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Application of the Statistical Theory of Extreme Values to Spacecraft Receivers

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ABSTRACT

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This Report considers the problem of error probability estimation in systems involving a threshold detector. The estimation procedure which merely observes the occurrence of errors is too slow, whereas the procedure which attempts to find a theoretical distribution for amplitude deviations and then predicts error probabilities from the tails of this distribution is too unreliable. Extreme-value theory, a procedure in use in fields such as flood control, monitors not only the occurrence of errors but also how close the detection scheme comes to making errors even when no error is made. Thus, the maximum amplitude deviation in a large number of bits is recorded as a new datum. The distribution of these extreme data then has an extreme-value distribution. The parameters of the extreme-value distribution are estimated, and this in turn yields an estimate of the error probability. There is a saving of a factor of ten in test time over mere bit error testing. The distributions obtained from actual threshold systems are shown to fit the theoretical distribution very well. The method was used to test the command receivers in *Rangers VII, VIII,* and *IX.*

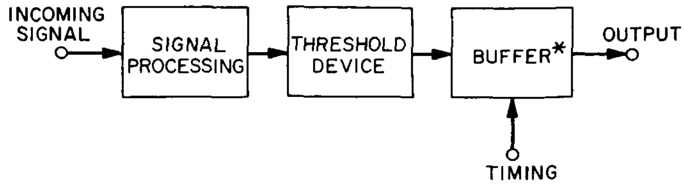
Author

I. INTRODUCTION

In the applications expounded in this paper, it is not proposed to delve into the sources of noise or errors in communication systems or into any of the associated philosophical arguments. Suffice it to say that man cannot at present physically manufacture the components assumed to exist during theoretical system analyses. If, in an attempt to make theory and practice agree, he modifies his models of these devices to more accurately represent components he can manufacture, the analysis becomes so cumbersome that he cannot generally predict precisely the behavior of the resulting model. This is one of the basic reasons to test systems or parts of systems; other reasons may be to prove capability of designs apart from the theoretical predictions, to check for degradation of performance with time, etc. For these applications, it is assumed that the need to test a portion of a communication system—especially the receiver—has been established. Furthermore, the receiver type is assumed to be restricted to those used in binary communication systems, and the examples discussed are slanted toward space communications.

II. BIT ERROR TESTING

In nearly all binary communication systems, information is ultimately conveyed by the use of some form of a decision or threshold device. At this point it becomes convenient to restrict the topic to systems whose receiver is basically a threshold detection device. This may take the very simple form of a telegraph receiver responding (i.e., the arm closing) if sufficient current passes through the coil, and the converse (non-response) if sufficient current is lacking. Here the receiver acts as its own threshold device and is simplicity itself among receivers. Threshold receivers, in general, are all basically "signal present" receivers, whether it be the presence of a certain amount of current, as in the above example, or the presence of a particular frequency, phase, amplitude, etc. They are widely used in communication—for example in telegraphy, remote control, and frequently in command and telemetry systems aboard both deep space probes and orbital spacecraft. In this type of system the question of reliability of received information eventually can be and frequently is reduced to the concept of a bit error, i.e., the probability of incorrect reception on a particular bit (binary *one* or *zero*). Thus, given a binary *one* (*zero*) and noise as the incoming signal of a threshold type receiver (Fig. 1), the basic question becomes "What is the probability of failing to receive a binary *one* (*zero*) at the output?"



*THE FUNCTION OF THE "BUFFER" AS USED HERE IS TO EXAMINE THE OUTPUT OF THE THRESHOLD DEVICE AS DICTATED BY THE TIMING SIGNAL AND TO TRANSLATE THE INFORMATION DETERMINED BY THE THRESHOLD DEVICE AS REQUIRED BY NEEDED OUTPUT CHARACTERISTICS.

Fig. 1. Threshold receiver

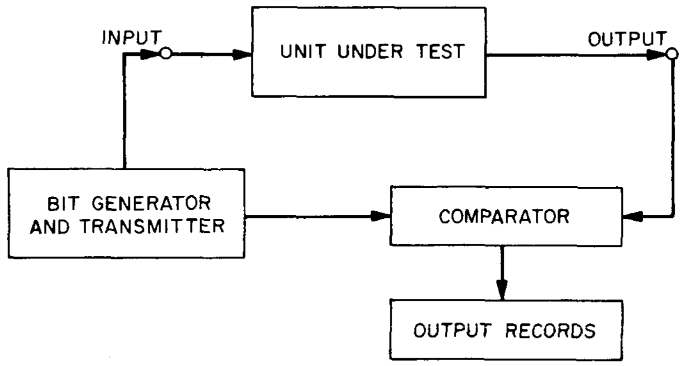


Fig. 2. Bit error testing

One of the more obvious attacks on this problem is the standard concept of *bit error testing* (Fig. 2). In this approach, the receiver under test is supplied with a given signal-to-noise ratio (SNR), a known bit is transmitted to it, and the output is examined and compared with the type of bit transmitted. The error rate is defined simply as the ratio of bits in error to total bits transmitted during the test. If either error rates or bit rates are high so that errors accumulate at the rate of 10 to 20 per hour of test time, this approach can give accurate results with high confidence levels in a "reasonable" length of time. However, if error rates are low (say 10^{-5}) and bit rates are also relatively low (say 1 bit/sec), then the test time required to establish such an error rate experimentally is about 45 hr to obtain an 80% confidence level. And as bit rates decrease, and/or error rates being measured decrease, the required test time increases.

In present-day deep space probes, bit rates used in communication with the spacecraft are normally low (1 bit/sec, for example, on both *Ranger* and *Mariner* command systems). Too, reliability of transmitted commands must be high. The defined threshold on these two systems is a bit error rate of 10^{-5}. Test time required to establish whether or not the required error rates are obtained at the specified SNR is on the order of several hours, as previously mentioned. If it is further desired to not only obtain this one point of data, but also to establish an actual experimental curve of bit error rates as a function of SNR (perhaps at several temperatures), test time can become prohibitive. Furthermore, long periods of testing allow variables, some known and some unknown, to influence the system under test. This, in turn, leads to highly instrumented test complexes involving large amounts of equipment, manpower, and operating time. A different approach would obviously be welcome.

III. EXTREME-VALUE THEORY

Refer again to Fig. 1. It is not uncommon for information to be presented to the threshold device in analog fashion. There is much information available in the analog signal that is not used in bit error testing as previously described. Thus, one can determine not only whether an error occurred or not, but also how close it came to occurring. This implies that knowledge of the amplitude distribution of the information presented to the threshold device at the time the threshold detector's output is examined will allow prediction of the probability that any one bit will be in error (Fig. 3). However, conventional measurements of amplitude distribution again present problems in test time unless the shape of the distribution is used to extrapolate beyond observed data. If the shape of the distribution under test is known, this approach would hold some promise. Although theory will again predict the shape of these curves for some systems, the theory generally disagrees with physical measurements at the extremes of the distribution—precisely where it becomes of most value in problems of this type. Thus, ideas of curve fitting by standard distributions (i.e., Rayleigh, Gaussian, etc.) are not as powerful as is needed. It is in defining and extrapolating this "tail" of a possibly unknown amplitude distribution function that the use of extreme value statistics becomes of aid.

First, we shall summarize extreme-value theory, taking much of the results from Gumbel (Ref. 1). The extreme-value distribution referred to is of the form $\exp\{-\exp[-\alpha(x-u)]\}$, where α, u are positive parameters. This distribution is the asymptotic distribution for large n of the extreme positive value among n independent random variables x_i chosen from a "nice" distribution. One is willing to assume that the unknown distribution of voltage fluctuation is of the "nice" type, since this assumption is satisfied for distributions having right-hand tails qualitatively like a normal or negative exponential distribution.

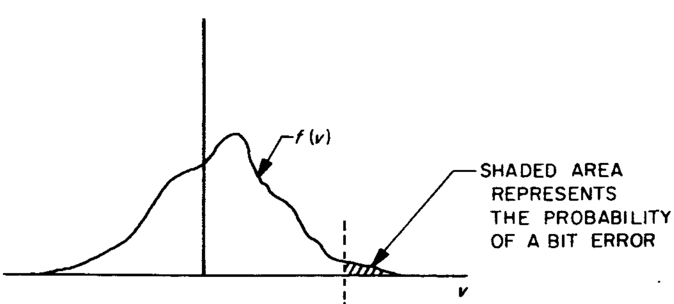


Fig. 3. Probability density of signal to threshold device

The way extreme-value theory is used in estimating error probabilities is as follows: Consider a threshold detection system as discussed. Assume the bit rate is slow enough so that deviations in different bits are independent random variables, as is true in the *Ranger* command detector. Examine a run of $M = Nn$ successive bits, with for example a *one* as the transmitted symbol. Look at the maximum of the (negative of) this deviation in each block of n bits. If n is large enough, these maximum deviations will approximately have the extreme-value distribution with unknown parameters α, u. Then estimate α, u from the N samples of the distribution of the maxima.

Using α and u, then estimate the error probability from "the probability of exceeding the fixed threshold in an extreme-value distribution with parameters α and u," for we can estimate the probability that the maximum out of n exceeds the threshold. Simple algebra transforms this probability into the probability that a given *one* observation exceeds the threshold; for small error probabilities, division by n accomplishes this. We thus obtain an estimate of the probability of the error "$1 \rightarrow 0$," given that a *one* was sent. In practice, we typically use $M = 3000$, $n = 100$, $N = 30$. That is, extremes were taken from successive groups of 100 samples, giving 30 independent samples from, hopefully, an extreme-value distribution.

A so-called "goodness-of-fit" test was applied to typical *Ranger* data (Fig. 4; the difference P^{+}_{30} is a certain statistic used in the test). The result was an exceptionally good fit—so encouraging that it was decided to use the method on succeeding *Rangers* that were flown.

The parameters α and u, and so the error probability, were estimated using Gumbel's technique. Confidence intervals of probability ⅔ were similarly obtained, and are shown on the figures.

It is interesting to ask what the increase in the number of samples taken would have to be to obtain comparably good estimates by the bit error testing method. The improvement is rather dramatic at error probabilities of

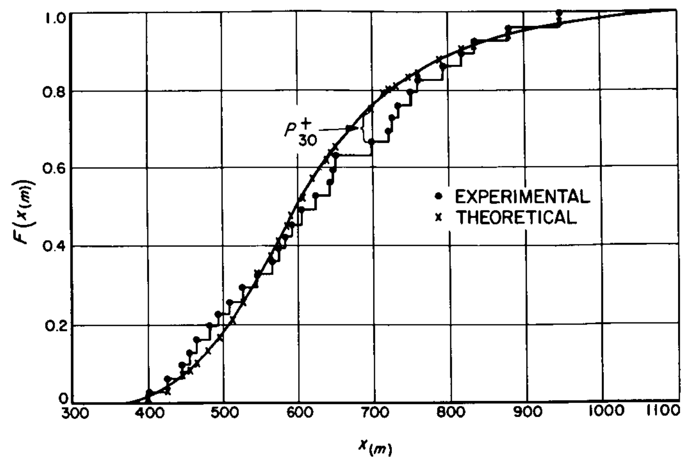


Fig. 4. Goodness-of-fit

10^{-7} or less, but even at 10^{-5}, savings on the order of a factor of 10 in test time can be demonstrated.

This opportunity should be taken to remark that extreme-value theory is not new in engineering. It has previously been applied in such areas as dam building (Ref. 1). What is done is to study the maximum flood on a river over a period of years, and use extreme-value theory to find the height of dam necessary to hold back all floods in the next, say, 50 years with high probability. Applications here are really different, however. What is done is study the floods (maximum amplitudes) and see how close they come to overflowing the dam (threshold). If this overflow (error) probability is too high, another river (detector) is used.

IV. EXPERIMENTAL RESULTS

Rangers VII, VIII, IX, and type approval command detectors were tested, but only data on the type approval unit will be presented here.

Measurements were made using a Beckman 4040B digital-to-analog converter and an associated printer on the *Ranger* command detector output (Fig. 5) at a number of different signal-to-noise ratios on the detector input. A processed form of the data obtained is given in Table 1; the same data is plotted in Fig. 6. Final results (i.e., estimated bit error rates) are presented in Table 2 along with results of bit error tests on the same system.

In evaluating Table 1, it should be pointed out that the signal-to-noise ratios indicated are accurate only to about ± 2 db with a relative accuracy of approximately ± 0.5 db. The major source of inaccuracy is the fact that an RF loop (modulator, transmitter, and transponder receiver) was used in the test with all the attendant problems that such a loop poses. It might be interesting to note that the error rate near 25-db SNR had long been suspected, mostly on intuitive grounds, of being 10^{-7} to 10^{-8}, but had not previously been measured.

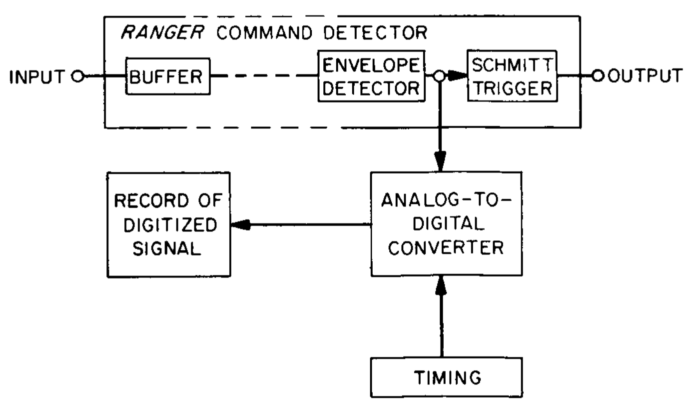


Fig. 5. Test setup

Table 1. Ordered voltages

Number per group	100	105	105	100
Signal-to-noise ratio (db in 1 cps)	$-\infty$ (noise)	At 20.6 db	At 22.8 db	At 25 db
Group number	Ordered voltage	Ordered voltage	Ordered voltage	Ordered voltage
1	−0.036	−0.541	0.304	1.587
2	0.010	−0.496	0.308	1.555
3	0.016	−0.332	0.407	1.552
4	0.019	−0.268	0.465	1.548
5	0.030	−0.235	0.586	1.535
6	0.049	0.142	0.708	1.532
7	0.058	0.152	0.731	1.523
8	0.065	0.164	0.772	1.516
9	0.081	0.164	0.798	1.513
10	0.107	0.171	0.820	1.510
11	0.116	0.194	0.827	1.510
12	0.129	0.197	0.830	1.497
13	0.129	0.203	0.859	1.481
14	0.139	0.206	0.872	1.474
15	0.149	0.206	0.881	1.471
16	0.149	0.212	0.965	1.471
17	0.155	0.252	0.965	1.471
18	0.172	0.268	0.994	1.468
19	0.175	0.284	1.026	1.465
20	0.181	0.306	1.033	1.465
21	0.194	0.312	1.035	1.432
22	0.200	0.322	1.096	1.432
23	0.207	0.326	1.102	1.429
24	0.214	0.358	1.106	1.426
25	0.220	0.361	1.112	1.397
26	0.256	0.378	1.112	1.377
27	0.259	0.419	1.135	1.374
28	0.272	0.422	1.244	1.352
29	0.334	0.432	—	1.345
30	0.618	0.438	—	1.258
31	—	0.442	—	—
32	—	0.458	—	—
33	—	0.468	—	—
34	—	0.487	—	—
35	—	0.491	—	—
36	—	0.497	—	—
37	—	0.513	—	—
38	—	0.526	—	—
39	—	0.532	—	—
40	—	0.532	—	—
41	—	0.535	—	—
42	—	0.538	—	—
43	—	0.542	—	—
44	—	0.545	—	—
45	—	0.558	—	—
46	—	0.564	—	—
47	—	0.586	—	—
48	—	0.586	—	—
49	—	0.694	—	—

Table 2. Results of tests

Signal-to-noise ratio (db in 1 cps)	Bit error rates			Method used[a]
	Lower limit	Nominal	Upper limit	
20.6	—	2.7×10^{-2}	—	Extreme-value theory
21.1	1.03×10^{-2}	1.26×10^{-2}	1.41×10^{-2}	Bit error test
22.8	1.85×10^{-3}	2.6×10^{-3}	3.45×10^{-3}	Extreme-value theory
23.0	2.4×10^{-4}	4.24×10^{-4}	6.9×10^{-4}	Bit error test
25.0	$4. \times 10^{-7}$	1.6×10^{-7}	$5. \times 10^{-8}$	Extreme-value theory
25.1	No errors in 230, 269 bits.			Bit error test

[a]Error rate limits are 68% confidence limits for extreme-value theory and 70% limits for measured values.

An additional application of the theory of extreme values to *Ranger* has been to the "start time" of the command detector. The *Ranger* command system is an asynchronous frequency-shift-keyed system. As such, it relies for program start on the known, or at least repeatable, delay in the detector between the initial application of a bit and the response at the detector output. This delay is typically on the order of 80 msec, but will vary as a function of signal strength. Of course, at a given signal-to-noise ratio, the delay will vary because of noise. If this variation is sufficient, causing the detector to start its program late enough, it could result in the second bit being interpreted as the first one, the third as the second one, etc. Hence, the start time and its variation become very much of interest.

Measurements of the start time have been made on the *Ranger* Block III type approval detector at various signal-to-noise ratios. Only two of these test results will be presented here; they are summarized in Table 3 and plotted in Fig. 7. It is indicated in Fig. 7 that, at 21-db SNR, there is a probability of $(1.51^{+2.37}_{-1.19}) \times 10^{-4}$ of sampling the wrong bit throughout the received word; this probability has never previously been measured or even estimated. Note that, as might be expected, this is well below the 25-db SNR threshold that is used in practice.

A certain hesitation may be experienced by many engineers when extrapolation over large values of the variate is required to obtain the desired goal, as in Fig. 6(d). Whether or not such extreme extrapolation is justified is not known; this application of the technique is new and most certainly not fully explored. The final answer must await further testing. Nonetheless, in mitigation, it is felt that the results so obtained do appear reasonable.

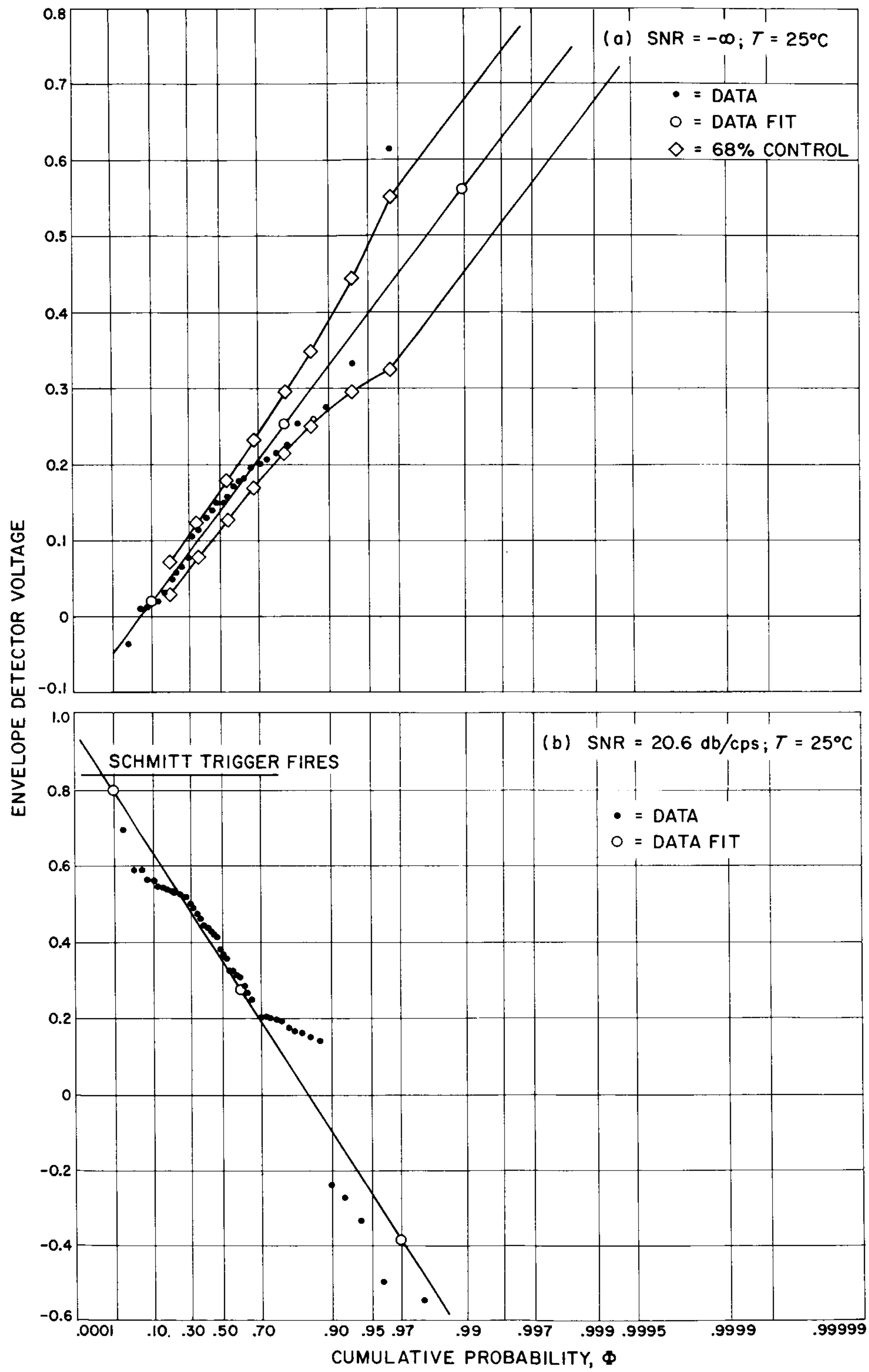


Fig. 6. ***Ranger*** **command detector curves**

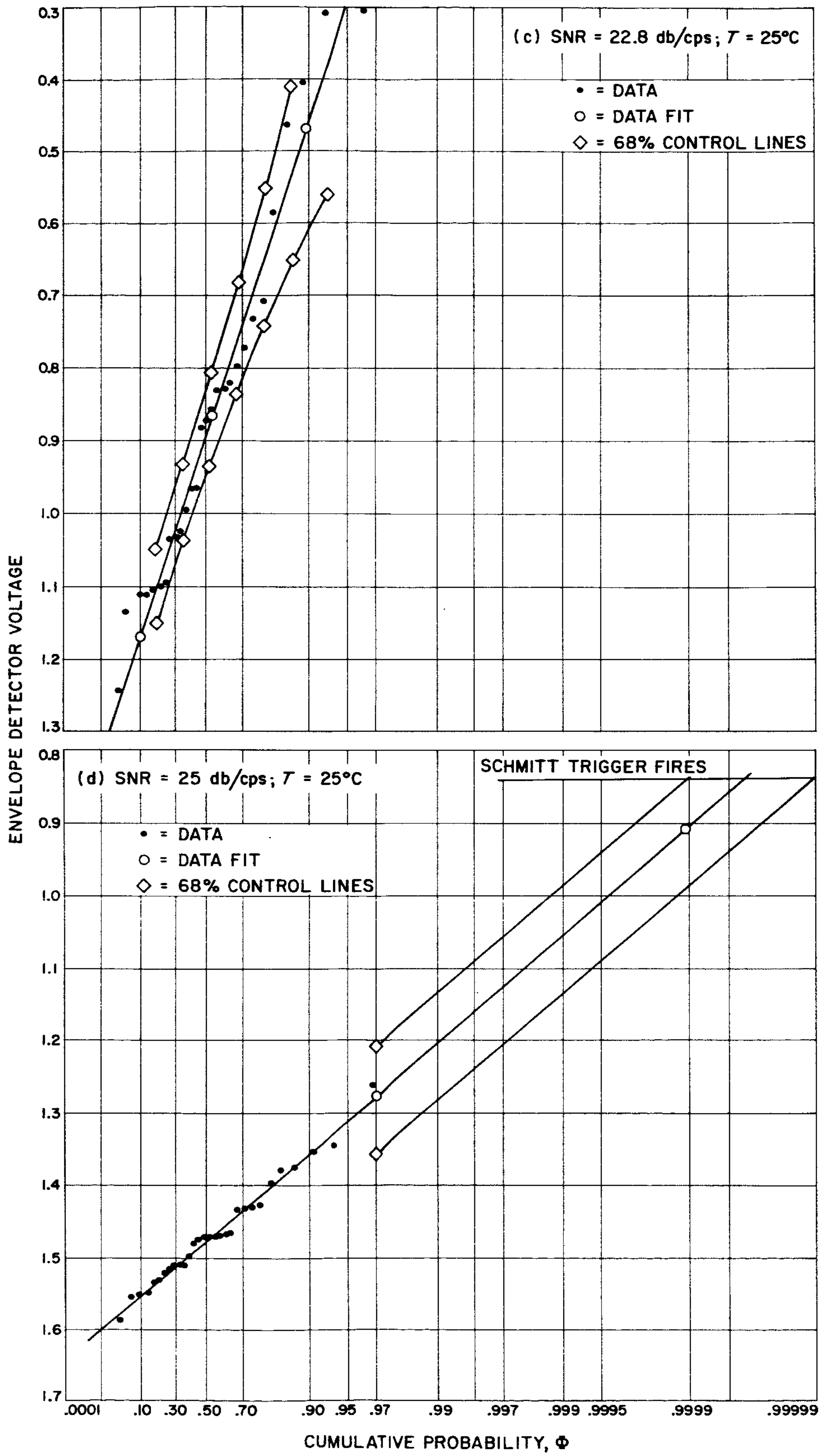


Fig. 6. *Ranger* command detector curves

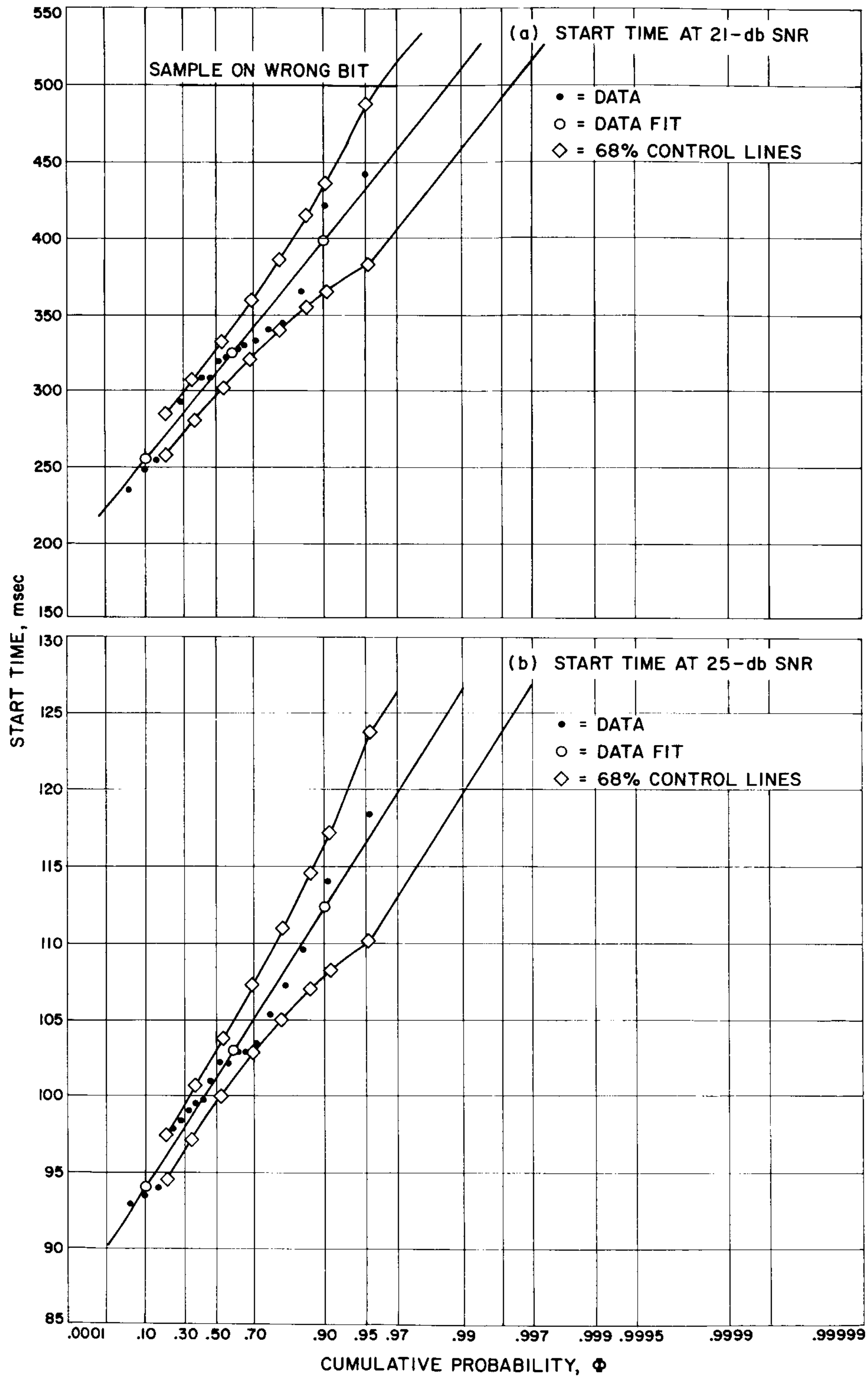


Fig. 7. Start time curves

Table 3. Application to start time

Signal-to-noise ratio (db in 1 cps)	At 21	At 27
Group number	Start time, msec	Start time, msec
1	234.67	92.86
2	245.29	93.42
3	254.06	93.93
4	282.85	97.82
5	287.80	97.84
6	292.77	98.44
7	305.27	99.01
8	307.17	99.72
9	307.82	99.74
10	308.37	100.95
11	318.39	102.19
12	320.29	102.21
13	327.79	102.86
14	328.47	102.87
15	332.18	103.40
16	339.66	105.31
17	343.46	107.18
18	366.54	109.73
19	429.91	114.08
20	442.80	118.46

Note: All groups had 105 samples.

REFERENCE

1. Gumbel, E. J., "Statistics of Extremes," Columbia University Press, New York, 1958.