 1, 1964) one finds that the GHA of the sun at 17 hours Greenwich Mean Time is 73° 26.8', and its declination is $+17^{\circ}$ 53.6'. Since the time of observation, 12h 38m 22s EST, corresponds to 17.64 hours Greenwich time, one obtains the tabular differences for the hour angle and declination. The differences are then multiplied by the factor of 0.64 and added in case of the hour angle and substracted in case of the declination (since GHA is increasing, but declination is decreasing). The result is

GHA at 18 h GMT:	88° 26'.9
GHA at 17 h GMT:	73 26'.8
Tabular Difference:	15° 0'.1

GHA at 17h GMT:	73° 26'.8
Correction: 0.64 × Tab. Diff.:	9 36
GHA at 17.64h GMT:	83° 2'.8

hence, LHA = 83° 2.8' - 83° 02' 30'' = 0° 0'.3.

Declination at 18h GMT:	17° 53'.0
Declination at 17h GMT:	17° 53'.6
Tabular difference	-0.6'

Declination at 17 hours GMT:	17° 53'.6
Correction: 0.64 × Tab. Diff.:	- 0'.38
Declination at 17.64h GMT:	17° 53'.2 .

Hence, declination is 17° 53' 13''. In view of the linear interpolations, the above results for LHA and declination when using the Ephemeris and the Almanac are in excellent agreement.

If one is not interested in extreme precision but is satisfied with accuracies on the order of 0.5 to 1 degree and is more interested in saving time required for the computation of the position of a celestial body, such as would be the case when one wishes to receive radio noise from the sun in the course of routine daily checks during a prevailing overcast with an antenna whose beamwidth is one degree or more, then one can profitably use tables of pre-computed look angles. A well-known one is "Tables of Computed Altitude and Azimuth, Vol. V, Latitudes 40° to 49°, Inclusive" (Publication No. 214 of the U.S. Navy Hydrographic Office). It is also for sale by the Superintendent of Documents, U.S. Government Printing Office, Washington 25, D.C. Other volumes, each covering a decade of latitudes, are also available.

Having found the declination and the LHA, either from the Ephemeris or from the Almanac, one refers to the pages tabulated for a latitude closest to the point of observation. If LHA is found in time measure it must be converted to degrees by using the relationship of $15^{\circ} = 1$ hour. For the example above the latitude is 40°. Pages are labeled "Declination same name as latitude" and "Declination contrary name to latitude". Since the latitude of observation was northerly and since the declination was also found to be northerly, the proper selection is a page for latitude 40, declination same name as latitude. The increment of the declination in the tabulation is 30'. Hence, for the example above, under the column headed declination 18° 00' and opposite to the hour angle 00°, the altitude and azimuth are read directly at 68° and 180°, respectively. This is in very good agreement with values calculated above. Interpolation to fractions of one degree in hour angle and declination is possible by using the tabular differences; interpolation to fractions of one degree in latitude is also possible, although the procedure becomes cumbersome and subject to error.

One method which is both swift and precise employs a small-size computer such as the IBM 1620. A program in the OSU version of Fortran 2 has been written for such a computer at The Ohio State University Numerical Computation Laboratory. The program, used by the author for almost a year with excellent results, is given in the Appendix.

B. Moon

Equations (4) and (7) are useful not only for finding the position of the sun and of distant stars, but also for finding the position of near heavenly bodies such as the moon and the planets. Information about the position of these objects on the celestial sphere can again be obtained from either the Ephemeris or the Almanac. For example, the position of the moon must be known accurately on August 1, 1964, at 8 A.M., EST, at the location of the OSU Satellite Communication Center.

The first step is to determine whether the moon will be visible for observation. From the Almanac for the prescribed date it is found that at latitude 40°N moon-rise is 23h 44m EST and moon-set is at 12h 54m EST, hence the moon will be above the horizon. Since 8 A.M. EST corresponds to 13h GMT, the table for the moon is entered at 13h GMT and the following values are obtained:

$$\text{GHA} = 102^\circ\ 02'.9\ ; \qquad \text{Declination} = +\ 12^\circ\ 31'.9\ .$$

Hence, LHA = $102^\circ\ 02'.9 - 83^\circ\ 02'.5 = 19^\circ\ 00'.4$, or-LHA = 1h 16m.

Using these values of LHA and declination Eqs. (4) and (7) can be used.

$$\text{Elevation} = \text{Arcsine}\ [\sin(40^\circ\ 00'\ 10'')\sin(12^\circ\ 31'.9)$$

$$+ \cos(40^\circ\ 00'\ 10'')\cos(12^\circ\ 31'.9)\cos(19^\circ\ 00'.4)]$$

$$= \text{Arcsine}\ (.84639) = 57^\circ.821 = 57^\circ\ 49'\ 16''\ .$$

$$\text{Azimuth} = \text{Arccosine}\left[\frac{\sin(12^\circ\ 31'.9)}{\cos(40^\circ\ 00'\ 10'')\cos(EL)}\right.$$

$$\left. - \tan(40^\circ\ 00'\ 10'')\tan(EL)\right]$$

$$= \text{Arccosine}\ [-.80235] = 216^\circ.645 = 216^\circ\ 38'\ 42''\ .$$

These results show that the position of the moon at 8 A.M. EST at the location of the OSU Satellite Communication Center is azimuth = $216^\circ\ 38'\ 42''$ and elevation = $57^\circ\ 49'\ 16''$. This position, however, was obtained from data referred to the center of the earth, whereas the observation is to be made from the surface of the earth. These two references are called geocentric and geodetic, respectively. The difference between the true elevation and the observed elevation is immediately obvious from Fig. 2. It can also be seen from Fig. 2 that the maximum parallax occurs when nearby objects are on the horizon, hence the descriptive name, horizontal parallax. The effect of the parallax is to decrease the elevation of the body, that is, the geocentric (true) and geodetic (observed) elevation angles are related by the following equation:

(10) True Elevation - Parallax Correction = Observed Elevation.

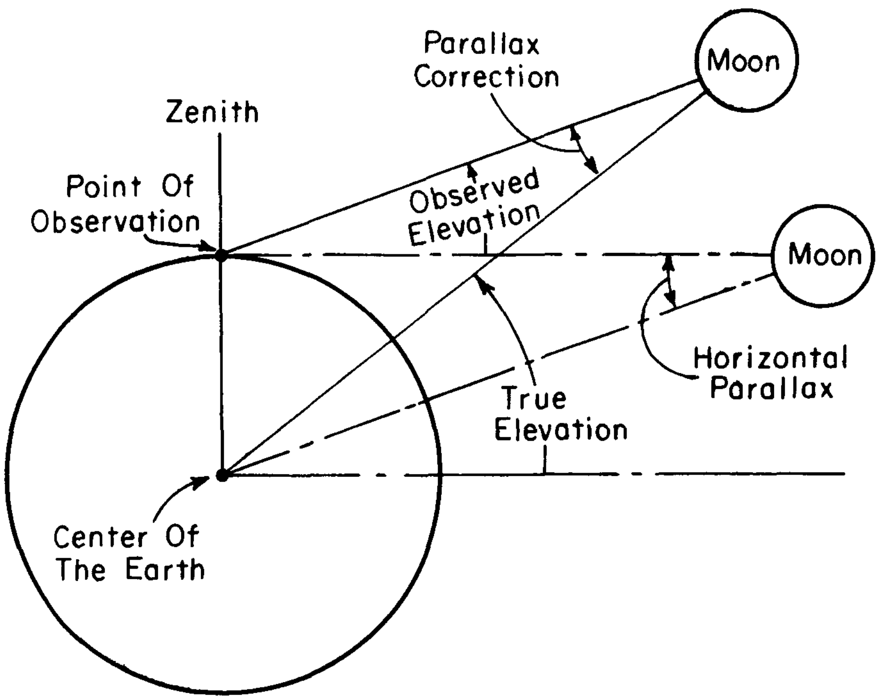


Fig. 2. Horizontal parallax.

From elementary trigonometry and Fig. 2 the parallax correction is

(11) Parallax Correction = (Horizontal Parallax)(Cosine of true elevation).

The Horizontal Parallax of the moon for the above time and date is 58'.9. The true elevation computed is 57° 49' 6", hence the parallax correction is (58.'9) cos(57° 49' 16") = 58'.9 $\times$ 0.5326 = 31'.368 = 31' 22". From Eq. (10), the observed elevation is 57° 49' 16" - 31' 22" = 57° 17' 54"; and the observed azimuth is 216° 38' 42". These results give the useful pointing information for the observation site at the prescribed date and time. These are the actual elevation and azimuth angles given to the antenna operator as steering information for bouncing radar signals off the moon.

The Horizontal Parallax presents a vexing problem in the case of the moon but it is practically negligible in case of the sun. For the sun the value of Horizontal Parallax is about 9 seconds of arc, which is negligible in most cases. Other corrections, such as the one for the refraction, are tabulated in the Almanac for optical observations and are applicable under various operating conditions; but the magnitudes of these are rather small and are best not treated here.

C. Polaris

A rather useful celestial reference is the pole star, or Polaris. Figure 3 shows its position relative to the constellation Ursa Major (Big Dipper). Actually "Polaris" is a misnomer, since Polaris is a circumpolar star. To first order approximation the altitude of Polaris equals the northerly latitude of the observer; at latitude 40°N the error is one degree. The initial alignment of the antennas was accomplished by sighting on Polaris. The procedure was amazingly simple as well as precise. During the day cross-hairs of nylon string were stretched across the aperture of the antenna reflector. An ordinary flashlight provided illumination of the cross-hairs. A telescope was mounted at the back of the dish and one on the bottom edge of the dish, then two points were chosen on a distant object (one leg of a TV antenna tower) so as to eliminate the parallax between the telescopes. In this manner the axis of the center telescope was coincident with the mechanical axis

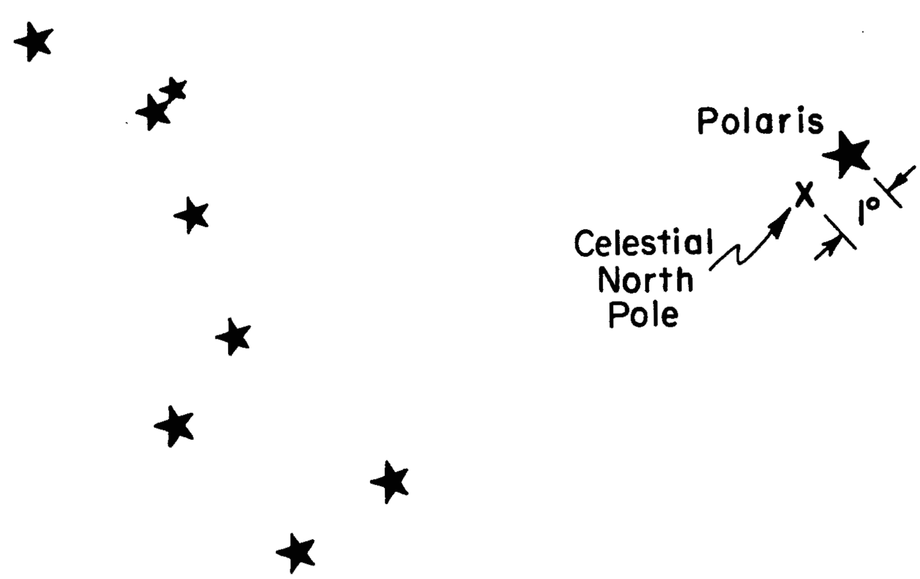


Fig. 3. Position of Polaris relative to the Celestial North Pole and the constellation Ursa Major.

of revolution of the dish, and the optical axis of the bottom telescope was parallel to the axis of the center one. The sighting was repeated several times with the observers trading places to insure maximum accuracy; also the alignments of the cross-hairs across the dish and in the center telescope were checked with the dish positioned at different elevations. Naturally, a very calm day was picked to carry out this sighting procedure. The TV antenna tower was 2150 feet from the antenna. The tower is triangular shaped; two-inch O. D. pipes are located at the corners of the triangle; 40-feet-tall sections, alternately painted red and white, are connected with zig-zagging welded webs which touch the corners of the triangle every five feet. By sighting on one edge of the tower at points where the webbing touched it, it was easy to obtain an accuracy of a fraction of an inch in the azimuth direction and similar accuracy in the elevation direction over the distance of 2150 feet. Thus, the mechanical axis of the reflector and the optical axes of both telescopes were coincident; Fig. 4 shows the physical arrangement.

At night a sighting was made on Polaris. The star was in the center of the cross-hairs of both telescopes; the cross-hairs of the center one were lined up with the cross-hairs across the aperture of the dish, and the indicated position of the antenna at the operator's console as well as the exact time were carefully noted. Next day the R. F. feed was mounted in place and the sun was tracked in automatic and monopulse modes.

The antenna was directed toward the sun and the cross-hairs of the bottom telescope were centered on it. Then the position of the feed was adjusted for accurate tracking; thus, the electrical axis of the reflector was lined up with an optical axis, which in turn was collimated to the mechanical axis of revolution of the paraboloid.

The next problem is to set the zero positions for the antenna position-indicating servos. There is a servo connection between the antenna mount and the operator's console; the servo generators on the mount are geared to the elevation and azimuth shafts; and the servo receivers drive the indicators at the operator's console. The position of Polaris was computed at the time of observation. From the noted indicator reading and the computed value for the star's position a correction was readily obtained and physically applied to the indicator synchros by turning them in their sockets while keeping the antenna stowed. Periodic checks of the pointing accuracy of the antenna were obtained by tracking the sun in monopulse mode and comparing computed look angles with indicated ones.

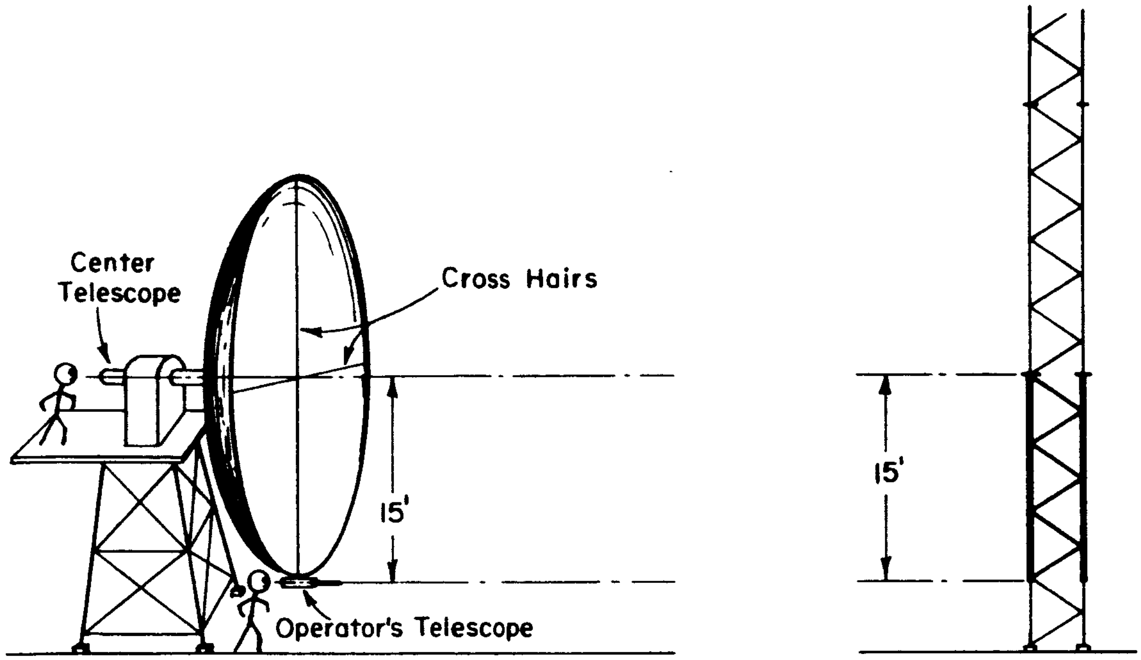


Fig. 4. Collimating arrangement for mechanical axis of paraboloid and optical axis of operator's telescope.

As an example, suppose we wish to know the position of Polaris at the OSU Satellite Communications Center on August 1, 1964, at 10 P.M. EST. This EST time corresponds to 22 + 5 = 27 = 03h GMT, hence the GHA of Aries at the time of observation is $355^\circ 44'.3$. By using Eq. (9), LHA = $355^\circ 44'.3 - 083^\circ 02'.5 = 272^\circ 41'.8$. At the back of the Almanac is a set of tables, "Polaris (Pole Star) Tables, 1964," showing the values as functions of the LHA of Aries, Latitude, and month of the year. These must be added to observed altitude to obtain latitude information. The tables also show azimuth as function of latitude and LHA of Aries. Knowing the latitude one can readily obtain pointing information by using the relationship

$$\text{Elevation} = \text{Latitude} + 1^\circ - (\text{tabulated values}).$$

For the present example,

$$\text{Elevation} = 40^\circ\ 00'\ 10'' + 1^\circ\ 00'\ 00'' - (1^\circ\ 23'\ 44'' + 0^\circ\ 00'\ 30'' + 0^\circ\ 00'\ 54'') = 39^\circ\ 35'\ 02''.$$

The azimuth is read from the table directly as

$$\text{Azimuth} = 001^{\circ}\ 00'\ 00''.$$

The first order alignment of the antenna was accomplished with the aid of a magnetic compas and the antenna looking on the horizon. The error was less than half a degree in elevation and about two degrees in azimuth. The azimuth correction bears out the fact that the magnetic correction in central Ohio is about two degrees westerly.

IV. PRACTICAL EXAMPLE

During the extensive tracking of Echo II shortly after it was launched, some minor tracking inaccuracies were noted and it became imperative to check the alignment and tracking accuracy of the master antenna to which the other three elements were slaved. On February 28, 1964, the sun was tracked in automatic monopulse mode. The solar radio noise level was computed for an approximate surface temperature to insure that the sun was on the electrical axis of the antennas. The maximum peak-to-peak monopulse error was determined to be 0.08 degree, which is less than the one-tenth of the beam-width figure customarily regarded as acceptable, since the beam width of the antenna is $1^{\circ}.2$. Figure 5 shows the computed and the measured positions. The measured values were plotted from picture records of the indicator dials obtained at one-minute intervals. The computed values were obtained with the help of the computer program given in the Appendix. The printout is given below.

ZOLNAY, S. JOBN FGJ032 06/05/64

1 640228.01 12 00 01 065 23 15 08 3590.6 -08 -08 -37 +056.6 40 00 10 000.1 001

2 999

END

LA-1 Look Angles for a Celestial Body RUN No. +640228.01

Elevation Parallax Correction = +.1 Min.

HOUR ANGLE				DECLINATION				N LATITUDE		
Hr	Min	Sec	Sec/Hr	Deg	Min	Sec	Sec/Hr	Deg	Min	Sec
+23	+15	+8	+3590.6	-8	-8	-37	+56.6	+40	+0	+10

LOCAL TIME		AZIMUTH ANGLE	ELEVATION ANGLE
Hr	Min	Deg E of N	Degrees
+12	+0	+165.275	+40.747
+12	+1	+165.594	+40.795
+12	+2	+165.914	+40.842
+12	+3	+166.235	+40.889
+12	+4	+166.556	+40.934
+12	+5	+166.878	+40.978
+12	+6	+167.200	+41.021
+12	+7	+167.522	+41.063
+12	+8	+167.845	+41.104
+12	+9	+168.169	+41.144
+12	+10	+168.492	+41.183
+12	+11	+168.817	+41.221
+12	+12	+169.141	+41.257
+12	+13	+169.466	+41.293
+12	+14	+169.792	+41.328
+12	+15	+170.117	+41.361
+12	+16	+170.443	+41.394
+12	+17	+170.770	+41.425
+12	+18	+171.097	+41.456
+12	+19	+171.424	+41.485
+12	+20	+171.751	+41.513
+12	+21	+172.079	+41.540
+12	+22	+172.407	+41.566
+12	+23	+172.735	+41.591
+12	+24	+173.064	+41.615
+12	+25	+173.393	+41.638
+12	+26	+173.722	+41.660
+12	+27	+174.051	+41.680
+12	+28	+174.380	+41.700
+12	+29	+174.710	+41.718

Hr	Min	Deg E of N	Degrees
+12	+30	+175.040	+41.735
+12	+31	+175.370	+41.752
+12	+32	+175.700	+41.767
+12	+33	+176.031	+41.781
+12	+34	+176.362	+41.794
+12	+35	+176.692	+41.806
+12	+36	+177.023	+41.816
+12	+37	+177.354	+41.826
+12	+38	+177.685	+41.834
+12	+39	+178.016	+41.842
+12	+40	+178.348	+41.848
+12	+41	+178.679	+41.853
+12	+42	+179.010	+41.858
+12	+43	+179.342	+41.861
+12	+44	+179.673	+41.862
+12	+45	+180.011	+41.863
+12	+46	+180.337	+41.863
+12	+47	+180.668	+41.862
+12	+48	+180.999	+41.859
+12	+49	+181.330	+41.855
+12	+50	+181.662	+41.851
+12	+51	+181.993	+41.845
+12	+52	+182.324	+41.838
+12	+53	+182.655	+41.830
+12	+54	+182.986	+41.821
+12	+55	+183.317	+41.810
+12	+56	+183.648	+41.799
+12	+57	+183.979	+41.787
+12	+58	+184.309	+41.773
+12	+59	+184.640	+41.758
+13	+0	+184.970	+41.743
+13	+1	+185.300	+41.726
+13	+2	+185.630	+41.708
+13	+3	+185.959	+41.689
+13	+4	+186.289	+41.669

END

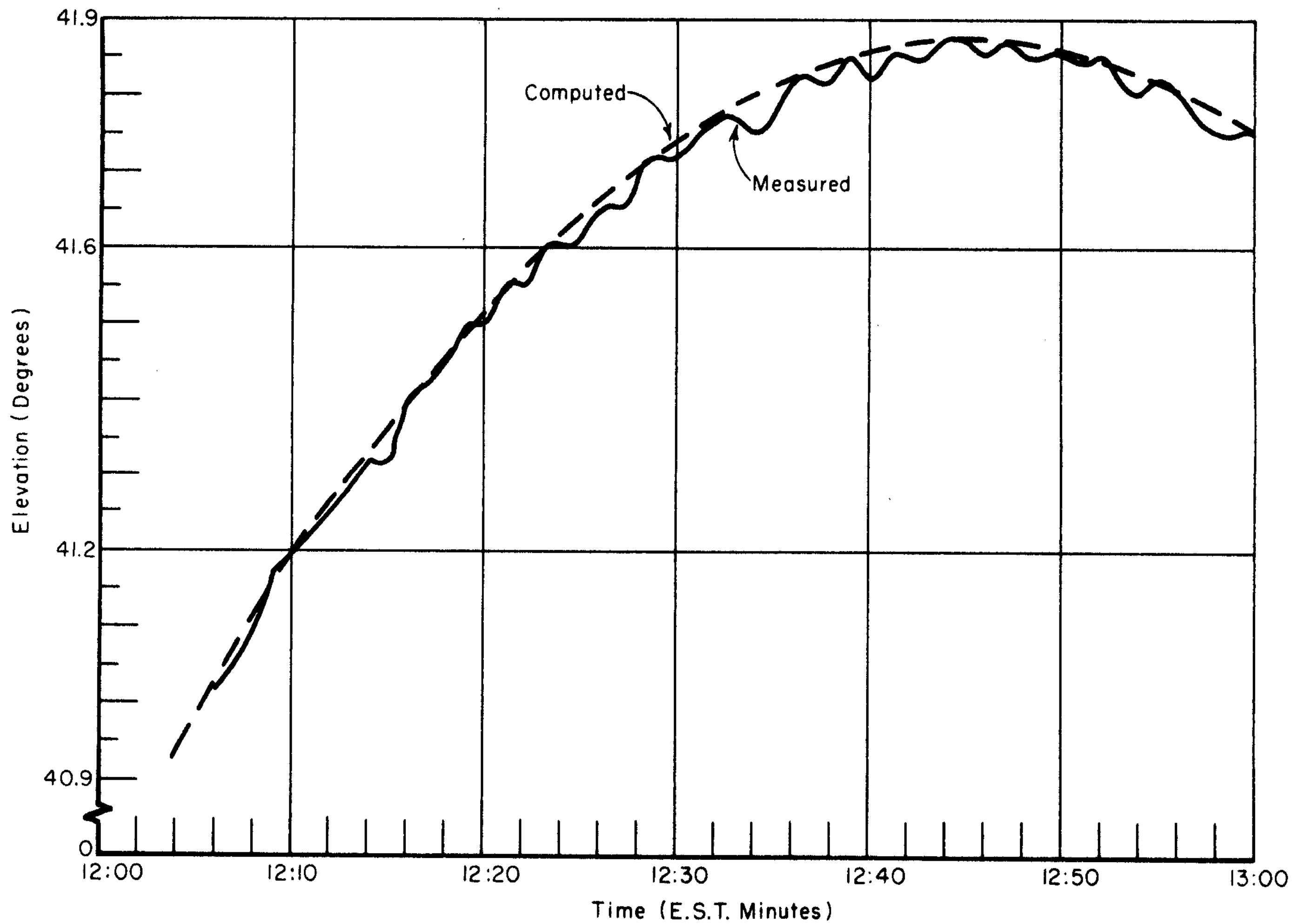


Fig. 5a. Computed and measured relative position of the Sun, February 28, 1964. (a) Elevation.

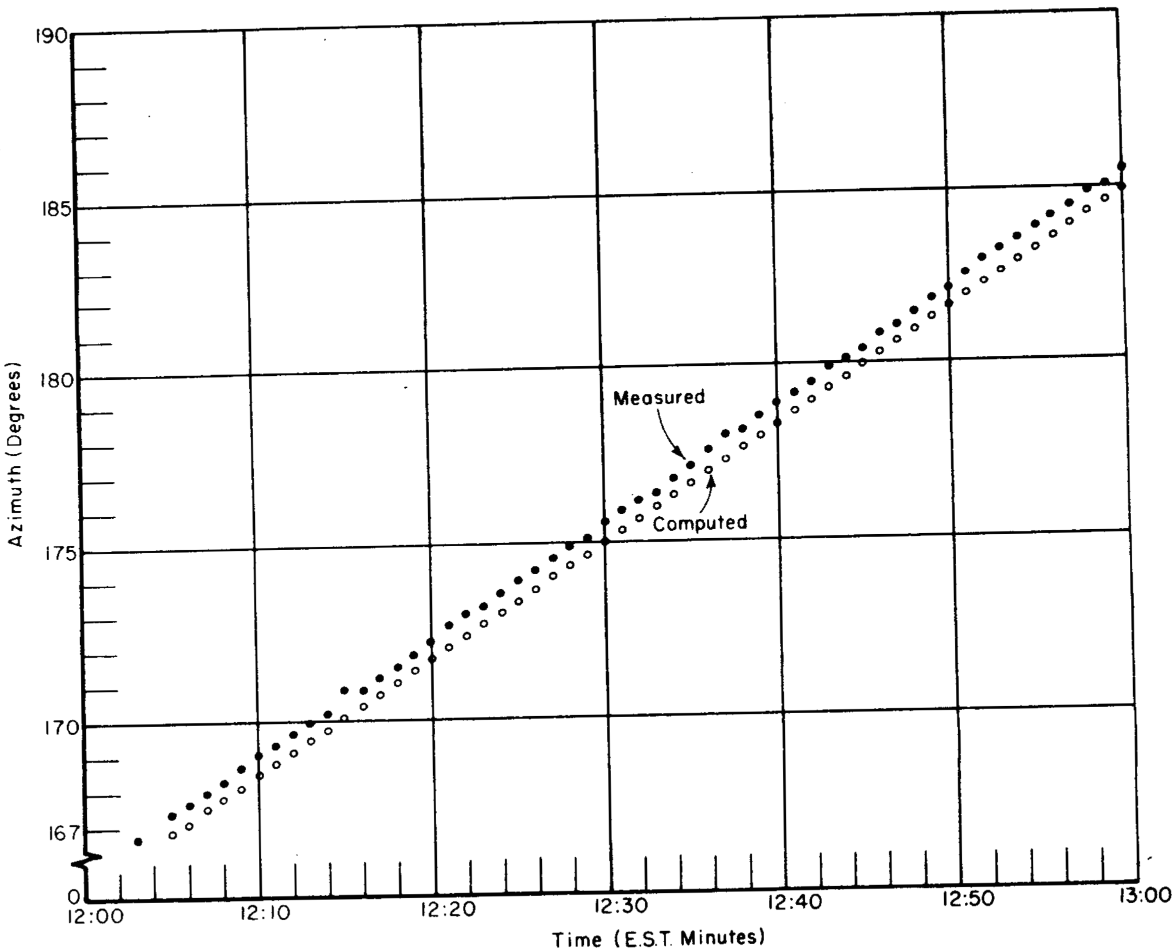


Fig. 5b. Computed and measured relative position of the Sun, February 28, 1964. (b) Azimuth.

As a check on the accuracy one finds that the computation predicts the occurrence of the local solar noon (azimuth of sun 180^{o}) at 12h 44m 51.6s, which is obtained by linear interpolation of the azimuth values between 12h 44m and 12h 45m. The equation of time, extrapolated to the longitude of the observing site, is

$$-12m\ 50.4s + 10.88\ \frac{17.532}{24} = -12m\ 42.4s.$$

Hence, the occurrence of the local solar noon is at 12h 32m 10s + 0h 12m 42.4s = 12h 44m 52.4 s, which is within 0.8 second of time to the predicted value. This 0.8 second time inaccuracy corresponds to 16.2 seconds of arc inaccuracy in azimuth. The elevation angle can also be computed by adding the declination at the time of observation to the co-latitude. The declination extrapolated to the observing time is

$$-08^{o}\ 24'38.''1 + \frac{1355.7}{24} \times 17.75 = -08^{o}\ 07'\ 56''.1.$$

By addition of this value to the co-latitude, the result is

$$49^{o}\ 59'\ 50'' - 08^{o}\ 07'\ 56''.1 = 41^{o}\ 51'\ 54''.$$

The horizontal parallax correction is $8''.88 \cos 41^{o}\ 51'\ 54'' = 6''.6$; hence the correct elevation is $41^{o}\ 51'\ 54'' - 6'' = 41^{o}\ 51'\ 48'' = 41^{o}.864$, which compares favorably with the computed figure at 12h 45m. This $0^{o}.001$ error in elevation is 3.6 seconds of arc. It can be stated with confidence that the accuracy of the predictions far exceeds the accuracy needed for the precise alignment of the antenna.

APPENDIX

The Computer Program in OSU version 2 of Fortran used to compute the look angles for a celestial body with the IBM 1620 is given on the following pages.

The inputs to this program are the first two lines (excluding the name, job number, and date) of the computer read-out given on the following page and the outputs are elevation and azimuth angles as functions of local time, along with the title and identifying data. The output is self explanatory, hence only the input will be translated here.

1 identifies a data card.

640228.01 is the run number: first card for station 0, February 28, 1964.

12 00 is the starting time for the computation.

01 is the time increment between computed points in minute of time.

065 is the number of increments to be taken.

23 15 08 is the Local Hour Angle in time measure at the start of computation.

3590.6 is the increment of LHA in seconds of time per one hour.

-08 -08 -37 is the declination at the start of computation.

+ 056.6 is the declination increment in seconds of arc per one hour time.

40 00 10 is the latitude of the observer.

000.1 is the Horizontal Parallax in minutes of arc.

001 is card serial number.

Proceeding to the second line, the digit 2 indicates end of computation and 999 is the maximum number of input cards and the serial number of the stop card.

```
C   STEPHEN ZOLNAY.  21 JAN 64.  LOOK ANGLES FOR A CELESTIAL BODY                  LA1  01
 10 READ1. JCT. RN. KH. KM. KMI. NI. KAH. KAM. KAS. HIS. JD. JM. JS. DIS. LD. LM. LS. EC LA1 02
    GO TO (11.30) JCT                                                              LA1  03
 11 PUNCH 2                                                                        LA1  04
    PUNCH 3. RN                                                                    LA1  05
    PUNCH 32. EC                                                                   LA1  06
    EC = EC / 60.                                                                  LA1  07
    PUNCH 4                                                                        LA1  09
    PUNCH 5                                                                        LA1  10
    PUNCH 6                                                                        LA1  11
    PUNCH 4. KAH. KAM. KAS. HIS. JD. JM. JS. DIS. LD. LM. LS                       LA1  12
    HAR = KAH                                                                      LA1  13
    T1 = KAM                                                                       LA1  14
    T2 = KAS                                                                       LA1  15
    HAR = (HAR + (T1 + T2 / 60.) / 60.) * .26179939                                LA1  16
    PUNCH 4                                                                        LA1  17
    TMI = KMI                                                                      LA1  18
    HIS = HIS * TMI * .12120342E-5                                                 LA1  19
    PUNCH 7                                                                        LA1  20
    DR = JD                                                                        LA1  21
    T1 = JM                                                                        LA1  22
    T2 = JS                                                                        LA1  23
    DR = (DR + (T1 + T2 / 60.) / 60.) * .17453293E-1                               LA1  24
    PUNCH 8                                                                        LA1  25
    DIS = DIS * TMI * .80802280E-7                                                 LA1  26
    ALR = LD                                                                       LA1  27
    T1 = LM                                                                        LA1  28
    T2 = LS                                                                        LA1  29
    ALR = (ALR + (T1 + T2 / 60.) / 60.) * .17453293E-1                             LA1  30
    CL = COS (ALR)                                                                 LA1  31
    SL = SIN (ALR)                                                                 LA1  32
```

```
      TL = SL / CL                                              LA1   33
      DO 29 I = 1.NI                                            LA1   34
      SD = SIN (DR)                                             LA1   35
      T2 = SL * SD + CL * COS (DR) * COS (HAR)                  LA1   36
      T1 = SQRT (1. - T2 * T2)                                  LA1   37
   14 IF (T1) 15.16.19                                          LA1   38
   15 ANG = 3.1415926 + ATAN (T2 / T1)                          LA1   39
      GO TO 20                                                  LA1   40
   16 IF (T2) 17.18.18                                          LA1   41
   17 ANG = -1.5707963                                          LA1   42
      GO TO 20                                                  LA1   43
   18 ANG = 1.5707963                                           LA1   44
      GO TO 20                                                  LA1   45
   19 ANG = ATAN (T2 / T1)                                      LA1   46
   20 GO TO (21.22). JCT                                        LA1   47
   21 EAD = ANG / .17453293E-1 - EC                             LA1   48
      JCT = 2                                                   LA1   49
      CE = COS (ANG)                                            LA1   50
      T1 = SD / (CL * CE) - TL * SIN (ANG) / CE                 LA1   51
      T2 = SQRT (1. - T1 * T1)                                  LA1   52
      IF (HAR - 3.1415926) 12.14.14                             LA1  A52
   12 T2 = - T2                                                 LA1  B52
      GO TO 14                                                  LA1   53
   22 ANG = ANG / .17453293E-1                                  LA1   54
      IF (ANG) 23.24.24                                         LA1   55
   23 ANG = ANG + 360.                                          LA1   56
   24 IF (KM - 60) 26.25.25                                     LA1   57
   25 KM = KM - 60                                              LA1   58
      KH = KH + 1                                               LA1   59
      GO TO 24                                                  LA1   60
   26 IF (KH - 24) 28.27.27                                     LA1   61
   27 KH = KH - 24                                              LA1   62
```

```
      GO TO 26                                                          LA1   63
   28 PUNCH 9. KH.KM. ANG. EAD                                          LA1   64
      KM = KM + KMI                                                     LA1   65
      HAR = HAR + HIS                                                   LA1   66
      JCT = 1                                                           LA1  A66
      IF (HAR - 6.2831853) 29.13.13                                     LA1  B66
   13 HAR = HAR - 6.2831853                                             LA1  C66
   29 DR = DR + DIS                                                     LA1   67
      GO TO 10                                                          LA1   68
   30 PUNCH 2                                                           LA1   69
      PUNCH 4                                                           LA1   70
      PRINT 31                                                          LA1   71
      STOP                                                              LA1   72
 1FORMAT(I1.F10.2.13.13.13.14.13.13.13.F7.1.14.14.14.F7.1.13.13.13.F6.1)  LA1   73
    2 FORMAT (6X3HEND)                                                  LA1   74
 3 FORMAT(5X38HLA-1 LOOK ANGLES FOR A CELESTIAL BODY11X7HRUN NO..F11.2)  LA1   75
    4 FORMAT (19.14.14.F8.1.19.14.14.F8.1.19.14.14.11X2H--)             LA1   76
    5 FORMAT (11X10HHOUR ANGLE14X11HDECLINATION11X10HN LATITUDE)        LA1   77
  6FORMAT(7X18HHR MIN SEC  SEC/HR6X19HDEG MIN SEC  SEC/HR6X11HDEG MIN SEC)  LA1   78
    7 FORMAT (10X10HLOCAL TIMES5X13HAZIMUTH ANGLES5X15HELEVATION ANGLE)  LA1   79
    8 FORMAT (12X7HHRS MIN8X10HDEG E OF N11X7HDEGREES)                  LA1   80
    9 FORMAT (I15.14.F17.3.F19.3.24X1H-)                                LA1   81
   31 FORMAT (50H LIST OUTPUT CARDS WITH ALTERATION 4 SWITCH ON.  )     LA1   82
   32 FORMAT (11X31HELEVATION PARALLAX CORRECTION =.F6.1.5H MIN.)       LA1  A82
      END                                                               LA1   83
```

ACKNOWLEDGEMENTS

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