TR-EE65-4
JPL CONTRACT 950670 / SUBNAS 7-100
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PURDUE UNIVERSITY
SCHOOL OF ELECTRICAL ENGINEERING

INVESTIGATION OF OPTIMIZATION OF ATTITUDE CONTROL SYSTEMS

Volume II

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PREFACE

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This volume contains listings of the computer programs used in solving the various example problems. Explanations are given at the beginning of each Appendix concerning the programs contained therein; descriptions of the general makeup of the programs and their interrelations are provided. It is not intended that this volume be independent of the first volume. Rather, use of the program listings contained herein will require that reference be made to the appropriate portions of the first volume.

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APPENDIX G

This appendix contains the program for space vehicle simulation. The standard simulation program is Main Program 1 with the following subroutines:

1. SVAD
2. TRIG
3. TA
4. ATOB
5. TB
6. BTOA
7. EULER
8. CHECK
9. DOT
10. WDOT
11. ANGDOT
12. DATA
13. PARA
14. PRINT

The standard routine AMRK is also required. An option is available which requires the use of an IBM 1620 computer with a 1443 line printer. For this option Main Program 1 is replaced by Main Program 2 and the Fortran II Routine PUNCH is added. The output from this program is the same as for the standard system with the addition of some BCD punched hollerith cards. These cards are then read into an IBM 1620 under the control of the SPS program "punch routine". This 1620 program will plot the motion of the vehicle roll axis in the initial system and the motion of the sun in the body system.

Another option is available through the replacement of Main Program 1 by Main Program 3. The printout will be similar to Main 1 output except that a plot of the rates and angles will be included. SHARE routine PLT1 must be used with this option.

Conventional sun acquisition is accomplished by control with the following subroutines:

1. CL (conventional sun acquire)
2. SUN
3. GYRO
4. SGN

Cruise mode derived rate control is accomplished by control with:

1. CL (derived rate control)

2. SUN

3. EARTH

4. DR

5. SGN

```
*       LABEL
C MA1
C1
C2      MAIN PROGRAM NO. 1
C3
4       DIMENSION NOP(4),TOL(7),DUMMY(44),PARAM(15)
5       FORMAT(3I5)
6       COMMON DUMMY,PARAM
7       READ INPUTTAPE 5,5,MROLL,IRUN,NPAR
8       IF(NPAR) 11,11,10
9       FORMAT(3E15.0)
10      READ INPUTTAPE 5,9,(PARAM(I),I=1,NPAR)
11      CALL DATA(PERIOD,TOL,NOP,CODE)
12      CALL PARA(PERIOD,TOL,NOP,CODE)
13      DO 16 I=1,IRUN
14      IN=I-1
15      CALL SVAD(IN,PERIOD,TOL,NOP)
16      CALL PRINT(IN,NOP,MROLL,CODE)
17      CALL EXIT
        END
*       LABEL
CSVAD
C1
C2      SPACE VEHICLE ATTITUDE DYNAMICS
C3
4       SUBROUTINE SVAD(IN,PERIOD,TOL,NOP)
5       DIMENSION X(7),AN(3),AI(3,3),A(3,3),XD(7),TOL(7),NOP(4),TEMPS(65),
      1EUB(6),ELB(6)
6       COMMON NSEQ,X,AN,AI,A,COSPHI,COSTH,COSPSI,SINPHI,SINTH,SINPSI,XD,N
      1OP1,NOP2
C7
C8      F CARD
C9
F       DOT
C10
C11     FIRST TIME SET-UP
C12
13      IF(IN) 14,14,40
14      NOP1=NOP(1)
15      NOP2=NOP(2)
16      TOL1=TOL(1)
17      TOL2=TOL(2)
18      HMAX=TOL(3)
19      HMIN=TOL(4)
20      EU=TOL(5)
21      EL=TOL(6)
22      HIN=TOL(7)
23      XD(1)=HIN
24      DO 25 I=1,65
25      TEMPS(I)=0.
```

```
C26
C27    DATA CHECK
C28
29     FORMAT(15H1    DATA ERROR)
30     IF(HMAX-HMIN) 35,31,31
31     IF(HMAX-HIN) 35,32,32
32     IF(HIN-HMIN) 35,33,33
33     IF(PERIOD-HIN) 35,34,34
34     IF(EU-EL) 35,35,40
35     WRITE OUTPUT TAPE 6,29
36     CALL EXIT
C37
C38    INTEGRATE
C39
40     T=X(1)
41     IF(IN) 43,43,42
42     IF(NOP(3)) 43,43,44
43     CALL TRIG
44     CALL CHECK(TOL1,TOL2)
45     DO 47 I=1,6
46     EUB(I)=EU*ABSF(X(I+1)) + EU
47     ELB(I)=EL*ABSF(X(I+1)) + EL
48     IF(T+PERIOD-X(1)-XD(1)) 49,50,50
49     XD(1)=T+PERIOD-X(1)
50     IF(NOP1)52,52,51
51     CALL CL
52     CALL DOT
53     CALL AMRK(X,XD,DOT,6,0,EUB,ELB,HMAX,HMIN,TEMPS)
54     IF(T+PERIOD-X(1)) 58,58,43
C55
C56    OPTIONS
C57
58     IF(NOP(3)) 60,60,59
59     CALL TRIG
60     IF(NOP(4)) 950,950,61
61     IF(NSEQ) 62,62,64
62     CALL TA
63     GO TO 950
64     CALL TB
950    RETURN
       END
*      LABEL
CTRIG
C1
C2     SUBROUTINE FOR EULER ANGLE TRIG FUNCTIONS
C3
4      SUBROUTINE TRIG
5      DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7)
6      COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPH
      1I,SINTH,SINPSI,XD,NOP1,NOP2
```

```
C
      COSPHI=COSF(PHI)
      COSTH=COSF(THETA)
      COSPSI=COSF(PSI)
      SINPHI=SINF(PHI)
      SINTH=SINF(THETA)
      SINPSI=SINF(PSI)
950   RETURN
      END
*     LABEL
CTA
C
C     SUBROUTINE FOR MATRIX TRANSFORMATION , SEQ. A
C
      SUBROUTINE TA
      DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7)
      COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPH
     1I,SINTH,SINPSI,XD,NOP1,NOP2
C
      A(1,1)=COSPSI*COSPHI-SINPSI*COSTH*SINPHI
      A(1,2)=COSPSI*SINPHI+SINPSI*COSTH*COSPHI
      A(1,3)=SINPSI*SINTH
      A(2,1)=-SINPSI*COSPHI-COSPSI*COSTH*SINPHI
      A(2,2)=-SINPSI*SINPHI+COSPSI*COSTH*COSPHI
      A(2,3)=COSPSI*SINTH
      A(3,1)=SINTH*SINPHI
      A(3,2)=-SINTH*COSPHI
      A(3,3)=COSTH
950   RETURN
      END
*     LABEL
CATOB
C
C     CHANGE SEQUENCE A TO B
C
      SUBROUTINE ATOB(ER,TOL)
      DIMENSION W(3),AN(3),AI(3,3),B(3,3),XD(7)
      COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,B,COSPHI,COSTH,COSPSI,SINPH
     1I,SINTH,SINPSI,XD,NOP1,NOP2
C
      L=1
      SINTH=B(2,3)
      COSTH=1.-SINTH**2
      COSTH=SQRTF(COSTH)
      SINPSI=-B(1,3)/COSTH
      SINPHI=-B(2,1)/COSTH
      COSPSI=B(3,3)/COSTH
      COSPHI=B(2,2)/COSTH
      ER=COSPSI*SINPHI+SINPSI*SINTH*COSPHI
      ER=ABSF(ER-B(1,2))
```

```
18      GO TO (19,30),L
19      IF(TOL-ER) 23,950,950
C20
C21     WRONG QUADRANT
C22
23      SINPHI=-SINPHI
24      COSPHI=-COSPHI
25      SINPSI=-SINPSI
26      COSPSI=-COSPSI
27      COSTH=-COSTH
28      L=2
29      GO TO 16
30      IF(TOL-ER) 31,950,950
31      WRITE OUTPUTTAPE 6,33
32      CALL EXIT
33      FORMAT(16H1    ERROR A TO B)
950     RETURN
        END
*       LABEL
CEULER
C1
C2      QUADRANT ANALYSIS AND EULERANGLE
C3
4       SUBROUTINE EULER
5       DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7)
6       COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPH
       1I,SINTH,SINPSI,XD,NOP1,NOP2
C7
8       TPHI=ABSF(SINPHI/COSPHI)
9       TTH=ABSF(SINTH/COSTH)
10      TPSI=ABSF(SINPSI/COSPSI)
11      PHI=ATANF(TPHI)
12      THETA=ATANF(TTH)
13      PSI=ATANF(TPSI)
14      PI=3.1415927
15      S=SINPHI
16      C=COSPHI
17      ANG=PHI
18      L=1
19      IF(S) 23,20,20
20      IF(C) 21,27,27
21      ANG=PI-ANG
22      GO TO 27
23      IF(C) 26,24,24
24      ANG=-ANG
25      GO TO 27
26      ANG=-PI+ANG
27      GO TO (28,34,40),L
28      PHI=ANG
29      L=2
```

```
30    C=COSTH
31    S=SINTH
32    ANG=THETA
33    GO TO 19
34    THETA=ANG
35    L=3
36    C=COSPSI
37    S=SINPSI
38    ANG=PSI
39    GO TO 19
40    PSI=ANG
950   RETURN
      END
*     LABEL
CTB
C1
C2    SUBROUTINE FOR MATRIX TRANSFORMATION , SEQ B
C3
4     SUBROUTINE TB
5     DIMENSION W(3),AN(3),AI(3,3),B(3,3),XD(7)
6     COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,B,COSPHI,COSTH,COSPSI,SINPH
     1I,SINTH,SINPSI,XD,NOP1,NOP2
C7
8     B(1,1)=COSPSI*COSPHI-SINPSI*SINTH*SINPHI
9     B(1,2)=COSPSI*SINPHI+SINPSI*SINTH*COSPHI
10    B(1,3)=-SINPSI*COSTH
11    B(2,1)=-COSTH*SINPHI
12    B(2,2)=COSTH*COSPHI
13    B(2,3)=SINTH
14    B(3,1)=SINPSI*COSPHI+COSPSI*SINTH*SINPHI
15    B(3,2)=SINPSI*SINPHI-COSPSI*SINTH*COSPHI
16    B(3,3)=COSPSI*COSTH
950   RETURN
      END
*     LABEL
CBTOA
C1
C2    CHANGE SEQUENCE B TO A
C3
4     SUBROUTINE BTOA(ER,TOL)
5     DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7)
6     COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPH
     1I,SINTH,SINPSI,XD,NOP1,NOP2
C7
8     L=1
9     COSTH=A(3,3)
10    SINTH=1.-COSTH**2
11    SINTH=SQRTF(SINTH)
12    SINPSI=A(1,3)/SINTH
13    COSPSI=A(2,3)/SINTH
```

```
14      SINPHI=A(3,1)/SINTH
15      COSPHI=-A(3,2)/SINTH
16      ER=COSPSI*SINPHI+SINPSI*COSTH*COSPHI
17      ER=ABSF(ER-A(1,2))
18      GO TO (19,30),L
19      IF(TOL-ER)23,950,950
C20
C21     WRONG QUADRANT
C22
23      SINTH=-SINTH
24      COSPHI=-COSPHI
25      SINPHI=-SINPHI
26      COSPSI=-COSPSI
27      SINPSI=-SINPSI
28      L=2
29      GO TO 16
30      IF(TOL-ER) 31,950,950
31      WRITE OUTPUT TAPE 6,33
32      CALL EXIT
33      FORMAT(16H1   ERROR B TO A)
950     RETURN
        END
*       LABEL
CCHK
C1
C2      SEQUENCE MAINTENANCE AND NORMALIZATION
C3
4       SUBROUTINE CHECK(TOL1,TOL2)
5       DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7)
6       COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPH
       1I,SINTH,SINPSI,XD,NOP1,NOP2
C7
C8      NORMALIZE EULER ANGLES
C9
10      PI=3.1415927
11      L=1
12      ANG=PHI
13      IF(PI-ABSF(ANG)) 14,14,18
14      IF(ANG) 15,15,17
15      ANG=ANG+2.*PI
16      GO TO 18
17      ANG=ANG-2.*PI
18      GO TO (19,23,27),L
19      PHI=ANG
20      L=2
21      ANG=THETA
22      GO TO 13
23      THETA=ANG
24      L=3
25      ANG=PSI
```

```
      GO TO 13
      PSI=ANG
C
C     CHECK FOR SEQUENCE CHANGE
C
      IF(NSEQ) 35,35,49
C
C     SEQUENCE A
C
      DIF=ABSF(THETA)
      IF(TOL1-DIF) 37,42,42
      DIF=PI-DIF
      IF(TOL1-DIF) 950,42,42
C
C     SWITCH TO SEQ. B
C
      CALL TA
      CALL ATOB(ER,TOL2)
      CALL EULER
      NSEQ=1
950   RETURN
C
C     SEQUENCE B
C
      DIF=ABSF(THETA)
      DIF=ABSF(PI/2.-DIF)
      IF(TOL1-DIF) 950,55,55
C
C     CHANGE TO SEQ. A
C
      CALL TB
      CALL BTOA(ER,TOL2)
      CALL EULER
      NSEQ=0
      GO TO 950
      END
*     LABEL
CDOT
C
C
C     VECTOR AUXSUB FOR AMRK
C
C     NOTE--- X(1) = TIME , XD(1) = H
C
      SUBROUTINE DOT
      DIMENSION X(7),AN(3),AI(3,3),A(3,3),XD(7),WD(3),ADOT(3)
      COMMON NSEQ,X,AN,AI,A,COSPHI,COSTH,COSPSI,SINPHI,SINTH,SINPSI,XD,N
     1OP1,NOP2
C
C     DETERMINE TORQUES
```

```
C12
13      IF(NOP1) 14,14,18
14      CALL CL
C15
C16     ASSEMBLE BODY RATES
C17
18      CALL WDOT(WD)
19      DO 20 I=1,3
20      XD(I+4)=WD(I)
C21
C22     ASSEMBLE EULER RATES
C23
24      IF(NOP2) 28,28,32
C25
C26     UPDATE TRIG FUNCTIONS
C27
28      CALL TRIG
C29
C30     ENTER AT 32 FOR NO UPDATE
C31
32      CALL ANGDOT(ADOT)
33      DO 34 I=1,3
34      XD(I+1)=ADOT(I)
950     RETURN
        END
*       LABEL
CWDOT
C1
C2      BODY RATES
C3
4       SUBROUTINE WDOT(WD)
5       DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7),TS(3,3),WD(3),C(3),EL(3)
501     COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPH
       1I,SINTH,SINPSI,XD,NOP1,NOP2
6       DO 7 I=1,3
7       EL(I)=0.
8       DO 10 I=1,3
9       DO 10 J=1,3
10      EL(I)=EL(I)+AI(I,J)*W(J)
11      L=1
12      DO 14 I=1,3
13      DO 14 J=1,3
14      TS(I,J)=AI(I,J)
15      GO TO (47,32,38),L
16      DI=DET
C17
C18     COMPUTE COLUMN
C19
20      C(1)=AN(1)+W(3)*EL(2)-W(2)*EL(3)
21      C(2)=AN(2)+W(1)*EL(3)-W(3)*EL(1)
```

```
22      C(3)=AN(3)+W(2)*EL(1)-W(1)*EL(2)
C23
C24     WDOTS
C25
26      DO 27 I=1,3
27      TS(I,1)=C(I)
28      L=2
29      GO TO 47
30      WD(1)=DET/DI
31      GO TO 12
32      DO 33 I=1,3
33      TS(I,2)=C(I)
34      L=3
35      GO TO 47
36      WD(2)=DET/DI
37      GO TO 12
38      DO 39 I=1,3
39      TS(I,3)=C(I)
40      L=4
41      GO TO 47
42      WD(3)=DET/DI
950     RETURN
C43
C44
C45     DETERMINANT
C46
47      DET=TS(1,1)*TS(2,2)*TS(3,3)
48      DET=DET+TS(1,2)*TS(2,3)*TS(3,1)
49      DET=DET+TS(1,3)*TS(2,1)*TS(3,2)
50      DET=DET-TS(3,1)*TS(2,2)*TS(1,3)
51      DET=DET-TS(2,1)*TS(1,2)*TS(3,3)
52      DET=DET-TS(1,1)*TS(2,3)*TS(3,2)
53      GO TO (16,30,36,42),L
        END
*       LABEL
CADOT
C1
C2      EULER ANGLE DERIVATIVES (ANGDOT)
C3
4       SUBROUTINE ANGDOT(ADOT)
5       DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7),ADOT(3)
6       COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPH
       1I,SINTH,SINPSI,XD,NOP1,NOP2
7       IF(NSEQ) 11,11,17
C8
C9      SEQUENCE A
C10
11      ADOT(1)=(W(1)*SINPSI+W(2)*COSPSI)/SINTH
12      ADOT(2)=W(1)*COSPSI-W(2)*SINPSI
13      ADOT(3)=W(3)-ADOT(1)*COSTH
```

```
950    RETURN
C14
C15    SEQUENCE B
C16
17     ADOT(1)=(W(3)*COSPSI-W(1)*SINPSI)/COSTH
18     ADOT(2)=W(3)*SINPSI+W(1)*COSPSI
19     ADOT(3)=W(2)-ADOT(1)*SINTH
951    RETURN
       END
*      LABEL
CDATA
C1
C2     DATA INPUT ROUTINE
C3
4      SUBROUTINE DATA(PERIOD,TOL,NOP,CODE)
5      DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7),NOP(4),TOL(7)
6      COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPH
     1I,SINTH,SINPSI,XD,NOP1,NOP2
7      FORMAT(A4)
8      FORMAT(3E15.0)
9      FORMAT(4I5)
10     FORMAT(I5,2E15.0)
11     READ INPUT TAPE 5,7,CODE
12     READ INPUT TAPE 5,8,((AI(I,J),J=1,3),I=1,3)
13     READ INPUT TAPE 5,8,(W(I),I=1,3)
14     READ INPUT TAPE 5,8,PHI,THETA,PSI
15     READ INPUT TAPE 5,9,(NOP(I),I=1,4)
16     READ INPUT TAPE 5,8,TOL(3),TOL(4),TOL(7)
17     READ INPUT TAPE 5,8,TOL(5),TOL(6),TOL(1)
18     READ INPUT TAPE 5,10,NSEQ,TIME,PERIOD
19     TOL(2)=0.0001
950    RETURN
       END
*      LABEL
CPARA
C1
C2     PRINT OUT ROUTINE,PARAMETERS
C3
4      SUBROUTINE PARA(PERIOD,TOL,NOP,CODE,NPAR)
5      DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7),NOP(4),TOL(7),PARAM(15)
6      COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPH
     1I,SINTH,SINPSI,XD,NOP1,NOP2,PARAM
7      FORMAT(1H1/16X,15HJOB CODE NAME- ,A4///)
8      FORMAT(12X,27HINERTIA TENSOR (SLUG FT**2)//)
9      FORMAT(8X,3E11.4)
10     FORMAT(//17X,18HINITIAL CONDITIONS//)
11     FORMAT(53H TIME NSEQ    PHI    THETA    PSI      W1      W2       W3//)
12     FORMAT(F5.1,I4,F8.3,F7.3,F7.3,1X,3F8.5)
13     FORMAT(//22X,7HOPTIONS//)
14     FORMAT(12X,27HNOP(1) NOP(2)NOP(3) NOP(4)/)
```

```
15     FORMAT(12X,I4,3I7//)
16     FORMAT(17X,18HCONTROL PARAMETERS//)
17     FORMAT(52H    HMAX     HMIN     HIN       EU        EL       DZ/)
18     FORMAT(1X,6(1X,E8.2)//)
19     FORMAT(15X,9HPERIOD = ,E14.8)
2Ø     WRITE OUTPUT TAPE 6,7,CODE
21     WRITE OUTPUT TAPE 6,8
22     WRITE OUTPUT TAPE 6,9,((AI(I,J),J=1,3),I=1,3)
23     WRITE OUTPUT TAPE 6,1Ø
24     WRITE OUTPUT TAPE 6,11
25     WRITE OUTPUT TAPE 6,12,TIME,NSEQ,PHI,THETA,PSI,(W(I),I=1,3)
26     WRITE OUTPUT TAPE 6,13
27     WRITE OUTPUT TAPE 6,14
28     WRITE OUTPUT TAPE 6,15,(NOP(I),I=1,4)
29     WRITE OUTPUT TAPE 6,16
3Ø     WRITE OUTPUT TAPE 6,17
31     WRITE OUTPUT TAPE 6,18,TOL(3),TOL(4),TOL(7),TOL(5),TOL(6),TOL(1)
32     WRITE OUTPUT TAPE 6,19,PERIOD
33     FORMAT(//9X,3E15.6)
34     IF(NPAR) 95Ø,95Ø,35
35     WRITE OUTPUT TAPE 6,33,(PARAM(I),I=1,NPAR)
95Ø    RETURN
       END
*      LABEL
CPRT
C1
C2     PROGRAM FOR PRINT OUT
C3
4      SUBROUTINE PRINT(IN,NOP,MROLL,CODE)
5      DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7),NOP(4),TOL(7),Q(3)
6      COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,COSPHI,COSTH,COSPSI,SINPH
      1I,SINTH,SINPSI,XD,NOP1,NOP2
7      FORMAT(1H1,17X,13HDATA FOR JOB ,A4//)
8      FORMAT(23X,A4,I3//)
9      FORMAT(52H ANGULAR VELOCITIES  EULER ANGLES   POSITION OF SUN )
1Ø     FORMAT(48H    (MRAD/SEC)          (RAD)              AXIS  //)
11     FORMAT(2ØX,11HSEQUENCE = ,I2/)
12     FORMAT(42H NO ROLL AXIS POSITION AVAILABLE, NOP(4)=Ø/)
13     FORMAT(7H  W1 = ,F9.4,4X,7HPHI   = ,F7.4)
14     FORMAT(7H  W2 = ,F9.4,4X,7HTHETA= ,F7.4)
15     FORMAT(7H  W3 = ,F9.4,4X,7HPSI   = ,F7.4//)
16     FORMAT(7H  W1 = ,F9.4,4X,7HPHI   = ,F7.4,5X,4HX = ,F6.3)
17     FORMAT(7H  W2 = ,F9.4,4X,7HTHETA= ,F7.4,5X,4HY = ,F6.3)
18     FORMAT(7H  W3 = ,F9.4,4X,7HPSI   = ,F7.4,5X,4HZ = ,F6.3//)
19     FORMAT(19X,7HTIME = ,F8.2)
19Ø1   DO 19Ø2 I=1,3
19Ø2   Q(I)=1ØØØ.*W(I)
2Ø     IF(IN) 21,21,3Ø
21     IPG=Ø
22     IF(MROLL) 23,23,25
```

```
23      ICK=Ø
24      GO TO 29
25      IF(NOP(4)) 26,26,28
26      ICK=-1
27      GO TO 29
28      ICK=1
29      ILC=Ø
29Ø1    GO TO 31
3Ø      IF(8-ILC) 31,31,4Ø
31      ISQ=2
32      IPG=IPG+1
33      ILC=Ø
34      WRITE OUTPUT TAPE 6,7,CODE
35      WRITE OUTPUT TAPE 6,8,CODE,IPG
36      WRITE OUTPUT TAPE 6,9
37      WRITE OUTPUT TAPE 6,1Ø
38      IF(ICK) 39,4Ø,4Ø
39      WRITE OUTPUT TAPE 6,12
4Ø      IF(ISQ-NSEQ) 41,43,41
41      WRITE OUTPUT TAPE 6,11,NSEQ
42      ISQ=NSEQ
43      IF(ICK) 44,44,49
44      WRITE OUTPUT TAPE 6,19,TIME
45      WRITE OUTPUT TAPE 6,13,Q(1),PHI
46      WRITE OUTPUT TAPE 6,14,Q(2),THETA
47      WRITE OUTPUT TAPE 6,15,Q(3),PSI
48      ILC=ILC+1
95Ø     RETURN
49      WRITE OUTPUTTAPE 6,19,TIME
5Ø      WRITE OUTPUT TAPE 6,16,Q(1),PHI,A(1,3)
51      WRITE OUTPUTTAPE 6,17,Q(2),THETA,A(2,3)
52      WRITE OUTPUT TAPE 6,18,Q(3),PSI,A(3,3)
53      GO TO 48
        END
```

```
*       LABEL
CCL1
C1
C2      CONVENTIONAL SUN ACQUIRE
C3
4       SUBROUTINE CL
5       DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7),DUMMY(6),ADZ(3),T(3)
6       COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,DUMMY,XD,NOP1,NOP2,
7      1 SLR,SSV,PYGG,RGG,T,ADZ
8       CALL GYRO(PRO,YRO,RRO)
9       CALL SUN(PSS,YSS)
10      L=1
11      ER=-(PRO+PSS)
12      IF(ABSF(ER)-ADZ(L)) 13,13,15
13      AN(L)=0.
14      GO TO 18
15      X=T(L)
16      CALL SGN(X,X,ER)
17      AN(L)=X
18      GO TO (19,22,950),L
19      L=2
20      ER=-(YRO+YSS)
21      GO TO 12
22      L=3
23      ER=-RRO
24      GO TO 12
950     RETURN
        END
*       LABEL
CSUN
C1
C2      SUN SENSOR ROUTINE
C3
4       SUBROUTINE SUN(PSS,YSS)
5       DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7),DUMMY(6),ADZ(3),T(3)
6       COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,DUMMY,XD,NOP1,NOP2,
       1 SLR,SSV,PYGG,RGG,T,ADZ
7       CALL TRIG
8       IF(NSEQ) 9,9,11
9       CALL TA
10      GO TO 12
11      CALL TB
12      X=A(1,3)
13      Y=A(2,3)
14      Z=A(3,3)
15      L=1
16      ARG=ABSF(Y/Z)
17      PA=ATANF(ARG)
18      ARG=ABSF(X/Z)
19      YA=ATANF(ARG)
```

```
20      IF(Z) 21,23,23
21      PA=3.1415927-PA
22      YA=3.1415927-YA
23      Q=-X
24      CALL SGN(PA,PA,Y)
25      CALL SGN(YA,YA,Q)
26      ANG=PA
27      IF(ABSF(ANG)-SLR) 31,28,28
28      IF(ABSF(ANG)-3.1415927+SLR) 29,29,33
29      CALL SGN(OUT,1.,ANG)
30      GO TO 36
31      OUT=(1./SLR)*ANG
32      GO TO 36
33      Q=3.1415927-ABSF(ANG)
34      CALL SGN(Q,Q,ANG)
35      OUT=(1./SLR)*Q
36      GO TO (37,41),L
37      L=2
38      ANG=YA
39      PSS=SSV*OUT*SQRTF(Y**2+Z**2)
40      GO TO 27
41      YSS=SSV*OUT*SQRTF(X**2+Z**2)
950     RETURN
        END
CGYRO
C1
C2      GYROSCOPE OUTPUTS
C3
4       SUBROUTINE GYRO(PRO,YRO.RRO)
5       DIMENSION W(3),AN(3),AI(3,3),A(3,3),XD(7),DUMMY(6),ADZ(3),T(3)
6       COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,DUMMY,XD,NOP1,NOP2,
7      1 SLR,SSV,PYGG,RGG,T,ADZ
        PRO=PYGG*W(1)
9       YRO=PYGG*W(2)
10      RRO=RGG*W(3)
950     RETURN
        END
*       LABEL
CSIGN
C1
C2      SUBROUTINE FOR SIGN TRANSFER
C3
4       SUBROUTINE SGN(X,Y,Z)
5       IF(Z) 7,6,6
6       X=ABSF(Y)
950     RETURN
7       X=-ABSF(Y)
951     RETURN
        END
```

```
*      LABEL
*      LABEL
CDRCL
C1
C2     DERIVED RATE CONTROL
C3
4      SUBROUTINE CL
5      DIMENSION W(3),AN(3),AI(3,3),A(3,3),DUMMY(6),XD(7),ER(3),ST(3),TC(
      13),TD(3),DRG(3),T(3),ADZ(3)
6      COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,DUMMY,XD,NOP1,NOP2,SLR,SS
      1V,ESV,ESLR,ESA,EAT,TC,TD,DRG,ER,ST,T,LT,ADZ,ONTM
7      CALL SUN(PSS,YSS)
8      IF(EAT-TIME) 9,9,11
9      CALL EARTH(ESO)
10     GO TO 12
11     ESO=0.
12     CALL DR(PRO,YRO,RRO)
13     NOP1=1
14     L=1
15     ER(L)=-(PRO+PSS)
16     IF(ABSF(ER(L))-ADZ(L)) 17,17,1801
17     AN(L)=ST(L)
1701   IF(IND) 22,22,1702
1702   XD(1)=RESTOR
1703   IND=0
18     GO TO 22
1801   IF(IND) 1802,1802,1803
1802   RESTOR=XD(1)
1803   IND=1
19     XD(1)=ONTM
20     CALL SGN(AN(L),T(L),ER(L))
21     AN(L)=AN(L)+ST(L)
22     GO TO (23,26,950),L
23     L=2
24     ER(L)=-(YRO+YSS)
25     GO TO 16
26     L=3
27     ER(L)=-(ESO+RRO)
28     GO TO 16
950    RETURN
       END
*      LABEL
CDR
C1
C2     DERIVED RATES
C3
4      SUBROUTINE DR(PRO,YRO,RRO)
5      DIMENSION W(3),AN(3),AI(3,3),A(3,3),DUMMY(6),XD(7),ER(3),ST(3),TC(
      13),TD(3),DRG(3),T(3),ADZ(3)
6      COMMON NSEQ,TIME,PHI THETA,PSI,W,AN,AI,A,DUMMY,XD,NOP1,NOP2,SLR,SS
```

```
      1V,ESV,ESLR,ESA,EAT,TC,TD,DRG,ER,ST,T,TLT,ADZ
7        L=1
8        VAR=PRO
9        DT=TIME-TLT
10       ERR=AN(L)-ST(L)
11       IF(ABSF(ERR)-0.0001) 12,12,14
12       VAR=VAR-DT*VAR/TD(L)
13       GO TO 18
14       IF(AN(L)) 15,17,17
15       VAR=VAR-VAR*(DT/TC(L))-DRG(L)*(DT/TC(L))
16       GO TO 18
17       VAR=VAR-VAR*(DT/TC(L))+DRG(L)*(DT/TC(L))
18       GO TO(19,23,27),L
19       PRO=VAR
20       L=2
21       VAR=YRO
22       GO TO 10
23       YRO=VAR
24       VAR=RRO
25       L=3
26       GO TO 10
27       RRO=VAR
28       TLT=TIME
950      RETURN
         END
*        LABEL
CEART    H
C1
C2       EARTH SENSOR
C3
4        SUBROUTINE EARTH(ESO)
5        DIMENSION W(3),AN(3),AI(3,3),A(3,3),DUMMY(6),XD(7),ER(3),ST(3),TC(
      13),TD(3),DRG(3),T(3),ADZ(3)
6        COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A,DUMMY,XD,NOP1,NOP2,SLR,SS
      1V,ESV,ESLR,ESA,EAT,TC,TD,DRG,ER,ST,T,LT,ADZ
7        ARG=ABSF(A(2,1)/A(1,1))
8        EA=ATANF(ARG)
9        IF(A(1,1)) 10,11,11
10       EA=3.1415927-EA
11       Q=-A(2,1)
12       CALL SGN(EA,EA,Q)
13       IF(ESA-ABSF(EA)) 14,16,16
14       ESO=0.
15       GO TO 950
16       IF(ABSF(EA)-ESLR) 19,17,17
17       CALL SGN(ESO,ESV,EA)
18       GO TO 950
19       ESO=(ESV/ESLR)*EA
950      RETURN
         END
```

```
*       LABEL
CMA2
C1
C2      MAIN PROGRAM NO 2
C3
4       DIMENSION NOP(4),TOL(7),DUMMY(44),PARAM(15)
5       FORMAT(3I5)
6       COMMON DUMMY,PARAM
7       READ INPUT TAPE 5,5,MROLL,IRUN,NPAR
8       IF(NPAR) 11,11,1Ø
9       FORMAT(3E15.Ø)
1Ø      READ INPUT TAPE 5,9,(PARAM(I),I=1,NPAR)
11      CALL DATA(PERIOD,TOL,NOP,CODE)
12      NOP(3)=1
13      NOP(4)=1
14      CALL PARA(PERIOD,TOL,NOP,CODE)
15      DO 19 I=1,IRUN
16      IN=I-1
17      CALL SVAD(IN,PERIOD,TOL,NOP)
18      CALL PRINT(IN,NOP,MROLL,CODE)
19      CALL PUNCH(IN,IRUN,CODE)
2Ø      CALL EXIT
        END
CMA3
C1
C2      MAIN PROGRAM NO. 3
C3
4       DIMENSION NOP(4),TOL(7),DUMMY(43),PARAM(15),T(1ØØ),X1(1ØØ),X2(1ØØ)
       1,X3(1ØØ),X4(1ØØ),X5(1ØØ),X6(1ØØ),TS(2Ø2Ø)
5       FORMAT(3I5)
6       COMMON NSEQ,DUMMY,PARAM
7       READ INPUT TAPE 5,5,MROLL,IRUN,NPAR
8       IF(NPAR) 11,11,1Ø
9       FORMAT(3E15.Ø)
1Ø      READ INPUT TAPE 5,9,(PARAM(I),I=1,NPAR)
11      CALL DATA(PERIOD,TOL,NOP,CODE)
12      CALL PARA(PERIOD,TOL,NOP,CODE,NPAR)
13      DO 23 I=1,IRUN
14      IN=I-1
15      CALL SVAD(IN,PERIOD,TOL,NOP)
16      CALL PRINT(IN,NOP,MROLL,CODE)
17      X1(I)=DUMMY(2)
18      X2(I)=DUMMY(3)
19      X3(I)=DUMMY(4)
2Ø      X4(I)=DUMMY(5)*1ØØØ.
21      X5(I)=DUMMY(6)*1ØØØ.
22      X6(I)=DUMMY(7)*1ØØØ.
23      T(I)=FLOATF(I)
24      FORMAT(1H1//5ØX,3ØHW1 = *, W2 = Ø,W3 = X, (MRAD)//)
25      WRITE OUTPUT TAPE 6,24
```

```
26      M=FLOATF(IRUN)/1Ø. + .5
27      CALL PLT1(IERR,TS,3,M,Ø.,1Ø.*FLOATF(M),-25.,25.,T,IRUN,X4,T,IRUN,X
       15,T,IRUN,X6)
28      FORMAT(1H1//47X,33HPHI = *, THETA = Ø, PSI = X (RAD)//)
29      WRITE OUTPUT TAPE 6,28
3Ø      CALL  PLT1(IERR,TS,3,M,Ø.,1Ø.*FLOATF(M),-5.,5.,T,IRUN,X1,T,IRUN,X2
       1,T,IRUN,X3)
31      CALL EXIT
        END
*       LABEL
CPCH
C1
C2      PUNCH ROUTINE 7Ø9Ø/94
C3
4       SUBROUTINE PUNCH(IN,IRUN,CODE)
5       DIMENSION W(3),AN(3),AI(3,3),A(3,3),N(18)
6       COMMON NSEQ,TIME,PHI,THETA,PSI,W,AN,AI,A
7       FORMAT(A4)
8       FORMAT(I3)
9       FORMAT(18I4)
1Ø      IF(IN) 11,11,15
11      PUNCH 7,CODE
12      PUNCH 8,IRUN
13      IC3=Ø
14      IC=Ø
15      IC=IC+1
16      IC3=IC3+1
17      K=6*(IC3-1)
18      N(K+1)=A(1,3)*25.+ .5
19      N(K+2)=A(2,3)*15.+ .5
2Ø      N(K+3)=A(3,3)*1ØØ. + .5
21      N(K+4)=A(3,1)*25.+ .5
22      N(K+5)=A(3,2)*15.+ .5
23      N(K+6)=N(K+3)
24      IF(IRUN-IC) 27,27,25
25      IF(3-IC3) 26,26,95Ø
26      IC3=Ø
27      L=K+6
28      PUNCH 9,(N(I),I=1,L)
95Ø     RETURN
        END
ØØØØ1*        PLOT ROUTINE
ØØØØ2*
ØØØØ3ENTRY RACDPA1
ØØØØ4         TFM A+49,16,1Ø
ØØØØ5         TFM A+54,ADR1
ØØØØ6         TDM PA1+7,Ø
ØØØØ7         TD  PA1+8,RM
ØØØØ8         TR  TITLE2-1,PA1-1
ØØØØ9         K   Ø,971
```

```
         K    Ø,953
         TDM  TITLE2+8,Ø
         WA   TITLE1,9ØØ
         TR   PA1-1,RESET-1
         TFM  CTR,Ø,1Ø
A        WA   PA1,9ØØ
         AM   CTR,1,1Ø
         CM   CTR,8,1Ø
         BNZ  A
         TFM  ADR1,YAW-1
         TFM  ADR2,CLR-1
         TFM  ADR3,R91-1
         BTM  SETUP,Ø
         B    B
*
         DS   5
SETUP    TF   S1+11,ADR1
         TF   S2+11,ADR2
         TF   S3+11,ADR1
         TF   S4+11,ADR3
S1       TR   PA1-1,*
S2       TR   PA31-1,*
S3       TR   PA61-1,*
S4       TR   PA91-1,*
         BB
*
B        WA   PA1,9ØØ
         TFM  ADR1,YONE-1
         BTM  SETUP,Ø
         WA   PA1,9ØØ
*
*        INITIALIZE PRINT AREA
*
         TFM  CTR,Ø,1Ø
         TFM  CTR1,Ø,1Ø
         TFM  ADR4,PRINT-1
         TFM  ADR1,YMK-1
C        BTM  SETUP,Ø
         TF   S5+6,ADR4
S5       TR   *,PA1-1
         AM   CTR,1,1Ø
         AM   CTR1,1,1Ø
         AM   ADR4,24Ø
         TFM  ADR1,Y-1
         CM   CTR,31,1Ø
         BZ   D
         CM   CTR1,5,1Ø
         BNZ  C
         TFM  CTR1,Ø,1Ø
         TFM  ADR1,YMK-1
```

```
00060      B    C
00061*
00062D     TFM ADR1,Y-1
00063      TFM ADR2,PITCH1-1
00064      TFM ADR3,PITCH2-1
00065      BTM SETUP,0
00066      TR   PRINT+3359,PA1-1
00067      TFM ADR1,XL-1
00068      TFM ADR2,XR1-1
00069      TFM ADR3,XR2-1
00070      BTM SETUP,0
00071      TR   PRINT+3599,PA1-1
00072      TFM ADR1,Y-1
00073      TFM ADR2,XONE1-1
00074      TFM ADR3,XONE2-1
00075      BTM SETUP,0
00076      TR   PRINT+3839,PA1-1
00077      TDM PRINT+4080,0
00078      B    G
00079E     TFM CTR,0,10
00080      TFM ADR4,PRINT
00081F     TF   S6+6,ADR4
00082S6    WA   *,900
00083      AM   ADR4,240
00084      AM   CTR,1,10
00085      CM   CTR,31,10
00086      BNZ F
00087      TFM A+49,49,10
00088      TFM A+54,*+24
00089      B    A-24
00090      K    0,952
00091      WA   AC2,900
00092      WA   AC3,900
00093      WA   AC4,900
00094      WA   AC5,900
00095      RCTY
00096      WATYAC
00097      H
00098      BC1 ENTRY
00099      H
00100AC    DAC 25,SWITCH ONE ON FOR REPEAT@
00101AC2   DAC 30,THE --BODY-- PLOT SHOWS THE BO,
00102      DAC 30,DY FIXED AXES WITH THE TIP OF ,
00103      DAC 30,A UNIT LENGTH SUN POINTING VEC,
00104      DAC 30,TOR PLOTTED. THE --INERTIAL-- ,
00105AC3   DAC 30,PLOT SHOWS SIMULTANEOUSLY THE ,
00106      DAC 30,FIXED INERTIAL AXES WITH A UNI,
00107      DAC 30,T LENGTH VECTOR ON THE BODY RO,
00108      DAC 30,LL AXIS PLOTTED. ROLL AXES ARE,
00109AC4   DAC 30,ASSUMED COMING OUT OF THE PAPE,
```

```
           DAC 3Ø,R. POINTS BELOW THE PITCH-YAW ,
           DAC 3Ø,PLANE ARE PLOTTED WITH AN   X ,
           DAC 3Ø, , POINTS ABOVE THIS PLANE    ,
AC5        DAC 3Ø,WITH A   *   . THE FIRST POINT ,
           DAC 3Ø,IS AN   S  AND THE LAST POINT,
           DAC 14, IS AN   E  .@
PA1        DAC 3Ø,                                 ,
PA31       DAC 3Ø,                                 ,
PA61       DAC 3Ø,                                 ,
PA91       DAC 31,                                @
RESET      DAC 3Ø,                                 ,
           DAC 3Ø,                                I,
           DAC 3Ø,                                 ,
R91        DAC 31,                                -@
CLR        DAC 31,                                I@
YAW        DAC 31,                         YAW    @
PITCH1     DAC 31,                       PITCH   I@
PITCH2     DAC 31,                       PITCH    @
Y          DAC 31,                           I    @
YMK        DAC 31,                           +   @
XL         DAC 31,+----+----+----+----+----+----+----@
XR1        DAC 31,+----+----+----+----+         I@
XR2        DAC 31,+----+----+----+----+          @
YONE       DAC 31,                            +1 @
XONE1      DAC 31,                      +1       I@
XONE2      DAC 31,                      +1        @
TITLE1     DAC 3Ø,                          BODY,
           DAC 3Ø,                          JOB ,
TITLE2     DAC 31,CODE                 INERTIAL@
RM         DC  1,@
PRINT      DAC 1,Ø
           DS  744Ø
ADR1       DS  5
ADR2       DS  5
ADR3       DS 5
ADR4       DS  5
CTR        DS  2
CTR1       DS  2
*
*
G          RNCDPA1
           SF  PA1
           TF  NO,PA1+2
           TFM CTR,Ø,1Ø
G1         C   NO,CTR
           BZ  E
           RNCDPA1
           BD  E,PA1,,              EXIT FOR STOP CODE
           TFM CTR1,Ø,1Ø
           TFM G2+6,PA1+1
```

```
00160        TFM G3+11,PA1
00161        TFM G4+6,PA1+3
00162        TFM CF+6,PA1
00163        TFM CFC,0,10
00164G3      BNF G5,*
00165G4      SF  *
00166G5      AM  CFC,1,10
00167CF      CF  *
00168        AM  CF+6,1
00169        AM  G3+11,1
00170        CM  CFC,3,10
00171        BNZ G3
00172G2      SF  *
00173        AM  CTR1,1,10
00174        AM  G2+6,4
00175        AM  G4+6,4
00176        AM  CF+6,1
00177        AM  G3+11,1
00178        TFM CFC,0,10
00179        CM  CTR1,18,10
00180        BNZ G3
00181*
00182*
00183*       SET SYMBOLS AND ADRS
00184*
00185        TFM CTR1,0,10
00186        TFM A1+11,PA1+11
00187        TFM A2+11,PA1+23
00188        TFM A3+11,PA1+3
00189        TFM A4+11,PA1+7
00190        TFM A5+11,PA1+15
00191        TFM A6+11,PA1+19
00192G6      AM  CTR,1,10
00193        AM  CTR1,1,10
00194        CM  CTR,1,10
00195        BNZ G7
00196        TFM TF1+11,S
00197        TFM TF2+11,S
00198        B   G10
00199G7      C   NO,CTR
00200        BNZ A1
00201        TFM TF1+11,END
00202        TFM TF2+11,END
00203        B   G10
00204A1      BNF G8,*
00205        TFM TF1+11,PER
00206        B   A2
00207G8      TFM TF1+11,STAR
00208A2      BNF G9,*
00209        TFM TF2+11,PER
```

```
       B    G10
G9     TFM  TF2+11,STAR
*
*
*           PLOT POINTS,SET ADRS
*
G10    TFM  ADR1,50
A3     TF   FAC,*
       TF   DV,FAC
       A    FAC,DV
       A    ADR1,FAC
A4     TF   FAC,*
       MM   FAC,240,9
       SF   95
       TF   DV,99
       TFM  FAC,PRINT+3600
       S    FAC,DV
       A   ADR1,FAC
       TFM  ADR2,170
A5     TF   FAC,*
       TF   DV,FAC
       A   FAC,DV
       A    ADR2,FAC
A6     TF   FAC,*
       MM   FAC,240,9
       SF   95
       TF   DV,99
       TFM  FAC,PRINT+3600
       S    FAC,DV
       A    ADR2,FAC
*
*      TRANSMIT SYMBOLS
*
       TF   TF1+6,ADR1
       TF   TF2+6,ADR2
TF1    TF   *,*
TF2    TF   *,*
       AM   A1+11,24
       AM   A2+11,24
       AM   A3+11,24
       AM   A4+11,24
       AM   A5+11,24
       AM   A6+11,24
       C    NO,CTR
       BZ   E
       CM   CTR1,3,10
       BNZ  G6
       B    G1
DV     DS   5
FAC    DS   5
```

```
ØØ26ØNO     DS  3
ØØ261CFC    DS  2
ØØ262STAR   DAC 1,*,
ØØ263PER    DAC 1,X,
ØØ264END    DAC 1,E,
ØØ265S      DAC 1,S,
ØØ266       DENDENTRY
```

APPENDIX H

The purpose of this appendix is to describe the computer programs used to obtain some of the data for examples 3.1 and 3.4.

These programs use the following standard subroutines.

INT, INTM: Floating point Adams-Moulton, Runge-Kutta Integration; IBM Share Program No. 602, FAP Coded Subroutine. This program has two distinct entries, INT to set up the necessary bookkeeping, and INTM for performing the integration steps. The user must supply a FORTRAN subroutine (with the name DAUX) which evaluates the derivatives.

MATINV: Matrix Inversion with Accompanying Solution of Linear Equations (FORTRAN II), IBM Share Program No. 664.

PLT1: Fortran Plotting Subroutine, FAP coded, New York University Library Routine No. 0-620424.

Program for Example 3.1

The main program performs the optimization as two one-dimensional problems. The subroutine DAUX describes the system equations. The subroutine PERIN intergrates the system equations for given boundary conditions. The SIMPSN integration subroutine is used to calculate the performance index. The subroutine PARFIT fits a parabolic curve $Y=AX^2+BX+C$ through the given three points and finds the minimum of the hill.

Program for Example 3.4

The main program sets up the problem. The subroutine OPTMAL finds the optimal solution. The subroutine INTCON obtains the constants in the linear equations in the differential approximation technique for the cases $U=bx_1$, $U=bx_1 + cx_1^3$ and $U=bx_1 + cx_2$. The subroutine UBX1 finds the differential approximation solution for the case $U=bx_1$ and obtains the exact solution by quasilinearization. The subroutines UBXCX3 and BX1CX2 do the same operations as the subroutine UBX1 for the cases $U=bx_1 + cx_1^3$ and $U=bx_1 + cx_2$ respectively. The subroutine DAUX describes the differential equations for all the cases mentioned above. The subroutine SKALE finds the scale factors for the graphs to be plotted. All the graphs are scaled to the interval -1 to +1. The subroutine PLOT sets up the proper variables for the PLT1 subroutine. The subroutine SIMPSN is the regular Simpson rule integration of a tabular function.

```
          PROGRAM LISTING FOR EXAMPLE 3.1  SPECIFIC OPTIMAL CONTROL
          PROBLEM      PARAMETER OPTIMIZATION METHOD
*       SYMBOL TABLE
*       LABEL
CGRADIENT TECHNIQUE MAIN PROGRAM
C         PLANT  DDX+3DX+2X=U U=AX+BDX
C         PARAMETER OPTIMIZATION USING PARABOLIC FITTING METHOD
C         THIS PROGRAM IS FOR TWO DIMENSIONAL HILL.
C         OPTIMIZATION IS CARRIED OUTAS TWO ONE-DIMENSIONAL PROBLEMS.
C
      COMMON T,API,BPI,CONE,CTWO,HGRID,TT,XONE,XTWO,U,PI,FUNC,NMAX
      DIMENSION T(50),XONE(501),XTWO(501),U(501),FUNC(501)
C
1     READ INPUTTAPE 5,500,CONA,CONB,TT,NMAX,NOPT,NOIT,TOLL
      FN=NMAX-1
      HGRID=TT/FN
2     READ INPUTTAPE 5,510,CONE,CTWO,A,B
148   WRITE OUTPUT TAPE 6,710
      WRITE OUTPUT TAPE 6,520,CONE,CTWO,TT,NMAX,HGRID,A,B,CONA,CONB
      WRITE OUTPUT TAPE 6,740,NOPT,NOIT,TOLL
101   AFB=A
102   BFB=B
145   AMIN=A
146   BMIN=B
      INDEX=0
      API=A
      BPI=B
3     CALL PERIN
4     PIAB=PI
C
C         SEARCH MINIMUM IN A DIRECTION
5     IF(ABSF(A)-0.02) 6,7,7
6     AA=A+0.0004
      GO TO 8
7     AA=A+0.02*A
8     API=AA
      INA=1
      CALL PERIN
9     PIAAB=PI
C         CALCULATE PARTIAL DERIVATIVE OF PI WRT A
10    DPIA=(PIAAB-PIAB)/(AA-A)
      WRITE OUTPUTTAPE 6,670
11    WRITE OUTPUTTAPE 6,530,A,B,PIAB,AA,PIAAB,DPIA
12    IF(PIAAB-PIAB) 13,13,15
15    AA=A
16    PIAAB=PIAB
      GO TO 13
13    AB=AA-DPIA*CONA
      API=AB
      INA=INA+1
```

```
17     CALL . PERIN
18     PIABB=PI
       WRITE OUTPUT TAPE 6,54Ø,AB,B,PIABB
19     IF(PIABB-PIAAB) 2Ø,21,21
2Ø     DPIA=(PIABB-PIAAB)/(AB-AA)
22     AC=AB-DPIA*CONA
23     API=AC
       INA=INA+1
24     CALL PERIN
25     PIACB=PI
26     WRITE OUTPUT TAPE 6,55Ø,DPIA,AC,PIACB
       IF(INA-NOPT) 27,29,29
27     IF(PIACB-PIABB) 28,29,29
28     AA=AB
3Ø     PIAAB=PIABB
31     AB=AC
32     PIABB=PIACB
33     CONA=2.Ø*CONA
       WRITE OUTPUTTAPE 6,56Ø,CONA
34     GO TO 2Ø
C
21     CONA=CONA/2.Ø
35     AC=AA-DPIA*CONA
36     API=AC
       INA=INA+1
37     CALL PERIN
38     PIACB=PI
39     WRITE OUTPUT TAPE 6,57Ø,DPIA,AC,PIACB,CONA
       IF(INA-NOPT) 4Ø,29,29
4Ø     IF(PIACB-PIAAB) 29,41,41
41     AB=AC
42     PIABB=PIACB
       GO TO 21
C          PERFORM PARABOLIC FITTING
29     WRITE OUTPUT TAPE 6,58Ø,AA,PIAAB,AB,PIABB,AC,PIACB
43     CALL PARFIT(AA,PIAAB,AB,PIABB,AC,PIACB,ALPHA,BETA,GAMMA)
44     AMIN=-BETA/(2.Ø*ALPHA)
45     API=AMIN
46     CALL PERIN
47     PIAMB=PI
48     WRITE OUTPUT TAPE 6,59Ø,AMIN,BMIN,PIAMB
124    IF(PIAMB-PIAAB) 125,125,126
126    PIAMB=PIAAB
127    AMIN=AA
125    IF(PIAMB-PIABB) 128,128,129
129    PIAMB=PIABB
13Ø    AMIN=AB
128    IF(PIAMB-PIACB) 131,131,132
132    PIAMB=PIACB .
133    AMIN=AC
```

```
131    WRITE OUTPUT TAPE 6,690,AMIN,BMIN,PIAMB
C
C           SEARCH MINIMUM IN B DIRECTION
49     A=AMIN
50     PIAB=PIAMB
51     IF(ABSF(B)-0.02)52,53,53
52     BA=B+0.0004
       GO TO 54
53     BA=B+0.02*B
54     API=A
55     BPI=BA
       INB=1
56     CALL PERIN
57     PIABA=PI
C          CALCULATE PARTIAL DERIVATIVE OF PI WRT B
58     DPIB=(PIABA-PIAB)/(BA-B)
       WRITE OUTPUTTAPE 6,680
59     WRITE OUTPUT TAPE 6,600,A,B,PIAB,BA,PIABA,DPIB
60     IF(PIABA-PIAB) 61,61,62
62     BA=B
63     PIABA=PIAB
       GO TO 61
61     BB=BA-DPIB*CONB
64     BPI=BB
       INB=INB+1
65     CALL  PERIN
66     PIABB=PI
67     WRITE OUTPUTTAPE 6,610,A,BB,PIABB
68     IF(PIABB-PIABA) 69,70,70
69     DPIB=(PIABB-PIABA)/(BB-BA)
71     BC=BB-DPIB*CONB
72     BPI=BC
       INB=INB+1
73     CALL PERIN
74     PIABC=PI
75     WRITE OUTPUTTAPE 6,620,DPIB,BC,PIABC
       IF(INB-NOPT)76,78,78
76     IF(PIABC-PIABB) 77,78,78
77     BA=BB
79     PIABA=PIABB
80     BB=BC
81     PIABB=PIABC
82     CONB=2.0*CONB
83     WRITE OUTPUT TAPE 6,630,CONB
84     GO TO 69
C
70     CONB=CONB/2.0
85     BC=BA-DPIB*CONB
86     BPI=BC
       INB=INB+1
```

```
87      CALL PERIN
88      PIABC=PI
89      WRITE OUTPUT TAPE 6,640,DPIB,BC,PIABC,CONB
        IF(INB-NOPT) 90,78,78
90      IF(PIABC-PIABA) 78,91,91
91      BB=BC
92      PIABB=PIABC
93      GO TO 70
C
C           PERFORM PARABOLICFITTING
78      WRITE OUTPUTTAPE 6,650,BA,PIABA,BB,PIABB,BC,PIABC
95      CALL PARFIT(BA,PIABA,BB,PIABB,BC,PIABC,ALPHA,BETA,GAMMA)
96      BMIN=-BETA/(2.0*ALPHA)
97      BPI=BMIN
98      CALL PERIN
99      PIABM=PI
100     WRITE OUTPUT TAPE 6,660,AMIN,BMIN,PIABM
134     IF(PIABM-PIABA) 135,135,136
136     PIABM=PIABA
137     BMIN=BA
135     IF(PIABM-PIABB) 138,138,139
139     PIABM=PIABB
140     BMIN=BB
138     IF(PIABM-PIABC) 141,141,142
142     PIABM=PIABC
143     BMIN=BC
141     WRITE OUTPUT TAPE 6,700,AMIN,BMIN,PIABM
        INDEX=INDEX+1
C
C           TEST THE SOLUTION
103     ERRA=ABSF(AFB-AMIN)
104     ERRB=ABSF(BFB-BMIN)
105     DELA=TOLL*ABSF(AMIN)
106     DELB=TOLL*ABSF(BMIN)
107     IF(ERRA-DELA) 108,108,110
108     IF(ERRB-DELB) 200,200,110
C
C           NO GOOD  GO BACK
110     AFB=AMIN
111     BFB=BMIN
112     A=AMIN
113     B=BMIN
114     PIAB=PIABM
115     IF(INDEX-NOIT) 5,200,200
200     WRITE OUTPUTTAPE 6,720
        API=AMIN
        BPI=BMIN
        CALL  PERIN
201     NP=(NMAX-1)/20
202     DO 210 N=1,NMAX,NP
```

```
203    FN=N-1
204    TIME=FN*HGRID
205    WRITE OUTPUT TAPE 6,730,TIME,XONE(N),XTWO(N),U(N)
210    CONTINUE
       WRITE OUTPUTTAPE 6,750,AMIN,BMIN,PI
109    GO TO 1
500    FORMAT(2F10.0,F5.0,315,F10.0)
510    FORMAT(4F10.0)
520   0FORMAT(1H0/39H CONE CTWO TT NMAX HGRID A B CONA CONB /
      1 2F10.5,F8.3,15,5F15.8/)
530    FORMAT(/24HABPIAB AA PIAAB DPIA  /6F15.8/)
540    FORMAT(/12HAB B PIABB  /3F15.8/)
550    FORMAT(/15HDPIA ACPIACB  /3F15.8/)
560    FORMAT(/6HCONA  /F15.8/)
570    FORMAT(/20HDPIA ACPIACB CONA /4F15.8/)
580    FORMAT(/28HAA PIAAB AB PIABB AC PIACB   /6F15.8/)
590    FORMAT(/17HAMIN BMIN PIAMB   /3F15.8/)
600    FORMAT(/24HAB PIAB BA PIABA DPIB /6F15.8/)
610    FORMAT(/12HABB PIABB  /3F15.8/)
620    FORMAT(/15HDPIB BC PIABC  /3F15.8/)
630    FORMAT(/6HCONB  /F15.8/)
640    FORMAT(/20HDPIB BCPIABC CONB /4F15.8/)
650    FORMAT(/28HBA PIABA BBPIABB BCPIABC  /6F15.8/)
660    FORMAT(/17HAMIN BMIN PIABM  /3F15.8/)
670    FORMAT(///31HSEARCH MINIMUM IN A DIRECTION  //)
680    FORMAT(///31HSEARCH MINIMUM IN BDIRECTION  //)
690    FORMAT(/27HCORRECTED AMIN BMIN PIAMB /3F15.8/)
700    FORMAT(/27HCORRECTED AMIN BMIN PIABM /3F15.8/)
720   0FORMAT(//29HSPECIFIC OPTIMAL TRAJECTORY   //
      1   6X1HT11X4HXONE16X4HXTWO18X1HU/)
730    FORMAT(F10.5,3F20.8)
740    FORMAT(/16HNOPT NOIT TOLL   /2I10,F10.5/)
710   0FORMAT(1H1/10X92HGRADIENT TECHNIQUE   PARAMETEROPTIMIZATION USI
      1NG PARABOLICFITTING METHOD   G C AGARWAL            /
      258HSYSTEM DDX+3DX+2X=U,I=INT(0,1)(X**2+DX**2+U**2),U=AX+BDX /)
750    FORMAT(/8HABPI   /3F15.8/)
1000   END
*      SYMBOL TABLE
*      LABEL
CDAUX    DAUX SUBROUTINE FOR GRADIENT TECHNIQUE
       SUBROUTINE DAUX
C
       COMMON T,API,BPI,CONE,CTWO,HGRID,TT,XONE,XTWO,U,PI,FUNC,NMAX
       DIMENSION T(50),XONE(501),XTWO(501),U(501),FUNC(501)
C
1      UT=API*T(4)+BPI*T(5)
2      T(6)=T(5)
3      T(7)=-3.0*T(5)-2.0*T(4)+UT
4      RETURN
5      END
```

```
*       SYMBOL TABLE
*       LABEL
CPERIN    SUBROUTINE  TO CALCULATE PERFORMANCE INDEX
        SUBROUTINE PERIN
C
        COMMON T,API,BPI,CONE,CTWO,HGRID,TT,XONE,XTWO,U,PI,FUNC,NMAX
        DIMENSION T(50),XONE(501),XTWO(501),U(501),FUNC(501)
C
        DO 20 I=1,50
20      T(I)=0.0
1       T(2)=0.0
2       T(3)=HGRID
3       T(4)=CONE
4       T(5)=CTWO
5       CALL  INT(T,2,2,0.,0.,0.,0.,0.,0.)
6       XONE(1)=CONE
7       XTWO(1)=CTWO
8       DO 10 I=2,NMAX
9       CALL  INTM
11      XONE(I)=T(4)
12      XTWO(I)=T(5)
10      CONTINUE
14      DO 13 I=1,NMAX
15      U(I)=API*XONE(I)+BPI*XTWO(I)
13      FUNC(I)=XONE(I)**2+XTWO(I)**2+U(I)**2
16      CALL SIMPSN(FUNC,HGRID,NMAX,PI)
17      RETURN
18      END
*       SYMBOL TABLE
*       LABEL
CPARFIT   PARABOLIC FITTING SUBROUTINE THROUGH 3 POINTS
        SUBROUTINE PARFIT(A,X,B,Y,C,Z,ALPHA,BETA,GAMMA)
D       DIMENSION PAR(10,10),RPAR(10,1)
1       PAR(1,1)=A**2
2       PAR(1,2)=A
3       PAR(1,3)=1.0
4       PAR(2,1)=B**2
5       PAR(2,2)=B
6       PAR(2,3)=1.0
7       PAR(3,1)=C**2
8       PAR(3,2)=C
9       PAR(3,3)=1.0
10      RPAR(1,1)=X
11      RPAR(2,1)=Y
12      RPAR(3,1)=Z
D       CALL  MATINV(PAR,3,RPAR,1,DET)
13      ALPHA=RPAR(1,1)
14      BETA=RPAR(2,1)
15      GAMMA=RPAR(3,1)
16      RETURN
```

```
17      END
*       SYMBOL TABLE
*       LABEL
CSIMPSN  SIMPSON INTEGRATION SUBROUTINE
        SUBROUTINE  SIMPSN(FUNC,HGRID,NMAX,SIMP)
C           WHERE  HGRID=LENGTH OF SUBINTERVAL,NMAX=NUMBER OF POINTS
C           NMAX MUST BE ODD
C
        DIMENSION FUNC(501)
        K=(NMAX-1)/2
        SUMA=0.0
        DO 15 J=1,K
     15 SUMA=SUMA+FUNC(2*J)
        SUMB=0.0
        DO 16 J=2,K
     16 SUMB=SUMB+FUNC(2*J-1)
        SIMP=(4.*SUMA+2.*SUMB+FUNC(NMAX)+FUNC(1))*HGRID/3.
    100 RETURN
        END
```

THIS PROGRAM ALSO REQUIRES SUBROUTINES INT,INTM,MATINV

```
          PROGRAM LISTING FOR EXAMPLE 3.4  SPECIFIC OPTIMAL CONTROL PROB.
          METHOD OF DIFFERENTIAL APPROXIMATION AND QUASILINEARIZATION
*       SYMBOL TABLE
*       LABEL
CMAIN    PROGRAM  THIS PROGRAM SOLVES THE OPTIMAL SOLUTION THEN FINDS
C   THE FEEDBACK COEFFICIENTS USING DIFF APPROX   FINALLY CHECKSTHE
C       FEEDBACK COEFFICIENTS USING QUASILINEARIZATION  ALL THE RESULTS
C       ARE PLOTTED FOR COMPARISION
C        PLANT  DDX+3DX+2X+Ø.5X**3=U
C
      ØCOMMON  T,W,H,P,HGRID,NMAX,PREV,CON,IFLAG,NPRNT,XONE,XTWO,BFC,CFC,
      1ALMD,AMU,ZETA,ETA,FUNC,XO,YO,UO,PERIN,Y1,Y2,Y3,MBL,PLT,TEMP,SCALE,
      2AMAX,IERR,YL,YU,TI,TT,NP,MCON,CSOC,SIMP,CONE,CTWO,ALMDP,AMUP
      ØDIMENSION  T(9ØØ),W(8,2Ø1),H(8,8,2Ø1),P(8,2Ø1),PREV(8),CON(8),
      1FUNC(2Ø1),XO(2Ø1),YO(2Ø1),UO(2Ø1),Y1(51),Y2(51),Y3(51),PLT(51),
      2TEMP(1Ø25),SCALE(3),AMAX(3),CSOC(1Ø)
C
        NP=51
        TI=Ø.Ø
        TT=1.Ø
        HGRID=Ø.ØØ5
        MBL=5
        NPRNT=2Ø
        MCON=4
        NMAX=2Ø1
1ØØ     READ INPUT TAPE 5,1Ø,XONE XTWO ALMD AMU
1Ø      FORMAT(18X,2F6.2,2E2Ø.8)
        WRITE OUTPUT TAPE 6,2Ø,XONE,XTWO,ALMD,AMU
2Ø     ØFORMAT(1H1/99H SOC PROBLEM SENSITIVITY ANALYSIS PLANT DDX+3DX+2X+Ø
      1.5X**3=U  OPTIMAL SOLUTION AND DIFFERENT SOC      ///5H XONE,F6.2/
      2  5H XTWO,F6.2/5HALMD,E2Ø.8/5H AMU ,E2Ø.8////)
C
        CONE=XONE
        CTWO=XTWO
        CALL  OPTMAL
        ALMDP=ALMD
        AMUP=AMU
C
        CALL  INTCON
        CALL  UBX1
        CALL  UBXCX3
        CALL  FX1CX2
        GO TO 1ØØ
        END
*       SYMBOL TABLE
*       LABEL
COPTMAL    INTEGRATION OF EULER LAGRANGE EQUATIONS WITH GIVEN BOUNDARY
C          CONDITIONS USING QUASILINEARIZATION FOR TPBVP TO FIND OPTIMAL
C          SOLUTION
C        PLANT  DDX+3DX+2X+Ø.5X**3=U  OPTIMAL SOLUTION
```

```
      SUBROUTINE OPTMAL
C
     ØCOMMON  T,W,H,P,HGRID,NMAX,PREV,CON,IFLAG,NPRNT,XONE,XTWO,BFC,CFC,
     1ALMD,AMU,ZETA,ETA,FUNC,XO,YO,UO,PERIN,Y1,Y2,Y3,MBL,PLT,TEMP,SCALE,
     2AMAX,IERR,YL,YU,TI,TT,NP,MCON,CSOC,SIMP,CONE,CTWO,ALMDP,AMUP
     ØDIMENSION  T(9ØØ),W(8,2Ø1),H(8,8,2Ø1),P(8,2Ø1),PREV(8),CON(8),
     1FUNC(2Ø1),XO(2Ø1),YO(2Ø1),UO(2Ø1),Y1(51),Y2(51),Y3(51),PLT(51),
     2TEMP(1Ø25),SCALE(3),AMAX(3),CSOC(1Ø)
C
C          INITIAL APPROXIMATION
C          INTEGRATE NONLINEAR DIFF EQS
C
      IFLAG=1
      DO 2 I=1,9ØØ
    2 T(I)=Ø.Ø
C          INTEGRATION  SETUP
  1Ø2 T(2)=TI
      T(3)=HGRID
      T(4)=XONE
      T(5)=XTWO
      T(6)=ALMD
      T(7)=AMU
      CALL INT(T,4,2,Ø.,Ø.,Ø.,Ø.,Ø.,Ø.)
C
C          SOLUTION OF NONLINEAR DIFF EQS
      DO 4 N=2,NMAX
      CALL  INTM
      DO 3 I=1,4
    3 W(I,N)=T(I+3)
    4 CONTINUE
C
C          HOLD GIVEN INITIAL CONDITIONS
      PREV(1)=XONE
      PREV(2)=XTWO
      PREV(3)=ALMD
      PREV(4)=AMU
      WRITE OUTPUT TAPE 6,97,(PREV(I),I=1,4),(W(I,2Ø1),I=1,4)
97   ØFORMAT(//52HFOR OPTIMAL SOLUTION INITIAL GUESS BOUNDARYPOINTS
     1/8H INITIAL    ,4E2Ø.8/9HTERMINAL    ,4E2Ø.8/)
C
C     QUASILINEARIZATION SOLUTION
  1Ø4 IFLAG=2
C
      ITER=1
35Ø   DO 6 I=1,9ØØ
    6 T(I)=Ø.Ø
      T(2)=TI
      T(3)=HGRID
      T(4)=1.Ø
      T(9)=1.Ø
```

```
      T(14)=1.Ø
      T(19)=1.Ø
      XONE=PREV(1)
      XTWO=PREV(2)
      ALMD=PREV(3)
      AMU=PREV(4)
C
      CALL INT(T,2Ø,2,Ø.,Ø.,Ø.,Ø.,Ø.,Ø.)
      N=1
      L=3
      DO 21 I=1,4
      DO 21 J=1,4
      L=L+1
   21 H(I,J,N)=T(L)
      DO 22 I=1,4
      L=L+1
   22 P(I,N)=T(L)
C
C         INTEGRATE P ANDH
C
      DO 7 N=2,NMAX
      XONE=W(1,N)
      XTWO=W(2,N)
      ALMD=W(3,N)
      AMU=W(4,N)
      CALL  INTM
      L=3
      DO 8 I=1,4
      DO 8 J=1,4
      L=L+1
    8 H(I,J,N)=T(L)
      DO 9 I=1,4
      L=L+1
    9 P(I,N)=T(L)
    7 CONTINUE
C
C          DETERMINE CONSTANTS TO MATCH THE BOUNDARY CONDITIONS
C     USES MATINV  DOUBLE PRECISION ACCURACY
C
D     DIMENSION  A(1Ø,1Ø),B(1Ø,1)
      N=NMAX
      CON(1)=PREV(1)
      CON(2)=PREV(2)
      DO 51 J=1,2
      DO 51 I=1,2
51    A(I,J)=H(I+2,J+2,N)
      DO 52 I=1,2
52    B(I,1)=-P(I+2,N)-H(I+2,1,N)*CON(1)-H(I+2,2,N)*CON(2)
D     CALL  MATINV(A,2,B,1,DET)
      DO 7Ø I=1,2
```

```
   70 CON(I+2)=B(I,1)
C
      DO 10 N=2,NMAX
      DO 11 I=1,4
      W(I,N)=P(I,N)
      DO 11 J=1,4
   11 W(I,N)=W(I,N)+H(I,J,N)*CON(J)
   10 CONTINUE
      WRITE OUTPUT TAPE 6,98,(CON(I),I=1,4),(W(I,201),I=1,4)
98   0FORMAT(/27H QL SOLUTION BOUNDARY PTS    /8HINITIAL   ,4E20.8/
     1  9H TERMINAL   ,4E20.8/)
C       CHECK THE SOLUTION
      ERROR=ABSF(PREV(3)-CON(3))
      IF(ERROR-0.000001) 71,71,72
71    ERROR=ABSF(PREV(4)-CON(4))
      IF(ERROR-0.000001) 73,73,72
72    IF(ITER-4)300,73,73
300   ITER=ITER+1
      DO 13 I=1,4
13    PREV(I)=CON(I)
C         DO NEXT ITERATION
      GO TO 350
73    W(1,1)=CON(1)
      W(2,1)=CON(2)
      UO(1)=-0.5*CON(4)
      WRITE OUTPUT TAPE 6,99,ITER
99    FORMAT(//20H ITERATION PERFORMED      ,I5///)
      TIME=TI
      WRITE OUTPUT TAPE 6,60,TIME,(CON(I),I=1,4),UO(1)
      M=1+NPRNT
      DO 61 N=2,NMAX
      UO(N)=-0.5*W(4,N)
      FN=N-1
      TIME=FN*HGRID+TI
      IF(N-M) 61,62,61
62    WRITE OUTPUT TAPE 6,90,TIME,(W(I,N),I=1,4),UO(N)
      M=M+NPRNT
61    CONTINUE
C
C         CALCULATE PERFORMANCE INDEX
C
      DO 26 N=1,NMAX
   26 FUNC(N)=W(1,N)**2+W(2,N)**2+UO(N)**2
      CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
  200 PERIN=SIMP
      WRITE OUTPUT TAPE 6,80,PERIN
C
C        SET NEW INITIAL CONDITIONS
      XONE=CON(1)
      XTWO=CON(2)
```

```
      ALMD=CON(3)
      AMU=CON(4)
C          HOLD OPTIMAL SOLUTION FOR DIFF APPROX
      DO 1000 N=1,NMAX
      XO(N)=W(1,N)
1000  YO(N)=W(2,N)
C          PLOT OPTIMAL SOLUTION
31    J=0
      DO 32 I=1,NMAX,MCON
      J=J+1
      FN=I-1
      PLT(J)=FN*HGRID+TI
      Y1(J)=XO(I)
      Y2(J)=YO(I)
      Y3(J)=UO(I)
32    CONTINUE
C
      CALL SKALE
C
      WRITE OUTPUT TAPE 6,100,XONE,XTWO,TT
C
      CALL  PLOT
C
C         FORMATS
100  0FORMAT(///82H SENSITIVITY ANALYSIS OPTIMAL SOLUTION GRAPHS IN ORDE
     1R XONE XTWO AND U OPTIMAL        /
     225X4HXONE,F8.4,5X4HXTWO,F8.4,5X2HTT,F8.4/)
60   0FORMAT(//17H OPTIMAL SOLUTION      /
     1  17X1HT10X4HXONE12X4HXTWO12X4HALMD12X3HAMU14X1HU//
     2   10XF10.4,5E16.6)
   80 FORMAT(/6HPERIN,F30.8/)
   90 FORMAT(10XF10.4,5E16.6)
  107 RETURN
      END
*     SYMBOL TABLE
*     LABEL
CINTCON   SUBROUTINE TO CALCULATE THE COEFFICIENTS IN THE LINEAR EQS
C         OBTAINED BY DIFF APPROX METHOD TO FIND THE FEEDBACK COEFFS
      SUBROUTINE INTCON
C
     0COMMON  T,W,H,P,HGRID,NMAX,PREV,CON,IFLAG,NPRNT,XONE,XTWO,BFC,CFC,
     1ALMD,AMU,ZETA,ETA,FUNC,XO,YO,UO,PERIN,Y1,Y2,Y3,MBL,PLT,TEMP,SCALE,
     2AMAX,IERR,YL,YU,TI,TT,NP,MCON,CSOC,SIMP,CONE,CTWO,ALMDP,AMUP
     0DIMENSION  T(900),W(8,201),H(8,8,201),P(8,201),PREV(8),CON(8),
     1FUNC(201),XO(201),YO(201),UO(201),Y1(51),Y2(51),Y3(51),PLT(51),
     2TEMP(1025),SCALE(3),AMAX(3),CSOC(10)
C
      DO 10 I=1,NMAX
10    FUNC(I)=UO(I)*XO(I)
      CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
```

```
11      CSOC(1)=SIMP
C
        DO 2Ø I=1,NMAX
2Ø      FUNC(I)=XO(I)**2
        CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
21      CSOC(2)=SIMP
C
        DO 3Ø I=1,NMAX
3Ø      FUNC(I)=XO(I)**4
        CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
31      CSOC(3)=SIMP
C
        DO 4Ø I=1,NMAX
4Ø      FUNC(I)=XO(I)**6
        CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
41      CSOC(4)=SIMP
C
        DO 5Ø I=1,NMAX
5Ø      FUNC(I)=UO(I)*XO(I)**3
        CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
51      CSOC(5)=SIMP
C
        DO 6Ø I=1,NMAX
6Ø      FUNC(I)=UO(I)*YO(I)
        CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
61      CSOC(6)=SIMP
C
        DO 7Ø I=1,NMAX
7Ø      FUNC(I)=YO(I)**2
        CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
71      CSOC(7)=SIMP
C
        DO 8Ø I=1,NMAX
8Ø      FUNC(I)=XO(I)*YO(I)
        CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
81      CSOC(8)=SIMP
        WRITE OUTPUT TAPE 6,2ØØ,(CSOC(I),I=1,8)
2ØØ    ØFORMAT(///68HCOEFFICIENTS IN THE LINEAR EQUATIONS TO FINDFEEDBAC
       1KCOEFFICIENTS    /4E2Ø.8/4E2Ø.8///)
9Ø      RETURN
1ØØ     END
*       SYMBOL TABLE
*       LABEL
CUBX1      CASE ONE U=BX1 DIFF APPROXIMATION AND QUASILINEARIZATION
        SUBROUTINE UBX1
C
       ØCOMMON  T,W,H,P,HGRID,NMAX,PREV,CON,IFLAG,NPRNT,XONE,XTWO,BFC,CFC,
       1ALMD,AMU,ZETA,ETA,FUNC,XO,YO,UO,PERIN,Y1,Y2,Y3,MBL,PLT,TEMP,SCALE,
       2AMAX,IERR,YL,YU,TI,TT,NP,MCON,CSOC,SIMP,CONE,CTWO,ALMDP,AMUP
       ØDIMENSION  T(9ØØ),W(8,2Ø1),H(8,8,2Ø1),P(8,2Ø1),PREV(8),CON(8),
```

```
      1FUNC(201),XO(201),YO(201),UO(201),Y1(51),Y2(51),Y3(51),PLT(51),
      2TEMP(1025),SCALE(3),AMAX(3),CSOC(10)
C
C          CASE ONE U=BX
1000  BFC=CSOC(1)/CSOC(2)
1500  WRITE OUTPUT TAPE 6,200,CONE,CTWO,TT,BFC
200   0FORMAT(////16H CASE ONE U=BX1    /5HXONE,F6.2/5H XTWO,F6.2/
      13H TT,F6.2/20H BY DIFF APPROX  BFC    ,E20.8/)
C          SPECIFIC OPTIMAL TRAJECTORY DIFF APP
C
      IFLAG=3
      DO 2 I=1,900
    2 T(I)=0.0
      ZETA=0.0
C          INTEGRATION  SETUP
      T(2)=TI
      T(3)=HGRID
      T(4)=CONE
      T(5)=CTWO
      T(6)=BFC
      T(7)=ALMDP
      T(8)=AMUP
      T(9)=ZETA
      CALL INT(T,6,2,0.,0.,0.,0.,0.,0.)
      U=BFC*T(4)
      WRITE OUTPUTTAPE 6,50,T(2),T(4),T(5),U
C
C          SOLUTION OF NONLINEAR DIFF EQS
      M=1+NPRNT
      DO 4 N=2,NMAX
      CALL  INTM
      DO 3 I=1,6
    3 W(I,N)=T(I+3)
      IF(N-M) 4,5,4
5     U=BFC*T(4)
      WRITE OUTPUTTAPE 6,70,T(2),T(4),T(5),U
      M=M+NPRNT
    4 CONTINUE
      W(1,1)=CONE
      W(2,1)=CTWO
      W(3,1)=BFC
      W(4,1)=ALMDP
      W(5,1)=AMUP
      W(6,1)=ZETA
      DO 45 N=1,NMAX
45    FUNC(N)=(1.0+BFC**2)*W(1,N)**2+W(2,N)**2
      CALL  SIMPSN(FUNC,HGRID,NMAX,SIMP)
      PERIN=SIMP
      WRITE OUTPUT TAPE 6,80,PERIN
C          PLOT SPECIFIC OPTIMAL TRAJECTORY
```

```
46     J=Ø
       DO 47 I=1, NMAX,MCON
       J=J+1
       Y1(J)=W(1,I)
       Y2(J)=W(2,I)
47     Y3(J)=BFC*W(1,I)
       WRITE OUTPUTTAPE 6,48,CONE,CTWO,TT,BFC
C
       CALL  PLOT
C
C          OBTAIN EXACT SOLUTION BY QUASILINEARIZATION
C
       IFLAG=4
       PREV(1)=CONE
       PREV(2)=CTWO
       PREV(3)=BFC
       PREV(4)=ALMDP
       PREV(5)=AMUP
       PREV(6)=ZETA
C     INTEGRATION  SETUP
C
       ITER=1
350    DO 20 I=1,900
20     T(I)=Ø.Ø
       T(2)=TI
       T(3)=HGRID
       T(4)=1.Ø
       T(11)=1.Ø
       T(18)=1.Ø
       T(25)=1.Ø
       T(32)=1.Ø
       T(39)=1.Ø
       XONE=PREV(1)
       XTWO=PREV(2)
       BFC=PREV(3)
       ALMD=PREV(4)
       AMU=PREV(5)
       ZETA=PREV(6)
       CALL  INT(T,42,2,Ø.,Ø.,Ø.,Ø.,Ø.,Ø.)
       N=1
       L=3
       DO 21 I=1,6
       DO 21 J=1,6
       L=L+1
    21 H(I,J,N)=T(L)
       DO 22 I=1,6
       L=L+1
    22 P(I,N)=T(L)
C
C     INTEGRATE  P  AND  H
```

```
C
      DO 24 N=2,NMAX
      XONE=W(1,N)
      XTWO=W(2,N)
      BFC=W(3,N)
      ALMD=W(4,N)
      AMU=W(5,N)
      ZETA=W(6,N)
      CALL  INTM
      L=3
      DO 23 I=1,6
      DO 23 J=1,6
      L=L+1
23    H(I,J,N)=T(L)
      DO 24 I=1,6
      L=L+1
24    P(I,N)=T(L)
C
C          DETERMINE CONSTANTS TO MATCH THE GIVEN BOUNDARY CONDITIONS
C     USES MATINV  DOUBLE PRECISION ACCURACY
C
D     DIMENSION  A(10,10),B(10,1)
      N=NMAX
      CON(1)=PREV(1)
      CON(2)=PREV(2)
      CON(6)=PREV(6)
      DO 31 J=1,3
      DO 31 I=1,3
31    A(I,J)=H(I+3,J+2,N)
      DO 32 I=1,3
      B(I,1)=-P(I+3,N)-H(I+3,6,N)*CON(6)
      DO 32 J=1,2
32    B(I,1)=B(I,1)-H(I+3,J,N)*CON(J)
D     CALL  MATINV(A,3,B,1,DET)
      DO 33 I=1,3
33    CON(I+2)=B(I,1)
C    NEW  VARIABLES
C
      DO 7 N=2,NMAX
      DO 6 I=1,6
      W(I,N)=P(I,N)
      DO 6 J=1,6
    6 W(I,N)=W(I,N)+H(I,J,N)*CON(J)
    7 CONTINUE
    9 DO 10 I=1,6
   10 PREV(I)=CON(I)
      WRITE OUTPUTTAPE 6,98,(CON(I),I=1,6),(W(I,201),I=1,6)
98   0FORMAT(/28HQL SOLUTION BOUNDARY PTS          /8H INITIAL,
     13E20.8/3E20.8/9H TERMINAL,3E20.8/3E20.8/)
C
```

```
C         CHECK THE SOLUTION
      ERROR=ABSF(BFC-CON(3))
      IF(ERROR-Ø.ØØØØ1)61,61,62
62    IF(ITER-4)3ØØ,61,61
3ØØ   ITER=ITER+1
      BFC=CON(3)
C  DO NEXT ITERATION
      GO TO 35Ø
C
C     WRITE AND PLOT QL TRAJ
61    BFC=CON(3)
      U=BFC*CON(1)
      WRITE OUTPUTTAPE 6,99,ITER
99    FORMAT(//2ØHITERATION PERFORMED   ,I5///)
      WRITE OUTPUT TAPE 6,63,TI,CON(1),CON(2),U
63   ØFORMAT(///24HSPE OPT TRAJ QL U=BX1     /
     117X1HT1ØX4HXONE12X4HXTWO12X5HU=BX1   //
     2 1ØXF1Ø.4,3E15.6)
      DO 64 N=21,NMAX,NPRNT
      FN=N-1
      TIME=FN*HGRID+TI
      U=BFC*W(1,N)
64    WRITE OUTPUT TAPE 6,65,TIME,W(1,N),W(2,N),U
65    FORMAT(1ØXF1Ø.4,3E15.6)
C         CALCULATE PERFORMANCE INDEX
C
      W(1,1)=CON(1)
      W(2,1)=CON(2)
      DO 26 N=1,NMAX
26    FUNC(N)=(1.Ø+BFC**2)*W(1,N)**2+W(2,N)**2
      CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
      PERIN=SIMP
      WRITE OUTPUT TAPE 6,8Ø,PERIN
   8Ø FORMAT(/6HPERIN,F3Ø.8/)
C
5Ø   ØFORMAT(/48HSPECIFIC OPTIMAL TRAJECTORY DIFF APPROX U=BX1   /
     117X1HT1ØX4HXONE12X4HXTWO12X5HU=BX1    //1ØXF1Ø.4,3E15.6)
7Ø    FORMAT(1ØXF1Ø.4,3E15.6)
48   ØFORMAT(///6ØHSPE OPT TRAJ DIFF APPU=BX1 GRAPHS XONE XTWO AND U S
     1PE OPT       /25X4HXONE,F8.4,5X4HXTWO,F8.4,5X2HTT,F8.4,5X3HBFC,
     2 F1Ø.6/)
C         PLOT QL TRAJ
71    J=Ø
      DO 72 I=1,NMAX,MCON
      J=J+1
      Y1(J)=W(1,I)
      Y2(J)=W(2,I)
      Y3(J)=BFC*W(1,I)
72    CONTINUE
      WRITE OUTPUT TAPE 6,73,CONE,CTWO,TT,BFC
```

```
C
      CALL   PLOT
73    0FORMAT(///55HSPE OPT TRAJ QL U=BX1 GRAPHS XONE XTWO AND U SPE OPT
      1        /25X4HXONE,F8.4,5X4HXTWO,F8.4,5X2HTT,F8.4,5X3HBFC,F10.6/)
      RETURN
      END
*     SYMBOL TABLE
*     LABEL
CUBXCX3   CASE TWO U=BX1+CX1**3  DIFF APPROX AND QL
      SUBROUTINE UBXCX3
C
      0COMMON  T,W,H,P,HGRID,NMAX,PREV,CON,IFLAG,NPRNT,XONE,XTWO,BFC,CFC,
      1ALMD,AMU,ZETA,ETA,FUNC,XO,YO,UO,PERIN,Y1,Y2,Y3,MBL,PLT,TEMP,SCALE,
      2AMAX,IERR,YL,YU,TI,TT,NP,MCON,CSOC,SIMP,CONE,CTWO,ALMDP,AMUP
      0DIMENSION  T(900),W(8,201),H(8,8,201),P(8,201),PREV(8),CON(8),
      1FUNC(201),XO(201),YO(201),UO(201),Y1(51),Y2(51),Y3(51),PLT(51),
      2TEMP(1025),SCALE(3),AMAX(3),CSOC(10)
C
C        CASE TWO U=BX+CX**3
1000  DENO=CSOC(2)*CSOC(4)-CSOC(3)**2
      BFC=(CSOC(1)*CSOC(4)-CSOC(3)*CSOC(5))/DENO
      CFC=(CSOC(2)*CSOC(5)-CSOC(3)*CSOC(1))/DENO
1100  WRITE OUTPUTTAPE 6,1200,CONE,CTWO,TT,BFC,CFC
1200 0FORMAT(///23HCASE TWO U=BX1+CX1**3   /5HXONE,F6.2/5H XTWO,
      1 F6.2/3H TT,F6.2/20H BY DIFF APPROX  BFC    ,E20.8,10X3HCFC,
      2  E20.8/)
C
C        SPECIFIC OPTIMAL TRAJECTORY DIFF APP
      IFLAG=5
      DO 2 I=1,900
    2 T(I)=0.0
C       INITIAL   APPROXIMATION
C         INTEGRATION  SETUP
      ZETA=0.0
      ETA=0.0
      T(2)=TI
      T(3)=HGRID
      T(4)=CONE
      T(5)=CTWO
      T(6)=BFC
      T(7)=CFC
      T(8)=ALMDP
      T(9)=AMUP
      T(10)=ZETA
      T(11)=ETA
      CALL INT(T,8,2,0.,0.,0.,0.,0.,0.)
      U=BFC*CONE+CFC*CONE**3
      WRITE OUTPUT TAPE 6,50,T(2),T(4),T(5),U
 50   0FORMAT(/39H SPE OPT TRAJ DIFF APPROX U=BX1+CX1**3      /
      1 17X1HT10X4HXONE12X4HXTWO12X13HU=BX1+CX1**3   //10XF10.4,3E15.6)
```

```
C
C        SOLUTION OF NONLINEAR DIFF EQS
C
      M=1+NPRNT
      DO 4 N=2,NMAX
      CALL  INTM
      DO 3 I=1,8
    3 W(I,N)=T(I+3)
      IF(N-M) 4,5,4
5     U=BFC*T(4)+CFC*T(4)**3
      WRITE OUTPUTTAPE 6,70,T(2),T(4),T(5),U
70    FORMAT(10XF10.4,3E15.6)
      M=M+NPRNT
    4 CONTINUE
      W(1,1)=CONE
      W(2,1)=CTWO
      W(3,1)=BFC
      W(4,1)=CFC
      W(5,1)=ALMDP
      W(6,1)=AMUP
      W(7,1)=ZETA
      W(8,1)=ETA
      DO 45 N=1,NMAX
45    FUNC(N)=W(1,N)**2+W(2,N)**2+(BFC*W(1,N)+CFC*W(1,N)**3)**2
      CALL   SIMPSN(FUNC,HGRID,NMAX,SIMP)
      PERIN=SIMP
      WRITE OUTPUT TAPE 6,80,PERIN
80    FORMAT(/ 6HPERIN,F30.8/)
C         PLOT SPE OPT TRAJ
46    J=0
      DO 47 I=1,NMAX,MCON
       J=J+1
      Y1(J)=W(1,I)
      Y2(J)=W(2,I)
      Y3(J)=BFC*W(1,I)+CFC*W(1,I)**3
47    CONTINUE
      WRITE OUTPUTTAPE 6,48,CONE,CTWO,TT,BFC,CFC
      CALL  PLOT
48   0FORMAT(///67HSPE OPT TRAJ DIFF APP U=BX1+CX1**3 GRAPHS XONE XTWO
     1AND U SPE OPT     /10X4HXONE,F8.4,5X4HXTWO,F8.4,5X2HTT,F8.4,5X3HBFC
     2,F10.6,5X3HCFC,F10.6/)
C         QUASILINEARIZATION SOLUTION
      IFLAG=6
      PREV(1)=CONE
      PREV(2)=CTWO
      PREV(3)=BFC
      PREV(4)=CFC
      PREV(5)=ALMDP
      PREV(6)=AMUP
      PREV(7)=ZETA
```

```
      PREV(8)=ETA
C
C     INTEGRATION  SETUP
      ITER=1
350   DO 20 I=1,900
20    T(I)=0.0
      T(2)=TI
      T(3)=HGRID
      T(4)=1.0
      T(13)=1.0
      T(22)=1.0
      T(31)=1.0
      T(40)=1.0
      T(49)=1.0
      T(58)=1.0
      T(67)=1.0
      XONE=PREV(1)
      XTWO=PREV(2)
      BFC=PREV(3)
      CFC=PREV(4)
      ALMD=PREV(5)
      AMU=PREV(6)
      ZETA=PREV(7)
      ETA=PREV(8)
      CALL  INT(T,72,2,0.,0.,0.,0.,0.,0.)
      N=1
      L=3
      DO 21 I=1,8
      DO 21 J=1,8
      L=L+1
   21 H(I,J,N)=T(L)
      DO 22 I=1,8
      L=L+1
   22 P(I,N)=T(L)
C
C     INTEGRATE  P  AND  H
C
      DO 24 N=2,NMAX
      XONE=W(1,N)
      XTWO=W(2,N)
      BFC=W(3,N)
      CFC=W(4,N)
      ALMD=W(5,N)
      AMU=W(6,N)
      ZETA=W(7,N)
      ETA=W(8,N)
      CALL  INTM
      L=3
      DO 23 I=1,8
      DO 23 J=1,8
```

```
      L=L+1
23    H(I,J,N)=T(L)
      DO 24 I=1,8
      L=L+1
24    P(I,N)=T(L)
C
C          DETERMINE CONSTANTS TO MATCH THE GIVEN BOUNDARY CONDITIONS
C     USES MATINV  DOUBLE PRECISION ACCURACY
D     DIMENSION  A(10,10),B(10,1)
      N=NMAX
      CON(1)=PREV(1)
      CON(2)=PREV(2)
      CON(7)=PREV(7)
      CON(8)=PREV(8)
      DO 31  J=1,4
      DO 31  I=1,4
31    A(I,J)=H(I+4,J+2,N)
      DO 32 I=1,4
      B(I,1)=-P(I+4,N)-H(I+4,7,N)*CON(7)-H(I+4,8,N)*CON(8)
      DO 32 J=1,2
32    B(I,1)=B(I,1)-H(I+4,J,N)*CON(J)
D     CALL  MATINV(A,4,B,1,DET)
      DO 33 I=1,4
33    CON(I+2)=B(I,1)
C
C    NEW  VARIABLES
C
      DO 7 N=2,NMAX
      DO 6 I=1,8
      W(I,N)=P(I,N)
      DO 6 J=1,8
    6 W(I,N)=W(I,N)+H(I,J,N)*CON(J)
    7 CONTINUE
    9 DO 10 I=1,8
   10 PREV(I)=CON(I)
      WRITE OUTPUTTAPE 6,98,(CON(I),I=1,8),(W(I,201),I=1,8)
98   0FORMAT(/28HQL SOLUTION BOUNDARY PTS           /8H INITIAL,
     14E20.8/4E20.8/9HTERMINAL,4E20.8/4E20.8/)
C
C          CHECK THE SOLUTION
      ERROR=ABSF(BFC-CON(3))
      IF(ERROR-0.00001) 61,61,62
61    ERROR=ABSF(CFC-CON(4))
      IF(ERROR-0.00001) 63,63,62
62    IF(ITER-4) 300,63,63
300   ITER=ITER+1
      BFC=CON(3)
      CFC=CON(4)
C  DO NEXT ITERATION
      GO TO 350
```

```
63    BFC=CON(3)
      CFC=CON(4)
      WRITE OUTPUTTAPE 6,99,ITER
99    FORMAT(//20HITERATION PERFORMED    ,I5///)
C      WRITE QL TRAJ
      U=BFC*CONE+CFC*CONE**3
      WRITE OUTPUT TAPE 6,64,TI,CONE,CTWO,U
64   0FORMAT(///30HSPE OPT TRAJ QL U=BX1+CX1**3     /17X1HT10X4HXONE12X
     14HXTWO12X13HU=BX1+CX1**3    /10XF10.4,3E15.6)
      DO 65 N=21,NMAX,NPRNT
      FN=N-1
      TIME=FN*HGRID+TI
      U=BFC*W(1,N)+CFC*W(1,N)**3
65    WRITE OUTPUT TAPE 6,66,TIME,W(1,N),W(2,N),U
66    FORMAT(10XF10.4,3E15.6)
C
C         PERFORMANCEINDEX
C
      W(1,1)=CON(1)
      W(2,1)=CON(2)
      DO 26 N=1,NMAX
26    FUNC(N)=W(1,N)**2+W(2,N)**2+(BFC*W(1,N)+CFC*W(1,N)**3)**2
      CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
      PERIN=SIMP
      WRITE OUTPUT TAPE 6,80,PERIN
C
C         PLOT QL TRAJ
71    J=0
      DO 72 I=1,NMAX,MCON
      J=J+1
      Y1(J)=W(1,I)
      Y2(J)=W(2,I)
72    Y3(J)=BFC*W(1,I)+CFC*W(1,I)**3
      WRITE OUTPUT TAPE 6,73,CONE,CTWO,TT,BFC,CFC
      CALL   PLOT
73    FORMAT(///61HSPE OPT TRAJ QL U=BX1+CX1**3 GRAPHS XONE XTWOAND U
     1SPE OPT     /10X4HXONE,F8.4,5X4HXTWO,F8.4,5X2HTT,F8.4,5X3HBFC,
     2 F10.6,5X3HCFC,F10.6/)
      RETURN
      END
*     SYMBOL TABLE
*     LABEL
CBX1CX2   CASE THREE U=BX1+CX2  DIFF APP AND QL
      SUBROUTINE  BX1CX2
C
C
     0COMMON  T,W,H,P,HGRID,NMAX,PREV,CON,IFLAG,NPRNT,XONE,XTWO,BFC,CFC,
     1ALMD,AMU,ZETA,ETA,FUNC,XO,YO,UO,PERIN,Y1,Y2,Y3,MBL,PLT,TEMP,SCALE,
     2AMAX,IERR,YL,YU,TI,TT,NP,MCON,CSOC,SIMP,CONE,CTWO,ALMDP,AMUP
     0DIMENSION  T(900),W(8,201),H(8,8,201),P(8,201),PREV(8),CON(8),
```

```
     1FUNC(201),XO(201),YO(201),UO(201),Y1(51),Y2(51),Y3(51),PLT(51),
     2TEMP(1025),SCALE(3),AMAX(3),CSOC(10)
C
C          CASE THREE  U=BX1+CX2
2000  DENO=CSOC(2)*CSOC(7)-CSOC(8)**2
      BFC=(CSOC(1)*CSOC(7)-CSOC(6)*CSOC(8))/DENO
      CFC=(CSOC(2)*CSOC(6)-CSOC(8)*CSOC(1))/DENO
2100  WRITE OUTPUTTAPE 6,2200,CONE,CTWO,TT,BFC,CFC
2200  FORMAT(///22HCASE THREE U=BX1+CX2    /5HXONE,F6.2/5HXTWO,
     1 F6.2/3H TT,F6.2/20HBY DIFF APPROX  BFC    ,E20.8,10X3HCFC,
     2  E20.8/)
C
C         SPECIFIC OPTIMAL TRAJECTORY DIFF APP
C         INTEGRATE NONLINEAR DIFF EQS
C
      IFLAG=7
      DO 2 I=1,900
    2 T(I)=0.0
C        INTEGRATION  SETUP
      ZETA=0.0
      ETA=0.0
  102 T(2)=TI
      T(3)=HGRID
      T(4)=CONE
      T(5)=CTWO
      T(6)=BFC
      T(7)=CFC
      T(8)=ALMDP
      T(9)=AMUP
      T(10)=ZETA
      T(11)=ETA
      CALL  INT(T,8,2,0.,0.,0.,0.,0.,0.)
      U=BFC*CONE+CFC*CTWO
      WRITE OUTPUT TAPE 6,50,T(2),T(4),T(5),U
50   0FORMAT(/39HSPE OPT TRAJ DIFF APPROX U=BX1+CX2        /
     1 17X1HT10X4HXONE12X4HXTWO12X13HU=BX1+CX2       //10XF10.4,3E15.6)
C
C         SOLUTION OF NONLINEAR DIFF EQS
      M=1+NPRNT
      DO 4 N=2,NMAX
      CALL  INTM
      DO 3 I=1,8
    3 W(I,N)=T(I+3)
      IF(N-M) 4,5,4
5     U=BFC*T(4)+CFC*T(5)
      WRITE OUTPUT TAPE 6,70,T(2),T(4),T(5),U
70    FORMAT(10XF10.4,3E15.6)
      M=M+NPRNT
    4 CONTINUE
C
```

```
C         HOLD GIVEN INITIAL CONDITIONS
      W(1,1)=CONE
      W(2,1)=CTWO
      W(3,1)=BFC
      W(4,1)=CFC
      W(5,1)=ALMDP
      W(6,1)=AMUP
      W(7,1)=ZETA
      W(8,1)=ETA
      DO 45 N=1,NMAX
45    FUNC(N)=W(1,N)**2+W(2,N)**2+(BFC*W(1,N)+CFC*W(2,N))**2
      CALL   SIMPSN(FUNC,HGRID,NMAX,SIMP)
      PERIN=SIMP
      WRITE OUTPUT TAPE 6,8Ø,PERIN
8Ø    FORMAT(/ 6HPERIN,F3Ø.8/)
C          PLOT SPE OPT TRAJ
46    J=Ø
      DO 47 I=1,NMAX,MCON
       J=J+1
      Y1(J)=W(1,I)
      Y2(J)=W(2,I)
      Y3(J)=BFC*W(1,I)+CFC*W(2,I)
47    CONTINUE
      WRITE OUTPUT TAPE 6,48,CONE,CTWO,TT,BFC,CFC
      CALL  PLOT
48   ØFORMAT(///67HSPE OPT TRAJ DIFF APP U=BX1+CX2    GRAPHS XONE XTWO
     1AND U SPE OPT    /1ØX4HXONE,F8.4,5X4HXTWO,F8.4,5X2HTT,F8.4,5X3HBFC
     2,F1Ø.6,5X3HCFC,F1Ø.6/)
C
C     QUASILINEARIZATION SOLUTION
      IFLAG=8
      PREV(1)=CONE
      PREV(2)=CTWO
      PREV(3)=BFC
      PREV(4)=CFC
      PREV(5)=ALMDP
      PREV(6)=AMUP
      PREV(7)=ZETA
      PREV(8)=ETA
      ITER=1
C
35Ø   DO 6 I=1,9ØØ
    6 T(I)=Ø.Ø
      T(2)=TI
      T(3)=HGRID
      T(4)=1.Ø
      T(13)=1.Ø
      T(22)=1.Ø
      T(31)=1.Ø
      T(4Ø)=1.Ø
```

```
      T(49)=1.0
      T(58)=1.0
      T(67)=1.0
      XONE=PREV(1)
      XTWO=PREV(2)
      BFC=PREV(3)
      CFC=PREV(4)
      ALMD=PREV(5)
      AMU=PREV(6)
      ZETA=PREV(7)
      ETA=PREV(8)
C
      CALL INT(T,72,2,0.,0.,0.,0.,0.,0.)
      N=1
      L=3
      DO 21 I=1,8
      DO 21 J=1,8
      L=L+1
   21 H(I,J,N)=T(L)
      DO 22 I=1,8
      L=L+1
   22 P(I,N)=T(L)
C
C         INTEGRATE P AND H
C
      DO 7 N=2,NMAX
      XONE=W(1,N)
      XTWO=W(2,N)
      BFC=W(3,N)
      CFC=W(4,N)
      ALMD=W(5,N)
      AMU=W(6,N)
      ZETA=W(7,N)
      ETA=W(8,N)
      CALL  INTM
      L=3
      DO 8 I=1,8
      DO 8 J=1,8
      L=L+1
    8 H(I,J,N)=T(L)
      DO 9 I=1,8
      L=L+1
    9 P(I,N)=T(L)
    7 CONTINUE
C
C         DETERMINE CONSTANTS TO MATCH THE GIVEN BOUNDARY CONDITIONS
C     USES MATINV  DOUBLE PRECISION ACCURACY
C
D     DIMENSION  A(10,10),B(10,1)
      N=NMAX
```

```
      CON(1)=PREV(1)
      CON(2)=PREV(2)
      CON(7)=PREV(7)
      CON(8)=PREV(8)
      DO 31 J=1,4
      DO 31 I=1,4
31    A(I,J)=H(I+4,J+2,N)
      DO 32 I=1,4
      B(I,1)=-P(I+4,N)-H(I+4,7,N)*CON(7)-H(I+4,8,N)*CON(8)
      DO 32 J=1,2
32    B(I,1)=B(I,1)-H(I+4,J,N)*CON(J)
D     CALL MATINV(A,4,B,1,DET)
      DO 33 I=1,4
33    CON(I+2)=B(I,1)
C
C    NEW  VARIABLES
      DO 10 N=2,NMAX
      DO 11 I=1,8
      W(I,N)=P(I,N)
      DO 11 J=1,8
   11 W(I,N)=W(I,N)+H(I,J,N)*CON(J)
   10 CONTINUE
C
C          HOLD NEW INITIAL CONDITIONS
      DO 13 I=1,8
13    PREV(I)=CON(I)
      WRITE OUTPUT TAPE 6,98,(CON(I),I=1,8),(W(I,201),I=1,8)
98   0FORMAT(/28HQL SOLUTION BOUNDARY PTS          /8H INITIAL,
     14E20.8/4E20.8/9HTERMINAL,4E20.8/4E20.8/)
C
C           CHECK THE SOLUTION
      ERROR=ABSF(BFC-CON(3))
      IF(ERROR-0.00001)61,61,62
61    ERROR=ABSF(CFC-CON(4))
      IF(ERROR-0.00001) 63,63,62
62    IF(ITER-4) 300,63,63
300   ITER=ITER+1
      BFC=CON(3)
      CFC=CON(4)
C  DO NEXT ITERATION
      GO TO 350
63    BFC=CON(3)
      CFC=CON(4)
      WRITE OUTPUTTAPE 6,99,ITER
99    FORMAT(//20HITERATION PERFORMED    ,I5///)
C      WRITE QL TRAJ
      U=BFC*CONE+CFC*CTWO
      WRITE OUTPUTTAPE 6,64,TI,CONE,CTWO,U
64   0FORMAT(///30HSPE OPT TRAJ QL U=BX1+CX2        /17X1HT10X4HXONE12X
     14HXTWO12X13HU=BX1+CX2         /10XF10.4,3E15.6)
```

```
      DO 65 N=21,NMAX,NPRNT
      FN=N-1
      TIME=FN*HGRID+ TI
      U=BFC*W(1,N)+CFC*W(2,N)
65    WRITE OUTPUT TAPE 6,66,TIME,W(1,N),W(2,N),U
66    FORMAT(10XF10.4,3E15.6)
C         CALCULATE PERFORMANCE INDEX
C
      W(1,1)=CON(1)
      W(2,1)=CON(2)
      DO 26 N=1,NMAX
26    FUNC(N)=W(1,N)**2+W(2,N)**2+(BFC*W(1,N)+CFC*W(2,N))**2
      CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
  200 PERIN=SIMP
      WRITE OUTPUT TAPE 6,80,PERIN
C
C         PLOT THE QL TRAJ
C
71    J=0
      DO 72 I=1,NMAX,MCON
      J=J+1
      Y1(J)=W(1,I)
      Y2(J)=W(2,I)
72    Y3(J)=BFC*W(1,I)+CFC*W(2,I)
      WRITE OUTPUT TAPE 6,73,CONE,CTWO,TT,BFC,CFC
      CALL   PLOT
73    FORMAT(///61H SPE OPT TRAJ QL U=BX1+CX2    GRAPHS XONE XTWO AND U
     1SPE OPT      /10X4HXONE,F8.4,5X4HXTWO,F8.4,5X2HTT,F8.4,5X3HBFC,
     2  F10.6,5X3HCFC,F10.6/)
  107 RETURN
      END
*     SYMBOL TABLE
*     LABEL
CDAUX     SUBROUTINE DAUX DESCRIBING DIFFERENTIAL EQUATIONS FOR PLANT
C         DDX+3DX+2X+0.5X**3=U OPTIMAL U=BX1 U=BX1+CX1**3 U=BX1+CX2
      SUBROUTINE  DAUX
C
     0COMMON  T,W,H,P,HGRID,NMAX,PREV,CON,IFLAG,NPRNT,XONE,XTWO,BFC,CFC,
     1ALMD,AMU,ZETA,ETA,FUNC,X0,Y0,U0,PERIN,Y1,Y2,Y3,MBL,PLT,TEMP,SCALE,
     2AMAX,IERR,YL,YU,TI,TT,NP,MCON,CSOC,SIMP,CONE,CTWO,ALMDP,AMUP
     0DIMENSION  T(900),W(8,201),H(8,8,201),P(8,201),PREV(8),CON(8),
     1FUNC(201),X0(201),Y0(201),U0(201),Y1(51),Y2(51),Y3(51),PLT(51),
     2TEMP(1025),SCALE(3),AMAX(3),CSOC(10)
C
C       PHASE DECISION
      GO TO (1,2,3,4,5,6,7,8),IFLAG
C
C         OPTIMAL SOLUTION
C        NONLINEAR  EQUATIONS
    1  T(8)=T(5)
```

```
      T(9)=-2.*T(4)-3.*T(5)-Ø.5*T(7)  -Ø.5*T(4)**3
      T(1Ø)=-2.*T(4)+2.*T(7) +1.5*T(7)*T(4)**2
      T(11)=-2.*T(5)+3.*T(7)-T(6)
1ØØ   RETURN
C
C       LINEAR  EQUATIONS     OPTIMAL SOLUTION
2     AA=-1.5*XONE**2
      AB=3.Ø*AMU*XONE
      DO 2Ø I=1,4
      T(I+23)=T(I+7)
      T(I+27)=-2.*T(I+3)-3.*T(I+7)-Ø.5*T(I+15) +AA*T(I+3)
      T(I+31)=-2.*T(I+3)+2.*T(I+15)+AB*T(I+3)-AA*T(I+15)
2Ø    T(I+35)=-2.*T(I+7)-T(I+11)+3.*T(I+15)
      T(4Ø)=T(21)
      T(41)=-2.*T(2Ø)-3.*T(21)-Ø.5*T(23)+XONE**3  +AA*T(2Ø)
      T(42)=-2.*T(2Ø)+2.*T(23) -3.Ø*AMU*XONE**2 +AB*T(2Ø)-AA*T(23)
      T(43)=-2.*T(21)-T(22)+3.*T(23)
2ØØ   RETURN
C
C     CASE ONE U=BX
C        NONLINEAR  EQUATIONS
3     T(1Ø)=T(5)
      T(11)=T(6)*T(4)-2.Ø*T(4)-3.Ø*T(5)-Ø.5*T(4)**3
      T(12)=Ø.Ø
     ØT(13)=-2.Ø*T(4)*T(6)**2-2.Ø*T(4)-T(6)*T(8)+2.Ø*T(8)+
     1  1.5*T(8)*T(4)**2
      T(14)=-2.Ø*T(5)-T(7)+3.Ø*T(8)
      T(15)=-2.Ø*T(6)*T(4)**2-T(8)*T(4)
3ØØ   RETURN
C
C       LINEAR  EQUATIONS  U=BX1
4     AA=BFC-2.Ø -1.5*XONE**2
      BB=-2.Ø*BFC**2-2.Ø+3.Ø*AMU*XONE
      CC=-4.Ø*BFC*XONE-AMU
      DD=-2.Ø*XONE**2
      DO 4Ø I=1,6
      T(I+45)=T(I+9)
      T(I+51)=AA*T(I+3)-3.Ø*T(I+9)+XONE*T(I+15)
      T(I+57)=Ø.Ø
      T(I+63)=BB*T(I+3)+CC*T(I+15)-AA*T(I+27)
      T(I+69)=-2.Ø*T(I+9)-T(I+21)+3.Ø*T(I+27)
4Ø    T(I+75)=CC*T(I+3)+DD*T(I+15)-XONE*T(I+27)
      T(82)=T(41)
      T(83)=AA*T(4Ø)-3.Ø*T(41)+XONE*T(42)-BFC*XONE +XONE**3
      T(84)=Ø.Ø
      T(85)=BB*T(4Ø)+CC*T(42)-AA*T(44)-CC*BFC -3.Ø*AMU*XONE**2
      T(86)=-2.Ø*T(41)-T(43)+3.Ø*T(44)
      T(87)=CC*T(4Ø)+DD*T(42)-XONE*T(44)-CC*XONE
4ØØ   RETURN
C
```

```
C      CASE TWO U=BX+CX**3
C         NONLINEAR  EQUATIONS
5      XONEQ=T(4)**3
       T(12)=T(5)
       T(13)=-2.*T(4)-3.*T(5)+T(6)*T(4)+T(7)*XONEQ-Ø.5*XONEQ
       T(14)=Ø.
       T(15)=Ø.
      ØT(16)=-2.*T(4)-2.*T(4)*T(6)**2-8.*T(6)*T(7)*XONEQ+2.*T(9)
      1 -(6.*T(7)**2)*XONEQ*T(4)**2-3.*T(9)*T(7)*T(4)**2-T(6)*T(9)
      2  +1.5*T(9)*T(4)**2
       T(17)=-2.*T(5)-T(8)+3.*T(9)
       T(18)=-2.*T(6)*T(4)**2-2.*T(7)*T(4)*XONEQ-T(9)*T(4)
       T(19)=-2.*T(6)*T(4)*XONEQ-2.*T(7)*XONEQ**2-T(9)*XONEQ
5ØØ    RETURN
C
C       FOR  U=B*XONE+C*XONE**3
C        LINEAR  EQUATIONS
6      XONEQ=XONE**3
       AA=-2.+BFC+3.*CFC*XONE**2 -1.5*XONE**2
      ØBB=-2.-2.*BFC**2-24.*BFC*CFC*XONE**2-3Ø.*CFC**2*XONE**4
      1  -6.*AMU*CFC*XONE  +3.Ø*AMU*XONE
       CC=-4.*BFC*XONE-8.*CFC*XONEQ-AMU
       DD=-8.*BFC*XONEQ-12.*CFC*XONEQ*XONE**2-3.*AMU*XONE**2
       EE=-2.*XONE**2
       FF=EE*XONE**2
       GG=FF*XONE**2
       DO 6Ø I=1,8
       T(I+75)=T(I+11)
       T(I+83)=AA*T(I+3)-3.*T(I+11)+XONE*T(I+19)+XONEQ*T(I+27)
       T(I+91)=Ø.Ø
       T(I+99)=Ø.Ø
       T(I+1Ø7)=BB*T(I+3)+CC*T(I+19)+DD*T(I+27)-AA*T(I+43)
       T(I+115)=-2.*T(I+11)-T(I+35)+3.*T(I+43)
       T(I+123)=CC*T(I+3)+EE*T(I+19)+FF*T(I+27)-XONE*T(I+43)
6Ø     T(I+131)=DD*T(I+3)+FF*T(I+19)+GG*T(I+27)-XONEQ*T(I+43)
       T(14Ø)=T(69)
      ØT(141)=AA*T(68)-3.*T(69)+XONE*T(7Ø)+XONEQ*T(71)-BFC*XONE-
      1  3.*CFC*XONEQ +XONEQ
       T(142)=Ø.Ø
       T(143)=Ø.Ø
      ØT(144)=BB*T(68)+CC*T(7Ø)+DD*T(71)-AA*T(73)
      1    +(32.*BFC*CFC+36.*CFC**2*XONE**2)*XONEQ
      2 +9.*AMU*CFC*XONE**2+4.*BFC**2*XONE+AMU*BFC-3.Ø*AMU*XONE**2
       T(145)=-2.*T(69)-T(72)+3.*T(73)
      ØT(146)=CC*T(68)+EE*T(7Ø)+FF*T(71)-XONE*T(73)+AMU*XONE+
      1  (4.*BFC+8.*CFC*XONE**2)*XONE**2
      ØT(147)=DD*T(68)+FF*T(7Ø)+GG*T(71)-XONEQ*T(73)+
      1  (8.*BFC*XONE+12.*CFC*XONEQ+3.*AMU)*XONEQ
6ØØ    RETURN
C
```

```
C          CASE THREE   U=BX1+CX2
C         NONLINEAR  EQUATIONS
C
7      T(12)=T(5)
       T(13)=-2.*T(4)-3.*T(5)+T(6)*T(4)+T(7)*T(5) -Ø.5*T(4)**3
       T(14)=Ø.Ø
       T(15)=Ø.Ø
      ØT(16)=-2.*T(4)-2.*T(6)**2*T(4)-2.*T(6)*T(7)*T(5)+2.*T(9)-T(6)*T(9)
      1  +1.5*T(9)*T(4)**2
      ØT(17)=-2.*T(5)-2.*T(6)*T(7)*T(4)-2.*T(7)**2*T(5)-T(8)+3.*T(9)
      1  -T(7)*T(9)
       T(18)=-2.*T(6)*T(4)**2-2.*T(7)*T(5)*T(4)-T(9)*T(4)
       T(19)=-2.*T(6)*T(4)*T(5)-2.*T(7)*T(5)**2-T(9)*T(5)
7ØØ    RETURN
C
C        LINEAR  EQUATIONS  U=BX1+CX2
8      AA=-2.+BFC    -1.5*XONE**2
       AB=-3.+CFC
       AC=-2.-2.*BFC**2 +3.Ø*AMU*XONE
       AD=-2.*BFC*CFC
       AE=-4.*BFC*XONE-2.*CFC*XTWO-AMU
       AF=-2.*BFC*XTWO
       AG=-2.-2.*CFC**2
       AH=-2.*CFC*XONE
       AI=-2.*BFC*XONE-4.*CFC*XTWO-AMU
       AJ=-2.*XONE**2
       AK=-2.*XONE*XTWO
       AL=-2.*XTWO**2
       AM=-BFC*XONE-CFC*XTWO
       AN=4.*BFC**2*XONE+4.*BFC*CFC*XTWO+AMU*BFC
       AO=4.*BFC*CFC*XONE+4.*CFC**2*XTWO+AMU*CFC
       AP=4.*BFC*XONE**2+AMU*XONE+4.*CFC*XONE*XTWO
       AQ=4.*BFC*XONE*XTWO+4.*CFC*XTWO**2+AMU*XTWO
       DO 8Ø I=1,8
       T(I+75)=T(I+11)
       T(I+83)=AA*T(I+3)+AB*T(I+11)+XONE*T(I+19)+XTWO*T(I+27)
       T(I+91)=Ø.Ø
       T(I+99)=Ø.Ø
       T(I+1Ø7)=AC*T(I+3)+AD*T(I+11)+AE*T(I+19)+AF*T(I+27)-AA*T(I+43)
      ØT(I+115)=AD*T(I+3)+AG*T(I+11)+AH*T(I+19)+AI*T(I+27)-T(I+35)
      1   -AB*T(I+43)
       T(I+123)=AE*T(I+3)+AH*T(I+11)+AJ*T(I+19)+AK*T(I+27)-XONE*T(I+43)
8Ø     T(I+131)=AF*T(I+3)+AI*T(I+11)+AK*T(I+19)+AL*T(I+27)-XTWO*T(I+43)
       T(14Ø)=T(69)
       T(141)=AA*T(68)+AB*T(69)+XONE*T(7Ø)+XTWO*T(71)+AM    +XONE**3
       T(142)=Ø.Ø
       T(143)=Ø.Ø
      ØT(144)=AC*T(68)+AD*T(69)+AE*T(7Ø)+AF*T(71)-AA*T(73)+AN
      1  -3.Ø*AMU*XONE**2
       T(145)=AD*T(68)+AG*T(69)+AH*T(7Ø)+AI*T(71)-T(72)-AB*T(73)+AO
```

```
      T(146)=AE*T(68)+AH*T(69)+AJ*T(70)+AK*T(71)-XONE*T(73)+AP
      T(147)=AF*T(68)+AI*T(69)+AK*T(70)+AL*T(71)-XTWO*T(73)+AQ
800   RETURN
      END
*     SYMBOL TABLE
*     LABEL
CSKALE    FIND THE SCALE FACTORS FOR THE GRAPH TO BE PLOTTED USING
C         PLOTTING SUBROUTINE PLOT
C         ALL GRAPHS ARE SCALED TO THE INTERVAL -1 TO +1  SCALEFACTORS
C         ARE 1,2,5,10,10**N AND RECIPROCALS OF THESE NUMBERS
      SUBROUTINE SKALE
C
     0COMMON  T,W,H,P,HGRID,NMAX,PREV,CON,IFLAG,NPRNT,XONE,XTWO,BFC,CFC,
     1ALMD,AMU,ZETA,ETA,FUNC,XO,YO,UO,PERIN,Y1,Y2,Y3,MBL,PLT,TEMP,SCALE,
     2AMAX,IERR,YL,YU,TI,TT,NP,MCON,CSOC,SIMP,CONE,CTWO,ALMDP,AMUP
     0DIMENSION  T(900),W(8,201),H(8,8,201),P(8,201),PREV(8),CON(8),
     1FUNC(201),XO(201),YO(201),UO(201),Y1(51),Y2(51),Y3(51),PLT(51),
     2TEMP(1025),SCALE(3),AMAX(3),CSOC(10)
C
      AMAX(1)=ABSF(Y1(1))
      DO 10 I=1,NP
      IF(ABSF(Y1(I))-AMAX(1)) 10,10,11
   11 AMAX(1)=ABSF(Y1(I))
   10 CONTINUE
      AMAX(2)=ABSF(Y2(1))
      DO 20 I=1,NP
      IF(ABSF(Y2(I))-AMAX(2)) 20,20,21
   21 AMAX(2)=ABSF(Y2(I))
   20 CONTINUE
      AMAX(3)=ABSF(Y3(1))
      DO 30 I=1,NP
      IF(ABSF(Y3(I))-AMAX(3)) 30,30,31
   31 AMAX(3)=ABSF(Y3(I))
   30 CONTINUE
   90 DO 40 I=1,3
      XMUL=1.0
      DIV=1.0
      SFI=0.0
      Y=AMAX(I)
      IF(10.-Y) 50,51,52
   51 SCALE(I)=10.0
      GO TO 40
   50 DIV=DIV*10.0
      Y=Y/DIV
      IF(1.0-Y) 53,54,54
   54 SCALE(I)=DIV
      GO TO 40
   53 Y=AMAX(I)
      GO TO 50
   52 IF(1.0-Y) 41,42,43
```

```
   42 SCALE(I)=1.Ø
      GO TO 4Ø
   41 SFI=SFI+1.Ø
      DIV=SFI**2+1.Ø
      Y=Y/DIV
      IF(1.Ø-Y) 44,45,45
   44 Y=AMAX(I)
      GO TO 41
   45 SCALE(I)=DIV
      GO TO 4Ø
   43 IF(Ø.ØØ1-Y)61,6Ø,6Ø
   6Ø SCALE(I)=Ø.ØØ1
      GO TO 4Ø
   61 IF(Ø.1-Y) 62,63,64
   63 SCALE(I)=Ø.1
      GO TO 4Ø
   64 XMUL=XMUL*1Ø.Ø
      Y=Y*XMUL
      IF(1.Ø-Y) 65,67,66
   66 Y=AMAX(I)
      GO TO 64
   67 SCALE(I)=1.Ø/XMUL
      GO TO 4Ø
   65 SCALE(I)=1Ø.Ø/XMUL
      GO TO 4Ø
   62 SFI=SFI+1.Ø
      XMUL=SFI**2+1.Ø
      Y=Y*XMUL
      IF(1.Ø-Y) 7Ø,71,72
   72 Y=AMAX(I)
      GO TO 62
   71 SCALE(I)=1.Ø/XMUL
      GO TO 4Ø
   7Ø SFI=SFI-1.Ø
      XMUL=SFI**2+1.Ø
      SCALE(I)=1.Ø/XMUL
      GO TO 4Ø
   4Ø CONTINUE
   8Ø RETURN
      END
*     SYMBOL TABLE
*     LABEL
CPLOT       PLOTTINGSUBROUTINE TO PLOT THREE CURVES WITH GIVEN SCALE
C           FACTORSUSING NYU PLT1 SUBROUTINE
      SUBROUTINE PLOT
C
     ØCOMMON  T,W,H,P,HGRID,NMAX,PREV,CON,IFLAG,NPRNT,XONE,XTWO,BFC,CFC,
     1ALMD,AMU,ZETA,ETA,FUNC,XO,YO,UO,PERIN,Y1,Y2,Y3,MBL,PLT,TEMP,SCALE,
     2AMAX,IERR,YL,YU,TI,TT,NP,MCON,CSOC,SIMP,CONE,CTWO,ALMDP,AMUP
     ØDIMENSION  T(9ØØ),W(8,2Ø1),H(8,8,2Ø1),P(8,2Ø1),PREV(8),CON(8),
```

```
      1FUNC(201),XO(201),YO(201),UO(201),Y1(51),Y2(51),Y3(51),PLT(51),
      2TEMP(1025),SCALE(3),AMAX(3),CSOC(10)
C
    5 WRITE OUTPUTTAPE 6,10,(SCALE(I),I=1,3)
   10 FORMAT(/25HSCALE FACTOR MULTIPLY BY   /
     1 15HFIRST CURVE *   ,E15.6,10X,15HSECOND CURVE 0   ,E15.6,10X,
     2  14HTHIRD CURVEX   ,E15.6///)
      DO 20 I=1,NP
      Y1(I)=Y1(I)/SCALE(1)
      Y2(I)=Y2(I)/SCALE(2)
      Y3(I)=Y3(I)/SCALE(3)
   20 CONTINUE
   30 YL=-1.0
      YU=1.0
   40 CALL  PLT1(IERR,TEMP,3,MBL,TI,TT,YL,YU,PLT,NP,Y1,PLT,NP,Y2,PLT,
     1  NP,Y3)
   50 RETURN
      END
*     SYMBOL TABLE
*     LABEL
CSIMPSN  SIMPSON INTEGRATION SUBROUTINE
      SUBROUTINE  SIMPSN(FUNC,HGRID,NMAX,SIMP)
C           INTEGRATION OF TABULAR FUNCTION,FUNC,BY SIMPSON RULE
C           WHERE  HGRID=LENGTH OF SUBINTERVAL,NMAX=NUMBER OF POINTS
C           NMAX MUSTBE ODD
      DIMENSION FUNC(201)
      K=(NMAX-1)/2
      SUMA=0.0
      DO 15 J=1,K
   15 SUMA=SUMA+FUNC(2*J)
      SUMB=0.0
      DO 16 J=2,K
   16 SUMB=SUMB+FUNC(2*J-1)
      SIMP=(4.*SUMA+2.*SUMB+FUNC(NMAX)+FUNC(1))*HGRID/3.
  100 RETURN
      END
```

THIS PROGRAM ALSO REQUIRES SUBROUTINES INT,INTM,MATINV,PLT1

APPENDIX I

The program listings contained in this appendix have been used to produce the data for examples in chapter 4. The first program listed produces the optimum control function u*(t) and the optimum control trajectory x*(t) by the "approximation in policy space" approach for example 4.3. Similar programs can be utilized for examples 4.1, 4.2, and 4.4 to produce the optimal control and trajectories. This program also requires the subroutines INT, INTM (integration routines, details in Appendix H) and MATINV (matrix inversion). The main program uses quasi linearization and "approximation in policy space" to solve the optimum control problem. The subroutine INSTRT starts the procedure by providing good initial approximations for the trajectories to start the quasilinearization technique. Subroutine CNSTNT uses MATINV to solve for the missing initial conditions. DAUX is a required subroutine for INT and INTM. SIMSPN is a simple Simspon's rule integration routine for calculating the performance index.

The second program listed produces the specific controller parameters for examples 4.2 & 4.3. by a gradient technique. The main program optimizes the controller parameters by a gradient search. This program requires also INT, INTM and MATINV. DAUX is a derivative routine. PARFIT is a parabolic fitting technique to find a minimum. PERIN calculates the performance index and uses SIMSPN.

The last program listed produces the parameters of specific controller (ii) for Examples 4.2 and 4.3. The main program performs the quasilinearization operations for solving the two-point boundary value problem resulting from the optimization of the controller. INSTRT starts the quasilinearization by providing good initial guesses. CNSTNT solves for the unknown initial conditions. DAUX is the usual derivative routine. SIMSPN is again for calculating performance index. This program also needs INT, INTM and MATINV.

```
*     LABEL
CRM000SECOND ORDER OPTIMAL SOLUTION USING P.M.P. AND Q.L.
C     OPTIMAL SOLUTION
C     MAIN PROGRAM
      COMMON T,W,H,P,KMAX,TI,HGRID,NMAX,N1,IFLAG,NPRNT,PREV,CON,
     1XONE,XTWO,ALMD,AMU,FUNC,NT,NU,DELU
      DIMENSION T(250),W(5,101),H(4,4,101),P(4,101),PREV(4),CON(4),
     1FUNC(101)
C
C     INPUT AND START
    1 CALL INSTRT
C
C     K ITERATIONS
C
      DO 97 N=1,NMAX
      W(5,N)=0.0
97    CONTINUE
      NT=1
      DO 99 K=1,KMAX
C     INTEGRATION SETUP
C
      DO 2 I=1,250
2     T(I)=0.0
      T(2)=TI
      T(3)=HGRID
      T(4)=1.0
      T(9)=1.0
      T(14)=1.0
      T(19)=1.0
      XONE=PREV(1)
      XTWO=PREV(2)
      ALMD=PREV(3)
      AMU=PREV(4)
      CALL INT(T,20,N1,0.,0.,0.,0.,0.,0.)
      N=1
      L=3
      DO 21 I=1,4
      DO 21 J=1,4
      L=L+1
21    H(I,J,N)=T(L)
      DO 22 I=1,4
      L=L+1
22    P(I,N)=T(L)
C
C     INTEGRATE P AND H
C
      DO 4 N=2,NMAX
      NT=N
      XONE=W(1,N)
      XTWO=W(2,N)
```

```
      ALMD=W(3,N)
      AMU=W(4,N)
      CALL INTM
      L=3
      DO 3 I=1,4
      DO 3 J=1,4
      L=L+1
3     H(I,J,N)=T(L)
      DO 4 I=1,4
      L=L+1
4     P(I,N)=T(L)
C
C     DETERMINE CONSTANTS,
C     OR INITIAL VALUES
C
      CALL CNSTNT
      TIME=TI
      IF(ABSF(Ø.5*CON(4))-1.Ø) 6Ø,6Ø,62
6Ø    W(5,1)=-Ø.5*CON(4)
      GO TO 76
62    IF(CON(4))64,6Ø,66
64    W(5,1)=1.Ø
      GO TO 76
66    W(5,1)=-1.Ø
      WRITE OUTPUTTAPE 6,5Ø,K,TIME,(CON(I),I,1,4),W(5,1)
C
C     NEW VARIABLES
C
      M=1+NPRNT
      DO 7 N=2,NMAX
      DO 6 I=1,4
      W(I,N)=P(I,N)
      DO 6 J=1,4
6     W(I,N)=W(I,N)+H(I,J,N)*CON(J)
      IF(ABSF(Ø.5*W(4,N))-1.Ø) 3Ø,3Ø,4Ø
3Ø    W(5,N)=-Ø.5*W(4,N)
      GO TO 28
4Ø    IF(W(4,N)) 32,3Ø,42
32    W(5,N)=1.Ø
      GO TO 28
42    W(5,N)=-1.Ø
28    FN=N-1
      TIME=FN*HGRID+TI
      IF(N-M) 7,15,7
15    WRITE OUTPUTTAPE 6,7Ø,TIME,(W(I,N),I=1,5)
      M=M+NPRNT
7     CONTINUE
9     DO 1Ø I=1,4
1Ø    PREV(I)=CON(I)
      W(1,1)=CON(1)
```

```
      W(2,1)=CON(2)
      DO 26 N=1,NMAX
26    FUNC(N)=W(1,N)**2+W(2,N)**2+W(5,N)**2
      CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
      PERIN=SIMP
      WRITEOUTPUT TAPE 6,8Ø,PERIN
C
C     DO NEXT ITERATION
C
99    CONTINUE
      GO TO 1
C     FORMATS
5Ø    FORMAT(1HØ//59X9HITERATION,I3///
     117X1HT1ØX4HXONE12X4HXTWO12X4HALMD12X3HAMU14X1HU//
     21ØXF1Ø.4,5E16.6)
7Ø    FORMAT(1ØXF1Ø.4,5E16.6)
8Ø    FORMAT(/6HPERIN,F3Ø.8/)
      END
*     LABEL
CRMØØ1  INPUT-STARTSECOND ORDER OPTIMAL SOLUTION
      SUBROUTINE INSTRT
      COMMON T,W,H,P,KMAX,TI,HGRID,NMAX,N1,IFLAG,NPRNT,PREV,CON,
     1XONE,XTWO,ALMD,AMU,FUNC,NT,NU,DELU
      DIMENSION T(25Ø),W(5,1Ø1),H(4,4,1Ø1),P(4,1Ø1),PREV(4),CON(4),
     1FUNC(1Ø1)
C
C     INPUT
      READ INPUT TAPE 5,11Ø,N1,KMAX,NMAX,NPRNT,TI
C
C     INITIAL CONDITIONS
      READ INPUT TAPE 5,12Ø,HGRID,XONE,XTWO,ALMD,AMU
      WRITE OUTPUT TAPE 6,1Ø,N1,KMAX,NMAX,NPRNT,HGRID
C
C     START
      IFLAG=1
      DO 2 I=1,25Ø
2     T(I)=Ø.Ø
C     INITIAL APPROXIMATION
C     INTEGRATION SETUP
      T(2)=TI
      T(3)=HGRID
      T(4)=XONE
      T(5)=XTWO
      T(6)=ALMD
      T(7)=AMU
      CALL INT(T,4,N1,Ø.,Ø.,Ø.,Ø.,Ø.,Ø.)
      WRITE OUTPUT TAPE 6,5Ø,T(2),(T(I),I,4,7)
C
C     SOLUTION OF NONLINEAR DIFF EQNS
C
```

```
      M=1+NPRNT
      DO 4 N=2,NMAX
      CALL INTM
      DO 3 I=1,4
3     W(I,N)=T(I+3)
      IF(N-M) 4,5,4
5     WRITE OUTPUT TAPE 6,70,T(2),(T(I),I=4,7)
      M=M+NPRNT
4     CONTINUE
C     INTIALIZATION FOR SECOND STAGE
      IFLAG=2
      PREV(1)=XONE
      PREV(2)=XTWO
      PREV(3)=ALMD
      PREV(4)=AMU
      RETURN
10    FORMAT(1H1/43HSECOND ORDER LINEAR SYSTEM OPTIMAL PROBLEM//
     13HN1,I10/5HKMAX,I10/5HNMAX,I10/6H NPRNT,I10/6H HGRID,F15.6///
     220X16HOPTIMAL SOLUTION///)
110   FORMAT(4I10,F10.0)
120   FORMAT(5F10.0)
50    FORMAT(//22HINITAL APPROXIMATION/
     117X1HT10X4HXONE12X4HXTWO12X4HALMD12X3HAMU//
     210XF10.4,4E16.6)
70    FORMAT(10XF10.4,4E16.6)
      END
*     LABEL
CRM002 SOLVE CONSTANTSOR INITIAL CONDITION
C     USES MATINV DOUBLE PRECISION ACCURACY
      SUBROUTINE CNSTNT
      COMMON T,W,H,P,KMAX,TI,HGRID,NMAX,N1,IFLAG,NPRNT,PREV,CON,
     1XONE,XTWO,ALMD,AMU,FUNC,NT,NU,DELU
      DIMENSION T(250),W(5,101),H(4,4,101),P(4,101),PREV(4),CON(4),
     1FUNC(101)
D     DIMENSION A(10,10.,B(10,1)
      N=NMAX
      CON(1)=PREV(1)
      CON(2)=PREV(2)
      DO 1 J=1,2
      DO 1 I=1,2
1     A(I,J)=H(I+2,J+2,N)
      DO 2 I=1,2
2     B(I,1)=-P(I+2,N)-H(I+2,1,N)*CON(1)-H(I+2,2,N)*CON(2)
      WRITE OUTPUT TAPE 6,80,((A(I,J),J=1,2),I=1,2)
      WRITE OUTPUT TAPE 6,90,(B(I,1),I=1,2)
D     CALL MATINV(A,2,B,1,DET)
      WRITE OUTPUT TAPE 6,100,DET
      DO 70 I=1,2
70    CON(I+2)=B(I,1)
      RETURN
```

```
80     FORMAT(/9H MATRIX A/(3ØX,2E16.6))
9Ø     FORMAT(/9HCOLUMN B/3ØX,2E16.6)
1ØØ    FORMAT(/12HDETERMINANT/3ØX,E16.6/)
       END
*      LABEL
CRMØØ3  DAUX SECOND ORDER LINEAR SYSTEM OPTIMAL SOLUTION
       SUBROUTINE DAUX
       COMMON T,W,H,P,KMAX,TI,HGRID,NMAX,N1,IFLAG,NPRNT,PREV,CON,
      1XONE,XTWO,ALMD,AMU,FUNC,NT,NU,DELU
       DIMENSION T(25Ø),W(5,1Ø1),H(4,4,1Ø1),P(4,1Ø1),PREV(4),CON(4),
      1FUNC(1Ø1)
C
C      PHASE DECISION
       GO TO (1,2),IFLAG
C
C      NONLINEAR EQUATIONS
1      T(8)=T(5)
       T(9)=-2.*T(4)-3.*T(5)-Ø.5*(T(4)**3)
       T(1Ø)=-2.*T(4)+2.*T(7)+1.5*T(7)*(T(4)**2)
       T(11)=-2.*T(5)-T(6)+3.*T(7)
       RETURN
C
C      LINEAR EQUATIONS
2      DO 1Ø I=1,4
       T(I+23)=T(I+7)
       T(I+27)=-2.*T(I+3)-3.*T(I+7)-1.5*(XONE**2)*T(I+3)
       T(I+31)=-2.*T(I+3)+2.*T(I+15)+3.*AMU*XONE*T(I+3)+1.5*(XONE**2)
      1T(I+15)
1Ø     T(I+35)=-2.*T(I+7)-T(I+11)+3.*T(I+15)
       T(4Ø)=T(21)
       T(41)=-2.*T(2Ø)-3.*T(21)+W(5,NT)-1.5*(XONE**2)*T(2Ø)+XONE**3
       T(42)=-2.*T(2Ø)+2.*T(23)
      1+3.*AMU*XONE*T(2Ø)+1.5*(XONE**2)*T(23)-3.*AMU*(XONE**2)
       T(43)=-2.*T(21)-T(22)+3.*T(23)
       RETURN
       END
*      LABEL
CRMØØ4  SIMPSON INTEGRATION
       SUBROUTINE SIMPSN(FUNC,HGRID,NMAX,SIMP)
C      INTEGRATION OF TABULAR FUNCTION ,FUNC,BY SIMPSON RULE
C      WHERE HGRID=LENGTH OF SUBINTERVAL,NMAX=NUMBER OF POINTS
       DIMENSON FUNC(1Ø1)
       K=(NMAX-1)/2
       SUMA=Ø.Ø
       DO 15 J=1,K
15     SUMA=SUMA+FUNC(2*J)
       SUMB=Ø.Ø
       DO 16 J=2,K
16     SUMB=SUMB+FUNC(2*J-1)
       SIMP=(4.*SUMA+2.*SUMB+FUNC(NMAX)+FUNC(1))*HGRID/3.
```

```
RETURN
END
```

```
*       LABEL
CGRADIENT TECHNIQUE MAIN PROGRAM
C            U=SAT(AX+BDX)
C            PARAMETER OPTIMIZATION USING PARABOLIC FITTING METHOD
C            THIS PROGRAM IS FOR TWO DIMENSIONAL HILL.
C            OPTIMIZATION IS CARRIED OUT AS TWO ONE-DIMENSIONAL PROBLEMS.
        COMMON T,API,BPI,CONE,CTWO,HGRID,TT,XONE,XTWO,U,PI,FUNC,NMAX
        DIMENSION T(5Ø),XONE(5Ø1),XTWO(5Ø1),U(5Ø1),FUNC(5Ø1)
C
1       READ INPUT TAPE 5,5ØØ,CONA,CONB,TT,NMAX,NOPT,NOIT,TOLL
        FN=NMAX-1
        HGRID=TT/FN
2       READ INPUT TAPE 5,51Ø,CONE,CTWO,A,B
148     WRITE OUTPUT TAPE 6,71Ø
        WRITE OUTPUT TAPE 6,52Ø,CONE,CTWO,TT,NMAX,HGRID,A,B,CONA,CONB
        WRITE OUTPUT TAPE 6,74Ø,NOPT,NOIT,TOLL
1Ø1     AFB=A
1Ø2     BFB=B
145     AMIN=A
146     BMIN=B
        INDEX=Ø
        API=A
        BPI=B
3       CALL PERIN
4       PIAB=PI
C
C            SEARCH MINIMUM IN A DIRECTION
5       IF(ABSF(A)-Ø.Ø2) 6,7,7
6       AA=A+Ø.ØØØ4
        GO TO 8
7       AA=A+Ø.Ø2*A
8       API=AA
        INA=1
        CALL PERIN
9       PIAAB=PI
C            CALCULATEPARTIAL DERIVATIVE OF PI WRT A
1Ø      DPIA=(PIAAB-PIAB)/(AA-A)
        WRITE OUTPUT TAPE 6,67Ø
11      WRITE OUTPUT TAPE 6,53Ø,A,B,PIAB,AA,PIAAB,DPIA
12      IF(PIAAB-PIAB) 13,13,15
15      AA=A
16      PIAAB=PIAB
        GO TO 13
13      AB=AA-DPIA*CONA
        API=AB
        INA=INA+1
17      CALL  PERIN
18      PIABB=PI
        WRITE OUTPUT TAPE 6,54Ø,AB,B,PIABB
19      IF(PIABB-PIAAB) 2Ø,21,21
```

```
20    DPIA=(PIABB-PIAAB)/(AB-AA)
22    AC=AB-DPIA*CONA
23    API=AC
      INA=INA+1
24    CALL PERIN
25    PIACB=PI
26    WRITE OUTPUT TAPE 6,550,DPIA,AC,PIACB
      IF(INA-NOPT) 27,29,29
27    IF(PIACB-PIABB) 28,29,29
28    AA=AB
30    PIAAB=PIABB
31    AB=AC
32    PIABB=PIACB
33    CONA=2.0*CONA
      WRITE OUTPUTTAPE 6,560,CONA
34    GO TO 20
C
21    CONA=CONA/2.0
35    AC=AA-DPIA*CONA
36    API=AC
      INA=INA+1
37    CALL PERIN
38    PIACB=PI
39    WRITE OUTPUT TAPE 6,570,DPIA,AC,PIACB,CONA
      IF(INA-NOPT) 40,29,29
40    IF(PIACB-PIAAB) 29,41,41
41    AB=AC
42    PIABB=PIACB
      GO TO 21
C         PERFORM PARABOLIC FITTING
29    WRITE OUTPUT TAPE 6,580,AA,PIAAB,AB,PIABB,AC,PIACB
43    CALL PARFIT(AA,PIAAB,AB,PIABB,AC,PIACB,ALPHA,BETA,GAMMA)
44    AMIN=-BETA/(2.0*ALPHA)
45    API=AMIN
46    CALL PERIN
47    PIAMB=PI
48    WRITE OUTPUT TAPE 6,590,AMIN,BMIN,PIAMB
124   IF(PIAMB-PIAAB) 125,125,126
126   PIAMB=PIAAB
127   AMIN=AA
125   IF(PIAMB-PIABB) 128,128,129
129   PIAMB=PIABB
130   AMIN=AB
128   IF(PIAMB-PIACB) 131,131,132
132   PIAMB=PIACB
133   AMIN=AC
131   WRITE OUTPUT TAPE 6,690,AMIN,BMIN,PIAMB
C
C         SEARCH MINIMUM IN B DIRECTION
49    A=AMIN
```

```
50     PIAB=PIAMB
51     IF(ABSF(B)-0.02) 52,53,53
52     BA=B+0.0004
       GO TO 54
53     BA=B+0.02*B
54     API=A
55     BPI=BA
       INB=1
56     CALL PERIN
57     PIABA=PI
C          CALCULATE PARTIAL DERIVATIVE OF PI WRT B
58     DPIB=(PIABA-PIAB)/(BA-B)
       WRITE OUTPUT TAPE 6,680
59     WRITE OUTPUT TAPE 6,600,A,B,PIAB,BA,PIABA,DPIB
60     IF(PIABA-PIAB) 61,61,62
62     BA=B
63     PIABA=PIAB
       GO TO 61
61     BB=BA-DPIB*CONB
64     BPI=BB
       INB=INB+1
65     CALL  PERIN
66     PIABB=PI
67     WRITE OUTPUT TAPE 6,610,A,BB,PIABB
68     IF(PIABB-PIABA) 69,70,70
69     DPIB=(PIABB-PIABA)/(BB-BA)
71     BC=BB-DPIB*CONB
72     BPI=BC
       INB=INB+1
73     CALL PERIN
74     PIABC=PI
75     WRITE OUTPUT TAPE 6,620,DPIB,BC,PIABC
       IF(INB-NOPT)76,78,78
76     IF(PIABC-PIABB) 77,78,78
77     BA=BB
79     PIABA=PIABB
80     BB=BC
81     PIABB=PIABC
82     CONB=2.0*CONB
83     WRITE OUTPUT TAPE 6,630,CONB
84     GO TO 69
C
70     CONB=CONB/2.0
85     BC=BA-DPIB*CONB
86     BPI=BC
       INB=INB+1
87     CALL PERIN
88     PIABC=PI
89     WRITE OUTPUT TAPE 6,640,DPIB,BC,PIABC,CONB
       IF(INB-NOPT) 90,78,78
```

```
90    IF(PIABC-PIABA) 78,91,91
91    BB=BC
92    PIABB=PIABC
93    GO TO 70
C
C        PERFORM PARABOLIC FITTING
78    WRITE OUTPUT TAPE 6,650,BA,PIABA,BB,PIABB,BC,PIABC
95    CALL PARFIT(BA,PIABA,BB,PIABB,BC,PIABC,ALPHA,BETA,GAMMA)
96    BMIN=-BETA/(2.0*ALPHA)
97    BPI=BMIN
98    CALL  PERIN
99    PIABM=PI
100   WRITE OUTPUT TAPE 6,660,AMIN,BMIN,PIABM
134   IF(PIABM-PIABA) 135,135,136
136   PIABM=PIABA
137   BMIN=BA
135   IF(PIABM-PIABB) 138,138,139
139   PIABM=PIABB
140   BMIN=BB
138   IF(PIABM-PIABC) 141,141,142
142   PIABM=PIABC
143   BMIN=BC
141   WRITE OUTPUT TAPE 6,700,AMIN,BMIN,PIABM
      INDEX=INDEX+1
103   ERRA=ABSF(AFB-AMIN)
104   ERRB=ABSF(BFB-BMIN)
105   DELA=TOLL*ABSF(AMIN)
106   DELB=TOLL*ABSF(BMIN)
107   IF(ERRA-DELA) 108,108,110
108   IF(ERRB-DELB) 200,200,110
110   AFB=AMIN
111   BFB=BMIN
112   A=AMIN
113   B=BMIN
114   PIAB=PIABM
115   IF(INDEX-NOIT) 5,200,200
200   WRITE OUTPUT TAPE 6,720
      API=AMIN
      BPI=BMIN
      CALL  PERIN
201   NP=(NMAX-1)/20
202   DO 210 N=1,NMAX,NP
203   FN=N-1
204   TIME=FN*HGRID
205   WRITE OUTPUT TAPE 6,730,TIME,XONE(N),XTWO(N),U(N)
210   CONTINUE
      WRITE OUTPUT TAPE 6,750,AMIN,BMIN,PI
109   GO TO 1
500   FORMAT(2F10.0,F5.0,3I5,F10.0)
510   FORMAT(4F10.0)
```

```
520  ØFORMAT(1HØ/39H CONE CTWO TT NMAX HGRID A B CONA CONB  /
     1 2F1Ø.5,F8.3,I5,5F15.8/)
530   FORMAT(/24H A B PIAB AA PIAAB DPIA  /6F15.8/)
540   FORMAT(/12HAB B PIABB  /3F15.8/)
550   FORMAT(/15HDPIA AC PIACB  /3F15.8/)
560   FORMAT(/6HCONA  /F15.8/)
570   FORMAT(/2ØHDPIA AC PIACB CONA /4F15.8/)
580   FORMAT(/28HAA PIAAB AB PIABB AC PIACB   /6F15.8/)
590   FORMAT(/17H AMIN BMIN PIAMB   /3F15.8/)
600   FORMAT(/24HA B PIAB BA PIABA DPIB  /6F15.8/)
610   FORMAT(/12HA BB PIABB  /3F15.8/)
620   FORMAT(/15HDPIB BC PIABC  /3F15.8/)
630   FORMAT(/6HCONB  /F15.8/)
640   FORMAT(/2ØHDPIB BC PIABC CONB  /4F15.8/)
650   FORMAT(/28H BA PIABA BBPIABB BC PIABC  /6F15.8/)
660   FORMAT(/17HAMIN BMIN PIABM  /3F15.8/)
670   FORMAT(///31HSEARCH MINIMUM IN A DIRECTION  //)
680   FORMAT(///31H SEARCH MINIMUM IN B DIRECTION  //)
690   FORMAT(/27HCORRECTED AMIN BMIN PIAMB /3F15.8/)
700   FORMAT(/27HCORRECTED AMIN BMIN PIABM /3F15.8/)
720  ØFORMAT(//29HSPECIFIC OPTIMAL TRAJECTORY   //
     1   6X1HT11X4HXONE16X4HXTWO18X1HU/)
730   FORMAT(F1Ø.5,3F2Ø.8)
740   FORMAT(/16HNOPT NOIT TOLL   /2I1Ø,F1Ø.5/)
710  ØFORMAT(1H1/1ØX92HGRADIENT TECHNIQUE   PARAMETER OPTIMIZATION  USI
     1NG PARABOLIC FITTING METHOD    R.MUKUNDAN.             /
     262H SYSTEM DDX+3DX+2X=U,I=INT(Ø,1)(X**2+DX**2+U**2),U=SAT(AX+BDX)
     3  /)
750   FORMAT(/8HA B PI   /3F15.8/)
1ØØØ  END
*     SYMBOL TABLE
*     LABEL
CDAUX   DAUX SUBROUTINE FOR GRADIENT TECHNIQUE
      SUBROUTINE DAUX
      COMMON T,API,BPI,CONE,CTWO,HGRID,TT,XONE,XTWO,U,PI,FUNC,NMAX
      DIMENSION T(5Ø),XONE(5Ø1),XTWO(5Ø1),U(5Ø1),FUNC(5Ø1)
C
      Z=API*T(4)+BPI*T(5)
      IF(ABSF(Z)-1.Ø) 2Ø,2Ø,21
2Ø    UT=Z
      GO TO 2
21    IF(Z) 22,23,24
22    UT=-1.Ø
      GO TO 2
23    UT=Ø.Ø
      GO TO 2
24    UT=1.Ø
      GO TO 2
2     T(6)=T(5)
3     T(7)=-3.*T(5)-2.*T(4)-Ø.5*(T(4)**3)+UT
```

```
4       RETURN
5       END
*       SYMBOL TABLE
*       LABEL
CPARFIT    PARABOLIC FITTING SUBROUTINE THROUGH 3 POINTS
        SUBROUTINE PARFIT(A,X,B,Y,C,Z,ALPHA,BETA,GAMMA)
D       DIMENSION PAR(1Ø,1Ø),RPAR(1Ø,1)
1       PAR(1,1)=A**2
2       PAR(1,2)=A
3       PAR(1,3)=1.Ø
4       PAR(2,1)=B**2
5       PAR(2,2)=B
6       PAR(2,3)=1.Ø
7       PAR(3,1)=C**2
8       PAR(3,2)=C
9       PAR(3,3)=1.Ø
1Ø      RPAR(1,1)=X
11      RPAR(2,1)=Y
12      RPAR(3,1)=Z
D       CALL  MATINV(PAR,3,RPAR,1,DET)
13      ALPHA=RPAR(1,1)
14      BETA=RPAR(2,1)
15      GAMMA=RPAR(3,1)
16      RETURN
17      END
*       SYMBOL TABLE
*       LABEL
CPERIN    SUBROUTINE  TO CALCULATEPERFORMANCE INDEX
        SUBROUTINE PERIN
        COMMON T,API,BPI,CONE,CTWO,HGRID,TT,XONE,XTWO,U,PI,FUNC,NMAX
        DIMENSION T(5Ø),XONE(5Ø1),XTWO(5Ø1),U(5Ø1),FUNC(5Ø1)
C
        DO 3Ø I=1,5Ø
3Ø      T(I)=Ø.Ø
1       T(2)=Ø.Ø
2       T(3)=HGRID
3       T(4)=CONE
4       T(5)=CTWO
5       CALL  INT(T,2,2,Ø.,Ø.,Ø.,Ø.,Ø.,Ø.)
6       XONE(1)=CONE
7       XTWO(1)=CTWO
8       DO 1Ø I=2,NMAX
9       CALL  INTM
11      XONE(I)=T(4)
12      XTWO(I)=T(5)
1Ø      CONTINUE
14      DO 13 I=1,NMAX
        Z=API*XONE(I)+BPI*XTWO(I)
        IF(ABSF(Z)-1.Ø) 2Ø,2Ø,21
2Ø      U(I)=Z
```

```
      GO TO 13
21    IF(Z) 22,23,24
22    U(I)=-1.Ø
      GO TO 13
23    U(I)=Ø.Ø
      GO TO 13
24    U(I)=1.Ø
      GO TO 13
13    FUNC(I)=XONE(I)**2+XTWO(I)**2+U(I)**2
16    CALL  SIMPSN(FUNC,HGRID,NMAX,PI)
17    RETURN
18    END
```

```
*       LABEL
CRMØØØ  SECOND ORDER SOFT CONSTRAINT SYSTEM
C       SOLUTION
C       MAIN PROGRAM
        COMMON T,W,H,P,KMAX,TI,HGRID,NMAX,N1,IFLAG,NPRNT,PREV,CON,
       1XONE,XTWO,BTRI,AMOO,ALAM,SIGM,U,FUNC,ALPHA
        DIMENSION T(6ØØ),W(6,251),H(6,6,251),P(6,251),PREV(6),U(251),
       1FUNC(251),CON(6)
C
C       INPUT AND START
1       CALL INSTRT
C
C       DO K ITRS.
C
        DO 99 K=1,KMAX
C       INTEGRATION SETUP
C
        DO 2 I=1,6ØØ
2       T(I)=Ø.Ø
        T(2)=TI
        T(3)=HGRID
        T(4)=1.Ø
        T(11)=1.Ø
        T(18)=1.Ø
        T(25)=1.Ø
        T(32)=1.Ø
        T(39)=1.Ø
        XONE=PREV(1)
        XTWO=PREV(2)
        BTRI=PREV(3)
        AMOO=PREV(4)
        ALAM=PREV(5)
        SIGM=PREV(6)
        CALL INT(T,42,N1,Ø.,Ø.,Ø.,Ø.,Ø.,Ø.)
        N=1
        L=3
        DO 21 I=1,6
        DO 21 J=1,6
        L=L+1
21      H(I,J,N)=T(L)
        DO 22 I=1,6
22      P(I,N)=T(L)
C
C       INTEGRATE P AND H
C
        DO 4 N=2,NMAX
        XONE=W(1,N)
        XTWO=W(2,N)
        BTRI=W(3,N)
        AMOO=W(4,N)
```

```
      ALAM=W(5,N)
      SIGM=W(6,N)
      CALL INTM
      L=3
      DO 3 I=1,6
      DO 3 J=1,6
      L=L+1
3     H(I,J,N)=T(L)
      DO 4 I=1,6
      L=L+1
4     P(I,N)=T(L)
C
C     DETERMINE CONSTANTS
C
      CALL CNSTNT
      TIME=TI
      U(1)=CON(1)*CON(3)
      WRITE OUTPUT TAPE 6,5Ø,K,TIME,(CON(I),I=1,6),U(1)
C
C     NEW VARIABLES
C
      M=1+NPRNT
      DO 7 N=2,NMAX
      DO 6 I=1,6
      W(I,N)=P(I,N)
      DO 6 J=1,6
6     W(I,N)=W(I,N)+H(I,J,N)*CON(J)
      U(N)=W(1,N)*W(3,N)
      FN=N-1
      TIME=FN*HGRID+TI
      IF(N-M) 7,15,7
15    WRITE OUTPUT TAPE 6,7Ø,TIME,(W(I,N),I=1,6),U(N)
      M=M+NPRNT
7     CONTINUE
9     DO 1Ø I=1,6
1Ø    PREV(I)=CON(I)
      W(1,1)=CON(1)
      W(2,1)=CON(2)
      W(6,1)=CON(6)
      DO 26 N=1,NMAX
26    FUNC(N)=W(1,N)**2+W(2,N)**2+U(N)**2
      CALL SIMPSN(FUNC,HGRID,NMAX,SIMP)
      PERIN=SIMP
      WRITE OUTPUT TAPE 6,8Ø,PERIN
C
C     DO NEXT OPERATION
C
99    CONTINUE
      GO TO 1
C     FORMATS
```

```
50      FORMAT(1H0//59X9HITERATION,I3///
       14X1HT9X4HXONE9X4HXTWO9X4HBTRI9X4HAMOO10X4HALAM9X4HSIGM10X1HU///
       2F8.4,1P7E14.6)
70      FORMAT(F8.4,1P7E14.6)
80      FORMAT(//19H PERFORMANCE INDEX=,F30.8//)
        END
*       LABEL
CRM001     INPUT START SOFT CONSTRAINT SYSTEM
        SUBROUTINE INSTRT
        COMMON T,W,H,P,KMAX,TI,HGRID,NMAX,N1,IFLAG,NPRNT,PREV,CON,
       1XONE,XTWO,BTRI,AMOO,ALAM,SIGM,U,FUNC,ALPHA
        DIMENSION T(600),W(6,251),H(6,6,251),P(6,251),PREV(6),U(251),
       1FUNC(251),CON(6)
C
C       INPUT
        READ INPUT TAPE 5,110,N1,KMAX,NMAX,NPRNT,TI
C
C       INITIAL CONDITIONS
        READ INPUT TAPE 5,120,HGRID,XONE,XTWO,BTRI
        READ INPUT TAPE 5,130,AMOO,ALAM,SIGM,ALPHA
        WRITE OUTPUT TAPE 6,10,N1,KMAX,NMAX,NPRNT,HGRID,ALPHA
C
C       START
        IFLAG=1
        DO 2 I=1,600
2       T(I)=0.0
C       INITIAL APPROXIMATION
C       INTEGRATION SETUP
        T(2)=TI
        T(3)=HGRID
        T(4)=XONE
        T(5)=XTWO
        T(6)=BTRI
        T(7)=AMOO
        T(8)=ALAM
        T(9)=SIGM
        CALL INT(T,6,N1,0.,0.,0.,0.,0.,0.)
        WRITE OUTPUT TAPE 6,50,T(2),(T(I),I=4,9)
C
C       SOLUTION OF NONLINEAR DIFFERENTIAL EQUATIONS
C
        M=1+NPRNT
        DO 4 N=2,NMAX
        CALL INTM
        DO 3 I=1,6
3       W(I,N)=T(I+3)
        IF(N-M) 4,5,4
5       WRITE OUTPUT TAPE 6,70,T(2),(T(I),I=4,9)
        M=M+NPRNT
4       CONTINUE
```

```
C      INITIALIZATION OF SECOND STAGE
       IFLAG=2
       PREV(1)=XONE
       PREV(2)=XTWO
       PREV(3)=BTRI
       PREV(4)=AMOO
       PREV(5)=ALAM
       PREV(6)=SIGM
       RETURN
10     FORMAT(1H1/36H SECOND ORDER SOFT CONSTRAINT SYSTEM//
      13H N1,I10/5HKMAX,I10/5HNMAX,I10/6H NPRNT,I10/6H HGRID,F15.6/
      26H ALPHA,F15.6///20X18HSPECIFIC SOLUTION///)
110    FORMAT(4I10,F10.0)
120    FORMAT(4F10.0)
130    FORMAT(4F10.0)
50     FORMAT(//22HINITIAL APPROXIMATION/
      14X1HT9X4HXONE9X4HXTWO9X4HBTRI10X4HAMOO9X4HALAM10X4HSIGM///
      2F8.4,1P6E14.6)
70     FORMAT(F8.4,1P6E14.6)
       END
*      LABEL
CRM002    SOLVE CONSTANTS
C      USES DOUBLE PRECISION ACCURACY
       SUBROUTINE CNSTNT
       COMMON T,W,H,P,KMAX,TI,HGRID,NMAX,N1,IFLAG,NPRNT,PREV,CON,
      1XONE,XTWO,BTRI,AMOO ALAM,SIGM,U,FUNC,ALPHA
       DIMENSION T(600),W(6,251),H(6,6,251),P(6,251),PREV(6),U(251),
      1FUNC(251),CON(6)
D      DIMENSION A(10,10),B(10,1)
       N=NMAX
       CON(1)=PREV(1)
       CON(2)=PREV(2)
       CON(6)=PREV(6)
       DO 1 J=1,3
       DO 1 I=1,3
1      A(I,J)=H(I+3,J+2,N)
       DO 2 I=1,3
2      B(I,1)=-P(I+3,N)-H(I+3,1,N)*CON(1)-H(I+3,2,N)*CON(2)
       WRITE OUTPUT TAPE 6,80,((A(I,J),J=1,3),I=1,3)
       WRITE OUTPUT TAPE 6,90,(B(I,1),I=1,3)
D      CALL MATINV(A,3,B,1,DET)
       WRITE OUTPUT TAPE 6,100,DET
       DO 70 I=1,3
70     CON(I+2)=B(I,1)
80     FORMAT(/9H MATRIX A/(30X,3E16.6))
90     FORMAT(/9HCOLUMN B/30X,3E16.6)
100    FORMAT(/12HDETERMINANT/30X,E16.6)
       RETURN
       END
*      LABEL
```

```
CRMØØ3   DAUX2
      SUBROUTINE DAUX
      COMMON T,W,H,P,KMAX,TI,HGRID,NMAX,N1,IFLAG,NPRNT,PREV,CON,
     1XONE,XTWO,BTRI,AMOO,ALAM,SIGM,U,FUNC,ALPHA
      DIMENSION T(6ØØ),W(6,251),H(6,6,251),P(6,251),PREV(6),U(251),
     1FUNC(251),CON(6)
C
C     PHASE DECISION
      GO TO (1,2),IFLAG
C
C     NONLINEAREQUATIONS
1     T(1Ø)=T(5)
      T(11)=-(2.-T(6))*T(4)-3.*T(5)-Ø.5*(T(4)**3)
      T(12)=Ø.Ø
      T(13)=-(2.+4.*ALPHA*(T(4)**2)*(T(6)**4))*T(4)+(2.-T(6))*T(8)
     1+1.5*T(8)*(T(4)**2)
      T(14)=-2.*T(5)-T(7)+3.*T(8)
      T(15)=-4.*ALPHA*(T(6)**3)*(T(4)**4)-T(4)*T(8)
      RETURN
C
C     LINEAR EQUATIONS
2     DO 1Ø I=1,6
      AA=ALPHA*(BTRI**4)*(XONE**2)
      BB=AA*XONE/BTRI
      T(I+45)=T(I+9)
      T(I+51)=-(2.-BTRI)*T(I+3)-3.*T(I+9)+XONE*T(I+15)-1.5*(XONE**2)*T
     1(I+3)
      T(I+57)=Ø.Ø
      T(I+63)=-(2.+12.*AA)*T(I+3)-(ALAM+16.*BB)*T(I+15)+(2.-BTRI)
     1*T(I+27)+3.*ALAM*XONE*T(I+3)+1.5*(XONE**2)*T(I+27)
      T(I+69)=-2.*T(I+9)-T(I+21)+3.*T(I+27)
1Ø    T(I+75)=-(ALAM+16.*BB)*T(I+3)-12.*XONE*(BB/BTRI)*T(I+15)-XONE
     1*T(I+27)
      T(82)=T(41)
      T(83)=-(2.-BTRI)*T(4Ø)-3.*T(41)+XONE*T(42)-XONE*BTRI
     1-1.5*(XONE**2)*T(4Ø)+XONE**3
      T(84)=Ø.Ø
      T(85)=-(2.+12.*AA)*T(4Ø)-(ALAM+16.*BB)*T(42)+(2.-BTRI)*T(44)+
     124.*AA*XONE+ALAM*BTRI+3.*ALAM*XONE*T(4Ø)+1.5*(XONE**2)*T(44)
     2-3.*ALAM*(XONE**2)
      T(86)=-2.*T(41)-T(43)+3.*T(44)
      T(87)=-(ALAM+16.BB)*T(4Ø)-12.*XONE*(BB/BTRI)*T(42)-XONE*T(44)
     1+ALAM*XONE+24.*BB*XONE
      RETURN
      END
```

```
*      LABEL
CRMØØ4    SIMPSON@S RULE
       SUBROUTINE SIMPSN(FUNC,HGRID,NMAX,SIMP)
       DIMENSON FUNC(251)
       K=(NMAX-1)/2
       SUMA=Ø.Ø
       DO 15 J=1,K
15     SUMA=SUMA+FUNC(2*J)
       SUMB=Ø.Ø
       DO 16 J=2,K
16     SUMB=SUMB+FUNC(2*J-1)
       SIMP=(4.*SUMA+2.*SUMB+FUNC(NMAX)+FUNC(1))*HGRID/3.
       RETURN
       END
```

APPENDIX J

The purpose of this appendix is to give a brief description of the programs which were used to produce some of the data displayed in Chapter 5. Specifically this appendix will limit itself to those programs concerned with estimanation of state variables and control, i.e., rate reduction, using the estimated state for the rotational motion of a rigid body.

The program labeled MJPL8 is the main program. The program labeled DVJPL8 is a subroutine used to compute the derivative for the system and state estimator equations. The program labeled DT8 is a subroutine used to generate the disturbance inputs.

The following variables are of interest:

VAR (2 to 4) - true angular velocities about the body principal axes, stored in X (1 to 3, I) where I is for the time increment

VAR (5 to 7) - estimated angular velocities, stored in XE (1 to 3, I)

VAR (8 to 16)- the P_{ij}'s with P_{11} = VAR(8), P_{21} = VAR(9), etc., stored in P(L,N,I)

CC (1 to 3) - the constants in the system equations (5.8.2) i.e. $CC(1) = \frac{I_2 - I_3}{I_1}$ etc.

CC (4 to 6) - the corresponding constants in the state estimate equations (5.8.3)

HV (1 to 3) - the actual linear combination measured, i.e. output = $h_1 \omega_1 + h_2 \omega_2 + h_3 \omega_3$

HV (4 to 6) - the corresponding constants in the estimator equations

ALPHA (1 to 3) - used to adjust the acceleration constants for the control portion for example considering axis 1, then the control input is

$u_1 = -$ ALPHA (1) • SGN (ω_1)

ALPHA (4 to 6) - used to adjust the error in the input, for example for axis 1 the error is = ALPHA(4) * SGN(ω_1)

TIC - the initial value of time

XO (1 to 3) - the initial conditions for the system equations

XOE (1 to 3) - the initial estimates for the states

PO (I, J) - the initial condition for the P equations

H	-	step size used to integrate the differential equations
NS	-	the number of steps, of size H, taken between points plotted
NOPT	-	(a) if = 0 then 171 points will be pointed out, (aa) if ≠ 0 then NOPT points will be printed out (usually NOPT = 0 or = 101
BETA (I)	-	the constant forcing term in the equation for P_{ii}
KSTART	-	the starting integer for the random sequence used for the noise
PER1, PER2	-	the parameters P1 and P2 in the noise model given by equation (5.8.1)
YL, YU	-	the lower and upper values of the dependent variable for the plotting routine PLT1.
XM (I)	=	stored value of the measured output
XLC (I)	=	stored value of the actual output
XLCE (I)	=	stored value of the estimated output

The variables X, XE, P, XM, XLC, and XLCD are used to store the data for use in the plotting routine.

The main program uses the standard subroutines AMRK and PLT1. The function RRN is used to generate sequences of random numbers uniformly distributed between 0 and 1.

```
*      SYMBOL TABLE
*      LABEL
CMJPL8
C
C      CONTROL OF ROTATIONAL MOTION OF RIGID BODY USING ESTIMATED STATE
C
       COMMON VAR,DER,CC,U1,U2,U3,UD1,UD2,UD3,Y,BETA,HV,HM
      ØDIMENSION VAR(16),DER(16),CC(6),ALPHA(6),EU(15),EL(15),TEMPS(35ØØ)
      1,X(3,171),XE(3,171),P(3,3,171),T(171),Y1(171),Y2(171),Y3(171),
      2Y4(171),PO(3,3),XO(3),XOE(3),UD(3,172),XM(171)
      3,XLC(171),XLCE(171),HV(6),HM(3,3)
F      DVJPL
C
C      READ IN PROBLEM PARAMETERS
C
 1     WRITE OUTPUT TAPE 6,98
       READ INPUT TAPE 5,99,PER1,PER2,KSTART,NS,H,YL,YU,BETA,NOPT
       WRITE OUTPUT TAPE 6,1ØØ,PER1,PER2,KSTART,NS,H,BETA
       READ INPUT TAPE 5,1Ø1,(CC(J),J=1,6)
       READ INPUT TAPE 5,1Ø1,(HV(J),J=1,6)
       WRITE OUTPUT TAPE 6,121,(CC(J),J=1,6)
       WRITE OUTPUT TAPE 6,122,(HV(J),J=1,6)
       READ INPUT TAPE 5,1Ø1,(ALPHA(J),J=1,6)
       WRITE OUTPUT TAPE 6,123,(ALPHA(J),J=1,6)
C
C      GENERATE DISTURBANCE TORQUES
C
       CALL TORQUE(H,NS,UD)
       Y=1.Ø
       DV1=SIGNF(Y,ALPHA(4))
       DV2=SIGNF(Y,ALPHA(5))
       DV3=SIGNF(Y,ALPHA(6))
       Y=NS
       HS=Y*H
C
C      READ IN I.C.
C
       READ INPUT TAPE 5,1Ø1,XO(1),XO(2),XO(3),TIC
       READ INPUT TAPE 5,1Ø1,(XOE(J),J=1,3)
       WRITE OUTPUT TAPE 6,115,(XO(J),J=1,3)
       WRITE OUTPUT TAPE 6,116,(XOE(J),J=1,3)
       DO 2 J=1,3
 2     READ INPUT TAPE 5,1Ø1,(PO(J,K),K=1,3)
       NBLKS=17
       NPTS=171
       IF(NOPT) 3Ø1,3Ø1,3
 3     NPTS=NOPT
       NBLKS=1Ø
 3Ø1   CONTINUE
       Y=RRN(KSTART)
```

```
      KRUN=Ø
      NE=15
      DO 4 I=1,3
      DO 4 J=1,3
 4    HM(I,J)=HV(I+3)*HV(J+3)
C
C     SET UP I.C.
C
      DO 1Ø J=1,3
      VAR(J+1)=XO(J)
      VAR(J+4)=XOE(J)
      DO 1Ø I=1,3
      JK=3*J+I+4
 1Ø   VAR(JK)=PO(I,J)
      DER(1)=H
      VAR(1)=Ø.Ø
      Z=HV(1)*VAR(2)+HV(2)*VAR(3)+HV(3)*VAR(4)
      Y=Z+2.Ø*PER1*(RRN(KRUN)-Ø.5)*ABSF(Z)+2.Ø*PER2*(RRN(KRUN)-Ø.5)
      DO 11 J=1,2Ø
 11   TEMPS(J)=Ø.Ø
      DO 2Ø I=1,NPTS
C
C     STORE SOLUTIONS
C
      DO 12 K=1,3
      X(K,I)=VAR(K+1)
      XE(K,I)=VAR(K+4)
      DO 12 L=1,3
      JK=3*L+K+4
 12   P(K,L,I)=VAR(JK)
      XM(I)=Y
      XLC(I)=HV(1)*VAR(2)+HV(2)*VAR(3)+HV(3)*VAR(4)
      XLCE(I)=HV(4)*VAR(5)+HV(5)*VAR(6)+HV(6)*VAR(7)
C
C     INTEGRATE EQUATIONS
C
      TT=VAR(1)
      DO 2Ø J=1,NS
      UINT=(VAR(1)-TT)/HS
      U1=-SIGNF(ALPHA(1),VAR(5))
      U2=-SIGNF(ALPHA(2),VAR(6))
      U3=-SIGNF(ALPHA(3),VAR(7))
      UD1=UD(1,I)+(UD(1,I+1)-UD(1,I))*UINT+DV1*SIGNF(ALPHA(4),VAR(5))
      UD2=UD(2,I)+(UD(2,I+1)-UD(2,I))*UINT+DV2*SIGNF(ALPHA(5),VAR(6))
      UD3=UD(3,I)+(UD(3,I+1)-UD(3,I))*UINT+DV3*SIGNF(ALPHA(6),VAR(7))
      CALL DVJPL
      CALL AMRK(VAR,DER,DVJPL,NE,Ø,EU,EL,H,H,TEMPS)
      Z=HV(1)*VAR(2)+HV(2)*VAR(3)+HV(3)*VAR(4)
      Y=Z+2.Ø*PER1*(RRN(KRUN)-Ø.5)*ABSF(Z)+2.Ø*PER2*(RRN(KRUN)-Ø.5)
 2Ø   CONTINUE
```

```
C
C     DATA PRINTOUT
C
      T(1)=TIC
      DO 22 J=2,171
 22   T(J)=T(J-1)+HS
      WRITE OUTPUT TAPE 6,117,(X(J,NPTS),J=1,3)
      WRITE OUTPUT TAPE 6,118,(XE(J,NPTS),J=1,3)
      WRITE OUTPUT TAPE 6,112
      M=1
      DO 28 J=1,18
      WRITE OUTPUT TAPE 6,113,(P(1,K,M),P(2,K,M),P(3,K,M),K=1,3)
 28   M=M+10
C
C     PLOTTING ROUTINE
C
      XL=TIC
      XU=T(NPTS)
      NPLOT=4
      DO 40 J=1,3
      DO 30 I=1,NPTS
      Y1(I)=X(J,I)
      Y2(I)=P(J,1,I)*HV(4)+P(J,2,I)*HV(5)+P(J,3,I)*HV(6)
      Y3(I)=XE(J,I)
 30   Y4(I)=P(J,J,I)
      M=J
      WRITE OUTPUTTAPE 6,114,M
     0CALL PLT1(IERR,TEMPS,NPLOT,NBLKS,XL,XU,YL,YU,T,NPTS,Y1,T,NPTS,Y2,
     1T,NPTS,Y3,T,NPTS,Y4)
 40   CONTINUE
      WRITE OUTPUT TAPE 6,124
     0CALL PLT1(IERR,TEMPS,3,NBLKS,XL,XU,YL,YU,T,NPTS,XLC,T,NPTS,XM,
     1T,NPTS,XLCE)
      IF(NOPT) 42,42,50
 42   WRITE OUTPUT TAPE 6,119
      WRITE OUTPUTTAPE 6,120
      DO 44 J=1,57
 44  0WRITE OUTPUT TAPE 6,113,T(J),XLC(J),XM(J),T(J+57),XLC(J+57),
     1XM(J+57),T(J+114),XLC(J+114),XM(J+114)
 50   GO TO 1
C
 98  0FORMAT(60H1SATELLITEATTITUDE CONTROL WITH N.L. FILTER STATE ESTIM
     1ATOR//)
 99   FORMAT(2F10.6,2I5,4F10.6,I5)
 100  FORMAT(30HPER1 PER2 KSTART NS H BETA = 2F7.3,2I5,2F10.5//)
 101  FORMAT(6F10.6)
 112 0FORMAT(100HP11           P21           P31           P12           P22
     1        P32           P13           P23           P33//)
 113  FORMAT(9E12.4//)
 114  FORMAT(46H1*=XN  O = HV(J)*PNJ  X = XEN  H = PNN  N = I5//)
```

```
 115  FORMAT(10HXO(1) =  3F10.6//)
 116  FORMAT(10H XOE(1) = 3F10.6//)
 117  FORMAT(10HXO(T) =  3F10.6//)
 118  FORMAT(10HXOE(T) = 3F10.6//)
 119 0FORMAT(55H1X1 AND X1 MEASURED CORRESPONDING TO THE POINTS PLOTTED/
     1/)
 120 0FORMAT(107H TIME        X1         X1 MEASURED TIME        X1
     1      X1 MEASURED TIME        X1            X1 MEASURED//)
 121  FORMAT(9H CC(J) = 6F10.6//)
 122  FORMAT(9HHV(J) = 6F10.6//)
 123  FORMAT(12H ALPHA(J) = 6F10.6//)
 124  FORMAT(28H1*= XLC  O = XLCM  X = XLCE//)
      END
*     SYMBOL TABLE
*     LABEL
CDVJPL8
      SUBROUTINE DVJPL
      COMMON VAR,DER,CC,U1,U2,U3,UD1,UD2,UD3,Y,BETA,HV,HM
     0DIMENSION VAR(16),DER(16),CC(6),A(3,3),B(3,3),C(3,3),G(3,3)
     1,HV(6),HM(3,3)
      DER(2)=CC(1)*VAR(3)*VAR(4)+U1+UD1
      DER(3)=CC(2)*VAR(2)*VAR(4)+U2+UD2
      DER(4)=CC(3)*VAR(2)*VAR(3)+U3+UD3
      Z=Y-HV(4)*VAR(5)-HV(5)*VAR(6)-HV(6)*VAR(7)
     0DER(5)=CC(4)*VAR(6)*VAR(7)+2.0*(HV(4)*VAR(8)+HV(5)*VAR(11)+
     1HV(6)*VAR(14))*Z+U1
     0DER(6)=CC(5)*VAR(5)*VAR(7)+2.0*(HV(4)*VAR(9)+HV(5)*VAR(12)+
     1HV(6)*VAR(15))*Z+U2
     0DER(7)=CC(6)*VAR(5)*VAR(6)+2.0*(HV(4)*VAR(10)+HV(5)*VAR(13)+
     1HV(6)*VAR(16))*Z+U3
      G(1,1)=0.0
      G(1,2)=CC(4)*VAR(7)
      G(1,3)=CC(4)*VAR(6)
      G(2,1)=CC(5)*VAR(7)
      G(2,2)=0.0
      G(2,3)=CC(5)*VAR(5)
      G(3,1)=CC(6)*VAR(6)
      G(3,2)=CC(6)*VAR(5)
      G(3,3)=0.0
      DO 2 I=1,3
      DO 2 J=1,3
      A(I,J)=0.0
      B(I,J)=0.0
      C(I,J)=0.0
      DO 2 L=1,3
      DO 2 M=1,3
      JK1=3*M+4+I
      JK2=3*J+4+L
 2    A(I,J)=A(I,J)+HM(M,L)*VAR(JK1)*VAR(JK2)
      DO 4 I=1,3
```

```
      DO 4 J=1,3
      DO 4 K=1,3
      JK1=3*J+K+4
      B(I,J)=B(I,J)+G(I,K)*VAR(JK1)
      JK2=3*K+I+4
 4    C(I,J)=C(I,J)+G(J,K)*VAR(JK2)
      DO 6 I=1,3
      DO 6 J=1,3
      JK=3*J+I+4
 6    DER(JK)=-2.Ø*A(I,J)+B(I,J)+C(I,J)
      DER(8)=DER(8)+BETA
      DER(12)=DER(12)+BETA
      DER(16)=DER(16)+BETA
 5Ø   RETURN
      END
*     SYMBOL TABLE
*     LABEL
CDT8
      SUBROUTINE TORQUE(H,NS,UD)
      COMMON VAR,DER,CC,U1,U2,U3,UD1,UD2,UD3,Y,BETA,HV,HM
      DIMENSION VAR(16),DER(16),CC(6),HV(6),HM(3,3),UD(3,172)
      WRITE OUTPUT TAPE 6,1ØØ
      DO 1Ø I=1,3
      DO 1Ø J=1,172
 1Ø   UD(I,J)=Ø.Ø
      NNS=NS
      HH=H
 1ØØ  FORMAT(34HDISTURBANCE TORQUES UD1,2,3 = Ø.Ø//)
 5Ø   RETURN
      END
```

APPENDIX K

This appendix contains listings of the Fortran programs used to solve the problem of example 6.2. The main program is listed first; it is divided into several different parts for performing the various steps in the min-max solution.

The first part of the main program generates the optimum solution to the problem as posed, using quasilinearization. The SHARE subroutine AMRK is used for the integration, and DERR is the derivative subroutine used by AMRK. The matrix inversion is done by SHARE subroutine MATINV. Next the initial guess on the solution is obtained by integrating the exact system equations with assumed initial conditions. The derivative routine used here is YDOT. The next part of the main program generates the approximation to the solution using the "quasilinear" equations. AMRK is used for the integration, along with derivative subroutines DERH and DERIH.

The next part of the program sets up the linear programming problem in standard form for a simplex method solution. The subroutines SMPLX, PIVOT, and PRINT are used to solve the linear programming problem.

A plot of the results is obtained in the final part of the main program. The SHARE subroutine PLT1 is used for plotting.

Listings of the programs follow.

```
*      SYMBOL TABLE
*      LABEL
C      TEST PROGRAM FOR SOC SOLUTION USING MIN-MAX CRITERION IN DIFF.
C      APPROX. USE 1Ø POINTS ON X1.
       DIMENSION ARRAY(52,253),IROW(5Ø),XOPT(4,1Ø1),Y(2Ø,1Ø1),VAR(21),DER
      1(21),EU(2Ø),EL(2Ø),TEMPS(1Ø2Ø),POINTS(1Ø),X(3,1Ø1),PART(3,1Ø1),
      2XONE(51),YONE(51),YTWO(51)
       COMMON ARRAY,IROW,AFC,X,M,VAR,DER
D      DIMENSION A(1Ø,1Ø),B(1Ø,1)
F      DERR,YDOT,DERH,DERIH
C      READ IN INITIAL CONDITIONS,TIME INTERVAL, AND STEP SIZE.
       READ INPUT TAPE 5,1,C1,C2,T,H
     1 FORMAT (2F1Ø.6/F1Ø.6/F1Ø.6)
C      READ IN NUMBER OF ITERATIONS OF OL/LP DESIRED.
       READ INPUT TAPE 5,2,NUMI
     2 FORMAT (I2)
C
C      SOLVE FOR OPTIMUM OPEN-LOOP SOLUTION WITH Q.L.
C
       DO 3 I= 1,191
     3 TEMPS(I) = Ø.
       DO 4 I = 1,2Ø
     4 Y(I,1)= Ø.
       DO 5 I = 1,16,5
     5 Y(I,1) = 1.
       VAR(1) = Ø.
       DER(1) = H
       IT = T/H+Ø.5
       DO 8 M = 1,IT
       DO 6 I = 1,2Ø
     6 VAR(I+1) = Y(I,M)
       CALL DERR
       CALL AMRK(VAR,DER,DERR,2Ø,Ø,EU,EL,H,H,TEMPS)
       DO 7 I= 1,2Ø
     7 Y(I,M+1) = VAR(I+1)
     8 CONTINUE
C      LINEAR EQUATIONS
       DO 9 I = 1,4
       IND = 1+(I-1)*4
       A(1,I) = Y(IND,1)
     9 A(2,I) = Y(IND+1,1)
       DO 1Ø I = 1,4
       IND = 3+(I-1)*4
       A(3,I) = Y(IND,1Ø1)
    1Ø A(4,I) = Y(IND+1,1Ø1)
       B(1,1)= C1-Y(17,1)
       B(2,1) = C2-Y(18,1)
       B(3,1) = -Y(19,1Ø1)
       B(4,1) = -Y(2Ø,1Ø1)
D      DET = Ø.
```

```
      CALL MATINV (A 4,B,1,DET)
      DO 11 I= 1,4
      DO 11 J= 1,101
   11 XOPT(I J) = B(1)*Y(I,J)+B(2)*Y(I+4,J)+B(3)*Y(I+8,J)+B(4)*Y(I+12,J)
     1+Y(I+16,J)
      WRITE OUTPUTTAPE 6,110
  110 FORMAT(1H1,4X,26HOPTIMUM OPEN-LOOP SOLUTION//)
      WRITE OUTPUT TAPE 6,111,((XOPT(I,J),I=1,4),J=1,101,2)
  111 FORMAT (5X,24HX1, X2, LAMBDA1, LAMBDA2//(1P4E16.7))
      DO 112 I = 1,51
      IND = 1 + (I-1)*2
  112 YONE(I) = XOPT(1,IND)
      XONE(1) = 0.
      DO 113 I= 2,51
  113 XONE(I) = XONE(I-1)+2.*H
      WRITE OUTPUT TAPE 6,114
  114 FORMAT(5X,53HPOINTS TO BE FITTED ARE TAKEN EVERY TENTH OFASECOND
     1)
C     OPTIMAL SOLUTION NOW STORED IN XOPT.  TAKE SELECTED POINTS AND PLA
C     CE IN POINTS.
      DO 12 I = 1,10
      IND = 11+(I-1)*10
   12 POINTS(I) = XOPT(1,IND)
C     PHASE ONE COMPLETED.  OBSERVED POINTS STORED IN POINTS.
C
C     BEGIN PHASE TWO.
C     GENERATE INITIAL GUESSUSING ASSUMED VALUE OF PARAMETER, WHICH
C     IS READ IN.
      INDEX = 0
      READ INPUT TAPE 5,13,AFC
   13 FORMAT (F10.6)
      DO 14 I = 1,38
   14 TEMPS(I) = 0.
      VAR(1)= 0.
      DER(1) = H
      X(1,1)= C1
      X(2,1) = C2
      X(3,1) = AFC
      DO 17 M = 1,IT
      DO 15 I = 1,3
   15 VAR(I+1) = X(I,M)
      CALL YDOT
      CALL AMRK(VAR,DER,YDOT,3,0,EU,EL,H,H,TEMPS)
      DO 16 I = 1,3
   16 X(I,M+1) = VAR(I+1)
   17 CONTINUE
C     INITIAL GUESS NOW STORED IN X.  SUBSEQUENT RESULTS OF ITERATIONS
C     WILL BE PLACED IN X.
C
C     PROCEED TO Q.L. SOLUTION.
```

```
C     FIRST GENERATE HOMOGENEOUS SOLUTION.
   18 DO 19 I = 1,92
   19 TEMPS(I) = Ø.
      INDEX = INDEX+1
      VAR(1) = Ø.
      DER(1) = H
      DO 2Ø I = 1,9
   2Ø Y(I,1) = Ø.
      DO 21 I = 1,9,4
   21 Y(I,1) = 1.
      DO 24 M = 1,IT
      M = M
      DO 22 I = 1,9
   22 VAR(I+1) = Y(I,M)
      CALL DERH
      CALL AMRK(VAR,DER,DERH,9,Ø,EU,EL,H,H,TEMPS)
      DO 23 I = 1,9
   23 Y(I,M+1) = VAR(I+1)
   24 CONTINUE
C     GENERATE PARTICULAR SOLUTION.
      DO 25 I = 1,38
   25 TEMPS(I) = Ø.
      VAR(1)= Ø.
      DER(1) = H
      DO 27 I= 1,3
   27 PART(I,1)= Ø.
      DO 3Ø M= 1,IT
      M = M
      DO 28 I = 1,3
   28 VAR(I+1)= PART(I,M)
      CALL DERIH
      CALL AMRK(VAR,DER,DERIH,3,Ø,EU,EL,H,H,TEMPS)
      DO 29 I= 1,3
   29 PART(I,M+1)= VAR(I+1)
   3Ø CONTINUE
C
C     FUNDAMENTAL MATRIX AND PARTICULAR SOLUTION NOW STORED IN Y AND PAR
C     T.  INITIATE PHASE THREE.  FIRST PUT APPROPRIATENUMBERS INTO
C     ARRAY.
      DO 31 I = 1,31
      DO 31 J = 1,74
   31 ARRAY(I,J) = Ø.
      DO 32 I = 1,1Ø
      IND = 11+(I-1)*1Ø
      ARRAY(I,1) = Y(7,IND)
      ARRAY(I,2) = -Y(7,IND)
      ARRAY(I,I+2)= -1.Ø
      ARRAY(I+1Ø,I+2) = 1.Ø
      ARRAY(I,I+12) = 1.Ø
      ARRAY(I+2Ø,I+12)= 1.Ø
```

```
      ARRAY(I+10,I+22) = 1.0
   32 ARRAY(I+20,I+32)= 1.0
      DO 33 I = 11,30
   33 ARRAY(I,72) = 1.0
      DO 34 I = 1,10
      IND = 11+(I-1)*10
   34 ARRAY(I,74)=POINTS(I)-PART(1,IND)-C1*Y(1,IND)-C2*Y(4,IND)
C     MATRIX OF ORIGINAL EQUATIONS NOW IN ARRAY.  ELIMINATE Z FROM ALL
C     EQUATIONS BUT LAST.
      DO 35 I = 1,29
      DO 35 J= 1,74
   35 ARRAY(I,J) = ARRAY(I,J)-ARRAY(30,J)
C     CHECK TO INSURE NON-NEGATIVITY OF CONSTANTS
      DO 351 I = 1,29
      IF(ARRAY(I,74))350,351,351
  350 DO 352 J = 1,74
  352 ARRAY(I,J) = -ARRAY(I,J)
  351 CONTINUE
C     ALL CONSTANTS ARE NOW NON-NEGATIVE
C     NUMBERS IN ARRAY ARE NOW IN PROPER FORM FOR APPLICATION OF
C     SIMPLEX METHOD.
      NADM = 42
      NEQ = 29
      WRITE OUTPUT TAPE 6,36
   36 FORMAT(72H1  IN OUTPUT FROM SIMPLEX METHOD SOLUTION, VALUE OF FEED
     1BACK COEFFICIENT/72H   WILL BE X(1) MINUS X(2).  THE VALUE OF THE
     2OBJECTIVE FORM WILL BE THE /34H  VALUE OF THE MAXIMUM DEVIATION.
     3///)
      CALL SMPLX (NADM,NEQ)
      AP = 0.
      APP = 0.
      DO 40 I = 1,NEQ
      IF (IROW(I)-1) 38,37,38
   37 AP = ARRAY(I,74)
      GO TO 40
   38 IF (IROW(I)-2) 40,39,40
   39 APP = ARRAY(I,74)
   40 CONTINUE
      A1 = AP-APP
      WRITE OUTPUT TAPE 6,41,INDEX,A1
   41 FORMAT(23H0  FOR ITERATION NUMBER,I3/16H   VALUE OF A IS,1PE16.7)
   42 AFC = A1
C     CONSTRUCT NEW X.
      DO 43 I = 1,3
      DO 43 J= 1,101
   43 X(I,J) = C1*Y(I,J)+C2*Y(I+3,J)+AFC*Y(I+6,J)+PART(I,J)
      WRITE OUTPUT TAPE 6,430,INDEX
  430 FORMAT(29H1    RESULTS OF ITERATION NO.,I3)
      WRITE OUTPUT TAPE 6,431,((X(I,J),I=1,2),J=1,101,2)
  431 FORMAT(5X,6HX1, X2//(1P2E16.7))
```

```
      DO 432 I = 1,51
      IND = 1 + (I-1)*2
  432 YTWO(I) = X(1,IND)
      WRITE OUTPUT TAPE 6,430,INDEX
      WRITE OUTPUT TAPE 6,433
  433 FORMAT(5X,54H* IS OPTIMAL TRAJECTORY   Ø IS TRAJECTORY FROM MIN-MA
     1X//)
      IERR = Ø
      CALL PLT1(IERR,TEMPS,2,5,Ø.,1.,-6.,Ø.,XONE,51,YONE,XONE,51,YTWO)
      IF(NUM1-INDEX) 44,44,18
   44 CALL EXIT
      END
*     SYMBOL TABLE
*     LABEL
CYDOT
      SUBROUTINE YDOT
C     DERIVATIVES FOR SYSTEM WITH FEEDBACK
      DIMENSION VAR(21),DER(21),ARRAY(52,253),IROW(5Ø),X(3,1Ø1)
      COMMON ARRAY,IROW,A,X,M,VAR,DER
      DER(2) = VAR(3)
      DER(3) = (-2.+VAR(4))*VAR(2)-3.*VAR(3)
      DER(4) = Ø.
      RETURN
      END
*     SYMBOL TABLE
*     LABEL
CDERR
      SUBROUTINE DERR
C     DERIVATIVES FOR EXACT OPTIMAL SOLUTION
      DIMENSION VAR(21),DER(21),ARRAY(52,253),IROW(5Ø),X(3,1Ø1)
      COMMON ARRAY,IROW,A,X,M,VAR,DER
      DO 1 I = 2,18,4
      DER(I) = VAR(I+1)
      DER(I+1) = -2.*VAR(I)-3.*VAR(I+1)-Ø.5*VAR(I+3)
      DER(I+2) = -2.*VAR(I)+2.*VAR(I+3)
    1 DER(I+3)= -2.*VAR(I+1)-VAR(I+2)+3.*VAR(I+3)
      RETURN
      END
*     SYMBOL TABLE
*     LABEL
CDERH
      SUBROUTINE DERH
C     HOMOGENEOUS DERIVATIVES FOR Q.L. SOLUTION
      DIMENSION VAR(21),DER(21),ARRAY(52,253),IROW(5Ø),X(3,1Ø1)
      COMMON ARRAY,IROW,A,X,M,VAR,DER
      DO 1 I= 2,8,3
      DER(I) = VAR(I+1)
      DER(I+1) = (-2.+X(3,M))*VAR(I)-3.*VAR(I+1)+X(1,M)*VAR(I+2)
    1 DER(I+2) = Ø.
      RETURN
```

```
      END
*     SYMBOL TABLE
*     LABEL
CDERIH
      SUBROUTINE DERIH
C     INHOMOGENEOUS DERIVATIVES FOR Q.L. SOLUTION
      DIMENSION VAR(21),DER(21),ARRAY(52,253),IROW(50),X(3,101)
      COMMON ARRAY,IROW,A,X,M,VAR,DER
      DER(2) = VAR(3)
      DER(3) = (-2.+X(3,M))*VAR(2)-3.*VAR(3)+X(1,M)*VAR(4)-X(1,M)*X(3,M)
      DER(4) = 0.
      RETURN
      END
```

```
*      SYMBOL TABLE
*      LABEL
CSMPLX
       SUBROUTINE SMPLX(NADM,NEQ)
C      MAIN PROGRAM OF PACKAGE CONTAINING SMPLX, PIVOT, AND PRINT FOR
C      SOLUTION OF LINEAR PROGRAMMING PROBLEM.  BASIC CAPABILITY IS 5Ø
C      EQUATIONS IN 2ØØ VARIABLES.  PROGRAM EMPLOYS SIMPLEX ALGORITHM
C      A LA DANTZIG TO OBTAIN A BASIC FEASIBLE SOLUTION, THEN PROCEEDS TO
C      MINIMIZE THE OBJECTIVE FORM USING SIMPLEX ALGORITHM.
C      IN THIS PROGRAM, ARRAY IS THE MATRIX OF COEFFICIENTS AND CONSTANTS
C      NADM IS THE NUMBER OF ADMISSIBLE VARIABLES, NEQ IS THE NUMBER OF
C      EQUATIONS, WHICH IS ALSO THE NUMBER OF ARTIFICIAL VARIABLES TO BE
C      INTRODUCED.  IROW IS A ONE-DIMENSIONAL ARRAY CONTAINING THE SUB-
C      SCRIPTS OF THE ELEMENTS IN THE BASIC SOLUTION.
C
       DIMENSION ARRAY(52,253),IROW(5Ø)
C
       COMMON ARRAY, IROW
C      MAIN PROGRAM PLACES COEFFICIENTS OF PROBLEM IN STANDARD FORM IN
C      ARRAY.
C      DEFINE CERTAIN USEFUL CONSTANTS
       K1 = NEQ+1
       K2 = NEQ+2
       K3 = NADM + NEQ
       K4 = K3 +3
C      SET IN COEFFICIENTS OF ARTIFICIAL VARIABLES, -Z, AND -W
       DO 1 I = 1,K2
       K5 = NADM + I
     1 ARRAY(I,K5) = 1.
C      MAKE SYSTEM CANONICAL IN ARTIFICIAL VARIABLES, -Z, AND -W.
       DO 3 I = 1,NADM
       SUM = Ø.
       DO 2 J = 1,NEQ
     2 SUM = SUM + ARRAY(J,I)
     3 ARRAY(K2,I) = -SUM
       SUM = Ø.
       DO 4 J = 1,NEQ
     4 SUM = SUM + ARRAY(J,K4)
       ARRAY(K2,K4) = -SUM
C      STORE SUBSCRIPTS OF BASIS ELEMENTS IN IROW.
       DO 5 I = 1,NEQ
     5 IROW(I) = NADM + I
C
C      BEGIN PHASEONE.
C
    51 M = 1
       D = ARRAY(K2,1)
C      SEARCH FOR COLUMN CONTAINING PIVOT ELEMENT.
       DO 7 I= 2,K3
       IF(D-ARRAY(K2,I))7,7,6
```

```
    6 D = ARRAY(K2,I)
      M = I
    7 CONTINUE
C     CHECK TO SEE IF D IS NON-NEGATIVE.  IF YES, CHECK INFEASIBILITY FO
C     RM.  IF NO, PROCEED WITH ITERATION.
      IF (D+Ø.ØØØØ1) 11,8,8
    8 W = ABSF(ARRAY(K2,K4))
      IF (Ø.ØØØØ1-W) 9,12,12
    9 WRITE OUTPUTTAPE 6,1Ø
   1Ø FORMAT (5X,29HTHERE IS NO FEASIBLE SOLUTION)
      RETURN
C     CHECK INDICATES IMPROVEMENT IS POSSIBLE.  NOW NECESSARY TO
C     REPLACE ONE ELEMENT IN BASIC SOLUTION WITH ANOTHER.  SUBROUTINE
C     PIVOT DOES THIS.
   11 CALL PIVOT(2,M,NADM,NEQ)
      GO TO 51
C
C     BEGIN PHASETWO.  CEASE CONSIDERATION OF ARTIFICIAL VARIABLES.
   12 M = 1
      D = ARRAY(K1,1)
      DO 14 I = 2,NADM
      IF(D-ARRAY(K1,I)) 14,14,13
   13 D = ARRAY(K1,I)
      M = I
   14 CONTINUE
C    D NOW CONTAINS MINIMUM COEFFICIENT IN OBJECTIVE FORM.  CHECK SIGN
C     OF D.
      IF (D+Ø.ØØØØ1) 16,15,15
   15 CALL PRINT(NADM,NEQ)
      RETURN
   16 II = Ø
      DO 18 I = 1,NEQ
      IF(ARRAY(I,M)) 17,17,18
   17 II = II+1
   18 CONTINUE
      IF(NEQ - II) 19,19,21
   19 WRITE OUTPUT TAPE 6,2Ø
   2Ø FORMAT (35H OBJECTIVE FORM HAS NO LOWER BOUND)
      RETURN
   21 CALL PIVOT (1,M,NADM,NEQ)
      GO TO 12
      END
*     SYMBOL TABLE
*     LABEL
CPIVOT
C
      SUBROUTINE PIVOT(JJ,M,NADM,NEQ)
C
C     PROGRAM FOR FINDING PIVOT ELEMENT AND REPLACING A BASIC VARIABLE
C     WITH A NEW BASIC VARIABLE.
```

```
C     JJ = 1 FOR PHASE TWO, JJ = 2 FORPHASE ONE.
C
      DIMENSION ARRAY(52,253),IROW(50),TEMP1(253),TEMP2(52)
      COMMON ARRAY,IROW
C
C     M IS THE COLUMN CONTAINING PIVOT ELEMENT, BROUGHT IN FROM CALLING
C     PROGRAM.  M1 WILL BE ROW OF PIVOT ELEMENT.
C     FIRST CHOOSEROW M1.
      D1 = 0.1E06
      M1 = 1
      K1 = NADM + NEQ + 3
      IJ = 0
      K3 = NEQ + 2
      DO 1 I= 1,K1
    1 TEMP1(I) = 0.
      DO 6 I = 1,NEQ
      IF(ARRAY(I,M)) 6,6,2
    2 A = ARRAY(I,K1)/ARRAY(I,M)
      IF (A-0.000001) 3,3,4
    3 IJ = IJ + 1
    4 IF (D1-A) 6,6,5
    5 D1 = A
      M1 = I
    6 CONTINUE
C     CHECK FOR MULTIPLE DEGENERACY.  IF MORE THAN ONE CONSTANT IS ZERO,
C     PICK FOR PIVOT ELEMENT THE ONE WITH LARGEST ARRAY(I,M) AND ZERO
C     CONSTANT.
      IF (IJ-1) 12,12,7
    7 DO 11 I = 1,NEQ
      IF (ARRAY(I,K1)-0.000001) 8,8,11
    8 IF (ARRAY(I,M)) 11,11,9
    9 IF (ARRAY(I,M)-A) 11,11,10
   10 M1 = I
      A = ARRAY(I,M)
   11 CONTINUE
C     ARRAY(M1,M) IS PIVOT ELEMENT.
   12 A= ARRAY(M1,M)
      IROW(M1) = M
      DO 13 I= 1,K1
   13 TEMP1(I) = ARRAY(M1,I)/A
      DO 14 I= 1,K3
   14 TEMP2(I) = ARRAY(I,M)
      K2 = NEQ + JJ
      DO 15 I = 1,K2
      DO 15 J = 1,K1
   15 ARRAY(I,J) = ARRAY(I,J)-TEMP2(I)*TEMP1(J)
      DO 16 I = 1,K1
   16 ARRAY(M1,I) = TEMP1(I)
      RETURN
      END
```

```
*     SYMBOL TABLE
*     LABEL
CPRINT
C
      SUBROUTINE PRINT(NADM,NEQ)
C
C     PROGRAM FOR PRINTING RESULTS OF SIMPLEX METHOD SOLUTION OF
C     LINEAR PROGRAMMING PROBLEM.
C
      DIMENSION ARRAY(52,253),IROW(5Ø)
      COMMON ARRAY, IROW
      K1 = NADM + NEQ + 3
      WRITE OUTPUT TAPE 6,1
    1 FORMAT(61H1    SOLUTION TO LINEAR PROGRAMMING PROBLEM BYSIMPLEX M
     1ETHOD////)
      WRITE OUTPUT TAPE 6,2,NEQ
    2 FORMAT (3X,22HNUMBER OF EQUATIONS = , I3///)
      WRITE OUTPUT TAPE 6,3,NADM
    3 FORMAT (3X,22HNUMBER OF VARIABLES = , I3///)
      WRITE OUTPUTTAPE 6,3Ø
   3Ø FORMAT(5X,92HVALUES OF BASIC VARIABLES IN OPTIMUM SOLUTION FOLLOW.
     1  SOLUTION APPEARS AS PAIRS OF NUMBERS./5X,91HTHE FIRST NUMBER IS
     2THE SUBSCRIPT OF THE COMPONENT APPEARING IN THE BASIC SOLUTION, AN
     3D THE/5X,88HSECOND IS ITS VALUE.  THE CORRESPONDENCE BETWEEN THE S
     4UBSCRIPTS AND THE VARIABLES OF THE/5X,55HORIGINAL PROBLEMMUST BE
     5ESTABLISHED BY THE PROGRAMMER.///)
      WRITE OUTPUT TAPE 6,4,(IROW(I),ARRAY(I,K1),I=1,NEQ)
    4 FORMAT (7(I4,1H,,1PE12.5))
      WRITE OUTPUT TAPE 6,5
    5 FORMAT(1HØ,39H    ALL OTHER VARIABLES HAVE VALUE ZERO///)
      K2 = NEQ+1
      Z = -ARRAY(K2,K1)
      WRITE OUTPUT TAPE 6,6,Z
    6 FORMAT (39H    OPTIMUM VALUE OF OBJECTIVE FORM = ,1PE12.5///)
      RETURN
      END
```