FACILITY FORM 602

N65-21334
(ACCESSION NUMBER)

19
(PAGES)

CR-57902
(NASA CR OR TMX OR AD NUMBER)

1
(THRU)

(CODE)

09
(CATEGORY)

PROGRESS REPORT NO. 5 ON

TV TRANSMITTER-EXCITERS AND POWER AMPLIFIERS

REPORT PERIOD 1 NOVEMBER 1964 TO 1 DECEMBER 1964

Prepared for

GEORGE C. MARSHALL SPACE FLIGHT CENTER

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Huntsville, Alabama

CONTRACT NO. NAS8-11609

CONTROL NO. ICN 2-4-10-001985

Prepared by

A. G. Van der Capellen

OF

RESDEL ENGINEERING CORPORATION

Pasadena, California

GPO PRICE $ ________

OTS PRICE(S) $ ________

Hard copy (HC) $1.10

Microfiche (MF) .50

15 December 1964

Sept 27 1964

INTRODUCTION

This is the fifth in a series of monthly reports prepared for the George C. Marshall Space Flight Center, NASA, Huntsville, Alabama, covering the period 1 November 1964 to 1 December 1964, as specified in the subject contract.

WORK ACCOMPLISHED

1) System test completed (Prototype).

2) Environmental test 80% completed (Prototype).

3) Documentation 90% completed.

4) Fabrication of transmitter-exciters 70% completed.

5) Fabrication of power amplifiers (Model 91064) 80% completed.

6) Writing of instruction manual completed.

DISCUSSION

A block diagram is included for reference.

Subassembly testing is almost completed on the first production unit. Second test of the RF and X16 multiplier section has been completed. First test of the AFC and Video section has been completed. This section is now ready for clean up, inspection and then potting, after which a second test follows. Subassembly testing on subsequent units has been done on power supplies, cavities, RF amplifier sections and AFC & Video sections.

A formal quality acceptance test procedure is ready for approval and will be submitted. Results on the study of $\sin^2$ pulse testing are discussed under problem areas.

PROBLEM AREAS

The problem of proving out the high frequency video response of the transmitter is a matter of:

1. Defining which technique is to be used.

2. Correlating the measurements with the specification.

Three techniques are known that will yield information about high frequency video response.

1. Measurement of horizontal resolving power which can be related to the 3 db video bandwidth. A camera and test pattern are used for test signal.

2. Measurement of $\sin^2$ pulse degradation. Relating this information to bandwidth is a difficult task.

3. **Direct measurement of video response.** A multiburst or a continuous sweep signal is used in this case.

From the standpoint that reproduction of an undistorted picture is the ultimate criterion, measurements taken with camera and test pattern would seem valid. However, this is a subjective test and it does not yield information relatable to the specification. (The specification calls for a modulation index of 6 at 1 mc and a modulation index of 1 at 6 mc which is equivalent to a flat response up to 6 mc. The 3 db bandwidth is not defined).

The only measurement that yields information directly relatable to the specification is the direct measurement of video response. However this method does not indicate the presence of overshoot or ringing. These are observed by employing the $\sin^2$ pulse method.

The conclusion then would be to use both two methods:

1. The $\sin^2$ pulse to obtain information on overshoot or ringing.

2. The sweep (or multiburst) signal to obtain information on response in general and bandwidth in particular.

Since it was indicated to us on this contract that the $\sin^2$ pulse method was the preferred way of testing the transmitter, an analysis has been made to relate $\sin^2$ pulse width degradation to bandwidth.

The analysis showed that $\sin^2$ pulse width measurements mainly yields <u>qualitative</u> information with regards to system bandwidth. The measurement indicates whether the bandwidth is within certain limits, but no <u>quantitative</u> information about the bandwidth is obtained.

On the basis of measurements on two units, a limit has been established which would assure meeting the specifications of this contract.

Extreme difficulty was encountered in proving out the test equipment, in particular the receiver. Since no standard could readily be found to independantly prove out the receiver bandwidth, a technique of comparing was used. This proved that some degradation of the $\sin^2$ pulse was caused by the receiver. The first measurements were taken with a down converter (1710 mc to 100 mc).and a 100 mc discriminator. Since the bandwidth of this system was under suspicion, a new discriminator was constructed with a center frequency of 1710 mc in order to obtain a larger bandwidth. Measurements with the transmitter and this discriminator showed an improvement of approximately 4 db at 6 mc.

Comparing the $\sin^2$ pulse degradation when the transmitter bandwidth was varied, a limit has been determined. This limit is .105 microsecond for an input of .080 microsecond. (see Figure 1)

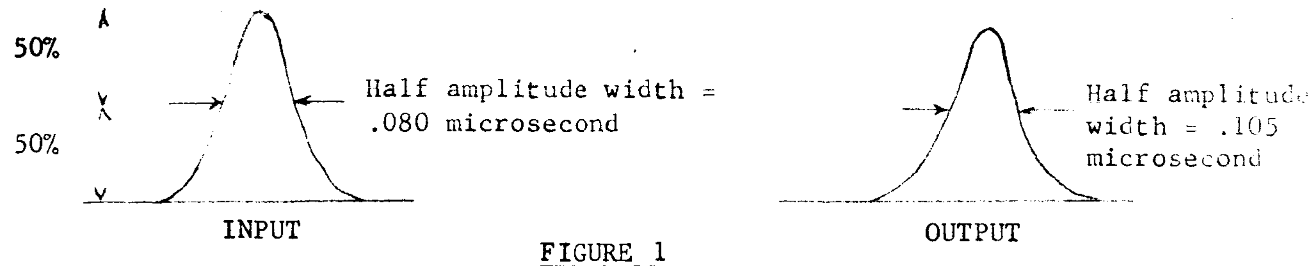


FIGURE 1

On the basis of this limit a graticule was made up that will be used in the acceptance test. (see Figure 2) This graticule also defines the maximum allowable ringing (10% first lobe, 5% second lobe; see Figure 3).

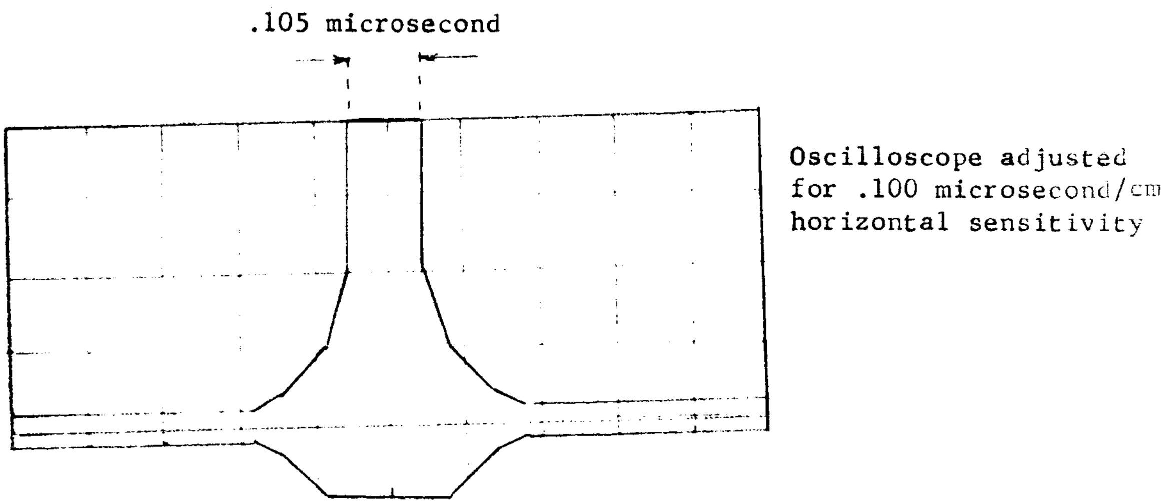


FIGURE 2

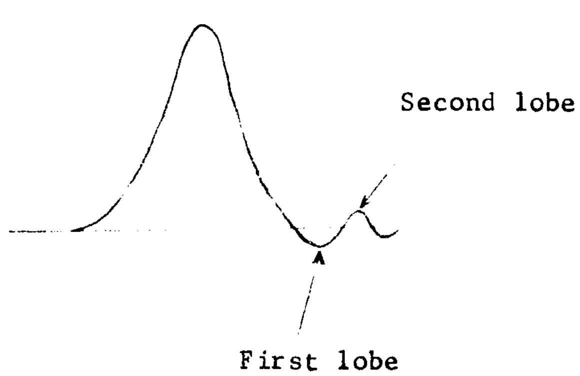


FIGURE 3

ANTICIPATED EFFORT FOR NEXT PERIOD

1) Finish environmental test on modified unit.

2) Continue production transmitter exciter units.

3) Continue production of power amplifiers.

4) Continue documentation.

5) Ship first transmitter-exciter and power amplifier.

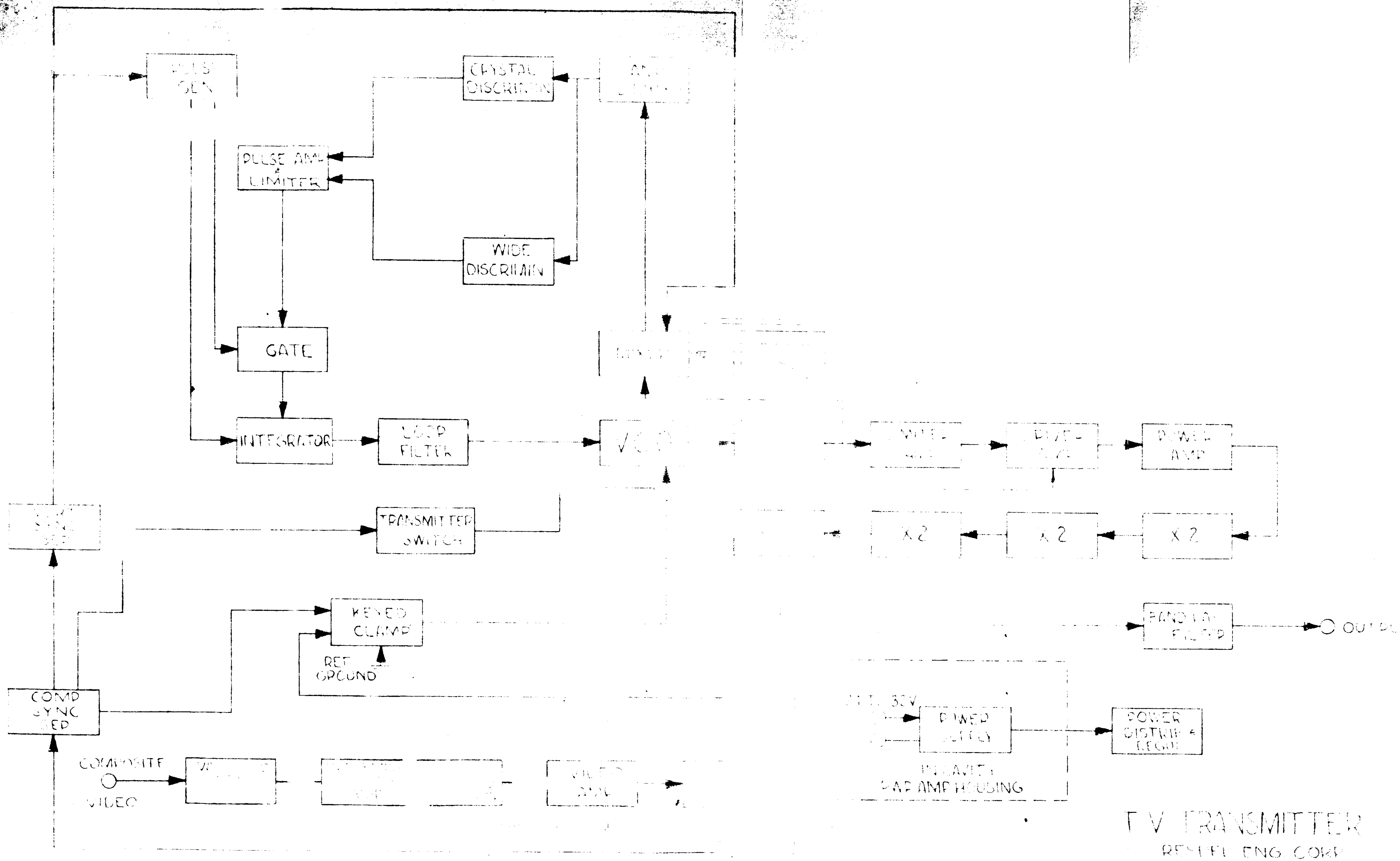
RESDEL ENGINEERING CORPORATION

Prepared By ____________

A. G. Van der Capellen

Approved By ____________

M. F. Brown, Jr.
Vice President



T V TRANSMITTER

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EFFECT OF BAND-PASS FILTER ON AN FM SIGNAL

Consider the following FM signal:

$$v = \sin\left(\omega_c t + \beta \sin \omega_m t\right) \tag{1}$$

$$= \mathrm{IM}\left\{e^{j\left(\omega_c t + \beta \sin \omega_m t\right)}\right. \tag{2}$$

If we drop the imaginary part:

$$v = e^{j\left(\omega_c t + \beta \sin \omega_m t\right)} = e^{j\omega_c t}\, e^{j\beta \sin\omega_m t} \tag{3}$$

But by Guillemin, <u>Mathematics of Circuit Analysis</u> p 509 eq 318,

$$e^{j\beta \sin\omega_m t} = \sum_{-\infty}^{\infty} J_n(\beta)\, e^{jn\omega_m t} \tag{4}$$

so,

$$v = e^{j\omega_c t} \sum_{-\infty}^{\infty} J_n(\beta)\, e^{jn\omega_m t} \tag{5}$$

$$= \sum_{-\infty}^{\infty} J_n(\beta)\; e^{j\left(\omega_c + n\omega_m\right)t} \tag{6}$$

Taking the imaginary part:

$$v = \sum_{-\infty}^{\infty} J_n(\beta)\; \sin\left(\omega_c + n\omega_m\right)t \tag{7}$$

but because

$$J_{-n}(\beta) = (-1)^n\, J_n(\beta)$$

$$v = J_o(\beta) \sin\omega_c t + \sum_{n=1}^{\infty} J_n(\beta) \left[\sin\left(\omega_c + n\omega_m\right)t + (-1)^n \sin\left(\omega_c - n\omega_m\right)t\right] \tag{8}$$

separating into odd and even parts:

(1)

$$v = J_o(\beta) \sin\omega_c t + \sum_{n=0}^{\infty} J_{2n+1}(\beta)\left[\sin\left(\omega_c \cancel{+} (2n+1)\omega_m\right)t - \sin\left(\omega_c + (2n+1)\omega_m\right)t\right]$$

$$+ \sum_{n=1}^{\infty} J_{2n}(\beta)\left[\sin\left(\omega_c + 2n\omega_m\right)t + \sin\left(\omega_c - 2n\omega_m\right)t\right] \tag{9}$$

(2)

$$(1) = \sin\left(\omega_c + (2n+1)\ \omega_m\right)t - \sin\left(\omega_c - (2n+1)\omega_m\right)t$$

$$= \sin\ \omega_c t\ \cos\ (2n+1)\omega_m t + \cos\ \omega_c t\ \sin(2n+1)\omega_m t \tag{10}$$

$$- \left[\sin\omega_c t\ \cos(2n+1)\omega_m t - \cos\omega_c t\ \sin\ (2n+1)\omega_m t\right] \tag{11}$$

$$= 2\ \cos\omega_c t\ \sin(2n+1)\omega_m t \tag{12}$$

$$(2) = \sin\left(\omega_c + 2n\omega_m\right)t + \sin\left(\omega_c - 2n\omega_m\right)t \tag{13}$$

$$= \sin\ \omega_c t\ \cos 2n\omega_m t + \cos\omega_c t\ \sin 2n\omega_m t$$

$$+ \sin\ \omega_c t\ \cos 2n\omega_m t - \cos\omega_c t\ \sin 2n\omega_m t \tag{14}$$

$$= 2\ \sin\ \omega_c t\ \cos 2n\omega_m t \tag{15}$$

thus

$$v = J_o\left(\beta\right)\sin\omega_c t + 2\sum_{n=0}^{\infty} J_{2n+1}\left(\beta\right)\cos\omega_c t\ \sin(2n+1)\omega_m t$$

$$+ 2\sum_{n=1}^{\infty} J_{2n}\left(\beta\right)\sin\omega_c t\ \cos 2n\omega_m t \tag{16}$$

as an example let $\beta = 1$

$J_o\ (1) = .80$

$2J_1\ (1) = .82$

$2J_2\ (1) = .28$

$2J_3\ (1) = .09$

From equation (1) we see that the instantaneous phase angle at $\omega_m t = \pi/2$ will be the modulation index β. We also note from eq. 16 that the phasor contributions due to the odd harmonics of the modulation will be perpendicular to the carrier, and the phasors due to the even harmonics will be parallel to the carrier.

Thus at $\omega_m t = \pi/2$

$J_o\ (1) = .80$

$2J_1\ (1)\ \sin\omega_m t = 2J_1\ (1)\ \sin\pi/2 = .82$

$2J_2\ (1)\ \cos 2\omega_m t = 2J_2\ \cos\pi = -.28$

etc.

The resultant phasor is shown in Figure 1 as $v/\pi/2, 4$ which is the resultant at $\omega_m t = \pi/2$ and the first four harmonics. We can see that if less than four harmonics were summed in the resultant that the result would be just the vector sum of the n phasors.

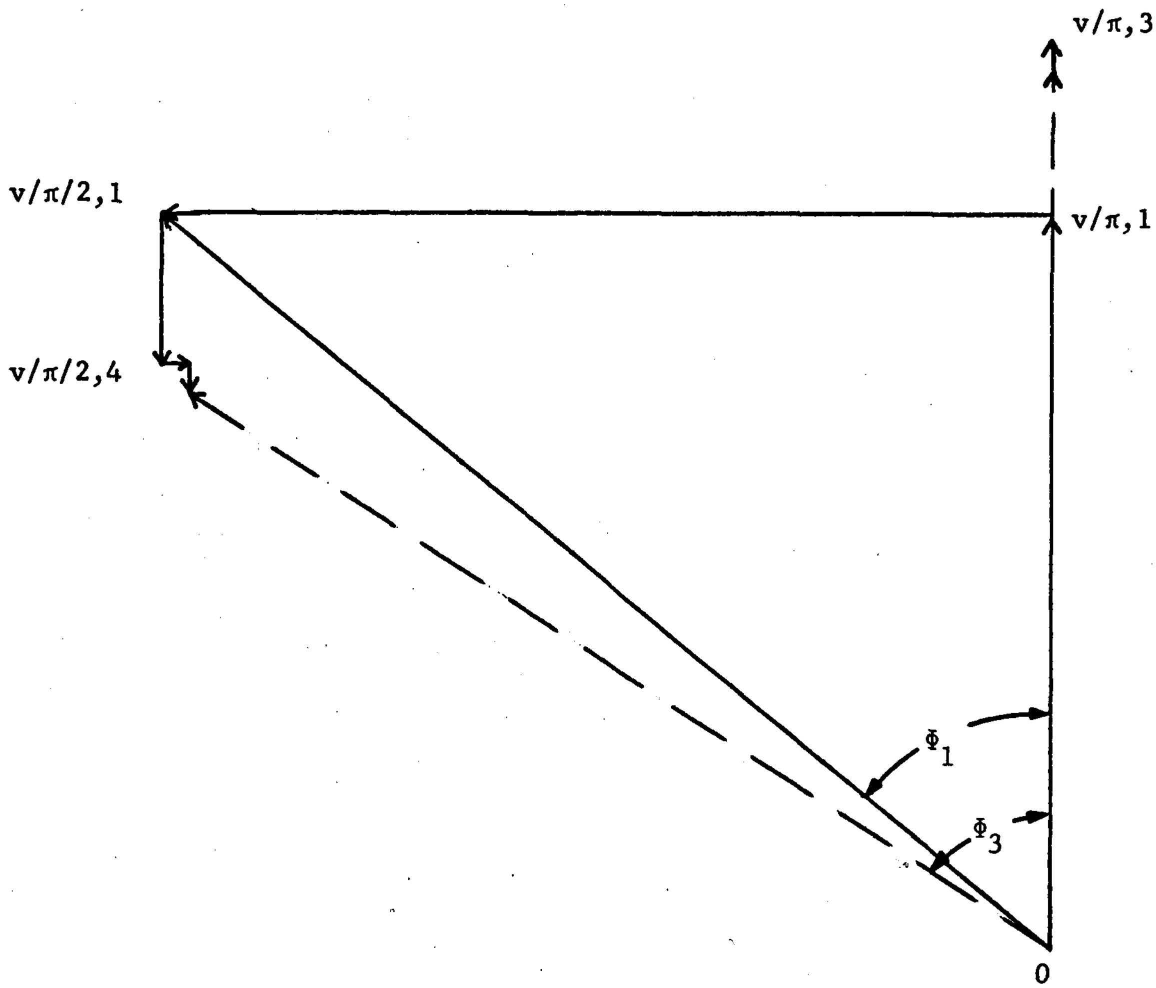


Figure 1

For example, if only one harmonic were considered, the resultant would be $v/\pi/2, 1$ as shown in Figure 1. This example might correspond to $\omega_m = \Delta\omega = 6$ mc. and a 6 mc. ideal filter.

If we consider the same example with a different modulating frequency $\omega_n = 1/3\ \omega_m$ and the same amplitude then $\beta_n = \frac{\Delta f_c}{2\pi\omega_m} = 3\beta_m = 3$ the same expansion is valid so we have:

$$e = J_o \beta_n \sin\omega t + 2J_1 \beta_n \sin\omega_m t \cos\omega t + 2J_2 \beta_b \cos 2\omega_n t \sin\omega t + \text{etc.}$$

$\Delta f = 6$ mc

$f_n = 2$ mc

$\beta_n = 3$

at $\omega_m t = \pi/2$

$J_o(3) \simeq -.25$

$2J_1(3) \sin \omega_m t \simeq .68$

$2J_2(3) \cos 2\omega_m t \simeq -.96$

$2J_3(3) \sin 3\omega_m t \simeq -.64$

$2J4(3) \cos 4\omega_m t \simeq .34$

$2J_5(3) \sin 5\omega_m 5 \simeq .14$

This is plotted in Figure 2 and the resultant is shown as $v/\pi/2, 5$ which is read as the instaneous voltage at $\omega_m t = \pi/2$ with 5 side bands. The angle Φ 5 is the maximum phase angle and is equal to β.

If we now pass this signal through a bandpass filter which admits only the first 3 sidebands (the same filter which admitted only the first sideband of $\omega_m = 6$ mc), we see that the resultant vector is $e/\pi/2$, 3 and phase angle is Φ 3. We see at once that $\Phi 3 > \Phi 5$ or the modulation index is apparently increased.

We also notice that the angle will no longer be sinusoidal in time but will be affected by the absence of the higher sidebands.

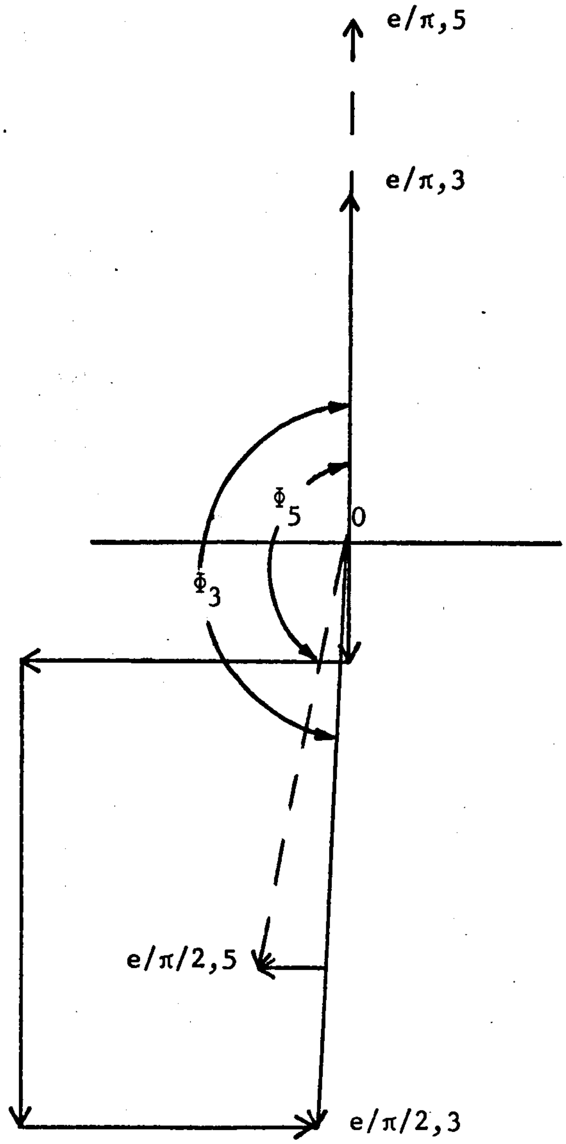


Figure 2

From equation 16 we can analytically express the output of a bandpass filter in terms of the number of harmonics passed, N. To do this we proceed as follows. We try to put v(t) into the following form,

$$v = V(t)\ \sin\omega_i t + \Phi\ (t)$$

but by inspection of the phasor diagrams we see that

$$V(t) = \left\{\left(J_o(\beta) + 2\sum_{n=1}^{\infty} J_{2n}(\beta)\cos 2n\omega_m t\right)^2 + \left[2\sum_{n=0}^{\infty} J_{2n+1}(\beta)\sin(2n+1)\ \omega_m t\right]^2\right\}^{\frac{1}{2}}$$

and

$$\Phi(t) = \mathrm{Tan}^{-1}\left\{\frac{2\sum_{0}^{\infty} J_{2n+1}(\beta)\sin(2n+1)\omega_m t}{J_o(\beta) + 2\sum_{1}^{\infty} J_{2n}(\beta)\cos 2n\omega_m t}\right\}$$

when all side bands are passed. If only N side bands are passed, we need only replace the upper limits summation by 2n + 1 N and 2n N.

As an example of this general case we may let N = 1 so

$$(t) = \mathrm{Tan}^{-1}\left[\frac{2J_1(\beta)\sin\omega_m t}{J_o(\beta)}\right]$$

We can draw some simple conclusions from this. If $\beta << 1$ then $J_1(\beta) = \beta/2$ and $J_o(\beta) = 1$ so $\Phi(t) = \beta \sin\omega_m t$ which is a standard result.

2) No matter what value β has $/\Phi(t)/\leq\pi/2$

This can be seen by considering the band pass filter as a nonlinear transfer function for phase angles as shown in Figure 3.

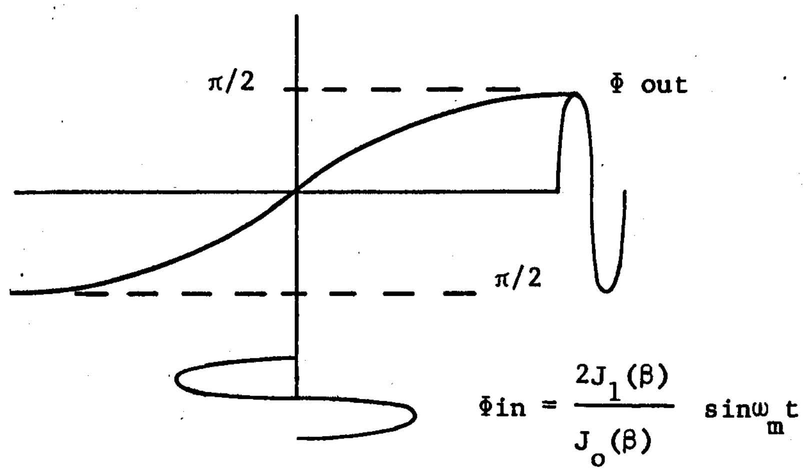


Figure 3

The amount of harmonic distortion in the output is governed by the ratio $\frac{2J_1(\beta)}{J_o(\beta)}$.

THE SINE SQUARED PULSE IN FM

A) To determine the spectrum of an FM carrier of frequency f_o, modulated by a sine squared pulse of half amplitude time t_o we first consider modulation of the carrier by a sine squared wave such that its frequency f is the following:

$$f = f_o + \Delta f_c \sin^2 \frac{\pi t}{2t_o} \tag{1}$$

but the instanious phase angle is

$$\frac{\Phi}{2\pi} = \int f dt = f_o t + \int \Delta f_c \sin^2 \frac{\pi t}{2t_o} dt \tag{2}$$

but $\sin^2 \alpha = \frac{1}{2} \left[1 - \cos 2\alpha\right]$ (3)

so we may write

$$\frac{\Phi}{2\pi} = f_o t + \frac{\Delta f_c}{2} \left\{ \int \left[1 - \cos \frac{\pi t}{t_o}\right] dt \right\} \tag{4}$$

$$\frac{\Phi}{2\pi} = f_o t + \frac{\Delta f_c}{2} t - \frac{\Delta f_c t_o}{\pi} \sin \frac{\pi t}{t_o} + \frac{\Phi_o}{2\pi} \tag{5}$$

or $$e_o = E_o \cos\Phi\ (t) = E_o \cos \left[2\pi \left\{ \left(f_o + \frac{\Delta f_c}{2}\right) t - 2\Delta f_c t_o \sin \frac{\pi t}{t_o} \right\} + \Phi_o \right] \tag{6}$$

Equation (6) can be expanded into a Fourier series with the aid of any standard reference such as Fink, Television Engineering, p. 413, eq. 220 and the result is:

$$\frac{e_o}{E_o} = J_o\left(-2\Delta f_c t_o\right) \cos 2\pi \left(f_o + \frac{\Delta f_c}{2}\right) + J_1 \left(-2\Delta f_c t_o\right)$$

$$\left[\cos 2\pi \left(f_o + \frac{\Delta f_c}{2} + \frac{1}{2t_o}\right) t - \cos 2\pi \left(f_o + \frac{\Delta f_c}{2} - \frac{1}{2t_o}\right) t \right] + \ldots \tag{7}$$

letting $m = -2\Delta f_c t_o$, $f_m = \frac{1}{2t_o}$, and $f_c = f_o + \frac{\Delta f_c}{2}$ and using the fact

that $J_n(-m) = (-1)^n J_n(m)$ we get:

$$\frac{e_o}{E_o} = J_o(m)\ \cos 2\pi\, f_c t + \sum_{n=1}^{\infty} J_n\ (m)\ \left[\cos 2\pi\left(f_c + nf_m\right)\ t + (-1)^n\ \cos 2\pi\left(f_c - nf_m\right)\ t\right] \tag{8}$$

This series can be interpreted as a one sided voltage spectrum as follows:

$$G\omega(f) = J_o(m)\ \delta\left(f-f_c\right) + \sum_{n=1}^{\infty} J_n(m)\ \left[\zeta\left(f-f_c-nf_m\right) + (-1)^n \delta\left(f-f_c+nf_m\right)\right] \tag{9}$$

If $f_c = 8\text{ mc}$, $t_o = 62.5$ ns. Then $m = -2\ \Delta f_c t_o = -1$ and $f(m) = \frac{1}{2t_o} = 8\text{mc}$.

$J_o(-1) \simeq .8$

$J_1(-1) \simeq -.45$

$J_2(-1) \simeq .15$

These numbers are plotted in Figure 1.

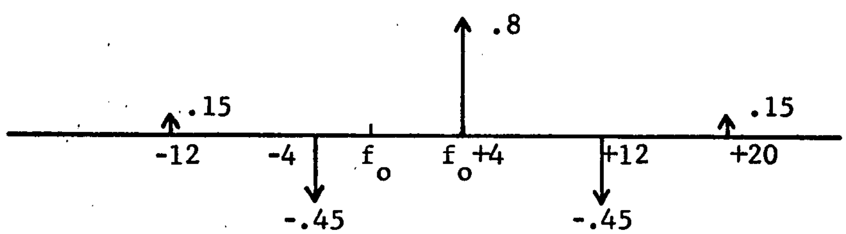


Figure 1. Voltage Spectrum

B) We can now make up the time wave form of the carrier modulated by a pulse instead of a wave in the following way: 1. multiply the wave form of the wave modulated carrier by a pulse of height 1 and duration 0 to $2t_o$, and add to this the unmodulated carrier multiplied by a zero pulse in the same interval. Since multiplication of time wave form results in convolution of frequency spectra, then we can write down the expression of the total spectrum.

Wave Form	Spectrum
Carrier	$G_c(f) = \delta\left(f-f_o\right)$
Sin^2 wave modulated	$G_\omega(f) = J_o(m)\ \delta\left(f-f_c\right) + \sum_{n=1}^{\infty} J_n(m)\ \left[\delta\left(f-f_c-nf_m\right) + (-1^n)\ \delta\left(f-f_c+nf_m\right)\right]$
Positive Pulse	$G_+(f) = t_o \frac{\sin 2\pi t_o f}{\pi t_o f}\ \underbrace{e\text{-}2j\pi t_o f}_{\text{Phase Factor}}$

negative pulse $\quad G_-(f) = \delta(f) - G_+(f) \quad G_T = G_\omega * G_+ + G_c * G_-$

Therefore, $\quad G_T e^{2j\pi t_o f} = J_o(m) \left[t_o \frac{\sin 2\pi t_o (f-f_c)}{\pi t_o (f-f_c)} \right]$

$$+ \sum_{n=1}^{\infty} J_n(m) \left[t_o \frac{\sin 2\pi t_o \left(f-f_c-nf_m\right)}{\pi t_o \left(f-f_c-nf_m\right)} + (-1)^n t_o \frac{\sin 2\pi t_o \left(f-f_c+nf_m\right)}{\pi t_o \left(f-f_c+nf_m\right)} \right] \tag{10}$$

$$+ \delta\left(f-f_o\right) e^{2j\pi t_o f} - t_o \frac{\sin 2\pi t_o \left(f-f_o\right)}{\pi t_o \left(f-f_o\right)}$$

To draw conclusions from the above equation we proceed as follows. The G_+ spectrum is the sin α/α curve with its first zeros at $\pm$ 8 mc and its shape is imposed on each of the lines of the sine square wave modulated carrier. The carrier itself is present with this shape also on it. With a modulation index of 1 or less only the first pair of side bands is important so the composite spectrum is as shown in Figure 2.

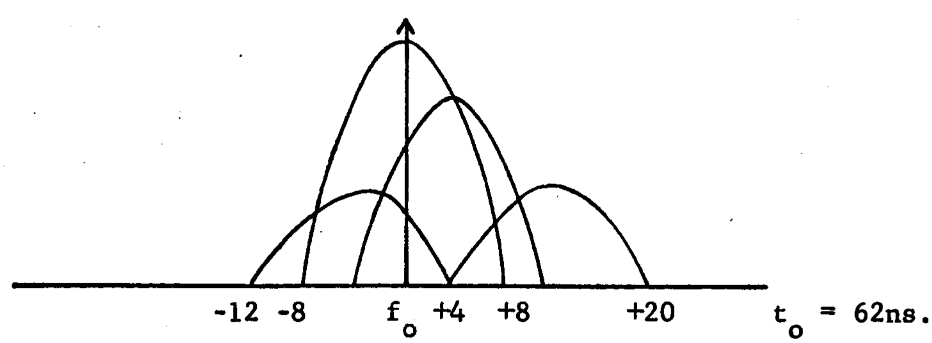


Figure 2

C) If we wish to consider a pulse train with time T between pulses, we know that the envelope of the spectrum would remain unchanged and that the spectrum would be composed of discrete lines separated by frequency $f = \frac{1}{T}$.

Analysis of the Effects of a Band Pass Filter on a Sine Squared Pulse of Half Amplitude Duration τ.

A) Frequency Spectrum of the Pulse - As the first step in the analysis we shall find the frequency spectrum of the pulse by evaluating its bilateral Fourier transform. The transform will be easier to evaluate if we pick $t = 0$ at the center of the pulse.

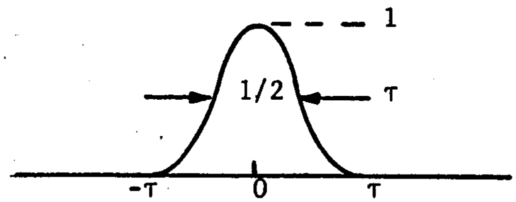


Figure 1

$$v(t) = 0, \; /t/ \geq \tau \tag{1}$$

$$v(t) = \tfrac{1}{2} \left\{ 1 + \cos\frac{\pi t}{\tau} \right\}, \; /t/ < \tau \tag{2}$$

$$V(\omega) = \int_{-\infty}^{\infty} v(t) e^{-j\omega t}\, dt = \int_{-\tau}^{\tau} v(t) e^{-j\omega t} dt \tag{3}$$

$$= \int_{-\tau}^{\tau} \cos\omega t \; v(t)\, dt - j \int_{-\tau}^{\tau} \underset{\text{even}}{v(t)} \; \underset{\text{odd} \Rightarrow \int = 0}{\sin\omega t}\, dt \tag{4}$$

$$= \tfrac{1}{2} \int_{-\tau}^{\tau} \left(1 + \cos\frac{\pi t}{\tau} \right) \cos\omega t \; dt \tag{5}$$

$$= \tfrac{1}{2} \int_{\tau}^{\tau} \cos\omega t \; dt + \tfrac{1}{2} \int_{-\tau}^{\tau} \cos\frac{\pi t}{\tau} \cos\omega t \; dt \tag{6}$$

$$= \int_{0}^{\tau} \cos\omega t \; dt + \tfrac{1}{2} \int_{0}^{\tau} \cos\left(\omega + \frac{\pi}{\tau}\right) t \; dt + \tfrac{1}{2} \int_{0}^{\tau} \cos(\omega - \pi/\tau) t \; dt \tag{7}$$

$$= \frac{\sin\omega t}{\omega} \Big/_{0}^{\tau} + \tfrac{1}{2} \frac{\sin(\omega + \pi/\tau) t}{\omega + \pi/\tau} \Big/_{0}^{\tau} + \tfrac{1}{2} \frac{\sin(\omega - \pi/\tau) t}{\omega + \pi/\tau} \Big/_{0}^{\tau} \tag{8}$$

$$V(\omega) = \frac{\sin\omega\tau}{\omega} + \tfrac{1}{2} \frac{\sin(\omega\tau + \pi)}{\omega + \pi/\tau} + \tfrac{1}{2} \frac{\sin(\omega\tau - \pi)}{\omega - \pi/\tau} \tag{9}$$

We can now visualize these terms.

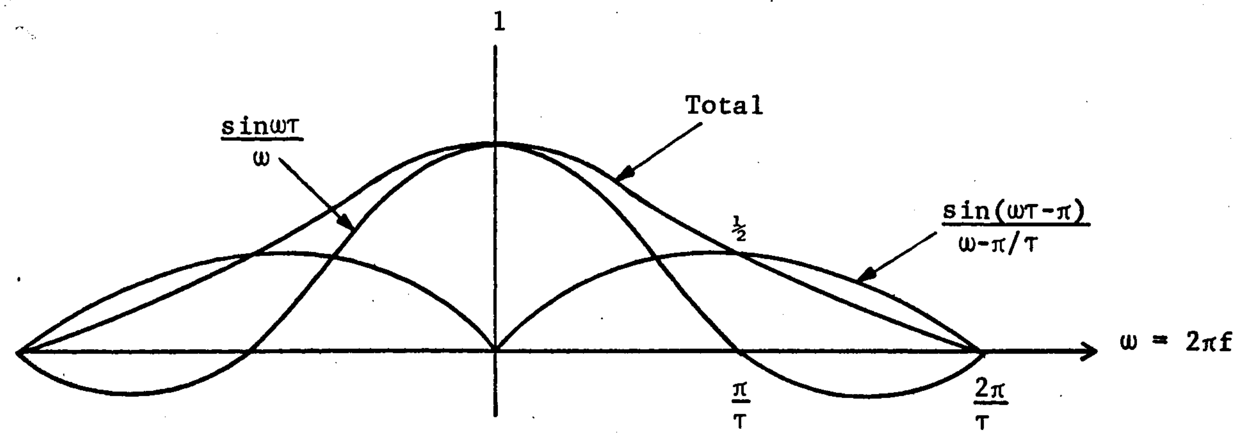


Figure 2

To interpret the spectrum as shown in Figure 2, it would be handy to insert some numbers.

If, $\tau = 62.5$ns then the 6 db point, $2\pi f = \omega = \pi/\tau$ or $f = \frac{1}{2}\pi = 8$ mc. The frequency beyond which the spectrum is essentially zero if $f = 1/\tau = 16$ mc.

B) Effect of a Low Pass Filter on the Pulse - As the second step in the analysis we pass the spectrum of the pulse through an ideal filter whose cutoff is the 6 db point of the pulse spectrum.

For example:
$$H(\omega) = 1, /\omega/ \leq \pi/\tau$$
$$H(\omega) = 0, /\omega/ > \pi/\tau \tag{10}$$

So the spectrum of the output pulse will be $V'(\omega) = H(\omega)\ V\ (\omega)$

$$V'(\omega) = V(\omega),\ /\omega/ \leq \pi/\tau$$
$$V'(\omega) = 0,\ /\omega/ > \pi/\tau \tag{11}$$

Taking the inverse Fourier transform we have:

$$v(t) = \tfrac{1}{2}\pi \int_{-\infty}^{\infty} e^{j\omega t}\, V'(\omega)\, d\omega \tag{12}$$

$$= 1/2\pi \left\{ \int_{-\pi/\tau}^{\pi/\tau} V(\omega)\cos\omega t\, d\omega + j\int_{-\pi/\tau}^{\pi/\tau} \underset{\text{even}}{V\ (\omega)}\ \underset{\text{odd}}{\sin\omega t}\, d\omega \right\} \tag{13}$$

$$\text{even} \quad \text{odd} \Rightarrow \int = 0$$

$$v(t) = 1/\pi \int_{0}^{\pi/\tau} V(\omega)\ \cos\omega t\, d\omega \tag{14}$$

$$= 1/\pi \int_{0}^{\pi/\tau} \frac{\sin\omega}{\omega}\ \cos^{(1)}\omega t\, d\omega + 1/2\pi \int_{0}^{\pi/\tau} \frac{\sin\omega\tau + \pi}{\omega + \pi/\tau} \cos^{(2)}\omega t d\omega$$
$$+ 1/2\pi \int_{0}^{\pi/\tau} \frac{\sin\omega\tau - \pi}{\omega - \pi/\tau}\ \cos^{(3)}\ \omega t\, d\omega \tag{15}$$

taking the terms one at a time.

$$(1) \quad = 1/\pi \, \frac{\sin\omega\tau}{\omega} \cos\omega t d\omega = 1/2\pi \left\{ \frac{\sin\omega(\tau+t)}{\omega} d\omega + \int_0^{\pi/\tau} \frac{\sin\omega(\tau-t)}{\omega} d\omega \right\} \quad (16)$$

$$=1/2\pi \; \mathrm{Si}\left(\frac{\pi(\tau+t)}{\tau}\right) + \mathrm{Si}\left(\frac{\pi(\tau-t)}{\tau}\right) \quad (17)$$

$$(2) \quad = 1/2\pi \int_0^{\pi/\tau} \frac{\sin\omega\tau+\pi}{\omega+\pi/\tau} \quad \cos\omega t \, d\omega \quad (18)$$

$$= 1/4\pi \left\{ \int_0^{\pi/\tau} \frac{\sin(\omega(\tau-t)+\pi)}{\omega+\pi/\tau} \; d\omega + \int_0^{\pi/\tau} \frac{\sin(\omega(\tau-t)+\pi)}{\omega+\pi/\tau} d\omega \right\} \quad (19)$$

If we introduce the variable $x = \omega + \pi/\tau \quad dx = d\omega$ and use the identity $\int_0^x \frac{\sin ax+b}{x} \; dx = \cos b \; \mathrm{Si}(ax) + \sin b \quad \mathrm{Ci}\;(ax) \Big/_0^x$

$$(2) = 1/4\pi \left\{ \cos \frac{\pi\tau}{\tau} \, \mathrm{Si} \, \frac{(\tau+t)}{\tau} - \frac{\sin\pi t}{\tau} \mathrm{Ci} \; (t+\tau)x \Big/_0^{\pi/\tau} + \frac{\cos\pi t}{\tau} \, \mathrm{Si}\left(\frac{(\tau-t)}{\tau}\right) \right.$$

$$\left. + \frac{\sin\pi t}{\tau} \; \mathrm{Ci} \; ((\tau-t)x) \Big/_0^{\pi/\tau} \right\} \quad (20)$$

$$(3) = 1/2\pi \int_0^{\pi/\tau} \frac{\sin\omega\tau-\pi}{\omega-\pi/\tau} \quad \cos \omega t \; dt \quad (21)$$

expanding as before and letting $x = \omega - \pi/\tau$.

$$(3) \quad = 1/4\pi \left\{ \frac{\cos\pi t}{\tau} \, \mathrm{Si}\left(\left(\frac{(\tau+t)\pi}{\tau}\right) + \frac{\sin\pi t}{\tau} \mathrm{Ci} \; (\; (\tau+t)x) \Big/_0^{\pi/\tau} \right. \right.$$

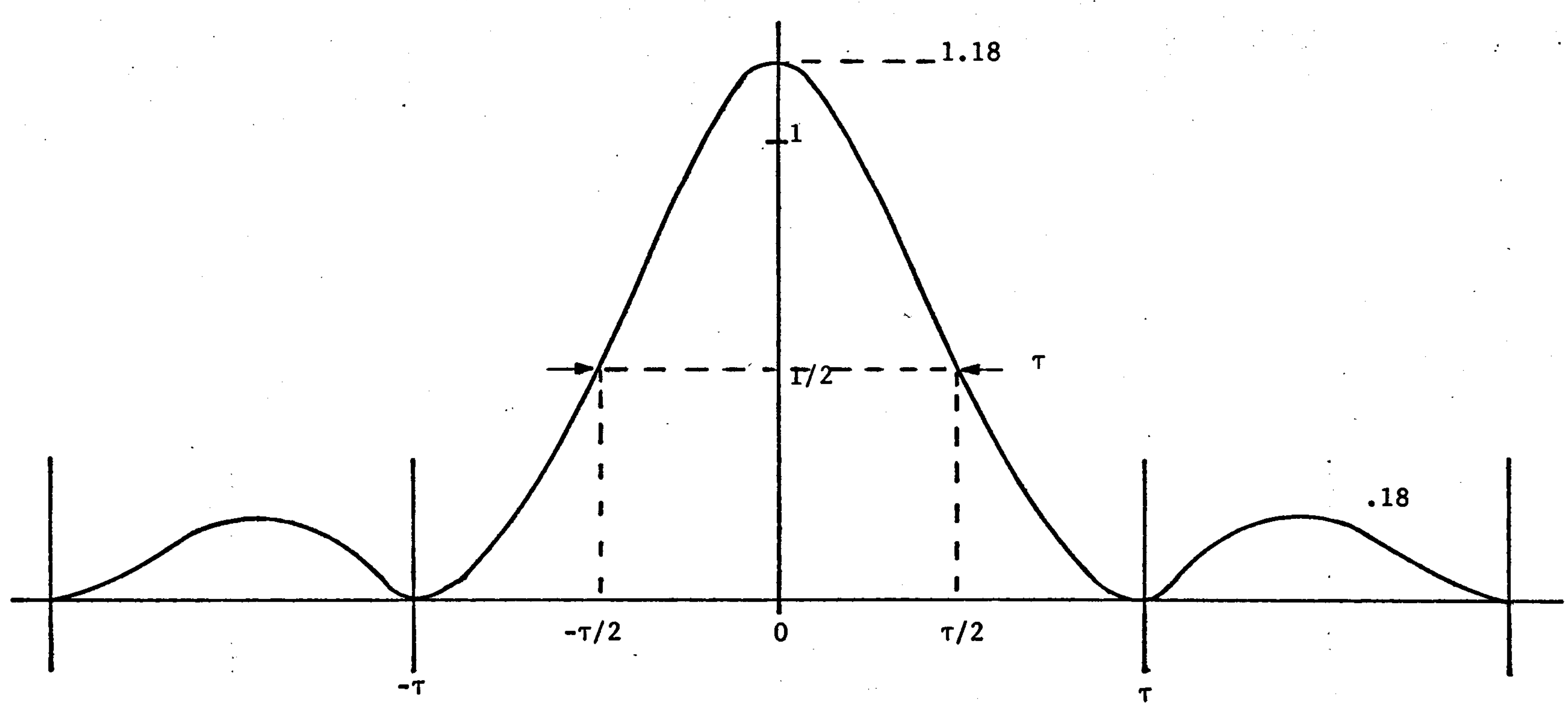
$$\left. + \; \frac{\cos\pi t}{\tau} \, \mathrm{Si}\left(\frac{(\tau-t)\pi}{\tau}\right) - \frac{\sin\pi t}{\tau} \quad \mathrm{Ci} \; (\; (\tau-t)x) \Big/_0^{\pi/\tau} \right\} \quad (22)$$

Adding (1), (2), and (3), we see that the terms with cosine integrals cancel and we have

$$v(t) = 1/2\pi \left\{ 1 + \frac{\cos\pi t}{\tau} \right\} \left\{ \mathrm{Si}\left(\frac{(t+\tau)\pi)}{\tau} \right. - \mathrm{Si} \; \left(\frac{(t-\tau)\pi}{\tau} \right) \right\} \quad (23)$$

A plot of this function is shown in Figure 3.

We see that the pulse is about 18% higher than the input pulse and the half amplitude duration of it is a little less than τ. The first lobes of the output pulse are about 18% of the input.



<u>Figure 3</u> Plot of the Response of Ideal Low Pass Filter to $\sin^2$ Pulse