

FINAL REPORT


## FEASIBILITY STUDY FOR DEVELOPMENT

OF A HYPERVELOCITY GUN
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## I. INTRODUCTION

The defining features of a rail gun are (See Figure l) a pair of conducting rails and across them a conducting armature. Electric current is passed through the rails and armature so that the magnetic field producad by the current in the rails (and possibly in auxiliary field coils) interacts with the current in the armature. The resulting Lorentz force ( $\mathrm{j} \times \mathrm{B}$ ) tends to accelerate the armature away from the end of the rails at which the current is introduced.

Practically, two kinds of armature have been used, electric arc plasma $(1,2)$ and solid conductors $(3-7)$. In most projectile accelerators, the projectile is itself conductive and serves as the armature. In our gun, the projectile is nonconductive (usually nylon) and the armature that drives it is an arc plasma ${ }^{(8)}$. This circumvents the probiem of ohmic heating in the projectile.

In our experiments, the gun itself is anywhere from three to eight inches or so in length and consists of a pair of metai rails, usually copper, sandwiched between iwo insulating slabs. The ?suiting barrel can be made to have either a square or circular cross section.

With guns of this kind and no auxiliary field, we have caccelerated rylon projectiles to the velocities listed below:

| Weight of Projectile | Shape | Velocity |  |
| :---: | :--- | :---: | :---: |
|  |  |  |  |
| 0.01 mg |  | Roughly Spherical | $10.3 \mathrm{~km} / \mathrm{sec}$ |
| 0.6 mg |  | Cubic | $3.7 \mathrm{~km} / \mathrm{sec}$ |
| 2.4 mg |  | Spherical | $6.0 \mathrm{~km} / \mathrm{sec}$ |
| 5 mg |  | Cube | $5.8 \mathrm{~km} / \mathrm{sec}$ |
| 31 mg |  | Cube | 5 to $6 \mathrm{~km} / \mathrm{sec}$ |
| 37 mg | Spherical | $4.8 \mathrm{~km} / \mathrm{sec}$ |  |

In each case except the first, the projectiles were single and fitted snugly in the barrel. The $10.3 \mathrm{~km} / \mathrm{ser}$, was achieved by a drag technique.

The system used consists of a 28 k joule, $142 \mu \mathrm{fd}$ condenser bonk, which is discharged by a triggered spark gap either directly into the gun or into an impedance matching pulse transformer giving peak currents up to 700 k sinp at ringing frequencies as high as 25 kc . The are is initiated by a small bit of aluminum foil behind the projectile and travels the entire length of the gun during the first half-cycle of the discharge. When a pulse transformer is used, about three fousths of the bank energy

Figure 1. The Arc Discharge Between Parallel Rails
is dissipated in that time. Of the total bank energy, however, at most about $3 \%$ goes into the kinetic energy of the projectile.

Besides this simple system, we have also tried two-stage and acxiliary field systems, for which we have a second independent, 28 k joule, $144 \mu \mathrm{fd}$ condeser bank and a second pulse transformer. Although high velocities were achieved by these means, in no case did they equal those quated above.

Routine diagnostics included magnetic fiux loops very close to the barrel to record the progress of the arc, a photoelectric muzzle-watcher to record the appearance of the luminous are plasma at the muzzle (this always coincides with the current front as deternined by the flux loop at the muzzle as has been observed by $c$ ther researchers $(9)$ ), a flux loop to record the total instantaneous current, and transit time and crater depth measurements to determine velocity. In a few experiments, we measured directly the voltage across the muzzle and breech of the gun to determine the resistive voltage of the arc and the rate of change of flux in the gun.

In the rail gun acceleration of plasmas, velocities upwards of $100 \mathrm{~km} / \mathrm{sec}$ have been achieved $(1,2)$. Offhand, one would, with some modesty, hope to use the same techniques and more energy to achieve somewhat lower velucities for small proiectiles. However, because even a 1 mg nylon sphere has an areal mass density $10^{6}$ times that of the usual rail gun plasma bodies ( $10 \mathrm{gm} / \mathrm{cm}^{2}$ versus $10 \mu \mathrm{gm} / \mathrm{cm}^{2}$ ) the magnetic pressure required to produce the same acceleration in the projectiles as in the plasmas is correspondingly higher.

The magnetic fields needed for such large accelerations are of the order of a megagauss. The ohmic heat per unit volume produced by such high fields (furned on fast enough to be contained by good conductors) is sufficient to melt the current carrying portion of the rails $(4)$, and momentum then goes into the molten rail material. Also the pressures are so high that the rails suffer plastic flow.

Smaller accelerations of longer duration over greater distance leads to: (1) greater loss of energy into mass spuiterad off the rails by current carrying ions, and (2) heating of even a nonconducting projectile by the are through thermal condu-in.

The auxiliary field systems was an aitempr to solve this dilemma.
Another limitation is that, for high magretic pressures, current begins to flow in front of the projectile as well as behind. The forward arc runs away from the p:ojectite and ot the same time jrows at the :ypense of the one behind. The force on the rear are is therefore diminished with the net effect of a lowet projectile velocity.

There have been two general approaches to this problem. One has been to try to understarid and control this phenomenon, the other to concede its inevitability and to' accelerate the projectile by drag.

In the following sections, we present the theory behind both approaches, the relevarit experimental results, and, based upon these, our recommendations for future work. Appended to this is a detailed descriotion of the novel equipment and techniques developed in the course of these experiments.

## SYMBOLS

$\sim$ is of the order of
$\theta$ something of the order of
$b, a, r, z, P, \Delta P, R, \& \quad$ see Figure 1
B magnetic induction
I instantaneous current
Ip peak current
j current density
L' inductance per unit length
$M$ mass of the projectile
$m_{\text {I }}$ mass-input per unit charge
router radius of the outer electrode of a coaxial rail gun
$S$ surface of integration
$\dagger$ time
$\checkmark$ velocity of the arc
$V$ volume of integration
1 unit tensor
$\delta$ skin depth
$\sigma$ electrical conductivity
$\omega$ angular frequency

## II. OPERATION OF THE RAIL GIJN

## A. THE LORENTZ FORCE ON THE ARC

The Lorentz force on an are in a rail gun without an auxiliary field can always $\mathfrak{y}$ written as $\frac{1}{2} L^{\prime} I^{z}$, where $I$ is the instantaneous total current in the rails, and $L^{\prime}$ is some inductance per unit length. The total momentum produced by this force will then be $\int \frac{1}{2} \angle^{\prime} I^{2} d t$. If $L^{\prime}$ is nearly constant in time, ar. $d$ if we know its value, we have a very useful expression for the momentum in terms of an easily measured quantity, $I$. In general this will not be the case, because $L^{\prime}$ is dependent on the current distribution in the rails, and, except in special cases, for example, the case of infinitely thin wire rails, this current distribution will vary with time in an unknown way.

Fortunately, the rail gun considered here falls under one of those special cases, af least to an approximation, and we can get an estimate of $L^{\prime}$ by mans of the Maxwell stress tensor, $B B-\frac{1}{2}|B|^{2} 1$ (1) is the unit tensor) in the following way.

The Lorentz force on an arbitrary volume, $V$, is given by

$$
\begin{align*}
F & =\int j \times B d V \\
& =\oint_{S} \frac{1}{\mu_{0}}\left(B B-\frac{1}{2}|B|^{2} 1\right) \cdot d S \tag{i}
\end{align*}
$$

where $S$ is the surface enclosing $V$, and $d S$ is outward. For $S$ (See Figure 1) we take the truncated spherical surface $\&$ of radius $r$ centered about the arc, and the disc $P$ perpendicular to the axis of symmetry at a distance $z$ from the arc. The integral over $S$ is the Lorentz force on the portion of the rails enclosed by 5 and upon the are.

We now let $\not \subset$ go to infinity. On $\not \perp, B$ is proportional to V re. Therefore, the integral on $\chi$ is proportional to $\ell r^{2}$ and goes to zero. $p$ is now the entire plane. For $z^{2}>g^{2}$, the forward component of the Lorentz force will be given by the integral of $\frac{1}{2 \alpha_{0}}|B|^{2}$ over this plane. The integral of $B_{2} \mathcal{B} \cdot d S$ will be negligible for the following reasons. $B_{z}$ is due solely to the current in and near the arc, so that

$$
\begin{align*}
& \left.B_{2}\right|_{0} \sim \mu_{1} I g / z^{2} \\
& A_{1}^{t_{0}} B_{x} B \cdot 15 \sim \mu \cdot I_{g=12}^{*}  \tag{2}\\
& \int_{\rho} \frac{1}{A_{0}} B_{2} B \cdot d S \sim \mu_{0} \cdot T^{2} g \cdot / z^{2}
\end{align*}
$$

On the other hand, in the gap

$$
\begin{align*}
& : B / \sim N=I / g \text { so that }  \tag{3}\\
& \int_{p} \frac{1}{2} N_{v}|<|^{2} / \pi-1 \sim /+I \cdot
\end{align*}
$$

If one compares the approximate expressions in (2) and (3), one sees that for $z^{2}>\boldsymbol{q}^{2}$, $\int_{D} \frac{i}{M_{0}} K_{2} B \cdot a, \quad$ can be ignored, and that the total forward force on the rails and arc is given by

$$
F_{t_{1},}=\int_{\left.R^{\prime}(x) \pi g_{y}\right)} \frac{1}{z_{i}}\left|E_{i}\right| / \mid-1
$$

The forward force on the rails alone is given by
where $\angle f$ is the cross section of the rails, $R$ is the surface of the rails, and $d S$ is now outward from the rails. The forward force on the arc, $F_{1, i}$ is just $F_{\text {ta }}-F_{\text {rails }}$ or,

The first integral in Equation (5) is the external magnetic energy per unit length behind the arc.

In the case of an azimuthally symmetrical discharge in a coaxial-cylinder rail gun, the first integral is just $\frac{4}{4}=\frac{1}{7}$ rimier , which is just $\frac{1}{2} I^{2}$ times the geometric external inductance. The second integral is zero in this case, because both $\mathcal{B}_{z}$ and $A$. is $^{2}$ are zero. Therefore, for azimuthal symmetry, $L^{\prime}$ is just the geometrical external inductance per unit length, independent of the radial distribution of the turgent ir the electrodes.

The same result holds for the previously mentioned special case of infir:tely thin wire rails and for the case of infinite conductivity. This can be derived from Equation (4) or from the conservation of energy and Faraday's Law. Both of these derivations depend on the fact that, in these special cases, the volume of the current
carrying region of the rails is zero．In one case we have a line current，in the other a surface current distribution．This means that $\mathcal{B}$ will be parallel to $R\left(B \cdot d \mathbb{R}_{R}=C\right.$ ）． $\Delta \mathcal{f}$ will be zero，and，therefore，both integrals in Equation（4）will be zero．From the point of view of Faraday＇s Low，it means that we can speak of the magnetic flux through the circuit，which is rot possible if flux penetrates a current carrying volume of finite extent．
in the cause of interest，the discharge is not azimuthally symmetrical，the cross sectional size of the rails is comparable to the size of the gap so that the rails can－ not be considered inf aitely thin，ard the electrical skin depth in the rails for typical transit times is consider－bl compared to the other dimensions so that the rails cannot offhand be considered or infinite conductivity．Near the arc，lines of induction which cut the rails because of their finite conductivity have components both normal to the surface of the rails and along the z－direction．Therefore，the second integral in Equation（5）may no longer be negligible．

In order to estimate this integral，we again make use of the fact that， away from the arc，only the current in the arc and not the current in the rails contributes to $B_{z}$ ．多 will，therefore，be roughly proportional to the inverse square distance from the arc，$/ / \pi^{2}$ ，and to the sine of the angle between the rait－surfuce normal and the radius vector from the surface to the center of the arc．This sine is approximately $g / z z$ so that $B_{z}$ is proportional to $1 / Z \mathbb{Z}$ ．At a given distance，$z$ ，from the arc，the width of the area on the rails cut by $\mathcal{B}$ will be the skin depth，$\delta$ ，corresponding to the time for the arc to travel that distance．Letting $v$ be the velocity of the arc and $\sigma$ the conductivity of the rails，we have

$$
\begin{align*}
& \partial \sim \sqrt{t \mu_{0} J} \quad \text { T~z/V } \\
& \int_{R} \frac{1}{\mu_{2}} E_{2} B \cdot d S \\
& \sim \int_{0}^{g} \frac{1}{\mu_{0}}|B|=\sqrt{\mu_{0}-r} d z+\int_{g}^{\infty} \frac{1}{\mu_{0}}|E| \frac{q^{2}}{L^{2}} \frac{q}{2} \sqrt{\frac{z}{\mu_{0} \sigma}} d z  \tag{6}\\
& \sim \frac{1}{\mu_{0}}|B|^{2} g \sqrt{\frac{q}{\mu v}}
\end{align*}
$$

If we let the first integral ir Equation（5）be $\frac{1}{2} \angle^{\prime} I^{2}$ and the second integral a correction to it，then，whatever the exact value of $L^{\prime}$ is，it will be roughly true that $\angle$＇$\sim \mu_{0}$ ， and near the gap， $\mid B / \sim L^{\prime}$＇IV．
Substituting these expressions into Equation（6），we have

$$
\frac{1}{\pi} S_{R} B_{x} B \cdot d S \sim\left(\frac{1}{x} L^{\prime} I I^{*}\right) \sqrt{1 / F \cdot \sigma \cdot \sigma g}
$$

Putting in the typical values

$$
\begin{aligned}
& g=1.5 \mathrm{~mm} \\
& v=5 \mathrm{~km} / \mathrm{sec} \\
& \sigma=5.8 \times 10^{7} \mathrm{mho} / \mathrm{in} \text { for copper }
\end{aligned}
$$

we have $1 / \sqrt{4} 5 \sqrt{5 g}-0.04$.
This is the order of magnitude of the fractional correction that will have to be made in the expression for the Larentz force derived from the first integral of Equation (5). From the above calculation, we see that, for copper, it is perhaps small enough to be neglected. For steel with ol conductivity of, say, $5.8 \times 10^{6} \mathrm{mho} / \mathrm{m}$, the correction is about 0.12, perhaps large enough to be considerable. (See Table 1 for electrical resistivities of rail materialsi.

As long as this correction is not too large, the skin depth at will be small enough so that we can get a fair approximation to $L^{\prime}$ from the high frequency inductance per unit length. Even for $\delta \ll g$, this may still be somewhat inaccurate because the surface current distribution in this problem is not exactly that of the steady state alternating current problem. However, when it is accurate enough, the high frequency inductance per unit length can be measured directly in a ringing circuit or indirectly by means of a two-dimensional electrical analog.

To summarize, the Lorentz force on the arc is given by

$$
\begin{equation*}
\left.F_{\text {arc }}=\frac{1}{2} L^{\prime} I^{2}\left[1-母_{( } \frac{1}{\sqrt{\mu_{c} \sigma v g}}\right)\right] \tag{7}
\end{equation*}
$$

where $L^{\prime}$ is roughly the high-frequency inductunce per unit length and is a correction of the order of the ratio between the skin depth near the arc to the width of the gap.

## B. MASS-INPUIT LIMITATION ON THE PROJECTILE VELOCITY

The momentum as calculated from Equation (7) using the measured values of $L^{\prime}$ and $I$ is actually about three times greater than the mass of the projectile times its measured velocity: even when oniy a single arc filament is observed. As a tentative explanation for this we proposed a mass-input to the are proportional to the total charge through the arc. This is the sort of thing one would expect if inn sputtering ware toking place(11).

Table 1

## IMPORTANT PHYSICAI. CHARACTERISTICS OF ELEMENTAL RAIL MATERIALS

| Eler.ent | Atumic Number | Total hea: <br> content $20^{\circ} \mathrm{C}$ <br> through iA. P. <br> $\frac{\mathrm{k} \text { joule }}{\mathrm{cm}^{3}}$ | Self sputtering yield at $200 \mathrm{ev}^{*}$ $\frac{a \mathrm{mu}}{\mathrm{ion}}$ | Electrical Resistivity $\mu \Omega \mathrm{cm}$ | Tensile <br> Strength <br> $10^{3} \mathrm{psi}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Be | 4 | 6.9 | 11 | 4.3 | 50 |
| C | 6 | $>15.0$ | 0.4 | 800 | - |
| Mg | 12 | 1.9 | 8 | 4.6 | 30 |
| Al | 13 | 2.7 | 8 | 2.8 | 40 |
| Ti | 22 | $>6.0$ | 11 | 3.2 | 100 |
| $\checkmark$ | 23 | $>6.7$ | 17 | 25 | 100 |
| Cr | 24 | $>12.0$ | 37 | 13 | 60 |
| Fe | 26 | 9.2 | 34 | 10 | 100 |
| Co | 27 | :0.5 | 39 | 9.8 | 100 |
| Ni | 28 | 9.2 | 43 | 7.8 | 160 |
| Cu | 29 | 5.6 | 61 | 1.7 | 70 |
| Zn | 30 | 2.1 | 3.5 | 5.8 | 30 |
| Y | 39 | $>3.5$ | 2.4 | 65 | 20 |
| Zr | 40 | $>5.1$ | $2 \hat{0}$ | 39 | 100 |
| Nb | 41 | $>10.0$ | 20 | 14 | 50 |
| Mo | 42 | $>11.0$ | 31 | 5.7 | 60 |
| Ru | 44 | $>10.0$ | 38 | 7.1 | - |
| Rh | 45 | $>8.5$ | 52 | 4.5 | 100 |
| Pd | 46 | 0.8 | 98 | 10.8 | 40 |
| Ag | 47 | $>3.3$ | 127 | 1.59 | 40 |
| Hf | 72 | $>5.0$ | 33 | 36 | 100 |
| Ta | 73 | 11.0 | 33 | 12.4 | 150 |
| W | 74 | $>12.0$ | 28 | 5.5 | 200 |
| Re | 75 | 15.0 | 46 | 19.1 | 150 |
| Os | 76 | 11.0 | 54 | 9.5 | 150 |
| Ir | 77 | 11.0 | 88 | 5.3 | - |
| Pt | 78 | 8.2 | 97 | 9.8 | 50 |
| Au | 79 | 5.4 | 171 | 2.2 | 20 |

*Not much data exists on the sputtering of cathodes by ions of the same element; therefore, the self-sputtering yields listed here have beer based on data for sputtering by noble gases(10). Since sputtering at those energies is thought to be predominantly a momentum transfer process, the yield for each elenient inas been takien from the data for the noble gas of most nearly some atomic wisight (11).

For experiments without an impedance matching transformer, the current had the form of a slightly damped sine wave. Puttina $I_{\mu} \sin \omega t$ for the current, $M$ for the mass of the projectile, and $m_{I}$ for the mass-input per unit charge, we get the following expression for velocity

$$
\begin{equation*}
v=\frac{L^{\prime} I_{A}}{4 m_{r}} \frac{\omega t-\frac{1}{2} \sin 2 \omega t}{\frac{a m}{I_{H} m_{I}}+1-\cos \omega t} \tag{8}
\end{equation*}
$$

by equating the momentum of the projectile and are plasma io the mimentum calculared from Equation (7).

Figure 2 shows a plot of position versus time based on Equation (8). The curve was made to fit through a set of experimental points by setting $m_{r}=4,6 \mathrm{Cu}$ atoms/ ${ }^{\prime}$ ion. In order to show the seriousness of inc!uding mass input, another curve with $m_{I}=0$ was arbitrarily made to fit through the experimental point at $30 \mu \mathrm{sec}$ by setting $L^{\prime}$ equal to about one-third its measured value. It is very unlikely that the measurement of $L^{\prime}$ could be so much in error, but, even if it were, the uurve for $m_{I}=0$ sti! l has the wrong shape. It, therefore, seems that phenomenologically, at least, a charge propotional mass-input describes the situation. The following section indicates how the description may be more than phenomenclogical.

## 1. Ion Sputtering

According to Thom, Norwood, cind Jalufka, the current in a plasma rail gun is carried equally by ions and electrons ${ }^{(12)}$. When the ions impinge on the cathode, they dislodge atoms of the cathode metal (this process is called sputtering), some of which are ionized near the anode and then serve as charge carriers. Therefore, regardless of what kind of atoms were initially present in the are, it eventually becomes loaded un with ntems of cathode material.

The number of atoms spu'tered per ion impact is roughly proportional to the translational energy of the impactirig ion. An estimate can be made from the total resistive voltage across the crc. This has an overage : alue of about 200 v by actual measurement. Current carrying copper ions impinging on the copper cathode with the corresponding energy of 200 ev would sputter roughly 0.9 copper atoms/ion ( 61 atomic mass units; see Table 1).


Figure 2. Distance vs Time For the Projectile Theoreticai and Experimental

## 2. Ohmic Skin Heating

During the transit of the arc down the rail, curent and magnetic field diffuse into the rail due to the finite conductivity oi the rail material. As we will show, the average energy density deposited by ohmic heating in the region of the arc is approximately the magnetic energy density in the gap. If this energy density is greater than the heat content of the rail material from its initial temperature, say, room temperature to just above its melting point, then rail material will be melted in that region ${ }^{(4)}$. Since there is a for ward component of the Lorentz force on the rails near the arc, this molten material will be carried forward and will. therefore, contribute to the mass of the arc-projectile system. (Although heating continues in the rails behind the arc, the Lorentz force is outward, and the molten material there will only be pressed against the rails, not carried forward)

The heating in the region of the arc is determined as follows. The average power per unit volume is $j^{2 / T}$ where $;$ is the average current density and 5 the coriductivity. The volume under consideration has the thickness of the rails, $g$, and an average depth $\sqrt{2+/ \mu_{c} \sigma}$, the electrical skin depth corresponding to the length of time, $\mathcal{F}$, for the arc to travel its own length. The current density is, therefore $(I / g) \sqrt{\mu_{0} J / e t}$, where $I$ is the total current. The average power is $\mu_{2} I^{2} / 2 g^{2} t$, which is roughly the magrietic energy density in the gap, as stated.
For

$$
\begin{aligned}
& I=200 \mathrm{kamp} \text { and } \\
& \eta=1.5 \mathrm{~mm} \\
& \frac{\mu_{0} I^{2}}{2 g^{2}} \sim 10 \mathrm{k} \text { joule } / \mathrm{cm}^{3}
\end{aligned}
$$

The averages above are very loosely defined, and the consequent results are only good to an order of magnitude. The actual heating will depend upon the details of the current distribution in the rails. Even so, one can see from Table 1, that this is the right order of magnitude to melt the current carrying part of the rail.

From flux loop data, we have a typical value for $t$ of $10 \mu \mathrm{sec}$. The corresponding electrical skin depth is 0.7 mm , and the thermal depth is 0.03 mm . It would, therefore, be impossible for the heat generated to dissipate by conduction during the passage of the arc. The appearance of the rails after the shot bears this cut. In fact, for a steel rail with 8 mil copper cladding, the entire copper face was me! ted in the region of highest current. The total amount of copper melted over a 6 cm length has a mass of from 0.1 to 1 gm . Ten or so milligrams of this carried forward against the projectile would account for the mass-input effect.

For a given energy density the amount of mare ial melted will increase in direct proportion to the skin depth and hence the sruare root of the resistivity in the rail material. In addition to the decrease in inductance, this may have contributed to the poorer performance of steel compared to copper even though the steel rails showed less deformation.

## C. MECHANICAL EFFECTS <br> 1. Lorentz Force On The Rails

In the region behind the arc, the Lorentz torce density is outward from the gaf in the plane of the rails. The equivalent pressure at the gap surface of the rails is just the magnetic energy density in the gap For the case described in the last section, this is a pressure of about $10^{6} \mathrm{psi}$, enough to cause plastic flow in the solid rail material.

In addition to plastic flow at the gap su-face, the Lorentz force causes gross motion of the rails in the lateral direction. Since the rails are restrained in this direction by bolts or steel dowel pins, this motion couses plastic flow of the rail around the bolts.

The appearance of the rails atter the shot shows that considerable plastic flow and gross motion of the rails do occur. Depending upon the rail material, the peak current, and upon the inssiator, which serves to prevent relief perpendicular to the plane of the rails, the gap may be enlarged by a factor of 2 or 3. The effect of this spreading is to lower the Lorentz force on the arc. This can be seer, either as ~ decrease in inductance or, equivalently, a drop in nagnetic pressure due to expansion.

## 2. The Pressure And Length Of The Arc

The are is contained at the front by the inertial forces of the projectile and of its own mass, from the sides by the rails and insulator, and from behind by what may be thought of as a magnetic pressure, (typically $10^{6} \mathrm{psi}$ ). In the steady state, the ordinary kinetic pressure in the are will just balance this magnetic pressure. For temperatures of 10 to 100 ev this corresponds to the following:

$$
\begin{aligned}
& 10^{22} \text { to } 10^{21} \text { atoms } / \mathrm{cm}^{3} \text { particle density } \\
& 1 \text { to } .1 \mathrm{gm} / \mathrm{cm}^{3} \text { mass density }
\end{aligned}
$$

Given the mass of the arc and the cross section of the barrel, the arc length is completaly specified by this density. Flux loop measurements
give this length as very roughly 1 cm . The correspondirg mass range is 2 to 20 mg , consistent with the ohserved mass-input limitation.

The mechanical effect of the arc pressure is to stress the insulator over the acc. This stress is followed by another due to the plastic deformation of the rails, as described above. One-half inch thick cloth-phenolic ins.letors have been broken into two pieces along the barrel by this shock. Melamine-glass cioth laminate blocks, which we nc w use, show some separation of the laminations but seem to be more than strong enough to withstand the pressures in the present current regime. This sort of failure heips to lower the Lorentz force on the arc by allowing the rails to spread.

## D. SPLITTING OF THE ARC

In the ideal operation of the rail gun, as we first imagined it, the projectile would fit :ightly in the barrel, the arc would be confined behind the projectile, and, therefore, the projectile would have to move at least as fast as the center of mass of the whale arc-projectile system. By increasing the current, the velocity of the center of mass and, consequently, of the projectile would have to increase.

Early in the last contract, it became apparent that even as little as a $3 \%$ looseness of fit could reduce the velocity by one-half. All projectiles were thereafter made to fit tightly, with the result of velocities as high as $6.0 \mathrm{~km} / \mathrm{sec}$. The position of the arc versus time as determined from magnetic flux loop data showed that the are was remaining behind the projectile.

On the basis of these results, an impediance maiching transformer was built to increase the current and, we expected, the veiocity. Instead, what happened was that, after a point, an increase in current led to a decrease in velocity (See Figures 3 and 4).

An improved magnetic flux loop technique (See Appendix A) has reveuled a phenomena which may explain inis. Figures 5 through 20 are plots of the position of the are versus time, the flux loop data from which they were taken, and the corresponding projectile-velocity data. They show, in every case except experiment 2.10-1 (Figures 13 and 14), that two arcs were present, ane moving faster than the projectile and the ather, beinind it, moving at the same speed as the projectile or slightly slower. The velocity dato is consistent with the interpretation that the projectile is somewhot in front ai ine slower are.

In all but experiment 3-1 (Figures 19 and 20), the projectile was initially some few centimeter, in front of the aluminum foil used to start the arc. This was done


Figure 3. Projectile Velocity vs
Peak Current


Figure 4. Projectile Velocity vs Peak Current
$1.27-2$

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Figure 6. Magnetic Flux Data

Position of the Arc Front versus Time

Figure 7.



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$$






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Figure 8. Magnetic Flux and Projectile Velocity Data

Figure 9. Position of the Arc Front versus Time
(As determined from the following magnetic flux data)
Figure 9. Position of the Arc Front versus Time









$$
\therefore \ldots \neq m \quad \therefore \quad \because
$$



Figure 10. Magnetic Flux and Projectile Velocity Data

(w) LNOEX TOH HHL to Mollised


Figure 12. Magnetic Flux and Projectile Velocity Data
$2.10-1$

TIME (MSEC)
Figure 13. Position of the Arc Front versus Tim.
(As determined from the following magnetic flux dara)



Figure 14. Magnetic Flux and Projectile Velocity Data









Figure 15. Magnetic Flux and Projectile Velocity Data

Figure 17. Position of the Arc Front versus Time
(As determined from the following magnetic flux data)


$$
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$$

$$
\cdots
$$

$$
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\cdots & -
\end{array}
$$

$$
\begin{array}{ccc}
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i \\
i & \vdots & \cdots \\
\hline & \\
\hline
\end{array}
$$

$=\operatorname{Hecc}=\mathrm{Sa}^{-}$


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The rem

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Figure 18. Magnetic flux Data

3,1-1


Figure 19.
(As determined from the following magnetic flux data)

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$5.1-1$

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$$


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$\because 1-1$


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Figure 20．Magnetic Flux and Projectile Velocity Data
so that we could observe clearly any change in behavior when the are reached the projectile.

The observed result is that, at first, a well defined front, behind which current flowed more or less uniformily over several centimeters behind the front, moved forward with c velocity of 10 or $12 \mathrm{~km} / \mathrm{sec}$ just as in experiments withou, a projectile. When this frort reached the position of the projectile, a second arc formed in front of the projectile. The velocity of the first are drcpped to zero ond gradualiy increased to a velocity of the order of $4 \mathrm{~km} / \mathrm{sec}$. At the same time the second arc accelerated to a higher velocity, and the current in it increased from a very small fraction of the total to roughly half or more by the time it reached the muzzle. Furthermore, instead of a uniform current distribution, both arcs became rather well defined filaments of less than a centimeter in extent. The separation of the two arcs grew from less than 1 cm to between 3 and 4.5 cm . This phenomenon is especially clear in Figures 5 through 7.

The effect of this second arc is to decrease the Lorentz force on the first and hence on the projectile. The magnitude of the effect can be estimated as follows. The total force on both arcs is the same as on a single arc, namely, $\frac{1}{2} L^{\prime} I^{2}$. If the forward are is far enough from the other, the force on it is just $\frac{1}{2} L^{\prime} I_{1}^{*}$ where $I$, is the current through the forward arc. The force on the rear arc is, therefore, $\frac{1}{2} L^{\prime}\left(I^{2}-I_{1}^{2}\right)$ and the fraction of the force lost to the rear arc just $\left(I_{1} / I_{2}\right)^{2}$. For $I_{1}=\frac{1}{2} I_{2}$ as observed, the fraction of the force lost is one-fourth.

An explanation for the inverse dependence of velocity on current . terms of this phenomenon is that before a certain critical current is reached no s . ad arc forms and the projectile is accelerated according to the considerations of Section II. B. After this critical point, a second arc grows, and the net effect of increasing the current is to increase the predominance of the first arc over the second at the expense of velocity. Experiments $2.10-1$ at $250 \mathrm{k} \mathrm{amp}, 2.19-4$ at 310 k amp , and 2.12-2 at 390 k amp (See Figures 13-18) are consistent with this. In the experiment at 250 k amp, only one arc appears, at the high currents two.

Two kinds of explanation have been offered for the appearance of the second arc. In one, the initial conductive path in front of the projectile is provided by the ionized shock front due to the motion of the projectile or by the front surface of the projectile itself, in the other, by plasma leaking around the projectile. The evidence is ambiguous but favors the latter explanation.

In the series of experiments iepresented in Figure 2! various amounts of aluminum foil were used to initiate the arc, and an optimum of 7 mg was found for a given gun at a given peak current. The time required for current to diffuse from the back to front of a wad of this mass due to the finite conductivity of aluminum is of the order of 40 msec . One can imagine that until current diffuses through the


Figure 21. Projectile Velocity versus the Amount of Aluninum Used to Initiate the Are
wad, it acts as an impermeable sabot against the arc plasma. However, beyond the optimum mass of aluminum, nothing further is gained in sealing the barrel, and any additional mass decreases the velocity by absorbing momentum.

Experiments 1.27-2 and 1.29-1 (Figures 5-8) were done with the gun of Appendix D without epoxy sealant. This left a small channel at each corner of the projectile, through which plasma could leak. Experiment 2.10-1 (Figures 13 and 14) was with the same gun except with the epoxy and, so, with the projectile completely sealing the barrel. In 1.27-2 and 1.29-1, we see two arcs, in 2.10-1, only one. Also, the projectile velocity is about $10 \%$ higher with the sealed barrel.

These two groups of experiments tend to support the plasma leakage explanation. The following experiments are less clear.

Experiments 1.29-1, 2.4-2, and 2.4-1 (Figures 7-12) were done at pressures of 760,38 and .14 mm Hg , respectively. In all of them, two arcs appeared. The single arc before reaching the projectile and the forward arc afterwards had progressively higher velocity with decreasing pressure, in agreement with a series of experiments under the previous contract. The projectile velocities had the opposite dependence on pressure.

If the ionized shock front were providing the initial conductive path before the projectile, one would expect that the number of ions and hence the conductivity would go down with pressure, that the formation of the second arc would be inhibited, and that the projectile velocity would increase. On the othe hand, it is hard to explain the decrease in velocity by the plasma leakage. Possibly the air before the projectile reduces the rate of plasma leakage.

In experiment 3.1-1 (Figures 19 and 20) the foil wad was placed directly behind the projectile as is usually done. In 2.19-4 (Figures 15 and 16), there was a space of 3 cm between projectile and foil. Neither mechanism seems to explair: the differer.ze in projectile velocity.

## E. VOLTAGE ACROSS THE MUZZLE AND BREE:CH

The voltage differences shown in Figure 22 are related in the following way:

$$
\begin{equation*}
V_{b}=V_{m}+\frac{d}{d t}(L I) \tag{9}
\end{equation*}
$$



Figure 22. Voltage Measurements Across The Rails
where $I$ is the total current through the arc and $L$ is the inductance included between the breech and the arc. By decomposing $\frac{\&}{Q t}(L I)$ we have:

$$
\begin{equation*}
V_{b}=V_{m}^{\prime}+L^{\prime} \dot{x}+\angle I \tag{10}
\end{equation*}
$$

where $L^{\prime}$ is the inductance per unit length of the rails. (Vra, which is the valtage across the arc, may be measured anywhere across the rails in front of the aic.)

The following actual measurement of $I, \angle^{\prime}, V_{6}, V_{m}$, and $X$ taken ot peak current $(\dot{I}=0)$ were consistent with Equation 10:

$$
\begin{aligned}
& I=290 \mathrm{kamp} \\
& L^{\prime}=.28 \mu \mathrm{~h} / \mathrm{m} \\
& \dot{X}=3.2 \mathrm{~km} / \mathrm{sec} \\
& V_{L}=700 \mathrm{v} \\
& V_{m}=440 \mathrm{v}
\end{aligned}
$$

The total electric power into the gun is $V_{\swarrow} I$. Of this, $V_{\infty} I$ is irretrievably dissipated in the arc, $\angle I I$ goes into increasing the magnetic field in the region between the breech and the orc, $\frac{1}{2} L^{\prime} I^{*} \dot{x}$ into creating field in the region being uncuvered by the motion of the arc, and $\frac{1}{2}\left\langle^{\prime} I^{*} \dot{x}^{-}\right.$into the kinetic energy of the arc-projectile system. The fraction $V_{m} / V_{\infty}$ is, therefore, a minimum measure of the energy inefficiency of the gun.

In experiments with no projectile, $V_{m} / V_{b}$ appeared to be greater than $90 \%$ indicating a very poor energy efficiency of the gun. When the experiment was repeated with a projectile (a $1 / 8$ inch nylon cube), $V_{m} / V_{0}$ was found to be roughly $60 \%$ during almost the entire acceleration. Since the magnetic field energy is always at least as great ns the projectile kinetic energy, the net energy efficiency must be less than half the complement of $60 \%$, i.e., less than $20 \%$. The gross energy efficiency computed from the projectile mass and velocity and the condenser bank capocity and voltoge wes $2.0 \%$ - less than the $20 \%$ upper limit on the gun's efficiency, as it must be.

## F. ADDITIONAL EXPERIMENTS

1. Measurement of the Mass Removed from the Rails During a S'hot

The appearance of the rails shows that several mils of metal are removed from the face of the rails forming the barrel during the shot. Some of this is found redeposited as a very fine film on the side faces of the rails and the adjacent insulatol as well as upon all exposed surfaces in front of the gun. The theory of Section II.B. requires that the total amount be of the order of 20 milligrams. We have four estimates of the mass which are consistent with this: the apparent amount removed from the rail, the amount deposited on the insulator, the amount deposited on a ballistic pendulum, and the momentum delivered to the pendulum together with the known velocity of the arc.

The film deposited on the insulators appears as streaks running away from the barrel. In the region near where the arc first strikes, the streaks are nearly straight and perpendicular to the barrel. Further down the barrel, they slant more and more towards the muzzle, eventually making angles of say, $15^{\circ}$, and curve away from the barrel. This suggests that the metal vapor leaves the barrel at first with only lateral momentum and, further down the barrel, with more and more forward mumentum, so that in the region of high arc velocity the forward components becomes of the order of four times greater than the lateral.
2. Tests with Various Rail MetalsExperiments with rails of various metals gove the following results:
Rail Metal
Brass
Projestile Velocity (km/sec)
Soft aluminum ..... 2.12.0
Mild steel ..... 3.1
Untempered tooi steel ..... 3.2
Cupper ..... 3.7
Aluminum 7075 ..... 3.7
Magnesium
$1 / 6^{\prime \prime}$ copper strip silver soldered to mild steel ..... 4.7
$.008 "$ cold rolled annealed copper sheath on untempered tool steel ..... 5.2
$\frac{\text { Arc Velocity }}{13}$
Copper
2\% thoriaterd tungsten rod silver solderedto untempered tool steel10

These experiments, especially me last two, seem to show that sputtering yield and heat content are not so important as electrical conductivity and strength (See Table 1). The dependence on electrical conductivity is in agreement with the conclusions of Se.tion II.A.

## 3. Tests with Various Insulating Linings

Experiments with guns of identical construction except for the thin insulating liner next to the rails ware done ut a relatively low current to minimize mechanical effecis. The esults were as follows:

> Insulator

Glass
Melamine -fiberglass !aminate
Epoxy-fiberglass laminate

Projectile Velocity ( $\mathrm{km} / \mathrm{sec}$ )
3.8
3.8
2.0

The glass and melamine showed much less erosion compared with the epoxy.
4. Auxiliary Field Guns.

Two kinds of auxiliary-field guns have been used by us. In one kind, the auxiliary-field turns are in series with the rails. The net effect of this arrangement is to increase the effective inductance per unit length ${ }^{(4)}$. Experiments using a series auxiliary-field gave an arc-frent velocity of $4 \mathrm{~km} / \mathrm{sec}$. The same gun using no cuxiliary-field gave $2.3 \mathrm{~km} / \mathrm{sec}$.

A projectile gun based on this principle was built (See Figure 23) and geve an immeasurobly low velocity. On the corijecture that this was due to increased ohmic skin heating (See Section l., B. 2), a gun with irdependerit auxiliary-field rurns (See Figure 24) 3nd a separate condenser-bank-transformer system was built (See Appendices C, D and E). The resulis were that, with the cuxi.'iary-field, the gun gave $4.3 \mathrm{~km} / \mathrm{sec}$ and, without, $4.5 \mathrm{~km} / \mathrm{sec}$. This is understandable under the consicerations of Section II.C.2, in which, after a critical current, or, in the case, a critical magnetic prassure nas been reacined, the velocity decreases.

Two experiments we performed in which the ausiliary-field was reversed to oppose the fiel. siue 'ר the rails. The result was, in both cases, very low velccities, lest than $0.4 \mathrm{~km} / \mathrm{sec}$. Both projectiles were recovered hardly damaged. The impo-tant conclusion to be drawn from this is that the exploding foil eifect is relarively unimportant in accelerating the projectile compared to the Lorentz force.


Figure 23. Exploded View of Series Type Auxiliary Field Gun


Figure 24. Exploded Viev ot the Rail-Gun with Seprarare Auxiliary-field Turns


## III. THE DRAG APPROACH

The drag approach assumes that the diameter of the projectile is smaller than the bore of the rail gun. The projectile is accelerated by the drag force of the plasma. This approach does not hare the velocity limitations found in other approaches, although it is inefficient from an energy standpoint.

## A. MODEL OF THE ARC IN A RAIL GUN AT VERY HIGH CURRENTS

Most of the reported work on plasma rail guns uses relatively low density parma (<<1 milligram/cc). For the purpose of accelerating a projectile; a high density ( > 1 milligram/cc), high velocity plasma is desirable. This can be achieved with small rail spacings and high current densities.

The theoretical results discussed in Section II. B. gave the arc velocity as proportional to the peak current in the gun. This result was based upon the assumption that the skin friction was negligible. However, it has been found that as the peak current is increased a point is reached where the arc velocity no longer increases as rapidly. Including the skin friction in the equation for the arc velocity gives an equation which more nearly explains the experimental data. The equation for the arc in a rail gun becomes

$$
\frac{d}{d t}(m v)+\frac{2 C_{f}}{D}\left(m v^{2}\right)=F
$$

where
$m \quad$ is the mass of the arc
D is the diameter of the channel
$C_{f}$ is the skin friction coefficient
$F \quad$ is the force on the arc $=1 / 2 L^{\prime} 1^{2}$ where $L^{\prime}$ is the rate of change of inductance with unit length

We will assume the velocity is essentially constant particularly in the region of peak current, i.e., where


$$
\begin{aligned}
& \frac{2 c_{f} m}{D} v^{2}+\dot{m} v-F=0 \\
& \dot{m}=m^{\prime} I \quad m=m^{\prime} Q \\
& \frac{2 c_{f} m^{\prime} Q}{D} V^{2}+m^{\prime} I v-\frac{1}{2} L^{\prime} I^{2}=0 \\
& \text { Let } \frac{D I}{2 C_{f} Q}=\alpha \text { and } \frac{1}{4} \frac{L^{\prime} D I^{2}}{m^{\prime} C_{f} Q}=\beta \\
& v=\frac{1}{2}\left(-\alpha \pm \sqrt{\alpha^{2}+4 \beta}\right. \\
& =\frac{1}{2} \alpha\left(-1 \pm \sqrt{1+4 \beta / \alpha^{2}}\right) \\
& =\frac{D I}{4 C_{f} Q}\left(-1 \pm \sqrt{1+\frac{4 \angle Q C}{m^{\prime} D}}\right.
\end{aligned}
$$

For $t=\frac{\pi}{2}$

$$
\begin{aligned}
& t=\frac{\pi}{2} \\
& Q=I_{p} \int_{0}^{\pi / 2} \sin \omega t d t=\frac{I_{p}}{\omega} \\
& V=\frac{D \omega}{4 C_{f}}\left[\sqrt{1+\frac{4 C^{\prime} I_{p} C_{f}}{m^{\prime D \omega}}}-1\right]
\end{aligned}
$$

so

Assuming the following

$$
\begin{aligned}
& L^{\prime}=0.3 \mu \mathrm{~m} / \mathrm{m} \\
& I_{\omega}=5 \times 10^{5} \mathrm{amps} \times \frac{20}{\pi} \mu^{\mathrm{sec}}=3.2 \text { coolombs } \\
& m^{\prime}=6 \text { atoms } / e \times \frac{e}{1.6 \times 10^{-19} \text { coul }} \times \frac{0,0635 \mathrm{Kg} \text { imote }}{6 \times 10^{23} \mathrm{atom} / \mathrm{m}} \\
& =4 \times 10^{-6} \mathrm{rg} / \text { coul. } \\
& c_{f}=0.0044 \text { ie } c_{f}=\frac{0,074}{R_{e}^{1 / 5}} \text { and } R_{e}=10^{7}
\end{aligned}
$$

Then

$$
\begin{aligned}
& \frac{4 L^{\prime} \frac{I p}{\omega} C_{f}}{m^{\prime} D}=\frac{4 \times 0.3 / \frac{\mu \mathrm{m}}{4 \mathrm{~kg} / \mathrm{coul} \times 3.2 \mathrm{cou} / \times 0.0044} \times 0.0063}{} \\
& =0.75 \\
& \frac{D \omega}{4 C_{f}}=\frac{0.0063}{4 \times 0.0044} \times \frac{\pi}{20}=56 \mathrm{~km} / \mathrm{sec} \\
& V
\end{aligned}
$$

For $1 / 8$ inch channel the result is $16 \mathrm{~km} / \mathrm{sec}$ and for the $1 / 16$ inch channel the velocity is $14 \mathrm{~km} / \mathrm{sec}$. The velocity would increase approximately as the square root of the peak current for large currents. The arc velocity according to this equation increases inversely as the square root of the rise i me, thus short rise times are desirable, from the standpoint of arc velocity although the density of the plasma is lower under these conditions.
B. UNIFORM DENSITY - UNIFORM VELOCITY PLASMA STREAM

The equation for the drag on a projectile is given by

$$
\frac{d V}{d t}=\frac{C_{D} p A}{2 g m}\left(V_{P 1}-V\right)^{2}
$$

where
$C_{D}=$ the drag coefficient $\sim 0.9$ for a sphere at Mach 7
$A=$ density of the stream of gas or plasma
$A=$ the projected area of the projectile
$V_{\mathrm{pl}}=$ velocity of the plasma
$V=$ velocity of the projectile

The solution of the above equation for a uniform density uniform velocity plasma stream is

$$
V=v_{0}\left[1-\exp \left(-\frac{x}{\ell}-\frac{V}{V_{0}-v}\right)\right]
$$

where x is the distance the projectile moves in the gun and


This curve is plotted as Figure 25.


Figure 25. Fellet Velocity as a Function of Relaxation Lengths in the Gun

A typical example would be for a 10 mil diameter nylon projectile with a average plasma density of $2 \mathrm{mg} / \mathrm{cm}^{3}$

$$
\begin{aligned}
\ell & =\frac{B}{3}-\frac{P_{B} \sqrt{2}}{P_{P /} C_{D}} \\
& =\frac{8}{3 \times 0.9} \times \frac{1.1 \times 1 / 2 \times 10^{-2} \text { inches }}{2 \times 10^{-3}}=8.1 \text { inches }
\end{aligned}
$$

With a $1 / 4$ inch square cross section channel at $20 \mathrm{~km} /$ /sec, the total mass ejected in $30 \mu \mathrm{sec}$ is

$$
\begin{aligned}
& =\quad 2 \times i \mathrm{mg} / \mathrm{cc} \times(2.54 / 4)^{2} \times 20,000 \times 30,4 \mathrm{sec} \\
& =\quad 0.5 \mathrm{milligrams}
\end{aligned}
$$

The actual quantity of material ejected appears to be about 50 times this. The initial density of the first plasma to emerge from the gun is probably about the above value. The density would be expected to increase and velocity decrease with tim ?. By using a nozzle on the end of the gun the pellet would see more nearly constant density and constant velocity along its path in the gun. This is because as the pellet moves into the nozzle section, a later more dense part of the plasma column would have reached the nozzle and would expand and increase in velocity.

## C. DISPERSION MEASUREMENTS

Several of the two stage experiments resulted in no projectile emerging from the second stage so measurements of the dispersion of the projectile on the target were made under a number of different conditions. This data is shown as Table 2. Use of the dispersion concept assumes the distribution about the point of aim is a circular normal distribution. The data obtained is insufficient to give a accurate measurement of dispersion but it does indicate that the dispersion is about 150 milliradians. It would appear that the smaller the initial loading of foil the lower the dispersion.

Dispersion is probably not a highly accurate way of looking at the probability of a projectile emerging from a long channel without striking the walls. However, it should give a first approximation to the results if it is recognized that the value used for the dispersion is somewhat sensitive to the length of the channel, the velocity and density of the plasma, etc. The probability of a projectile emerging from the channel becomes

$$
P=1-\exp -\left[\frac{N}{2 \sigma^{2}}\left(\frac{\Omega}{L}\right)^{2}\right]
$$

where
$\sigma$ is the dispersion
$\Omega \quad$ is the radius of the channel
$L \quad$ is the length of the channel
$N$ is the number of projectiles
For a $90 \%$ probability

$$
\frac{N}{2 \sigma^{2}}\left(\frac{\Omega}{L}\right)^{2}=2.3
$$

For $1 / 8$ inch radius channel and 150 milliradians dispersion

$$
\begin{aligned}
N & =2.3 \times 2 \times 0.15^{2} \times 4\left(\frac{1}{\mathrm{D}}\right)^{2} \\
& =0.41\left(\frac{\mathrm{~L}}{\mathrm{D}}\right)^{2}
\end{aligned}
$$

Thus for a L/D of 20 (1/4 inch diameter channel 5 inches long) about 160 projectiles would be required for a $90 \%$ probability of one emerging.

To use a single pe!'et, a taper in the channel of 150 milliradians ( $\sim 8^{\circ}$ ) would be required to give a reasonable probability of one pellet emerging.

## D. ABLATION OF THE PROJECTILE

The ablation problem is very similar to the reentry heating problem. The technique used for calculating the thickness ablated is to estimate the heating rate and make a heat balance, or

$$
q \Delta t=h_{A} p_{m} x
$$

where
$t=$ length of time the projectile is in the stream
$x=$ thickness ablated
$h_{A}=$ effective heat of ablation
$q=$ heating rate
$P_{m}=$ material density

One of the equations commonly used for convective heat flux at the stagnation point is

$$
70=865\left(\frac{V}{10^{4}}\right)^{3.15} \sqrt{\frac{\rho_{a}}{p_{a}}} \sqrt{\frac{1}{\Omega}}
$$

(See Reference (13))
where

$$
\begin{array}{ll}
v & =\text { velocity } f i / s e c \\
\rho & =\text { density } \\
\rho_{a} & =\text { atmospheric density } \\
\Omega & =\text { radius, } \mathrm{ft}
\end{array}
$$

Assuming a 5 mil radius particle, atmospheric density, and $10 \mathrm{~km} / \mathrm{sec}$ relative velocity between the stream and particle, $\mathrm{q}_{\mathrm{o}}=1.8 \times 10^{6} \mathrm{Btu} / \mathrm{ft} 2 \mathrm{sec}$. The heat transfer along the side of the body is given from

$$
\text { Stanton }=\frac{q}{p \vee h}=\frac{1}{p_{n}^{2 / 3}} \frac{C_{f}}{2}
$$

where

$$
\begin{aligned}
& P=\text { density } \\
& h=\frac{x^{2}}{2 g} \\
& C_{f} \cong 0.003 \\
& f_{\mathrm{r}}^{2 / 3} \cong 1
\end{aligned}
$$

or

$$
9=3700\left(\frac{v^{3}}{10^{4}}\right)
$$

which is about $1 / 10$ the above result. There is almost no heat transfer to the rear surface of tine pellet. The pellet will tumble in the stream, so all surfaces would be ablated similarly, although corners would ablate first. On the basis of the above
Table 2 DISPERSION MEASUREMENTS Distance Measured From Muzzle



| $E$ | $E$ | $E E E E E$ | $E$ |
| :--- | :--- | :--- | :--- |
| $U$ | $E$ | $E$ |  |
| $N$ | $N$ | $N$ | $N$ |
| $N$ | $N$ | $n$ | $N$ |





arg:ments, the average heat ng will be assumed to be $20 \%$ of the stagnation point heating rate.

T'ae effect of the high tempe, ature of the plasma stream can be neglected. in the calcularion of correcfive heat transfer to the projectile since the kinetic energy of the stream is tigh compared to the thermal energy fossuming the thermal temperature is 5 ev$)_{i}-i . e .$,

$$
\begin{aligned}
& \frac{v^{2}}{2 \mathrm{~g}}=\frac{\left(10 \times 3.3 \times 10^{3}\right)^{2}}{2 \times 32.2 \times 778}=21,000 \mathrm{Btu} / \mathrm{t} \\
& E_{\mathrm{fh}}=\frac{5 \mathrm{ev} \times 1.6 \times 10^{-12} \times 6.02 \times 10^{23} \times 453.6 \mathrm{~g} / 1 \mathrm{~b}}{63 \mathrm{~g} / \text { mole } \times 10^{7} \times 1054 \text { joules } / \mathrm{Btu}}=3,200 \mathrm{Btu} / \mathrm{H}
\end{aligned}
$$

The effective heat of ablation is dependent upon the enthalpy of the stream. For these heat rates an estimate of $5000 \mathrm{Btu} / \mathrm{lb}$ has been used.

The ablation is $f$ ind from the heat balance

$$
\begin{aligned}
& x= \frac{g \Delta t}{h \rho}= \\
& \\
&=\quad 0.2 \times 1.8 \times 10^{6} \times 20 \times 10^{-6} \times 12 \text { inches } / \mathrm{ft} \\
& 5000 \mathrm{Btu} / \mathrm{lb} \times 1.2 \times 62.4 \mathrm{lb} / \mathrm{ft}^{3}
\end{aligned}
$$

The stagnation point radiative heat transfer, if the plasma is assumed to be similar to air, is negligible - just as it is in the earth reentry case. This is because of the low emissivities. The equation shown below assumes atmospheric density

$$
\begin{aligned}
\frac{\text { grad }}{R} & =100\left(\frac{v}{10^{4}}\right)^{8.5} \\
& =100\left(\frac{3.3 \times 10^{4}}{10^{4}}\right)^{8.5} \\
& =100 \times 21,000 \times 0.0004 \mathrm{ft} \\
& =840 \mathrm{Btu} / 8 \mathrm{t}^{2} \mathrm{sec}
\end{aligned}
$$

From these calculations the ablation loss would appear to be negligible for 10 mil diameter projectiles with an average relative velocity of $10 \mathrm{~km} / \mathrm{sec}$. With densities greater than atmospheric, and with relative velocites of $20 \mathrm{~km} / \mathrm{sec}$, an ablation loss of as much a 1 mil could be expected.

## E. $\therefore$ EXPERIMENTAL RESULTS

Table 3 shows the experimental results obtained using the experimental configtration stown in Figure. 26. The tirne of flight is measured by the time at which an 8 riil oluminum sheet is perforated by the pellet. It is necessary to correct the time of flight by the length of time it takes the arc te reach the pillet and for the fact that the pellet does not immediately reach full velocity. Two velocities are quoted for each of the experiments. One velocity is coleclated on the following basis

$$
V_{\mathrm{av}}=\frac{\text { Distance from pellet to impact plate }}{\text { time of impact }-9 \mathrm{issec}}
$$

The nine microsecond correction allows fo the time for the arc to reach the pellet. Thus this velocity is in reality the average velocity of the pellet over its total path length. This should be corrected for the time required to accelerate the pellet to velocity. The second velocity shown corrects for the time to accelerate to full velocity by assuming that the average velocity over the gun length is $2 / 3$ of the final velocity. Thus

$$
\begin{aligned}
V & =\frac{\text { Distance from pellet to impact plate }}{\text { time of impact }-9 \mu \mathrm{sec}-\left[\frac{l}{\frac{2}{3} v}-\frac{l}{V}\right]} \text { where } l=\text { gun length } \\
& =\frac{\text { Distance from pellet to : :mpact plate }+0.5 l}{t-9}
\end{aligned}
$$

The relatively small increase in velocity which resulted when the peak current was increased is expected because the rise time also increased and the plasma velocity should be proportional to $\sqrt{\frac{I_{p}}{t_{a}}}$. The increase that did result is due to the increased plasma density.

These measurements were made with nyion particles which varied between 0.004 and 0.010 inch in diameter. Inspection under a optical comparator showed few particles outside this size range. The gun which was used is shown as Figure 27

## F. IMPROVED DESIGN

Figure 28 and Table 4 show the characteristics of an improved design which should give an improved velocity. The ratio of $\sqrt{\frac{I P}{\hbar \eta}}$ has been increased by $40 \%$. In addition the increase in IP should double the density of the plasma. The increase in average density will incrase the value of $x / \ell$ (gun length/relaxation length) by a factor of 2. The combination of increased plasma density and velocity should give a velocity of $20 \mathrm{~km} / \mathrm{sec}$ for a 7 mil diameter cylinder and $15 \mathrm{~km} / \mathrm{sec}$ for a 15 mil . diameter cylinder.

Table 3

## RESULTS' WITH DRAG TEST



* Distances measured from riuzzzle end of rails
** Velocity computed as average over total flight path
*** Gun had 15 cm total nozzle length


Figure 26. Experimental Configl:ation for Drag Tests

Figure 27. Standard Gun Used for Drag Experiments


Figure 28. Improved Gun Design

## PROPOSED. DESIGN CHARACTERISTICS



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## APPENDIX A

## DIAGNOSTICS

## 1. Velocity Measurements

Figures $A-1$ and $A-2$ show the system used to measure the projectile velocity and to observe the motion of the arc. The puncturing of the two light fight boxes and the impact provide three independent measures of the velocity of the projectile. "Figure A-3 is a set of typical traces from this system. Figure A-4 shows the construction of the mylar window assembly through which the projectile passes.

In the drag techniques of acceleration the entire gut assembly is placed within the vocuum tank and one photomultiplier is used to record the time of penetration of the 8 mil aluminum sheet which covers the light tight bois. This wasshown previously on Figure 26.

## 2.: Magnetic Flux Loops

With the high currents achieved with the transformers built unde. this contract and the resulting high pressure along the barrel (See Séction II.C.1), we found that flux loops of the old design placed between the expendable liner and the supporting blocks (See Figure B-1) were destroyed during the shot and had tc be rebuilt for each shot. In order to avoid this problem, we designed and built the flux, loop assembly shown in Figures A-5 and A-6. Its position in the gun is shown in Figure B-1.

The chief viriwes of this device is that, except for the small regions occupied by the coils themselves, that part of the structure exposed to the pressure of the arc is of high impact melamine fiberglass laminate. Since the epoxy resin combines with the melamine, the structure has little tendency to delaminiate. Repeafed use of this device has shown only little more damage than solid pieces of melamine-fiberglass.

Figure A-7 shows a typical tries from one of these loops, the zams signal isitegrated, and the trace of rail current for the same shot. The iategrated trace represents total flux through the loop, and, since the arc is treveli g with nearly constant velocity over the duration of the pulse, the intec;ated trace represents approximately the current distribution in the arc. The long overshoot following the spike in the integrated trace is probably dise to a slight disorientation of the loup, which causes it to couple with the current in the rails. At a velocity of $5 \mathrm{~km} / \mathrm{sec}$ a current distribution like the one shown has a length of about one centimeter.


Figure A-2. Photograph of Diggnotics Sysfem


Figure A-3. Typical Velocity Dara

Figure A-4. Construction of Mylar Window

Figure A.ß. Flux Loop Sandwich Before Cementing


Figure A-6. Component Layers of the Flux Loop Sindwich


Figure A-7. Typical Flux Loop and Current Traces

## APPENDIX B

## DEMOUNTABLE GUN

Figures B-1 and B-2 show the construction details of the most recent and most successful demountable gun. This design enables one to use rail cores and insuiators of any arbitrary material available in slabs, and rail sheaths of any malleable material available in thin sheets.

The melamine-fiberglass laminate (Panelyie "146) was chosen for the supporting blocks and clamps because of its hardness, density, and high impact strength. It was also chosen for the expendable insulating lining becau- of its resistance to erosion by the arc (See Section II.F.3).

The epoxy resin used to cement the barrel assembly acturlly combines with the melamine as well as bonds with the rail sheath to give additional strength in the plane of the rails. By using a mandrel of the appropriate cross section, either a square or a circular barrel can be fabricated with the epoxy.

The reason for not using steel for the clamp is that, for times of the order of 20 usec, magnetic fields due to eddy currents in the clamp would terid to cancel the fields due to the rail current. This was also the reason for placing the large bolts of the clamp far from the rails.


Figure $B-1$. Exploded View of Demountable $\bar{k}_{r} \cdot$ : Gun


Figure B-2. Demountable Gun in Clamp

## APPENDIXC

## TRANSFORMERS

Two air-core transformers with low leakage inductance were built to bring the condenser bank voltage (up to 20 kv ) closer to the breech voltage of the gun or the voltage of the auxiliary field turns (about 1 kv ) and, thereby, increase the current. Figures $\mathrm{C}-1, \mathrm{C}-2$ and $\mathrm{C}-3$ and Table $\mathrm{C}-1$ give the performance and construction specifications of the transformers as well as the construction of the lecids used to connect the secondary side to various equipment.

## Table C-1

Pulse Transformer Specifications

C $6^{\prime \prime} \quad 3^{\prime \prime}$

Number of Primary Turns
Dimension A

B

D $30^{\prime \prime}$

E
Secondary Induction
Effective admittance*

Ringing period with typical load
4.6
10.6

26" 32"
$13-3 / 4^{\prime \prime}$ $12^{\prime \prime}$
$10^{\prime \prime}$
7"
$.13 \mu h$
$.087 \mu h$
26 mho
36 mho
$60 \mu \mathrm{sec} \quad 200^{\circ} \mu \mathrm{sec}$

* The ratio of paak secondary current to the peak voltage of the 142 fo condenser bank with a typical load on the secondary.
RETURN LEADS FOR PRIMARY WINDING-


## (3)

2 THEES AF 0.04


Figure C-2. Construction of Secondary Output Terminals of Puise Tiansformer:


Figure C-3. Brass Leads from Transformer into Vacuum System

## かPPENDIX こ

## AUXILIARY FIELD AND TWO-STAGE SYSTEMS

In order to perform experiments with auxiliary-field turns and with a two-stage gun the system shown in Figures D-1, D-2 ar.d D-3 was built. Figure D-4 shows the two-stage gun.

The new condenser bank consists of ten $\mu \mathrm{fd}, 20 \mathrm{kv}$ capacitors of the same type as used in the old bank (Sangamo, type EDC, Class B). (In addition to these, an eleventh capacitors were bought to replace a damaged one in the old bank.) These ten capacitors were strapped together in pairs and each pair placed on a separate, four wheel dolly. Each capacitor is connected to the spark gap switch by separate coaxial cables. (See Figure D-5). This bank was used with the old bank in experiments with auxiliary-field turns and with two-stage guns.

A remote contrul charging and automatic crowbar mechonism was attached to the new bank. The charging leads are connected to the same power supply as the old bank so that both banks may be charged in paralle:l, each being disconnected from the charging supply as it reaches the desired veltage. The banks are discharged through their respective spark gaps in a controlled time sequence provided by an Abtronix delay chassis. Figure $D-6$ shows the construction of the triggered spark gap.


Figure D-2. Two-Stage Gun System


Figure D-3. Brass Leads from Transformers to Secoid Stage of Two-Stage Gun or to Auxiliary Field Turns



Figure D-5. Construction of Witness Plate and Impact
Microphone


Figure D-6. Construrtion Details of Spark Gap

