COMPUTER SOLUTIONS OF THE VLASOV EQUATIONS

FINAL REPORT

Period: June 1, 1963 thru November 30, 1964

By

Dr. D. C. Prince, Jr.

N. P. Jeffries

prepared for

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

CONTRACT NAS 8-5214

SPACE POWER AND PROPULSION SECTION

MISSILE AND SPACE DIVISION

GENERAL ELECTRIC

CINCINNATI, OHIO 45215

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Program Manager

NASA - George C. Marshall Space Flight Center
Huntsville, Alabama
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SPACE POWER AND PROPULSION SECTION
MISSILE AND SPACE DIVISION
GENERAL ELECTRIC COMPANY
CINCINNATI, OHIO 45215

SUMMARY

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This is the final report of the program "Computer Solutions of the Vlasov Equations", performed for the National Aeronautics and Space Administration under Contract NAS 8-5214. The work was executed by a combined effort of the Applied Research and Aerodynamics Operation, Flight Propulsion Division, and the Space Power and Propulsion Section, Missile and Space Division of the General Electric Company. The period of performance was from 1 June, 1963 to 30 November, 1964. The NASA Program Manager was Dr. Robert N. Seitz, George C. Marshall Space Flight Center, Huntsville, Alabama.

Computer programs were written to determine solutions of the Vlasov equations for plane, cylindrical and spherical geometries. Potential distributions and charge density distributions were found for each of these configurations under a range of conditions. Values of the independent parameters of bias potential, electrode spacing (in Debye lengths), and charge density ratio, were chosen so as to yield a wide variety of types of solutions. The fundamental physical characteristics of each of these systems were established.

author

The participants in this program were Dr. D. C. Prince, Jr., and N. P. Jeffries as the Principal Investigators, with A. Wikoff and L. Cumbers as Engineering Assistants.

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I. Introduction and Physical Picture

Two related physical problems provide the inspiration for studying solutions of the collisionless Boltzmann or Vlasov equations[1]. In the first problem (Fig. I-1), designated the "diode problem", the objective is to predict current-voltage characteristics between two electrodes, an emitter and a collector, in the presence of a plasma. Primary attention is given to an emitter on which charged particles are formed. The particles may have either positive or negative charges. In principle, the electron current, leaving the emitter, of positively charged particles and negatively charged particles may be varied quite independently. For example, if the emitter is a hot oxide-coated surface in the presence of a metallic cesium vapor, the electron emission might vary with temperature according to the Richardson-Dushman relation, (Ref. 2) while the ion current (assuming that all neutral atoms striking the hot surface are ionized) would be a simple function of the vapor pressure. Facing the emitter there is a collector at some electrostatic bias potential relative to the emitter. The bias potential might be applied externally or it might be a result of the Fermi effect. The motion of the charged particles after leaving the emitter is determined by the electrostatic field distribution, which in turn is the result of both the distribution of space charge in the field and the applied bias potential. It is assumed that any charged particle reaching the collector, or touching the emitter upon returning, disappears, either into the external circuit or by losing its charge and forming a neutral atom.

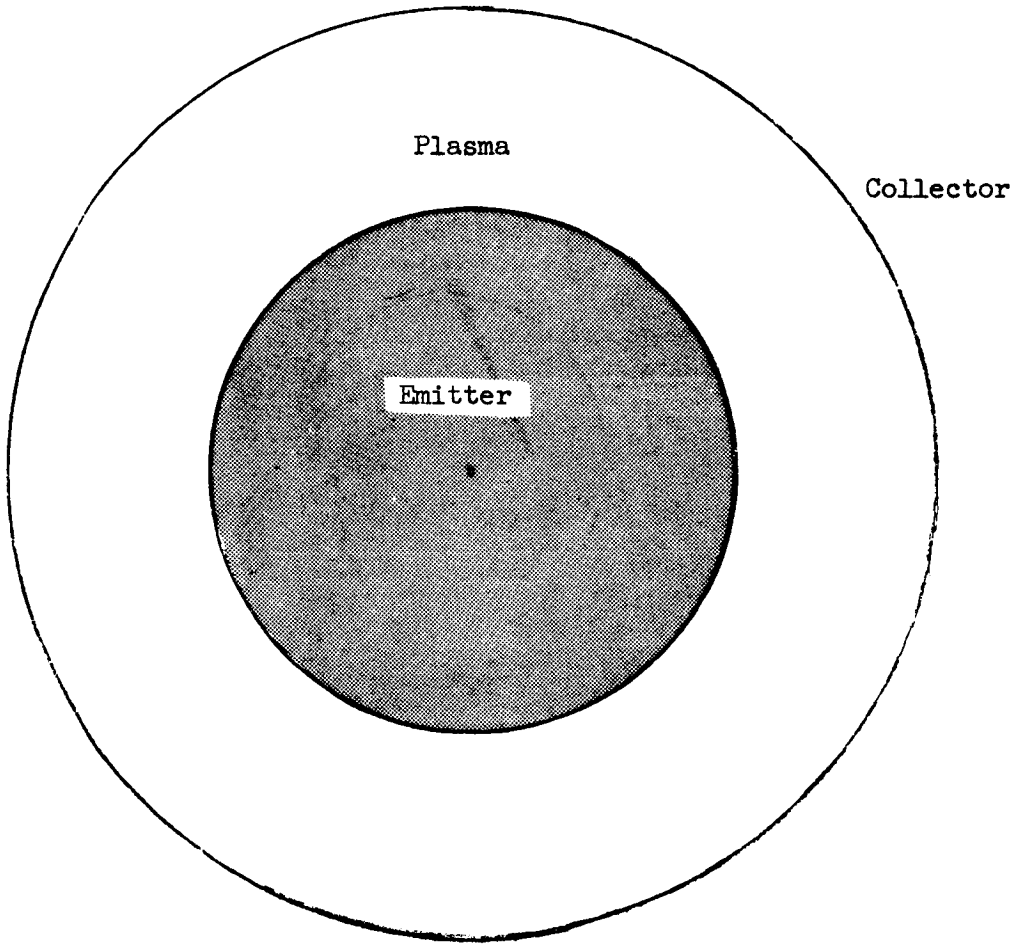

Figure I-1: The Plasma Diode Configuration

It is further assumed that the emitted particles, both ions and electrons, possess velocity distributions satisfying a half-Maxwell-Boltzmann relation, while the complete velocity distribution at any point in the configuration is the Maxwell-Boltzmann relation modified by appropriate momentum and energy relations.

Diode configurations which are considered in this study either have an electrode spacing which is small compared to the radius of curvature (including the plane case), or are concentric cylindrical or spherical situations with the collector external to the emitter.

The second problem is designated as the "probe problem" (Fig. II-2). The intent here is to predict the disturbance caused by a small object at some enforced bias potential relative to a plasma inside a much larger (or near infinite) container. The disturbance involves, in general, both a local departure from an essentially neutral plasma and a charged particle current of one sign or the other from the plasma to the probe. Cylindrical or spherical symmetry is implied by the definition of the problem. The distinguishing feature of the probe problem as compared with the diode problem is that most particles of either sign leave the emitter with a velocity component perpendicular to the radius, which, under the action of constancy of angular momentum, prevents them from negotiating the bias potential barrier to the collector or probe. Instead, these particles follow trajectories which become entirely tangential at some radius and finally return to the emitter. The net charged particle current of either sign is a small fraction of the emitted current. A special case of the cylindrical plasma diode, the

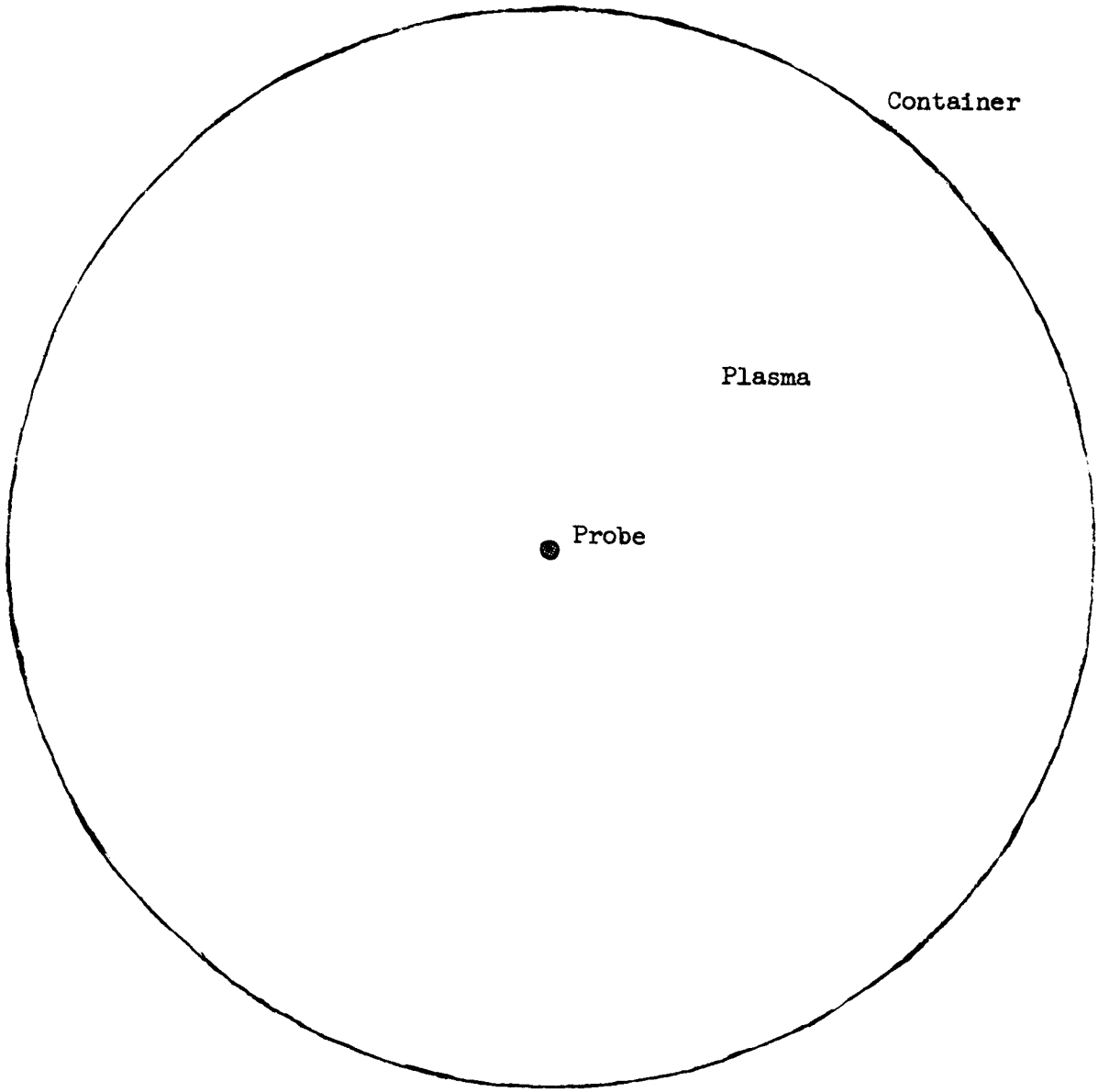

Figure I-2: The Plasma Probe Configuration

cylindrical monode or plasma inside a cylindrical container, was treated at some length in Ref. 3.

At the start of investigation under this contract, it was expected that the effort would be placed on cylindrical and spherical geometries, rather than on the plane geometry. Fairly early in the program, however, it became apparent that peculiarities in the behavior of cylindrical geometry solutions (primarily instability of a near-neutral charge density distribution) were present. It seemed likely that study of the plane situation would provide considerable insight into the nature of the problems of the curved geometries. After this work was well along, several references to other studies on planar plasma diodes were identified (Refs. 4, 5, & 6). The systematic analysis and presentation of this investigation does seem to be a contribution to understanding the behavior of planar plasma diodes.

A. Philosophy and General Discussion of Analysis Methods

The potential distribution in a plasma diode satisfies Poisson's Equation, including a complex space charge density. This charge density at any point is supposed to be the result of an initial formation of charged particles with a thermal distribution of particle velocities, and a subsequent history of the particle motion consistent with appropriate laws of motion. Altogether, five successive mathematical integration operations must be performed in the process of arriving at the solution. Summation of particles, identified by their velocity components in three mutually perpendicular coordinate directions, to get the charge density, accounts for three of the five integrations. The remaining two integrations appear in the solution of the second order ordinary differential equation (made ordinary by symmetry assumptions).

Depending on the particular facet of the general problem, one or more of the five integrations may be carried out analytically and disposed of. One or two more of the integrations may require numerical evaluation, but do allow such numerical evaluation to be carried out explicitly, once and for all. Most of the time, however, at least one of the integrations must be carried out by some trial-and-error or iteration scheme. Most of the effort under this contract has been concerned with the development of workable iteration schemes for those cases where means for explicit integration have not been found.

In the case of the planar diode, the summations of charged particles over two out of the three perpendicular velocity components reduce to evaluation of probability integrals from zero to infinity, and consequently lead to a particularly simple form. The summation in the third direction can always be expressed in terms of the error function (which, however, may require numerical evaluation). The complete analytical expression for the second derivative of the potential involves two Kronecker deltas and plus-or-minus options. The plus or minus, or the zero or one, depends on whether the particular point lies on the emitter or the collector side of a potential extremum controlling transmission or reflection of the one or the other charge particle species. In the planar case the second order differential equation can always be expressed in exact differential form and integrated once analytically, subject to an undetermined constant of integration. At this point, the variables are always separable, and a formula is set up for use in explicit numerical evaluation of the final integral which, however, retains the constant of integration of the earlier integration as a parameter. The initial boundary condition can be satisfied automatically by definition of the reference potential at the emitter, zero distance. With the bias potential as the upper limit of integration of a definite integral (the emitter at reference potential is the lower limit), the parameter is varied until the final boundary condition is satisfied.

Classes of Solution

Solutions of the planar diode problem turned out to fall into three main classes: monotonic potential solutions, solutions with at least one extremum in the direction of the bias potential, and solutions with an extremum in the opposite direction from the bias potential.

Each of these classes may be further subdivided, depending on whether the number of Debye lengths in the electrode spacing is large or small, on the number of inflection points in the potential distribution (which may depend on both the number of Debye lengths and on the emitted electron-ion density ratio), and on the electron-ion density ratio.

The most convenient choice of iteration parameter for the monotonic case proved to be the slope of the potential distribution curve at the inflection point closest to the collector. A second inflection point may be present if the charge density near the emitter due to attracted particles is greater than the total charge density, including reflected particles, of the species repelled by the bias potential, but this excess of particles is not great enough to produce a potential extremum opposite to the bias potential. This second inflection point does not affect the computation. The main subdivision of monotonic cases occurs between situations where changes in the slope of the potential distribution curve at the inflection point do or do not produce changes in the slope at the emitter and the collector (i.e., slope changes which can be detected by the eight significant figure

accuracy of the computer used for evaluation). The situation when the slope at the inflection point does not significantly influence the slopes at emitter and collector, is referred to as the plateau situation. An additional case requiring special consideration occurs when the slope at the emitter is positive and less than the slope at the inflection point.

The plateau situation may also arise where there is a potential extremum in the opposite direction from the bias potential. In this case the convenient iteration parameter is the value of the potential at the extremum. The value of the potential at the extremum is also the convenient iteration parameter for the case of an extremum in the same direction as the bias potential. In this situation, however, the plateau does not appear but is replaced by a tendency toward oscillatory solutions. This distinction between electrode spacings which are a small number of Debye lengths and those which are a large number of Debye lengths is made when the oscillation first appears. Altogether, it appeared that there were about ten different situations which had to be distinguished and given special numerical handling to give complete coverage to the planar plasma diode problem.

Many of the contingencies investigated in detail in the planar problem have also been found in the cylindrical and spherical problems, but with modifications introduced by the curved geometry. It has not yet been possible to demonstrate ability to find cylindrical or spherical solutions comparable to all the planar solutions. The simplest situation for which a generally satisfactory solution seems to have been found is the situation

where the proportions of charged particles passing from the emitter to the collector and returning to the emitter are determined entirely by the collector bias potential. In contrast to the planar case, the charge density of any point may be determined not only by the value of the potential there, but also by the location relative to the emitter. This is a consequence of the influence of the momentum effect in the curved geometry on the limits of admissible velocities. We have found solutions to curved geometry problems with both internal and external collectors, analogous to the planar situation where the slope at the inflection point does make a difference to the slopes at emitter and collector, provided the electrode spacing is a fairly small number of Debye lengths. Under conditions where the planar solution approaches a slope small enough not to influence the slope at emitter and collector, the curved geometry solutions begin to show oscillations. These oscillations complicate the problems of arriving at solutions considerably. As the number of Debye lengths and the electrode spacing increases some more, it is found that the collector bias potential is no longer the only criterion determining the charge density. Any time there is a potential extremum in the opposite direction from the bias potential, this extremum will serve as a barrier to some particles which could otherwise make the transit to the collector. The extremum itself is the barrier for particles with zero tangential velocity. Particles with some tangential velocity will be repelled before reaching the extremum in external collector situations, by radii beyond the extremum for internal collector situations. In external collector situations where the electrode spacing is a fairly large number of Debye lengths, the repelling slope close to the emitter may easily be large enough to repel

some particles which would otherwise convert enough kinetic energy of tangential velocity to radial velocity, through the angular momentum effect, to make the transit to the collector. In internal collector situations where there is an effective sheath close to the collector, the potentials close to the edge of the sheath away from the collector may control the reflection of some particles.

In cylindrical diodes, integration of velocity components parallel to the axis of symmetry results in the complete probability integral. A second integration of the three associated with charge density calculation becomes possible if a transformation is made so as to express the integral in terms of the resultant velocity and the angle made by the trajectory with the tangential direction. The third integration for charge density has always required numerical evaluation.

No practical procedure has been found for performing either of the two integrations in the differential equation analytically. Some success has been found with two techniques: (1) double numerical integration from an assumed initial slope using charge densities evaluated from past history and a future history introduced by a small number of assumed input points; and (2) assuming a complete potential distribution, calculating a charge density distribution using this assumed potential distribution for both past and future history at all points and then using Simpson's rule integration twice together with appropriate constants of integration to obtain a new potential distribution. The numerical integration of the assumed initial slope is quite effective for

those cases where a single future history point is sufficient to define all charge densities, and almost as effective when the influence of some other repelling potential besides the collector is only a minor modification to the major influence of the collector potential. The use of the entire assumed potential distribution and Simpson's rule integration has been effective for monotonic solutions and for solutions in which the net charge density never changes sign. This method has been found to be unsatisfactory for obtaining solutions in which there is a tendency toward oscillation. It is not clear at this time that any of the solutions obtained by this technique could not have been obtained more easily with the technique of numerical integration from the assumed initial slope.

Two of the three integrations of the calculation of charge density in the spherical case may also be carried out analytically. The first one is carried out by observing that complete polar symmetry exists for velocity components in a plane perpendicular to the radius of the sphere. Thus, the problem may be reduced to integration over velocity components along the radius of the sphere and perpendicular to that radius. The second integration may be made analytically by expressing the integral over the radial velocity components in terms of appropriate error functions. The third integration, over the range of negative expotentials of the square of this velocity component, must be made numerically. All spherical results have been obtained using the first described procedure for the cylindrical problem.

II THE PLANAR PLASMA DIODE PROBLEM

A. Introduction

The planar plasma diode was originally investigated under this contract because of difficulties encountered in trying to solve for the cylindrical diode configuration. The difficulties involved the establishing of possible conditions for neutrality or near-neutrality in the neighborhood of a potential disturbance. It was assumed that an understanding of the planar diode would be helpful in obtaining solutions for the cylindrical and spherical configurations.

It was found that the planar diode was a quite interesting problem in itself and several aspects of its behavior were later observed also in the cylindrical and spherical diode solutions. Some direct comparisons will be made between these configurations which demonstrate the effects of the diode geometry.

This chapter will outline the method of solution used for the planar diode problems, and then the various types of solutions which have been found will be discussed. Some comparisons will be made between the planar solutions and cylindrical solutions in order to demonstrate the effects of different geometries.

B. Basic Equations

The basic problem is to find the potential and charge density distributions in a diode with flat electrodes. If we omit consideration of the edge effects (or assume infinite electrodes), then the problem becomes one-dimensional. The potential in an electrostatic field is known to obey Poisson's equation, which can be written as follows, for the MKS system of units:

$$\nabla^2 \phi = -\frac{4\pi\rho}{\varepsilon_o} \qquad \text{(II-1)}$$

where ϕ = potential, volts

ρ = charge density, coulombs/m^3

ε_o = dielectric constant of the medium, farad/m

Here the medium is taken as a vacuum, so ε_o is unity. The net charge density is the difference between the densities of ions and electrons,

$$\rho = e(n_i - n_e) \qquad \text{(II-2)}$$

where e = charge on a particle, coulombs

n_i = number density of ions, 1/cm^3

n_e = number density of electrons, 1/cm^3

Substituting and introducing the subscript o denoting conditions at the emitter electrode, we have

$$\nabla^2\phi = \frac{4\pi\epsilon n_{io}}{\epsilon_o}\left(\frac{n_e}{n_{eo}}\frac{n_{eo}}{n_{io}} - \frac{n_i}{n_{io}}\right) \qquad \text{(II-3)}$$

The Debye length represents the maximum distance over which a given charge can exert an influence on other particles. It is a parameter that can be used to non-dimensionalize the equation. It is given by:

$$\lambda_D = \sqrt{\frac{\epsilon_o kT}{4\pi n_i e^2}} \qquad \text{(II-4)}$$

where k = Boltzmann's constant, 1.38×10^{-23} joule/^oK

T = Temperature of particles, ^oK

This distance parameter can be normalized by dividing by the electrode spacing L, and the potential can be expressed in electron-volts by using the parameter $\Phi = \frac{e\phi}{kT}$. Substituting these quantities into equation (II-3), we obtain a convenient form for the potential variation,

$$\nabla^2 \frac{e\phi}{kT} = \frac{L^2}{\lambda_D^2}\left(\frac{n_e}{n_{eo}}\frac{n_{eo}}{n_{io}} - \frac{n_i}{n_{io}}\right) \qquad \text{(II-5)}$$

where $\nabla^2 = \frac{d^2}{d(x/L)^2}$ and the Debye length is here based on total ion density at the emitter (although emitted ion density is probably preferable because it is an independent variable).

It has been shown in Reference 1 that the velocity distribution of the particles at the emitter may be taken as a half-Maxwellian, that is the velocities have the Maxwellian distribution but all are directed away from the emitter. Therefore, the probability function of the velocities is given by:

$$f = K e^{\frac{e\phi - \frac{m}{2}(v_x^2 + v_y^2 + v_z^2)}{kT}} \qquad \text{(II-6)}$$

where v_x, v_y, v_z are velocities in each direction and K is a normalizing constant. The number density of particles is then the integral of all possible velocity vectors for a cube of the phase space

$$n_e(x, y, z) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x, y, z, v_x, v_y, v_z)\, dv_x\, dv_y\, dv_z \qquad \text{(II-7)}$$

When the problem involves only two dimensions, such as when there is no variation with z, the integration may be carried out immediately:

$$n_e(x, y) = \sqrt{\frac{2\pi kT}{m}}\, K e^{\frac{e\phi}{kT}} \iint e^{-\frac{m}{2kT}(v_x^2 + v_y^2)}\, dv_x\, dv_y \qquad \text{(II-8)}$$

or this can be normalized with the emitter charge density

$$\frac{n_e(x, y)}{n_{eo}} = \frac{e^{\frac{e\phi}{kT}} \iint e^{-\frac{m}{2kT}(v_x^2 + v_y^2)}\, dv_x\, dv_y}{\iint e^{-\frac{m}{2kT}(v_{x_o}^2 + v_{y_o}^2)}\, dv_{x_o}\, dv_{y_o}} \qquad \text{(II-9)}$$

For the planar diode, the variation is in the x-direction only

$$\frac{n_e}{n_{eo}} = \frac{e^{\frac{e\phi}{kT}} \int e^{-\frac{m v_x^2}{2kT}}\, dv_x}{\int e^{-\frac{m v_{x_o}^2}{2kT}}\, dv_{x_o}} \qquad \text{(II-10)}$$

If we consider a monotonic distribution of potential, then the limits of integration can be determined. In some cases, the potential distribution will not be monotonic but rather it may have a potential peak or valley between the emitter and collector. These cases will be dealt with later in this chapter.

It should be mentioned that we are being sufficiently general when we consider only bias potentials which are attractive to electrons, because the solutions for bias potentials attractive to ions will simply be mirror images of the solutions we obtain here.

Under the conditions of such a monotonic potential variation attractive to electrons, all the electrons are formed at the emitter and make the transit to the collector. This means that all the electrons found at a point in the electrode spacing which has a potential ϕ have been accelerated through that potential. Therefore, their velocity is at least as large as the kinetic energy appropriate for that potential acceleration, and the lower limit of velocities is thus given by:

$$\frac{1}{2} m v_x^2 = e\phi \qquad \text{(II-11)}$$

or

$$v_x \sqrt{\frac{m}{2kT}} = \sqrt{\frac{e\phi}{kT}} \qquad \text{(II-12)}$$

The upper limit on these velocities is infinity. These conditions allow equation (II-10) to be written as:

$$\frac{n_e}{n_{eo}} = e^{\frac{e\phi}{kT}} \left(1 - \operatorname{erf} \sqrt{\frac{e\phi}{kT}}\right) \qquad \text{(II-13)}$$

For the ions, particles are traveling in both directions because many of them are decelerated and turned around by the opposing potential gradient. The particles traveling in the positive direction may have all energies, but the minimum initial energy is that which is removed by deceleration through the potential difference. The lower limit for returning particles is also zero, but there is an upper limit above which the particles had enough energy to negotiate the bias potential difference. Therefore the expression becomes:

$$\frac{n_i}{n_{io}} = \frac{e^{\frac{e\phi}{kT}} \int_{-\sqrt{e(\phi_c-\phi)/kT}}^{\infty} e^{-\frac{mv_x^2}{2kT}} d\left(v_x\sqrt{\frac{m}{2kT}}\right)}{\int_{-\sqrt{e\phi_c/kT}}^{\infty} e^{\frac{-m}{2kT}v_{x_o}^2} d\left(v_{x_o}\sqrt{\frac{m}{2kT}}\right)} \qquad \text{(II-14)}$$

which integrates to become

$$\frac{n_i}{n_{io}} = e^{-\frac{e\phi}{kT}} \frac{1+erf\sqrt{\frac{e(\phi_c-\phi)}{kT}}}{1+erf\sqrt{\frac{e\phi_c}{kT}}} \qquad \text{(II-15)}$$

Now we introduce the symbols

$$\Phi = \frac{e\phi}{kT}$$

$$\mathcal{L} = \frac{L}{\lambda_D}$$

$$\alpha = \frac{n_{eo}}{n_{io}}$$

and equation (5) becomes

$$\frac{d^2\Phi}{dx^2} = \mathcal{L}^2\left[\alpha e^{\Phi}(1-erf\sqrt{\Phi}) - e^{-\Phi}\frac{1+erf\sqrt{\Phi_c-\Phi}}{1+erf\sqrt{\Phi_c}}\right] \qquad \text{(II-16)}$$

The electron current density leaving the emitter may be determined from the charge density there multiplied by the average particle velocity away from the emitter. This average velocity may be found by integrating the particle velocity component in the positive direction times the probability function over the full range of admissible velocity vectors,

$$j_{eo} = n_{eo} e \bar{v}_x = n_{eo} e \frac{\sqrt{2kT/m}\, e^{\Phi} \int e^{-\frac{m v_x^2}{2kT}} \left(v_x \sqrt{\frac{m}{2kT}} \right) d\left(v_x \sqrt{\frac{m}{2kT}} \right)}{\int e^{-\frac{m v_x^2}{2kT}} d\left(v_x \sqrt{\frac{m}{2kT}} \right)} \qquad \text{(II-17)}$$

$$j_{eo} = n_{eo} e \sqrt{2kT/\pi m}$$

or since

$$n_{oe} = \frac{\varepsilon_o kT}{4\pi \lambda_D^2 e^2} \qquad \text{(II-18)}$$

the electron current can be written as

$$j_{eo} = \frac{\varepsilon_o}{\lambda_D^2} \left(\frac{kT}{2\pi e} \right)^{3/2} \sqrt{\frac{e}{m}} \qquad \text{(II-19)}$$

This current density was evaluated at the emitter but it could have been evaluated at any other point, with the same result. Thus, j is constant with Φ (or x) as it should be.

The net ion current is conveniently integrated at the emitter. In contrast with the situation for electrons, the limits of integration depend on the bias potential. Using the appropriate limits of integration, we can find the emitter electron current density.

$$j_{io} = n_{io} e \sqrt{\frac{2kT}{m}} \frac{1}{\sqrt{\pi}} \frac{e^{-\Phi_c}}{1+\operatorname{erf}\sqrt{\Phi_c}} \qquad \text{(II-20)}$$

The equation for the potential distribution, equation (II-16), can be simplified by integrating it once. This is done by first multiplying by Φ'. Then we have:

$$\int \Phi' \Phi'' = \mathcal{L}^2 \left[\int \alpha e^{\Phi} (1-\operatorname{erf}\sqrt{\Phi}) \Phi' - \int e^{-\Phi} \frac{1+\operatorname{erf}\sqrt{\Phi_c - \Phi}}{1+\operatorname{erf}\sqrt{\Phi_c}} \Phi' \right] \qquad \text{(II-21)}$$

Then

$$\int \Phi' \Phi'' = \frac{1}{2} (\Phi')^2$$

and

$$\int e^{\Phi} (1-\operatorname{erf}\sqrt{\Phi}) \Phi' = (1-\operatorname{erf}\sqrt{\Phi}) e^{\Phi} - \int e^{\Phi} \left(-\frac{2}{\sqrt{\pi}} e^{-\Phi}\right) \left(\frac{\Phi^{-\frac{1}{2}}}{2}\right) \Phi'$$

$$= (1-\operatorname{erf}\sqrt{\Phi}) e^{\Phi} + \frac{2}{\sqrt{\pi}} \sqrt{\Phi}$$

also

$$\int e^{-\Phi} \frac{1+\operatorname{erf}\sqrt{\Phi_c - \Phi}}{1+\operatorname{erf}\sqrt{\Phi_c}} \Phi' = -\frac{1+\operatorname{erf}\sqrt{\Phi_c - \Phi}}{1+\operatorname{erf}\sqrt{\Phi_c}} e^{-\Phi} + \int \frac{e^{-\Phi} \left(\frac{2}{\sqrt{\pi}} e^{\Phi - \Phi_c}\right) (-\Phi')}{(1+\operatorname{erf}\sqrt{\Phi_c})(2\sqrt{\Phi_c - \Phi})}$$

$$= -\frac{1+\operatorname{erf}\sqrt{\Phi_c - \Phi}}{1+\operatorname{erf}\sqrt{\Phi_c}} e^{-\Phi} + \frac{\frac{2}{\sqrt{\pi}} e^{-\Phi_c} \sqrt{\Phi_c - \Phi}}{1+\operatorname{erf}\sqrt{\Phi}}$$

Thus, we have

$$\frac{1}{2}(\Phi')^2 = \mathcal{L}^2\left\{\alpha\left[(1-\operatorname{erf}\sqrt{\Phi})e^{\Phi}+\frac{2}{\sqrt{\pi}}\sqrt{\Phi}\right]+\frac{1+\operatorname{erf}\sqrt{\Phi_c-\Phi}}{1+\operatorname{erf}\sqrt{\Phi_c}}e^{-\Phi}\right.$$
$$\left.-\frac{2}{\sqrt{\pi}}\frac{e^{-\Phi_c}\sqrt{\Phi_c-\Phi}}{1+\operatorname{erf}\sqrt{\Phi_c}}\right\}+\frac{C}{2} \qquad \text{(II-22)}$$

The variables can then be separated

$$dx = \frac{d\Phi}{\sqrt{2\mathcal{L}^2\left\{\alpha\left[(1-\operatorname{erf}\sqrt{\Phi})e^{\Phi}+\frac{2}{\sqrt{\pi}}\sqrt{\Phi}\right]+\frac{1+\operatorname{erf}\sqrt{\Phi_c-\Phi}}{1+\operatorname{erf}\sqrt{\Phi}}e^{-\Phi}-\frac{\frac{2}{\sqrt{\pi}}e^{-\Phi_c}\sqrt{\Phi_c-\Phi}}{1+\operatorname{erf}\sqrt{\Phi_c}}\right\}+C}} \qquad \text{(II-23)}$$

This equation can then be integrated numerically on a computer. The constant C is determined so as to give the value unity when the right-hand side is integrated from $\Phi = 0$ to $\Phi = \Phi_c$.

For large values of Φ, the evaluation of the first term on the right-hand side of Equation II-16 is facilitated by using an expansion,

$$e^{\Phi}(1-\operatorname{erf}\sqrt{\Phi}) = \frac{1}{\sqrt{\pi\Phi}}\left[1-\frac{1}{2\Phi}+\frac{1\cdot 3}{(2\Phi)^2}-\frac{1\cdot 3\cdot 5}{(2\Phi)^3}+\cdots\right] \qquad \text{(II-24)}$$

The preceding equations are the basic forms which are used to find solutions for the planar plasma diodes. These equations are modified for the cases of a potential maximum or potential minimum between the electrodes, and these cases will be accounted for by altering the limits of integration on the expression for charge densities.

C. Types of Solutions

The potential distributions in a planar plasma diode can be generally classified into the following types:

1. Monotonic Solutions
2. Potential Minimum Solutions
3. Potential Maximum Solutions
4. Potential Oscillations with Amplitude Greater Than the Bias Potential

The first three of these classes have been investigated extensively under this Contract; the fourth type has been proven to exist but no exact solutions were obtained.

The particular type of solutions obtained for a planar diode depends on the physical parameters involved, especially the ratio of electron density to ion density at the emitter. This parameter is denoted as α and it has been varied from 0.05 to 5.0 in these investigations. The bias potential has been chosen as 2 kT/e and an electrode spacing of nine Debye lengths is used here. For a value of α between about 0.2 and 1.25 a monotonic variation of potential is found. For α greater than 1.25, a minimum in the potential is found near the emitter. For α less than 0.2 a maximum in the potential is observed, and oscillations in the potential occur between the potential maximum and the collector. As α is decreased below 0.2, the amplitude of the oscillations grows until, at some value of somewhat less than 0.1, the amplitude of the oscillations becomes larger than the bias potential. Under these conditions, there exists at least one potential

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The particular type of solutions obtained for a planar diode depends on the physical parameters involved, especially the ratio of electron density to ion density at the emitter. This parameter is denoted as α and it has been varied from 0.05 to 5.0 in these investigations. The bias potential has been chosen as 2 kT/e and an electrode spacing of nine Debye lengths is used here. For a value of α between about 0.2 and 1.25 a monotonic variation of potential is found. For α greater than 1.25, a minimum in the potential is found near the emitter. For α less than 0.2 a maximum in the potential is observed, and oscillations in the potential occur between the potential maximum and the collector. As α is decreased below 0.2, the amplitude of the oscillations grows until, at some value of somewhat less than 0.1, the amplitude of the oscillations becomes larger than the bias potential. Under these conditions, there exists at least one potential

maximum and one potential minimum outside the range of the bias potential.

Each of these types of solutions will be discussed in this section, and some examples of each will be given. The modifications in the basic equations needed for the potential maximum and potential minimum cases will be explained.

1. Monotonic Solutions

Solutions have been obtained for a planar plasma diode configuration under a wide range of parametric conditions. The electrode spacing has been nondimensionalized by dividing by the Debye length. Since the Debye length is a function of ion density, dielectric constant, and temperature, this means that the (L/λ_D) parameter actually combines a number of variables. Usually it is considered as a function of electrode spacing or space charge density since these are often the variables of interest.

The following ranges were investigated in this study (although not simultaneously):

Bias potential = ϕ: 2 to 1000 kT/e

Electrode spacing/Debye length = L/λ_D : 0 to 500

Emitter electron/Ion density = α : 0.05 to 5.0

A computer program was written to find these solutions. It required revision several times to account for unexpected phenomena. The listing for this program is given in Appendix A of this report. The input quantities for this program are listed in Tables I and II, where Table I shows the necessary input to obtain a solution and Table II has additional input quantities which

TABLE II-1

MINIMUM INPUT FOR PLANAR PLASMA DIODE PROGRAM

4	MONTH, three numbers of two digits each describing month, day, and year, of the calculation.
3	DEBYE, the number of Debye lengths in the electrode spacing.
3	POT, the collector bias potential (only positive values may be used).
3	ETA, the ratio of emitted electron density to total ion density at the emitter.

TABLE II-2

OPTIONAL INPUT FOR FLEXIBILITY IN SOLUTIONS OF PLANAR DIODE PROBLEMS

Col. 1	Col. 2
4	LPRINT, should be input as 2 if a printout after each iteration is desired.
4	KPRINT, should be input as 1 if the output data are to be edited to put on a single page. This feature is not often used because it would be likely to destroy the detailed presentation of data in potential minimum and/or plateau regions.
3	DEBYY, an alternative to DEBYE if it is desired to use a Debye length as the unit of length replacing the electrode spacing. This has occasionally proved to be useful in cases where the electrode spacing is a large number of Debye lengths (greater than 200).
3	PHIPRM, may be input if desired as a starting point for the iteration of monotonic type solutions. This value will be taken as the slope of the potential distribution curve at the inflection point, where the net charge density changes from ion rich to electron rich going toward the collector, if there is such a point. If there is no inflection point, this will be the slope of the potential curve at the emitter. If PHIPRM is not input, the initial value chosen will be the straight line value for very low plasma density.
3	PHIM, may be input as a starting value for potential minimum type solutions. The value must be negative; its absolute value should be less than the absolute magnitude of the true minimum value.
3	XLOOP, may be input to give a maximum number of iterations different from 15. Our experience here is that the solution for any configuration will converge in fewer than 15 loops if it converges at all.

can be used to speed up the solution or obtain additional information.

The results of these computer runs have been graphed, usually in the form of potential as a function of distance from the emitter and charge density as a function of distance. In Figure II-1a is shown the variation of potential with distance for ratios of electrode spacing to Debye length from 0 to 100. The curves for 30 and 100 become plateaus at a potential of about 0.75 kT/e as will be shown on later curves. All the curves on this graph are calculated for a bias potential of 2 kT/e. In Figures II-1b, II-1c, and II-1d, the bias potential has been increased to 5, 10, and 20 kT/e respectively, but the shape of the potential curve has not been greatly altered. A closer look at the variation of potential in the regions near the emitter and collector may be obtained from Figures II-2a, 2b, 2c, and 2d. The characteristic nature of the potential distribution in the sheath for distance measured in Debye lengths is apparent. However, the coincidence of $L/\lambda_D = 30$ and $L/\lambda_D = 100$ for $\Phi_c = 20\ \frac{kT}{e}$ may be questionable. The effect of bias potential as an independent parameter may be seen in Figures II-3a, 3b, 3c, and 3d. Then in Figures II-4a, 4b, and 4c, the variation of densities of ions and electrons are shown as a function of distance from the emitter. These charge density distributions correspond to the same parametric variations as applied to the preceding potential distribution curves. Note that all these solutions begin with a neutral plasma at the emitter.

Some interesting observations may be made from a study of these curves. It is apparent that the influence of small disturbances on a neutral plasma is to reduce the charge density faster by acceleration than by retardation, when starting from a reference potential condition where all velocities are present.

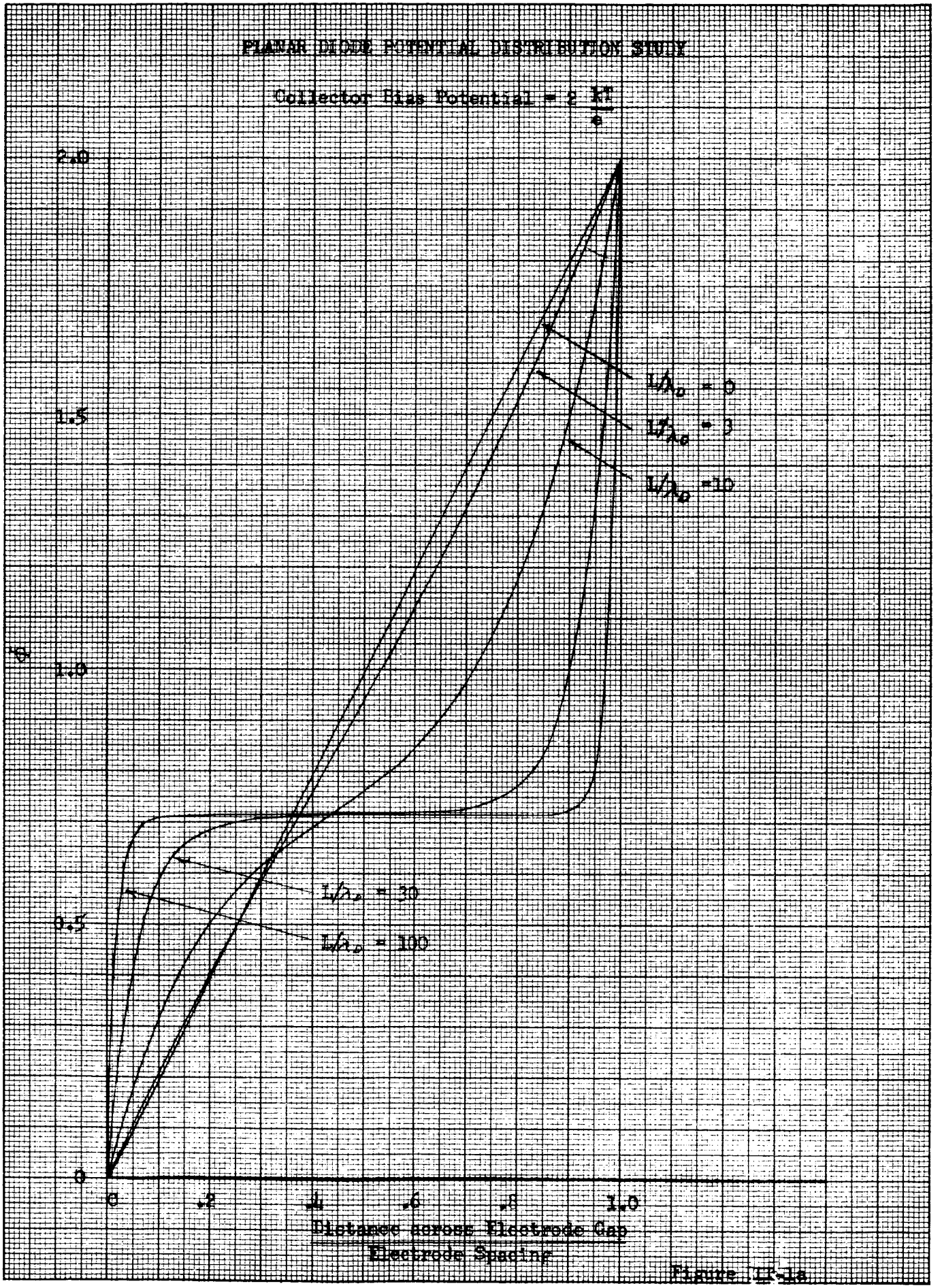

PLANAR DIODE POTENTIAL DISTRIBUTION STUDY
Collector Bias Potential = 2 kT/e
2.0
1.5
1.0
0.5
0
Φ
L/λD = 0
L/λD = 3
L/λD =10
L/λD = 30
L/λD = 100
0
.2
.4
.6
.8
1.0
Distance across Electrode Gap / Electrode Spacing

Figure II-1a

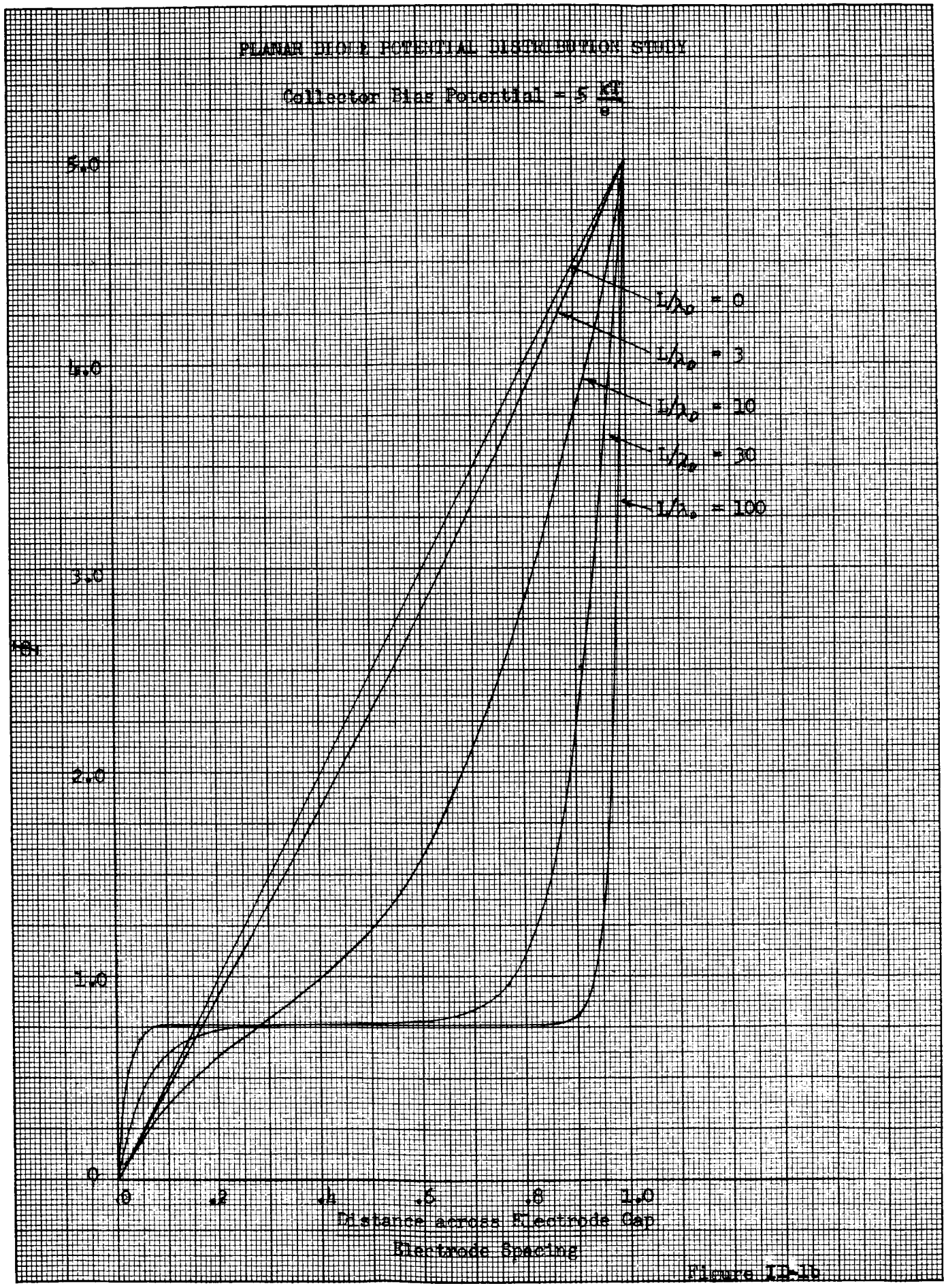

PLANAR DIODE POTENTIAL DISTRIBUTION STUDY
Collector Bias Potential = 5 $\frac{kT}{e}$
5.0
4.0
3.0
2.0
1.0
0
L/λ_D = 0
L/λ_D = 3
L/λ_D = 10
L/λ_D = 30
L/λ_D = 100
0
.2
.4
.6
.8
1.0
Distance across Electrode Gap
Electrode Spacing

Figure II-1b

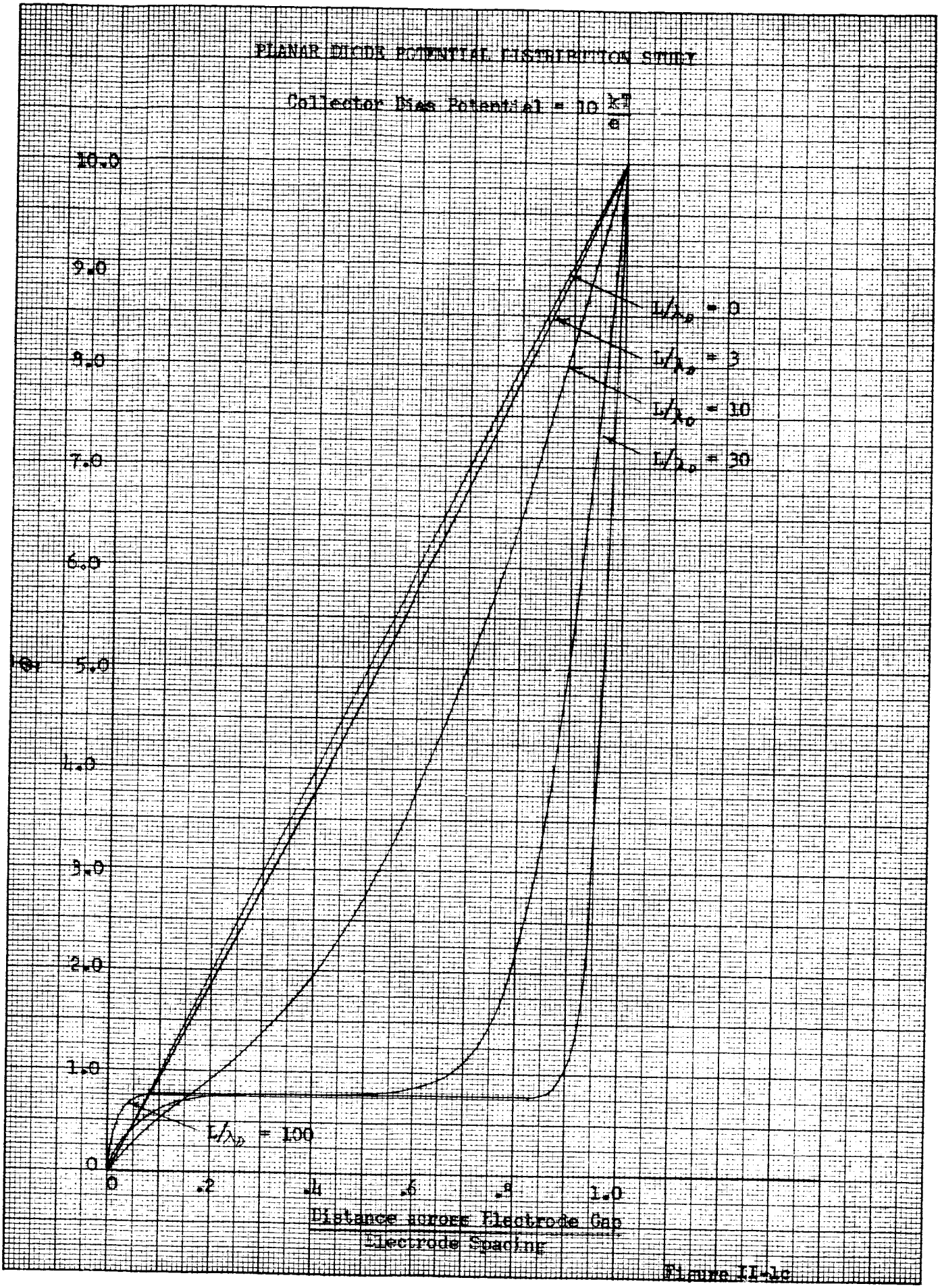

PLANAR DIODE POTENTIAL DISTRIBUTION STUDY
Collector Bias Potential = 10 kT/e
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
L/λD = 0
L/λD = 3
L/λD = 10
L/λD = 30
L/λD = 100
0
.2
.4
.6
.8
1.0
Distance across Electrode Gap / Electrode Spacing

Figure II-1c

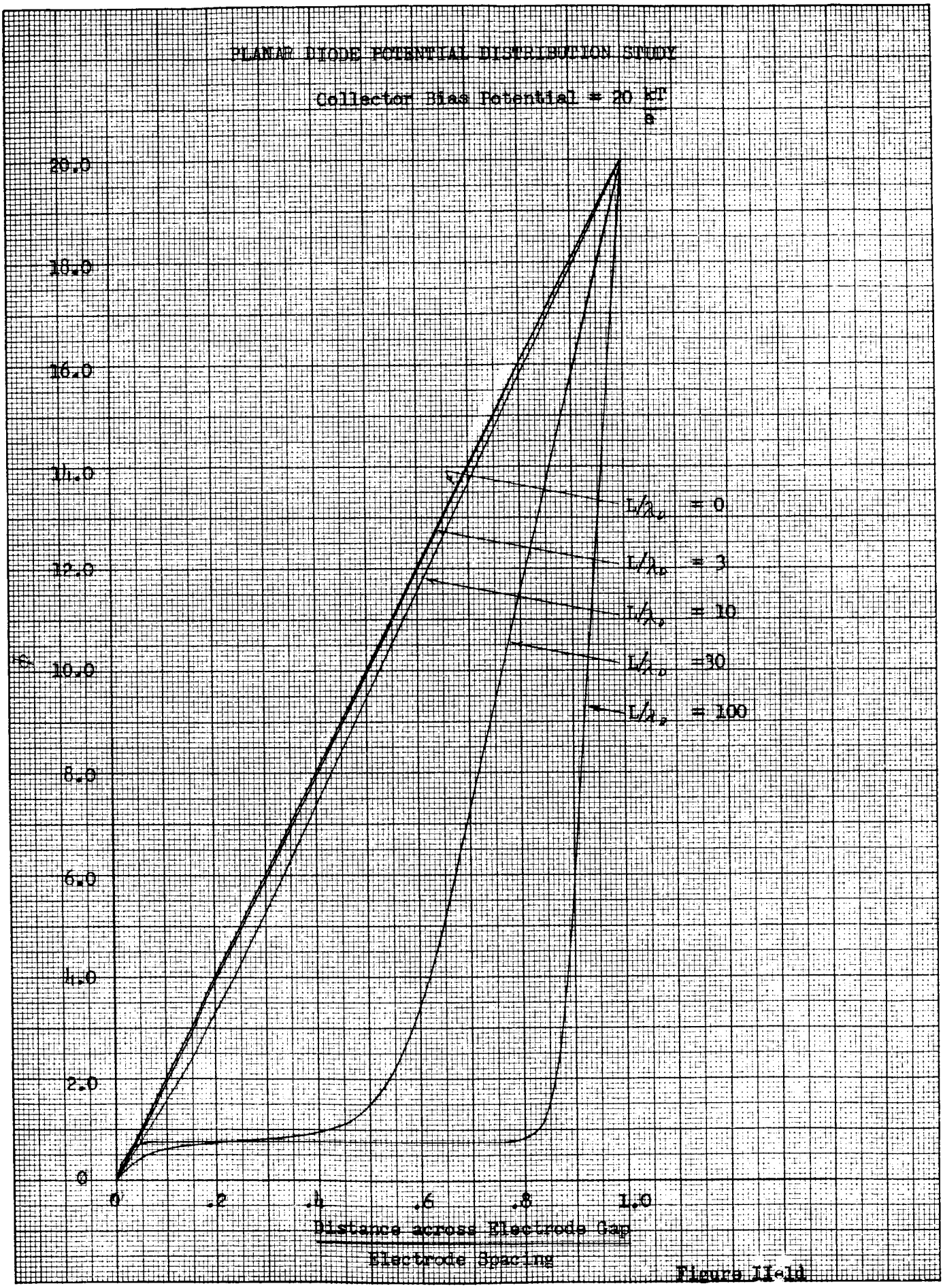

PLANAR DIODE POTENTIAL DISTRIBUTION STUDY
Collector Bias Potential = 20 kT/e
20.0
16.0
12.0
8.0
4.0
0
0
.2
.4
.6
.8
1.0
L/λD = 0
L/λD = 3
L/λD = 10
L/λD = 30
L/λD = 100
Distance across Electrode Gap
Electrode Spacing

Figure II-1d

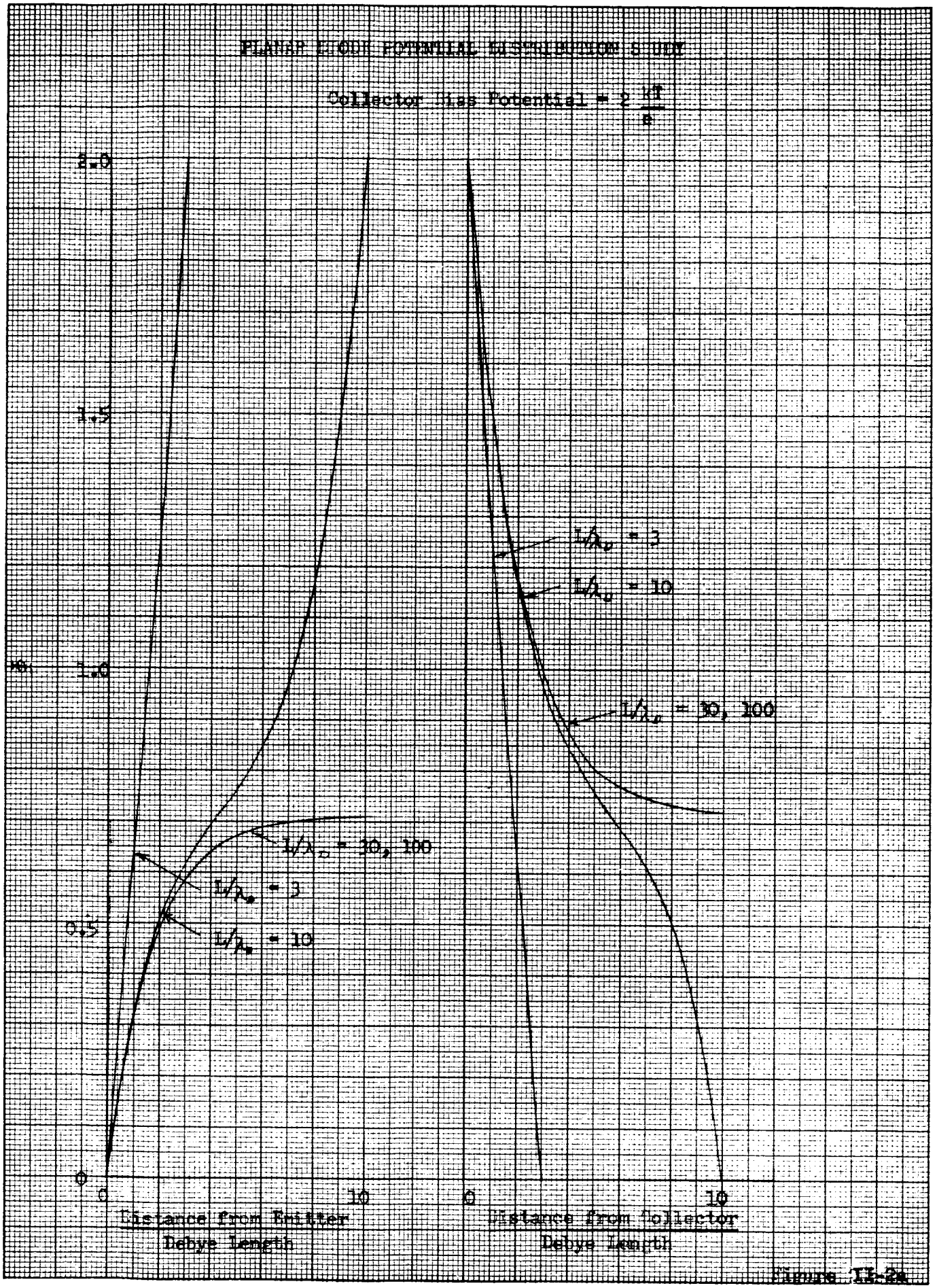

PLANAR E-CUR POTENTIAL DISTRIBUTION STUDY
Collector Bias Potential = 2 kT/e
2.0
1.5
1.0
0.5
0
φ/φ₀
L/λ_D = 3
L/λ_D = 10
L/λ_D = 30, 100
L/λ_D = 30, 100
L/λ_D = 3
L/λ_D = 10
0
10
0
10
Distance from Emitter
Debye Length
Distance from Collector
Debye Length

Figure II-2a

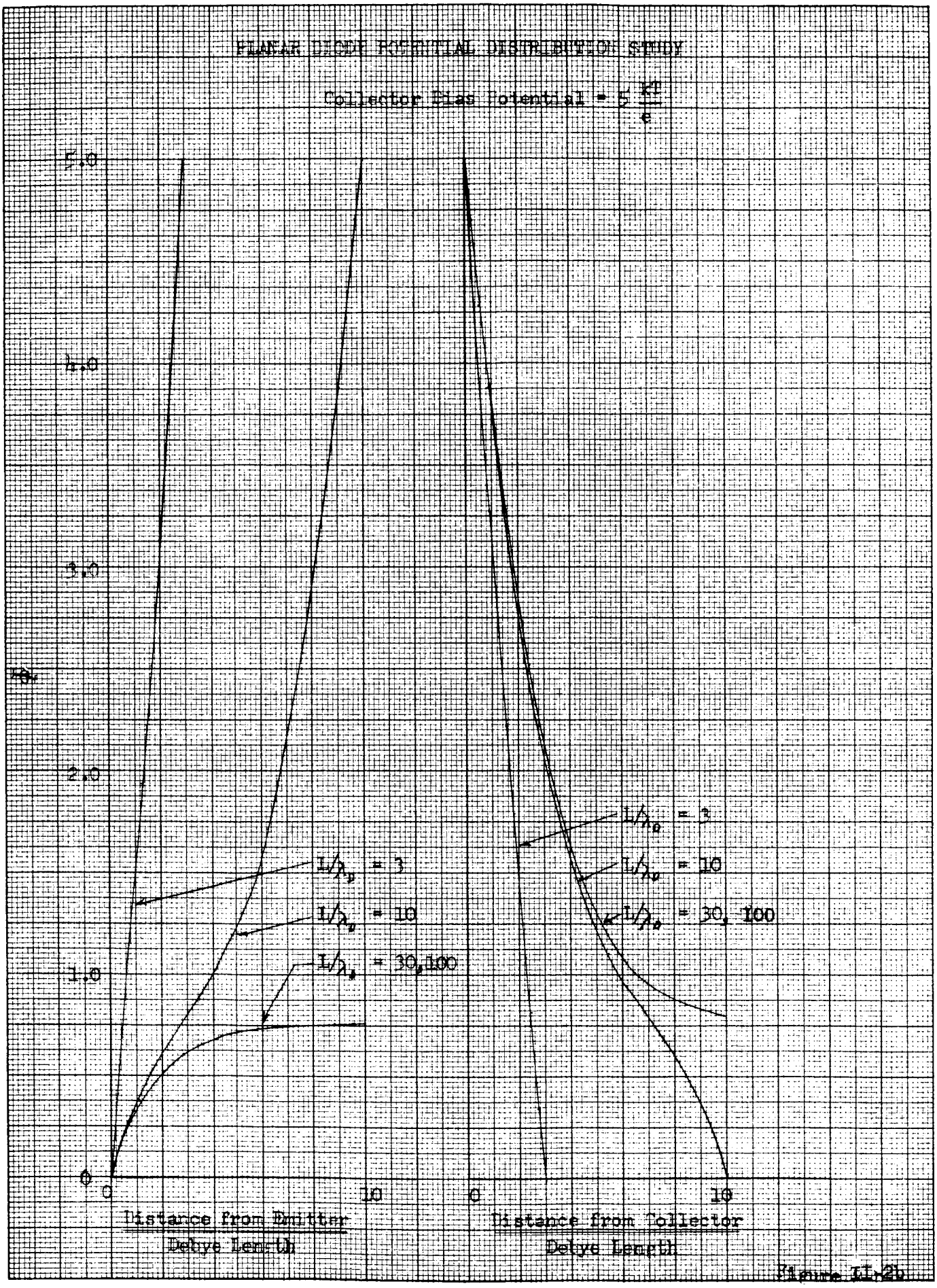

PLANAR DIODE POTENTIAL DISTRIBUTION STUDY
Collector Bias Potential = 5 $\frac{kT}{e}$
5.0
4.0
3.0
2.0
1.0
0
L/λ_D = 3
L/λ_D = 10
L/λ_D = 30, 100
L/λ_D = 3
L/λ_D = 10
L/λ_D = 30, 100
0
10
0
10
Distance from Emitter
Debye Length
Distance from Collector
Debye Length

Figure II-2b

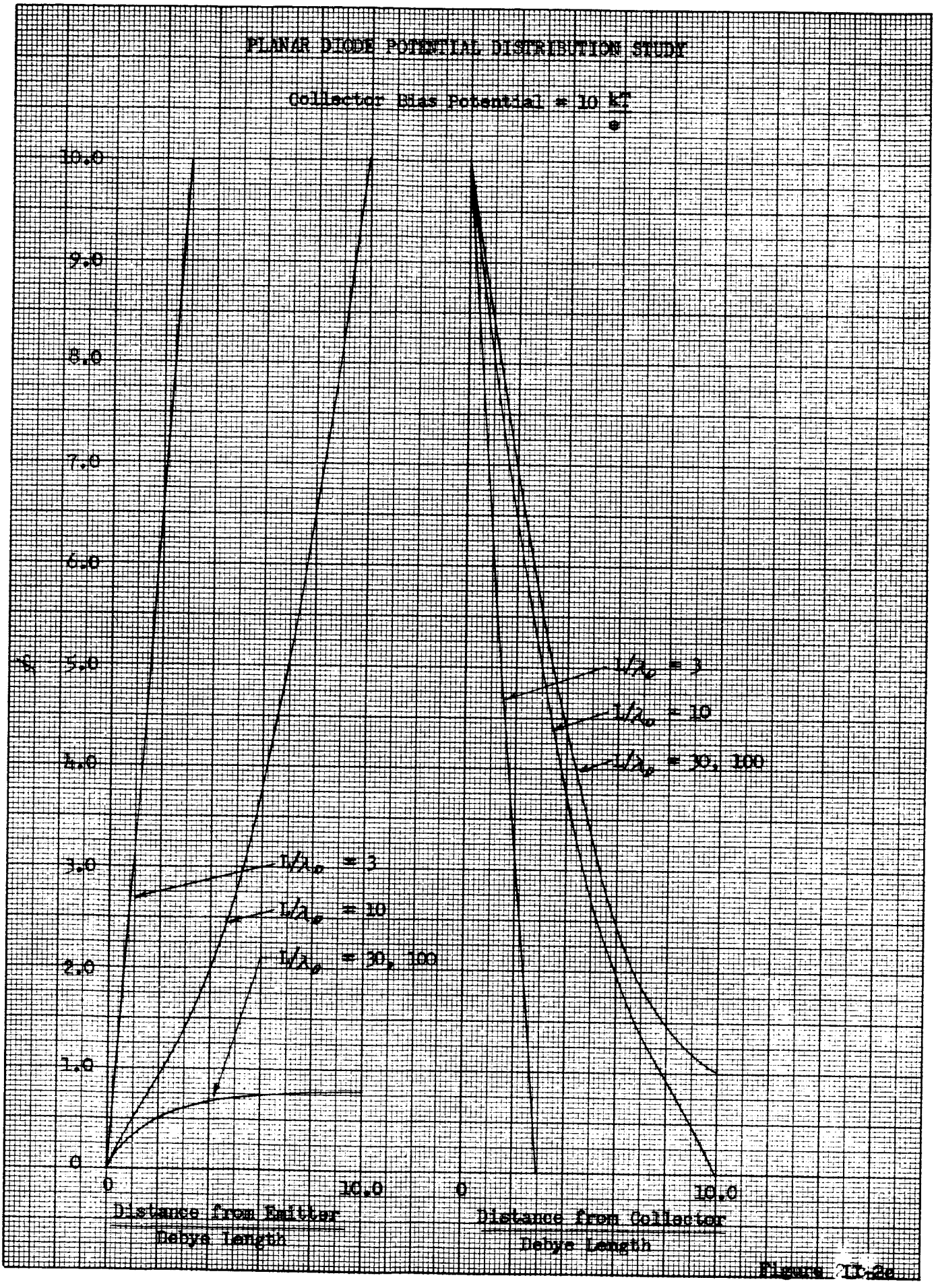

PLANAR DIODE POTENTIAL DISTRIBUTION STUDY
Collector Bias Potential = 10 kT
φ
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
φ
L/λ_φ = 3
L/λ_φ = 10
L/λ_φ = 30, 100
L/λ_φ = 3
L/λ_φ = 10
L/λ_φ = 30, 100
0
10.0
Distance from Emitter
Debye Length
0
10.0
Distance from Collector
Debye Length

Figure II.2e

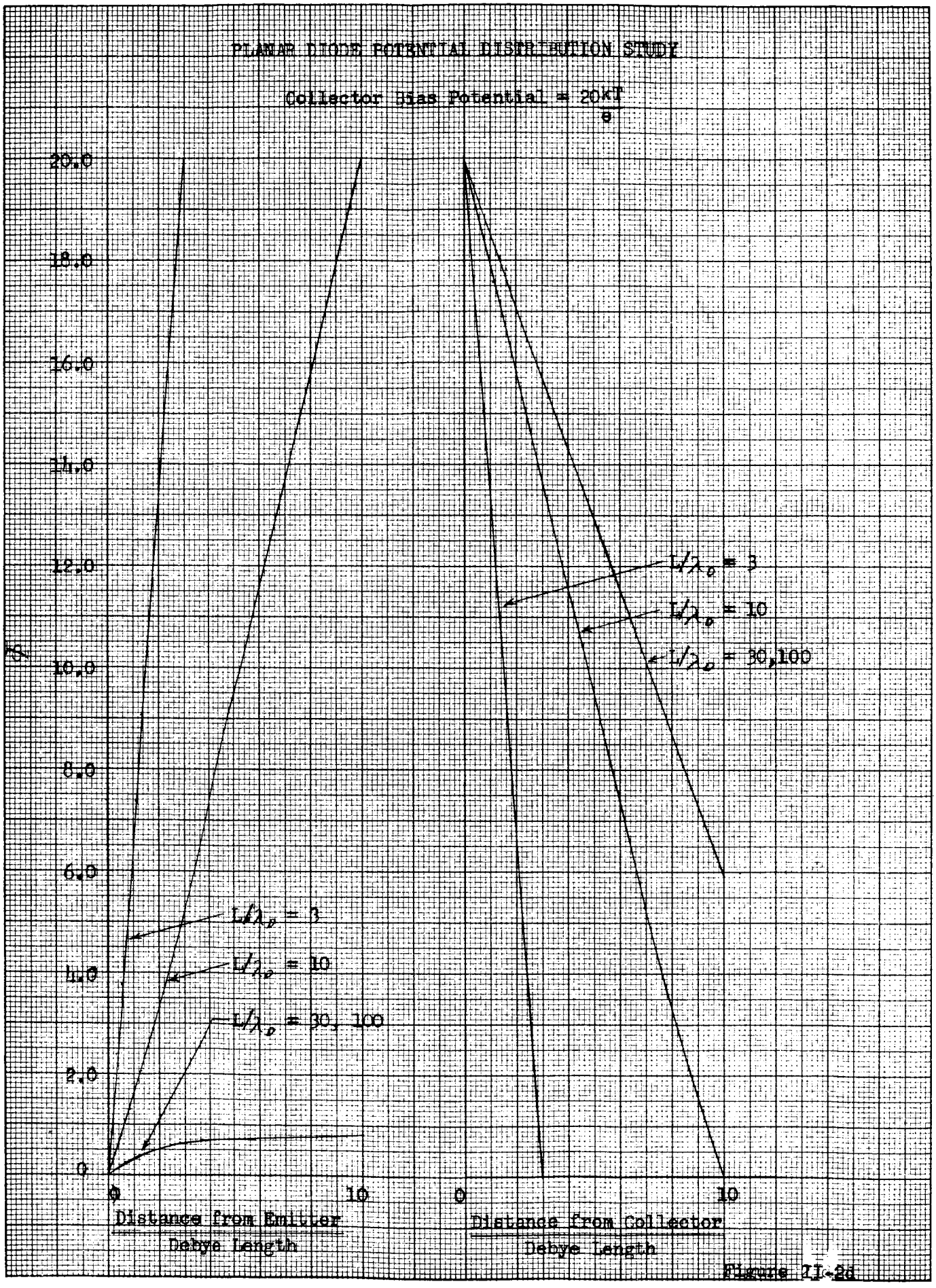

PLANAR DIODE POTENTIAL DISTRIBUTION STUDY
Collector Bias Potential = 20 KT/e
20.0
18.0
16.0
14.0
12.0
10.0
8.0
6.0
4.0
2.0
0
φ
L/λ_D = 3
L/λ_D = 10
L/λ_D = 30, 100
0
10
Distance from Emitter
Debye Length
0
10
Distance from Collector
Debye Length

Figure II-2a

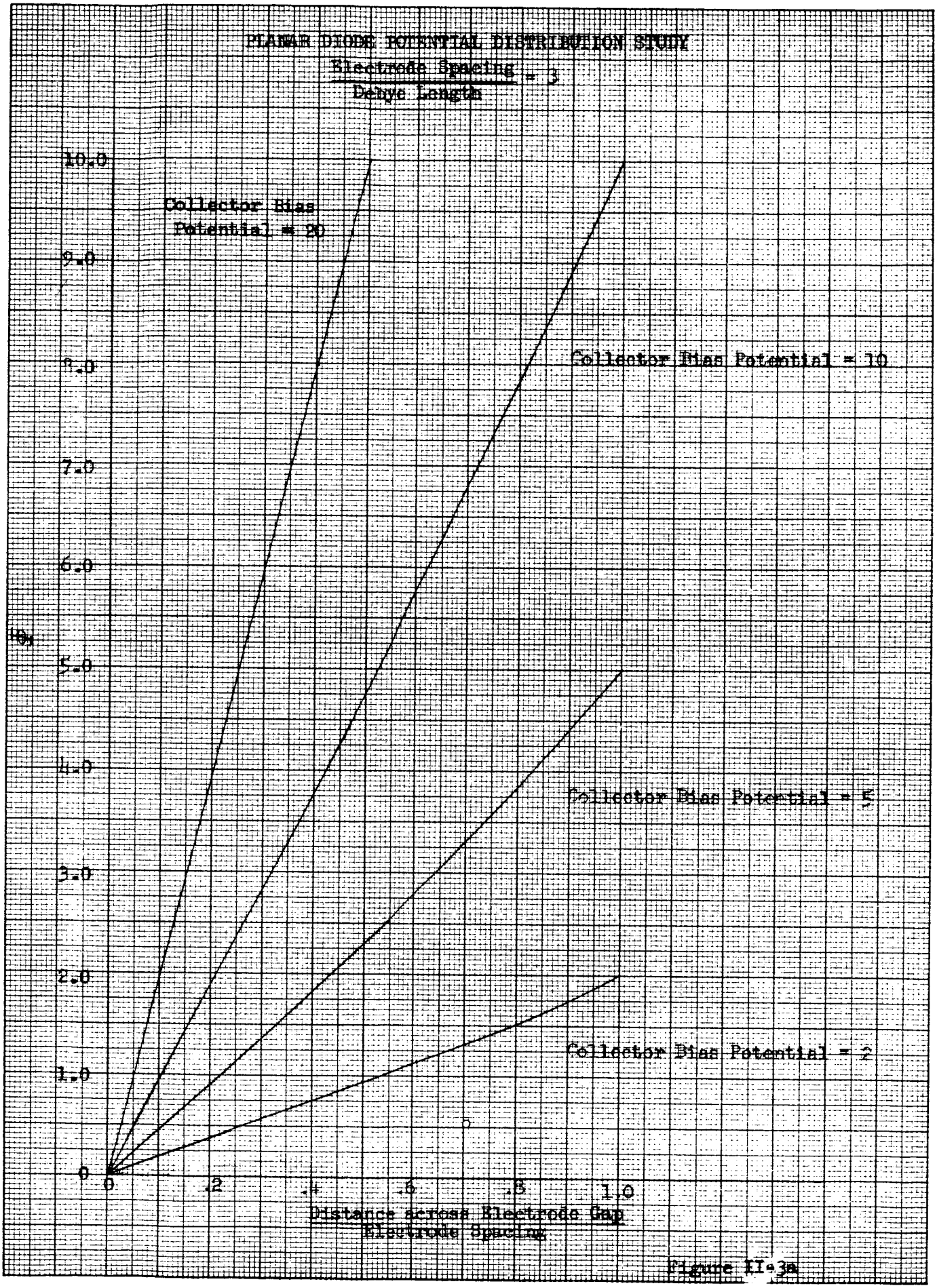

PLANAR DIODE POTENTIAL DISTRIBUTION STUDY
Electrode Spacing / Debye Length = 3
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
Collector Bias Potential = 20
Collector Bias Potential = 10
Collector Bias Potential = 5
Collector Bias Potential = 2
0
.2
.4
.6
.8
1.0
Distance across Electrode Gap / Electrode Spacing

Figure II-3a

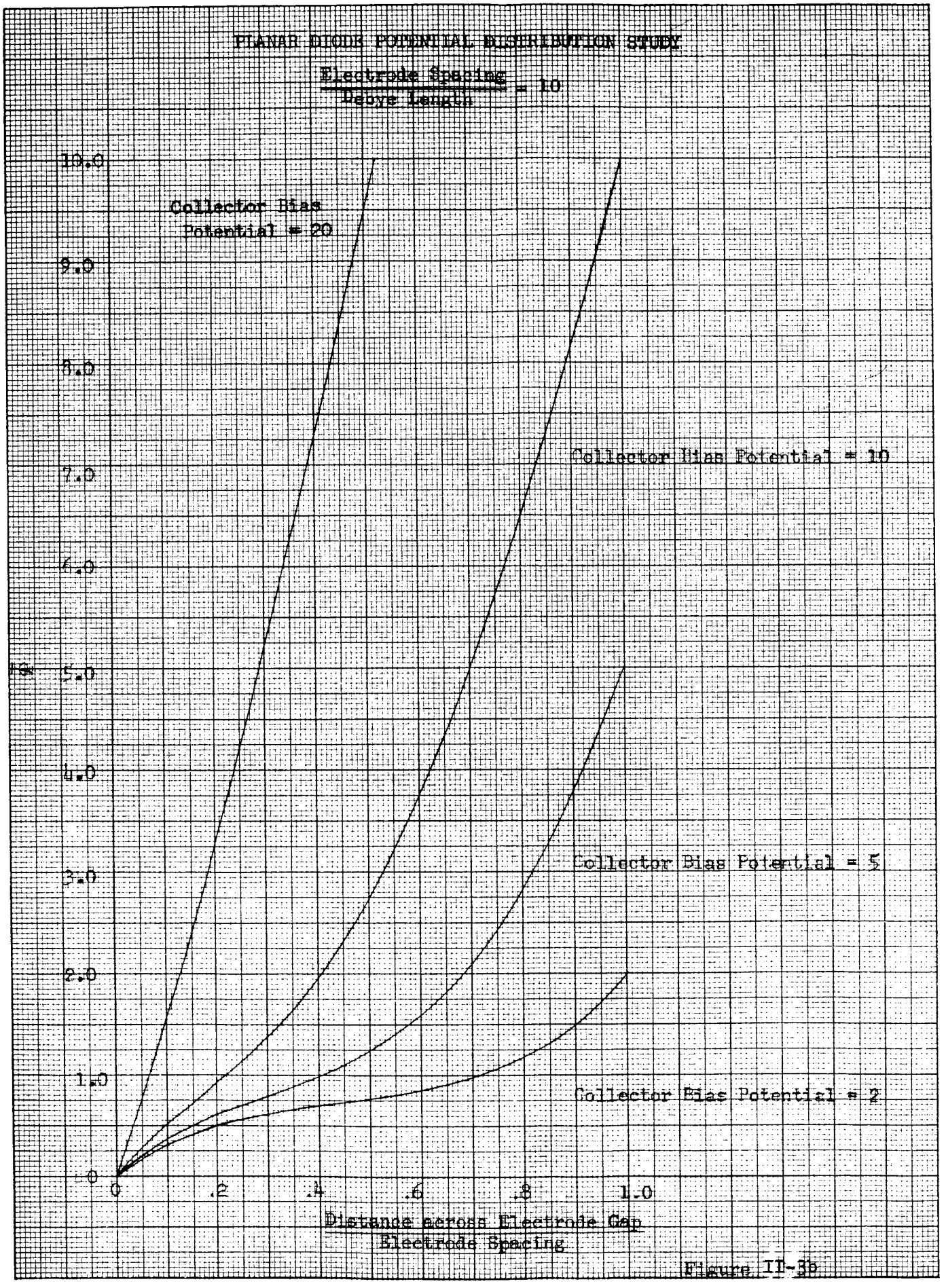

PLANAR DIODE POTENTIAL DISTRIBUTION STUDY
Electrode Spacing / Debye Length = 10
Collector Bias Potential = 20
Collector Bias Potential = 10
Collector Bias Potential = 5
Collector Bias Potential = 2
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
0
.2
.4
.6
.8
1.0
Distance across Electrode Gap / Electrode Spacing

Figure II-3b

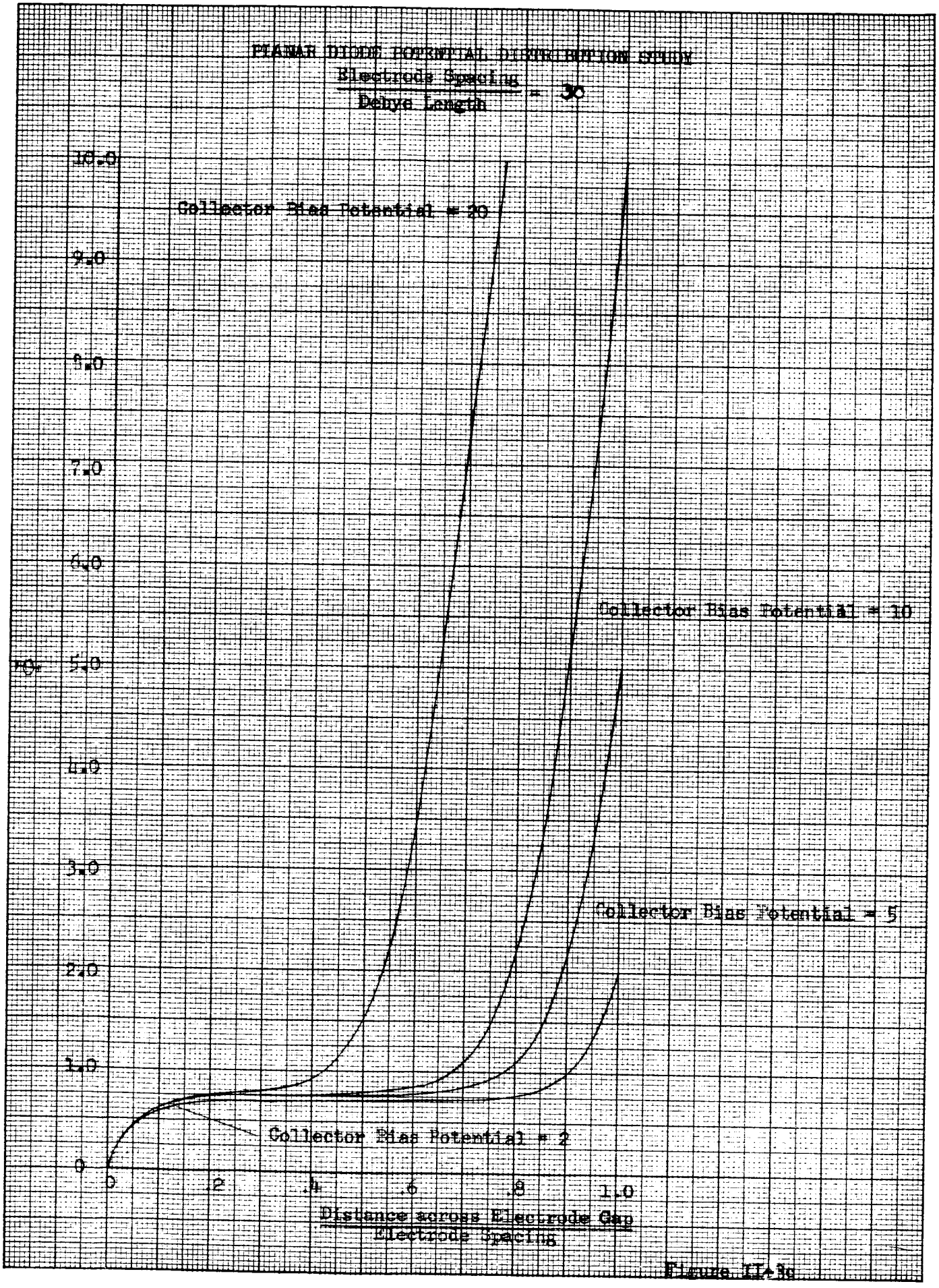

PLANAR DIODE POTENTIAL DISTRIBUTION STUDY
Electrode Spacing / Debye Length = 30
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
Collector Bias Potential = 20
Collector Bias Potential = 10
Collector Bias Potential = 5
Collector Bias Potential = 2
0
.2
.4
.6
.8
1.0
Distance across Electrode Gap / Electrode Spacing

Figure II-3a

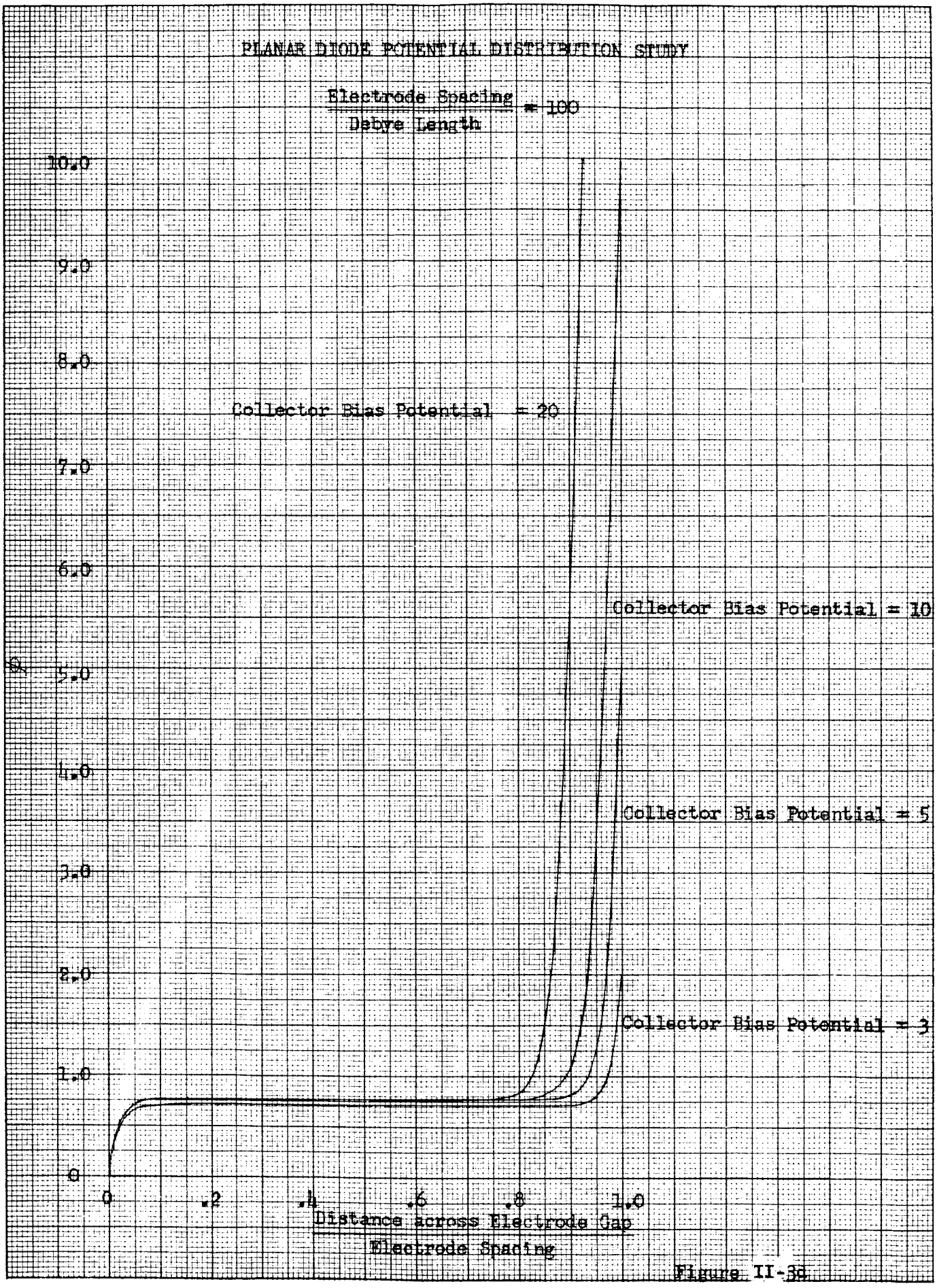

PLANAR DIODE POTENTIAL DISTRIBUTION STUDY
Electrode Spacing / Debye Length = 100
10.0
9.0
8.0
Collector Bias Potential = 20
7.0
6.0
Collector Bias Potential = 10
5.0
4.0
Collector Bias Potential = 5
3.0
2.0
Collector Bias Potential = 3
1.0
0
0
.2
.4
.6
.8
1.0
Distance across Electrode Gap / Electrode Spacing

Figure II-3E

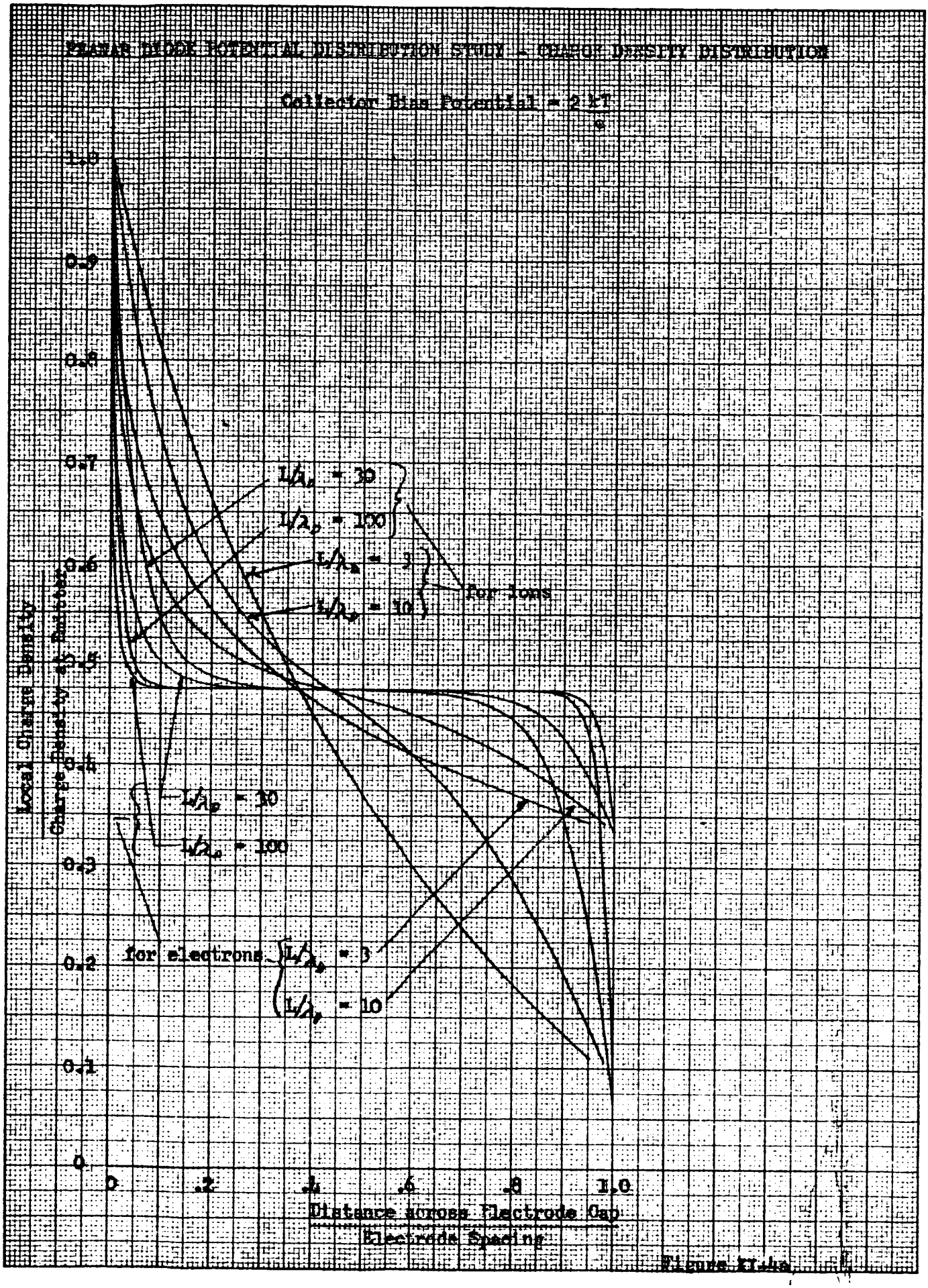

PLANAR DIODE POTENTIAL DISTRIBUTION STUDY - CHARGE DENSITY DISTRIBUTION
Collector Bias Potential = 2 kT/e
Local Charge Density / Collector Charge Density
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
L/λD = 30
L/λD = 100
L/λD = 3
L/λD = 10
for ions
L/λD = 30
L/λD = 100
for electrons
L/λD = 3
L/λD = 10
0
.2
.4
.6
.8
1.0
Distance across Electrode Gap / Electrode Spacing

Figure KI-4a

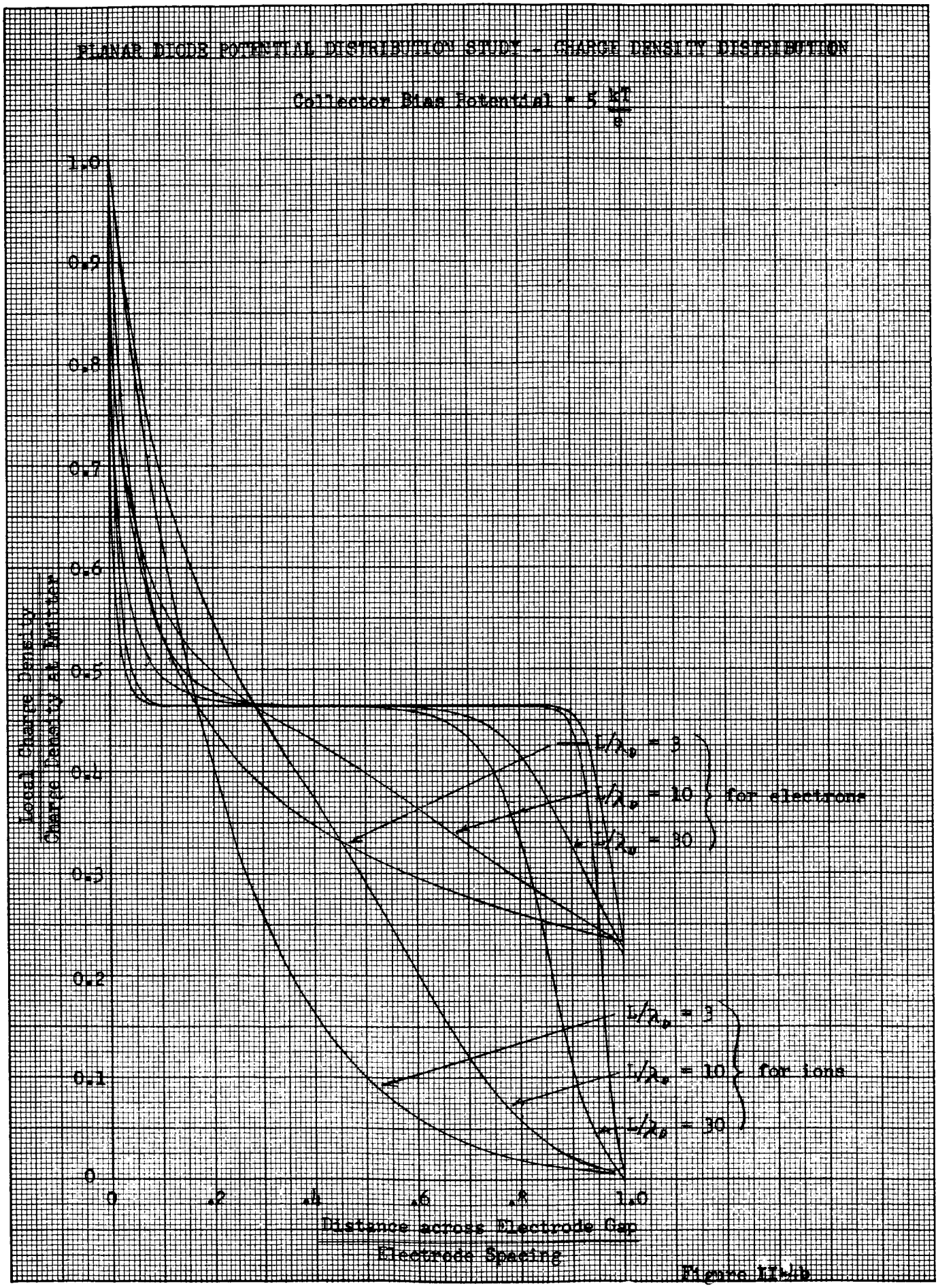

PLANAR DIODE POTENTIAL DISTRIBUTION STUDY - CHARGE DENSITY DISTRIBUTION
Collector Bias Potential = 5 kT/e
Local Charge Density / Emitter Charge Density at Emitter
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
.2
.4
.6
.8
1.0
L/λ0 = 3
L/λ0 = 10
L/λ0 = 30
for electrons
L/λ0 = 3
L/λ0 = 10
L/λ0 = 30
for ions
Distance across Electrode Gap / Electrode Spacing

Figure II-4b

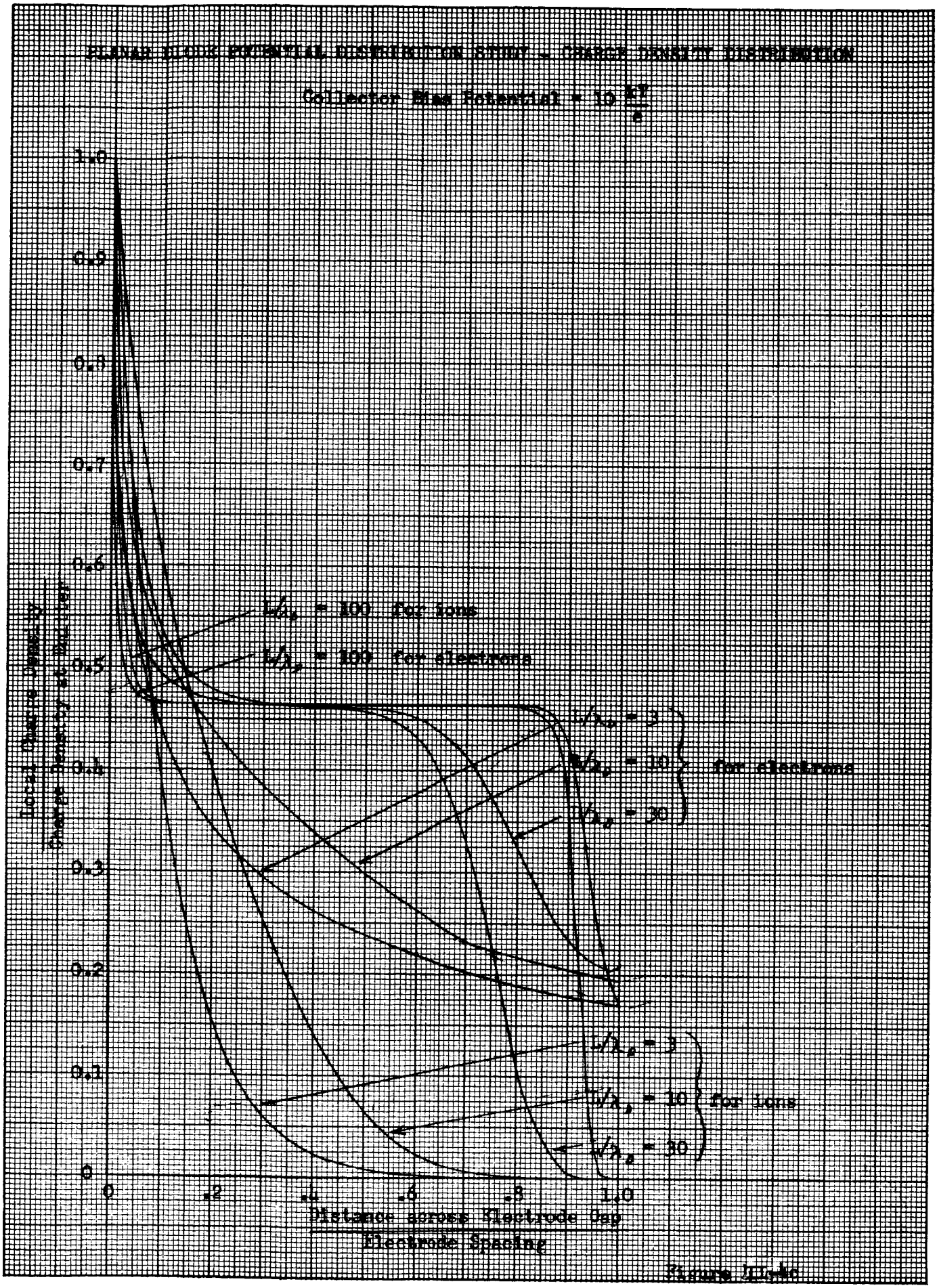


Figure III-4c

Therefore, an ion-rich condition appears in the region of the emitter. Farther away from the emitter, the electron density decrease slows down enough that the decrease in ion density catches up and a point of neutral plasma density is attained. For a sufficiently large electrode spacing (or ion density), the potential will enter a plateau region at that point. The minimum value of (L/λ_D) needed for establishing such a region has been found to be about 20. The value of the plateau potential is independent of the plasma density level and it is independent of bias potential for bias potentials larger than approximately 2 kT/e. On the downstream side of the plateau region (or the neutral plasma point if no plateau exists) the charge density of ions continues to be reduced quite rapidly and reaches a level at the collector which depends on the bias potential. For bias potentials of about 7 or greater, the ion density will be practically zero at the collector. Meanwhile, the electron density decreases relatively slowly downstream of the neutral plasma until it reaches some finite value at the collector. Meanwhile, the electron density decreases relatively slowly downstream of the neutral plasma until it reaches some finite value at the collector. The extent of this collector sheath region depends on the collector potential, although the region of non-neutrality near the emitter appears usually to extend about 10 Debye lengths into the electrode spacing.

A. The Characteristic Potential Distribution

Some of the results from these graphs may be generalized and this has been done in Figure II-5. The solid curve represents data for electrode spacings of 30 to 100 Debye lengths and collector bias potentials of 2, 5, 10, and 20 kT/e, within the ability to plot on the scale. Therefore, it may be concluded that this potential distribution is generally characteristic of the phenomenon.

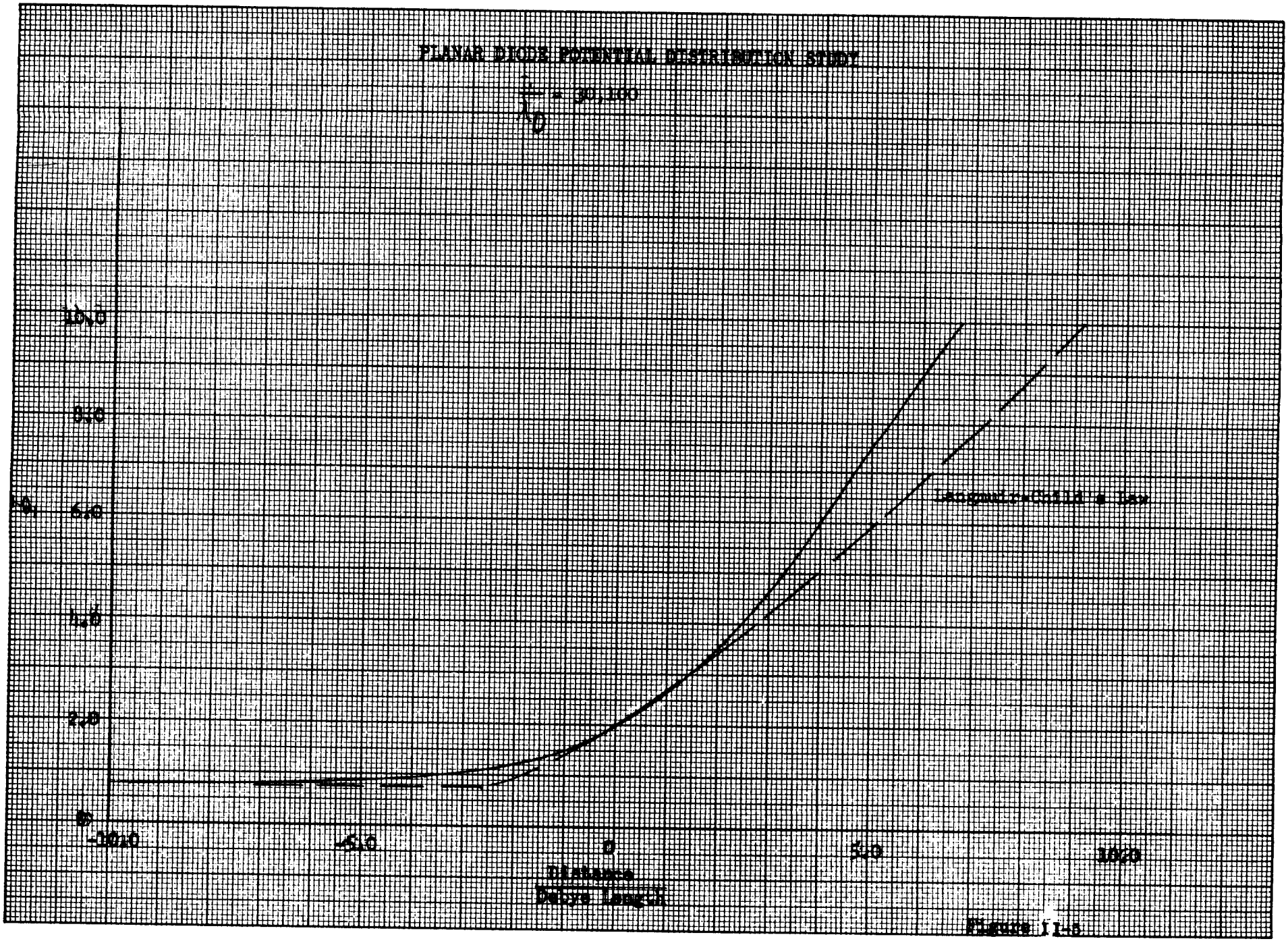

PLANAR DIODE POTENTIAL DISTRIBUTION STUDY
$\frac{x}{\lambda_D}$ = 30,100
10,0
8,0
6,0
4,0
2,0
0
-10,0
-5,0
0
5,0
10,0
Distance
Debye Length
Langmuir-Child's Law

Figure II-5

Also shown on this graph for a comparison is a sample curve along which the potential varies as the 4/3 power of distance, as would be required by the Langmuir-Child law for space charge limited flow. That these two curves do not coincide is expected, because the infinite charge density of space charge limited flow is not necessarily associated with the extraction of a charged particle current from a neutral plasma. However, the computed variation of potential does begin to approximate a 4/3 power variation as the bias potential is increased to very large values. This is shown in Figure II-6, which gives the potential distributions for bias potentials of 200 and 1000 kT/e over an electrode spacing of 500 Debye lengths. The potential gradient in the plateau was assumed at values from 10^{-8} to 10^{-5} kT/e per Debye length. Some program adaption was necessary to obtain these results and these were incorporated as optional features of the program.

The results showed that the potential distribution above the plateau is truly characteristic within the accuracy of the calculations. The electron current passed by the plasma depends on the electron temperature and the number density, according to equation (II-19) and is independent of the collector potential. This equation is similar in form to the electron current of the Langmuir-Child analysis (see equations 2-11 and 2-24 of the NAS 8-623 Contract Report) and leads to a relation between the Langmuir-Child electrode spacing and the Debye length of the planar plasma diode analysis:

$$\left(\frac{L}{\lambda_0}\right)^2 = \frac{4}{9}\sqrt{\pi}\left(\frac{V}{kT/e}\right)^{3/2} \qquad \text{(II-25)}$$

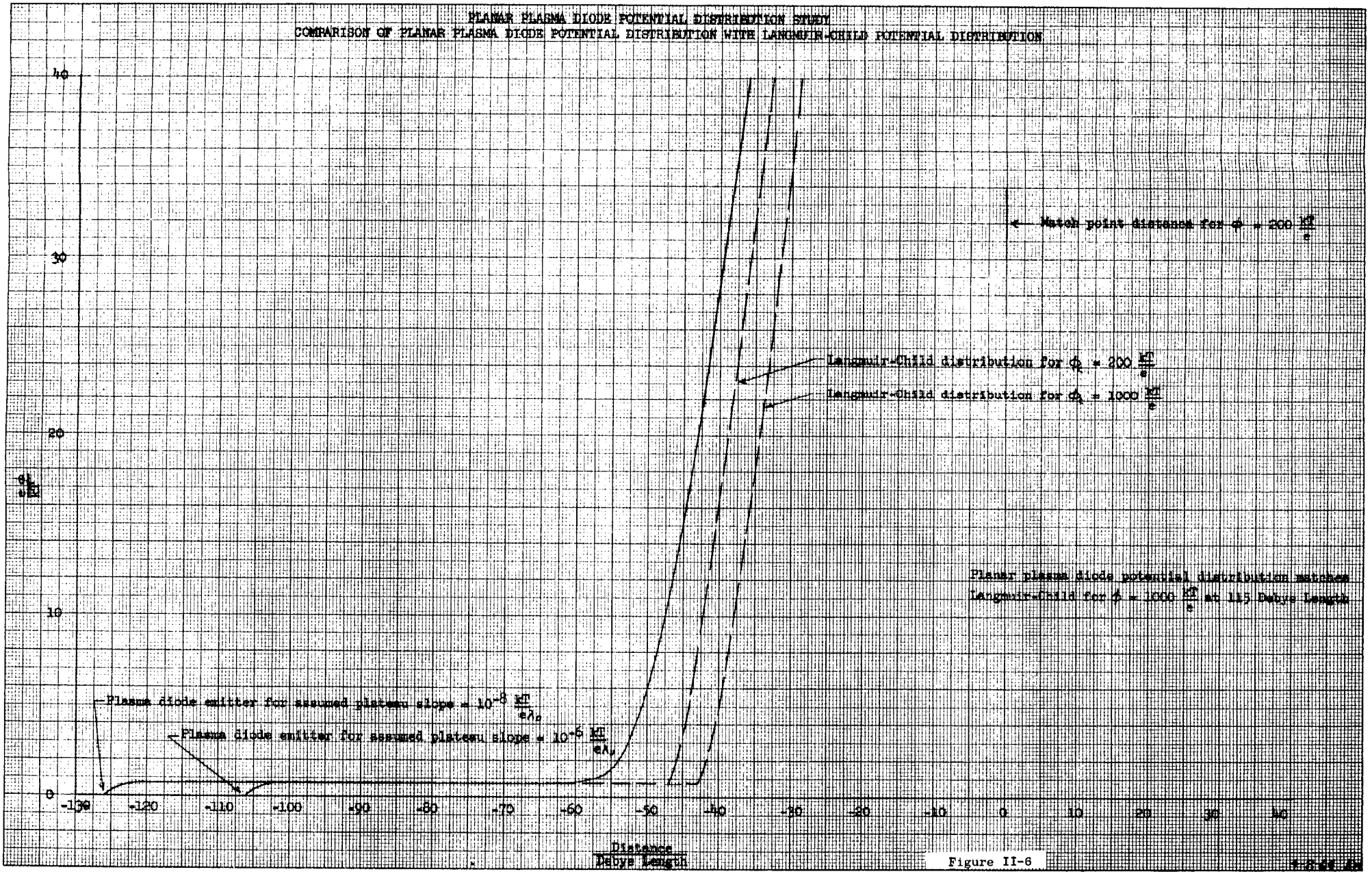

PLANAR PLASMA DIODE POTENTIAL DISTRIBUTION STUDY
COMPARISON OF PLANAR PLASMA DIODE POTENTIAL DISTRIBUTION WITH LANGMUIR-CHILD POTENTIAL DISTRIBUTION
40
30
20
10
0
-130
-120
-110
-100
-90
-80
-70
-60
-50
-40
-30
-20
-10
0
10
20
30
40
Match point distance for $\phi = 200 \frac{kT}{e}$
Langmuir-Child distribution for $\phi_L = 200 \frac{kT}{e}$
Langmuir-Child distribution for $\phi_L = 1000 \frac{kT}{e}$
Planar plasma diode potential distribution matches Langmuir-Child for $\phi = 1000 \frac{kT}{e}$ at 115 Debye Length
Plasma diode emitter for assumed plateau slope = $10^{-8} \frac{kT}{e\lambda_D}$
Plasma diode emitter for assumed plateau slope = $10^{-6} \frac{kT}{e\lambda_D}$
$\frac{\phi}{\frac{kT}{e}}$
Distance / Debye Length

Figure II-6

This relation was used to match the Langmuir-Child potential curves for 200 and 1000 kT/e bias potentials to the characteristic distribution of the planar plasma diode analysis. It is interesting to observe that the influence of the collector potential penetrates significantly farther (but not qualitatively farther) into the plasma than would be predicted by considering a simple space charge limited condition for particles of one species.

B. Effects of Charge Density Ratio

The ratio of electron density to ion density at the emitter may be varied as an independent parameters. Some results from variation of this parameter from 0.6 to 1.2 are shown in Figures II-7a, 7b, and 7c. In the cases where the electron density at the emitter is less than the ion density, a higher repelling potential is necessary before the ion density drops to a level equal to the electron density, than for the case of unity α. Conversely, for values of α greater than 1.0, the neutral plasma potential is reduced from its value for equal emitter charge densities. Other than these differences, the potential behavior is rather similar for the variations of charge density ratio within this range.

Some results of monotonic potential distributions were also obtained at a value of (L/λ_D) equal to 60. These are shown in Figures II-8 with charge density variations given in Figure II-9. It can be observed, by comparison of results for various electrode spacings, that the effect of increasing (L/λ_D) is simply to increase the relative extent of the plateau region.

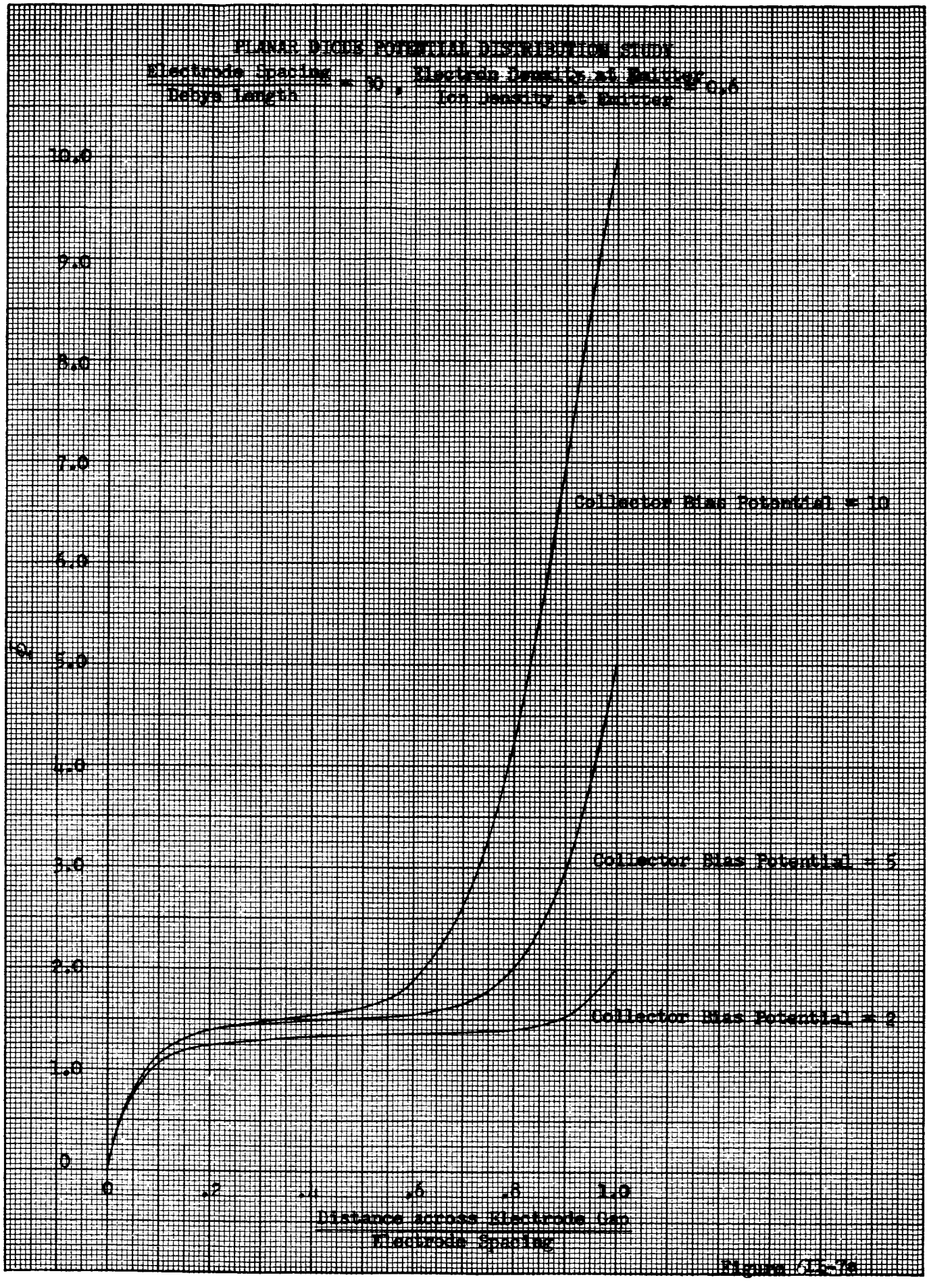

PLANAR DIODE POTENTIAL DISTRIBUTION STUDY
Electrode Spacing = 50 λ Debye Length
Electron Density at Emitter = 0.6
Ion Density at Emitter
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
Collector Bias Potential = 10
Collector Bias Potential = 5
Collector Bias Potential = 2
0
.2
.4
.6
.8
1.0
Distance across Electrode Gap / Electrode Spacing

Figure 15-7a

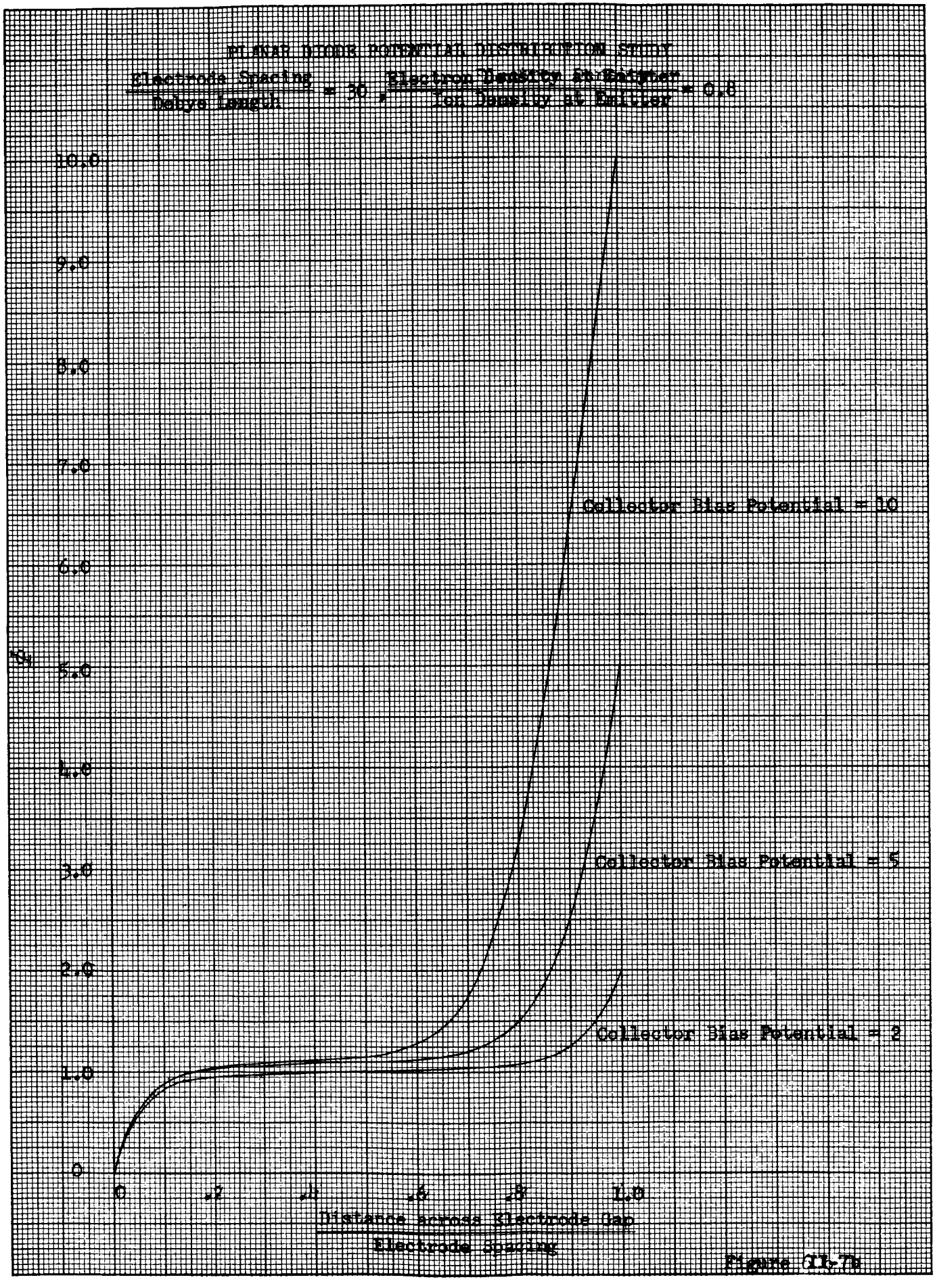


Figure III-7b

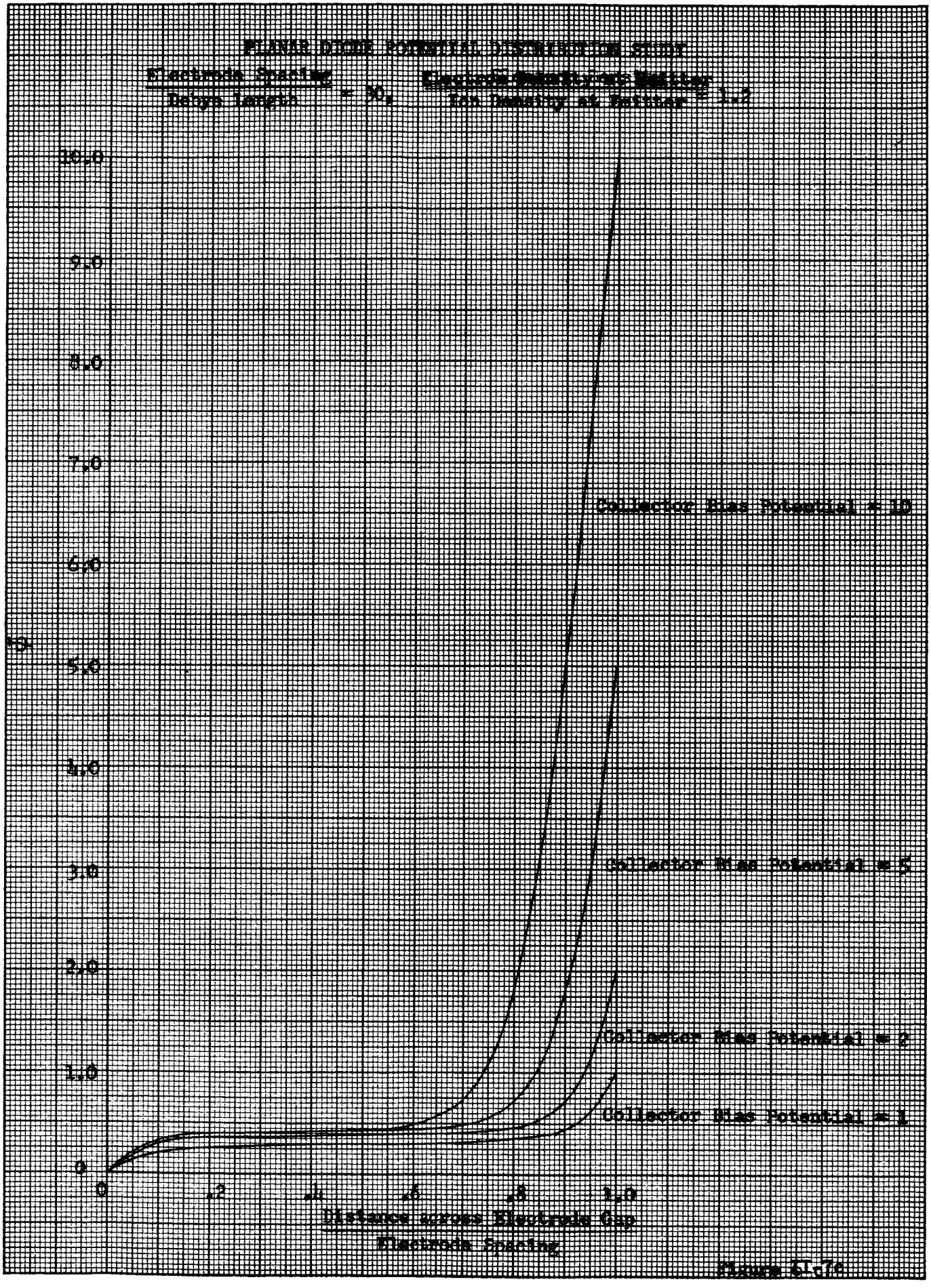

PLANAR DIODE POTENTIAL DISTRIBUTION STUDY
Electrode Spacing / Debye Length = 50
Electrode Potential / Ion Density at Emitter = 1.2
10.0
9.0
8.0
7.0
6.0
5.0
4.0
3.0
2.0
1.0
0
Collector Bias Potential = 10
Collector Bias Potential = 5
Collector Bias Potential = 2
Collector Bias Potential = 1
0
.2
.4
.6
.8
1.0
Distance across Electrode Gap / Electrode Spacing

Figure 5I.7e

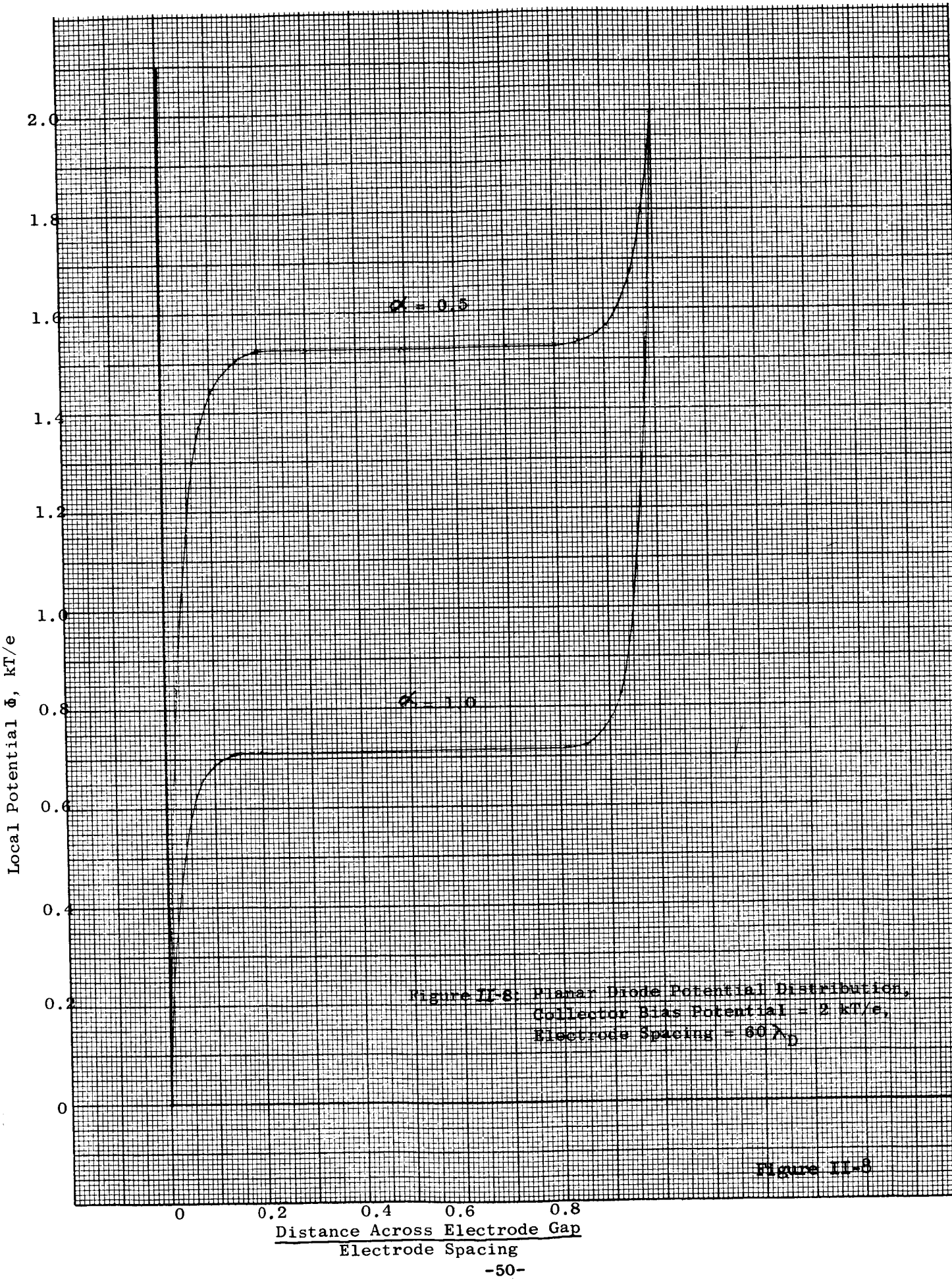


Figure II-8: Planar Diode Potential Distribution, Collector Bias Potential = 2 kT/e, Electrode Spacing = 60 λ_D

Figure II-8

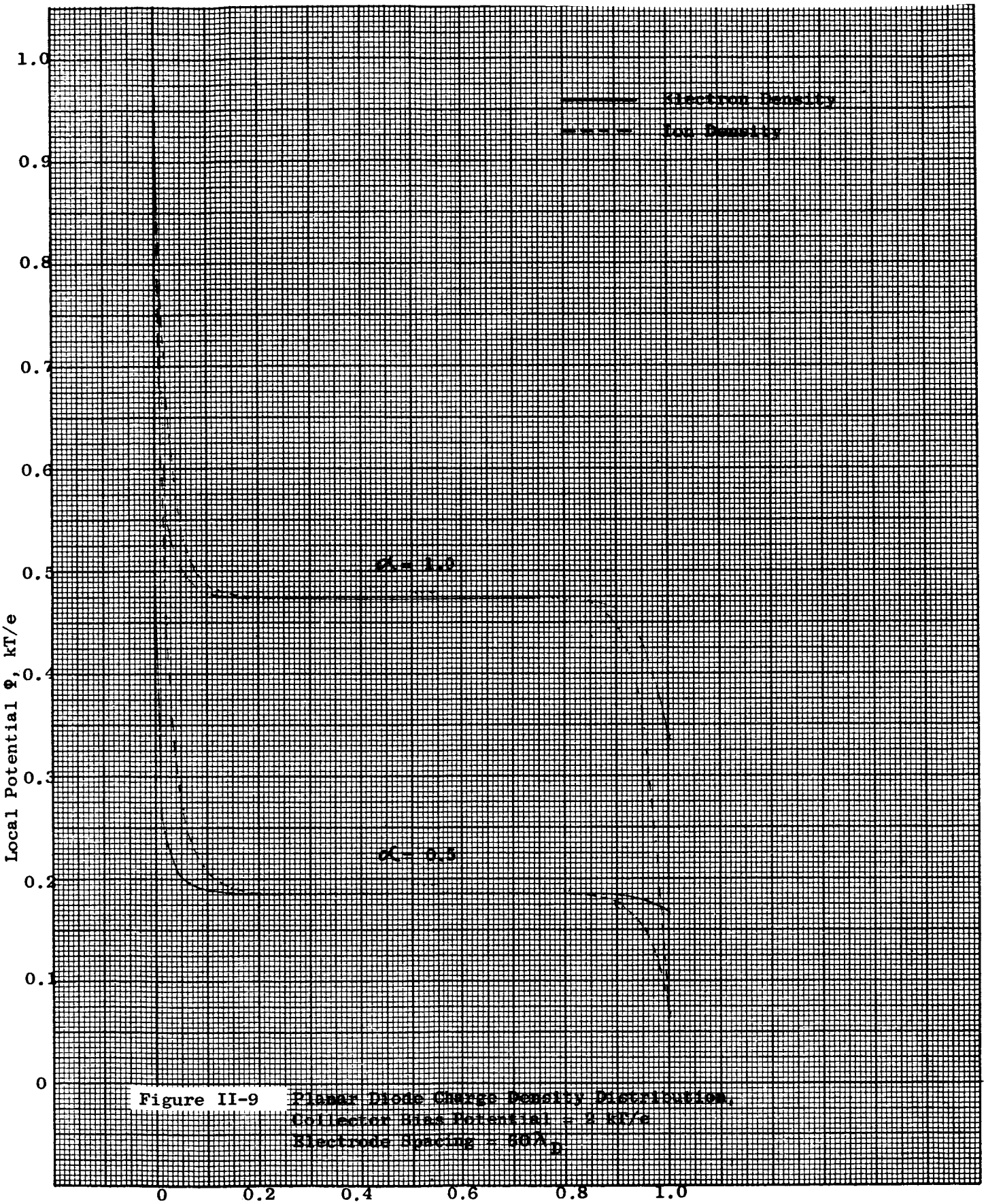


Figure II-9 Planar Diode Charge Density Distribution.
Collector Bias Potential = 2 kT/e
Electrode Spacing = $60\lambda_D$

2. Potential Minimum Solutions

For certain conditions in a planar plasma diode the potential distribution incurs a minimum which is below the potential at the emitter. This will occur when there is a large excess of electrons near the emitter and a negative potential is necessary to repel most of the excess. Therefore, such a condition may be obtained by increasing the value of α to somewhat greater than unity.

A good example of this type of phenonenon is shown in Figures II-10, 11, and 12, with the accompanying charge density distributions shown in Figures II-13, 14, and 15. As the value of α is increased from 0.4 to 1.2, the plateau potential is monotonically reduced. Then, in the result for $\alpha = 2.0$, a potential well is observed near the emitter. From the charge density curve for this case it can be seen that the electron density drops rapidly near the emitter due to deceleration. It continues to drop on the collector side of the minimum but this is now caused by acceleration of the electrons. A neutral plasma condition occurs but it is unstable and an ion rich region with rising potential then occurs. Eventually, a neutral plasma plateau level is reached when the ion density reduction from repulsion catches up to the electron density reduction from acceleration. From here on, the potential variation is similar to the monotonic potential distribution discussed previously.

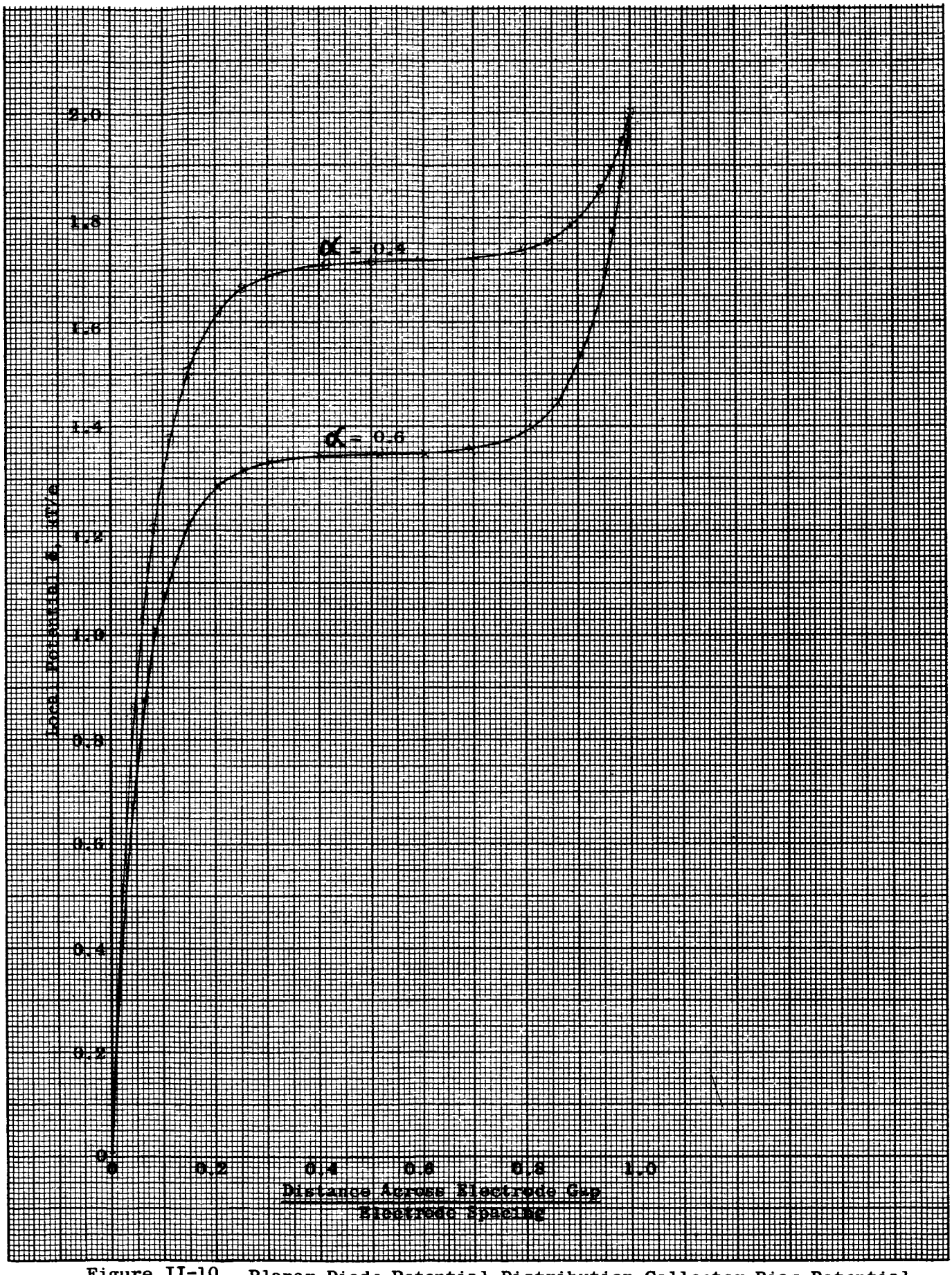


Figure II-10 Planar Diode Potential Distribution, Collector Bias Potential = 2 kT/e, Electrode Spacing = 30 λ_D

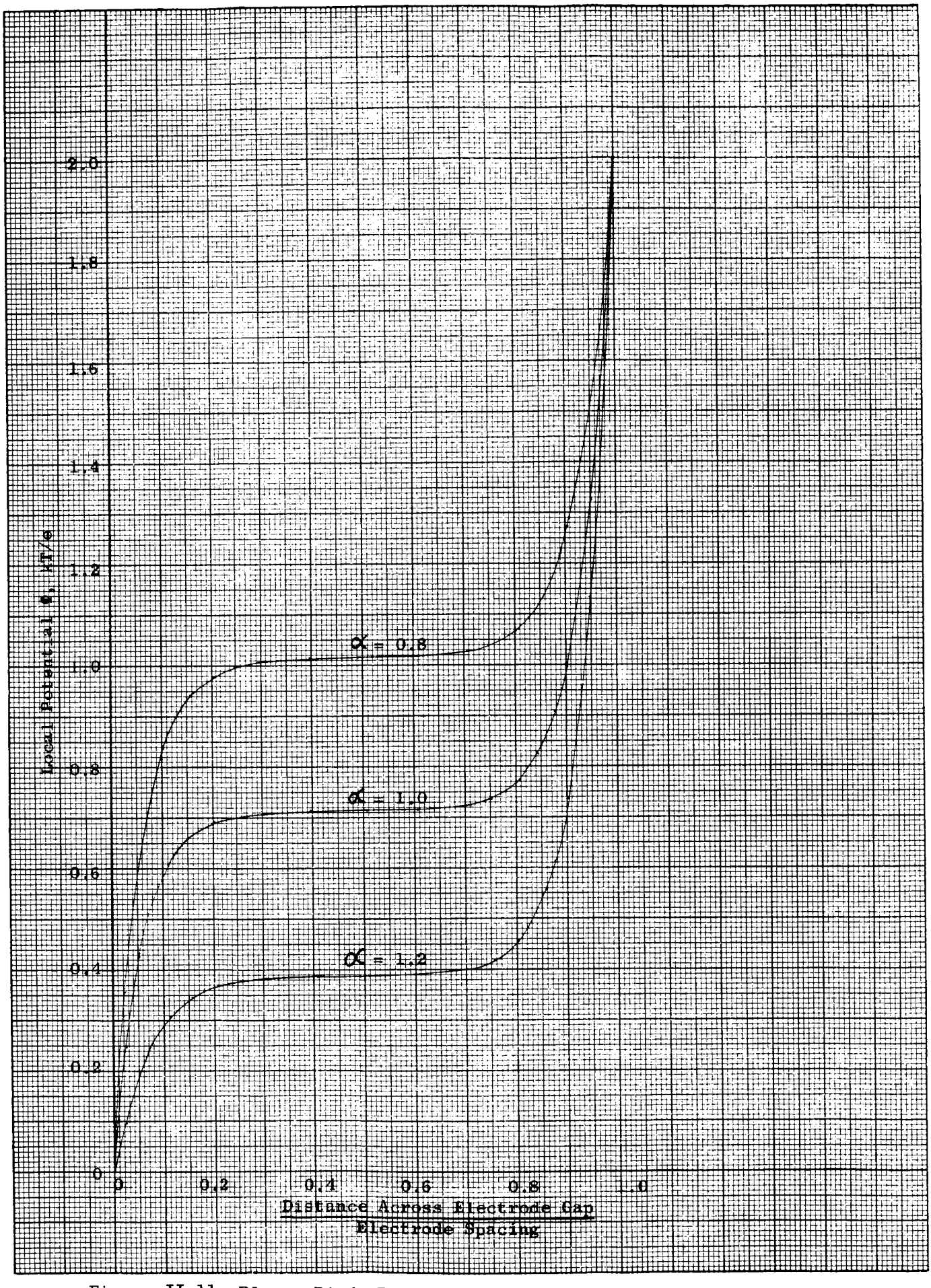


Figure II-11 Planar Diode Potential Distribution, Collector Bias Potential = 2 kT/e, Electrode Spacing = $30\lambda_D$

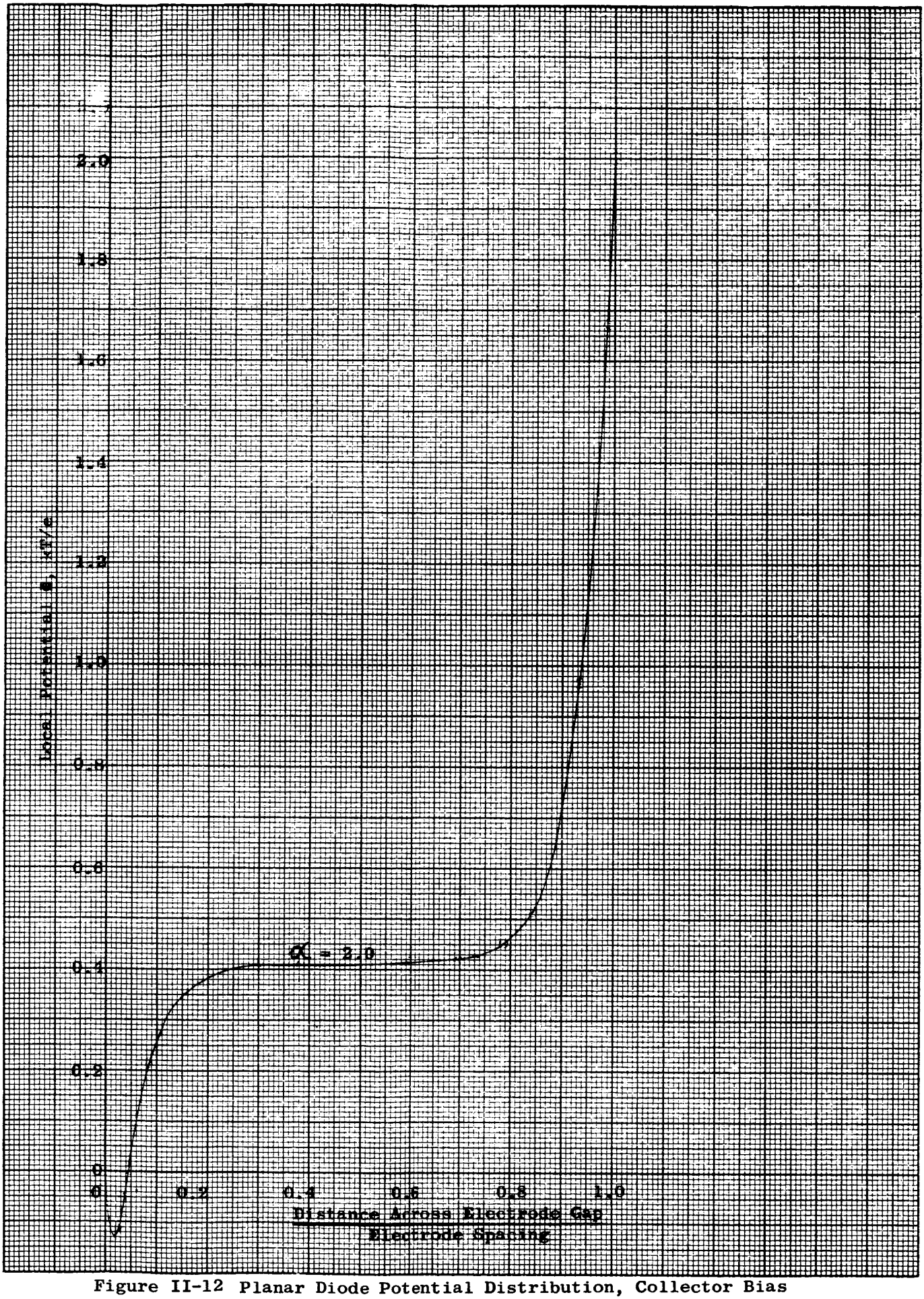


Figure II-12 Planar Diode Potential Distribution, Collector Bias Potential = 2kT/e, Electrode Spacing = $30\lambda_D$

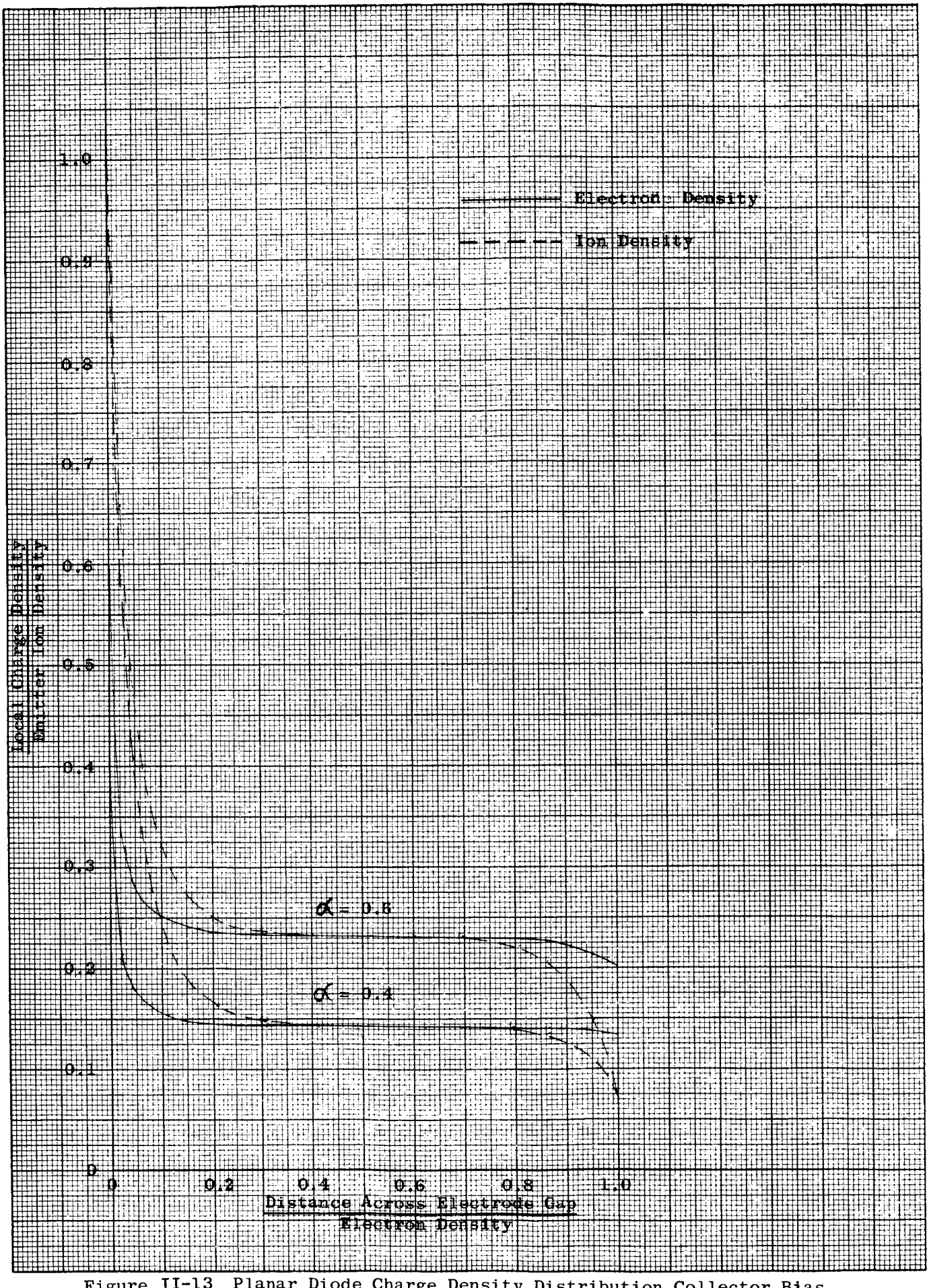


Figure II-13 Planar Diode Charge Density Distribution, Collector Bias Potential = 2 kT/e. Electrode Spacing = 30 λ_D

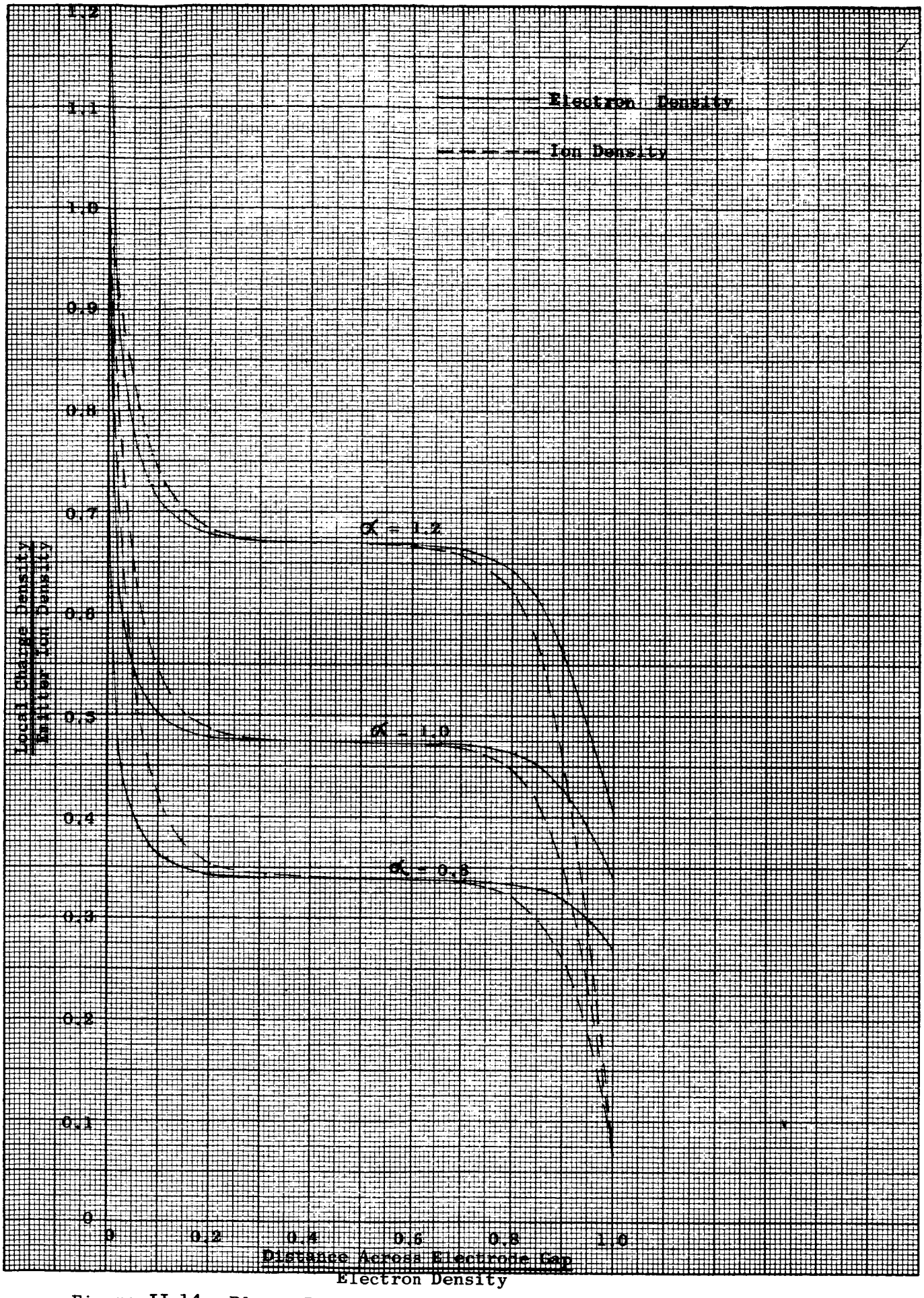


Figure II-14 Planar Diode Charge Density Distribution, Collector Bias Potential = 2 kT/e, Electrode Space = 30 λ_D

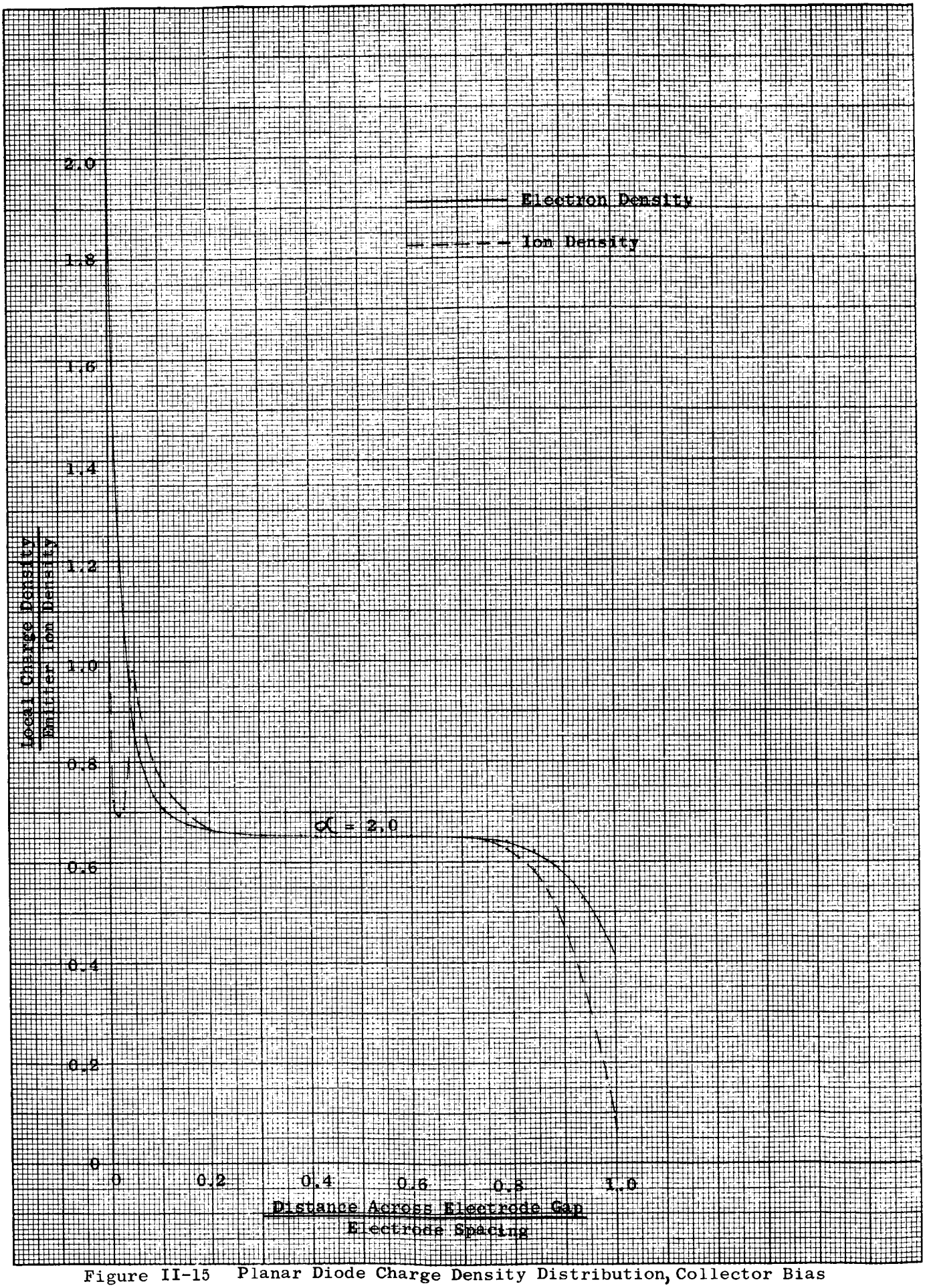


Figure II-15 Planar Diode Charge Density Distribution, Collector Bias Potential = 2 kT/e, Electrode Spacing = $30\lambda_D$

A. Equations for the Potential Minimum Solution

The development of the governing equations for a potential minimum condition is similar to the development for the monotonic potential distribution except for the limits on electron and ion densities. The revised expressions will be derived here and Poisson's equation will be integrated once.

In the case of a single potential minimum close to the emitter, some electrons fail to negotiate the retarding potential and return to the emitter. Returning electrons occupy the range from zero velocity to that velocity acquired by acceleration from the minimum potential back to the local potential. All velocities are possible for outward bound electrons.

$$\frac{n_e}{n_{eo}} = \frac{\int_{v_x=-\sqrt{\Phi-\Phi_M}}^{\infty} e^{\Phi - v_x^2}\, dv_x}{\int_{v_{ox}=-\sqrt{\Phi_M}}^{\infty} e^{-v_{xo}^2}\, dv_{xo}} \qquad \text{(II-26)}$$

$$= e^{\Phi}\, \frac{1 + \operatorname{erf}\sqrt{\Phi-\Phi_M}}{1 + \operatorname{erf}\sqrt{-\Phi_M}}$$

Between the potential minimum and the collector there are no returning electrons. The minimum velocity is that corresponding to acceleration away from the minimum. The electron density at the emitter was evaluated in connection with (II-26).

$$\frac{n_e}{n_{eo}} = \frac{\int_{v_x=\sqrt{\Phi-\Phi_m}}^{\infty} e^{\Phi - v_x^2}\, dv_x}{1 + \operatorname{erf}\sqrt{-\Phi_M}}$$

$$\frac{n_e}{n_{eo}} = e^{\Phi} \frac{1 - \operatorname{erf} \sqrt{\Phi - \Phi_M}}{1 + \operatorname{erf} \sqrt{-\Phi_M}} \qquad \text{(II-27)}$$

It may be observed that (II-26) and (II-27), taken with Φ_M approaching zero, are consistant with the expression for electron density in monotonic distributions (see equation II-13).

The ion density formulation must be altered also to account for the potential minimum. In the region of positive potential toward the collector, ions are present with all positive velocities. Admissible negative velocities run from zero to the level corresponding to acceleration from zero velocity at the collector.

$$\frac{n_i}{n_{i_o}} = \frac{\int_{v_x = -\sqrt{\Phi_c - \Phi}}^{\infty} e^{-\Phi - v_x^2} dv_x}{1 + \operatorname{erf} \sqrt{\Phi_c}}$$

$$= e^{-\Phi} \frac{1 + \operatorname{erf} \sqrt{\Phi_c - \Phi}}{1 + \operatorname{erf} \sqrt{\Phi_c}} \qquad \text{(II-28)}$$

Equations (II-26 to II-28) may be included in Poisson's equation so as to relate the local charge density to the potential behavior.

$$\Phi'' = \lambda^2 \left\{ \alpha e^{\Phi} \frac{1 \pm \operatorname{erf} \sqrt{\Phi - \Phi_M}}{1 + \operatorname{erf} \sqrt{-\Phi_M}} - e^{-\Phi} \frac{1 + \operatorname{erf} \sqrt{\Phi_c - \Phi} - 2\delta \operatorname{erf} \sqrt{-\Phi}}{1 + \operatorname{erf} \sqrt{\Phi_c}} \right\} \qquad \text{(II-29)}$$

where $\delta = 1$ for negative Φ, 0 for positive Φ, and the positive sign in the $\pm$ applies between the emitter and the potential minimum, if any.

Poisson's equation may be integrated once if both sides are multiplied by $\underline{\Phi}'$.

$$\int \underline{\Phi}'\underline{\Phi}'' = \tfrac{1}{2}(\underline{\Phi}')^2$$

$$\int e^{\underline{\Phi}} \frac{1 \pm \operatorname{erf}\sqrt{\underline{\Phi}-\underline{\Phi}_M}}{1+\operatorname{erf}\sqrt{\underline{\Phi}_M}}\,\underline{\Phi}' = e^{\underline{\Phi}} \frac{1 \pm \operatorname{erf}\sqrt{\underline{\Phi}-\underline{\Phi}_M}}{1+\operatorname{erf}\sqrt{-\underline{\Phi}_M}} - \int e^{\underline{\Phi}} \frac{\left(\pm\frac{2}{\sqrt{\pi}}\right) e^{\underline{\Phi}_M-\underline{\Phi}} \left(\frac{1}{2}\right)\left(\underline{\Phi}-\underline{\Phi}_M\right)^{-\frac{1}{2}} \underline{\Phi}'}{1+\operatorname{erf}\sqrt{-\underline{\Phi}_M}} \qquad \text{(II-30)}$$

$$= e^{\underline{\Phi}} \frac{\left(1 \pm \operatorname{erf}\sqrt{\underline{\Phi}-\underline{\Phi}_M} \mp \frac{2}{\sqrt{\pi}} e^{\underline{\Phi}_M}\sqrt{\underline{\Phi}-\underline{\Phi}_M}\right)}{1+\operatorname{erf}\sqrt{-\underline{\Phi}_M}} \qquad \text{(II-31)}$$

$$\int e^{-\underline{\Phi}} \frac{1+\operatorname{erf}\sqrt{\underline{\Phi}_c-\underline{\Phi}} - 2\delta\sqrt{\underline{\Phi}}}{1+\operatorname{erf}\sqrt{\underline{\Phi}_c}}\,\underline{\Phi}' = e^{-\underline{\Phi}} \frac{1+\operatorname{erf}\sqrt{\underline{\Phi}_c-\underline{\Phi}} - 2\delta \operatorname{erf}\sqrt{-\underline{\Phi}}}{}$$

$$+ \int \frac{e^{-\underline{\Phi}}\left(\frac{2}{\sqrt{\pi}} e^{\underline{\Phi}-\underline{\Phi}_c}\right)\frac{1}{2}\left(\underline{\Phi}_c-\underline{\Phi}\right)^{-\frac{1}{2}}\left(-\underline{\Phi}'\right)}{1+\operatorname{erf}\sqrt{\underline{\Phi}_c}} - 2\delta \int \frac{e^{-\underline{\Phi}}\left(\frac{2}{\sqrt{\pi}} e^{\underline{\Phi}}\right)\frac{1}{2}\left(-\underline{\Phi}\right)^{-\frac{1}{2}}\left(-\underline{\Phi}'\right)}{1+\operatorname{erf}\sqrt{\underline{\Phi}_c}} \qquad \text{(II-32)}$$

$$= \frac{-e^{-\underline{\Phi}}\left\{1+\operatorname{erf}\sqrt{\underline{\Phi}_c-\underline{\Phi}} - 2\delta \operatorname{erf}\sqrt{\underline{\Phi}}\right\} + \frac{2}{\sqrt{\pi}} e^{-\underline{\Phi}_c}\sqrt{\underline{\Phi}_c-\underline{\Phi}} - \frac{4\delta}{\sqrt{\pi}}\sqrt{-\underline{\Phi}}}{1+\operatorname{erf}\sqrt{\underline{\Phi}_c}}$$

Therefore

$$(\underline{\Phi}')^2 = 2\mathcal{L}^2 \left\{ \alpha \frac{e^{\underline{\Phi}}\left(1 \pm \operatorname{erf}\sqrt{\underline{\Phi}-\underline{\Phi}_M}\right) \mp \frac{2}{\sqrt{\pi}} e^{\underline{\Phi}_M}\sqrt{\underline{\Phi}-\underline{\Phi}_M}}{1+\operatorname{erf}\sqrt{-\underline{\Phi}_M}} \right.$$

$$\left. + \frac{e^{-\underline{\Phi}}\left[1+\operatorname{erf}\sqrt{\underline{\Phi}_c-\underline{\Phi}} - 2\delta \operatorname{erf}\sqrt{-\underline{\Phi}}\right] - \frac{2}{\sqrt{\pi}} e^{-\underline{\Phi}_c}\sqrt{\underline{\Phi}_c-\underline{\Phi}} + \frac{4\delta}{\sqrt{\pi}}\sqrt{-\underline{\Phi}}}{1+\operatorname{erf}\sqrt{\underline{\Phi}_c}} \right\} + C \qquad \text{(II-33)}$$

When (II-33) is used to obtain a monotonic solution, the procedure is to assume a value of $\underline{\Phi}'$ at a value of $\underline{\Phi}$ which causes $\underline{\Phi}''$ to vanish; iterating on this assumed $\underline{\Phi}'$ to make the second integration gives values of x covering the range 0 to 1 as $\underline{\Phi}$ ranges from 0 to $\underline{\Phi}_c$. The net charge density is always electron rich near the collector. If the charge density is ion rich near the emitter, there will be only one such value, $\underline{\Phi}_I$, which gives a zero value of $\underline{\Phi}''$. If, however, the region close to the emitter is electron rich, there may be two

such values. The desired $\underline{\phi}_I$ is the one closer to the collector potential. For this case

$$C = \left(\underline{\phi}_I'\right)^2 - 2\alpha^2 \left\{ \alpha \left[e^{\underline{\phi}_I} \left(1 - \operatorname{erf} \sqrt{\underline{\phi}_I}\right) + \frac{2}{\sqrt{\pi}} \sqrt{\underline{\phi}_I} \right] + \frac{e^{-\underline{\phi}_I} \left[1 + \operatorname{erf} \sqrt{\underline{\phi}_c - \underline{\phi}_I}\right] - \frac{2}{\sqrt{\pi}} e^{-\underline{\phi}_c} \sqrt{\underline{\phi}_c - \underline{\phi}_I}}{1 + \operatorname{erf} \sqrt{\underline{\phi}_c}} \right\} \quad \text{(II-34)}$$

Solutions with a potential minimum near the emitter are obtained by assuming a value for the minimum potential and iterating on this assumption to get the desired range of x. For this purpose $\underline{\phi}'$ vanishes at the minimum.

$$C = -2\alpha^2 \left\{ \alpha \frac{e^{\underline{\phi}_M}}{1 + \operatorname{erf} \sqrt{-\underline{\phi}_M}} + \frac{e^{-\underline{\phi}_M} \left[1 + \operatorname{erf} \sqrt{\underline{\phi}_c - \underline{\phi}_M} - 2 \operatorname{erf} \sqrt{-\underline{\phi}_M}\right] - \frac{2}{\sqrt{\pi}} e^{-\underline{\phi}_c} \sqrt{\underline{\phi}_c - \underline{\phi}_M} + \frac{4}{\sqrt{\pi}} \sqrt{-\underline{\phi}_M}}{1 + \operatorname{erf} \sqrt{\underline{\phi}_c}} \right\} \quad \text{(II-35)}$$

The process of obtaining a second integral of Poisson's equation (or an integral of II-33) usually must be carried through regions where $\underline{\phi}'$ is very small, so that a large variation of x results from a small change in $\underline{\phi}$. It is therefore necessary for the program to have the ability to describe the slope of the potential curve in the plateau region, particularly in the potential minimum case, with sufficient accuracy to arrive at a definitive solution. A means for doing this has been identified which should in principle produce the desired results without resorting to double-precision calculations.

The principle of this analysis depends on recognition of the fact that the constant C, defined by equation II-34 is not affected by the slope of the plateau at the neutral plasma inflection point of the potential curve, within the eight significant figure accuracy of single precision calculations, if that slope is less than 10^{-4} kT/e per Debye length. For practical purposes,

the ability to detect the influence of the inflection point slope on the rest of the potential distribution ends when the slope drops below 10^{-3} kT/e per Debye length. This result can be put to work.

We also observe that

$$\bar{\Phi}'' = \frac{d\bar{\Phi}'}{d\bar{\Phi}}\frac{d\bar{\Phi}}{dx} = \bar{\Phi}'\frac{d\bar{\Phi}'}{d\bar{\Phi}} = \frac{1}{2}\frac{d}{d\bar{\Phi}}\left(\bar{\Phi}'^2\right) \tag{II-36}$$

This, therefore, defines the first coefficient needed for a Taylor's series expansion for $(\bar{\Phi}')^2$ in terms of $\bar{\Phi}$. Subsequent coefficients are obtained by differentiating again:

$$\frac{d^2(\bar{\Phi}'^2)}{d\bar{\Phi}^2} = 2\lambda^2\left\{\alpha e^{\bar{\Phi}}\frac{1\pm \operatorname{erf}\sqrt{\bar{\Phi}-\bar{\Phi}_M}}{1+\operatorname{erf}\sqrt{-\bar{\Phi}_M}} \pm \alpha\frac{e^{\bar{\Phi}}\left(\frac{2}{\sqrt{\pi}}e^{\bar{\Phi}_M-\bar{\Phi}}\right)\frac{1}{2}\left(\bar{\Phi}-\bar{\Phi}_M\right)^{-\frac{1}{2}}}{1+\operatorname{erf}\sqrt{-\bar{\Phi}_M}}\right.$$

$$+e^{-\bar{\Phi}}\frac{1+\operatorname{erf}\sqrt{\bar{\Phi}_c-\bar{\Phi}}-2\delta\operatorname{erf}\sqrt{-\bar{\Phi}}}{1+\operatorname{erf}\sqrt{\bar{\Phi}_c}} - \frac{e^{-\bar{\Phi}}}{1+\operatorname{erf}\sqrt{\bar{\Phi}_c}}\frac{2}{\sqrt{\pi}}\left[\frac{-e^{\bar{\Phi}-\bar{\Phi}_c}(\bar{\Phi}_c-\bar{\Phi})^{-\frac{1}{2}}}{2}+\frac{2\delta e^{\bar{\Phi}}(-\bar{\Phi})^{-\frac{1}{2}}}{2}\right]\Bigg\}$$

$$= \frac{d(\bar{\Phi}'^2)}{d\bar{\Phi}} + 2\lambda^2\left\{2e^{-\bar{\Phi}}\frac{1+\operatorname{erf}\sqrt{\bar{\Phi}_c-\bar{\Phi}}-2\delta\operatorname{erf}\sqrt{-\bar{\Phi}}}{1+\operatorname{erf}\sqrt{\bar{\Phi}_c}}\right. \tag{II-37}$$

$$\left.\pm\frac{\alpha}{\sqrt{\pi}}\frac{e^{\bar{\Phi}_M}(\bar{\Phi}-\bar{\Phi}_M)^{-\frac{1}{2}}}{1+\operatorname{erf}\sqrt{-\bar{\Phi}_M}}+\frac{1}{\sqrt{\pi}}\frac{e^{-\bar{\Phi}_c}(\bar{\Phi}_c-\bar{\Phi})^{-\frac{1}{2}}-2\delta(-\bar{\Phi})^{-\frac{1}{2}}}{1+\operatorname{erf}\sqrt{\bar{\Phi}_c}}\right\}$$

$$\frac{d^3(\bar{\Phi}'^2)}{d\bar{\Phi}^3} = \frac{d^2(\bar{\Phi}'^2)}{d\bar{\Phi}^2} + 2\lambda^2\left\{-2e^{-\bar{\Phi}}\frac{1+\operatorname{erf}\sqrt{\bar{\Phi}_c-\bar{\Phi}}-2\delta\operatorname{erf}\sqrt{-\bar{\Phi}}}{1+\operatorname{erf}\sqrt{\bar{\Phi}_c}}\right.$$

$$+\frac{2e^{-\bar{\Phi}}}{1+\operatorname{erf}\sqrt{\bar{\Phi}_c}}\left[\frac{2}{\sqrt{\pi}}e^{\bar{\Phi}-\bar{\Phi}_c}\left(-\tfrac{1}{2}\right)\left(\bar{\Phi}_c-\bar{\Phi}\right)^{-\frac{1}{2}}+2\delta e^{\bar{\Phi}}\left(\tfrac{1}{2}\right)(-\bar{\Phi})^{-\frac{1}{2}}\right] \tag{II-38}$$

$$\pm\frac{\alpha}{\sqrt{\pi}}\left(-\tfrac{1}{2}\right)\frac{e^{\bar{\Phi}_M}(\bar{\Phi}-\bar{\Phi}_M)^{-\frac{3}{2}}}{1+\operatorname{erf}\sqrt{-\bar{\Phi}_M}}+\frac{1}{2}\frac{1}{\sqrt{\pi}}\frac{e^{-\bar{\Phi}_c}(\bar{\Phi}_c-\bar{\Phi})^{-\frac{3}{2}}-2\delta(-\bar{\Phi})^{-\frac{3}{2}}}{1+\operatorname{erf}\sqrt{\bar{\Phi}_c}}$$

$$= \frac{d(\bar{\Phi}'^2)}{d\bar{\Phi}} + 2\lambda^2\left\{\pm\frac{\alpha}{\sqrt{\pi}}\frac{e^{\bar{\Phi}_M}}{1+\operatorname{erf}\sqrt{\bar{\Phi}_M}}\left[(\bar{\Phi}-\bar{\Phi}_M)^{-\frac{1}{2}}-\frac{1}{2}(\bar{\Phi}-\bar{\Phi}_M)^{-\frac{3}{2}}\right]\right.$$

$$\left.-\frac{1}{\sqrt{\pi}}\frac{e^{-\bar{\Phi}_c}(\bar{\Phi}_c-\bar{\Phi})^{-\frac{1}{2}}-2\delta(-\bar{\Phi})^{-\frac{1}{2}}}{1+\operatorname{erf}\sqrt{\bar{\Phi}_c}}+\frac{1}{2}\frac{1}{\sqrt{\pi}}\frac{e^{-\bar{\Phi}_c}(\bar{\Phi}_c-\bar{\Phi})^{-\frac{3}{2}}-2\delta(-\bar{\Phi})^{-\frac{3}{2}}}{1+\operatorname{erf}\sqrt{\bar{\Phi}_c}}\right\}$$

$$\frac{d^4(\Phi'^2)}{d\Phi^4} = \frac{d^2(\Phi'^2)}{d\Phi^2} + 2\mathcal{L}^2\left\{\pm\frac{\alpha}{\sqrt{\pi}}\ \frac{e^{\Phi_M}}{1+\operatorname{erf}\sqrt{-\Phi_M}}\left[-\frac{1}{2}\left(\Phi-\Phi_M\right)^{-\frac{3}{2}}+\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(\Phi-\Phi_M\right)^{-\frac{5}{2}}\right]\right.$$

$$-\frac{1}{\sqrt{\pi}}\ \frac{e^{-\Phi_c}}{1+\operatorname{erf}\sqrt{\Phi_c}}\left[\frac{1}{2}\left(\Phi_c-\Phi\right)^{-\frac{3}{2}}-\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(\Phi_c-\Phi\right)^{-\frac{5}{2}}\right]$$

$$\left.+\frac{2\delta}{\sqrt{\pi}}\ \frac{\left[\frac{1}{2}\left(-\Phi\right)^{-\frac{3}{2}}-\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(-\Phi\right)^{-\frac{5}{2}}\right]}{1+\operatorname{erf}\sqrt{\Phi_c}}\right\} \qquad \text{(II-39)}$$

$$\frac{d^5(\Phi'^2)}{d\Phi^5} = \frac{d^3(\Phi'^2)}{d\Phi^3} + 2\mathcal{L}^2\left\{\pm\frac{\alpha}{\sqrt{\pi}}\ \frac{e^{\Phi_M}}{1+\operatorname{erf}\sqrt{-\Phi_M}}\left[\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(\Phi-\Phi_M\right)^{-\frac{5}{2}}-\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)\left(\Phi-\Phi_M\right)^{-\frac{7}{2}}\right]\right.$$

$$-\frac{1}{\sqrt{\pi}}\ \frac{e^{-\Phi_c}}{1+\operatorname{erf}\sqrt{\Phi_c}}\left[\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(\Phi_c-\Phi\right)^{-\frac{5}{2}}-\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)\left(\Phi_c-\Phi\right)^{-\frac{7}{2}}\right.$$

$$\left.\left.+\frac{2\delta}{\sqrt{\pi}}\ \frac{1}{1+\operatorname{erf}\sqrt{\Phi_c}}\left[\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(-\Phi\right)^{-\frac{5}{2}}-\left(\frac{1}{2}\right)\left(\frac{3}{2}\right)\left(\frac{5}{2}\right)\left(-\Phi\right)^{-\frac{7}{2}}\right]\right\} \qquad \text{(II-40)}$$

The Taylor's Series expansion for $\bar{\phi}'^2$, making use of the above coefficients, is valid in the neighborhood of the inflection point (but not in the neighborhood of a potential minimum or maximum, where the charge density distribution has singular derivatives). Sufficiently close to the inflection point the quadratic term of the Taylor's Series is sufficiently precise to define the slope.

Then:

$$\bar{\Phi}'^2 = \bar{\Phi}_I'^2 + \left[\frac{d^2}{d\bar{\Phi}^2}\left(\bar{\Phi}'^2\right)\right]_I \left(\bar{\Phi}-\bar{\Phi}_I\right)^2 \qquad \text{(II-41)}$$

Separating variables:

$$dx = \frac{d\bar{\Phi}}{\sqrt{\bar{\Phi}_I'^2 + \left[\frac{d^2}{d\bar{\Phi}^2}\left(\bar{\Phi}'^2\right)\right]_I \left(\bar{\Phi}-\bar{\Phi}_I\right)^2}} \qquad \text{(II-42)}$$

$$= \frac{d\bar{\Phi}}{\sqrt{\left[\frac{d^2}{d\bar{\Phi}^2}\left(\bar{\Phi}'^2\right)\right]_I}\sqrt{\frac{\bar{\Phi}_I'^2}{\left[\frac{d^2}{d\bar{\Phi}^2}\left(\bar{\Phi}'^2\right)\right]_I} + \left(\bar{\Phi}-\bar{\Phi}_I\right)^2}}$$

$$x - x_I = \left[\frac{d^2}{d\bar{\Phi}^2}\left(\bar{\Phi}'^2\right)\right]_I^{-\frac{1}{2}} \sinh^{-1}\left\{\frac{\sqrt{\left[\frac{d^2}{d\bar{\Phi}^2}\left(\bar{\Phi}'^2\right)\right]_I}}{\bar{\Phi}_I}\left(\bar{\Phi}-\bar{\Phi}_I\right)\right\} \qquad \text{(II-43)}$$

$$\bar{\Phi} = \bar{\Phi}_I + \frac{\bar{\Phi}_I}{\sqrt{\left[\frac{d^2}{d\bar{\Phi}^2}\left(\bar{\Phi}'^2\right)\right]_I}} \sinh\left\{\sqrt{\left[\frac{d^2}{d\bar{\Phi}^2}\left(\bar{\Phi}'^2\right)\right]_I}\left(x-x_I\right)\right\} \qquad \text{(II-44)}$$

If now a situation is identified in which the assumed slope at the inflection point is too small to affect the constant of the monotonic solution, or in which the difference between two large quantities (based on the difference in potential between this inflection point and the minimum) from which the inflection point slope is to be calculated disappears by comparison with those quantities, the formulation in (II-44) may be used to define the slope of the potential distribution in the plateau region. Integration is performed from the emitter (in the monotonic case) or the potential minimum (if applicable) towards the inflection point, and then backwards from the collector towards the inflection point: in these regions the subsequent determination of the plateau slope will not alter the potential distribution. At two potentials differing from the inflection point potential by the same amount (chosen arbitrarily at about 10^{-6}) a gap in the coordinates appears. This gap, together with the potential difference, is inserted in (II-44) to determine $\bar{\phi}_I'$. Some adjustment in some values of $\bar{\phi}'$ for potentials close to the inflection point potential may be required, which will in turn affect the gap width to be bridged, but these adjustments should be small.

B. Graphical Results for Potential Minimum Solutions

In Figure II-16 is shown a family of curves representing potential distributions for a planar diode with electrode spacing equal to nine Debye lengths. It is seen that no plateau region is present which is to be expected for electrode spacings less than 20 Debye lengths. The emitter charge density ratio has been raised in successive steps from 0.4 to 1.6. Between 1.2 and 1.4, the potential has become negative near the emitter and a potential minimum has formed. The potential at the inflection point corresponds to the plateau potential for larger ratios of electrode spacing to Debye length. This potential has decreased monotonically down to the case for $\alpha = 1.4$; then the curve for $\alpha = 1.6$ has a higher value of inflection point potential. It will be shown later that the minimum inflection point potential is found for an emitter charge density ratio of about 1.33.

It should be mentioned that the potential curves in Figure II-16 and the corresponding electron and ion densities given in Figures II-17, and II-18, have been computed for a bias potential of 2 kT/e. This value of the bias potential has been used for most of the planar diode computations because it is the lowest bias potential which does not affect the value of the inflection point potential.

Results for $(L/\lambda_D) = 200$

It was of interest in this study to obtain results for the case of an electrode spacing of 200 Debye lengths. These are shown in Figures II-19 and II-20. For $\alpha = 1.5$, a very steep potential minimum condition is observed

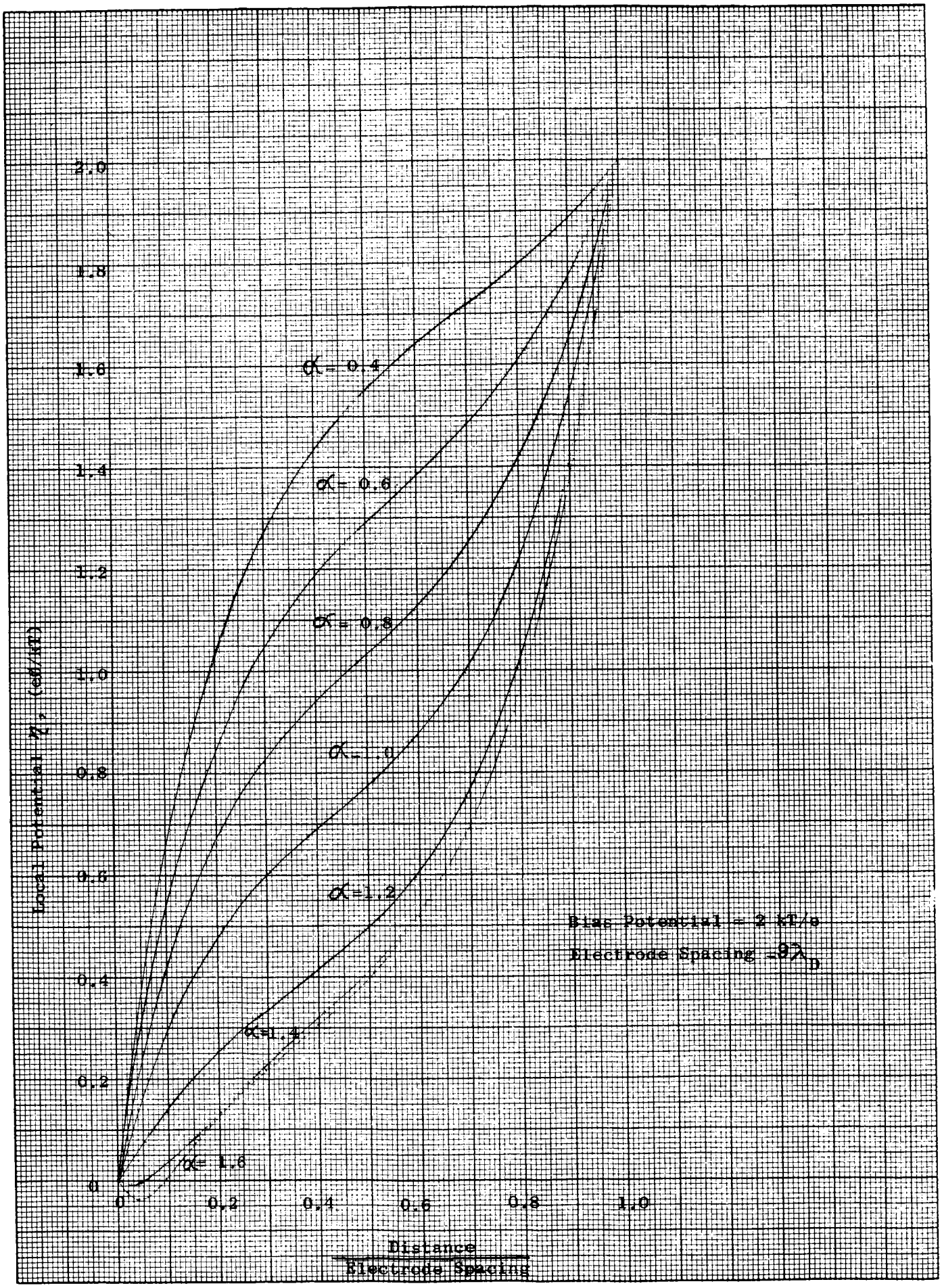


Figure II-16: Potential Variation Across a Planar Plasma Diode

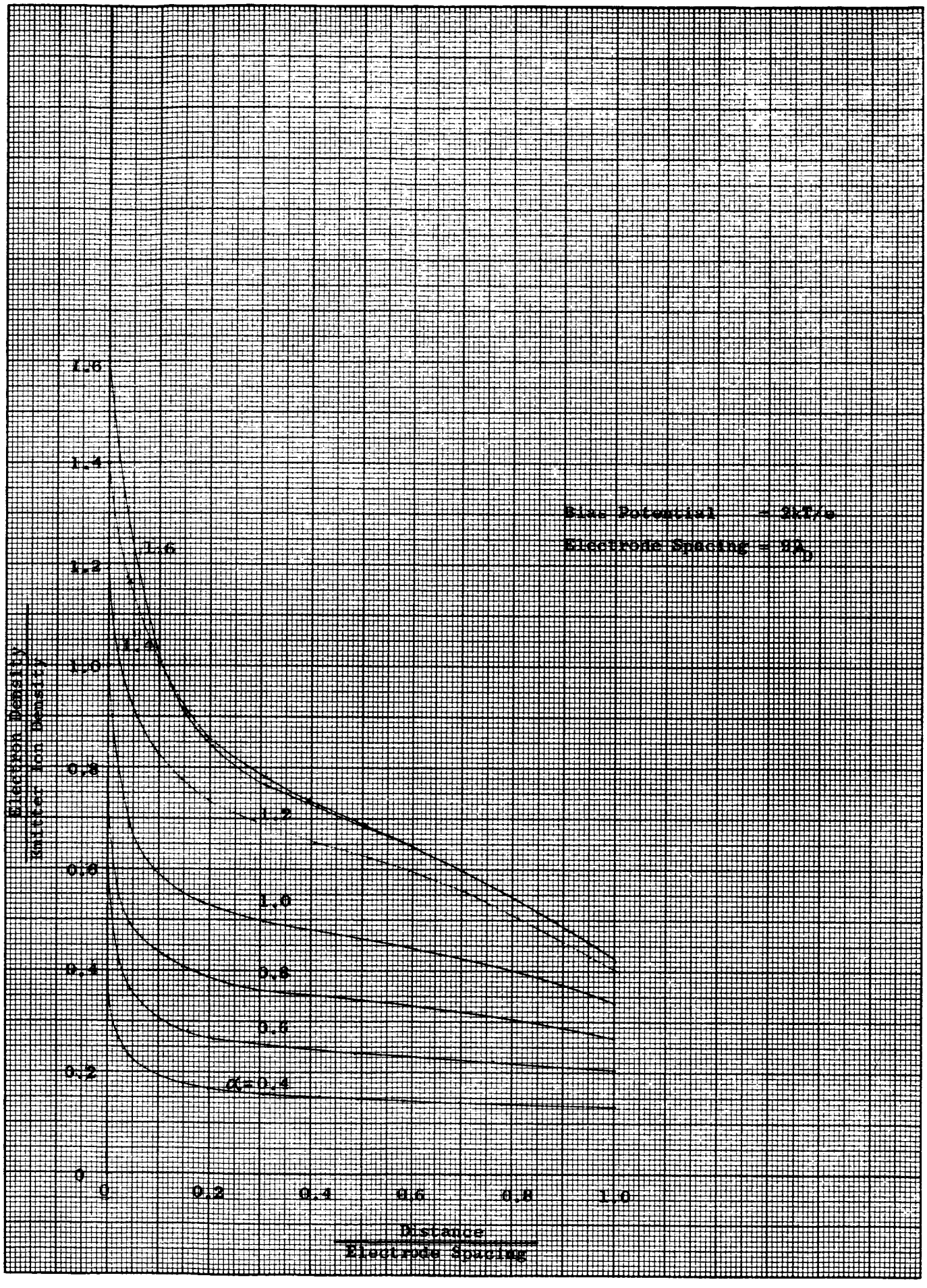


Figure II-17: Electron Density Variation in a Planar Diode

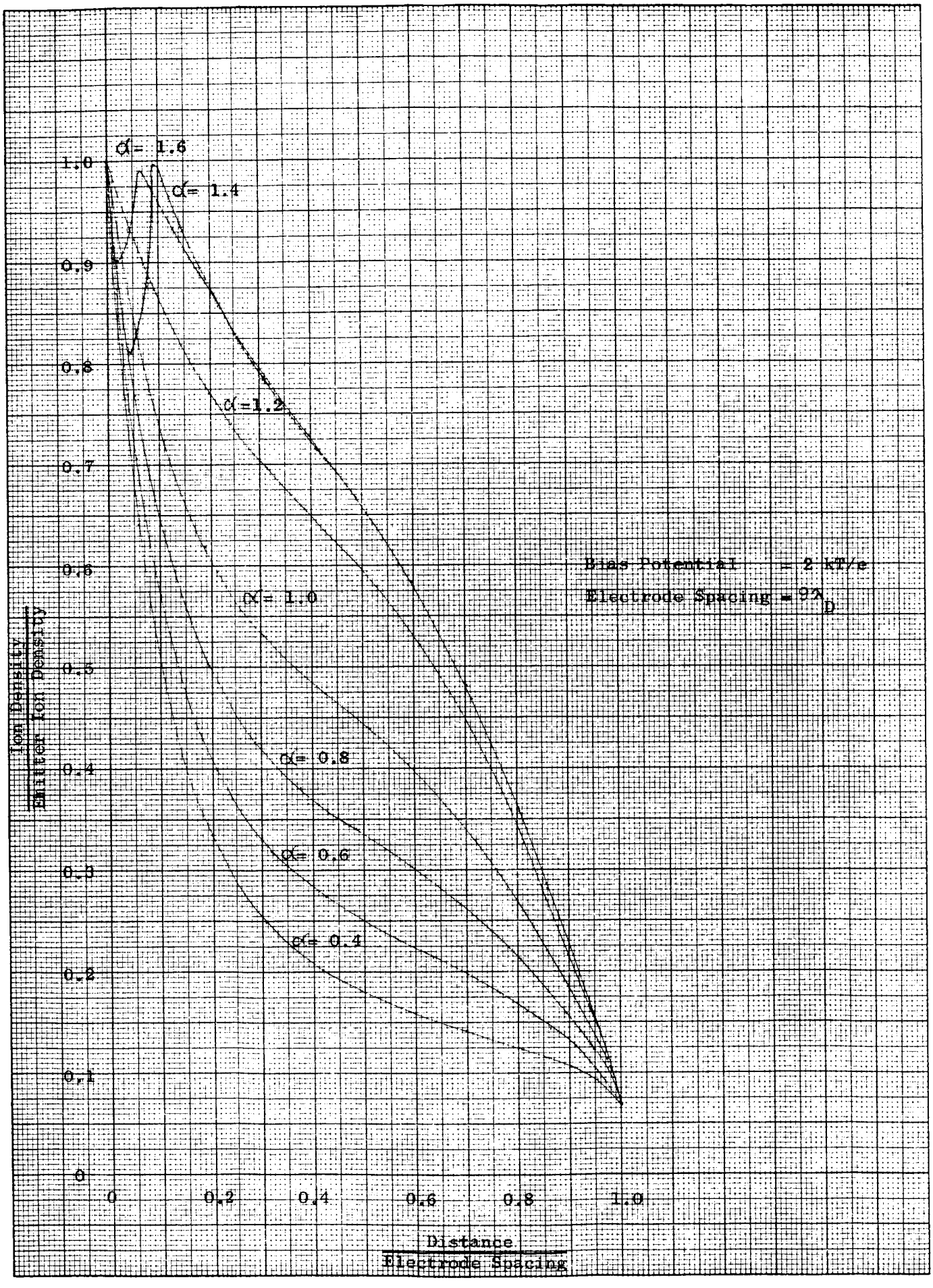


Figure II-18: Ion Density Variation in a Planar Diode

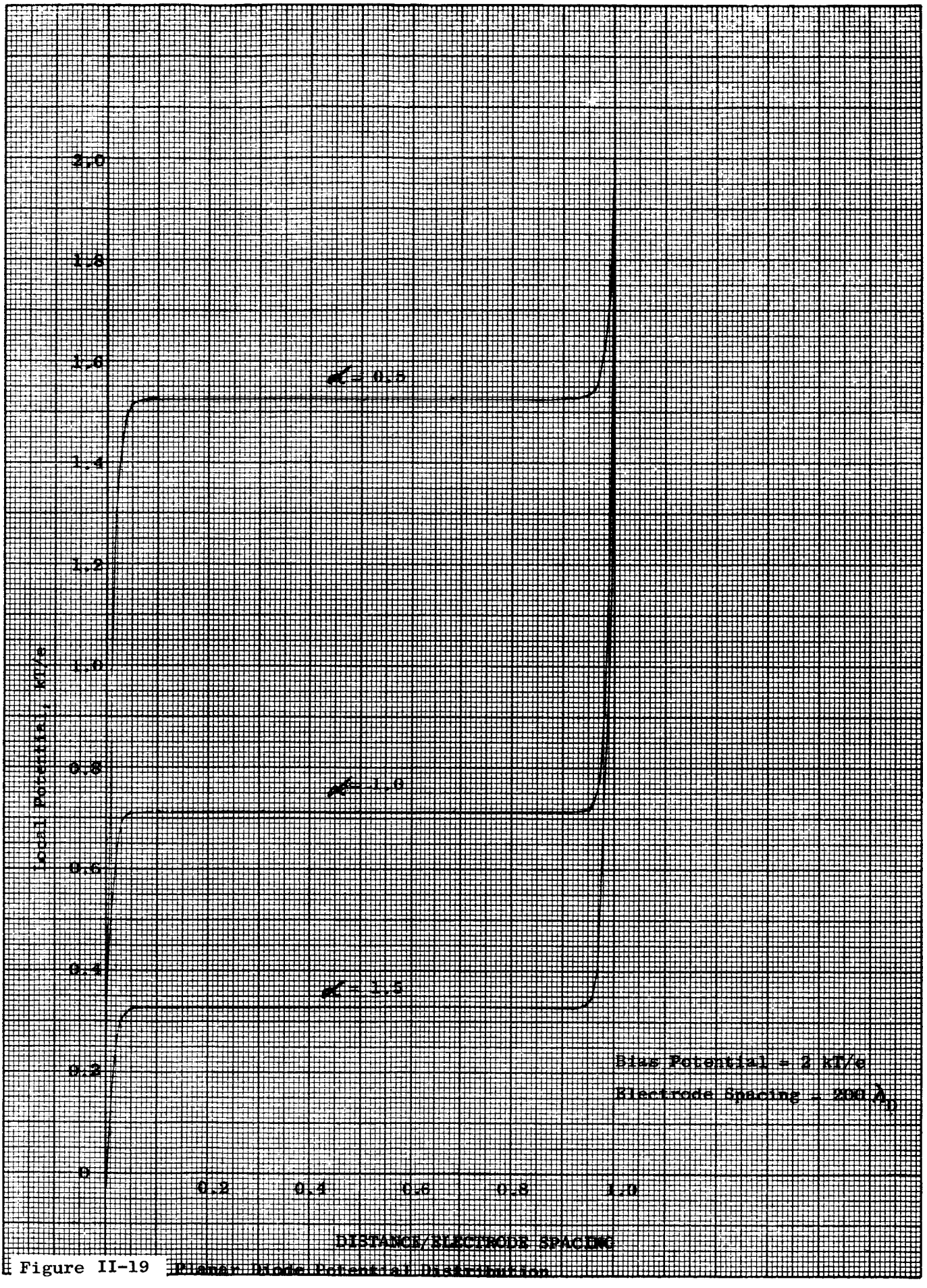


Figure II-19 Planar Diode Potential Distribution

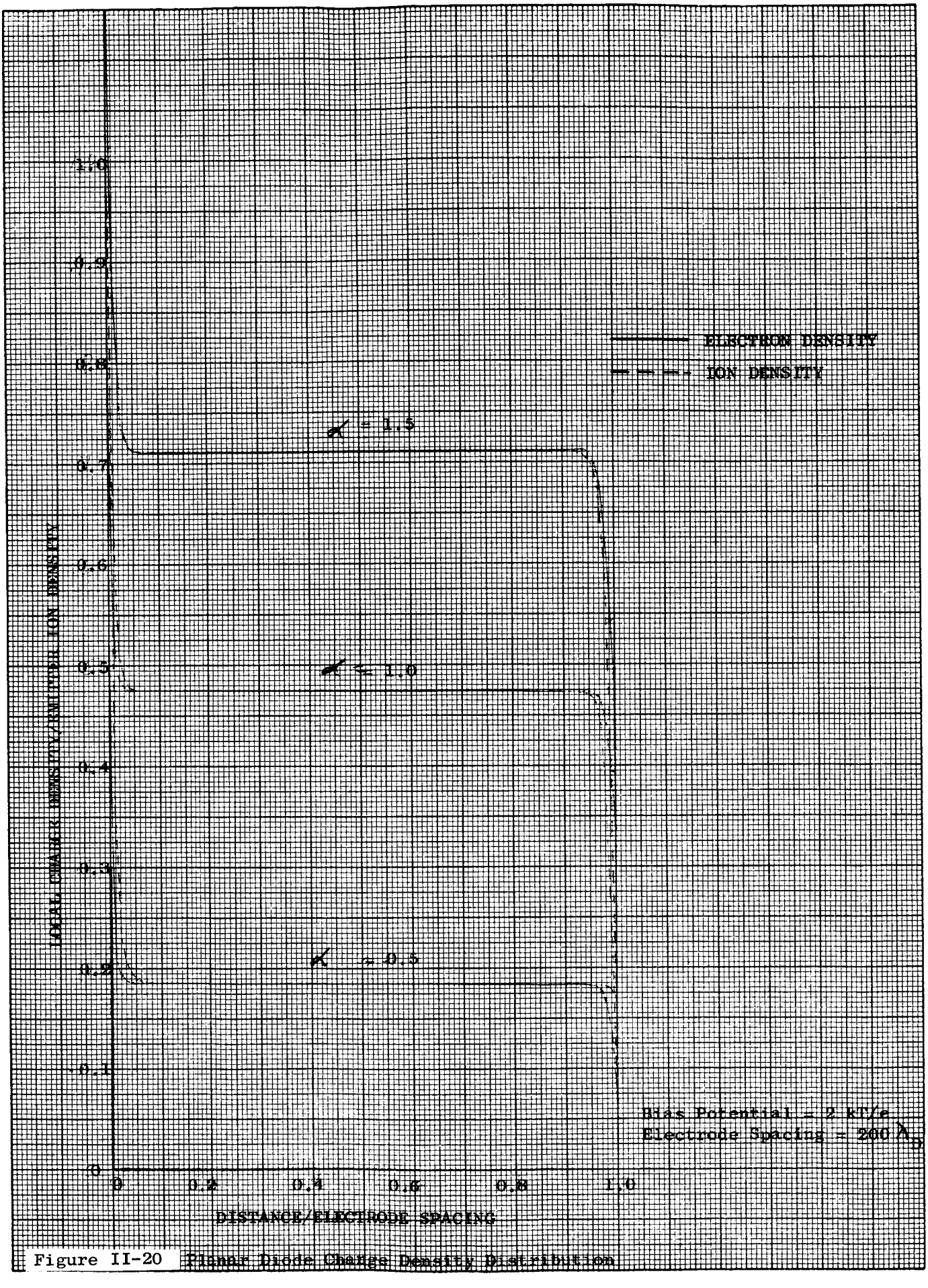


Figure II-20 Planar Diode Charge Density Distribution

because of the large value of (L/λ_D). Similarly the plateau region has increased its relative extent.

C. Investigations of the Emitter Potential Slope

The slope of the potential at the emitter of a planar plasma diode exhibits an unusual property near the point of zero slope. This problem was investigated because of difficulties that arose when trying to find solutions that had a very small slope of potential at the emitter. This problem was first encountered in the cylindrical diode investigations and it was then also found to occur in the planar diode calculations.

There are actually two difficulties that arise in this problem. One is the fact that the potential slope at the emitter varies continuously but not smoothly with the emitter charge density ratio. This may be seen from Figure II-21, and also from Figure II-22 which is an enlarged plot of some of the data on the previous graph. It can be observed that the potential slope is everywhere continuous but there is a kink at the point of zero slope. This behavior is actually not so unusual when it is understood that the expression used to calculate the emitter potential slope for potential minimum cases is different from the expression for monotonic cases. For a monotonic distribution the emitter potential slope is given by

$$\Phi_o' = \sqrt{2\mathcal{L}^2\left\{\alpha + \frac{1+\operatorname{erf}\sqrt{\Phi_c} - \frac{2}{\sqrt{\pi}}e^{-\Phi_c}\sqrt{\Phi_c}}{1+\operatorname{erf}\sqrt{\Phi_c}}\right\} + C} \qquad \text{(II-45)}$$

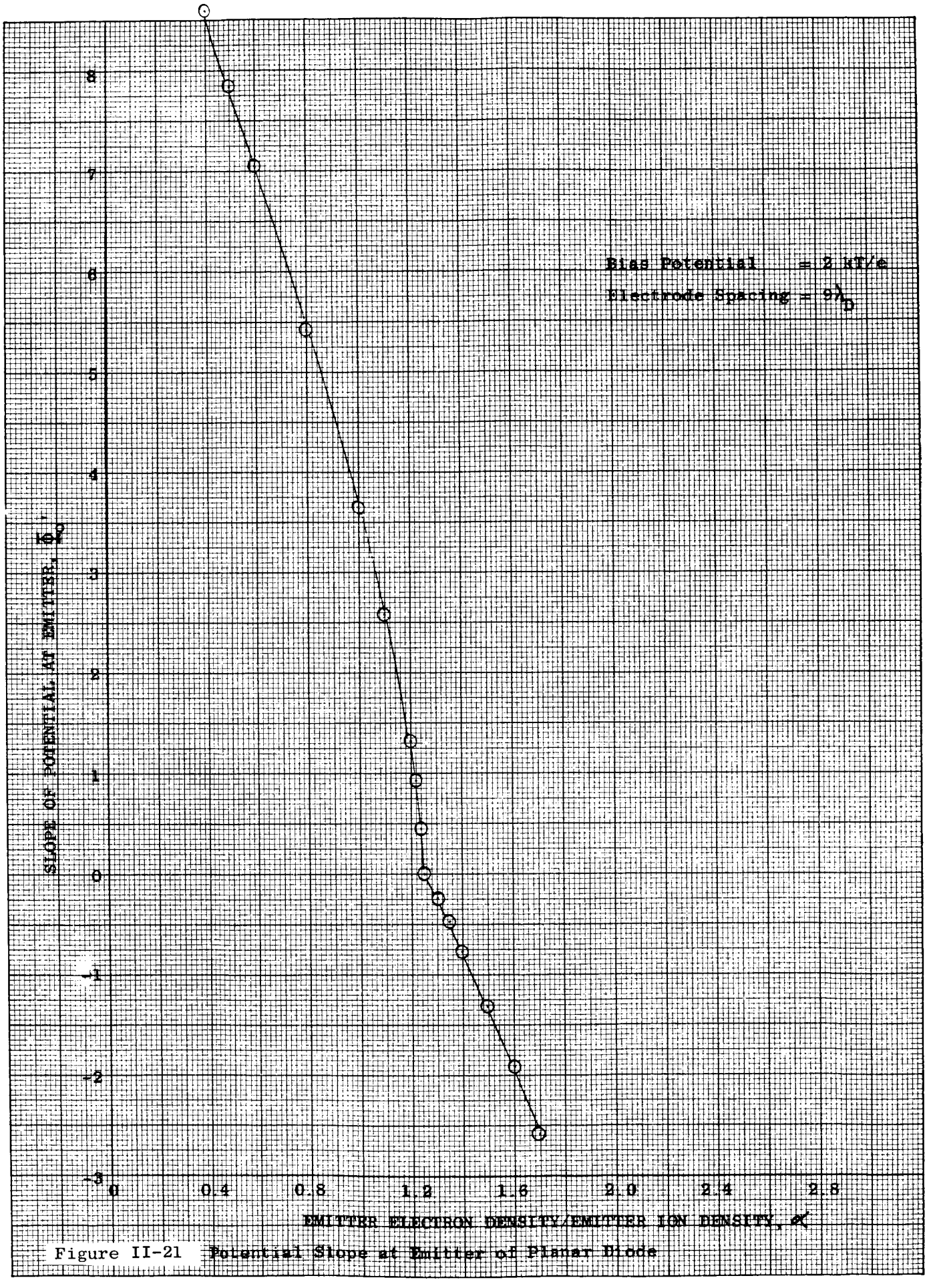


Figure II-21 Potential Slope at Emitter of Planar Diode

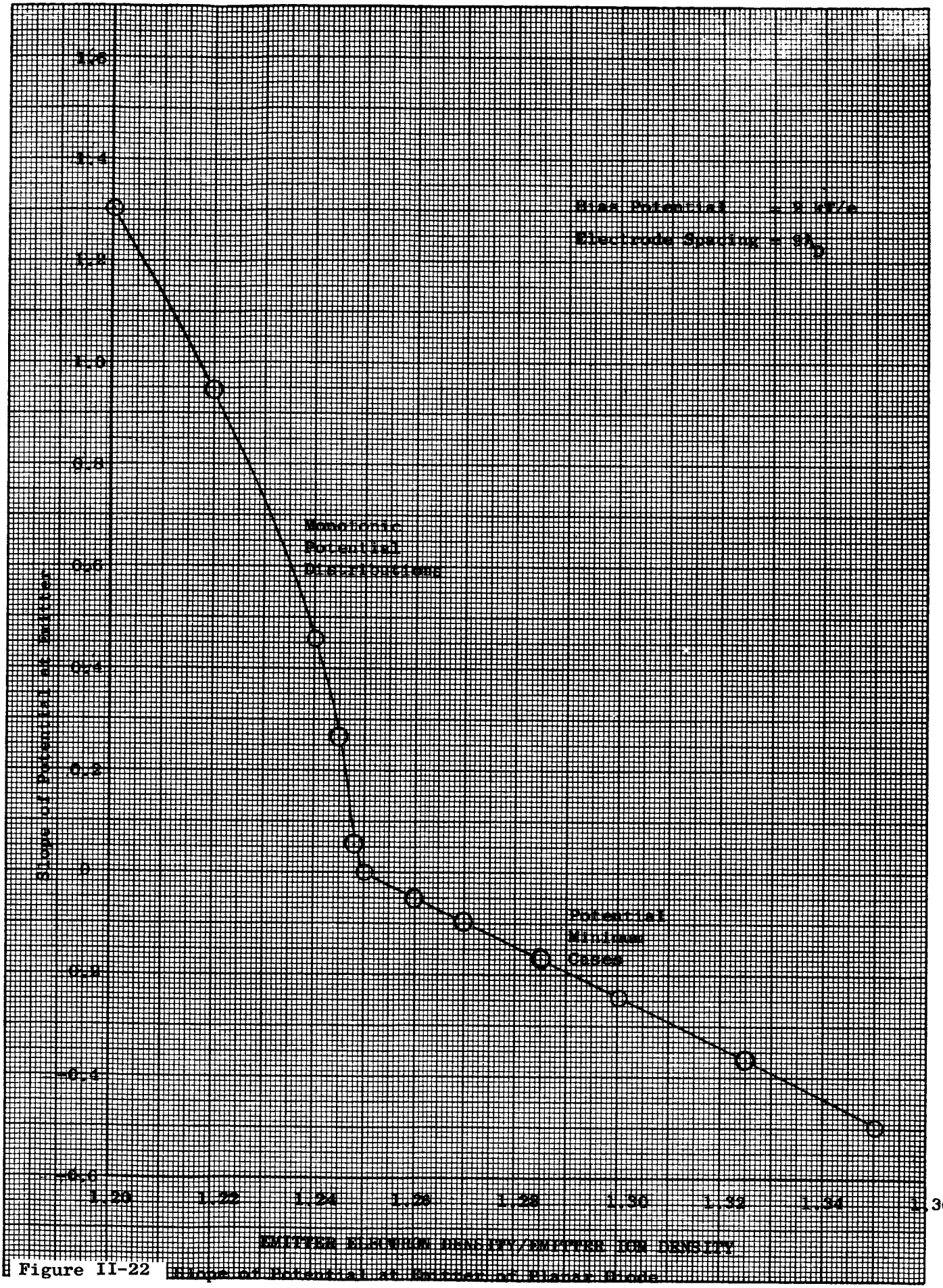


Figure II-22 Shape of Potential at Surface of Plasma Sheath

where

$$C = \Phi_I'^2 - 2\mathcal{L}^2\left\{\alpha\left[e^{\Phi_I}\left(1-\operatorname{erf}\sqrt{\Phi_I}\right)+\frac{2}{\sqrt{\pi}}\sqrt{\Phi_I}\right]\right.$$

$$\left. + \frac{e^{-\Phi_I}\left[1+\operatorname{erf}\sqrt{\Phi_c-\Phi_I}\right]-\frac{2}{\sqrt{\pi}}e^{-\Phi_c}\sqrt{\Phi_c-\Phi_I}}{1+\operatorname{erf}\sqrt{\Phi_c}}\right\}$$

from Equations II-33 and II-34. On the other hand, for potential minimum solutions the emitter potential slope is found from

$$\Phi_o' = \sqrt{2\mathcal{L}^2\left\{\alpha\frac{1+\operatorname{erf}\sqrt{-\Phi_M}-\frac{2}{\sqrt{\pi}}e^{\Phi_M}\sqrt{-\Phi_M}}{1+\operatorname{erf}\sqrt{-\Phi_M}}+\frac{1+\operatorname{erf}\sqrt{\Phi_c}-\frac{2}{\sqrt{\pi}}e^{-\Phi_c}\sqrt{\Phi_c}}{1+\operatorname{erf}\sqrt{\Phi_c}}\right\}+C} \qquad \text{(II-46)}$$

where

$$C = -2\mathcal{L}^2\left\{\alpha\frac{e^{\Phi_M}}{1+\operatorname{erf}\sqrt{-\Phi_M}}+\frac{e^{-\Phi_M}\left[1+\operatorname{erf}\sqrt{\Phi_c-\Phi_M}-2\operatorname{erf}\sqrt{-\Phi_M}\right]-\frac{2}{\sqrt{\pi}}e^{-\Phi_c}\sqrt{\Phi_c-\Phi_M}+\frac{4}{\sqrt{\pi}}\sqrt{-\Phi_M}}{1+\operatorname{erf}\sqrt{\Phi_c}}\right\}$$

Therefore, the variation of Φ' with α should not be expected to be the same for both types of distributions since different expressions are used to calculate the emitter potential slope.

As a check on the continuity of the slope, the two expressions for the slope should give the same value at the point of zero slope. This means that equations (II-45) and (II-46) should both indicate a zero slope for $\alpha = 1.25$. Therefore,

$$\alpha + \frac{1+\operatorname{erf}\sqrt{\Phi_c}-\frac{2}{\sqrt{\pi}}e^{-\Phi_c}\sqrt{\Phi_c}}{1+\operatorname{erf}\sqrt{\Phi_c}} + \frac{\Phi_I'^2}{2\mathcal{L}^2} - \alpha\left[e^{\Phi_I}\left(1-\operatorname{erf}\sqrt{\Phi_I}\right)+\frac{2}{\sqrt{\pi}}\sqrt{\Phi_I}\right]$$

$$+\frac{-e^{-\Phi_I}\left[1+\operatorname{erf}\sqrt{\Phi_c-\Phi_I}\right]-\frac{2}{\sqrt{\pi}}e^{-\Phi_c}\sqrt{\Phi_c-\Phi_I}}{1+\operatorname{erf}\sqrt{\Phi_c}}=\frac{\alpha\left[1+\operatorname{erf}\sqrt{-\Phi_M}-\frac{2}{\sqrt{\pi}}e^{\Phi_M}\sqrt{-\Phi_M}\right.}{1+\operatorname{erf}\sqrt{-\Phi_M}}$$

$$+\frac{1+\operatorname{erf}\sqrt{\Phi_c}-\frac{2}{\sqrt{\pi}}e^{-\Phi_c}\sqrt{\Phi_c}}{1+\operatorname{erf}\sqrt{\Phi_c}}-\frac{\alpha e^{\Phi_M}}{1+\operatorname{erf}\sqrt{-\Phi_M}}-\frac{e^{-\Phi_M}\left[1+\operatorname{erf}\sqrt{\Phi_c-\Phi_M}-2\operatorname{erf}\sqrt{-\Phi_M}\right]}{1+\operatorname{erf}\sqrt{\Phi_c}}$$

$$+\frac{-\frac{2}{\sqrt{\pi}}e^{-\Phi_c}\sqrt{\Phi_c-\Phi_M}+\frac{4}{\sqrt{\pi}}\sqrt{-\Phi_M}}{1+\operatorname{erf}\sqrt{\Phi_c}}=0$$

Substitution of appropriate values, $\Phi_C = 2$, $\Phi_M = 0$, $\Phi_I = .2719$, $\Phi_I' = .818$ and $\mathcal{L} = 9$, gives the result that both slopes are indeed calculated to be zero for $\alpha = 1.25$. Therefore, the emitter slope is proven to be everywhere continuous, but to have a different variation for potential minimum solutions than for monotonic solutions.

Another problem was encounted during the investigation of the emitter potential slope. At very small values of the emitter potential slope, the computer program fails to function properly. The reason is that it expects the slope at the emitter to be greater than the slope at the (second) inflection point. For sufficiently large values of (L/λ_D) the latter value is the plateau potential slope and this problem is not encountered. However, for the case studied herein, the electrode spacing was only nine Debye lengths and the inflection point slope was larger than the emitter slope for charge density ratios between 1.22 and 1.25. In this case, the procedure found effective was to determine the difference between the squares of the slopes at the inflection point and at the emitter, (for which the computer run will

end with an error in square root). Then we re-input a value of PHIPRM which is slightly greater than this value. Using this technique for several successive computer runs will eventually provide a correct solution.

D. Other Effects

During the investigation of the emitter potential slope problem, some other interesting effects were noticed. In Figure II-23 is shown the variation of emitter potential slope with electrode spacing (or ion density) for a charge density ratio of unity. The variation is linear for values of (L/λ_D) greater than about 10 and it approaches a finite value as (L/λ_D) approaches zero.

The variation of plasma potential (or inflection point potential) has an interesting behavior as a function of charge density ratio. This may be seen in Figure II-24, which is a compilation of results for various values of (L/λ_D). The implications are that the plateau potential is not a function of electrode spacing and that it is a double-valued function of charge density ratio. It should also be noted that the charge density ratio for minimum plasma potential is about 1.33, and this does not coincide with the point of zero emitter slope, which is about 1.25.

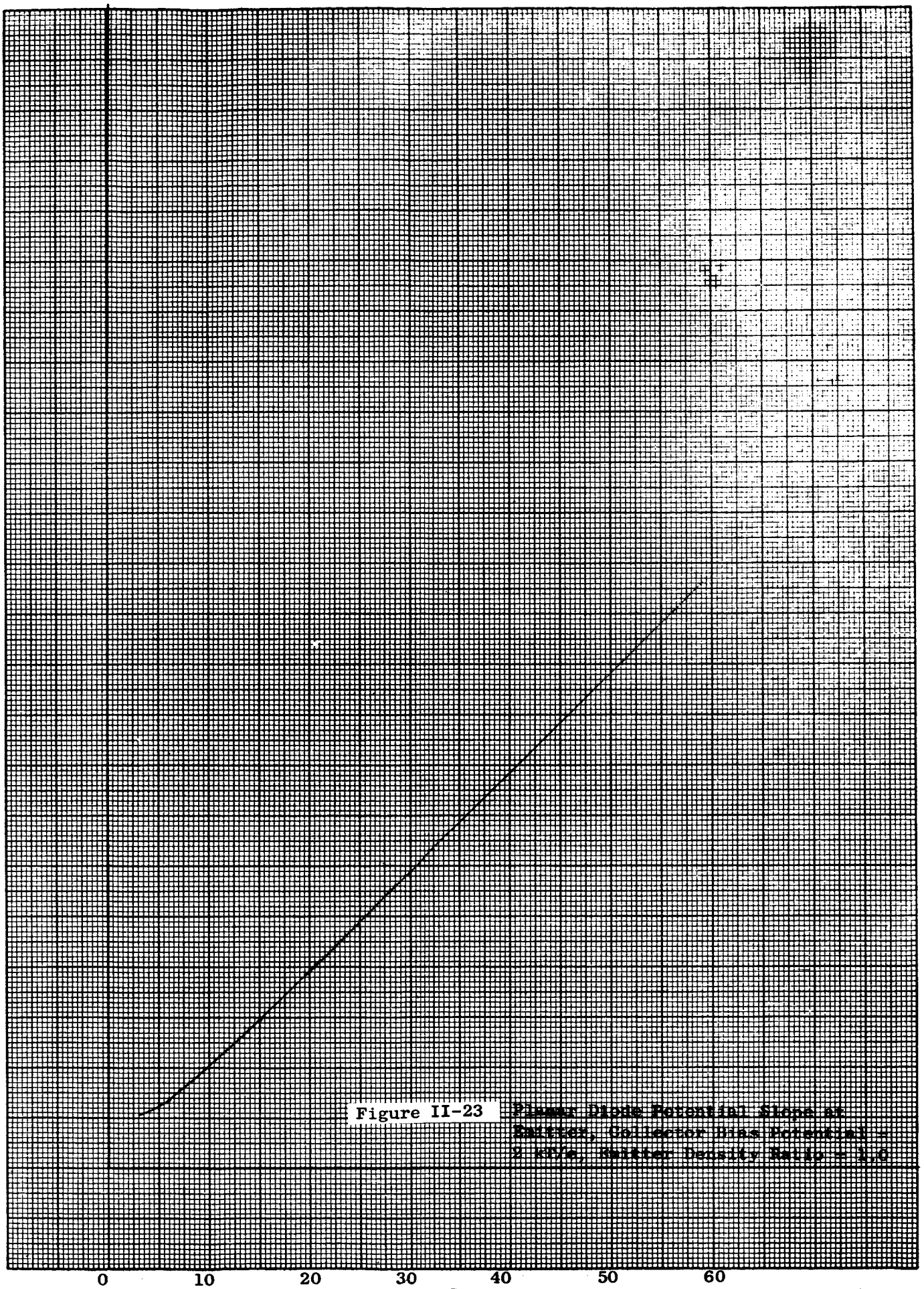


Figure II-23 Planar Diode Potential Slope at Emitter, Collector Bias Potential = 2 kTe/e, Emitter Density Ratio = 2.0

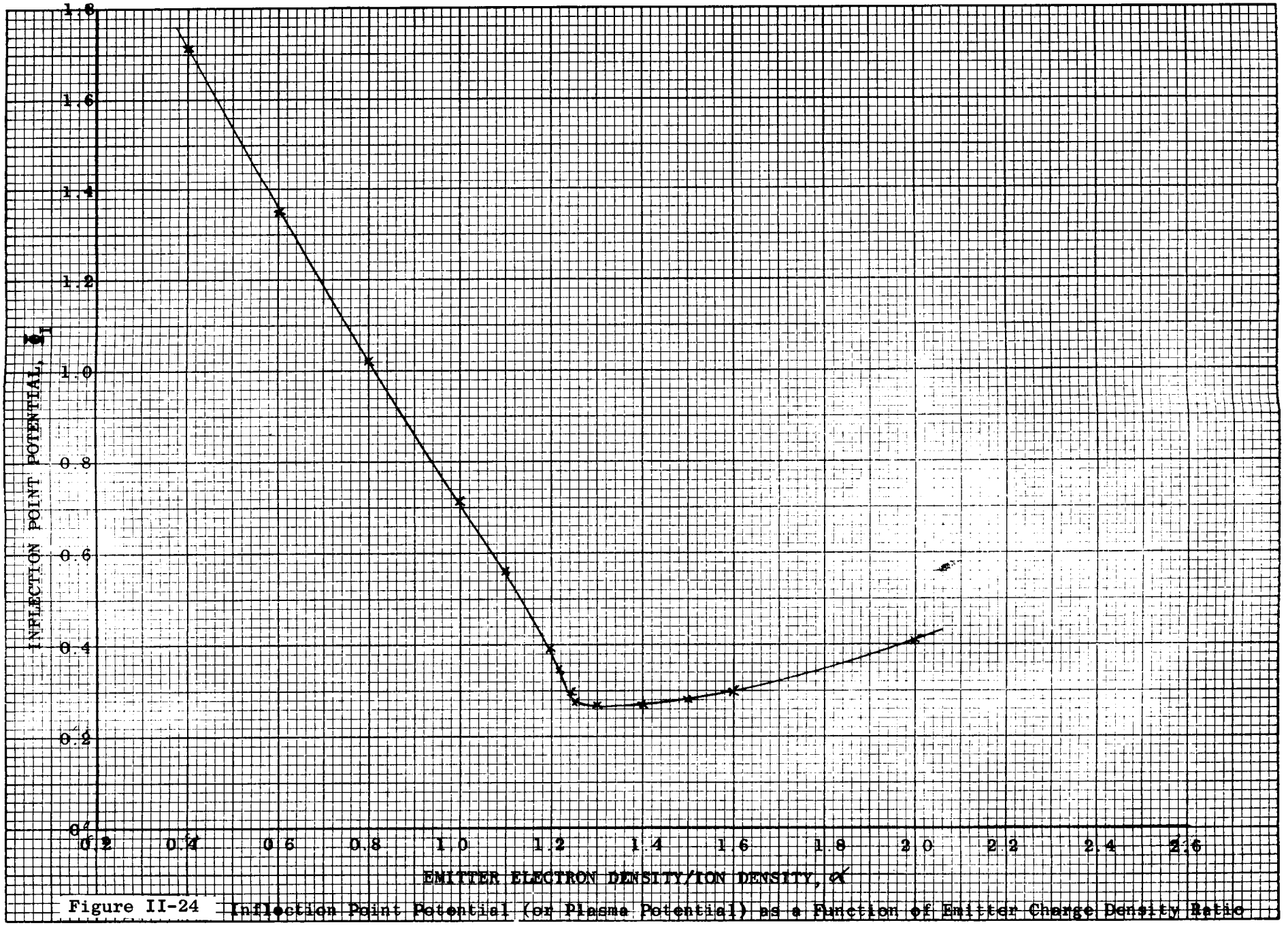


Figure II-24 Inflection Point Potential (or Plasma Potential) as a Function of Emitter Charge Density Ratio

3. Potential Maximum Solutions

The preceding sections of this chapter have dealt with monotonic solutions and potential minimum solutions. The potential minimum solutions were obtained when the charge density ratio at the emitter was increased above 1.25. For smaller values of α the solutions were monotonic distributions of potentials.

Another type of solution is obtained when the emitter charge density ratio is decreased to a value of about 0.2 or less. Under these conditions the diode is heavily ion-rich in the region of the emitter and a maximum is attained in the potential. This requires revision in the computer program to account for the presence of this potential hill.

The formulation of the potential maximum problem is similar to the equations previously developed for the potential minimum case. In this case, Poisson's equation has the form:

$$\frac{\Phi'^2}{2\mathcal{L}^2} = \alpha\left\{e^{\Phi}\left[1-\operatorname{erf}\sqrt{\Phi}\right]+\frac{2}{\sqrt{\pi}}\sqrt{\Phi}\right\}+e^{-\Phi}\frac{1\pm\operatorname{erf}\sqrt{\Phi_M-\Phi}}{1+\operatorname{erf}\sqrt{\Phi_M}}\mp\frac{2}{\sqrt{\pi}}e^{-\Phi_M}\frac{\sqrt{\quad}}{1+\operatorname{erf}\sqrt{\Phi_M}}$$
$$-\alpha\left\{e^{\Phi_M}\left[1-\operatorname{erf}\sqrt{\Phi_M}\right]+\frac{2}{\sqrt{\pi}}\sqrt{\Phi_M}\right\}-\frac{e^{-\Phi_M}}{1+\operatorname{erf}\sqrt{\Phi_M}} \qquad \text{(II-47)}$$

where the upper sign of a double sign applies to conditions between the emitter and the maximum; the lower sign applies to conditions between the maximum and the collector. The similarity of this equation to equation II-33 of the preceeding section with $\delta = 0$, should be evident. The ambiguous signs follow directly from analysis of the limits of integration in equation II-14 of the monotonic analysis, the lower limit of admissible particle velocities on

either side of the maximum being determined by acceleration of particles just brought to rest at the maximum.

The terms in (II-47) which arise from the electron population can be developed in a Taylor's Series in $(\Phi - \Phi_M)$ without difficulty.

$$E = e^{\Phi}\left[1-\operatorname{erf}\sqrt{\Phi}\right] + \frac{2}{\sqrt{\pi}}\sqrt{\Phi} - e^{\Phi_M}\left[1-\operatorname{erf}\sqrt{\Phi_M}\right] - \frac{2}{\sqrt{\pi}}\sqrt{\Phi_M}$$

$$= e^{\Phi_M}\left[1-\operatorname{erf}\sqrt{\Phi_M}\right]\left[\Phi-\Phi_M\right] + \left[e^{\Phi_M}\left(1-\operatorname{erf}\sqrt{\Phi_M}\right) - \frac{1}{\sqrt{\pi\Phi_M}}\right]\frac{(\Phi-\Phi_M)^2}{2} \qquad \text{(II-48)}$$

$$+ \left[e^{\Phi_M}\left(1-\operatorname{erf}\sqrt{\Phi_M}\right) - \frac{1}{\sqrt{\pi\Phi_M}} + \frac{1}{2}\frac{1}{\Phi^{3/2}\sqrt{\pi}}\right]\frac{(\Phi-\Phi_M)^3}{6} + \cdots$$

The terms in (II-47) which come from the ion population cannot be developed in a Taylor's Series in $(\Phi - \Phi_M)$, because derivatives beyond the first are undefined. A substitution

$$t = \left(\Phi_M - \Phi\right)^{1/2}$$

$$t' = -\frac{1}{2}\left(\Phi_M - \Phi\right)^{-\frac{1}{2}}\Phi' \qquad \text{(II-49)}$$

$$\Phi' = -2tt'$$

does however make a series development possible:

$$I = e^{-\Phi}\frac{1 \pm \operatorname{erf}\sqrt{\Phi_M - \Phi}}{1+\operatorname{erf}\sqrt{\Phi_M}} \mp \frac{2}{\sqrt{\pi}}e^{-\Phi_M}\frac{\sqrt{\Phi_M-\Phi}}{1+\operatorname{erf}\sqrt{\Phi_M}} - \frac{e^{-\Phi_M}}{1+\operatorname{erf}\sqrt{\Phi_M}} \qquad \text{(II-50)}$$

$$= \frac{e^{-\Phi_M}}{1+\operatorname{erf}\sqrt{\Phi_M}}\left[e^{t^2}\left(1 \pm \operatorname{erf} t\right) \mp \frac{2}{\sqrt{\pi}}t - 1\right]$$

Now if

$$T = e^{t^2}(1 \pm \operatorname{erf} t) \mp \frac{2}{\sqrt{\pi}}\, t - 1$$

$$\frac{dT}{dt} = 2t e^{t^2}(1 \pm \operatorname{erf} t)$$

$$\frac{d^2T}{dt^2} = 2e^{t^2}(1 \pm \operatorname{erf} t) + 4t^2 e^{t^2}(1 \pm \operatorname{erf} t) \pm \frac{4t}{\sqrt{\pi}}$$

$$= 2(1+2t^2)e^{t^2}(1 \pm \operatorname{erf} t) \pm \frac{4t}{\sqrt{\pi}}$$

$$\frac{d^3T}{dt^3} = \pm \frac{4}{\sqrt{\pi}} + 8te^{t^2}(1 \pm \operatorname{erf} t) + 4t(1+2t^2)e^{t^2}(1 \pm \operatorname{erf} t) \pm (1+2t^2)$$

$$= \pm \frac{8}{\sqrt{\pi}}(1+t^2) + (12t+8t^3)e^{t^2}(1 \pm \operatorname{erf} t)$$

$$\frac{d^4T}{dt^4} = \pm \frac{16}{\sqrt{\pi}}\, t + (12+24t^2)e^{t^2}(1 \pm \operatorname{erf} t)$$

$$+ (24t^2+16t^4)e^{t^2}(1 \pm \operatorname{erf} t) + (12t+8t^3)\frac{2}{\sqrt{\pi}}$$

$$= \pm \frac{40}{\sqrt{\pi}}\, t \pm \frac{16}{\sqrt{\pi}}\, t^3 + (12+48t^2+16t^4)e^{t^2}(1 \pm \operatorname{erf} t)$$

$$\frac{d^5T}{dt^5} = \pm \frac{40}{\sqrt{\pi}} \pm \frac{48}{\sqrt{\pi}}\, t^2 + (96t+64t^3)e^{t^2}(1 \pm \operatorname{erf} t) \qquad \text{(II-51)}$$

$$+ (24t+96t^3+32t^5)e^{t^2}(1 \pm \operatorname{erf} t) \pm \frac{2}{\sqrt{\pi}}(12+48t^2+16t^4)$$

$$= \pm \frac{64}{\sqrt{\pi}} \pm \frac{144}{\sqrt{\pi}}\, t^2 \pm \frac{32}{\sqrt{\pi}}\, t^4 + (120t+160t^3+32t^5)e^{t^2}(1 \pm \operatorname{erf} t)$$

With

$$A = \frac{e^{-\mathcal{I}_M}}{1 + \operatorname{erf}\sqrt{\mathcal{I}_M}} \qquad \text{(II-52)}$$

$$\frac{I}{A} = t^2 \pm \frac{4}{3\sqrt{\pi}}\, t^3 + \frac{1}{2}\, t^4 \pm \frac{8}{15}\, t^5 + \frac{1}{6}\, t^6 \qquad \text{(II-53)}$$

Combining (II-53) and (II-48)

$$\frac{\Phi'^2}{2\mathcal{L}^2} = \alpha e^{\Phi_M}\left(1-\operatorname{erf}\sqrt{\Phi_M}\right)\left(-t^2+\frac{t^4}{2}-\frac{t^6}{6}\right)$$
$$-\frac{\alpha}{\sqrt{\pi\Phi_M}}\left(\frac{t^4}{2}+\left[1-\frac{1}{2\Phi_M}\right]\frac{t^6}{6}\right) \qquad \text{(II-54)}$$
$$+A\left(t^2\pm\frac{4}{3\sqrt{\pi}}t^3+\frac{1}{2}t^4\pm\frac{8}{15\sqrt{\pi}}t^5+\frac{1}{6}t^6\right)$$

The net charge density ρ_M at the maximum may be identified as

$$\rho_M = \alpha e^{\Phi_M}\left(1-\operatorname{erf}\sqrt{\Phi_M}\right)-A \qquad \text{(II-55)}$$

With the use of (II-55) and (49), (54) can be written as

$$t^2t'^2 = \frac{\mathcal{L}^2t^2}{2}\left\{-\rho_M\pm\frac{4}{3\sqrt{\pi}}At+\left(\alpha\left[e^{\Phi_M}\left(1-\operatorname{erf}\sqrt{\Phi_M}\right)-\frac{1}{\sqrt{\pi\Phi_M}}\right]+A\right)\frac{t^2}{2}\right.$$
$$\left.\pm\frac{8}{15\sqrt{\pi}}t^3-\left[\rho_M+\frac{\alpha}{\sqrt{\pi\Phi_M}}\left(1-\frac{1}{2\Phi}\right)\right]\frac{t^4}{6}\right\} \qquad \text{(II-56)}$$

If the parameter values in (II-56) are such that the first three series terms dominate the expansion for t', (56) may be integrated in closed form, (Ref. 7).

$$\chi-\chi_M = \int_0^t \frac{dt}{\sqrt{\frac{\mathcal{L}^2}{2}}\sqrt{-\rho_M\pm\frac{4}{3\sqrt{\pi}}At+\left[\alpha e^{\Phi_M}\left(1-\operatorname{erf}\sqrt{\Phi_M}\right)-\frac{\alpha}{\sqrt{\pi\Phi_M}}+A\right]\frac{t^2}{2}}} \qquad \text{(II-57)}$$

Letting

$$B = \frac{\mathcal{L}}{2}\sqrt{\alpha e^{\Phi_M}\left(1-\operatorname{erf}\sqrt{\Phi_M}\right)-\frac{\alpha}{\sqrt{\pi\Phi_M}}+A}$$
$$C = \frac{4}{3\sqrt{\pi}}\,\frac{A}{\alpha e^{\Phi_M}\left(1-\operatorname{erf}\sqrt{\Phi_M}\right)-\frac{\alpha}{\sqrt{\pi\Phi_M}}+A} \qquad \text{(II-58)}$$
$$D = \frac{-\rho_M}{\alpha e^{\Phi_M}\left(1-\operatorname{erf}\sqrt{\Phi_M}\right)-\frac{\alpha}{\sqrt{\pi\Phi_M}}+A}$$

$$x - x_M = \frac{1}{B} \int_0^t \frac{dt}{\sqrt{D \pm Ct + t^2}}$$

$$= \frac{1}{B} \ln \left| \frac{\sqrt{D \pm Ct + t^2} + t \pm C/2}{\sqrt{D} \pm C/2} \right| \qquad \text{(II-59)}$$

There may be many cases when (II-56) with the negative sign yields a zero value for t'^2 when t is still small enough for the three terms to be an excellent appropriation. In such cases, (II-59) gives a definite distance from the potential maximum to a minimum. Since the negative sign for the square root could equally well have been used in (II-57), (II-59) may be used with a changed sign to give another distance from the minimum to a subsequent maximum. Following additional steps, an oscillatory solution of this sort, extending indefinitely, may be constructed.

a. Results of Potential Maximum Computations

Using the planar diode computer program, solutions were found for some cases of potential maximum distributions. For the case of a 200 Debye length spacing with a 0.1 ratio of electron/ion density at the emitter, a total of five manual iterations produced the correct computer solutions. This solution is shown in Fig. II-25a along with two of the iterations that were obtained. The iterations were made by varying the value of maximum potential between 2.0 and 2.07. A potential maximum of 2.048 gave the correct solution for this case, which has an oscillation amplitude of 0.961 kT/e with a wavelength of 39.3 Debye lengths.

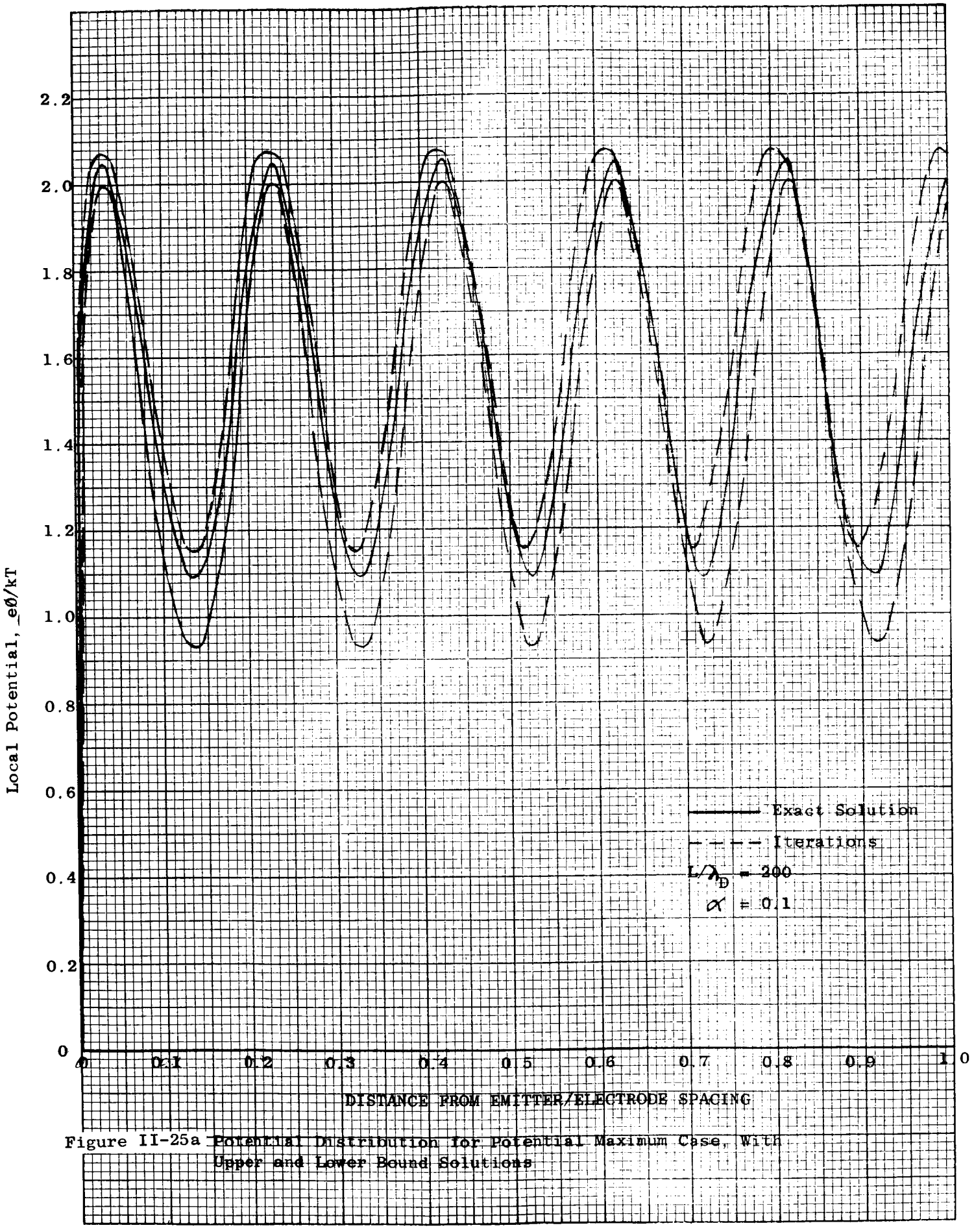


Figure II-25a Potential Distribution for Potential Maximum Case, With Upper and Lower Bound Solutions

b. Potential Distributions with Two or More Extrema

The monotonic distribution may be more generally classed as a potential distribution with no extrema in potential between the emitter and collector. The potential minimum case is then a distribution with an extremum opposite to the bias potential, and the potential maximum case has an extremum in the same direction as the bias potential.

There is another case which has been considered as a possible distribution. This is the existence of two (or more) extrema; it can occur when the amplitude of the oscillations in a potential maximum condition exceeds the bias potential. During this investigation, it was not considered within the scope of effort to examine this type of solution in detail. In fact, the computer program would require some revisions to be able to calculate the charge densities after passing through each potential maximum and minimum. Furthermore, the oscillation is not of constant amplitude so the computation must proceed throughout the electrode space rather than just duplicate the first oscillation as was done in the potential maximum solutions.

In spite of these obstacles, it has still been possible to obtain some conclusions with the program in its present form. The graph of Figure II-25b shows the results of several runs that were made for a bias potential of 2 kT/e with an electron/ion charge density ratio at the emitter of 0.05. Upper and lower bounds to the solution have been found, but the lower bound becomes negative and the program is unable to complete the calculations. This indicates that the correct solution will probably have amplitude larger than the bias

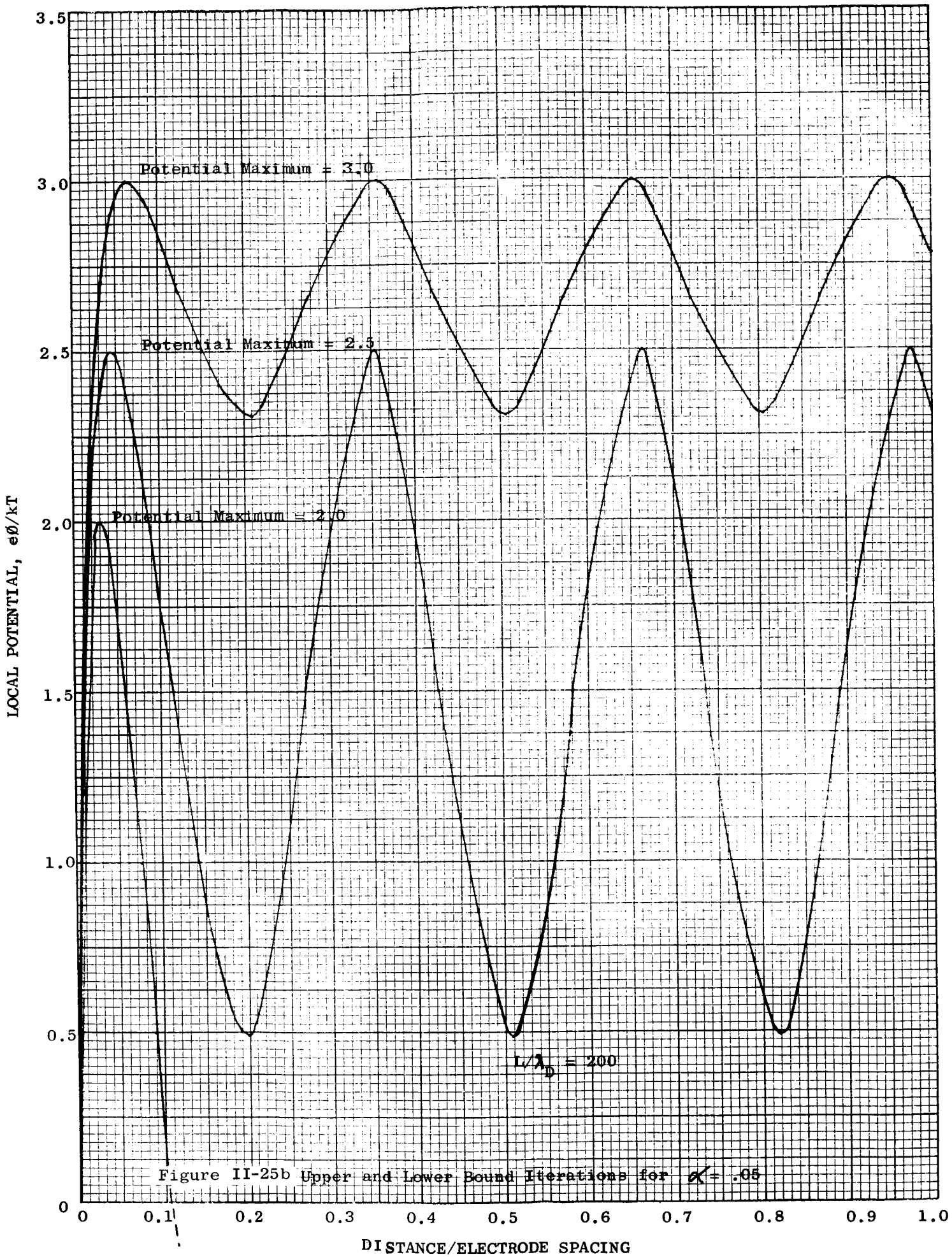


Figure II-25b Upper and Lower Bound Iterations for $\alpha = .05$

potential and the solution becomes unavailable.

It should be observed that the amplitude of oscillation increases sharply as the value of the potential at the maximum is decreased. For instance, the amplitude is about 0.7 for a potential maximum of 3.0, and about 2.0 for a potential maximum of 2.5. Now the correct solution will have a potential maximum between 2.0 and 2.5 so its amplitude will be somewhat larger than 2.0 and the curve will probably pass below zero. Therefore, the conclusions that may be reached are that solutions of this type are possible, and that a considerable modification in the program is required to make it capable of finding these solutions.

D. Summary of Planar Diode Solutions

A complete set of the various types of planar diode potential solutions found during this program is shown in Figure II-26. The corresponding charge density distributions are given in Figure II-27.

For emitter charge density ratios greater than about 1.25 (with Φ_C = 2kT/e and $L/\lambda_D = 9$), the potential curve has a sharp minimum close to the emitter which traps a portion of the electrons. Downstream of this minimum the potential curve experiences a sharp rise up to the plateau potential (or the inflection point potential) and then there is another sharp rise in the "sheath" at the collector.

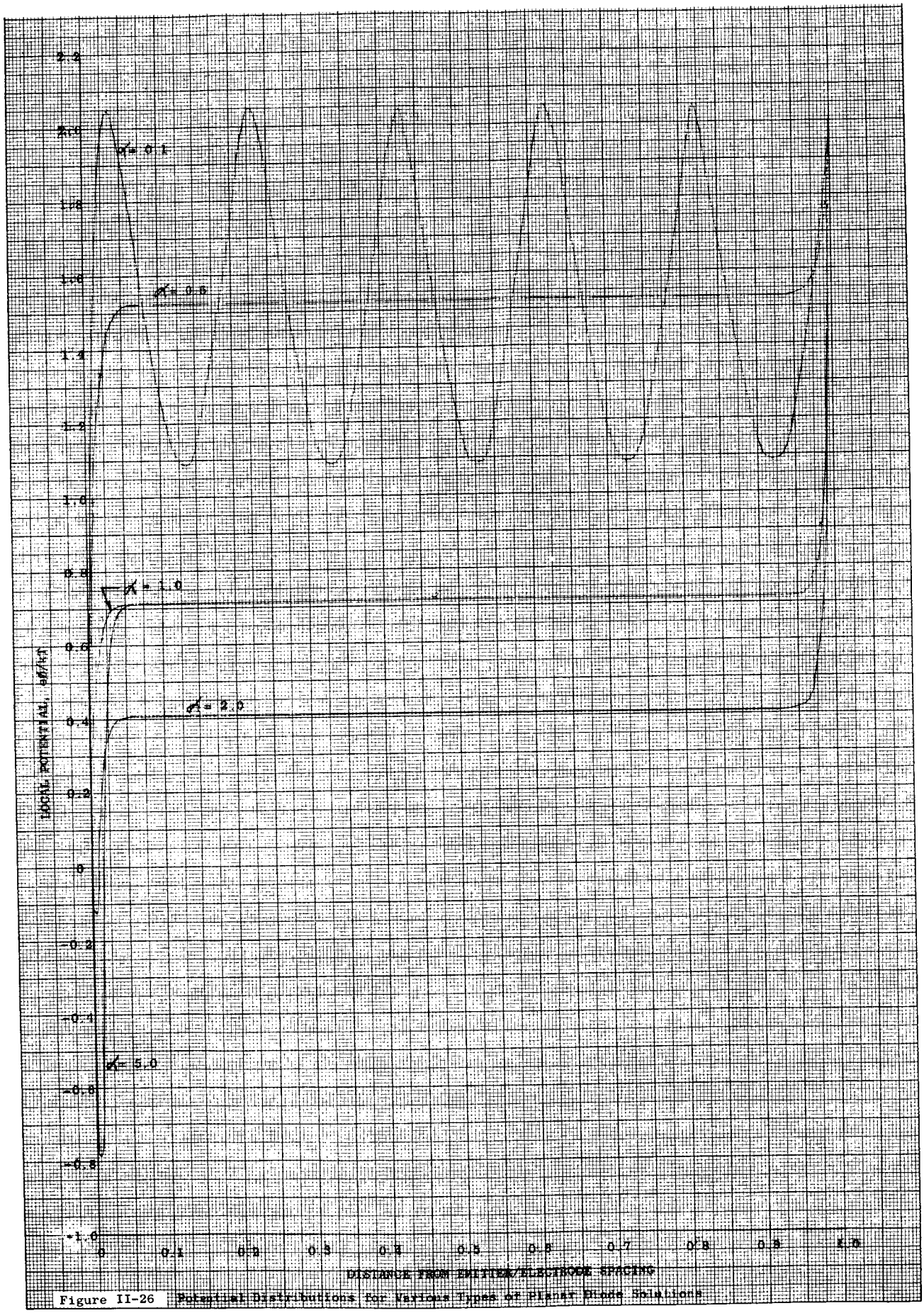


Figure II-26 Potential Distributions for Various Types of Planar Diode Solutions

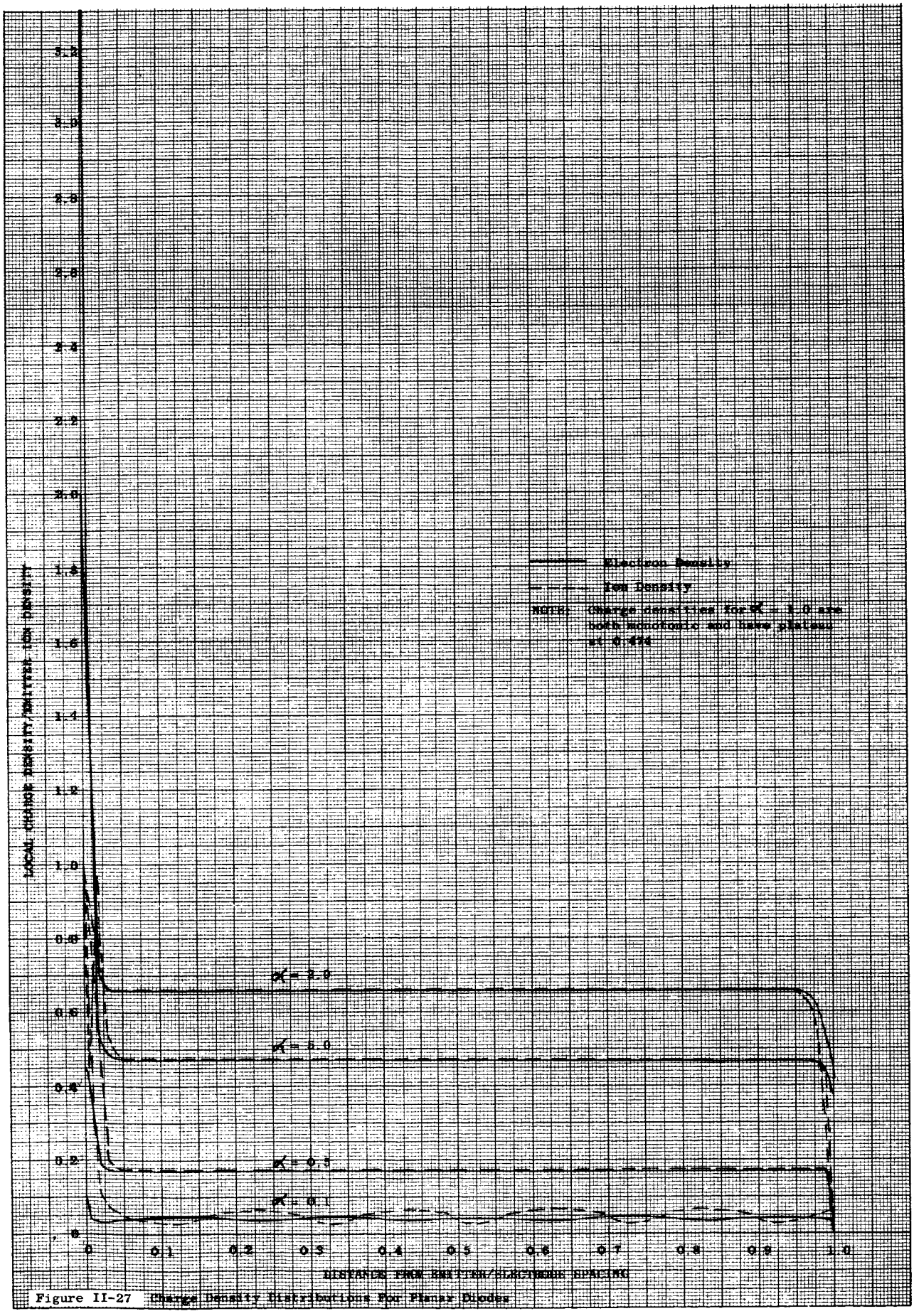


Figure II-27 Charge Density Distributions For Planar Diodes

For ratios of electron/ion density at the emitter in the range of about 0.2 to 1.25 the potential curve is monotonic. These curves are represented in Figures II-26 and 27 by the curves for $\alpha = 0.5$ and $\alpha = 1.0$. It should be noted that the plateau potential is the same for $\alpha = 1.0$ as for $\alpha = 5.0$. This is because the plateau potential is a double-valued function of α, as was shown previously.

The third type of potential distribution that can occur in a planar diode occurs for $\alpha = 0.2$. This excess of ions causes a potential maximum to occur near the emitter which will trap most of the ions from the emitter. Then the potential decreases and the ions are accelerated while the electrons are now decelerated. The charge densities soon become equal and the process is reversed. This oscillation is continued until the collector is reached.

E. Comparison of Planar and Cylindrical Diode Solutions

It is interesting to compare solutions for planar and cylindrical diodes under similar conditions. For the case of diodes with a bias potential of 2 kT/e, a spacing of 6.6 Debye lengths (based on total ion density), and a ratio of electron/ion densities at the emitter of 0.83, the graphs in Figure II-28 show the planar and cylindrical distributions. The planar curve for $L/\lambda_D = 6$ is also shown; it was found by mistake because of a confusion which is likely to occur in comparing distributions for negative and positive bias potential. It should be remembered at all times that the Debye length is always based on the total ion density at the emitter and such problems should not arise. From a comparison of these curves, some of the effects of geometry may be observed.

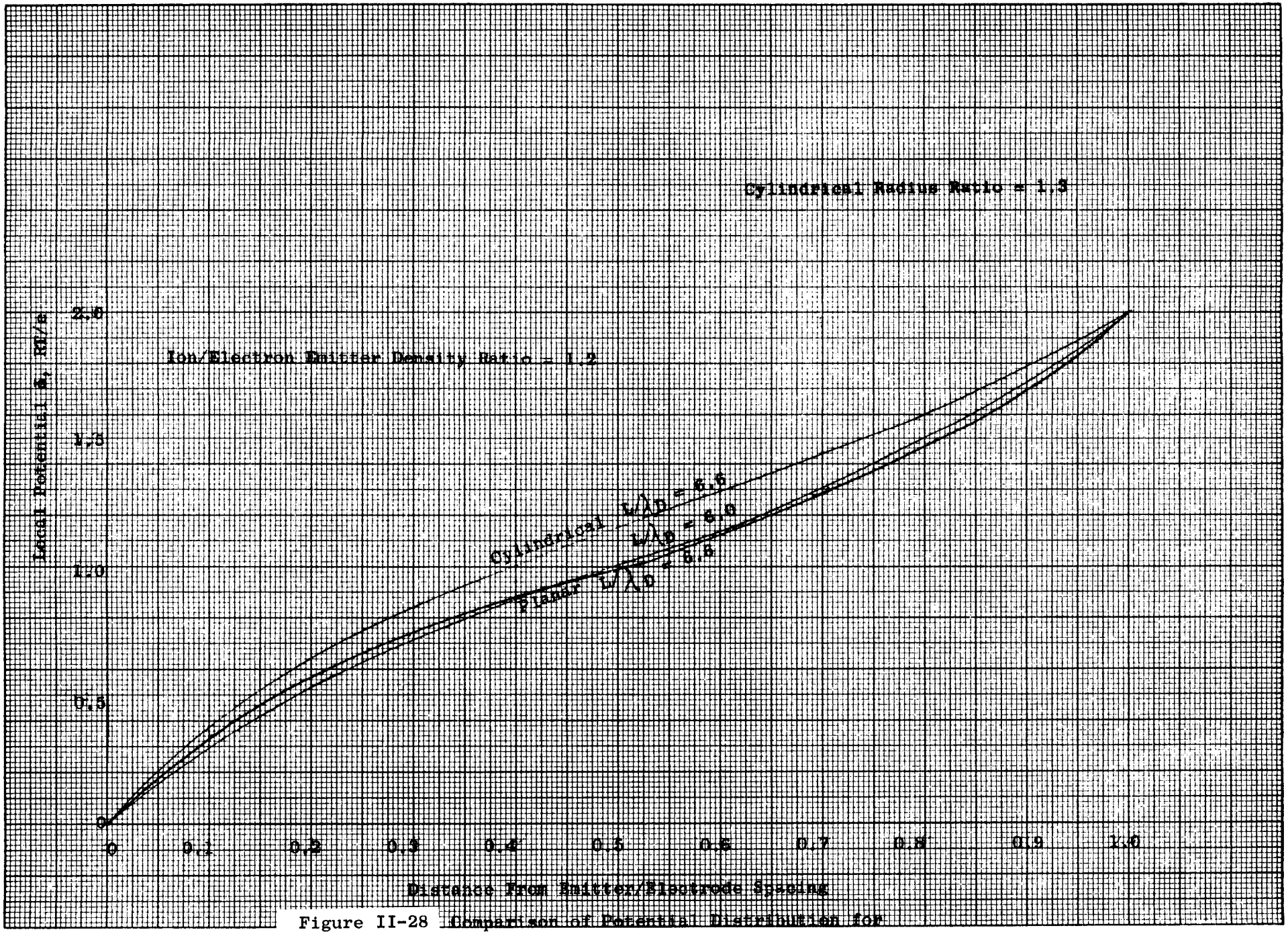


Figure II-28 Comparison of Potential Distribution for Planar and Cylindrical Diodes

At the collector (which is attractive to electrons) the field (i.e., potential gradient) is smaller in the cylindrical case than in the planar situation. This is because, for a space charge limited region, the current density is approximately expressed by the following equations:

$$j_{PLANAR} = K \frac{V^{3/2}}{x^2}$$

$$j_{CYLINDRICAL} = \beta K \frac{V^{3/2}}{x^2}$$

where x is the distance from the emitter, and β is greater than unity for an inside emitter.

In general, therefore,

$$V = \left(\frac{j x^2}{\beta K} \right)^{2/3}$$

where β is unity for the planar geometry. We can then differentiate

$$\frac{dV}{dx} = \left(\frac{j}{\beta K} \right)^{2/3} \left(\frac{4}{3} \right) x^{1/3}$$

Therefore, at a point where the current densities are equal, the slope is smaller for the cylindrical case than for the planar case because $\beta > 1$ for cylindrical and $\beta = 1$ for planar.

At the inflection point, the absolute value of the potential is smaller for planar than for cylindrical diodes. This is because, at a given potential in the cylindrical diode, there is a larger number density of ions due to the conversion of angular velocity to radial velocity. Therefore the point of equal number densities of ions and electrons (i.e., the inflection point) occurs at a higher potential for the cylindrical situation.

Near the emitter the field is larger for a cylindrical diode than for a planar diode. This is apparently caused by the larger current density at the cylindrical emitter than at the planar emitter, necessitating a larger field in that vicinity to carry the current.

In Figure II-29, a comparison is shown for planar and cylindrical potential distributions for minima. These solutions were obtained for 8.5 Debye length spacing with a bias potential of 2 kT/e and emitted electron/ion density ratio of 2. It may be seen that the plateau is less apparent in the cylindrical case because of the geometry and angular momentum effects. Also the potential minimum falls farther from the emitter in the cylindrical case because of the conversion of tangential to radial kinetic energy. The graph also includes a planar curve for $L/\lambda_D = 12$; this again was obtained because of confusion in comparison of Debye lengths for negative and positive potentials with emitter density ratios other than unity.

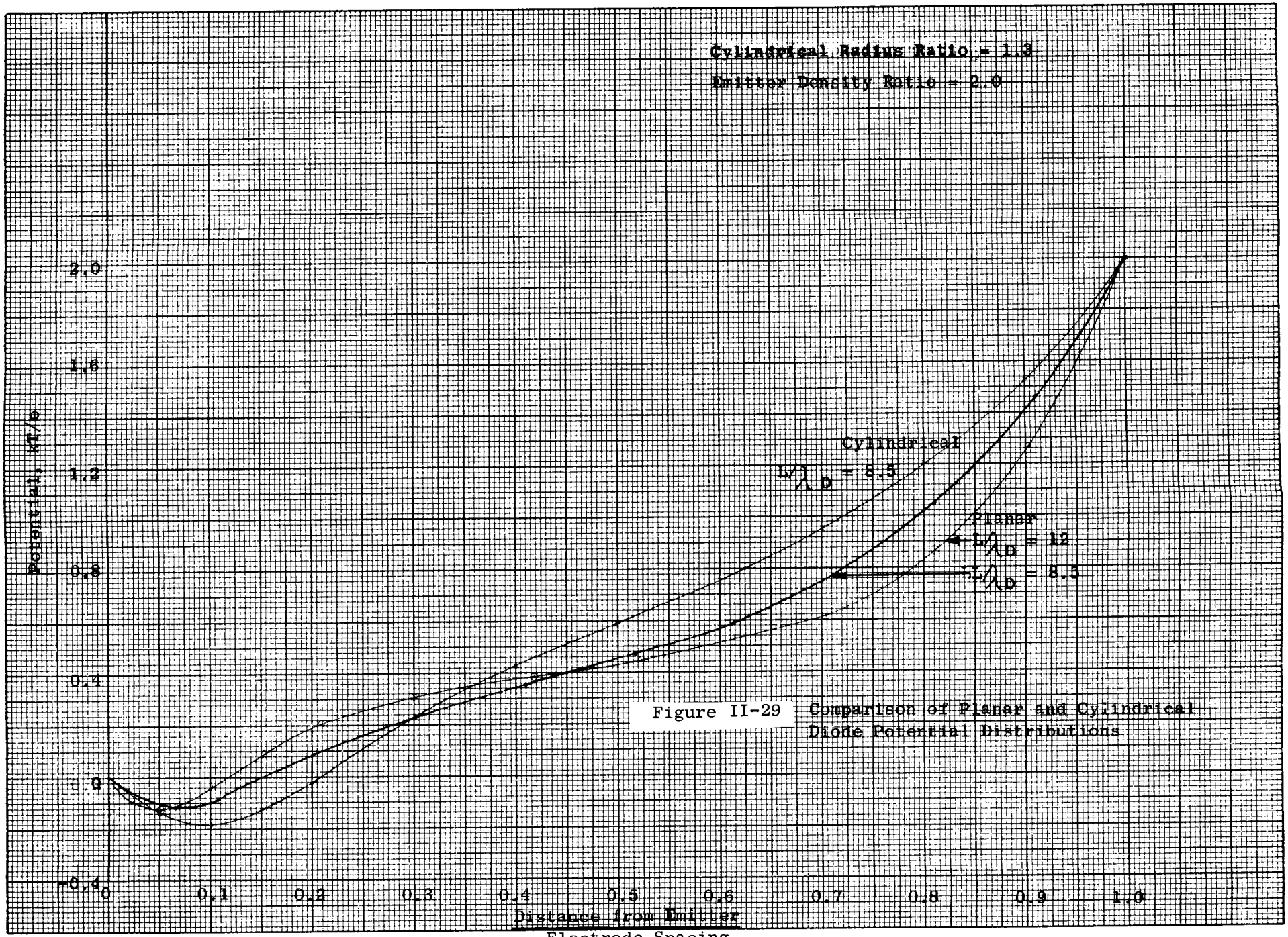


Figure II-29 Comparison of Planar and Cylindrical Diode Potential Distributions

III. INVESTIGATIONS OF CYLINDRICAL AND SPHERICAL PLASMA DIODES

A. Comparison of Governing Equations for Cylindrical and Spherical Diodes

1. Energy and Momentum Relationships for Charged Particles

These relations are identical for problems of cylindrical and spherical symmetry.

a. Momentum relations:

For electrons

$$V_{Te} r = \text{const}$$

For ions

$$V_{Ti} r = \text{const}$$

b. Energy relations:

For electrons

$$e\phi = m_e (v_e^2 - v_{oe}^2)/2$$

For ions

$$e\phi = m_i (v_{oi}^2 - v_i^2)/2$$

The energy and momentum relations are used to determine the limits of admissible velocities at all points in the diode. Probability distributions for all particles within the admissible velocity range are assumed to be governed by the distribution of these particles at the emitter.

2. Charge Density

The general expression for charge density was given in the NAS 8-623 Contract Report, equations 7-7 and 7-8:

$$n(x,y,z) = \int^{+\infty} \int^{+\infty} \int^{+\infty} f(x,y,z,v_x,v_y,v_z)dv_x dv_y dv_z \qquad \text{(III-1)}$$

The distribution function f in (1) is assumed to be the Maxwell-Boltzmann distribution:

$$f = \left[\frac{m}{2\pi kT}\right]^{3/2} e^{-\frac{m}{2kT}\left(v_{ox}^2 + v_{oy}^2 + v_{oz}^2\right)}$$

$$= \left[\frac{m}{2\pi kT}\right] e^{\frac{1}{n}\ln n} e^{-\frac{m}{2kT}\left(v_x^2 + v_y^2 + v_z^2\right)} \qquad \text{(III-2)}$$

provided the velocity is within the range that is admissible subject to energy and momentum restrictions; otherwise the distribution function vanishes. The choice of $\pm$ depends on whether the particular species is being accelerated or decelerated. In the following discussion, limits of integration are shown for the entire distribution function; the algebraic expression for the integrand will be the expression applicable to admissible velocities (non-zero distribution function).

For cylindrical symmetry problems:

It will prove convenient to make the transformation

$$\sqrt{\frac{m}{2kT}}\ v_x = v\sin\theta \qquad (3C)$$

$$\sqrt{\frac{m}{2kT}}\ v_y = v\cos\theta$$

$$\frac{m}{2kT}\,dv_x\,dv_y = v\,dv\,d\theta$$

Then (1) and (2) become (4C)

$$n(x,y,z) = \frac{e^{\frac{e\phi}{kT}}}{\pi^{3/2}} \int_{v_z=-\infty}^{v_z=+\infty} \int_{v=0}^{v=+\infty} \int_{\theta=0}^{\theta=2\pi} e^{-v^2} - \frac{m}{2kT} v_z^2\, v\,dv\,d\theta\,d\left[\sqrt{\frac{m}{2kT}}\,v_z\right]$$

For cylindrical diodes we observe that the distribution in the z direction is independent of x, y, v_x, v_y, so that a first integration may be carried out immediately,

$$\left[\frac{m}{2\pi kT}\right]^{1/2} \int_{-\infty}^{+\infty} e^{-\frac{m}{2kT}v_z^2}\,dv_z = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-\left[\sqrt{\frac{m}{2kT}}v_z\right]^2} d\left[\sqrt{\frac{m}{2kT}}v_z\right] \qquad (5C)$$

$$= 1$$

so that (4C) becomes

$$n(r) = \frac{1}{\pi} e^{\frac{e\phi}{kT}} \int_{\theta=0}^{\theta=2\pi} \int_{v=0}^{v=+\infty} e^{-v^2}\, v\,dv\,d\theta \qquad (6C)$$

For spherical diodes any particle remains in a plane defined by the radius vector from the center of the sphere to its starting point and the initial velocity vector of the particle.

It is therefore appropriate to make the transformation:

$$\left[\frac{m}{2kT}\right]\left[v_x^2 + v_y^2\right] = v_T^2 \qquad (3S)$$

$$\sqrt{\frac{m}{2kT}}\ v_z = v_R$$

$$\frac{m}{2kT}\,dv_x\,dv_y = v_T\,dv_T\,d\theta$$

In this case (1) and (2) become

$$n(x,y,z) = \frac{e^{\frac{e\phi}{kT}}}{\pi^{3/2}} \int_{\theta=0}^{\theta=2\pi} \int_{v_R=-\infty}^{v_R=+\infty} \int_{v_T=0}^{v_T=+\infty} e^{-(v_T^2+v_R^2)}\, v_T\,dv_T\,dv_R\,d\theta \qquad (4S)$$

Integration of (4S) with respect to θ may be carried out immediately, yielding

$$n(r) = \frac{2}{\sqrt{\pi}} e^{\frac{e\phi}{kT}} \int_{v_R=-\infty}^{v_R=+\infty} \int_{v_T=0}^{v_T=+\infty} e^{-(v_T^2+v_R^2)}\, v_T\,dv_T\,dv_R \qquad (6S)$$

In each case, the result of applying the symmetry argument is to leave an expression for n depending on r alone.

It is convenient to recognize an expression equivalent to (6C):

$$n(r) = \frac{-e^{\frac{e\phi}{kT}}}{\pi} \int_{\theta=-\frac{\pi}{2}}^{\theta=+\frac{\pi}{2}} \int_{V=0}^{V=+\infty} d\left(e^{-V^2}\right) d\theta \qquad (7C)$$

An expression equivalent to (6S) is:

$$n(r) = \frac{-e^{-\frac{e\phi}{kT}}}{2} \int_{V_R=-\infty}^{V_R=+\infty} \int_{V_T=0}^{V_T=+\infty} d\left(e^{-V_T^2}\right) d(\mathrm{erf}\, V_R) \qquad (7S)$$

To carry out integration of (7C) or (7S) numerically, the procedure is to determine a table of values giving a lower limit to the velocity component parallel to the radius for particles headed away from the emitter (the upper limit for these particles is always infinity) and upper and lower limits to this velocity component for particles returning toward the emitter for a suitable collection of velocity components perpendicular to the radius to give adequate definition of the integrand. (The tangential velocity components are chosen from a table with values ranging from 10^{-4} to 10 by the ratio $10^{1/100}$, so as to give successive values of θ differing by less than 3°, successive values of v_R lying between .667 and 1.5 of the preceding value, and lying as close to critical corners as the table permits).

In the cylindrical case it is convenient to express the integral in terms of the initial kinetic energy:

$$n(r) = \frac{1}{\pi}\left[\int_{\theta=0}^{\theta=\pi/2} e^{-V_0^2} d\theta + \int_{\theta=-\pi/2}^{\theta=0} e^{-V_{10}^2} d\theta - \int_{\theta=-\pi/2}^{\theta=0} e^{-V_{20}^2} d\theta \right] \qquad (8C)$$

Here the first term gives the charge density arising from particles bound away from the emitter. The second term gives the charge density arising from particles returning toward the emitter with energy above the level corresponding to the lower limit of admissible radial velocities. The third term describes the charge density arising from particles returning toward the emitter with energies above the level corresponding to its upper limit of admissible radial velocities which is the part of the second term which is not present. θ in (8C) is the trajectory angle from the tangential at the actual radius, not the angle at the start of the trajectory, as might be inferred from the use of the initial kinetic energy.

The spherical case does not permit convenient expression of the integrand in terms of the initial kinetic energy. The form of (7S) which is used for obtaining charge density is:

$$n(r) = \frac{e^{\pm\frac{e\phi}{kT}}}{2}\left[1 - \int_{e^{-V_T^2}=0}^{e^{-V_T^2}=1} (\mathrm{erf}\, V_R)\, d\left(e^{-V_T^2}\right) + \int_{e^{-V_T^2}=0}^{e^{-V_T^2}=1} \left[\mathrm{erf}(V_{2R}) - \mathrm{erf}(V_{1R})\right] d\left(e^{-V_T^2}\right) \right] \qquad (8S)$$

Here the first two terms describes charge density arising from particles bound away from the emitter. The remaining terms describe charge density arising from particles returning toward the emitter. For bias potentials greater than 7 or 8 kT/e it may be more convenient to use:

$$n(r) = \int e^{\left(\frac{e\phi}{kT} - V_R^2\right)} e^{\left(V_R^2\right)} \left(1 - \mathrm{erf}\, V_R\right) d\left(e^{-V_T^2}\right)$$

$$= \frac{1}{\sqrt{\pi}} \int \frac{e^{\left(\frac{e\phi}{kT} - V_R^2\right)}}{V_R} \left\{ 1 - \frac{1}{2V_R^2} + \frac{1\cdot 3}{\left(2V_R^2\right)^2} - \cdots \right\} d\left(e^{-V_T^2}\right)$$

It is commonly convenient to express the local charge density in terms of the charge density at the emitter attributable to emitted particles only. Then:

$$\frac{n(r)}{n_{o+}} = \int_0^1 e^{-v_0^2} d\left[\frac{\theta}{\pi/2}\right] + \int_{-1}^{0} e^{-v_{10}^2} d\left[\frac{\theta}{\pi/2}\right] - \int_{-1}^{0} e^{-v_{20}^2} d\left[\frac{\theta}{\pi/2}\right] \qquad (9C)$$

$$\frac{n(r)}{n_{o+}} = e^{\pm \frac{e\phi}{kT}} \left[1 - \int_0^1 erf(v_R)\, d\left(e^{-v_T^2}\right) + \int_0^1 \left[erf(v_{2R}) - erf(v_{1R})\right] d\left(e^{-v_T^2}\right)\right] \qquad (9S)$$

C. Current Transmission

The net current density leaving the emitter is obtained by modifying the charge density integral, multiplying each distribution probability by the associated component of velocity away from the emitter. It is generally desirable to relate the net current to the emitted current. The emitted current is:

$$j_+ = n_e \overline{v_R} \qquad (10)$$

(11C)

$$\frac{j_+}{n_e} = \sqrt{\frac{2kT}{m}} \; \frac{\int_{\theta=0}^{\theta=\pi} \int_{v=0}^{v=\infty} e^{-v^2} v^2 \sin\theta \, dv \, d\theta}{\int_{\theta=0}^{\theta=\pi} \int_{v=0}^{v=\infty} e^{-v^2} v \, dv \, d\theta}$$

(11S)

$$\frac{j_+}{n_e} = \frac{\sqrt{\frac{2kT}{m}} \; \frac{2}{\sqrt{\pi}} \int_{v_R=0}^{v_R=\infty} \int_{v_T=0}^{v_T=\infty} e^{-(v_T^2 + v_R^2)} v_T v_R \, dv_T \, dv_R}{\frac{2}{\sqrt{\pi}} \int_{v_R=0}^{v_R=\infty} \int_{v_T=0}^{v_T=\infty} e^{-(v_T^2 + v_R^2)} v_T \, dv_T \, dv_R}$$

Without sacrifice these integrals may be limited to the range of θ from 0 to $\pi/2$.

$$\sqrt{\frac{m}{2kT}} \; \frac{j}{n_e} = \frac{\int_{\theta=0}^{\theta=\pi/2} \int_{v=0}^{v=\infty} v \, d\left(-e^{-v^2}\right) d(-\cos\theta)}{\int_{v=0}^{v=\infty} \int_{\theta=0}^{\theta=\pi/2} d\left(-e^{-v^2}\right) d\theta} \qquad (12C)$$

$$= \frac{\sqrt{\frac{2kT}{m}} \; \frac{1}{2\sqrt{\pi}} \int_{v_R=0}^{v_R=\infty} \int_{v_T=0}^{v_T=\infty} d\left(-e^{-v_T^2}\right) d\left(-e^{-v_R^2}\right)}{\frac{1}{2} \int_{v_R=0}^{v_R=\infty} \int_{v_T=0}^{v_T=\infty} d\left(-e^{-v_T^2}\right) d\left(erf\, v_R\right)}$$

$$= \frac{1}{\sqrt{\pi}} \sqrt{\frac{2kT}{m}}$$

$$\sqrt{\frac{m}{2kT}}\ \frac{j}{n_e} = \frac{\int_{\theta=0}^{\theta=\pi/2}\left(-ve^{-v^2}\right)_{v=0}^{v=\infty} d(-\cos\theta) - \int_{\theta=0}^{\theta=\pi/2}\int_{v=0}^{v=\infty} e^{-v^2}\,dv\,d(-\cos\theta)}{\pi/2}$$

$$= \frac{-1}{\sqrt{\pi}} \int_{v=0}^{v=\infty}\int_{\theta=0}^{\theta=\pi/2} d(\operatorname{erf} v)\, d(-\cos\theta)$$

$$= \frac{1}{\sqrt{\pi}}$$

Similarly the net current transmitted is:

$$\sqrt{\pi}\sqrt{\frac{m}{2kT}}\ \frac{j_N}{n_e} = 1 + \frac{\int_{\theta=-\pi/2}^{\theta=0}\int_{v=v_1}^{v=v_2} e^{-v^2} v^2 \sin\theta\, dv\, d\theta}{\frac{\sqrt{\pi}}{2}} \qquad (13C)$$

$$= 1 + \frac{\int_{\theta=-\pi/2}^{\theta=0} v_1 e^{-v_1^2} d(-\cos\theta) - \int_{\theta=-\pi/2}^{\theta=0} v_2 e^{-v_2^2} d(-\cos\theta) - \int_{\theta=-\pi/2}^{\theta=0}\int_{v=v_1}^{v=v_2} e^{-v^2}\, dv\, d(\cos\theta)}{\sqrt{\pi}}$$

$$= 1 + \frac{2}{\sqrt{\pi}}\left[\int_{\theta=-\pi/2}^{\theta=0} v_1 e^{-v_1^2} d(-\cos\theta) - \int_{\theta=-\pi/2}^{\theta=0} v_2 e^{-v_2^2} d(-\cos\theta)\right] +$$

$$- \left[+\int_{\theta=-\pi/2}^{\theta=0} \operatorname{erf} v_2\, d(-\cos\theta) - \int_{\theta=-\pi/2}^{\theta=0} \operatorname{erf} v_1\, d(-\cos\theta)\right]$$

The net current transmitted is:

$$\sqrt{\pi}\sqrt{\frac{m}{2kT}}\ \frac{j_N}{n_e} = 1 - \int_{v_R=v_{1R}}^{v_R=v_{2R}} \int_{v_T=0}^{v_T=\infty} d\left(-e^{-v_T^2}\right) d\left(-e^{-v_R^2}\right) \qquad (13S)$$

$$= 1 - \int_{v_T=0}^{v_T=\infty} \left(e^{-v_{1R}^2} - e^{-v_{2R}^2}\right) d\left(-e^{-v_T^2}\right)$$

Note here, that the minus sign connected with the integration from v_{1R}, the minimum radial velocity for returning particles to v_{2R}, the maximum radial velocity for returning particles, is the result of reversing the direction of integration, which was originally from $v_R = -\infty$ to $v_R = 0$.

D. Poisson's Equation and Its Integration

It has been useful to consider versions of Poisson's equation for cylinders in which the variable of integration is the fractional distance across the interelectrode space as well as versions with the radius as variable of integration. It is expected that the same technique will be valuable for spherical situations as for cylindrical situations. We will try to present, therefore, four sets of equations for comparison. Equations using distance x across the interelectrode spacing L are related to equations using radius r (non-dimensional, but with emitter radius R implied)

Poisson's Equation:

$$\nabla^2\left(\frac{e\phi}{kT}\right) = \left(\frac{R}{\lambda_D}\right)^2\left(\frac{n_{eo}}{n_{io}}\frac{n_e}{n_{eo}} - \frac{n_i}{n_{io}}\right)$$

Debye Length Definition:

$$\lambda_D = \left(\frac{\varepsilon_o\, kT}{4\pi\, n_i e^2}\right)^{1/2}$$

Independent Variable Transformations:

$$\begin{cases} r = 1 - \frac{L}{R}x \\ x = \frac{1-r}{L/R} \\ dr = \left[-\frac{L}{R}\right]dx \end{cases} \qquad (14)$$

Charge Density Conversions:

$$\frac{f(r)}{R^2} = \frac{F(x)}{L^2}$$

Poisson's equation for cylindrical coordinates takes the forms:

$$\frac{1}{r}\frac{d}{dr}\left[r\phi'\right] = f(r) \qquad (15C1)$$

$$\frac{1}{1-\frac{L}{R}x}\frac{d}{dx}\left[\left(1-\frac{L}{R}x\right)\frac{d\phi}{dx}\right] = F(x) \qquad (15C2)$$

Which may also be written:

$$\phi'' + \frac{\phi'}{2} = f(r) \qquad (16C1)$$

$$\phi'' - \frac{\frac{L}{R}\phi'}{1-\frac{L}{R}x} = F(x) \qquad (16C2)$$

Poisson's equation for spherical coordinates takes the forms:

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\phi'\right) = f(r) \qquad (15S1)$$

$$\frac{1}{\left(1-\frac{L}{R}x\right)^2}\frac{d}{dx}\left[\left(1-\frac{L}{R}x\right)^2\frac{d\phi}{dx}\right] = F(x) \qquad (15S2)$$

Which may also be written:

$$\phi'' + \frac{2}{R}\phi' = f(r) \qquad (16S1)$$

$$\phi'' - \frac{2\frac{L}{R}}{1-\frac{L}{R}x}\phi' = F(x) \qquad (16S2)$$

These equations may be integrated once to give:

$$r\phi' - r_o\phi_o' = \int_{r_o}^{r} r f(r)\,dr \qquad (17C1)$$

$$\left(1 - \frac{L}{R}x\right)\phi' - \left(1 - \frac{L}{R}x_o\right)\phi_o' = \int_{x_o}^{x}\left(1 - \frac{L}{R}x\right)F(x)\,dx \qquad (17C2)$$

$$r^2\phi' - r_o^2\phi_o' = \int_{r_o}^{r} r^2 f(r)\,dr \qquad (17S1)$$

$$\left(1 - \frac{L}{R}x\right)^2\phi' - \left(1 - \frac{L}{R}x_o\right)^2\phi_o' = \int_{o}^{x}\left(1 - \frac{L}{R}x\right)^2 F(x)\,dx \qquad (17S2)$$

Then:

$$\phi' = \frac{r_o}{r}\phi_o' + \frac{1}{r}\int_{r_o}^{r} r f(r)\,dr \qquad (18C1)$$

$$\phi' = \frac{1 - \frac{L}{R}x_o}{1 - \frac{L}{R}x}\phi_o' + \frac{1}{1 - \frac{L}{R}x}\int_{x_o}^{x}\left(1 - \frac{L}{R}x\right)F(x)\,dx \qquad (18C2)$$

$$\phi' = \frac{r_o^2}{r^2}\phi_o' + \frac{1}{r^2}\int_{r_o}^{r} r^2 f(r)\,dr \qquad (18S1)$$

$$\phi' = \frac{\left(1 - \frac{L}{R}x_o\right)^2}{\left(1 - \frac{L}{R}x\right)^2}\phi_o' + \frac{1}{\left(1 - \frac{L}{R}x\right)^2}\int_{x_o}^{x}\left(1 - \frac{L}{R}\right)^2 F(x)\,dx \qquad (18S2)$$

Equations 18 may now be integrated again:

$$\phi - \phi_o = r_o\phi_o' \ln\frac{r}{r_o} + \int_{r_o}^{r}\left[\frac{1}{r}\int_{r_o}^{r} r f(r)\,dr\right]dr \qquad (19C1)$$

$$\phi - \phi_o = \left[1 - \frac{L}{R}x_o\right]\phi_o' \ln\frac{1 - \frac{L}{R}x}{1 - \frac{L}{R}x_o} + \int_{x_o}^{x}\left[\frac{1}{1 - \frac{L}{R}x}\int_{x_o}^{x}\left[1 - \frac{L}{R}x\right]F(x)\,dx\right]dx \qquad (19C2)$$

$$\phi - \phi_o = r_o\phi_o'\left[1 - \frac{r_o}{r}\right] + \int_{r_o}^{r}\left[\frac{1}{r^2}\int_{r_o}^{r} r^2 f(r)\,dr\right]dr \qquad (19S1)$$

$$\phi - \phi_o = \left[1 - \frac{L}{R}x_o\right]\phi_o'\left[1 - \frac{1 - \frac{L}{R}x_o}{1 - \frac{L}{R}x}\right] + \int_{x_o}^{x}\left[\frac{1}{\left[1 - \frac{L}{R}x\right]^2}\int_{x_o}^{x}\left[1 - \frac{L}{R}x\right]^2 F(x)\,dx\right]dx \qquad (19S2)$$

Typically, in the use of equations (17) to (19) the second boundary condition is a potential ϕ_1, at r_1 or x_1, rather than ϕ'_o. Equations (19) may be used to obtain ϕ'_o

$$r_o\phi_o' = \frac{\phi_1 - \phi_o - \int_{r_o}^{r_1}\left[\frac{1}{r}\int_{r_o}^{r_1} r f(r)\,dr\right]dr}{\ln\frac{r_1}{r_o}} \qquad (20C1)$$

$$r_o\phi_o' = \frac{\phi_1 - \phi_o \int_{r_o}^{r_1}\left[\frac{1}{r^2}\int_{r_o}^{r_1} r^2 f(r)\,dr\right]dr}{1 - \frac{r_o}{r}} \qquad (20S1)$$

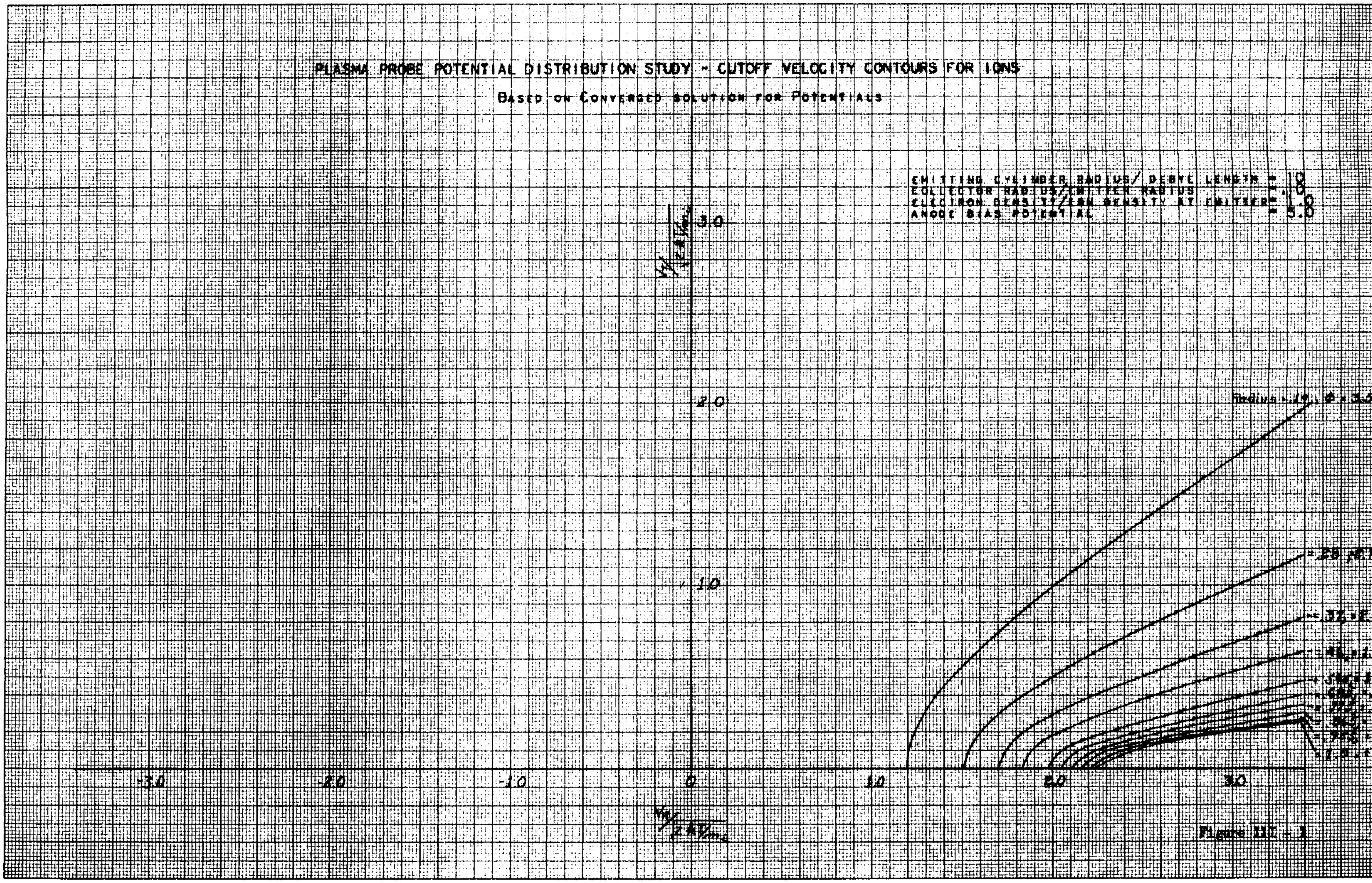
$$\left[1-\frac{L}{R}x_0\right]\phi_0' = \frac{\phi_1-\phi_0\int_{x_0}^{x_1}\left[\frac{1}{1-\frac{L}{R}x}\int_{x_0}^{x}\left[1-\frac{L}{R}x\right]F(x)dx\right]}{\ln\left[\frac{1-\frac{L}{R}x_1}{1-\frac{L}{R}x_0}\right]} \qquad (20C2)$$

$$\left[1-\frac{L}{R}x_0\right]\phi_0' = \frac{\phi_1-\phi_0-\int_{x_0}^{x}\left[\frac{1}{\left(1-\frac{L}{R}x\right)^2}\int_{x_0}^{x}\left[1-\frac{L}{R}x\right]^2F(x)dx\right]dx}{1-\frac{1-\frac{L}{R}x_0}{1-\frac{L}{R}x}} \qquad (20S2)$$

B. Charge Density Calculations for Cylindrical and Spherical Diodes

Equations 9C and 9S of Section A give formal expressions of the problem of evaluating charge density throughout the cylindrical and spherical diodes. Before the numerical evaluation of these expressions can be carried out it is necessary to establish a correspondence between the initial kinetic energy v_o^2 and the local trajectory angle with the tangential velocity in the spherical case. The application of the momentum and energy relations in Section A is the same for cylindrical and spherical problems, and means that both of these sets of correspondences are conveniently established using the same process.

The problems which must be faced in establishing the correspondence between variables for integration, and in carrying out the integration with sufficient accuracy, varies somewhat according to the particular situation. Figs. III-1 and 2 are typical of a probe situation with a monotonic potential distribution. Fig. 1 represents the particles which are repelled by the bias potential. At the emitter, all possible velocity vectors with components toward the collector are included. Since the local potentials away from the emitter are repulsive, these velocity vectors represent a loss in kinetic energy for each particle from its initial state. Some velocity vectors for returning particles will not be admissible. For example, particles with zero tangential velocity and initial kinetic energy higher than the bias potential will be able to make the transit to the collector. The boundary of the region of admissible velocities is essentially hyperbolic with an asymptote at the direction such that particles with very large tangential velocity moving under


PLASMA PROBE POTENTIAL DISTRIBUTION STUDY - CUTOFF VELOCITY CONTOURS FOR IONS
BASED ON CONVERGED SOLUTION FOR POTENTIALS
EMITTING CYLINDER RADIUS/ DEBYE LENGTH =
COLLECTOR RADIUS/EMITTER RADIUS =
ELECTRON DENSITY/ION DENSITY AT EMITTER =
ANODE BIAS POTENTIAL = 5.0
3.0
2.0
1.0
-3.0
-2.0
-1.0
0
1.0
2.0
3.0
Figure III - 1

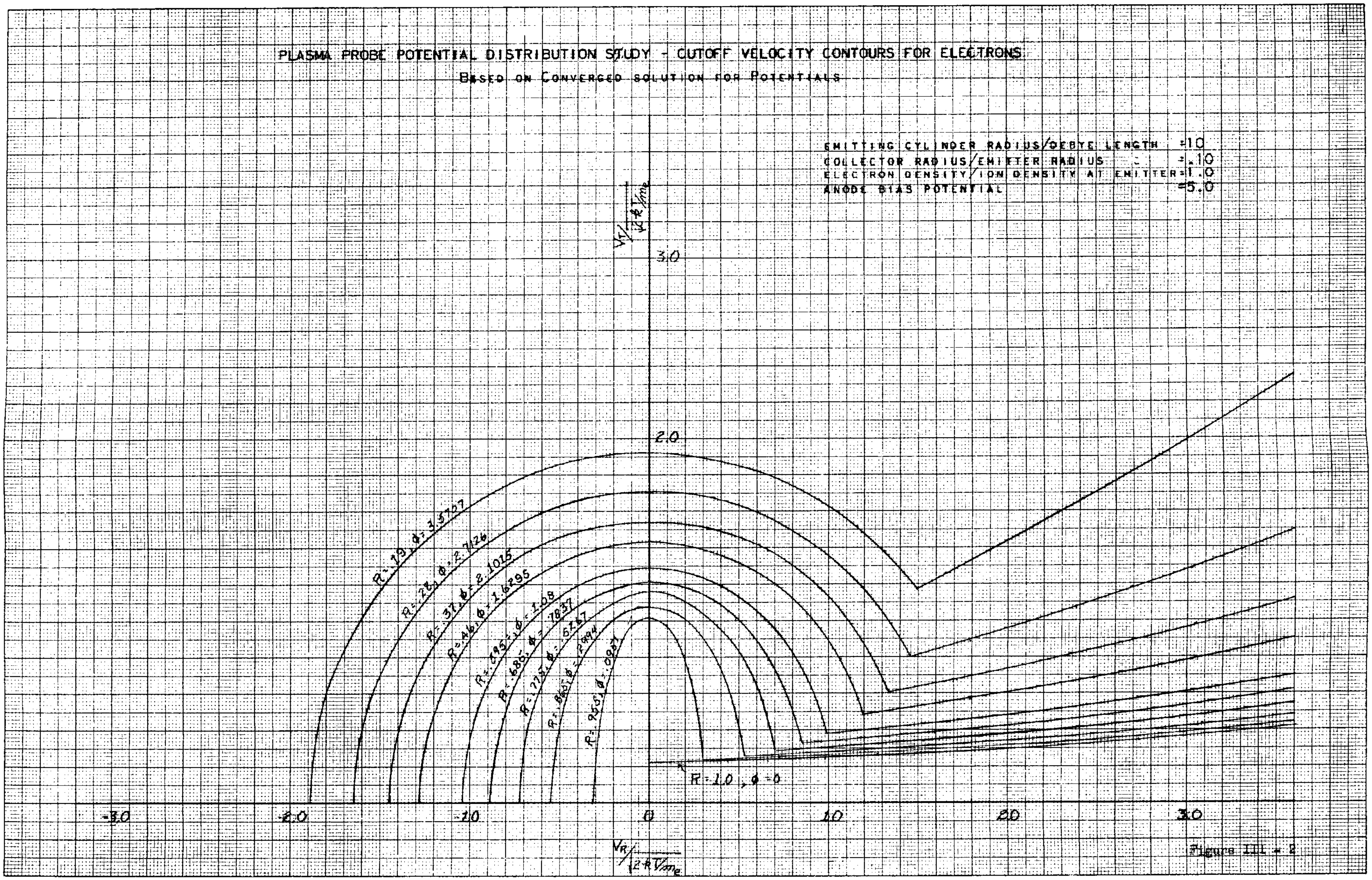

PLASMA PROBE POTENTIAL DISTRIBUTION STUDY - CUTOFF VELOCITY CONTOURS FOR ELECTRONS
BASED ON CONVERGED SOLUTION FOR POTENTIALS
EMITTING CYLINDER RADIUS/DEBYE LENGTH =10
COLLECTOR RADIUS/EMITTER RADIUS =.10
ELECTRON DENSITY/ION DENSITY AT EMITTER =1.0
ANODE BIAS POTENTIAL =5.0
R: .19, φ: 3.5707
R: .28, φ: 2.7126
R: .37, φ: 2.1015
R: .46, φ: 1.6295
R: .595, φ: 1.08
R: .685, φ: .7837
R: .775, φ: .5267
R: .865, φ: .2994
R: .955, φ: .0987
R: 1.0, φ: 0
3.0
2.0
-3.0
-2.0
-1.0
0
1.0
2.0
3.0

Figure III - 2

the influence of constancy of angular momentum have trajectories just tangent to the collector. Fig. 2 shows the corresponding situation for those particles which are attracted by the collector. The boundaries of the regions of admissible velocities in this case are determined by two different criteria. A particular velocity vector is admissible in the first place only if its kinetic energy is greater than the potential difference through which the particle has been accelerated: negative initial kinetic energies are not possible. This condition gives rise to an essentially elliptical inner boundary to the region of admissible velocity vectors. The second boundary is a boundary on particles returning from some maximum penetration toward the collector, which separates particles which had trajectories tangential to the collector and just missing it from particles which actually reached the collector. This boundary is essentially hyperbolic, with the same asymptote as was the case for the repelled particles, but with its major axis in the perpendicular direction. These boundaries are described as "essentially elliptic" and "essentially hyperbolic", although they may in some cases be rigorously elliptic and hyperbolic respectively, because the previous history in non-monotonic cases can cause small departures from the true quadratic curve.

In order to carry out the integration of 9C and 9S with reasonable accuracy, it is necessary to obtain points along the curves of Figs. 1 and 2 spaced quite close together. Ideally, points should be obtained with equal spacings of θ in 9C or of exponential $\left(-v_t^2\right)$ in 9S so that Simpson's rule could be used in the integration. Alternatively, some other means for getting regular point spacing as required for other curve fit integration methods might be considered. In this case, however, the variable of integration

is relatively easy to obtain as a dependent variable but much more difficult as an independent variable. It has seemed much easier to choose the tangential velocity at the emitter as the independent variable, to follow a particle with a given tangential velocity through the appropriate past and future histories, adjusting the initial radial velocity if necessary to satisfy conservation of energy, and to accept the value of the independent variable for integration that results. Following this procedure implies that integration will be performed by the trapezoid rule, and that increased accuracy if necessary will be achieved by using more points. The scheme as used has been to establish a table of 502 possible values of the initial tangential velocity starting with 0, and ranging from 10^{-4} to 10 by ratios of $(10)^{.01}$. The three values 0, 10^{-4}, and 10 are always used as part of the definition of the integrand curve. Intermediate points are introduced until adjacent points are spaced within some tolerance (typically 3°) and have successive radial velocities along the curve within a ratio of 1.5, and until points are spaced as close as possible to critical corners such as the vertex of the ellipse and the intersection between ellipse and hyperbola in Fig. III-2, subject to the closeness of spacing permitted by the table of initial tangential velocities. This has been a fairly satisfactory procedure. In some typical cases, the normal tolerances have seemed to permit a scatter in the resulting charge densities on the order of 0.1 - 0.2%. In at least one case, however, the density of points in the initial tangential velocity table has seemed to cause a jump of nearly 15° between the adjacent points along the integrand curve. In this case, there seems to be a possible error in the charge density

calculation of nearly 1%. These errors do not seem to have been the factors limiting the success of the investigations, but additional refinement may be needed in the future.

Fig.III-3 shows a typical set of cutoff velocity contours for a monotonic diode or external collector situation. The contours for the particles attracted by the collector, the electrons, are essentially hyperbolic. The asymptote is determined by the proportion of the kinetic energy of initial tangential velocity which is converted to radial velocity by the constancy of angular momentum. All the electrons make the transit to the collector; there are no returning electrons. Ions with zero initial tangential velocity are constrained to return to the emitter unless the kinetic energy of the initial radial velocity is greater than the bias potential. As the initial tangential velocity is increased, the conversion of kinetic energy of tangential velocity to kinetic energy of radial velocity makes it progressively easier for ions to negotiate the adverse potential. At some tangential velocity, the conversion is great enough so that all ions with this tangential velocity or more regardless of their initial radial velocity are able to make the transit to the collector. Beyond this point, there are no returning ions. The boundaries of the admissible velocity region are essentially hyperbolic for outward bound particles, essentially elliptic for returning particles. A special case arises at the spacing which is one-tenth of the electrode spacing. This point is close to a potential minimum at a radius greater than one where there was a potential maximum. In this case, an additional hyperbolic boundary to the regions of admissible velocity vectors, representing a locus of lower

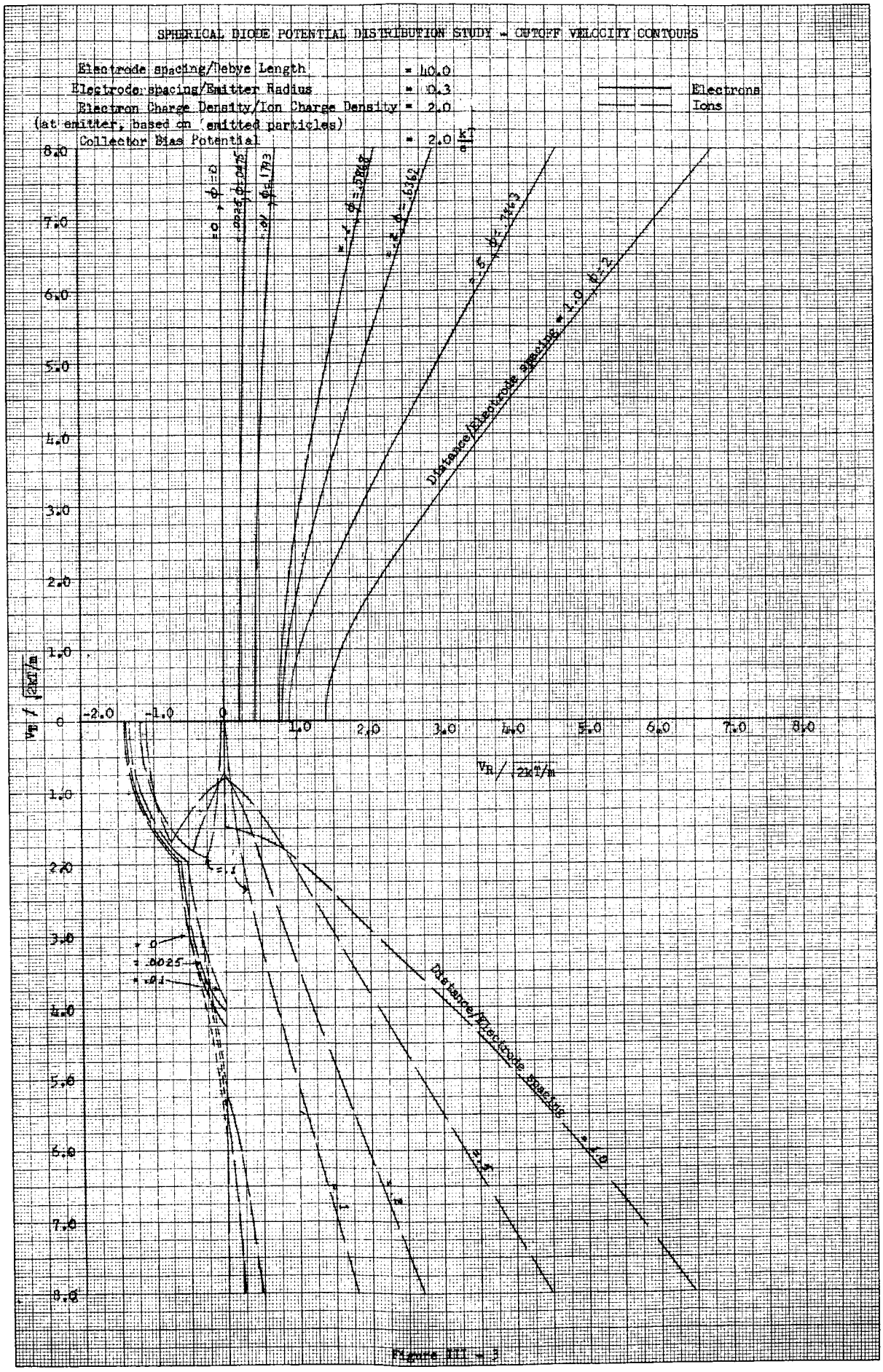

SPHERICAL DIODE POTENTIAL DISTRIBUTION STUDY - CUTOFF VELOCITY CONTOURS
Electrode spacing/Debye Length = 40.0
Electrode spacing/Emitter Radius = 10.3
Electron Charge Density/Ion Charge Density = 2.0
(at emitter, based on emitted particles)
Collector Bias Potential = 2.0 kT/q
Electrons
Ions
Distance/Electrode spacing = 1.0
Distance/Electrode spacing = .1
$V_R / \sqrt{2kT/m}$
$V_T / \sqrt{2kT/m}$

Figure III - 3

limits of kinetic energies of ions which just barely succeeded in passing the maximum and have then undergone acceleration, appears.

The configuration from which the cutoff velocity contours of Fig. III-3 were taken appears to be one in which some ions find that the critical barrier potential is the collector potential, while others are unable to pass some intermediate barrier. The contours for distance close to the emitter show corners, which appear to separate portions of the contour determined by the collector potential from the portion determined by the intermediate barrier. The solid curves were those obtained with a particular representation of the barrier. It seems likely that the barrier representation was not complete, and that the dashed contour would have been obtained with the complete barrier definition. The actual barrier potential increased the computed charge density by 1-3% at small distances, as compared with the initial assumption of charge density determined entirely by the collector potential. The remaining area is in a less dense region of the velocity distribution spectrum so it is believed that the charge density represented there is a small fraction of 1%. This should, of course, be checked for ultimate accuracy in a solution.

C. Step by Step Integration of the Differential Equation Using an Assumed Initial Slope

The most successful means for solving Poisson's equation for the plasma diode problem, at least for the external collector case, seems to have been the step by step numerical integration starting with an assumed initial slope. This technique has been used when the charge density distribution is determined only by the past history (the potential distribution between the emitter and the local point), and the collector potential. It has also been used effectively in cases where a potential extremum in the opposite direction from the collector potential, close to the emitter, serves as a barrier to some of the particles, and in the case there is a potential extremum in the same direction as the collector potential. Situations evaluated so far have been limited to those where the proportion of charged particles reflected by a barrier close to the emitter is only a few percent of the particles whose motion would otherwise be controlled by the collector potential. Consequently, it is not yet known whether the procedure would be equally effective for cases where the local barrier is the dominant feature. An example of this situation might be a case where the collector bias potential is attractive to electrons and the emitted electron current is very small compared to the emitted ion current, so that there sould be a potential maximum close to the emitter at a level greater than the bias potential.

In this investigation, the Blaess method for integrating an ordinary differential equation has been used in preference to the more common Runge-Kutta,

Adams-Moulton, or Adams-Sturmer methods, primarily because of its simplicity and its flexibility for arbitrary variation of the interval of integration. The basis for any of these methods of integration of ordinary differential equations is the manner in which higher order derivatives are used for extrapolation and for improving the solutions. Most of these methods evaluate the higher order derivatives from previous history and, consequently, are more or less dependent on retaining the same interval of integration. In the Blaess method, higher derivatives are determined from data acquired in a series of five integration steps and used to improve the integral values at these steps. The interval of integration may be chosen independently for each sequence of five steps. The author's introduction to this method came by third-hand contact with a scientist who had been engaged in studying captured German documents after World War II. No derivation was given at the original contact. Later efforts to produce such a derivation was given at the original contact. Later efforts to produce such a derivation led instead to the conclusion that there were albegraic errors in the formulas provided. Consequently, the formulas were derived again from scratch; the derivation is produced in the Appendix.

The logic behind the use of the assumed initial slope and subsequent step by step integration is described in simplest form for those cases where the only subsequent history that need to be considered is the bias potential at the collector. In this exposition the bias potential will be assumed to be positive or attractive to electrons.

The range of validity of the assumption about future history lies roughly in the range where the charge density attributed to emitted electrons is greater than 0.4 times the charge density due to emitted ions (since almost all ions are repelled by the collector bias potential, the total charge density due to ions is roughly double this level) and less than 2.0 times the emitted ion density. Within this range the electron charge density decreases more rapidly under the acceleration of a small potential difference than the decrease of the ion density due to repulsion. If the net charge density is not initially positive (ion rich) it quickly becomes so. Fig. III-4 illustrates the problem of obtaining the solution.

The solution is known to lie entirely within the rectangular area defined by the spacing from the emitter to the collector and by the potential range from the emitter potential to the bias potential. At very low plasma densities (electrode spacing a fraction of a Debye length) the space charge has a negligible influence on the potential distribution. In the absence of better information the initial slope may be assumed to be that appropriate for a linear potential distribution. It is always desirable, of course, to use the best available estimate for the initial slope.

As the net charge density becomes positive, the potential distribution curve has negative curvature and the calculation of the potential distribution falls below the projection of the initial slope. Characteristically, the calculated curve oscillates, remaining positive throughout and rising slowly as the average charge density decreases slowly with increasing radius. The

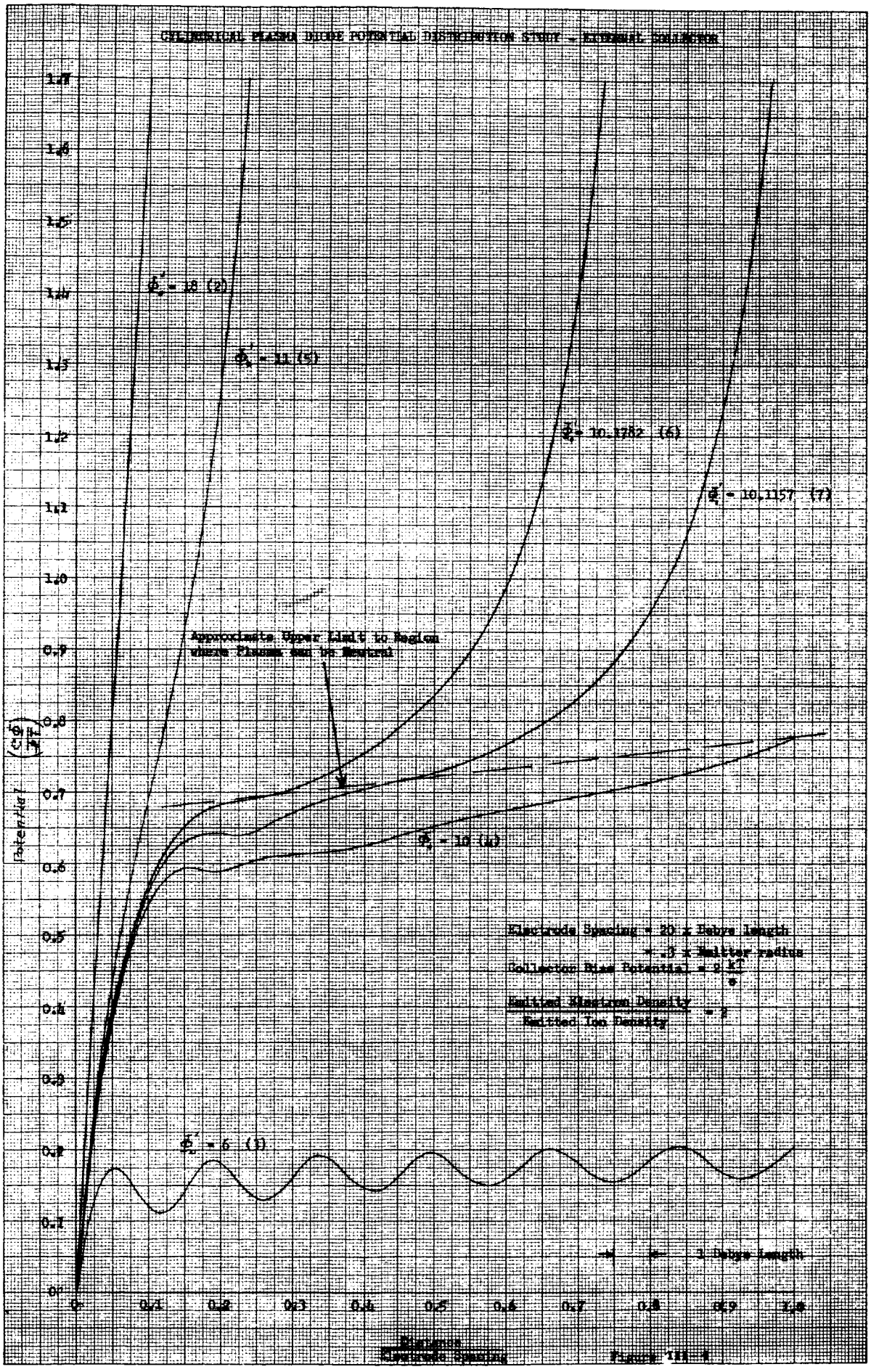

CYLINDRICAL PLASMA DIODE POTENTIAL DISTRIBUTION STUDY - NORMAL COLLECTOR
$\phi_c' = 18$ (2)
$\phi_c' = 11$ (5)
$\phi_c' = 10.1782$ (6)
$\phi_c' = 10.1157$ (7)
Approximate Upper Limit to Region where Plasma can be Neutral
$\phi_c' = 10$ (4)
$\phi_c' = 6$ (1)
Electrode Spacing = 20 x Debye Length
= .3 x Sheller radius
Collection Bias Potential = 9 $\frac{kT}{e}$
$\frac{\text{Emitted Electron Density}}{\text{Emitted Ion Density}} = 9$
$\frac{\phi}{\phi_c}$
Potential
1 Debye Length
Distance
Electrode Spacing
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

Figure III-4

calculation is discontinued when the integration has been carried past the distance associated with the electrode spacing. Curve # 3 on Fig. III-4 represents this situation. The integration interval must be kept small enough so that several of the five-step sequences of the Blaess method of integration fall within each wave length. Since the wave length usually starts out being in the neighborhood of three Debye lengths, satisfactory results have been obtained by choosing 0.1 Debye lengths as the initial step, giving 6 five-step sequences in each wave length. The wave length tends to increase as the radius increases the size of the integration step.

If the initial slope is too large, the integrated potential distribution curves look like curves 2 and 5 of Fig. III-4. Curves 2 and 3 of Fig. III-4 cross opposite sides of the rectangular area and indicate outer limits to the true solution. Subsequent slopes are assumed in between the previously considered limiting values. Thus, curve # 4 has too low an initial slope but passes closer to the desired end point than curve # 3. Curves 5 and 6 have initial slopes that are too large; these curves show progressive improvement. Curve 7 passes very close to the correct end point. Curve # 4 shows quite clearly how the initial wave length of the first oscillation is between 2 and 3 Debye lengths while the wave length of the second oscillation is approximately 5 Debye lengths, illustrating the potential value of ability to adapt to variable integration steps. For the final solution under these conditions just one oscillation remains. It appears to be characteristic that some locus exists, suggested by the dashed line, above which the net charge density must necessarily be negative (or electron rich). Below this level either positive or negative net charge densities may exist depending

upon whether retardation of the ions in the average overall adverse potential gradient or acceleration in a locally favoring potential gradient happens to be dominant. The correct solution has progressively less net positive charge density at each following potential maximum until the distribution curve passes through the locus, after which an electron rich sheath extends the remaining distance to the collector.

Once a trial solution of the type demonstrated in Figure III-4 has been obtained it is appropriate to check on the validity of the assumption that the collector potential was actually the only feature of the potential distribution curve to determine ion repulsion. The general regions which may impose a more severe repulsion than the collector on the ions can easily be found. These are the regions close to the emitter where the adverse potential gradient is most severe. Provisions are made for testing several points along this portion of the curve for a possible repelling influence on various portions of the ion distribution spectrum.

D. Use of Cylindrical Diode Computer Program

CYLDIODE is a computer program developed at the Evendale, Ohio installation of the General Electric Company for the IBM 7094 computer. It has been written in Fortran II language. Since, however, it makes use of several features of the Fortran compiler that are apparently peculiar to the Evendale installation, some special precautions are required by any user who desires to make modifications. In particular, decimal input is handled using GE DING, the TABLE statement is used to present the list of input variables and for various other purposes, FORMAT GENERATOR is used to generate the format for printed output, and GE ERROR is used for various error checking purposes. In addition, one subroutine, CUTOFF, has been extensively modified in machine language, resulting in an improvement in operating efficiency by a factor of 2-3. The program is available in self-contained form for use with any IBM Fortran II monitor. Normal practice at the Evendale installation is to use tape A2 for input and to write output, both printed and punched card, on tape A3. Modification for other systems is readily accomplished. In particular, operation at the NASA Marshall Space Flight Center is carried out with input on A3, printed output on A2, punched card output on A5. Provisions have been made for program exit at the end of an iteration by depressing Sense Switch 4. Summary write-ups on the features of the DING, ERROR, TABLE are included as Appendices.

Description of the use of CYLDIODE will be presented, giving first the minimum input for a cylindrical plasma diode (with external collector) using the technique of iterating on the assumed initial slope under the assumption that

the potential at the collector is the only item of future history which is needed for charge density evaluation. Second, the use of additional features which are helpful in expediting convergence and/or allowing for intermediate potentials which serve as barriers to some particles will be described. Third, some problems connected with program operation for internal collectors (a problem which has not been solved in a very satisfactory manner) will be discussed. Fourth, some discussions will be given of the technique of assuming the entire potential distributions, calculating the charge density distribution appropriate for the assumed potential distribution, double integration of the charge density to satisfy the boundary conditions, and, based on the results, choice of a new assumed charge distribution for another iteration.

1. Minimum Input for Cylindrical Plasma Diode Configurations

Table III-1 summarizes the minimum input required for obtaining cylindrical plasma diode solutions. Symbols designating input variables are capitalized with comma's separating data fields. Symbols are listed on separate lines for clarity but may be punched on a single line up to the card capacity of columns 2-72. The = in column 1 for the end of the data record may be punched in column 1 of the last card of data in the set. With this minimum input given the program will normally assume that the initial slope is the same as the average potential gradient from the emitter to the collector. Integration will then be carried out with 1/10 Debye length steps. This step size will then be maintained at least until the first time when the net charge density

TABLE III-1

MINIMUM INPUT FOR CYLINDRICAL PLASMA DIODE CONFIGURATION

Col. 1	Col. 2
4	KEND, 7, identification of the manner of obtaining the solution.
4	INSIDE, 4, designates the configuration, diode or external collector with electrode spacing as a fraction of emitter radius and plasma density given as a number of Debye shielding lengths in the electrode spacing.
	MONTH, ____, ____, ____, month, day, and year (2 digits each) of the calculation.
	KPRINT, 2, asks for printout of all integration steps, otherwise a maximum of 40 to fit on a single page would be printed.
	POT, ____, the collector bias potential in units of kT/e.
	DEBYE, ____, the number of Debye lengths in the electrode spacing.
	SPACE, ____, the ratio of electrode spacing to emitter radius, must be negative for external collectors.
	ETA, ____, the ratio of charge density due to emitted electrons to charge density due to emitted ions.
=	end of data record.

changes sign from positive to negative (in the case of the positive bias potential; it would be negative to positive in the case of a negative bias potential). After this point the step size will be increased at any time when more than 15 integration steps are carried out between changes of size of the net charge density. Integration is carried out until either the distance from the emitter equals the electrode spacing or the potential reaches the bias potential. Then the direction of the error is observed and an appropriate adjustment to the initial slope made. The magnitude of the adjustment will be 30% of the curve slope or 30% of the bias potential whichever is the larger. After 2 successive potential distribution curves have straddled the desired input, successive initial slopes will be chosen by interpolation. An error estimate is given by the intercept of the potential distribution curve with the distance of the electrode spacing. This intercept will be determined by extrapolation of a straight line through the last two calculated points for potential distribution curves passing beyond the prescribed end point. Interpolation will be linear after the first pair of curves has straddled the desired answer, by a parabolic curve fit thereafter. The program will normally stop after ten iterations if the desired end point potential has not been matched within 10^{-5} at that time. The actual practice has always been to use some means for improving the estimate of the initial slope, so there has been no serious effort to determine whether the minimum input would be sufficient to obtain solutions. It is difficult to give a good estimate of the computer running time since this depends strongly on the number integration intervals. With 40 Debye lengths electrode spacing and with 2.0 $\frac{kT}{e}$ bias potential, a potential distribution curve falling below the bias potential has been found to use about 200 integration intervals requiring approximately 3 minutes. Curves passing above the desired potential

tend to have much larger integration intervals since waves in the solution are much less likely to be present, and the integration intervals are rapidly increased. Also the distribution curve passes beyond the assigned limits at an earlier distance, which further reduces the number of integrations which must be carried out. The running time is proportionately less when the electrode spacing is a smaller number of Debye lengths.

2. Optional Supplementary Input Data for Increased Flexibility in Obtaining Cylindrical and Spherical Diode Solutions

Table III-2 gives a summary description of optional input which may be added before the end of record designation in Table III-1 for increased flexibility. Many of the optional input values listed in Table III-2 should be self explanatory.

Provision for varying ANGTOL, VELTOL was made at a time when some erratic behavior of the net charge density, an extremely small difference between very nearly equal ion and electron charge densities was observed. Some subsequent investigation suggested that the erratic behavior might be a consequence of the minimum spacing in the Table of values for initial tangential velocity, even though that spacing is very small. In the investigation, up to the time of preparation of this report, the slightly erratic behavior has not been a serious obstacle to the progress of the investigation.

It is almost always desirable to use an input slope for the initial slope. The standard value is a lower limit, a rather remote lower limit, for those

TABLE III-2

OPTIONAL INPUT FOR FLEXIBILITY IN SOLUTION OF CYLINDRICAL AND SPHERICAL DIODE PROBLEMS.

KYLSPH,2, designates a spherical diode solution.

NLIM, ____, defines a maximum allowable number of iterations different from 10.

SLOPE, ____, is an initial slope to be used for the first iteration, if a different choice from the standard is desired.

DSLOPX,____, is an arbitrary increment for changing the initial slope which may be specified in preference to the normal 30% change.

POT1, ____, ____, ...,, Arrays defining a previously calculated potential distribution passing beyond the desired bias
XPOT1,____, ____,.....,, potential. POT1 refers to the potentials, XPOT1 to the distances at which these potentials are found. The array inputs must end with two commas.

SLOPEU, ____, is an initial slope for the potential distribution array, POT1 if the normal numerical derivative formula is not expected to be sufficiently accurate.

POT2, ____, ____, ...,, arrays defining a previously calculated potential
XPOT2,____,____, ...,, distribution passing inside the desired potential.

SLOPEL, _____, is the initial slope for the POT2 distribution

JFIRST, _____, is a number, 1 + 5n, of values in the POT1 array which are to be exempted from further iteration.

TOL, _____, is a convergence tolerance different from 10^{-5}.

POT, _____, 1, ..., ____, ____,, is an array of pairs of values, potential and distance from the emitter defining barrier potentials to be considered in evaluating charge density. These value pairs are to be in order of decreasing distance from the emitter. A maximum of 30 pairs is permissible. Note that the double comma ending the array is essential here, but is not to be used in connection with the single bias potential of the minimum input.

ANGTOL, ____, is an angular tolerance on successive points along cutoff velocity contours if this should be different from 3°.

VELTOL, ____, is a radial velocity ratio tolerance between successive points along cutoff velocity contours, if this should be different from 1.5.

Table III-2, Continued

WRITE, ____, ____, is a series of up to 20 distances at which printout of the cutoff velocity contours is desired.

NWRMAX, _____, is the number of WRITE values.

FMULT, ____, is the number of integration intervals in a Debye length for use at the start of the integration, if different from 10.

cases where the charge density close to the emitter is nearly neutral. As was mentioned in connection with the minimum input, the computer running time for a potential distribution passing beyond the desired bias potential tends to be much shorter than that for a potential distribution passing inside the bias potential. Consequently, it is desirable to specify the first initial slope in such a way that the exploratory iterations prior to straddling of the solution shall be carried on in the regime outside of the desired end point.

It may happen that a series of iterations has previously been performed without arriving at satisfactory convergence, either because the specified number of iterations was insufficient or because the estimated running time was insufficient. In such a case, it is desirable to estimate the next value of SLOPE to be used from the previous trend. It is also desirable to put in some record of the best available approximation to the solution on either side of the correct solution. This is the function of the inputs for POT1, POT2, XPOT1, XPOT2, SLOPEU, and SLOPEL. As far as the mechanics of the program are concerned, skeleton values of the arrays are sufficient. The skeleton consists of three values: The zero initial potential and distance, and two values defining the end of the distribution curve for use in measuring the error. The remainder of these arrays, if input, is reserved for use in making a print-out comparison of the difference in potential between curves which pass above and below the desired end point.

It was anticipated in the course of development of the computer program that situations might arise such that curves passing inside and outside of the desired end point might eventually produce series of points close to the emitter such that the difference between them could not be distinguished within the eight significant figure accuracy of the computer. The variable JFIRST was introduced as a means for specifying that such points should be considered as fixed, and iteration continued from a point where the difference between the inside and outside curve could be detected. This feature has not been tested in the investigations to date.

The provision for input of pairs of values, potential and distance, so that the calculation would not be restricted to cases where the collector bias potential was the sole factor determining how many charge particles are returned to the emitter, appears to be generally satisfactory. The feature has served its purpose for cases in which a steep potential gradient close to the emitter serves to return 2-3% more of the emitted particles than are turned back by the collector. The feature has also been tested and found to work in situations with an extremum near the emitter opposite in sign to the bias potential. The program will discontinue the iteration, if it finds a local potential extremum beyond the specific extremum by more than 10% of the bias potential. It is reasoned that any curve, which has an extremum more than 10% of the bias potential, beyond the specified extremum and in the opposite direction from the bias potential and then still passes outside of the bias potential, already shows that the specified potential curve definition is not adequate and that a new distribution should be input which act as a barrier to more of the particles.

It has been considered beyond the scope of the present investigation to make the choice of the series of barrier potentials automatic in the program.

3. Solutions for Probes or Internal Collectors Using the Assumed Initial Slope

The manner of execution of CYLDIODE is intended to make its use possible for internal collectors or probes. The probe problem has turned out to be substantially more difficult than the diode problem. In contrast to the diode problem, where approximate solutions can be obtained in many cases by assuming that the charge density is determined by the past history of integration from the emitter and a single future history point at the collector, and where considerably greater refinement is possible by the use of three of four additional defining points which are generally located close to the emitter, the critical barrier potentials in the probe case are usually to be found close to the collector. Experience in getting the solutions of this type has been exploratory in nature without conspicuous success, so discussion of it must be speculative.

Table III-3 lists inputs for the probe investigation different from those input variables contained in Tables III-2 and III-2. The chances of obtaining solutions without the initial input of several points defining a prediction of the potential distribution curve seem to be slight. Figure III-5 shows input quantities that were attempted for one representative case. Apparently this prediction is not very good. With this general slope of the predicted potential

TABLE III-3

INPUT FOR CYLINDRICAL AND SPHERICAL PLASMA PROBES

Col. 1
Col. 2

INSIDE, 3, designates the configuration, probe or internal collector, with electrode spacing given as a fraction of emitter radius and plasma density given as a number of Debye lengths in the electrode spacing.

SPACE, ____, the ratio of electrode spacing to emitter radius, positive for internal collectors.

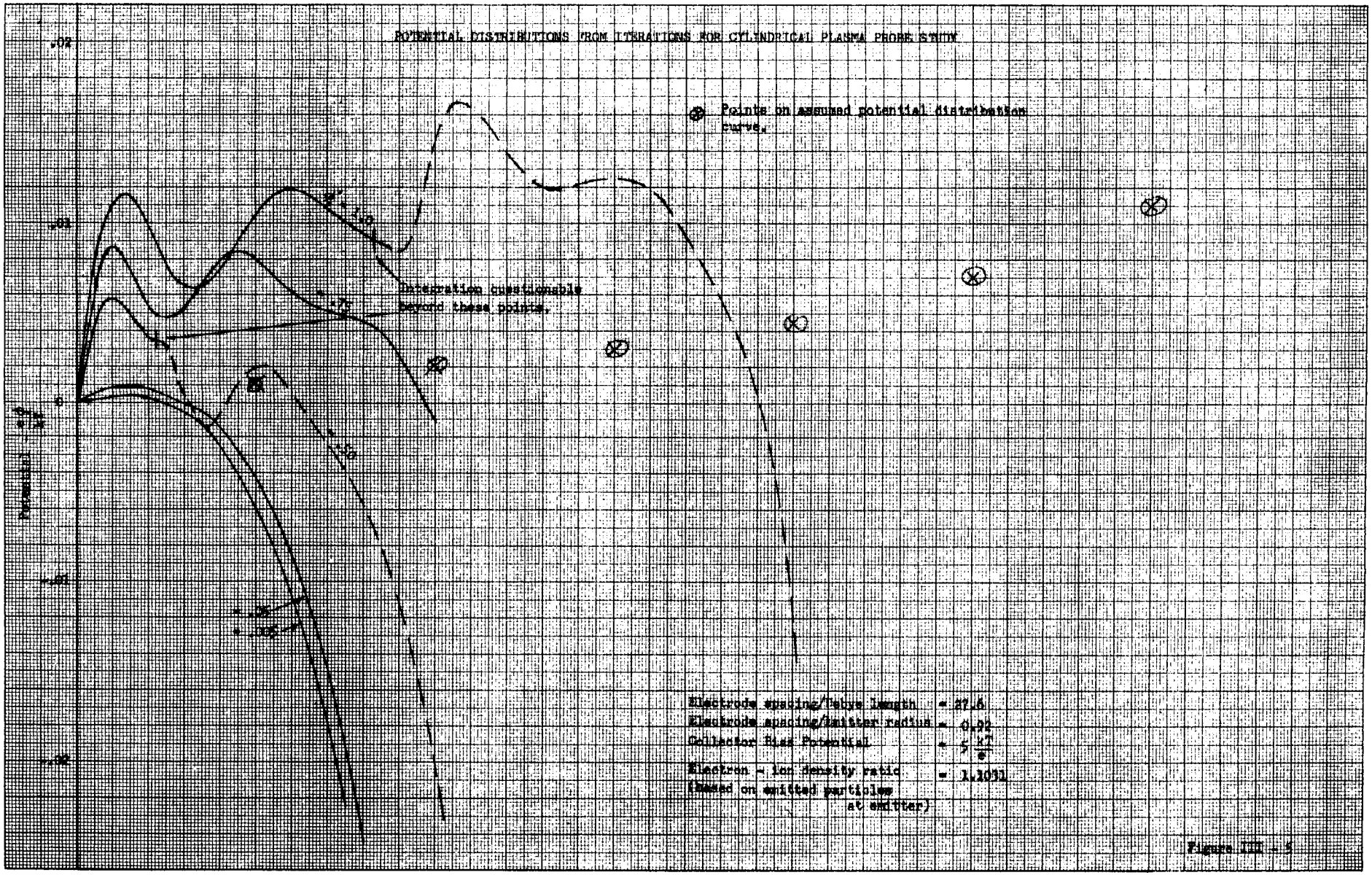

POTENTIAL DISTRIBUTIONS FROM ITERATIONS FOR CYLINDRICAL PLASMA PROBE STUDY
Points on assumed potential distribution curve.
Integration questionable beyond these points.
Electrode spacing/Debye length = 27.6
Electrode spacing/Emitter radius = 0.02
Collector Bias Potential = 5 kT/e
Electron - ion density ratio = 1.1011
(Based on emitted particles at emitter)
Potential
Distance / Electrode Spacing
0
0.1
0.2
0.3
0.4
0.5
0.6

Figure III - 5

distribution it is found that a progressively larger proportion of the electrons reaching particular radii are able to negotiate the barriers and reach the collector. In the presence of small magnitude repulsive potentials the ion density remains nearly constant. In every attempt so far, the electron density drops below the ion density, however, leading to negative curvature of the calculated potential distribution curve. It is, of course, necessary for the ion density to drop finally below the electron density to give the positive curvature which can lead to the bias potential. It is believed that the lack of success to this point reflects lack of understanding as to the nature of the potential distribution curve which should be expected. A possible procedure in continuing the investigation is to use the predictions on Fig. III 5 and raise the initial slope until one is found large enough so that the net charge density does become and remain electron rich. The shape of the curve obtained in that way should give a much better clue than has yet been available, as to the real nature of the barrier potentials for electrons. Past investigations have shown that oscillatory type solutions are probably to be expected. There is no information to permit deductions as to whether the wave lengths of such oscillations increase with distance from the emitter toward the collector, as was the case for the diodes, or whether, as is perhaps more likely, wave lengths decrease and amplitudes increase in this direction. In the probe situation, conditions near the emitter are expected and desired to simulate an infinite plasma which should have only infinitesimal disturbances. If then, disturbances are present they should at least increase in magnitude as the collector is approached. By analogy with probe results it seems likely that an increase in magnitude will be accompanied by a decrease in wave length.

4. Solution by Iteration on an Assumed Potential Distribution

Many of the results were obtained during the course of this study with the technique of iterating on an assumed potential distribution. At the time of writing this report, it is believed that this technique is obsolete and that all results obtained with it could have been obtained more easily by the initial slope technique. Since, however, some future application may be treated more easily with its use, some discussion of it may prove rewarding. Input variables are listed in Table III-4. Some of these have been discussed in previous paragraphs, but have broader significance or specialized requirements and are repeated for this reason.

The simplest way to get started on any problem using this technique is to assume that the input points are to be equally spaced and to use a linear potential variation as the assumed input potential distribution. This would be accomplished by using the minimum set of inputs as listed in part A of Table 4. For this purpose, KEND should be input as 2. With the data set up in this manner, one iteration will be performed. The results will be printed out (if KPRINT has been input as 2) and the program will return to read more data. It will quit if the additional data is not available.

The next added sophistication in the use of the program comes by using KEND = 3 or 6. The first iteration will be performed in the manner described in the previous paragraph. The input of the value for WEIGHT and KEND = 3 allows the program to choose a new potential distribution for use in the second iteration by combining the assumption for the first iteration and the

TABLE III-4

INPUT FOR ITERATION ON ASSUMED POTENTIAL DISTRIBUTIONS

A. Necessary Input Variables

4 POT, ____, collector bias potential.

ETA, ____, ratio of charge density near emitter due to emitted electrons to charge density due to emitted ions.

INSIDE, __, = 1 for probe with collector radius as fraction of emitter radius, emitter radius in Debye lengths.

= 2 for diode with collector radius as fraction of emitter radius, emitter radius in Debye lengths.

= 3 for probe with electrode spacing as fraction of emitter radius, spacing in Debye lengths.

= 4 for diode with electrode spacing as fraction of emitter radius, spacing in Debye lengths.

DEBYE, __, number of Debye lengths in emitter radius for INSIDE equals 1 or 2. Number of Debye lengths in electrode spacing for INSIDE equals 3 or 4.

RAD, ____, ratio of collector radius to emitter radius for INSIDE equals 1 or 2.

SPACE, ___, ratio of electrode spacing to emitter radius, positive for INSIDE = 3, negative for INSIDE = 4.

JMAX, ___, number of points for charge density evaluation and potential calculation, must be odd.

KEND, ___, = 1 erase potential history after completing current iteration, read more data.

= 2 retain potential history after completing current iteration, read more data.

Table III-4, Cont'd

= 3 perform NLIM iterations using WEIGHT on POT1 array, (1 - WEIGHT) on POT2 array, then read more data.

= 6 perform NLIM iterations choosing weight factor to eliminate error between POT1 and POT2 at midpoint.

= end of record

B. Optional Input Variables

ANGTOL, KPRINT, KYLSPH, MONTH, NLIM, NWRMAX, TOL, VELTOL, WRITE are optional input variables with the same significance as in Table I or Table II.

POT1, ____, _____, ...,, is an array of potentials, thought of as the input assumption for the previous iteration (the double commas signal the end of the array).

POT2, ____, _____, ...,, is an array of potentials, thought of as the output of the last previous iteration.

WEIGHT, _____, is the weight factor assigned to the POT1 array in establishing the assumed distribution for the new array; (1-WEIGHT) is the factor assigned to the POT2 array.

KPOT, 1, is to be input to make use of the WEIGHT feature.

KRAD, 1, is to be input if it is desired to use an array of calculation point distances which are not equally spaced.

RADII, ____,_____, ...,, is an array of radii or distances, called for by KRAD = 1, when variable spacing between points is desired. Points must be in pairs with equal spacings from the previous points.

JMIN, ____, an odd number (>1) if it is desired to restrict iteration to a portion of the interelectrode spacing starting with position no. JMIN.

result of that iteration in the proportions WEIGHT for the assumption and (1-WEIGHT) for the result. After a second iteration, the process is repeated, applying the factor WEIGHT to the input assumption for the second iteration, and again, (1-WEIGHT) to the result of that second iteration. This process is repeated NLIM times or until the convergence test has been satisfied before returning to read more data.

With the option KEND = 6 the first and second iterations are performed in the same manner as under option KEND = 3. Subsequent iterations are carried out using a process shown schematically in Fig.III-6. Here the assumed potential at the mid point between electrodes for each iteration is plotted as abscissa; the calculated potential at that point is the ordinate. It is supposed that a smooth curve connects all possible pairs of values of this type, so that choice of a proper weight will serve to choose an input potential which can be carried through the process and result in returning the same value. It is further supposed that all other potentials deserve modification in the same proportion as the mid point potentials. The use of this procedure, when and if the suppositions are justified, frees the calculations from dependence on the suitability of an arbitrary choice of weight factor.

Unfortunately, the supposition that potentials away from the mid point behave in a manner proportional to the potential at the mid point is often quite far from the truth. In particular, it often seems that there may be a sort of neutral plasma locus (like the dashed line of Fig.III-4). If the potential distribution could be imagined to lie along this locus and then be subjected

$$WEIGHT = \frac{\Phi_{MA3} - \Phi_{MA2}}{\Phi_{MC2} - \Phi_{MA2}}$$

$$\Phi_{MA2} = \frac{\Phi_{MC1}(\Phi_{MA2} - \Phi_{MA1}) - \Phi_{MA1}(\Phi_{MC2} - \Phi_{MC1})}{1 - \Phi_{MC2} + \Phi_{MC1}}$$

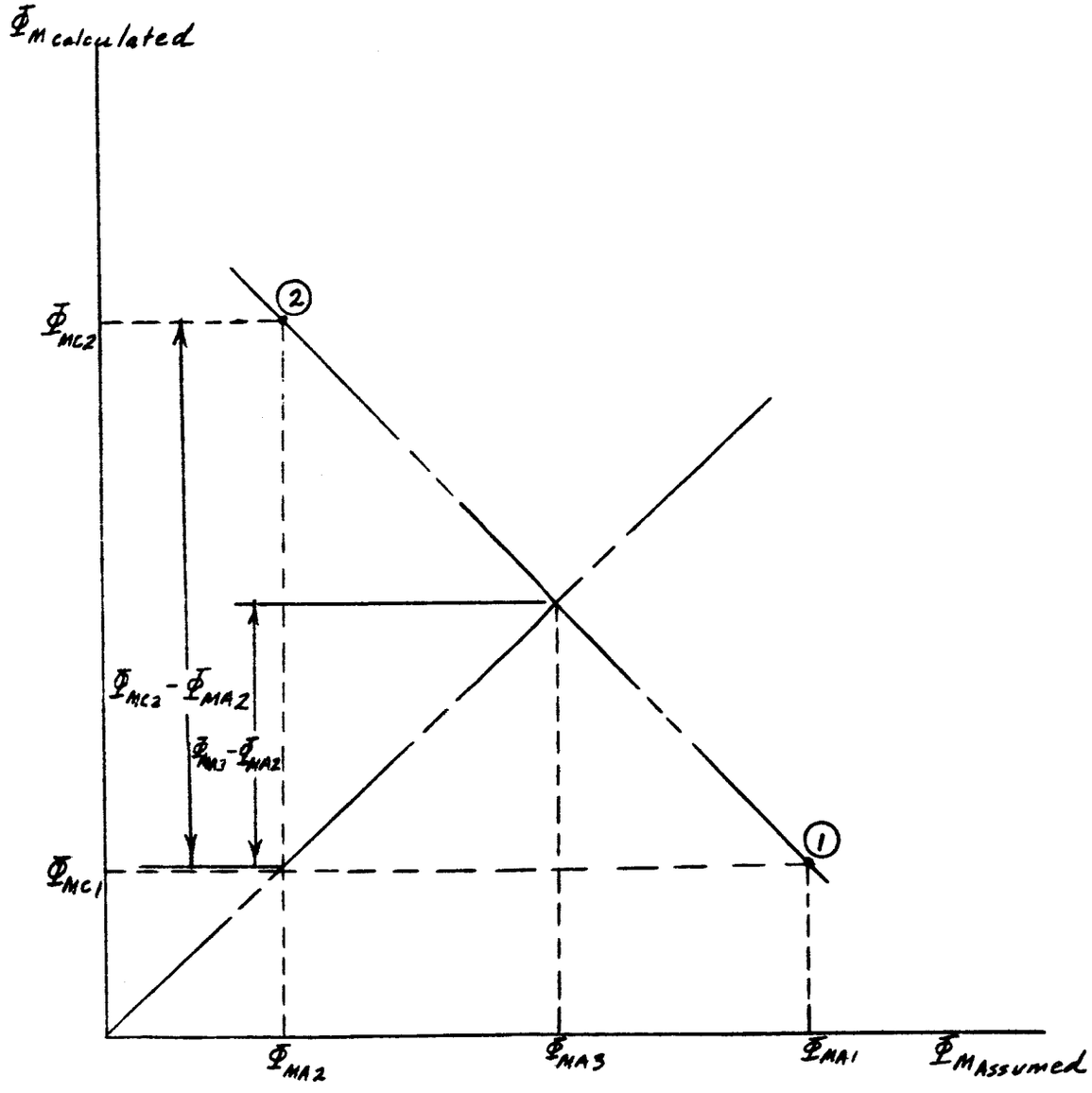


Figure III-6: Schematic of Procedure for Choosing Weight Factor with KEND = 6.

to a local disturbance, either positive or negative, it would be found that the net charge density changes sign in the same direction and leads at the end of the next iteration to an erratically different estimate of the potential distribution. In that respect one many conclude that this technique of iteration on the assumed potential distribution worked as long as plasma densities are low enough so that the actual potential distribution curve crosses the neutral plasma locus at a sufficient large angle, so that most potential disturbances of reasonable size do not result in change of sign of the net charge density over an extended range. This condition appears to be satisfied if the electrode spacing is 10 Debye lengths or less, and is definitely violated for an electrode spacing of 20 Debye lengths or more.

A part of the success or lack of success in using this technique depends on the ability to choose suitable values for the weight factor. The most appropriate weight factor changes rapidly with the plasma density. With electrode spacing of perhaps 6 Debye lengths a weight factor of 0.5 may be suitable. At 10 Debye lengths electrode spacing, the best weight factor is ordinarily at the level of 0.9, and at 15 Debye lengths electrode spacing, the best weight factor is apparently around 0.97. This optimum probably also depends on the electron-ion density ratio and perhaps on the level of the collector bias potential.

Some experience showed that the tailoring of weight factors to meet the immediate situation as carried out as KEND = 6, works well for the first several iterations and then starts giving erratic results. This situation arises when

proportional displacements of the assumed potential curves are fairly good for large errors at the mid point but inappropriate for small errors. It has been found appropriate to use 3 or 4 iterations using the KEND = 6 technique followed by several other iterations using the standard, arbitrary weight factor technique (KEND = 3). To follow this technique, the number of iterations, NLIM, is specified with the input for KEND = 6 after which the program is to read more data which will be a new value of NLIM and the new designation of KEND.

It sometimes happens that it is desired to use a potential distribution from a previous case, with a small change in one or two of the parameters. This potential distribution cannot be input directly for use with KEND = 6 because there will be no way to inform the program with KEND = 6 option that the input data do not constitute a legitimate set of assumed input and calculated result. Using the KEND = 3 technique, it would be permissible to give the same distribution as input for POT1 and POT2 using whatever weight factor is desired. If, however, it is desired to perform several iterations using KEND = 6 it is suitable to start with one iteration using KEND = 2, the assumed distribution input with POT2, and WEIGHT = 0. After the first iteration is complete, the program will read more data which now can consist of an appropriate value of WEIGHT and the designation KEND = 6, with NLIM also input as desired.

One hazard in the operation of the program is met if rapid change in the slope of the potential distribution curve occurs in some portions of the inter-electrode space. In meeting the requirements for the solution in this region, it is at least necessary for the calculation points to be speced closely

enough together to define the slopes in the region of sharp curvature. This is the kind of situation under which the optimum of variable point spacing may become useful. The points may be spaced farther apart in the regions where the slope changes only slowly and close together in the regions of sharp curvature.

5. THE THERMIONIC DIODE

The thermionic diode is a plasma diode in which only one species of particles is present. Since there is no change of sign in charge density, the curvature of the potential distribution curve is always in the same direction. Solutions of thermionic diode problems are found to fit into two classes depending on whether there is or is not a potential extremum in the opposite direction from the bias potential. If there is no such extremum all emitted particles succeed in making the transit to the collector. The charge density of any point can be determined by knowing the potential there relative to the emitter potential. The differential equation is one which can be solved directly with the slope of the potential distribution curve at the emitter as a parameter. If, however, there is an extremum in the opposite direction of the bias potential, this extremum will serve as a barrier potential, sending some particles back toward the emitter. Between the emitter and the extremum, the entire history of the potential distribution contributes to determining the local charge density. It is necessary to iterate to obtain a consistent distribution in this region. Once the extremum has been passed, an extrapolation or a simple step-by-step integration gives a uniquely defined curve which then gives the local potential as a function of radius ratio. In principle, thermionic diode potential distributions for all radius ratios can be obtained in terms of two independent parameters: the ratio of Debye length to emitter radius close to the emitter, and the slope of the potential distribution curve at the emitter.

Table III-5 describes input variables so that the CYLDIODE program may be used to carry out the step-by-step integration/extrapolation for regions of the thermionic diode in which the future history no longer has any influence on the charge density. For the cases having the extremum in the opposite direction from the bias potential, the distribution from the emitter to the extremum would be obtained by either the method of the integration with an additional slope and isolated varying potential definitions, or by the method of iterating on an assumed distribution. Provisions have been made for automatic transition from the integration on an assumed distribution. Transition may be made automatically, if the potential distribution to the extremum has been obtained by iteration on the assumed distribution, by simply reading the appropriate values of KEND, J1MAX, and XRAD.

TABLE III-5

INPUT FOR EXTRAPOLATION OF THERMIONIC DIODE SOLUTIONS

KEND, 5,	designates this mode of operation.
XRAD, _____,	denotes the radius increment to be used for extrapolation, must be negative for INSIDE = 2, not permissible for other values of INSIDE.
J1MAX, _____,	the number of the last point of the extrapolation, must be greater than previous JMAX or J1MAX by a multiple of 5.

E. Results of Cylindrical and Spherical Diodes and Probes

The useful numerical results which have been obtained from CYLDIODE computer program can be described in three groups. First, a fairly extensive collection of data has been obtained for cylindrical diodes where the electrode spacing is not greater than 15 Debye lengths. Second, sample results have been obtained for both cylindrical and spherical diodes for Debye lengths ranging from 1/15 to 1/40 of the electrode spacing. Third, results have been obtained for cylindrical probes for Debye lengths not shorter than 1/10 of the emitter radius, with some additional experimentation at Debye lengths as small as 1/30 of the emitter radius.

The simplest case of the cylindrical plasma diode is that when only one species is present, the thermionic diode. Figure III-7 shows a series of potential distributions for a thermionic diode with 1.3 radius ratio, at fixed bias potential, and for varying charged particle density. Unfortunately, the data for Figure III-7 were obtained early in the development of the computer program. At that time the program was not given a valid current calculation. Consequently, current calculations were not available for inclusion in this report. It is clear from Figure III-7 that increasing the number of emitted particles results first of all in increasing the magnitude of the potential barrier against transmitting current between the electrodes. The absolute magnitude of the current depends, of course, on the effective temperature of the particle velocities and the reference Debye length. (See equations III-13 and 14). In making a study of the absolute level of current density it should be noted that Figure III-7 refers to a thermionic diode transmitting ions. In

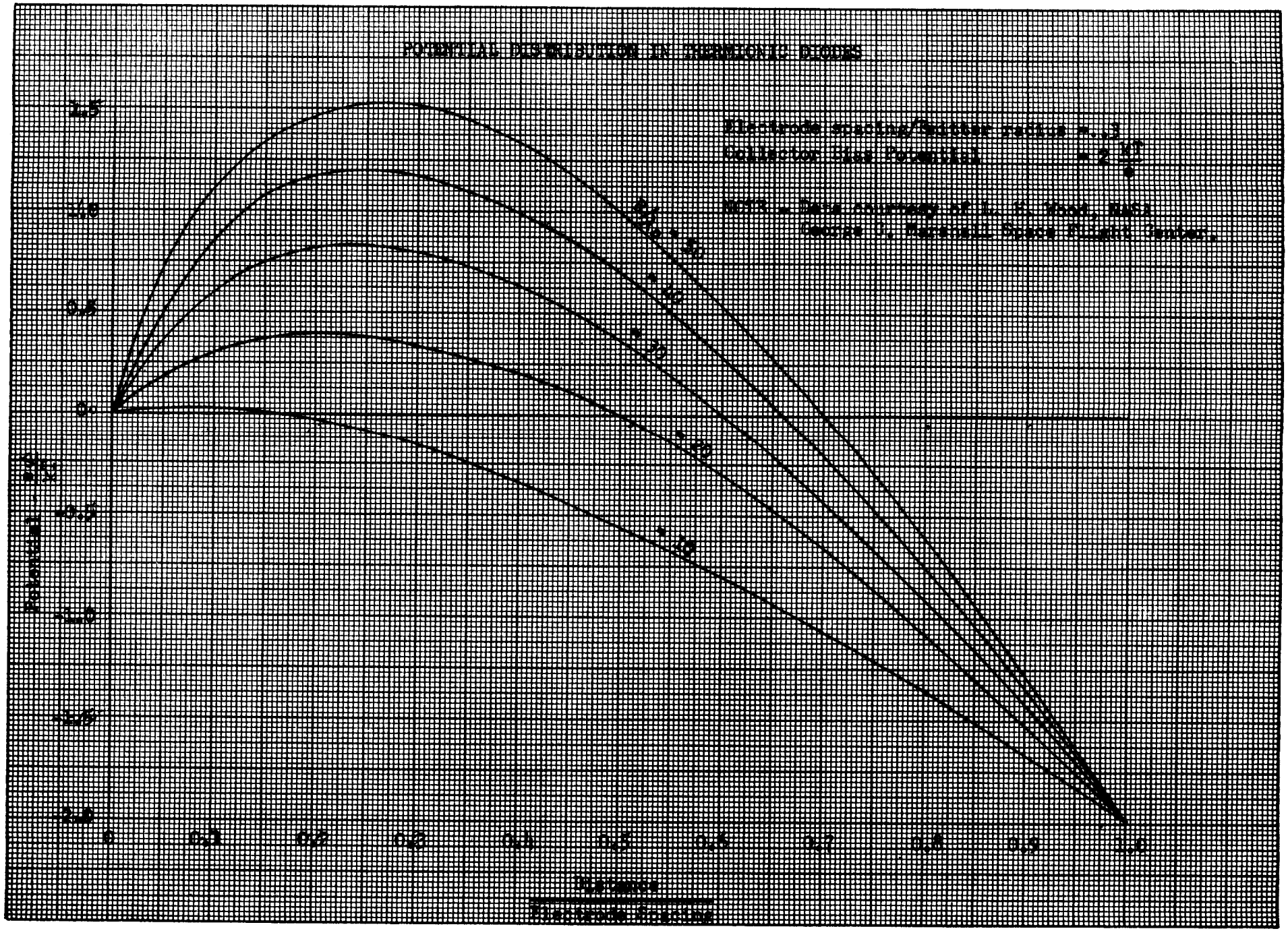

POTENTIAL DISTRIBUTION IN THERMIONIC DIODES
Electrode spacing/Emitter radius = .13
Collector bias potential = 2 volts
NOTE - Data courtesy of L. C. Wood, NASA
George C. Marshall Space Flight Center.
Potential
Distance
Electrode Spacing

Figure III - 7

its present form the program takes ion densities as reference, making no special provision for treating the case where only electrons are present. For a diode transmitting electrons the curves should be inverted.

Figure III-8a to III-8e show influences of an electron background on the potential distribution and consequently on the ability of the diode to transmit ions. All curves in this group are for plasma densities low enough so that the Debye length based on ion density is greater than 1/15 of the electrode spacing. This is the regime in which net charge densities change sign no more than once, a situation which simplifies the problem of obtaining solutions considerably. The large excess of ions always tends to produce a barrier potential for ions which will drive a sufficient proportion of these particles back to the emitter so that the plasma can stay near-neutral throughout most of the interelectrode space. The higher the base density of the plasma is, the more closely localized the region of excess ions is in the neighborhood of the emitter. Figure III-8c may be compared with Figure II-16 to show the similarity of the phenomena to be found in planar and cylindrical plasma diodes, when the curved geometry does not become a dominating influence on the characteristics.

When the plasma density gets high enough so that the Debye length is less than 1/15 of the electrode spacing the extent of the near-neutral plasma seems to become large enough for vulnerability to instabilities. Figure III-9 for spherical diodes shows this situation very clearly. When the Debye length is 1/10 of the electrode spacing there is one inflection point in the potential distribution showing that the net charge density changes sign only once.

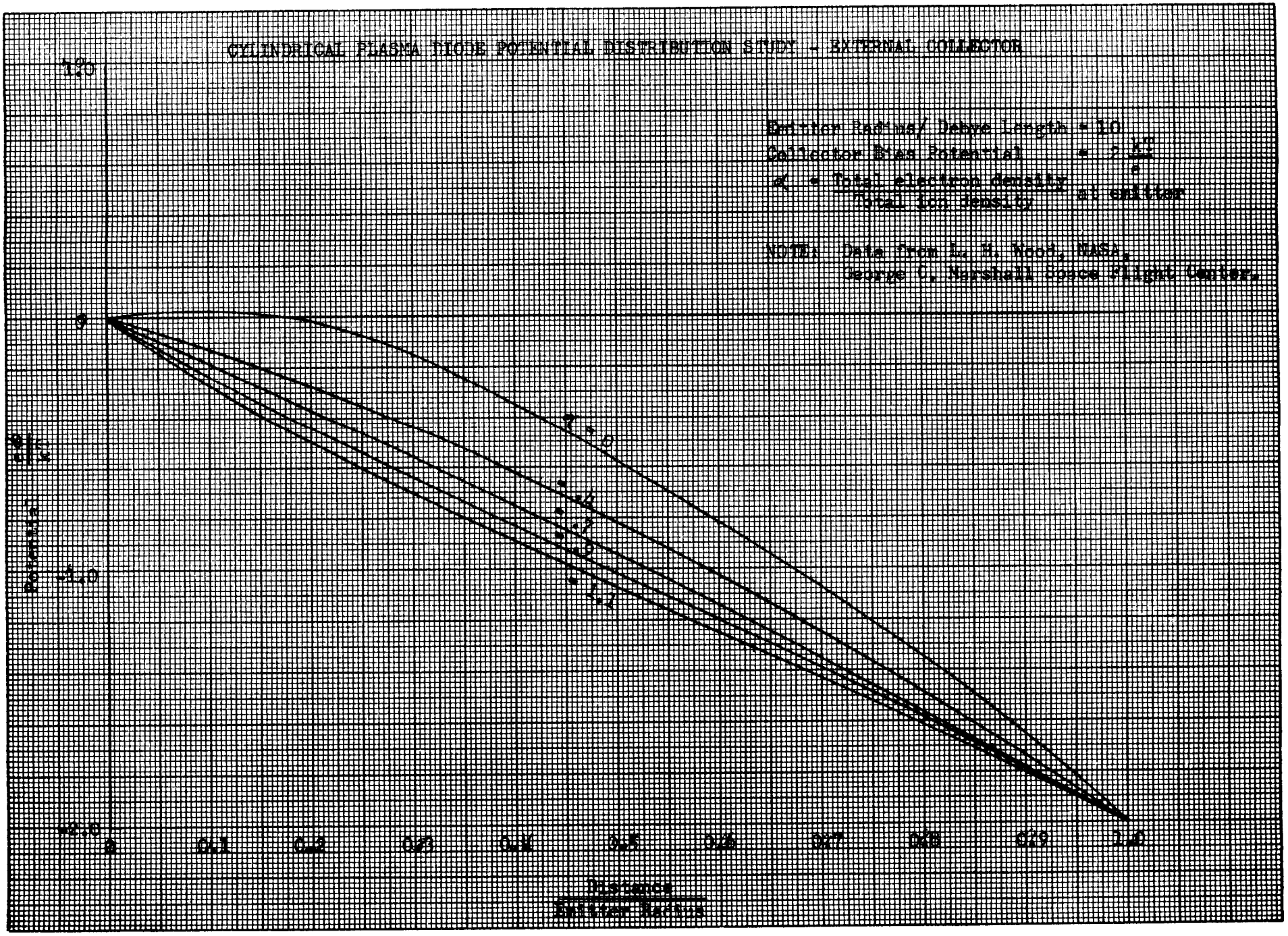


Figure III - 8a

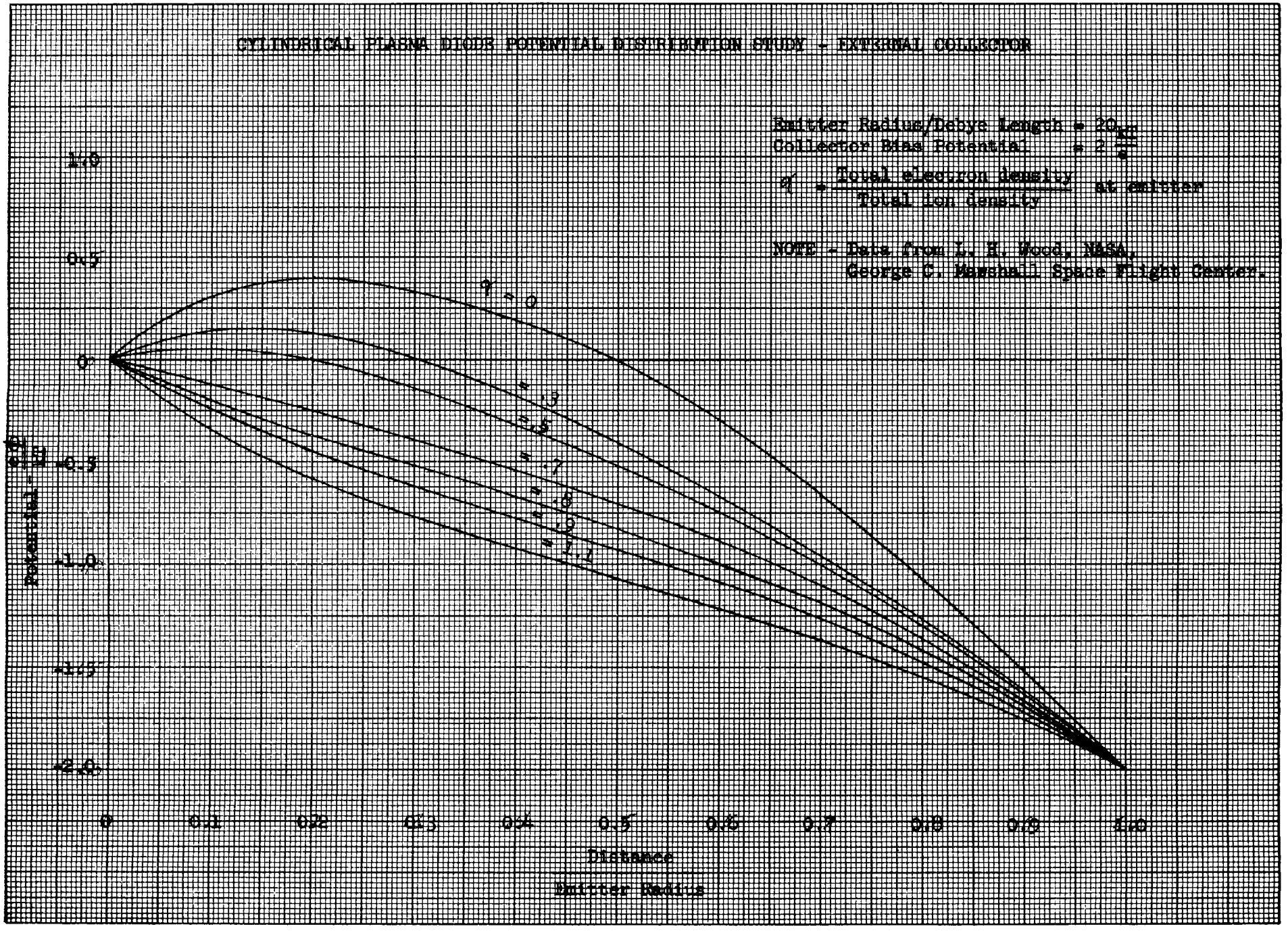


Figure III-8b

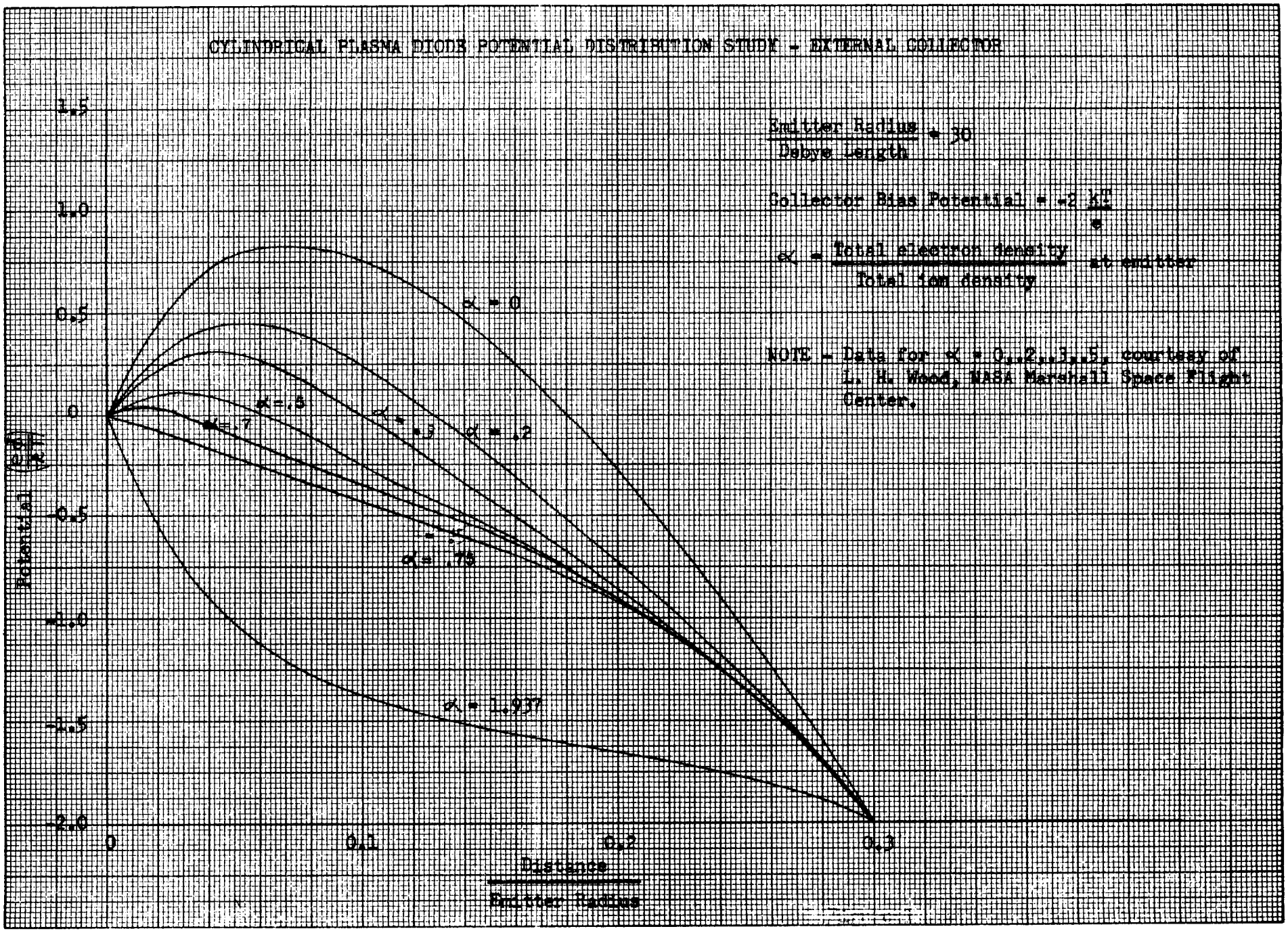


Figure III-8c

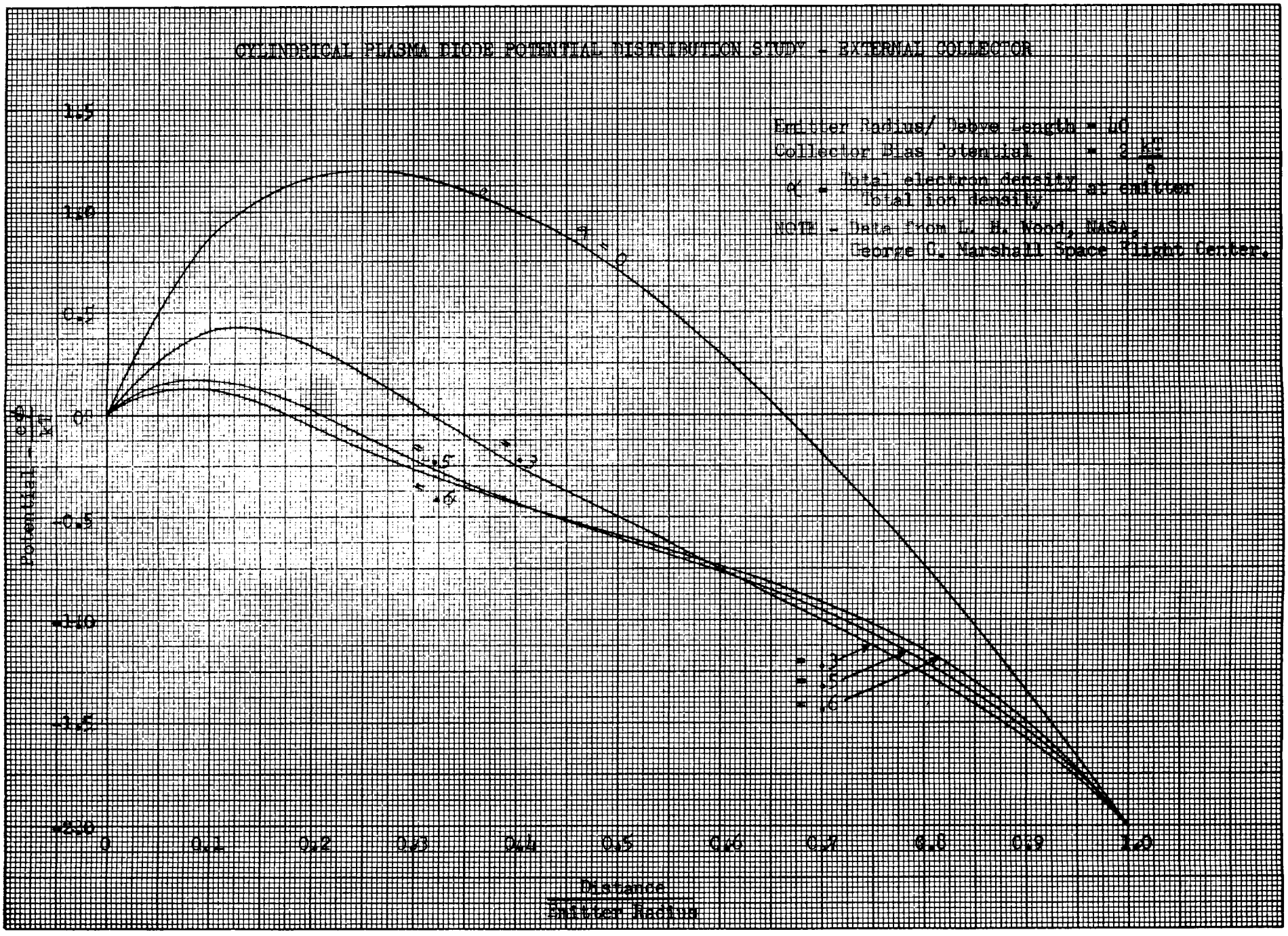


Figure III - 8d

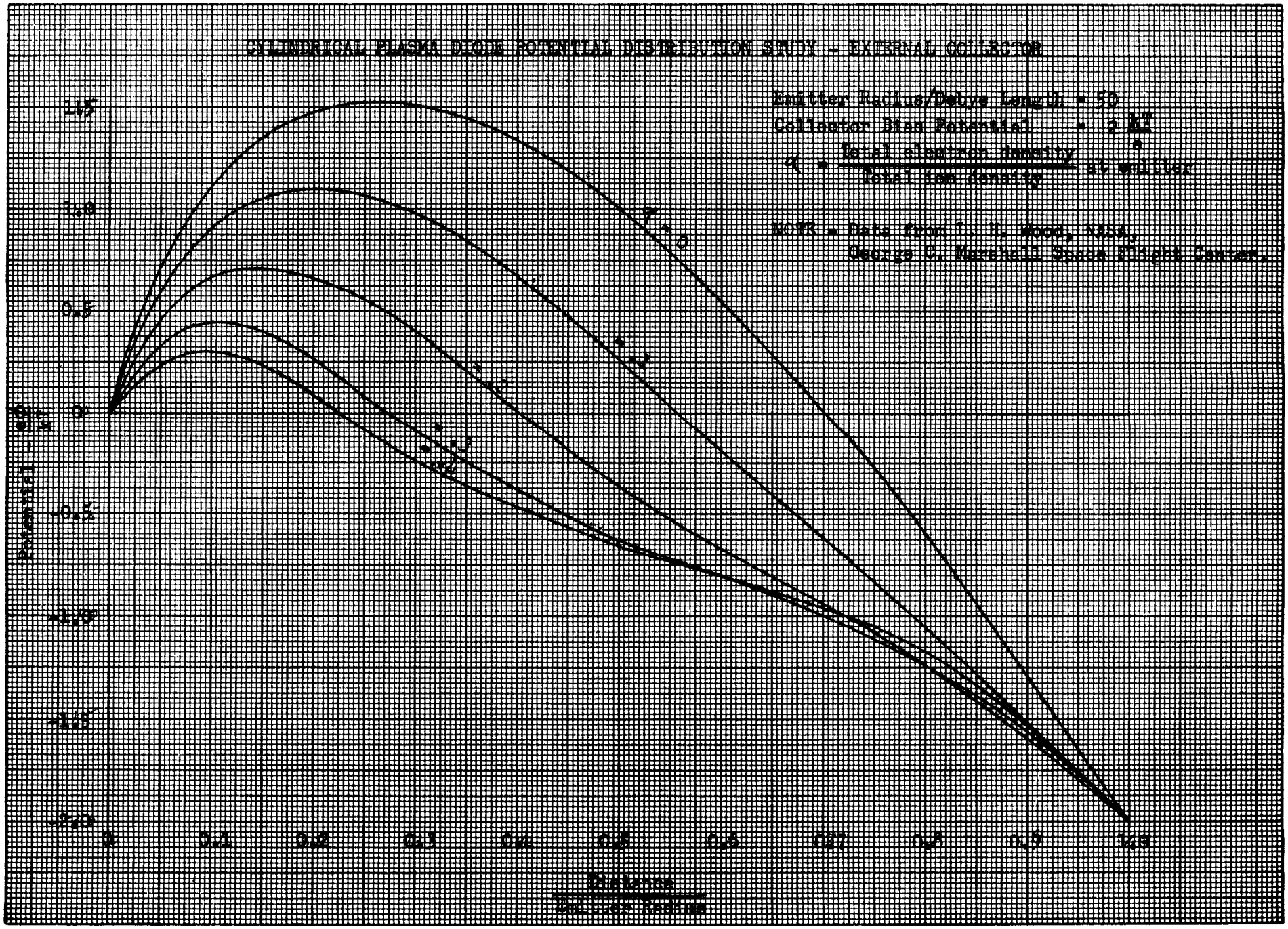


Figure III-8e

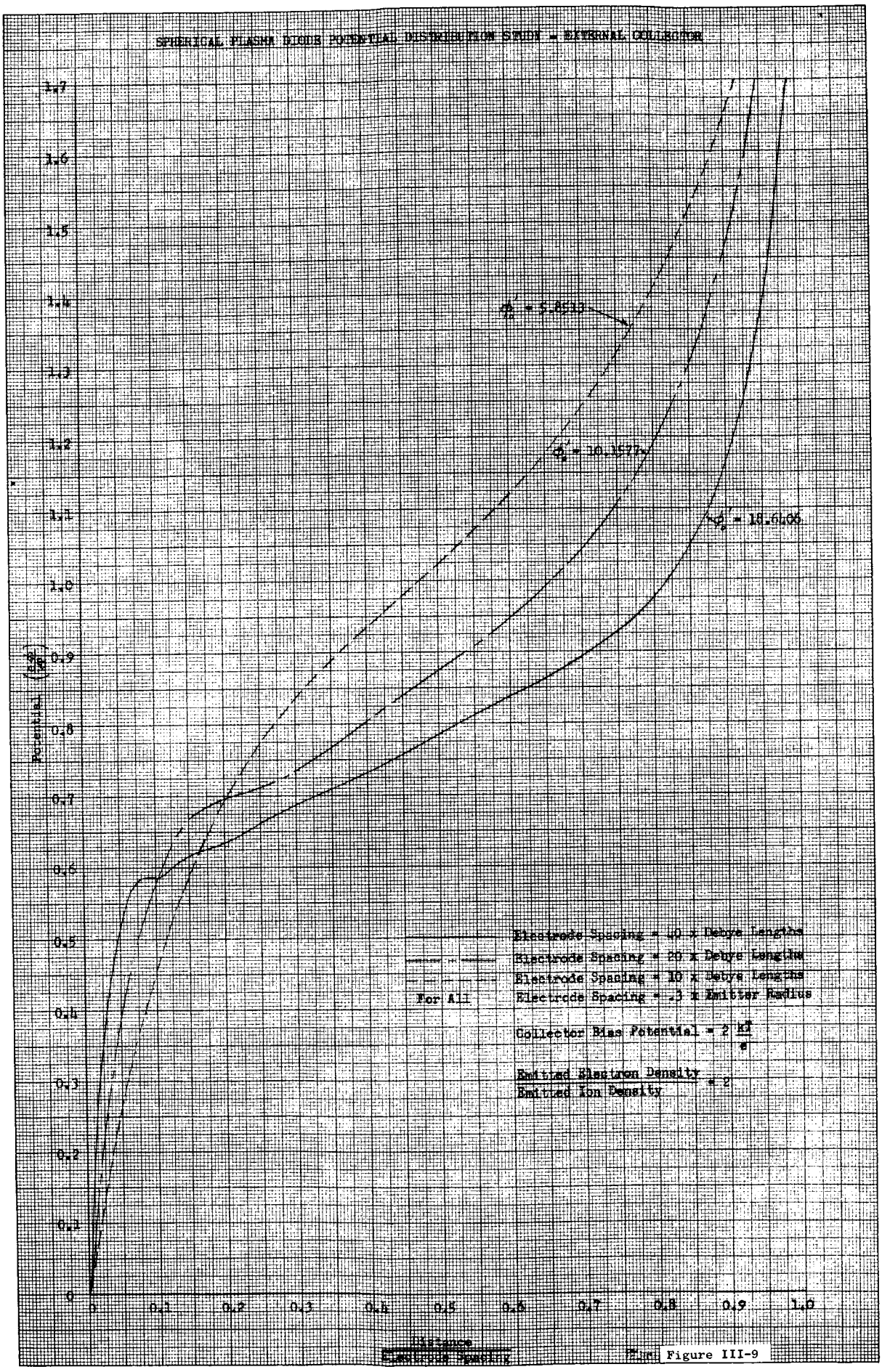

SPHERICAL PLASMA DIODE POTENTIAL DISTRIBUTION STUDY - EXTERNAL COLLECTOR
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
Distance / Electrode Spacing
Potential / Collector Bias Potential
$\phi_c = 5.8513$
$\phi_c = 10.1577$
$\phi_c = 18.6406$
Electrode Spacing = 40 x Debye Lengths
Electrode Spacing = 20 x Debye Lengths
Electrode Spacing = 10 x Debye Lengths
Electrode Spacing = .3 x Emitter Radius
For All
Collector Bias Potential = $2\frac{kT}{e}$
$\frac{\text{Emitted Electron Density}}{\text{Emitted Ion Density}} = 2$

Figure III-9

Reference to Figure III-10 shows that the distance over which the net charge density is less than 1% of the charge density of the individual species is approximately 1/2 Debye length. When the Debye length is 1/20 of the electrode spacing, there are approximately 5 Debye lengths in which the net charge density is less than 1% of the density of the individual constituents. Three inflection points showed clearly in the potential distribution curve of Figure III-9. Carrying on one step further, when the Debye length is 1/40 of the electrode spacing, the distance over which near-neutrality is found is approximately 25 Debye lengths. At least seven inflcection points are visible in the potential distribution curve. Figure III-11 shows results for cylindrical plasma diodes in the region where the range of near-neutral plasma is extensive: the general appearance of the cylindrical and spherical results are quite similar.

The problems of obtaining potential distributions in cylindrical plasma probes have turned out to be considerably more difficult than those for plasma diodes. Results have been obtained for cases where the plasma density is low enough so that the Debye length is greater than 1/10 of the emitter radius. Representative results are presented in two alternative forms, Figure III-12 and 13. The second form, Figure III-13 was prepared in an effort to investigate whether there might be an electron rich sheath which will show a characteristic dimension is measured in Debye lengths. It seems possible that such a trend should be present but the plasma densities represented in Figure III-13 are all too small to establish the existence of that situation. The probe configurations with Debye lengths greater than 1/10 of the emitter radius and the diode situations with Debye lengths greater than 1/15 of the electrode spacing seem to show the common characteristic that

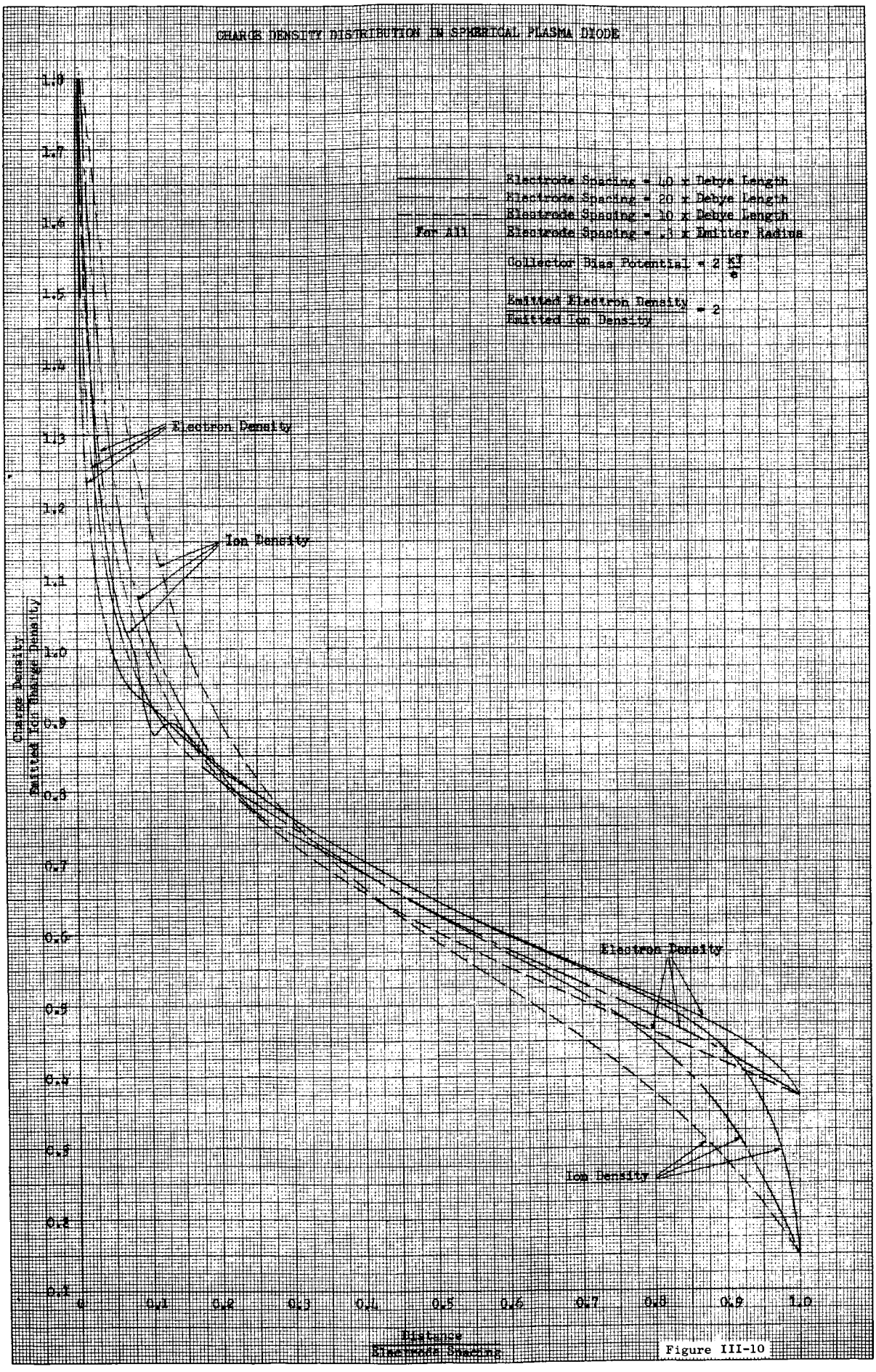


Figure III-10

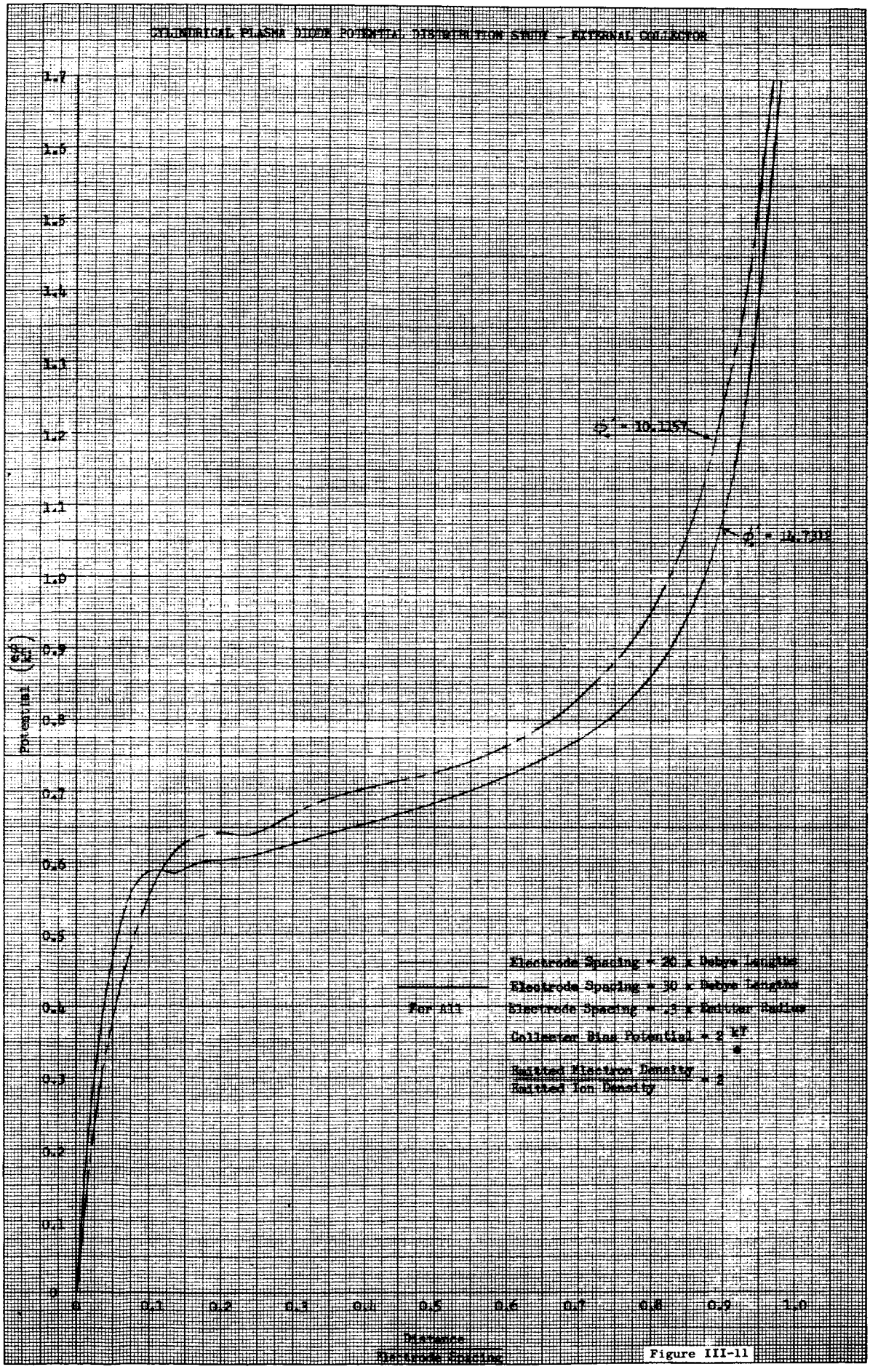


Figure III-11

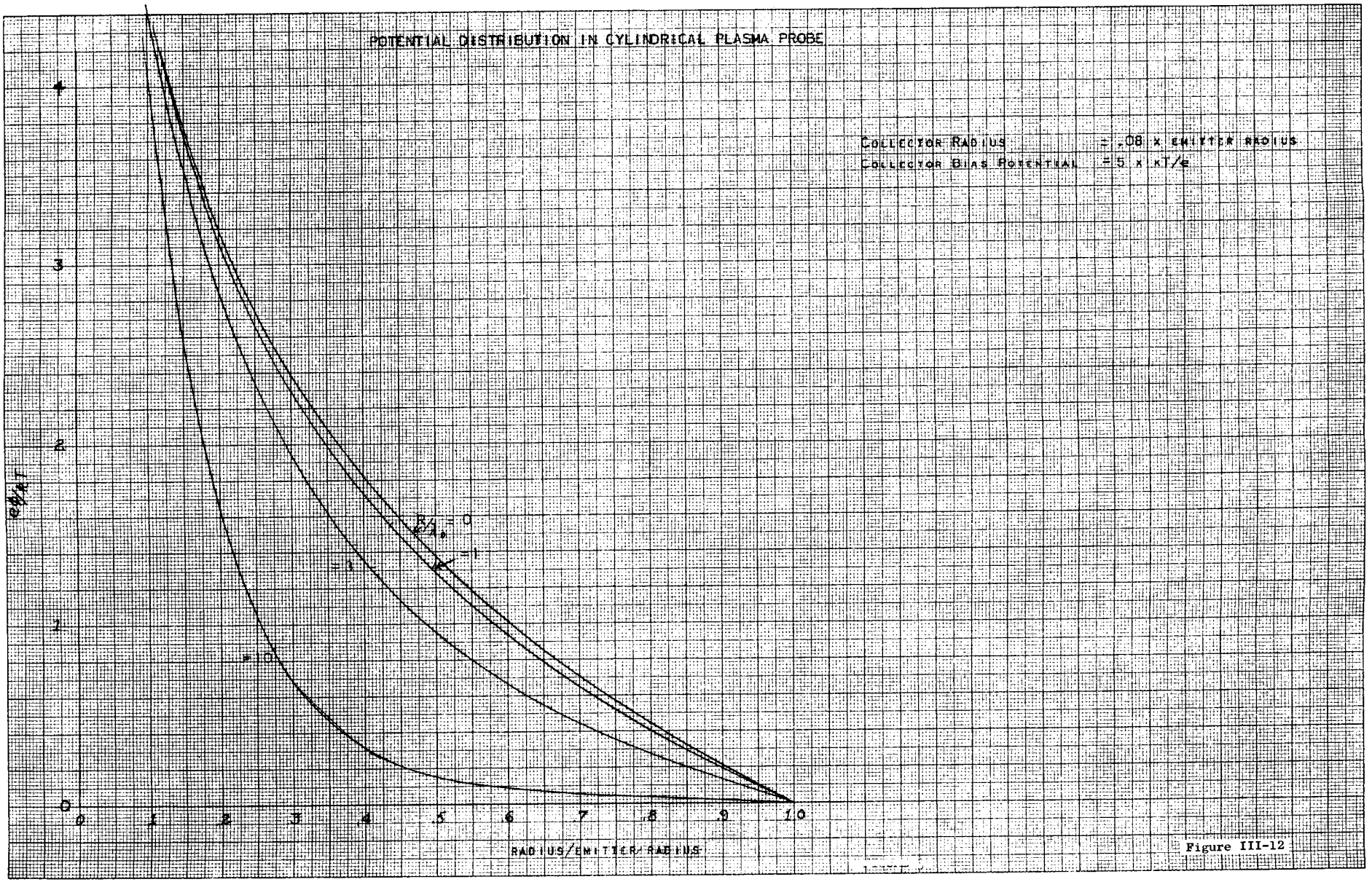


Figure III-12

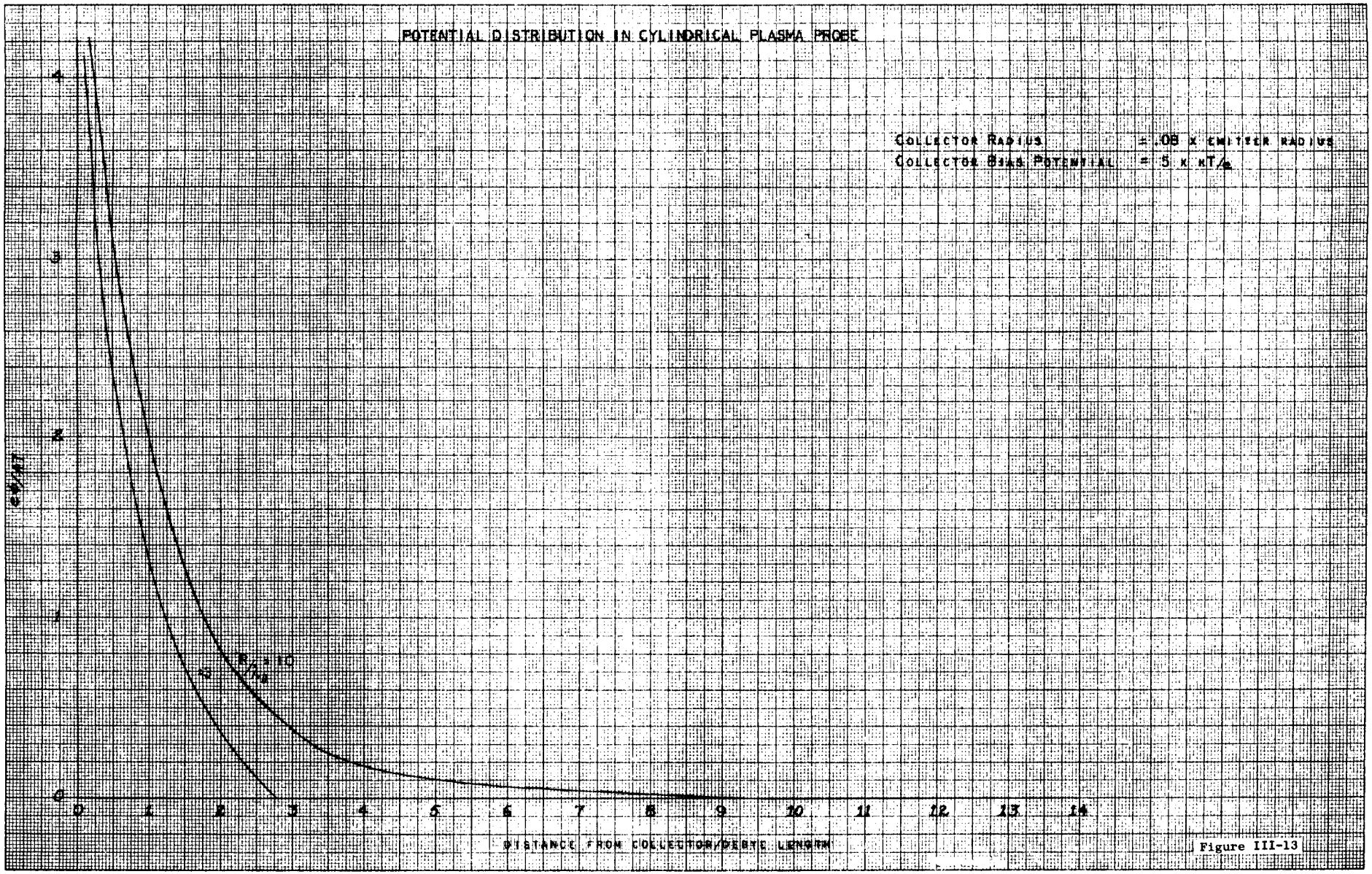


Figure III-13

there is no tendency to have a region of near-neutral plasma extending more than a fraction of a Debye length.

The probe analysis runs into difficulties, for which no solutions have been found as yet, as soon as any extended region of near-neutral plasma seems to be demanded. The difficulties may be demonstrated by considering the series of iterations on an assumed potential distribution in Fig. III-14.

The 30 Debye length electrode spacing was chosen on the basis of planar diode results as about the lowest at which plateau phenomena could be expected to dominate the potential distribution. The ratio of electrode spacing to emitter radius, 0.1, the collector bias potential, 5 kT/e, and the choice of the external collector were all arbitrary: the problem of satisfying these conditions proved to be a more than sufficient challenge.

The investigations consisted of a series of assumed potential distributions used to determine the charge density distribution, and then the potential distribution implied by the charge density distribution. The series of assumed and calculated potential distributions, and the errors between those, is shown in Fig. III-14. It does not seem practical to describe the thinking behind all of the trials individually. Instead, the approach will be to try to illustrate the lessons learned and the possible future approach to an automatic solution.

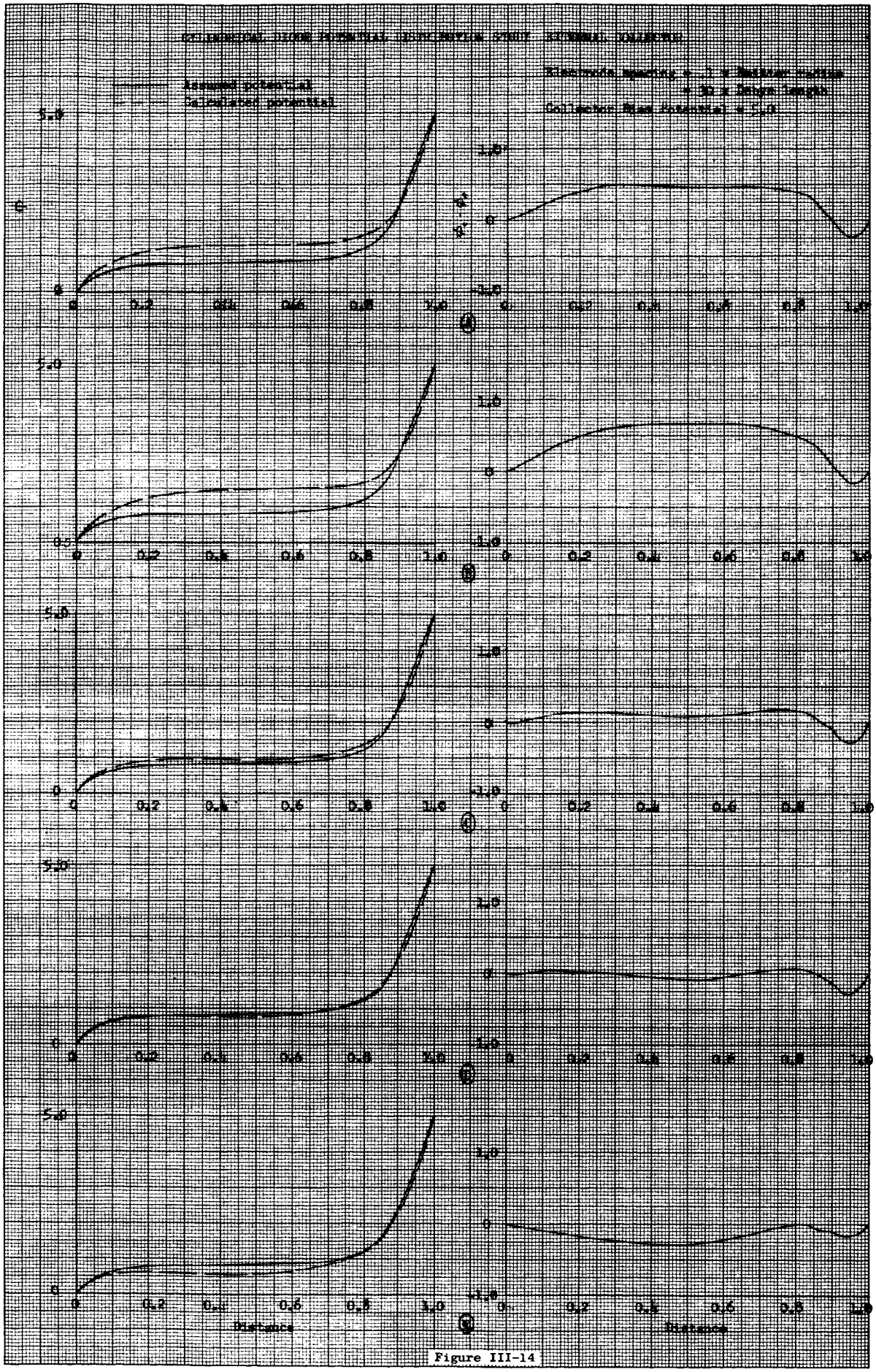


Figure III-14

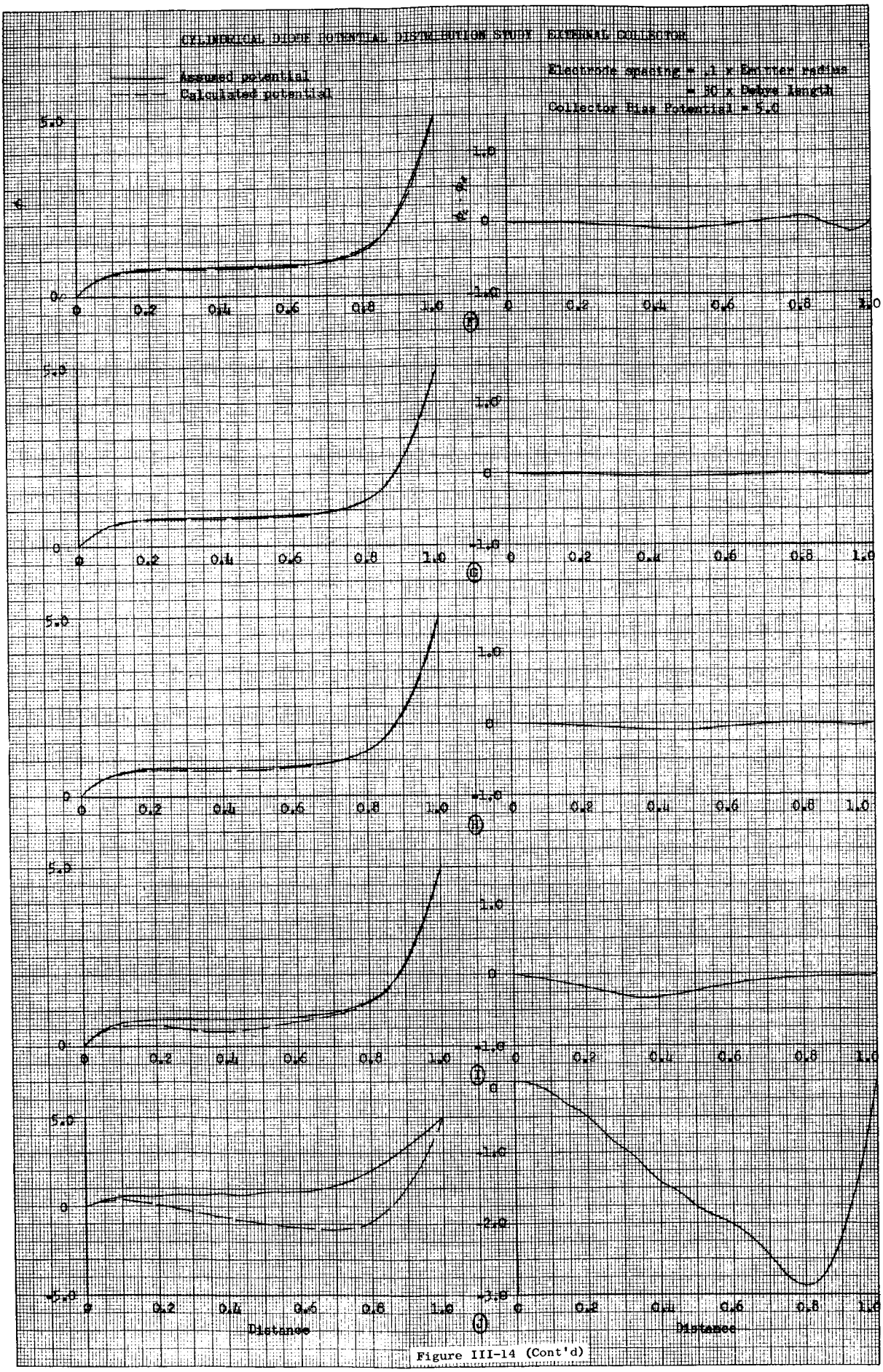


Figure III-14 (Cont'd)

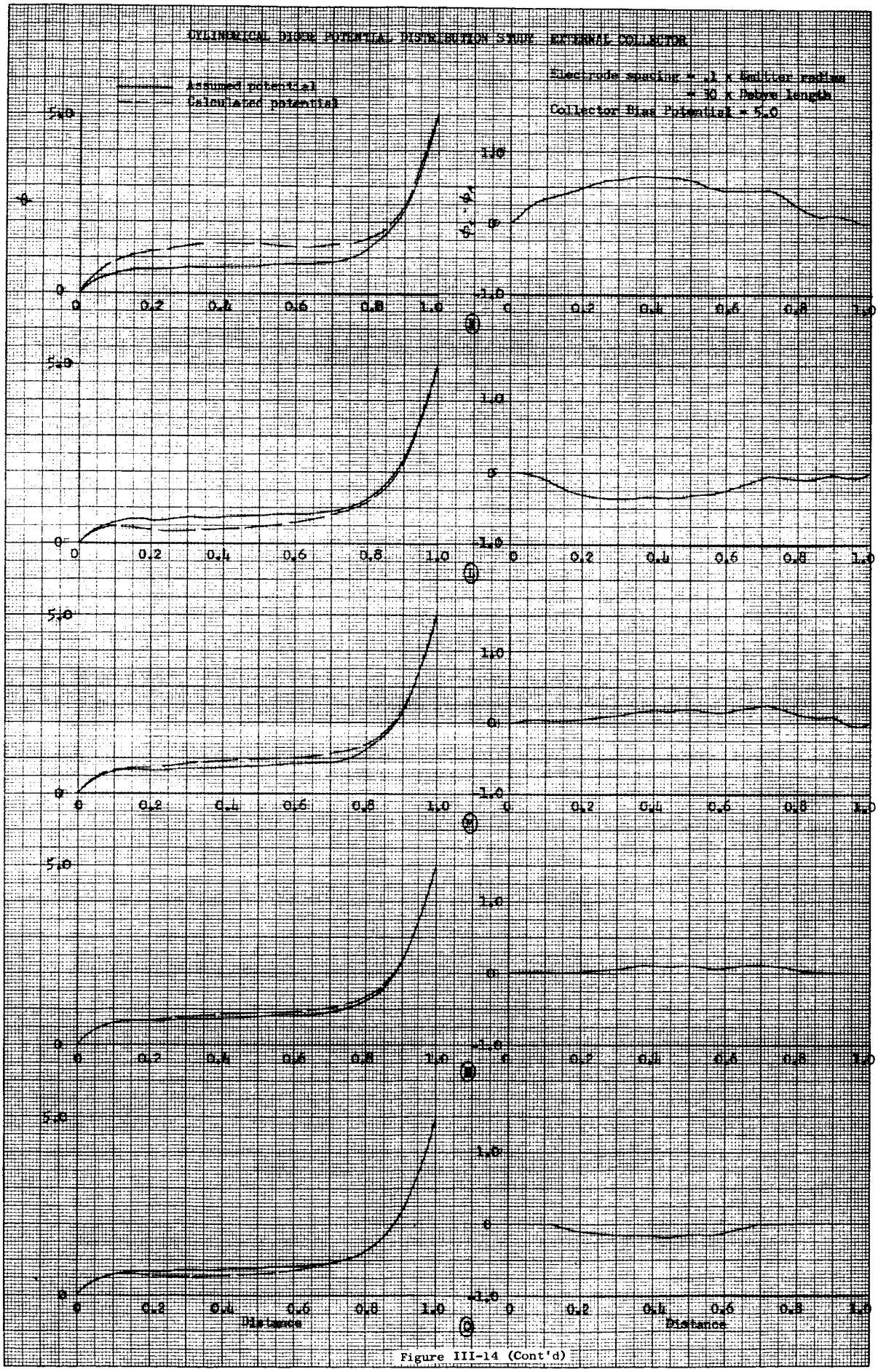


Figure III-14 (Cont'd)

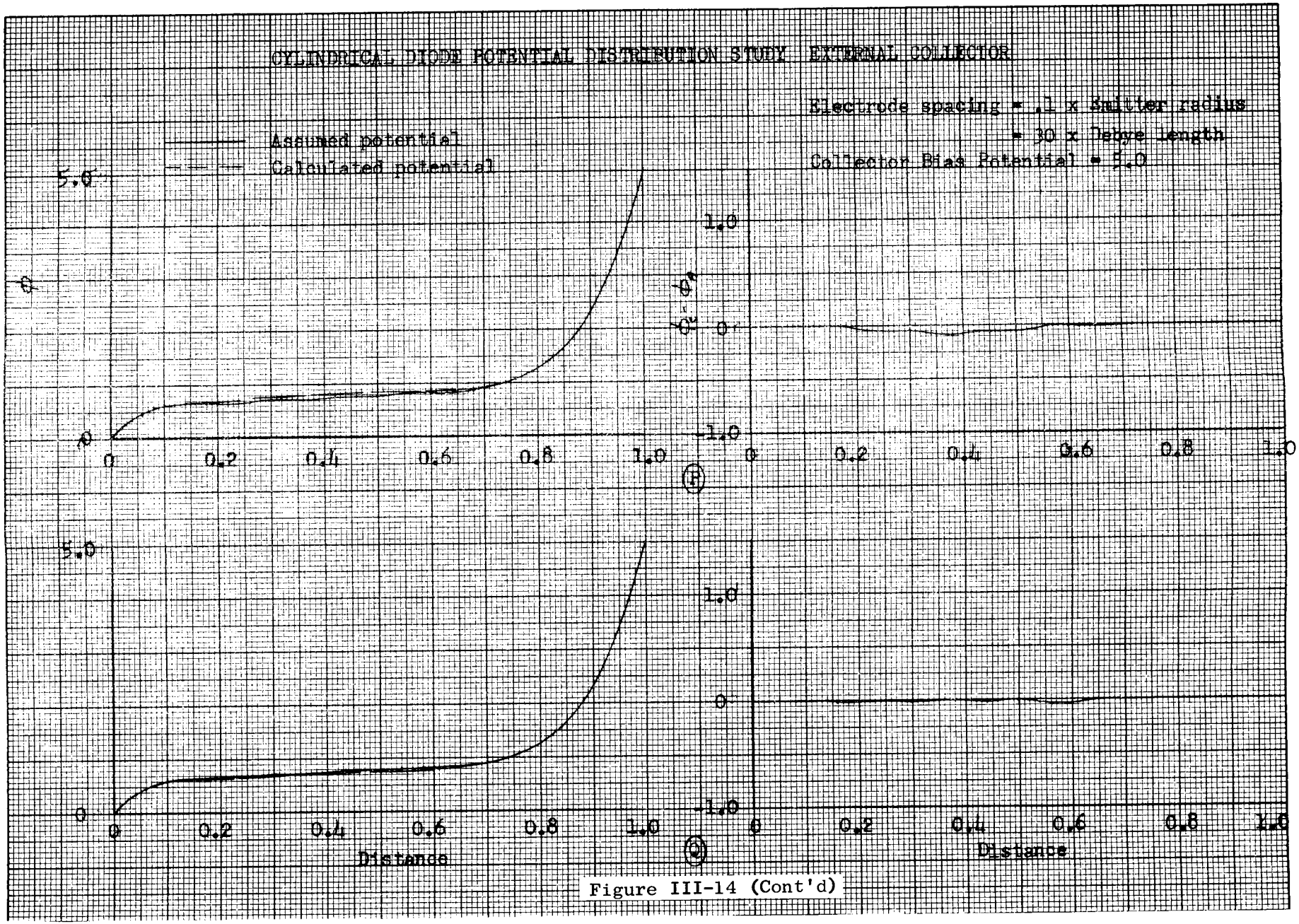


Figure III-14 (Cont'd)

The initial sequence of trials a to i represents the portion of the investigation aimed at finding a possible solution of the cylindrical diode equation. (Trial b actually belongs ahead of trial a in the sequence). The starting point was the planar diode solution for the first 8 Debye lengths. Then the predicted slope for the next 12 Debye lengths was drawn from previous experience. Finally the planar diode results were adapted to provide an estimate of the situation close to the collector. Trials a and b demonstrate that monotonic charge density remains ion rich to a higher potential than the .765 kT/e value for the inflection point of the planar diode solution. The potential level in the plateau region was allowed to rise gradually by modifying the previously assumed potential with a small proportion of the difference between assumed and calculated distribution until the trial c resulted. At this point a tendency appears for the charge density on the knee at the emitter end of the plateau to be more ion-rich than the density closer to the collector, with the result that more correction to the potential distribution is demanded at $x = .25$ than at $x = .5$. In retrospect it appears that the error at $x = .95$ could have been liquidated without significant influence on the situation from $x = .1$ to $x = .75$, which is the critical region. This was actually done after trial f. Trials g, h, and i repeated the tendency found in trials f, d, and e, (the correct order) but not accepted in the earlier group. Here the indicated change is to raise the potential around $x = .25$ and to lower the potential around $x = .5$. Doing this, however, sets up an acceleration situation for the ions which get past $x = .2$, and causes the net charge density to move rapidly in the electron-rich direction. This in turn amplifies the indicated error. The conclusion as found in trials e and i could no longer be ignored: there were no stable monotonic potential distribution to be found.

Starting the trial j the investigation was directed toward the possibility that a stable solution might take a wavy form. In the cylindrical case the obstacle met in the study of oscillatory solutions of the planar equation was no longer insuperable. The potential distribution would no longer have to be symmetrical around each extremum, so successive extrema could have different heights. Then, the average charge density over the wave length might be close to neutral, and stably so, even though the local density would not be for any significant distance. Following up this approach, the initial wave length and amplitude were based on the planar oscillatory results of the January report. Unfortunately, those results were not definitive as to whether either the wave length or the amplitude might vary with the magnitude of the ion-rich charge density at the local maximum. Trials k, l, and m are illustrative of the possibility of producing any desired average charge density for fixed potential maximum by varying the wave amplitude. It is not so easy to see whether a stable wave amplitude is implied by the relations between the waves in the calculated distributions and the waves in the assumed distributions. The evidence from various trials appeared to be conflicting in this respect. Trials p and q demonstrate that fairly good accuracy could be obtained by the rather hit-or-miss procedure used. There is, however, no evidence available as to whether the wave length used is a particularly good one or whether the results could be this good for any wave length, and whether a limiting accuracy will always be found unless all wave lengths, variable or not, are exactly right.

If it is assumed that the results to date are encouraging about the existence of stable wavy solutions, it is possible to speculate on more detailed characteristics of the solutions, and how these might be used to expedite actual production of the solutions. In the first place the charge densities do not vary very rapidly with potential for potentials more than 0.1 or so above the level at which the charge density finally becomes electron-rich permanently, or more than 0.1 below the first potential maximum and between the emitter and the first maximum. Consequently, the potential distribution in both of these regions should be amenable to a direct procedure for enforcing conformity with the charge density. Perhaps it may be feasible to establish a catalog of solutions from the emitter to points approaching a first maximum when the local charge density depends on the slope of the distribution curve as well as the actual potential level there. These would be solutions having the initial slope as a parameter. Presumably this has to be done for each ratio of distance from emitter to maximum to emitter radius, but perhaps this portion of the potential curve is not affected much by cylindrical geometry.

In the plateau region the average net charge density over several wave lengths is affected both by the amplitude of the wave and by the proximity of a potential envelope across the maxima to the monotonic neutral plasma curve. Perhaps the monotonic neutral plasma curve can be found and used as a reference curve for the wave-type solutions.

IV. CONCLUSIONS AND RECOMMENDATIONS

This report has presented the entire scope of work performed on planar, cylindrical and spherical diodes during the contracting period. A number of conclusions can be reported:

1. Methods were developed for finding potential distributions and current-voltage characteristics for planar, cylindrical and spherical diodes.

2. Results were demonstrated for planar diodes with arbitrarily large plasma densities and bias potentials.

3. Solutions were found for cylindrical and spherical diodes up to a density such that the reference Debye length was 1/40 of the electrode spacing, and to bias potentials of 5 kT/e. There is no apparent reason why the range of applicability should be limited, except for computer running time and possibly also some adaptation of equations for large bias potentials in the same manner as was used for the planar case.

4. Neutral plasmas, at least as described by the mathematical model used in these investigations, appear to be unstable to small potential disturbances.

It is believed that this work has established many of the fundamental characteristics of plasma diodes. However, problem areas that have appeared which warrant further investigation do include the following:

a) Calculation of potential distributions with two or more extrema, i.e. with oscillation amplitude larger than the bias potential.

b) Calculation of current-voltage curves for plasma diodes of various geometries.

c) Applications of theory to experimental conditions of interest.

d) Consideration of two-dimensional plasma configurations.

e) Satisfactory methods of analysis for probes in high-density plasmas.

APPENDIX A

PLANAR DIODE COMPUTER PROGRAM

APPENDIX A: Planar Diode Computer Program

```
*PLANDIOD
      DIMENSION POTL(8,1001),ARRAY(2,501),MONTH(3),XJ(8),XJ1(8),LABEL(3,
     12),CNS(6,3),CC(10),CHG2(50),PHIM2(50),XJ2(8)
      DIMENSION XJ3(8),PHI(6),PHIX(6),PHIMAX(6)
      COMMON POTL,ARRAY,DEBY2,CONST,CNS,POT,PHIM,ETA,CC,TEMP
     1    ,LANE6,DPHIXX,PHIMAX,LANE
      DIMENSION LABEL1(4,2),LABEL2(3,2),LABEL3(2),BITS(1)
      TABLE LABEL(36H    / (R/LAMBDA)SQ                    )
      TABLE LABEL1(48HEMITTING CYLINDER RADIUS       ELECTRODE SPACING)
      TABLE LABEL2(36H COLLECTOR RADIUS ELECTRODE SPACING )
      TABLE LABEL3(12H POSIT NEGAT)
      TABLE BITS (0-377777777777)
C  KEND = 1 FOR MONOTONIC SITUATION OR ION RICH SITUATION AT COLLECTOR
C  KEND = 2 FOR POTENTIAL MINIMUM SITUATION
C  KEND = 3 DURING PRINTOUT BEFORE NEXT ITERATION FOR MONOTONIC SITUATION
C  KEND = 4 DURING PRINTOUT BEFORE NEXT ITERATION FOR POTENTIAL MINIMUM
C            SITUATION
C  KEND = 5 OR 6 FOR PRINTOUT AFTER ERROR CALL
C  KEND = 7 FOR OSCILLATION SITUATION
C     KEND=8 FOR POTENTIAL MAXIMUM SOLUTION
      IF(SENSE LIGHT 1)34,27
   34 KEND=XMINOF(6,KEND+4)
      J=J+2
      GOTO 166
   27 XLOOP=15.
      TOL=1.E-4
      TOL1=1.E-7
      ETA=1.
      JBELOW=0
      JABOVE=0
      KEND=1
      LANE6=1
      LBL2=2
      LOOP = 15
      LOOP1  =0
      LPRINT=1
      KPRINT=2
      CUREL=1.
      PHIMAX=0.
      PHIPRM = BITS
      DEBYY = 1.
   33 READ DIP DEBYE,MONTH,LPRINT,POT,PHIPRM,ETA,
     1    XLOOP,KPRINT,KEND,PHIM,TOL1,DEBYY,LOOP
      IF(SENSE LIGHT 1)33,28
   28 LANE4 = 1
      LANE 5  = 1
      CC(1) = 1.
      CALL ERF(SQRTF(MAX1F(0.,POT)),CC(3))
      CC(3) = CC(3) + 1.
      IF(DEBYY -1. )152,151,152
  151 DEBY3 = DEBYE
      DEBY2 = DEBYE **2
      GOTO 153
  152 DEBYE = 1.
      DEBY2 = 1.
      DEBY3 = DEBYY
  153 IF(POT-20.)37,38,38
   37 CURION=EXPF(-POT)/CC(3)
      GOTO 39
 38   CURION=0.
*XJ1 IS QIRE VECTOR FOR ITERATION ON PHIPRM
```

```
   39 IF(KEND-2) 150,482,150
  150 XJ1=0.
      PHIM = MIN1F(0.,1.001*POT)
      IF(PHIM)328,155,155
  155 CHG2(1)=ETA-1.
      NCOUNT=1
*LOOP TO FIND INFLECTION POINT OF POTENTIAL CURVE, EFN 58 TO EFN 12
*XJ IS QIRE VECTOR FOR LOCATING INFLECTION POINT
   58 XJ=1.
      XJ(4)=0.
      XJ(8)=0.
      PHI=0.
   64 LANE = 1
      CALL CHARGE(ETA,POT,PHIM,POT,CHGELC,CHGION,CHGNET,-1.)
      IF(CHGNET)1,1,2
    2 XJ(2)=POT
      XJ(6)=CHGNET
      IF(CHG2(NCOUNT))3,4,4
    3 XJ(3)=0.
      XJ(7)=CHG2(NCOUNT)
      PHI=.1*POT
      GO TO 11
    4 DPHI=.001
      CHG1=CHG2(NCOUNT)
    5 PHI=PHI+DPHI
    8 CALL CHARGE(ETA,POT,PHIM,PHI,CHGELC,CHGION,CHG3,-1.)
      IF(CHG3-CHG1)6,9,9
    6 DPHI=MIN1F(2.*DPHI,.1*POT)
      CHG1=CHG3
      PHI=PHI+DPHI
      IF(PHI-POT)8,7,7
    7 WRITE(3,1003)LABEL3(KEND),POT,ETA,DEBYE,DEBYY,PHIM
      GO TO (108,111,112,112,112,112,34),KEND
  108 PHI=0.
      GO TO 65
  111 PHI = PHIM
      XJ2=1.
      XJ2(2)=PHIM
      XJ2(6)=POT-PHIM
      LANE2=1
      GOTO 66
  112 CALL ERROR
    9 IF(CHG1)10,7,7
   10 XJ(3)=PHI-DPHI
      XJ(7)=CHG1
      GO TO          12
   11 CALL CHARGE(ETA,POT,PHIM,PHI,CHGELC,CHGION,CHG3,-1.)
   12 CALL QIRE(PHI,CHG3        ,0.,0.,0.,0.,TOL1,XLOOP,XJ,GO)
      IF(GO-6HGOBACK)13,11,13
   13 GO TO (65,66,324,324,324,324,185),KEND
*PHIPRM IS POTENTIAL GRADIENT AT NEUTRAL PLASMA INFLECTION POINT
C  PHIPRM IS ITERATION VARIABLE FOR MONOTONIC SITUATION
   65 IF(PHIPRM-BITS)14,149,14
  149 PHIPRM=1./DEBYE
   14 LANE=3
      CALL PHIDRV(M,PHI,PHIPRM,-1.,0.)
*LOOP FOR INTEGRATION
C  OF THE POSITIVE SLOPE STARTS HERE FOR THE MONOTONIC CASE
      LANE6=LANE6
      LANE1=1
      DO 41 I=1,8008
```

```
   41   POTL(I)=0.
        POTL(1,1)=0.
        POTL(2,1)=0.
        POTL(3,1)=0.
        POTL(4,1)=ETA
        POTL(5,1)=1.
        POTL(6,1)=ETA-1.
        POTL(7,1)=DEBY2*POTL(6,1)
        J=1
        LANE = 1
        CALL PHIDRV(M,POTL(3,1),POTL(8,1),-1.,0.)
        IF(POTL(8,1))161,160,139
  161   CALL ERROR
  160   CNS(2,2)=DEBY2*(ETA-1.)
        CNS(3,2)=DEBY2*ETA*CC(2)*SQRTF(2.*(ETA-1.))
        DDX = MIN1F(.25/DEBYE,.01)
        POTL(8,1) = PHIPRM
  469   FACT2 = DDX*CNS(2,2)
        FACT3 = DDX**2/2.*CNS(3,2)
        POTL(8,2) = POTL(8,1)+FACT2-FACT3
        POTL(8,3) = POTL(8,1)+2.*FACT2-4.*FACT3
        FACT2 = FACT2*DDX/2.
        FACT 3 = FACT3*DDX/3.
        POTL(3,2) = FACT2 - FACT3
        POTL(3,3) = 4.*FACT2-8.*FACT3
        POTL(1,2)= DDX
        POTL(1,3) = 2.*DDX
        CALL CHARGE(ETA,POT,PHIM,POTL(3,2),POTL(4,2),POTL(5,2),POTL(6,2),
       1    -1.)
        CALL CHARGE(ETA,POT,PHIM,POTL(3,3),POTL(4,3),POTL(5,3),POTL(6,3),
       1    -1.)
        J = 3
        POTL(7,2) = DEBY2*POTL(6,2)
        POTL(7,3) = DEBY2*POTL(6,3)
        IF(ABSF(POTL(7,3)/POTL(7,1)-1.)-.05)470,470,467
  467   IF(POTL(3,2)-POTL(3,1)-1.E-6)470,470,468
  468   DDX = DDX/2.
        GOTO 469
  470   LANE1=5
  139   IF(PHI-PHIM)109,109,110
  109   DPHI=POT/10.
        PHITST=POT
        LANE1 = XMAXOF(3,LANE1)
        LANE = 1
        GO TO 15
  110   PHITST=PHI                                                    MAIN
  148   DPHI = MAX1F(POTL(3,J),PHI,.1)/10.
C STATMENTS 15-21 INVOLVE INTEGRATION IN THE POSITIVE SLOPE REGION
   15   IF(PHITST-POTL(3,J)- DPHI  )16,16,17
   16   DPOT=(PHITST-POTL(3,J))/2.                                   MAIN
        LANE1=LANE1+1                                               MAIN
        POTL(3,J+2)=PHITST
        POTL(3,J+1)=(POTL(3,J+2)+POTL(3,J))/2.
        GO TO 491
   17   DPOT=MAX1F(1.E-6,MIN1F(POTL(8,J)/DEBYE,DPHI/2.))
   20   POTL(3,J+1)=POTL(3,J)+DPOT
        POTL(3,J+2)=POTL(3,J+1)+DPOT
C       LANE1=1 NORMALLY BETWEEN EMITTER OR POTENTIAL
C        MINIMUM AND INFLECTION POINT
C       LANE1=2 FOR LAST INTERVAL BEFORE INFLECTION
C        POINT
```

```
C      LANE1=3 NORMALLY AFTER INFLECTION POINT
C      LANE1=4 FOR LAST INTERVAL BEFORE COLLECTOR
  491 LANE=1
      CALL PHIDRV(M,POTL(3,J+1),POTL(8,J+1),-1.,0.)
      IF(POTL(8,J+1))85,164,135
  135 CALL PHIDRV(M,POTL(3,J+2),POTL(8,J+2),-1.,0.)
      IF(POTL(8,J+2))85,164,134
  164 LANE 5  = 2
      IF(POTL(8,J)**2/CONST+1.E-7)42,42,165
C  THIS SEQUENCE IS INTENDED TO GET US PAST AN INFLECTION POINT
C  FOR BOTH MONOTONIC AND POTENTIAL MINIMUM CASES
C  IF PHIDRV IS UNABLE TO CALCULATE NON-ZERO PHIPRIME
  165 POTL(3,J+1) = PHI
      POTL(3,J+2) = 2. * POTL(3,J+1) - POTL (3,J)
      POTL(8,J+2) = POTL(8,J)
      POTL(8,J+1) = 0.
      POTL(1,J+2) = POTL(1,J)+2.*(POTL(3,J+2)-POTL(3,J))/POTL(8,J)
      J = J+2
      GOTO 48
  134 IF(POTL(8,J+2)/POTL(8,J)-.5)42,43,43
   43 IF(POTL(8,J+2)/POTL(8,J)-2.)44,44,42
   42 IF(DPOT-1.E-6)44,44,426
  426 DPOT=DPOT/2.
      GO TO (20,35,20,35,20,35),LANE1
   35 LANE1=LANE1-1
      GOTO 20
   44 IF(LANE1-2)432,433,432
  433 IF(LANE6-2)432,434,103
  432 DX=(.333333/POTL(8,J)+1.333333/POTL(8,J+1)+.333333/POTL(8,J+2))*DP
     1OT
      POTL(1,J+2)=POTL(1,J)+DX
      POTL(1,J+1)=POTL(1,J)+DX/2.
      IF(J-999)52,52,51
   51 KEND=XMINOF(KEND+4,6)
      GOTO(24,122,24,122,24,122,34),KEND
   52 J=J+2
      GO TO (15,48,50,21,50,22),LANE1
   50 GOTO(148,15,412,412,412,412,412,412),LANE
  412 CALL ERROR
   48 PHITST=POT                                                    MAIN
      LANE1=3                                                       MAIN
      GO TO 20
   21 J2MAX = J
  495 GO TO (22,36),LPRINT
   36 KEND1=KEND
      KEND=KEND+2
      GO TO (477,122),KEND1
  221 KEND=KEND1
      IF (LANE6-2)22,323,22
   22 GOTO(82,83,84,84,84,84,84,411),KEND
   84 CALL ERROR
   82 CALL QIRE(PHIPRM,POTL(1,J)-DEBYY,0.,9.*PHIPRM,0.,-.9*PHIPRM,TOL,
     1    XLOOP,XJ1,GO)
C  SEE SUBROUTINE PHIDRV FOR SIGNIFICANCE OF LANE6 VALUES
      GO TO(23,477,122,24,122),LANE6
   23 IF(GO-6HGOBACK)128,14,128
  128 GO TO (24,122,129,129,324,324),KEND
  129 CALL ERROR
C  STATEMENTS EFN 24 TO EFN 29 PROVIDE FOR INTERPOLATION TO GET
C  THE ODD VALUES OF X FOR THE MONOTONIC CASE
C  DECIDE WHETHER TO ADJUST DISTANCES IN PLATEAU SITUATION
```

```
   24 LANE4 = 2
  477 DO 25 JJ=1,J,2
      I=(JJ+1)/2
      ARRAY(1,I)=POTL(3,JJ)/PHIPRM
   25 ARRAY(2,I)=POTL(1,JJ)
      I1=2
      I4=4
      DO 26 II=2,I
      JJ=2*II-2
      IF(JJ-JBELOW)465,31,31
   31 IF(JJ-JABOVE)45,465,465
  465 CALL LSP(ARRAY(1,II-2),I1,I4,POTL(3,JJ)/PHIPRM,POTL(1,JJ),DUMMY)
   45 IF(II+1-I)26,40,40
   40 I4=3
   26 I1=1
      DO 29 II=1,J
   29 POTL(2,II)=POTL(1,II)*DEBYE
  217 GOTO 166
  437 DX = DEBYY-POTL(1,J)
C  STATEMENTS EFN 434 TO EFN 461 RECOGNIZE THE PLATEAU SITUATION
C  AND JUMP TO THE COLLECTOR, WORKING BACK FROM THERE BEFORE USING
C  THE SINH FORMULATION FOR THE PLATEAU
   30 IF(POTL(8,1)-1.1E-6)438,438,437
  438 IF(KEND-2)166,439,444
  439 CALL ERROR
  440 KEND=KEND1
      GO TO 166
  434 JBELOW=J
      JABOVE=J+4
      J2MAX=JABOVE+2
      POTL(3,J2MAX)=POT
      POTL(1,J2MAX)=1.
      CALL PHIDRV(M,POTL(3,J2MAX),POTL(8,J2MAX),-1.,0.)
      DPOT=(POT-PHI)/20.
  435 POTL(3,JABOVE+1)=POTL(3,JABOVE+2)-DPOT
      POTL(3,JABOVE)=POTL(3,JABOVE+1)-DPOT
      CALL PHIDRV(M,POTL(3,JABOVE),POTL(8,JABOVE),-1.,0.)
      IF(POTL(8,JABOVE)/POTL(8,JABOVE+2)-.5)436,442,442
  436 IF(DPOT-1.E-6)443,443,441
  441 DPOT=DPOT/2.
      GO TO 435
  443 IF(POTL(3,JABOVE)-PHI*2.+POTL(3,JBELOW))450,450,442
  442 CALL PHIDRV(M,POTL(3,JABOVE+1),POTL(8,JABOVE+1),-1.,0.)
      DX=(.333333/POTL(8,JABOVE)+1.333333/POTL(8,JABOVE+1)
     1    +.333333/POTL(8,JABOVE+2))*DPOT
      POTL(1,JABOVE)=POTL(1,JABOVE+2)-DX
      POTL(1,JABOVE+1)=POTL(1,JABOVE+2)-DX/2.
      GOTO (452,481),LANE1
  481 IF(J2MAX-999)447,447,444
  444 KEND=KEND+4
  445 DO 446 I=1,J2MAX
  446 POTL(2,I)=POTL(1,I)*DEBYE
      GO TO 314
  447 JJJ=J2MAX*8
      J2MAX=J2MAX+2
      JJABV=(JABOVE-1)*8+1
  448 POTL(JJJ+16)=POTL(JJJ)
      IF(JJJ-JJABV)435,435,449
  449 JJJ=JJJ-1
      GO TO 448
  450 GO TO(442,451),LANE1
```

```
  451 DPOT = (POTL(3,JABOVE+2)-2.*PHI+POTL(3,JBELOW))/2.
      LANE 1  =  1
      GO TO 435
  452 DPHIX2=DPHIXX
  162 DX=(POTL(1,JABOVE)-POTL(1,JBELOW))/2.
      IF(DX)492,492,496
  492 DO 493 I=JABOVE,J2MAX
  493 POTL(1,I)=POTL(1,I)-DX*2.
      J=J2MAX
      GO TO 495
  496 POTL(1,JBELOW+2)=POTL(1,JBELOW)+DX
      TEMP1=SQRTF(CNS(2,1))
      TEMP2=EXPF(TEMP1*DX)
      POTL(8,JBELOW+2)=(PHI-POTL(3,JBELOW))*TEMP1*2./(TEMP2-1./TEMP2)
      DPHIXX=POTL(8,JBELOW+2)**2
      IF(DPHIXX/DPHIX2-2.)453,453,454
  453 IF(DPHIX2/DPHIXX-2.)460,460,454
C     EFN 454-459 CORRECT SLOPES WHICH MAY BE
C      CHANGED BY A NEW INFLECTION POINT SLOPE
  454 DELT=(DPHIXX-DPHIX2)/2.
      JBEL=JBELOW
      JABV=JABOVE
  455 DDPHI=DELT/POTL(8,JBEL)**2
      IF(ABSF(DDPHI)-.01)457,457,456
  456 POTL(8,JBEL)=POTL(8,JBEL)*(1.+DDPHI)
      POTL(8,JABV)=POTL(8,JABV)*(1.+DELT/POTL(8,JABV)**2)
      JABV=JABV+1
      JBEL=JBEL-1
      GO TO 455
  457 N=(JBELOW-JBEL)/2
  458 JBEL=JBELOW-2*N
      JABV=JABOVE+2*N
      POTL(1,JBEL)=POTL(1,JBEL-2)+(POTL(3,JBEL)-POTL(3,JBEL-2))*
     1    (.166667/POTL(8,JBEL-2)+.666667/POTL(8,JBEL-1)+.166667/POTL(8,
     2JBEL))
      POTL(1,JABV)=POTL(1,JABV+2)-(POTL(3,JABV+2)-POTL(3,JABV))*
     1    (.166667/POTL(8,JABV+2)+.666667/POTL(8,JABV+1)+.166667/POTL(8,
     2JABV))
      IF(N)452,452,459
  459 N=N-1
      GO TO 458
  460 DX=DX/2.
      POTL(1,JBELOW+1)=POTL(1,JBELOW)+DX
      POTL(1,JABOVE-1)=POTL(1,JABOVE)-DX
      DPOTL=POTL(8,JBELOW+2)/TEMP1*(EXPF(TEMP1*DX)-1./EXPF(TEMP1*DX))/2.
      POTL(3,JBELOW+2)=PHI
      POTL(3,JBELOW+1)=PHI-DPOTL
      POTL(3,JBELOW+3)=PHI+DPOTL
      JBEL=JBELOW+1
      JABV=JABOVE-1
      DO 461 I=JBEL,JABV
  461 CALL PHIDRV(M,POTL(3,I),POTL(8,I),-1.,0.)
      PHIPRM=(POTL(3,JABOVE)-POTL(3,JBELOW))/DX
      LANE6=1
      J = J2MAX
      GO TO(24,122,166,166,166,166,166,166),KEND
  166 J2MAX=J
  314 LBL=2
  317 CUREL = CC(6)/CC( 1)
      WRITE OUTPUT TAPE 3,1000,(MONTH(I),I=1,3),(LABEL1(I,LBL2),I=1,4),
     1DEBY3,                              ETA,(LABEL(I,LBL),I=1,3),POT,
```

```
     2CUREL,CURION,(LABEL(I,LBL),I=2,3)
      GOTO(341,318),KPRINT
  341 IF(J2MAX-41)318,318,319
  318 N1=1
      N2=J2MAX
      LSKIP=1
      LGO=1
      GO TO320
  319 LSKIP=J2MAX/40+1
      N1=1
      N2=J2MAX
      LGO=2
  320 WRITE OUTPUT TAPE 3,1001,((POTL(I,J),I=1,8),J=N1,N2,LSKIP)
      GOTO(323,322),LGO
  322 N1=J2MAX
      N2=J2MAX
      LSKIP=1
      LGO=1
      GO TO320
  323 IF(SENSE SWITCH 4)324,327
  327 GO TO (328,27,221,221,324,324,185),KEND
  328 KEND=2
      LANE6 = 1
      LANE2=1
      GO TO (157,27),LANE4
C  ITERATION FOR POTENTIAL MINIMUM SITUATION STARTS HERE
C  PHIM IS ITERATION VARIABLE FOR POTENTIAL MINIMUM SITUATION
  482 XJ1 = 0.
      NCOUNT = 1
      DPHIM=-MIN1F(.005,MAX1F(.0005,.1*ABSF(POT)))
      GOTO 483
  157 XJ1(3) = 0.
      XJ1(7) = POTL(1,J)-POTL(1,1)-DEBYY
      ERR1 = XJ1(7)
      XJ1(1) = 1.
      XJ1(2) = 0.
      XJ1(6) = 0.
      XJ1(4) = 0.
      XJ1(8) = 0.
      NCOUNT=2
      DPHIM=-MIN1F(.005,MAX1F(.0005,.1*ABSF(POT)))
      PHIM2(2)=PHIM+DPHIM
  483 PHIM2(1) = PHIM
      LANE7=1
      LANE = 4
      DO 158 I = 1,8
  158 XJ2(I) = 0.
  175 XJ = 1.
      XJ(4)=0.
      XJ(8)=0.
      CHG2(1)=ETA-1.
      XK4 = DPHIM
      PHIM = PHIM2(NCOUNT)
      PHI=PHIM
      CALL ERF(SQRTF(-PHIM),CC(1))
      CC(1) = CC(1) + 1.
   63 CALL CHARGE(ETA,POT,PHIM,PHIM,POTL(4,2),POTL(5,2),CHG2(NCOUNT),0.)
      IF(CHG2(NCOUNT))60,60,64
   60 IF(NCOUNT-2)147,62,61
   61 IF(CHG2(NCOUNT)-CHG2(NCOUNT-1))147,147,62
  147 WRITE (3,1002)POT,ETA,DEBYE,(PHIM2(I),CHG2(I),I=1,NCOUNT)
```

```
      GO TO 327
   62 PHIM2(NCOUNT+1)=PHIM2(NCOUNT)+DPHIM
      PHIM=PHIM2(NCOUNT+1)
      DPHIM = DPHIM*2.
      CALL ERF(SQRTF(-PHIM),CC(1))
      CC(1) = CC(1) + 1.
      NCOUNT=NCOUNT+1
      GO TO 63
   66 DX=MIN1F(.25/DEBYE,.002*DEBYY)
      DPHI=MAX1F((PHI-PHIM)/10.,.02)
      PHITST=PHI
      DDX=DX
      DO 140 I=14,8008
  140 POTL(I)=0.
      J2=2
      J=3
      J2MIN=1
      LANE = 4
      CALL PHIDRV(M,PHI,DUMMY,-1.,CHG2(NCOUNT))
      LANE6=LANE6
C     THIS IS SUPPOSED TO BE CHARGE DENSITY AT PHIM
  171 IF(PHI-PHIM)170,86,85
  170 CALL ERROR
   85 PHIMX=PHIM2(NCOUNT)
C  THIS IS AN OVERRIDE TEST, DESIGNED TO LOCATED PHIM GIVING ZERO PHIPRIME
C  AT INFLECTION POINT AND TO AVOID TANGLING WITH SPURIOUS INFLECTION
C  POINTS, INDICATED BY DPHIXX = 0 AT NON-INFLECTION POINT
      CALL QIRE(PHIMX,DPHIXX,0.,0.,0.,0.,1.E-10,XLOOP,XJ2,GOONE)
      IF(SENSE LIGHT 1)173,414
  414 IF(GOONE-6HGOBACK)173,172,173
  172 IF(DPHIXX)174,86,86
  174 PHIM2(NCOUNT)=PHIMX
      GO TO 175
  173 LANE7=2
      DPHIXX = MAX1F(DPHIXX,0.)
C  WITH NO INFLECTION POINT, OR POSITIVE SLOPE AT INFLECTION POINT,
C  TRY CALCULATING POTENTIAL DISTRIBUTION
C  EFN 86 TO EFN 116 APPLY TO IMMEDIATE NEIGHBORHOOD OF POTENTIAL MINIMUM
   86 POTL(3,J2)=PHIM
      DUMMY=DUMMY**2
      POTL(6,J2) = CHG2(NCOUNT)
      POTL(7,J2) = POTL(6,J2)*DEBY2
      POTL(8,J2)=0.
      POTL(1,J2)=0.
   71 FACT2=CNS(2,2)*DDX
      FACT22=FACT2*DDX/2.
      FACT3=CNS(3,2)*DDX**2/2.
      FACT33=FACT3*DDX/3.
      POTL(1,1)=POTL(1,2)-DX
      POTL(1,J)=POTL(1,J-1)+DX
      POTL(3,1)=PHIM+FACT22+FACT33
      POTL(3,J)=PHIM+FACT22-FACT33
      POTL(8,1)=-FACT2-FACT3
      POTL(8,J)=FACT2-FACT3
      LANE = 2
      CALL CHARGE(ETA,POT,PHIM,POTL(3,1),POTL(4,1),POTL(5,1),POTL(6,1),1
     1.)
      CALL CHARGE(ETA,POT,PHIM,POTL(3,J),POTL(4,J),POTL(5,J),POTL(6,J),-
     11.)
      POTL(7,1)=POTL(6,1)*DEBY2
      POTL(7,J)=POTL(6,J)*DEBY2
```

```
      IF(ABSF(POTL(7,1)/POTL(7,J2)-1.)-.05)473,471,471
  471 IF(ABSF(POTL(3,1)-POTL(3,J2))-1.E-6)473,473,472
  472 DDX = DDX/2.
      GOTO 71
  473 PHIX=-PHIM
      C1=-.5
      C2=1.
      LANE=2
      LANEX=2
      IF(POTL(3,1)) 72,113,113
  113 XJ(2)=POTL(1,1)
      XJ(6)=POTL(3,1)
      XJ(3)=POTL(1,2)
      XJ(7)=POTL(3,2)
      XJ(4)=0.
      XJ(8)=0.
      XJ(1)=1.
      POTL(1,1)=POTL(1,2)-DX*POTL(3,2)/(POTL(3,2)-POTL(3,1))
  115 D3X=POTL(1,2)-POTL(1,1)
      FACT2=CNS(2,2)*D3X
      FACT22=FACT2*D3X/2.
      FACT3=CNS(3,2)*D3X**2/2.
      FACT33=FACT3*D3X/3.
      POTL(3,1)=PHIM+FACT22+FACT33
      POTL(8,1)=     -FACT2-FACT3
      CALL QIRE(POTL(1,1),POTL(3,1),0.,0.,0.,0.,TOL1,XLOOP,XJ,GO)
      IF(GO-6HGOBACK)116,115,116
  116 CALL CHARGE(ETA,POT,PHIM,POTL(3,1),POTL(4,1),POTL(5,1),POTL(6,1),1
     1.)
      POTL(7,1)=POTL(6,1)*DEBY2
      LANE1=1
      JJMAX=J
      GOTO 139
C 114 IF(FACT33-FACT22/20.)67,67,72
C  67 JJ=J
C     J2=J2+1
C     J=J+2
C  68 DO 69 I=1,8
C  69 POTL(I,JJ+1)=POTL(I,JJ)
C     JJ=JJ-1
C     IF(JJ)70,70,68
C  70 DDX=DDX+DX
C     GO TO 71
C  EFN 72 TO EFN 121 APPLY TO NEGATIVE SLOPE CALCULATION FOR POTENTIAL
C  MINIMUM, POSITIVE SLOPE FOR ION-RICH SOLUTION
   72 JJ=J
      J2=J2+2
      J2MIN=J2MIN+2
      J=J+2
   73 DO 74 I=1,8
   74 POTL(I,JJ+2)=POTL(I,JJ)
      JJ=JJ-1
      IF(JJ)75,75,73
   75 DPOT1=PHIX/10.
      DPOT2=C1*POTL(3,3)
      IF(DPOT1-DPOT2)80,79,79
   79 DPOT=DPOT2
      LANE1=2
      GO TO 77
   80 DPOT=DPOT1
      LANE1=1
```

```
   77 POTL(3,2)=POTL(3,3)+DPOT  * C2
      POTL(3,1)=POTL(3,2)+DPOT * C2
      CALL PHIDRV(M,POTL(3,1),POTL(8,1),C2,0.)
      IF(POTL(8,1)/POTL(8,3)-2.)78,78,76
   76 IF(DPOT-1.E-6)78,78,466
  466 DPOT=DPOT/2.
      LANE1=1
      GO TO 77
   78 CALL PHIDRV(M,POTL(3,2),POTL(8,2),C2,0.)
      DDX=(.333333/POTL(8,1)+1.333333/POTL(8,2)+.333333/POTL(8,3))
     1    *DPOT
      POTL(1,1)=POTL(1,3)+DDX * C2
      POTL(1,2)=POTL(1,3)+DDX/2. * C2
      GO TO (72,81),LANE1
   81 LANE1=1
      FACT=(POTL(1,J2MIN)-POTL(1,J2MIN-2))/(POTL(3,J2MIN-2)-POTL(3,J2MIN
     1))
      ARRAY(1,1)=POTL(3,J2MIN+1)*FACT
      ARRAY(2,1)=POTL(1,J2MIN+1)
      JJMAX=J
      JJJ=2
      JJJJ=J2MIN
  118 ARRAY(1,JJJ)=POTL(3,JJJJ)*FACT
      ARRAY(2,JJJ)=POTL(1,JJJJ)
      IF(JJJJ-1)119,119,117
  117 JJJJ=JJJJ-2
      JJJ=JJJ+1
      GO TO 118
  119 I1=1
      I4=3
  120 CALL LSP(ARRAY(1,JJJ-2),I1,I4,POTL(3,JJJJ+1)*FACT,POTL(1,JJJJ+1),D
     1UMM)
      IF(JJJJ+2-J2MIN)121,177,177
  121 I4=4
      JJJJ=JJJJ+2
      JJJ=JJJ-1
      GO TO 120
C  NOW GO TO POSITIVE SLOPE CALCULATION
  177 IF(LANE-5)479,178,178
  479 IF(PHI-PHIM)478,478,480
  478 PHITST = POT
      DPHI = (POT-PHIM)/10.
      LANE 1 = 3
      GOTO 15
  480 PHITST = PHI
      LANE1 = 1
      DPHI = (PHI-PHIM)/10.
      GOTO 15
  178 IF(POT-PHIMAX)182,179,184
C  THIS IS A LOOP FOR ION-RICH CHARGE EVERYWHERE WITH NO
C  POTENTIAL MAXIMUM
  179 ERR1=POTL(1,J2)-POTL(1,1)-DEBYY
      CALL QIRE(PHIPRM,ERR1                          ,0.,POT/(POTL(1,J2)
     1    -POTL(1,1)),0.,0.,TOL,XLOOP,XJ1,GOTWO)
      IF (PHIPRM)182,182,180
  180 IF(GOTWO-6HGOBACK)183,19,183
  183 J=J2
      GO TO 166
  181 XJ3(1)=1.
      XJ3(6)=0.
      XJ3(8)=0.
```

```
      XJ3(7)=POTL(6,J2)
      XJ3(2)=0.
      XJ3(3)=PHIMAX
      XJ3(4)=0.
      PHIMX2=PHIMAX+1.
  420 CALL ERF(SQRTF(PHIMX2),CC(3))
      CC(3)  = CC(3) + 1.
      PHIMX3 = PHIMAX
      PHIMAX = PHIMX2
      CALL CHARGE(ETA,POT,0.,PHIMX2,CHGELC,CHGION,CHGNET,-1.)
      DUMM=DUMM
      PHIMAX = PHIMX3
      CALL QIRE(PHIMX2,CHGNET,0.,0.,0.,PHIMX2-PHIMAX,TOL,XLOOP+5.,
     1   XJ3,GOFOUR)
      IF(GOFOUR-6HGOBACK)422,421,422
  421 IF(PHIMX2-1.E6)420,420,422
  422 XJ1(3)=PHIMAX
      XJ1(7)=ERR5
      XJ1(2)=0.
      XJ1(4)=0.
      XJ1(6)=0.
      XJ1(8)=0.
  423 CALL QIRE(PHIMAX,ERR5,0.,0.,0.,(PHIMX2-PHIMAX)/2.,TOL,XLOOP,XJ1,GO
     1TWO)
      IF(GOTWO-6HGOBACK)166,428,166
  428 CALL ERF(SQRTF(PHIMAX),CC(3))
      CC(3)  = CC(3) + 1.
      GOTO 19
  185 PHIMX1 = PHIMAX+PHI/20.
      LOOP1 = LOOP1+1
      IF(PHIMX1-PHI)429,430,430
  429 PHIMAX = PHIMX1
      GOTO 18
  430 IF(LOOP1-LOOP)431,431,27
  431 PHIMAX  =  (PHIMAX+PHI)/2.
      GOTO 18
C     EFN 184-400 CALCULATE NEGATIVE SLOPE
C     PORTION OF POTENTIAL MAXIMUM SOLUTION
  184 LANE = 8
  197 DPOT = -.005
      GOTO 193
  182 LANE=7
  191 IF(PHIMAX-POT)197,197,497
  497 DPOT=-MIN1F((POTL(3,J)-POT)/2.,.005)
  193 POTL(3,J+1)=POTL(3,J)+DPOT
      POTL(3,J+2)=POTL(3,J+1)+DPOT
      IF(POTL(3,J+2))425,490,490
  490 CALL PHIDRV(LDUM,POTL(3,J+2),POTL(8,J+2),-1.,0.)
      GO TO(186,425,425),LDUM
  186 IF(POTL(8,J+2)/POTL(8,J)-2.)187,187,192
  187 IF(POTL(8,J+2)/POTL(8,J)-.5)192,188,188
  188 CALL PHIDRV(LDUM,POTL(3,J+1),POTL(8,J+1),-1.,0.)
      DX=(.333333/POTL(8,J)+1.666667/POTL(8,J+1)+.333333/POTL(8,J+2))
     1    *DPOT
      POTL(1,J+1)=POTL(1,J)+DX/2.
      POTL(1,J+2)=POTL(1,J)+DX
      GO TO 194
  192 IF(DPOT+1.E-6)425,188,188
  425 DPOT = DPOT/2.
      GOTO 193
  194 J = J+2
```

```
      IF(POTL(8,J)-(POTL(3,J)-POTL(3,J2))/(POTL(1,J)-POTL(1,J2))
     1    /100.)197,195,195
  195 J = J+2
      D3PHI = (POTL(7,J-2)-POTL(7,J-4))/(POTL(1,J-2)-POTL(1,J-4))
      DX=(SQRTF(POTL(7,J-2)**2-2.*POTL(8,J-2)*D3PHI)-POTL(7,J-2))/D3PHI
      POTL(1,J)=POTL(1,J-2)+DX
      POTL(3,J)=POTL(3,J-2)+POTL(8,J-2)*DX+POTL(7,J-2)*DX**2/2.
     1    +D3PHI*DX**3/6.
      POTL(3,J-1)=POTL(3,J-2)+POTL(8,J-2)*DX/2.+POTL(7,J-2)*DX**2/8.
     1    +D3PHI*DX**3/48.
      CALL CHARGE(ETA,POT,0.,POTL(3,J-1),POTL(4,J-1),POTL(5,J-1),
     1    POTL(6,J-1),-1.)
      CALL CHARGE(ETA,POT,0.,POTL(3,J),POTL(4,J),POTL(5,J),POTL(6,J),
     1    -1.)
      POTL(7,J-1) = POTL(6,J-1)*DEBY2
      POTL(7,J  ) = POTL(7,J-1)*DEBY2
  400 FACT=(POTL(1,J2+3)-POTL(1,J2))/(POTL(3,J2)-POTL(3,J2+3))
      ARRAY(1,1) = POTL(3,J2)*FACT
      ARRAY(2,1) = POTL(1,J2)
      JJJ = 2
      JJJJ = J2+1
      JJMAX = J
  401 ARRAY(1,JJJ) = POTL(3,JJJJ)*FACT
      ARRAY(2,JJJ) = POTL(1,JJJJ)
      IF(JJJJ-JJMAX)402,403,403
  402 JJJ = JJJ+1
      JJJJ = JJJJ+2
      GOTO 401
  403 I1  = 1
      I4 = 3
  404 CALL LSP(ARRAY(1,JJJ-2),I1,I4,POTL(3,JJJJ-1)*FACT,POTL(1,JJJJ-1),
     1    DUMM)
      IF(JJJJ-1-J2)406,406,405
  405 I4 = 4
      JJJJ = JJJJ-2
      JJJ = JJJ-1
      GOTO 404
  406 IF(LANE-7)407,410,408
  407 CALL ERROR
      GOTO 83
  408 DO 409 I = 1,J
  409 POTL(2,I) = POTL(1,I)*DEBYE
      GOTO 166
  410 DO 512 I=1,J
  512 POTL(2,I)=POTL(1,I)*DEBYE
      WAVE=POTL(1,J)-POTL(1,J2)
      DIST=POTL(1,J)
      LDIR=-1
  499 IF(DIST-DEBYY)498,502,502
  498 IF(DIST+WAVE-DEBYY)500,501,501
  500 DIST=DIST+2.*WAVE
      GO TO 499
  501 LDIR=1
C     DIST IS THE DISTANCE FROM THE EMITTER OF
C      THE MINIMUM THAT IS CLOSEST TO THE
C      COLLECTOR
  502 DELTX=DIST-POTL(1,J)
      DELTXX=DELTX*DEBYE
      JJJJ=J
      IF(LDIR)505,508,508
  505 JJ1=J2-1
```

```
  506 JJ1=JJ1+1
      J=J+1
      POTL(1,J)=POTL(1,JJ1)+DELTX
      POTL(2,J)=POTL(2,JJ1)+DELTXX
      DO 507 I=3,8
  507 POTL(I,J)=POTL(I,JJ1)
      IF(POTL(1,J)-DEBYY)506,511,511
  508 JJ1=JJJJ+1
  509 JJ1=JJ1-1
      J=J+1
      POTL(1,J)=2.*POTL(1,JJJJ)-POTL(1,JJ1)+DELTX
      POTL(2,J)=2.*POTL(2,JJJJ)-POTL(2,JJ1)+DELTX
      DO 510 I=3,8
  510 POTL(I,J)=POTL(I,JJ1)
      IF(POTL(1,J)-DEBYY)509,511,511
  511 TEST=POTL(3,J-1)+(DEBYY-POTL(1,J-1))/(POTL(1,J)-
     1POTL(1,J-1))*(POTL(3,J)-POTL(3,J-1))
      ERR5=TEST-POT
      GO TO (411,413),LPRINT
  411 IF(XJ1)181,181,423
  413 KEND1=8
      KEND=3
      GOTO166
   83 ERR2 = POTL(1,J)-POTL(1,1)-DEBYY
      IF(ERR2-TOL)144,144,133
  144 GOTO(92,24,122,24,122),LANE6
   92 IF(XJ2(7))87,145,145
   87 XKK4 = PHIMX-PHIM
      GOTO 138
  133 LANE6 = 1
  145 XKK4 = DPHIM
      DPHIM = DPHIM*2.
  138 PHIMX = PHIM
      CALL QIRE(PHIMX,ERR2,0.,0.,0.,XKK4,TOL,XLOOP,XJ1,GOTWO)
      IF(GOTWO-6HGOBACK)122,136,122
  136 NCOUNT=NCOUNT+1
      PHIM2(NCOUNT)=PHIMX
   89 PHIM=PHIM2(NCOUNT)
      CALL ERF(SQRTF(-PHIM),CC(1))
      XJ=1.
      XJ(4)=0.
      XJ(8)=0.
      PHI=PHIM
      CC(1) = CC(1) + 1.
  107 CALL CHARGE(ETA,POT,PHIM,PHIM,CHGELC,CHGION,CHG2(NCOUNT),0.)
      IF(CHG2(NCOUNT))88,88,64
   88 PHIM2(NCOUNT)=(PHIM2(NCOUNT)+PHIM2(NCOUNT-1))/2.
      GO TO 89
C  STATEMENTS EFN 122 TO EFN 132 PROVIDE FOR INTERPOLATION TO GET
C  THE ODD VALUES OF X FOR THE POTENTIAL MINIMUM CASE
C  INITIAL DISTANCE ADJUSTED TO ZERO
  122 FACT=(POTL(1,JJMAX+2)-POTL(1,JJMAX))/(POTL(3,JJMAX+2) -POTL
     1(3,JJMAX))
      ARRAY(1,1)=POTL(3,JJMAX-1)*FACT
      ARRAY(1,2)=POTL(1,JJMAX-1)
      JJJ=2
      JJJJ=JJMAX
  123 ARRAY(1,JJJ)=POTL(3,JJJJ)*FACT
      ARRAY(2,JJJ)=POTL(1,JJJJ)
      IF(JJJJ-J)124,125,125
  124 JJJ=JJJ+1
```

```
      JJJJ=JJJJ+2
      GO TO 123
  125 I1=1
      I4=3
  126 IF(JJJ-JBELOW)462,463,463
  463 IF(JJJ-JABOVE)464,464,462
  462 CALL LSP(ARRAY(1,JJJ-2),I1,I4,POTL(3,JJJJ-1)*FACT,POTL(1,JJJJ-1),
     1DUMM)
  464 IF(JJJJ-JJMAX-2)130,127,127
  127 I4=4
      JJJJ=JJJJ-2
      JJJ=JJJ-1
      GOTO 126
  130 DX=-POTL(1,1)
      DO 132 II=1,J
      POTL(1,II)=POTL(1,II)+DX
  132 POTL(2,II)=POTL(1,II)*DEBYE
      GO TO 166
C  STATEMENTS EFN 96 TO EFN 105 ADJUST DISTANCES IN THE POTENTIAL
C  MINIMUM SITUATION TO MATCH THE ELECTRODE COORDINATES
  103 DX=-POTL(1,1)
      JJJ=J
  104 JJJ=JJJ-1
      IF(JJJ)513,513,105
  105 POTL(1,JJJ)=POTL(1,JJJ)+DX
      GO TO 104
  513 GO TO (162,434,434),LANE6
C  STATEMENTS EFN 1 TO EFN 176 SET UP POTENTIAL MAXIMUM IF NET CHARGE
C  AT COLLECTOR IS NEGATIVE.  FIRST TRIAL IS WITH ZERO SLOPE AT COLLECTOR
C  THE POSITIVE SLOPE REGION TOWARD THE COLLECTOR MAY NOT HAVE AN
C  INFLECTION POINT
    1 LANE=5
      PHI=0.
      NCOUNT=1
      CHG2(1)=CHGNET
      PHIMAX=POT
      XJ1=0.
      IF(PHIPRM-BITS)19,18,19
   18 PHIPRM=0.
   19 LANE=5
      DO 427 I=1,8008
  427 POTL(I)=0.
      J2=2
      J=3
      J2MIN=1
      DDX=MIN1F(.002*DEBYY,.25/DEBYE)
      POTL(3,J2)=PHIMAX
      CALL PHIDRV(M,POTL(3,2),PHIPRM,-1.,CHG2(NCOUNT))
      POTL(8,J2)=0.
  474 FACT2=CNS(2,2)*DDX
      POTL(1,1) = POTL(1,2)-DDX
      POTL(1,3) = POTL(1,2)+DDX
      FACT22=FACT2*DDX/2.
      POTL(8,1)=-FACT2
      POTL(8,J)=FACT2
      POTL(3,1)=PHIMAX+FACT22
      POTL(3,J)=POTL(3,1)
      LANE=6
      CALL CHARGE(ETA,POT,0.,POTL(3,1),POTL(4,1),POTL(5,1),POTL(6,1),-1.
     1)
      LANE = XMAXOF(7,KEND+1)
```

```
      CALL CHARGE(ETA,POT,0.,POTL(3,3),POTL(4,3),POTL(5,3),POTL(6,3),-1.
     1)
      POTL(7,1)=POTL(6,1)*DEBY2
      POTL(7,3)=POTL(6,3)*DEBY2
      IF(ABSF(POTL(7,1)/POTL(7,2)-1.)-.05)176,176,475
  475 IF(ABSF(POTL(3,2)-POTL(3,1))-1.E-6)176,176,476
  476 DDX = DDX/2.
      GOTO 474
  176 PHIX=PHIMAX
      C1=.5
      C2=-1.
      LANEX=7
      LANE=6
      GO TO 72
  324 CALL EXIT
 1000 FORMAT
      RESTORE
       PLANAR DIODE POTENTIAL DISTRIBUTION ANALYSIS - CALCULATION DATE
X-I/-I/-I
      SPACE
-X                        -A / DEBYE LENGTH     =       -F4
  ELECTRON DENSITY / ION DENSITY AT EMITTER     =       -F4
  COLLECTOR BIAS POTENTIAL                   -A =       -F4
  ELECTRON CURRENT                              =       -F4
  ION CURRENT                                   =          -1PG4
      SPACE
 DISTANCE  DISTANCE CALCULATED ELECTRON      ION    NET CHARGE     PHI
X   PHI
 /SPACING   /DEBYE   POTENTIAL  DENSITY    DENSITY    DENSITY     DOUBLE
X  PRIME
            LENGTH              /EMITTER   /EMITTER *           -A PRIME
                                  ION        ION
                                 DENSITY    DENSITY
      SPACE
      END OF FORMAT
 1001 FORMAT
-X  -0PF4       -F2        -F6     -1PG3        -G3        -G3        -G3
X      -G3
      END OF FORMAT
 1003 FORMAT
      RESTORE
      NO INFLECTION POINT WAS FOUND FOR    -AIVE INITIAL POTENTIAL GRADI
XENT WITH
                                POT =      -F3
                                ETA =      -F3
                              DEBYE =      -F3
                              DEBYY =      -F3
                               PHIM =      -F3
      END OF FORMAT
 1002 FORMAT
      RESTORE
      NO NEGATIVE VALUE OF PHIM GAVE AN ELECTRON-RICH SITUATION FOR
      SPACE
                                POT =      -F3
                                ETA =      -F3
                              DEBYE =      -F3
      SPACE
                               PHIM       CHG2
                                 -F5        -F5
      REPEAT 1
      END OF FORMAT
```

```
      END
*ERF1 IS THE ASYMPTOTIC EXPANSION FOR (1-ERF(SQRTF(X)))*EXPF(X)
      SUBROUTINE ERF1(ARG,RESULT,TOL)
      SUM=1.
      COUNT=1.
      TERM1=1.
    1 TERM2=-COUNT/2./ARG*TERM1
      IF(ABSF(TERM2/SUM)-TOL)4,4,2
    2 IF(ABSF(TERM2)-ABSF(TERM1))3,4,4
    3 SUM=SUM+TERM2
      TERM1=TERM2
      COUNT=COUNT+2.
      GOTO 1
    4 RESULT=0.5641875/SQRTF(ARG)*SUM
      RETURN
*ERF   00
      SUBROUTINE ERF(X,Y)
      DIMENSION XS(8),VERX(8),DERIV(8)
      TABLE XS(0.0,0.5,1.0,1.5,2.0,2.5,3.0,3.5)
      TABLE VERX(.00000000,.52049987,.84270079,.96610514,.99532226,
     1.99959304,.99997790,.99999925)
      TABLE DERIV(1.1283791,.87878257,.41510749,.11893028,
     1.02066698,.00217828,.00013925,.00000539)
      IF (X)10,11,12
   11 Y=0.0
      GO TO 100
   10 X=ABSF(X)
      SIGN=-1.
      GO TO 13
   12 SIGN=1.
   13 IF (XS(1)-X)16,11,99
   16 DO 19 I=2,8
      IF (XS(I)-X)19,20,21
   19 CONTINUE
   14 IF (SIGN)18,99,17
   17 Y=1.
      GO TO 100
   18 Y=-1.
      GO TO 100
   20 IF (SIGN)22,99,23
   22 Y=-VERX(I)
      GO TO 100
   23 Y=VERX(I)
      GO TO 100
   21 H1=X-XS(I-1)
      H2=X-XS(I)
      IF (H1-ABSF(H2))24,24,25
   24 H=H1
      I=I-1
      GO TO 26
   25 H=H2
   26 UX=XS(I)
      UXU=UX**2
      HSQ=H**2
      Y=VERX(I)+DERIV(I)*(H+HSQ*(-2.*UX)/2.+H*
     1HSQ*(4.*UXU-2.)/6.+HSQ**2.*(-8.*UXU*UX
     2+12.*UX)/24.+HSQ**2.*H*(16.*UXU**2.-48.
     3*UXU+12.)/120.)
      IF (SIGN)27,99,100
   99 CALL ERROR
   27 Y=-Y
```

```
  100 RETURN
      END
*PHIDRV
      SUBROUTINE PHIDRV(LDUMMY,PHI,DPHI,SIGN,CHG)
      DIMENSION POTL(8,1001),ARRAY(2,501),CNS(6,3),CC(10)
     1    ,PHIMAX(6),PHI(6),PHIMIN(6)
      COMMON POTL,ARRAY,DEBY2,CONST,CNS,POT,PHIM,ETA
     1    ,CC,TEMP,LANE6,DPHIXX,PHIMAX,LANE
CLANE = 1  NORMALLY FOR MONOTONIC SOLUTIONS
CLANE = 2  NORMALLY FOR NEGATIVE SLOPE OF POTENTIAL MINIMUM SOLUTIONS
CLANE = 3  TO SET UP CONSTANTS FOR MONOTONIC SOLUTIONS
CLANE = 4  TO SET UP CONSTANTS FOR POTENTIAL MINIMUM SOLUTIONS
CLANE = 5  TO SET UP CONSTANTS FOR POTENTIAL MAXIMUM SOLUTIONS
CLANE = 6  ON POSITIVE SLOPE OF POTENTIAL MAXIMUM SOLUTION
CLANE = 7  ON NEGATIVE SLOPE OF POTENTIAL MAXIMUM SOLUTION
CLANE = 8  ON NEGATIVE SLOPE OF OSCILLATORY SOLUTION
C  LANE6 = 1 NORMALLY (OR TO START)
C  LANE6 = 2 IF PHIPRIME AT A INFLECTION POINT OF A MONOTONIC
C  SOLUTION IS TOO SMALL TO AFFECT CONST
C  LANE6 = 3 IF PHIPRIME AT AN INFLECTION POINT OF A POTENTIAL
C   MINIMUM SOLUTION IS TOO SMALL TO AFFECT CONST
C  LANE6 = 4 OR 5 ON THE FINAL PASS FOR WITH THE CORRECT INFLECTION
C   POINT PHIPRIME FOR MONOTONIC OR POTENTIAL MINIMUM
CC    SOLUTION RESPECTIVELY
C  PHIPRIME**2 = 2.*DEBY2*((ELECTRON CHARGE DENSITY)-(ION CHARGE DENSITY)
C    -SIGN*1.128*EXPF(PHIM)/(1.+ERF(SQRTF(-PHIM)))*SQRTF(PHI-PHIM)*ETA
C    -(1.128*C1*EXPF(-MAX1F(POT,PHIMAX))*SQRTF(MAX1F(POT,PHIMAX)-PHI)
C    -2.256*C2*EXPF(-PHIMAX)*SQRTF(PHIMAX-PHI))/(1.+ERF(MAX1F(PHIMAX,
C    POT))))+CONST
C  C1 = +1.  IF POT IS GREATER THAN PHIMAX
C  C1 = -1.  IF POT IS LESS THAN PHIMAX
C  C2 =  1.  IF POT IS GREATER THAN PHIMAX AND PHI IS LESS THAN PHIMAX
C  C2 =  0.  OTHERWISE
    3 X2 = 1.
      LDUMMY = 1
      CALL CHARGE(ETA,POT,PHIM,PHI(1),PHI(2),PHI(3),PHI(4),SIGN)
      PHI(5) = DEBY2*PHI(4)
      TERM5  =  0.
    1 GOTO(8,8,2,2,2,8, 8, 8),LANE
    8 FACT=PHI(1)-PHINFL
      IF(ABSF(FACT)-.1)20,20,13
   13 GOTO(41,10,41,10,11,11,46,48),LANE
    2 DO 4 I=1,18
    4 CNS(I)=0.
      GOTO(8,8,6,5,6,8,46,46),LANE
    5 CC(6)=EXPF(PHIM)
      CC(2)=CC(6)/CC(1)*.564189
      GO TO 7
6     CC(6) = 1.0
      CC(2)=.564189
    7 X1 = 1.
   40 CC(4)=.564189/CC(3)
      IF(POT-100.)36,35,35
   35 CC(5) = 0.
      GOTO 38
   36 CC(5) = CC(4)*EXPF(MAX1F(-MAX1F(POT,PHIMAX),-POT))
      CC(7)=CC(4)*EXPF(-PHIMAX)
   38 PHINFL=PHI
      IF(PHIM)24,9,9
   24 IF(PHI-PHIM)25,25,9
   25 PHINFL=PHIM-.2
```

```
9    GOTO(57,10,41,10,11,11,46,48),LANE
 57  IF(PHI)10,41,41
 46  TRY = SQRTF(PHIMAX-PHI)
     TERM4 = 2.*CC(5)*TRY
     GOTO 42
 48  TRY = SQRTF(PHIMAX-PHI)
     TERM5 = 4. * CC(7)*TRY
     TRY = SQRTF(POT-PHI)
     TERM4 = -2.*CC(5)*TRY
     GOTO 42
 11  TRY = SQRTF(MAX1F(PHIMAX,POT)-PHI)
     TERM4 = -2.*CC(5)*TRY
     GOTO 42
 10  TRY = SQRTF(-PHI)
     TERM5 = 4.*CC(4)*TRY
 41  TRY=SQRTF(POT-PHI)
     TERM4 = -TRY*CC(5)*2.
 42  TERMYY=PHI(3)+TERM4+TERM5
 43  TRY1 = SQRTF(PHI-PHIM)
     TERMX = -SIGN*2.*CC(2)*TRY1 *ETA + PHI(2)
28   TEMP=2.*DEBY2*(TERMX+TERMYY)
 14  GOTO(19,19,15,16,49,50,51,52),LANE
 15  DPHIXX=DPHI**2
     CONST=DPHIXX-TEMP
     IF(DPHIXX-1.E-7*TEMP)56,56,17
56   LANE6 = XMAXOF(2,LANE6)
     GO TO 17
 16  CNS(2,2)=CHG*DEBY2
     CNS(3,2)= DEBY2*ETA*CC(2)*SQRTF(2.*CHG)
     PHIMIN(1) = PHIM
     CALL CHARGE(ETA,POT,PHIM,PHIMIN(1),PHIMIN(2),PHIMIN(3),PHIMIN(4),
    1     0.)
     CONST = -2.*DEBY2*(PHIMIN(2)+PHIMIN(3)-2.*CC(5)*SQRTF(POT-PHIM)
    1     +4.*CC(4)*SQRTF(-PHIM))
     DPHIXX=TEMP+CONST
     IF(DPHIXX)22,23,23
 22  DPHI=DPHIXX
     GOTO 18
 23  DPHI=SQRTF(DPHIXX)
     IF(DPHIXX+CONST*1.E-7+PHI-PHINFL)44,44,17
 44  LANE6 = MAXOF(3,LANE6)
     DPHIXX = 1.E -12
 17  CNS(1,1)= 0.
     CNS(2,1)=DEBY2*(2.*PHI(3)-ETA*CC(2)/TRY1+CC(5)/TRY)+CNS(1,1)/2.
     CNS(3,1)=DEBY2*(-ETA*CC(2)*(1./TRY1-.5/TRY1**3)-CC(5)*
    1     (1./TRY-.5/TRY**3))/3.+CNS(1,1)/6.
     CNS(4,1)=(DEBY2*(ETA*CC(2)*(.5/TRY1**3-.75/TRY1**5)
    1     -CC(5)*(.5/TRY**3-.75/TRY**5))+CNS(2,1))/12.
     CNS(5,1)=(CNS(3,1)-DEBY2*(ETA*CC(2)*(.75/TRY1**5-1.875/TRY1**7)
    1     +CC(5)*(.75/TRY**5-1.875/TRY**7))/3.)/20.
     CNS(6,1)=(CNS(4,1)+DEBY2*(ETA*CC(2)*(1.875/TRY1**7-6.5625/TRY1**9)
    1     -CC(5)*(1.875/TRY**7-6.5625/TRY**9))/12.)/30.
 18  RETURN
 19  DPHI=-SIGN*SQRTF(TEMP+CONST)
     GO TO 18
 20  IF(PHI)11,37,37
37   IF(PHINFL)9,9,21
 21  TENT = DPHIXX+FACT*CNS(1,1)
    1            +FACT**2*CNS(2,1)+FACT**3*CNS(3,1)
    2     +FACT**4*CNS(4,1)+FACT**5*CNS(5,1)+FACT**6*CNS(6,1)
     DPHI=SQRTF(TENT)
```

```
      GO TO 18
   49 CNS(2,2) = PHI(5) * DEBY2
      CONST = -TEMP
      DPHIXX = 0.
      DPHI = 0.
      PHINFL = MAX1F(POT,PHIMAX)+.2
      GOTO 18
   50 DPHI = SQRTF(TEMP+CONST)
      GOTO 18
   51 IF(TEMP+CONST)55,54,54
   54 DPHI = -SQRTF(TEMP+CONST)
      GOTO 18
   52 IF(TEMP+CONST)53,54,54
   53 LDUMMY = 3
      GOTO 18
   55 LDUMMY = 2
      GOTO 18
      END
*CHARGE
      SUBROUTINE CHARGE(ETA,PHIC,PHIM,PHIX,CHGELC,CHGION,CHGNET,SIGN)    CHARGE
      DIMENSION POTL(8,1001),ARRAY(2,501),CNS(6,3),CC(10),PHIMAX(6)
      COMMON POTL,ARRAY,DEBY2,CONST,CNS,POT,PHIM,ETA
     1    ,CC,TEMP,LANE6,DPHIXX,PHIMAX,LANE
    1 PHI=PHIX
      IF(PHI-40.)17,16,16
   16 X1 = 1.E20
      GOTO 18
   17 X1=EXPF(PHI)
   18 X4=0.
      X6=0.
C    CHGELC = ETA * EXPF(PHI)* (1. + SIGN * ERF(SQRTF(PHI - PHIMIN)))/
C             (1. + ERF(SQRTF(-PHIMIN)))
C     SIGN IS NEGATIVE BETWEEN XPHIMIN AND COLLECTOR
C     SIGN IS POSITIVE BETWEEN EMITTER AND XPHIMIN
    6 IF(PHI-PHIM-9.)8,9,9                                               CHARGE
    8 CALL ERF(SQRTF(PHI-PHIM),X5)
      Y5=X1*(1.+X5*SIGN)/CC(1)
      GO TO 10
    9 CALL ERF1(PHI-PHIM,Y5,1.E-5)                                       CHARGE
      Y5=Y5*EXPF(PHIM)/(1.+X6)                                           CHARGE
   10 CHGELC=ETA*Y5
C  CHGION = EXPF(-PHI)/(1.+ERF(SQRTF(MAX1F(POT,PHIMAX)))) *
C             (1.+C1*ERF(SQRTF(MAX1F(PHIMAX,POT)-PHI))
C             + C2*ERF(SQRTF(PHIMAX-PHI)))
C     C1 =  1. IF POT IS GREATER THAN PHIMAX
C             OR FOR XPHI BETWEEN EMITTER AND XPHIMAX
C  C1 = -1. IF POT IS LESS THAN PHIMAX AND XPHI IS BETWEEN XPHIMAX AND
C        COLLECTOR
C     C2 = -2. FOR POT GREATER THAN PHIMAX, XPHI BETWEEN XPHIMAX
C         AND COLLECTOR, AND PHI LESS THAN PHIMAX
C     C2 =  0. OTHERWISE
      GOTO(12,21,12,20,12,12,12,20),LANE
   21 IF(PHI)20,12,12
 20   CALL ERF(SQRTF(PHIMAX-PHI),X4)
      X4=X4*2.
   12 CALL ERF(SQRTF(MAX1F(POT,PHIMAX)-PHI),X3)
      IF(LANE-7)13,19,13
   19 X3 = -X3
 13   CHGION=(1.+X3-X4)/CC(3)/X1
      CHGNET=CHGELC-CHGION
    7 RETURN
```

```
*LSP                    LEAST SQUARES PARABOLA
C       SUBROUTINE ROTATES AND TRANSLATES COORDINATES SO THAT THE NEW
C      ORIGIN IS  X(3),X(4) AND THE X-AXIS PASSES THROUGH X(5),X(6).      LSP
C      THE PARABOLA Y=B*(1-X/(X3-X2))*X  IS DETERMINED BY A LEAST SQUARE
C      DEVIATION OF Y1 AND Y4. ON RETURN THE INTERPOLATED VALUE, YP, IS
C      AVAILABLE.
       SUBROUTINE LSP(X,I1,I2,XP,YP,DYDXP)
       DIMENSION X(10),GOBACK(1)                                          LSP
       TABLE GOBACK(6HGOBACK)                                             LSP
       DIMENSION XJ(8)
       EQUIVALENCE (S1,SY),(S2,SX),(XR,XX),(YR,YY),(B,Y4),(XJ(4),X1),(XJ(LSP
      18),Y1)                                                             LSP
C
       II1=2*I1                                                           LSP
       II2=2*I2                                                           LSP
       XJ=1.                                                              LSP
       XJ(2)=1.                                                           LSP
       X12=X(3)                                                           LSP
       X22=X(4)                                                           LSP
       SX=X(5)-X12                                                        LSP
       XJ(6)=X(5)-XP                                                      LSP
       SY=X(6)-X22                                                        LSP
       XJ(7)=X12-XP                                                       LSP
       F=-XJ(7)/SX                                                        LSP
       S=SQRTF(SX**2 SY**2)                                               LSP
       S1=SY/S                                                            LSP
       S2=SX/S                                                            LSP
       YY=X(II1)-X22                                                      LSP
       XX=X(II1-1)-X12                                                    LSP
       X1=XX*S2+YY*S1                                                     LSP
       Y1=YY*S2-XX*S1                                                     LSP
       X1=X1*(S-X1)                                                       LSP
       XX=X(II2-1)-X12                                                    LSP
       YY=X(II2)-X22                                                      LSP
       X4=XX*S2+YY*S1                                                     LSP
       Y4=YY*S2-XX*S1                                                     LSP
       X4=X4*(S-X4)                                                       LSP
       B=S*(Y1*X1+Y4*X4)/(X1**2+X4**2)                                    LSP
       XJ(8)=0.                                                           LSP
C      PATCH IN XJ(3)=0.,XJ(4)=0.                                         LSP
    90 XR=F*S                                                             LSP
       YR=B*(1.-F)*XR                                                     LSP
       CALL QIRE(F,X12+XR*S2-YR*S1-XP,0.,0.,0.,0.,5.E-6,10.,XJ,GO)        LSP
C      THIS LSP VERSION IS INTENDED TO BE USED WITH DC PRINCE QIRE        LSP
   100 IF(GO-GOBACK)101,90,101                                            LSP
   101 YP=X22+YR*S2+XR*S1                                                 LSP
       X1=B*(1.-F-F)                                                      LSP
       Y1=SY/SX                                                           LSP
       DYDXP=(X1+Y1)/(1.-X1*Y1)                                           LSP
   102 RETURN                                                             LSP
       END                                                                LSP
```

APPENDIX B

CYLINDRICAL DIODE COMPUTER PROGRAM

APPENDIX B: Cylindrical Diode Computer Program

```
*CYLDIODE
      COMMON POINT,POTL,VTO,MONTH,J,WRITE,NWRITE,NWRMAX,KPLOT,
     1    DNSEL,DNSION,CONST1,CONST2,      RAD,POT1,POT2,CUREL,CURION
     2    ,JMAX,INSIDE,DEBY2,DEBY1,LGOTO,J2MAX,RADVAL
     3    ,ENRGMN
      COMMON KEND,       CYLSPH,KYLSPH,ANGTOL,VELTOL,VELTLL,POT,NPOT
      DIMENSION POINT(20,150),POTL(8,1001),          MONTH(3)
      DIMENSION WRITE(20),POT1(1001),POT2(1001),XPOT1(1001),XPOT2(1001)
      DIMENSION VECTOR(8),RADII(1001)
      DIMENSION VTO(2,502),RADVAL(1001),POT(3,30)
      DIMENSION LABEL(3,2),ARRAY(2,501),XTEST(2,20),LABEL3(2,2)
      DIMENSION BITS(1),LABEL1(4,2),LABEL2(3,2),LABEL4(2),LABEL5(2)
      DIMENSION LABEL6(2),LABEL7(2,2),LIST(125)
      TABLE LABEL(36H    / (R/LAMBDA)SQ                    )
      TABLE LABEL1(48HEMITTING CYLINDER RADIUS       ELECTRODE SPACING)
      TABLE LABEL2(36H COLLECTOR RADIUS ELECTRODE SPACING )
      TABLE LABEL3(24HRADIUS      DIST- ANCE  )
      TABLE LABEL4 (12HUPPER LOWER )
      TABLE LABEL5(12H XPOT1 XPOT2)
      TABLE LABEL6(12HPOT1  POT2  )
      TABLE LABEL7(24HCYLINDRICAL   SPHERICAL )
      TABLE LIST(39,$(ANGTOL,DEBYE,DEBY2,DSLOPX,ENRGMN,ETA,
     1    INSIDE,JFIRST,JMAX,J1MAX,J2MAX,JMIN,KEND,KFLAT,KPLOT,
     2    KPOT,KPRINT,KRAD,KYLSPH,MONTH,NLIM,NWRMAX,POT,POT1,POT2,
     3    POTL,RAD,RADII,SLOPE,SLOPEL,SLOPEU,SPACE,TOL,VELTOL,WEIGHT,
     4    WRITE,XPOT1,XPOT2,XRAD))
      TABLE BITS(O-377777777777)
      DIMENSION DLTPHI(1001)
      EQUIVALENCE(XPOT1,RADII),(XPOT2,DLTPHI),(ARRAY,POINT)
*KEND = 1  TO CALL FOR REINITIALIZATION OF ALL DATA BEFORE
*           STARTING NEW CASE.
*     = 2  TO RETAIN INITIAL DATA FOR USE WITH NEW INPUT.
*     = 3  TO ITERATE USING PRESCRIBED WEIGHTING OF TWO
*           INPUT POTENTIAL DISTRIBUTIONS, OR PREVIOUS ASSUMPTION
*           AND CALCULATED RESULT.
*     = 4  AFTER ERROR CALL, PRINT RESULTS AND EXIT.
*     = 5  TO EXTRAPOLATE SINGLE SPECIES, EXTERNAL COLLECTOR SOLUTION.
*     = 6  TO ITERATE, CALCULATING WEIGHT FACTOR FROM PREVIOUS HISTORY.
*     = 7  FOR ITERATION ON INITIAL SLOPE, ASSUMING COLLECTOR
*           POTENTIAL CONTROLS CUTOFF CONTOURS.
      IF(SENSE LIGHT 1)27,1
   27 KEND=4
      GOTO 26
    1 KPLOT=0
      KPOT=0
      KYLSPH=1
      ASSIGN 27 TO LGOTO
      NWRMAX=0
      DSLOP=0.
      TOL=1.E-5
      ANGTOL=3.
      POT(2)=1.
      POT(3)=BITS
      SPACE = BITS
      VELTOL=1.5
      ERR = BITS
      CONST1=.5
      CONST2=.5
      DSLOPX=BITS
      SLOPEU=BITS
      SLOPEL=BITS
```

```
      SLOPE = BITS
      JFIRST=1
      NLIM=10
      JMIN=2
      KEND=1
*KPRINT = 1  TO PRINT OUT 40 MAXIMUM DISTANCE INCREMENTS (1 PAGE).
*       = 2  TO PRINT OUT ALL DISTANCES.
      KPRINT=1
*KRAD = 0  FOR EQUAL DISTANCE INCREMENTS.
*     = 1  FOR ARBITRARY DISTANCE INPUT.
      KRAD=0
      VTO(1,1)=0.
      VTO(2,1)=0.
      DO 15 I=2,101
      FACTOR=10.**(FLOATF(I-2)/100.)
      VTO(1,I)=.0001*FACTOR
      VTO(1,I+100)=.001*FACTOR
      VTO(1,I+200)=.01*FACTOR
      VTO(1,I+300)=.1*FACTOR
   15 VTO(1,I+400)=FACTOR
      VTO(1,502)=10.
      DO 188 I = 2,502
  188 VTO(2,I)=VTO(1,I)**2
      DO 207 I=2,1001
      XPOT1(I)=BITS
      XPOT2(I)=BITS
      POT1(I)=BITS
  207 POT2(I)=BITS
      DEBY2=1.
*SPACE, XRAD ARE POSITIVE FOR INTERNAL COLLECTOR
    2 CALL DING (LIST,1)
      IF(SENSE LIGHT 1)25,51
   51 INSIDE=INSIDE
      KYLSPH=KYLSPH
      KPRINT=KPRINT
      KEND=KEND
      CTEST = SIGNF(1.,POT)
      JFIRST=JFIRST
      J2MAX=J2MAX
      JMIN=JMIN
      JMAX=JMAX
      NCOUNT=0
      CYLSPH=KYLSPH
      VELTLL=1./VELTOL
      NPOT=LCOUNT(1,30,POT,2)
      POTMIN=0.
      IF(NPOT-1)255,255,252
  252 II=3*NPOT
      III=2*NPOT
  253 POT(II-1)=POT(III)
      POT(II-2)=POT(III-1)
      IF(III-4)255,255,254
  254 III=III-2
      II=II-3
      GO TO 253
  255 DO 248 II=1,NPOT
  248 POTMIN=MIN1F(POTMIN,POT(1,II)*CTEST)
      POTMIN = 3. * POTMIN*CTEST-1.*POT
      IF(KEND-5)3,50,3
*INSIDE = 1  FOR INTERNAL COLLECTOR, EMITTER RADIUS SPECIFIED IN
*            DEBYE LENGTHS.
```

```
*       = 2  FOR EXTERNAL COLLECTOR, EMITTER RADIUS SPECIFIED IN
*            DEBYE LENGTHS.
*       = 3  FOR INTERNAL COLLECTOR, ELECTRODE SPACING SPECIFIED IN
*            DEBYE LENGTHS.
*       = 4  FOR EXTERNAL COLLECTOR, ELECTRODE SPACING SPECIFIED IN
*            DEBYE LENGTHS.
    3 GO TO(122,122,123,123),INSIDE
  122 POTL(1,1)=1.
      POT(2,1)=RAD
      GO TO 124
  123 POTL(1,1)=0.
  124 IF((JMAX+1)/2-JMAX/2)28,28,29
   28 JMAX=JMAX+1
   29 XJMAX=FLOATF(JMAX)-1.
      J2MAX=XMAX0F(JMAX,J2MAX,LCOUNT(1,1001,POT1,1),LCOUNT(1,1001,POT2,1
     1))
   97 NWRITE = 1
      XJMAX = FLOATF(J2MAX)-1.
      DO 500 I = 1,8
  500 VECTOR(I) = 0.
      CUREL=0.
      CURION=0.
      J=1
      X=1./XJMAX
      GOTO(72,72,73,73),INSIDE
   72 LBL2=1
      SPACE=RAD
      XRAD=X*(1.-RAD)
      GOTO 74
   73 LBL2=2
      RAD=SPACE
      SPACEX=1./SPACE
      XRAD=-X
   74 POTRAT=X*POT
      XRAD2=XRAD**2/2.
      DEBY1=SQRTF(DEBY2)
      DEBY3=DEBYE**2/DEBY2
      POTL(2,1)=0.
      POTL(3,1)=0.
      IF(KEND-7)126,131,126
  126 IF(KPOT-4)66,56,66
   66 DO 6 J=2,J2MAX
      IF(KRAD)69,69,68
   68 POTL(1,J)=RADII(J)
   69 RATIO=J2MAX-1
      CALL RADFCN(KRAD+1,J2MAX,1,RATIO)
   70 IF(KPOT-1)4,5,6
    4 POTL(2,J)=POTL(2,J-1)+POTRAT
      GO TO 6
    5 POTL(2,J)=WEIGHT*POT1(J)+(1.-WEIGHT)*POT2(J)
    6 CONTINUE
      IF(KEND-6)56,57,56
57    IF(KPOT-1)56,58,56
58    IF(WEIGHT)59,56,59
59    NCOUNT=NCOUNT+1
      JTEST=(JMAX+JMIN)/2
      XTEST(2,NCOUNT)=POT2(JTEST)
      XTEST(1,NCOUNT)=POT1(JTEST)
   56 GOTO(81,81,82,82),INSIDE
81    POTL(1,J2MAX)=RAD
      GOTO 83
```

```
   82 POTL(1,J2MAX)=1.
83    POTL(2,J2MAX)=POT
    7 CONTINUE
      CALL CUTOFF
      POTL(4,1)=DNSEL
      POTL(5,1)=DNSION
      POTL(6,1)=DEBY3*(ETA*DNSEL-DNSION)
  132 DO 8 J=JMIN,JMAX
      J=J
      CALL CUTOFF
      POTL(4,J)=DNSEL
      POTL(5,J)=DNSION
    8 POTL(6,J)=DEBY3*(ETA*DNSEL-DNSION)
      POTL(8,1)=0.
      IF(JMIN-2)106,106,107
  106 JJMIN=3
      J2MIN=1
      J3MIN=2
      GO TO 108
  107 JJMIN=JMIN+2
      POTL(8,JMIN)=0.
      J2MIN=JMIN
      J3MIN=JMIN+1
      IF(((JMAX-JMIN)/2)*2-(JMAX-JMIN))121,108,121
  121 CALL ERROR
  108 DO 227 J=JJMIN,JMAX,2
      GOTO(75,75,76,76),INSIDE
* REF1, CF EQ. 17C1
   75 R1=POTL(1,J-2)
      R2=POTL(1,J-1)
      R3=POTL(1,J)
      GOTO 9
* REF1, CF EQ. 17C2
   76 R1=(1.-POTL(1,J-2)*SPACE)
      R2=(1.-POTL(1,J-1)*SPACE)
      R3=1.-POTL(1,J)*SPACE
    9 GO TO (227,226),KYLSPH
* REF1, CF EQS. 17S1, 17S2
  226 R1=R1**2
      R2=R2**2
      R3=R3**2
* USING SIMPSONS RULE INTEGRATION, ALTERNATE INTEGRAL VALUES
*     ARE FOUND
  227 POTL(8,J)=(POTL(6,J-2)*R1+4.*POTL(6,J-1)*R2+POTL(6,J)*
     1R3)/6.*(POTL(1,J)-POTL(1,J-2))+POTL(8,J-2)
      JJMAX=(JMAX+J2MIN)/2
      RR1=1.
      DO 114 J=JJMIN,JMAX
      RR=ABSF((POTL(8,J)-POTL(8,J-2))/(POTL(1,J)-POTL(1,J-2)))
  100 IF(RR/RR1-3.)114,114,99
   99 RR1=RR1*10.
      GOTO 100
  114 CONTINUE
  101 DO 10 J=     1,JJMAX
      GOTO(89,90,90,90),INSIDE
   89 ARRAY(1,J)=1.-POTL(1,2*J-1)
      GOTO 10
   90 ARRAY(1,J)=POTL(1,2*J-1)
   10 ARRAY(2,J)=POTL(8,2*J-1)/RR1
      I1=2
      I4=4
```

```
      JXMIN=J3MIN/2+1
      DO 12 J=JXMIN  ,JJMAX
      JJ=2*(J-1)
      GOTO(91,92,92,92),INSIDE
   91 ENTRY=1.-POTL(1,JJ)
      GOTO 93
   92 ENTRY=POTL(1,JJ)
*  INTERPOLATE FOR INTERMEDIATE INTEGRAL VALUES
   93 CALL LSP(ARRAY(1,J-2),I1,I4,ENTRY,POTL(8,JJ),DUMMY)
      POTL(8,JJ)=POTL(8,JJ)*RR1
      IF(J+1-JJMAX)12,11,11
   11 I4=3
   12 I1=1
      POTL(7,1)=0.
      POTL(7,J2MIN)=0.
      GO TO (110,110,111,111),INSIDE
  110 POTL(8,J2MIN)=POTL(8,J2MIN)/POTL(1,J2MIN)** CYLSPH
      GO TO 112
  111 POTL(8,J2MIN)=POTL(8,J2MIN)/(1.-POTL(1,J2MIN)*SPACE) ** CYLSPH
  112 DO 30 J=JJMIN,JMAX,2
      GOTO(77,77,78,78),INSIDE
*  REF1, CF EQ. 19C1
   77 POTL(8,J-1)=POTL(8,J-1)/POTL(1,J-1)
      POTL(8,J)=POTL(8,J)/POTL(1,J)
      GO TO (30,228),KYLSPH
*  REF1, CF EQ. 19S1
  228 POTL(8,J-1)=POTL(8,J-1)/POTL(1,J-1)
      POTL(8,J)=POTL(8,J)/POTL(1,J)
      GO TO 30
*  REF1, CF EQS. 19C2, 19S2
   78 POTL(8,J-1)=POTL(8,J-1)/(1.-POTL(1,J-1)*SPACE) ** CYLSPH
      POTL(8,J)=POTL(8,J)/(1.-POTL(1,J)*SPACE) ** CYLSPH
*  INTERMEDIATE INTEGRAL VALUES ARE MISSING AGAIN
   30 POTL(7,J)=POTL(7,J-2)+(POTL(8,J-2)+4.*POTL(8,J-1)+POTL(8,J))/6.
     1     *(POTL(1,J)-POTL(1,J-2))
      RR1=1.
      DO 104 J=JJMIN,JMAX
      RR=ABSF((POTL(7,J)-POTL(7,J-2))/(POTL(1,J)-POTL(1,J-2)))
  102 IF(RR/RR1-3. )104,104,103
  103 RR1=RR1*10.
      GOTO 102
  104 CONTINUE
      DO 31 J=     1,JJMAX
   31 ARRAY(2,J)=POTL(7,2*J-1)/RR1
      I1=2
      I4=4
      DO 33 J=JXMIN,JJMAX
      JJ=2*(J-1)
      GOTO (94,95,95,95),INSIDE
   94 ENTRY=1.-POTL(1,JJ)
      GO TO 105
   95 ENTRY=POTL(1,JJ)
*  INTERPOLATE FOR MISSING INTEGRAL VALUES
  105 CALL LSP(ARRAY(1,J-2),I1,I4,ENTRY,POTL(7,JJ),DUMMY)
      POTL(7,JJ)=POTL(7,JJ)*RR1
      IF(J+1-JJMAX)33,32,32
   32 I4=3
   33 I1=1
      GOTO (79,79,80,80),INSIDE
   79 POTLL=POTL(2,JMAX)-POTL(2,JMIN)-POTL(7,JMAX)
      GO TO (229,230),KYLSPH
```

```
* REF1, CF EQ. 20C1
  229 POTLL=POTLL/LOGF(POTL(1,JMAX)/POTL(1,JMIN))
      GO TO 231
* REF1, CF EQ. 20S1
  230 POTLL=POTLL/    (1.-POTL(1,JMIN)/POTL(1,JMAX))
  231 POTL(8,J2MIN)=POTLL/POTL(1,J2MIN)
      POTL(3,J2MIN)=POTL(2,J2MIN)
      DO 38 J=J3MIN,JMAX
      IF(J-J2MAX)36,35,35
   35 POTL(3,J2MAX)=POT
      GO TO (235,236),KYLSPH
   36 POTL(3,J)=POTL(7,J)+POTL(2,J2MIN)
      GO TO (232,233),KYLSPH
  232 POTL(3,J)=POTL(3,J)+POTLL*LOGF(POTL(1,J)/POTL(1,J2MIN))
  235 POTL(8,J)=POTL(8,J)+POTLL/POTL(1,J)
      GO TO 38
  233 POTL(3,J)=POTL(3,J)+POTLL*(1.-POTL(1,J2MIN)/POTL(1,J))
  236 POTL(8,J)=POTL(8,J)+POTLL/POTL(1,J)**2*POTL(1,J2MIN)
   38 POTL(7,J)=POTL(6,J)-POTL(8,J)/POTL(1,J)  * CYLSPH
      POTL(7,1)=POTL(6,1)-POTL(8,1)    * CYLSPH
      IF(JMIN-2)117,117,109
  109 POTL(7,JMIN)=POTL(6,JMIN)-POTL(8,JMIN)/POTL(1,JMIN) * CYLSPH
  120 DO 116 J=2,JMIN
  116 POTL(3,J)=POTL(2,J)
  117 IF(J2MAX-JMAX)26,26,118
  118 DO 119 J=JMAX,J2MAX
  119 POTL(3,J)=POTL(2,J)
      GOTO 26
   80 POTLL=POTL(2,JMAX)-POTL(2,J2MIN)-POTL(7,JMAX)
      GO TO (237,238),KYLSPH
* REF1, CF EQ. 20C2
  237 POTLL=POTLL/LOGF((1.-POTL(1,JMAX)*SPACE)/(1.-POTL(1,J2MIN)
     1    *SPACE))
      GO TO 239
* REF1, CF EQ. 20S2
  238 POTLL=POTLL/(1.-(1.-POTL(1,J2MIN)*SPACE)/(1.-POTL(1,JMAX)*SPACE))
* REF1, CF EQ. 19S2
   85 GO TO (240,241),KYLSPH
  241 POTL(3,J)=POTL(2,J2MIN)+POTL(7,J)+POTLL*(1.-(1.-POTL(1,J2MIN)
     1    *SPACE)/(1.-POTL(1,J)*SPACE))
* REF1, CF EQ. 17S2
  242 POTL(8,J)=POTL(8,J)+POTLL/(1.-POTL(1,J)*SPACE)**2*(1.-POTL(1,J2MIN
     1)    *SPACE)
      GO TO 87
  239 POTL(8,J2MIN)=POTLL/(1.-SPACE*POTL(1,J2MIN))
      POTL(3,J2MIN)=POTL(2,J2MIN)
      DO 87 J=J3MIN,JMAX
      IF(J-J2MAX)85,84,84
   84 POTL(3,J2MAX)=POT
      GO TO (86,242),KYLSPH
* REF1, CF EQ. 19C2
  240 POTL(3,J)=POTL(2,J2MIN)+POTL(7,J)+POTLL*LOGF
     1    ((1.-POTL(1,J)*SPACE)/(1.-POTL(1,J2MIN)*SPACE))
* REF1, CF EQ. 17C2
   86 POTL(8,J)=POTL(8,J)+POTLL/(1.-POTL(1,J)*SPACE)
* REF1, CF EQS. 16C2, 16S2
   87 POTL(7,J)=POTL(6,J)+POTL(8,J)/(1.-POTL(1,J)*SPACE)*SPACE * CYLSPH
      POTL(7,1)=POTL(6,1)+POTL(8,1)*SPACE * CYLSPH
      IF(JMIN-2)117,117,115
  115 POTL(7,JMIN)=POTL(6,JMIN)+SPACE/(1.-POTL(1,JMIN)*SPACE)*
     1POTL(8,JMIN) * CYLSPH
```

```
      GOTO 120
   26 IF(DEBY2-1.)16,14,16
   14 LBL=2
      LBL1=1
      GO TO 17
   16 LBL=1
      LBL1=2
   17 WRITE(3,1000)LABEL7(1,KYLSPH),LABEL7(2,KYLSPH),(MONTH(I),I=1,3),
     1  (LABEL1(I,LBL2),I=1,4),DEBYE,(LABEL2(I,LBL2),I=1,3),RAD,ETA,
     2  (LABEL(I,LBL),I=1,3),POT(1),CUREL,CURION
      IF(KEND-7)210,211,210
  210 WRITE(3,1003)LABEL3(1,LBL2),LABEL3(2,LBL2)
     1             ,(LABEL(I,LBL),I=1,3),(LABEL(I,LBL1),I=2,3)
      GO TO 212
  211 WRITE (3,1004)LABEL4(  LBL4),(LABEL(I,LBL1),I=2,3)
  212 J2MAX=XMAXOF(J2MAX,JMAX)
      IF(KEND-7)219,220,219
  219 DO 125 I=1,J2MAX
  125 DLTPHI(I)=POTL(3,I)-POTL(2,I)
  220 GOTO(41,18),KPRINT
   41 IF(J2MAX-41)18,18,19
   18 N1=1
      N2=J2MAX
      LSKIP=1
      LGO=1
      GO TO 20
   19 LSKIP=J2MAX/40+1
      N1=1
      N2=J2MAX
      LGO=2
   20 IF(KEND-7)214,213,214
  214 WRITE(3,1001)((POTL(I,J),I=1,8),DLTPHI(J),J=N1,N2,LSKIP)
      GO TO 217
  213 WRITE (3,1005)(XPOT1(J),POTL(2,J),XPOT2(J),(POTL(I,J),I=3,8),
     1J=N1,N2,LSKIP)
      NWRITE = 1
  217 GO TO (23,22),LGO
   22 N1=J2MAX
      N2=J2MAX
      LSKIP=1
      LGO=1
      GO TO 20
   23 DO 24 I=1,J2MAX
      POT1(I)=POTL(2,I)
   24 POT2(I)=POTL(3,I)
  215 PUNCH 1006,LABEL6(2  ),(POT2(I),I = 1,J2MAX)
      PUNCH 1006,LABEL6(1  ),(POT1(I),I = 1,J2MAX)
      IF(KEND-7)218,216,218
  216 PUNCH 1006,LABEL5(1),(XPOT1(I),I=1,J2MAX)
      PUNCH 1006,LABEL5(2),(XPOT2(I),I=1,J2MAX)
  218 IF(SENSE SWITCH 4)25,98
   98 GOTO(1,2,39,25,39,55,156),KEND
  156 IF(NCOUNT+1-NLIM)256,1,25
  256 IF(ABSF(ERR)-TOL)1,1,157
  157 NCOUNT=NCOUNT+1
      I3 = JFIRST + 1
      DO 728 I1 = I3,1001
      DO 728 I2=4,8
  728 POTL(I2,I1)=BITS
      GO TO 131
   55 JTEST=(JMAX+JMIN)/2
```

```
      IF(NCOUNT-NLIM)71,2,2
   71  NCOUNT=NCOUNT+1
      XTEST(1,NCOUNT)=POTL(2,JTEST)
      XTEST(2,NCOUNT)=POTL(3,JTEST)
      IF(ABSF(XTEST(1,NCOUNT)-XTEST(2,NCOUNT))-TOL)60,61,61
   60 NCOUNT=NCOUNT-1
      KEND=3
      GOTO 39
   61 IF(NCOUNT-1)62,62,67
   67 ANGL=ATANF((XTEST(2,NCOUNT)-XTEST(2,NCOUNT-1))/(XTEST(1,NCOUNT)
     1    -XTEST(1,NCOUNT-1)))
      CALL COEFFT(XTEST(1,NCOUNT),ANGL,UN,VN,CN)
      UN1=1.
      VN1=-1.
      CN1=0.
      CALL DETER(CN,CN1,VN,VN1,UN,UN1,XX,YY)
      WGHT=(YY-XTEST(2,NCOUNT))/(XTEST(1,NCOUNT)-XTEST(2,NCOUNT))
      IF(WGHT+5.)62,62,63
   63 IF(WGHT-1.)64,62,62
   62 WGHT=WEIGHT
   64 DO 65 I=1,J2MAX
   65 POTL(2,I)=WGHT*POT1(I)+(1.-WGHT)*POT2(I)
      KPOT=4
      GOTO 29
   39 ERR=0.
      DO 40 J=2,JMAX
   40 ERR  =MAX1F(ERR  ,ABSF(POTL(2,J)-POTL(3,J)))
      NCOUNT=NCOUNT+1
      IF(NCOUNT-NLIM)42,2,2
   42 IF(ERR  -TOL)2,3,3
   25 CALL EXIT
      GOTO LGOTO,(27,1)
*  EFN 50 TO EFN 49 PROVIDE FOR EXTRAPOLATING SINGLE SPECIES
*      EXTERNAL COLLECTOR SOLUTIONS.
   50 J2=JMAX+1
      DO 43 J=J2,J1MAX
   43 POTL(1,J)=POTL(1,J-1)-XRAD
   48 J3=J2+4
      DO 44 J=J2,J3
      JMAX=J
      POTL(8,J)=POTL(8,J-1)-XRAD*POTL(7,J-1)
      POTL(3,J)=POTL(3,J-1)-XRAD*POTL(8,J-1)+XRAD**2/2.*POTL(7,J-1)
   45 POTL(2,J)=POTL(3,J)
      CALL CUTOFF
      POTL(4,J)=DNSEL
      POTL(5,J)=DNSION
      POTL(6,J)=DEBY3*(ETA*DNSEL-DNSION)
      POTL(7,J)=POTL(6,J)-POTL(8,J)/POTL(1,J)* CYLSPH
      GOTO(54,44),LANE
   54 IF(J-J3)44,46,46
   46 CALL BLAESS(-XRAD)
      LANE=2
      GOTO 45
   44 CONTINUE
      IF(J3-J1MAX)47,49,49
   47 J2=J3+1
      LANE=1
      GOTO 48
   49 KEND=2
      RAD=POTL(1,JMAX)
      POT=POTL(3,JMAX)
```

```
      GOTO 26
      J2MAX = JFIRST+1
      JMAX = J2MAX
      POTL(2,J2MAX) = POT
      POTL(1,J2MAX) = 1.
      J = 1
      DX = 1./FMULT/DEBYE
      IF(SLOPEU-BITS)128,189,128
  189 IF(POT1(JFIRST+1))150,128,150
  150 SLOPEU = (2.*POT1(JFIRST+1)- .5*POT1(JFIRST+2)-1.5*POT1(JFIRST))
     1   /DX
  128 IF(SLOPEL-BITS)127,190,127
  190 IF(POT2(JFIRST+1))152,127,152
  152 SLOPEL = (2.*POT2(JFIRST+1)- .5*POT2(JFIRST+2)-1.5*POT2(JFIRST))/
     1    DX
*  POT1 IS A SET OF POTENTIALS FOR A DISTRIBUTION PASSING ABOVE THE END POINT
*  POT2 IS A SET OF POTENTIALS FOR A DISTRIBUTION PASSING BELOW THE END POINT
* WE START HERE FOR CALCULATION WITH INITIAL SLOPE AND
*     COLLECTOR CONTROLLING CUTOFF CONTOURS.
  127 I=FMULT*DEBYE*(1.-XPOT1(JFIRST))
      RATIO=FLOATF(I)
      DX=(1.-XPOT1(JFIRST))/RATIO
      I=I+JFIRST
      JXMAX=I+6
      POTL(1,JFIRST)=XPOT1(JFIRST)
      CALL RADFCN(1,I,JFIRST,RATIO)
      IF(NCOUNT)181,181,222
  181 IF(JFIRST-1)130,130,223
  223 DO 224 J=2,JFIRST
  224 POTL(1,J)=XPOT1(J)
      CALL RADFCN(2,JFIRST,1,FLOATF(JFIRST-1))
  130 DO 133 J=1,JFIRST
      POTL(1,J)=XPOT1(J)
      POTL(2,J)=(POT1(J))
      CALL CUTOFF
      POTL(4,J)=DNSEL
      POTL(5,J)=DNSION
      POTL(8,J)=(POTL(2,J+1)-POTL(2,J-1))/(POTL(1,J+1)-POTL(1,J-1))
  133 POTL(6,J)=DEBY3*(POTL(4,J)*ETA-POTL(5,J))
  222 IF(POTL(6,JFIRST) * CTEST)196,196,195
  195 ITEST1=-1
      GO TO 197
  196 ITEST1=0
  197 IF(VECTOR(8))172,153,172
  172 SLOPE = SLOPEX
      GOTO 168
  153 IF(SLOPE-BITS)134,160,134
  160 IF(SLOPEU-BITS)151,163,151
  151 IF(VECTOR(3))191,175,191
  191 SLOPE=SLOPEX
      GO TO 163
  175 VECTOR(3) = SLOPEU
      SLOPE = SLOPEU
      I = LCOUNT(1,1001,XPOT1 ,1)-1
  159 VECTOR(1) = MAX1F(VECTOR(1),1.)
  162 VERT=POT1(I-1)+(POT1(I)-POT1(I-1))/(XPOT1(I)-XPOT1(I-1))
     1    *(1.-XPOT1(I-1))
      VECTOR(7)=(POT-VERT)*CTEST
      ERR = VECTOR(7)
  163 IF(SLOPEL-BITS)164,165,164
  164 IF(VECTOR(6))172,176,172
```

```
 176  VECTOR(1) = MAX1F(VECTOR(1),1.)
      DO 192 I=1,JXMAX
      IF(XPOT2(I)-.9999)192,193,193
  192 CONTINUE
      CALL ERROR
  193 VERT=POT2(I-1)+(POT2(I)-POT2(I-1))/(XPOT2(I)-XPOT2(I-1))
     1    *(1.-XPOT2(I-1))
      ERR=(POT-VERT)*CTEST
      SLOPE = SLOPEL
  165 IF(ERR-BITS)168,167,168
*  ITEST1 = -1  IF EMITTER CHARGE DENSITY IS ELECTRON-RICH.
*         = 0   UNTIL FIRST TRANSITION FROM ION-RICH CHARGE
*               DENSITY TO ELECTRON-RICH CHARGE DENSITY, THEN
*  ITEST1 IS NUMBER OF LATEST CHANGE OF SIGN OF CHARGE DENSITY.
  168 IF(VECTOR(8))243,243,244
  244 VECTOR(6) = VECTOR(8)
      VECTOR(8) = 0.
      VECTOR(2) = VECTOR(4)
      VECTOR(4) = 0.
  243 IF(DSLOPX-BITS)249,250,249
  249 DSLOP=DSLOPX*CTEST
      GO TO 251
  250 DSLOP=.3*MAX1F(SLOPE*CTEST,POT*CTEST)*CTEST
  251 CALL QIRE(SLOPE,ERR,0.,DSLOP,0.,-DSLOP,TOL,3.+FLOATF(NLIM),
     1     VECTOR,GO)
      IF(SENSE LIGHT 1)27,166
  166 IF(GO-6HGOBACK)1,134,1
  167 IF(SLOPE-BITS)134,170,134
  170 SLOPE = POT
  134 POTL(8,JFIRST)=SLOPE
      POTL(7,JFIRST)=POTL(6,JFIRST)+SPACE/(1.-POTL(1,JFIRST)*SPACE)
     1    *POTL(8,JFIRST) * CYLSPH
      J2=JFIRST+1
  135 J3=J2+4
      LANE=1
      DO 140 J=J2,J3
      J2MAX=J+1
      JMAX = J2MAX
      POTL(1,J)=POTL(1,J-1)+DX
      POTL(8,J)=POTL(8,J-1)+DX*POTL(7,J-1)
      POTL(3,J)=POTL(2,J-1)+DX*POTL(8,J-1)+DX**2/2.*POTL(7,J-1)
      IF((POTL(3,J)-POTMIN)*CTEST)145,137,137
  137 POTL(2,J)=POTL(3,J)
      POTL(3,J+1)=POT
      POTL(2,J+1) = POT
      POTL(1,J+1) = 1.
      CALL CUTOFF
      POTL(4,J)=DNSEL
      POTL(5,J)=DNSION
      POTL(6,J)=(ETA*DNSEL-DNSION)*DEBY3
      IF(ITEST1)198,200,200
  198 IF(POTL(6,J)*CTEST)199,199,136
  199 ITEST1=0
      GO TO 136
  200 IF(POTL(6,J)/POTL(6,J-1))201,201,136
  201 ITEST1=J
  136 POTL(7,J)=POTL(6,J)+SPACE/(1.-POTL(1,J)*SPACE)*POTL(8,J)
     1    *CYLSPH
      GO TO(138,171),LANE
  138 IF(J-J3)140,139,139
  139 CALL BLAESS(DX)
```

```
      LANE=2
      GO TO 137
  140 CONTINUE
  171 IF((POTL(3,J)-POT)*CTEST)141,182,182
  182 IF(POTL(1,J)-1.001)143,143,183
  183 JJ = J
      IF(POTL(1,J-1)-1.)246,245,245
  245 JJ = JJ-1
      IF(POTL(1,JJ-1)-1.)246,245,245
  246 POTL(3,JJ) = POTL(3,JJ-1)+(POTL(3,JJ)-POTL(3,JJ-1))
     1    /(POTL(1,JJ)-POTL(1,JJ-1))*(1.-POTL(1,JJ-1))
      POTL(2,JJ) = POTL(3,JJ)
      POTL(8,JJ) = POTL(8,JJ-1)+(POTL(8,JJ)-POTL(8,JJ-1))
     1    /(POTL(1,JJ)-POTL(1,JJ-1))*(1.-POTL(1,JJ-1))
      POTL(1,JJ) = 1.
      IF((POTL(3,JJ)-POT)*CTEST)185,187,247
  185 J = JJ
      GOTO 145
  247 J = JJ
      GOTO 143
  187 KEND = 1
      GOTO 155
  141 IF(POTL(1,J)-.9999)142,183,183
  142 J2=J+1
      IF(ITEST1)135,135,202
  202 IF(J-ITEST1-15)135,135,203
  203 DXX=(POTL(1,J)-POTL(1,ITEST1))/15.
      ITEST2=(1.-POTL(1,J))/DXX
  204 IF(XMODF(ITEST2,5))205,206,205
  205 ITEST2=ITEST2+1
      GO TO 204
  206 DXX=(1.-POTL(1,J))/FLOATF(ITEST2)
      DX=MAX1F(DX,DXX)
      J2MAX=J+ITEST2+1
      JXMAX=J2MAX
      CALL RADFCN(1,J2MAX-1,J,1./DX)
      GO TO 135
  143 JXMAX = J+1
      DO 144 I=1,JXMAX
      XPOT1(I)=POTL(1,I)
  144 POTL(3,I)=POT2(I)
      I=JXMAX+1
      LBL4=1
      DO 208 II=I,1001
      POTL(2,II)=BITS
      POTL(3,II)=POT2(II)
  208 XPOT1(II)=BITS
      SLOPEU=POTL(8,JFIRST)
  174 VERT = POTL(2,J)+(1.-POTL(1,J))*POTL(8,J)
      ERR=(POT-VERT)*CTEST
      SLOPEX = SLOPEU
      GO TO 147
  145 JXMAX=J+1
      DO 146 I=1,JXMAX
      XPOT2(I)=POTL(1,I)
  146 POTL(2,I)=POT1(I)
      I=JXMAX+1
      LBL4=2
      DO 209 II=I,1001
      POTL(2,II)=POT1(II)
      POTL(3,II)=BITS
```

```
  209 XPOT2(II)=BITS
      SLOPEL=POTL(8,JFIRST)
      VERT = POTL(3,J)
      ERR=(POT-VERT)*CTEST
      SLOPEX = SLOPEL
  147 SLOPE = BITS
      IF(ABSF(POTL(2,JFIRST+5)-POTL(3,JFIRST+5))-1.E-6)148,155,155
  148 J2=JFIRST+1
      IF(ERR)178,177,177
  177 VECTOR(6)=ERR
      GOTO 179
  178 VEC TOR(7)=ERR
  179 VECTOR(4) = 0.
      VECTOR(8) = 0.
      J3=J2+4
      SLOPEU=(POTL(2,J3+1)-POTL(2,J3-1))/2./DX
      SLOPEL=(POTL(3,J3+1)-POTL(3,J3-1))/2./DX
      DO 149 J4=J2,J3
      POTL(2,J4)=(POTL(2,J4)+POTL(3,J4))/2.
  149 POTL(3,J4)=POTL(2,J4)
      JFIRST=JFIRST+5
      IF(ABSF(POTL(2,JFIRST+5)-POTL(3,JFIRST+5))-1.E-6)148,180,180
  180 SLOPE=VECTOR(6)/(VECTOR(6)-VECTOR(7))*(VECTOR(3)-VECTOR(2))
     1     +VECTOR(2)
  155 J2MAX=XMAXOF(LCOUNT(1,1001,XPOT1,1),LCOUNT(1,1001,XPOT2,1))
      GO TO 26
 1000 FORMAT
      RESTORE
-X          -ADIODE POTENTIAL DISTRIBUTION ANALYSIS - CALCULATION DATE
X-I/-I/-I
      SPACE
-X                      -A / DEBYE LENGTH     =       -F4
-X                -A / EMITTER RADIUS         =       -F4
  ELECTRON DENSITY / ION DENSITY AT EMITTER   =       -F4  (EMITTED PARTIC
XLES)
  COLLECTOR BIAS POTENTIAL                  -A =      -F4
  ELECTRON CURRENT                            =       -F4
  ION CURRENT                                 =       -F4
      END OF FORMAT
 1003 FORMAT
      SPACE
 -X    -A  ASSUMED  CALCULATED ELECTRON      ION    NET CHARGE     PHI
X   PHI   PHIC - PHIA
 -X    -A POTENTIAL  POTENTIAL  DENSITY    DENSITY    DENSITY     DOUBLE
X  PRIME
          (                  -A) /EMITTED  /EMITTED *            -A PRIME
                                 ELECTRON     ION
                                  DENSITY   DENSITY
      SPACE
      END OF FORMAT
 1001 FORMAT
-X  -0PF4       -F6         -F6       -F5        -F5        -F4        -F4
X    -F4       -1PG3
      END OF FORMAT
 1004 FORMAT
                                                    THESE DATA ARE CONSISTENT
X WITH-X    -A BOUND
     UPPER BOUND           LOWER BOUND       ELECTRON       ION
   DIST-                 DIST-               DENSITY/    DENSITY    NET C
XHARGE       PHI        PHI
   ANCE     POTENTIAL    ANCE    POTENTIAL  EMITTED      EMITTED      DEN
```

```
XSITY       DOUBLE      PRIME
                                                        ELECTRON        ION       *
X       -A      PRIME
                                                        DENSITY       DENSITY
        SPACE
        END OF FORMAT
 1005 FORMAT
-X  -0PF4           -F6         -F4          -F6        -F5           -F5
X   -F4           -F4          -F4
        END OF FORMAT
 1006 FORMAT
 -X     -A
            -F6          -F6          -F6         -F6          -F6          -F6
        REPEAT 1
        END OF FORMAT
        END
```

```
*CUTOFF00CALCULATES CUTOFF VELOCITY CONTOURS
      SUBROUTINE CUTOFF
      COMMON POINT,POTL,VTO,MONTH,J,WRITE,NWRITE,NWRMAX,KPLOT,
     1    DNSEL,DNSION,CONST1,CONST2,      RAD,POT1,POT2,CUREL,CURION
     2    ,JMAX,INSIDE,DEBY2,DEBY1,LGOTO,J2MAX,RADVAL
     3    ,ENRGMN,KEND,CYLSPH,KYLSPH,ANGTOL,VELTOL,VELTLL,POT,NPOT
      DIMENSION POINT(20,150),POTL(8,1001),          MONTH(3)
      DIMENSION VTO(2,502),RADVAL(1001),NNX(150),GRAND(4,150),POT(3,30)
      DIMENSION WRITE(20),POT1(1001),POT2(1001)
      TABLE LABEL1(24HCYLINDRICAL   SPHERICAL )
      DIMENSION LABEL(2,2),BITS(1)
      DIMENSION LABEL1(2,2)
      DIMENSION X(2),TEMP(3)
      TABLE BITS(0-377777777777)
      TABLE LABEL(24H    ELECTRON          ION)
      EQUIVALENCE (POTL,POINT(3001)),(GRAND,POINT(8001))
      EXPPF(X)=EXPF(-MIN1F(10.,X))
      LANE = 1
      CDIR=1.
      IF(KEND-7)654,653,654
  653 LANE = 2
  654 N = 1
      NN=1
      NN1=1
      NNX(1)=1
      DO 171 I=1,3000
  171 POINT(I)=0.
      DO 231 I=2,150
  231 NNX(I)=0
      GO TO (125,126,284,285,284,285),INSIDE
  284 TCRIT=ATANF(SQRTF(((1.-POTL(1,J)*RAD)/(1.-RAD))**2-1.))
      GOTO 109
  285 TCRIT=ATANF(SQRTF((1.-POTL(1,J)*RAD)**2-1.))+1.E-6
      GOTO 109
  125 TCRIT=ATANF(SQRTF((POTL(1,J)/RAD)**2-1.))
      CDIR=-1.
      GOTO 109
  126 TCRIT=ATANF(SQRTF(POTL(1,J)**2-1.))+1.E-6
* HERE WE SCAN FOR CUTOFF CONTOUR FOR PARTICLES BOUND AWAY FROM EMITTER
  109 VRJION=0.
      VRJEL=0.
      POTMAX = 0.
      DO 501 I=1,J
  656 TAN=VTO(2,N)*RADVAL(I)
      TAN1=TAN-POTL(2,I)
      VRJEL=MAX1F(VRJEL,TAN1)
      TAN2=TAN+POTL(2,I)
      VRJION=MAX1F(VRJION,TAN2)
  501 CONTINUE
      VRXION=VRJION
* HERE WE SCAN FOR CUTOFF CONTOURS FOR PARTICLES RETURNING TO
*     EMITTER FROM BEYOND LOCAL POINT.
* STORED CUTOFF CONTOUR DATA
      VRXEL=VRJEL
      GOTO (662,660),LANE
  662 DO 502 I=J,J2MAX
      TAN=VTO(2,N)*RADVAL(I)
      TAN11=TAN-POTL(2,I)
      VRXEL=MAX1F(VRXEL,TAN11)
      TAN12=TAN+POTL(2,I)
```

```
      VRXION=MAX1F(VRXION,TAN12)
  502 CONTINUE
      GOTO 661
  660 DO 741 II=1,NPOT
      IF((POT(2,II)-POTL(1,J))*CDIR)661,740,740
  740 TAN=VTO(2,N)*POT(3,II)
      TAN11=TAN-POT(1,II)
      VRXEL=MAX1F(VRXEL,TAN11)
      TAN12=TAN+POT(1,II)
  741 VRXION=MAX1F(VRXION,TAN12)
  661 POINT(1,NN)=VTO(1,N) /(1.-RAD*POTL(1,J))
      P=VRJEL-TAN1
      IF(P)742,742,745
  742 POINT(2,NN)=0.
      IF(NN-1)743,743,744
  743 POINT(5,NN)=90.
      GO TO 747
  744 POINT(5,NN)=0.
      GO TO 747
  745 POINT(2,NN)=SQRTF(P)
      IF(NN-1)743,743,746
  746 POINT(5,NN)=57.296*ATANF(POINT(2,NN)/POINT(1,NN))
  747 P=VRJION-TAN2
      IF(P)748,748,751
  748 POINT(12,NN)=0.
      IF(NN-1)749,749,750
  749 POINT(15,NN)=90.
      GO TO 505
  750 POINT(15,NN)=0.
      GO TO 505
  751 POINT(12,NN)=SQRTF(P)
      IF(NN-1)749,749,752
  752 POINT(15,NN)=57.296*ATANF(POINT(12,NN)/POINT(1,NN))
  505 GO TO (730,731),KYLSPH
  730 POINT(8,NN)=EXPPF(VTO(2,N)+VRJEL)
      POINT(18,NN)=EXPPF(VTO(2,N)+VRJION)
      GOTO 732
  731 CALL ERF(POINT(2,NN),POINT(8,NN))
      CALL ERF(POINT(12,NN),POINT(18,NN))
  725 POINT(11,NN)=EXPPF( POINT(1,NN)**2)
  732 IF(VRXEL-VRJEL)506,506,507
*  THERE ARE NO RETURNING ELECTRONS
  506 POINT(3,NN)=BITS
      POINT(4,NN)=BITS
      POINT(6,NN)=BITS
      POINT(7,NN)=BITS
      POINT(9,NN)=BITS
      POINT(10,NN)=BITS
      GO TO 511
*  SOME ELECTRONS RETURN
  507 POINT(3,NN)=-POINT(2,NN)
      POINT(4,NN)=-SQRTF(VRXEL-TAN1)
      POINT(6,NN)=-POINT(5,NN)
      IF(NN-1)508,508,509
  508 POINT(7,NN)=-90.
      GO TO 510
  509 POINT(7,NN)=57.296*ATANF(POINT(4,NN)/POINT(1,NN))
  510 POINT(9,NN)=POINT(8,NN)
      GO TO (733,734),KYLSPH
  733 POINT(10,NN)=EXPPF(VRXEL+VTO(2,N ))
      GO TO 511
```

```
  734 CALL ERF(-POINT(4,NN),POINT(10,NN))
  511 IF(VRXION-VRJION)512,512,513
*  THERE ARE NO RETURNING IONS
  512 POINT(13,NN)=BITS
      POINT(14,NN)=BITS
      POINT(16,NN)=BITS
      POINT(17,NN)=BITS
      POINT(19,NN)=BITS
      POINT(20,NN)=BITS
      GO TO 517
*  SOME IONS RETURN
  513 POINT(13,NN)=-POINT(12,NN)
      POINT(14,NN)=-SQRTF(VRXION-TAN2)
      POINT(16,NN)=-POINT(15,NN)
      IF(NN-1)514,514,515
  514 POINT(17,NN)=-90.
      GO TO 516
  515 POINT(17,NN)=57.296*ATANF(POINT(14,NN)/POINT(1,NN))
  516 POINT(19,NN)=POINT(18,NN)
      GO TO (735,736),KYLSPH
  735 POINT(20,NN)=EXPPF(VRXION+VTO(2,N ))
      GO TO 517
  736 CALL ERF(-POINT(14,NN),POINT(20,NN))
  517 IF(NN-2)518,536,537
  518 NN=2
      NN1=2
      NNX(2)=502
      N=502
      GO TO 109
*  WE DO NOT SET A MINIMUM ANGLE INCREMENT BETWEEN THE FIRST
*      TWO POINTS.  IT MAY BE 90 DEGREES
  536 IF(POINT(5,NN))537,538,537
  538 IF(POINT(15,NN))520,525,520
*  MINIMUM ANGLE IS 3 DEGREES
  537 IF(ABSF(POINT(5,NN)-POINT(5,NN-1))-ANGTOL)539,539,540
  539 IF(NN-2)538,538,520
  520 IF(ABSF(POINT(15,NN)-POINT(15,NN-1))-ANGTOL)550,550,540
*  RADIAL VELOCITIES ON SUCCESSIVE POINT SHOULD BE BETWEEN 2/3 AND
*      1.5 TIMES LAST POINT.
  550 IF(POINT(2,NN-1))521,400,521
  400 IF(POINT(2,NN))540,551,540
  521 IF(POINT(2,NN)/POINT(2,NN-1)-VELTOL)522,522,540
  522 IF(POINT(2,NN)/POINT(2,NN-1)-VELTLL)540,551,551
  551 IF(POINT(12,NN-1))523,401,523
  401 IF(POINT(12,NN))540,525,540
  523 IF(POINT(12,NN)/POINT(12,NN-1)-VELTOL)524,524,540
  524 IF(POINT(12,NN)/POINT(12,NN-1)-VELTLL)540,525,525
*  CRITICAL CORNER TESTS ON ELECTRONS
  525 IF(POINT(4,NN)-BITS)526,534,526
*  MINIMUM ANGLE INCREMENT FOR RETURNING ELECTRONS IS 3 DEGREES.
  534 IF(POINT(4,NN-1)-BITS)540,529,540
  526 IF(ABSF(POINT(7,NN)-POINT(7,NN-1))-ANGTOL)527,527,540
*  RADIAL VELOCITY RATIO TEST ON RETURNING ELECTRONS
  527 IF(POINT(4,NN)/POINT(4,NN-1)-VELTOL)528,528,540
  528 IF(POINT(4,NN)/POINT(4,NN-1)-VELTLL)540,529,529
  529 IF(POINT(14,NN)-BITS)530,535,530
  535 IF(POINT(14,NN-1)-BITS)540,600,540
*  ANGLE INCREMENT TEST ON RETURNING IONS
  530 IF(ABSF(POINT(17,NN)-POINT(17,NN-1))-ANGTOL)531,531,540
*  RADIAL VELOCITY RATIO TEST ON RETURNING IONS
  531 IF(POINT(14,NN)/POINT(14,NN-1)-VELTOL)532,532,540
```

```
  532 IF(POINT(14,NN)/POINT(14,NN-1)-VELTLL)540,600,600
*  CRITICAL CORNER TESTS ON IONS
*  CAN ANOTHER POINT BE INSERTED IF TESTS ARE NOT SATISFIED
  540 IF(NNX(NN)-NNX(NN-1)-1)600,600,541
*  IF ALL TESTS ARE SATISFIED FOR ALL INTERVALS CALCULATE CHARGE DENSITY
  600 IF(NN-NN1)601,610,609
*  ALL TESTS ARE SATISFIED FOR THIS INTERVAL, TRY THE NEXT.
  601 NN=NN+1
      GOTO 537
  609 CALL ERROR
*  MAKE ROOM FOR INSERTION OF POINT
  541 IF(NN1-150)552,610,610
  552 NLAST=20*NN1
      NFIRST=20*NN-19
  542 POINT(NLAST+20)=POINT(NLAST)
      NLAST=NLAST-1
      IF(NLAST-NFIRST)543,542,542
  543 NNN1=NN1
  544 NNX(NNN1+1)=NNX(NNN1)
      NNN1=NNN1-1
      IF(NNN1-NN)545,544,544
  545 NN1=NN1+1
      IF(J+NN-3)553,553,554
  553 N=2
      GO TO 555
  554 N=(NNX(NN-1)+NNX(NN+1))/2
  555 NNX(NN)=N
      GO TO 109
*  CHARGE DENSITY CALCULATION
  610 DNSEL=0.
      DNSION=0.
      GO TO (698,699),KYLSPH
  698 DO 628 I=2,NN1
      DTHETA=(POINT(5,I-1)-POINT(5,I))/180.
      TERM1=DTHETA*(POINT(8,I)+POINT(8,I-1))
      GO TO (611,615,611,615),INSIDE
  611 IF(POINT(4,I)-BITS)612,618,612
  612 IF(POINT(4,I-1)-BITS)614,613,614
  613 TERM1=TERM1+(POINT(10,I)+POINT(9,I))/180.*(POINT(6,I)-POINT(7,I))
      GO TO 618
  614 TERM1=2.*TERM1+(POINT(10,I)+POINT(10,I-1))/180.*(POINT(7,I-1)
     1    -POINT(7,I))
      GO TO 618
  615 IF(POINT(4,I)-BITS)614,616,614
  616 IF(POINT(4,I-1)-BITS)617,618,617
  617 TERM1=TERM1+(POINT(10,I-1)+POINT(9,I-1))/180.*(POINT(6,I-1)-
     1    POINT(7,I-1))
  618 DNSEL=DNSEL+TERM1
      TERM2=(POINT(18,I)+POINT(18,I-1))/180.*(POINT(15,I-1)-POINT(15,I))
      GO TO (621,625,621,625),INSIDE
  621 IF(POINT(14,I)-BITS)622,628,622
  622 IF(POINT(14,I-1)-BITS)624,623,624
  623 TERM2=TERM2+(POINT(20,I)+POINT(19,I))/180.*(POINT(16,I)-POINT(17,I
     1))
      GO TO 628
  624 TERM2=2.*TERM2+(POINT(20,I)+POINT(20,I-1))/180.*(POINT(17,I-1)
     1    -POINT(17,I))
      GO TO 628
  625 IF(POINT(14,I)-BITS)624,626,624
  626 IF(POINT(14,I-1)-BITS)627,628,627
  627 TERM2=TERM2+(POINT(20,I-1)+POINT(19,I-1))/180.*(POINT(16,I-1)
```

```
   1     -POINT(17,I-1))
628 DNSION=DNSION+TERM2
    IF(J-1)629,629 ,652
629 CURION=1.
    CUREL=1.
    DO 635 I=1,NN1
    IF(POINT(4,I)-BITS)631,630,631
630 GRAND(1,I)=0.
    GRAND(2,I)=0.
    GO TO 632
631 V=SQRTF(POINT(1,I)**2+POINT(3,I)**2)
    CALL ERF(V,VV)
    GRAND(1,I)=( - VV+1.12837*V*POINT(9,I))
    V=SQRTF(POINT(1,I)**2+POINT(4,I)**2)
    CALL ERF(V,VV)
    GRAND(2,I)=( - VV+1.12837*V*POINT(10,I))
632 IF(POINT(14,I)-BITS)634,633,634
633 GRAND(3,I)=0.
    GRAND(4,I)=0.
    GO TO 635
634 V=SQRTF(POINT(1,I)**2+POINT(13,I)**2)
    CALL ERF(V,VV)
    GRAND(3,I)=( - VV+1.12837*V*POINT(19,I))
    V=SQRTF(POINT(1,I)**2+POINT(14,I)**2)
    CALL ERF(V,VV)
    GRAND(4,I)=( - VV+1.12837*V*POINT(20,I))
635 CONTINUE
    DO 651 I=2,NN1
    GO TO (636,644,636,644),INSIDE
636 IF(POINT(4,I)-BITS)637,640,637
637 IF(POINT(4,I-1)-BITS)639,638,639
638 CUREL=CUREL-(GRAND(1,I)+GRAND(2,I))/2.*(COSF(POINT(6,I)/57.296)
   1     -COSF(POINT(7,I)/57.296))
    GO TO 640
639 CUREL=CUREL-((GRAND(1,I)+GRAND(1,I-1))*(COSF(POINT(6,I)/57.296)
   1     -COSF(POINT(6,I-1)/57.296))-(GRAND(2,I)+GRAND(2,I-1))
   2     *(COSF(POINT(7,I)/57.296)-COSF(POINT(7,I-1)/57.296)))/2.
640 GO TO (645,648,645,648),INSIDE
645 IF(POINT(14,I)-BITS)641,651,641
641 IF(POINT(14,I-1)-BITS)643,642,643
642 CURION=CURION-(GRAND(3,I)+GRAND(4,I))/2.*(COSF(POINT(16,I)/57.296)
   1     -COSF(POINT(17,I)/57.296))
    GO TO 651
643 CURION=CURION-((GRAND(3,I)+GRAND(3,I-1))*(COSF(POINT(16,I)/57.296)
   1     -COSF(POINT(16,I-1)/57.296))-(GRAND(4,I)+GRAND(4,I-1))*
   2     (COSF(POINT(17,I)/57.296)-COSF(POINT(17,I-1)/57.296)))/2.
    GO TO 651
644 IF(POINT(4,I)-BITS)639,646,639
646 IF(POINT(4,I-1)-BITS)647,648,647
647 CUREL=CUREL+(GRAND(1,I-1)+GRAND(2,I-1))/2.*(COSF(POINT(6,I-1)
   1     /57.296)-COSF(POINT(7,I-1)/57.296))
648 IF(POINT(14,I)-BITS)643,649,643
649 IF(POINT(14,I-1)-BITS)650,651,650
650 CURION=CURION+(GRAND(3,I-1)+GRAND(4,I-1))/2.*(COSF(POINT(16,I-1)
   1     /57.296)-COSF(POINT(17,I-1)/57.296))
651 CONTINUE
    CUREL = CUREL - (1.-COSF(TCRIT))
    CURION = CURION - (1.-COSF(TCRIT))
    GO TO 652
699 DO 700 I=2,NN1
    TERME=2.-(POINT(8,I)+POINT(8,I-1))
```

```
      TERMI=2.-(POINT(18,I)+POINT(18,I-1))
      IF(POINT(4,I)-BITS)701,703,701
  701 IF(POINT(4,I-1)-BITS)702,703,702
  702 TERME=TERME+POINT(10,I)+POINT(10,I-1)-POINT(9,I)-POINT(9,I-1)
  703 IF(POINT(14,I)-BITS)704,706,704
  704 IF(POINT(14,I-1)-BITS)705,706,705
  705 TERMI=TERMI+POINT(20,I)+POINT(20,I-1)-POINT(19,I)-POINT(19,I-1)
  706 DX= .5*(POINT(11,I-1)-POINT(11,I))
      DNSEL=DNSEL+TERME*DX
  700 DNSION=DNSION+TERMI*DX
      FACT=EXPF(POTL(2,J))
      DNSEL=DNSEL*FACT
      DNSION=DNSION/FACT
      IF(J-1)707,707,652
  707 CUREL=1.
      CURION=1.
      IF(POINT(4,1)-BITS)710,711,710
  710 TERMEE=EXPPF(+POINT(4,1)**2)
      GO TO 712
  711 TERMEE=0.
  712 IF(POINT(14,1)-BITS)713,714,713
  713 TERMII=EXPPF( POINT(14,1)**2)
      GO TO 727
  714 TERMII=0.
  727 DO 724 I=2,NN1
      DX= .5*(POINT(11,I-1)-POINT(11,I))
      IF(POINT(4,I)-BITS)708,717,708
  708 IF(POINT(4,I-1)-BITS)709,715,709
  709 TERME1=EXPPF( POINT(4,I)**2)
  719 TERME = 2.-TERME1+TERMEE
      TERMEE=TERME1
      CUREL=CUREL-TERME*DX
  717 IF(POINT(4,I-1)-BITS)718,715,718
  718 TERME1 = 0.
      GOTO 719
  715 IF(POINT(14,I)-BITS)716,721,716
  716 IF(POINT(14,I-1)-BITS)720,724,720
  720 TERMI1 = EXPPF( POINT(14,I)**2)
      GOTO 723
  721 IF(POINT(14,I-1)-BITS)722,652,722
  722 TERMI1 = 0.
  723 TERMI = 2.-TERMI1-TERMII
      TERMII = TERMI1
      CURION = CURION-TERMI*DX
  724 CONTINUE
  652 TCRIT = TCRIT * 57.296
  127 IF(NWRITE-NWRMAX)89,89,98
   89 IF(ABSF(POTL(1,J)-POTL(1,1))-ABSF(WRITE(NWRITE)-POTL(1,1)))
     1    98,90,90
   90 DO 95 L = 1,2
      K = (L-1)*10+1
      N1 = 1
      N2 = NN
      WRITE(3,1000)LABEL1(1,KYLSPH),LABEL1(2,KYLSPH),(LABEL(I,L),I =1,2)
     1POTL(1,J),(MONTH(I),I=1,3),POTL(2,J),TCRIT,DNSEL,DNSION
      IF(NN-40)91,91,92
   91 KPRINT=1
      LPRINT=1
      GO TO 93
   92 KPRINT=NN/40+1
      LPRINT=2
```

```
   93 WRITE OUTPUT TAPE 3,1001,(POINT(K+1,I),POINT(1,I),POINT(K+4,I),
     1     POINT(K+7,I),POINT(K+2,I),POINT(1,I),POINT(K+5,I),POINT(K+8,I)
     2,     POINT(K+3,I),POINT(1,I),POINT(K+6,I),POINT(K+9,I),I=N1,N2,KPR
     3INT)
      GO TO (95,94),LPRINT
   94 N1=NN
      N2=NN
      LPRINT=1
      GO TO 93
   95 CONTINUE
      IF(KPLOT)96,97,96
   96 CALL PLOT
   97 NWRITE=NWRITE+1
   98 RETURN
 1000 FORMAT
      RESTORE
-X          -ADIODE POTENTIAL DISTRIBUTION STUDY - CUTOFF VELOCITY CONTO
XUR DATA -          -A DISTRIBUTION
      SPACE
      RADIUS                    -F4       CALCULATION DATE  -I/-I/-I
      POTENTIAL                 -F4
      CRITICAL ANGLE           -F2
      ELECTRON DENSITY          -F4
      ION DENSITY               -F4
      SPACE
 LOWER LIMIT FOR INWARD BOUND PARTICLES  LOWER LIMIT FOR OUTWARD BOUND P
XARTICLES UPPER LIMIT FOR OUTWARD BOUND PARTICLES
      SPACE
     VR     VT       ANGLE  ENERGY              VR     VT       ANGLE  ENERG
XY             VR     VT       ANGLE  ENERGY
      SPACE
      END OF FORMAT
 1001 FORMAT
-X  -0PF4       -F5       -F2     -1PG3          -0PF4       -F5       -F2     -
X1PG3          -0PF4       -F5       -F2     -1PG3
      END OF FORMAT
      END
```

```
*RADFCN00
      SUBROUTINE RADFCN (KPOT,I,I1,RATIO)
      COMMON POINT,POTL,VTO,MONTH,J,WRITE,NWRITE,NWRMAX,KPLOT,
     1    DNSEL,DNSION,CONST1,CONST2,      RAD,POT1,POT2,CUREL,CURION
     2    ,JMAX,INSIDE,DEBY2,DEBY1,LGOTO,J2MAX,RADVAL
     3    ,ENRGMN,KEND,CYLSPH,KYLSPH,ANGTOL,VELTOL,VELTLL,POT,NPOT
      DIMENSION POINT(20,150),POTL(8,1001),          MONTH(3)
      DIMENSION WRITE(20),POT1(1001),POT2(1001)
      DIMENSION VTO(2,502),RADVAL(1001),POT(3,30)
      I2=I1+1
      GO TO (1,4,7,10),INSIDE
    1 POTL(1,1)=1.
      RADVAL(1)=0.
      DO 3 II=2,J2MAX
      GO TO (2,3),KPOT
    2 POTL(1,II)=1.-(1.-RAD)/RATIO*FLOATF(II-1)
    3 RADVAL(II)=1./POTL(1,II)**2-1.
      DO 18 II=1,NPOT
   18 POT(3,II)=1./POT(2,II)-1.
      GO TO 13
    4 POTL(1,1)=0.
      RADVAL(1)=0.
      DO 6 II=2,JMAX
      GO TO (5,6),KPOT
    5 POTL(1,II)=1.+(RAD-1.)/RATIO*FLOATF(II-1)
    6 RADVAL(II)=1./POTL(1,II)**2 -1.
      DO 17 II=1,NPOT
   17 POT(3,II)=1./POT(2,II)**2-1.
      GO TO 13
    7 POTL(1,1)=0.
      RADVAL(1)=0.
      DO 9 II=I2,I
      GO TO (8,9),KPOT
    8 POTL(1,II)=FLOATF(II-1)/RATIO
    9 RADVAL(II)=1./(1.- RAD *POTL(1,II))**2-1.
      DO 15 II=1,NPOT
   15 POT(3,II)=1./(1.-RAD*POT(2,II))**2-1.
      GO TO 13
   10 POTL(1,1)=0.
      RADVAL(1)=0.
      DO 12 II = I2,I
      GO TO (11,12),KPOT
   11 POTL(1,II)=FLOATF(II-I1)/RATIO+POTL(1,I1)
   12 RADVAL(II)=1./(1.- RAD *POTL(1,II))**2-1.
      DO 16 II=1,NPOT
   16 POT(3,II)=1./(1.-RAD*POT(2,II))**2-1.
   13 RETURN
      END
```

```
*BNN1
*                        B CALCULATION
*
*           B WILL BE CALCULATED ACCORDING TO THE
*           VALUE OF INDICATOR K.
*
*      B = BNN1(ALPHA,K)
*           WHERE ALPHA = (THETA +/- MU).
*           K MUST = 0 IF ALPHA LESS THAN 45 DEGREES.
*           K MUST = 1 IF ALPHA GREATER THAN 45 DEGREES.
*           K MUST = 2 IF ALPHA = 90 DEGREES
*
    1 FUNCTION BNN1(ALPHA,K)
*
    2 IF (K - 1)      4,3,5
*
    3 BNN1 = (COSF(ALPHA))/(SINF(ALPHA))
      GO TO 6
*
    4 BNN1 = (SINF(ALPHA))/(COSF(ALPHA))
      GO TO 6
*
    5 BNN1 = 0.0
*
    6 RETURN
*
      END (0,1,0)
```

```
*COEFT 00
      SUBROUTINE COEFT(Y,Y2DOT,DX,K,PX,QX,J)
      DIMENSION Y(1),Y2DOT(1)
      J5=J
      J4=J5-K
      J3=J4-K
      J2=J3-K
      J1=J2-K
      J0=J1-K
      CC1=(Y2DOT(J3)-Y2DOT(J1))/(Y(J3)-Y(J1))
      CC2=(Y2DOT(J5)-Y2DOT(J3))/(Y(J5)-Y(J3))
      DX2=DX**2
      A1=2.-1.333333*CC1*DX2
      B1=2.-.666667*CC1*DX2
      C1=Y2DOT(J2)-Y2DOT(J0)+(Y2DOT(J1)-Y2DOT(J0))*DX2/2.*CC1
      A2=4.-10.666667*CC2*DX2
      B2=8.-10.666667*CC2*DX2
      C2=Y2DOT(J4)-Y2DOT(J0)+DX2*(4.5*Y2DOT(J0)-2.5*Y2DOT(J1)-1.5*
     1    Y2DOT(J2)-.5*Y2DOT(J3))*CC2
      DEN=A1*B2-A2*B1
      PX=(C1*B2-C2*B1)/DEN
      QX=(A1*C2-A2*C1)/DEN
    1 RETURN
      END
```

```
*BLAESS  SUBROUTINE TO IMPROVE INTEGRATION BY BLAESS METHOD
      SUBROUTINE BLAESS(DTIME)
      COMMON POINT,POTL,VTO,MONTH,J,WRITE,NWRITE,NWRMAX,KPLOT,
     1     DNSEL,DNSION,CONST1,CONST2,       RAD,POT1,POT2,CUREL,CURION
     2     ,JMAX,INSIDE,DEBY2,DEBY1,LGOTO,J2MAX,RADVAL
     3     ,ENRGMN
      DIMENSION POINT(20,150),POTL(8,1001),          MONTH(3)
      DIMENSION WRITE(20),POT1(1001),POT2(1001)
      DIMENSION VTO(2,502),RADVAL(502)
      DIMENSION BITS(1)
      DX=POTL(3,J-2)-POTL(3,J-5)
      CALL COEFT(POTL(3,1),POTL(7,1),DTIME,8,PX,QX,(J-1)*8+1)
      PXX1=PX/2./DTIME
      PXX2=PXX1/3.
      QXX1=QX/6./DTIME**2
      QXX2=QXX1/4.
   10 I1=J-4
      DO 11 N=1,5
      DTIMEN=DTIME*FLOATF(N)
      POTL(8,I1)=POTL(8,J-5)+POTL(7,J-5)*DTIMEN+PXX1*DTIMEN**2
     1+QXX1*DTIMEN**3
      POTL(3,I1)=POTL(3,J-5)+POTL(8,J-5)*DTIMEN+POTL(7,J-5)/2.*
     1DTIMEN**2+PXX2*DTIMEN**3+QXX2*DTIMEN**4
   11 I1=I1+1
   16 RETURN
      END
```

```
*LCOUNT
*FUNCTION SUBPROGRAM TO COUNT ROWS IN A
*2- DIMENSIONAL ARRAY.
*           N19 = COLUMN TO BE COUNTED
*           N20 = MAXIMUM LENGTH OF ARRAY
*           XX  = NAME OF ARRAY
*           N21 = NUMBER OF COLUMNS IN ARRAY
     1 FUNCTION LCOUNT(N19,N20,XX,N21)
       DIMENSION XX(1)
       L17 = N21
       BITS = 1.7014118E+38
     3 LCOUNT = 0
       N22 = N20*N21
       DO 6 I=N19,N22,N21
       IF(XX(I)-BITS)6,7,7
     6 LCOUNT = LCOUNT+1
     7 RETURN
       END
```

```
*COEFFTOO CALCULATES COEFFICIENTS OF EQUATION OF LINE
      SUBROUTINE COEFFT(ARRAY,DIR,UN,VN,CN)
      DIMENSION ARRAY(2)
      IF(ABSF(DIR)-.7854)10,10,12
   10 UN=-BNN1(DIR,0)
      VN=1.
      CN=ARRAY(2)+UN*ARRAY(1)
   11 RETURN
   12 IF(ABSF(DIR)-2.3562)16,16,13
   13 IF(DIR)15,14,14
   14 DIR=DIR-3.1416
      GOTO 10
   15 DIR=DIR+3.1416
      GOTO 10
   16 VN=-BNN1(DIR,1)
      UN=1.
      CN=ARRAY(1)+VN*ARRAY(2)
      GOTO 11
      END
```

```
*QIREX03
      SUBROUTINE QIRE (X,E,XK1,XK2,XK3,XK4,TOL,XJMAX,XJ,GOWHER)
*     JULY 27, 1961
*     QUADRATIC INTERPOLATION ROOT EVALUATION
      DIMENSION XJ(8)
      DIMENSION XX(8),EE(4)
      EQUIVALENCE(XX(5),EE)
* X= THE GUESS OF THE ROOT
* E= ERROR (Y-YD)
* XK1,XK2,XK3,XK4, ARE CONSTANTS
* TOL= TOLERANCE
* XJMAX= MAX NUMBER OF TRIAL ROOTS TO TRY
* XJ= AN 8 ELEMENT VECTOR CONTAINING THE ITERATION CTR., THE 3 POINT
* HISTORY AND AN INDICATOR
* GOWHER= A FORK TO DETERMINE IF E LESS THAN OR GREATER THAN TOL
   10 IF(XJ)3300,11,90
* XJ=0 INITIALIZE
* EA= GREATEST POSITIVE ERROR
* EB= GREATEST NEGATIVE ERROR
* EC= ERROR FOR MIDDLE POINT
* XA= VALUE OF X CORRESPONDING TO EA
* XB= VALUE OF X CORRESPONDING TO EB
* XC= VALUE OF X CORRESPONDING TO EC
    11 EA=0.
    12 EB=0.
    13 EC=0.
    14 XA=0.
    15 XB=0.
    16 XC=0.
    80 GO TO 100
* XJ IS POSITIVE
    90 XA=XJ(2)
    91 XB=XJ(3)
    92 XC=XJ(4)
    93 EA=XJ(6)
    94 EB=XJ(7)
    95 EC=XJ(8)
       DO 96 N=1,3
       XX(N)=XJ(N+1)
    96 XX(N+4)=XJ(N+5)
       X1=X
       XTNT=X
       XX(4)=X
       EE(4)=E
* ADD 1 TO COUNTER
   100 XJ=XJ+1.
   101 IF (XJ-XJMAX) 102,3300,3300
   102 IF(ABSF(E)-TOL)3400,200,200
* E GREATER OR = TOL
   200 IF(EA*EB)1200,300,1200
* (EA)(EB)=0
   300 IF(E)700,400,400
* NEW GUESS AT ROOT WHEN E GREATER OR =0
   400 EA=E
       EE(1)=E
       XX(1)=X
       XA=X
   500 X1=X+XK1*E+XK2
   600 GO TO 900
* NEW GUESS AT ROOT WHEN E LESS THAN 0
```

```
  700 EB=E
      EE(2)=E
      XX(2)=X
      XB=X
  800 X1=X+XK3*E+XK4
  900 IF(EA*EB)1000,3200,1000
* NEW GUESS AT ROOT WHEN ROOT IS SPANNED
 1000 X1=QIREX(1,2,EE,XX)
 1100 GO TO 3200
* (EA)(EB) NOT ZERO
 1200 IF(XA-XB)1202,1201,1201
 1201 XK=1.
      LANE=1
      KK=-1
      GO TO 1203
 1202 XK=-1.
      LANE=2
      KK=1
 1203 IF(EC)1399,1300,1399
* EC=0
 1300 EC=E
      XC=X
      IF(E/EA)1301,1301,1302
 1301 XTNT=QIREX(4,1,EE,XX)
      GO TO 2700
 1302 XTNT=QIREX(4,2,EE,XX)
      GO TO 2700
 1399 GO TO(1400,1401),LANE
 1400 XX(1)=XB
      EE(1)=EB
      XX(4)=XA
      EE(4)=EA
      GO TO 1402
 1401 XX(1)=XA
      EE(1)=EA
      XX(4)=XB
      EE(4)=EB
 1402 IF(X-XC)1404,1403,1403
 1403 XX(3)=X
      EE(3)=E
      XX(2)=XC
      EE(2)=EC
      GO TO 1405
 1404 XX(3)=XC
      EE(3)=EC
      XX(2)=X
      EE(2)=E
 1405 IF(EE(1)/EE(2))1406,1407,1407
 1406 II=2-KK
      XTNT=QIREX(1,2,EE,XX)
      GO TO 1413
 1407 IF(EE(3)/EE(4))1408,1409,1409
 1408 II=3-KK
      XTNT=QIREX(3,4,EE,XX)
      GO TO 1413
 1409 IF(ABSF(EE(4))-ABSF(EE(1)))1411,1411,1410
 1410 II=2-KK
      GO TO 1412
 1411 II=3-KK
 1412 XTNT=QIREX(2,3,EE,XX)
 1413 XA=XX(II)
```

```
      EA=EE(II)
      II=II+KK
      XC=XX(II)
      EC=EE(II)
      II=II+KK
      XB=XX(II)
      EB=EE(II)
 2700 D1=(EC-EB)/(XC-XB)
 2800 D12=(((EA-EC)/(XA-XC))-D1)/(XA-XB)
 2810 IF(D12)2820,2821,2820
 2820 IF(1.E-3-ABSF((XA-XB)/(D1/D12-XC+XB)))2900,2821,2821
 2821 X1=XTNT
      GOTO 3200
 2900 D13=(D12*(XC-XB)-D1)**2-4.*D12*EB
      IF(D13)3103,3102,3102
 3102 X1=(D12*(XC-XB)-D1+XK*SQRTF(D13))/2./D12+XB
      IF(MAX1F(ABSF(X1-XA),ABSF(X1-XB))-ABSF(XA-XB))3200,3200,3108
 3108 X1=XTNT
 3103 ASSIGN 3105 TO MMM
 3104 WRITE OUTPUT TAPE 3,1,TOL,XJ(1),X,E,(XX(I),EE(I),I=1,4),X1,XJ(5)
      GO TO MMM,(3105,3302)
 3105 CALL ERRORA
 3106 X=XTNT
      GO TO 3400
 3300 IF(EA*EB)3103,3301,3103
 3301 ASSIGN 3302 TO MMM
      GO TO 3104
 3302 CALL ERROR
 3200 X=X1
      GOWHER=6HGOBACK
      GO TO 3500
*THE ROOT HAS BEEN FOUND
 3400 GOWHER=4HGOON
 3410 XJ(5)=XJ
 3420 XJ=0.
 3430 GO TO 3510
*THE ROOT HAS NOT BEEN FOUND
 3500 XJ(5)=XJ
*
 3510 XJ(2)=XA
      XJ(3)=XB
      XJ(4)=XC
      XJ(6)=EA
      XJ(7)=EB
      XJ(8)=EC
      RETURN
 1    FORMAT
      RESTORE
  SUBROUTINE QIRE FAILED TO CONVERGE WITHIN THE TOLERANCE LIMIT
         -1PE2 IN -0PF0 LOOPS
                                          ARGUMENT     ERROR
  ENTRY TO SUBROUTINE OCCURRED WITH          -1PE5        -E5
      SPACE
  RECENT CONVERGENCE HISTORY WAS              -E5         -E5
                                              -E5         -E5
                                              -E5         -E5
                                              -E5         -E5
      SPACE
  EXIT FROM SUBROUTINE OCCURRED WITH          -E5
      SPACE
      LOOP COUNT       -0PF0
```

```
      END OF FORMAT
* QIRE-LAST CARD
      END(0,1,0)
```

```
*QIREX03
      FUNCTION QIREX(I1,I2,EE,XX)
      DIMENSION EE(4),XX(4)
      QIREX=(EE(I1)*XX(I2)-EE(I2)*XX(I1))/(EE(I1)-EE(I2))
      RETURN
      END
```

```
*DETER
*                    DETER SUBROUTINE
*
*
*          THIS SUBROUTINE MAY BE USED TO
*          CALCULATE X, Y, MACH NO., AND THETA
*
*      CALL DETER(XA,XB,VA,VB,UA,UB,RESUL1,RESUL2)
*
*          WHERE RESUL1 = X OR MACH
*          AND RESUL2 = Y OR THETA
*
*
      SUBROUTINE DETER(XA,XB,VA,VB,UA,UB,RESUL1,RESUL2)
*
      RESUL1 = (XA*VB - XB*VA)/(UA*VB - VA*UB)
*
      RESUL2 = (UA*XB - UB*XA)/(UA*VB - VA*UB)
*
*
      RETURN
*
      END (0,1,0)
```

APPENDIX C

EXPLANATIONS OF GE ERROR, DING AND TABLE ROUTINES

ADDENDUM

IDENTIFICATION:

Title: Standard Error Procedure
Source Language:
Application: 7090 FORTRAN
Author: Dorothea S. Clarke
Unit: CTD
Installation: General Electric - Evendale
Date: April 1960

During the September 1958 SHARE XI meeting the FORTRAN Standards Committee proposed a standard error detection procedure for FORTRAN II. IBM modified the compiler and ECO has put it into effect. This allows the programmer to call an ERROR subroutine which will identify the location of the error, back trace the flow through subroutines to the main program and then continue either to the next statement or to the next case.

The calling sequences of the following subroutines will be affected.

1. Function definition subroutines compiled from function definition statements.
2. Library function subroutines.
3. FUNCTION subprograms.
4. SUBROUTINE subprograms.

Changes in the calling sequences to the following subroutines are not included:

1. Closed subroutines computing index values.
2. Library subroutines used to accomplish input and output.

The standard error detection feature will lengthen the calling sequence by the following two instructions at the end of the calling sequence:

```
NTR     *+2, 0, A
PZE     C, 0, B
```

where A equals the external formula number of the statement producing the calling sequence. If the statement producing the calling sequence was not

assigned a statement number, then A equals the external formula number of the last preceding source program statement with a statement number. If no external formula numbers were assigned to statements, then A equals 0.

B equals the internal formula number of the statement producing the calling sequence.

C depends on the manner of occurrence of the calling sequence. If the calling sequence occurs in a function definition statement, then C equals $(77777)_8$. If the calling sequence occurs in a subprogram headed by a SUBROUTINE or FUNCTION statement, or in a library subroutine, C equals $ + 2 which is the location in the prologue of the contents of index register 4. If neither of these conditions apply or the subroutine was called from a main program, then C equals 0.

The compilation of FORTRAN II subprograms headed by SUBROUTINE and FUNCTION statements will be altered in the prologue by the addition of one BCD word, the name of the subroutine. The prologue will then appear as follows:

```
                    $   HTR
                        HTR
                        HTR
                        BCD     1, NAME
(Entry point to Subroutine) SXD     $, 1
                        SXD     $+1, 2
                        SXD     $+2, 4
```

The program card preceding each subprogram will be adjusted to reflect both the subprogram's increased length and that the entry point is one location higher than the subprogram compiled without the standard error procedure.

The additional words compiled in the calling sequences and prologues will permit an error program, called at object execution time, to determine the internal and external formula numbers of the program statement calling the error program. If this statement calling the error program occurs in a SUBROUTINE or FUNCTION type subprogram or a library function subroutine, then the name of that subprogram is known. If the calling of the error program occurs in a subroutine or subprogram, the additionally compiled words give the needed information to determine the statement in a higher level program which called the subroutine or subprogram. This method is used to back trace through any number of levels of subprograms

out to the main program. It should be noted that, although the function definition subroutine does not have a name that can be printed, the statement's external formula number identifies the subroutine unambiguously.

In a FORTRAN II coded subprogram the programmer has two options:

(1) He may call the error program by the statement

CALL ERROR

which returns to the initial entry to calculate the next case, or (2)

CALL ERRORA

which returns to the next statement after the CALL. The proper calling sequence will be compiled and the name ERROR or ERRORA placed in the transfer list. If the CALL statement is assigned a statement number, the location of the error is easily located by the programmer.

Any SAP coded subprogram that can call the error program directly or indirectly should also call other programs by a calling sequence followed by the above discussed NTR (TXI) and PZE instructions. The A and B can be arbitrarily assigned by the programmer which may be used to designate the type of error if the subprogram has multi-error detection. The C should be zero if the SAP coded routine is the main program; otherwise, C should give the address of a location L where index register 4 of the TSX to the subprogram is saved. The BCD name of this SAP coded subprogram should be in L+1. It is suggested that the SAP coded program use a prologue the same as those used for FORTRAN subprograms. If the error program can be called directly, ERROR or ERRORA must be included in the transfer list. When ERRORA is used, provision for the return must be included.

ERROR SUBROUTINE

An error subroutine has been prepared which will back trace the flow of a problem utilizing the additional compiled instruction of the FORTRAN II standard error detection procedure. When an error is encountered and the error routine is called, comments as to flow are written on the peripheral output tape after restoring the page, for example,

(Addendum)

AN ERROR HAS BEEN FOUND AT OR NEAR

EXTERNAL FORMULA NO.				INTERNAL FORMULA NO.				
"	"	"	3157,	"	"	"	213,	OF
"	"	"	11,	"	"	"	4,	OF CBDA
"	"	"	125,	"	"	"	314,	OF MAIN

THE END

NOTE: The title of the subprogram is listed whenever possible. The space for the title is left blank when the subprogram is from a function definition statement. When it is a main program the title MAIN is supplied.

The ERROR subroutine will trace back to the main program or 15 subprograms, whichever occurs first. The comment THE END is printed, the page restored, and sense light 1 turned on. If entry was by way of ERROR, control is then returned to the transfer instruction in the BSS-load to re-enter the program for another case. If the programmer does not wish to restart from the beginning he must provide an indicator and a test at the beginning. The test must be to decide if this is a re-entry after an error was detected (sense light 1, etc.), in which case, transfer around the initialization. (Light 1 must be off before loading.) If entry was by way of ERRORA, control is returned to the next statement after the CALL ERRORA. It is then the programmer's responsibility to act accordingly. Library function subroutines arbitrarily use ERROR to return to the initial entry to the main program. If it is necessary to avoid this, the programmer must test any arguments for error before entering the subroutines.

ERROR is in the FORTRAN II library of subroutines. Output is on the peripheral output tape.

LIBRARY FUNCTION SUBROUTINES

All of the library routines which can utilize an error return have been modified to do so. Below are listed the subroutines, origin, space required, and timing. Internal and external statement numbers will be blank unless otherwise indicated to distinguish different types of errors.

(Addendum)

Name	Origin	Storage	Approx. Timing	Error Codes
LOG	FORTRAN	57	2.6 ms	IFN = Blank if X = 0 IFN = 1 if X<0
EXP	FORTRAN	56	2.0 ms	
EXP(1	FORTRAN	49	variable	IFN = 1 if 0**0 IFN = 2 if I**(-J)
EXP(2	FORTRAN	58	variable	IFN = 1 if 0**0 IFN = 2 if 0**(-A)
EXP(3	FORTRAN	125	5.1 ms	Same as EXP and LOG
SIN/COS	Modified GE	93	3.6 ms	IFN = 1 if COS IFN = blank if SIN
SQRT	GE	31	2.0 ms	
DIP	GE		variable	IFN = error code EFN = 1000 x card count + column number

SUBROUTINE

IDENTIFICATION:

Title: DING, Decimal Input
Source Language: FAP
Application: 7090 FORTRAN
Author: D. S. Clarke
Unit: Computer Techniques Development
Installation: General Electric - Evendale
Date: January 1, 1961

GENERAL INFORMATION:

The DING input program, which loads from the peripheral input tape, converts and stores integers, floating point data, and alphabetic data or octal instructions. Column 1, of each data card, is reserved for control information, which indicates whether the data is to be converted to numeric or binary-coded decimal. Other controls are end-of-record and end-of-file. Taking precedence over column 1 controls, variable names are examined to determine integer or floating point conversion. The data cards may be set up for any number of fields, limited only by the 72 columns on the card.

USAGE:

Calling Sequence:

CALL DING (SYMTAB, IOF)

SYMTAB is the input table generated by a FORTRAN TABLE statement. (See Example - page 2)

IOF is an end-of-file parameter with the following connotations:

for DING: IOF $\neq$ 0, DING calls EXIT on EOF
IOF = 0, DING stores $\pm$ 1 on EOF and returns to caller

for calling program upon return from DING:

IOF = + 1 if control card EOF encountered
IOF = - 1 if physical EOF encountered

Example:

```
DIMENSION SYMTAB (m)
TABLE SYMTAB (n, $(s1, s2, ... sn))
CALL DING (SYMTAB, IOF)
```

SYMTAB = table of input variables $s_1, s_2 \ldots$
n = number of variables in table
m ≥ 3n+1
IOF = EOF parameter

Data Card Format:

Column 1 is reserved for control punches. Listed below are the various control punches permitted by DING:

Control	Symbol	Designation
0 (Zero)	0	Octal Instruction
2	2	Data is Binary Coded Decimal stored ascending order
3	3	Data is Floating Point
4	4	Data is Fixed Point
5	5	Data is Binary Coded Decimal stored descending order
8	8	End-of-File when followed by 5 blanks
8-0	Y	Octal Instruction - End-of-Record
8-3	=	Data is Floating Point - End-of-Record
8-4	-	Data is Fixed Point - End-of-Record
9	9	This card is to be ignored
12	+	Conversion mode is continued from previous card--first number on card is absolute storage location
12-2	B	Data is Binary Coded Decimal --first number is an absolute storage location
12-3	C	Data is Floating Point -- first number is an absolute storage location
12-4	D	Data is Fixed Point -- first number is an absolute storage location
12-5	E	Data is Binary Coded Decimal -- first number is an absolute storage location
12-8	H	Conversion mode is continued from the previous card -- first number is an absolute storage location -- end-of-record

Control	Symbol	Designation
12-8-2	None	Data is Binary Coded Decimal -- first field is an absolute storage location -- end-of-record
12-8-3	.	Data is Floating Point -- first field is an absolute storage location -- end of record
12-8-4	)	Data is Fixed Point -- first field is an absolute storage location -- end of record
Blank		The conversion mode is continued from the previous card, with the exception of control 0 (Zero).

Absolute Location:

Whenever there is a 12 punch in column 1, the next number on the card must be an absolute location. DING will then store all the data that follows in descending order with the first piece of data stored at this location.

Variable Storage Reference:

Whenever a variable is found on the data card, DING searches through the List of Variables and equates it to an absolute storage location. The variable may appear anywhere on the card with the exception of column 1, and all data that follows are stored with reference to the absolute location.

In conjunction with the table look-up feature, the variable name is examined. If the first character is I, J, K, L, M, or N, the conversion mode is set for integer conversion. If the first character is not one of these special characters, the conversion mode is set for floating point. This procedure will take precedence over the "3" or "4" control punch in column 1. Normally, control punches (3, 4) will not be required except when it is desired to load numeric data following BCD data. This feature allows integer and floating point data to be loaded from the same card.

Data:

Floating Point: - A 3 punch in column 1 indicates that all the data on this card are to be converted to 704 floating point notation. There are a variety of acceptable ways to punch floating point data, e. g.:

8126
+8126.00
8.126/3
+8.126/+3
8.126 +3
+812600/-2
812600-2

The plus sign on the data is optional. The exponent is separated from the mantissa by either a/, +, or - sign. A blank or a comma tells DING that this is the end of the number.

Fixed Point: - A 4 punch in column 1 indicates that all data on this card are to be converted to a binary integer and stored in the decrement field of the current absolute storage location. The data on the card must be punched as an integer and may be preceded by a sign. The absolute value of the integer must not exceed 32767. The end of the integer is indicated by a blank or a comma.

Bypassing Storage Locations:- An equal sign immediately followed by a blank or a comma causes the storage location to be decreased by one. An integer (n) immediately preceding the equal sign causes the storage location to be decreased by n. This feature enables the loading of data into scattered locations without continually defining the storage location.

Octal Instruction Cards: - If column 1 contains a zero punch, this indicates to DING that this is an octal instruction card. Each octal instruction card must have a zero punch column 1, an octal address in columns 7 - 11, a sign and 12 octal numbers of the instruction in columns 18 - 30.

Binary Coded Decimal Data: - If column 1 contains a 2 or a 5 punch, the data on the card are stored in six-bit binary coded decimal form. If there is also a 12 punch in column 1, then the first information must be the absolute storage location. If there is no 12 punch in column 1, the first character of the next field is examined to determine if it is numeric or alphabetic. If alphabetic, the field is interpreted to be the variable defining the storage location. Following the storage reference, and separated from it by either a blank or a comma, is the integer N where N x 6 is the number of BCD characters to be stored.

If the first character of the first field is numeric, the field is interpreted as N and the storage location is sequential from the previous card. The integer N is followed by a blank or comma and the next character will be the first BCD character to be stored. DING will store BCD information in ascending locations if the column 1 control punch is a 2, or in descending order if the column 1 control punch is a 5. (When BCD information is being read for use as format by IOH, the control 5 must be used.)

When it is necessary to continue BCD information for more than one card, the following cards must have a blank or an 8 punch (EOR) in column 1 followed by the integer N, a blank or comma, and the BCD information. Note: N may not exceed the physical capacity of one card. Succeeding cards must each contain N.

End-of-File: - When either a physical or control punch end-of-file is encountered, the second parameter of the calling sequence is examined. As noted above, if the parameter value is non-zero, the standard EXIT subroutine is called. If the parameter value is zero, a ± 1 is stored in the location designated and control is returned to the calling program. A +1 is stored if a control card EOF was encountered; a -1 is stored if a physical EOF was encountered.

Error Codes: - When DING finds an error, it reads in, but does not convert the remaining cards until an end-of-record control punch occurs in column 1. The exception is that the storage location of each piece of datum is checked to determine if the location is within the stated dimension of the variable. If not, an error print-out is made, but DING will continue to store data. While bypassing the data cards in the first instance, DING will continue to search for errors, printing each error as it is found. The standard error procedure print-out is produced on the peripheral output tape as:

AN ERROR HAS BEEN FOUND AT OR NEAR

EXTERNAL FORMULA NO. α, INTERNAL FORMULA NO. β, OF DING

⋮

where α is the error code and β is (100 x card count + column number)$_{10}$ where the error occurred. The error codes are:

Code	1	Column 1 control not acceptable
	3	Variable location exceeds dimension
	5	Error in data character
	7	Integer is too large
	8	Variable not given in table
	9	Tape failure

CODING INFORMATION:

Restrictions

1. The control punch zero must be on every octal instruction card. In the octal fields, blanks are equivalent to zero.

2. A data field is terminated by either a blank or a comma. Any number of blanks may separate the data fields.

3. Absolute storage locations, if specified, must be designated in the first field of the card.

4. An end-of-file error exit may be obtained by either an end-of-file control card or if there is an attempt to read more data cards when no more are available.

5. A control character including a BCD 5 control and an EOR control is an illegal character unacceptable to the peripheral card reader.

6. If it is desired to load numeric data after loading BCD data, it is necessary to use a control punch "3" or "4" in column 1 to cause DING to revert to numeric conversion.

7. Two commas in succession will cause a dummy number of all 1's to be stored.

ADDENDUM

IDENTIFICATION:

Title: TABLE Statement
Source Language:
Application: 7090 FORTRAN
Author: Dorothea S. Clarke
Unit: Computer Techniques Development
Installation: General Electric - Evendale
Date: January 1961

TABLE	GENERAL FORM	EXAMPLES
	TABLE $V_1(v_1, v_2, v_3 \ldots v_i), \ldots$ $V_i(v_1, v_2, v_3 \ldots v_i), \ldots\ldots\ldots$ $\ldots V_j(v_1, v_2 \ldots\ldots v_i)$ where V is the name of a variable. $v_1, v_2 \ldots$ are the values contained in a table.	TABLE AESOP(3.79, -2.3, ...), IOST(3, 2, 6...), DATA(7, $(A, N, CEPH, ...)

The TABLE statement provides the tool for the programmer to compile tables of fixed information into a FORTRAN program or subprogram. The variable must have been defined in a DIMENSION statement. The value list must be in the usual "columnwise" order.

At least seven types of conversion are acceptable:

1. Integers
2. Floating point numbers
3. BCD information
4. Octal information
5. Locations of variables
6. Table generation
7. = symbol

Conversion is determined by the format of the field, as follows:

1. Integers simply designate integer conversion.

2. Floating point is designated by the acceptable FORTRAN and DIP forms with the exception that only the DIP form containing a decimal point or exponent is recognized. For example, the following forms are all acceptable:

4096.	+4.096E3
+4096.0	4.096E+3
4.096/3	+4.096+3
4.096/+3	40960/-1

3. BCD information is designated by preceding the information by nH, where n is the number of characters, including blanks, contained by the field. A single field may fill multiple words, that is, $n > 6$; however, $n < 6$ causes the n characters to be stored, left justified and followed by blanks in a full word of storage.

   As an example, information preceded by 8H indicates that the first 6 characters are stored in the first storage location, and the remaining 2 characters followed by 4 blanks are stored in the second storage location.

4. The character O followed by 1 to 12 octal digits, sign optional, designates octal conversion; for example,

   O37
   O377777777777
   O-3777
   O+700000

5. A field which contains only a variable name stores the initial address of the variable.

6. The symbol $ preceding a field enclosed in parentheses indicates that a table is to be generated from the enclosed list. Example: ,$(A, B, C, D),

   Each variable name causes three words to be generated in a table. The first word is the BCD variable name. The second word is the location, and the third word contains the dimensions of the variable. Each 12 bits of the third word contains a dimension, for example,

O 0200 0000 0000 defines i = 128
O 0200 0037 0000 defines i = 128, j=31
O 0200 0037 0123 defines i = 128, j=31, k=83

This table may be generated within a defined table by embedding the $(LIST) as a field among other fields. The following illustration

TABLE DATA (7, 3, $(A, B, C, D, E, F, G))

produces a 23 word table located at DATA.

7. The conversion of a field which contains n=, where n is an integer, depends upon the preceding field. If the preceding field is numeric, n words of zero are generated in the table. If the field is BCD, n words of blanks are generated. If n is not specified, a value of one is assumed.

If the tabular information is larger than can be contained on the initial plus 9 continuation cards, TABLE continuation cards may be used, that is, another TABLE statement occurs in the form

TABLE n (.....)

where n is a digit. The only sequence check is that n> previous n.

A single TABLE statement may be used to define several small tables.

TABLE statements may appear any place in the source program after the DIMENSION statements defining any variables named in a $(LIST).

The present loader does not permit the use of tables located in COMMON.

APPENDIX D

THE BLAESS METHOD FOR NUMERICAL INTEGRATION OF ORDINARY DIFFERENTIAL EQUATIONS

APPENDIX D

The Blaess Method for Numerical Integration of Ordinary Differential Equations

The purpose for this appendix is to put on record the derivation of equations used in the Blaess Method for integrating an ordinary, linear or non-linear, differential equation of the second order. This method is most useful for homogeneous equations, i.e., when the second derivative depends on the dependent variable, and only indirectly on the independent variable. The attractive feature of this method as compared with other more common methods, such as Adams-Moulton or Runge-Kutta, is that the integration and correction is carried out in self-contained sequences of five steps. No special techniques are required for starting the integration. The user has complete freedom to vary the interval of integration for successive sequences of steps.

This method came to the author's attention through Reference 8 and a lecture given to the General Electric Advanced Courses. Reference 8 did not give a derivation, and there seem to be some algebraic errors in the results. Consequently, a new derivation has been prepared and is presented here for record.

We are interested in a differential equation of the form,

$$\frac{d^2y}{dx^2} = f\left(x, y, \frac{dy}{dx}\right), \tag{1}$$

subject to initial conditions:

at
$$x = x_0 \tag{2}$$
$$y = y_0$$
$$\frac{dy}{dx} = \left(\frac{dy}{dx}\right)_0$$

Formal integration over a step may be carried out if we assume that d^2y/dx^2 does not change during the interval: $x_0 \quad x \quad x_0 + x$

$$\left(\frac{dy}{dx}\right)_1 = \left(\frac{dy}{dx}\right)_0 + \left(\frac{d^2y}{dx^2}\right)\Delta x \tag{3}$$

$$y_1 = y_0 + \left(\frac{dy}{dx}\right)_0 \Delta x + \left(\frac{d^2y}{dx^2}\right)_0 \frac{\Delta x^2}{2}$$

Then $(d^2y/dx^2)_1$ can be calculated using (1) and the formal integration process repeated.

The principle behind the Blaess Method is to carry out five steps according to (3), and then examine the results for error and determine a correction. The procedure is to look for representative values of d^3y/dx^3 and d^4y/dx^4 over the entire range x_o to $x_o + 5\Delta x$, which can be used to improve the integration. Approximate values may be found if the variation of d^2y/dx^2 with x is neglected in comparison with variation with respect to y. The five integration steps carried out so far provide a body of data on the variation of d^2y/dx^2 with y. It is supposed that the best values of d^3y/dx^3 and d^4y/dx^4 would be calculated from the differences between d^2y/dx^2 at the corrected values of y corresponding to $x_o + 4\Delta x$ and $x_o + 2\Delta x$ and the value $(d^2y/dx^2)_o$. In the neighborhood of $x_o + 4\Delta x$, d^2y/dx^2 is varying with y at approximately the rate

$$\frac{\left(\frac{d^2y}{dx^2}\right)_5 - \left(\frac{d^2y}{dx^2}\right)_3}{y_5 - y_3} \qquad (4)$$

In the neighborhood of $x_o + 2\Delta x$ the corresponding rate is

$$\frac{\left(\frac{d^2y}{dx^2}\right)_3 - \left(\frac{d^2y}{dx^2}\right)_1}{y_3 - y_1}$$

We are, of course, assuming here that the corrections to y_2 and y_4 will be small compared to $(y_3 - y_1)$ and $(y_5 - y_3)$.

Now, the corrected value of y_2 is to be given by its Taylor Series expansion, using $2\Delta x$,

$$y_{2c} = y_o + \left(\frac{dy}{dx}\right)_o (2\Delta x) + \left(\frac{d^2y}{dx^2}\right)_o \frac{(2\Delta x)^2}{2} + \left(\frac{d^3y}{dx^3}\right)_o \frac{(2\Delta x)^3}{6} + \left(\frac{d^4y}{dx^4}\right)_o \frac{(2\Delta x)^4}{24} \qquad (5)$$

The current estimate for y_2 is,

$$y_2 = y_o + \left[\left(\frac{dy}{dx}\right)_o + \left(\frac{dy}{dx}\right)_1\right]\Delta x + \left[\left(\frac{d^2y}{dx^2}\right)_o + \left(\frac{d^2y}{dx^2}\right)_1\right]\frac{(\Delta x)^2}{2} \qquad (6)$$

$$= y_o + \left[2\left(\frac{dy}{dx}\right)_o + \left(\frac{d^2y}{dx^2}\right)_o \Delta x\right]\Delta x + \left[\left(\frac{d^2y}{dx^2}\right)_o + \left(\frac{d^2y}{dx^2}\right)_1\right]\frac{(\Delta x)^2}{2}$$

$$= y_o + 2\left(\frac{dy}{dx}\right)_o \Delta x + \left[\frac{3}{2}\left(\frac{d^2y}{dx^2}\right)_o + \frac{1}{2}\left(\frac{d^2y}{dx^2}\right)_1\right]\frac{(\Delta x)^2}{2}$$

The corrected value of $(d^2y/dx^2)_2$ is now,

$$\left(\frac{d^2y}{dx^2}\right)_{2c} = \left(\frac{d^2y}{dx^2}\right)_2 + \frac{\left(\frac{d^2y}{dx^2}\right)_3 - \left(\frac{d^2y}{dx^2}\right)_1}{y_3 - y_1}\left(y_{2c} - y_2\right) \tag{7}$$

$$= \left(\frac{d^2y}{dx^2}\right)_2 + \left\{\frac{1}{2}\left[\left(\frac{d^2y}{dx^2}\right)_2 - \left(\frac{d^2y}{dx^2}\right)_1\right](\Delta x)^2 + \frac{1}{3}\frac{d^3y}{dx^3}(\Delta x)^3 + \frac{1}{3}\frac{d^4y}{dx^4}(\Delta x)^4\right\}\frac{\left(\frac{d^2y}{dx^2}\right)_3 - \left(\frac{d^2y}{dx^2}\right)_1}{y_3 - y_1}$$

The notation will be simplified by using

$$\left.\begin{aligned} y_0' &= \left(\frac{dy}{dx}\right)_0 \ ; \\ y_0'' &= \left(\frac{d^2y}{dx^2}\right)_0 \ ; \\ y_0''' &= \left(\frac{d^3y}{dx^3}\right)_0 \ ; \\ y_0^{IV} &= \left(\frac{d^4y}{dx^4}\right)_0 \ ; \end{aligned}\right\} \tag{8}$$

with similar quantities y_1, y_2, y_3, ..., y_1', y_2', ..., y_1'', y_2'', ... designating values after intervals Δx, $2\Delta x$, etc.

Then (7) becomes

$$y_{2c}'' = y_2'' + \left[\frac{1}{2}(y_0'' - y_1'')(\Delta x)^2 + \frac{1}{3}y_0'''(\Delta x)^3 + \frac{1}{3}y_0^{IV}(\Delta x)^4\right]\frac{y_3'' - y_1''}{y_3 - y_1} \tag{9}$$

y_{2c}'' may also be expressed in terms of a Taylor's Series based on conditions at 0.

$$y_{2c}'' = y_0'' + y_0''(2\Delta x) + y_0^{IV}\frac{(2\Delta x)^2}{2} \tag{10}$$

Equating (9) and (10), and collecting terms,

$$y_0'''\Delta x\left[2 - \frac{4}{3}\frac{y_3'' - y_1''}{y_3 - y_1}(\Delta x)^2\right] + y_0^{IV}(\Delta x)^2\left[2 - \frac{2}{3}\frac{y_3'' - y_1''}{y_3 - y_1}(\Delta x)^2\right] = y_2'' - y_0'' + \frac{(\Delta x)^2}{2}(y_0'' - y_1'')\frac{y_3'' - y_1''}{y_3 - y_1} \tag{11}$$

A similar expression is based on studying y_{4c}''

$$y_0'''\Delta x\left[4 - \frac{32}{3}\frac{y_5'' - y_3''}{y_5 - y_3}(\Delta x)^2\right] + y^{IV}(\Delta x)^2\left[8 - \frac{32}{3}\frac{y_5'' - y_3''}{y_5 - y_3}\right] \tag{12}$$

$$= y_4'' - y_0'' + (\Delta x)^2\left[\frac{9}{2}y_0'' - \frac{5}{2}y_1'' - \frac{3}{2}y_2'' - \frac{1}{2}y_3''\right]\frac{y_5'' - y_3''}{y_5 - y_3}$$

Equations (11) and (12) are the basic equations of the Blaess Method. If the solution is being carried out by computer, it is profitable to use these formulas as is, as may be seen from the numerical comparisons given on the last page. If the calculations are being done by hand or desk calculator however, it may be preferable to choose x small enough so that the terms involving $[(y_3'' - y_1'')(y_3 - y_1)]\Delta x^2$ and $[(y_5'' - y_3'')(y_5 - y_3)]\Delta x^2$ may be neglected. Then

$$\left.\begin{aligned} y_0'' &= \frac{y_2'' - .75y_0'' - .25y_4''}{\Delta x} \\ y_0''' &= \frac{.5y_2'' - .25y_0'' - .25y_4''}{(\Delta x)^2} \end{aligned}\right\} \qquad (13)$$

Equations (13) can give quite good results. They may be interpreted in the form of corrections to y_5 and y_5':

$$\Delta y_5 = \left[\frac{275}{96}(y_2'' - y_0'') + \frac{5}{96}(y_2'' - y_4'')\right](\Delta x)^2 \qquad (14)$$

$$\Delta y_5' = \left[\frac{39}{4}(y_0'' - y_2'') + 11(y_4'' - y_2'')\right]\Delta x$$

Equations (14) have been obtained by comparing the Taylor's Series values for y_5 and y_5' with the actual calculation, and replacing y_1'' by $(y_2'' + y_0'')/2$ and y_3'' by $(y_4 + y_2)/2$.

Application of the Blaess Method to Solution of $y'' + y = 0$, with $y_o = 0$, $y'_o = 1$.

x	sin x	$y = -y''$	y'	y'''	y^{IV}	y	y'''	y^{IV}	y	Δy_5	y
		From Equation (3)		From Equation (13)			From Equations (11) and (12)			From Equation (14) and (3)	
0	0	0	1	-1.01246	.1495		-1.01310	.1978			
.1	.099833	.100000	1								
.2	.198669	.199500	.990000								
.3	.295520	.297500	.970050								
.4	.389418	.393018	.940300								
.5	.479426	.485083	.900998			.479297			.479408	-.005614	.479469
Error		+.005657				-.000129			-.000018		+.000043
.5	.479426	.479426	.877582	-.88786	.6034		- .88815	.6421			
.6	.564642	.564787	.829639								
.7	.644218	.644927	.773160								
.8	.717356	.719018	.708667								
.9	.783327	.786290	.636765								
1.0	.841471	.846036	.558136			.841362			.841464	-.004667	.841369
Error		+.004565				-.000109			-.000007		-.000102

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