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UNPUBLISHED PRELIMINARY DATA

AN AVERAGE ATMOSPHERE FOR PARTICLES TRAPPED IN THE EARTH'S MAGNETIC FIELD.

by

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Abstract

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Particles trapped in the earth's magnetic field follow orbits which vary over all longitudes and over wide ranges of latitude and altitude. An efficient method of averaging any spatially dependent function over these orbits is described. A Fortran program which computes average atmospheres, using an Anderson and Francis (1964) atmosphere and the Jensen and Cain (1962) 48 term representation of the earth's magnetic field, is described. This program has been run on the CDC 3600 and IBM 7090 computers.

Author

Introduction

One of the major factors determining the distribution of particles trapped in the earth's magnetic field, is the collisions of the particles with the atmosphere. In considering the effects of the atmosphere it is necessary to integrate over the orbit of the particle. Ray (1960) has given an approximate expression for the integrals in a dipole field. Lenchek and Singer (1962a) and Hamlin et al. (1961) have refined part of the expression. However, it is known that the dipole approximation is not adequate. For example, Welch et al. (1963) has a graph which shows the variation of altitude with longitude on the line with a magnetic field of .235 gauss and an L value of 1.25. In a dipole field the altitude would be constant: in the earth's field it varies from 100 to 1300 kms. For comparison, the scale height of oxygen is about 120 km. Newkirk and Walt (1963) and Anderson, Crane, Francis, Newkirk and Walt (1964) have performed an accurate average over longitude, using a 48 term representation of the earth's magnetic field. In this paper we describe an accurate and comparatively fast method of computing averages over both latitude and longitude, and we describe a computer program which can be used to do further calculations.

The Mathematical Equations

It is well known that the motion of the particle can be decomposed into three separate components. The guiding center moves parallel to a line of force with velocity v_p. The guiding center also drifts perpendicular to the line of force with velocity v_d, where v_d denotes the instantaneous drift average over a longitudinal bounce. The particle rotates rapidly about the guiding center. Let $f(\overline{r})$ denote any property of the atmosphere; for example, f might be the density of oxygen or the total energy loss cross section. The problem is to compute the average value of f for a particle orbit, where the orbit is continued for an indefinite time. Assuming that the particle stays on a shell defined by B_m and I, where B_m is the mirror point magnetic field, and I is the usual adiabatic invariant, then

$$f_{av}(I, B_m) = S(f, I, B_m) / S(1, I, B_m) \qquad (1)$$

where
$$S(f, I, B_m) = \iint f(r) \frac{dx\, dy}{v_p\, v_d} \qquad (2)$$

dx is an element of arc in the direction of v_p (that is an element of the line of force), dy is an element of arc in the direction of v_d, and the integration is taken over the whole shell defined by I and B_m. We have assumed that $f(r)$ is evaluated at the guiding center: the effect of variations in f over distances comparable with the radius of gyration can be taken into account by the method of Lenchek and Singer (1962b). It has been shown by Hassitt (1964) that dy/v_d is independent of x, and thus that (2) is separable, we can therefore put

$$S(f, I, B_m) = \int U(f, I, B_m, \phi) \frac{d\phi}{\dot{\phi}} \qquad (3)$$

where
$$U(f, I, B_m, \phi) = \int f(r) \frac{dx}{v_p} \qquad (4)$$

The co-ordinate ϕ denotes the geographic longitude of the geomagnetic equator on the line of force. The co-ordinates (I, ϕ) completely define a line of force. The integration in (4) is along the line of force defined by (I, ϕ), the limits of integration are the mirror points defined by B_m. Equation (3) has made use of the relation

$$\frac{dy}{v_d} = \frac{dy}{dy/dt} = \frac{d\phi}{d\phi/dt} \qquad (5)$$

where $\dot{\phi}$ is the angular drift velocity projected along the ϕ co-ordinate. For certain applications it is useful to compute

$$W\,(f,\, I,\, B) = \int f(r)\,\frac{dy}{v_d} \qquad (6)$$

$$= \int f(r)\,\frac{d\phi}{\dot{\phi}} \qquad (7)$$

The integration in (6) is taken along the line of constant I and constant B. In fact there are two such lines, one at the northern and one at the southern end of the line of force; we take the average of the two results. The relation (7) follows from the fact that dy/v_d is independent of x. It is used as follows. We evaluate f at the point on the line (I, B), we take the line of force through this point, follow this line to the geomagnetic equator and then find the associated ϕ. This rather complicated process is in fact much faster and easier than calculating dx/v_d at the point itself.

Finding the Line of Force

Given I ϕ, before we can do any calculation, it is necessary to find the correct line of force. A series of computer programs written by Dr. C. E. McIlwain was available to compute the line of force and the L value at any point. The program currently uses the Jensen and Cain (1960) 48 term representation of the earth's field, but it can easily be changed to use any other field. Given I , ϕ we could have found the line of force by a three dimensional search; in fact, we took a modified approach which allowed a one dimensional search. Let

$$\phi_E = \text{geographic longitude at which a line of force crosses the geographic equator} \tag{8}$$

We used the co-ordinates (L, ϕ_E) to define a line of force, where L is defined by McIlwain (1960) and is to be evaluated at the geomagnetic equator. We will relate the (L, ϕ_E) system to the (I, ϕ) system at a later stage. Given ϕ_E we evaluate L for a series of points (r, θ, ϕ), with $\theta = o$, $\phi = \phi_E$ and r taking on several values. The correct line force is then found by an iterative process. Since the geomagnetic equator is close to the geographic equator and since the correct r is approximately equal to L, this process is quite rapid.

Integration Along a Line of Force

The use of the first invariant shows that

$$v_p = v\,(1 - B/B_m)^{\frac{1}{2}} \tag{9}$$

where v, the velocity of the particle, is a constant of the motion. The integrand in equation (4) is singular at the mirror points. Only in a dipole field is it possible to transform to a non-singular form. In some problems $f(r)$ may be a rapidly varying function of x. A numerical method capable of overcoming these difficulties must be used.

The standard program which finds the line of force uses a predictor-corrector method to find a set of points along the line of force. The program was modified so that it computes x, B and h at each point of the set; x is the distance along the line of force, B is the field value and h is the altitude above the surface of the earth. B and h are smooth functions of x, hence we can find $B(x)$ and $h(x)$ for any x by quadratic interpolation. The function $f(r)$ is, in practice, a function of h only. The relation

$$\int_c^d \frac{dx}{2\sqrt{a+bx}} = \frac{\sqrt{a+bd}-\sqrt{a+bc}}{b} = \frac{(d-c)}{\sqrt{a+bd}+\sqrt{a+bc}} \qquad (10)$$

is an exact one, even near the singular point $b = -a/c$. To evaluate the integral (4) we assume that, over a sufficiently small interval, B is a linear function of x and f is a constant, hence

$$\int_c^d \frac{f(r)dx}{(1-B(x)/B_m)^{\frac{1}{2}}} = \frac{[f(h(d)) + f(h(c))](d-c)}{(1-B(c)/B_m)^{\frac{1}{2}} + (1-B(d)/B_m)^{\frac{1}{2}}} \qquad (11)$$

The numerical integration proceeds as follows. Let x_j and x_{j+1} denote two successive values of x in the set of points chosen by predictor-corrector integration. Divide the range x_j to x_{j+1} into two parts, apply formula (11) on each part, using quadratic interpolation on B_{j-1},

B_j, B_{j+1} to find $B(x)$ and a similar interpolation for $h(x)$. Next divide the range into four parts and apply formula (11) four times. Compare the two part and four part results; if they do not agree to sufficient accuracy, use eight, sixteen, etc., parts. Now repeat this process for x_{j-1} to x_{j+2} and so on.

When comparing the two part, four part, etc., results we use two differing accuracy criteria. We require the separate results to differ by less than .1% or each result to be less than R/5000, where R is the result of the integration over the range x_1 up to x_j. In most problems the major contribution to the integral (4) comes from the part of the line of force near the southern mirror point. By starting the integration at the southern mirror point, R builds up rapidly and we avoid unnecessary accuracy at points where the contribution to the integral is small.

Integration in the ϕ Direction

The program chooses $\phi_E = 0^\circ, 60^\circ, \ldots 300^\circ$, and evaluates U and ϕ on each of these six lines of force. It then computes the sum of the six U values; this sum is a first estimate of S. We could now take $\phi_E = 0^\circ, 30^\circ$, etc. and get a better estimate of S; however, this would usually be a very wasteful process. U is often a rapidly varying function of ϕ : it can change by a factor of 10^{10} as ϕ is varied. What we would like to do is to compute $U(\phi)$ at a number of carefully chosen values of ϕ. This is done as follows.

The program takes the results for U at $\phi = 300^\circ, 60^\circ$, and 120°, it then computes U at $\phi = 0^\circ$ by quadratic interpolation on log U. The interpolated and the exact results for U at $\phi = 0^\circ$ are compared If they agree to sufficient accuracy, then we go on to 60°. If they do not agree then we compute U at $\phi = 330^\circ$ and $\phi = 30^\circ$ and repeat the interpolation. The accuracy test evaluates what effect the difference between the correct and interpolated value would make in S, and checks that this is less than .1% of the estimate of S. The estimate of S is re-computed every time an additional pair of points is found.

At the completion of this process we have a table of U(f), U(l) and ϕ at many values of ϕ_E. During this process we also computed $\dot{\phi}$. $\dot{\phi}$ is computed from

$$\dot{\phi} = \left(\frac{B \times \nabla B}{B^3 r \sin\theta} \right)_\phi$$

$$= \frac{1}{B^3 r \sin\theta} \left(B_\theta \frac{\partial B}{\partial r} - \frac{B_r}{r} \frac{\partial B}{\partial \theta} \right) \frac{m\gamma c v^2}{2e} \qquad (12)$$

where $m\gamma$ is the mass, v the velocity and e the charge on the particle. This is the drift of a particle mirroring at the equator. It has been shown elsewhere (Hassitt, 1964) that the value of $\dot{\phi}T$ for a particle mirroring at B_m is a constant (independent of ϕ) times the $\dot{\phi}$ of an equatorial particle. T is the bounce time and this is, by definition, a constant times $U(1, I, B_m, \phi)$. Finally we evaluate the integral (3) by numerical integration. Simpson's rule is used; each ϕ range is split into many parts and parabolic interpolation on log U and on T is used. At the same time the integral (7) is performed. Figure 1. illustrates the choice of ϕ points in a particular case where U varies by a factor of 10^4.

Accuracy

The models of the atmosphere, and to a lesser extent, of the earth's magnetic field, which are currently available contain appreciable errors. However, in order to compare one model with another, the calculation should proceed as though the physical data were completely accurate. The accuracy of the overall calculation depends on several factors.

(a) The line of force calculation. (b) Given L, finding the correct line of force. (c) The accuracy of equation (11). (d) The choice of interval in using equation (11). (e) The calculation of $\dot{\phi}$. (f) The integration in the ϕ direction.

The lines of force are computed by a standard set of routines which have been used extensively in other theoretical calculations and in co-ordinating data from many satellites. It can safely be assumed that the basic routines are accurate and we have only to select an appropriate value of the parameter which controls the accuracy. The accuracy of (b) depends on the accuracy of the line of force calculation only; L is a smooth function of altitude and there is no problem of accuracy in the iterative search for L. The accuracy of (11) can be analyzed using the mean value theorem; any desired accuracy can be achieved by a suitable choice of interval. The accuracy governed by factors (d) and (f) relies on the quadratic approximation of certain functions and on the efficiency of repeated interval splitting. It is possible to construct examples where splitting an interval in two will not improve the accuracy, and on which comparison tests will lead to spurious results; however, the functions employed here are monotonic over the interval of integration. The accuracy of the quadratic interpolation depends directly on the interval since the functions approximated are nowhere singular; the interval can be varied by changing one of the parameters in the program. Newkirk and Walt (1963) have shown that the variation of drift velocity can change the value of longitudinal averages by as much as 25%. However, much of this effect is due to a geometrical factor, the spreading of the lines of force. The variation of $\dot{\phi}$ has only a 5 or 10% effect on the final result. Comparison of equation (12) with an exact method of computing the drift has shown that the approximation results in a possible 2% error in $\dot{\phi}$; this is comparable with errors due to variations in L, which we will discuss later.

The Significance of I

It has been shown by Northrop and Teller (1960) and by others that I is an adiabatic invariant. In most applications of this invariant it is usual to make an approximation, namely that L is also invariant. McIlwain (1961) has shown that L varies by less than 1% on most lines of force. The importance of the L co-ordinate is that it gives the same label to particles which are on the same line of force.

The program described here computes f_{av} and W for a given L and B_m. It computes its L value on any line of force from the relation

$$L = (.311653/B_o)^{1/3} \tag{13}$$

where B_o is the value of B at the geomagnetic equator, this value will differ from the true L value by possibly 1%. At an L value of 1.5, a 1% error in L at the equator corresponds to 100 km; however, this does not necessarily imply a large error in f_{av}. There are several mitigating factors. For particles mirroring near the equator there will be no significant error in L. For particles mirroring a long way from the equator, the main contributor to f_{av} will come from high B values and at high B values the error in L does not have such a large effect on the altitude.

The are several economic reasons for using this approximation in the program. It speeds up the search for the line of force with a given L and ϕ. In addition, having found the line of force we can use it to compute results for several values of B_m. It is not necessary to compute a new line of force for each B_m. Furthermore, the function f has to be computed at many points on a line of force. We carry the calculations for several values of B_m in parallel, and whenever possible, use one f calculation for all B_m values which require that f. The program could be changed, by a minor modification, to use the correct L for each B_m, however, it would then be necessary to compute results for only one B_m in each calculation.

The Atmosphere

The program currently incorporates the atmosphere of Anderson and Francis (1964). Values for atmospheric properties near sunspot minimum were taken from the tables given by Anderson et al. (1964) and fitted by an exponential function of the reduced height. No attempt was made to fit values below 200 km. The atmosphere is computed in a self-contained Fortran subroutine and any other atmosphere can easily be substituted.

Using the Program

The program is controlled by setting some parameters in the main program. FL is the value of L required. B(I) gives the value, in gauss, of the values of B_m for which the calculation should be done. B(1), B(2),... must be in decreasing order and the last value must be zero. The program will construct a table from which values of B(I) which are less than B_o have been deleted and to which the value B_o has been added. If any of the higher B values cause the mirror point to dip below the surface of the earth, the program will set $f(r) = 10^{30}$ for these underground points. N is the number of the atmosphere constituent used in the routine ATMO, with the correspondence

N =	1	2	3	4	5	6	7	8	9	10	11	12
Constituent	e^-	O_2	N_2	O	N	He	H	O_2^+	No^+	O^+	He^+	H^+

N = 13 mixes the constituents in proportion to the energy loss cross section for that constituent. This mixture appears as a single Fortran statement in the routine ATMO, and can easily be changed. N = 14 sets f(r) equal to unity.

The program does not read any data, and uses no tapes other than the standard output tapes. The variable N TAPE should be set to the unit number of the print tape. The variable NP TAPE should be set to the number of the standard punch tape.

To Summarize

FL	L value
B(1), B(2),...	B_m values in decreasing order. Last value should be zero

N	Constituent to be selected for averaging.
N TAPE	Standard output tape
NP TAPE	Standard punch tape or zero if no cards required.
ERR1,ERR2,ERR3	Error control. See later description.

The program as distributed contains a main program with

FL = 1.25	B(3) = 0.
B(1) = 0.21892	N = 4
B(2) = 0.17822	

This problem takes 2 mins 56 secs on the CDC 3600.

This time include loading and other times spent in the monitor. For L = 1.25 and ten B values the CDC 3600 takes 5 minutes.

Output

The first page of output of the sample problem is shown in Figure 2. Ignoring for the moment the line which begins with a *, the results are

Longitude	ϕ_E						
L = 1	Equator	r	θ	ϕ	B_o	DPHI/DT	$\dot{\phi}$
B(1)	$\log_{10}(U)$	U	T	hn	f(hn)	hs	f(hs)
B(2)	"	"	"	"	"	"	"
B_o	"	"	"	"	"	"	"

ϕ_E, B_o, U and f have been defined previously. l is the value of L for the last point on the line of force: this point will have a B value slightly greater than B(1). r, θ, ϕ are the co-ordinates of the geomagnetic equator. $\dot{\phi}$ is the result of equation (12) with the factor of $m\gamma^4 cv^2/2eR_e^2$ omitted, where R_e is the radius of the earth. T is $U(1, I, B_m, \phi)$. This should be multiplied by $2R_e/v$ to give the bounce time in seconds. hn and hs are the height in kilometers, of the northern and southern mirror points. Any printed line marked by a * has the following significance. The process described following equation (11) encountered difficulty in convergence. For some x_j it was necessary to split the interval x_j to x_{j+1} into more than 64 parts. The results are usually correct in this situation but a sequence of * would indicate that the step size x_j to x_{j+1} is too large (see accuracy control) or more probably that you had changed routine ATMO and had made an error of some sort. This line of printing contains: the mirror point B, the height/R_e, the penultimate and ultimate values of the integral of $f(r)dx/v_p$, the relative error, followed by similar results for dx/v_p.

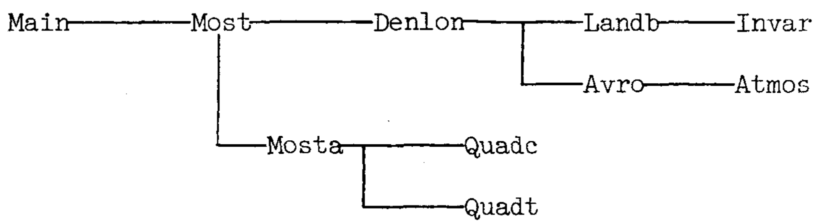
The line of print beginning "K = " refers to the quadratic interpolation process which was discussed in the section on integration in the ϕ direction. Finally at the top of a new page (see Figure 1) we have the results for one B value, namely, the number of the substance, the value of L, f_{av}, $W(f, L, B_m)/W(1, L, B_m)$, minimum height for this value of L and B_m and finally $\log_{10}(f_{av})$. The numbers, in the same format, are punched on a card unless NP TAPE = 0. Following this are the results for each ϕ value, namely

$$\phi \quad \dot{\phi}/\dot{\phi}_o \quad \log_{10}(U(f)) \quad U(1) \quad (f(hn) + f(hs))/2$$

This is followed by the results for the next B value, if any.

Organization of the Program

The program is written completely in Fortran and will compile under Fortran II or Fortran 63. The flow of control is as follows



Atmos is the routine which computes f(h). For convenience it actually calls another routine ATMO to compute the Anderson and Francis atmosphere.

Avro performs the integration described by equation (11). The arguments of Avro are values of B_m, and the coefficients which give a quadratic fit to B and L over the range x_j to x_{j+1}.

Invar computes a line of force.

Landb The arguments of this routine are L and ϕ_E. It does an iterative calculation to find the line of force with the given value of L and which cross the geographic equator at longitude ϕ_E. It then goes south along this line of force (using Invar) to a point with B greater than B_m. Starting from this point it goes north to find the complete line of force between the limits. $B > B_m$ in the southern hemisphere to $B > B_m$ in the northern hemisphere.

Denlon The arguments to this routine are L and ϕ_E and the table of B values. It uses Landb to compute the line of force. Then for each B value it uses Avro to compute S(f) and S(1): it also saves f(hn) and f(hs). The calculation is arranged so that one line of force suffices for all B values and usually Atmos is called once for each point and the result saved for the next B value.

Mosta controls the integration in the ϕ direction. It is entered when all the U values are available.

Most controls the major part of the calculation. It selects $\phi = 0^\circ$, 60°, etc., calling Denlon to do the U calculation. It performs the test of parabolic fit against true value and decides if an additional pair of ϕ values should be used. When all the data is ready it calls Mosta to do the integration.

Invar has a number of subsidiary routines, namely Start, Lines, Magnet, Bulge, Grado, Carmel, Integ. Most of these routines were written by Dr. C. E. McIlwain and have been used extensively in many computing centers; we will not describe them in detail. The routine, Magnet, computes the earth's field and is due to D. C. Jensen. We have made the following additions to these routines. Bulge computes the effect of the earth's oblateness; it was formerly used but was not a separate routine. Grado computes the ϕ component of ∇B. Lines has been modified so that one can start at a low B value and integrate until some specified higher B value has been passed. Lines has also been modified so that the coordinates of the geomagnetic equator are computed and also so that the

the co-ordinates of a point with a specified B value can be found. This latter feature is not used in the current calculations.

To change the computation of the earth's field the routine Magnet should be changed. To use a spherical earth, set Bulge equal to unity. To change the atmosphere, modify ATMO. To compute different combinations of components of the atmosphere, change Atmos.

If a clock is attached to the computer, the routine Mytime should be modified so that it calls the clock.

Control of Accuracy

The accuracy of the line of force calculation is controlled by the variable ERR1 in routine Landb. The iterative calculation to find a line of force with a given L value is also controlled by this variable; we iterate until the change in L is less than ERR1/10. The accuracy of integration along the line of force is controlled by ERR2 in Avro. Within the interval x_j to x_{j+1} the routine computes equation (11) for two, four, eight, etc. divisions of the interval; it stops dividing when the $\Delta U/U$ for the $U(1)$ integral is less than ERR2 or when $\Delta U(f)$ is less than ERR2/5 times (the value of U for steps x_1 to x_j). The accuracy of the ϕ integration is controlled primarily by ERR3 in the routine Most. The ϕ are chosen such that

$$(U(\phi_i) - \text{Fit}\ (U(\phi_{i-2}), U(\phi_{i-1}), U(\phi_{i+1}))) \times (\phi_{i+1} - \phi_{i-1}) < S \times \text{ERR3}$$

where S is the current estimate of S. The actual integration of U(f) is done in Quadc and repeated division and the use of Simpson's rule is used until the result is changing by less than one part in 5000. Changing the accuracy numbers produces the following changes in the result. For L = 1.25 and B = .21892

	f_{av}	W	Minimum Height
Standard Values	1.028609 10^6	5.452904 10^6	276.25
With ERR1 = .0005	1.028890 10^6	5.453579 10^6	276.26
With ERR2 = .0005	1.028638 10^6	5.452679 10^6	276.25
With ERR3 = .0005	1.028861 10^6	5.453576 .0^6	276.25

The standard case has ERR1 = ERR2 = ERR3 = .001.

The quantity W has been computed by Anderson et al. (1964). Their calculation was done in a different way. Their L value refers to the B_m point and not to the geomagnetic equator. They computed the drift velocity at many points on the line of force. For the test problem at B = .21892 we have W = 5.453 10^6 and minimum height 276.3 km. Anderson et al. in their table 4-3 have W = 5.597 10^6 and minimum height 272 km. The minimum height is the lowest height reached by a particle with the given B and L. The scale height of oxygen is 120 km so a discrepancy of 3% corresponds to a 4 km change in height. For the same L and B values using the concentration of helium we find W = 1.143 10^5 and Anderson et al. have W = 1.137 10^5.

The integration along the line of force can be checked for the case of f(r) equal to a constant. The integration routine is completely separate from the routine which computes the magnetic field. We have modified the routine Magnet to produce a dipole field, and the routine Bulge to use a spherical earth. The results for the time can be compared with the function $2LT(\mu)$ where $T(\mu)$ is the function tabulated by Hamlin et al. (1961). For the value $L = 1.25$ the results are shown in Table 1.

$(B_o/B)^{\frac{1}{2}}$	θ	$2LT(\mu)$	T
1.	90°	1.8512	1.8516
.5647	60°	2.4065	2.4090
.2812	45°	2.823	2.826
.09310	30°	3.160	3.163

Table 1. $2LT(\mu)$ is the bounce time computed by Hamlin et al. for a dipole field. T is the corresponding result for this program.

Acknowledgements

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Figure Captions

Figure 1. Results for one value of L and B.

Figure 2. Intermediate results for the test problem.

NSUBS	L	B	F AVER.	W	MIN. H	LOG10(F AVER.)
4	1.250	.21892	1.028609E 06	5.452904E 06	276.25458	6.012211

LONGITUDE	DRIFT	LOG(U)	I	(FS+FN)/2
359.068694	1.000000	4.530519E 00	1.970575E 00	9.380407E 04
14.008476	1.007574	3.789617E 00	1.964601E 00	1.672071E 04
21.712613	1.004847	3.506981E 00	1.970444E 00	8.607689E 03
29.426995	.997034	3.264608E 00	1.981397E 00	4.844294E 03
44.603004	.972323	2.709047E 00	2.005522E 00	1.289045E 03
52.065541	.959793	2.301512E 00	2.007011E 00	4.930448E 02
59.489444	.949219	1.826293E 00	1.999687E 00	1.618593E 02
89.595955	.922819	3.850061E-01	1.979821E 00	5.514690E 00
104.934713	.911683	1.905362E-01	1.990195E 00	3.457801E 00
120.169544	.902708	3.125461E-01	2.008945E 00	4.547910E 00
135.387685	.897934	7.049233E-01	2.033688E 00	1.127956E 01
150.756237	.894594	1.303747E 00	2.065524E 00	4.534815E 01
166.056251	.890877	1.885374E 00	2.089724E 00	1.752112E 02
173.538521	.890034	2.070172E 00	2.090506E 00	2.697948E 02
180.898420	.890359	2.174717E 00	2.084084E 00	3.451450E 02
209.881619	.898040	2.270008E 00	2.044653E 00	4.283854E 02
224.401559	.904719	2.350485E 00	2.028416E 00	5.115530E 02
238.997344	.916819	2.556983E 00	2.012453E 00	8.263953E 02
253.643804	.937340	2.958159E 00	1.993308E 00	2.143504E 03
268.501501	.967256	3.644524E 00	1.980877E 00	1.076579E 04
276.224219	.984504	4.163225E 00	1.985070E 00	3.599072E 04
284.335530	1.000498	4.863211E 00	2.003449E 00	1.819994E 05
288.571245	1.006692	5.295836E 00	2.020485E 00	4.944287E 05
292.919692	1.010832	5.782947E 00	2.043571E 00	1.521740E 06
295.125085	1.011988	6.042633E 00	2.057263E 00	2.770662E 06
297.341478	1.012499	6.308337E 00	2.072148E 00	5.113901E 06
299.558631	1.012392	6.574127E 00	2.087710E 00	9.440039E 06
300.664175	1.012119	6.704547E 00	2.095619E 00	1.275588E 07
301.215367	1.011935	6.768649E 00	2.099502E 00	1.479085E 07
301.765317	1.011719	6.831808E 00	2.103271E 00	1.711374E 07
303.948990	1.010572	7.071668E 00	2.117819E 00	2.979368E 07
306.098452	1.009042	7.282722E 00	2.130186E 00	4.857321E 07
308.204971	1.007226	7.454070E 00	2.139200E 00	7.235723E 07
309.239579	1.006237	7.521911E 00	2.142198E 00	8.482391E 07
310.261371	1.005205	7.576885E 00	2.143862E 00	9.657337E 07
312.263054	1.003058	7.647101E 00	2.144092E 00	1.143482E 08
313.244653	1.001947	7.662478E 00	2.142401E 00	1.189759E 08
314.212182	1.000834	7.665237E 00	2.139753E 00	1.203003E 08
316.107418	.998609	7.636876E 00	2.131941E 00	1.137863E 08
317.036715	.997505	7.607784E 00	2.127079E 00	1.069525E 08
317.497403	.996956	7.590006E 00	2.124415E 00	1.029243E 08
317.955220	.996412	7.570298E 00	2.121658E 00	9.860438E 07
319.759657	.994282	7.474828E 00	2.110014E 00	7.992256E 07
321.526855	.992275	7.358728E 00	2.097923E 00	6.174473E 07
323.262378	.990416	7.229413E 00	2.085842E 00	4.620806E 07
324.119400	.989551	7.161514E 00	2.080416E 00	3.965989E 07
324.971584	.988738	7.092064E 00	2.074863E 00	3.391348E 07
328.336427	.986036	6.807260E 00	2.055185E 00	1.779045E 07
331.658506	.984320	6.522522E 00	2.038938E 00	9.294770E 06
345.024201	.987402	5.461286E 00	1.997149E 00	8.093572E 05

```
LONGITUDE        0
L=         1.2456 EQUATOR      1.227290       1.439833       6.266931       .159556  DPHI/DT       12.806524
*  1.782200E-01  1.224679E-01  1.048289E 02  1.049818E 02  1.456328E-03  4.677528E-01  4.676441E-01  2.324496E-04
     .218920       4.530519    3.392724E 04    1.970575E 00    1007.83   2.694230E 03    718.40   1.849139E 05
     .178220       2.123688    1.329541E 02    1.858342E 00    1327.82   3.637448E 01   1152.26   3.688508E 02
     .159566       1.191559    1.554414E 01    1.801382E 00    1441.03   8.629008E 00   1441.03   8.629008E 00
LONGITUDE     60.000000
L=         1.2479 EQUATOR      1.292226       1.452766       1.038287       .159553  DPHI/DT       12.156199
*  1.782200E-01  1.958515E-01  4.078358E-01  4.082781E-01  1.083356E-03  4.712277E-01  4.711294E-01  2.085558E-04
     .218920       1.826293    6.703557E 01    1.999687E 00    1366.62   2.210873E 01   1167.20   3.016098E 02
     .178220       -.201308    6.290579E-01    1.883365E 00    1706.68   3.456366E-01   1594.38   1.312129E 00
     .159566       -.940854    1.145883E-01    1.824267E 00    1854.93   6.281332E-02   1854.93   6.281332E-02
LONGITUDE    120.000000
L=         1.2487 EQUATOR      1.325388       1.413935       2.097354       .159564  DPHI/DT       11.560546
     .218920        .312546    2.053753E 00    2.008345E 00    1485.78   4.943062E 00   1499.87   4.152758E 00
     .178220      -1.335901    4.614138E-02    1.896740E 00    1860.99   5.866488E-02   1871.51   5.210401E-02
     .159566      -1.949579    1.123075E-02    1.836308E 00    2066.45   6.115938E-03   2066.45   6.115938E-03
LONGITUDE    180.000000
L=         1.2533 EQUATOR      1.282431       1.482914       3.157273       .159555  DPHI/DT       11.402406
     .218920       2.174717    1.495308E 02    2.084084E 00    1111.49   6.415897E 02   1305.25   4.870024E 01
     .178220        .130093    1.349256E 00    1.960776E 00    1535.14   2.692699E 00   1643.26   7.308226E-01
     .159566       -.616103    2.420431E-01    1.891443E 00    1792.41   1.279674E-01   1792.41   1.279674E-01
LONGITUDE    240.000000
L=         1.2491 EQUATOR      1.259146       1.672253       4.171291       .159566  DPHI/DT       11.741268
     .218920       2.556983    3.605786E 02    2.012453E 00    1105.17   6.994288E 02   1082.59   9.533617E 02
     .178220        .784736    6.091737E 00    1.901514E 00    1459.16   6.880323E 00   1444.51   8.261483E 00
     .159566        .124870    1.333125E 00    1.842580E 00    1644.10   7.235099E-01   1644.10   7.235099E-01
LONGITUDE    300.000000
L=         1.2542 EQUATOR      1.186426       1.846452       5.266798       .159569  DPHI/DT       12.956604
     .218920       6.831808    6.789729E 06    2.103271E 00     823.37   3.835720E 04    391.00   3.418912E 07
     .178220       3.869023    7.396873E 03    1.971925E 00    1101.71   7.333404E 02    862.36   2.163384E 04
     .159566       2.673569    4.716132E 02    1.911329E 00    1182.15   2.467462E 02   1182.15   2.467462E 02
GUESS OF S    4.094541E 08
K=     1 LONG.=        0     LOGU   4.530519E 00 FIT   6.588245E 00 DS   1.125757E 00 DT   3.258975E-02
LONGITUDE    330.000000
L=         1.2502 EQUATOR      1.194358       1.666796       5.788533       .159562  DPHI/DT       12.605724
     .218920       6.522522    3.330923E 06    2.038938E 00     858.55   2.287316E 04    427.72   1.856667E 07
     .178220       3.567518    3.694378E 03    1.913748E 00    1150.26   3.789749E 02    905.12   1.162583E 04
     .159566       2.374878    2.370790E 02    1.848213E 00    1231.30   1.282748E 02   1231.30   1.282748E 02
LONGITUDE     30.000000
L=         1.2468 EQUATOR      1.258821       1.447548        .513598       .159558  DPHI/DT       12.768534
*  1.782200E-01  1.684717E-01  7.714113E 00  7.731953E 00  2.307350E-03  6.182592E-01  6.180266E-01  3.762239E-04
*  1.782200E-01  1.684717E-01  6.969721E 00  6.976962E 00  1.037797E-03  4.263477E-01  4.262659E-01  1.917749E-04
     .218920       3.264608    1.839199E 03    1.981397E 00    1187.32   2.302763E 02    919.45   9.458318E 03
     .178220        .985914    9.681001E 00    1.869105E 00    1516.58   3.380766E 00   1358.84   2.442019E 01
     .159566        .126940    1.339495E 00    1.808671E 00    1642.14   7.405961E-01   1642.14   7.405961E-01
GUESS OF S    4.065722E 08
K=     1 LONG.=        0     LOGU   4.530519E 00 FIT   5.333455E 00 DS   2.679877E-02 DT   3.519097E-03
LONGITUDE    345.000000
L=         1.2476 EQUATOR      1.210094       1.572275       6.021808       .159563  DPHI/DT       12.645185
     .218920       5.461286    2.892817E 05    1.997143E 00     923.82   8.882529E 03    578.89   1.609832E 06
     .178220       2.638190    6.889822E 02    1.882055E 00    1232.09   1.269416E 02   1027.69   2.040500E 03
     .159566       1.800503    6.317056E 01    1.817900E 00    1331.36   3.474920E 01   1331.36   3.474920E 01
LONGITUDE     15.000000
L=         1.2449 EQUATOR      1.244034       1.456056        .244494       .159559  DPHI/DT       12.903524
*  1.782200E-01  1.378309E-01  2.385696E 01  2.389190E 01  1.462696E-03  4.804906E-01  4.803728E-01  2.451891E-04
     .218920       3.789617    6.160856E 03    1.964601E 00    1099.61   7.547883E 02    834.22   3.268663E 04
     .178220       1.493872    3.118041E 01    1.852902E 00    1426.50   1.035560E 01   1265.18   8.211763E 01
     .159566        .616978    4.139824E 00    1.796542E 00    1547.90   2.304329E 00   1547.90   2.304329E 00
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