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A REVIEW OF SELECTED METHODS OF PREDICTING BASE FLOW ENVIRONMENT IN SUPERSONIC FLOW

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SUMMARY

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Methods for predicting the base flow environment in supersonic flow were reviewed. Only those methods which were analytical in approach were selected for discussion. The merits and the limitations of each selected method are discussed. In some cases, means of improvement are suggested, such as including heat-transfer effect or expanding two-dimensional-flow theory to three-dimensional-flow theory.

Of the methods selected, primary emphasis was placed on the two-dimensional turbulent wake methods of Korst, et al and Nash and the axisymmetric turbulent wake method of Zumwalt. Less emphasis is placed on the laminar theory of Chapman, et al. The method of Korst et al was developed earlier and with a less exact but more versatile theoretical approach than that of Nash. Zumwalt's approach to the axisymmetric turbulent wake is made by extensively modifying Korst's theory. The laminar theory of Chapman et al in which the flow is two-dimensional, considers the approach boundary layer thickness to be zero, and the laminar theory of Dennison and Baum takes into account the approaching boundary layer thickness and heat transfer to the base, and can be used either two-dimensionally or axisymmetrically. Discussed briefly is the semi-empirical turbulent wake method of Love, and the combination of Chapman's equivalent-cylinder flow-turning method with Zumwalt's method for predicting base pressures on cones and boattails.

Author

TABLE OF CONTENTS

INTRODUCTION

Many experiments have been performed and various analytical theories developed on the subject of base flow environment of bodies in supersonic flow. In January 1964 at the request of Aero-astrodynamics Laboratory of Marshall Space Flight Center of National Aeronautics and Space Administration, a review of literature on the base flow environment for bodies of revolution in supersonic turbulent flow was initiated by LMSC/Huntsville Research & Engineering Center. The purpose of this review is to determine methods for predicting the base flow environment, to recommend the best approach, and to develop a computer program which primarily computes base pressure.

A completely satisfactory method has not yet been found. However, several comprehensive approaches do exist, each of which has its own merits and limitations. A critique of these approaches is presented in this report.

TECHNICAL DISCUSSION

Numerous analytical approaches to solving the problem of base pressure have been postulated and examined. Of these approaches, four have been widely accepted. Each of them is self-sufficient and requires no experimental data as analytical input. These approaches, examined in this document, are: (1) theory of Chapman, Kuehn and Larson, (2) Korst's theory, (3) Nash's theory and (4) Zumwalt's method. The first three theories are for two-dimensional bodies. Zumwalt's method is for a cylinder with its axis of rotation parallel to the flow. All methods, except that of Chapman, et al, are for turbulent supersonic flow. The application of the theory of Chapman, et al, is primarily for laminar flow. Because it is less applicable to the present requirement than are the other three, this theory is discussed in less detail.

In addition to the four methods listed above, there is the one suggested by E. S. Love which is treated briefly. His method is semiempirical in nature and it represents a somewhat different school of thought from the previous four. For the purpose of comparison, it also will be presented.

Before these theories and methods are discussed, a flow model and a hypothesis proposed by Chapman which have played an important role in development of base pressure theory will be discussed.

<u>Chapman's Flow Model and Hypothesis</u> (Reference 1 and 2)

In 1951 Chapman proposed a flow model which dissects the base flow into four regions (Figure 1):

1. Viscous flow at the trailing edge of the body
2. Expansion fan at the corner of separation
3. Region behind the base between the point where the flow is separated (at point S in Figure 1), to the region where recompression occurs
4. Recompression region where the separated flow converges

Chapman concluded the following: At the corner of the base, the flow is divided by a mixing layer from the "dead-fluid" immediately behind the body. As the separated fluid flows downstream, it entrains some of the fluid from the dead-fluid region because of the scavenging effect at the boundary of the dead-fluid region. When the separated fluid reaches the recompression region, its low kinetic energy segment of fluid cannot penetrate the pressure-rise, so this amount of fluid reverses back to the dead-fluid region along the axis. A balance is maintained between the mass-flow scavenged from the dead-fluid region by the separated fluid (i.e. the mixing layer) and the mass-flow reversing back to the dead-fluid region. If this balance did not exist, fluid would be either continually injected into or removed from the dead-fluid region.

To simplify the solution of this balance concept, Chapman introduced the hypothesis of the "Dividing Streamline." As shown in Figure 1, the dividing streamline, j, divides the separated flow into two layers. The fluid in the layer between the streamline, j, and the streamline, IIa, (streamline IIa at the outer edge of the mixing layer) has sufficient velocity to negotiate the pressure-rise in the recompression region and continues downstream. The fluid in the lower layer, having lower velocity, reverses before the pressure-rise and flows back to the dead-fluid region.

Theory of Chapman, Kuehn and Larson (Reference 2)

The flow model and the "dividing streamline" concept lead to the necessity of finding a suitable mixing theory. As a result, Chapman derived a solution of the laminar mixing layer. In it he reasons that the growth of the thickness of the laminar mixing layer is similar to that of the laminar boundary layer; that is, the layer grows parabolically with distance from the origin of the layer. However, the rate of growth of mixing layer thickness is approximately three times that of boundary layer thickness. The velocity profile remains constant throughout the mixing layer, so the velocity ratio, u_j/u_{IIa}, is not affected by Reynolds number or the distance from the origin of layer. It is only slightly affected by Mach number and the temperature-viscosity relationship.

With the results of this laminar-mixing-layer theory, Chapman, Kuehn and Larson complete the base pressure theory for laminar, two-dimensional flow and show that it agrees well with experimental measurements carefully taken to meet the theoretical requirements.

In this theory, the following assumptions are made:

1. A Prandtl-Meyer expansion exists at the corner of separation.

2. The static pressure of the separated flow (i.e. mixing layer) is equal to that of the adjacent free stream.

3. Pressure along and across the separated flow is constant.

4. The temperature-viscosity relationship is linear.

5. The Prandtl number is unity.

6. In the dead-fluid region, the velocity of flow is slow, and the circulating motion is negligible. Hence the pressure is essentially constant and equal to the static pressure of the separated flow.

7. The compression in the recompression region is isentropic along the dividing streamline.

8. In the recompression region, the total pressure along the dividing streamline is equal to the static pressure behind the trailing shock and far downstream in the wake.

The base pressure is predicted by this equation,

$$\frac{P_b}{P_I} = \left\{ \frac{1 + \left(\frac{\gamma-1}{2}\right) M_{Ia}^2}{1 + \left(\frac{\gamma-1}{2}\right) \left[\frac{M_{Ia}^2}{\left(1 - \frac{u_j^2}{u_{IIa}^2}\right)} \right]} \right\}^{\frac{\gamma}{\gamma-1}}$$

with $\frac{u_j}{u_{IIa}} = 0.587$

As mentioned before, this theory is restricted to two-dimensional, pure laminar wake flow. In addition, the following limitations are imposed:

a. The approaching boundary layer thickness, δ_I, is zero.

b. The heat transfer effect at the base is not considered.

c. Besides jet mixing, there is no additional mass bleed from or into the dead-fluid region.

Although the assumptions mentioned before have made this theory less exact, within the theory's limitation it has adequately predicted the base pressure. The theory can be applied to both subsonic and supersonic flows.

Korst's Theory (References 3, 4, 5, 6 and 7)

Between 1954 and 1959 at the University of Illinois, Korst and his associates, using Chapman's flow model and the "Dividing Streamline" hypothesis, developed a theory to analyze the base environment of two-dimensional turbulent wake. To characterize the mixing phenomenon, they generate an approximate solution which gives the velocity distributions of the turbulent mixing layer.

$$\frac{u}{u_{IIa}} = \frac{1}{2}\left[1 + \operatorname{erf}(\eta-\eta_p)\right] + \frac{1}{\sqrt{\pi}} \int_{\eta-\eta_p}^{\eta} \frac{u_{II}}{u_{IIa}} \left(\frac{\eta-\beta}{\eta_p}\right) e^{-\beta^2}\, d\beta \qquad \text{(K1)}$$

The base pressure theory is then governed by the following assumptions:

1. The expansion of the free stream at the corner of separation follows the Prandtl-Meyer solution for the simple wave.

2. Static pressure between cross-sections II and III on Figure 2 is that of the adjacent free stream.

$$p_j = p_{IIa} \qquad\qquad p_b = p_{IIa}$$

3. Pressure along the mixing layer is constant.

4. Pressure rise in the recompression region is caused by the plane shock in the adjacent free stream.

5. The compression angle, θ_{IIIa}, is equal to the expansion angle, θ_{IIa} as shown on Figure 2.

6. In the recompression region, the total pressure along the reattachment streamline or the discriminating streamline, d, is equal to the static pressure behind the trailing shock and far downstream in the wake. The discriminating streamline is defined in Figure 2.

7. X = 0 is at the corner of separation.

By satisfying the momentum equation and the law of conservation of mass, Korst et al generalize the base pressure theory in these three equations:

$$\eta_m = \eta_R - \eta_p + (1-C_{IIa}^2)\left[\eta_p \int_0^1 \frac{\left(\frac{u_{II}}{u_{IIa}}\right)^2 d\left(\frac{y}{\delta_{II}}\right)}{\Lambda - C_{IIa}^2\left(\frac{u_{II}}{u_{IIa}}\right)^2} - \int_{-\infty}^{\eta_R} \frac{\left(\frac{u}{u_{IIa}}\right)^2 d\eta}{\Lambda - C_{IIa}^2\left(\frac{u}{u_{IIa}}\right)^2}\right] \tag{K2}$$

where η_R is a large arbitrary reference value.

$$\frac{G_d\ \eta_p}{(1 - C_{IIa}^2)\ \rho_{IIa}\ u_{IIa}\ \delta_{II}} = \int_{-\infty}^{\eta_d} \frac{\left(\frac{u}{u_{IIa}}\right) d\eta}{\Lambda - C_{IIa}^2\left(\frac{u}{u_{IIa}}\right)^2} \quad \int_{-\infty}^{\eta_j} \frac{\left(\frac{u}{u_{IIa}}\right) d\eta}{\Lambda - C_{IIa}^2\left(\frac{u}{u_{IIa}}\right)^2} \tag{K3}$$

and

$$\int_{-\infty}^{\eta_j} \frac{\left(\frac{u}{u_{IIa}}\right) d\eta}{\Lambda - C_{IIa}^2\left(\frac{u}{u_{IIa}}\right)^2} = \int_{-\infty}^{\infty} \frac{\left(\frac{u}{u_{IIa}}\right)\left(1 - \frac{u}{u_{IIa}}\right) d\eta}{\Lambda - C_{IIa}^2\left(\frac{u}{u_{IIa}}\right)^2} - \eta_p \int_0^1 \frac{\left(\frac{u_{II}}{u_{IIa}}\right)\left(1 - \frac{u_{II}}{u_{IIa}}\right) d\left(\frac{y}{\delta_{II}}\right)}{\Lambda - C_{IIa}^2\left(\frac{u}{u_{IIa}}\right)^2} \tag{K4}$$

Here, the effect of approaching boundary layer is included.

The base pressure can be solved by utilizing the above equations, when δ_{II}, $\frac{u_{II}}{u_{IIa}}$, η_p and G_d are known. Unfortunately, Korst et al did not develop any solution for δ_{II}, $\frac{u_{II}}{u_{IIa}}$ and η_p. Instead, Korst and associates restricted themselves to a special case in which the approaching boundary layer is thin, approaching a value of zero. The result of this restriction is that in the mixing layer

$$\frac{\delta_{II}}{x} \longrightarrow 0, \qquad \psi = \frac{x}{\delta_{II}} \longrightarrow \infty$$

so
$$\xi = \frac{1}{2\sigma^2} \int_0^{\psi} \psi \, f(\psi) \, d\psi \longrightarrow \frac{\psi^2}{2\sigma^2},$$

while
$$\eta_p = \frac{1}{2\sqrt{\xi}} \longrightarrow \frac{\sigma}{\psi} \longrightarrow 0$$

and
$$\eta = \zeta \, \eta_p = \left(\frac{y}{\delta_{II}}\right)\left(\frac{\sigma}{\psi}\right) = \left(\frac{y}{\delta_{II}}\right) \sigma \left(\frac{\delta_{II}}{x}\right) = \frac{\sigma y}{x}.$$

With $\eta_p = 0$, the importance of δ_{II} and $\frac{u_{II}}{u_{IIa}}$ is thereby diminished. The solution of $\eta_p = 0$ represents the fully developed turbulent velocity profile in mixing layer.

The equation (K1) now becomes

$$\frac{u}{u_{IIa}} = \frac{1}{2}(1 + \text{erf } \eta)$$

$$= \frac{1}{2}\left[1 + \text{erf}\left(\frac{\sigma y}{x}\right)\right] \qquad \text{(K5)}$$

The equations (K2), (K3) and (K4) become

$$\eta_m = \eta_R - (1 - C_{IIa}^2) \int_{-\infty}^{\eta_R} \frac{\left(\frac{u}{u_{IIa}}\right)^2 d\eta}{\Lambda - C_{IIa}^2 \left(\frac{u}{u_{IIa}}\right)^2} \tag{K6}$$

$$\frac{G_d \sigma}{x(1 - C_{IIa}^2)\, \rho_{IIa}\, u_{IIa}} = \int_{-\infty}^{\eta_d} \frac{\left(\frac{u}{u_{IIa}}\right) d\eta}{\Lambda - C_{IIa}^2 \left(\frac{u}{u_{IIa}}\right)^2} - \int_{-\infty}^{\eta_j} \frac{\left(\frac{u}{u_{IIa}}\right) d\eta}{\Lambda - C_{IIa}^2 \left(\frac{u}{u_{IIa}}\right)^2} \tag{K7}$$

$$\int_{-\infty}^{\eta_j} \frac{\left(\frac{u}{u_{IIa}}\right) d\eta}{\Lambda - C_{IIa}^2 \left(\frac{u}{u_{IIa}}\right)^2} = \int_{-\infty}^{\infty} \frac{\left(\frac{u}{u_{IIa}}\right)\left(1 - \frac{u}{u_{IIa}}\right) d\eta}{\Lambda - C_{IIa}^2 \left(\frac{u}{u_{IIa}}\right)^2} \quad . \tag{K8}$$

The total rate of energy transmitted to the wake now is

$$\Omega = \Omega_e + \Omega_d$$

$$= \frac{x}{\sigma} \rho_{IIa}\, u_{IIa}\, C_p\, T_{oa} (1 - C_{IIa}^2) \left[\int_{\eta_j}^{\infty} \frac{\left(\frac{u}{u_{IIa}}\right)(1-\Lambda)\, d\eta}{\Lambda - C_{IIa}^2 \left(\frac{u}{u_{IIa}}\right)^2} + \int_{\eta_j}^{\eta_d} \frac{\Lambda \left(\frac{u}{u_{IIa}}\right) d\eta}{\Lambda - C_{IIa}^2 \left(\frac{u}{u_{IIa}}\right)^2} \right] \tag{K9}$$

From empirical information, Korst and Tripp (Reference 8) express the jet-spreading parameter, σ, with the linear equation

$$\sigma = 12 + 2.758\, M_{IIa} \tag{K10}$$

The equations (K5) through (K10) cover a broad range of applications, namely,

1. Mass-bleeding effect.

2. Heat-transfer effect (i.e., heat addition).

3. Two-jet interference.

4. Any combination of the above three items.

The Korst theory is the most versatile one. It agrees well with experimental measurements so long as it is within its limitations.

The theory is limited by:

a. It is for two-dimensional, turbulent wake.

b. The approaching boundary layer must be thin, but not zero. The layer must be small and have a finite value; say, the lower limit of ratio of approaching-boundary-layer momentum thickness to base height is approximately 0.01; the upper limit is defined as "thin".

c. The velocity profile in the mixing layer is a fully developed, turbulent-flow profile.

When Korst et al elected to restrict their theory, their intention was to eliminate consideration of the approaching-boundary-layer effect. Both Nash (Reference 9) and Cooke (Reference 10) have applied Korst's theory to the condition of zero approaching-boundary-layer thickness where the approaching-boundary-layer effect truly does not exist. They found that the base pressure from Korst's theory is much over-predicted. It is concluded that some incorrectness exists in the theory.

By solving their equations at $\eta_p = 0$, Korst et al inevitably define

$$\psi = \infty$$

that is,

$$\frac{x}{\delta_{II}} = \infty$$

This means that either $x = \infty$ or $\delta_{II} = 0$. (x and δ_{II} are defined in Figure 2). In reality, it is not possible for $x = \infty$ so δ_{II} must be zero. Hence, the approaching-boundary-layer thickness must be zero. This is to say that actually Korst's theory is limited to the condition of the zero approaching-boundary-layer thickness instead of the thin approaching boundary layer.

The assumption that in the recompression region the total pressure is equal to the static pressure behind the shock, if taken literally, would indicate that the point of reattachment is at the peak of the pressure rise. This contradicts the observed phenomena in which reattachment has occurred before the recompression pressure reaches its maximum.

The error of this assumption together with the over-estimated amount of base pressure at the zero thickness tends to compensate for the necessary increase of base pressure to account for the thin-boundary-layer effect. Consequently, Korst's theory agrees well with experiments in which the approaching-boundary-layer is thin, but does not agree with experiments in which the approaching thickness is zero.

Nash's Theory (Reference 9)

Using a similar flow model (Figure 3) as Chapman, Nash contributes further improvement on base pressure theory. Not only does he include the appropriate treatment of the reattachment, but he also considers the effect of approaching boundary layer. He, too, adopts Chapman's "Dividing Streamline" concept.

He assumes that

1. The flow of the free stream expanding around the corner of separation, S, follows Prandtl-Meyer expansion.
2. Static pressure in the separated flow region is equal to the static pressure of the adjacent free stream after the expansion and before the recompression; i.e., $P_j = P_{IIa}$, $P_b = P_{IIa}$.
3. The pressure along the separated flow is constant.
4. In the dead-fluid region, the circulating motion is negligibly slow, so that the pressure is essentially constant.
5. The Prandtl number is unity.
6. The compression along the reattachment streamline, d, in the recompression region is isentropic.

For supersonic turbulent mixing, Nash assumes the velocity profile lying between the values given by the error function and the linear profile.

$$\left(\frac{u_{II}}{u_{IIa}}\right)_c = (0.348 + 0.018\, M_{IIa})^{\frac{1}{2}} \qquad \text{(N1)}$$

However, he tentatively accepts the jet-spreading parameter, σ, of Korst and Tripp. He then shifts the origin of boundary layer with the argument that the real mixing layer developing from the approaching boundary layer can be replaced by an equivalent asymptotic mixing layer growing over a greater distance from zero thickness. This is shown on Figure 4. The distance between the origin of this equivalent asymptotic mixing layer and the corner of separation is assumed to be proportional to the boundary-layer momentum thickness after separation. With this transformation of the origin of mixing layer, he takes into account the influence of approaching boundary layer on the separated flow.

The treatment of reattachment is based on the fact that the reattachment point lies before the maximum value of pressure rise, so the total pressure on the reattachment streamline, d, in Figure 3 is equal to the static pressure, P_r at the reattachment point, R , instead of equal to the static pressure behind the trailing shock. The pressure P_r can be related to the pressure at the separation corner and the base pressure by

$$\frac{P_r - P_b}{P_I - P_b} = 0.35 \qquad \text{(N2)}$$

The value of 0.35 is the rough mean of the results of experimental measurements.

With the resolving of turbulent mixing and reattachments point, the Nash's theory can be expressed by

$$\lambda_b = \frac{1 + \frac{\gamma-1}{2} M_{IIa}^2}{1 + \frac{\gamma-1}{2} M_{IIa}^2 \left[1 - \left(\frac{u_{II}}{u_{IIa}} \right)_c^2 \right]} \qquad \text{(N3)}$$

$$q = \frac{\rho_{IIa}\, u_{IIa} \sqrt{\pi}\, t \left\{ \ell n\, \lambda_b - \ell n \left(\frac{P_r}{P_b} \right)^{\frac{\gamma-1}{\gamma}} \right\}}{(\gamma-1)\, \sigma\, M_{IIa}^2 \sin\left(\nu_{IIa} - \nu_{Ia}\right)} - \frac{\rho_{Ia}\, u_{Ia}\, \vartheta\, M_{Ia}^2\, \ell n \left(\frac{P_r}{P_b} \right)^{\frac{\gamma-1}{\gamma}}}{M_{IIa}^2\, \ell n\, \lambda_b} \qquad \text{(N4)}$$

$$\frac{P_b}{P_I} = \left[\frac{1 + \frac{\gamma-1}{2} M_{Ia}^2}{1 + \frac{\gamma-1}{2} M_{IIa}^2} \right]^{\frac{\gamma}{\gamma-1}} \qquad \text{(N5)}$$

The equations shown here are for supersonic flow. A different set has also been developed for subsonic flow, but is not shown here since it does not apply to the requirement stated in the introduction.

The base pressure prediction given by this theory agrees well with experimental measurements. Its application, however, is limited by the facts that

a. The theory is for two-dimensional turbulent wake.

b. The ratio of approaching-boundary-layer thickness to base height must be small or zero.

4. There is no heat transfer effect; that is, no heat addition.

There is a possibility that Nash's theory may be used to predict the base pressure of bodies of revolution because the basic theory of Nash's is written in terms of the stream function, which can be expressed in either cylindrical coordinates or Cartesian coordinates. However, some changes of the basic theory inevitably must be made because it is very doubtful that assumptions (2) and (3) are applicable to bodies of revolution. The observed phenomena have already shown that the pressure along the separated flow behind bodies of revolution is similar to the one experienced on the surface of a conical afterbody (Reference 11). This means that there is a pressure gradient along the mixing layer in the case of bodies of revolution and the assumptions (2) and (3) therefore are not valid. The nullification of the assumptions (2) and (3) will somewhat effect the basic theory.

Since the value 0.35 of recompression parameter $\frac{p_r - p_b}{p_I - p_b}$ is empirical and mainly two-dimensional, the validity of using it on the body of revolution becomes questionable. It may need to be replaced by some other numerical values which have been obtained either empirically or theoretically.

Whether the velocity profile (Equation N1) of Nash's mixing theory and the jet-spreading parameter (Equation K10) can be applied to the prediction of base pressure of bodies of revolution also remains to be seen. Recent findings (Reference 12) in regard to the jet-spreading parameter suggest that a different expression may be in order.

Zumwalt's Method (References 12 and 13)

G. W. Zumwalt, one of Korst's associates, attempts to solve the base pressure problem of a cylinder by superimposing Korst's theory, the restricted version (i.e., $\eta_p = 0$), on a conical flow model. The basic mathematical derivation is presented in his Ph. D. thesis. This concept was then further developed by Zumwalt and Tang to include mass-bleed in the base region and change of base pressure with respect to time.

In this method, the following assumptions are used:

1. There is a Prandtl-Meyer expansion at the corner of separation.

2. The approaching-boundary-layer thickness before separation is negligible.

3. Instead of constant pressure and velocity-mixing, pressure and velocity gradient along the mixing layer is the same as that along a conical surface.

4. There is no pressure gradient normal to the mixing layer.

5. The pressure rise in the recompression region is caused by a plane shock in the adjacent free stream.

6. The compression angle, θ_{IIIa}, is equal to the expansion angle, θ_{IIa}, as shown on Figure 5.

In the recompression region, the total pressure along the discriminating streamline, d, is equal to the static pressure behind the trailing shock and far downstream in the wake.

8. X = 0 is at the corner of separation.

To reduce the great amount of trial and error required to obtain a proper combination of various parameters used in this method, Zumwalt and Tang, present a series of graphs of solution of their main equations. While these curves ease the tedious calculation, they also introduce the possibility of significant error caused by interpolation.

Differing from Korst's Theory, this method contains an improved jet-spreading parameter. Instead of the linear equations,

$$\sigma = 12 + 2.758\ M$$

suggested by Korst and Tripp, which has been shown to be good only for the flow up to and about M = 1.6, Zumwalt and Tang use

$$\sigma = 47.1\ C^2 \qquad \text{(Z1)}$$

based on good agreement with experimental data.

For the case of no base bleed, the mathematical equations actually describe only the external flow past a cylinder mounted on a sting. The lower limit is bounded by a sting-diameter-to-base-diameter ratio equal to 0.25. Hence, a blunt-base cylinder (no sting) is beyond the limitation of this method. However, because of the supposedly nearly constant base-pressure ratio in the region of small sting-diameter-to-base-diameter ratio, Zumwalt and Tang extrapolate the base pressure ratio to "no sting" condition, where the diameter ratio equals zero. They then replot this base pressure ratio as a function of Mach number as shown in Figure 6, and express the faired curve mathematically as

$$\frac{P_b}{P_1} = 0.906 - \ln \sqrt{M_1} \qquad \text{(Z2)}$$

for $1.2 \leq M_1 \leq 5.0$. This curve agrees relatively well with experimental measurements as shown in References 12, 13 and 14.

In Reference 14, some incorrect information is presented which should be discussed before results between theory and experiment are compared. In Reference 14 it is stated that the base pressure attains a minimum value when transition is at the wake throat. Following the reasoning of Crocco and Lees (Reference 15), the minimum base pressure occurs when the transition is ahead of the wake throat and not at the wake throat. The variation of base pressure with respect to the location of transition from laminar flow to turbulent flow on a model of fixed length is shown schematically in Figure 7. When the transition locates at E, the flow on the body and in the separated region is laminar, and is at low Reynolds number. When the transition with its accompanying turbulent flow moves into the separated flow region, the base pressure drops rapidly at the beginning, because the initial effect of turbulent flow on the mixing is more predominant than the increasing Reynolds number, which would have the opposite effect. Base pressure decreases more gradually than does the initial steep drop, as the transition moves forward to the base, and as Reynolds number increases. When the transition reaches the corner of separation, C, base pressure increases rapidly as Reynolds number increases, because of the increase in thickness of boundary layer at the corner of separation. Base pressure reaches a maximum as the transition moves upstream along the body. With further increase of Reynolds number, the transition moves toward A and the base pressure begins to decrease gradually. This phenomenon is caused by the decrease in thickness of the local turbulent-boundary-layer at the corner of separation with increasing Reynolds number.

In the case of turbulent wake, base pressure is at a minimum, while transition terminates and turbulence begins at the corner of separation. This would nearly satisfy Korst's and now Zumwalt's assumption of zero approaching boundary layer. However, it is incorrectly interpreted (and consequently stated in Reference 14) that Zumwalt's method is restricted to fully developed turbulent approaching boundary layer. This fixes the transition near point A instead of between B and C (Figure 7).

There is no doubt that transition occurs nearer the corner of separation (that is, between B and C in Figure 7) than to the forebody in the experiment of Reid and Hastings (Reference 16). The base pressure of a cylinder measured in this experiment is shown in Figure 8. Here the lowest base pressure ratio (which means that the transition, relatively speaking, is close to the corner of separation) is 0.538 and is less than the predicted 0.552. This is caused by the incorrectness of Korst's assumptions which Zumwalt has inherited in his method. As in Korst's theory, this method is restricted to a thin, but not zero, approaching boundary layer.

In addition to the restriction mentioned above, Zumwalt's method is limited to

1. Cylindrical afterbody with its rotating axis parallel to the flow.

2. Turbulent wake.

3. Fully developed velocity profile in mixing layer.

4. No heat-addition effect.

There is a possibility that the last restriction can be eliminated by some mathematical modification.

Love's Semiempirical Method (Reference 11)

E. S. Love suggested in 1957 that an analogy exists between the peak pressure rise associated with the separation of the boundary layer and the base pressure. Utilizing the method of characteristics and the experimentally compiled angle between free stream direction and the clearly defined outer boundary of the convergent wake, he developed a method of predicting base pressure of bodies of revolution. Love's method is for fully turbulent approaching boundary layer. It is shown in Reference 11 that the prediction from this method agrees well with experimental measurements.

In Figure 9, the base pressure of a cylinder predicted by this method and by Zumwalt' method are compared. The difference between the two predictions may be caused by the effect of approaching boundary layer. Some additional results of experiments with fully turbulent boundary layer are also shown in Figure 9. Generally speaking, they suggest that Love's prediction is good. Although in Reference 11 Love's method is shown to predict the base pressure up to Mach 8.6, the prediction above Mach 5.1 is not quantitatively substantiated by experimental results. Therefore, the applicability of this method above Mach 5.1 remains undetermined.

Comparisons of predicted and experimental results at various boattail angles are presented in Reference 11. As data become available, more comparisons should be made.

Love's method contains no direct provision to account for the effect of heat transfer and mass bleed.

Other Methods

Using Chapman's method as a basis, Denison and Baum at Electro-Optical Systems Inc., Pasadena, California, developed a method to include the initial boundary layer effect in a laminar wake flow (Reference 21). The method applies to wedges and cones and it takes into account the effect of heat transfer. Calculated values of base pressure ratio seem to agree well with experimental measurements. Although this method has been shown to predict base pressure from Mach 2.0 up to Mach 20.0, it has not yet been substantiated quantitatively by experimental results in the hypersonic region. The trend of increasing base pressure with increasing hypersonic Mach numbers, however, agrees with the suggestion of Whitfield and Potter (Reference 22).

Another method of approximating base pressure of cone and boattail (Reference 23) suggests combining the equivalent cylinder method of Chapman as modified by Reid and Hastings (Reference 16) with Zumwalt's method. The flow along the cone (or the boattail) is first turned to the equivalent free stream direction by using the appropriate isentropic compression (of Prandtl-Meyer expansion). This gives an equivalent cylinder value of local Mach number, M_1^* and pressure ratio, P_1^*/P_0. The equivalent cylinder value of base pressure ratio, P_b/P_1^*, can be found from Zumwalt's method by using M_1^*. Finally,

$$\frac{P_b}{P_1} = \frac{\left.\dfrac{P_1^*}{P_0}\right|_{f(M_1^*)}}{\left.\dfrac{P_1}{P_0}\right|_{f(M_1)}} \left(\frac{P_b}{P_1^*}\right)$$

The comparison of calculated results with experimental measurements is presented in Figure 10.

The method certainly is limited by the restrictions imposed on Zumwalt's method. It is further limited to cases in which there is no flow separation along the boattail.

CONCLUSIONS

A general method to predict the base flow environment in supersonic flow has not yet been developed. Much work still remains to be done in three-dimensional flow. Explicitly needed are a better understanding of the interrelationship between the trailing shock and the separated flow in the recompression region, and of the relationship of the pressure at reattachment with the pressure rise. Also necessary is further theoretical development of the effect of boattail upon base flow environment. Presently supersonic flow is being extensively studied. Further investigation in the hypersonic range is highly desirable.

LIST OF SYMBOLS

p	pressure
M	Mach number
γ	ratio of specific heats
u	velocity component in X or x direction
η	dimensionless coordinate
η_p	position parameter
β	dummy variable
C	Crocco number, $\left[1 + \frac{2}{(\gamma-1)M^2}\right]^{-\frac{1}{2}}$
x, y	coordinates in the intrinsic coordinate system
X, Y	coordinates in the reference coordinate system
δ	boundary layer thickness
Λ	stagnation temperature [illegible]
ρ	density
G	mass rate per unit width
σ	jet-spreading parameter
ζ	dimensionless y-coordinate
Ω	total energy rate per [illegible]
Ω_e	net rate of energy transfer [illegible] the outside flow to the dead-fluid region across the jet [illegible] streamline j
Ω_d	rate of energy carried into [illegible] at the downstream location x by mass flowing between [illegible] with local coordinates y_j and y_d
C_p	specific heat, constant pressure
T	temperature
λ	density ratio, $\frac{\rho_c}{\rho_b}$

t	step height
ν	Prandtl-Meyer angle
ϑ	momentum thickness of [illegible] boundary layer
r	radius

Subscripts

b	at base
I	boundary layer in Region I
Ia	adjacent free stream to boundary layer in Region I
j	streamline j
IIa	adjacent free stream to boundary layer in Region II
II	boundary layer in Region II
m	refers to coordinate shift due to momentum integral
d	streamline d
o	stagnation
[illegible]	conditions on median streamline of asymptotic free shear layer
[illegible]	at the attachment point, R

Superscripts

*	equivalent cylinder

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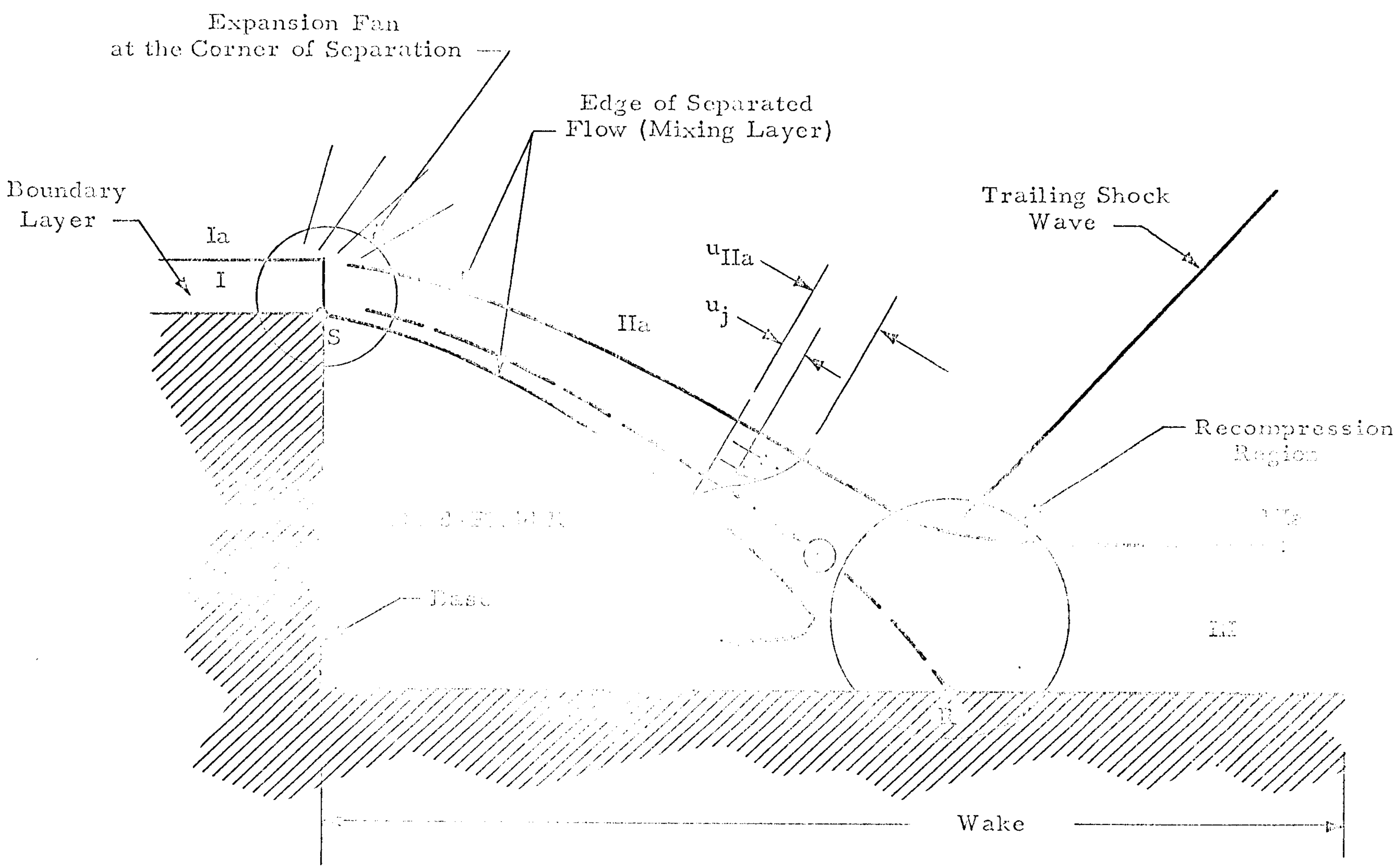
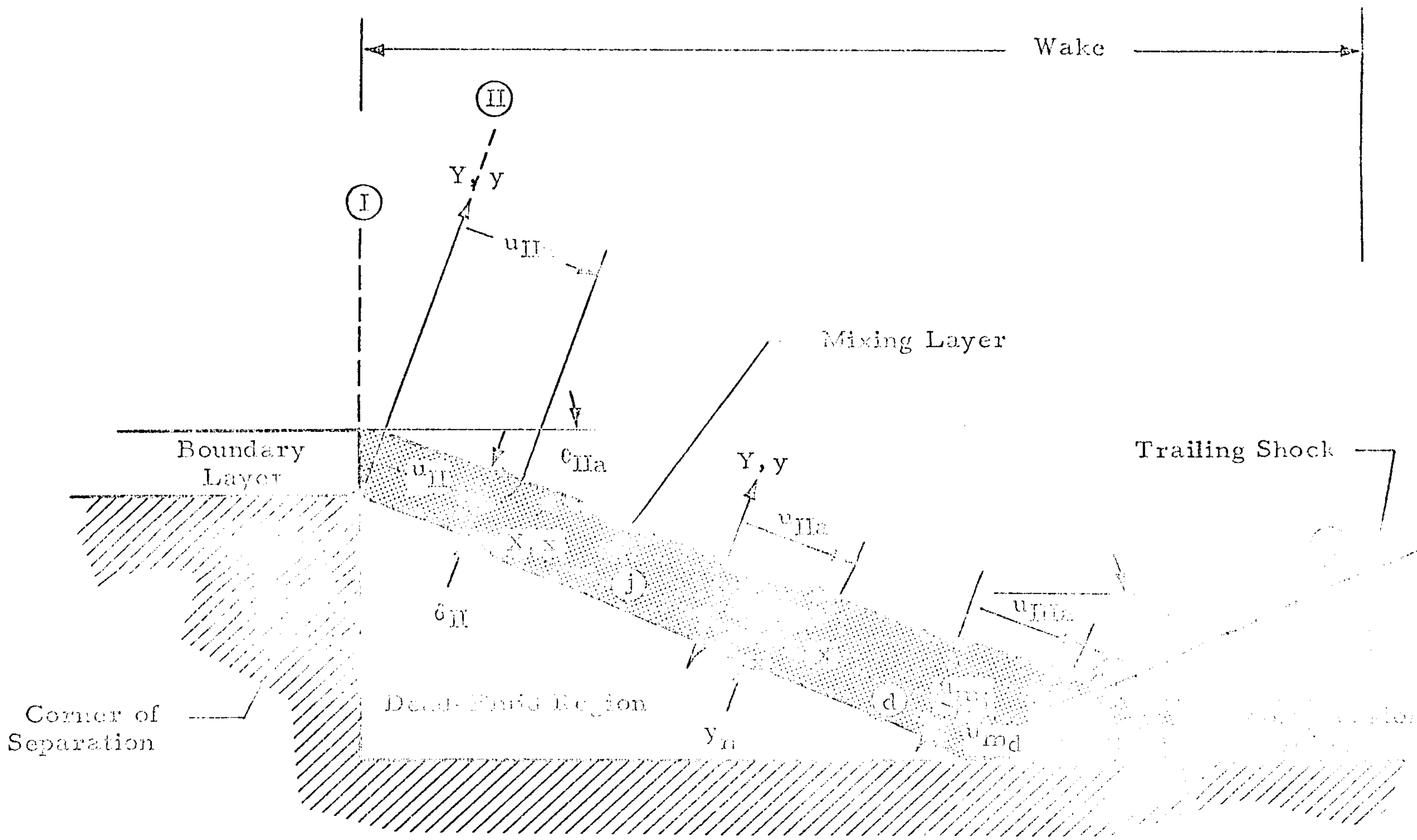


Figure 1 - Chapman's Flow Model



Note: j - The boundary streamline which divides the mass flow coming around the corner of separation from the mass flow entrained by the viscous action of the jet.

d - Recompression streamline or discriminating streamline above which streamlines have higher kinetic energies to penetrate the pressure rise at the recompression region and below which streamlines have lower kinetic energies so are turned back to dead-fluid region.

When there is no secondary mass bleed, streamline j coincides with streamline d.

Figure 2 - Korst's Flow Model

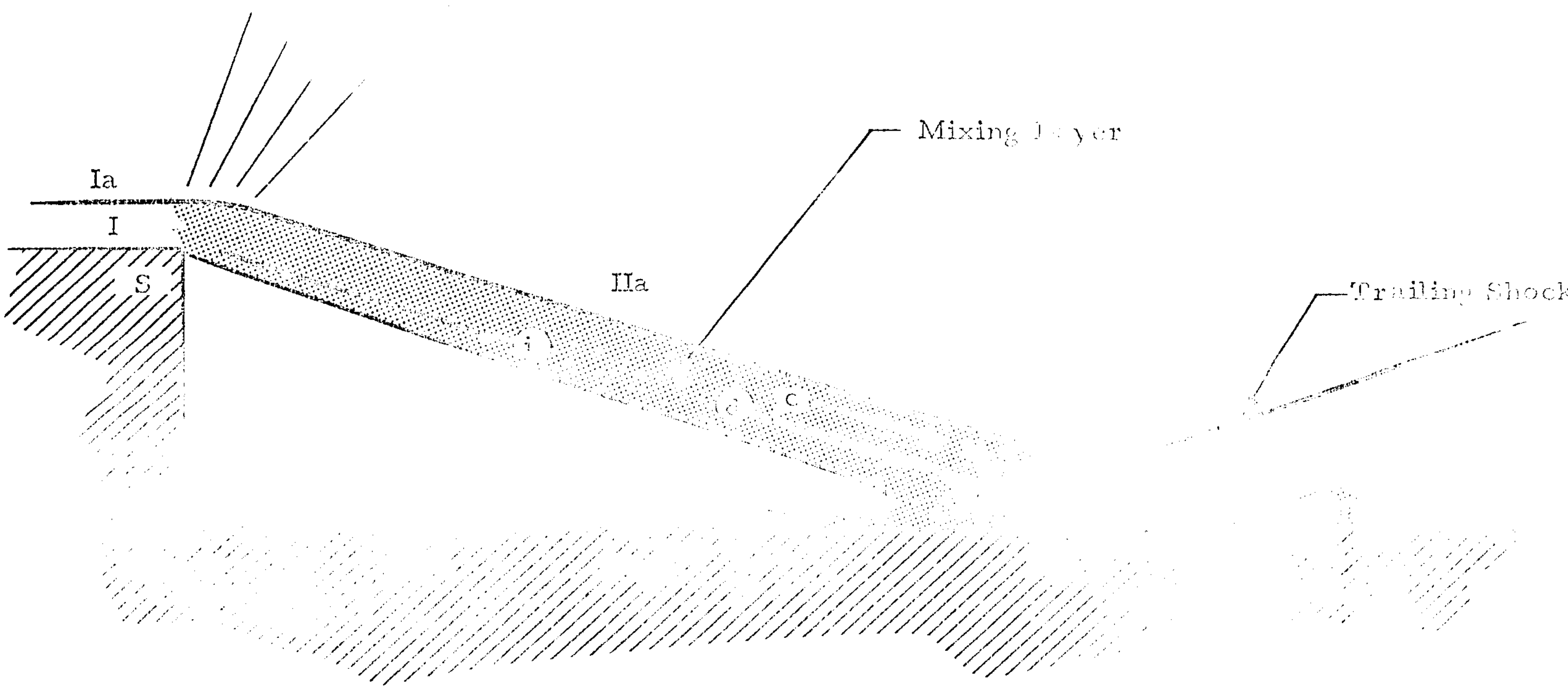


Figure 5 - Nash's Flow Model

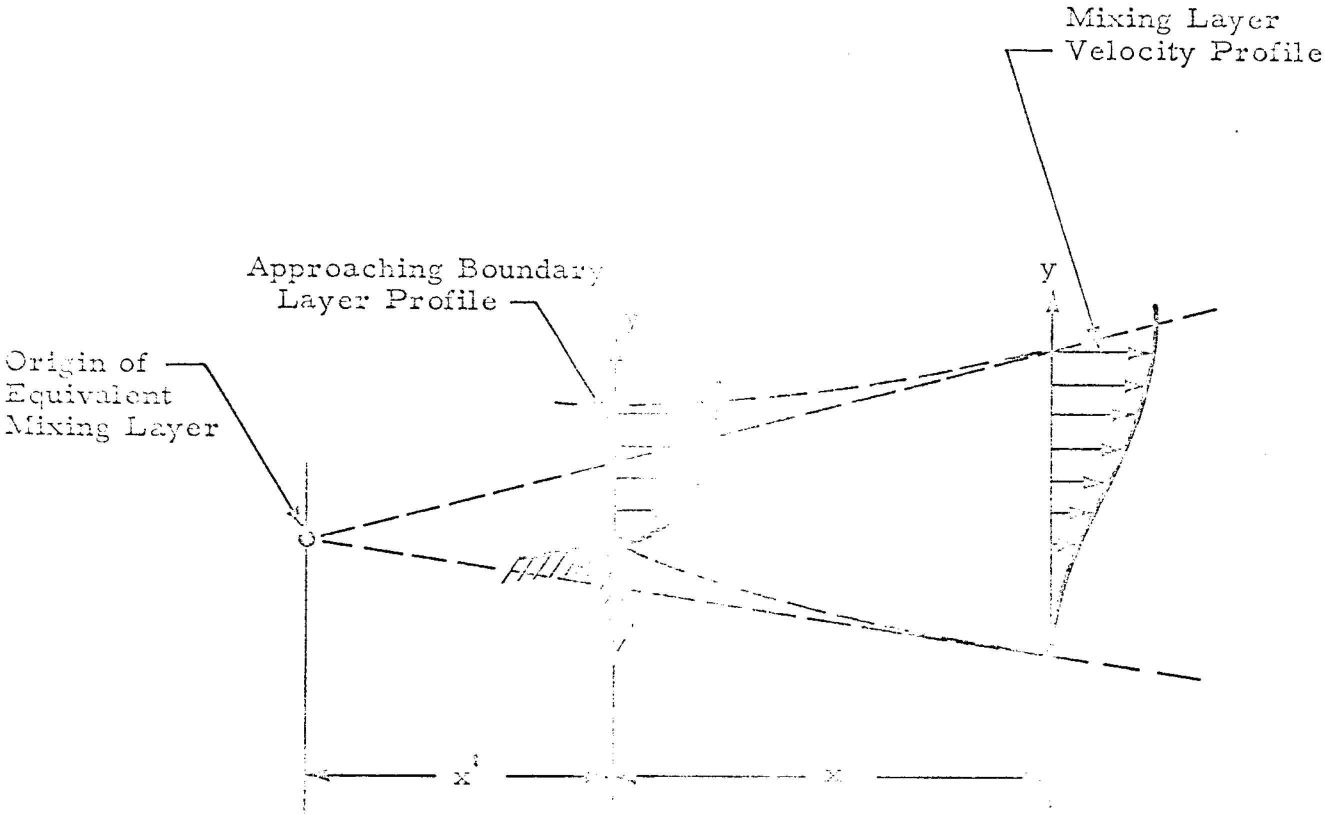


Figure 4 - Transformation of Mixing Layer

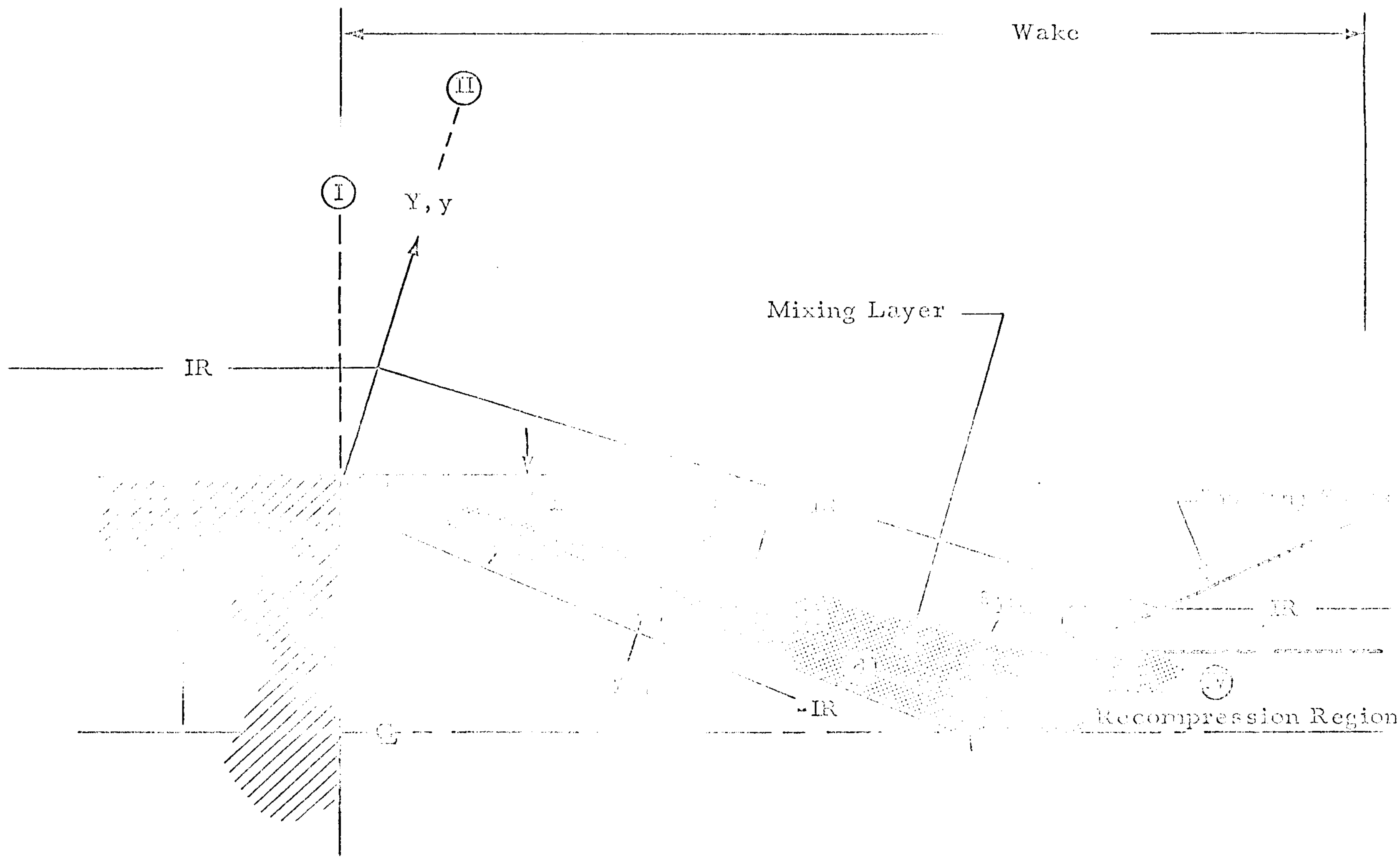


Figure 5 - Zumwalt's Conical Flow Model

+ Base Pressure Ratio Extrapolated to

——— $P_b/P_I = 0.906 - \ln \sqrt{M_I}$

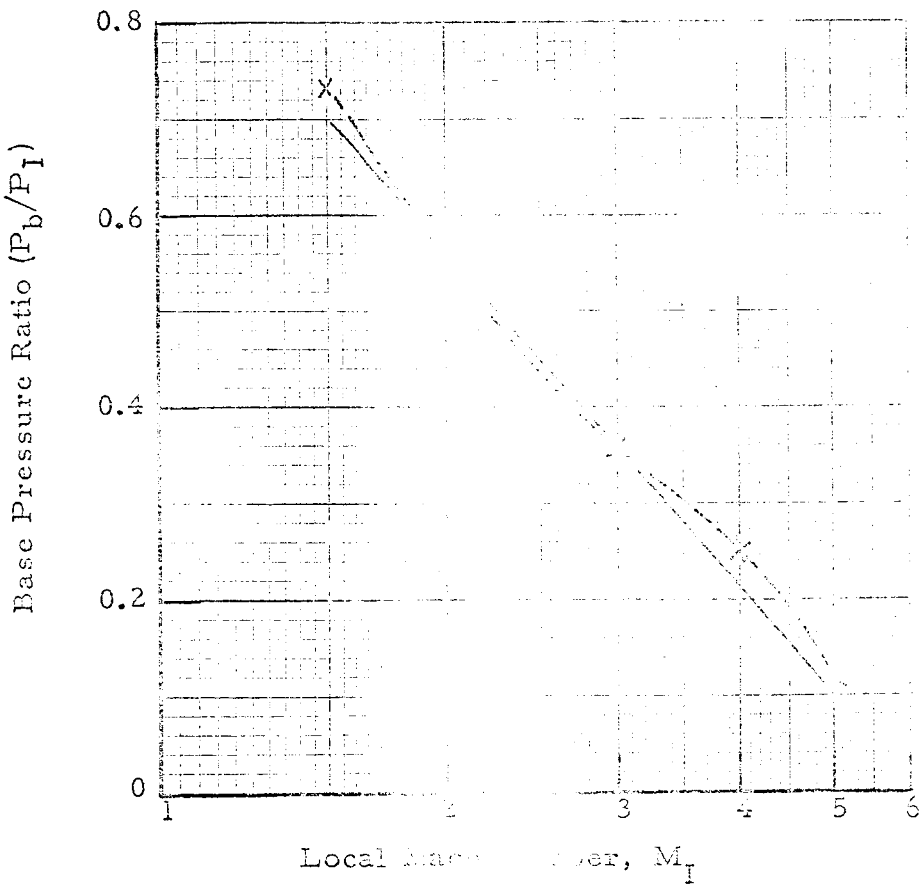


Figure 6 - Zumwalt's Method Base Pressure

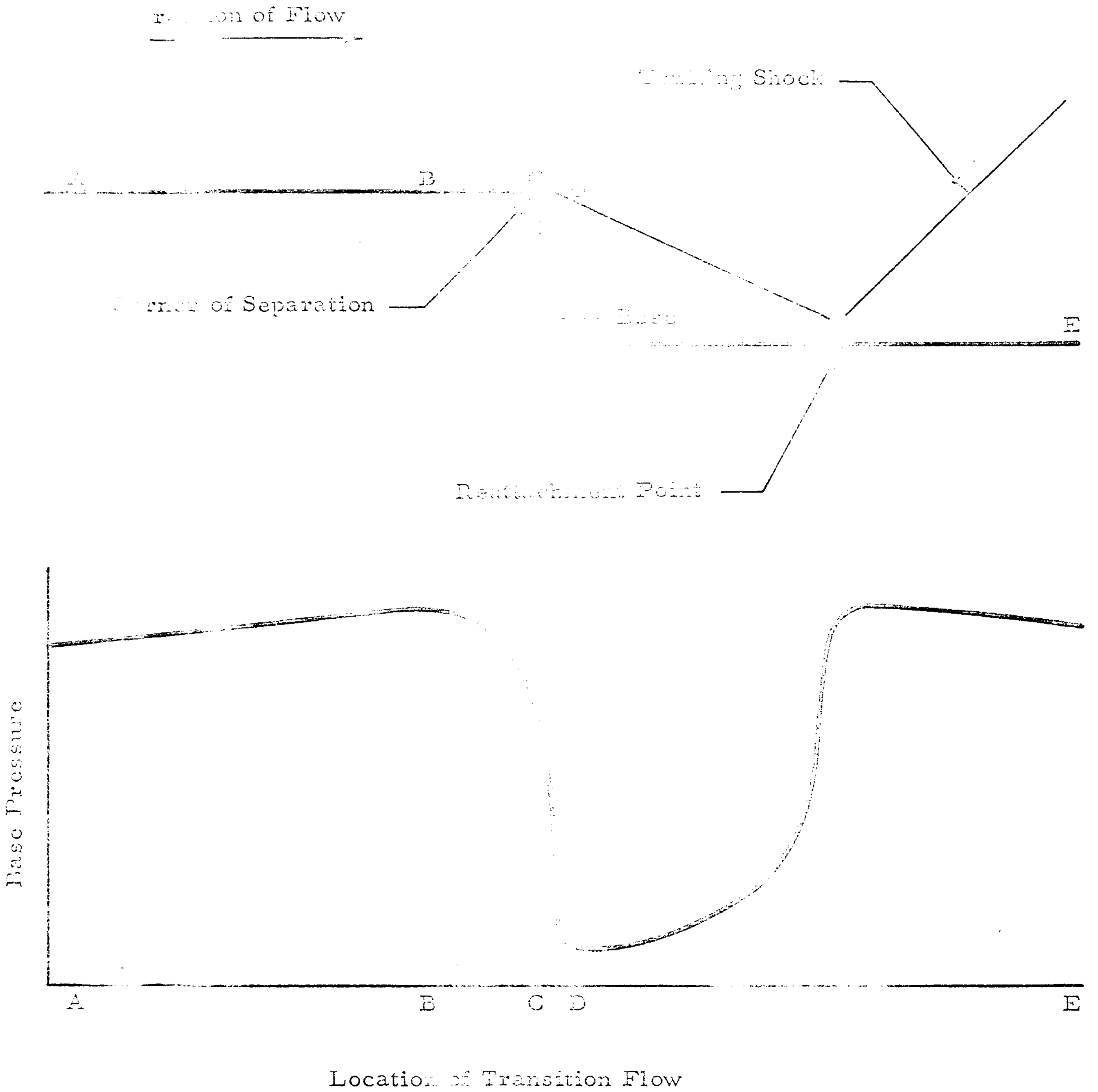


Figure 7 - Base Pressure with Respect to Variation of Flow

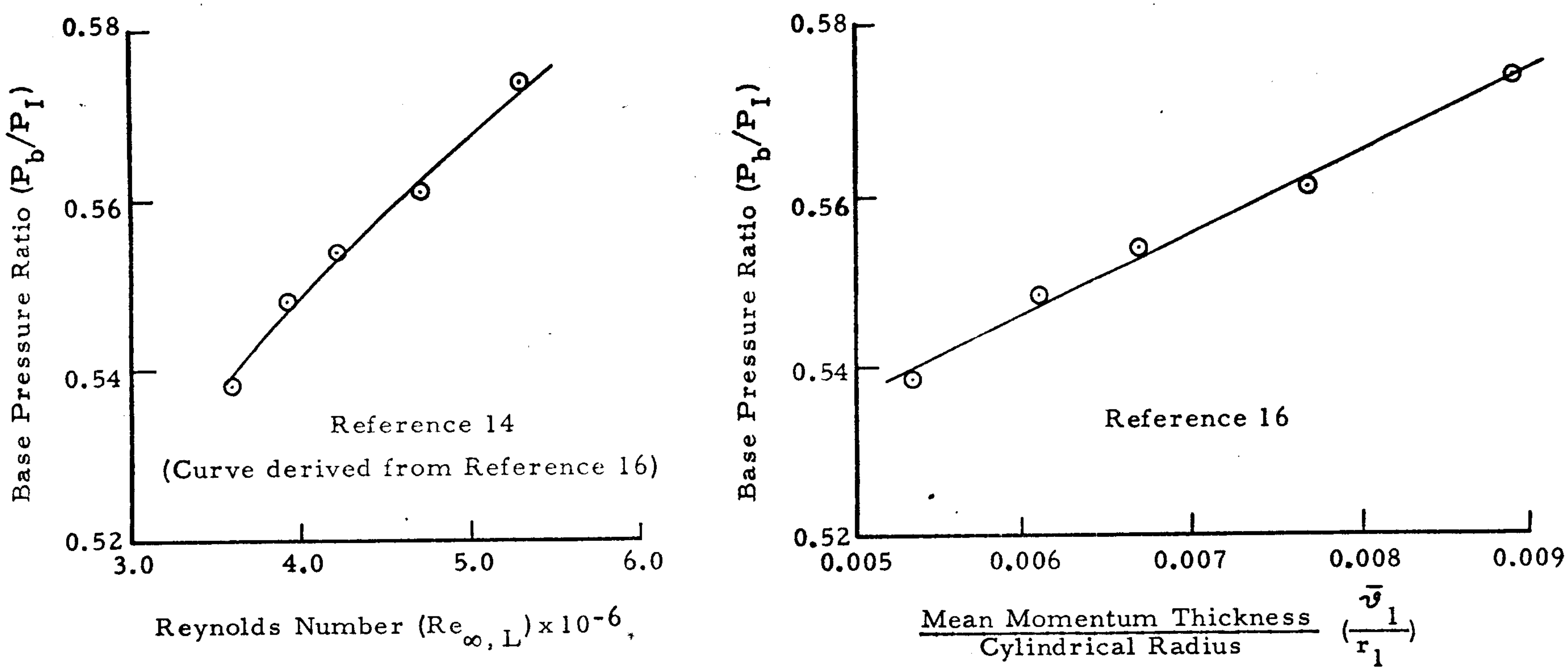


Figure 8 - Base Pressure of a Cylinder (M_1 = 2.03)

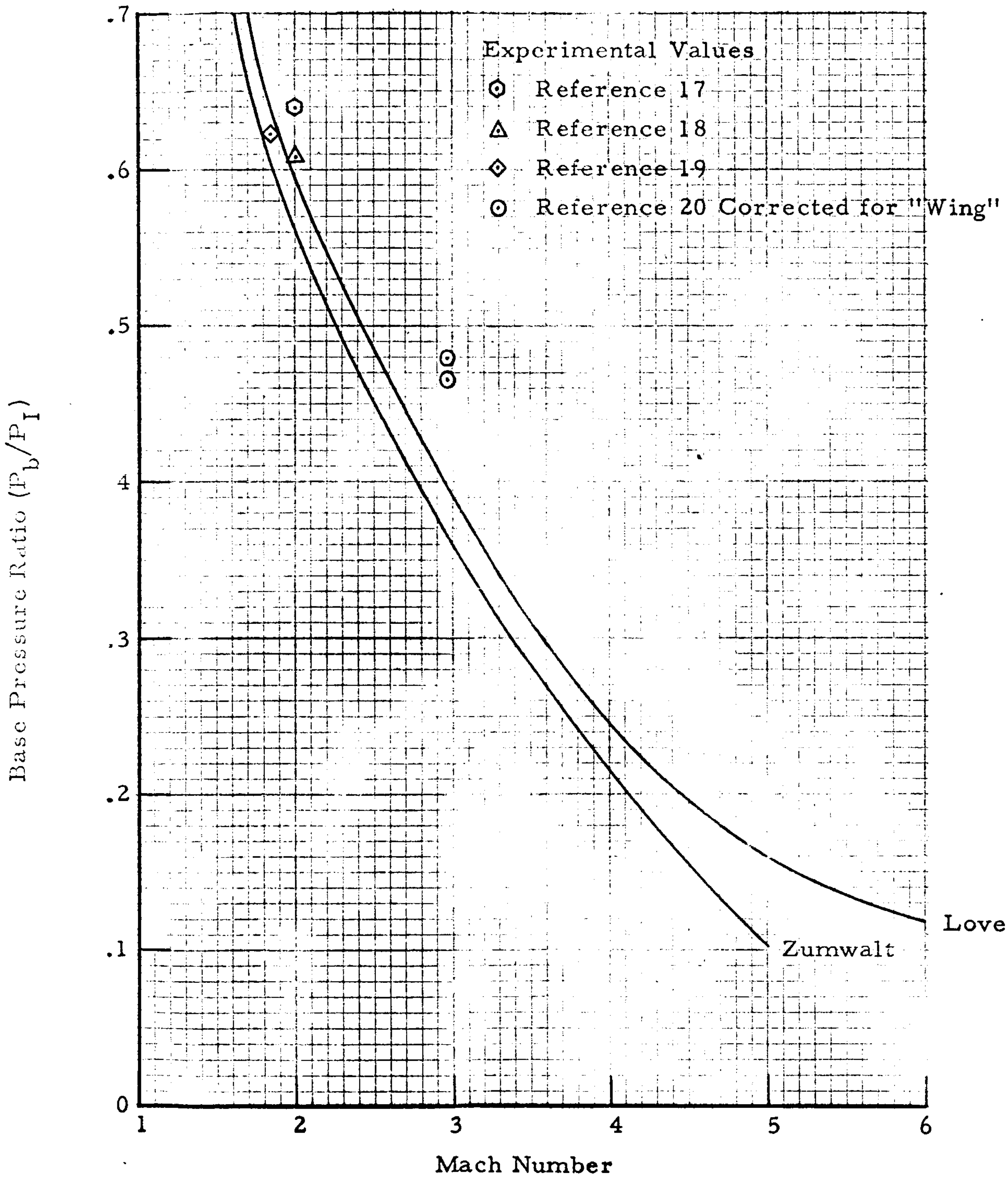


Figure 9 - Base Pressure of a Cylinder

Symbols	Reference	M_∞	β (in order)
——	Zumwalt-Chapman Method (23)		
◇	Reid & Hastings (16)	2.03	0, 5, 7.5, 10, 11, & 14
○	Baughman & Kochendorfer (24)	1.91	3, 5.63, 5.63, 5.63, 5.63, 7, & 11
◐	Baughman & Kochendorfer (24)	3.12	3, 5.63, 5.63, 5.63, 5.63, & 11
⬭	Whitfield & Potter (22)	3.00	-9
⬬	Whitfield & Potter (22)	2.00*	-9
△	Lehnert & Schermerhorn (25)	2.46	-10
□	Love (11)	2.90	-5, -10, & -15

* Extrapolated

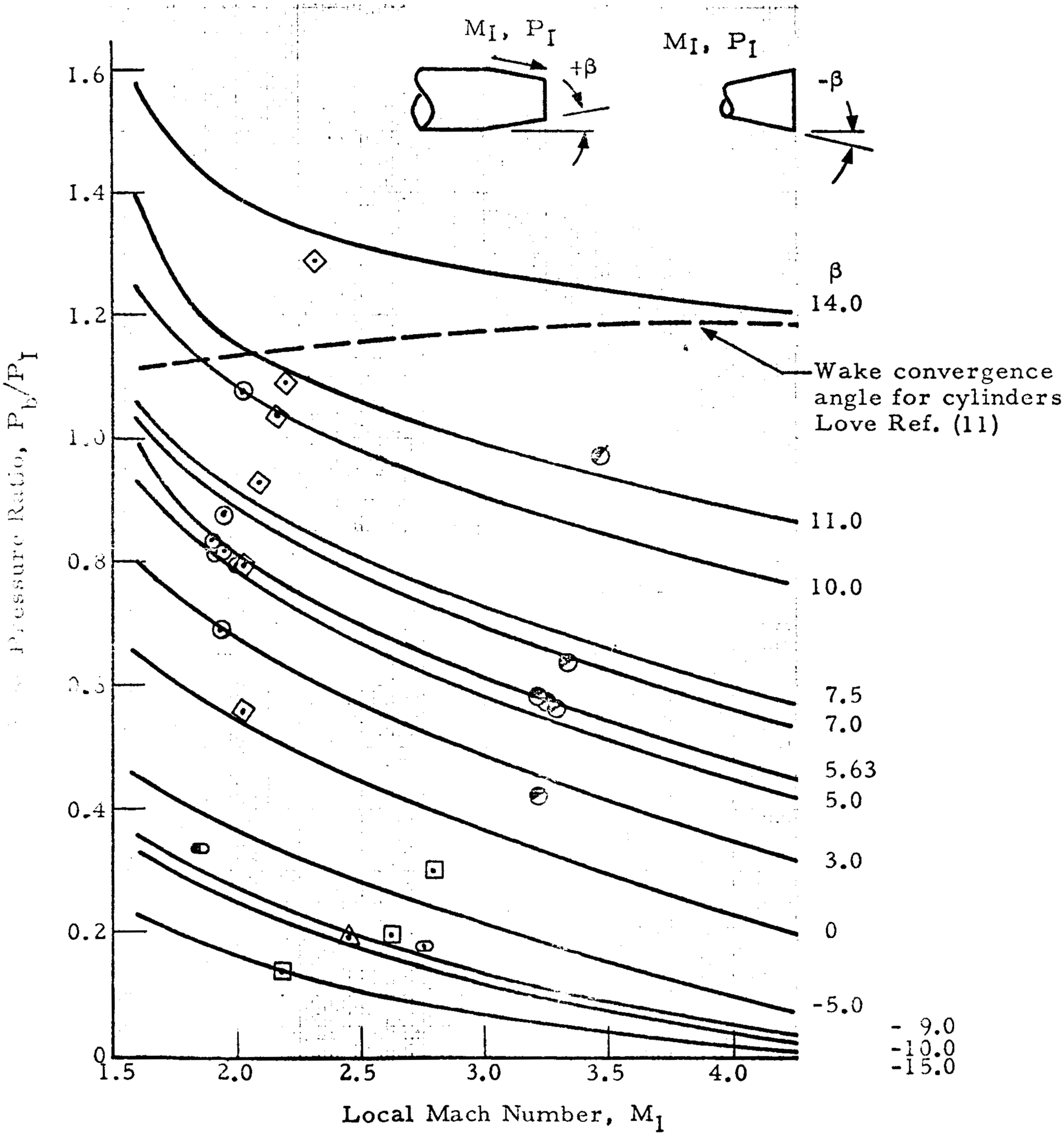


Figure 10 - Comparison of Zumwalt-Chapman Method with Experiment