ON THE DEPOLARIZATION OF DISCRETE RADIO SOURCES BY FARADAY DISPERSION

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SUMMARY

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A study is made of the implications of the recent polarization measurements for the structures of discrete radio sources and the source-observer media. Simple models of wavelength dependent depolarizing mechanisms are investigated and it is found that most are imcompatible with the observations of Gardner and Whiteoak. The models of internal Faraday dispersion predict a lower polarization at 30cm than is observed. It is suggested that the depolarization of the Crab nebula is produced by Faraday rotation in the filamentary shell that surrounds the nebula. Such filaments could also exist in the outer regions of extragalactic sources.

A complex number representation is used for the state of linear polarization and a Faraday dispersion function is defined to describe the distribution of polarized radiation with respect to Faraday depth. The persistence of polarization at 30cm, after partial depolarization between 10cm and 20cm, implies that the radiation is spread over a large range of Faraday depths. The observed linearity of the plot of the angle of polarization against wavelength squared implies that it is justifiable to make an assumption which enables one to calculate the Faraday dispersion function of a source from the dependence of its polarization on wavelength.

Upper limits are given for the possible densities of internal ionised gases in the sources for which we have polarization measurements.

INTRODUCTION

Within the last few years great interest has been shown in polarization measurements of discrete radio sources. Early observations (1, 2, 3, 4) were made to test the theory of Alfven and Herlofsen (5) that the radiation from the sources is due to the synchrotron emission of relativistic electrons spiralling in a magnetic field. The unexpected results of Cooper and Price (6), showing the wavelength dependence of the state of polarization of Centaurus A, revealed the exciting prospect of obtaining from such observations much information about the source and the media through which its radiation passes. Three further surveys (7, 8, 9) are now available and we have polarization measurements of about 30 sources at three or more wavelengths.

The first striking feature of the results to show up is the linearity of the plot of the angle of polarization against the square of the wavelength. The slope of this line for a given source is called the rotation measure of the source. Only three sources (3C-353, Taurus A and Pictor A) show reliable departures from such a law and in all cases these are small. Cooper and Price (6) suggested that this rotation of the plane of polarization is produced by the Faraday effect occurring in the vicinity of our Galaxy. This has been substantiated by the surveys of Gardner and Whiteoak (7) and Seielstad, Morris and Radhakrishnan (8) which showed that there is a correlation between the rotation measure and the galactic latitude of a source.

The other important feature is that the degree of polarization of most sources decreases with increasing wavelength, and in no case is there a significant increase. It was suggested (7) that the depolarization is produced by differential Faraday rotation of different lines of sight through the galaxy. However, in reference 8 it was found that there is no correlation between the rate of depolarization and galactic latitude. It is shown in this paper that it is likely that the depolarization is due to Faraday rotation in the outskirts of the sources themselves.

1. POLARIZATION OF THE RADIATION FROM AN EXTENDED SOURCE

The state of polarization of monochromatic electromagnetic radiation is defined by the four Stokes parameters $\mathcal{I}$, Q, U and V (10). This paper is concerned with linear polarization, which can be represented by P, the complex linear polarization, defined as

$$P = pe^{2i\chi} = \frac{Q + iU}{\mathcal{I}} , \tag{1}$$

where the parameters p and χ are the "degree" and "angle" of polarization. For radiation which covers a range of frequencies we may write

$$\mathcal{I} = \int_0^\infty I(\nu)\, d\nu , \tag{2}$$

and similar expressions for Q, U and V. The functions $\mathcal{I}(\nu)$, $Q(\nu)$, $U(\nu)$ $V(\nu)$, are the Stokes parameters for the radiation at frequency ν.

The complex polarization $P(\nu)$ can be defined by an equation similar to (1). Hence:

$$P = \frac{\int_0^\infty I(\nu)\, P(\nu)\, d\nu}{\int_0^\infty I(\nu)\, d\nu} \quad . \tag{3}$$

The Stokes parameters for the superposed radiation from incoherent beams are the sums of the parameters for the individual beams. The state of polarization of the total radiation arriving at an observer from an extended source may therefore be expressed as:

$$P = \frac{\int_{source} \int_0^\infty I(\nu,\underline{r})\, P(\nu,\underline{r})\, d\nu dV}{\int_{source} \int_0^\infty I(\nu,\underline{r})\, d\nu dV} \quad , \tag{4}$$

where $I(\nu,\underline{r})\,d\nu dV$ is the intensity of the radiation in the frequency range ν to $\nu+d\nu$ coming from a volume dV situated at the point $\underline{r}$, and $P(\nu,\underline{r})$ is its complex polarization. This is the average of $P\nu,\underline{r})$ taken over frequency and space with weighting factor $I(\nu,\underline{r})$.

The flux density and polarization of the radiation from a source depend on the properties of the source and the media through which its radiation passes. We write $\epsilon(\nu,\underline{r})\,dVd\nu$ for the power radiated per unit steradian in ν to $\nu+d\nu$ from volume dV at $\underline{r}$ in the direction of an observer at the origin, and $p(\nu,\underline{r})e^{2i\alpha(\underline{r})}$ for its intrinsic polarization. Assuming that the radiation is due to synchrotron emission it follows that the degree of polarization and the emissivity depend on the energy and pitch angle distribution of the relativistic electrons and on $H_\perp$, the magnetic field strength perpendicular to the line of sight. If the energy spectrum is a power law of index γ and the pitch distribution is isotropic then the emissivity is proportional to $H_{\perp}^{(\gamma+1)/2}$ and the intrinsic degree of polarization is $\frac{3\gamma+3}{3\gamma+7}$ (11). The polarization direction is $\underline{r}_{\wedge}\underline{H}_{\perp}$.

The polarization measurements have been made at high enough frequencies for the absorption of radiation by the media between source and observer to be negligible. If magnetic fields and free electrons exist in these media Faraday rotation will occur, the angle of rotation of the plane of polarization being given by the well known expression

$$f(\nu,\underline{r}) = \Phi(\underline{r})\,\lambda^2 = K\lambda^2\int_0^r n\,\underline{H}.\underline{k}ds \quad \text{radians,} \qquad (5)$$

where λ is the wavelength corresponding to the frequency ν, n and $\underline{H}$ are the density of free electrons and the magnetic field strength in the intervening media, $\underline{k}$ is a unit vector in the direction or $\underline{r}$, $\underline{k}ds$ is an element of path between the observer and $\underline{r}$, and $K = 2.62 \times 10^{-17}$ when c.g.s. units are used throughout. We call $\Phi(\underline{r})$ the Faraday depth of the point $\underline{r}$. Since the observed angle of polarization of the radiation from each part of the source is wavelength dependent so also is the state of polarization of the whole source. If all points of the source are at the same Faraday depth then the only effect is a rotation of the angle of polarization by an amount proportional to λ^2. When there are regions at different Faraday depths the relative angles of polarization vary by an amount proportional to λ^2, this

Faraday dispersion in general giving rise to depolarization which is more effective for larger than for smaller wavelengths. We shall discuss depolarization by Faraday dispersion in more detail in later sections.

The observed polarization is also dependent on the properties of the measuring system. When the apparent state of polarization varies over the source the use of aerials with different polar diagrams could give rise to different weightings in the averaging of polarization. This effect (Beamwidth effect) is quite noticeable in the optical surveys of the Crab nebula (12,13,14). For most radio observations the beamwidth has been much larger than the angular dimensions of the source and so all parts received equal weighting.

A receiver is not monochromatic but amplifies all the radiation with frequencies within a small band about the frequency of observation. If Faraday rotation occurs the apparent angle of polarization, which is frequency dependent, will be smeared out even for the radiation from a single point. Suppose that the bandwidth is $\Delta \nu$ about the frequency ν and that the Faraday rotation at this frequency is f. The degree of polarization is reduced by the factor $\frac{\sin \Delta f}{\Delta f}$ where $\Delta f = 2f \frac{\Delta\nu}{\nu}$ is the spread in angle of polarization across the band. The condition for this effect (Bandwidth effect) to be unimportant is $\Delta f \ll 1$ for all parts of the source. This condition is satisfied for each of the sources so far observed if the Faraday depths of all parts are of the same order as the rotation measure of the whole source.

Assuming that absorption, beamwidth and bandwidth effects are unimportant the expression for the observed polarization at frequency ν is

$$P(\nu) = p(\gamma) \frac{\iint_{source} \varepsilon(\nu, \underline{r}) e^{2i[\alpha(\underline{r}) + f(\nu, \underline{r})]} d\ell d\Omega}{\iint_{source} \varepsilon(\nu, \underline{r}) d\ell d\Omega} . \qquad (6)$$

Apart from Faraday dispersion there is one further process which can produce variations of the observed polarization with frequency.

2. SPECTRAL EFFECT

This effect is due to spatial variations of both the emission spectrum and the state of polarization. In order to estimate its importance we take the case where the source may be considered as consisting of two regions with different spectra and polarizations. Suppose that the polarizations of the two componets are p_1 and p_2 and that the spectra are $k_1 \nu^{-\beta_1}$ and $k_2 e^{-\beta_2}$. The polarization of the whole source is

$$P = \frac{r p_1 + p_2}{r + 1} \qquad (7)$$

where $r = \frac{k_1}{k_2} \nu^{-(\beta_1-\beta_2)}$ is the ratio of the fluxes from the two components. This situation has two main types.

A. The first is the case where the degrees of polarization are the same and the interaction is due to differing angles of polarization. If the degrees of polarization are about p and the the difference in polarization angles is θ then the minimum degree of polarization $p \cos \theta$, occurs when $r \approx 1$. In passing from small to large wavelengths the intrinsic angle of polarization is rotated by an angle θ, implying quite a significant departure from a λ^2 law of rotation. In general it seems unlikely that such deviations exist, though there are several sources for which this possibility cannot be ruled out.

The observed fact that the degree of polarization always decreases with λ increasing implies that it is always the component which is less intense at the frequencies of polarization observations which has the steeper spectrum. If the wavelength is increased still further until $r > 1$ the degree of polarization will rise again — this has not been observed. Fitting this model to the data we find that for most sources the required difference in spectral indices is greater than 0.2 and that $r \approx 1$ at a wavelength in the range 30cm — 60cm. The superposition of two such spectra produces a concave spectrum at radio wavelengths — this has not been observed (15).

B. The second case is where the polarization angles are about the same but the two regions have considerably different degrees of polarization. This is perhaps quite feasible in a source where relativistic particles are being continually produced in the central regions and energy loss mechanisms are important enough to modify their spectrum by the time they have diffused to the outer regions. Fitting this model to the data and taking care the total spectrum is not concave at radio wavelengths we find that for most sources the required difference in spectral indices is between 0.6 and 2.5. As it is the less intense component that has the flatter spectrum, in most cases its spectral index must be negative. Although this possibility cannot be ruled out it would seem that the required modifications of the spectra by energy losses are excessive.

3. RANDOM FLUCTUATIONS OF THE MAGNETIC FIELD

Before studying Faraday dispersion in greater detail we should consider the effect of fluctuations of the magnetic field within a source on the intrinsic polarization of the whole source. It has been noted for some time that the energy associated with an extragalactic radio source is so large as to indicate that some catastrophic event involving a whole galaxy or perhaps a super-star has taken place. It is to be expected then that the non-relativistic gas will be in a state of turbulent motion; this motion perhaps amplifying the magnetic field and/or accelerating the relativistic particles. The turbulent motion of the gas in which the field lines are frozen (assuming infinite conductivity) will produce random fluctuations of the magnetic field about any overall structure.

We assume that the magnetic field consists of two components, one uniform (H_x, H_y, H_z) and the other an isotropic random field which we represent by a Gaussian of variance $\frac{3}{2} H_r^2$. The probability that the total field at point lies in the range $\underline{H}$ to $\underline{H} + d\underline{H}$ is:

$$\text{Prob}\,(h_x, h_y, h_z) = \pi^{-3/2} \exp\left\{ -[(h_x - h_x^o)^2 + (h_y - h_y^o)^2 + (h_z - h_z^o)^2]\right\}. \tag{8}$$

where $h_x = H_x/H_r$, etc. The complex polarization of the radiation from such a point is:

$$P(h_x, h_y, h_z) = -p(\gamma)\,\frac{h_x^2 - h_y^2 + 2h_x h_y i}{h_x^2 + h_y^2} \;. \tag{9}$$

We assume that the emissivity at such a point is proportional to $(H_x^2 + H_y^2)^{(a+1)/2}$ where a is a constant (the emission spectral index). If the scale of the random component is much less than the dimensions of the source then the expression for the intrinsic polarization of the whole source reduces to:

$$P = \frac{a+3}{4}\, p(\gamma)\, h_o^2 \,\frac{{}_1F_1\left(\frac{a+5}{2}, 3, h_o^2\right)}{{}_1F_1\left(\frac{a+3}{2}, 1, h_o^2\right)}\, e^{(2\beta+\pi) i} \;, \tag{10}$$

where

$$\left.\begin{aligned} h_x^o &= h_o \cos\beta \\ h_y^o &= h_o \sin\beta \end{aligned}\right\} \quad , \tag{11}$$

and the functions ${}_1F_1(\cdot, \cdot, \cdot)$ are modified hyergeometric functions.

Equation (10) shows that the polarization is reduced from $p(\gamma)$ by a factor which depends on the two parameters h_o^2 and a. This factor is plotted in Figure 1 against $\log_{10}(h_o^2)$ with a as a parameter. When the density and spectrum of the relativistic electrons are independent of the magnetic field then $a = \frac{\gamma-1}{2}$ is the radio spectral index which for most sources lies in the range 0.7±0.2 (15). It is accurate enough to take $a \approx 1$ obtaining the simple result

$$p(h_o^2) = p(\gamma) \frac{h_o^2}{h_o^2 + 1} \quad . \tag{12}$$

The parameter h_o^2 is a measure of the ratio of the energies in the uniform and random fields. The average energy density in the random field is $\frac{3}{16\pi} H_r^2$. Hence if we assume that $H_x \approx H_y \approx H_z$ then

$$h_o^2 = \frac{\text{Energy in uniform field}}{\text{Energy in random field}} \quad . \tag{13}$$

4. FARADAY DISPERSION - GENERAL REMARKS

Before considering some simple models we shall make a few remarks on the Faraday effect in general. It is quite obvious from the observed linear dependence of χ on λ^2 that Faraday rotation is occuring and it is natural to ask whether the depolarization is due to Faraday dispersion. Clearly there must be some restrictions on the nature of the dispersion if it is not to disturb the λ^2 law of rotation.

We can simplify equation (6) by superposing all the radiation from the same Faraday depth. Let $E(\Phi)d\Phi$ be the fraction of the total radiation with Faraday depth between Φ and $\Phi+d\Phi$ and let its intrinsic polarization be $P(\Phi) = p(\Phi)e^{2i\chi(\Phi)}$. The observed polarization at wavelength λ is then

$$P(\lambda^2) = \int_{-\infty}^{\infty} E(\Phi)\ P(\Phi)\ e^{2i\Phi\lambda^2}\ d\Phi \quad , \tag{14}$$

which is the Fourier transform of $E(\Phi)\ P(\Phi)$. We call $F(\Phi) = E(\Phi)P(\Phi)$ the Faraday dispersion function of the source. It would be very convenient to be able to invert this transform and so obtain the relation

$$F(\Phi) = \pi^{-1} \int_{-\infty}^{\infty} P(\lambda^2)e^{-2i\Phi\lambda^2}\ d(\lambda^2) \ , \tag{15}$$

but unfortunately to evaluate this integral requires a knowledge of $P(\lambda^2)$ for $\lambda^2 < 0$, and this is not an observable quantity. It is seen from equation (14) that $P(-\lambda^2)$ is the polarization that we would observe at wavelength λ if all of the Faraday rotation were in the opposite sense (i.e. if all the magnetic fields were in the opposite sense).

To make use of equation (15) we must assume some property of the source which will enable us to predict the behavior of $P(-\lambda^2)$ from that of $P(\lambda^2)$. The simplest assumption to make is that $\alpha(\Phi)$ is the same for all Faraday depths, that is,

$$\alpha(\Phi) = \text{const.} \tag{16}$$

This does not require all points of the source to have the same intrinsic angle of polarization, but only that the superposition of all the radiation with the same Faraday depth has a polarization direction that is independent of the Faraday depth. Examples of situations where this assumption is valid are:

(i) The direction of the magnetic field is the same for all parts of the source.

(ii) The field of the source has random variations about a mean direction.

(iii) The rotation is external, the Faraday depths of different lines of sight varying at random.

Assuming the condition (16) and choosing coordinates such that $\alpha(\Phi) = 0$ we see from equation (14) that $P(-\lambda^2)$ is the complex conjugate of $P(\lambda^2)$ and equation (15) becomes

$$F(\Phi) = 2\pi^{-1} \int_0^{\infty} \text{Real} \; [P(\lambda^2) e^{-2i\Phi\lambda^2}] \; d(\lambda^2) \quad . \tag{17}$$

As has already been mentioned the observed plot of $\chi(\lambda^2)$ against λ^2 is normally a straight line and we can write

$$P(\lambda^2) = r(\lambda^2)e^{2iA\lambda^2} , \tag{18}$$

where $r(\lambda^2)$ and A are real. Equation (15) then yields

$$F(\Phi) = 2\pi^{-1}\int_0^\infty r(\lambda^2)\cos\left\{2(\Phi-A)\lambda^2\right\} d(\lambda^2) , \tag{19}$$

from which it is seen that $F(\Phi)$ is symetrical about $\Phi = A$.

There is another way of approaching the problem of deducing $P(-\lambda^2)$ from the observations. We have already pointed out that it is the polarization we would observe at wavelength λ if all the magnetic fields were in the opposite sense. The source would be dynamically similar if it contained such fields which are, in fact, those which would have been produced if the primary field had been in the opposite sense. It does not seem likely that the sense of the primary field could be in any way related to the occurrence of the phenomena leading to the production of the radio source. Assuming that there is no such relation we can deduce that, as $\alpha(\lambda^2)$ is proportional to λ^2 for $\lambda^2 > 0$, it is also proportional to λ^2 for $\lambda^2 < 0$. We can therefore extend assumption (16) to hold for both positive and negative λ^2.

We split $F(\Phi)$ into two parts as follows

$$F(\Phi) = F_s(\Phi) + F_p(\Phi) , \tag{20}$$

where

$$F_s(A-\Phi) = F^*(\Phi-A) \quad ,$$

$$F_p(\Phi) = 0 \quad \text{for all } \Phi < A. \tag{21}$$

Changing the variable from Φ to ψ, where $\Phi = A+\psi$, we get that for all λ^2

$$r(\lambda^2)e^{2iA\lambda^2} = \left[\int_0^\infty \left\{ F_s(\psi)e^{2i\psi\lambda^2} + F_s^*(\psi)e^{-2i\psi\lambda^2} \right\} d\psi \right] e^{2iA\lambda^2}$$
$$+ \left[\int_0^\infty F_p(\psi)e^{2i\psi\lambda^2}\, d\psi \right] e^{2iA\lambda^2}. \tag{22}$$

The part of the second term of the right hand side in square brackets must be real for both positive and negative λ^2. It follows that

$$\left.\begin{aligned} \int_0^\infty \text{Real}\left[F_p(\psi)\right] \sin 2\psi\lambda^2 \, d\psi &= 0 \\ \int_0^\infty \text{Imag}\left[F_p(\psi)\right] \cos 2\psi\lambda^2 \, d\psi &= 0 \end{aligned}\right\} \tag{23}$$

implying that $F_p(\psi) = 0$. In other words

$$F(\Phi - A) = F^*(A-\Phi). \tag{24}$$

The value of $|F(\Phi)|\, d\Phi$ represents the flux of linearly polarized radiation with Faraday depth between Φ and $\Phi+d\Phi$ expressed as a fraction of the total flux. This function is to be symetrical about $\Phi = A$, while the angles of polarization of these fractions are to be skew-symmetrical about the same Faraday depth A. This latter property would imply rather special relationships between the transverse and parallel components of the source's magnetic fields and also a rather special orientation of the source relative to the direction of the sun. It is therefore probable that the structure is such that $\alpha(\Phi)$ = const, as is the case for random fluctuations about a mean direction.

It seems then that, although equation (15) cannot be used directly from the observations, the common feature of the dependence of χ on λ^2 implies that we are able to use the assumption (16) which leads to the equation (17). Unfortunately this procedure cannot yet be used on the present data for most sources because of the large errors and the small number of wavelengths covered. The one exception is Taurus A which will be discussed in greater detail in section 7.

5. INTERNAL FARADAY DISPERSION

It is now generally believed that the radiation from a radio source is due to synchrotron emission. This occurs only in the presence of a strong magnetic field within the source so we might expect Faraday rotation effects associated with this internal field. The qualitative features of the polarization changes will depend on the structure of the source. The wavelength at which depolarization occurs will depend on the strength of the magnetic field, the dimensions of the source and the density of the thermal electron gas. For many sources it is possible to estimate the first two of these parameters. Detailed polarization measurements may enable us to identify the structures of sources and the densities of the thermal plasmas associated with them. It is possible to impose theoretical limits on many of the parameters related to the source structure from arguments concerning such things as the containment of the relativistic gas, the permanence of the magnetic field, the nature of the radio spectrum, etc., which are outside the scope of this paper. In this section we shall assume that the external depolarization is negligible and the only change in polarization possible after the radiation leaves the source is rotation of the angle of polarization by an amount proportional to λ^2.

We assume that the magnetic field of a source is of the type discussed in section 3 and that the scale of the fluctuations is d. Let $f(x)dx$ be the fraction of the source's radiation which traverses a pathlength between x and x+dx within the source. The distribution of Faraday depths of this fraction may be represented by a Gaussian with mean $KnH_z^o x$ and variance $(KnH_r d)^2 x/2d$.

Writing $m = knH_z^o$ and $v^2 = (KnH_r)^2 d/2$, the observed polarization at wavelength λ is:

$$P(\lambda^2) = p(h_o^2) \int_0^{\infty}\int_{-\infty}^{\infty} \frac{f(x)}{\sqrt{2\pi v^2 x}} \exp\left\{ - \frac{(\Phi - mx)^2}{2v^2 x} + 2i\Phi\lambda^2 \right\} d\Phi dx,$$

$$= p(h_o^2) \int_0^{\infty} f(x) e^{-2sx} dx \qquad ,$$

(25)

where $s = v^2\lambda^4 - im\lambda^2$ and we have chosen the coordinate system such that the intrinsic angle of polarization is zero.

The function $f(x)$ depends on the geometry of the source. The simplest function we can take is $f(x) = L^{-1}$ in $0 < x < L$, and 0 otherwise. This represents a slab such that the linear depth of each line of sight through the source is L . Equation (25) then yields:

$$P(\lambda^2) = p(h_o^2) \frac{1 - e^{-S}}{S} \qquad ,$$

(26)

where

$$S = (KnH_r)^2 dL\lambda^4 - 2iKnH_z^o L\lambda^2 .$$

(27)

It is perhaps more realistic to assume that the source is a uniform sphere of diameter L, when $f(x) = \frac{3}{2L}\left\{1 - \left(\frac{x}{L}\right)^2\right\}$ in $0 < x < L$, and 0 otherwise.

Equation (25) then yields:

$$P(\lambda) = p(h_o^2)\frac{3[(S+1)e^{-S} + \frac{1}{2}S^2 - 1]}{S^3} \quad . \qquad (28)$$

Figure 2 plots the polarizations of these models against a variable u which is chosen to be proportional to λ and such that $p/p(h_o^2) = 0.5$ at u = 1. The parameter μ is the ratio of the real and imaginary parts of S at u = 1.

The properties of these models depend on the ratio of the real and imaginary parts of S in the wavelength range where most of the depolarization occurs. (i.e. $|S| \sim 1$). It is readily seen that this is determined by the relative magnitudes of the ratios $(H_r/2H_z^o)^2$ and L/d (the number of cells, N say, cut by the longest line of sight through the source).

(1) $N \gg (H_r/2H_z^o)^2$. The imaginary part dominates; the spread in Faraday depths at the appropriate wavelengths is due to the z component of the uniform magnetic field. This corresponds to $\mu = 0$ in Figure 2. Hence for the slab the depolarization follows the familiar $p(h_o^2)\frac{\sin\delta}{\delta}$ law ($\delta = KnH_z^oL\lambda^2$) with the constant factor $p(h_o^2)$ due to the intrinsic depolarization produced by the random component of the magnetic field. The angle of polarization follows a λ^2 law of rotation except for discontinuities of $\frac{\pi}{2}$ at $\delta = n\pi$, where the polarization falls to zero. At much longer wavelengths the real part will become more important and finally dominate. However, at these wavelengths, the source is essentially completely depolarized.

For a spherical source the Faraday dispersion function is asymmetric and, as a result, there are departures from a λ^2 law of rotation. Such a law is followed very closely for $u < 1$. As the wavelength increases beyond this range the angle of rotation steadies down with damped oscillations about $\chi = \frac{\pi}{4}$.

(2) $N \ll (H_r/2H_z^0)^2$. The real part dominates at the appropriate wavelengths, corresponding to the case $\mu = \infty$ in Figure 2. The Faraday dispersion function is essentially symmetric so there are no deviations from a λ^2 law of rotation. For $u < 1$ the polarization falls in the same manner as the previous case. At larger wavelengths the polarization falls more quickly ($\propto \lambda^{-4}$ rather than λ^{-2}) and without the fluctuations present in the previous case.

Assuming that the 10 cm degree of polarization is close to the zero wavelength value we see that in most sources the random field has reduced the intrinsic polarization to about $\frac{1}{10}\,p(\gamma)$. Assuming that $H_x^2 \approx H_y^2 \approx H_z^2$ it follows that $(H_r/2H_z^0)^2 \sim 1$ and hence the former condition is normally satisfied. Faraday dispersion due to the random component of the magnetic field can only be important if the steady magnetic field is nearly perpendicular to the line of sight.

As we know the polarization of 18 sources at three or more wavelengths, it is possible to test if any of the above models are consistent with the observations. Assuming a model, we take as given the polarization at two wavelengths, predict the polarization at another wavelength, and test for agreement with the observed value. The three wavelengths chosen for each source are those which appear to give the least errors coupled with maximum spread in wavelength. The polarization at the two shortest wavelengths were used to predict that at the longest. To allow for the large errors, three values are taken at each wavelength; the observed value and the extremities of the error range. Table (1) gives the number of permutations out of the 27 possible for which the predicted polarization is greater than the assumed observed value. If the data are consistent with the model the number of high predictions may be expected to be in the range 9 - 18, less than 9 indicating that the polarization falls at a slower rate than the model, and more that 18, that it falls faster. The observations used are those of Gardner and Whiteoak (7) and Cooper and Price (6).

The first three models are those of a spherical source. We see that 9 of the 14 sources showing depolarization are more polarized at large wavelengths than this model predicts. One noticeable exception is 21-64 which is at the other extreme. The remaining sources can be fitted to all three cases. The source 3C-370 has a very large error in the 10 cm measurement and so the fit is not convincing. 12S6 A is also not significant as we used measurements at 19 and 21 cm and this spacing is not really large enough to differentiate between models. For Centaurus A (a) the errors are small but we must bear in mind that this is a double source with only one component polarized (18). The beamwidth effect is operating as the spacing of the components is of the same order as the beamwidth. This will tend to depolarize the 30 cm measurement and so give a false fit with these models. The only source that does give a reliable agreement is 3C-353 which is satisfied by all three models. It should be noted that the variations of the angle of polarization show deviations from λ^2 law that are consistent with the case $\mu \ll 1$.

In all of the above models most of the radiation comes from regions with Faraday depths of the same order of magnitude. In such models, if there is significant depolarization between 10 cm and 20 cm, then the polarization at 30 cm should be very much lower. However, the 30 cm observations of Gardner and Whiteoak showed that there is still quite appreciable polarization at this wavelength. This would imply that a considerable fraction of the radiation comes from a range of Faraday depths much smaller than the rest. To obtain such a Faraday dispersion function the author has calculated several models in which there were systematic variations of the emissivity, the magnetic field strength, or the electron density. It was found that very large deviations were required and that these produced a strong asymmetry in the Faraday dispersion function. Hence there should be very significant departures from a λ^2 law of rotation. As such departures have not been found we must look for other mechanisms which can produce the required Faraday dispersion functions without such asymmetry.

6. EXTERNAL FARADAY DISPERSION

Although there is, as yet, no evidence for a dependence of the rate of depolarization on galactic coordinates we should not overlook the possibility that the depolarization is produced in either the disk or the halo of our galaxy. In this section we consider a few models of such depolarization, showing that it is unlikely to be significant.

We first investigate the effects of random fluctuations in the magnetic field and/or electron density in a region extending for a distance R from the observer. If the scale of the fluctuations $d \ll \alpha R$, where α is the angular dimension of a radio source, the Faraday dispersion function of the source is well represented by a Gaussion with variance $K^2(nH_{\parallel})_f^2 dR$, where $(nH_{\parallel})_f^2$ is the variance of the product of the electron density and the line of sight magnetic field of a cell. The degree of polarization at wavelength λ is therefore

$$p(\lambda^2) = p_i \exp\left\{ - 2K^2(nH_{\parallel})_f^2 dR\lambda^4 \right\} \quad . \tag{29}$$

There are three objections to such a model.

(i) The dependence of the depolarization on R should produce a correlation with galactic coordinates. This objection is perhaps not quite so serious for fluctuations in the halo as for fluctuations in the disk.

(ii) Qualitatively, the polarization falls off much faster at large u than do the models of the previous section. The results of an attempt to fit the Gardner and Whiteoak results to this model are shown in the last column of Table 1. The only source to give satisfactory agreement is 21-64.

(iii) For there to be significant depolarization at $\lambda = 20$ cm we require

$$2K^2(nH_{\parallel})_f^2 dR \approx 20^{-4} \quad . \tag{30}$$

Applying the condition $d \ll \alpha R$ and taking $\alpha = 1'$ we obtain $(nH_{\parallel})_f > 4 \times 10^{-6}$ in the disk ($R = 10^{21}$cm), and $(nH_{\parallel})_f > 4 \times 10^{-8}$ in the halo ($R = 10^{23}$cm). If

$n = 0.1$ for the disk the magnetic field required is $H_f > 4 \times 10^{-5}$ which is much too large. This objection is even more serious for the halo.

Objection (iii) above is weaker for large d so we consider the other extreme $d \gg \alpha R$. In this case nearly all the lines of sight to the source pass through the same cells. The spread of Faraday depth across the source produced by one cell will be of the order $\alpha RK(nH_{\parallel})_f$. The polarization at wavelength λ is therefore well represented by

$$p(\lambda^2) = p_i \exp\left\{ -2K^2 (nH_{\parallel})_f^2 \frac{\alpha^2 R^3}{d} \lambda^4 \right\} . \tag{31}$$

The objection (i) above is now even stronger due to the R^3 dependence of the variance. Objection (ii) is still valid as the qualitative features of the depolarization are identical. The inequalities for $(nH_{\parallel})_f$ in (iii) are still true and now become more marked for larger d.

The above models assume that the dispersing cells fill the entire region. We now suppose that they are discrete clouds of dimension d with average spacing D, $(nH_{\parallel})_c^2$ being the variance of $nH_{\parallel}$ for the clouds. The probability of there being m clouds on a line of sight is

$$\text{Prob}(m) = \frac{\eta^m e^{-\eta}}{m!} , \tag{32}$$

where $\eta = \frac{d^2 R}{D^3}$ is the average number of cells on a line of sight. If $\eta \gg 1$ the depolarization is qualitatively identical to the above models and objections (i) and (ii) hold. When $\eta \ll 1$ most lines of sight will not pass through any clouds and there will be almost no depolarization.

When $\eta \sim 1$ we consider the two extremes for d. If $d \gg \alpha R$ all the lines of sight from a source pass through the same cloud which will produce a gradient in Faraday depth across the source. Simple models of this situation, e.g. a wedge-shaped cloud in front of a spherical source, exhibit rapid depolarization very similar to the Gaussian for random fluctuations. Therefore objection (ii) still holds. If $d \ll \alpha R$ the degree of polarization at wavelength λ is

$$P(\lambda^2) = p_i \exp\left[-\eta\left\{1 - e^{-2K^2 (nH_{\parallel})_c^2 d^2 \lambda^4}\right\}\right] \tag{33}$$

The fraction $e^{-\eta}$ of the source which is not covered by clouds is not affected while the rest is depolarized rather quickly. This could get around objection (ii) and explain the high polarization observed at 30 cm. It should be pointed out that objection (i) will still hold as η depends on R. To satisfy the observations the parameters must be such that $\eta \approx 1$ and $K(nH_{\parallel})_c d \approx 5 \times 10^{-3}$. Suppose that the clouds are within the disk. The following inequalities must hold.

$$d \lesssim 10^{18} \text{ cm} \tag{34}$$

$$d/D \lesssim 3 \times 10^{-2} \tag{35}$$

$$(nH)_c \gtrsim 2 \times 10^{-4} \text{ gauss cm}^{-3} \tag{36}$$

In the absence of containing forces such clouds would soon disrupt under their own thermal and magnetic pressures which would be at least 2 orders of magnitude greater than in the surrounding interstellar gas. We might wonder if the containment could be due to the gravitational attraction of stars. The distance to which a star of one solar mass can contain an ionised gas of temperature 10^4 °K is about 2×10^{14} cm. This is much too small for clouds which would have the interstellar spacing of about 10^{19} cm. It is extremely unlikely that there can exist in the interstellar region clouds having properties that could produce the observed depolarization. It is even more unlikely that the clouds can exist in the halo. However if future observations show that there is a correlation between depolarization and galactic latitude we may have to take the existence of such clouds seriously.

7. THE CRAB NEBULA

Evidently the Faraday dispersion functions of most sources must have two properties in common. For each source it must be symmetric about a mean Faraday depth, which is the rotation measure of the source, otherwise there will be departures from a λ^2 rotation law. Also comparable fractions of the radiation must be spread over ranges of Faraday depths of different orders of magnitude. The Faraday dispersion function may be calculated from equation 17 provided we have a large number of polarization measurements over a large range of wavelengths. Such information is available on the Crab nebula and is summarized in Figure 3.

It is obvious that there are departures from a λ^2 law of rotation at short wavelengths. The best fit using all the radio data is $\alpha = (150.5 - 0.147\lambda^2)^\circ$, giving a deviation at 3 cm of -6°, and at optical wavelengths of 9°. Ignoring the observations at wavelengths less than 9 cm the best fit is $\alpha = (154 - 0.155\lambda^2)^\circ$, giving a 3 cm deviation of -9.5°, and an optical deviation of 5.5°. It has been noted that the distributions of optical and radio emission differ (12) indicating that the spectral index is less in the outer regions by about 0.07. This would allow a spectral effect between optical and radio frequencies which could possibly explain the discrepancy between the extrapolated and observed angle of polarization of the optical continuum. To estimate the magnitude of possible discrepancies we use the surveys of Walraven (13) and Woltjer (14) to estimate the states of polarization of the central and outer regions. The nebula was divided into a series of concentric elliptic shells with major axes at angle 135° and with axial ratios 8 : 5

(the approximate shape of the nebula). The intensities and polarizations of the radiation from each shell were calculated and the results are given in Table 2.

The degree of polarization is less in the outer regions, but the angle of polarization does not change significantly. This means that although the spectral effect probably reduces the degree of polarization by about 2 per cent it will not produce any noticeable change in the angle, 2° - 3° at most. It is significant that the results of Walraven give lower degrees of polarization in the outer regions. This is perhaps because Woltjer subtracted the radiation due to the filaments, while Walraven did not make this correction. The polarization measurement of Oort and Walraven also did not allow for filamentary emission, and so it seems that the best value to take for the degree of polarization of the optical continuum is 14 per cent, and for the intrinsic polarization at radio frequencies 12 per cent. There also appears to be a discrepancy between the results of Oort and Walraven and the later surveys in measurements of the angle of polarization, which according to the later data is about 155°, in good agreement with the straight line fitted ignoring the short wavelength results.

It is quite easy to attribute the short wavelength deviations to Faraday rotation effects. The rotation measure at long wavelengths is the mean Faraday depth of the fraction of the source with a small spread of Faraday depths, as the radiation from regions with a large spread of Faraday depth is essentially unpolarized. If the average Faraday depth of the fraction with a large spread is not the same there will be deviations from λ^2 dependence of the angle of polarization at short wavelengths. We are able to use equation (17) to estimate the required Faraday dispersion function. For $P(\lambda^2)$ we assume the values shown in Table 3. The results are shown in Figure 4 where the fraction of the radiation in ranges of half an order of magnitude are shown. The rotation measure at long wavelengths is probably due to regions between the source and the observer.

A closer look at the structure of the nebula reveals the likely reason for this large range of Faraday depths. The most striking feature of the nebula is the filamentary shell which surrounds the central continuum. The filaments are threadlike dense ionised regions which, as Woltjer (17) pointed out, are probably due to the passage of electric currents on the surface of the nebula. These currents are necessary to match the force free field of the nebula to the interstellar field and are directed along the axes of the filaments. The magnetic fields due to these currents are circular about the filaments and so large fluctuations are to be expected in the Faraday depths of different lines of sight through the filaments. Assuming that the filaments are held together by a pinch effect we can estimate the magnetic field due to the currents. For a bright filament they are of the order 3.10^{-1} Gauss and so the maximum Faraday depth is the order 2.10^{-1} cm^{-2} which is to be of the required magnitude. The extent of the filaments is difficult to estimate but they are certainly more extensive than those visible in photographs taken in the light of emission lines (18). Between the individual filaments there are gaps, and the radiation from behind these will have a small spread in Faraday depth, as both the density and the magnetic field due to the filament currents will be very small. The variations of the mean Faraday depths of the fractions with different spreads is also to be expected if there is a net flow of lines of force across the surface; that is, if the external field is partially linked to source's field. If the linkage was present before the formation of the filaments it would be expected that the normal field is also greater in the filaments, as the lines of force will have been crowded together when they condensed. The variations of the mean Faraday depth can therefore also be explained by the filamentary structure.

We can calculate the mass of the filamentary shell from $E(\Phi)$. The density, the magnetic field and the thickness of the filaments (d) in front of regions with Faraday depth in Φ to $\Phi + d\Phi$ satisfy the relation $\Phi = KnH_{\parallel} d$. Assuming that $H_{\parallel}$ is constant and A is the apparent area of the source then the mass dm in these filaments is

$$dm = nAm_H d\Phi$$

$$= \frac{Am_H \Phi E}{KH_{\parallel}} d\Phi \quad , \tag{37}$$

where m_H is the mass of a hydrogen atom. Hence

$$m = \frac{Am_H}{K} \int \frac{\Phi E}{H_{\parallel}} d\Phi \quad . \tag{38}$$

The results of this calculation are shown in Table 4. The first three columns give the ranges of Faraday depth, the average Faraday depth and the fraction of the radiation in these ranges. Column 4 gives the masses of the filaments producing these Faraday depths assuming that $H_{\parallel}$ is 10^{-4} gauss and A is 1.85×10^{37} cm^2. The total mass of the shell is twice the sum of the masses in column 4 as the filaments at the back have very little effect on $E(\Phi)$. It is seen to be $1.1M_{\odot}$ which is in quite reasonable agreement with the value of $0.64M_{\odot}$ that was obtained by O'Dell (18) from photoelectric observations of the flux in $H(\beta)$ emission.

8. OTHER SOURCES

The absence of internal Faraday dispersion of the type of section 4 enables us to put an upper limit to the density of the thermal gas within a source once the magnetic field strength and the dimensions of the source are known. Both of these parameters may be estimated if an optical identification enables us to determine the distance of the source (19). Suppose that H_t is the total magnetic field and that H_o is the strength of the uniform component of it. Then

$$H_o^2 = \frac{h_o^2}{1+h_o^2} H_t^2 = \frac{p(h_o^2)}{p(\gamma)} H_t^2 \quad , \tag{39}$$

using equation (3-10). The value of $H_{\parallel}$ to be used in applying model D is therefore

$$H_{\parallel} \approx \left\{ \frac{p(h_o^2)}{3p(\gamma)} \right\}^{\frac{1}{2}} H_t \quad . \tag{40}$$

Assuming that the depolarization between the two longest wavelengths is due to internal dispersion we can estimate upper limits for the density and total mass of the ionised gas in the body of the source. Columns 4 and 5 of Table 5 give these limits for all of the sources in the polarization surveys which have been optically identified. The volumes and diameters in columns 2 and 3 are taken from reference 8. In the case of a double source we give the diameters of each component and the total volume.

The depolarization features of other sources are similar to those of the Crab, giving the same difficulty in interpreting the large polarization at long wavelengths. It is therefore natural to wonder whether these features are also produced by filamentary structures in the outskirts of these extragalactic sources. The above estimates of upper limits to the internal densities show that for all the sources if the temperature is less than 10^8 °K the magnetic and cosmic ray pressure are very much greater than the thermal pressure. The internal magnetic field is therefore force free and we may expect filamentary structure in the outskirts associated with surface currents.

To calculate the amount of radiation emitted by these filaments in emission lines we must know their density, temperature, and dimensions. We tentatively assume that within the filaments the temperature T is 2×10^4 °K as in the Crab and that the thermal and magnetic forces balance. Hence

$$2knT \approx \frac{H^2}{8\pi} \quad . \qquad (41)$$

A fraction $E(\Phi)\, d\Phi$ of the source is covered by filaments with Faraday depth Φ and these filaments we have thickness d where $\Phi \approx knH_{\parallel}d$. The emissivity in H_β is given by Burgess (20) as

$$j(H_\beta) = 1.2 \times 10^{-25} n^2 \quad . \quad \text{erg cm}^{-3}\ \text{sec}^{-1} \qquad (42)$$

It follows that the total flux in H_β received from a source of angular diameter α is

$$S(H_\beta) \approx 6 \times 10^{-7} \alpha^2 H \int |\Phi| E(\Phi)\, d\Phi \quad , \qquad (43)$$

For the extragalactic sources we have no knowledge of $E(\Phi)$ for $|\Phi| > 10^{-2}$ and the brightness could be down to 10^{-3} of that of the Crab. However, if the filaments are dense enough to produce depolarization at cm wavelengths there could be detectible line emission from the outskirts of these sources. It should be pointed out that the filaments are too small to be resolved. If the scale of the filaments were larger the density would need to be correspondingly less and the flux reduced in the same ratio.

CONCLUSIONS

In this paper we have shown that it is possible to obtain interesting information concerning the internal structure of radio sources from low resolution polarization observations, provided a large range of wavelengths is covered. The linearity of the plot of the angle of polarization against wavelength squared implies that we may deduce the Faraday dispersion function of a source from the dependence of its polarization on wavelength. This calculation requires the assumption that the Faraday dispersion function may be represented by a real function and that any departures from a λ^2 law of rotation is due to its asymmetry. As infinitely many source structures can give rise to the same Faraday dispersion function we must use other types of observation to determine the actual structure of the source.

There are sufficient data on the Crab nebula for its Faraday dispersion function to be calculated. It is found to be spread over a large range of Faraday depths with comparable fractions of the radiation being spread over ranges of different orders of magnitude. This is probably produced by the filamentary shell which surrounds the nebula, the fraction of the radiation from behind the filaments being depolarized at short wavelengths, while the radiation from behind the gaps remains polarized at much longer wavelengths. The high polarizations measured by Gardner and Whiteoak (7) and Cooper and Price (6) at 30 cm indicate that the Faraday dispersion functions of other sources are similar to that of the Crab. It should perhaps be mentioned at this point that there is tentative evidence (29) that there is still significant polarization at much longer wavelengths. It is therefore suggested that filamentary structures may also exist in the outskirts of extragalactic sources. It is interesting to note that there have recently been other indications of such structures in extragalactic objects (30, 31).

It is possible to estimate an upper limit to the density of the internal ionized gas of a source from its rate of depolarization once we know its dimensions and magnetic field strength.

In the near future the results of more complete surveys covering a larger range of wavelengths will be available. Such observation will be invaluable in determining the physical structure of the discrete radio sources and will perhaps also shed some light on the source observer media.

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TABLE 1

Source	Wavelength (cm)	$\mu = 0$	$\mu = 0.16$	$\mu = \infty$	Gaussian
Fornax A (a)	10, 21, 30	0	0	1	1
Pictor A	10, 21, 30	6	6	6	6
3C-161	10, 21, 30	0	0	0	0
3C-270	10, 21, 30	10	9	9	10
3C-273	10, 15, 21	2	2	0	0
Hercules A	10, 21, 30	0	0	0	0
3C-327 (a)	10, 21, 30	6	6	6	6
3C-353	10, 21, 30	9	18	15	1
21-64	10, 21, 30	25	27	27	18
Taurus A	10, 21, 30	2	0	0	0
Centaurus A (a)	10, 21, 30	17	17	17	3
Centaurus A (b)	15, 19, 30	0	0	0	0
Centaurus A (c)	15, 21, 30	2	2	1	1
1356 A	10, 19, 21	14	15	13	12

<u>TABLE 2</u>

	Major axes of bounding ellipses	Relative intensity	Degree of polarization	Angle of polarization
Walraven	0 - 2	12	19.5	155.5°
	2 - 4	29	16.0	155.5°
	4 - 6	32	9.0	158.5°
	6 -	27	7.5	144.0°
	Total	100	11.8	154.1°
Woltjer	0 - 1	4	21.9	148.0°
	1 - 2	9	17.8	159.5°
	2 - 3	15	18.8	157.5°
	3 - 4	16	16.8	153.0°
	4 - 5	19	12.5	155.5°
	5 - 6	15	7.0	171.5°
	6 - 7	12	8.8	158.8°
	7 - 8	10	18.0	148.0°
	Total	100	14.0	156.0°

Major axes in units of 0.835'.

TABLE 3

Wavelength squared (cm^2)	Degree of Polarization	Relative Degree of Polarization	Angle of Polarization
0	12.0	1.00	0°
4	8.6	0.72	-8°
10	7.0	0.58	-10°
12	6.5	0.54	-12°
30	5.2	0.43	-8°
100	3.5	0.29	-4°
130	2.6	0.22	-4°
225	2.2	0.18	1°
324	1.8	0.15	-8°
444	1.6	0.13	6°
900	0.9	0.075	0°

TABLE 4

Range of log|Φ|	Average Faraday Depth (cm^{-2})	Fraction of Radiation	Mass of Filaments (Solar Masses)
-0.5 ... 0.0	0.56	0.09	0.30
-1.0 ... -0.5	0.18	0.16	0.17
-1.5 ... -1.0	0.056	0.18	0.06
-2.0 ... -1.5	0.018	0.14	0.02
-2.5 ... -2.0	0.0056	0.14	0.01
-3.0 ... -2.5	0.0018	0.11	-
-3.5 ... -3.0	0.00056	0.11	-
... -3.5	0.00018	0.07	-

TABLE 5

Source	Volume (cm^3)	Diameter (kpc)	$B \times 10^5$ (Gauss)	Density (cm^{-3})	Mass ($M_\odot$)
3C-33	5×10^{68}	25, 25	10	3×10^{-5}	1×10^{7}
3C-78	4×10^{68}	29	3.0	1×10^{-4}	4×10^{7}
Fornax A (a)	1×10^{70}	89	0.8	3×10^{-4}	3×10^{9}
Fornax A (b)	1×10^{70}	89	0.8	3×10^{-5}	3×10^{8}
3C-98	6×10^{68}	34	3	2×10^{-4}	9×10^{7}
Pictor A	2×10^{70}	103	2.4	9×10^{-5}	1×10^{9}
Taurus A	6×10^{55}	0.0016	10	0.9	0.04
3C-270	2×10^{67}	8.6, 8.6	3	6×10^{-4}	1×10^{7}
3C-273	3×10^{67}	10, 10	15	3×10^{-4}	9×10^{6}
Centaurus A (a)	2×10^{66}	3.5, 3.5	8.0	5×10^{-4}	7×10^{5}
Centaurus A (b)	5×10^{70}	120, 120	0.6	9×10^{-5}	4×10^{9}
Hercules A	3×10^{70}	100, 100	5.0	4×10^{-5}	9×10^{8}
3C-327	1.6×10^{70}	25, 25	4.0	3×10^{-4}	1×10^{8}
3C-433	3×10^{68}	26	3.0	4×10^{-4}	1×10^{8}
3C-353	5×10^{68}	25	4.0	1×10^{-3}	4×10^{8} *

*This is an estimate of the density as model D does fit the data.

Legends to Figures

Figure 1. Polarization of a source containing a small scale random magnetic field.

Figure 2. Polarization of models of internal Faraday dispersion. (a) Degree of polarization, (b) Angle of polarization.

Figure 3. Summary of polarization data for the Crab nebula.

Key

O	Oort and Walraven	(12)
W	Woltjer	(14)
B	Boland et alia	(21)
M	Mayer et alia	(9)
H	Hollinger et alia	(22)
K	Kuzmin and Udalt'tsov	(23)
A	Altenhoff et alia	(24)
G	Gardner and Whiteoak	(7)
S	Seielstad et alia	(25)
D	Davies and Verschuur	(26)
U	Udalt'tsov	(27)
MR	Morris and Radhakrishnan	(28)

Figure 4. Faraday dispersion function of the Crab nebula. The rotation measure is taken as the zero of Faraday depth.

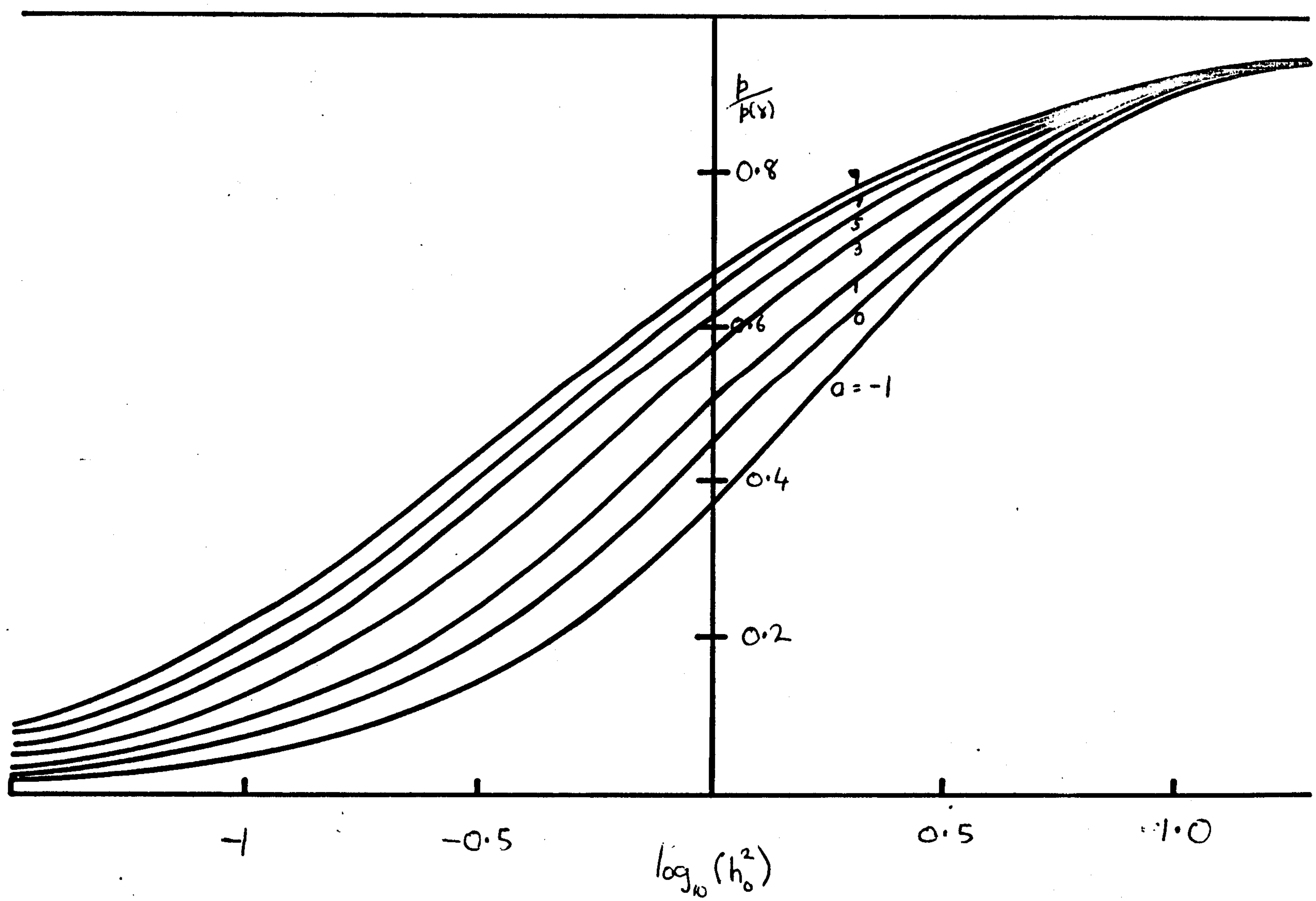

Figure 1

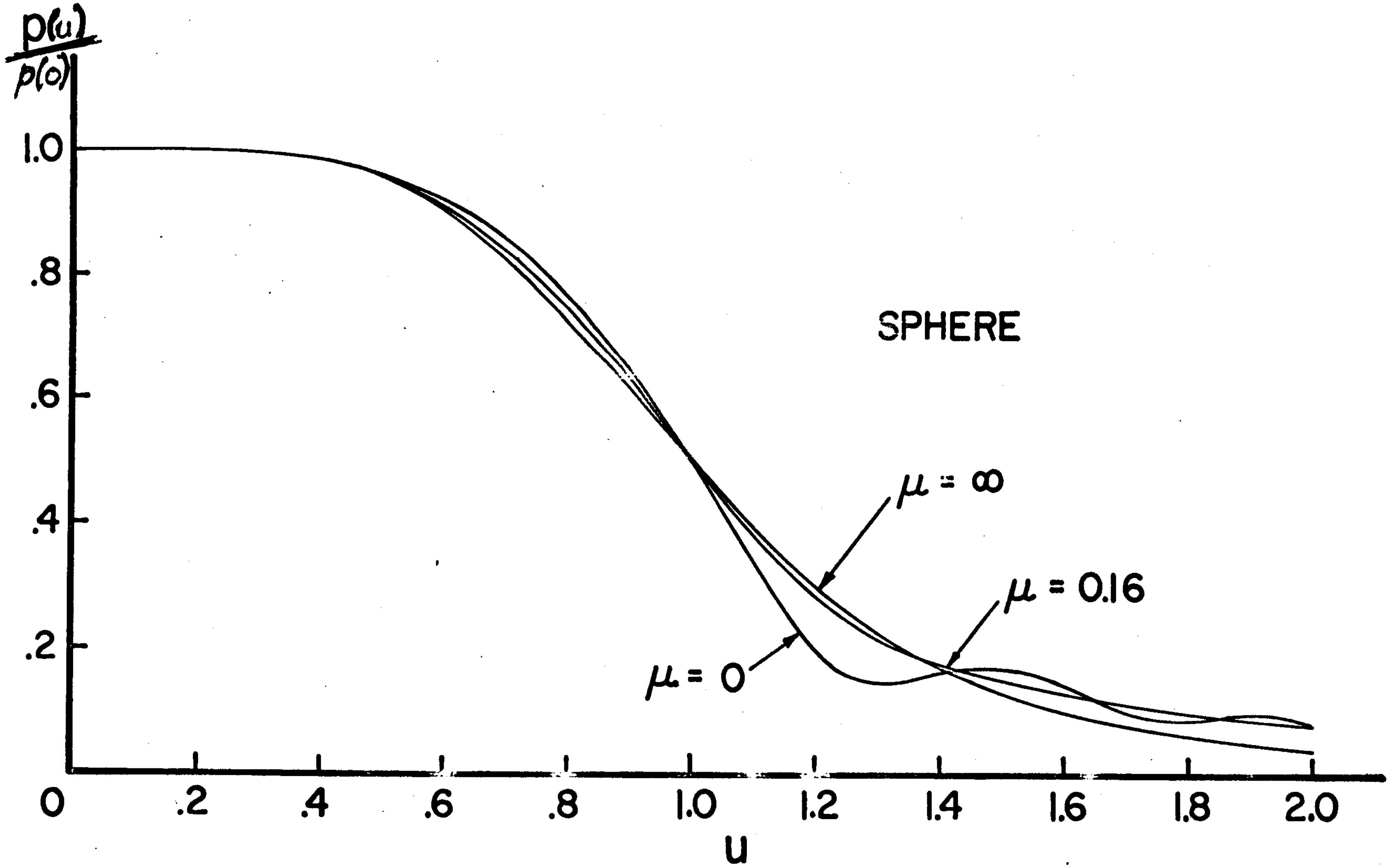

Figure 2a

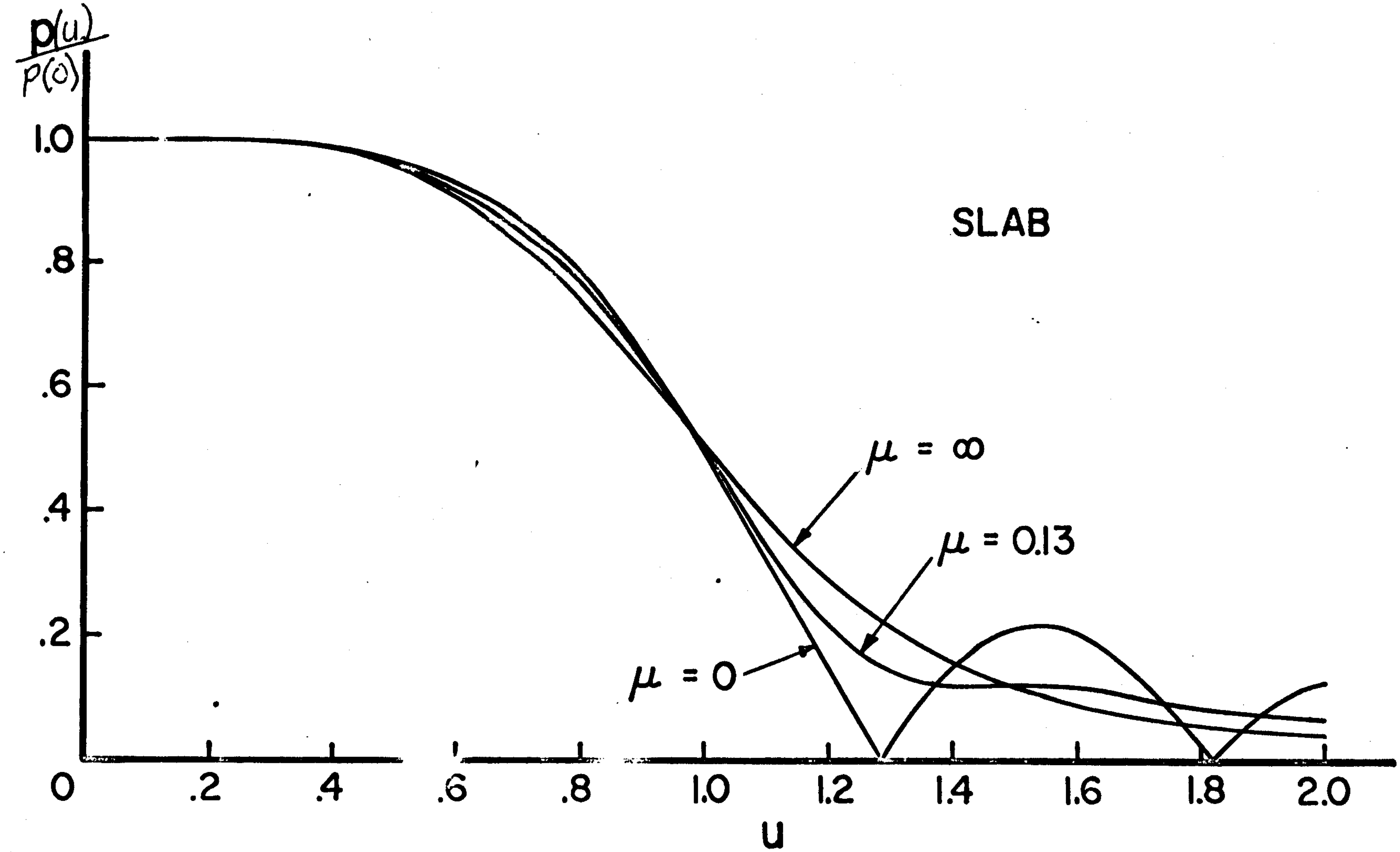

Figure 2a

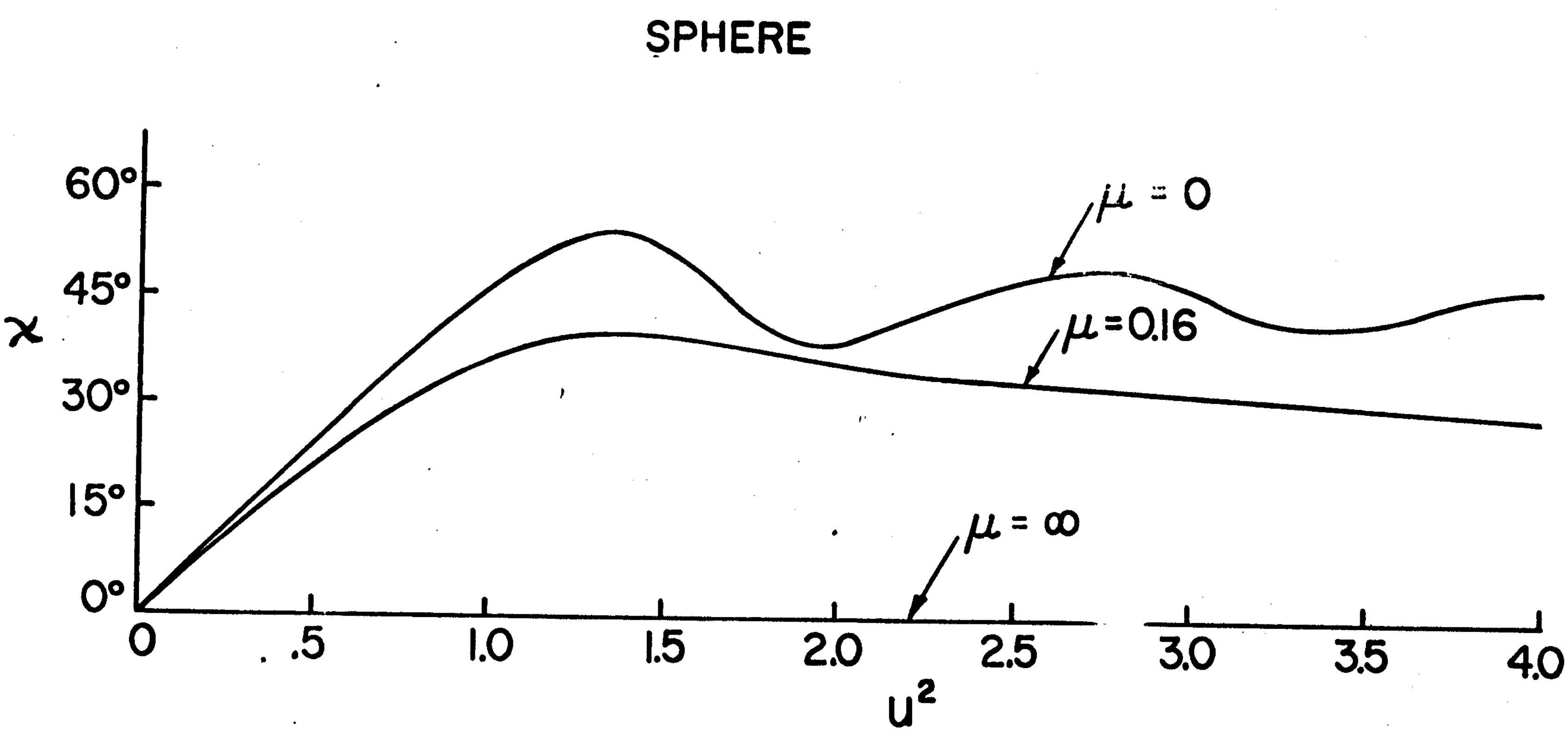

Figure 2b

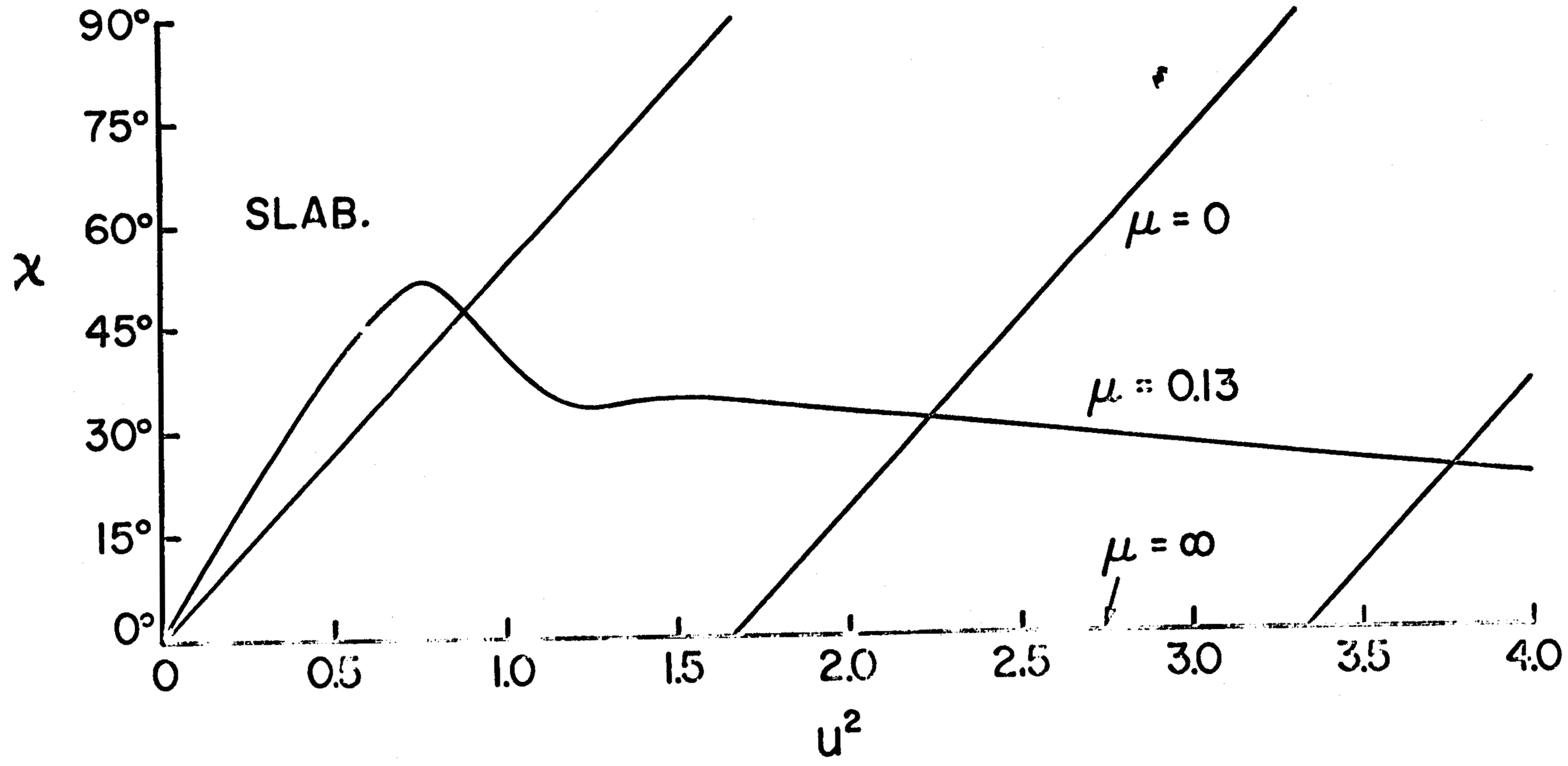

Figure 2b

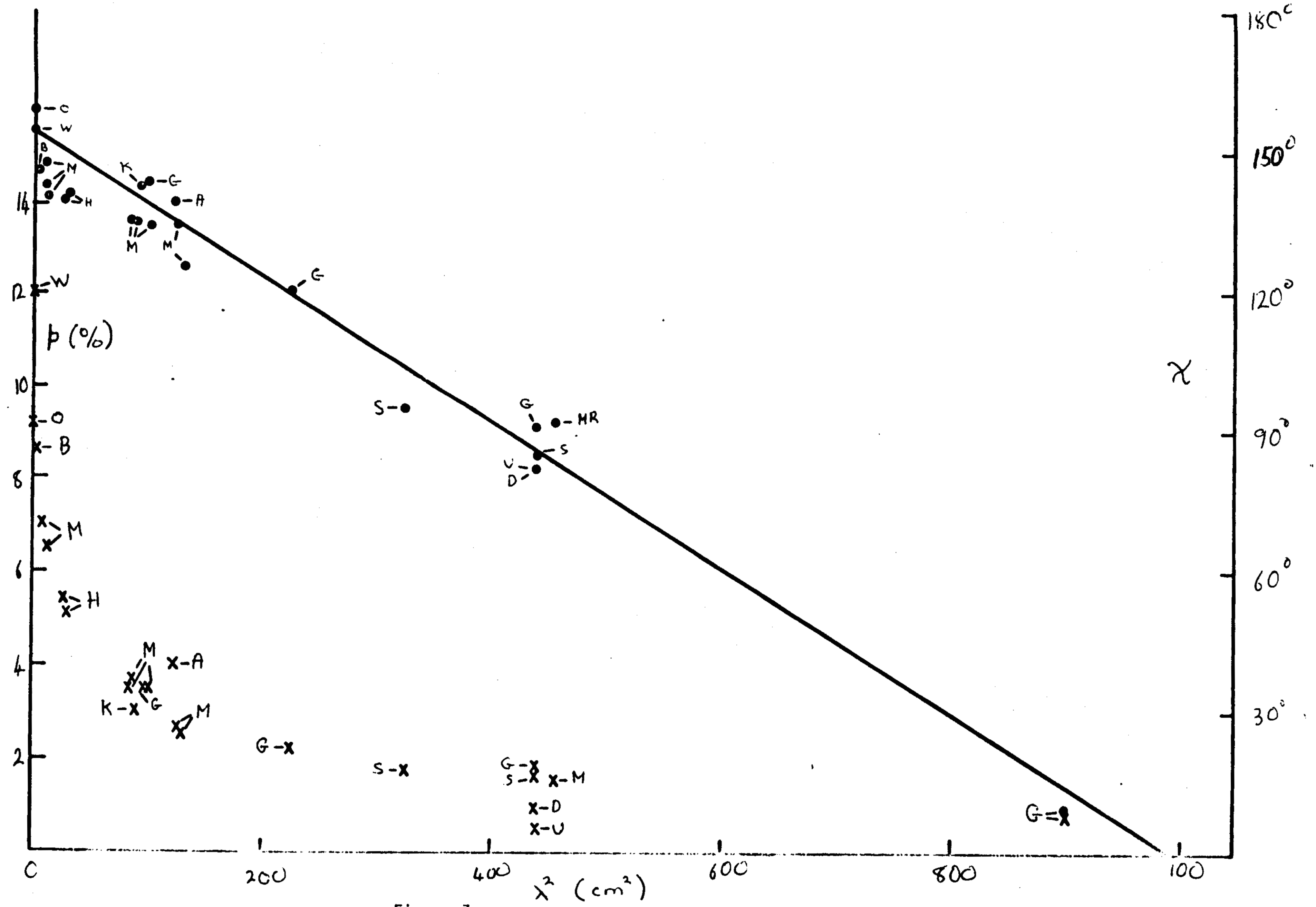

Figure 3

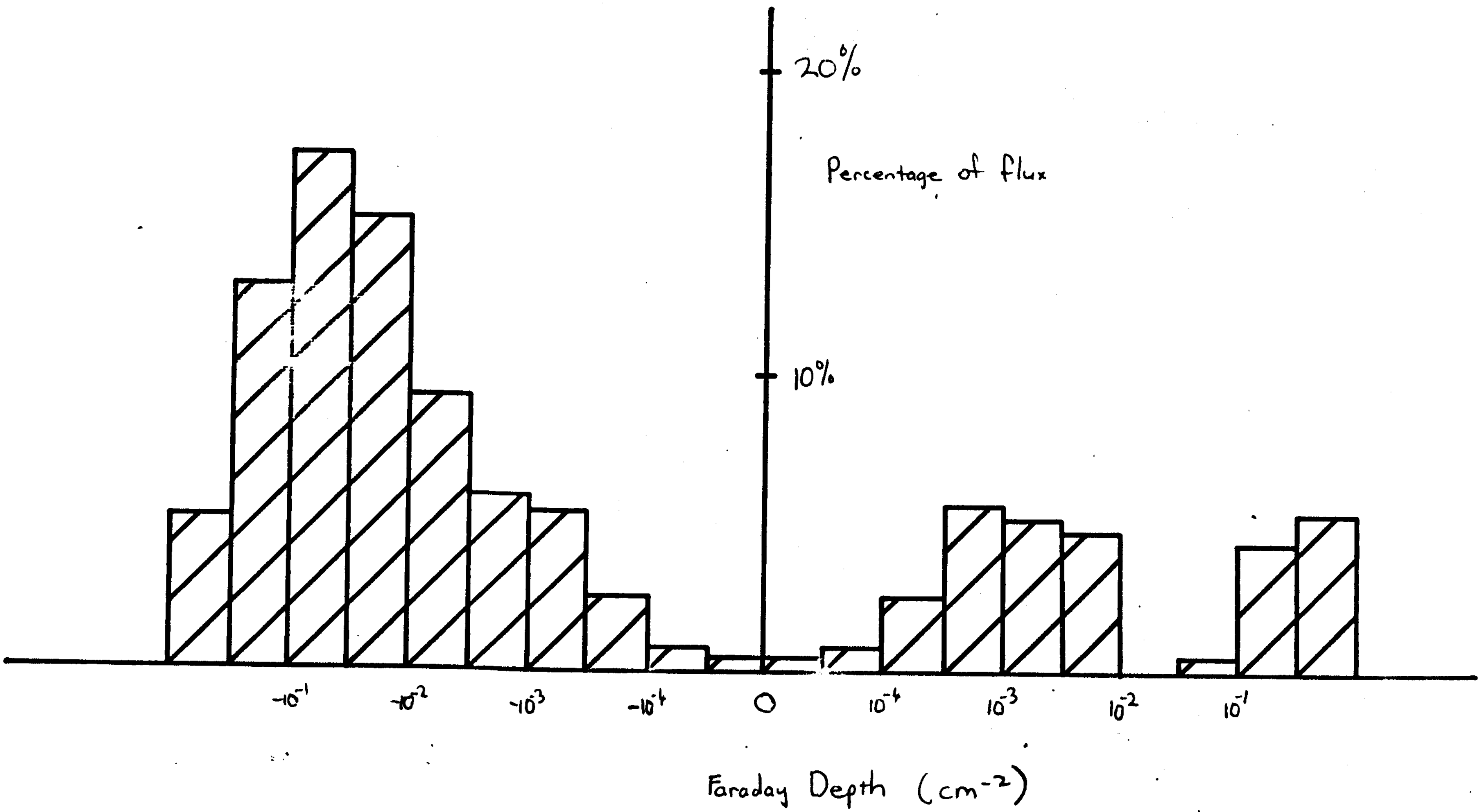
20%
Percentage of flux
10%
-10^-1
-10^-2
-10^-3
-10^-4
0
10^-4
10^-3
10^-2
10^-1
Faraday Depth (cm^-2)

Figure 4