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REPORT NO. SM-46346

THOR-AGENDA LONGITUDINAL

MODE COMPUTATIONS

10 DECEMBER 1964

MISSILE & SPACE SYSTEMS DIVISION
DOUGLAS AIRCRAFT COMPANY, INC.
SANTA MONICA, CALIFORNIA

FACILITY FORM 602

N65-26084 (ACCESSION NUMBER) ______ (THRU)

204 (PAGES) 1 (CODE)

Cr 63368 (NASA CR OR TMX OR AD NUMBER) 32 (CATEGORY)

DOUGLAS

ABSTRACT

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In conjunction with planned vibration testing at the Langley Research Center, the first three longitudinal frequencies and mode shapes of their Thor-Agena test vehicle are predicted in this study for several test configurations. A bar idealization of structure is employed and the iterative procedure of Myklestad is used to obtain numerical solutions. These calculations are made with a standard Douglas program (DE 11) commonly used for such analyses.

Author

DESCRIPTORS

Thor-Agena

Longitudinal Oscillations

Myklestad's Method

SM 46346

TABLE OF CONTENTS

NOMENCLATURE

X	Coordinate Axis
Y	Coordinate Axis
Z	Coordinate Axis
F_x, M_x	Coordinate Force and Moment
F_y, M_y	Coordinate Force and Moment
F_z, M_z	Coordinate Force and Moment
θ	Rotation Angle about X Axis
α	Rotation Angle about Y Axis
ϕ	Rotation Angle about Z Axis
EI	Bending Stiffness
A_sE	Shear Stiffness
GJ	Torsional Stiffness
$\bar{x}$, $\bar{y}$, $\bar{z}$	Beam Element Center of Gravity Coordinates with respect to Element Coordinates x_i, y_i, z_i
ω	A Circular Frequency
$\{S\}$	State Vector at a Section
$[A]$	Transfer Matrix
*	Indicates Matrix Transpose in Section 1.0
Root	Beam Boundary at Largest Value of X
Tip	Beam Boundary at Smallest Value of X

Other symbols are defined as they are introduced in the text.

SM 46346

INTRODUCTION

The Langley Research Center (LRC) is currently investigating certain aspects of the longitudinal oscillations associated with several liquid propellant booster combinations. In connection with this investigation LRC plans a series of vibration tests on the Thor, an Agena and a combined Thor-Agena test vehicle. One parameter of considerable interest in these tests will be the natural frequencies of the test vehicle (s) during longitudinal oscillation. A commonly used analytical method for predicting these frequencies idealizes the structure as a bar such that the one-dimensional wave equation applies for elements of the bar. Even with this idealization, numerical analysis techniques are required to obtain solutions since the structural properties are neither uniform nor linear in most cases. The purpose of this report is to provide natural frequencies for the LRC test vehicle (s) resulting from such an idealization. In addition to the natural frequencies, the associated normal modes of oscillation (mode shapes) are also provided. The numerical solutions are obtained using a standard production program (DB 11) developed by Douglas for use with the IBM 7094 digital computer. This is an existing Douglas program, widely used in the Missiles and Space Systems Division. The first portion of this report is a monograph describing the program. The monograph begins the mathematical model with the transfer matrix between elements of the bar. The rows of this matrix correspond to writing the one dimensional wave equation as two first order equations in their finite difference form.

The structural properties of the LRC test vehicle (s) were compiled as a part of this study for use in the computation of natural frequencies and mode shapes. A number of weight and stiffness configurations simulating various flight times were analyzed using these data and the bar idealization previously mentioned. This work was done under contract to LRC and is restricted to those cases and the idealization desired by the Langley Research Center.

1.0 PROGRAM MONOGRAPH

1.1 Introduction

The DAC digital program DB11, written for the IBM 7094 computer, is the principal numerical tool used for determining the mode shapes and natural frequencies of beam type structures by the Missile and Space Systems Division. The basic element or building block in the program is a Timoshenko beam element with provisions for particularizing this element to suit the needs of a given problem. The numerical technique employed in the program is Myklestad's method. This method, published for the Bernoulli-Euler beam by Myklestad in 1944, has received wide attention in the literature[1,2] of beam dynamics and will be outlined only briefly here. In essence the method utilizes the linear transformation that exists between adjacent elements of a beam to relate conditions at the beam boundaries in terms of a circular frequency. In the process of specifying known boundary conditions, a determinant containing this frequency is defined. The zeros of this determinant yield the natural frequencies.

1.2 Mathematical Model

A description of the basic element in the DB11 program will be provided in terms of the linear transformation relating the state of adjacent sections in the beam. Conditions at a section are given by a (12 x 1) state vector S which contains three orthogonal forces, moments, deflections, and rotations. The relation between sections is:

$$\left\{S_i\right\} = \left[A_i\right] \left\{S_{i+1}\right\} \qquad (1)$$

where A_i is the matric representation of the linear transformation between sections. The coordinate system used by the program in this formulation is:

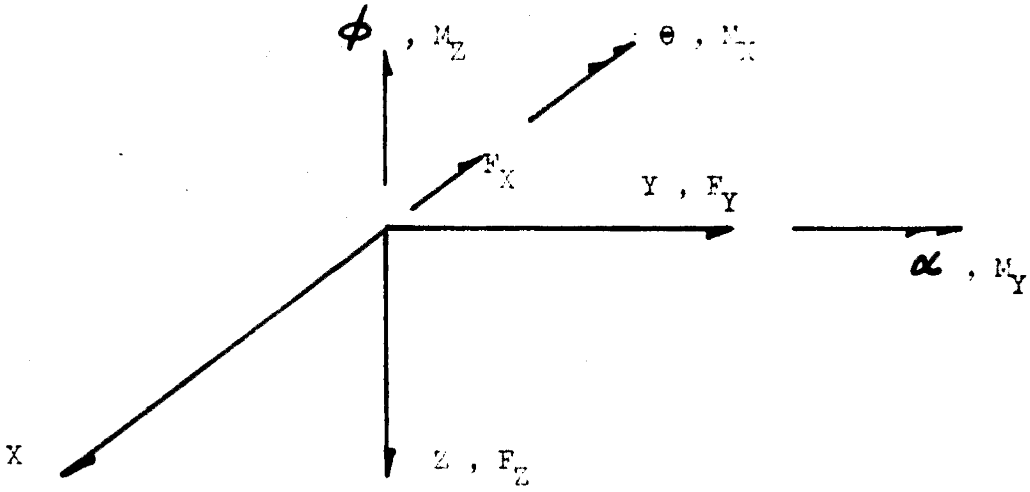


Figure 1

In this coordinate system the program defines the matrices of Equation (1) as:

$$A_i = \begin{bmatrix} L_{2i} & 0 \\ G_i & L_{1i} \end{bmatrix} + \omega^2 \begin{bmatrix} M_{Si} \\ 0 \end{bmatrix} \begin{bmatrix} G_i & L_{1i} \end{bmatrix}$$

and

$$\left\{ S_i \right\} = \left\{ S \right\}_i = \left\{ \begin{matrix} F_Z \\ M_Y \\ F_X \\ M_X \\ F_Y \\ M_Z \\ Z \\ \alpha \\ X \\ \theta \\ Y \\ \phi \end{matrix} \right\}_i$$

where

$$L_{1i} = \begin{bmatrix} 1 & L_i & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & L_i \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$
L_{2i} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -L_i & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -L_i & 1 \end{bmatrix}
$$

$$
C_i = \begin{bmatrix} C_{ZZ} & C_{Z\alpha} & 0 & 0 & 0 & 0 \\ C_{\alpha Z} & C_{\alpha\alpha} & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{XX} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{\theta\theta} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{YY} & C_{Y\phi} \\ 0 & 0 & 0 & 0 & C_{\phi Y} & C_{\phi\phi} \end{bmatrix}_i
$$

$$
G_i = -L_{1i} \quad C_i
$$

$$
M_{S_i} = \begin{bmatrix} m & S_{Z\alpha} & 0 & S_{Z\theta} & 0 & 0 \\ S_{\alpha Z} & I_{\alpha\alpha} & S_{\alpha X} & P_{\alpha\theta} & 0 & P_{\alpha\phi} \\ 0 & S_{X\alpha} & m & 0 & 0 & S_{X\phi} \\ S_{\theta Z} & P_{\theta X} & 0 & I_{\theta\theta} & S_{\theta Y} & P_{\theta\phi} \\ 0 & 0 & 0 & S_{Y\theta} & m & S_{Y\phi} \\ 0 & P_{\phi\alpha} & S_{\phi X} & P_{\phi\theta} & S_{\phi Y} & I_{\phi\phi} \end{bmatrix}_i
$$

The following element definitions apply.

L_i = length of element i

$$C_{ZZi} = \frac{L_i^3}{3\,(EI_Y)_i} + \frac{L_i\,\nu_i}{(A_S G)_i}$$

$$C_{Z\alpha_i} = C_{\alpha Z} = -\,L_i^2 \,/\, (2EI_Y)_i$$

$$C_{\alpha\alpha i} = L_i / (EI_Y)_i$$

$$C_{XXi} = L_i / (A_S E)_i$$

$$C_{\theta\theta i} = L_i / (GJ_X)_i \quad , \quad C_{\phi\phi} = L_i / (EI_Z)_i$$

$$C_{YYi} = \frac{L_i^3}{3(EI_Z)_i} + \frac{L_i\,\nu_i}{(A_S G)_i}$$

$$C_{Y\phi_i} = C_{\phi Y} = -\,\frac{L_i^2}{2(EI_Z)_i}$$

$$m\bar{X} = S_{Z\alpha} = S_{Y\phi} = S_{\alpha Z} = S_{\phi Y}$$

$$m\bar{Y} = S_{Z\theta} = S_{X\phi} = S_{\theta Z} = S_{\phi X}$$

$$m\bar{Z} = S_{X\alpha} = -S_{Y\theta} = S_{\alpha X} = -S_{\theta Y}$$

$P_{ab} = P_{ba}$ = product of inertia in the (a,b) plane

The program determines the size of the transfer matrix required for the degrees of freedom present in a given problem and works with this matrix rather than the (12x12) general case matrix. For example, if three degrees of freedom (Z, α, θ) are admitted the matrix equation (1) becomes;

$$\begin{Bmatrix} F_Z \\ M_Y \\ M_X \\ Z \\ \alpha \\ \theta \end{Bmatrix}_i = \begin{bmatrix} A_i \\ (6 \times 6) \end{bmatrix} \begin{Bmatrix} F_Z \\ M_Y \\ M_X \\ Z \\ \alpha \\ \theta \end{Bmatrix}_{i+1}$$

and

$$\{S\}_{Root} = \prod_i^R \left[A\right]_i \{S\}_{Tip} = \left[\bar{A}\right]\{S\}_{Tip}$$

If the boundary conditions are,

$$\text{Root} \qquad F_Z = M_Y = \theta = 0$$

$$\text{Tip} \qquad M_Y = Z = \theta = 0$$

the columns of $\bar{A}$ corresponding to (M_Y, Z, θ) and the rows of $\bar{A}$ corresponding to (M_Y, Z, α) may be deleted to form the frequency determinant.

$$\begin{Bmatrix} 0 \\ 0 \\ M_X \\ Z \\ \alpha \\ 0 \end{Bmatrix}_{Root} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} & a_{26} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} & a_{56} \\ a_{61} & a_{62} & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix} \begin{Bmatrix} F_Z \\ 0 \\ M_X \\ 0 \\ \alpha \\ 0 \end{Bmatrix}_{Tip}$$

The resulting equation is;

$$\begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{13} & a_{15} \\ a_{21} & a_{23} & a_{25} \\ a_{61} & a_{63} & a_{65} \end{bmatrix} \begin{Bmatrix} F_Z \\ M_X \\ \alpha \end{Bmatrix}_{Tip} = \left[\bar{\bar{A}} \right] \begin{Bmatrix} F_Z \\ M_X \\ \alpha \end{Bmatrix}_{Tip}$$

The determinant $\bar{\bar{A}}$ is a polynomial in the frequency ω and its roots correspond to the desired natural frequencies

Two variations of the basic A_i transfer matrix are possible with DF11. One of these variations permits a bend in the beam and the other provides for the attachment of a branch beam. In such cases only the transfer matrix at station (i) is affected. A bend in the beam requires a point transformation of the affected A_i denoted by T_i .

$$A_i = \begin{bmatrix} T_i & 0 \\ 0 & T_i \end{bmatrix} \begin{bmatrix} L_{2i} & 0 \\ G_i & L_{1i} \end{bmatrix} - \omega^2 \begin{bmatrix} M_{Si} \\ 0 \end{bmatrix} \begin{bmatrix} T_i \end{bmatrix} \begin{bmatrix} G_i & L_{1i} \end{bmatrix}$$

A branch beam requires this and the continuity equations at the intersection. The branch beam is treated exactly as the basic beam except that end conditions at the intersection are set by the continuity conditions. A matrix $\bar{A}$ denoted $\bar{A}_N$ is determined for the branch beam. The columns of $\bar{A}_N$ corresponding to zero conditions at the tip of the branch are deleted and the resulting matrix is partitioned into two square matrices denoted by B_N and C_N . The B_N matrix corresponding to the force equations and C_N corresponding to the deflection equations. Denoting the point transformation for the branch for T_{Ni} , the change to the affected basic beam transfer matrix A_i is:

$$A_i'' = \begin{bmatrix} I & M_{Ni} \\ 0 & I \end{bmatrix} A_i$$

where

$$K_{Ni} = T_{Ni} \quad B_N \quad C_N^{-1} \quad T^{*}_{Ni}$$

The frequency determinant involves only the coordinates of the basic beam. The transformation implicit in this formulation has a first order singularity for each branch.

1.3 Program Input Characteristics

The idealization of weight and stiffness characteristics for input to the program are on an element basis. The weight distribution, defined by a series of straight lines, is integrated by the program over each element and the total lumped at the element center of gravity. The stiffness distribution is considered constant over each element and only the magnitude for each element is input. Provisions are available for the input of concentrated weights and equivalent springs in addition to the distributed input.

The boundary conditions at the ends of the beam are specified as zero or free for each component of the state vector at the ends. The number of degrees of freedom (one to six) is also specified. The number of modes to be computed is an input variable. Normalization is permitted on a single variable at an arbitrary station and all other variables are computed with this same scaling of the transfer matrix.

1.4 Longitudinal Oscillations

A special case of the general transfer matrix is that for pure longitudinal oscillations. The only degree of freedom admitted is (X) and the resulting transfer matrix is;

$$\begin{bmatrix} A_i \end{bmatrix} = \begin{bmatrix} 1 - {}^{2}C_{XX}m \,, & \omega^2 m \\ -C_{XX} \,, & 1 \end{bmatrix}$$

or if an equivalent spring is substituted for C_{XX},

$$\begin{bmatrix} A_i \end{bmatrix} = \begin{bmatrix} 1 - \frac{\omega^2 m}{K} & , \ \omega^2 m \\ 1/K & , \ 1 \end{bmatrix}_i$$

The special case of longitudinal oscillations with an end spring boundary condition can be represented in several ways using the program with perhaps the most straight forward being a grounded branch beam attached arbitrarily close to the end of the beam. Five digit accuracy has been obtained in test cases with greater numerical accuracy possible if desired. The alteration to the affected transfer matrix in this formulation is;

$$\begin{bmatrix} A''_i \end{bmatrix} = \begin{bmatrix} 1 & , \ -\bar{K} \\ 0 & , \ 1 \end{bmatrix}_i \begin{bmatrix} 1 - \frac{\omega^2 m}{K} & , \ \omega^2 m \\ -1/K & , \ 1 \end{bmatrix}_i$$

$$\begin{bmatrix} A''_i \end{bmatrix} = \begin{bmatrix} 1 - \frac{\omega^2 m}{K} + \bar{K}/K & , \ \omega^2 m - \bar{K} \\ -1/K & 1 \end{bmatrix}_i$$

where $\bar{K}$ is the end spring. The representation of the sprung mass liquid propellants is also effected using the branch beam option. The alteration to the affected transfer matrix for this case is;

$$\begin{bmatrix} A_i'' \end{bmatrix} = \begin{bmatrix} 1 & , & \dfrac{\omega^2 \bar{m}}{1 - \dfrac{\omega^2 \bar{m}}{\bar{K}}} \\ 0 & , & 1 \end{bmatrix}_i \begin{bmatrix} 1 - \dfrac{\omega^2 \bar{m}}{K} & , & \omega^2 \bar{m} \\ -1/K & , & 1 \end{bmatrix}_i$$

$$\begin{bmatrix} A_i'' \end{bmatrix} = \begin{bmatrix} 1 - \dfrac{\omega^2}{K}\left(m + \dfrac{\bar{m}}{1 - \dfrac{\omega^2 \bar{m}}{\bar{K}}}\right) & , & \omega^2\left(m + \dfrac{\bar{m}}{1 - \dfrac{\omega^2 \bar{m}}{\bar{K}}}\right) \\ -1/K & , & 1 \end{bmatrix}_i$$

where $\bar{m}$ is the sprung mass and $\bar{K}$ is the branch spring. The transformation singularity mentioned earlier is evident in this A_i'' matrix. It produces an extraneous asymptote in the frequency determinant whos affect is entirely local.

1.5 Computation of Mode Shapes and Frequencies

The numerical solution for natural frequencies is the iterative procedure of Myklestad. The frequency range of interest is specified, a search increment given and the desired accuracy level set by the input. The frequency determinant is evaluated at each frequency located by the search increment. This procedure generates a "null plot" or locus of the determinant over a range of frequencies. The points at which the null plot crosses zero are the roots. When a crossing is detected between two search increments, the program iterates on frequency until the input accuracy level is satisfied. The program then continues the null plot until the desired number of modes have been located or until the frequency range input has been swept. After locating the natural frequencies, the state vector at each station is computed by substituting the natural frequencies into the transfer matrices. This procedure generates the modal forces and shapes. An othogonality check is made by computing an N x N "modal mass" matrix, where N is the number of modes. The elements of this matrix are $V_m^* M V_n$ where m and n range from 1 to N. The V matrices are formed from rows of the S_i matrices of a given mode where i ranges over the length of the beam. The magnitude of the off diagonal terms in the modal mass matrix indicate the degree of orthogonality achieved. Typical longitudinal oscillation cases have off diagonal terms that are on the order of 10^{-5} times the main diagonal terms or smaller.

SM 46346

2.0 TEST VEHICLE STRUCTURAL DATA

The weight and stiffness properties of the test vehicle(s) were determined from existing Douglas documentation and information obtained from the Langley Research Center. A representative of the Douglas Weight Control Section inspected the test vehicle(s) during the study and collected available data from Langley. These data include photographs of the vehicle components, results of load cell weight measurements, a partial set of drawings and information on planned ballasting. On the basis of this information the weight and stiffness distributions of this section were compiled.

2.1 Thor

The particular Thor booster being used in the LRC tests is a DM-18A booster. This vehicle has been partially stripped of components for one portion of the test and ballasted to simulate a DSV-2A booster for the remainder of the testing. In both cases, the transition section (Station 151 forward) is not present. For purposes of the test, the transition section has been incorporated in the Agena vehicle.

2.1.1 DM-18A Test Vehicle Weights

General Configuration:

No guidance section

No equipment in center section

No vernier engines or fireshields

No exhaust extension

No thrust chamber on engine (MB-3 Block I)

No engine actuators

No tunnels or wires

SM 46346

Distributed Weight Coordinates:

Station	Dead Load - lb/in.	Dead Load - lb/in.
151.0	0	2.568
336.0	4.196	5.606
369.0	6.023	3.657
600.0	3.368	.774
636.7	11.770	8.295
722.4	10.402	0

Distributed weight = 2,776 pounds at station 496.1.

Cantilevered Weights:

1. Fuel Tank Forward Bulkhead
2. Fuel Tank Aft Bulkhead
3. Liquid Oxygen Tank Forward Bulkhead
4. Liquid Oxygen Tank Aft Bulkhead
5. Main Engine without Thrust Chamber

Ident.	Wt.-lbs.	C.G. Sta-in.	Cant. Sta-in.	Moment	Direction
1	56	148.4	153.5	280	CCW
2	85	345.1	333.5	986	CW
3	129	365.0	371.5	639	CCW
4	124	610.8	600.0	1,350	CW
5	1,134	654.9	636.7	20,639	CW
	1,527	591.2			

Concentrated Weights:

HP Bottles = 131 pounds at station 620.7

Weight Summary:

	Weight - lbs.	Station - in.
Distributed Loads	[illegible]	[illegible]
Cantilevered Loads	1,3[illegible]7	591.2
Concentrated Loads	1[illegible]1	[illegible].7
	4,434	[illegible]

2.1.2 DSV-2A Test Vehicle Weights

General Configuration:

No guidance section

No equipment in center section*

No vernier engines or fireshields*

No exhaust extension

No thrust chamber on engine (MB-3 Block I)

No engine actuators (simulated by ballast)

Distributed Weight Coordinates:

Station	Dead Load - lb/in.	Dead Load - lb/in.
151.0	0	3.016
336.0	4.672	5.854
369.0	6.272	3.907
600.0	3.624	1.305
636.7	17.719	9.673
722.4	10.223	0

Distributed weight = 2,982 pounds at station 491.5.

***Simulated by Ballast.**

Cantilevered Weights:

1 Fuel Tank Forward Bulkhead

2 Fuel Tank Aft Bulkhead

3 Liquid Oxygen Tank Forward Bulkhead

4 Liquid Oxygen Tank Aft Bulkhead

5 Main Engine without Thrust Chamber plus Shaker Attachment

Ident.	Wt-Lbs.	C.G. Sta-in.	Cant. Sta-in.	Moment	Direction
1	56	[illegible]	123.5	280	CCW
2	85	345.1	333.5	986	CW
3	129	365.0	371.5	839	CCW
4	125	610.8	600.0	1,350	CW
5	1,271	657.6	636.7	26,564	CW
	1,665	598.4			

Concentrated Weights:

HP Bottles	-	131 pounds at Station	620.7
CS Ballast	-	[illegible] pounds at Station	391.7
Eng. Sec. Ballast	-	[illegible] pounds at Station	722.4
		608	626.1

Weight Summary:

	Weight-lbs.	Station-in.
Distributed Loads	2,982	491.5
Cantilevered Loads	1,665	598.4
Concentrated Loads	608	626.1
	5,255	540.9

2.1.3 Thor Test Vehicle Stiffness Properties

The same stiffness data applies to both the partially stripped DM-18A and DSV-2A boosters. Since the testing will be at room temperature, all data given are for room temperature conditions. Although the stiffness distribution is not everywhere linear, there are only small deviations from linearity. The distribution shown here is the piecewise linear distribution used in all numerical work of the report.

Stiffness Distribution Coordinates:

Station	AE - lbs/in.	AE - lbs/in.
151.0	0	$.196 \times 10^9$
158.08	$.196 \times 10^9$	$.174 \times 10^9$
175.0	$.174 \times 10^9$	$.178 \times 10^9$
200.0	$.178 \times 10^9$	$.1855 \times 10^9$
250.0	$.1855 \times 10^9$	$.201 \times 10^9$
300.0	$.201 \times 10^9$	$.214 \times 10^9$
325.0	$.214 \times 10^9$	$.222 \times 10^9$
333.5	$.222 \times 10^9$	$.224 \times 10^9$
335.0	$.224 \times 10^9$	$.189 \times 10^9$
365.0	$.189 \times 10^9$	$.200 \times 10^9$
600.0	$.200 \times 10^9$	$.184 \times 10^9$
636.7	$.184 \times 10^9$	$.354 \times 10^9$
670.0	$.354 \times 10^9$	$.303 \times 10^9$
687.5	$.303 \times 10^9$	$.304 \times 10^9$
710.0	$.304 \times 10^9$	$.334 \times 10^9$
722.4	$.334 \times 10^9$	0

The idealization used to compute natural frequencies and mode shapes in this report assumes the fuel and oxidizer mass are elastically coupled to the vehicle structure by linear springs. The fuel is mounted on one spring and the oxidizer on another, each representing elasticity in the particular tank (bottom) bulkhead. The calculations were made using four sets of spring constants; infinite, twice nominal, half nominal and nominal. The term nominal here refers to the spring constants normally used by Douglas in any Thor calculations. They are the results of a theoretical analysis which will not be developed in this report.

Spring Constant	Fuel (lb/in)	Oxidizer (lb/in)
Infinite	∞	∞
Nominal	1.357×10^6	1.528×10^6
Twice Nominal	2.714×10^6	3.056×10^6
Half Nominal	0.6785×10^6	0.764×10^6

2.2 Agena

The Agena test vehicle is a simulated Agena B. The tank section is from an Agena B however other sections differ significantly. The test vehicle is geometrically similar to the Agena B but the stiffness properties, especially in the adapter section, are quite different in magnitude.

2.2.1 Simulated Agena B Weights

General Configuration:

No Agena Payload
Forward Equipment Rack, Equipment*
Tank Section Complete
Aft Midbody Section Complete
Aft Rack and Engine, Equipment*
Thor-Agena Adapter - Complete

Thor Transition Section included is considered as part of the Agena for this test.

*Partially simulated by ballast.

Distributed Weight Coordinates:

Station	Dead Load-lb/in.	Dead Load-lb/in.
-138.7	0	7.886
-111.9	7.886	9.071
- 88.5	9.182	1.296
- 83.4	2.681	1.938
- 73.6	1.938	3.458
- 44.7	3.117	1.834
- 11.1	1.834	3.957
- 5.9	3.480	5.044
6.0	4.234	7.268
17.1	6.669	4.961
64.1	6.590	2.840
107.0	3.138	0

Total = 1,181 pounds at station -30.0

Thor Transition Section

107	0	3.537
141	14.847	13.635
151	11.595	0

439 pounds at Thor Station 132.8

Cantilevered Weights:

1 Engine
2 H_e and Bottle (simulated)
3 N_2 and Bottle (simulated)
4 Ullage Rockets (simulated)

	Weight	C.G. Station	Cant. Station
1	298	56.4	26.2
2	32	72.4	64.1
3	72	72.4	64.1
4	65	73.4	64.1
	467	62.3	

Concentrated Weights:

Weight	Station
55	-116.5
25	-111.5
104	-107.5
10	-106.5
194	-110.5

Weight Summary:

	Weight-lbs.	Station-in.
Distributed Load	1,181	-30.0
Cantilevered Load	467	62.3
Concentrated Load	194	-110.5
	1,842	- 15.1
	439	132.8
	2,281	13.4

2.2.2 Simulated Agena B Stiffness Properties

As for the Thor test vehicle, all properties given are room temperature values. The propellant tank arrangement on the Agena B is a tank within a tank such that only one tank bottom spring has been used in the idealization of the vehicle. The IRFNA oxidizer is mounted on this spring.

Stiffness Distribution Coordinates:

Station	AE - lbs/in.	AE - lbs/in.
-138.7	0	$.130 \times 10^9$
-137.1	$.130 \times 10^9$	$.1284 \times 10^9$
-114.8	$.1284 \times 10^9$	$.1406 \times 10^9$
- 87.8	$.1406 \times 10^9$	$.118 \times 10^9$
- 73.3	$.118 \times 10^9$	$.1135 \times 10^9$
- 52.6	$.1135 \times 10^9$	$.128 \times 10^9$
- 32.6	$.128 \times 10^9$	$.143 \times 10^9$
- 11.1	$.143 \times 10^9$	$.1474 \times 10^9$
- 5.8	$.1474 \times 10^9$	$.1772 \times 10^9$
5.9	$.1772 \times 10^9$	$.1986 \times 10^9$
88.9	$.1986 \times 10^9$	$.1533 \times 10^9$
90.0	$.1533 \times 10^9$	$.1575 \times 10^9$
107.0	$.1575 \times 10^9$	$.220 \times 10^9$
117.0	$.2220 \times 10^9$	$.190 \times 10^9$
130.0	$.190 \times 10^9$	$.197 \times 10^9$
140.0	$.197 \times 10^9$	$.248 \times 10^9$
151.0	$.248 \times 10^9$	0

The same set of tank bottom spring constants namely; infinite, twice nominal, half nominal and nominal were used for the IRFNA spring.

Spring Constant	IRFNA (lbs/in)
Infinite	∞
Nominal	9.000×10^5
Twice Nominal	1.800×10^6
Half Nominal	4.500×10^5

3.0 CONFIGURATIONS ANALYZED

A total of fifty three weight and stiffness configurations were analyzed in the study. The cases represent propellant tank loadings for various percent burntimes in a nominal flight using different sets of tank bottom spring constants. Four of the cases approximate the influence of the shaker and spring support system to be used in the planned LRC testing. In all other cases the vehicle suspension is considered free-free. A tabulation of these cases and pertinent descriptive material is provided in this section.

3.1 Thor

Seventeen weight and stiffness configurations of the Thor test vehicle were analyzed for the first three longitudinal modes of oscillation. Two of these cases were for a vehicle empty of all propellant. The first of these cases was the partially stripped DM-19A booster and the other was the DSV-2A booster. In both cases the tank bottom masses were sprung off the nominal springs for the fuel and oxidizer tanks. The remaining fifteen cases represent five propellant tank loadings, each analyzed for three sets of tank bottom spring constants. In all of these cases the DSV-2A booster weights were used. The five propellant tank loadings simulate 0, 25, 50, 75 and 100 percent burntimes in a nominal flight. The three sets of tank bottom spring constants were infinite, twice nominal and nominal as defined in paragraph 2.1.3.

3.1.1 DSV-2A Nominal Propellant Weights

0% BT	Flight Time (seconds)	Oxidizer (lbs)	Fuel (lbs)
0	0	67,675	32,270
25	37.13	50,679	24,307
50	74.26	33,711	16,343
75	111.39	16,831	8,319
100	148.52	132	263

3.1.2 Enumeration of Thor Cases

To provide some reference designation for these cases, they have each been assigned a case number. The empty DM-18A is case T1 and the empty DSV-2A is case T2. The remaining case numbers are given in the following table.

Springs	0% BT	25% BT	50% BT	75% BT	100% BT
Infinate	T 3	T 4	T 5	T 6	T 7
Twice Nominal	T 8	T 9	T 10	T 11	T 12
Nominal	T 13	T 14	T 15	T 16	T 17

These designations will be used in the report when referring to the cases.

3.2 Agena

Seven weight and stiffness configurations of the Agena test vehicle were analyzed for the first three longitudinal modes. These cases represent 0 and 50 percent burntimes each analyzed for an infinite, twice nominal and nominal IRFNA tank bottom spring, and an empty case with nominal springs.

3.2.1 Simulated Agena B Propellant Weights

% BT	Flight Time (seconds)	IRFNA (lbs)	UDMH (lbs)
0	0	9650	3776
50	117.95	4957	1939

3.2.2 Enumeration of Agena Cases

The case numbers designated for the Agena cases are given in the following table. The empty case is case A1.

Springs	0% BT	50% BT
Infinite	A 2	A 3
Twice Nominal	A 4	A 5
Nominal	A 6	A 7

3.3 Thor-Agena

A total of twenty-five weight and stiffness configurations were analyzed for the first three longitudinal modes. One case is for both the Thor and Agena empty of all propellant with nominal spring constants used for the tank bottom bulkheads. The remaining twenty-four cases represent six propellant tank loadings each analyzed for four sets of tank bottom spring constants. The Agena is fully loaded with propellant in these cases and the Thor loaded to simulate 0, 20, 40, 75, 90 and 100 percent burntimes in a nominal flight. The four sets of tank bottom spring constants are infinite, twice nominal, nominal and one half nominal. In all twenty-five cases the DSV-2A booster weights were used.

3.3.1 DSV-2A - Agena Nominal Propellant Weights

% BT	Flight Time (seconds)	Oxidizer (lbs)	Fuel (lbs)
0	0	67,675	32,270
20	20.70	54,077	25,908
40	59.40	40,492	19,547
75	111.39	16,831	8,319
90	133.67	6,776	3,494
100	143.52	132	266

It will be noted that some of these data are repetions of that in paragraph 3.1.1.

3.3.2 Enumeration of Thor-Agena Cases

The empty Agena empty Thor case is designated case TA 1. The remaining case numbers can be determined from the following table.

Springs	0% BT	20% BT	40% BT	75% BT	90% BT	100% BT
Infinite	TA 2	TA 3	TA 4	TA 5	TA 6	TA 7
Twice Nominal	TA 8	TA 9	TA 10	TA 11	TA 12	TA 13
Nominal	TA 14	TA 15	TA 16	TA 17	TA 18	TA 19
Half Nominal	TA 20	TA 21	TA 22	TA 23	TA 24	TA 25

3.4 Thor-Agena with Shaker

Four cases were analyzed which approximate the influence of the shaker and suspension systems on the natural frequencies. In these cases, a weight of 2,200 lbs was added to the gimbal block to simulate the inertia of the shaker system and a spring was introduced at the gimbal block, connected to ground, which represents the spring constant of the air suspension system. The value of the spring constant changes with the weight of the vehicle supported so that a different value is required for each of the four cases. The cases represent four propellant loadings with nominal tank bottom spring constants. These propellant loadings simulate empty Agena empty Thor, loaded Agena with the Thor loaded to 0, 75 and 100 percent burntime weights. Except for the shaker and suspension simulation these cases are exactly the same as cases TA 1, TA 14, TA 17 and TA 19.

3.4.1 Enumeration of Shaker Cases

The empty Agena empty Thor case is designated case TAS 1. The remaining three case numbers are given in the following table.

Springs	0% BT	75% BT	100% BT
Nominal	TAS 2	TAS 3	TAS 4

3.4.2 Suspension System Spring Constants

Case	Spring (lbs/in.)
TAS 1	1.4×10^{4}
TAS 2	1.1×10^{5}
TAS 3	5.8×10^{4}
TAS 4	4.0×10^{4}

The idealization of these four cases is shown schematically in Figure 2. It should be noted that any load introduced at the gimbal block is transmitted to primary structure through a truss which connects at station 636.7. Hence the 2,200 lbs and the air spring attach at this point in the one dimensional idealization of primary structure.

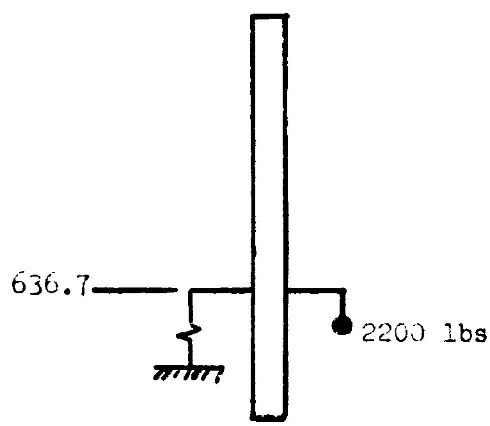


FIGURE 2

An inspection of the frequencies for these cases will show that the first mode has no counterpart in the free-free cases. This mode becomes the trivial zero mode as the grounded spring constant approaches zero. The second and third modes for these cases correspond to the first and second free-free modes.

4.0 NATURAL FREQUENCY PREDICTIONS

In preceding sections the test vehicle has been defined and the cases for analysis described. This section contains the results of the numerical solution (DB 11) of these cases for the first three longitudinal natural frequencies. The associated normal modes of oscillation (mode shapes) were placed in the Appendix because of the large number of pages required. The natural frequencies are tabulated with reference to the case designations assigned in Section 3.0. The numerical accuracy of all cases is several digits better than the five digits shown. This is required for accurate mode shapes since they do not in general converge as well as frequency. There was one case (T 17) in which it was not possible to obtain satisfactory convergence for second and third mode shapes. The frequency values for this case are presented since frequency did converge. The source of the problem was an asymptote between the second and third mode (cf. Section 1.0) coupled with a program limitation on the number of iterations permitted.

4.1 Thor

The first three natural frequencies for the Thor cases are given in the following table.

Case	f_1 (cps)	f_2 (cps)	f_3 (cps)
T 1	84.057	206.94	277.39
T 2	79.714	200.99	255.80
T 3	17.898	128.69	187.32
T 4	20.489	129.45	187.82
T 5	24.716	130.94	188.79
T 6	33.433	135.22	191.36
T 7	76.330	201.92	262.55

Case	f_1 (cps)	f_2 (cps)	f_3 (cps)
T 8	14.681	83.236	129.63
T 9	16.865	83.709	130.12
T 10	20.483	84.879	131.09
T 11	28.239	87.985	133.95
T 12	75.385	198.05	228.26
T 13	12.719	66.224	110.65
T 14	14.628	66.703	110.90
T 15	17.808	67.642	111.41
T 16	24.725	70.304	112.94
T 17	74.945	189.57	195.89

4.2 Agena

The first three natural frequencies for the Agena cases are given in the following table.

Case	f_1 (cps)	f_2 (cps)	f_3 (cps)
A 1	140.05	296.59	512.30
A 2	78.188	146.24	273.56
A 3	99.161	156.38	292.43
A 4	54.420	112.04	272.67
A 5	71.769	116.62	288.60
A 6	42.970	107.83	272.42
A 7	56.022	113.56	287.27

4.3 Thor-Agena

The first three natural frequencies for the Thor-Agena cases are given in the following table.

Case	f_1 (cps)	f_2 (cps)	f_3 (cps)
TA 1	48.687	109.54	159.92
TA 2	14.323	25.178	80.125
TA 3	15.110	26.919	80.143
TA 4	16.055	29.681	80.174
TA 5	18.841	41.303	80.357
TA 6	22.200	56.981	80.910
TA 7	30.715	77.921	96.288
TA 8	12.793	20.336	55.266
TA 9	13.672	21.530	55.277
TA 10	14.718	23.552	55.295
TA 11	17.596	33.001	55.410
TA 12	20.943	47.212	55.997
TA 13	29.358	55.106	87.417
TA 14	11.565	17.711	45.355
TA 15	12.400	18.585	45.394
TA 16	13.583	20.134	45.459
TA 17	16.490	27.946	45.856
TA 18	19.735	39.623	47.755
TA 19	27.618	45.900	86.095
TA 20	9.7968	14.727	36.063
TA 21	10.694	15.282	36.145
TA 22	11.821	16.313	36.284
TA 23	14.678	22.182	37.087
TA 24	17.605	31.356	39.839
TA 25	23.902	39.703	84.098

4.4 Thor-Agena with Shaker

The first three natural frequencies for the four cases approximating the influence of the shaker system are given in the following table.

Case	f_1 (cps)	f_2 (cps)	f_3 (cps)
TAS 1	3.7455	43.926	107.03
TAS 2	2.8505	11.596	17.712
TAS 3	3.3420	16.480	27.944
TAS 4	3.9614	25.098	45.219

5.0 SUMMARY

The purpose of this report, as stated in the introduction, has been to provide natural frequencies and mode shapes resulting from a particular type idealization of structure and propellant. The emphasis in the study has been on an accurate numerical analysis to this end. In this respect it is felt that the numerical solution of the mathematical model, embodied in the results, is an accurate one.

The more general question of applicability of idealization or accuracy of idealization for each of the many cases analyzed is beyond the scope of this study. Certainly such an important question could not be neglected in a design problem and it seems appropriate to stress this point in conclusion. The results of this study cannot be used in a design problem without considering the question of applicability. The experimental work now planned by the Langley Research Center should provide valuable information on the general question of idealization applicability and accuracy. In this regard, the results of this study provide a necessary reference for comparison. It was for this heuristic purpose that the study was undertaken. Recognizing such work as a requisite part of the learning process, the Douglas Aircraft Company will continue to actively support such undertakings.

APPENDIX

The normal mode shapes in graphical form plus the modal masses (cf. Section 1.0) associated with each shape are presented here in the Appendix. The modal mass would be required in a typical response problem if modal analysis is used. On the graphs, the modal mass is abbreviated as M.M. The graphs are labeled according to the case designations of Section 3.0.

REFERENCES

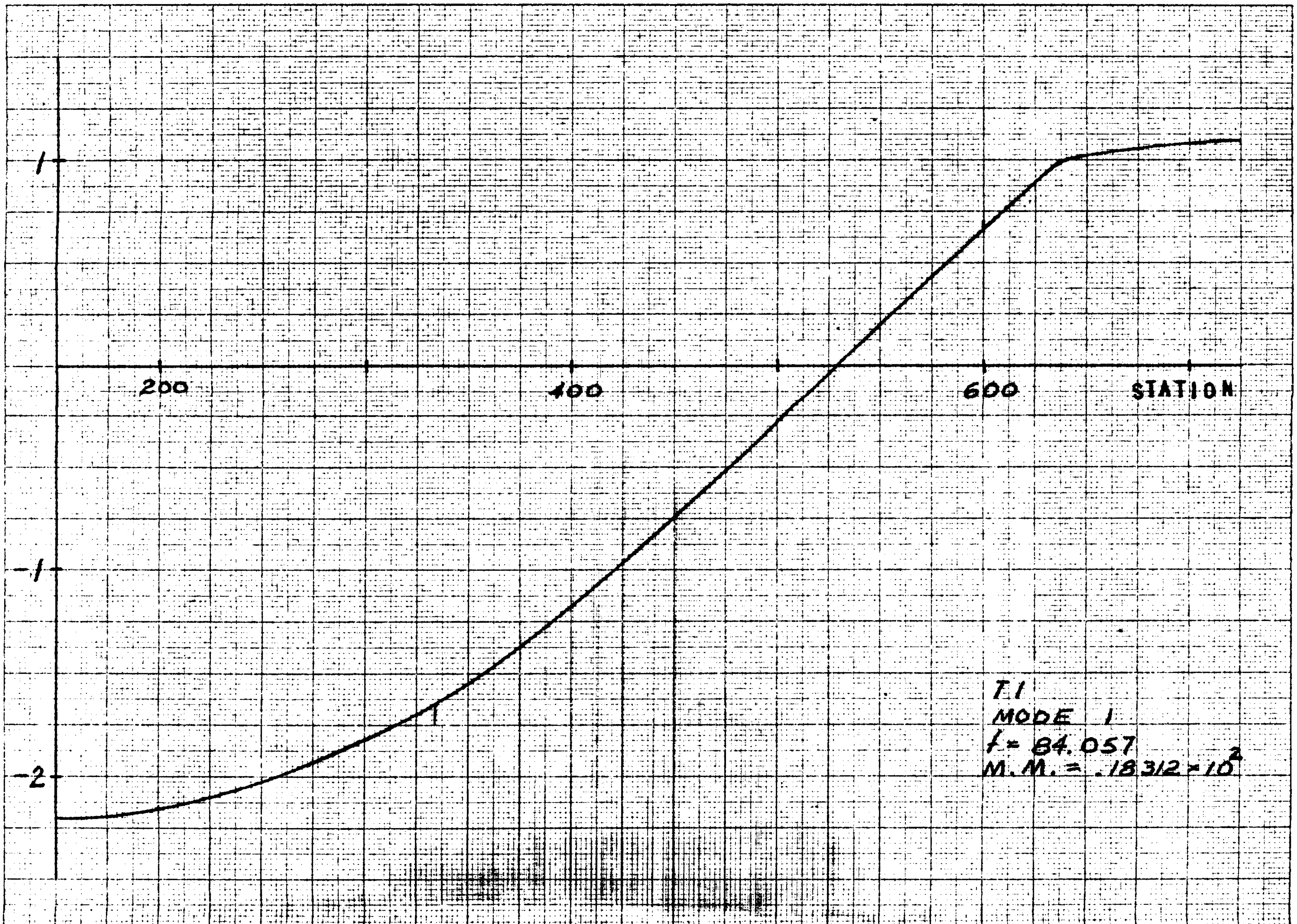
1. Myklestad, N. O., Vibration Analysis, McGraw Hill, 1944.

2. Jacobsen, L. S. and R. S. Ayre, Engineering Vibrations, McGraw Hill, 1958.

APPENDIX

A. Thor mode shapes

SM 46346



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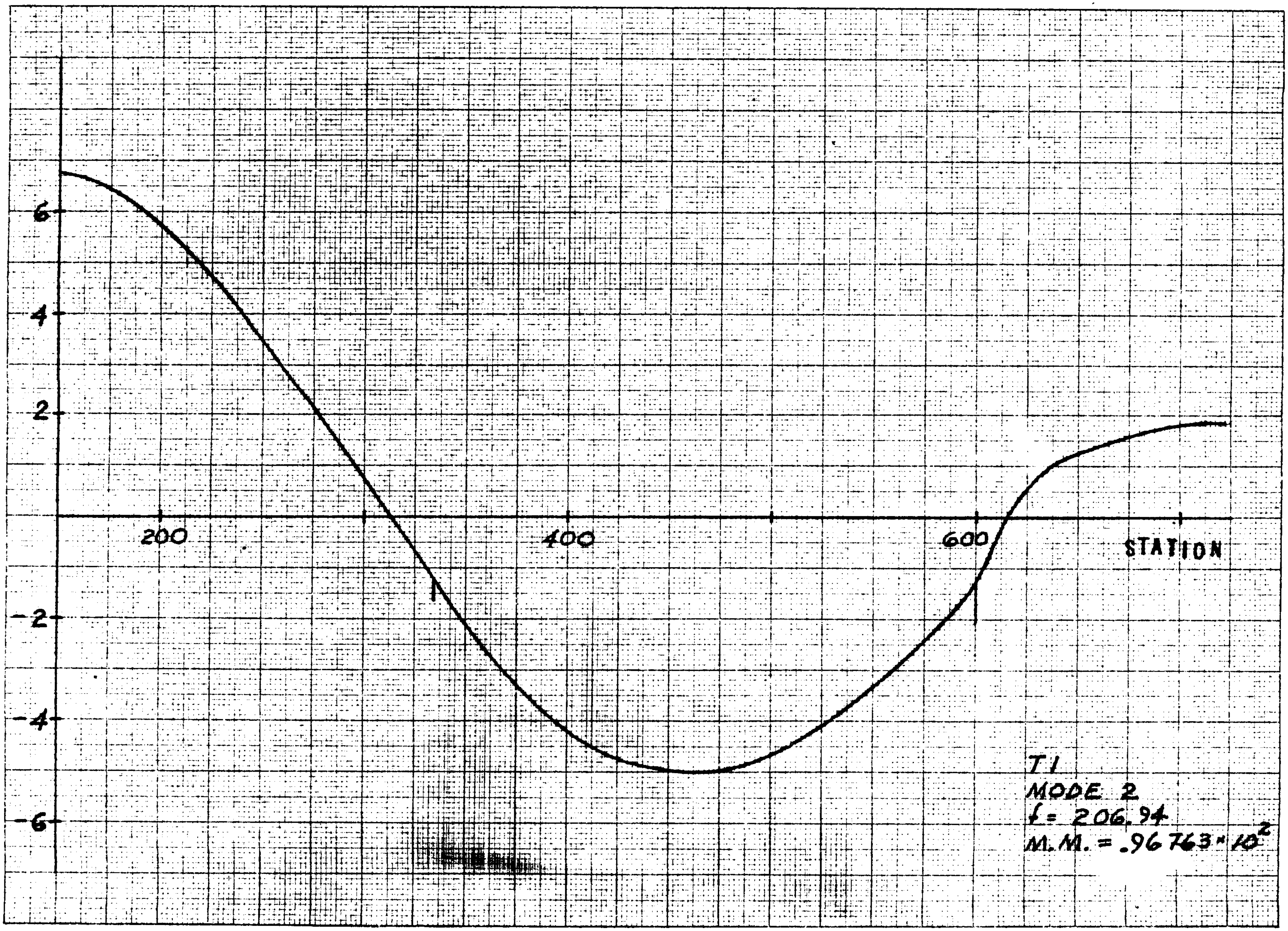
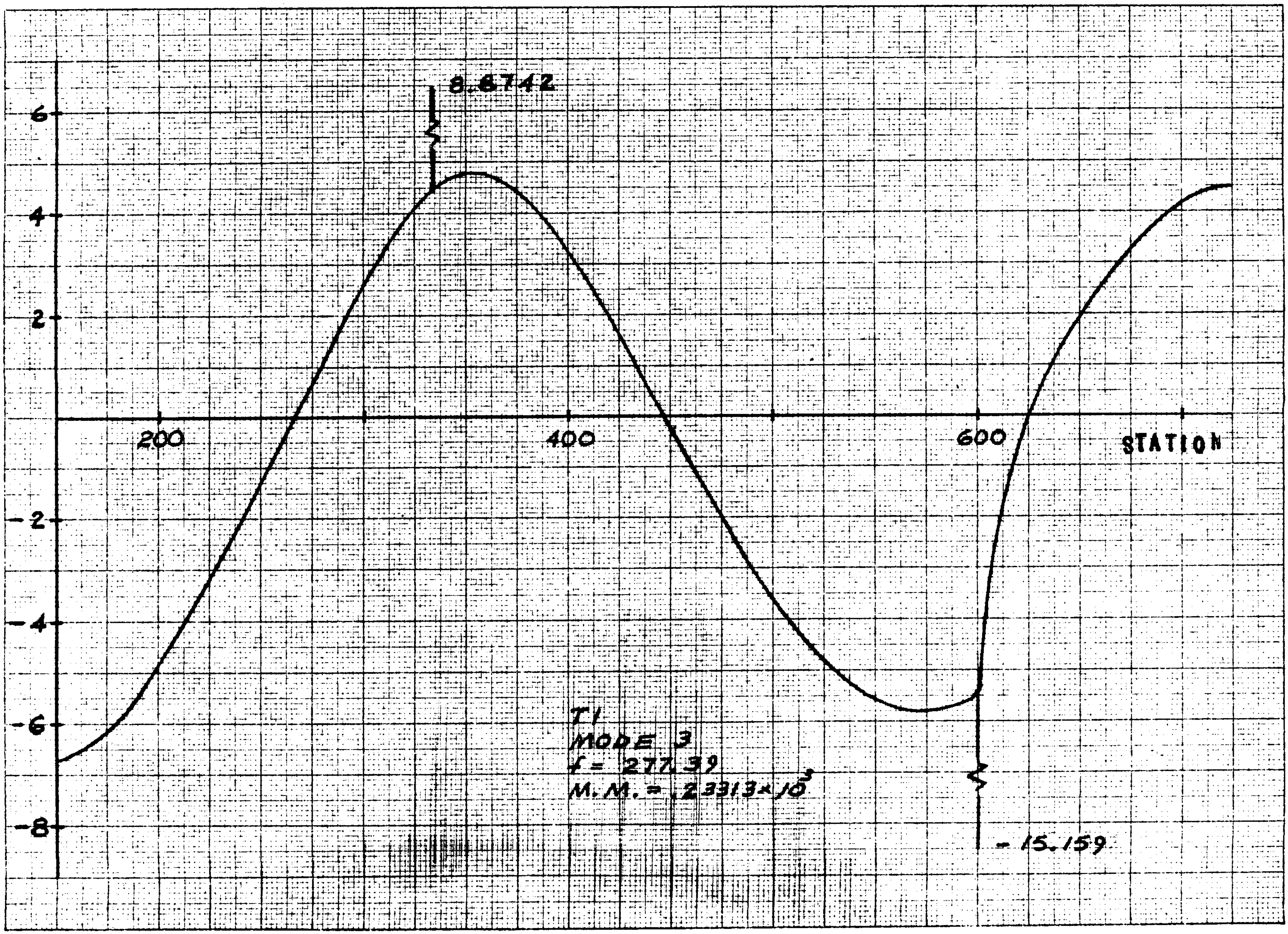
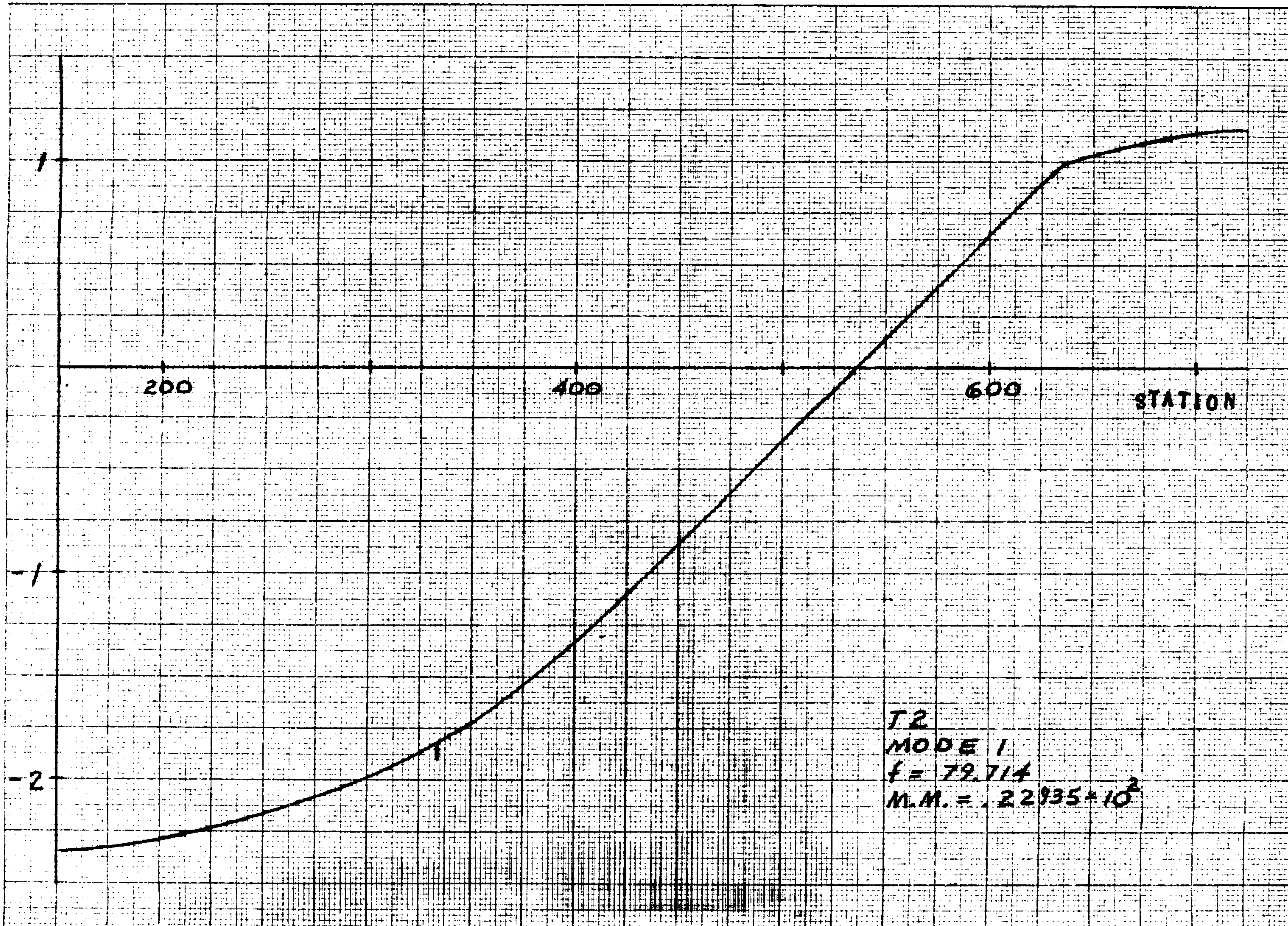



8.8742
6
4
2
-2
-4
-6
-8
200
400
600
STATION
T1
MODE 3
f = 277.39
M.M. = .23313×10³
- 15.159

SM 46346



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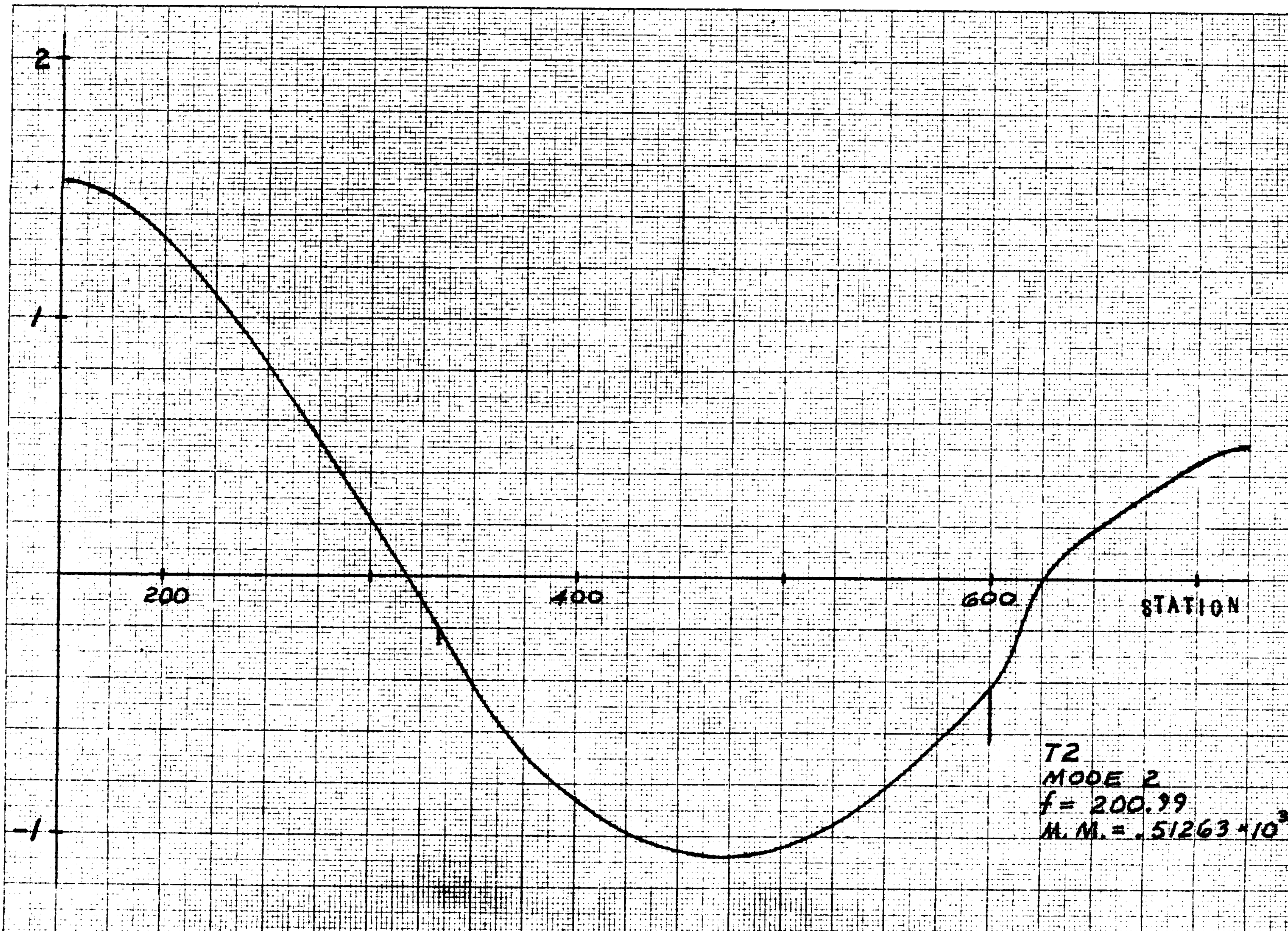

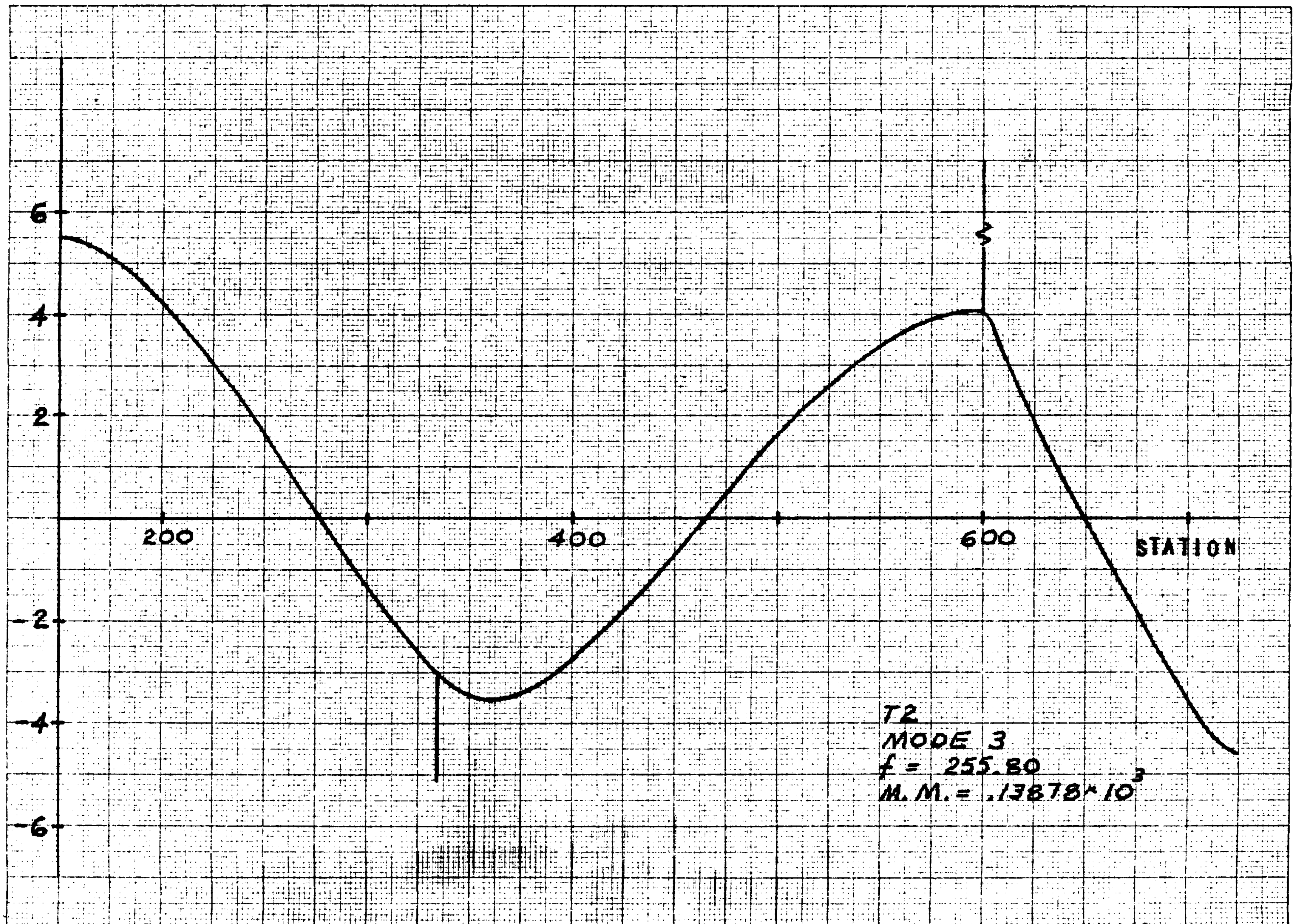
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4
2
-2
-4
-6
200
400
600
STATION
T2
MODE 3
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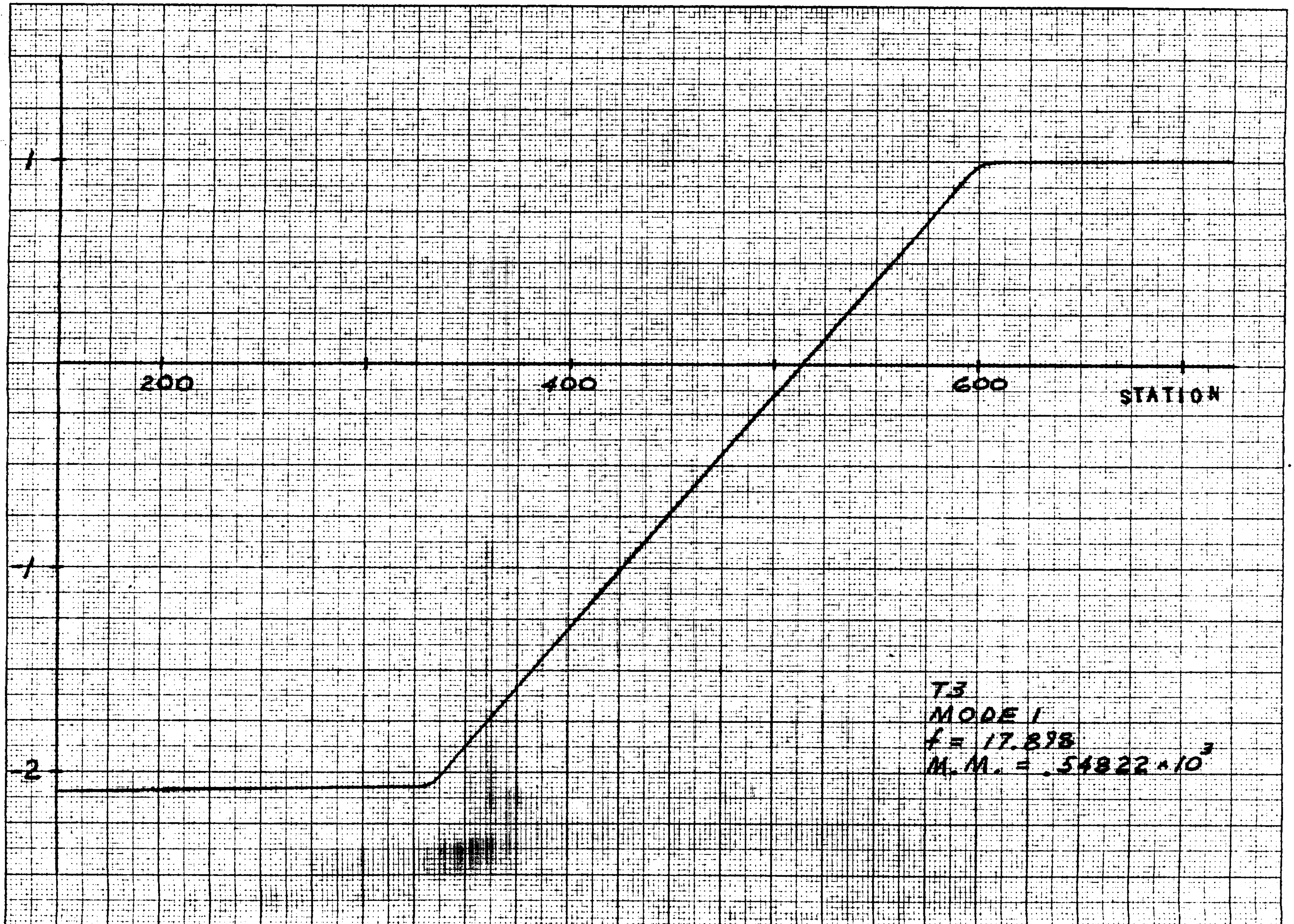
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-2
200
400
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T3
MODE I
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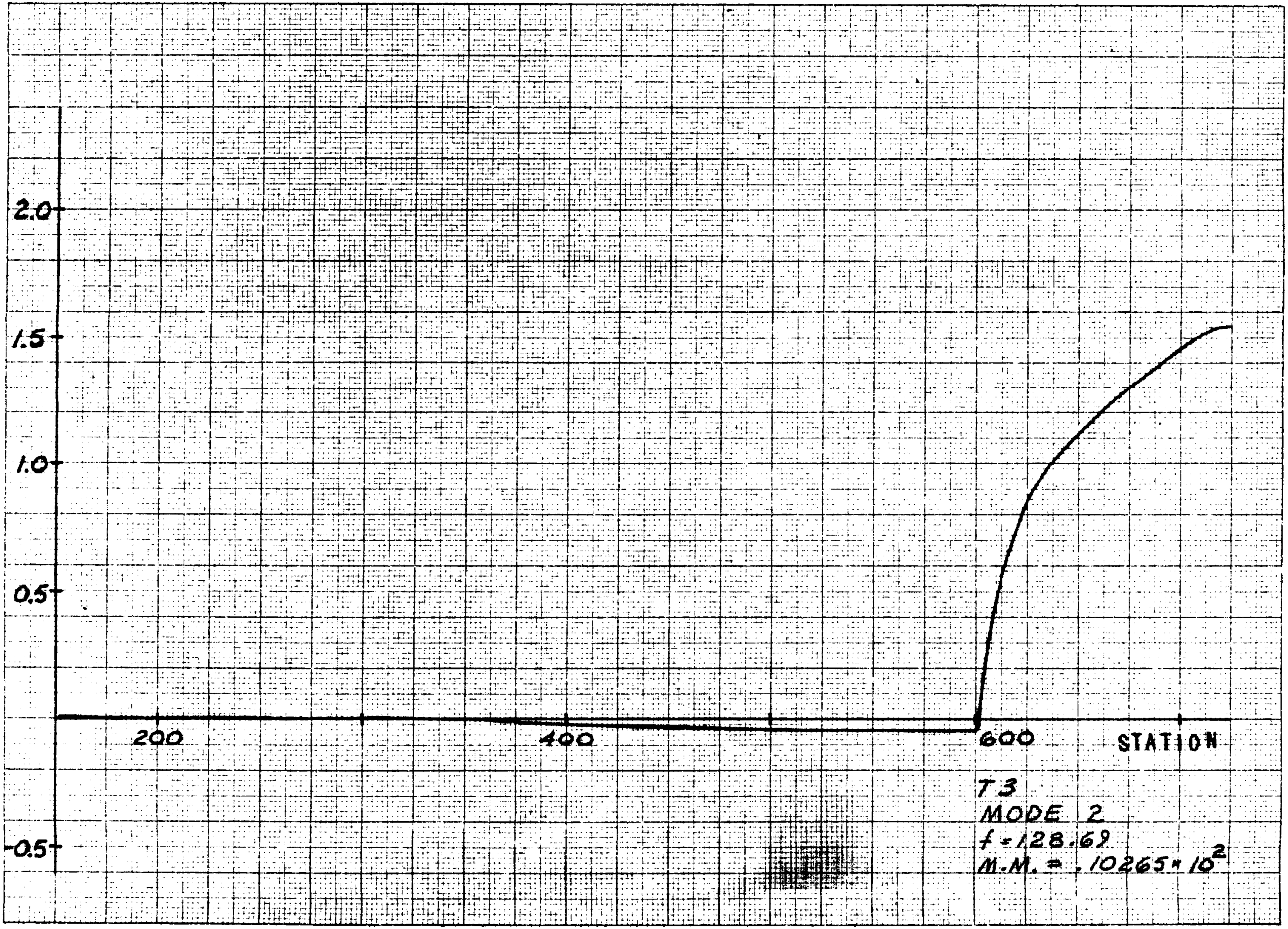
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1.5
1.0
0.5
-0.5
200
400
600
STATION
T3
MODE 2
f = 128.69
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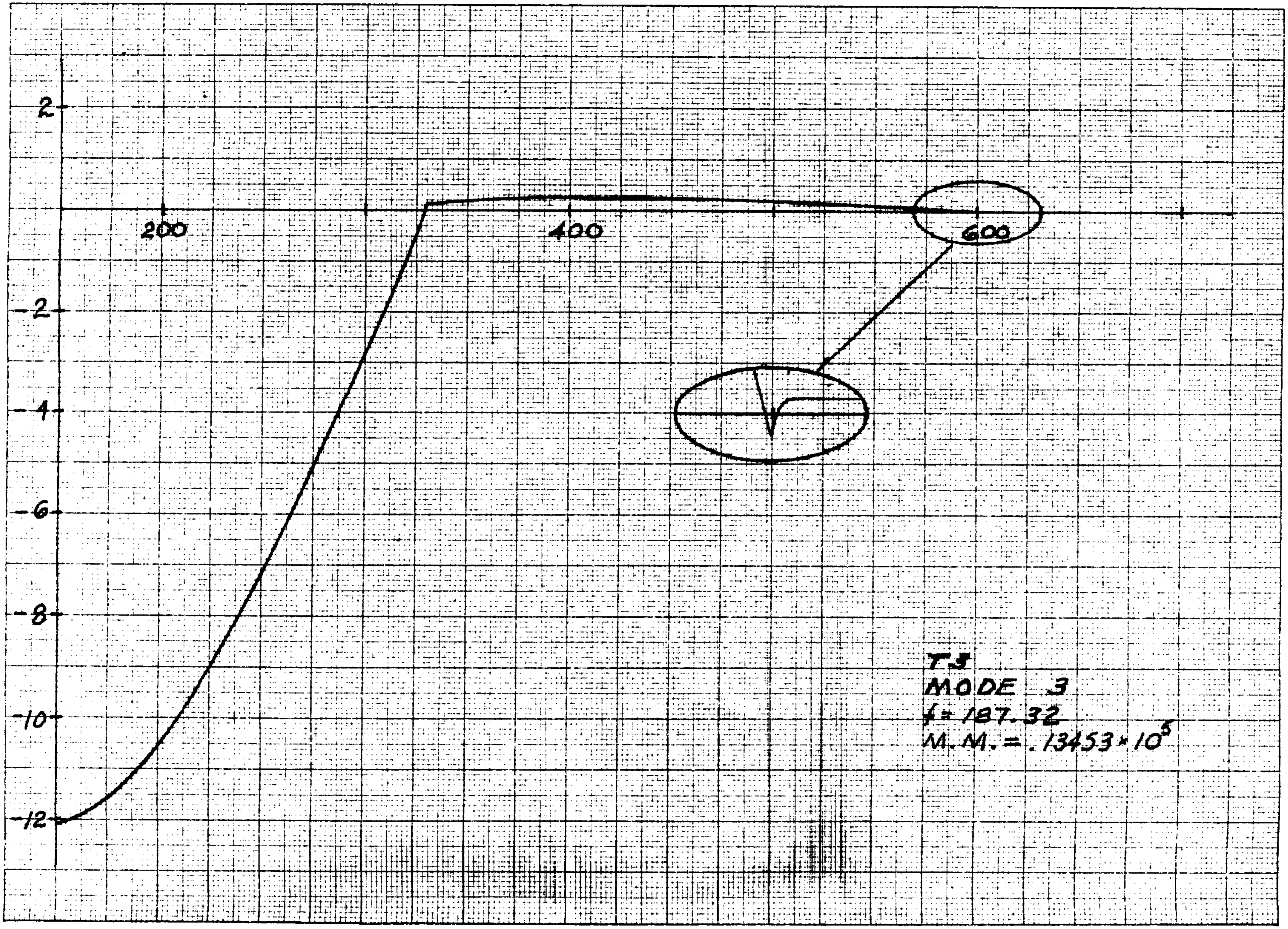
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-2
-4
-6
-8
-10
-12
200
400
600
T5
MODE 3
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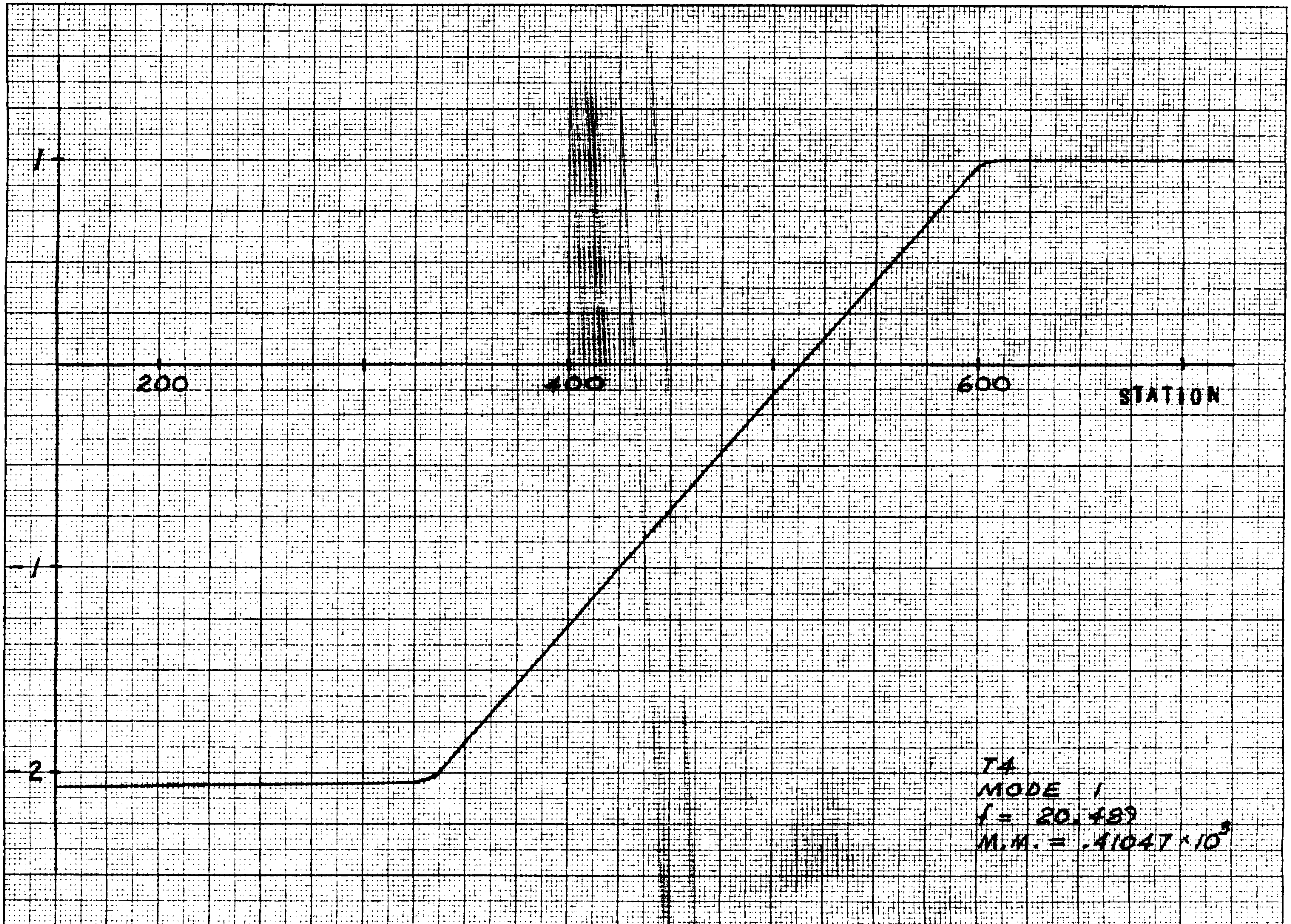
M.M. = .13453 × 10^5


1
-1
-2
200
400
600
STATION
T4
MODE 1
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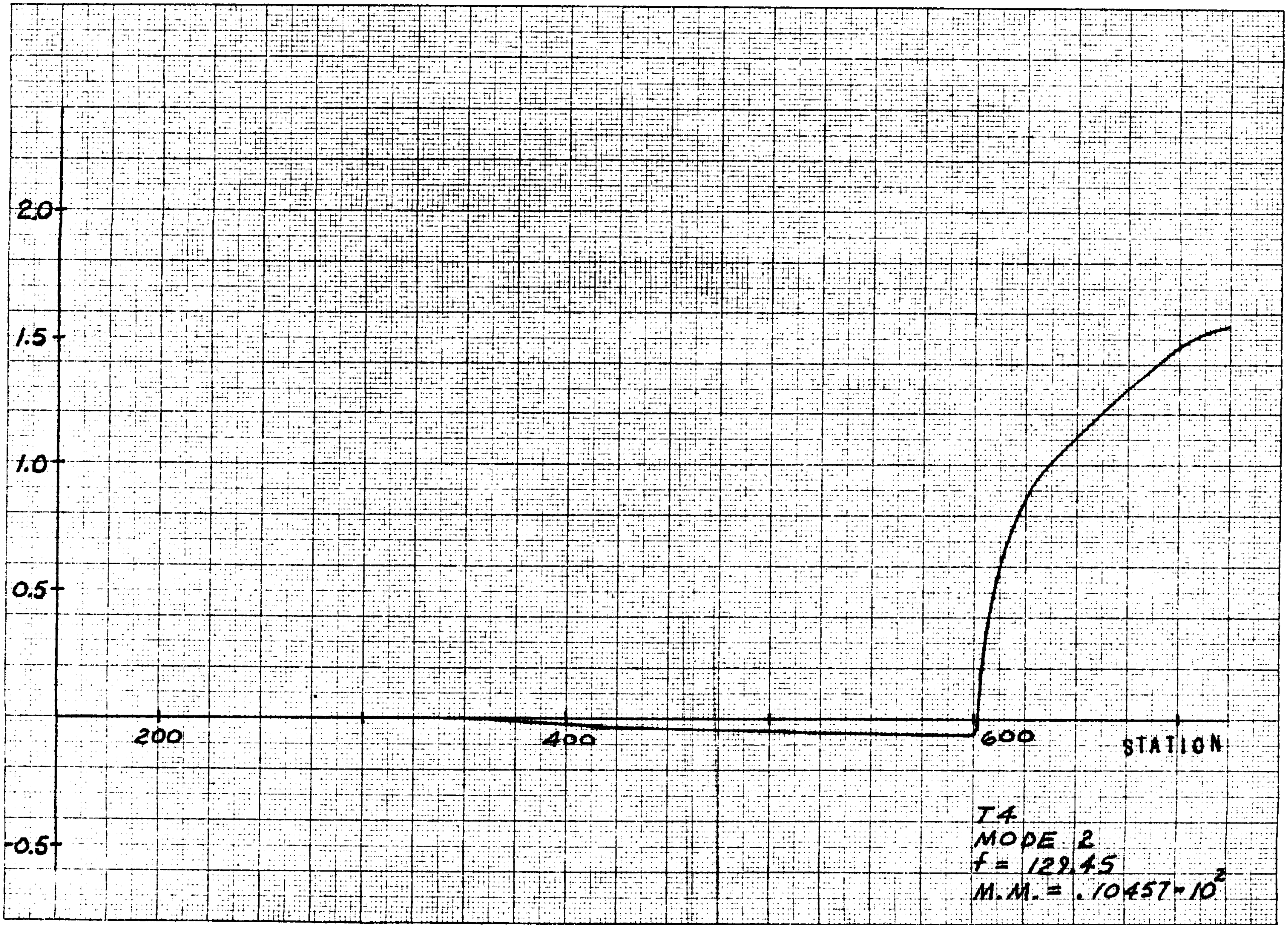
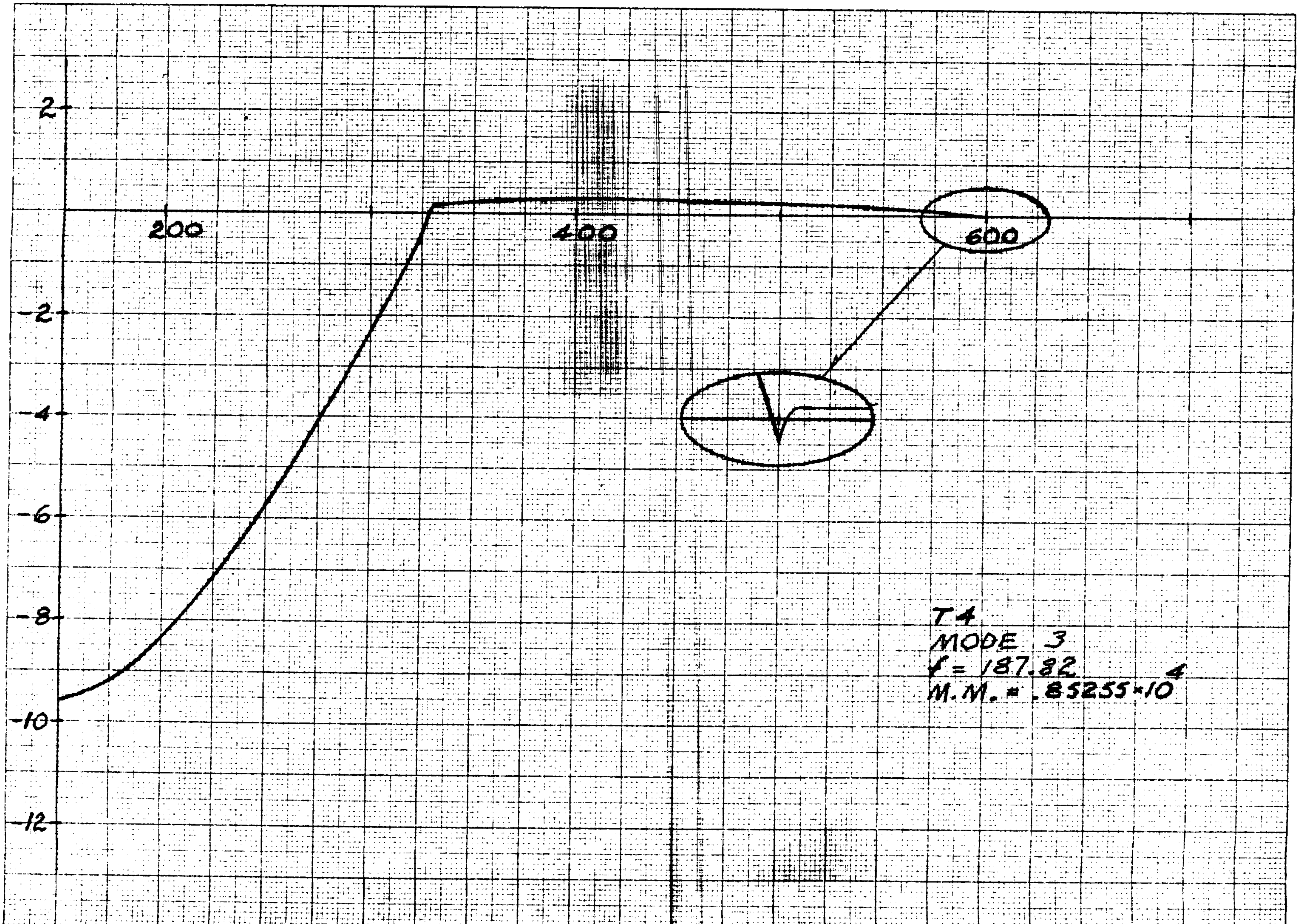
SM 46346

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200
400
600
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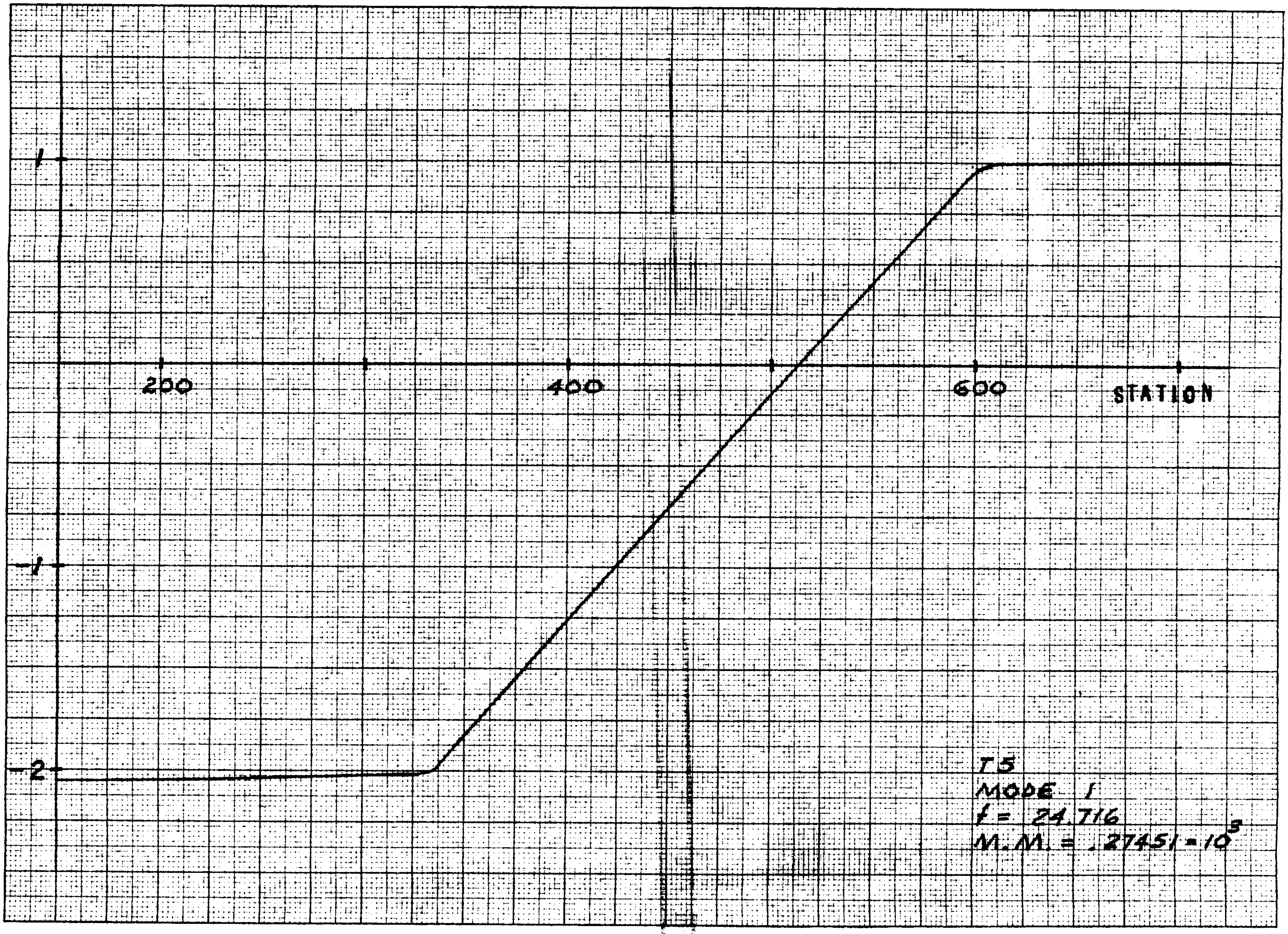
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200
400
600
STATION
T5
MODE I
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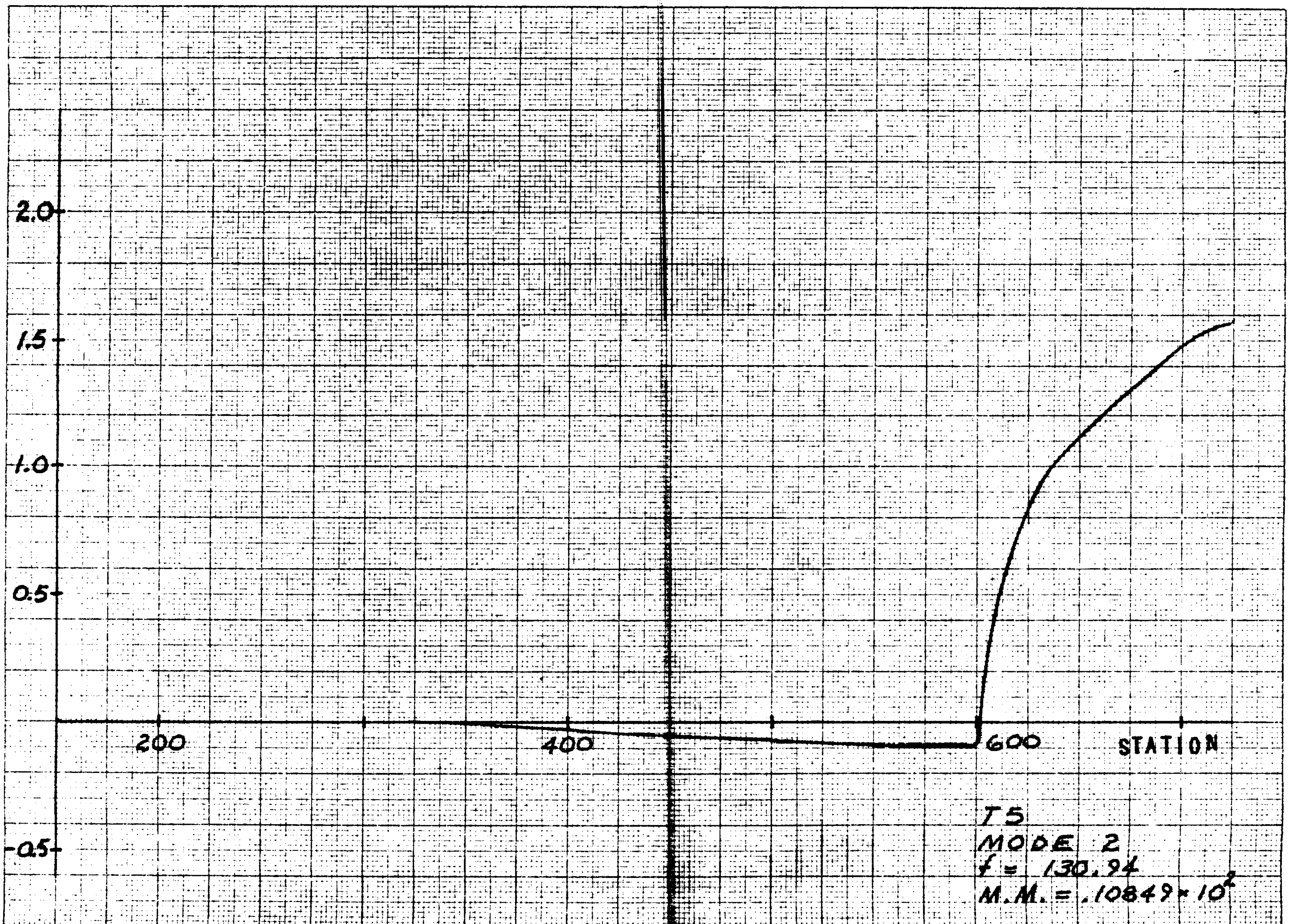
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200
400
600
STATION
T5
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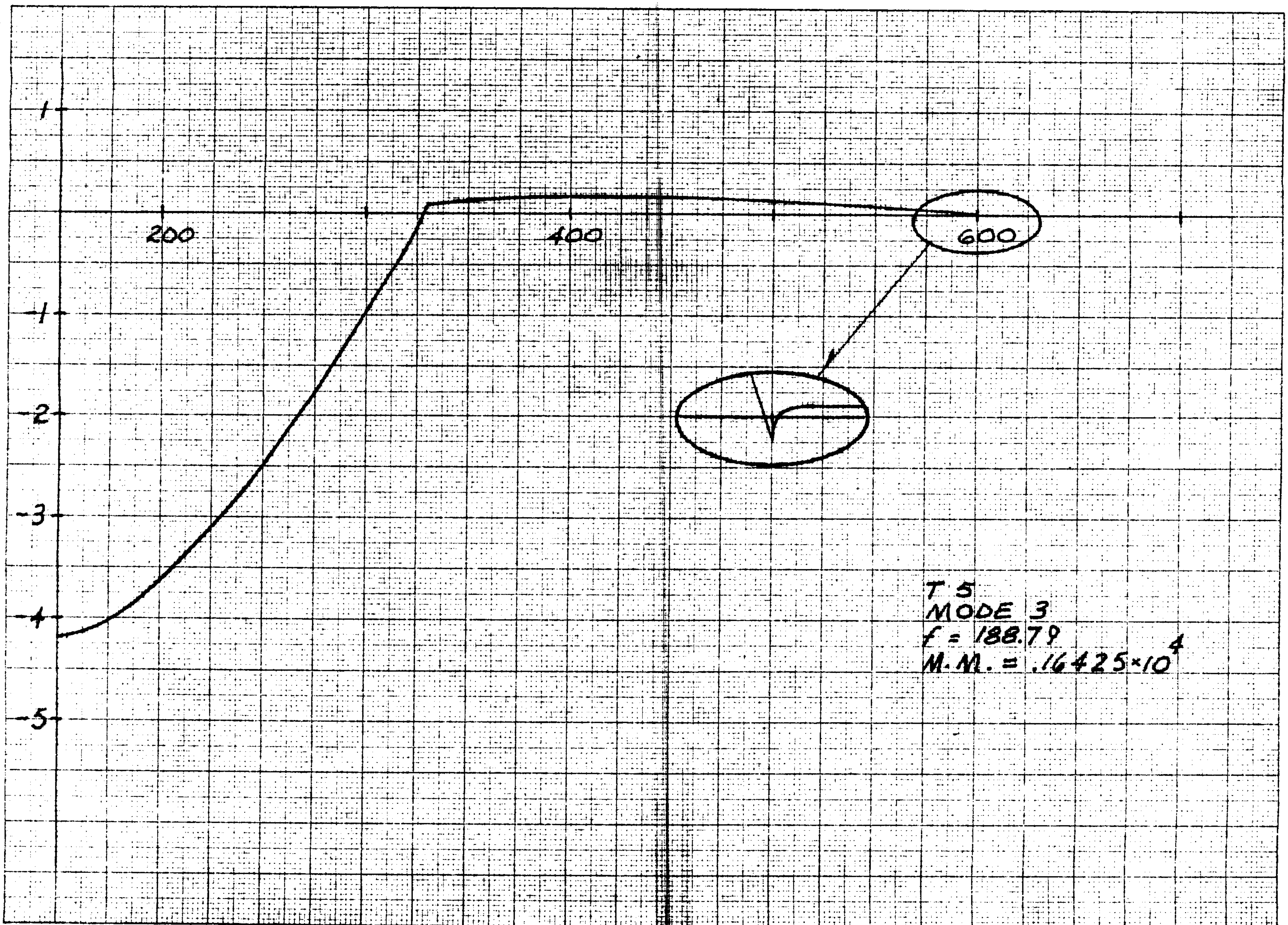
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-5
200
400
600
T 5
MODE 3
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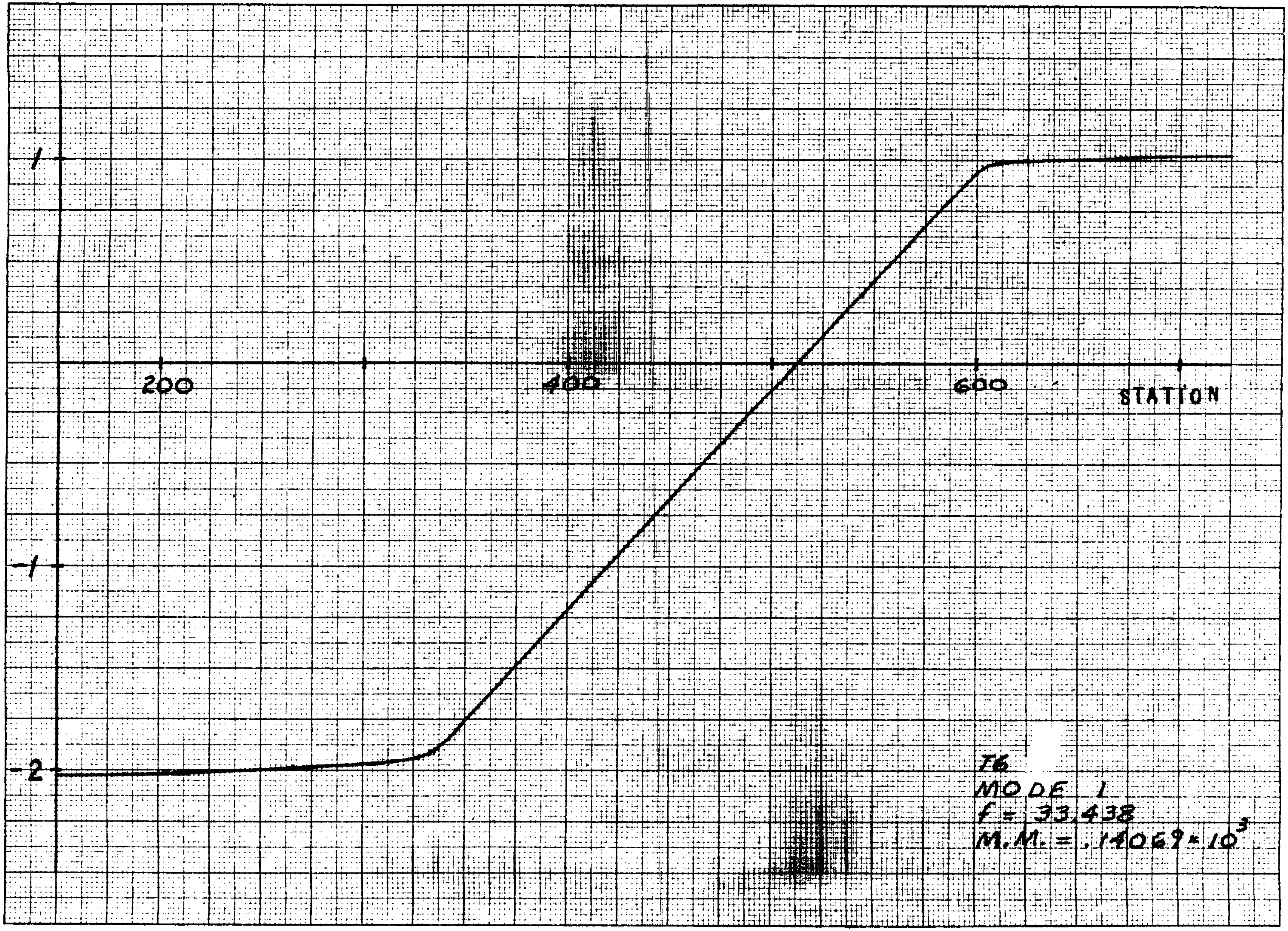
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200
400
600
STATION
T6
MODE 1
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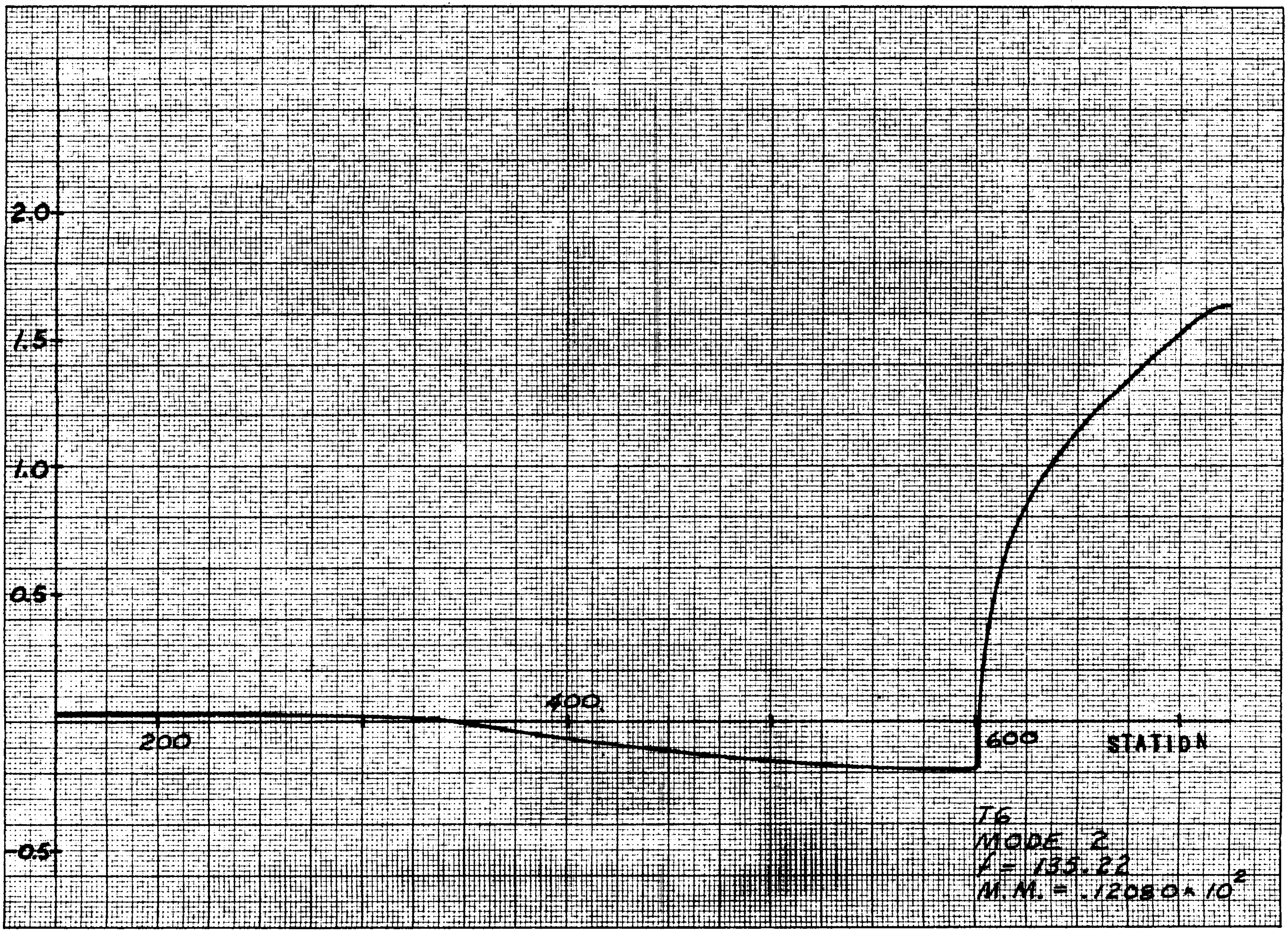
M.M. = .14069 × 10^3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200
400
600
STATION
T6
MODE 2
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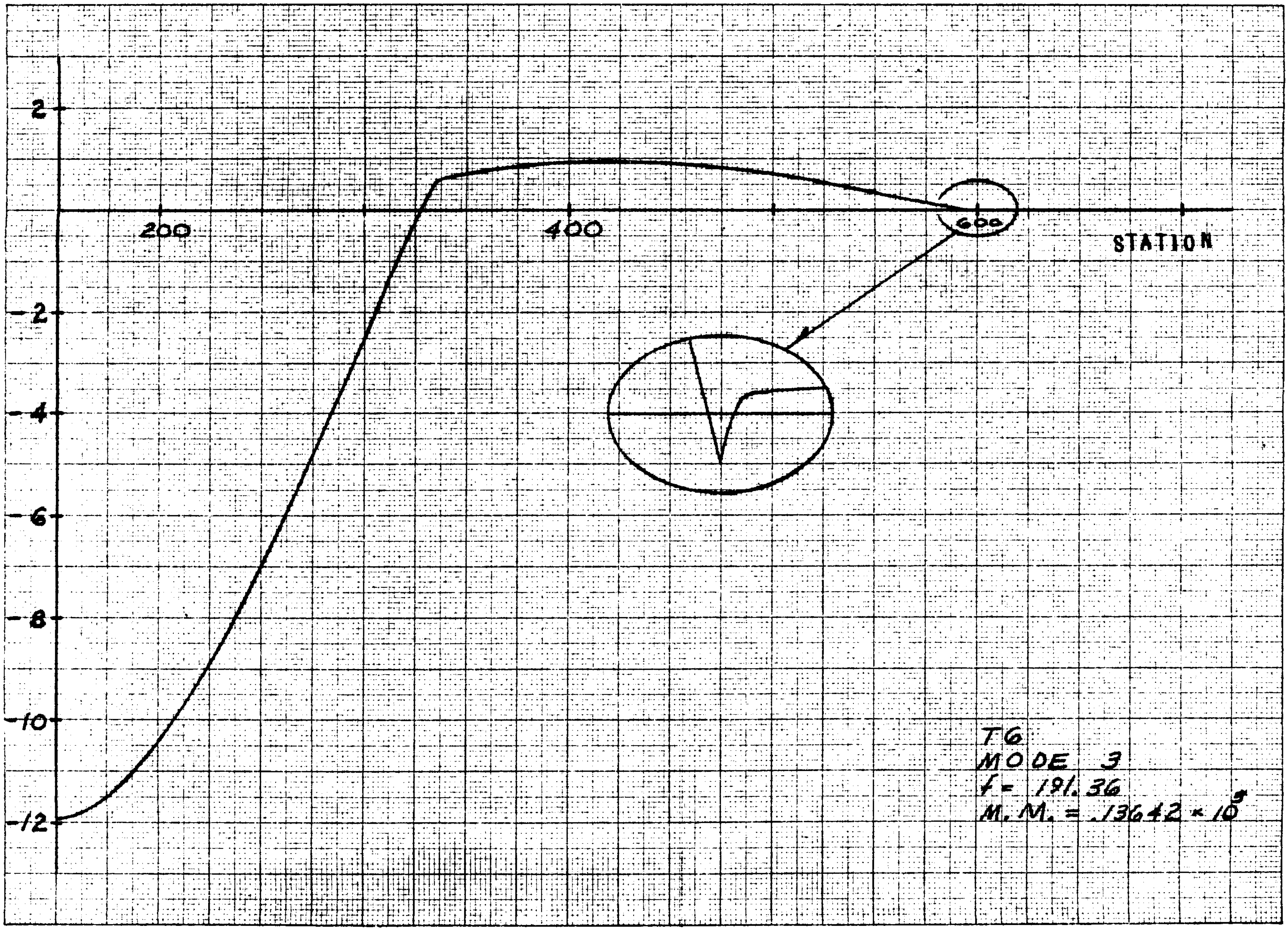
SM 46346



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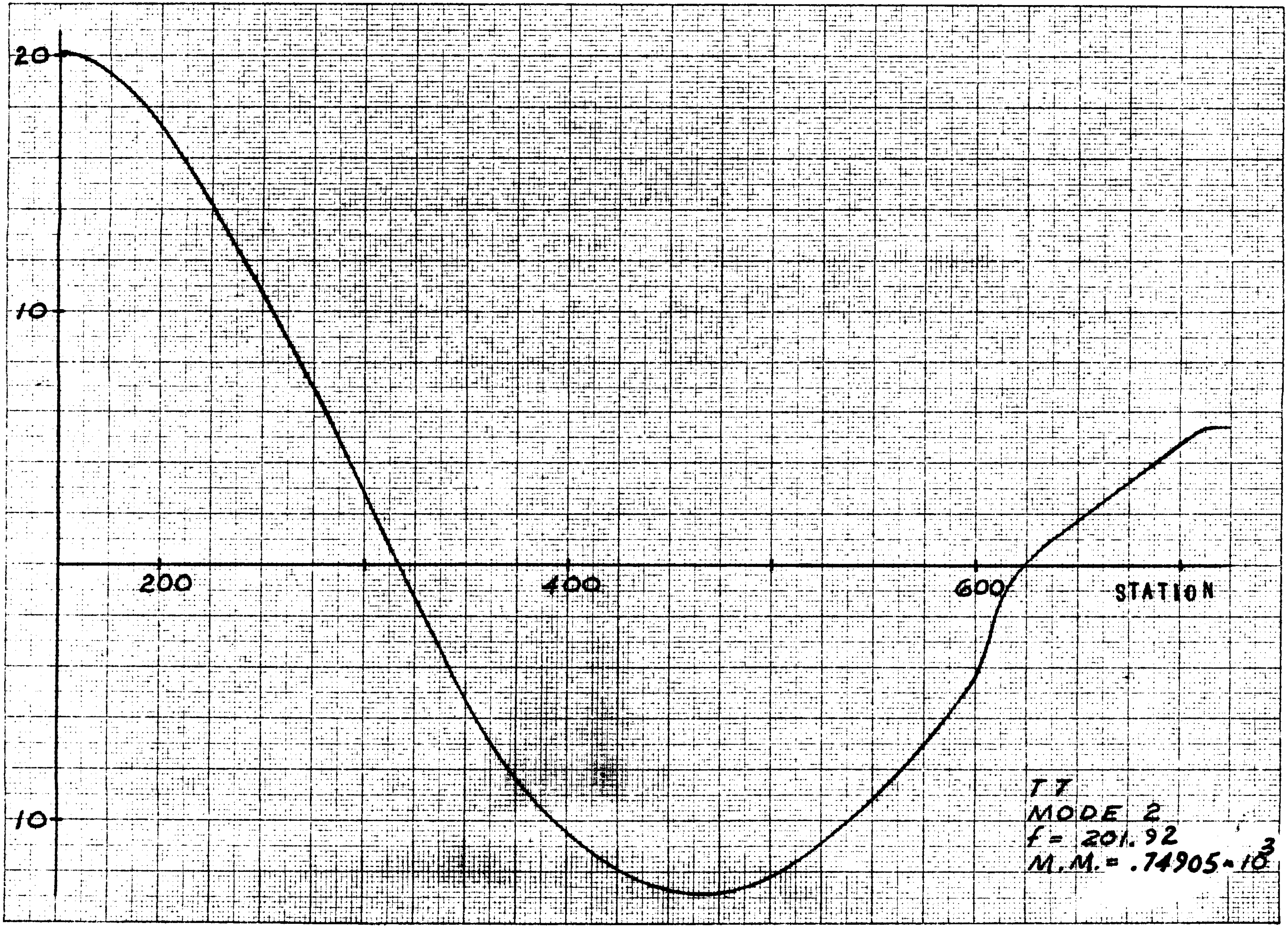
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10
10
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STATION
T.T
MODE 2
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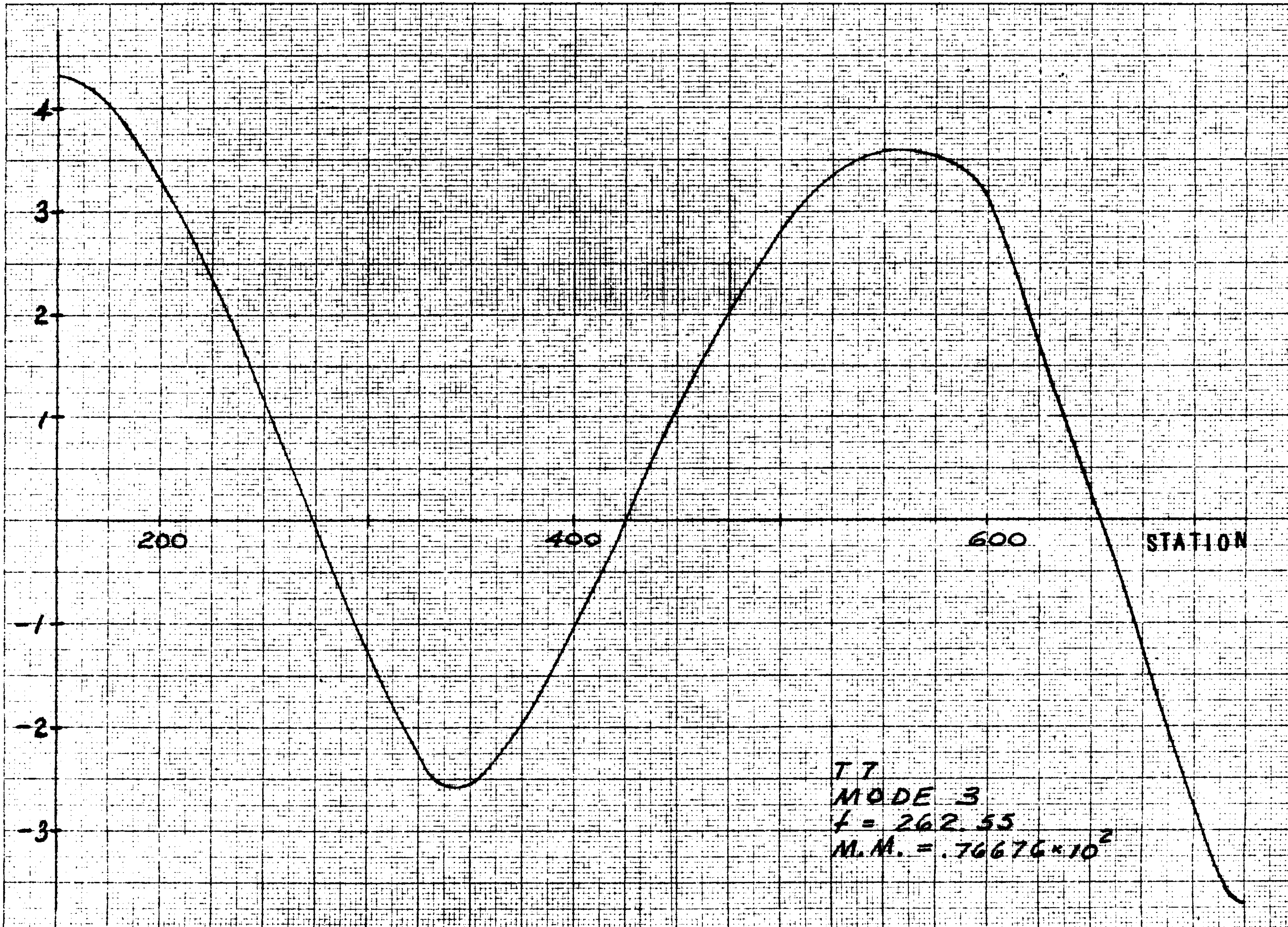
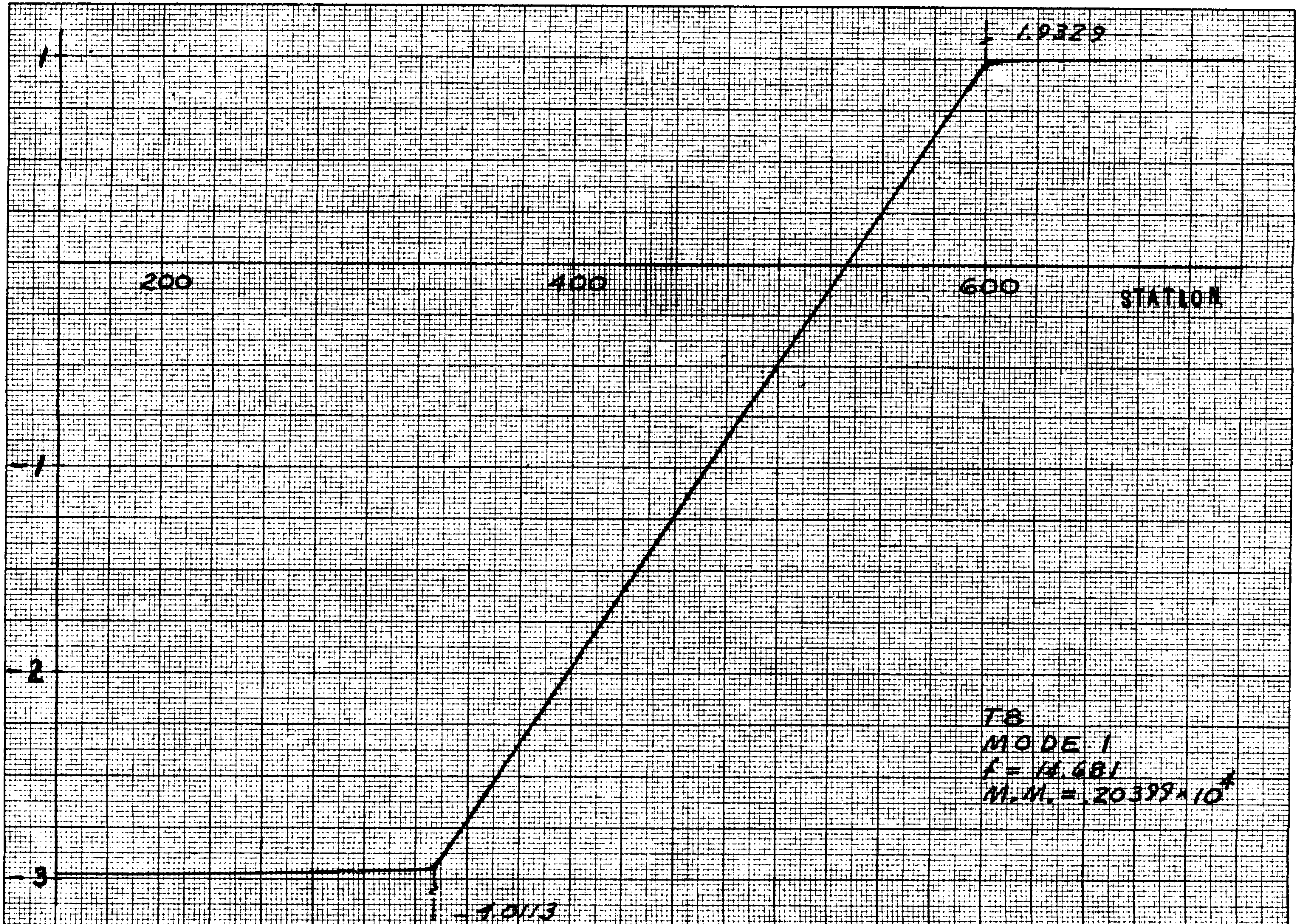
SM 46346

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200
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-2
T8
MODE 1
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-3
-1.0113


1.2
1.0
0.8
0.6
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200
400
600
STATION
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MODE 2
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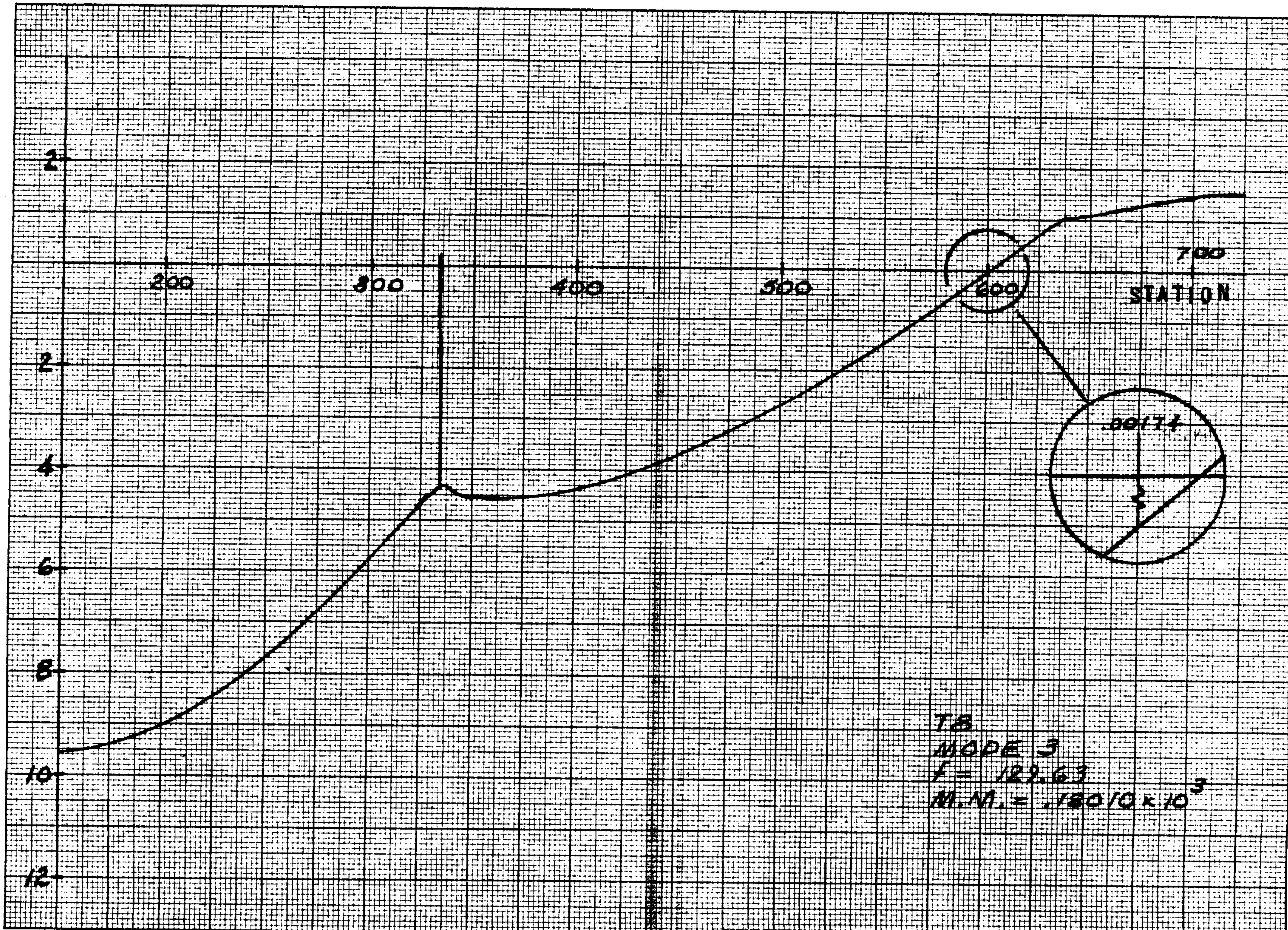
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2
4
6
8
10
12
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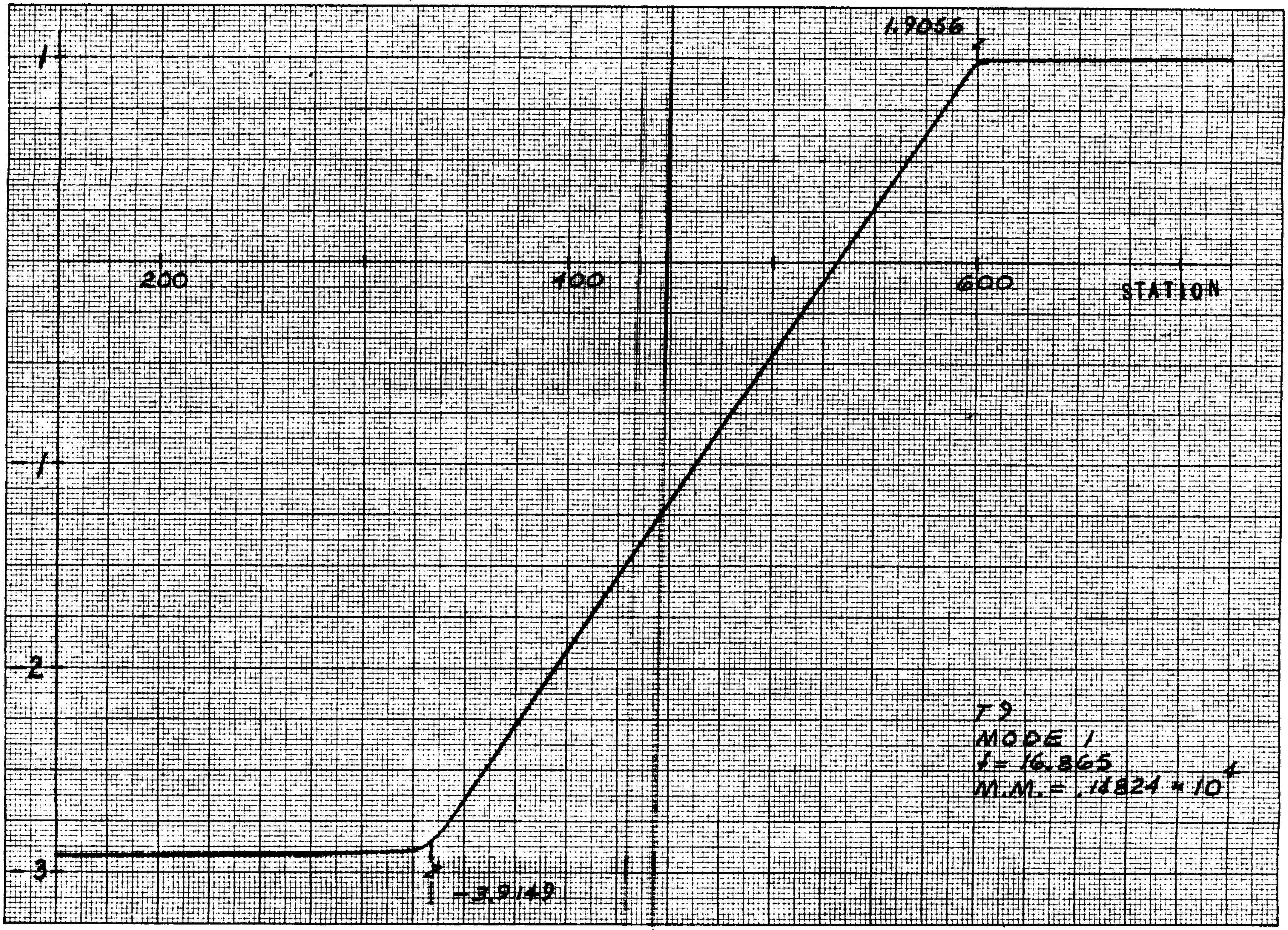
M.M. = .18010 x 10^3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1
2
3
200
400
600
STATION
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T 9
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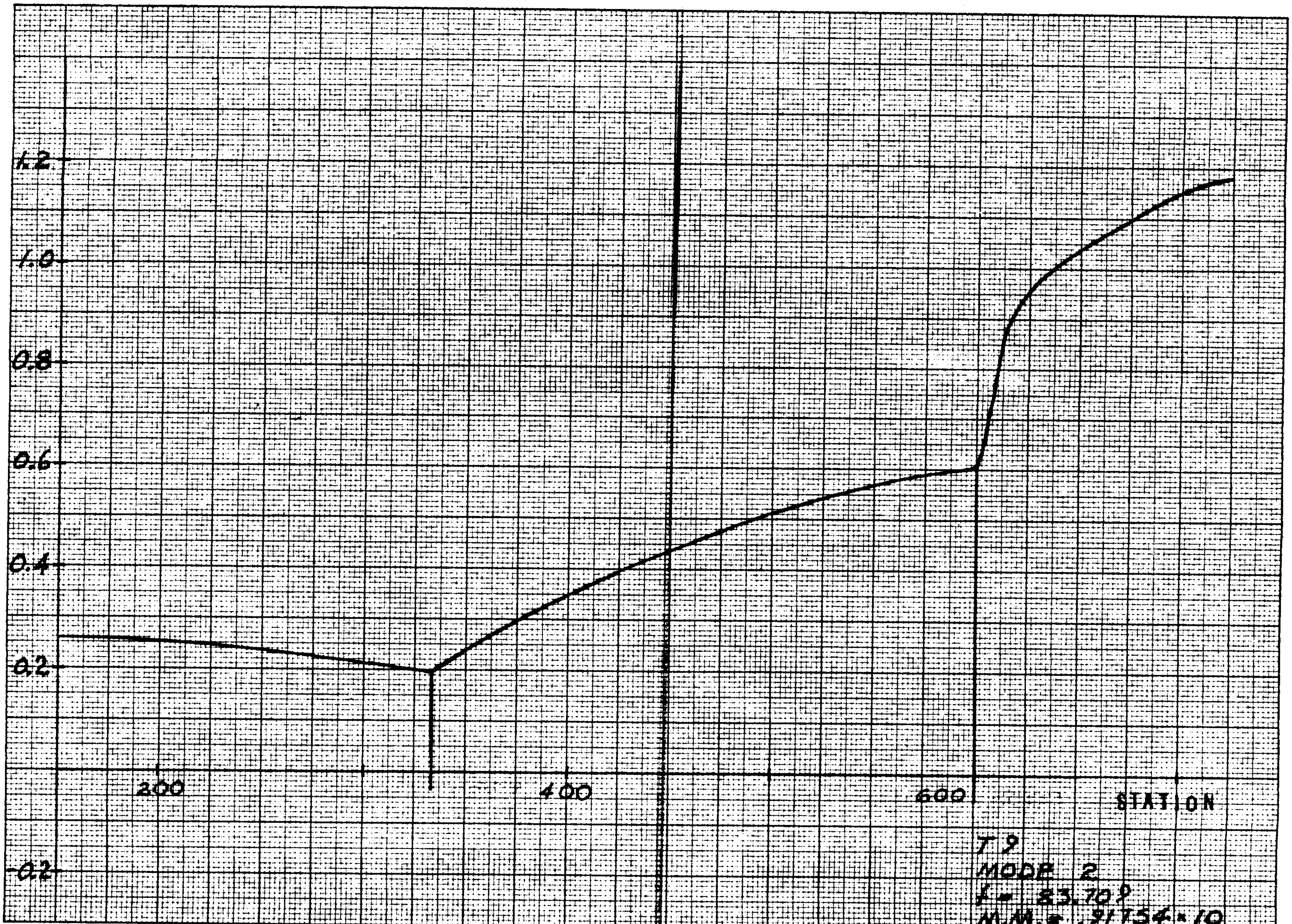
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200
400
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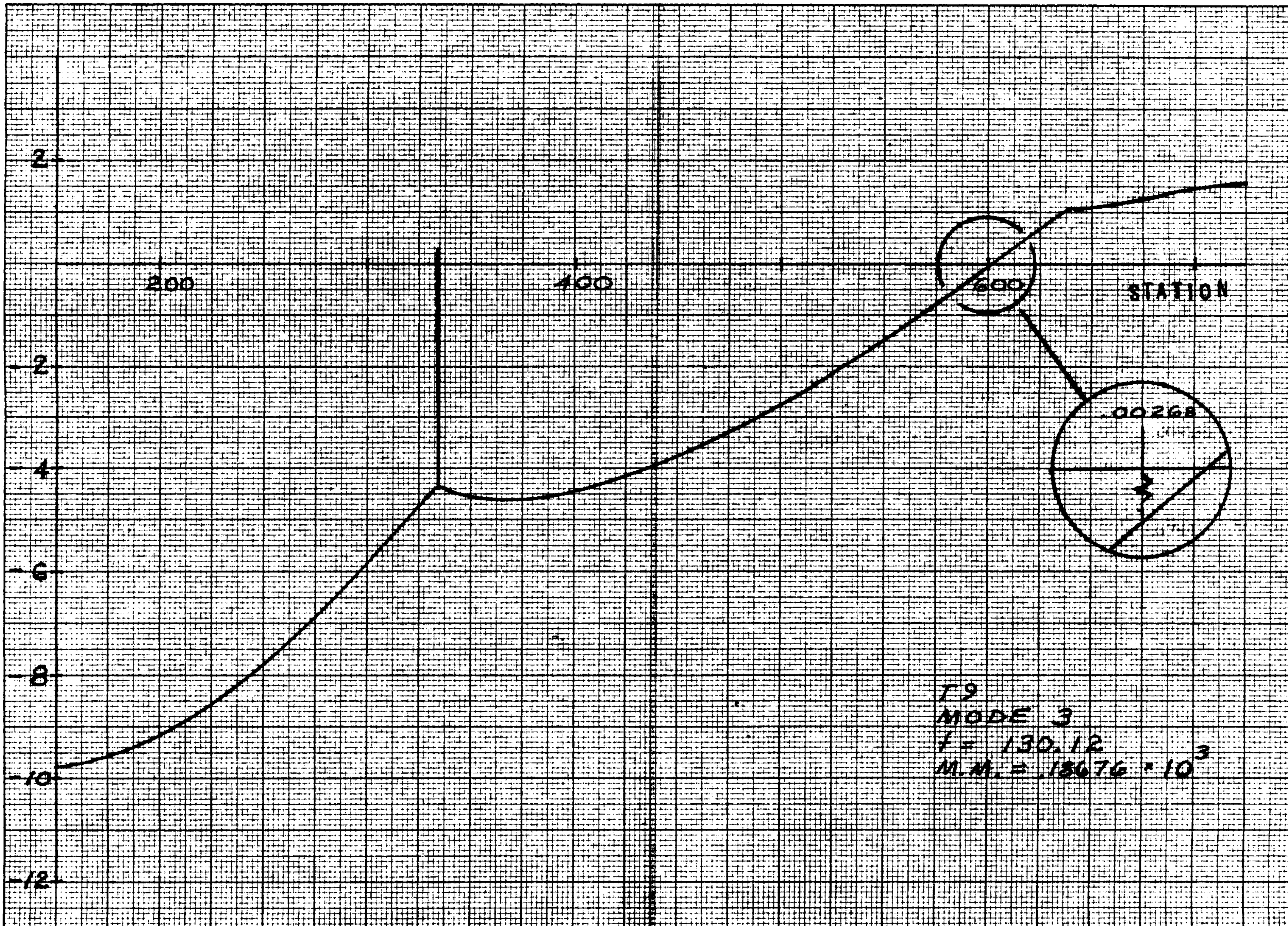
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200
400
600
STATION
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T9
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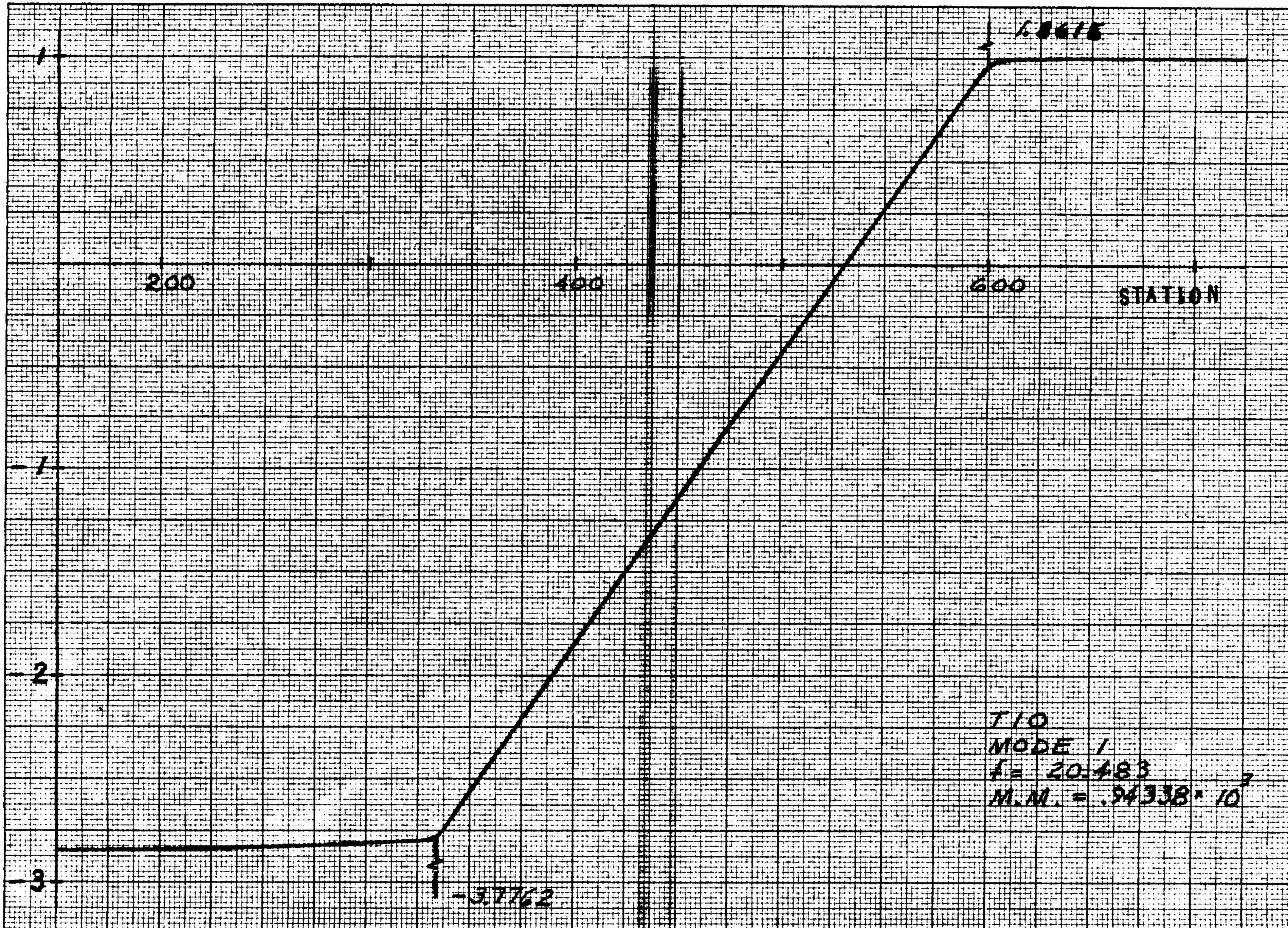
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-3
200
400
600
STATION
-.37762
T/O
MODE 1
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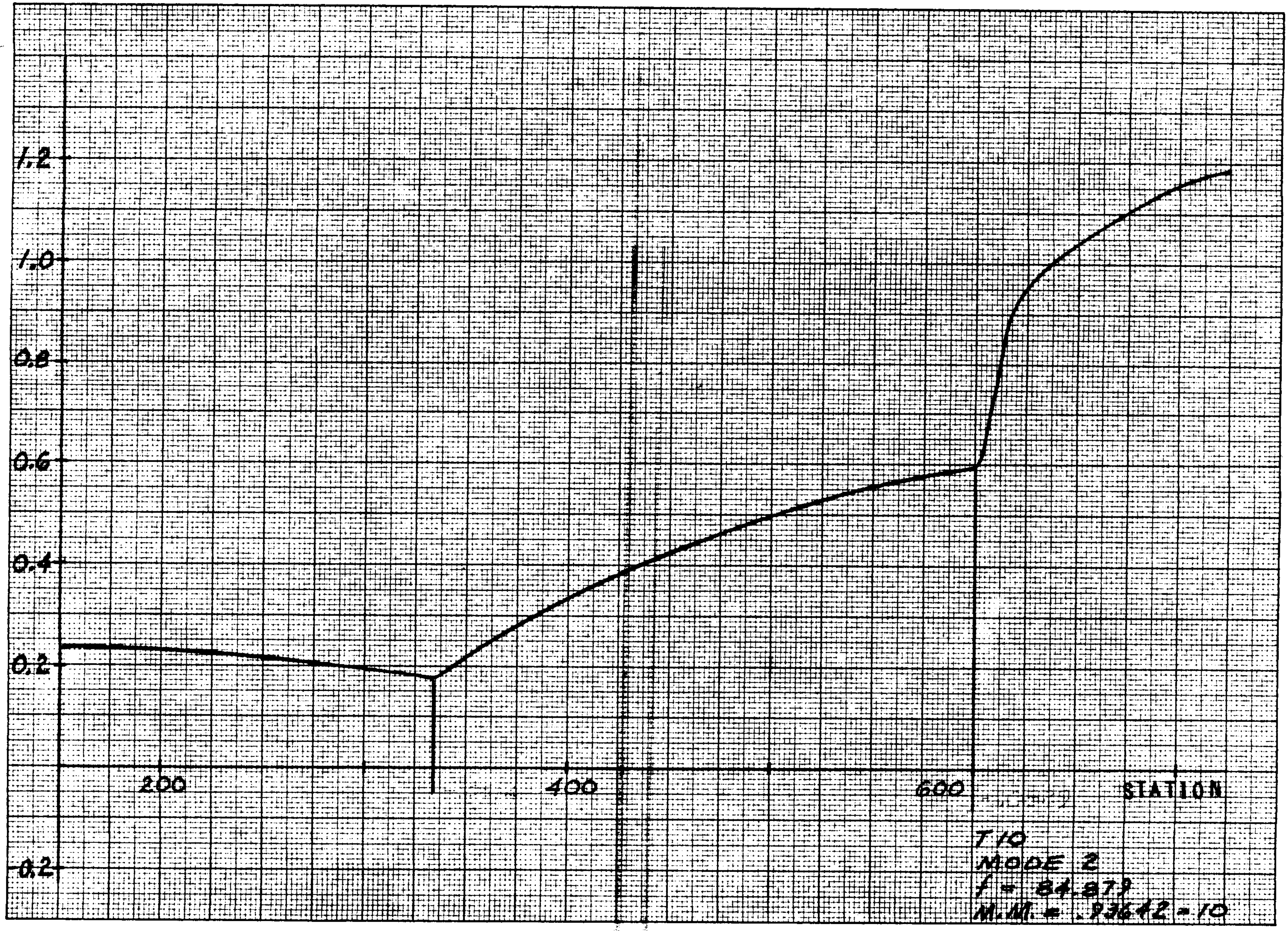
SM 46346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0.6
0.4
0.2
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200
400
600
STATION
T10
MODE 2
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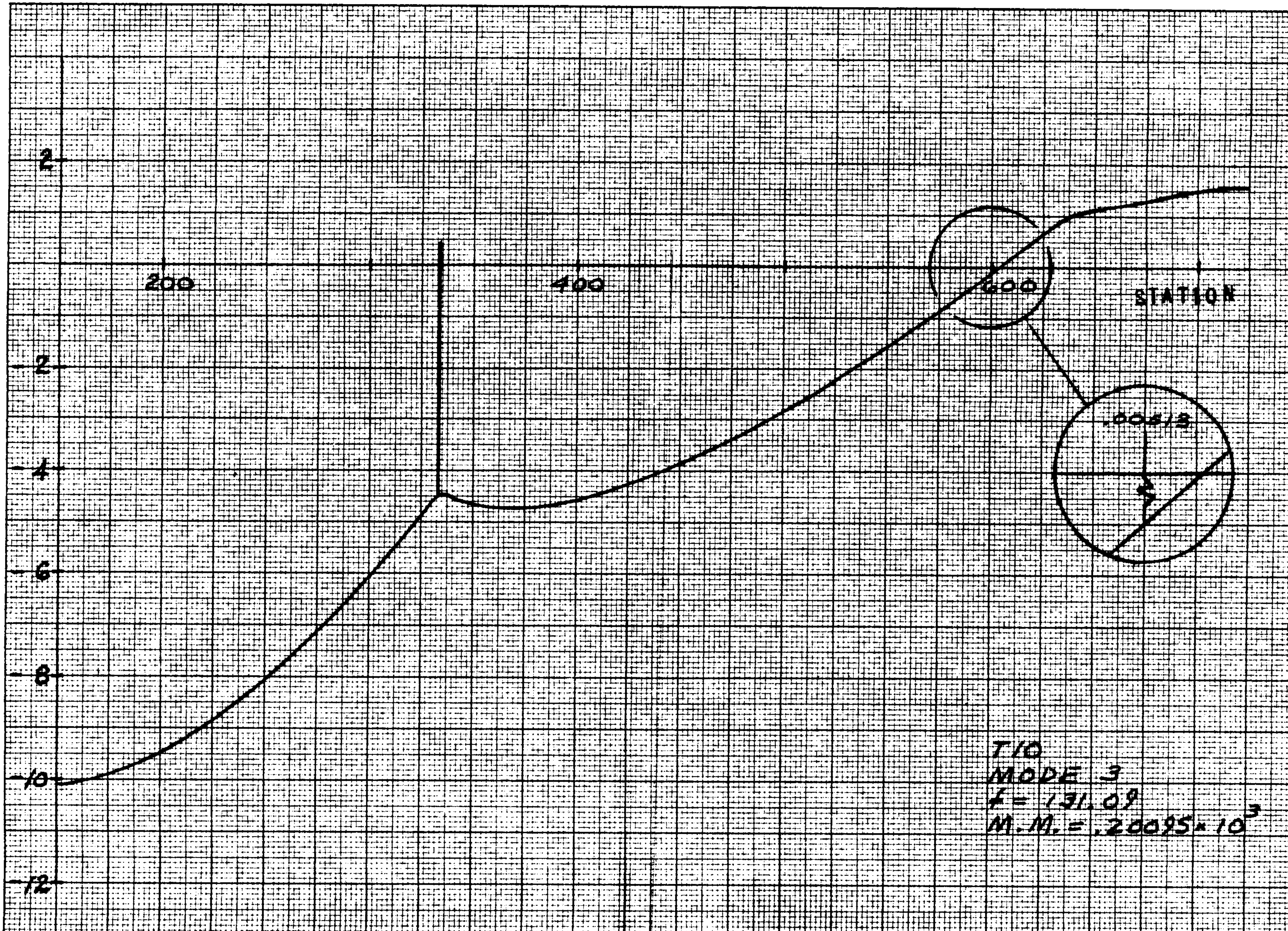
SM 46346


2
2
4
6
8
10
12
200
400
600
STATION
.00513
T/O
MODE 3
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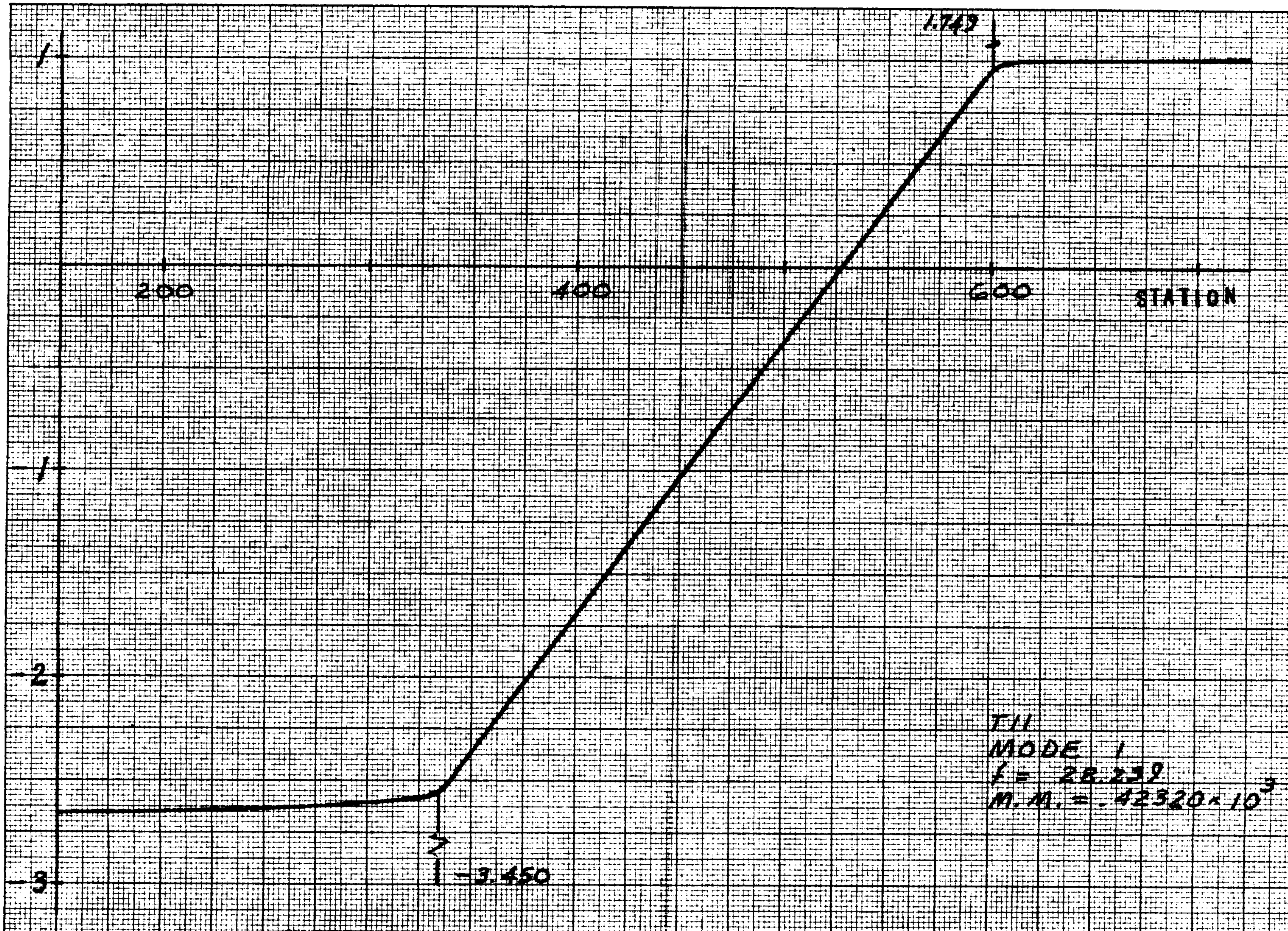
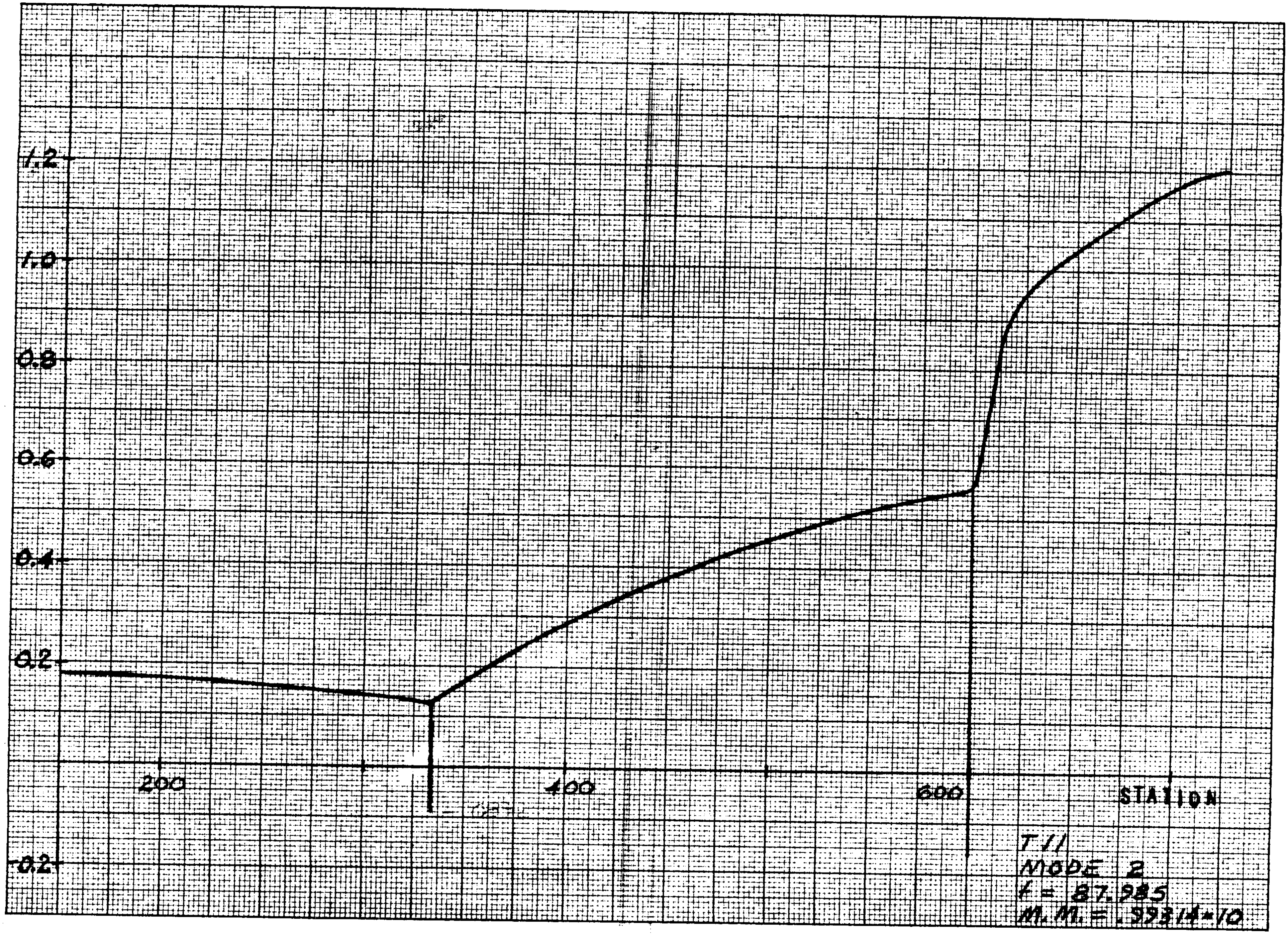
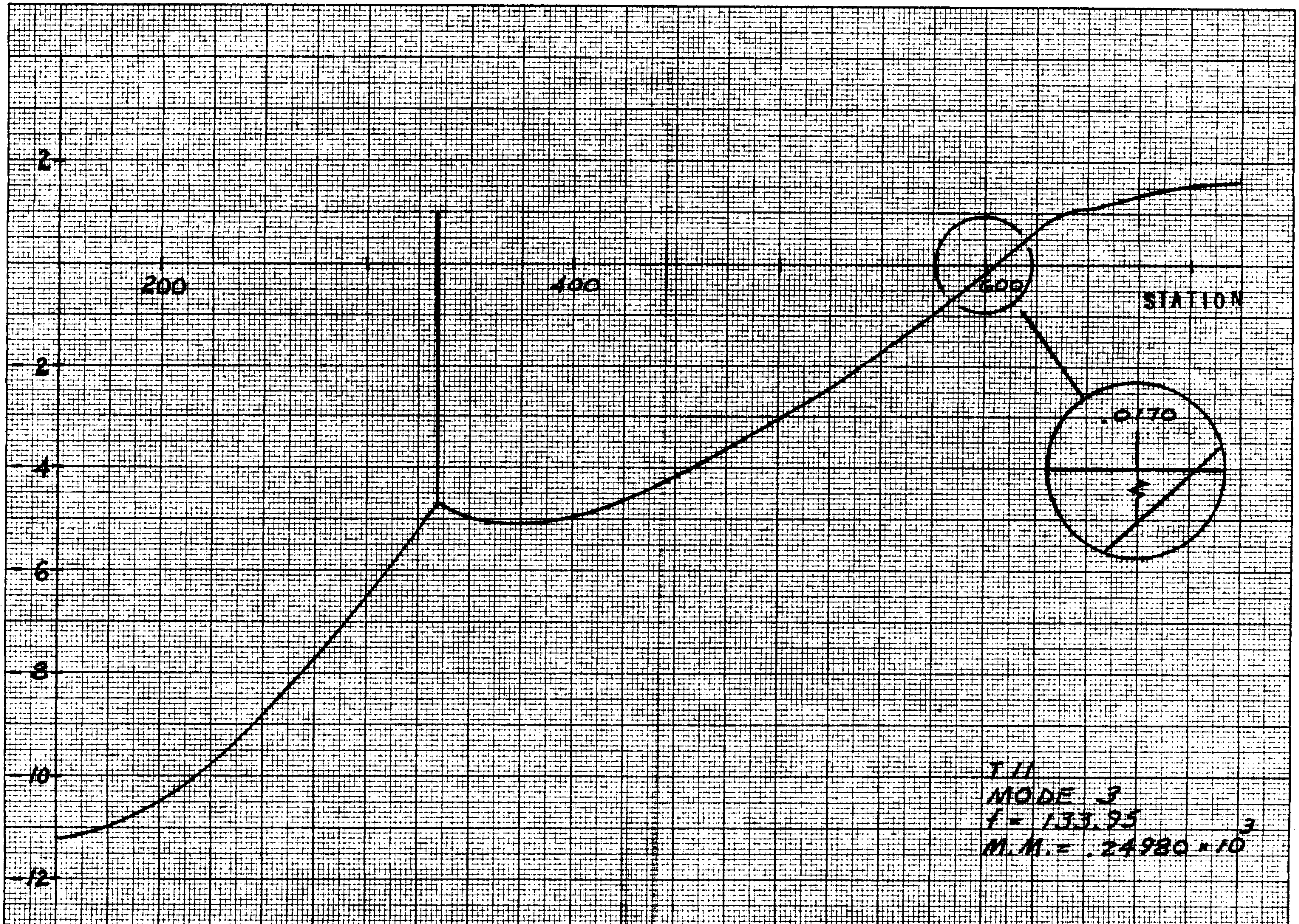

1.749
1
200
400
600
STATION
1
2
3
-3.450
T11
MODE 1
f = 28.239
M.M. = .42320 × 10^3

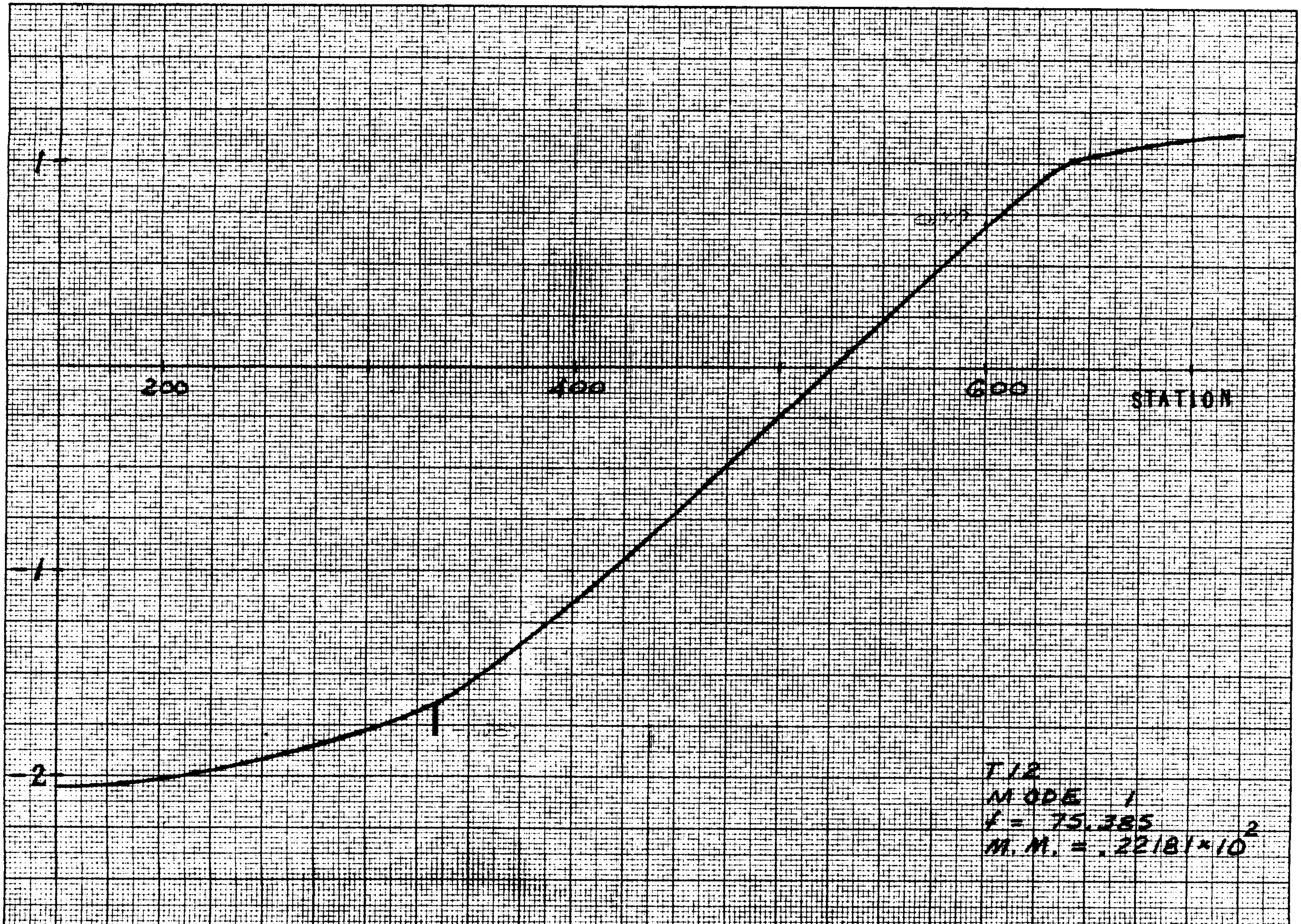

1.2
1.0
0.8
0.6
0.4
0.2
-0.2
200
400
600
STATION
T/1
MODE 2
f = 87.985
M.M. = .99314×10

SM 46346

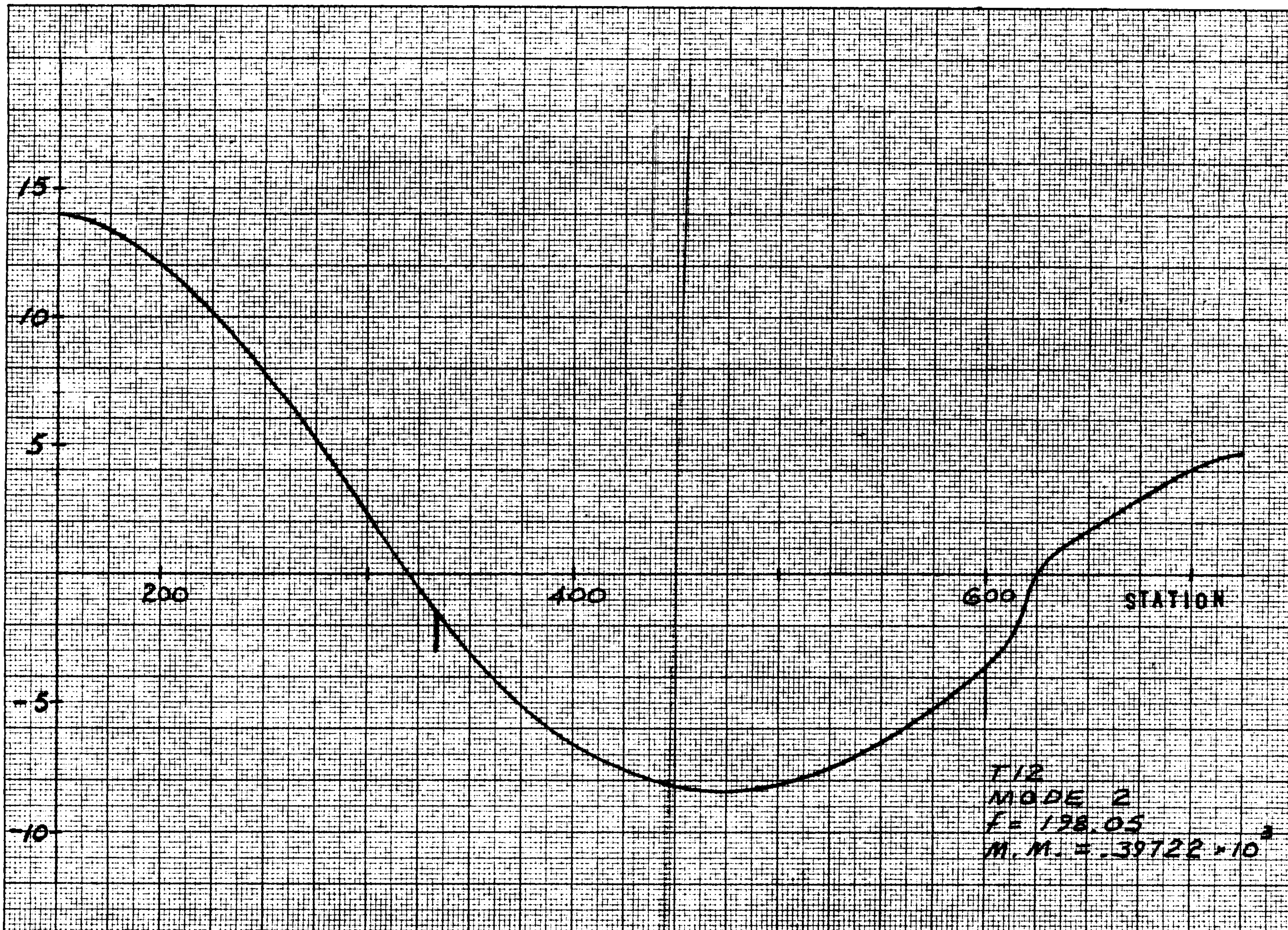

2
-2
-4
-6
-8
-10
-12
200
400
600
STATION
.0170
T 11
MODE 3
f = 133.95
M.M. = .24980 × 10^3

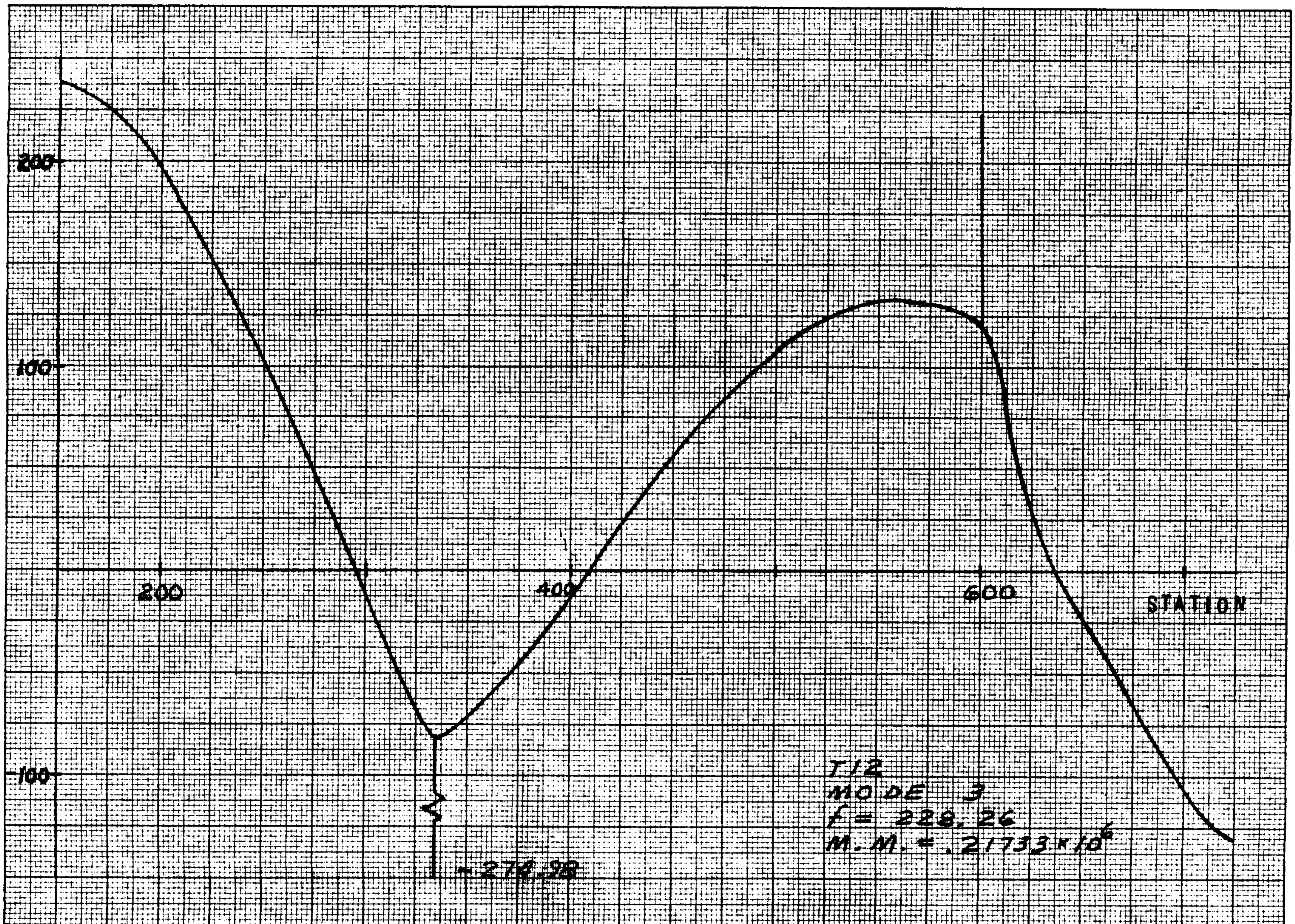
K+E 10 X 10 TO THE CENTIMETER 46 1517
18 X 25 CM. • ALBANENE® MADE IN U.S.A.
KEUFFEL & ESSER CO.

SM 46346

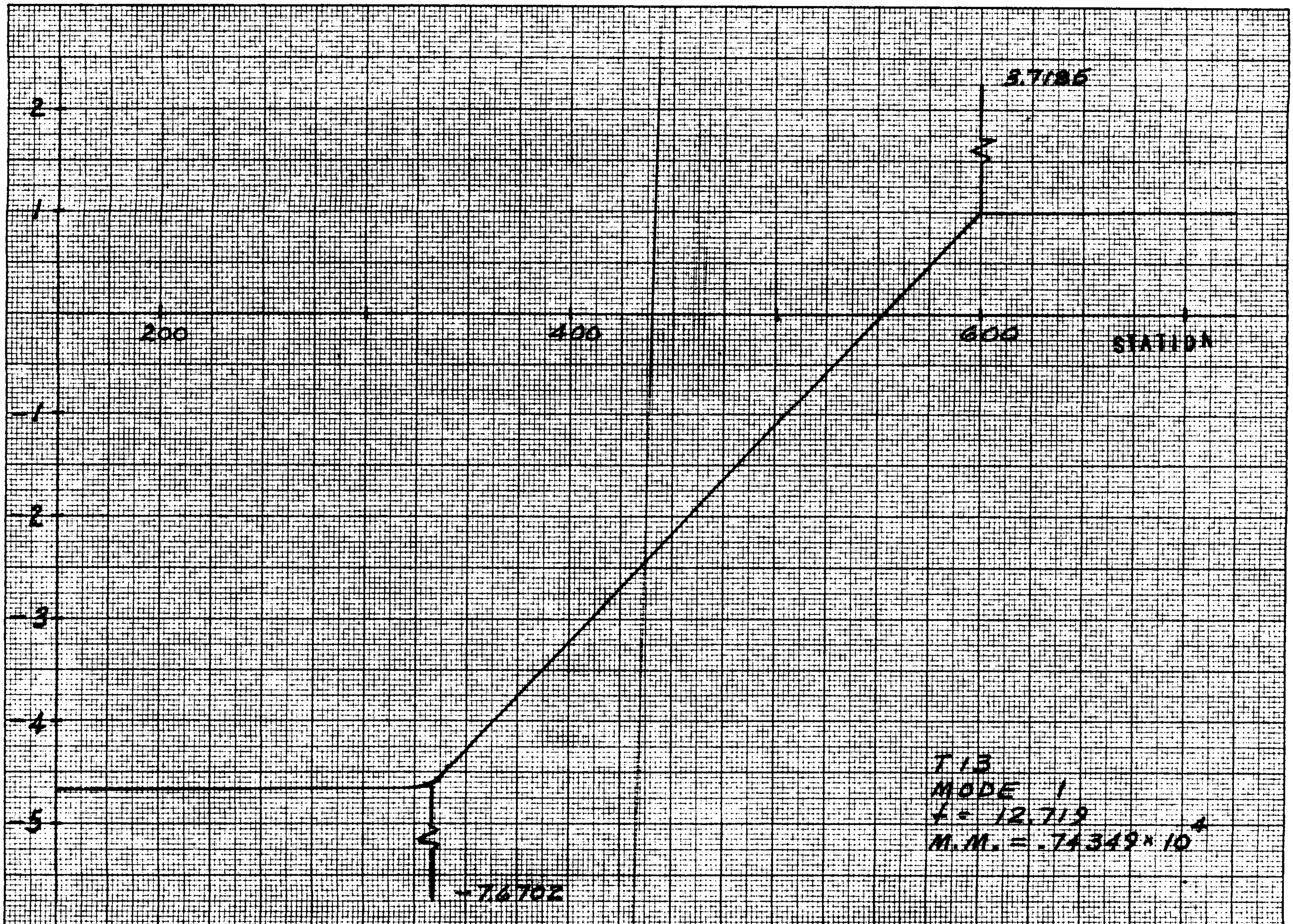


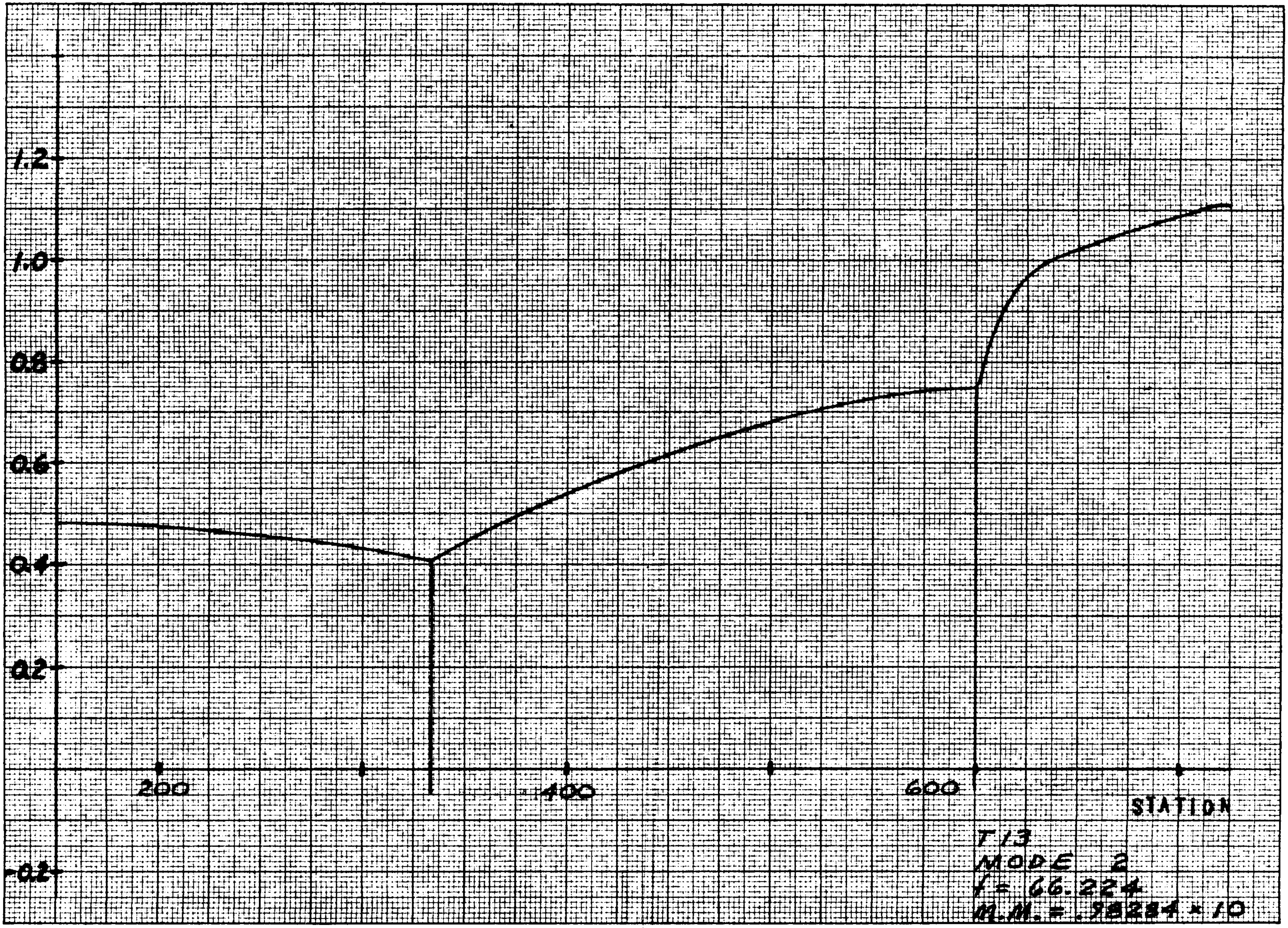
SM 46346



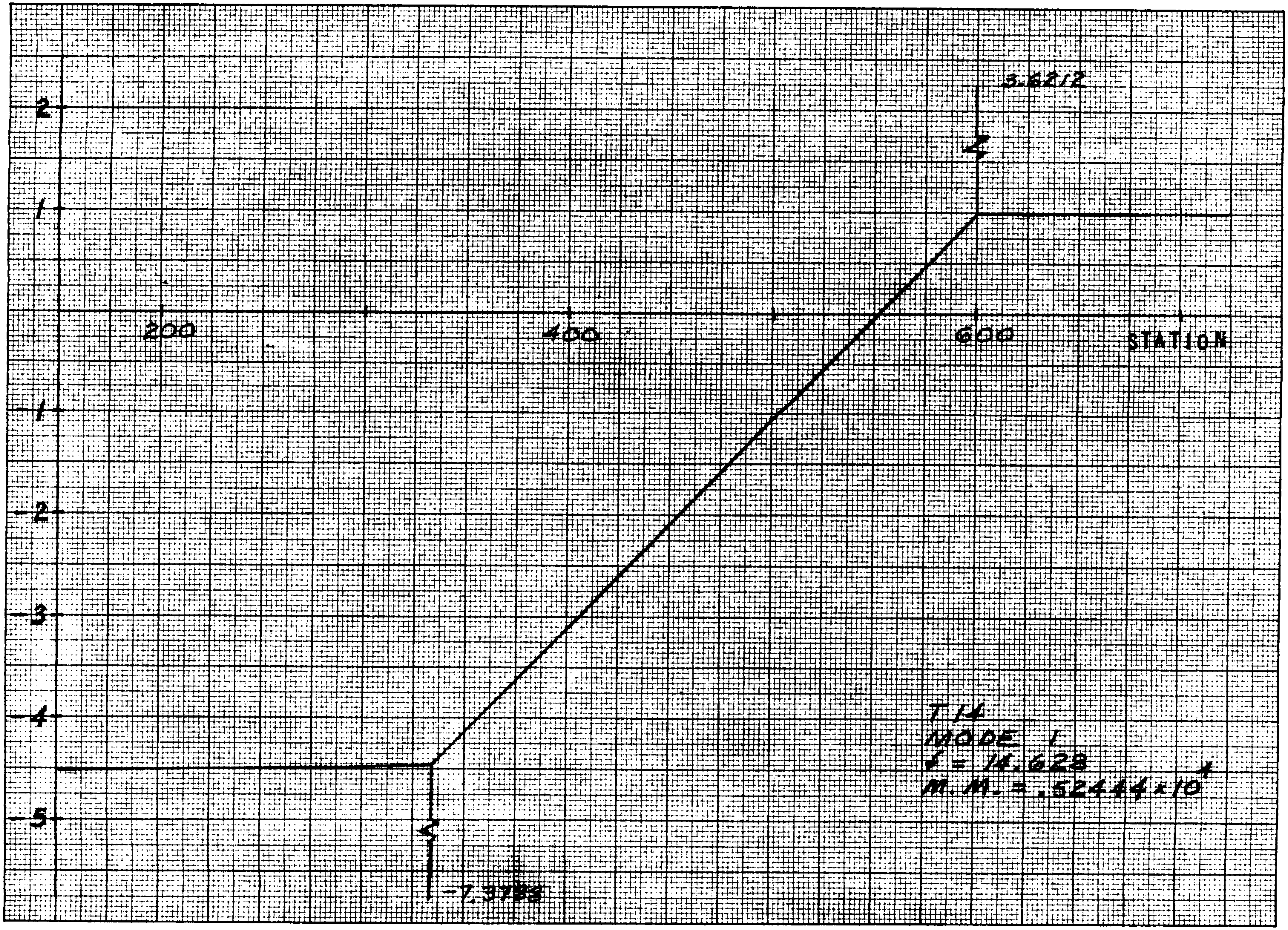

200
100
-100
200
400
600
STATION
-274.98
T12
MODE 3
f = 228.26
M.M. = .21733 × 10^6

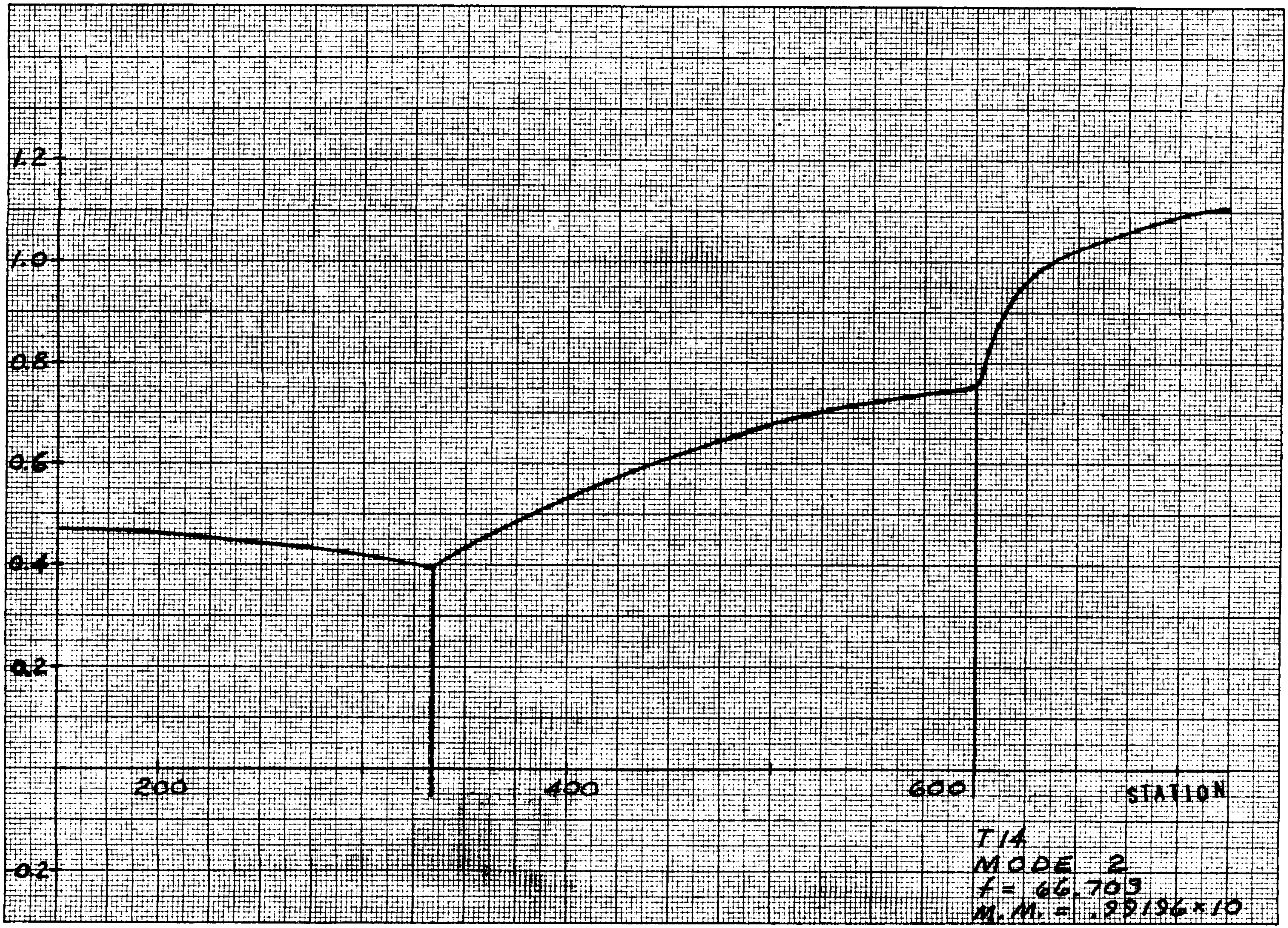
SM 46346

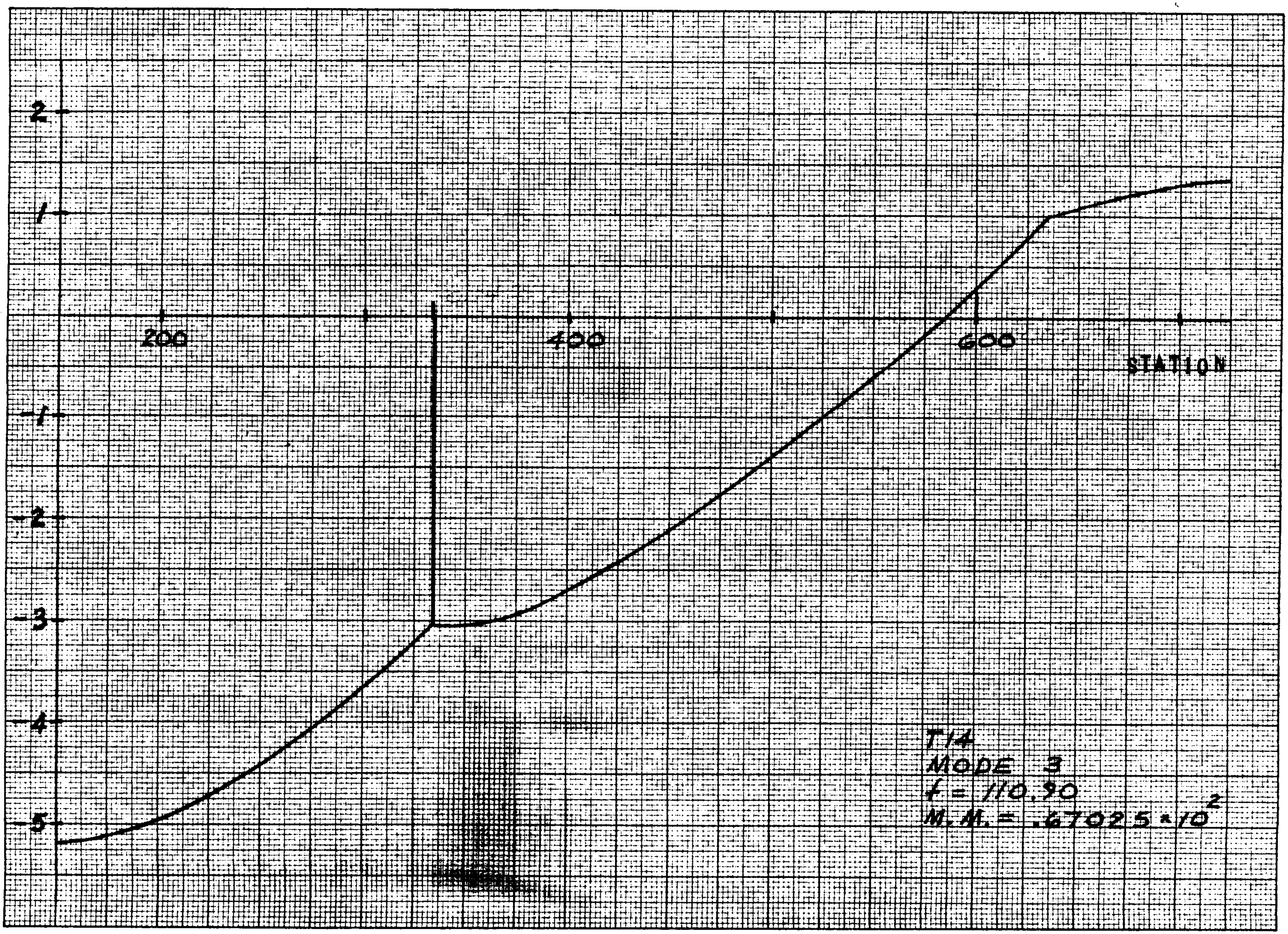

3.7185
2
1
200
400
600
STATION
-1
-2
-3
-4
-5
-7.6702
T 13
MODE 1
f = 12.719
M.M. = .74349 × 10^4


1.2
1.0
0.8
0.6
0.4
0.2
-0.2
200
400
600
STATION
T13
MODE 2
f = 66.224

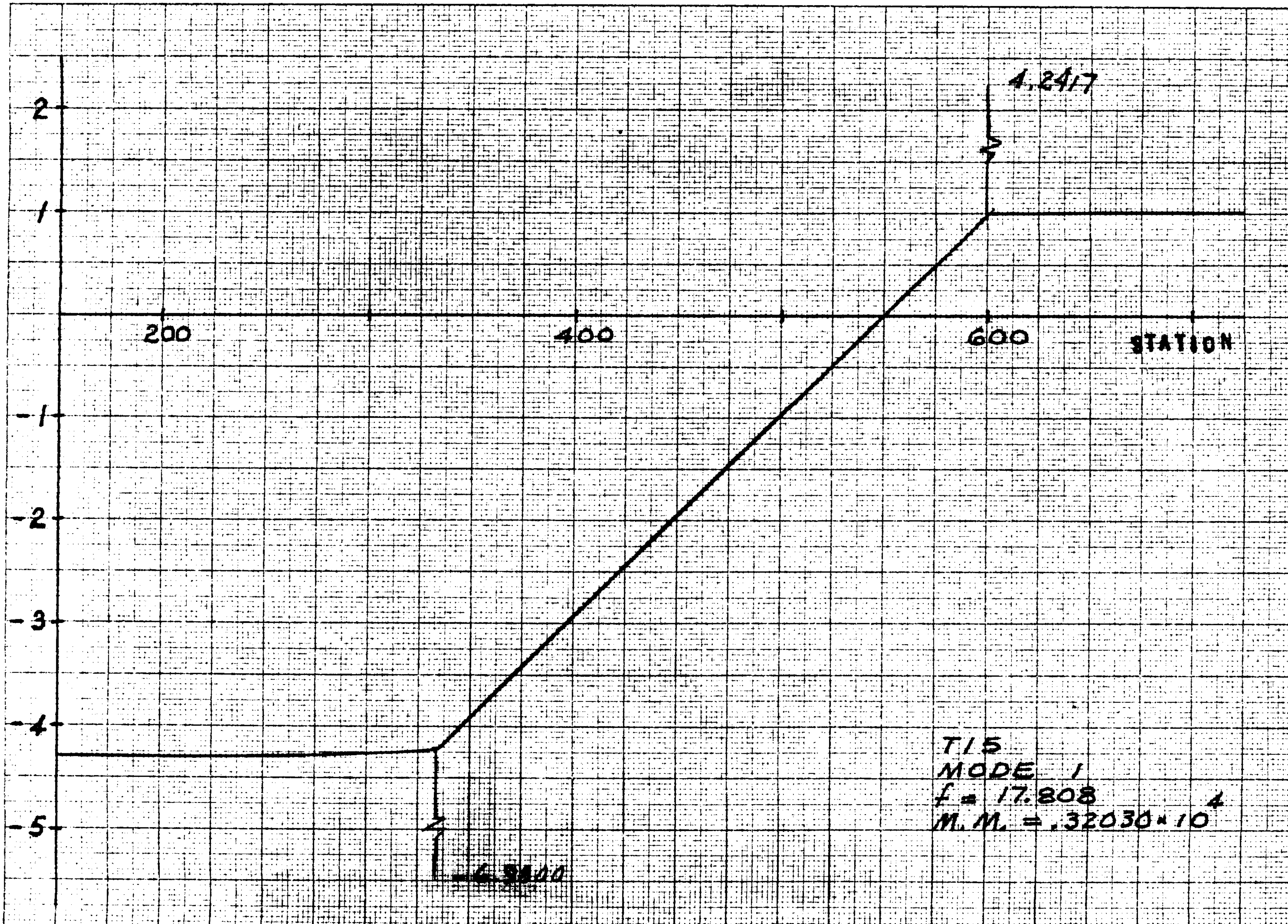

2
1
-1
-2
-3
-4
-5
200
400
600
STATION
T 13
MODE 3
f = 110.65
M.M. = .65865 × 10^2

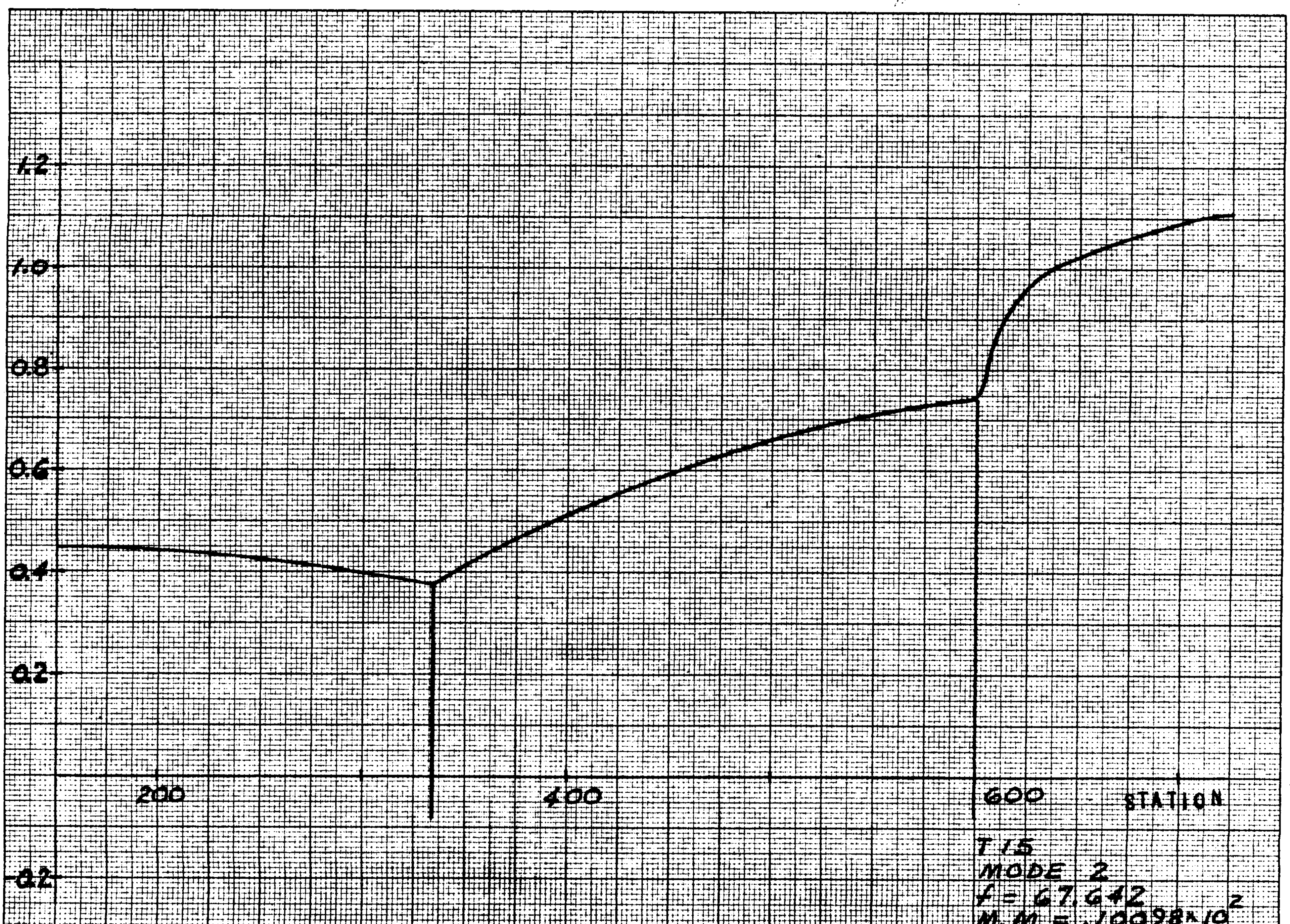

3.6212
2
1
-1
-2
-3
-4
-5
200
400
600
STATION
T14
MODE 1
f = 14.628
M.M. = .52444 x 10^4

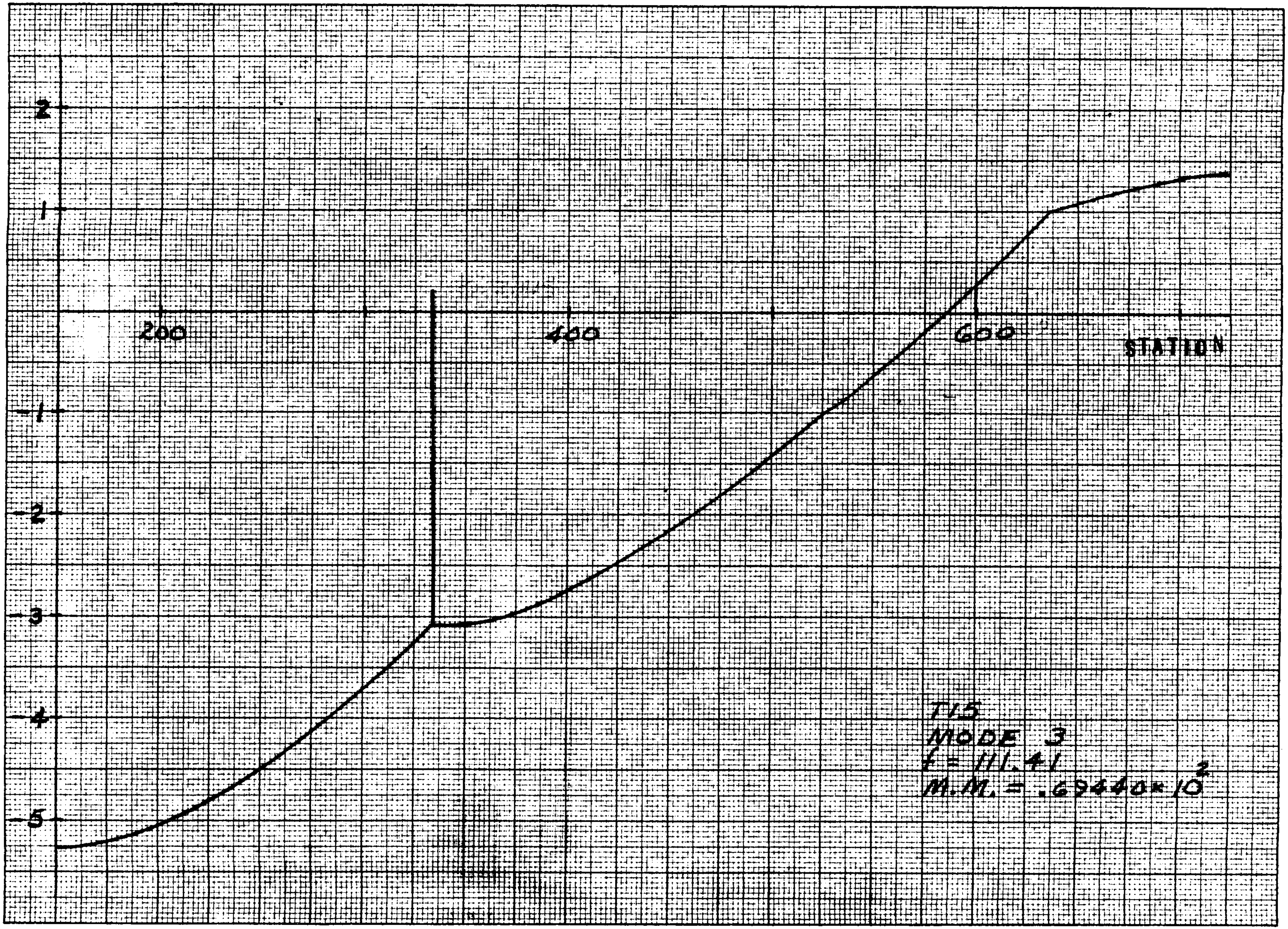

1.2
1.0
0.8
0.6
0.4
0.2
-0.2
200
400
600
STATION
T14
MODE 2
f = 66.703
M.M. = .99196 × 10

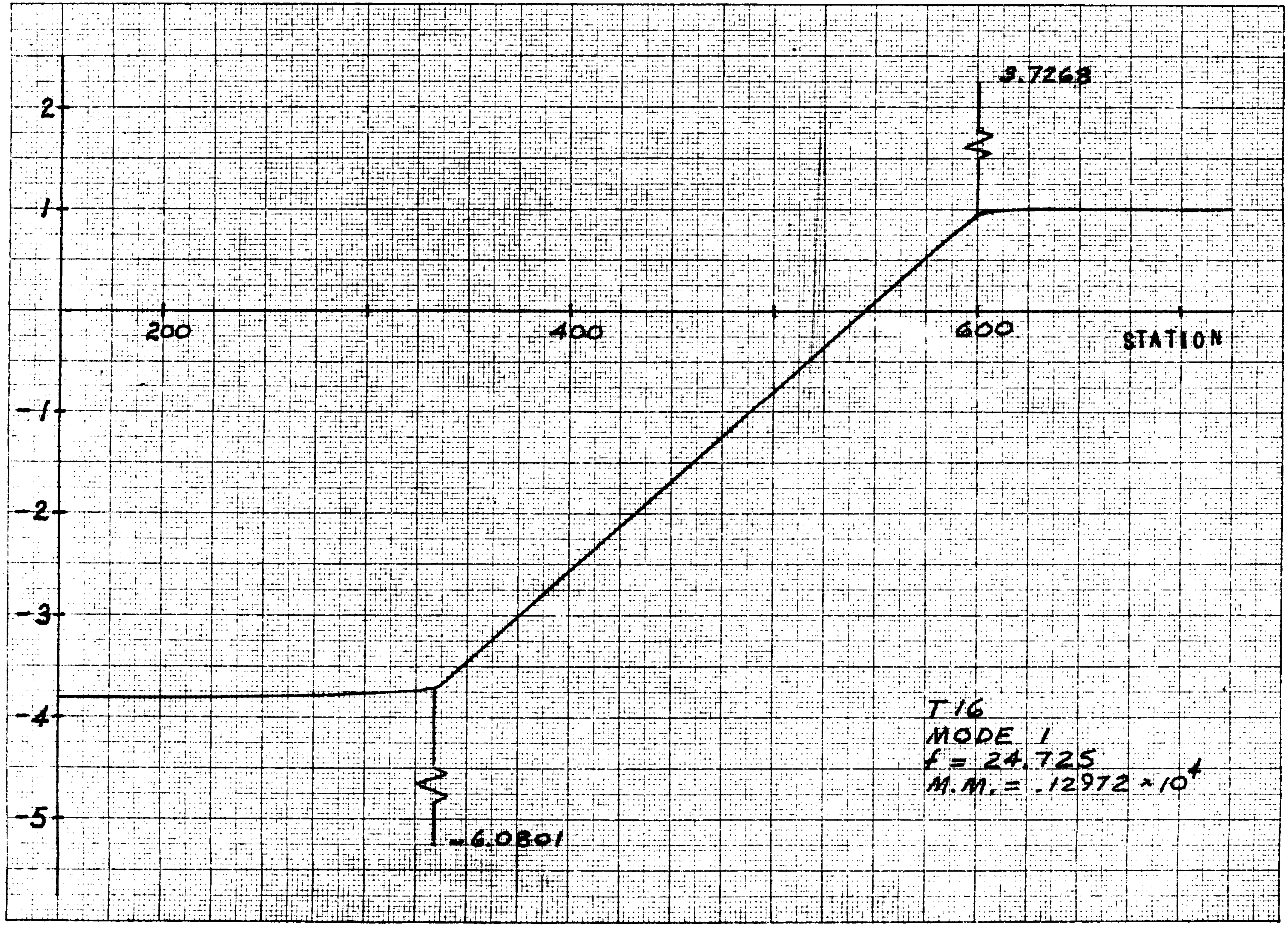

2
1
-1
-2
-3
-4
-5
200
400
600
STATION
T/4
MODE 3
f = 110.90
M.M. = .67025 × 10²

SM 46346

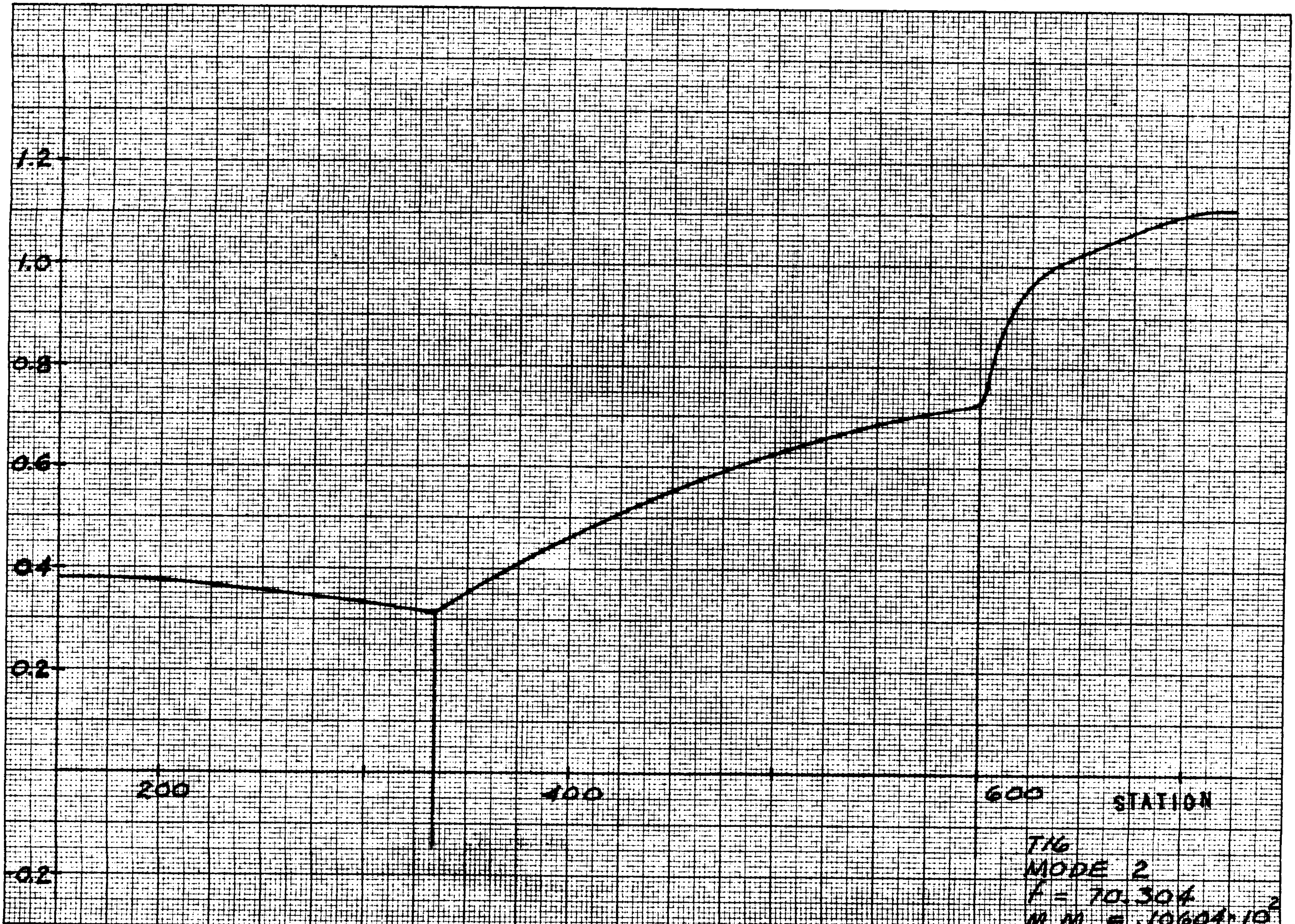

4.2417
2
1
200
400
600
STATION
-1
-2
-3
-4
-5
T15
MODE 1
f = 17.808
M.M. = .32030 × 10^4


1.2
1.0
0.8
0.6
0.4
0.2
-0.2
200
400
600
STATION
T 15
MODE 2
f = 67.642
M.M. = .10098×10²

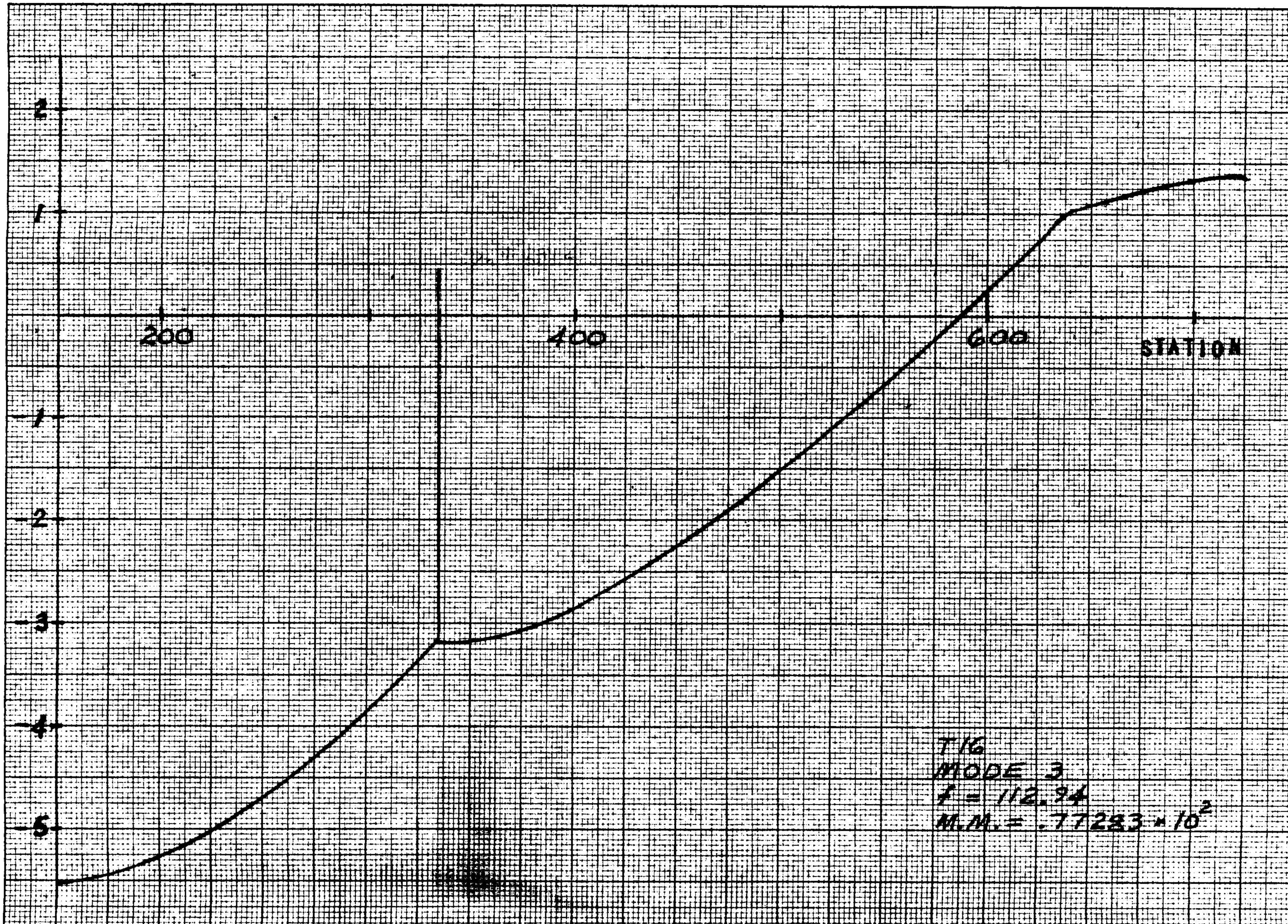

2
1
-1
-2
-3
-4
-5
200
400
600
STATION
T15
MODE 3
f = 111.41
M.M. = .69440×10²


3.7268
2
1
-1
-2
-3
-4
-5
200
400
600
STATION
-6.0801
T 16
MODE 1
f = 24.725
M.M. = .12972 × 10⁴

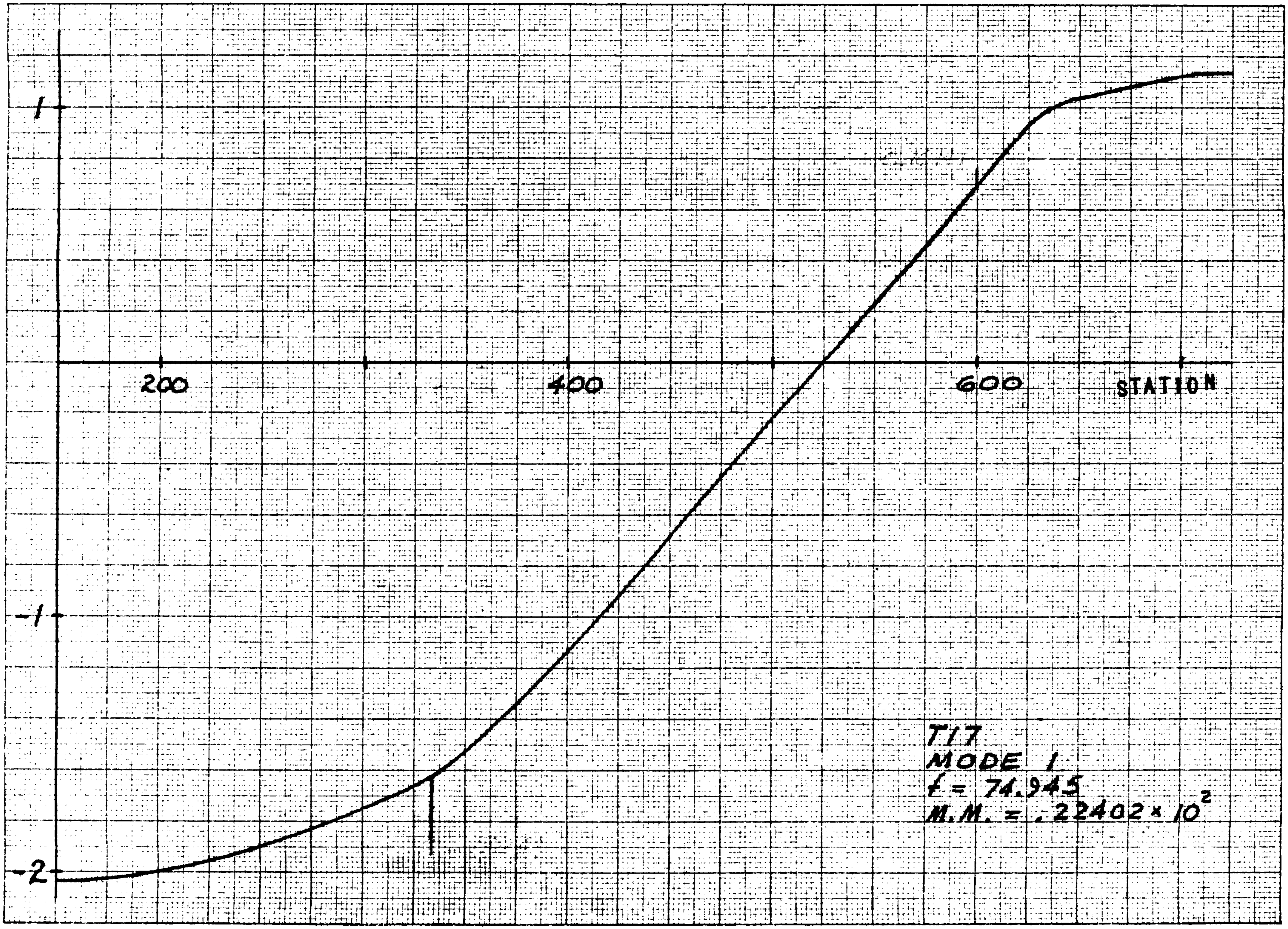
SM 46346


1.2
1.0
0.8
0.6
0.4
0.2
-0.2
200
400
600
STATION
T16
MODE 2
F = 70.304
M.M. = .10604·10²

SM 46346



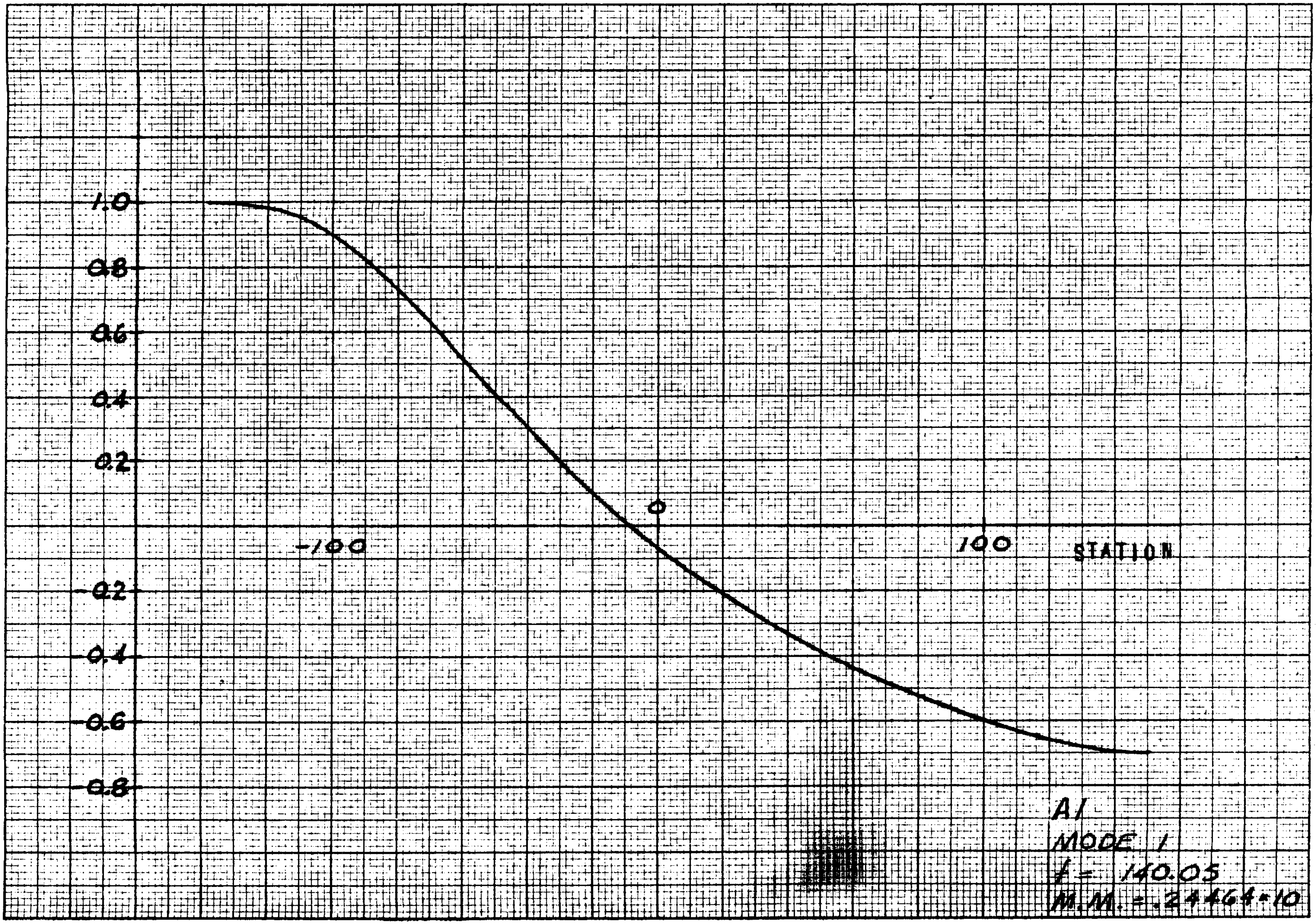
SM 46346



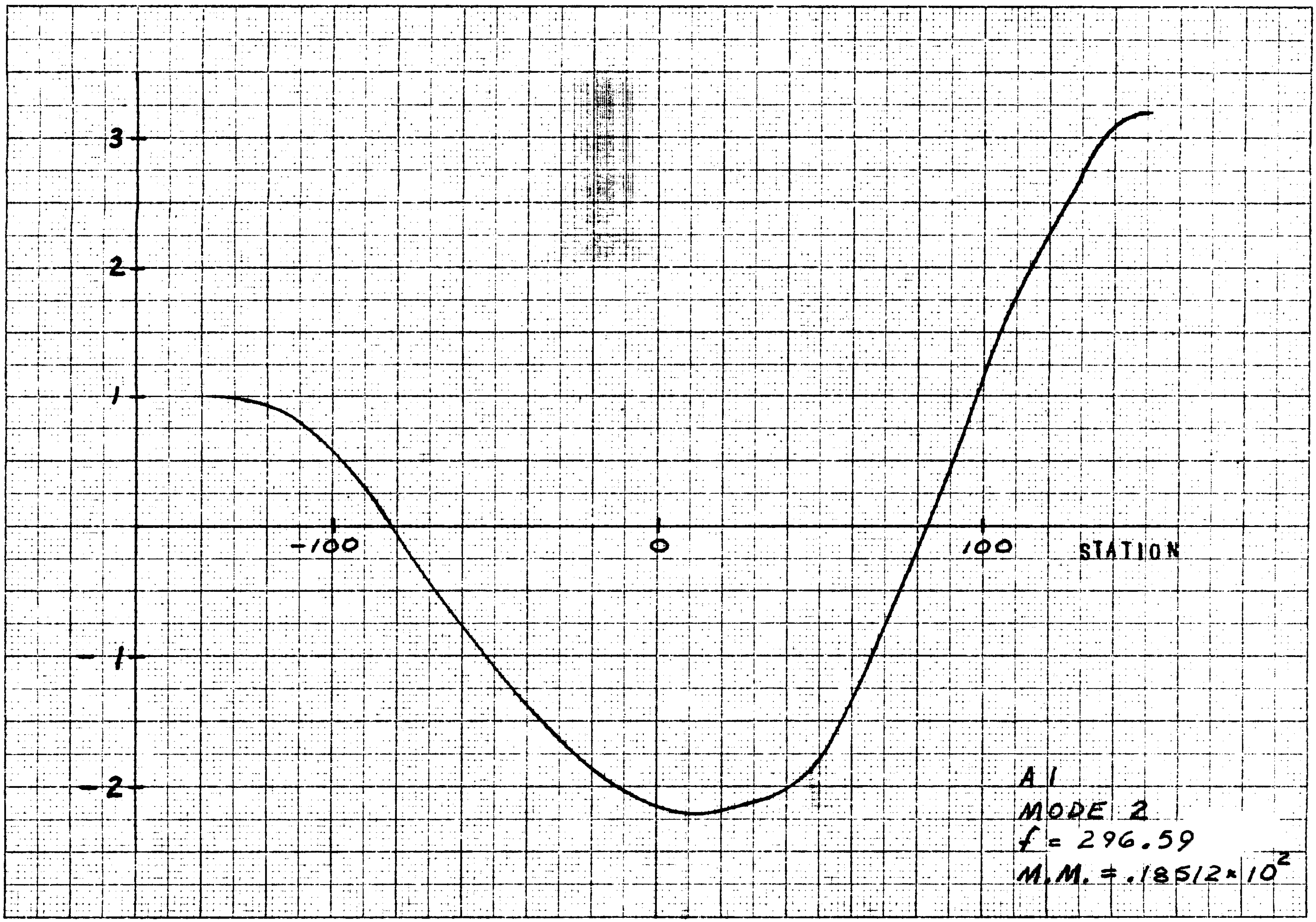
APPENDIX

B. Agena mode shapes

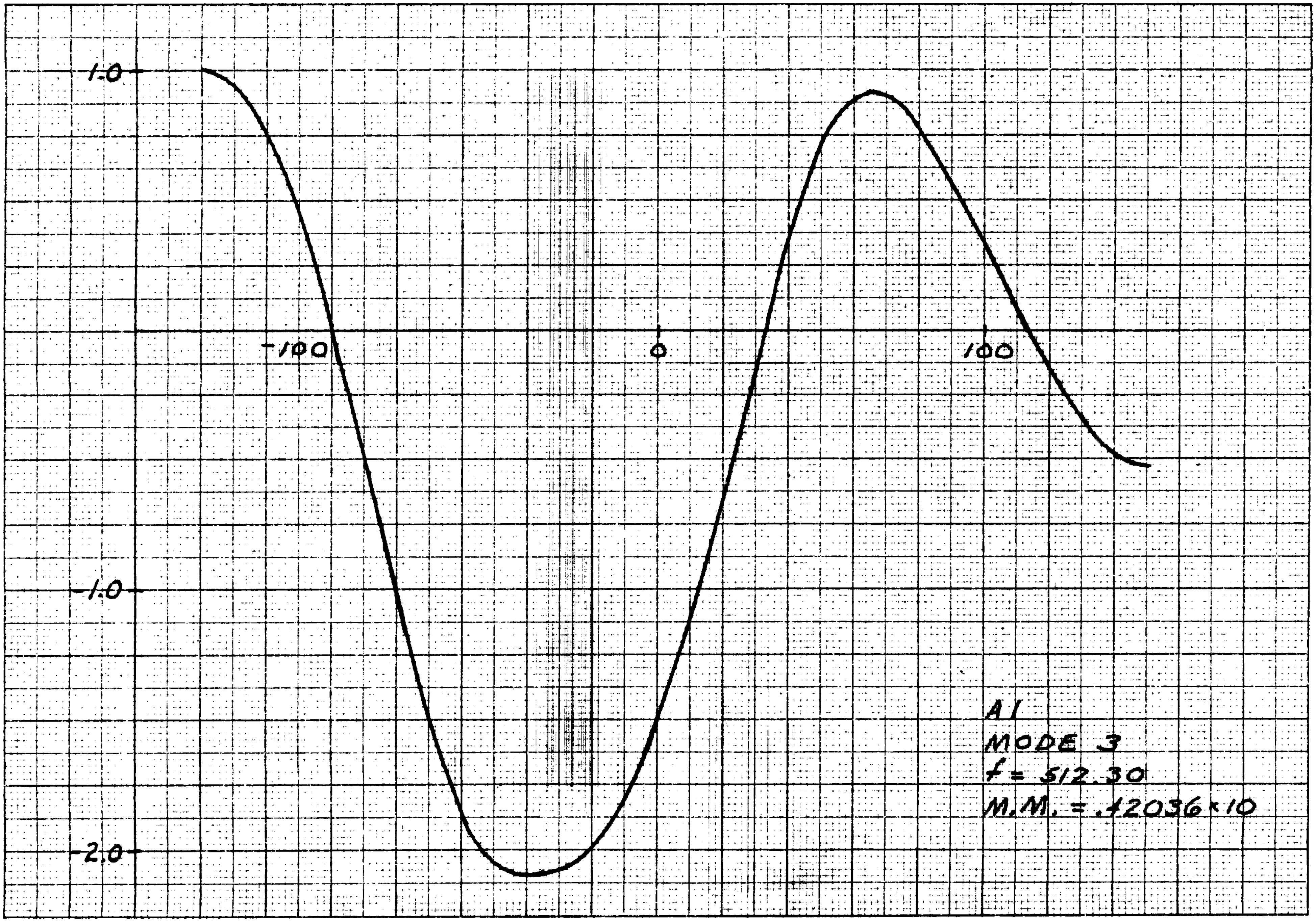
SM 46346


1.0
0.8
0.6
0.4
0.2
0
-100
100
STATION
-0.2
-0.4
-0.6
-0.8
A1
MODE 1
f = 140.05
M.M. = .24464•10

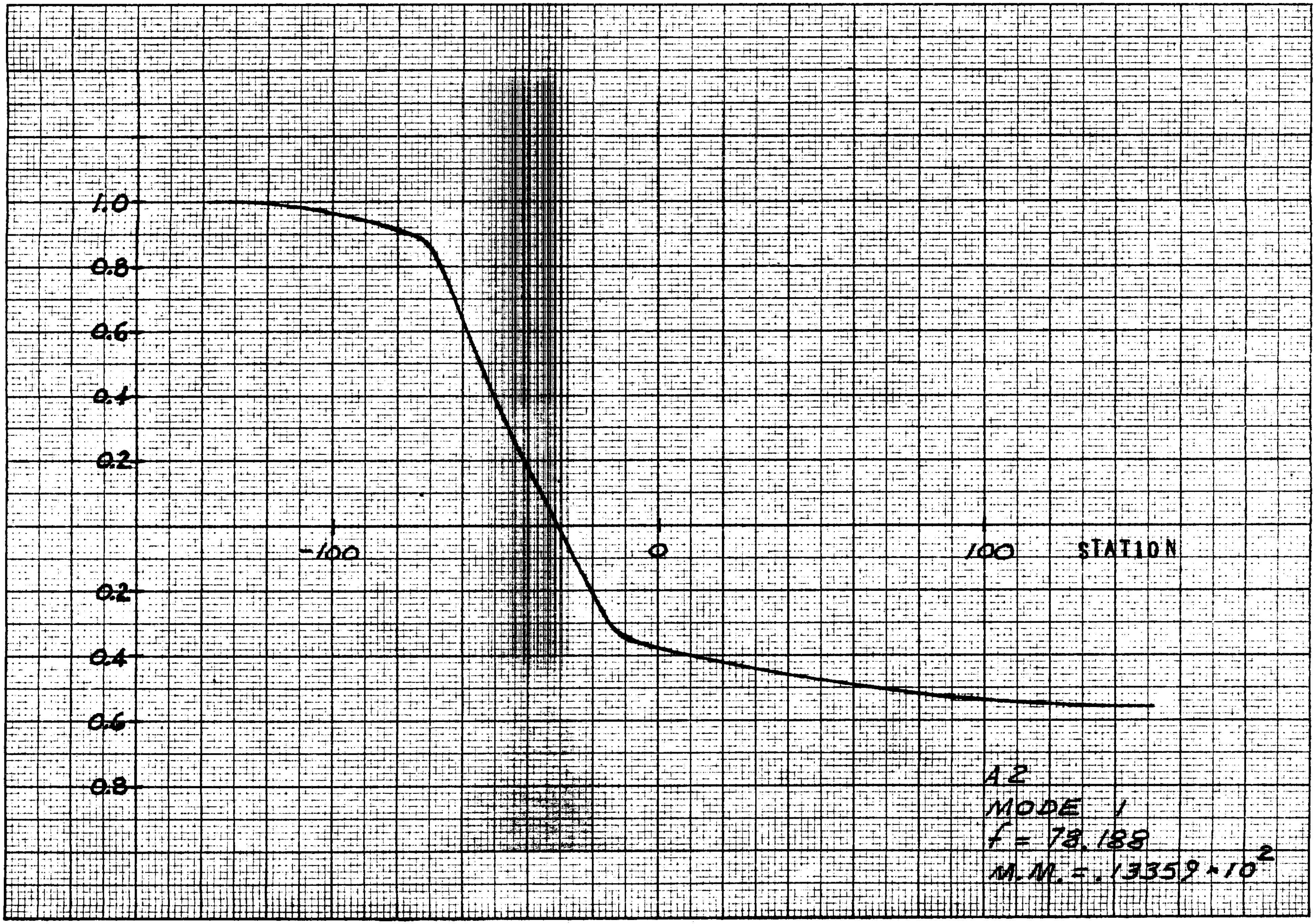
SM 46346



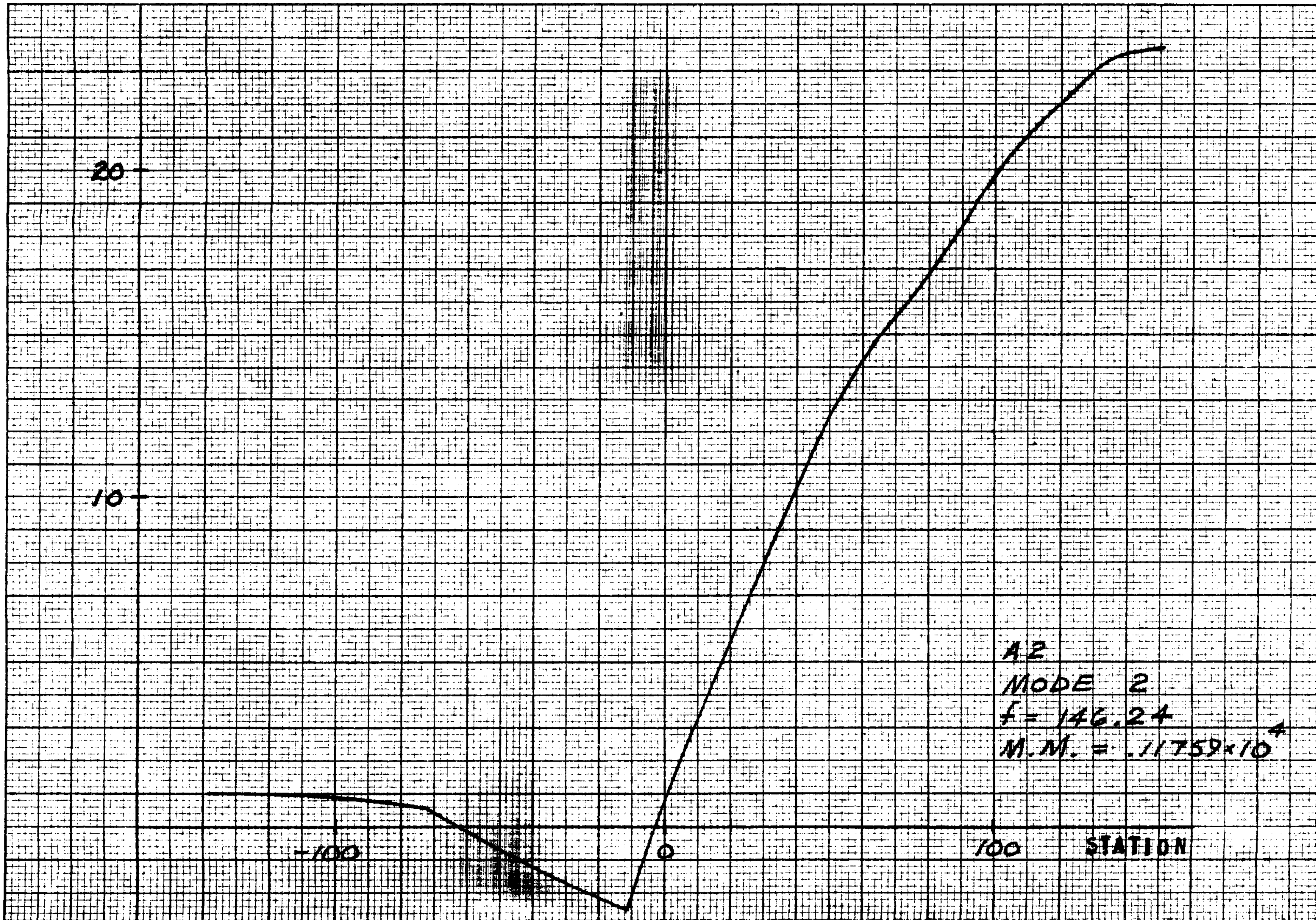
SM 46346

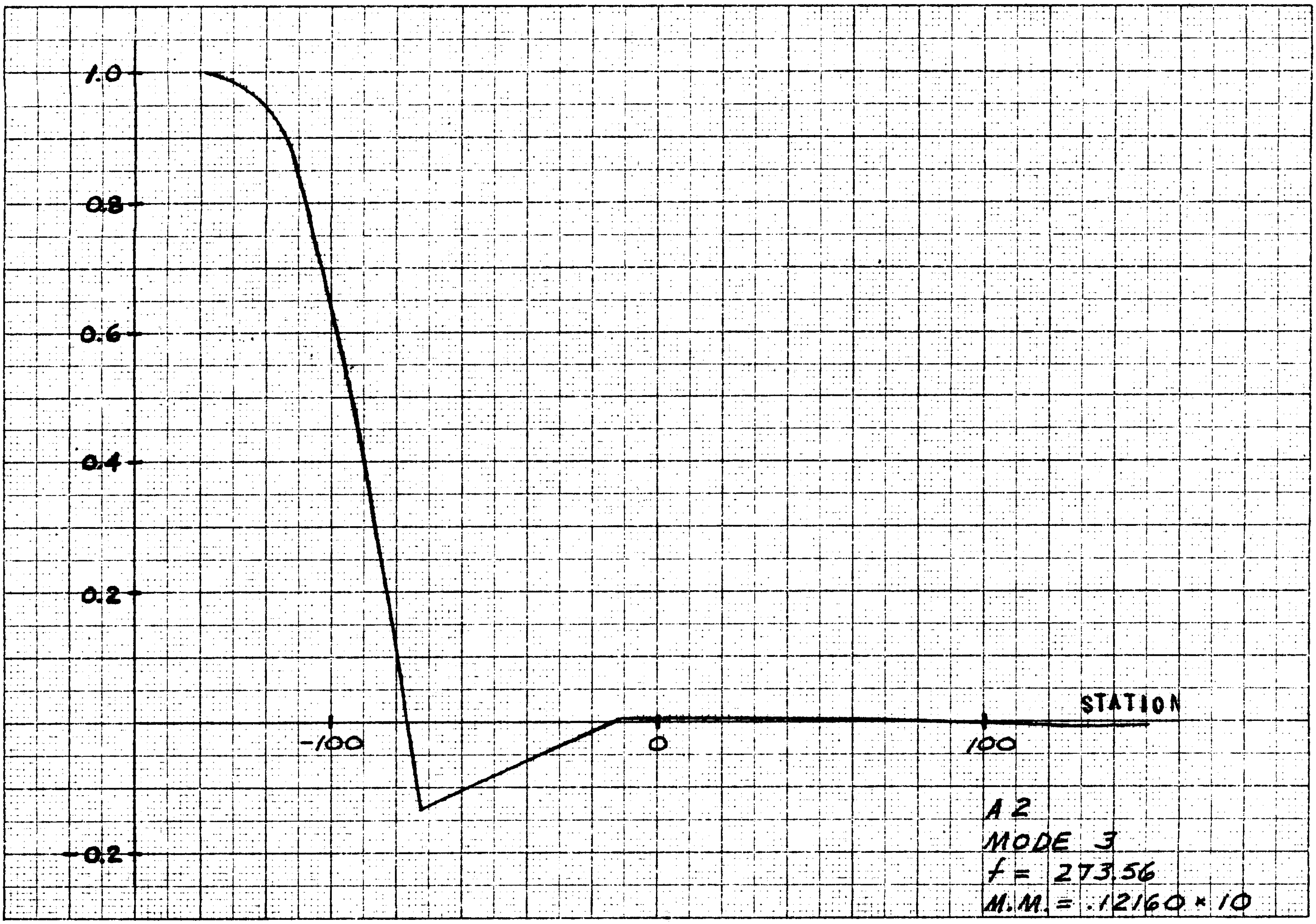


SM 46346

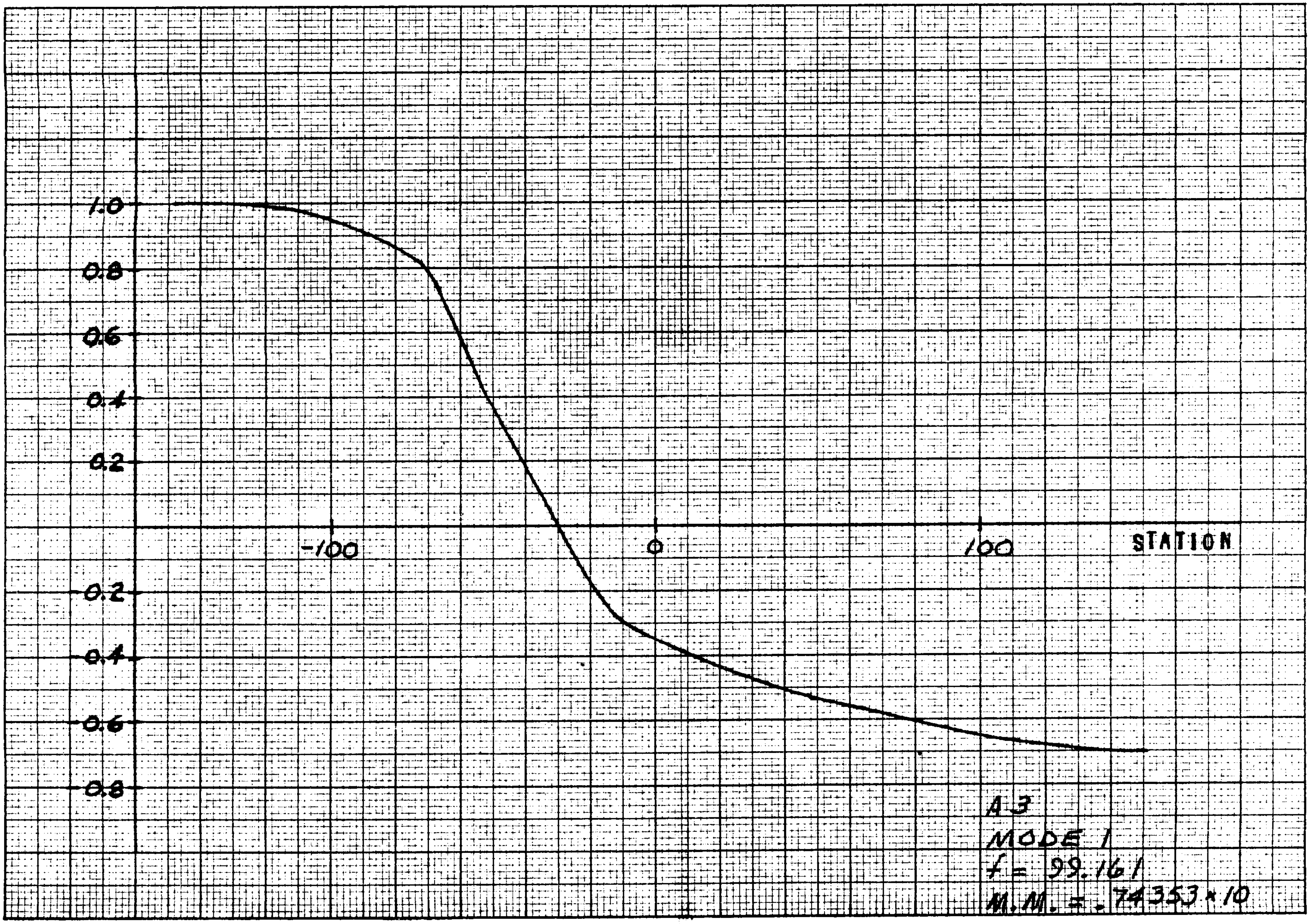

1.0
0.8
0.6
0.4
0.2
-100
0
100
STATION
0.2
0.4
0.6
0.8
A2
MODE 1
f = 78.188
M.M. = .13359 • 10^2

SM 46346

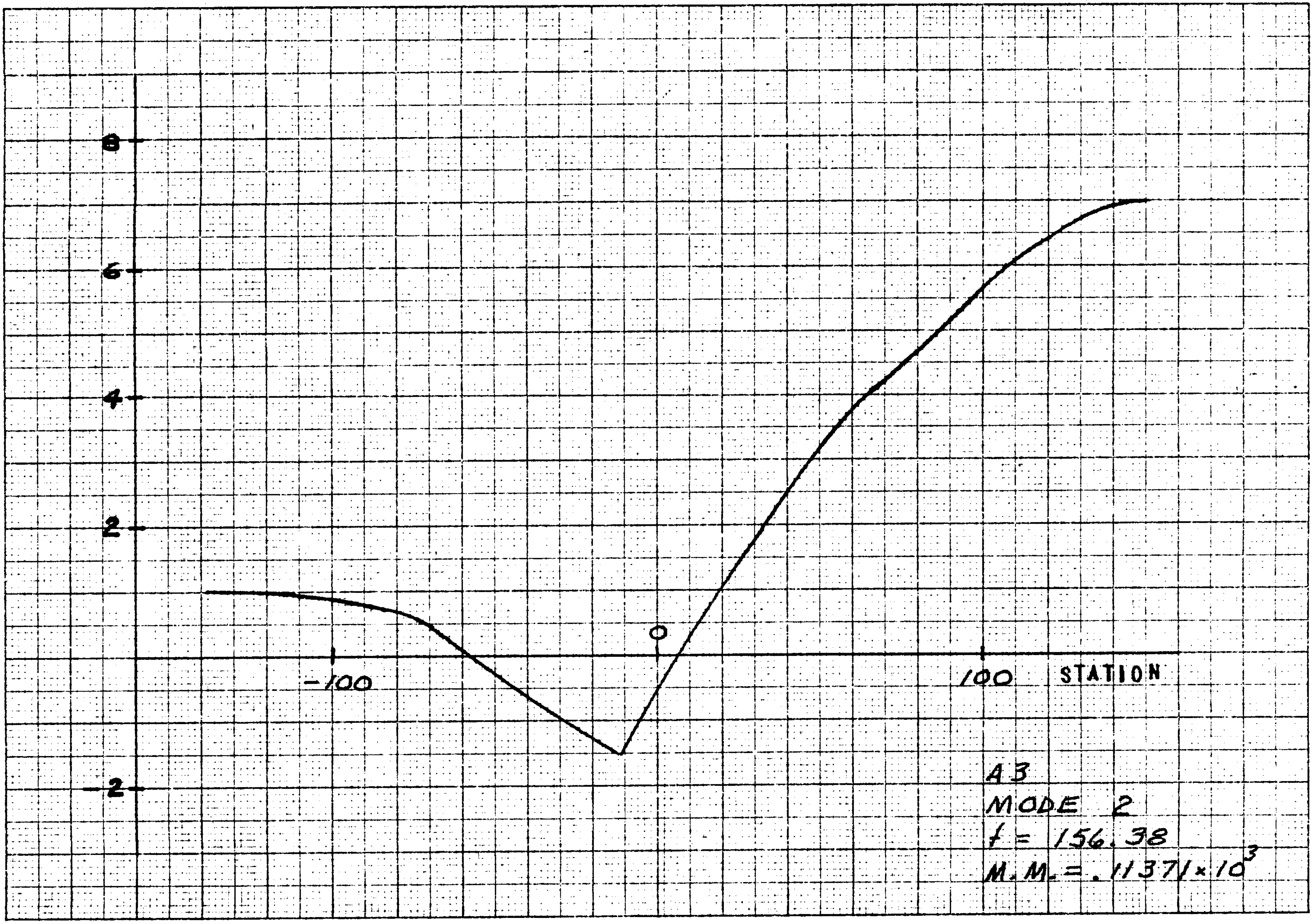


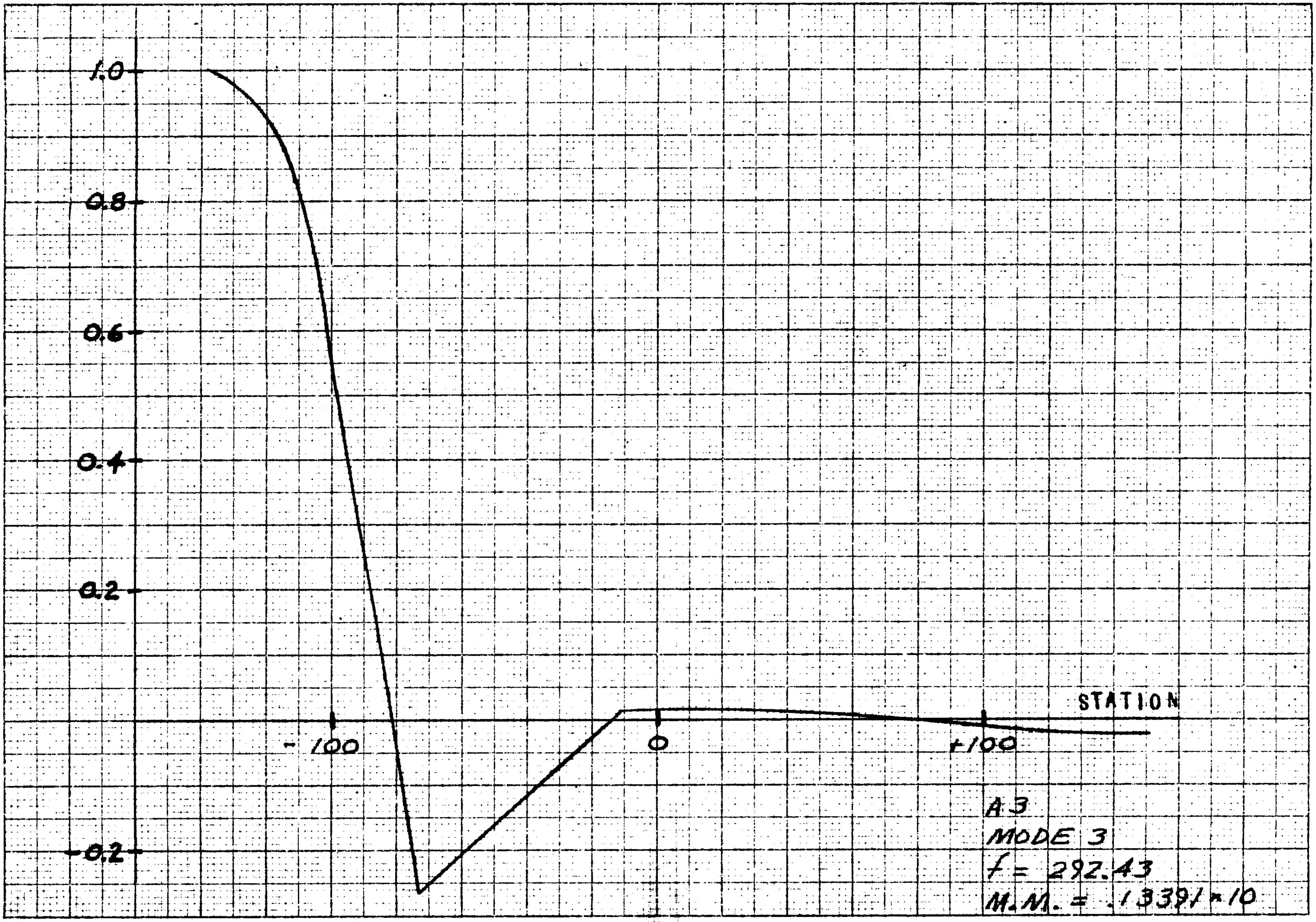

1.0
0.8
0.6
0.4
0.2
-0.2
-100
0
100
STATION
A 2
MODE 3
f = 273.56
M.M. = .12160 × 10

SM 46346

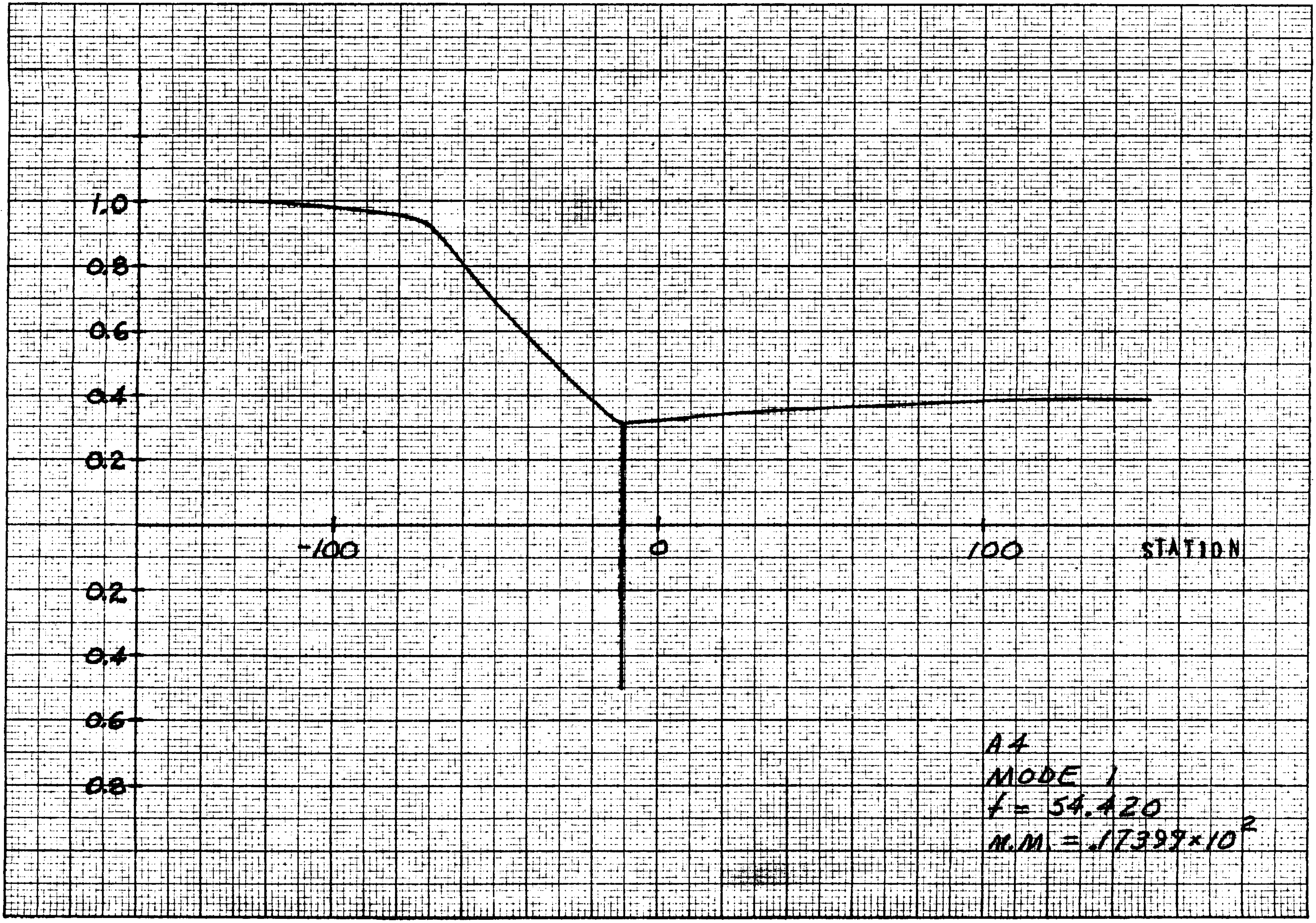

1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
-100
0
100
STATION
A 3
MODE 1
f = 99.161
M.M. = .74353 × 10

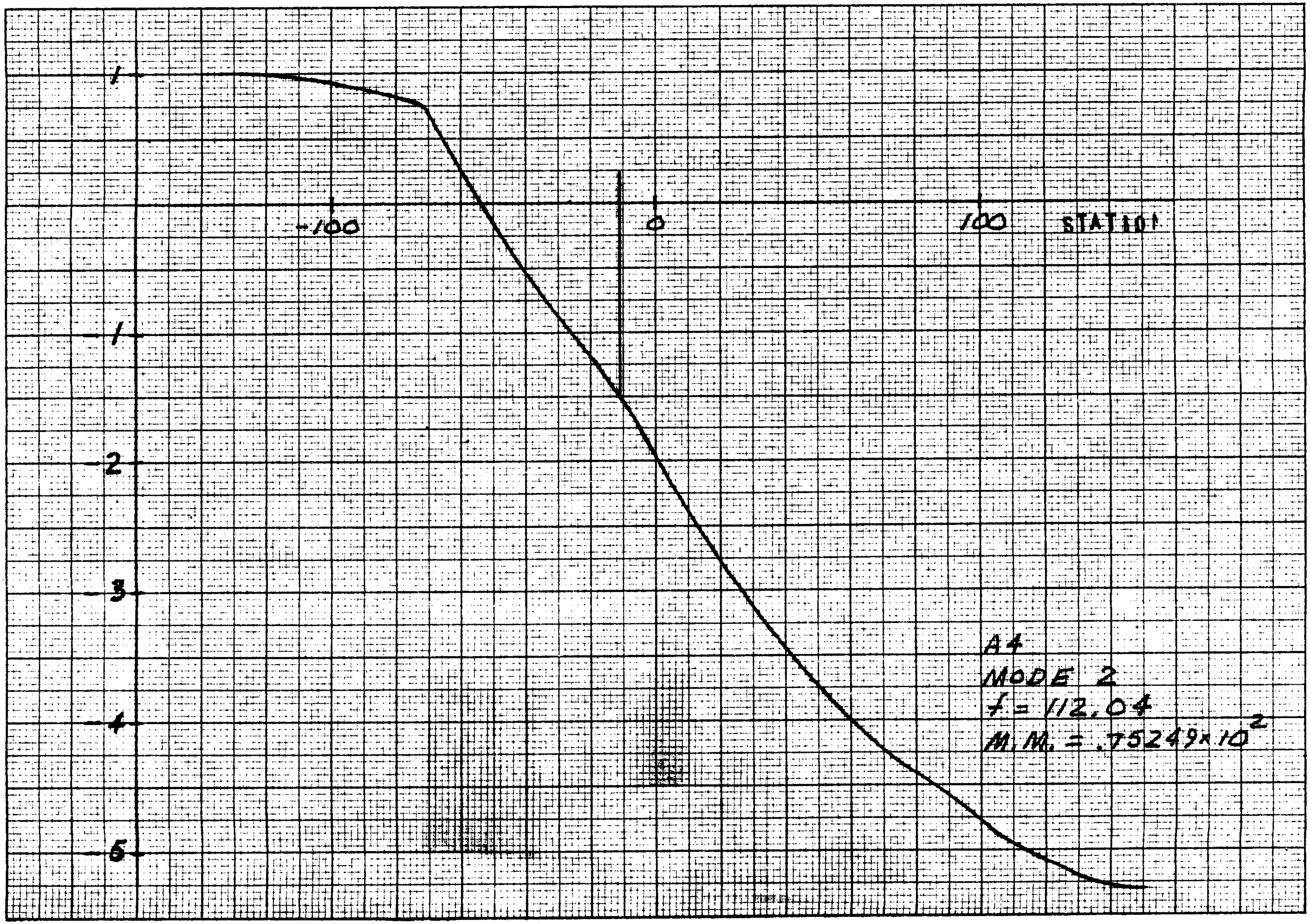
SM 46346



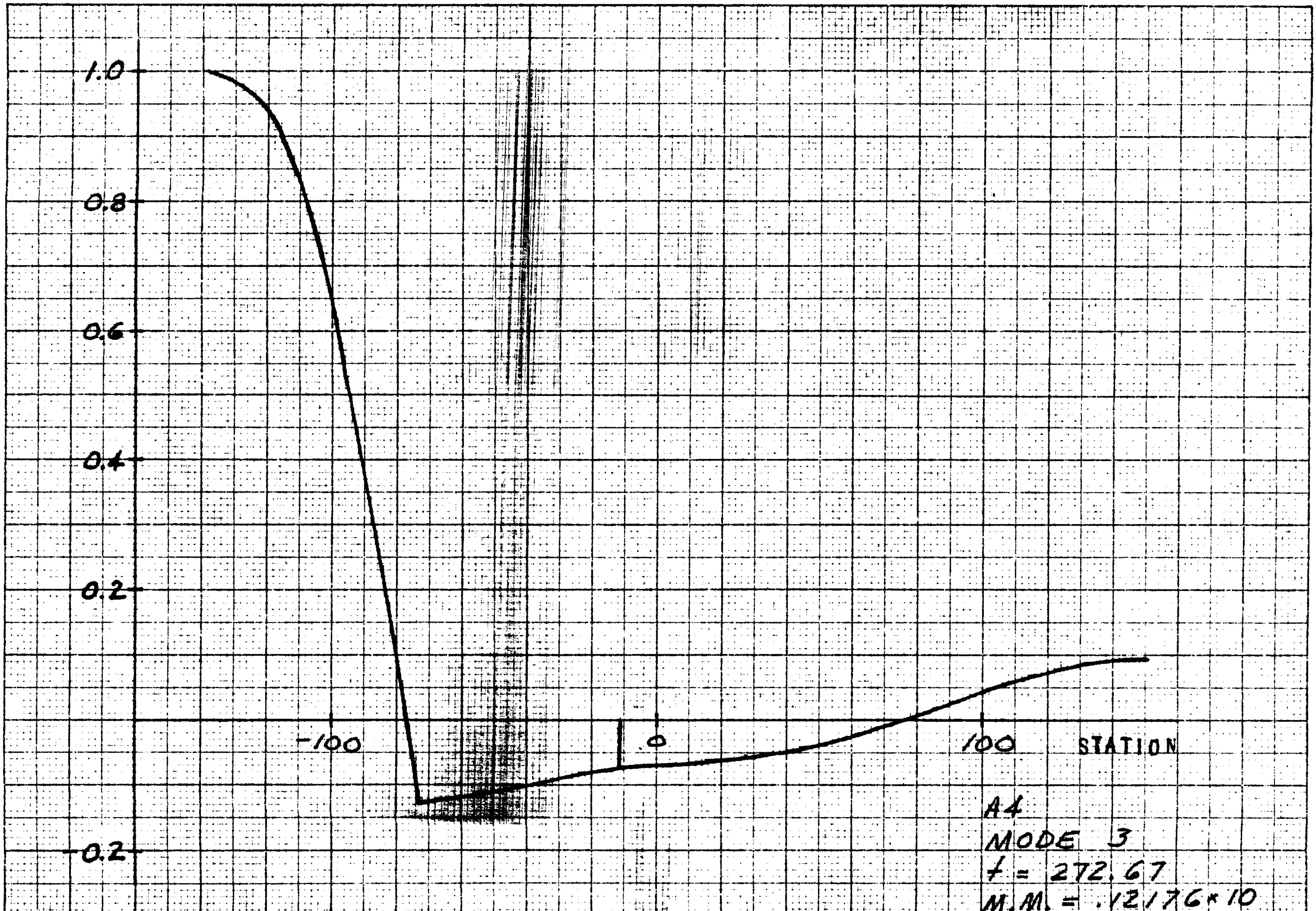

1.0
0.8
0.6
0.4
0.2
-0.2
-100
0
+100
STATION
A 3
MODE 3
f = 292.43
M.M. = .13391 × 10

SM 46346


1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
-100
0
100
STATION
A4
MODE 1
f = 54.420
M.M. = .17399×10²

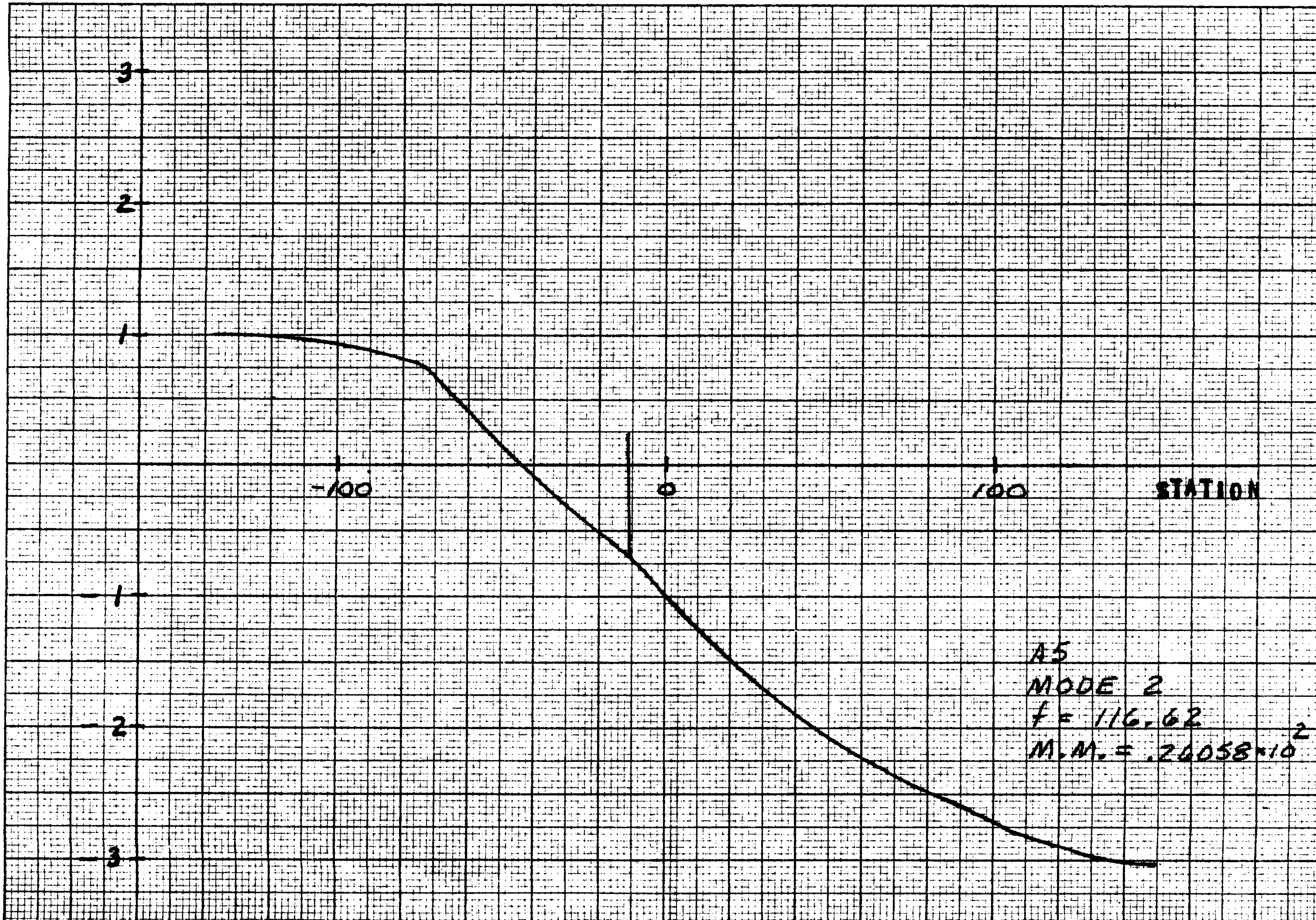

1
-100
0
100
STATION
1
2
3
4
5
A4
MODE 2
f = 112.04
M.M. = .75249×10²

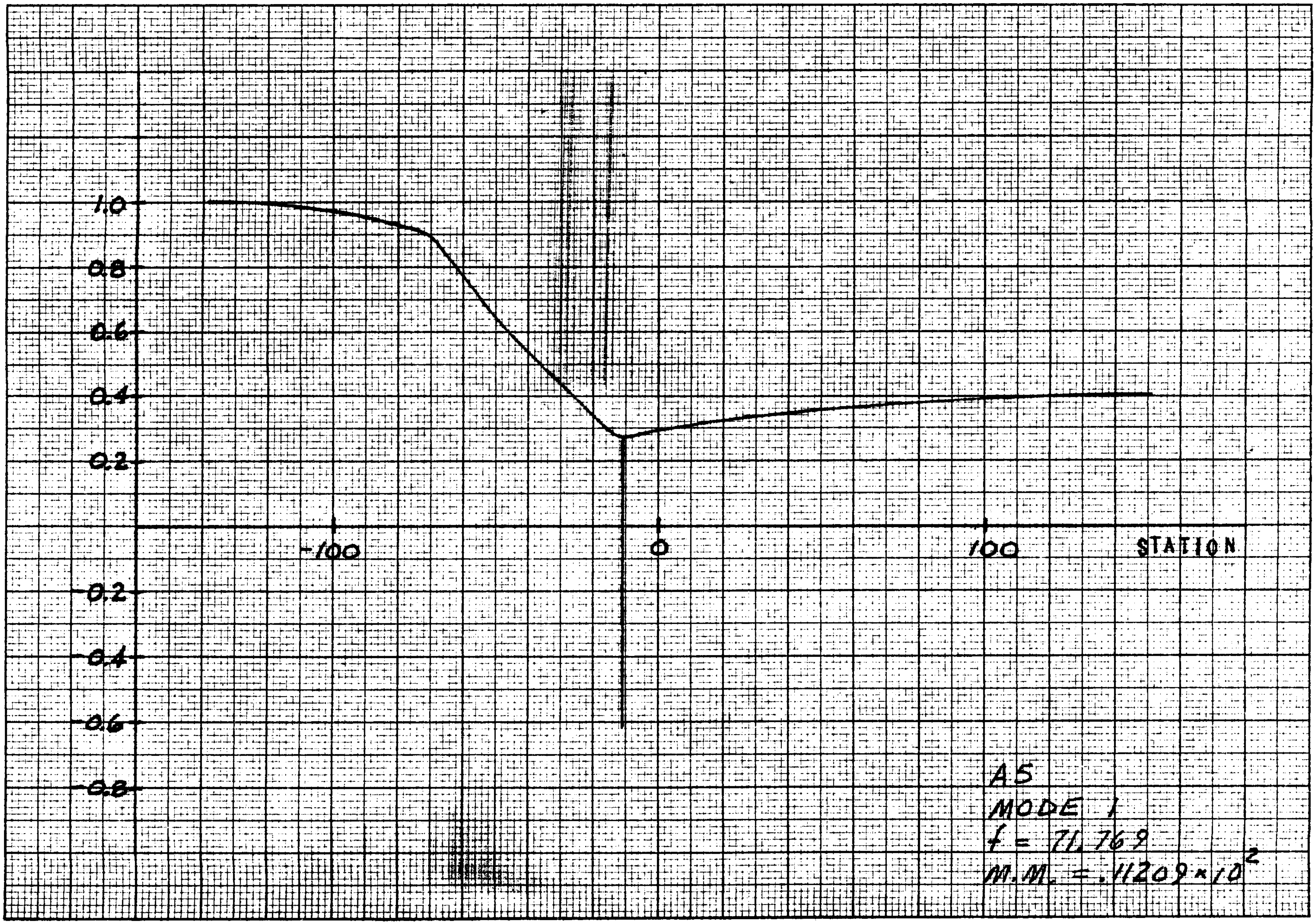
SM 46346



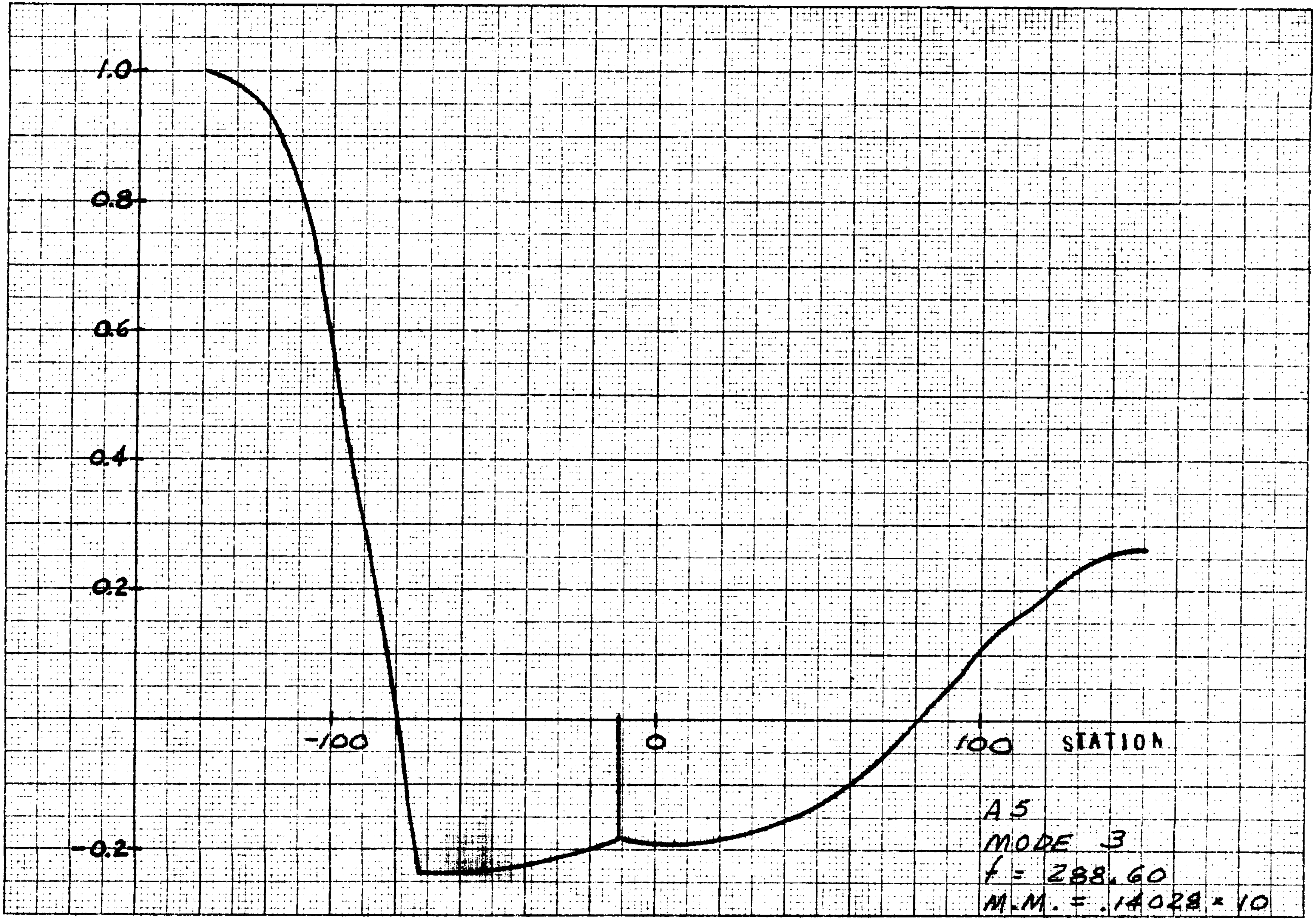
K+E 10 X 10 TO ½ INCH 7 X 10 IN. • ALBANENE 46 1327 MADE IN U.S.A. KEUFFEL & ESSER CO.

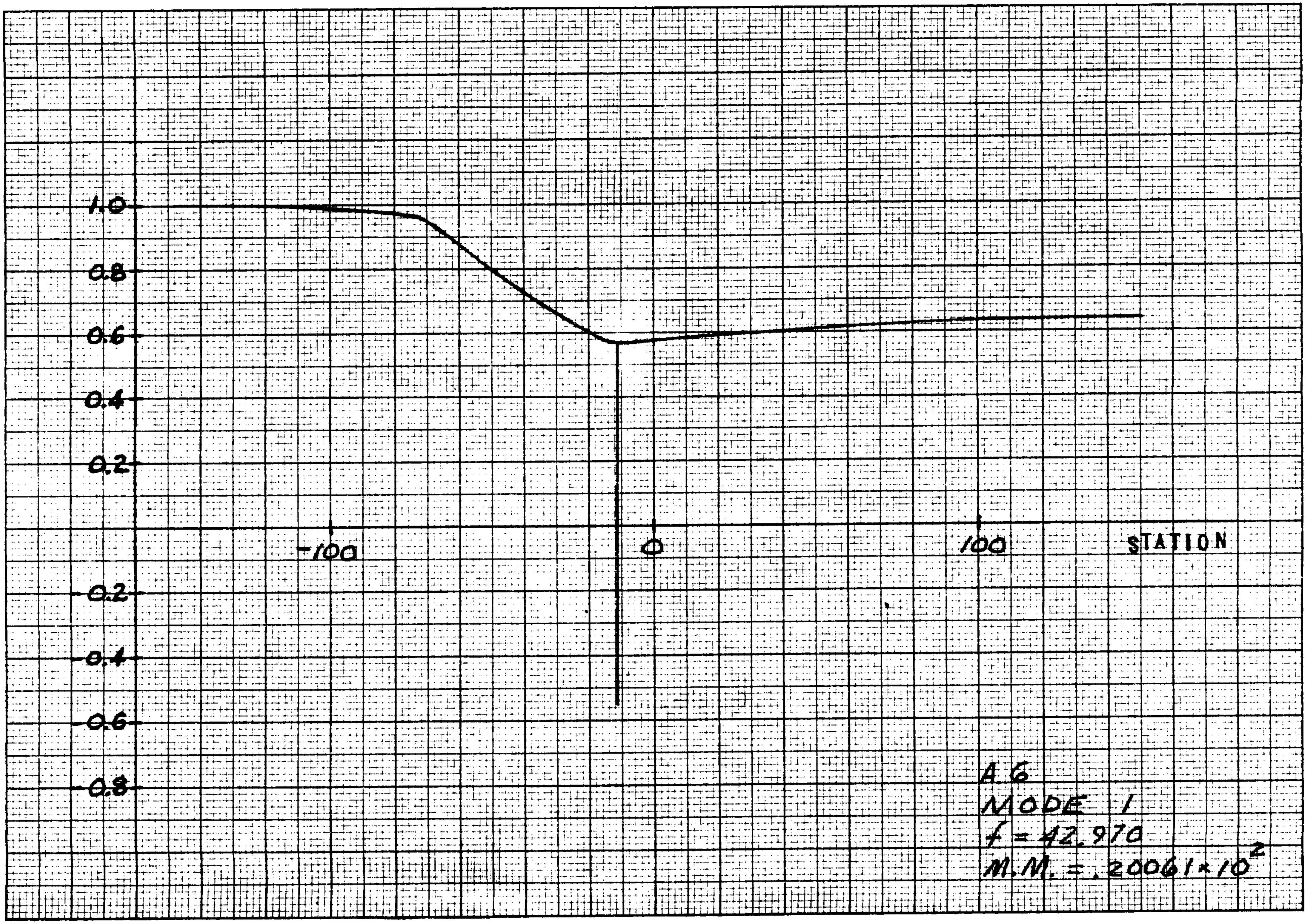
SM 46346

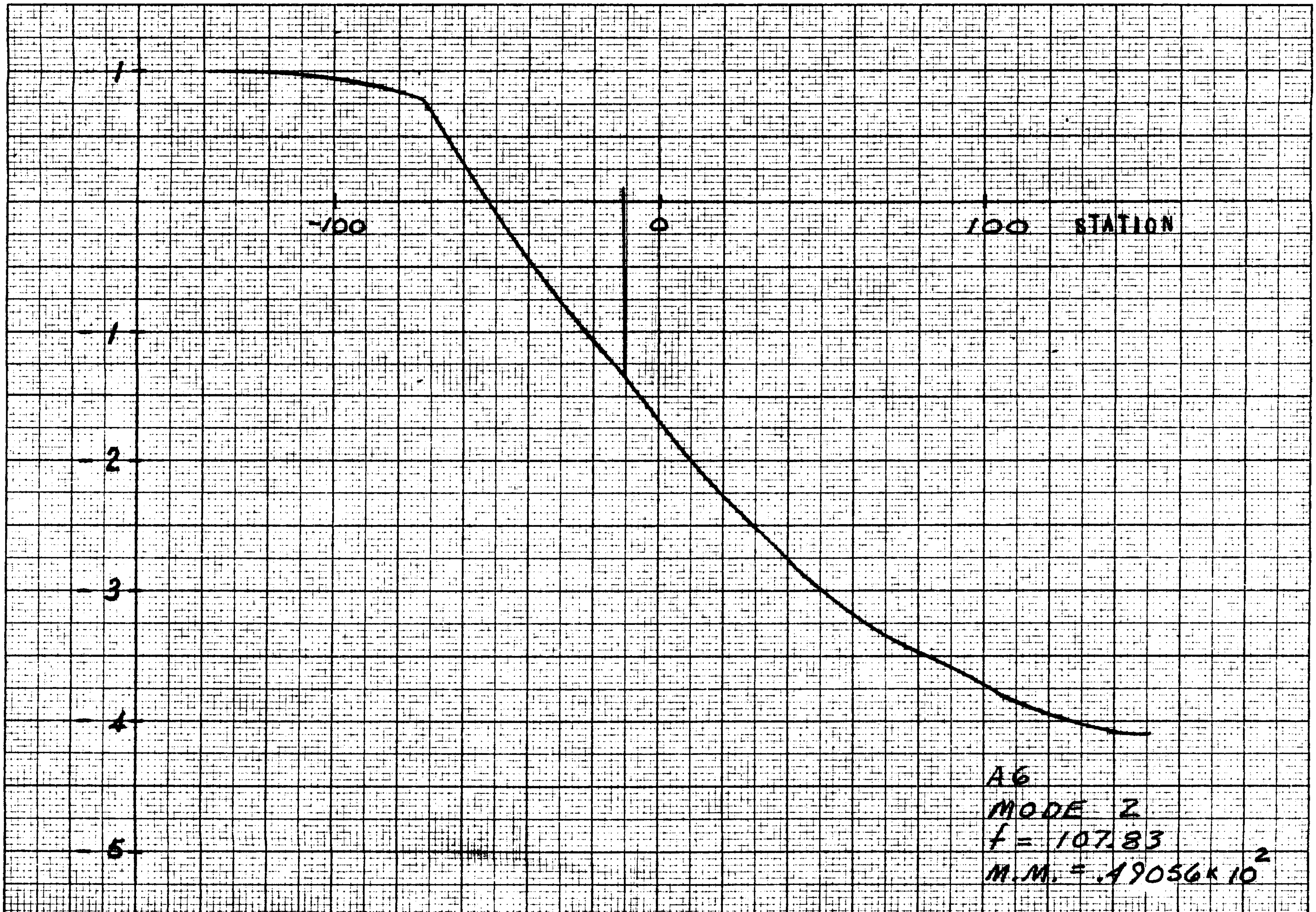



1.0
0.8
0.6
0.4
0.2
-0.2
-0.4
-0.6
-0.8
-100
0
100
STATION
A5
MODE 1
f = 71.769
M.M. = .11209×10²

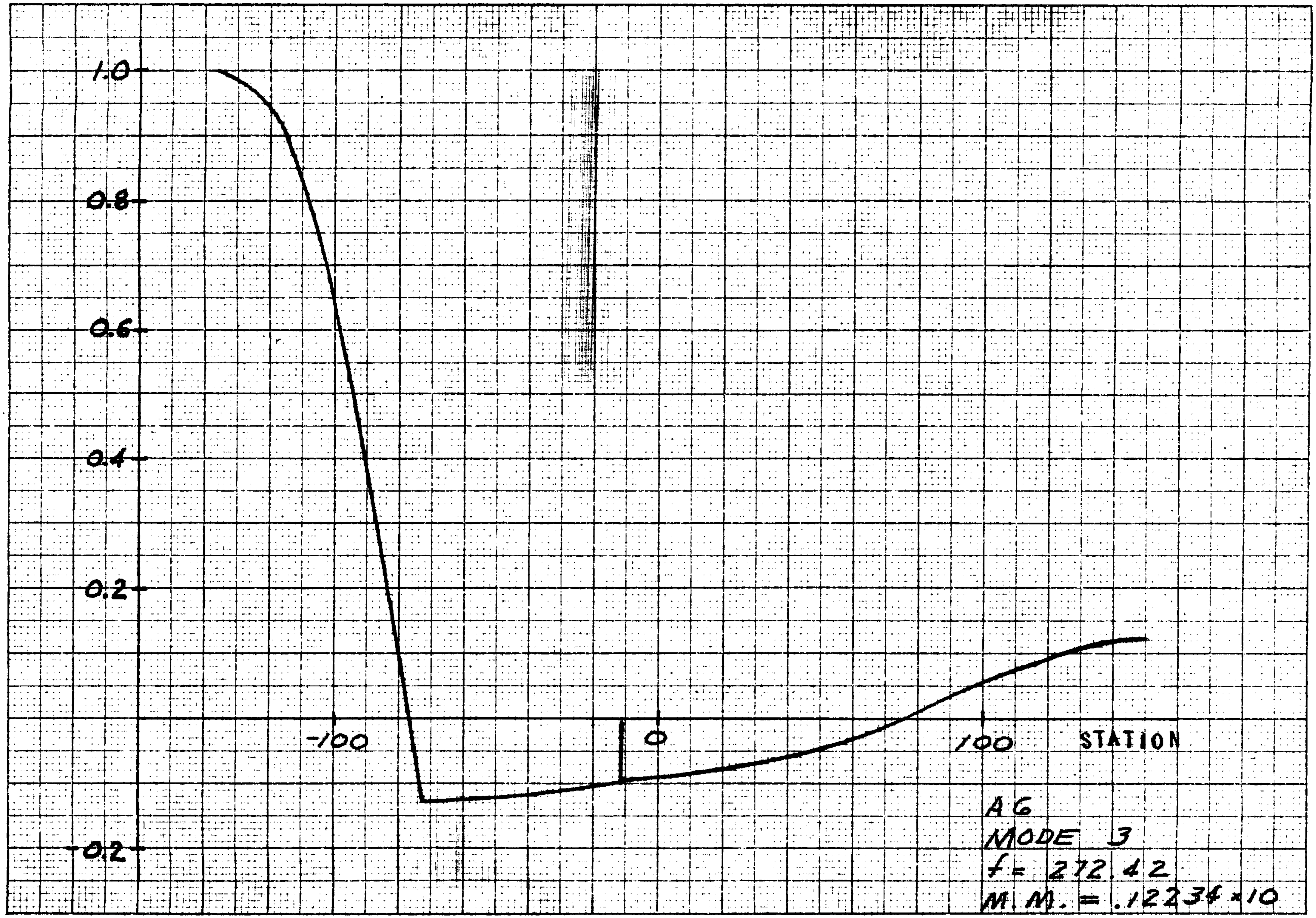
SM 46346




1.0
0.8
0.6
0.4
0.2
0.2
0.4
0.6
0.8
-100
0
100
STATION
A 6
MODE 1
f = 42.970
M.M. = .20061 × 10^2

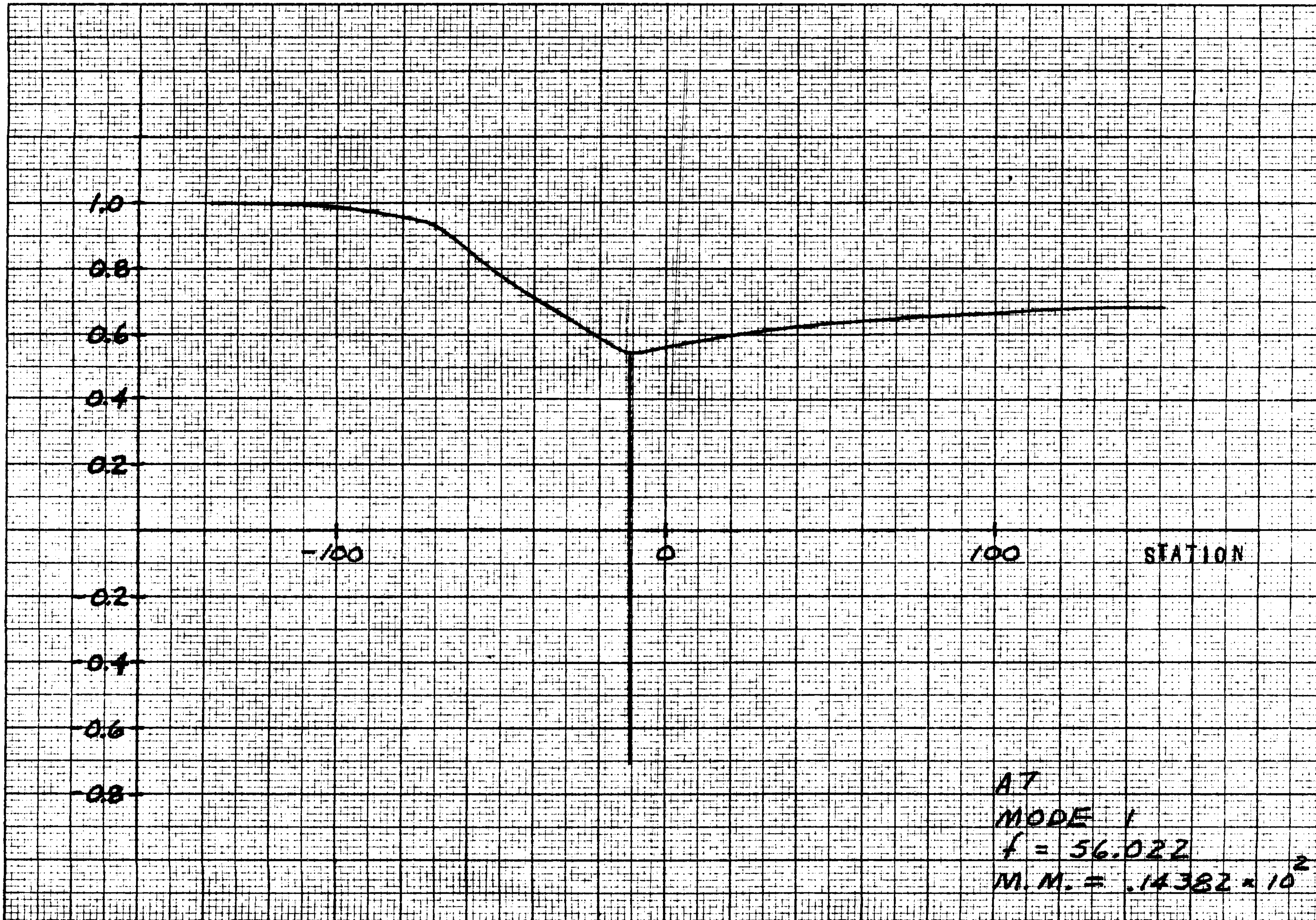

1
-100
0
100 STATION
1
2
3
4
5
A6
MODE Z
f = 107.83
M.M. = .49056 × 10²

SM 4 0 3 4 6

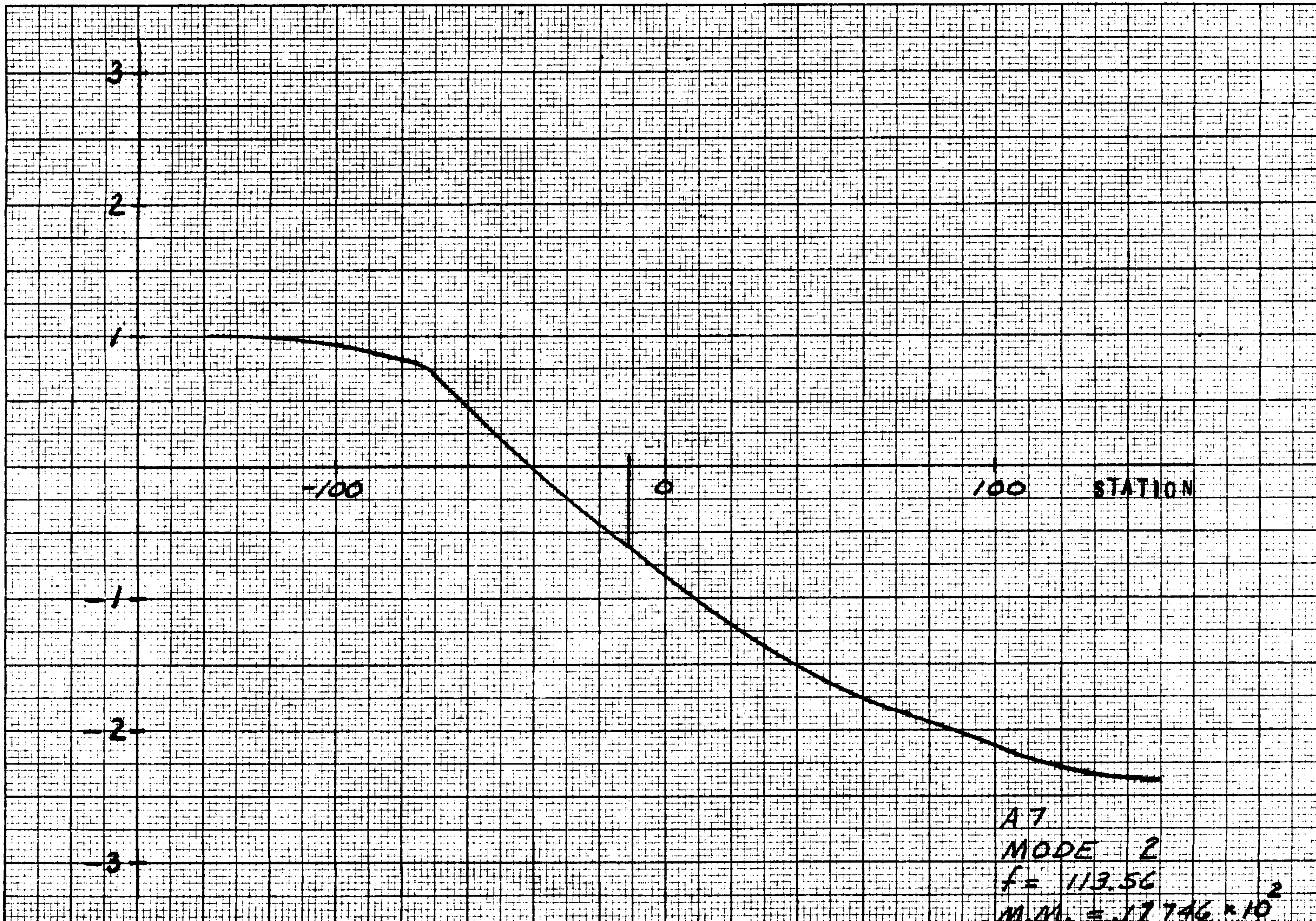


K+E 10 X 10 TO ½ INCH 7 X 10 IN. • ALBANENE® 46 1327 MADE IN U.S.A. KEUFFEL & ESSER CO.

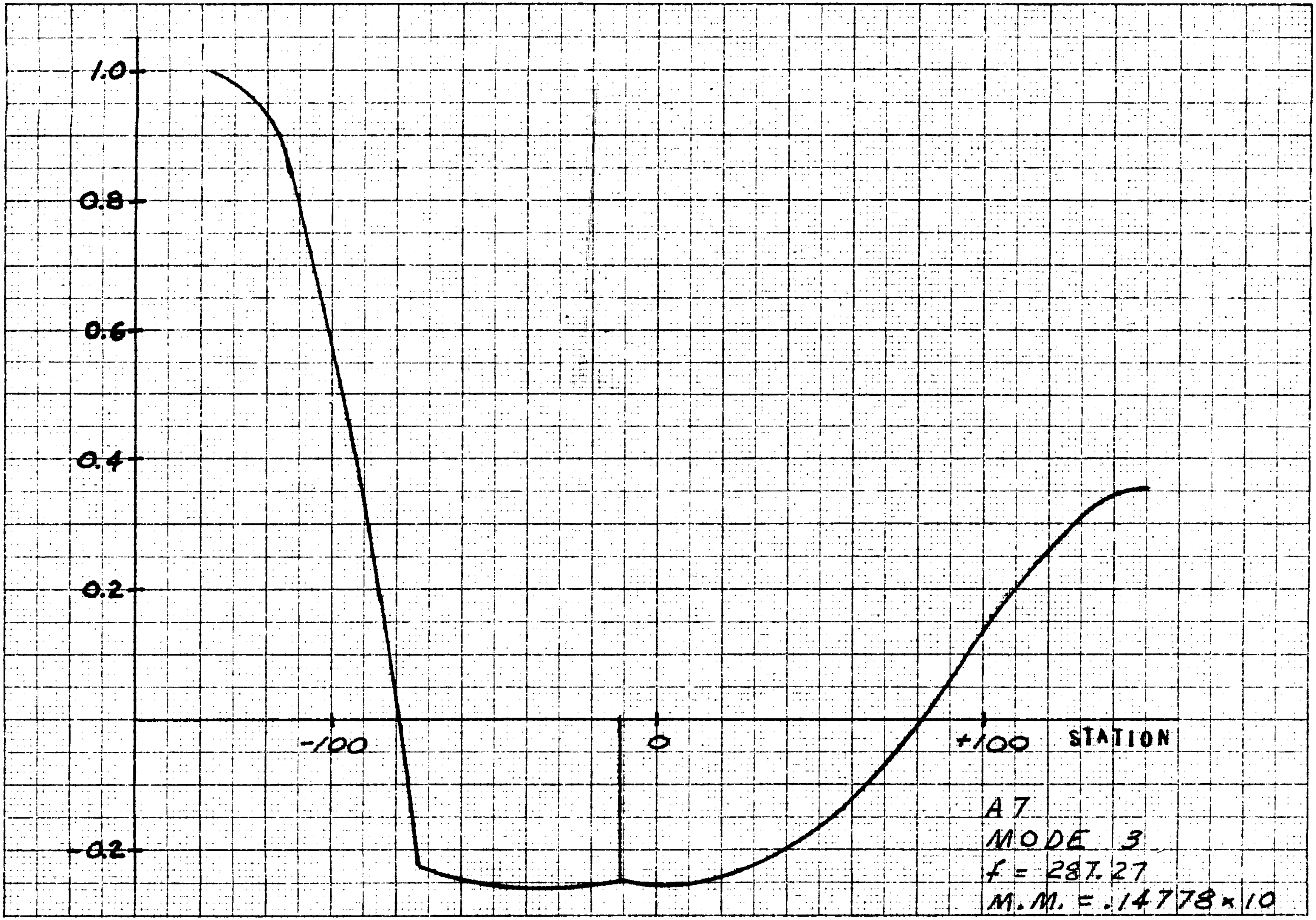
SM 46346



SM 46346



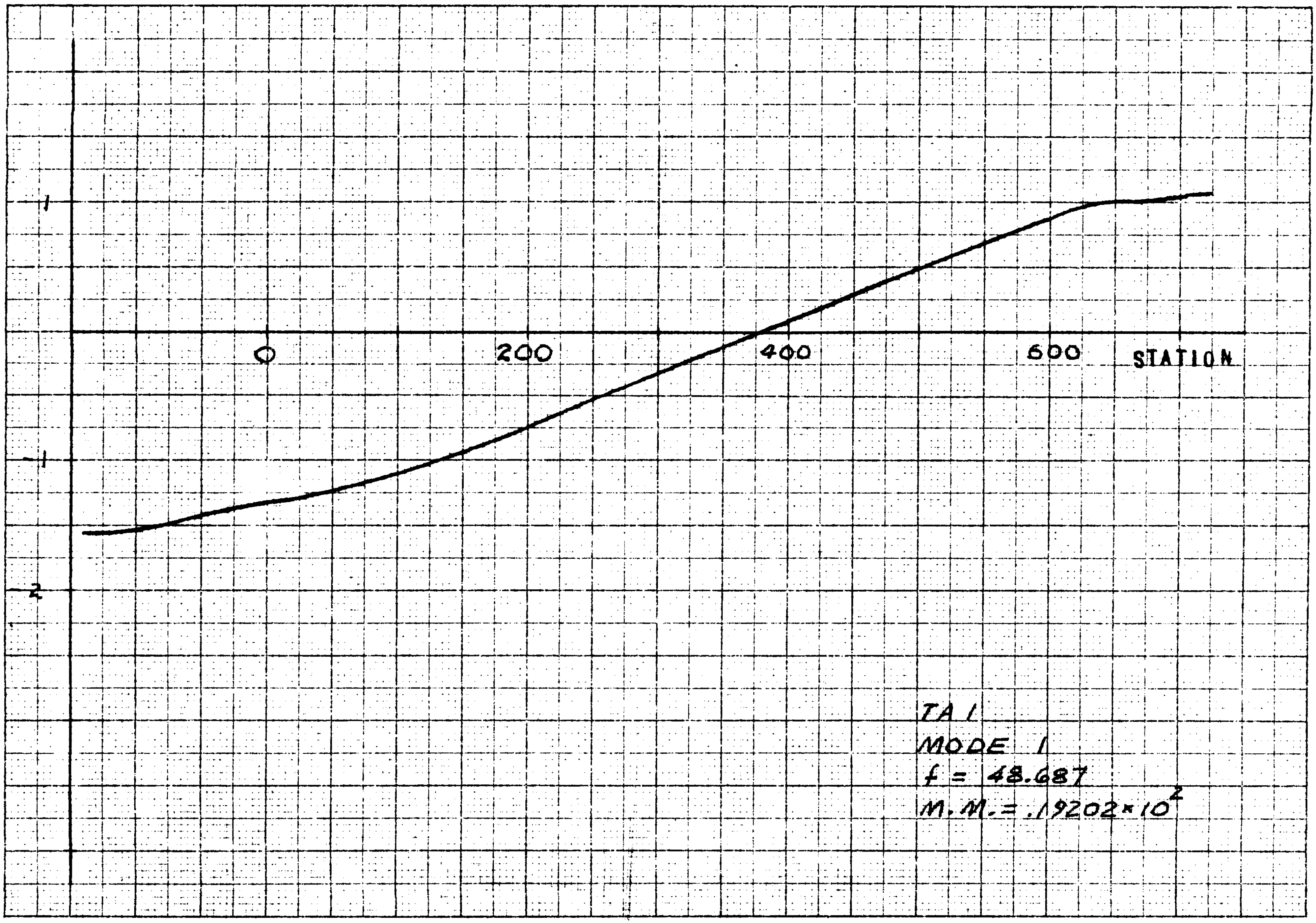
SM 46346



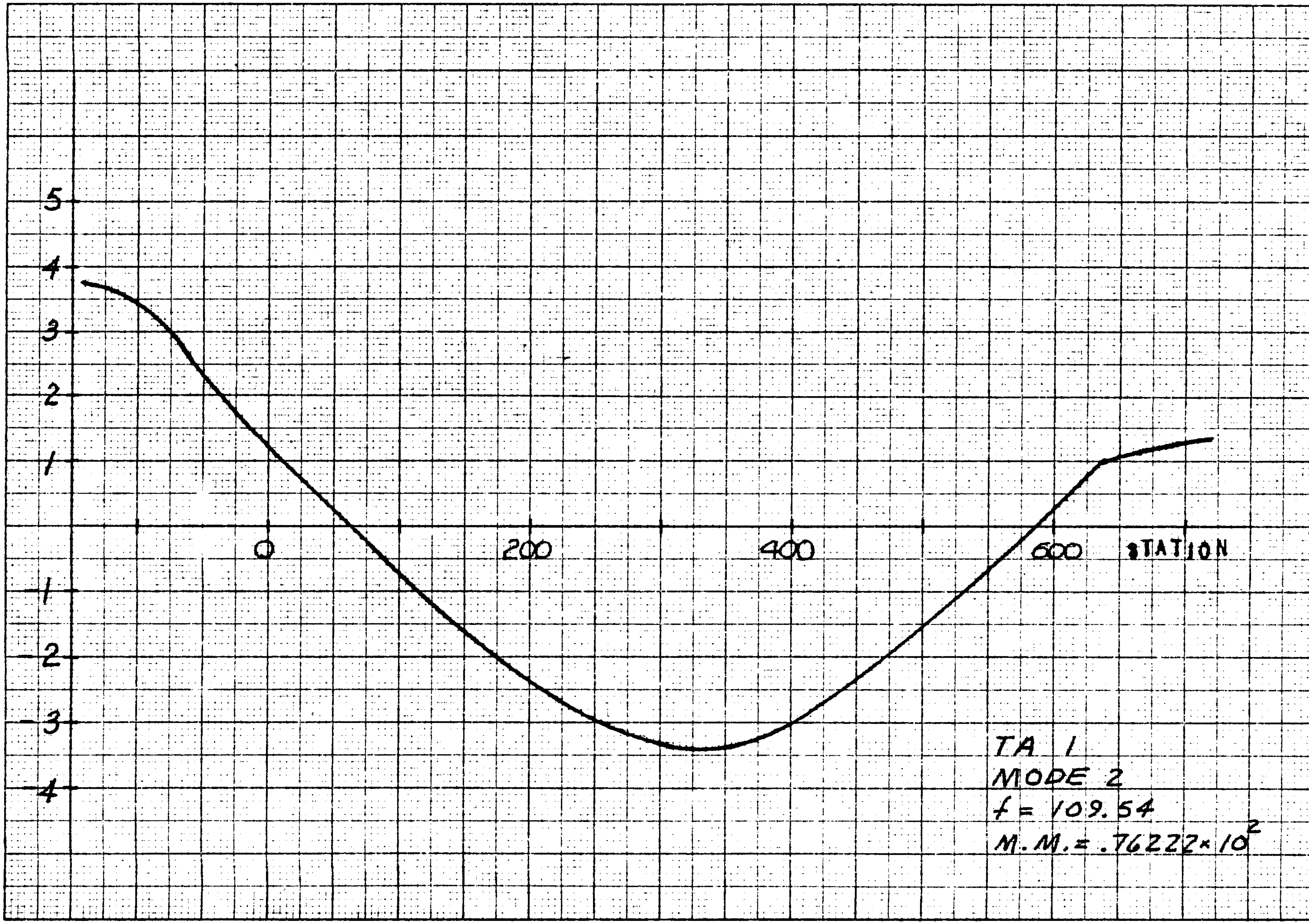
SM 46346

APPENDIX

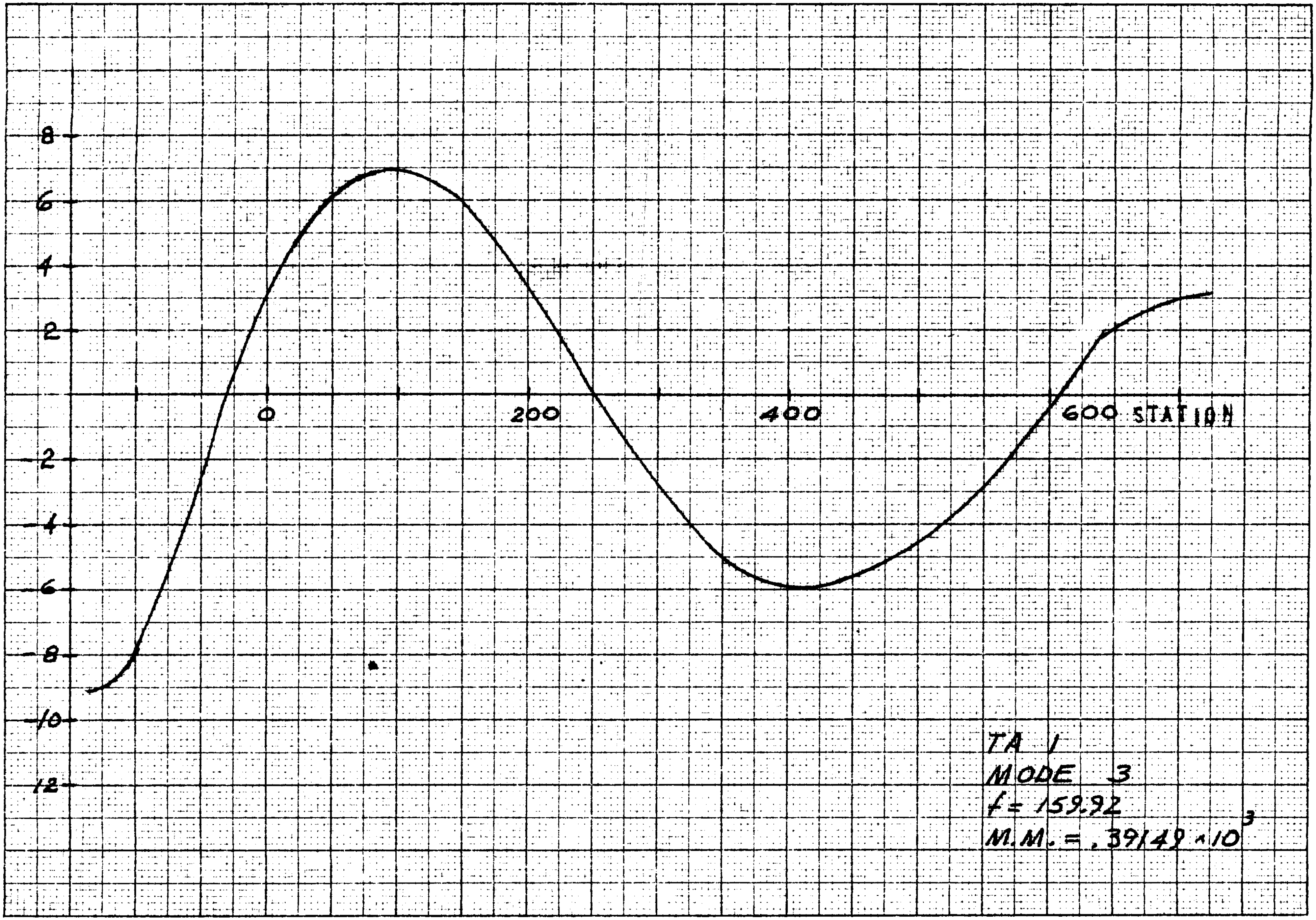
C. Thor-Agena mode shares


1
-1
-2
0
200
400
600
STATION
TA 1
MODE 1
f = 48.687
M.M. = .19202×10²

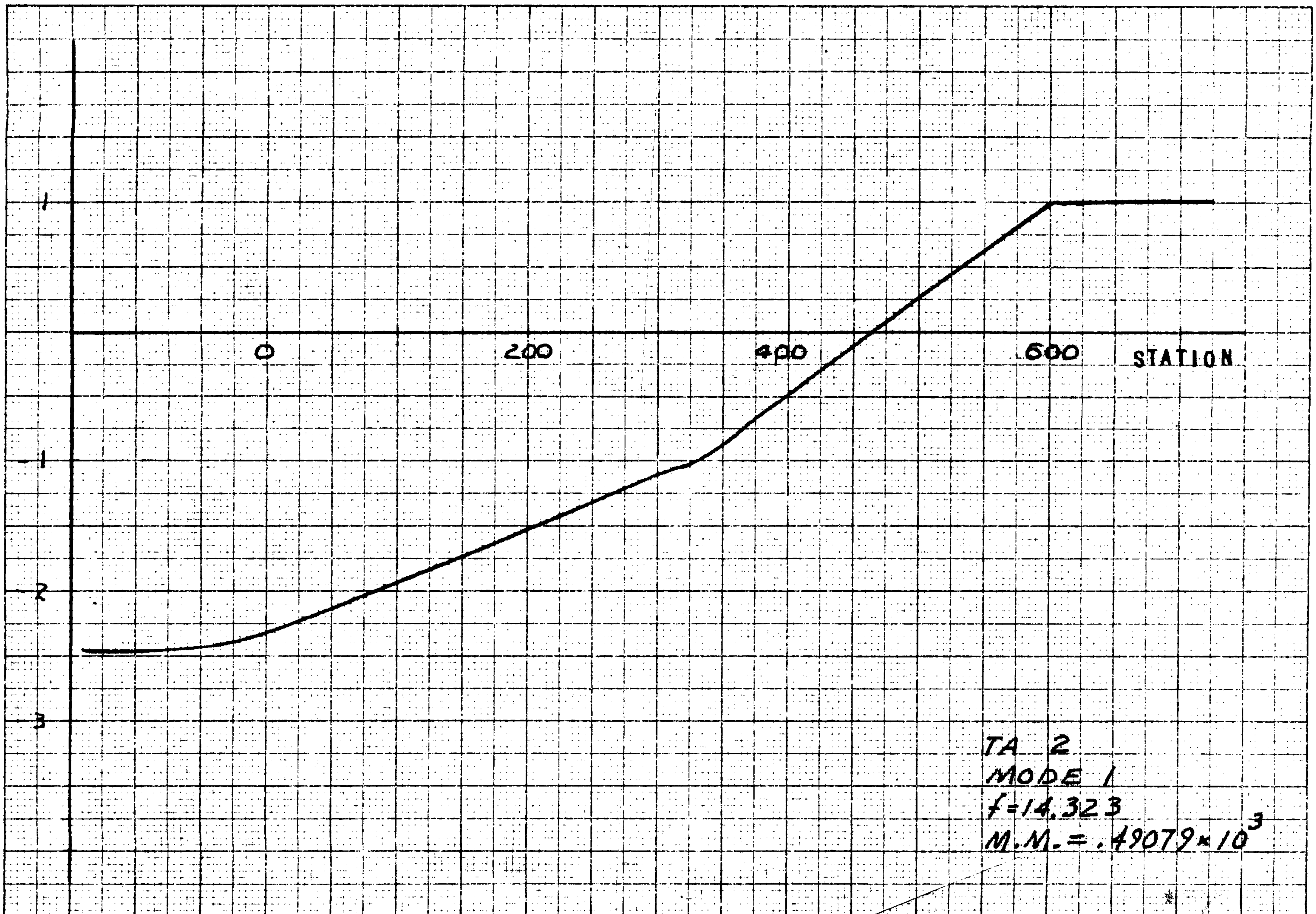
SM 46346

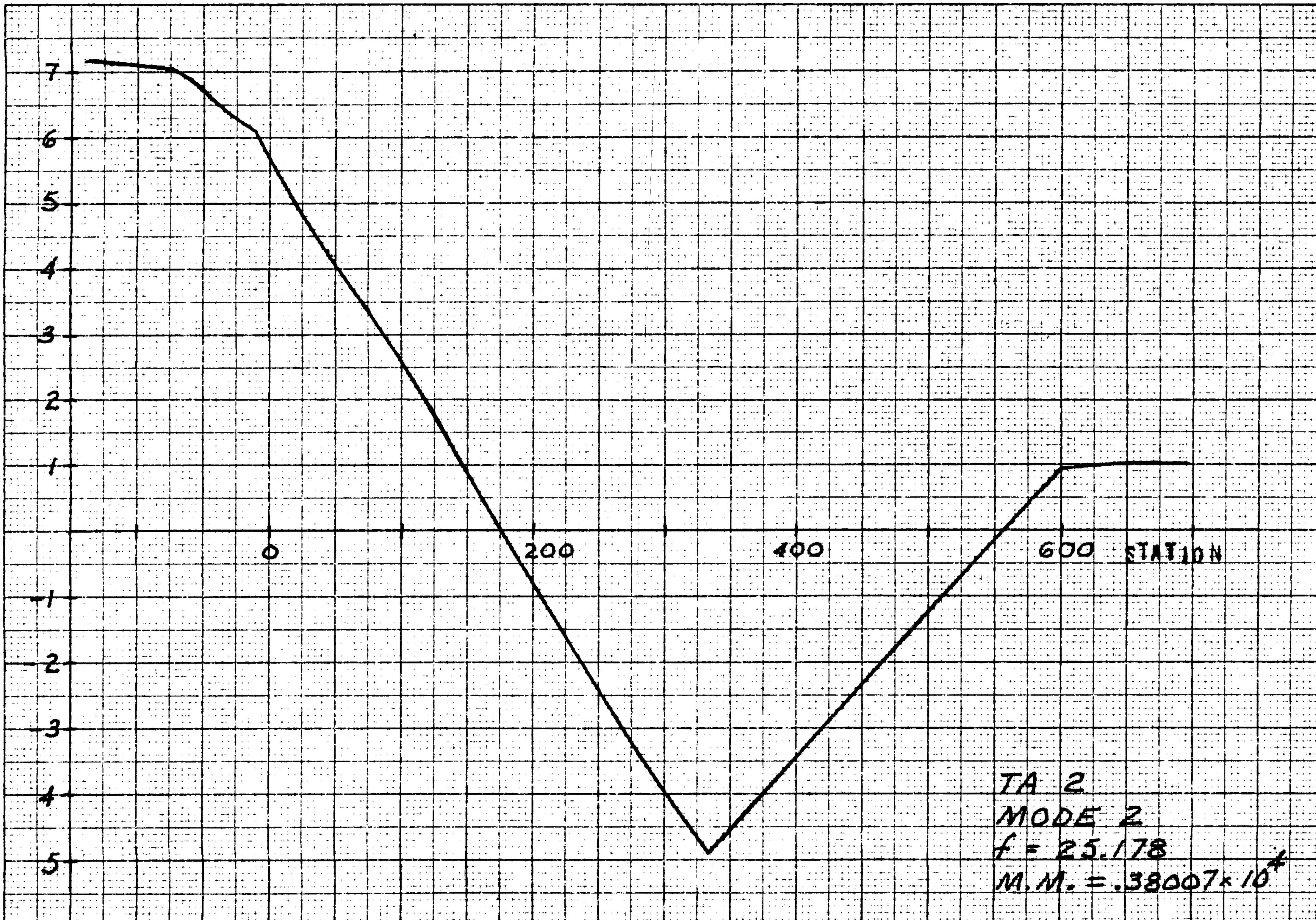


5
4
3
2
1
1
2
3
4
0
200
400
600
STATION
TA 1
MODE 2
f = 109.54
M.M. = .76222×10^2

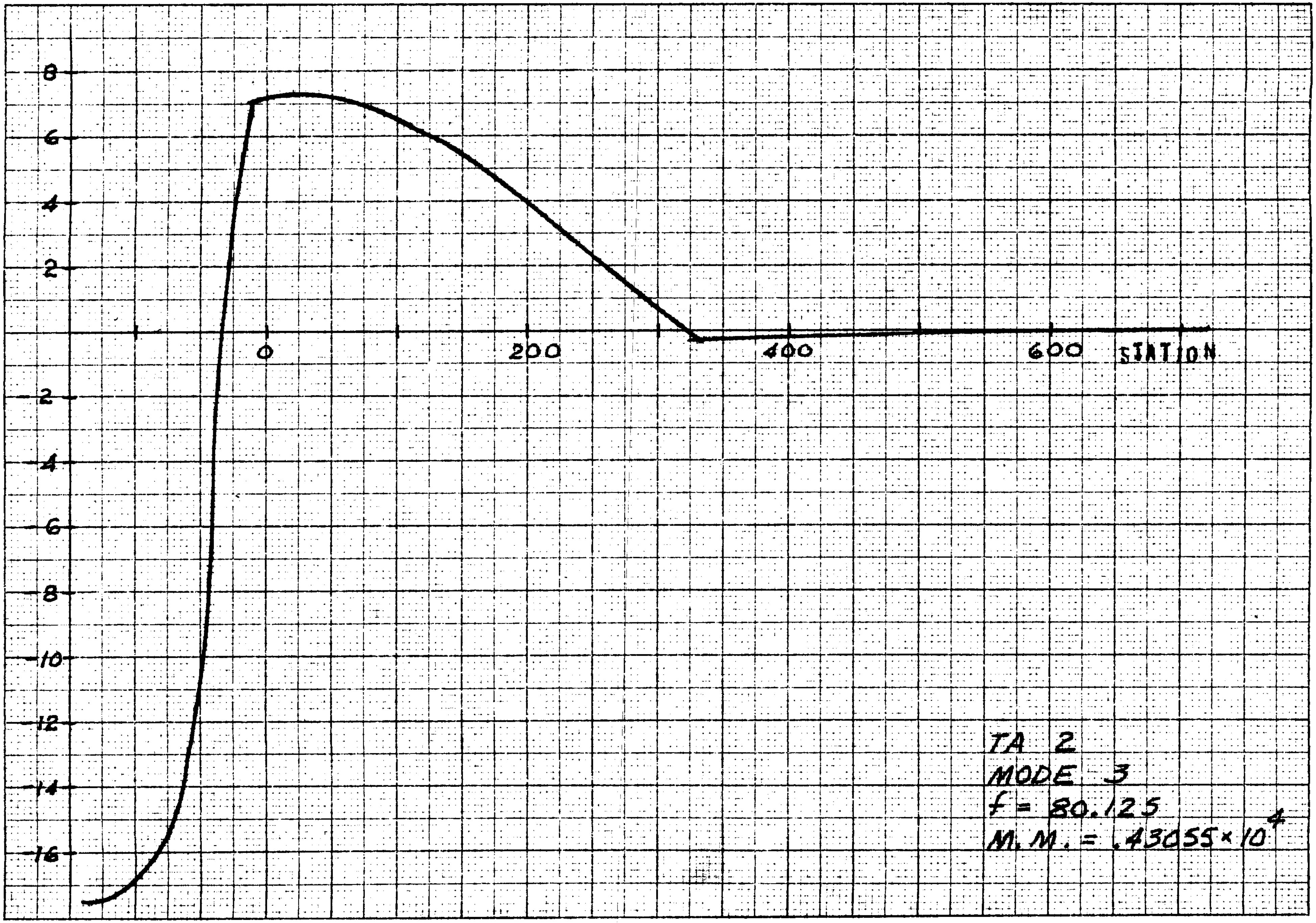

8
6
4
2
-2
-4
-6
-8
-10
12
0
200
400
600 STATION
TA 1
MODE 3
f = 159.92
M.M. = .39149 × 10^3

SM 46346

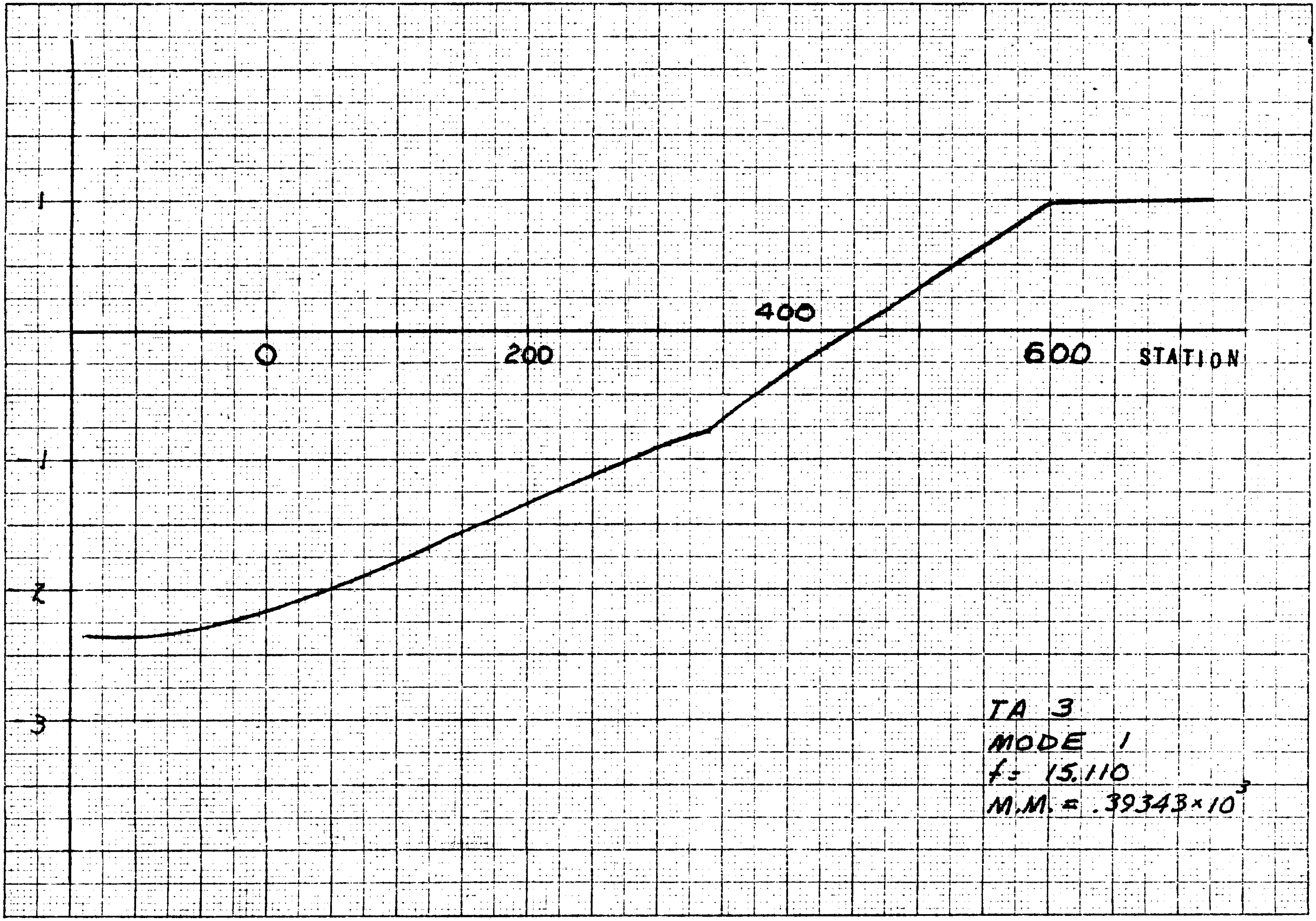


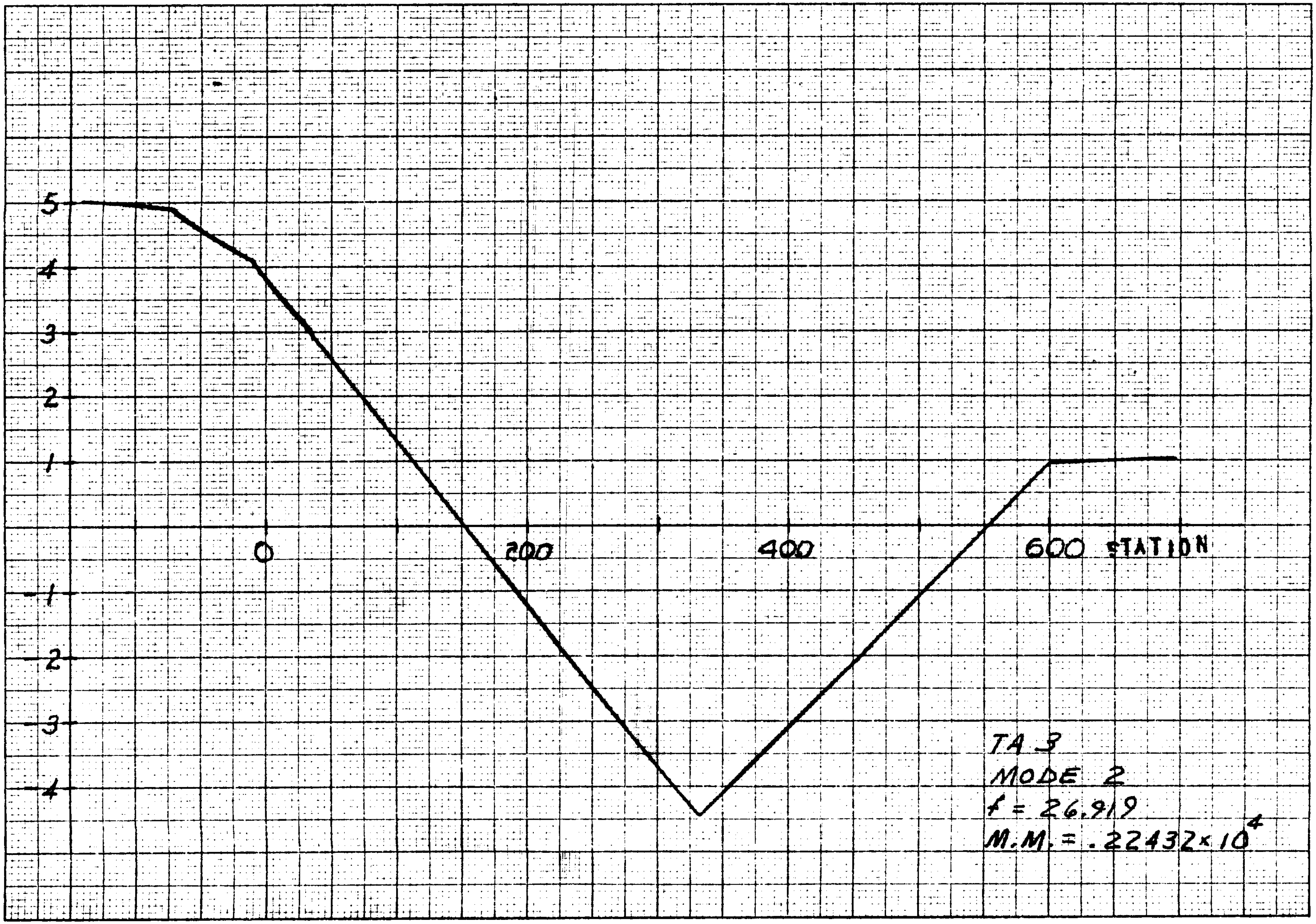

7
6
5
4
3
2
1
-1
-2
-3
-4
-5
0
200
400
600
STATION
TA 2
MODE 2
f = 25.178
M.M. = .38007×10⁴

SM 46346

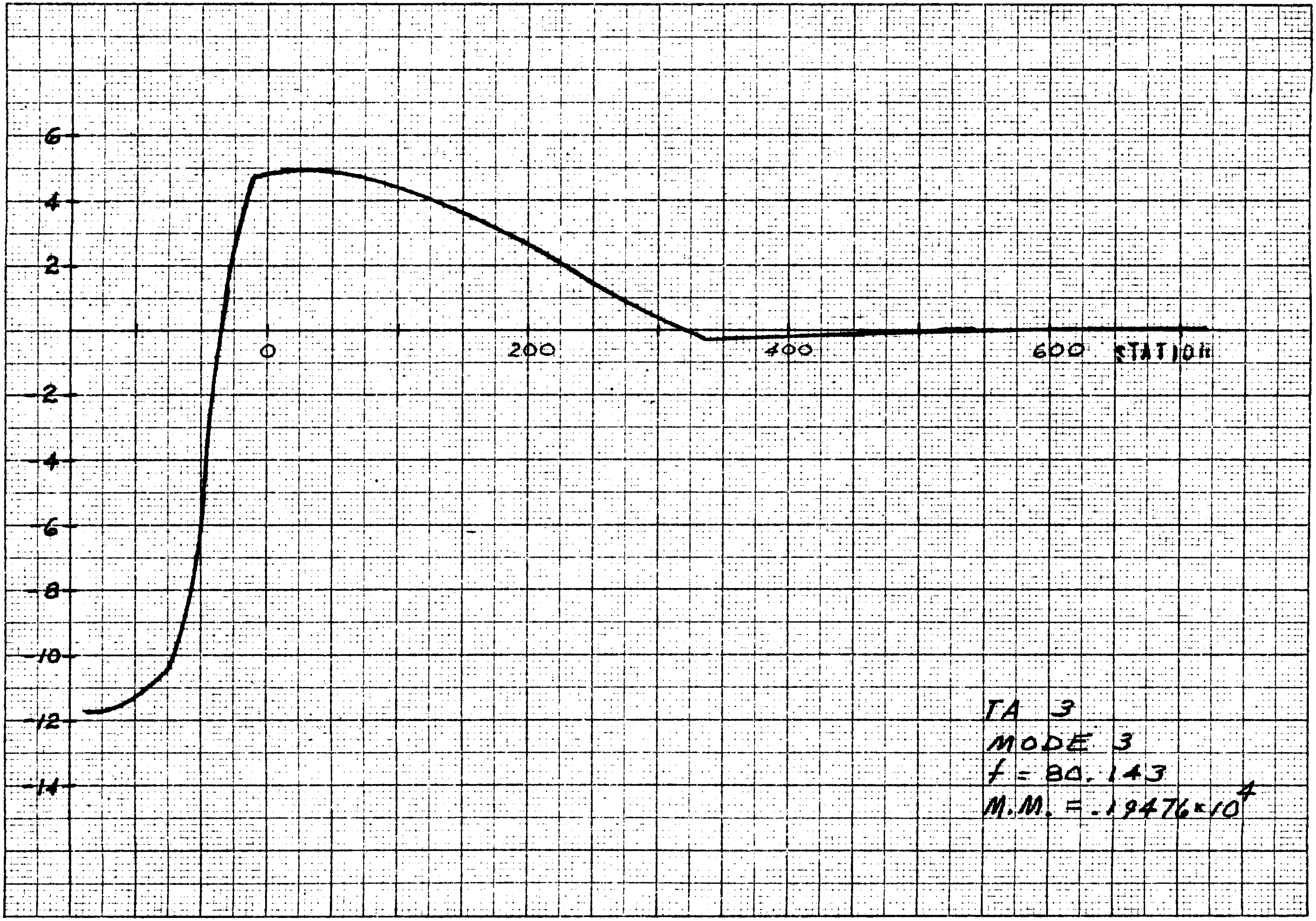

8
6
4
2
2
4
6
8
10
12
14
16
0
200
400
600
STATION
TA 2
MODE 3
f = 80.125
M. M. = .43055 × 10^4

SM 46346



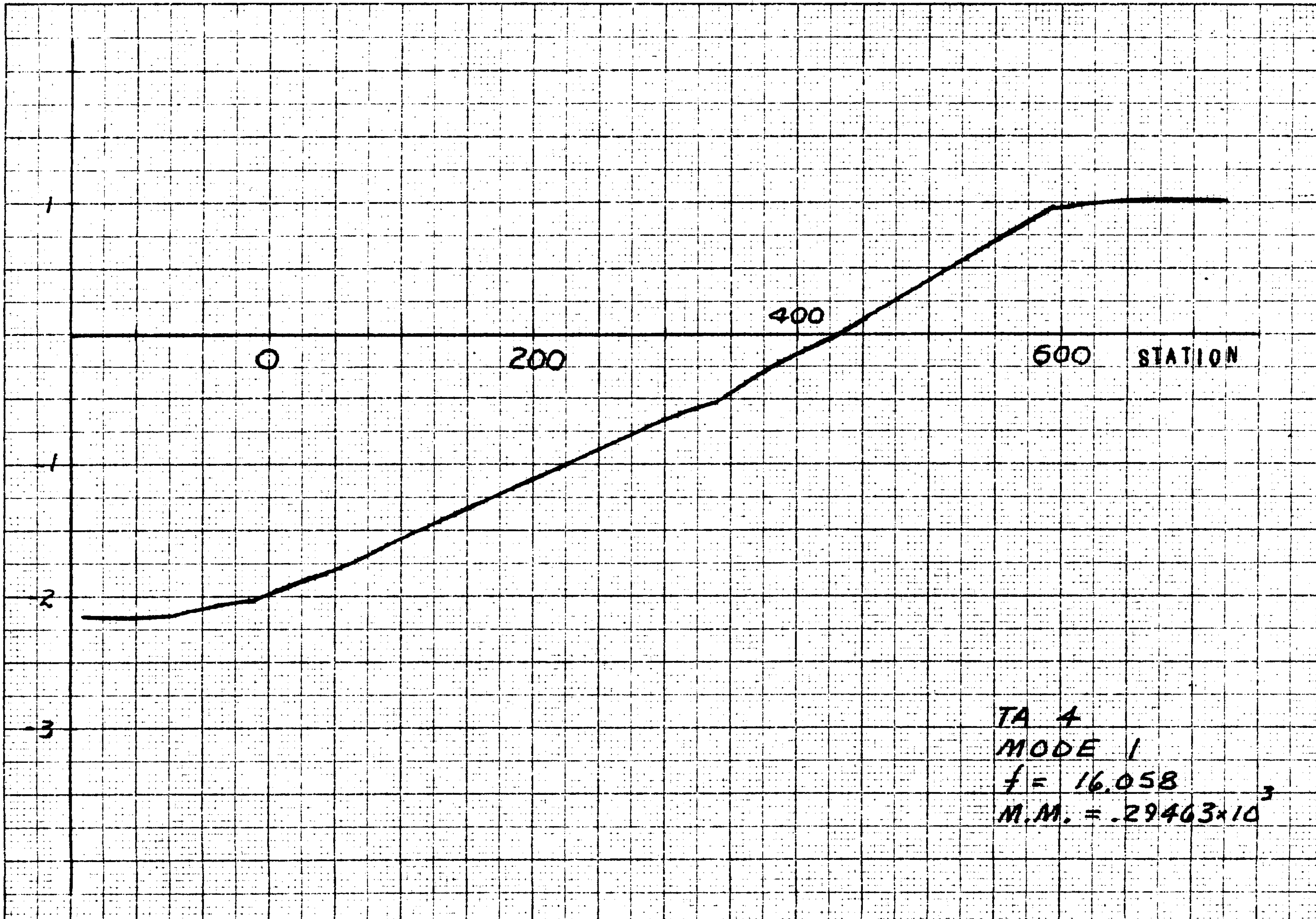

5
4
3
2
1
0
200
400
600 STATION
-1
-2
-3
-4
TA 3
MODE 2
f = 26.919
M.M. = .22432×10⁴

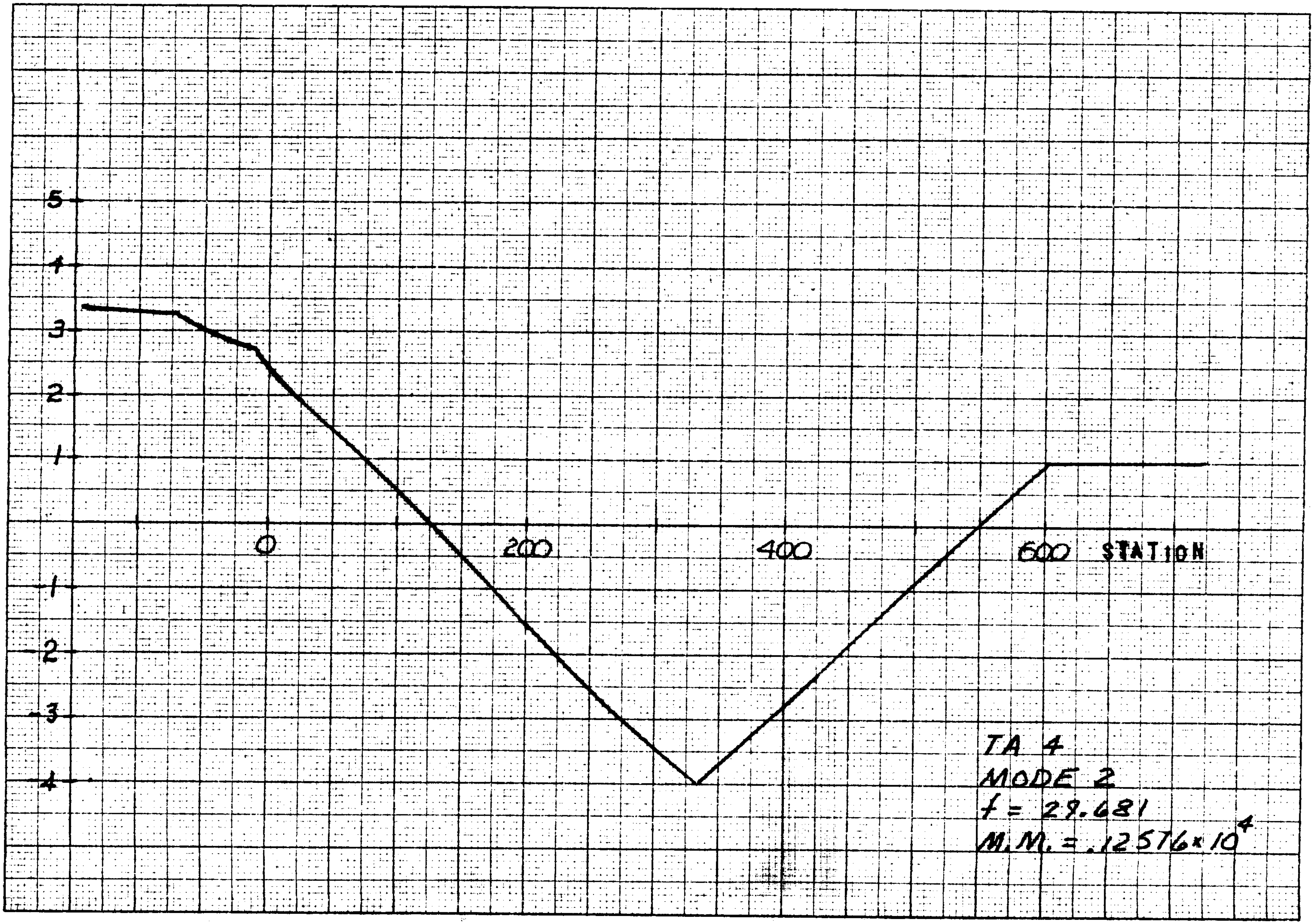
SM 40346

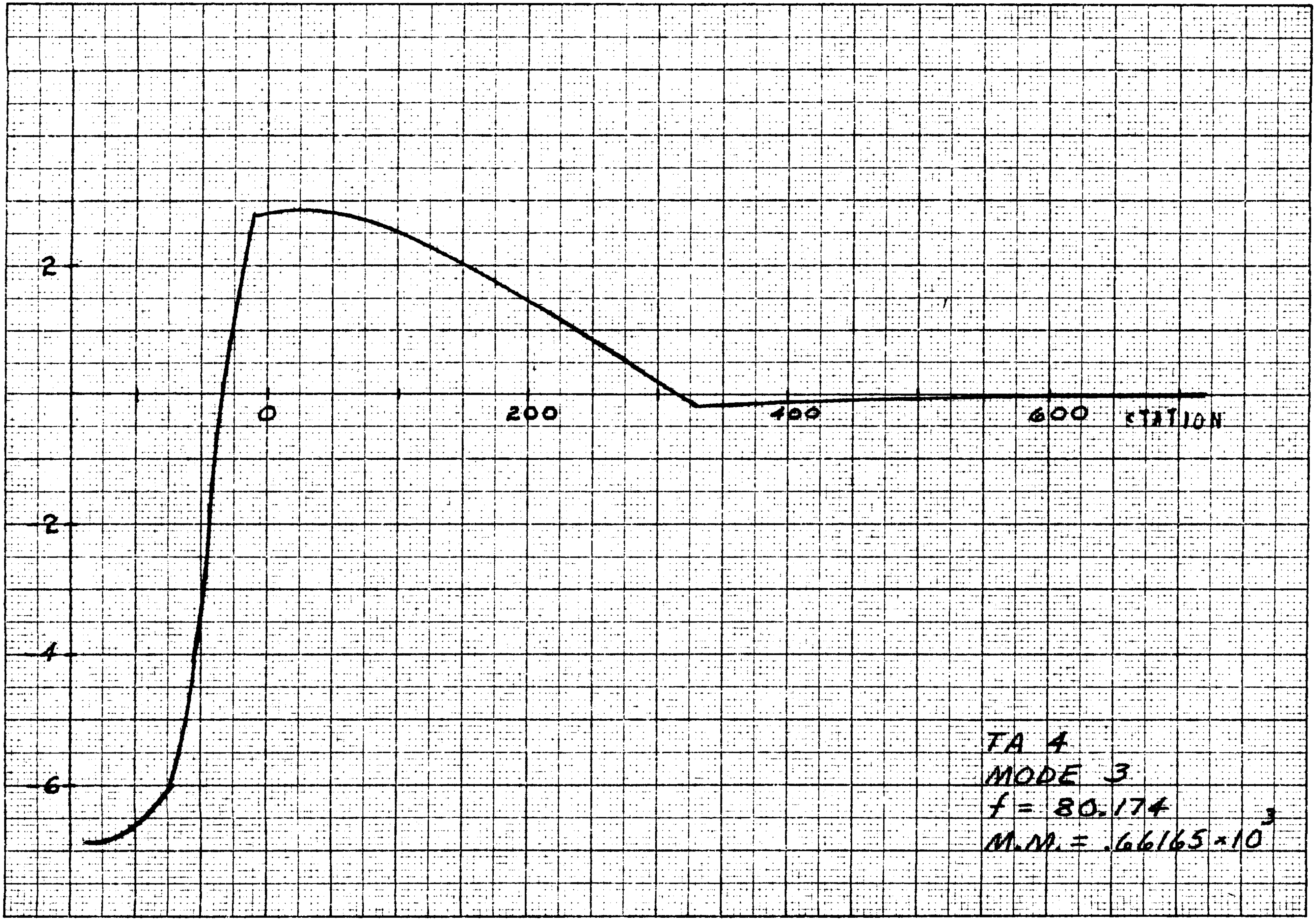


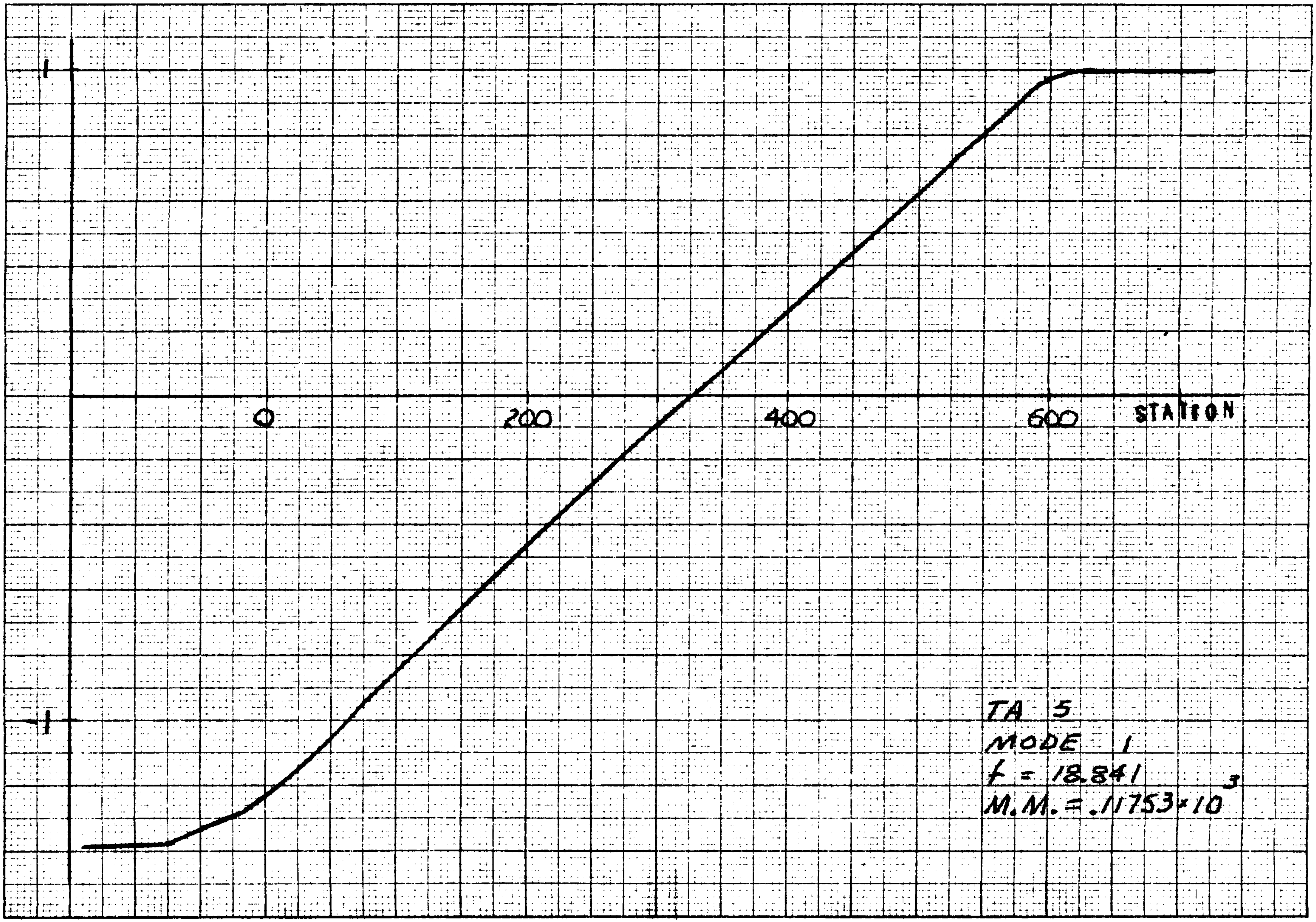
6
4
2
-2
-4
-6
-8
-10
-12
-14
0
200
400
600
STATION
TA 3
MODE 3
f = 80.143
M.M. = .19476×10^4

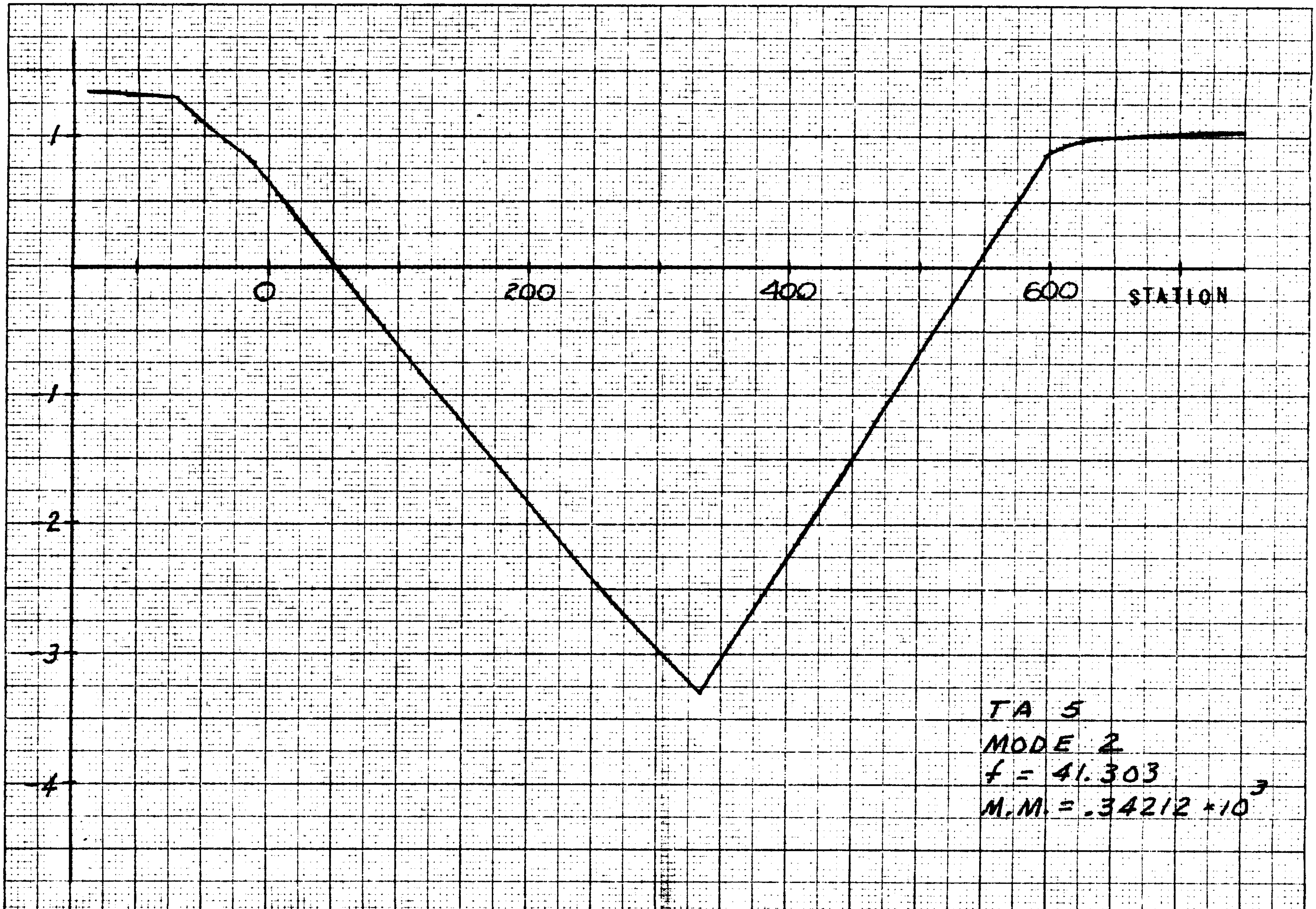
SM 46346

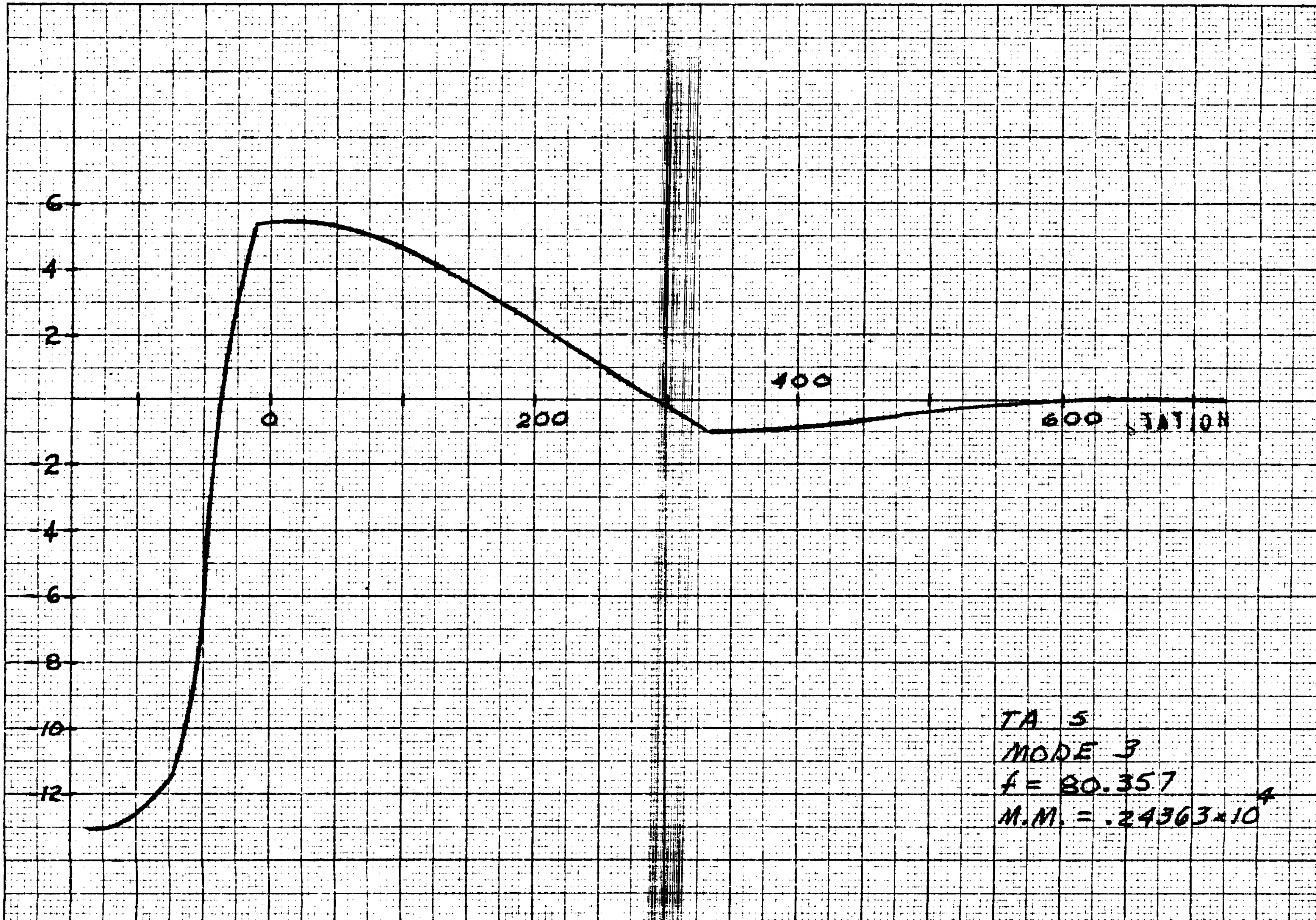



5
4
3
2
1
1
2
3
4
0
200
400
600
STATION
TA 4
MODE 2
f = 29.681
M.M. = .12576 x 10^4

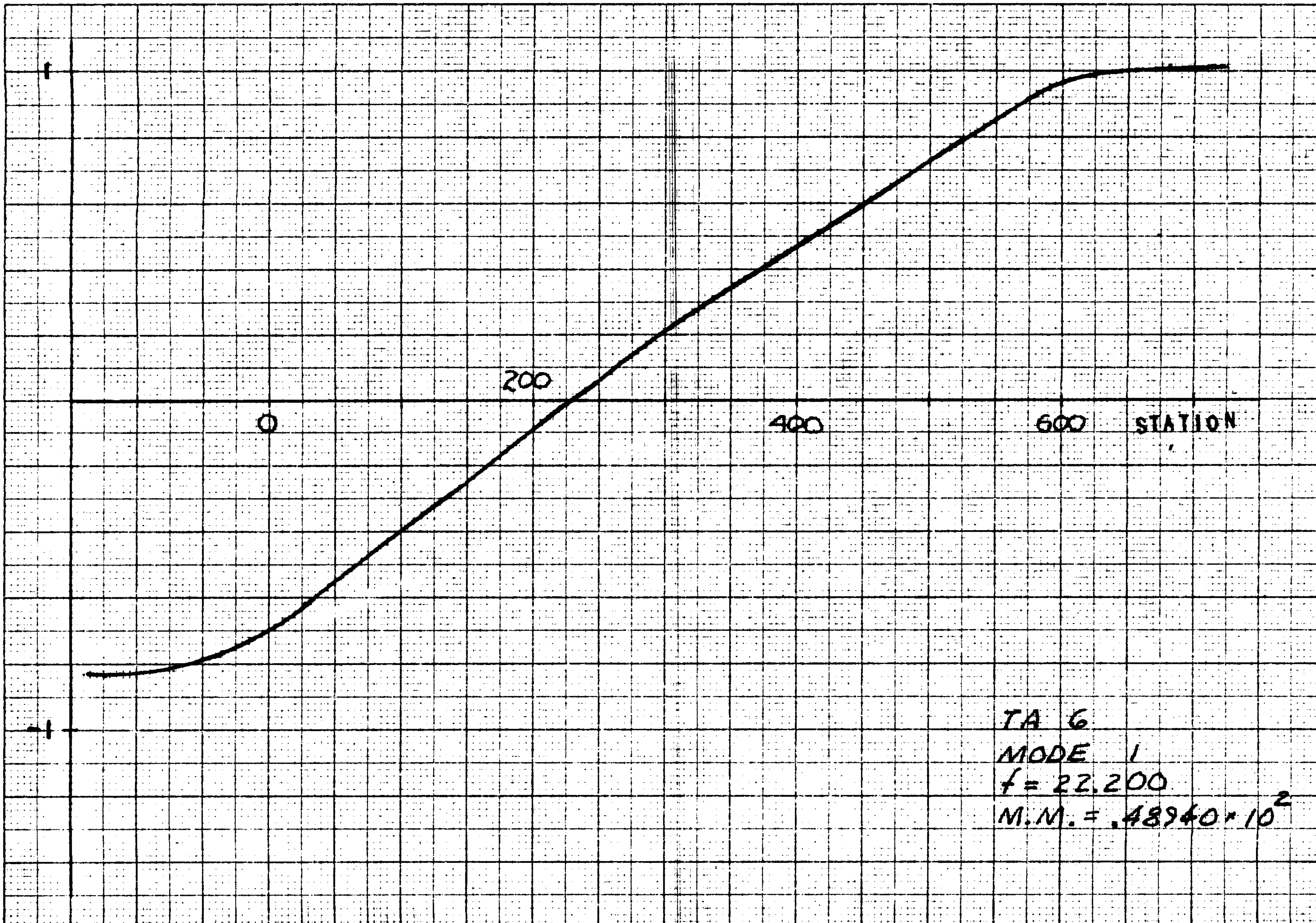

2
0
200
400
600
STATION
2
4
6
FA 4
MODE 3
f = 80.174
M.M. = .66165 × 10^3


1
-1
0
200
400
600
STATION
TA 5
MODE 1
f = 18.841
M.M. = .11753×10^3


1
0
200
400
600
STATION
1
2
3
4
TA 5
MODE 2
f = 41.303
M.M. = .34212 × 10
3


6
4
2
-2
-4
-6
-8
-10
-12
0
200
400
600
STATION
TA 5
MODE 3
f = 80.357
M.M. = .24363×10^4

SM 46340



SM 46346

K+E 10 X 10 TO ½ INCH 46 1327
7 X 10 IN. • ALBANENE® MADE IN U.S.A.
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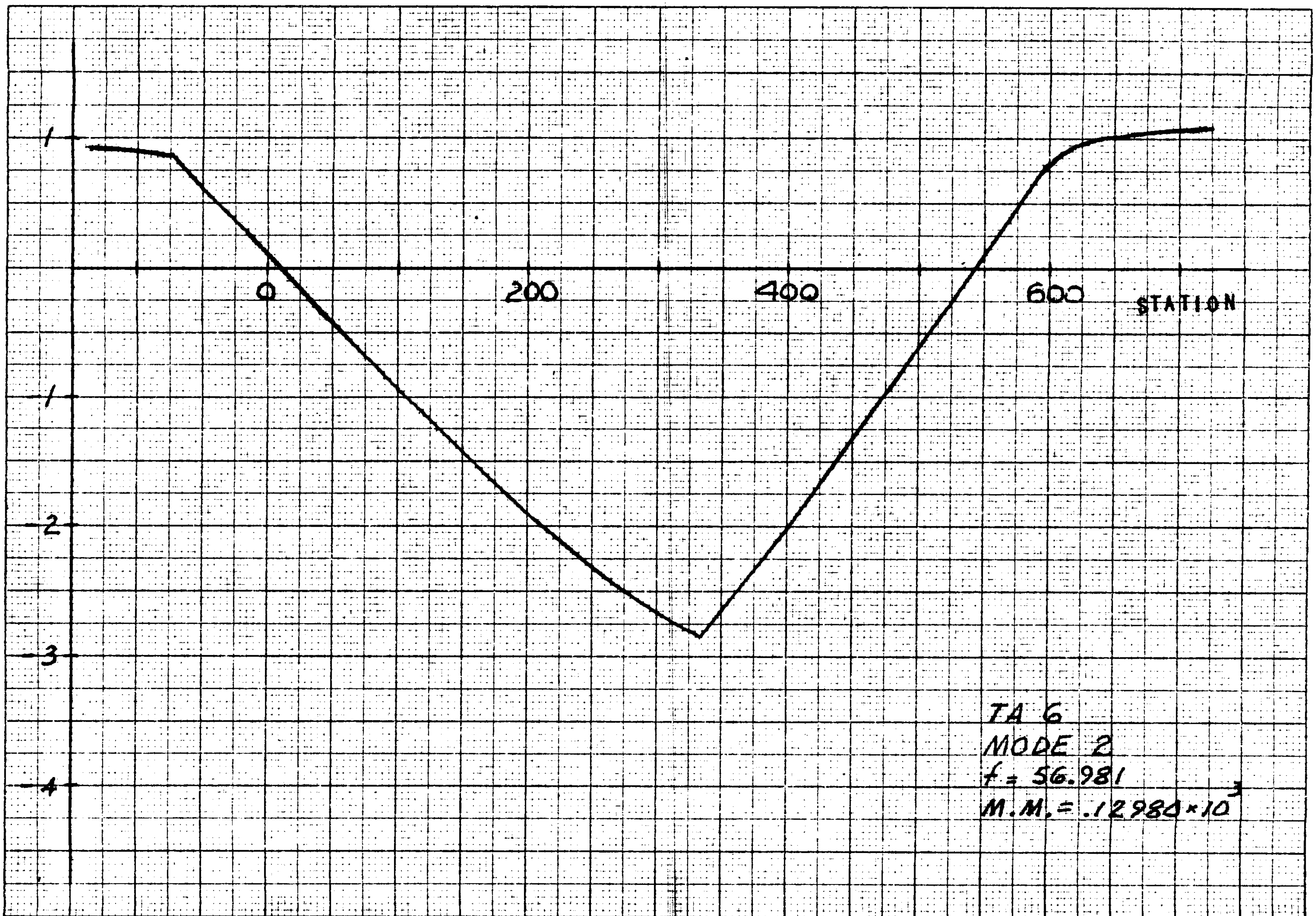
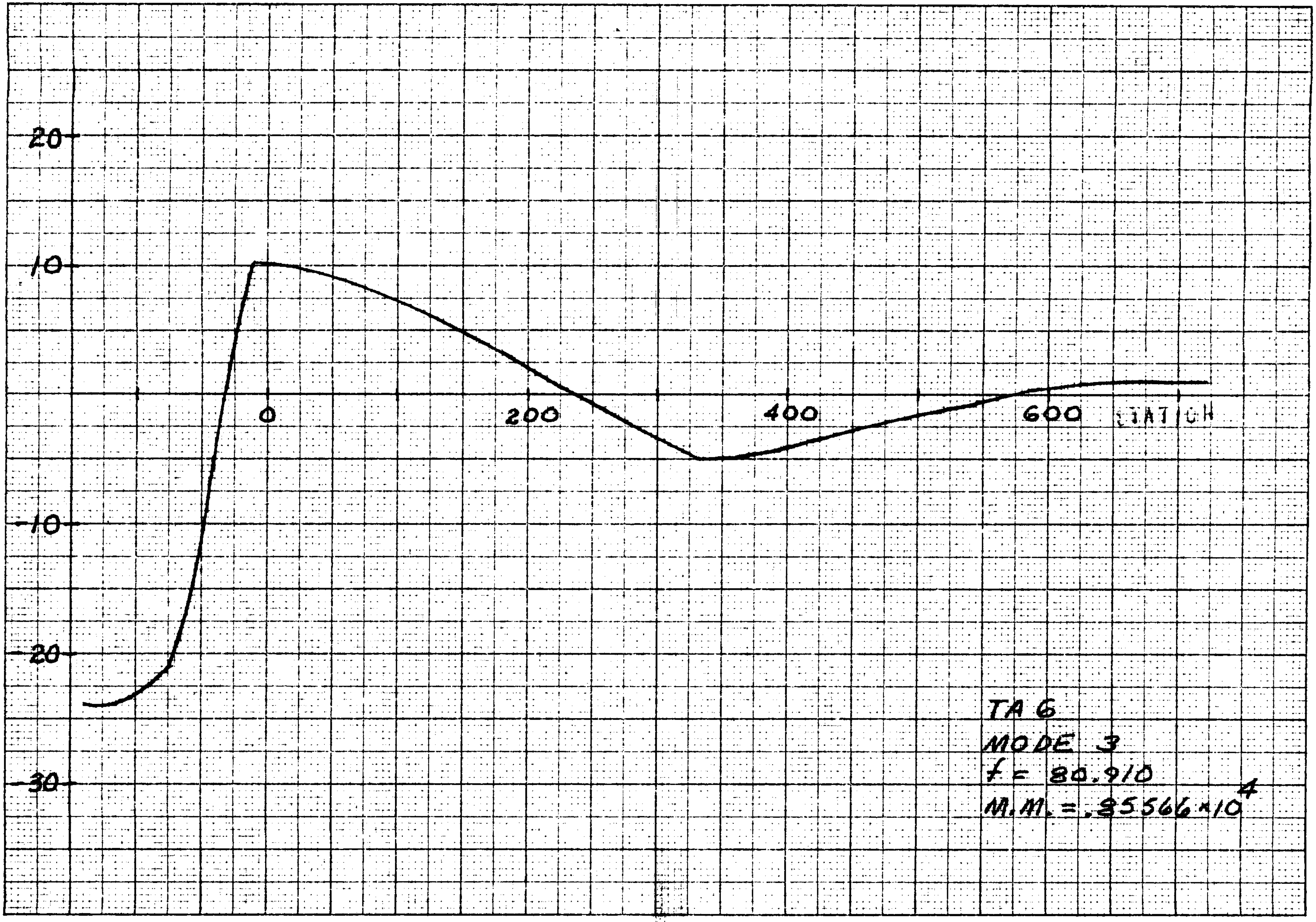
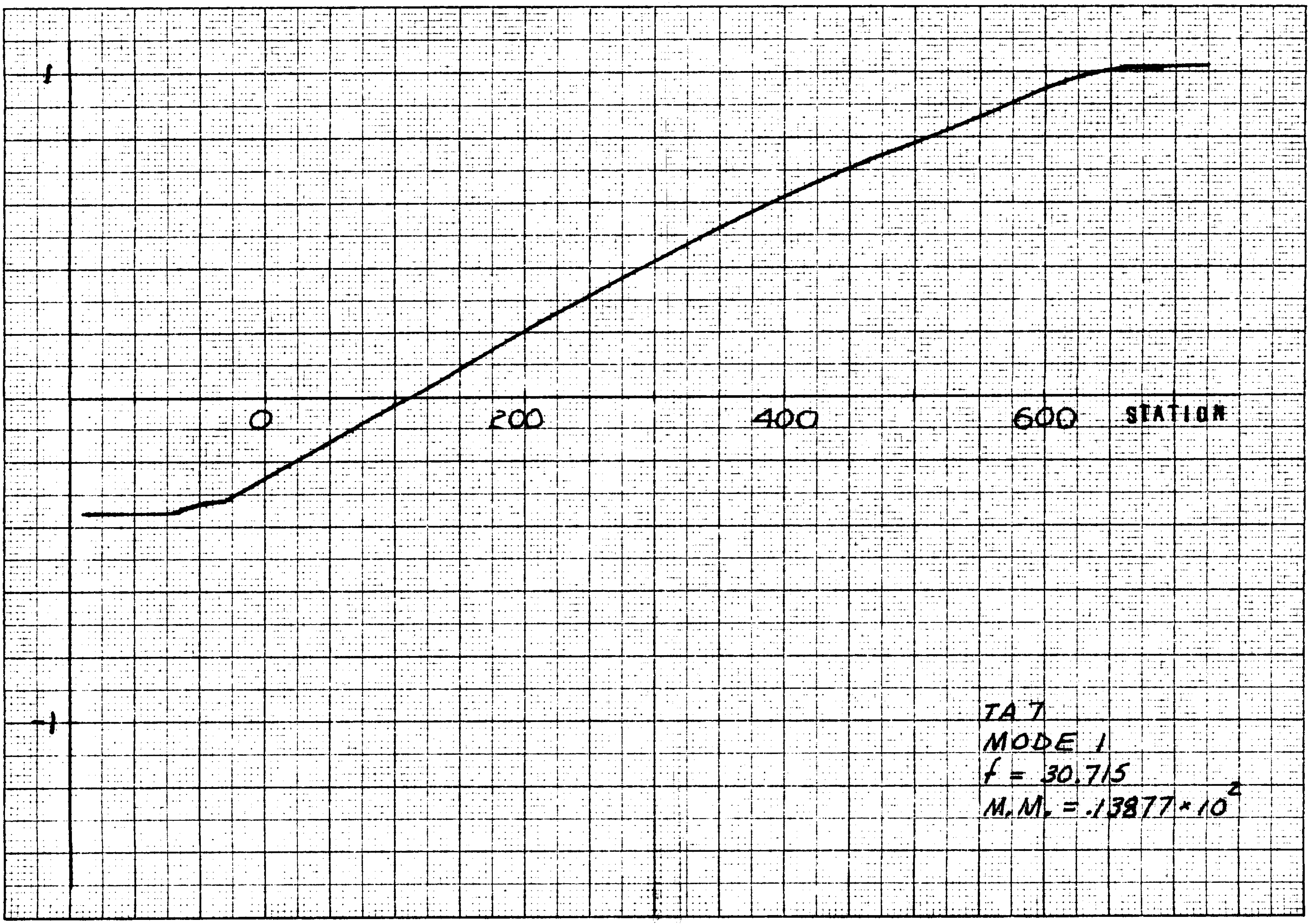


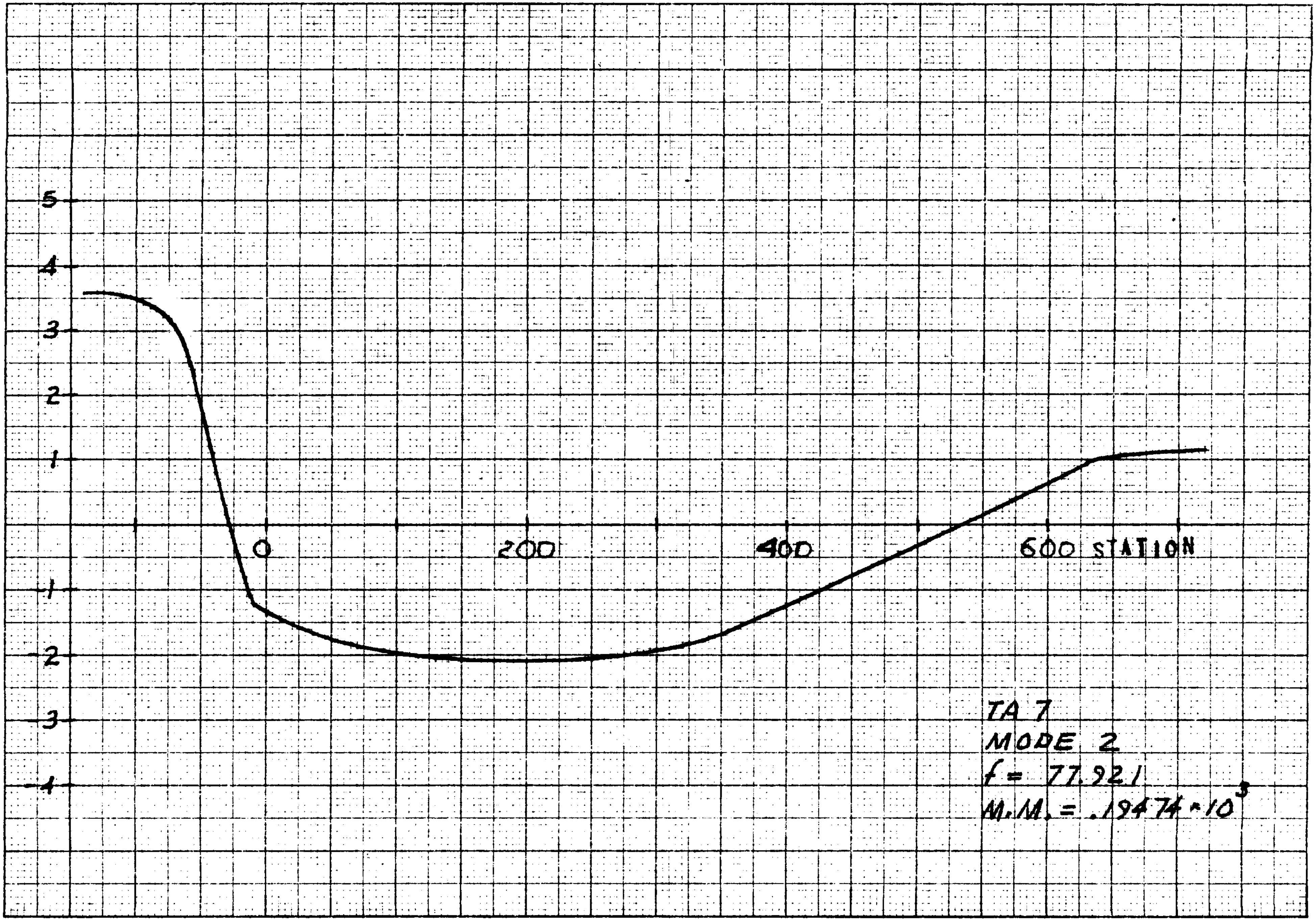
SM 46346


20
10
-10
-20
-30
0
200
400
600
STATION
TA 6
MODE 3
f = 80.910
M.M. = .85566 × 10^4

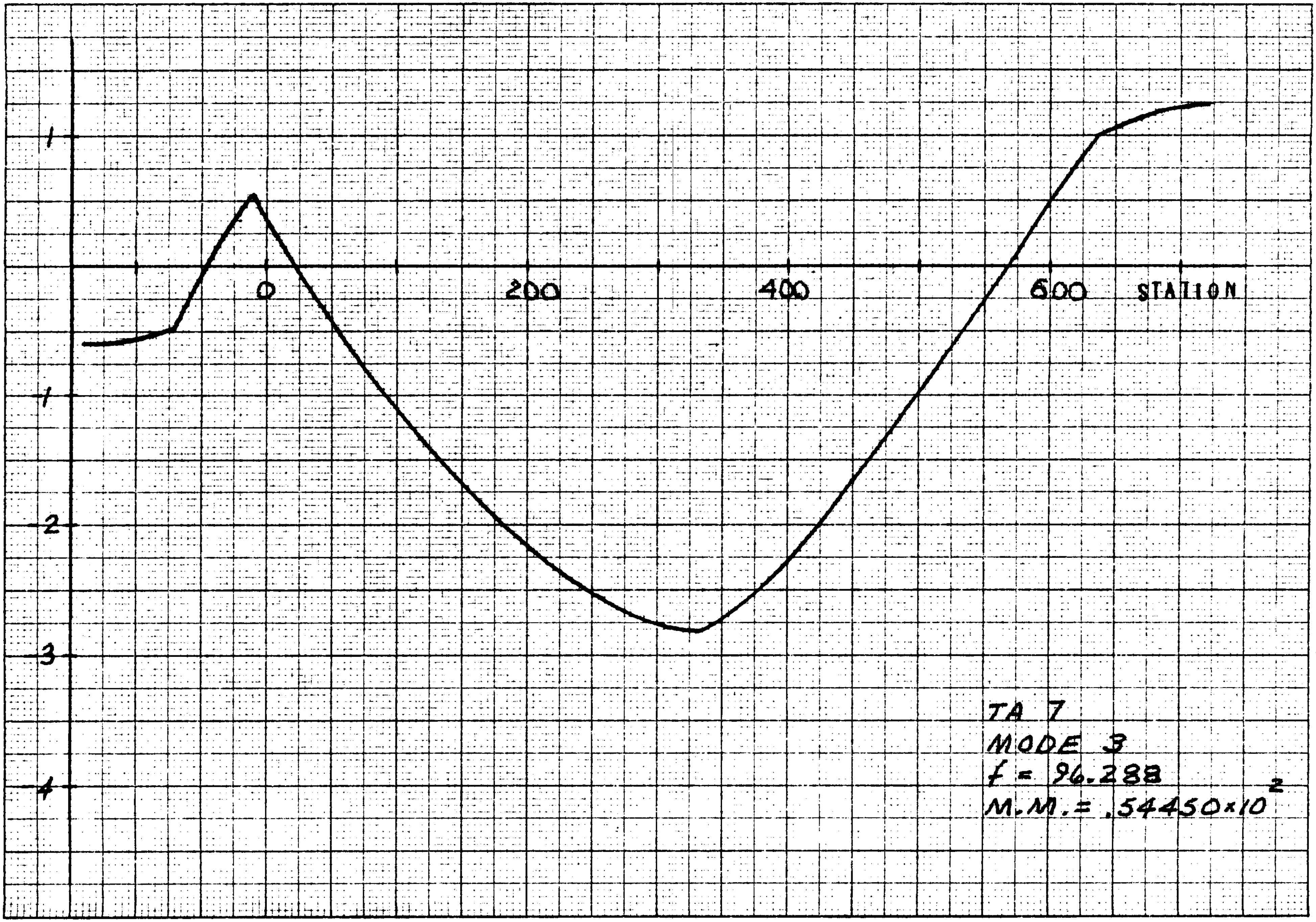
SM 46346



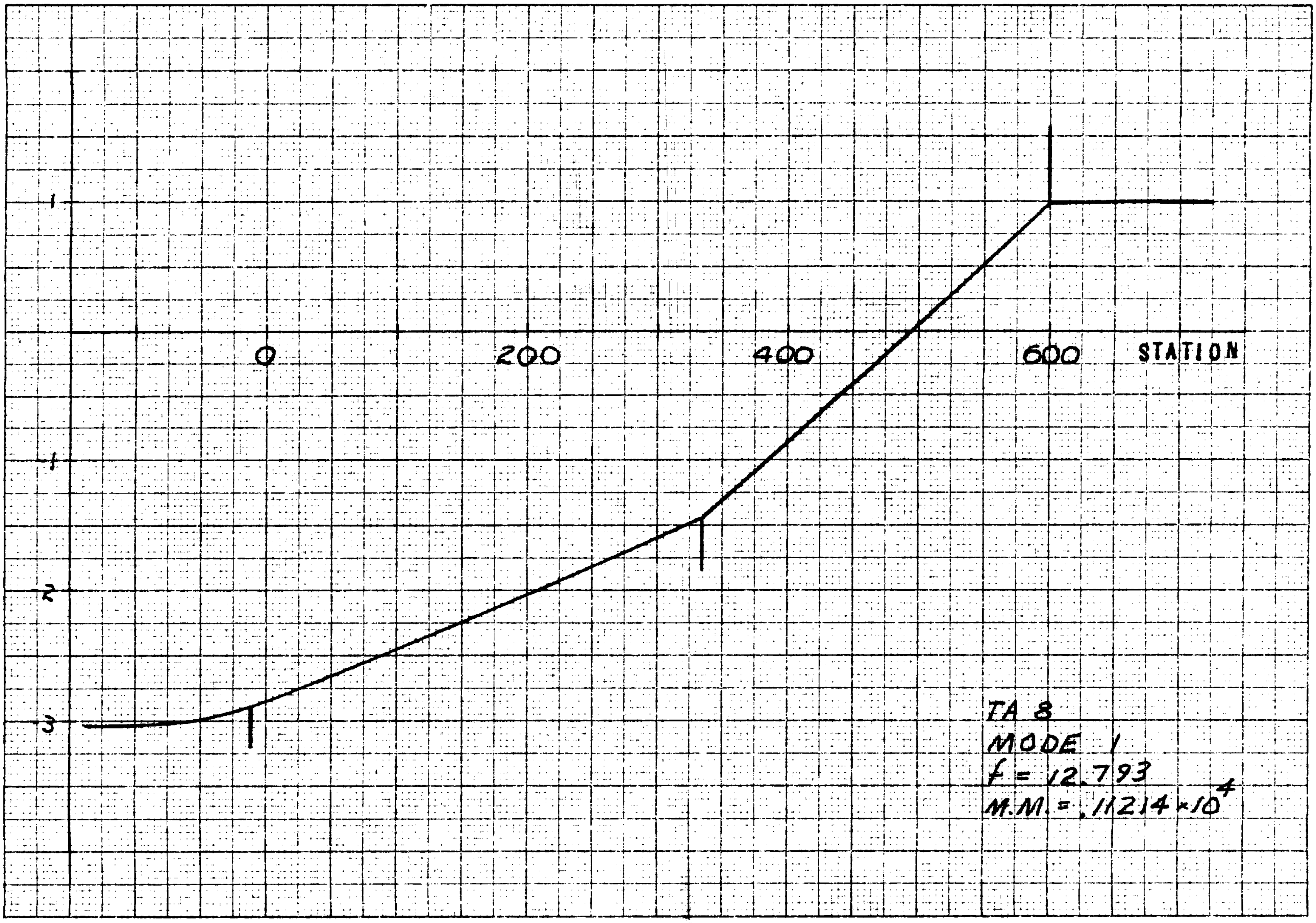
SM 46346

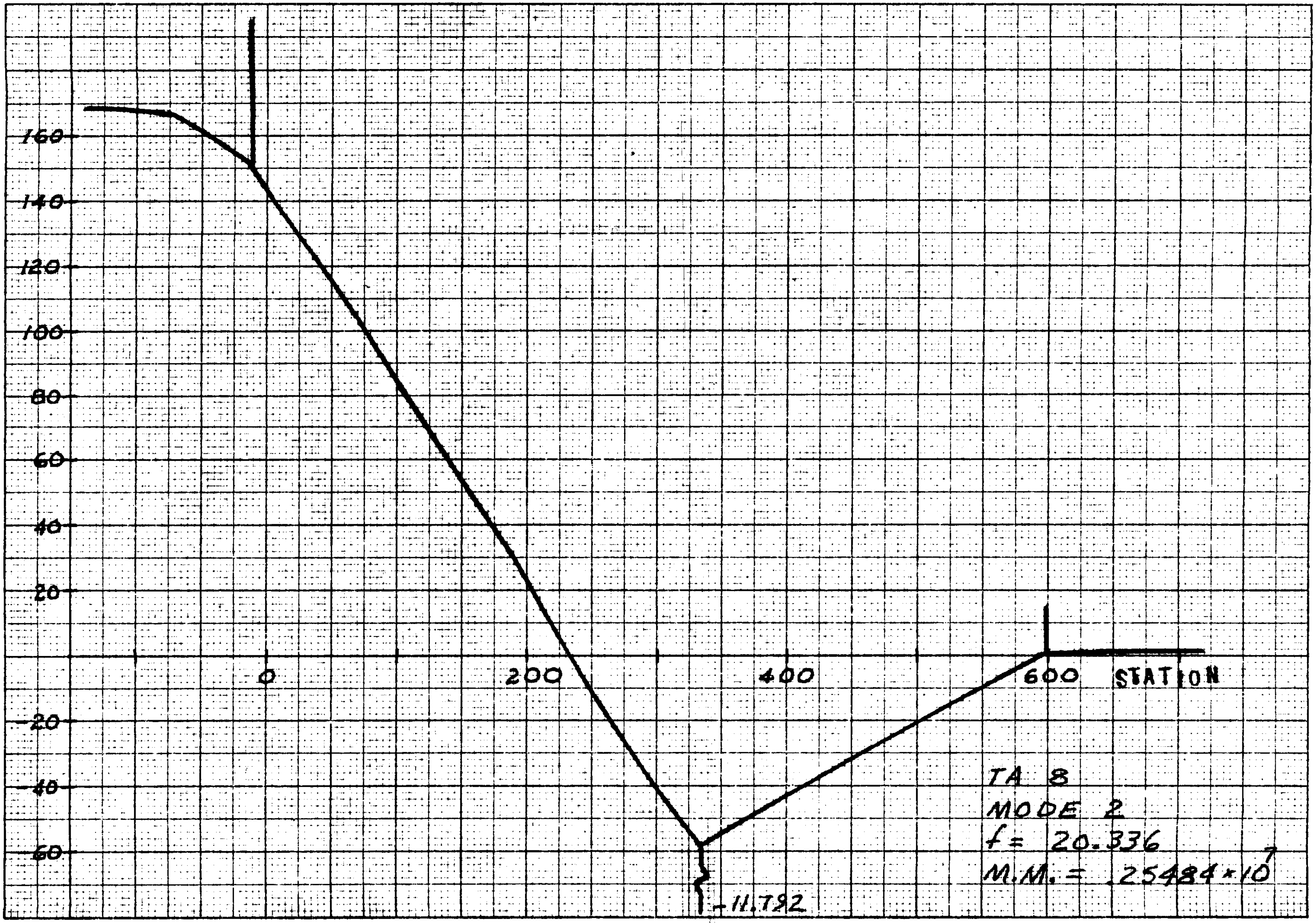

5
4
3
2
1
-1
-2
-3
-4
0
200
400
600 STATION
TA 7
MODE 2
f = 77.921
M.M. = .19474 · 10^3

SM 4634[illegible]

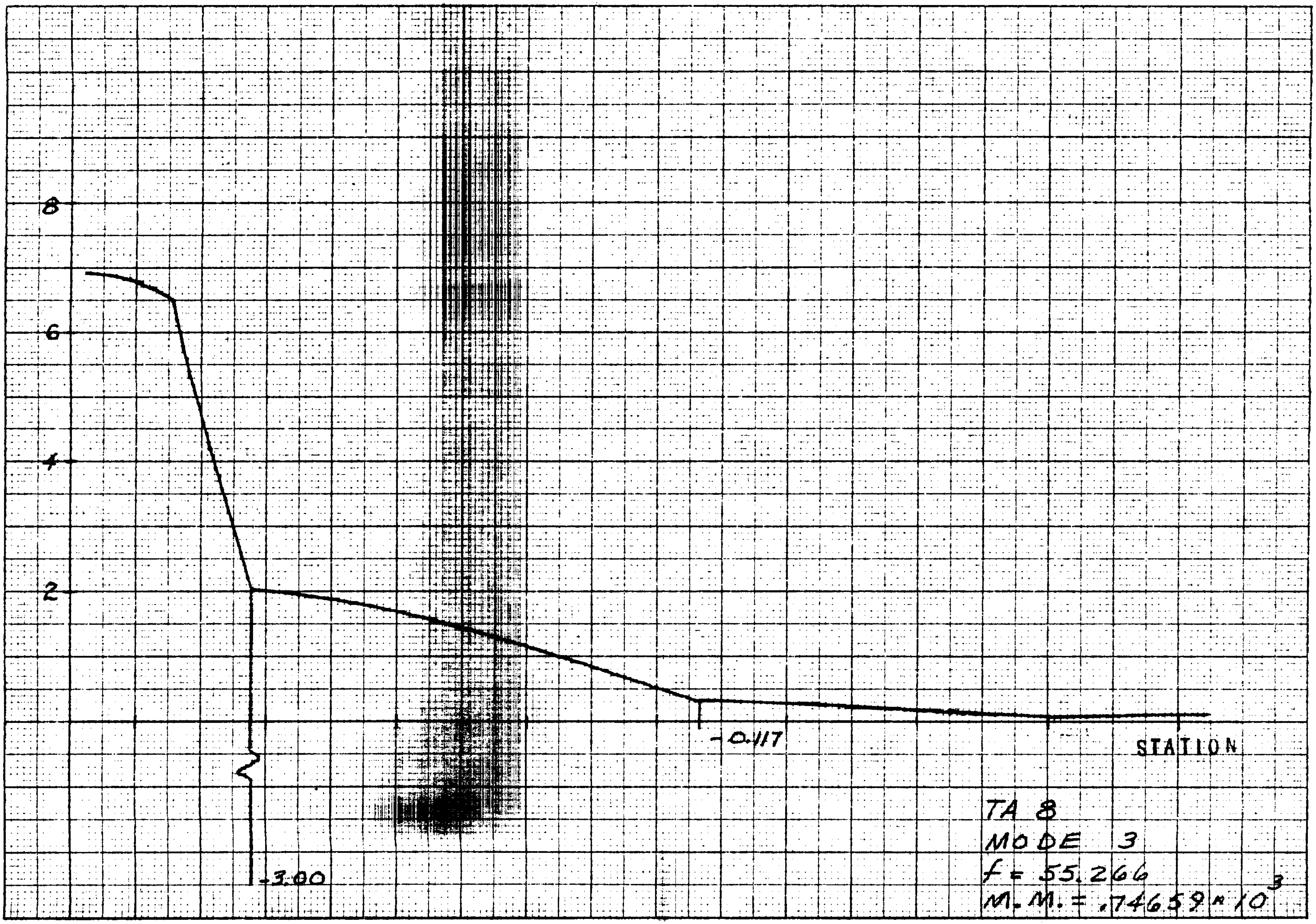


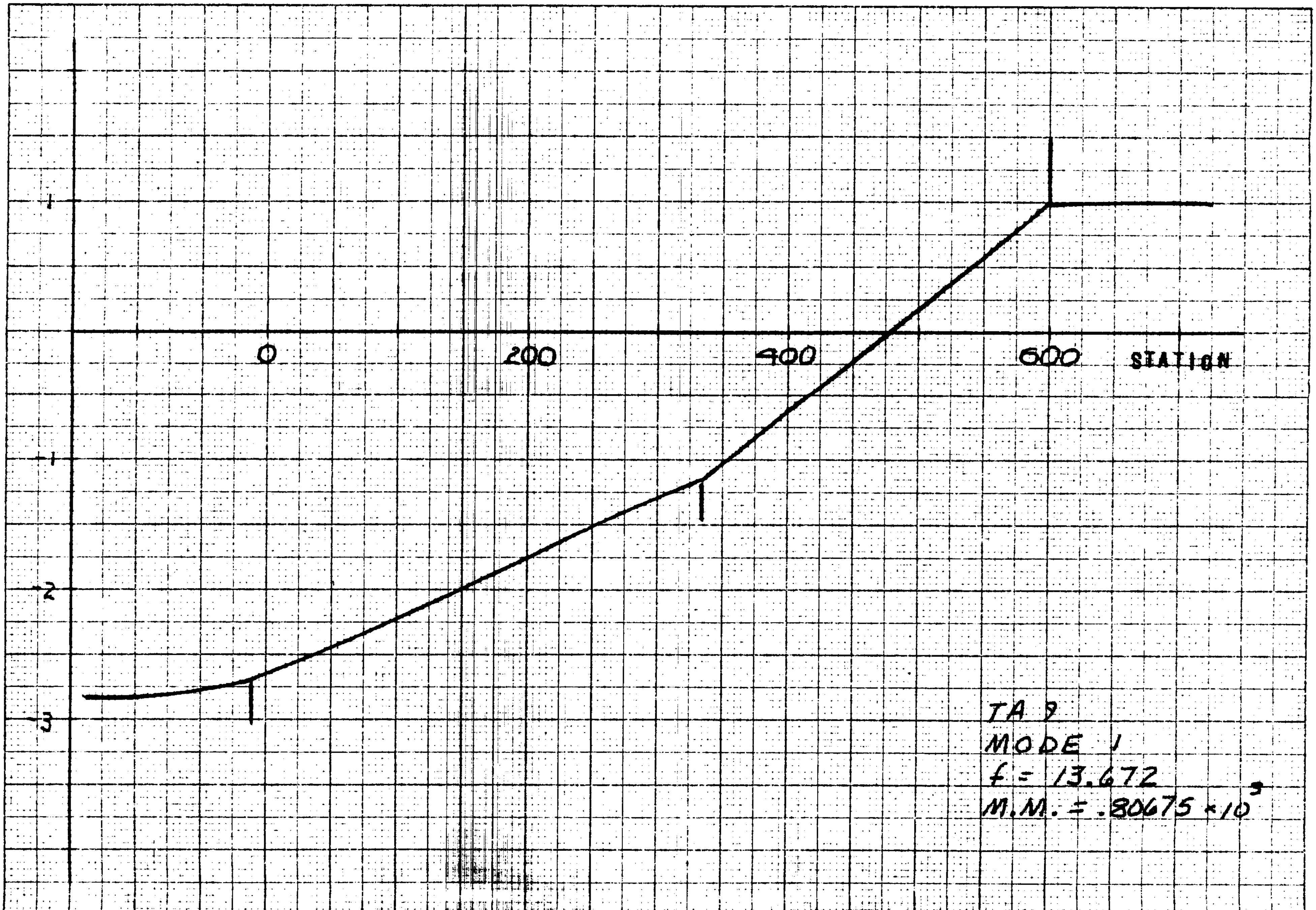
SM 46346



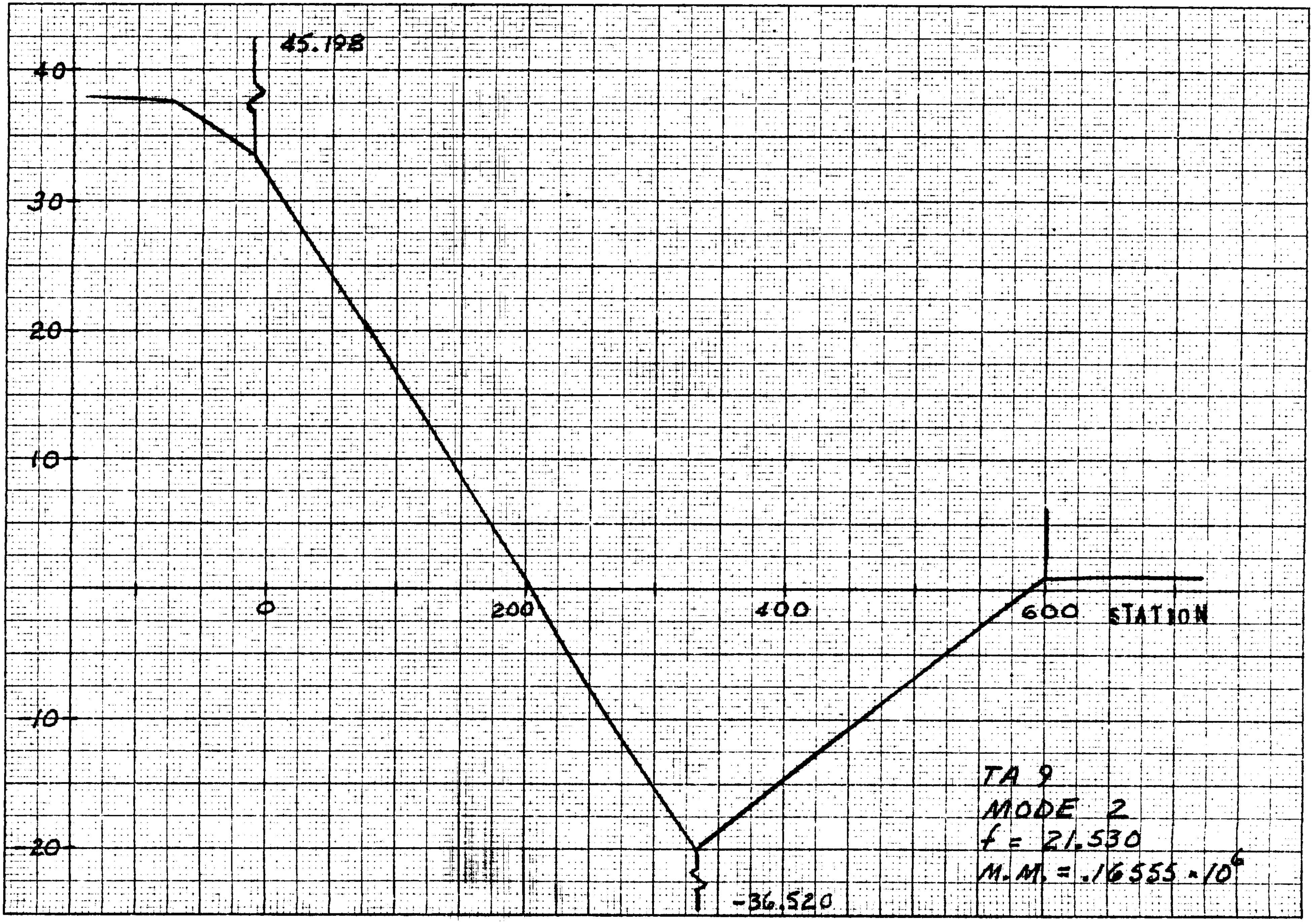

160
140
120
100
80
60
40
20
-20
-40
-60
0
200
400
600
STATION
-11.792
TA 8
MODE 2
f = 20.336
M.M. = .25484×10^7

SM 46346



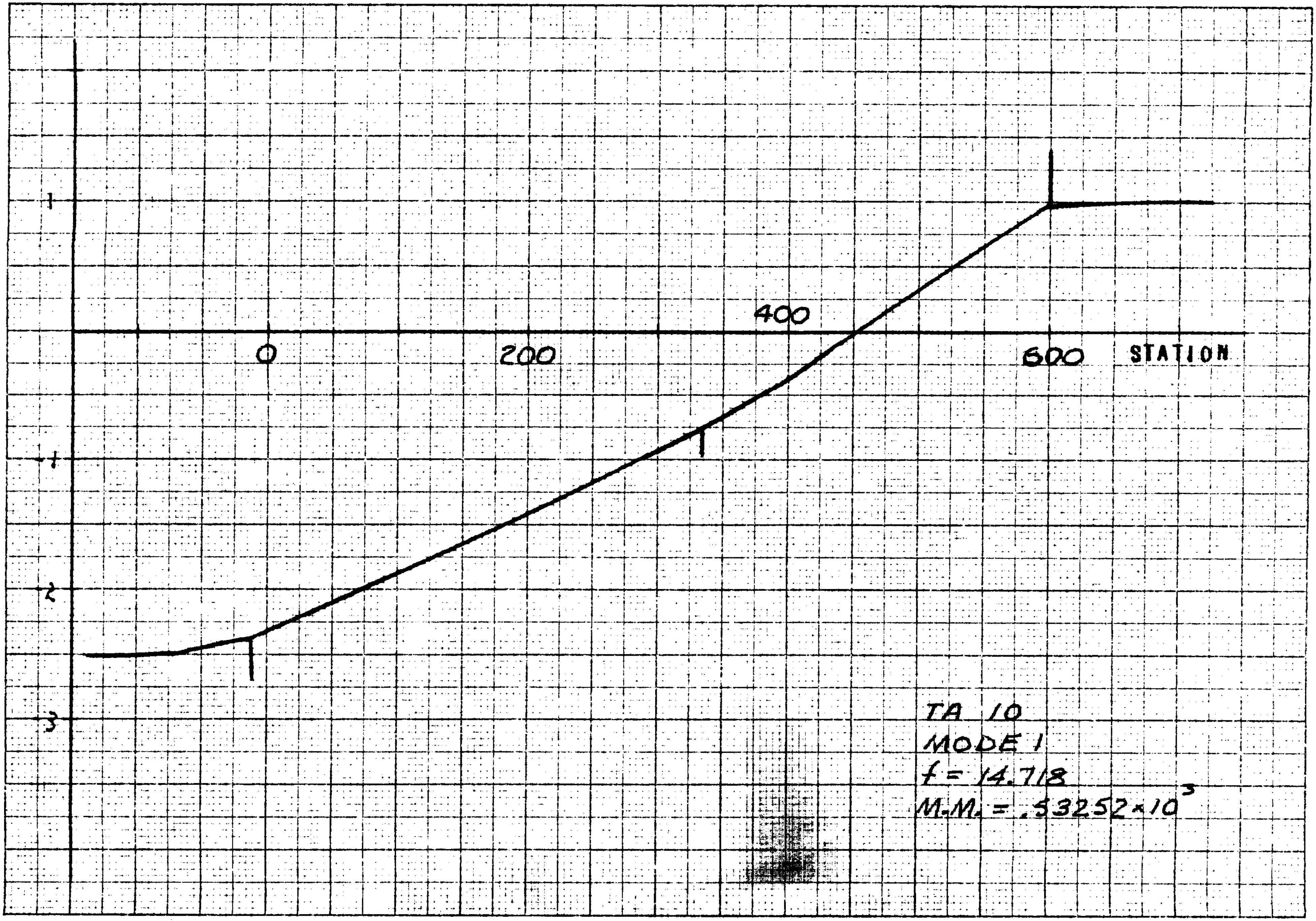

1
-1
-2
-3
0
200
400
600
STATION
TA 9
MODE 1
f = 13.672
M.M. = .80675 × 10^3

SM 46346

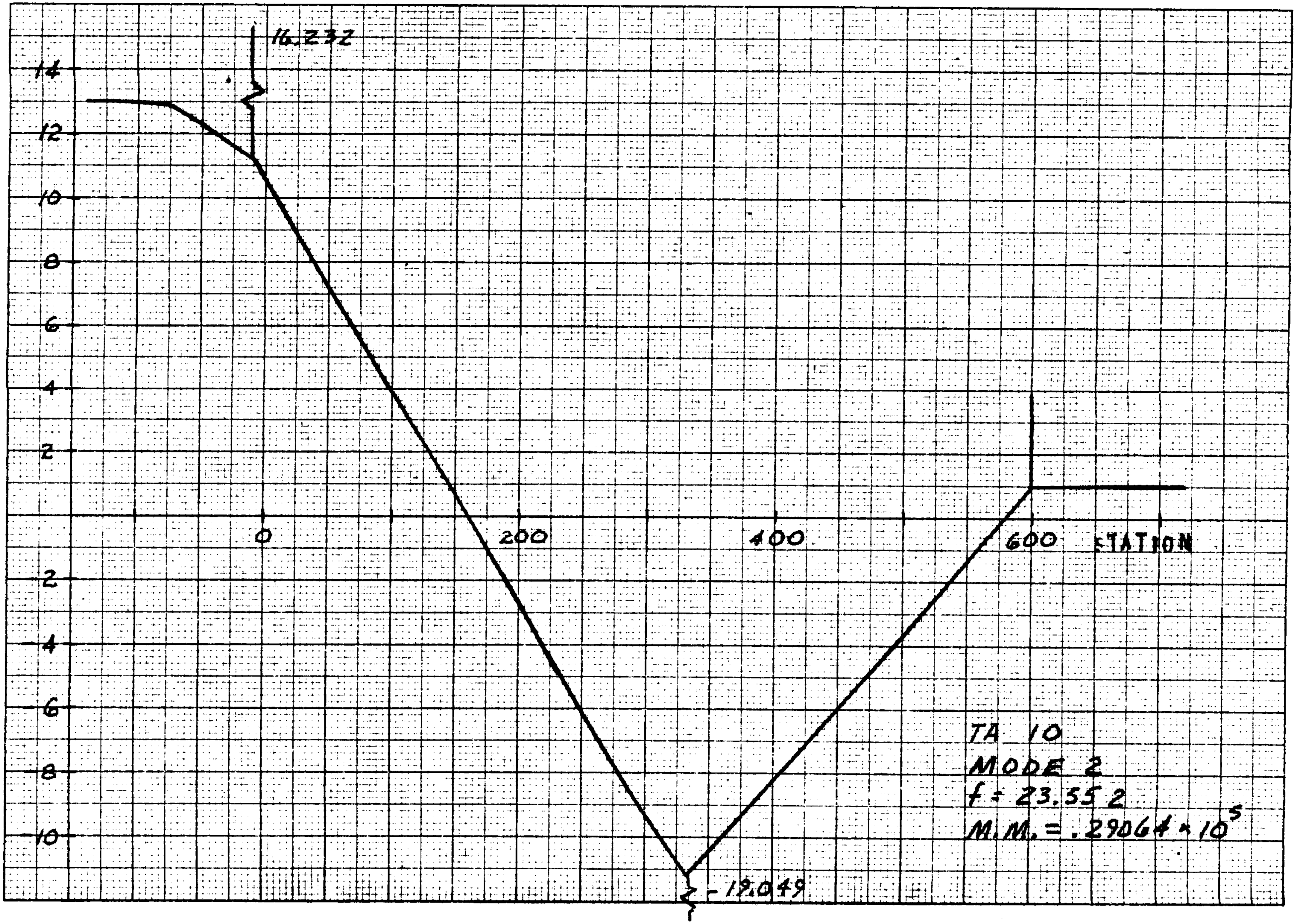

45.198
40
30
20
10
0
200
400
600
STATION
10
20
-36.520
TA 9
MODE 2
f = 21.530
M.M. = .16555 × 10^6

SM 46346

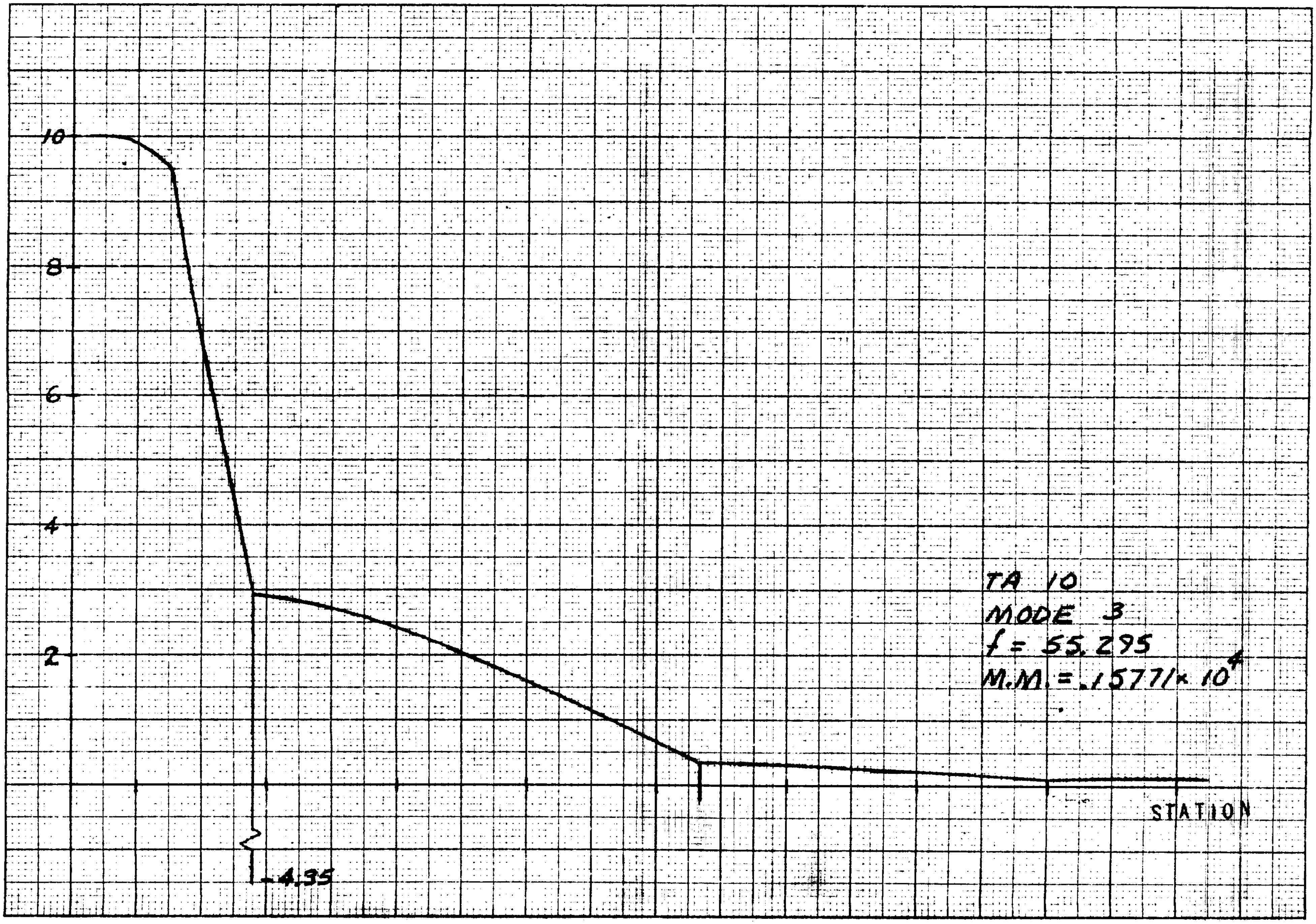
SM 46346



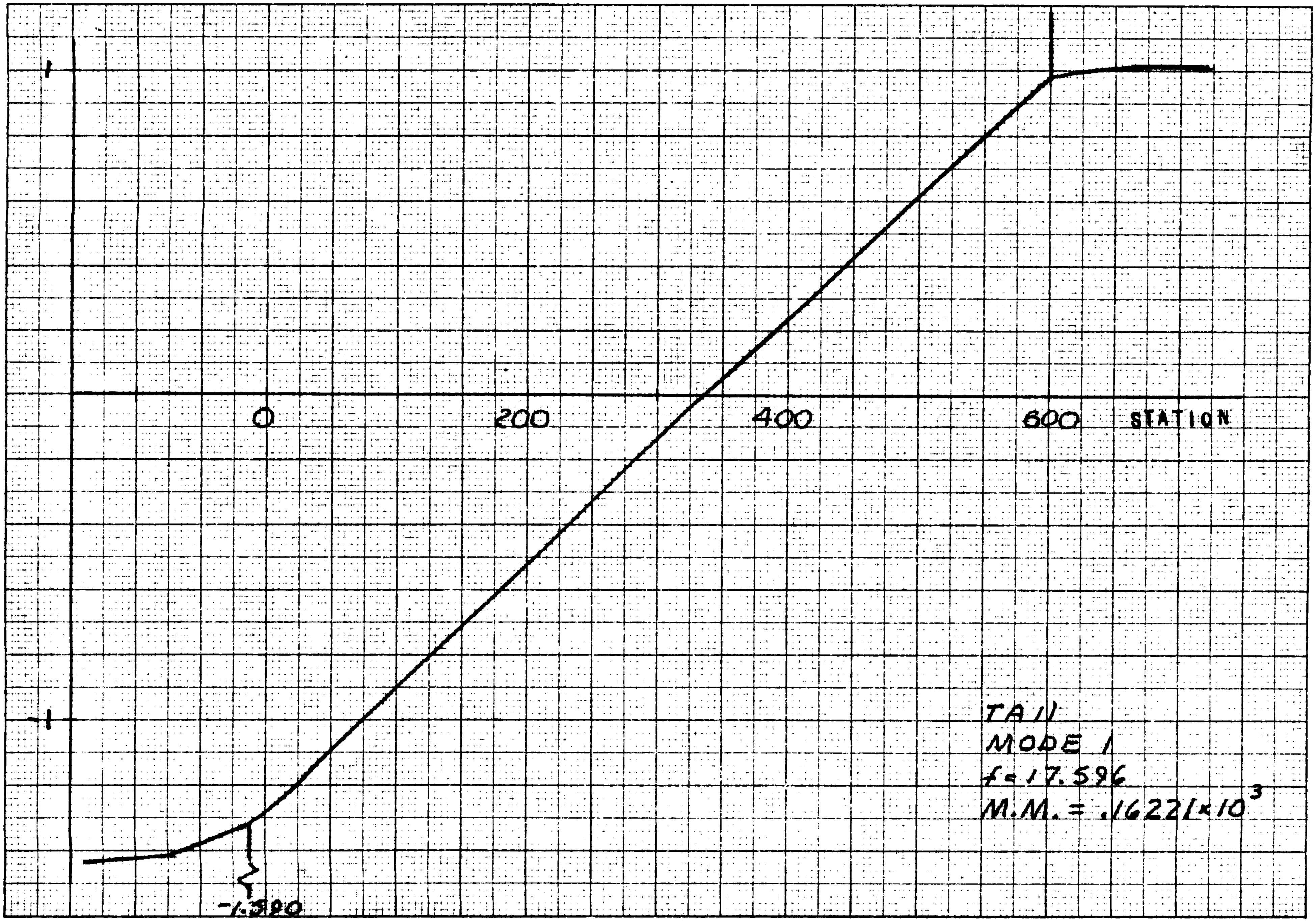
SM 46346


16.232
14
12
10
8
6
4
2
0
200
400
600
STATION
2
4
6
8
10
-19.049
TA 10
MODE 2
f = 23.552
M.M. = .29064 × 10^5

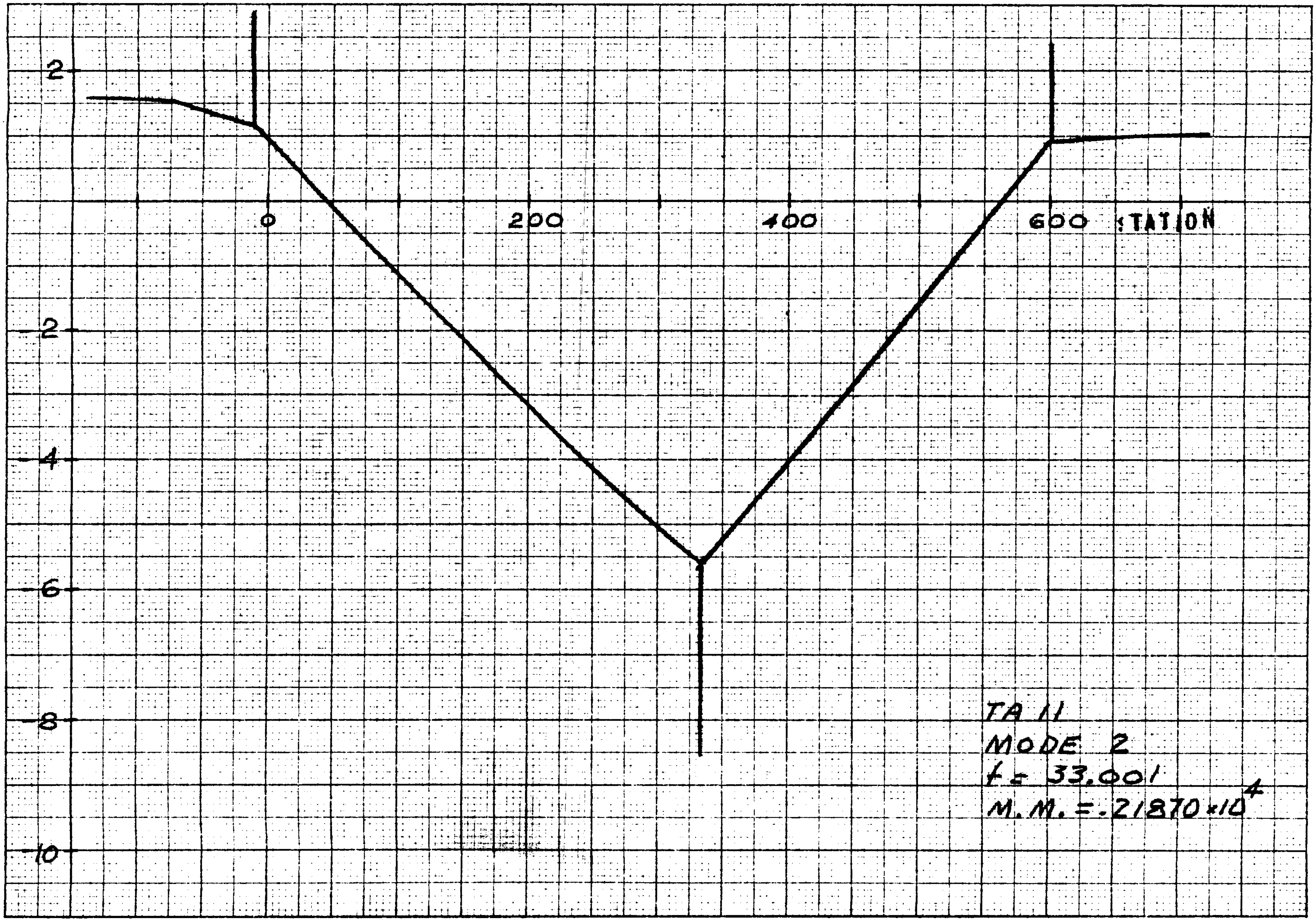
SM 46346



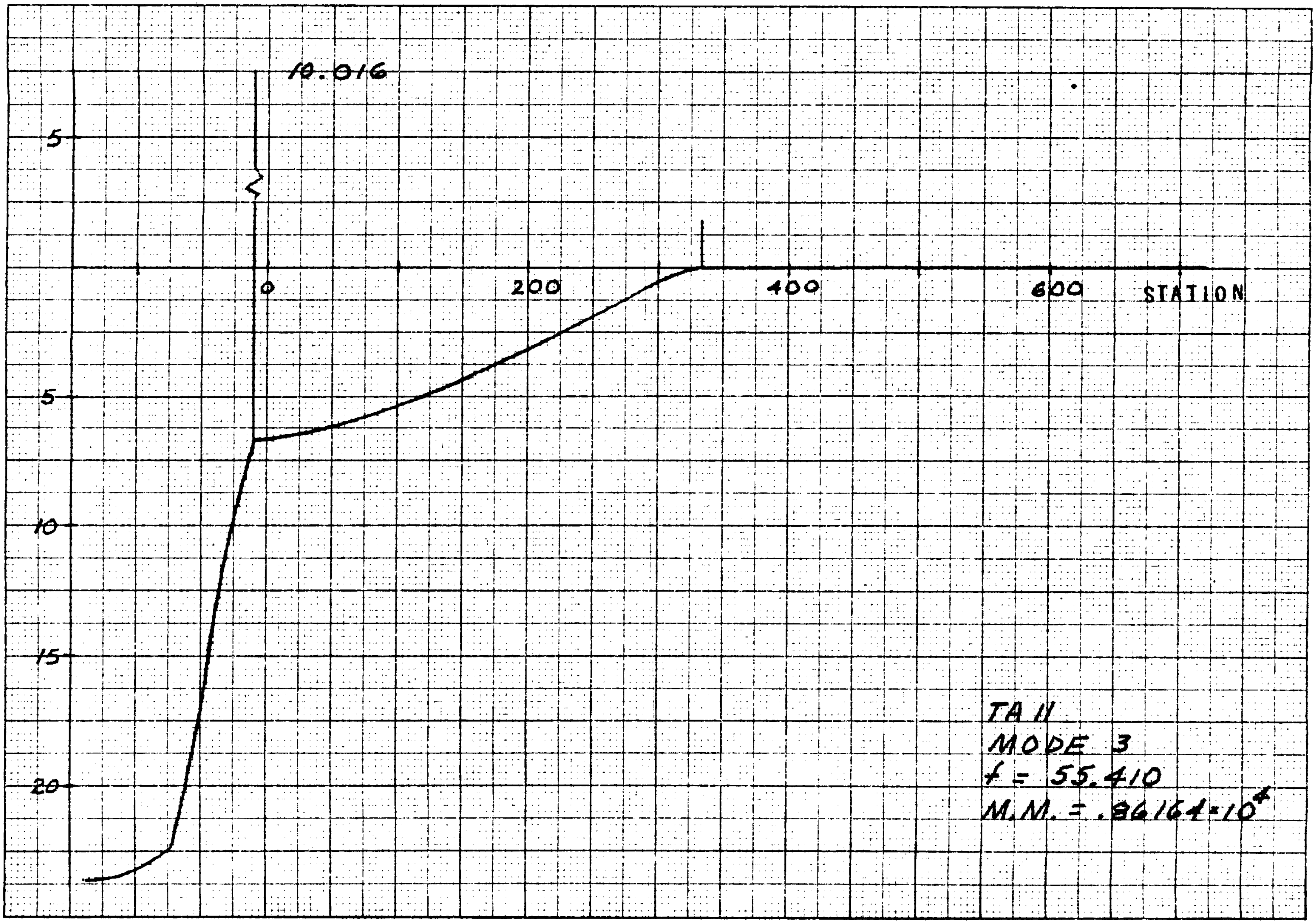
SM 46346



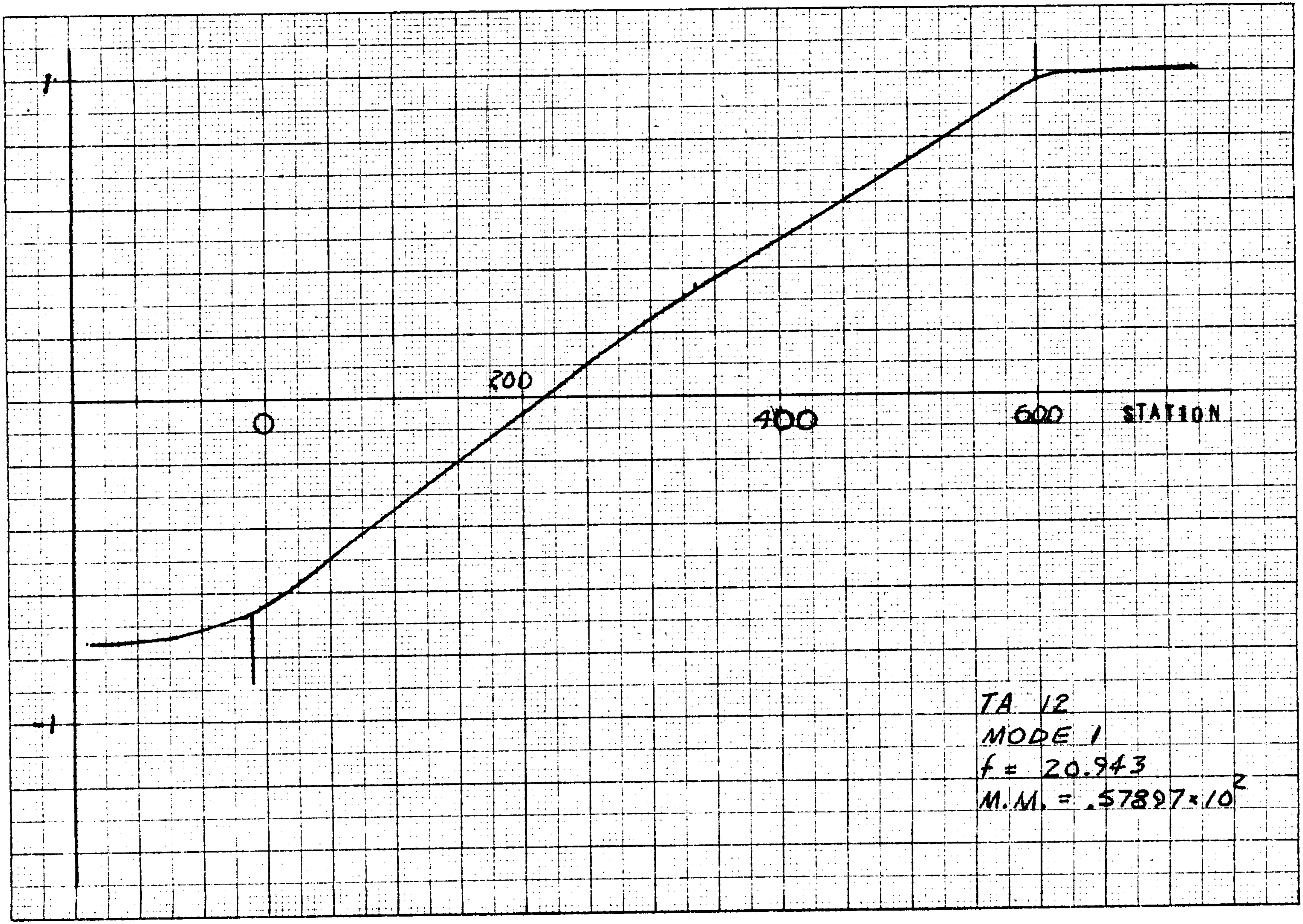
SM 46344



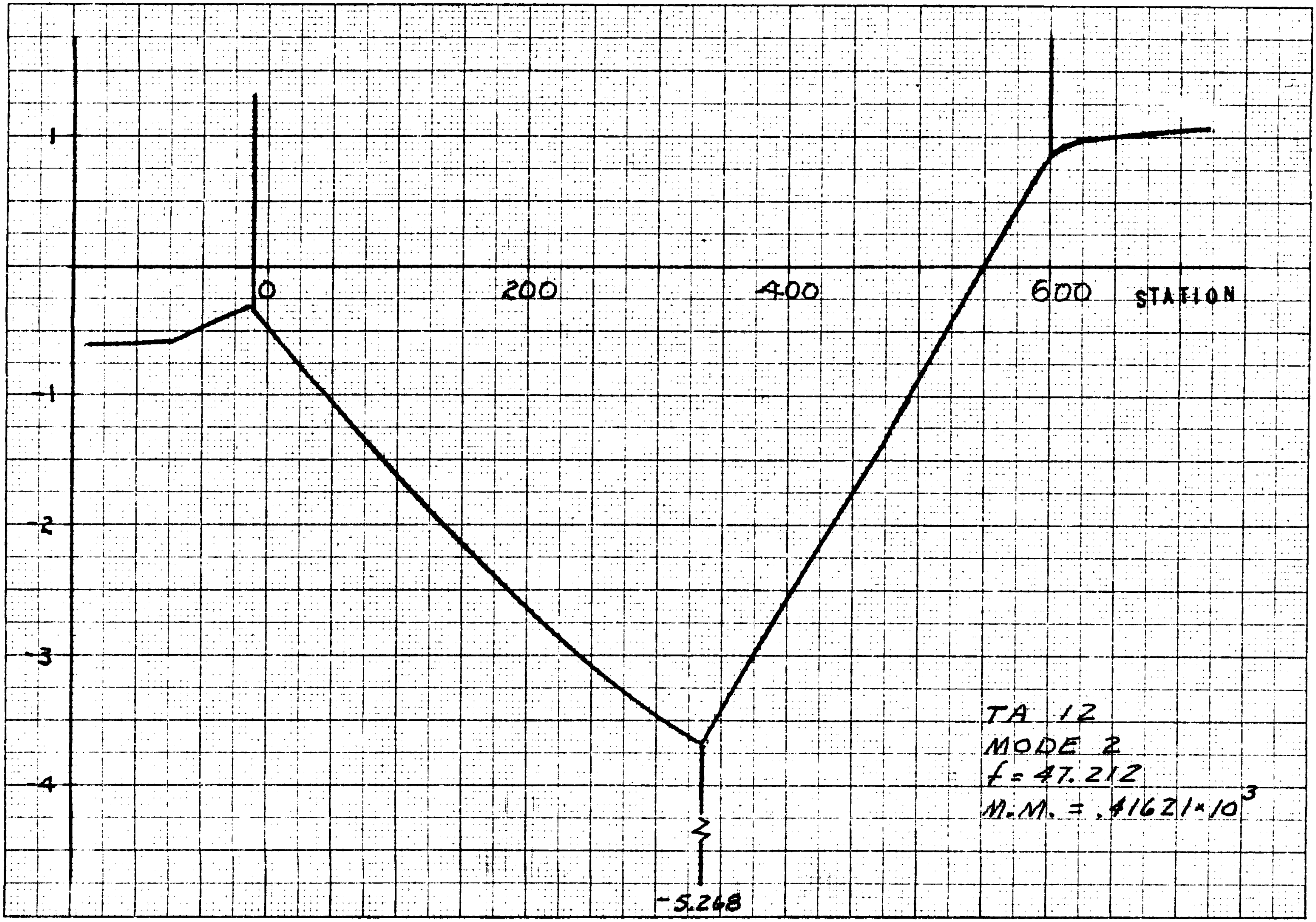
SM 46346

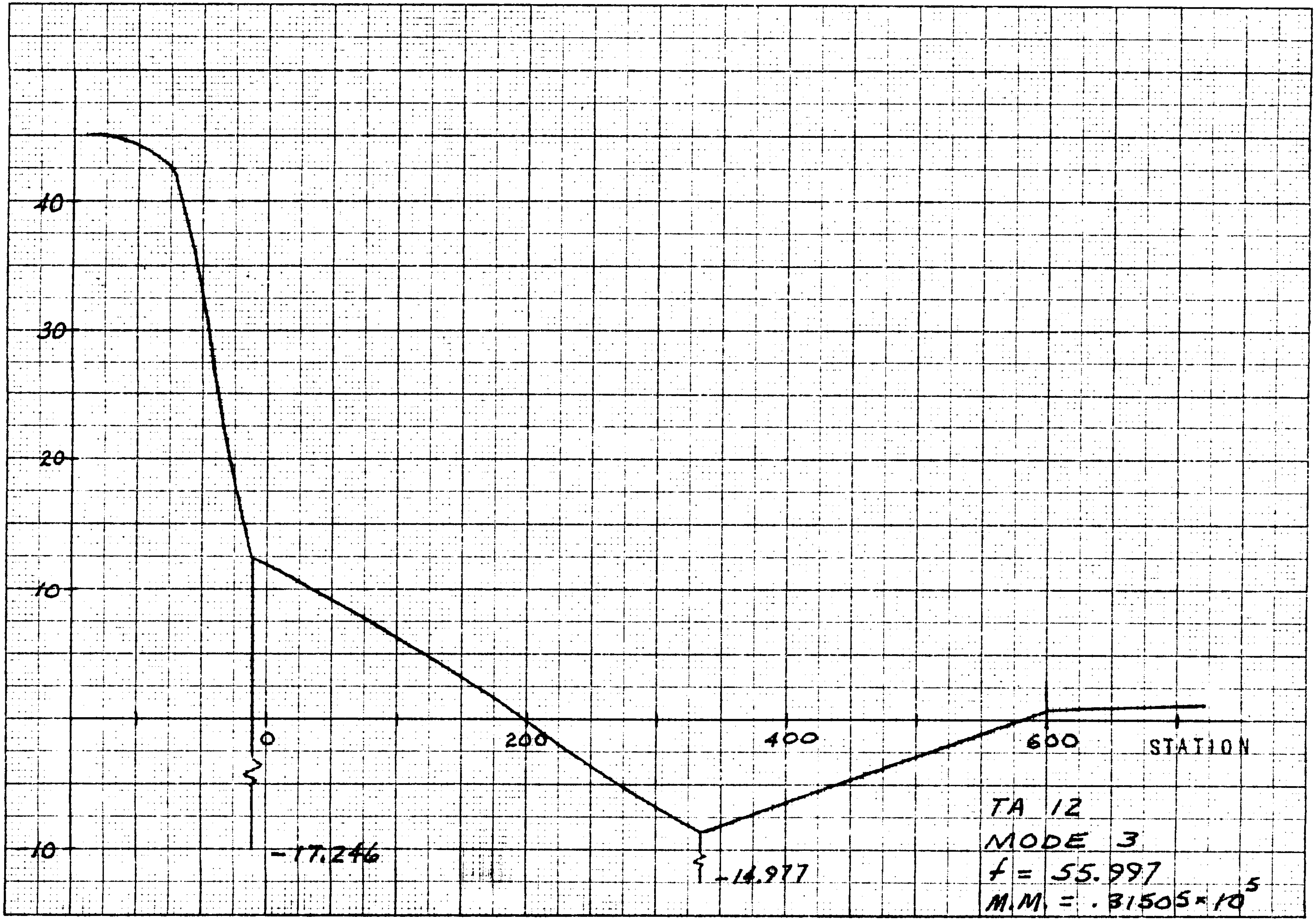

10.016
5
5
10
15
20
0
200
400
600
STATION
TA II
MODE 3
f = 55.410
M.M. = .86164×10⁴

SM 46346

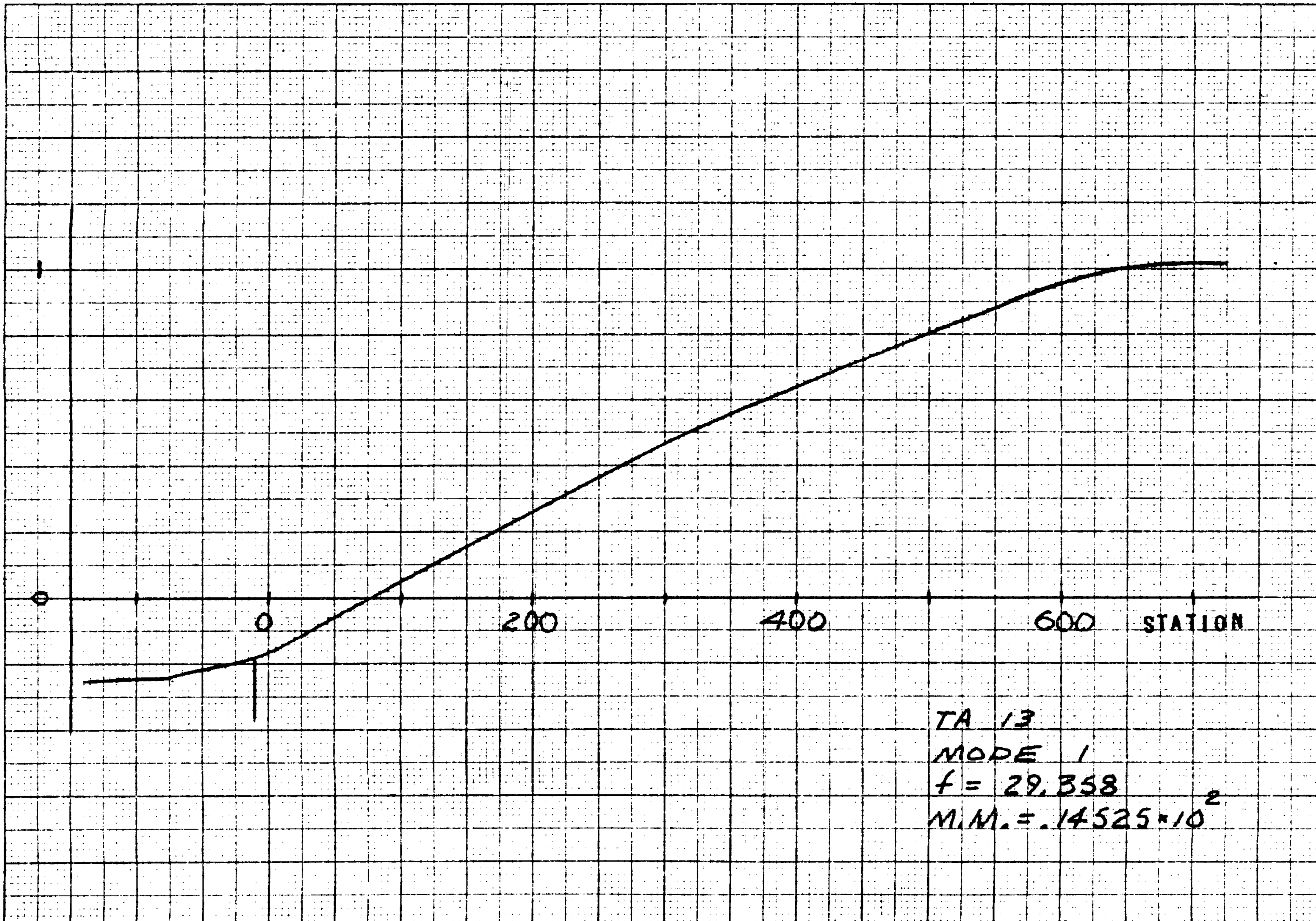


SM 46346

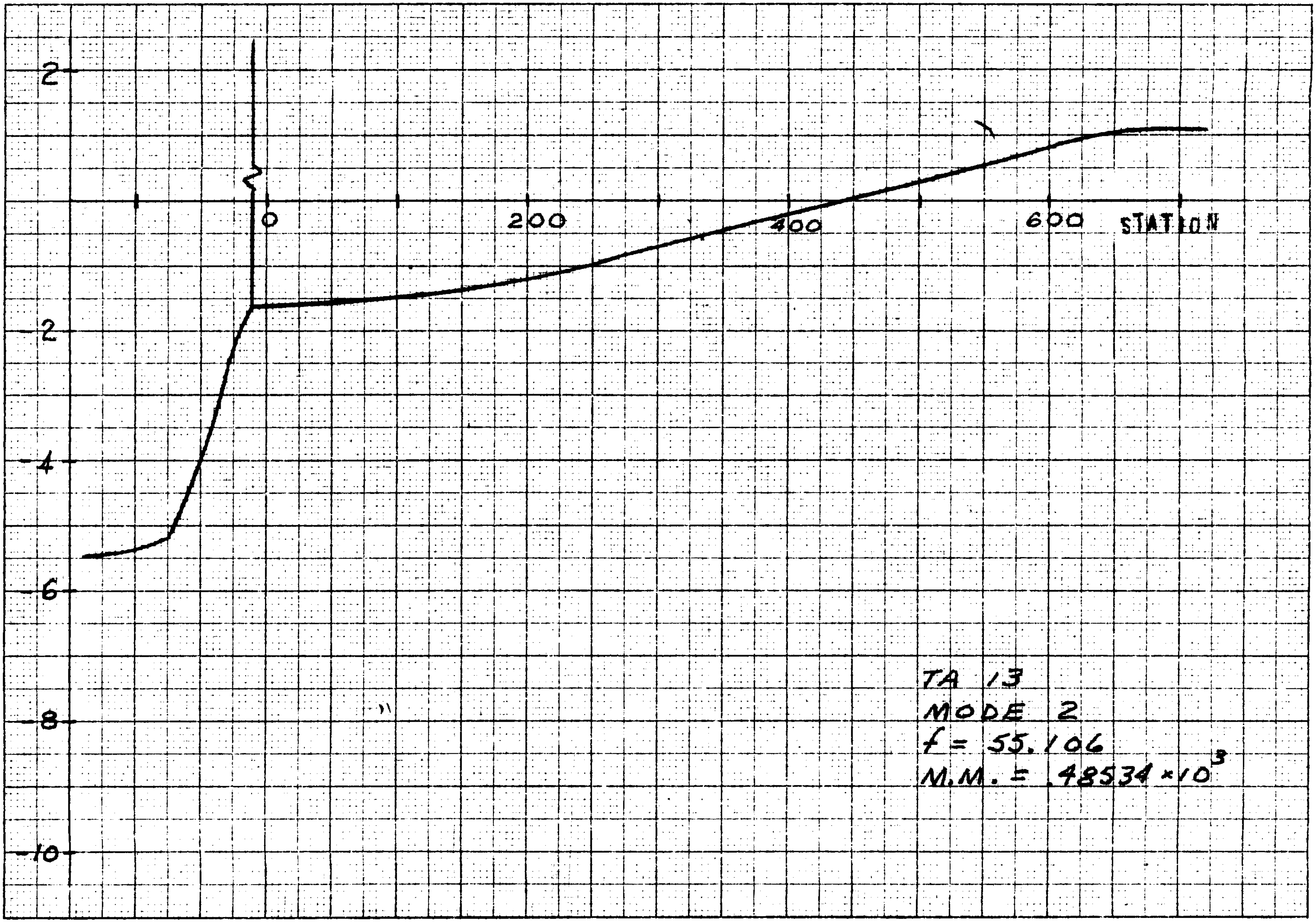

1
-1
-2
-3
-4
0
200
400
600
STATION
-5.268
TA 12
MODE 2
f = 47.212
M.M. = .41621×10^3

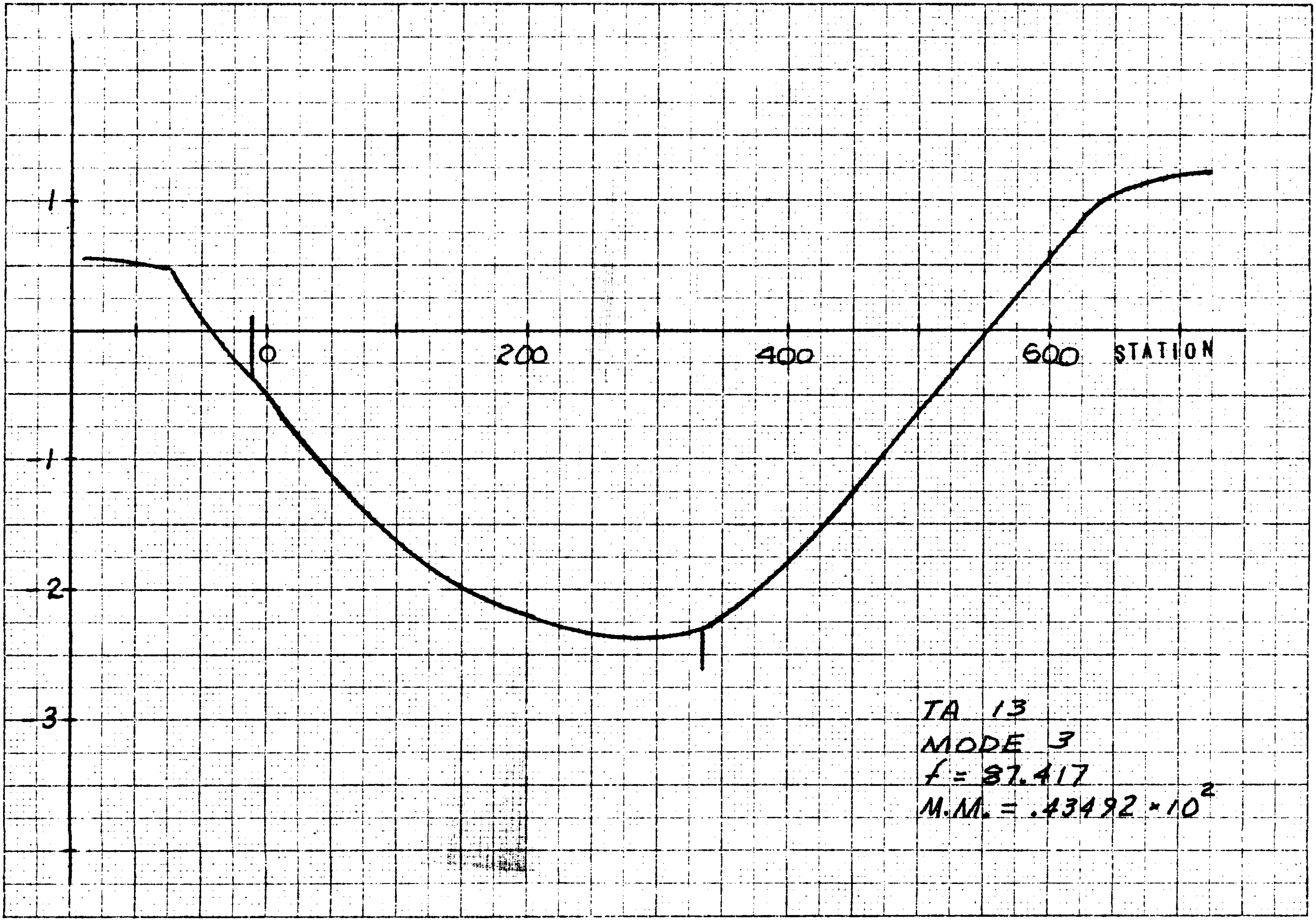

40
30
20
10
-10
0
200
400
600
STATION
-17.246
-14.977
TA 12
MODE 3
f = 55.997
M.M. = .31505 × 10^5

SM 46346

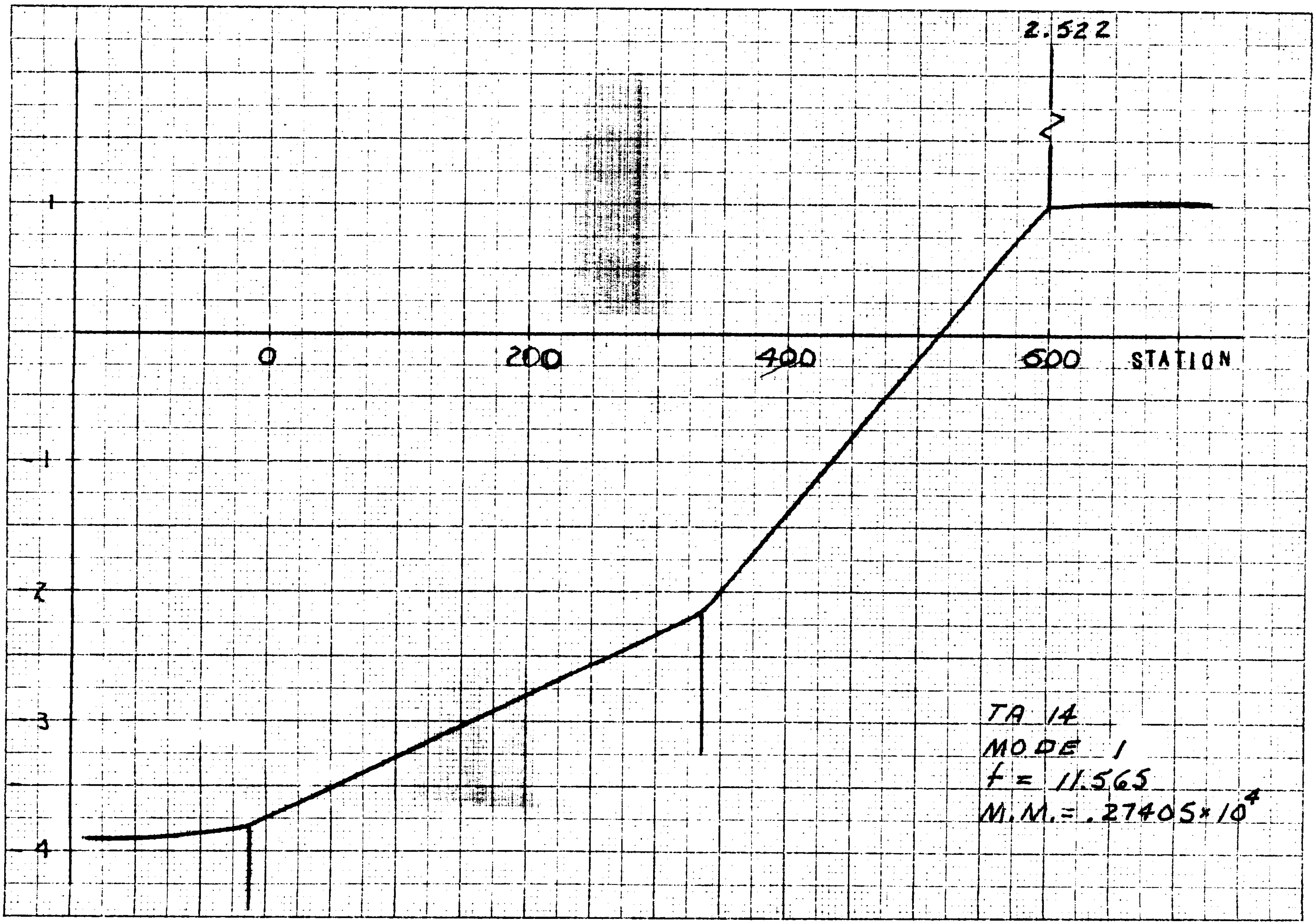


SM 46346

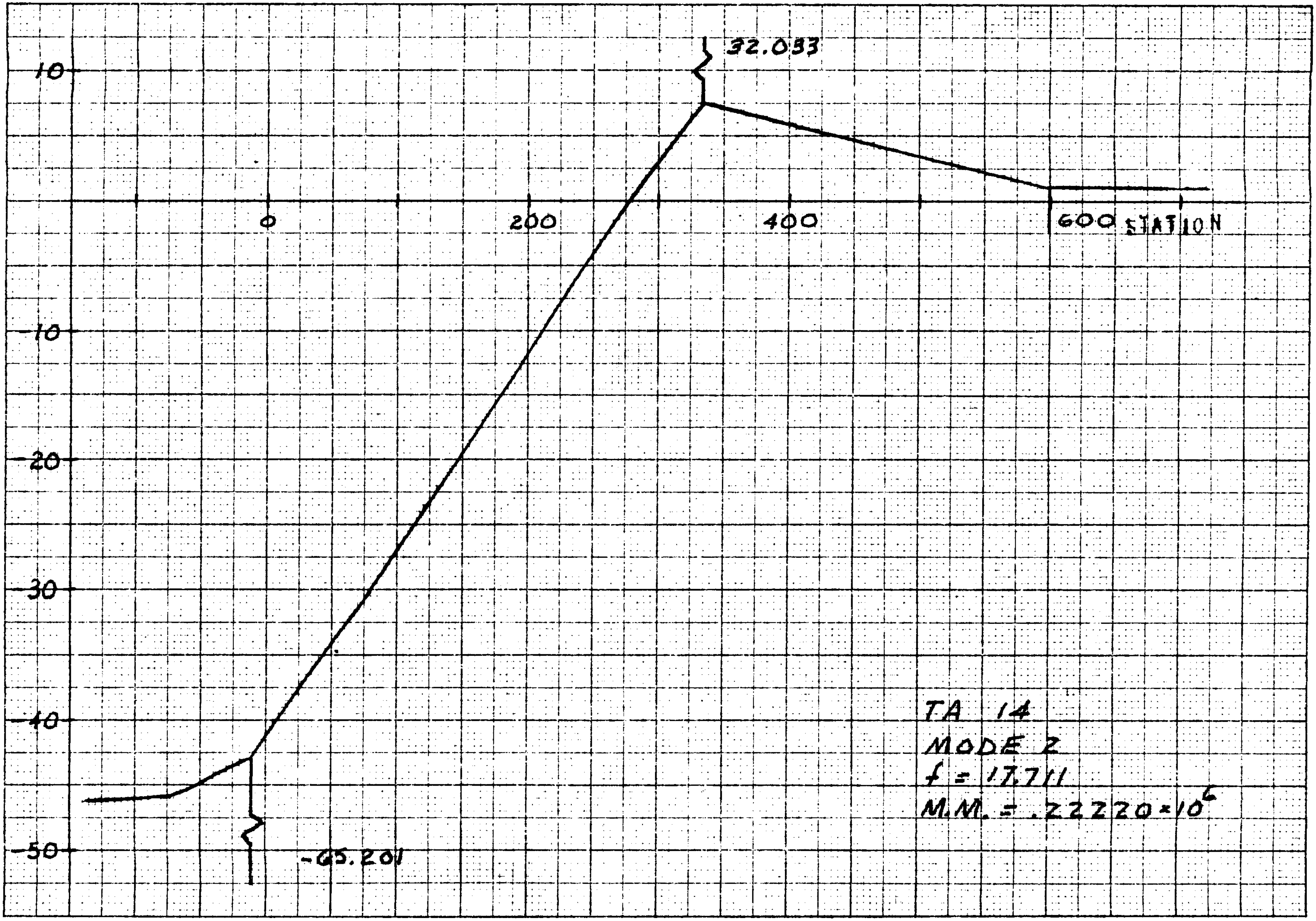



1
-1
-2
-3
0
200
400
600
STATION
TA 13
MODE 3
f = 87.417
M.M. = .43492 × 10^2

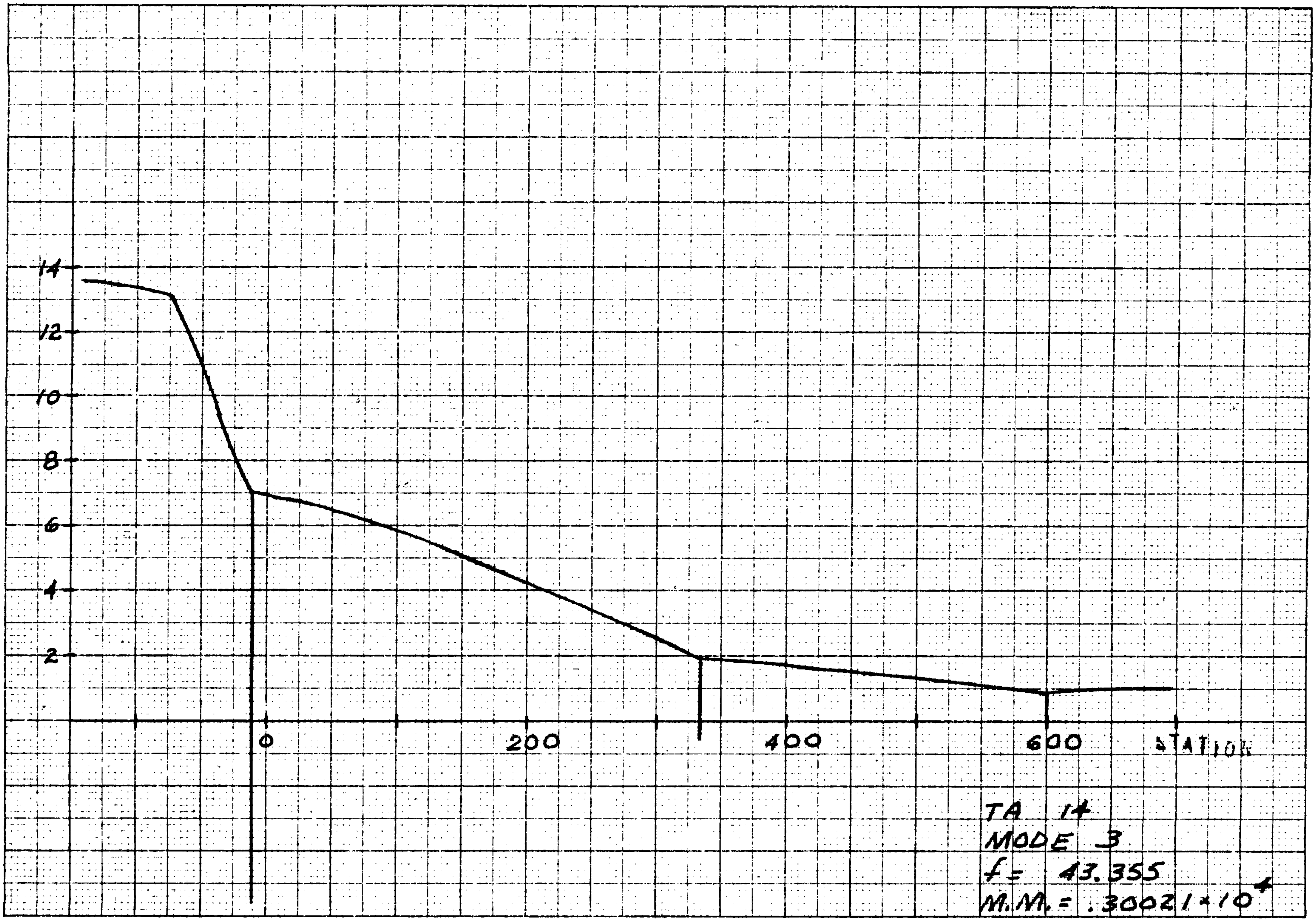
SM 46346


2.522
1
-1
2
3
4
0
200
400
600
STATION
TA 14
MODE 1
f = 11.565
M.M. = .27405×10^4

SM 46346

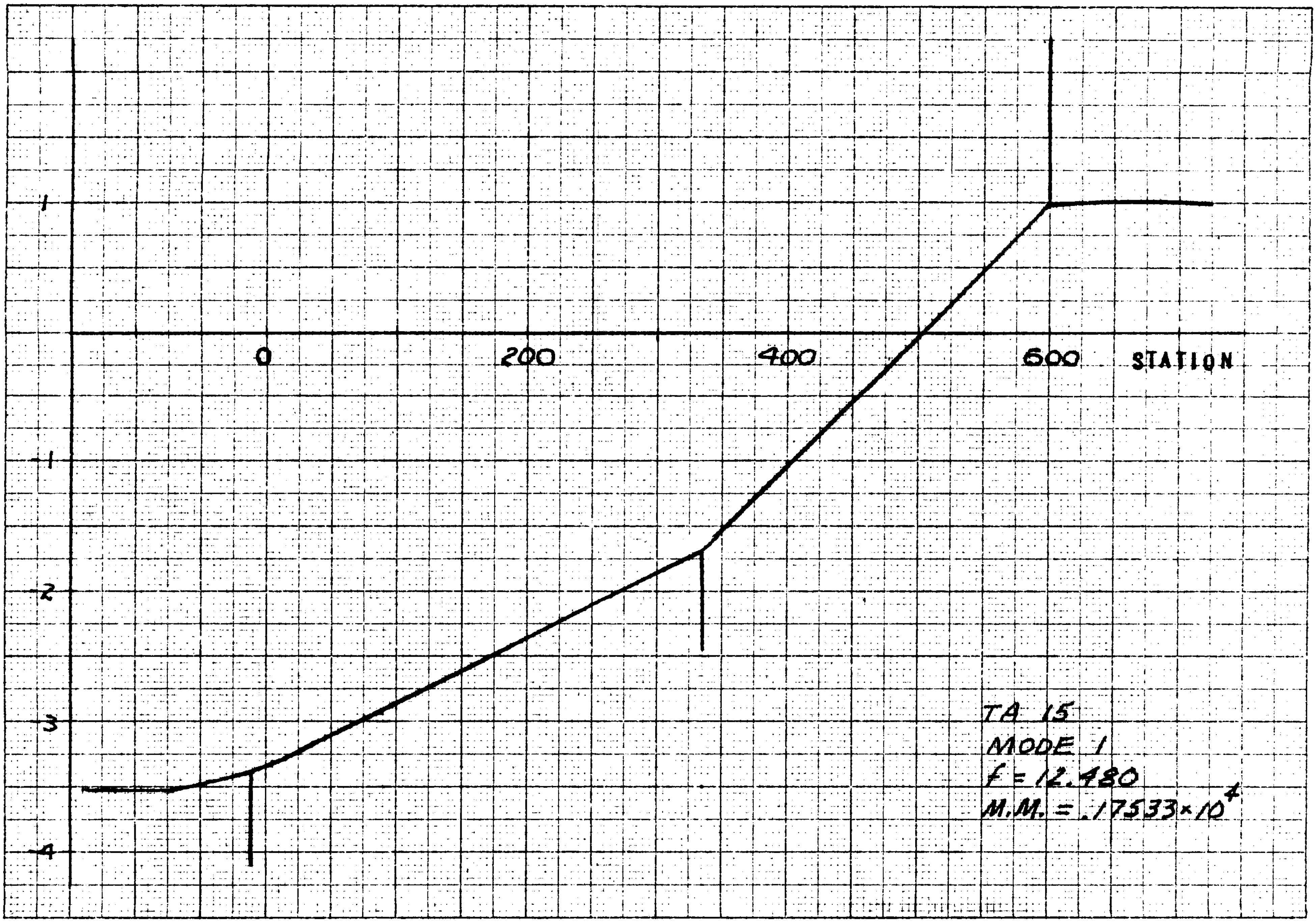

32.033
10
0
200
400
600 STATION
-10
-20
-30
-40
-50
-65.201
TA 14
MODE 2
f = 17.711
M.M. = .22220 × 10^6

SM 46346

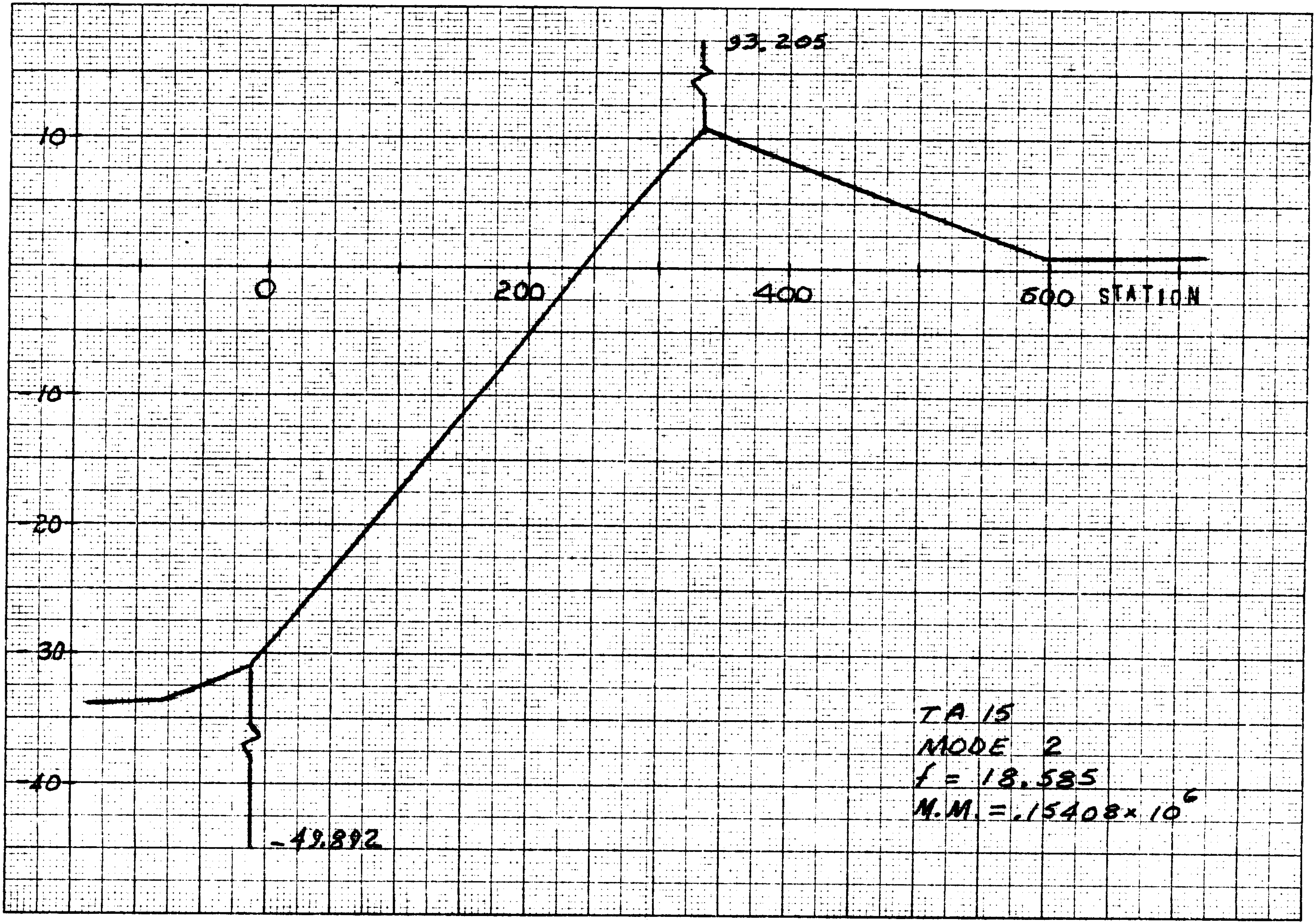


14
12
10
8
6
4
2
0
200
400
600
STATION
TA 14
MODE 3
f = 43.355
M.M. = .30021×10⁴

SM 46340



SM 46346


93.205
10
-10
-20
-30
-40
0
200
400
600
STATION
-49.892
TA 15
MODE 2
f = 18.585
M.M. = .15408 × 10^6

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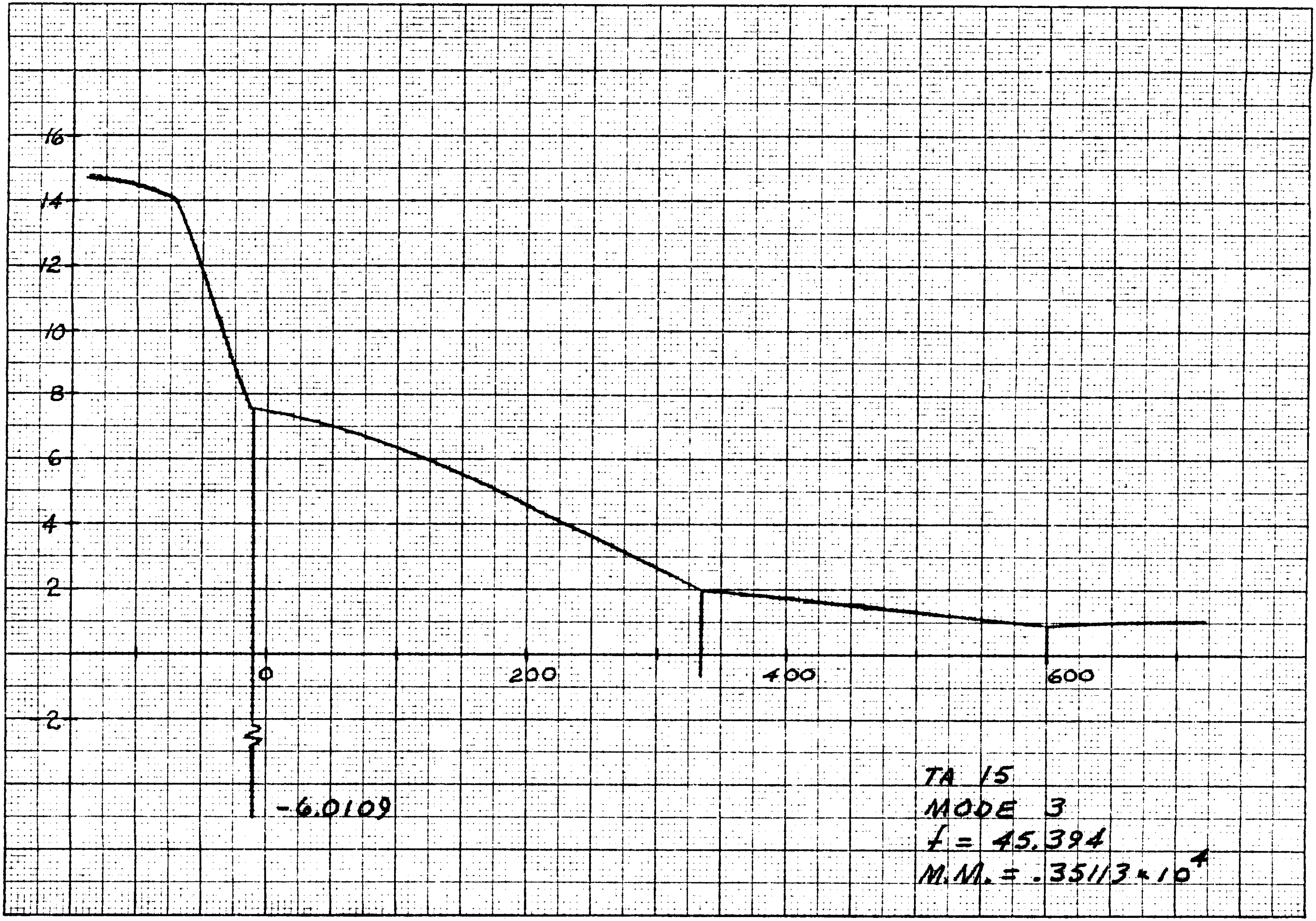
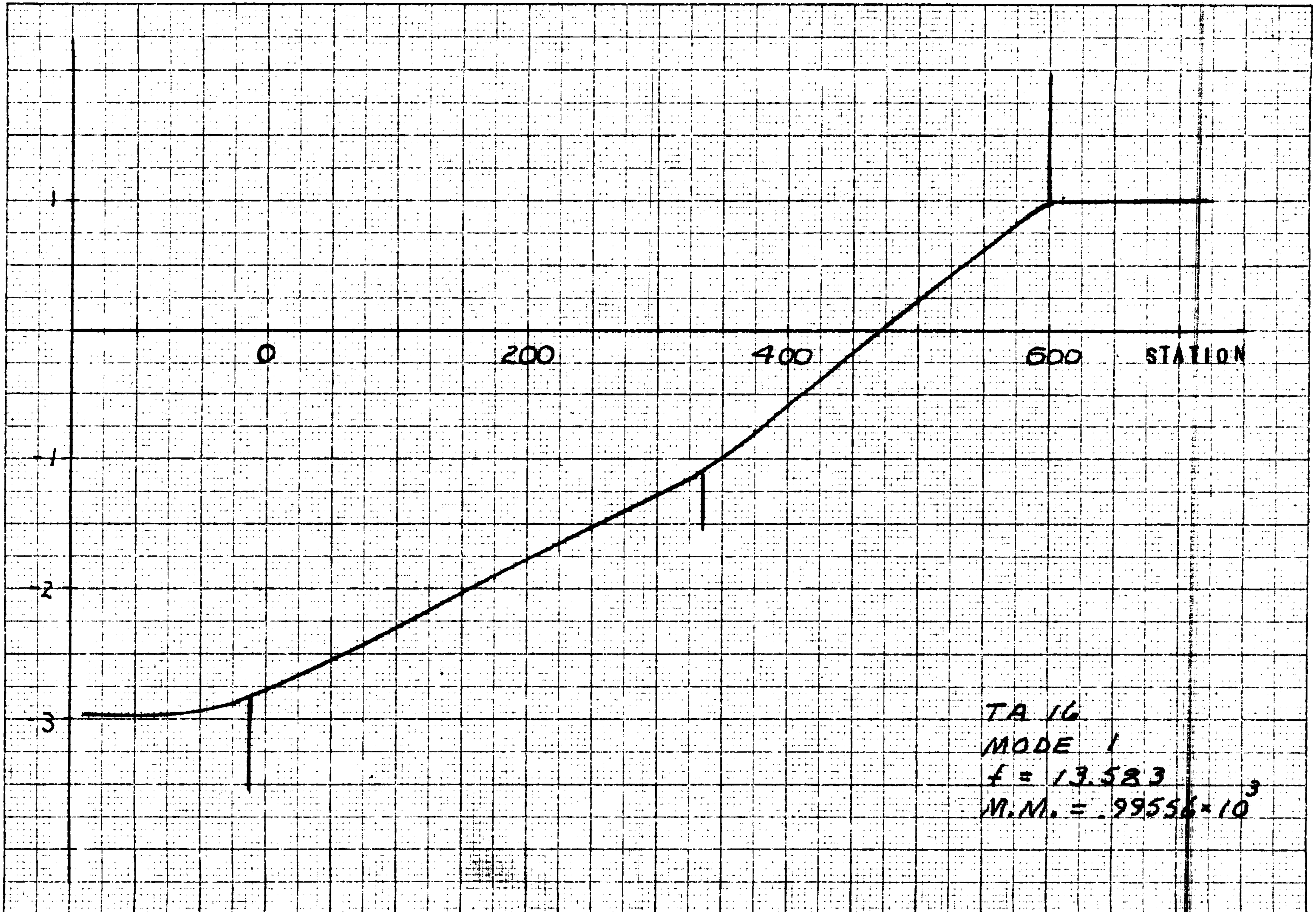
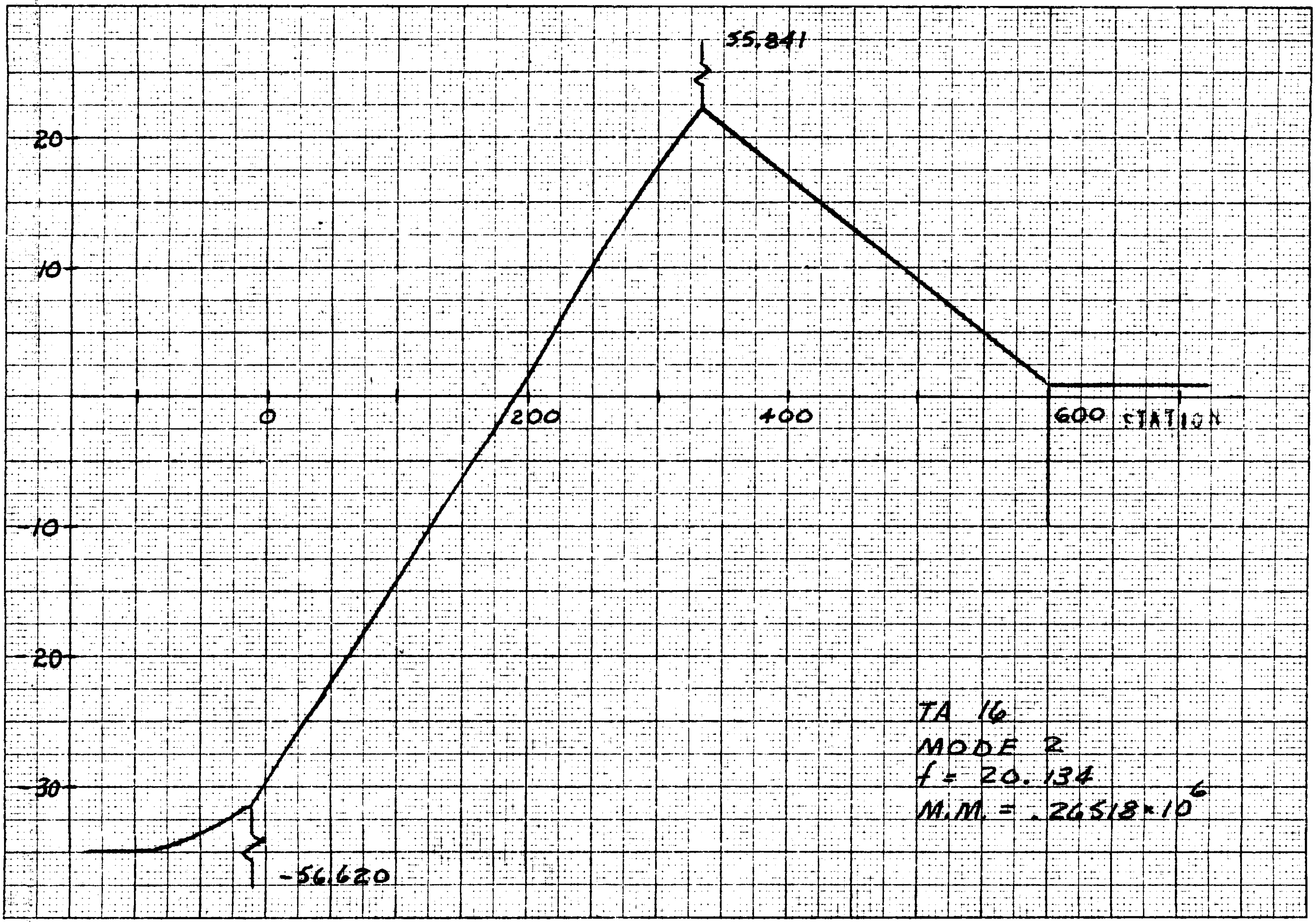
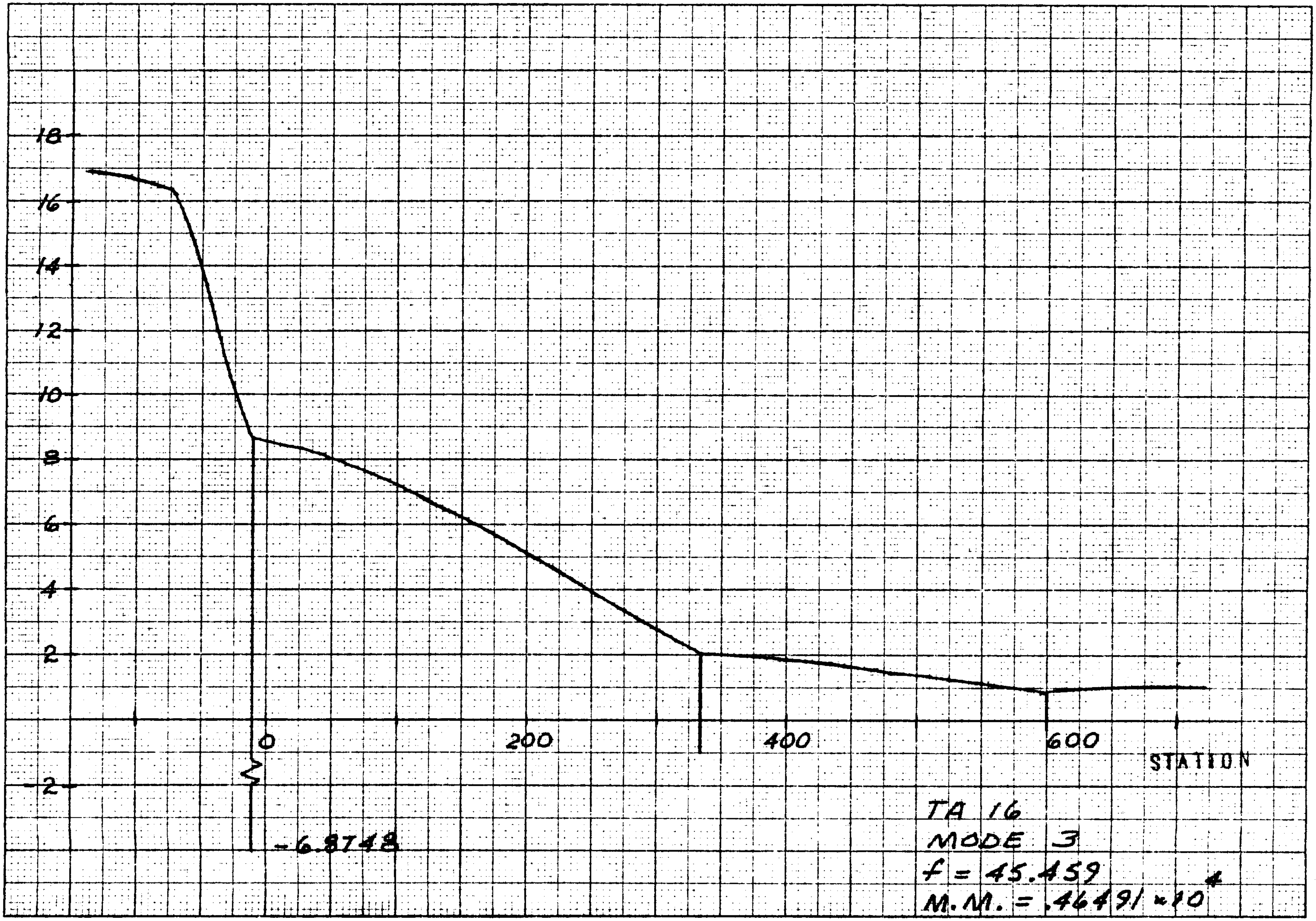


SM 46346




55.841
20
10
-10
-20
-30
0
200
400
600
STATION
-56.620
TA 16
MODE 2
f = 20.134
M.M. = .26518 × 10^6

SM 46346


18
16
14
12
10
8
6
4
2
-2
0
200
400
600
STATION
-6.8748
TA 16
MODE 3
f = 45.459
M.M. = .46491 × 10⁴

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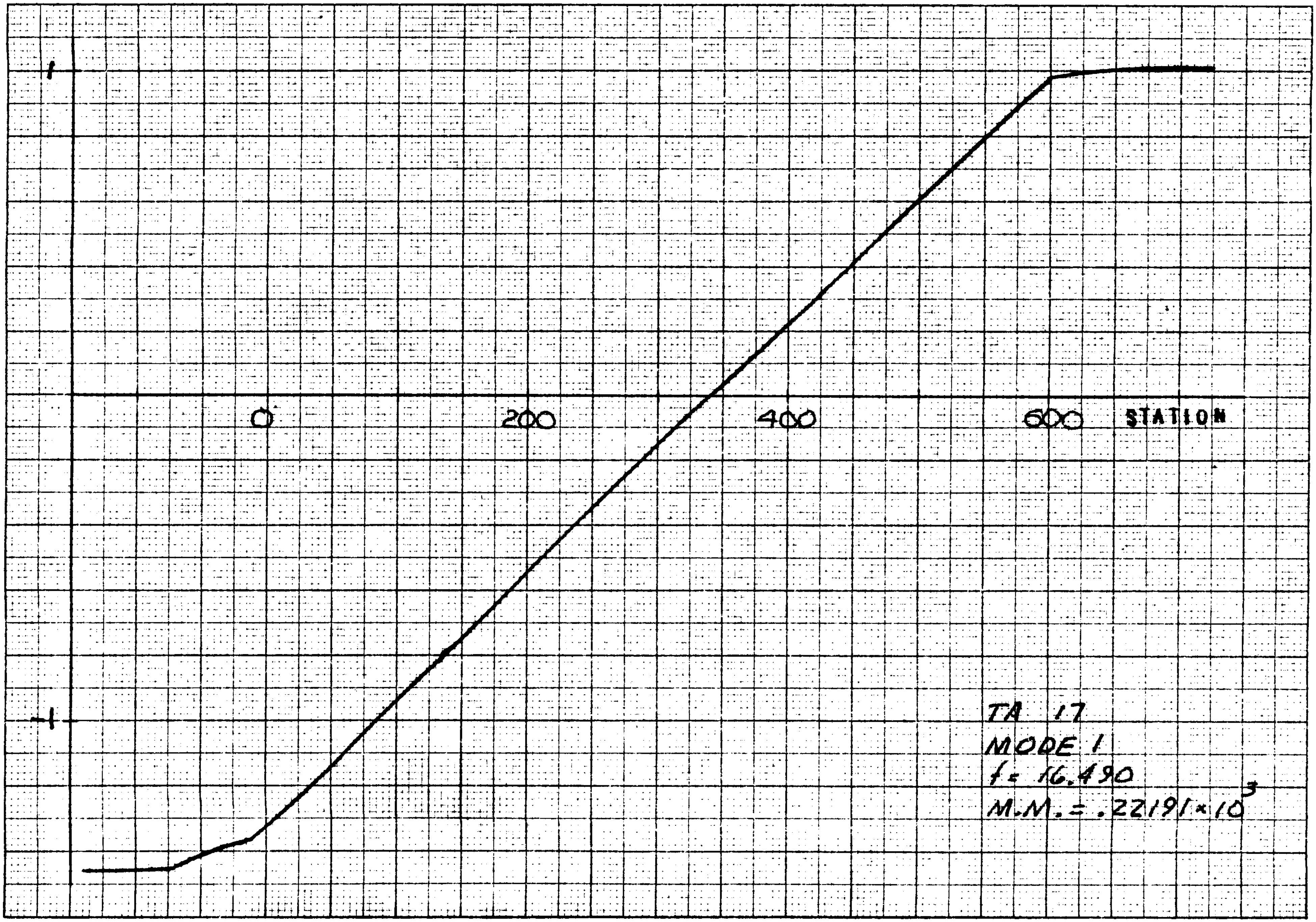
SM 4634(



K+E 10 X 10 TO 1/2 INCH 46 1327
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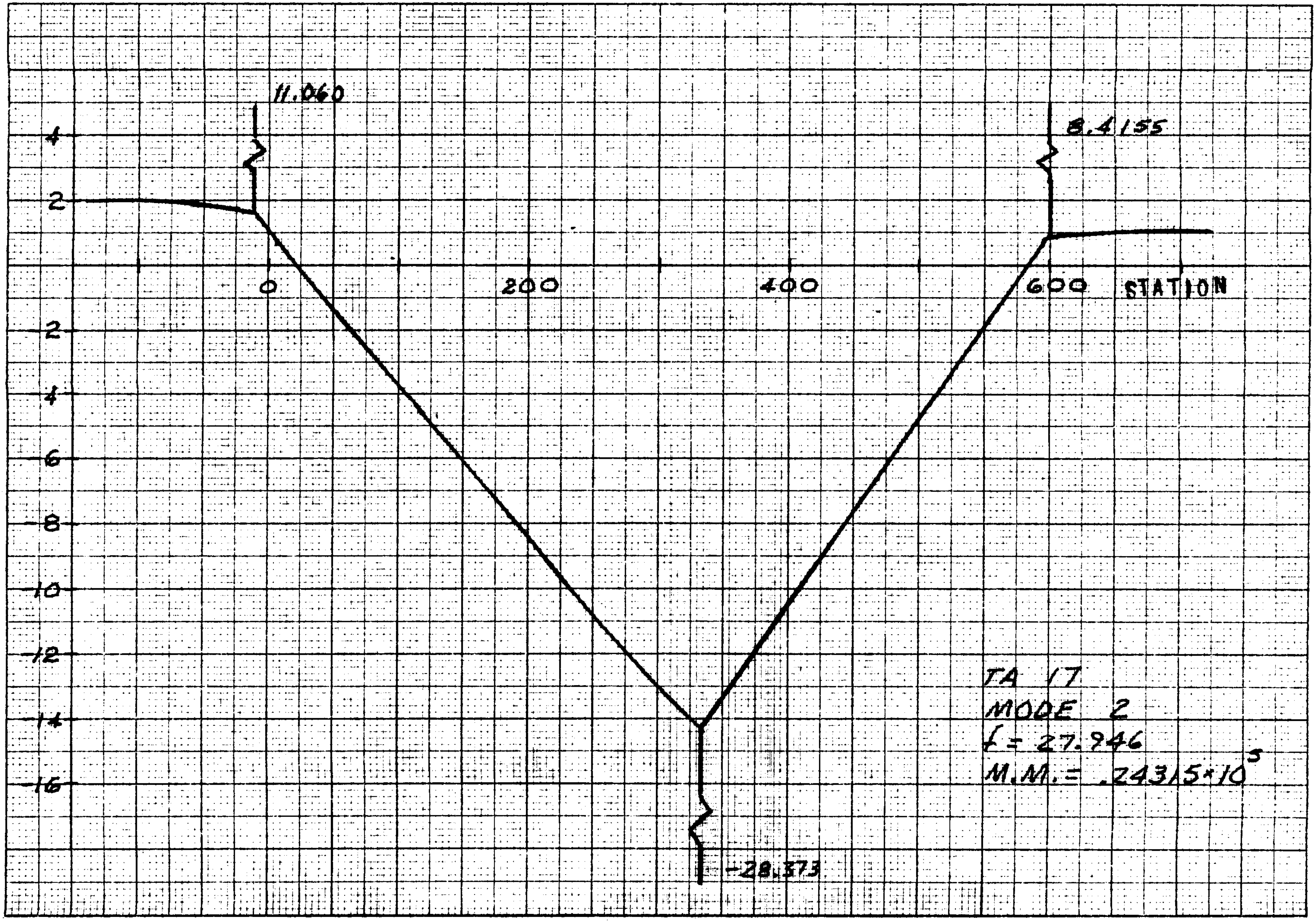
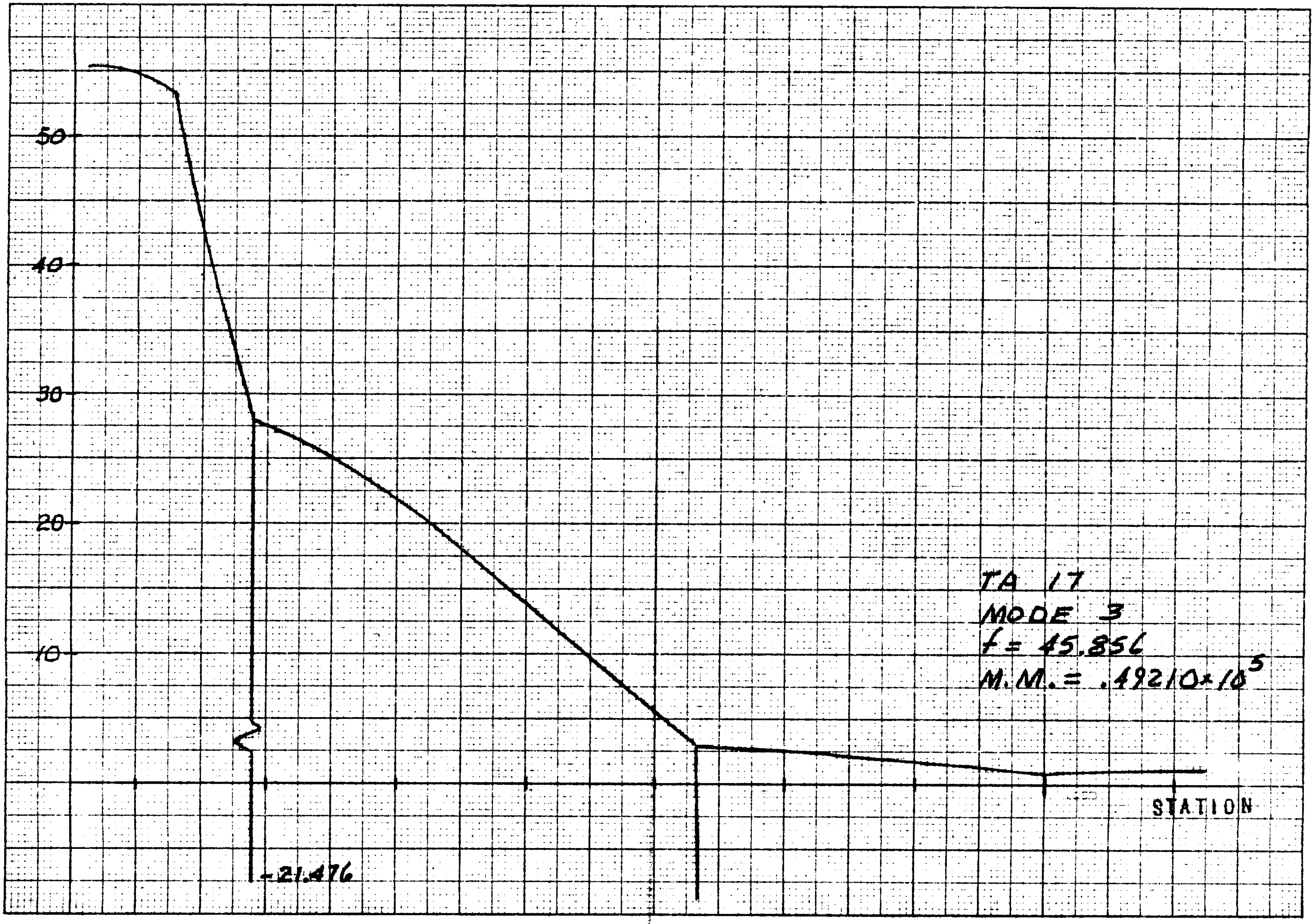


SM 46346



K+E 10 X 10 TO ½ INCH 46 1327
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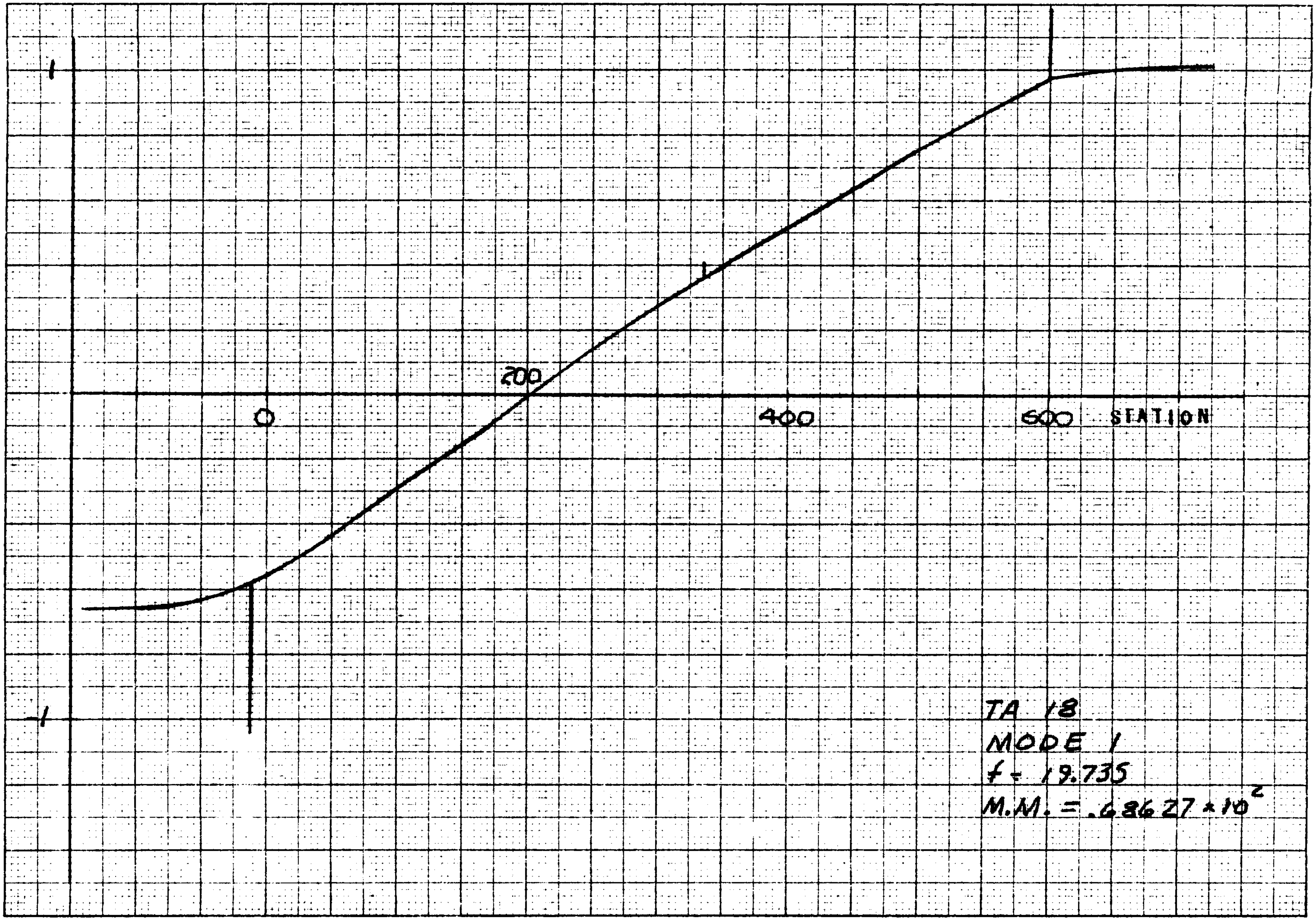
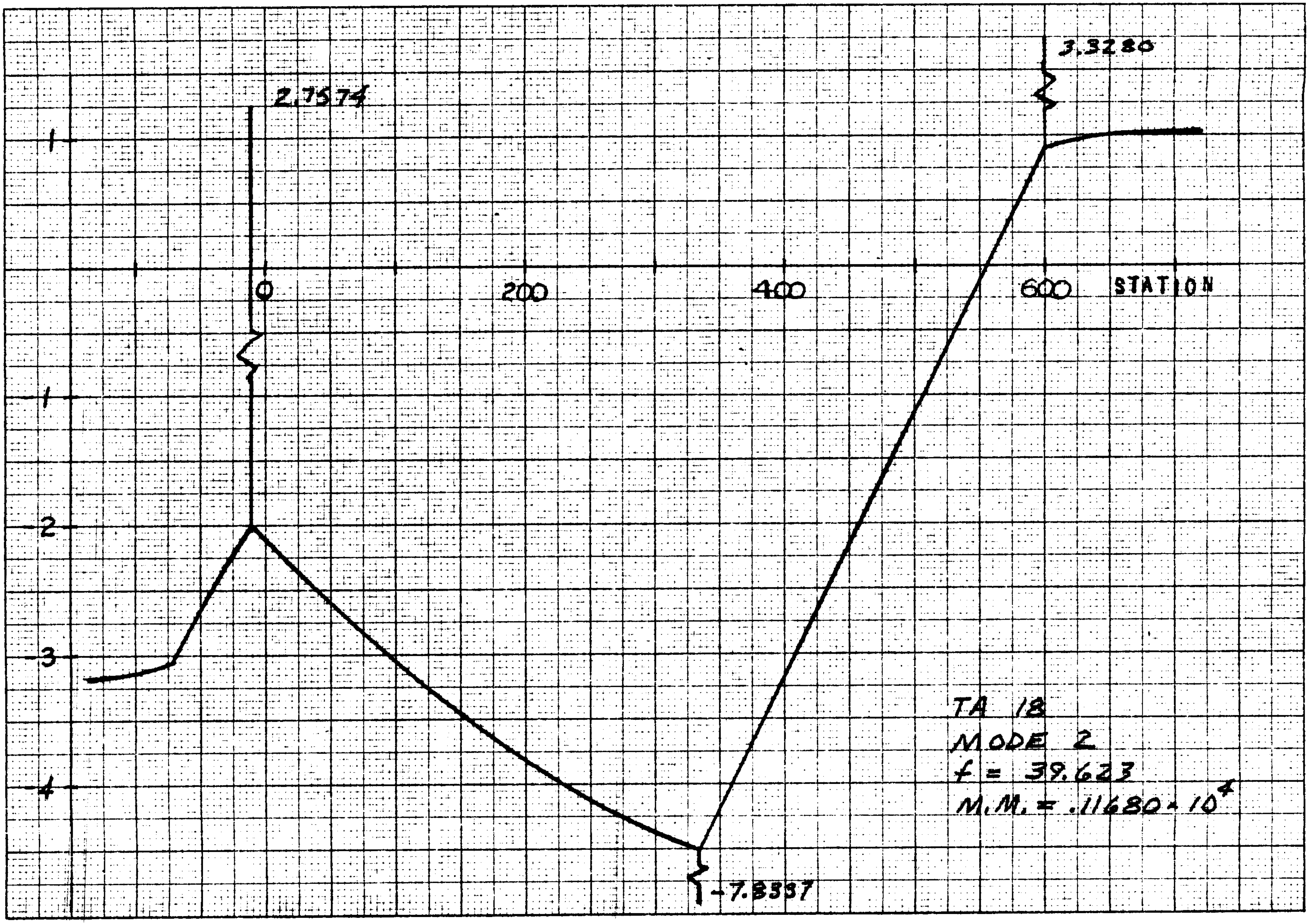
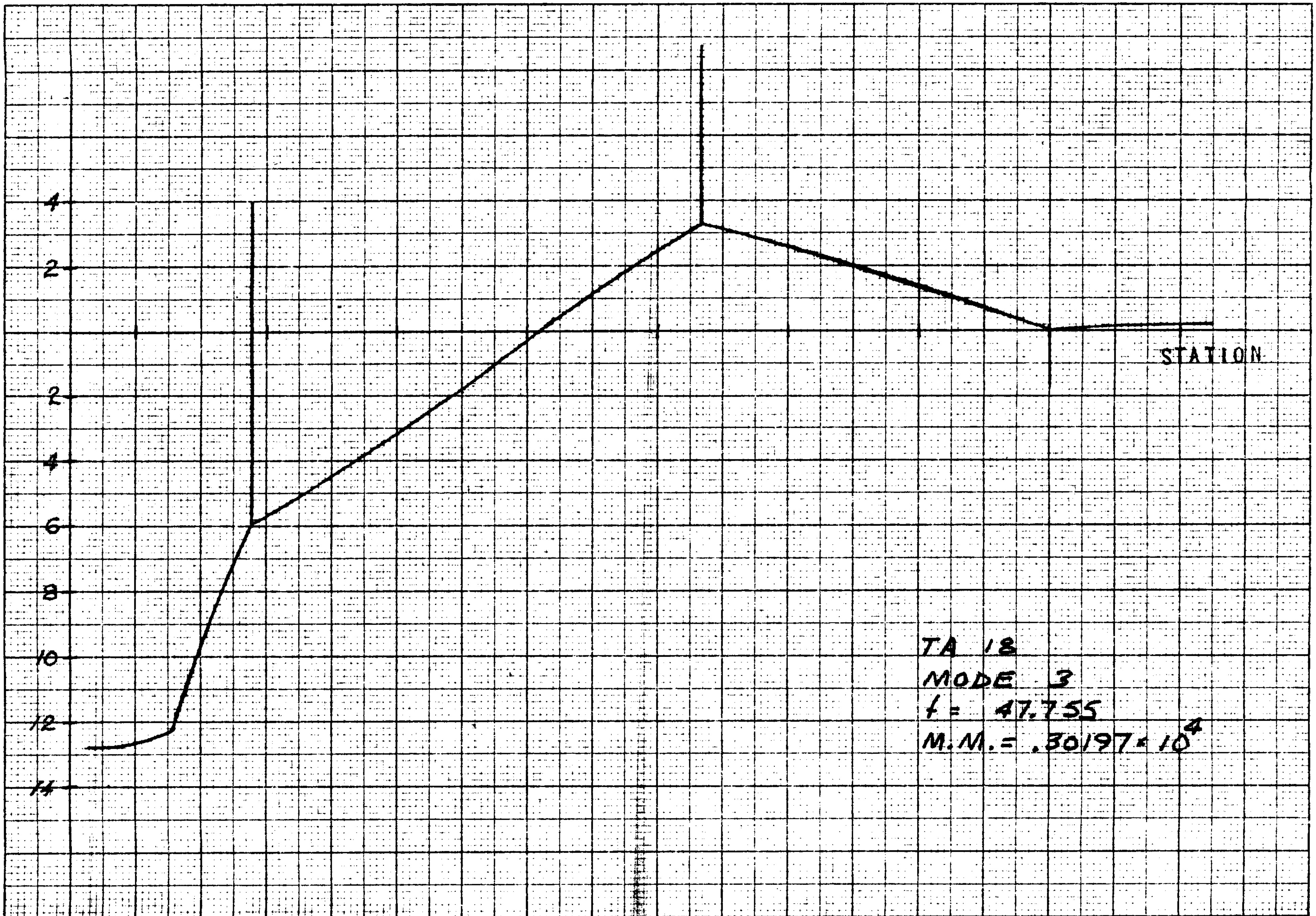
SM 46346



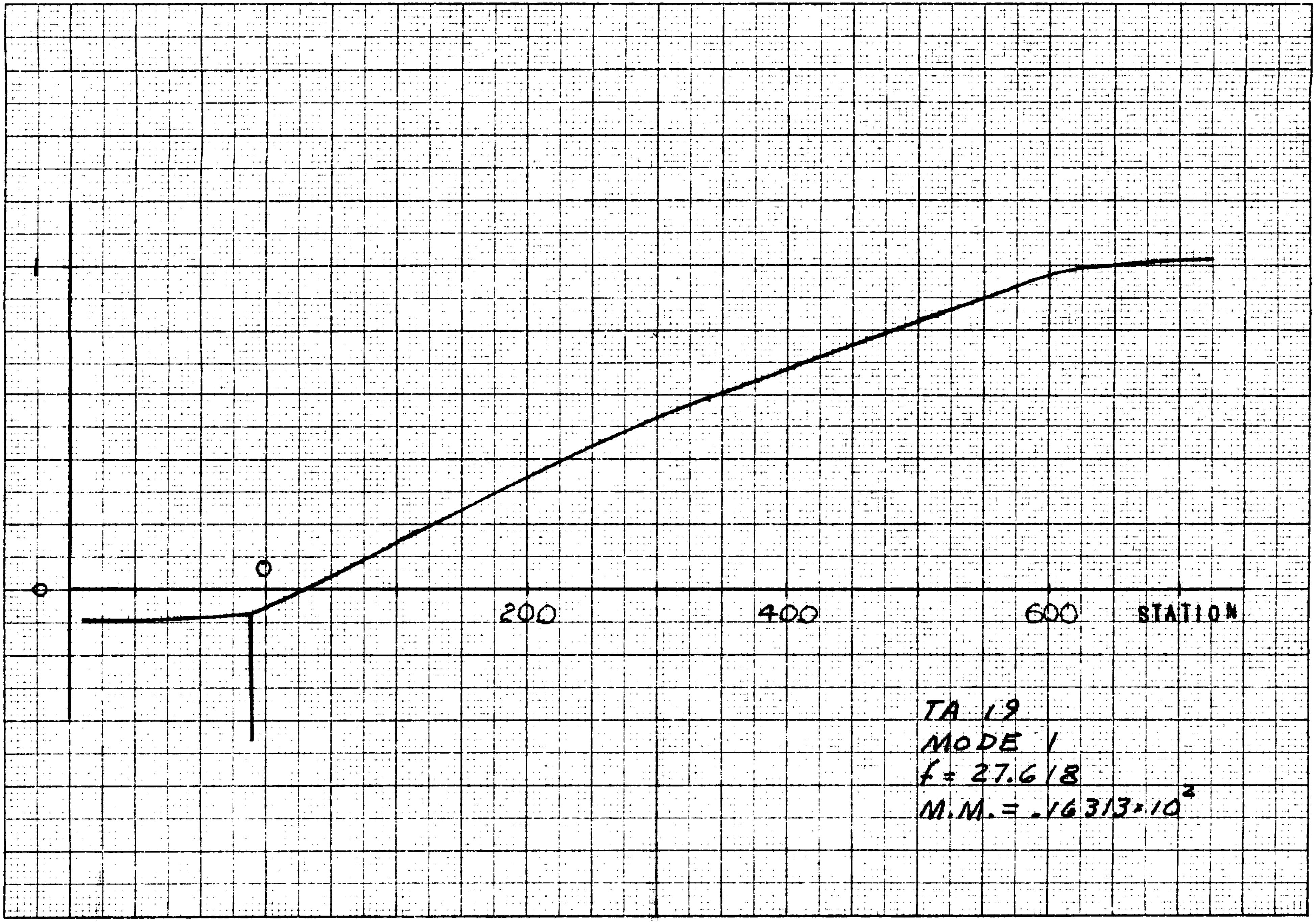
SM 46346


3.3280
2.7574
1
0
200
400
600
STATION
-1
-2
-3
-4
-7.8337
TA 18
MODE 2
f = 39.623
M.M. = .11680 × 10^4

SM 46346



SM 46346



SM 40346

10 X 10 TO ½ INCH 46 1327
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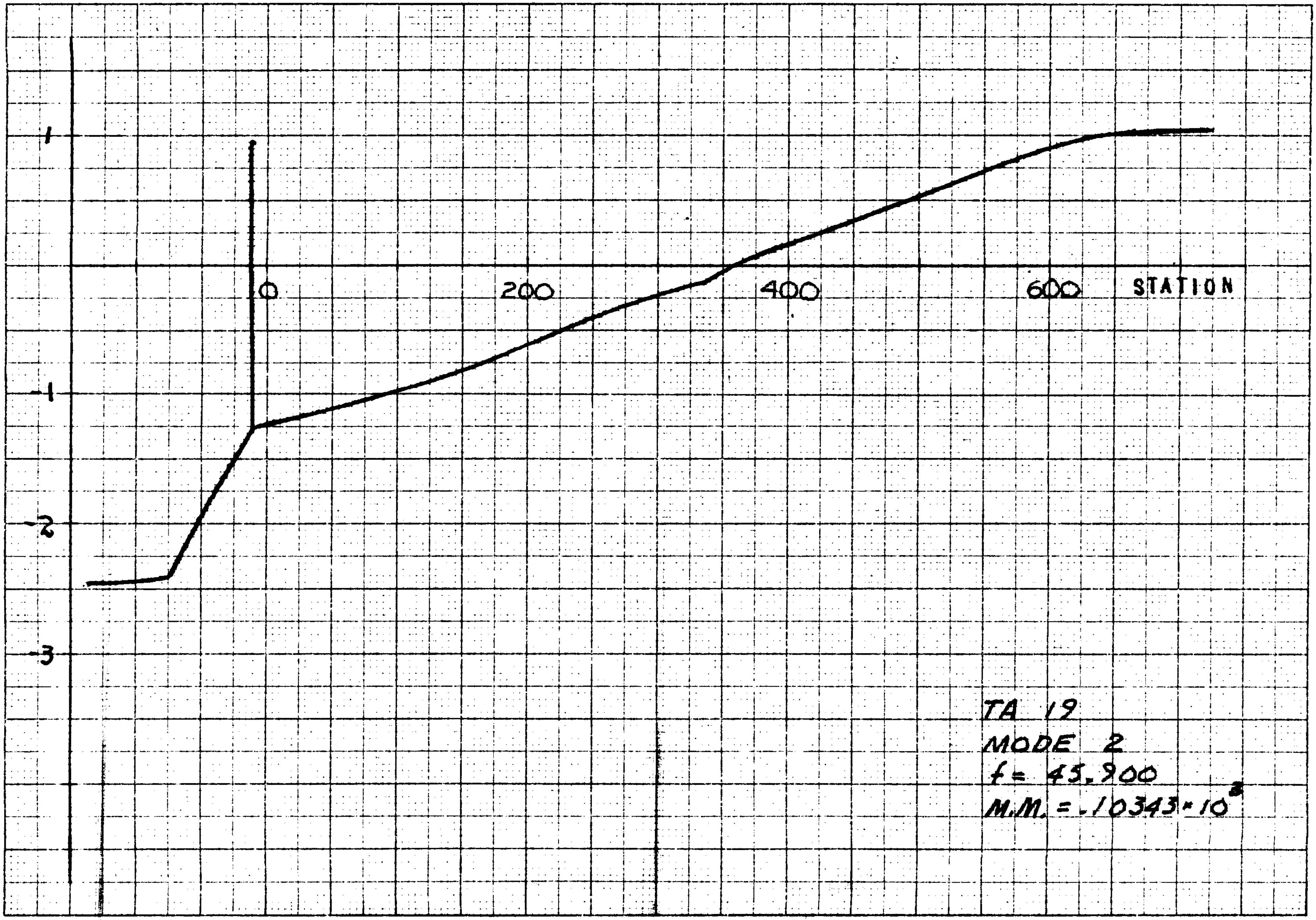
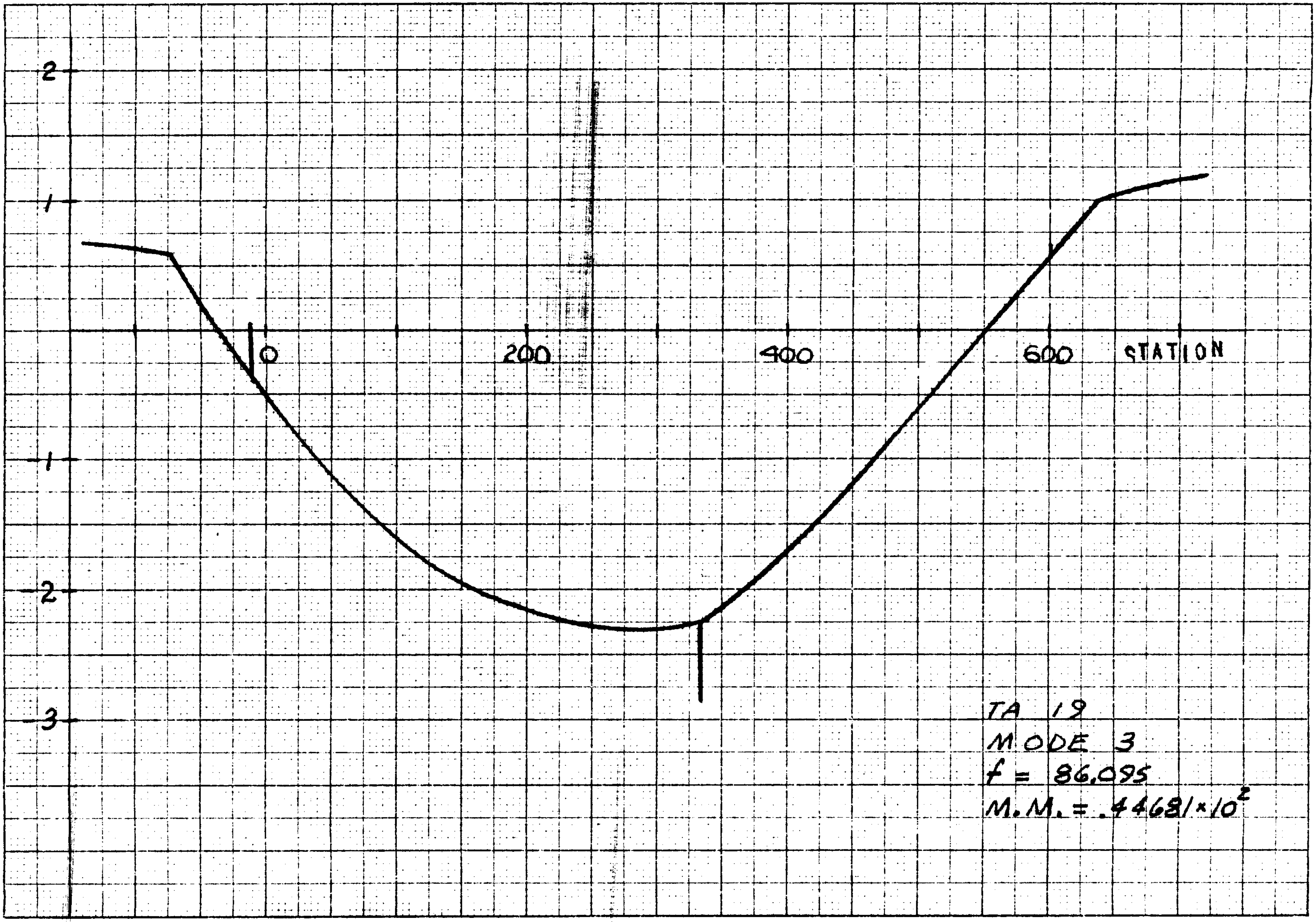
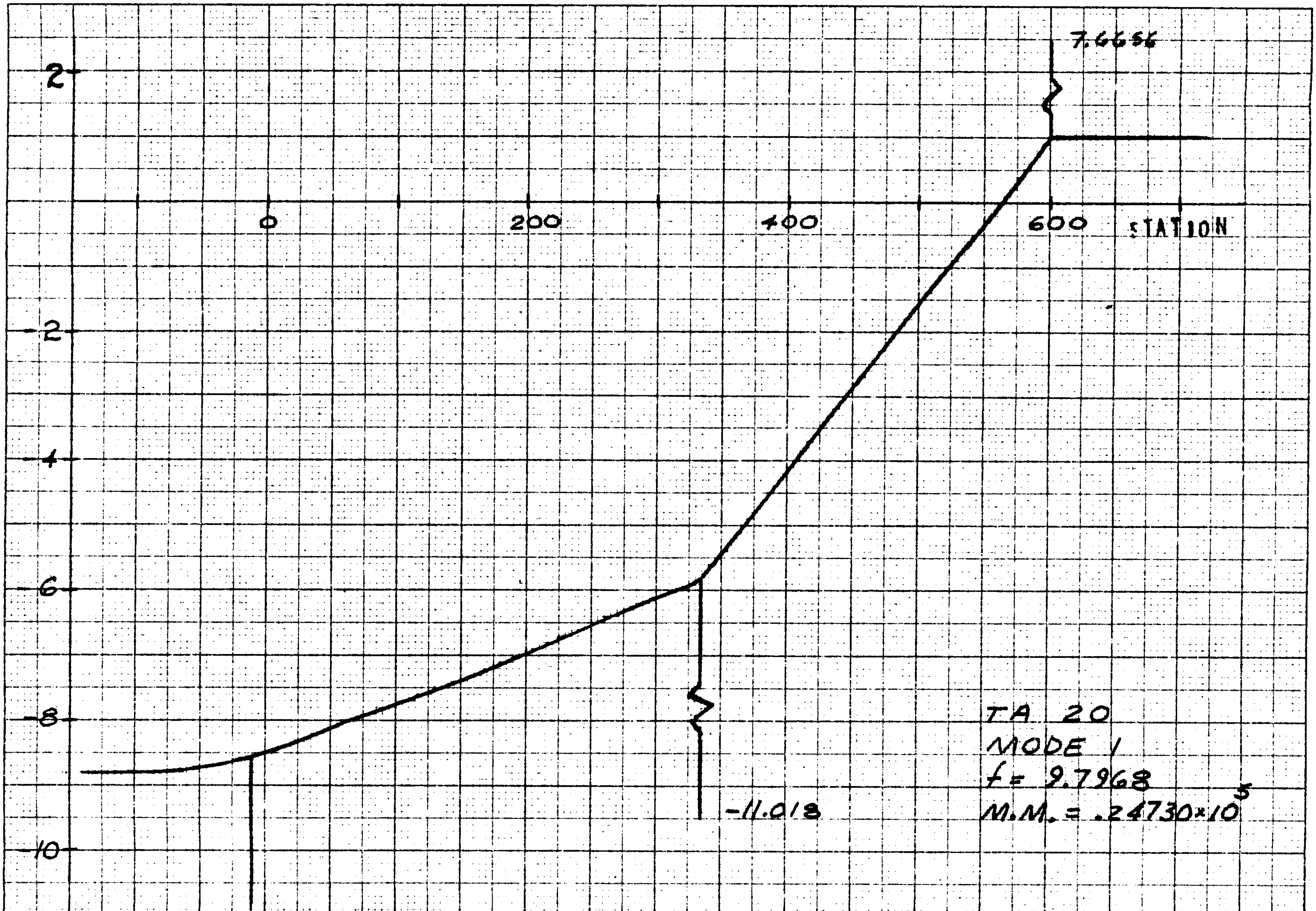


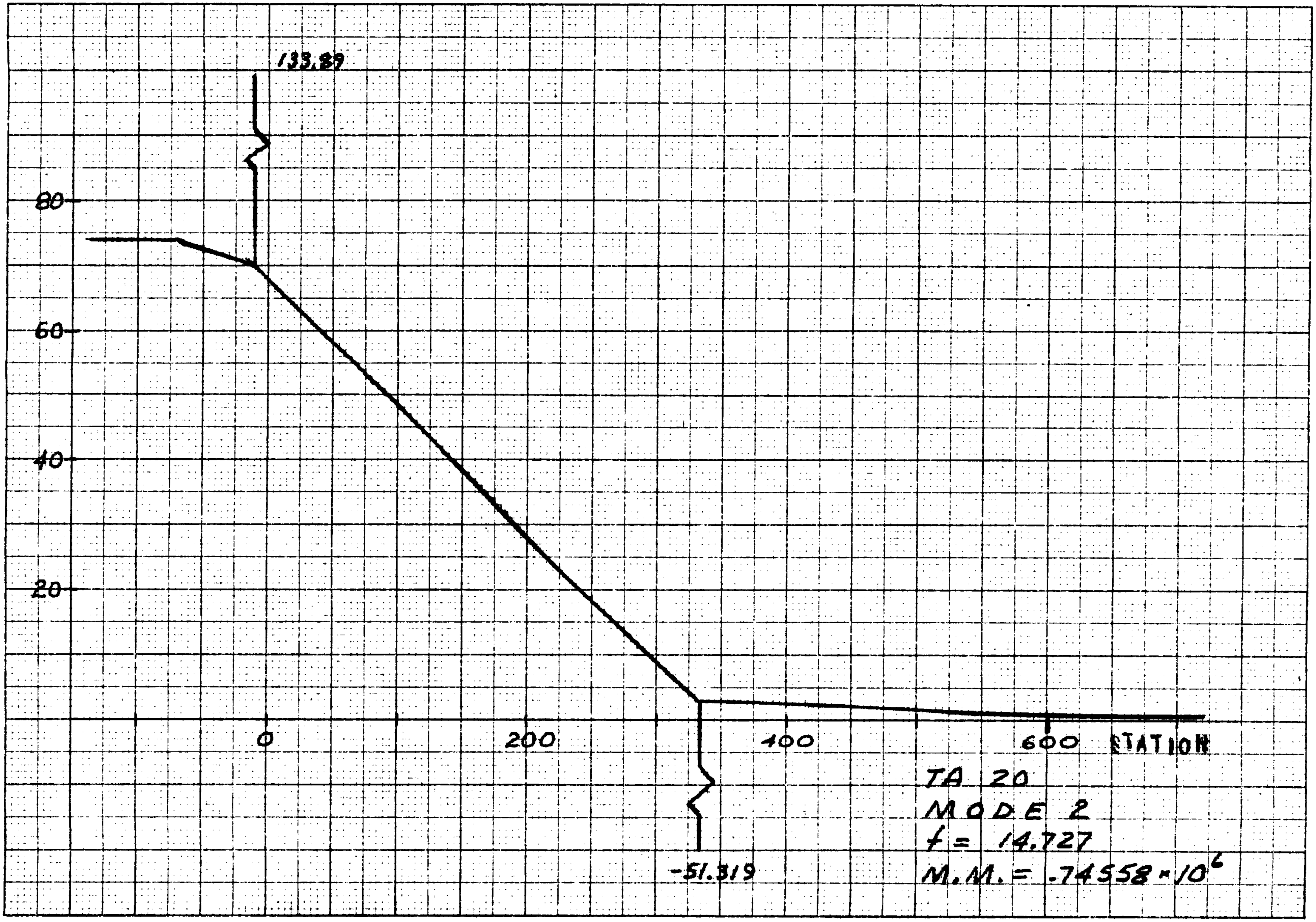
SM 46346



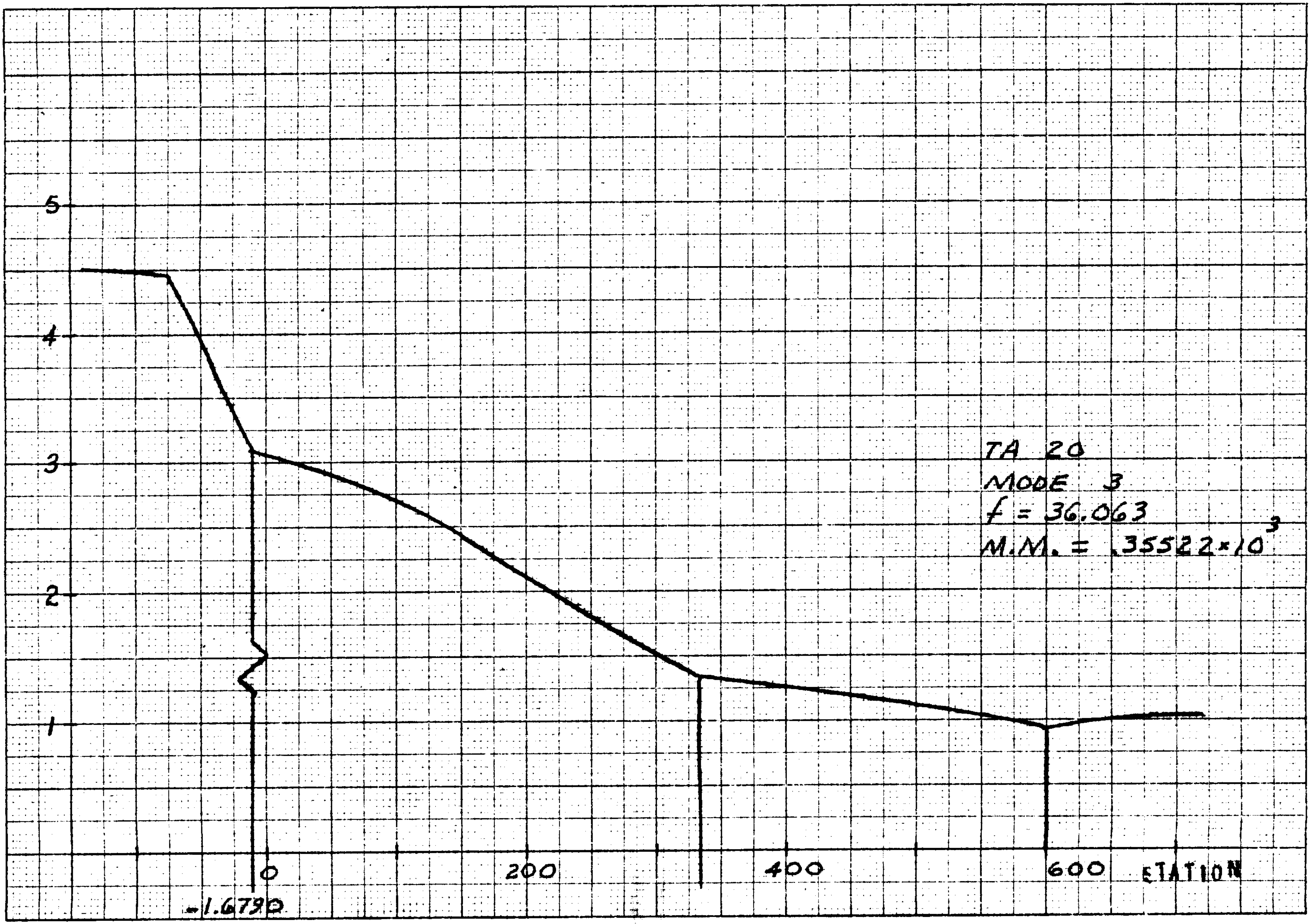
SM 46346


7.6656
2
0
200
400
600
STATION
-2
-4
-6
-8
-10
-11.018
TA 20
MODE 1
f = 9.7968
M.M. = .24730×10^5

SM 46346

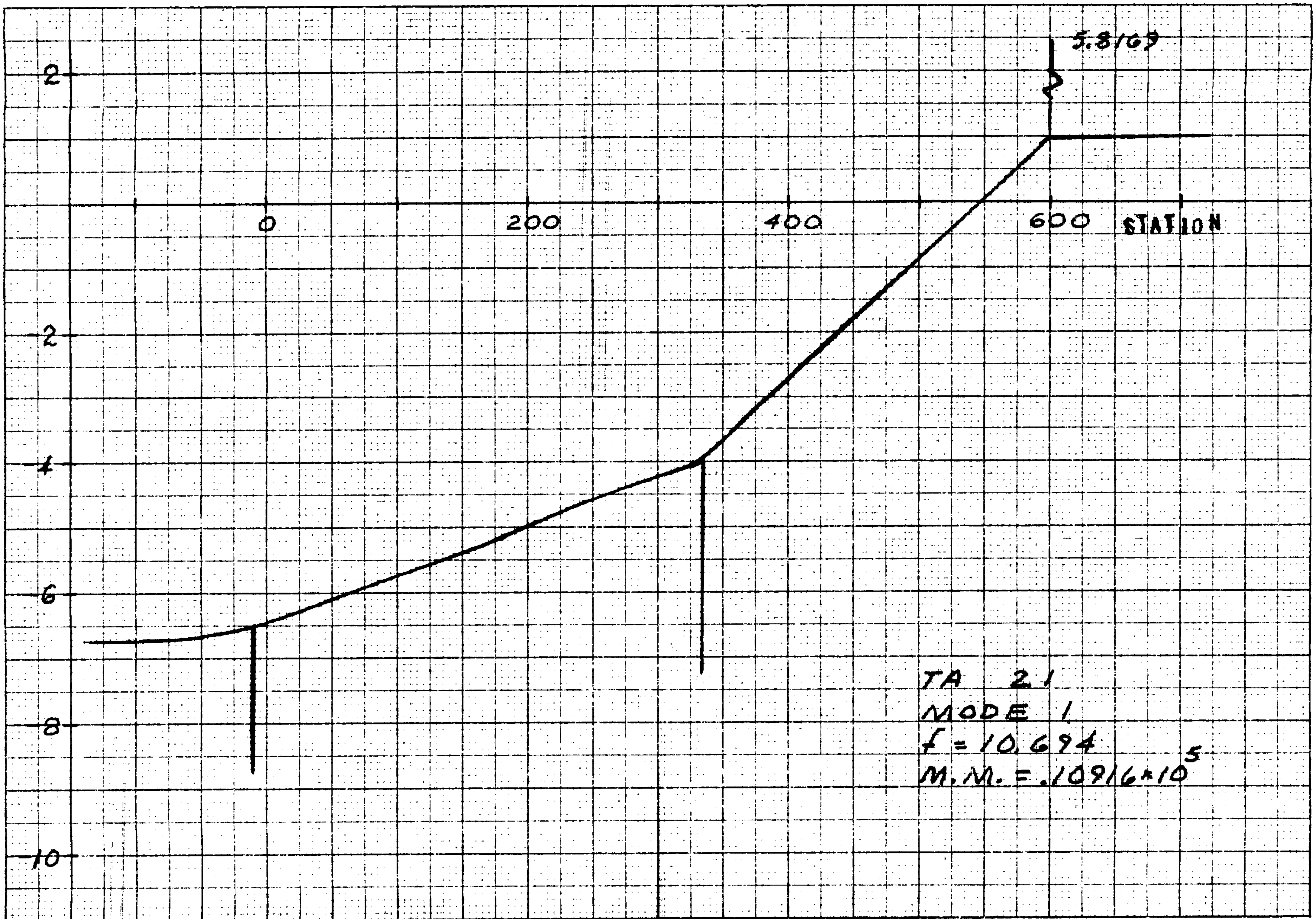

133.89
80
60
40
20
0
200
400
600
STATION
-51.319
TA 20
MODE 2
f = 14.727
M.M. = .74558 × 10^6

SM 46346



5
4
3
2
1
0
200
400
600
STATION
-1.6790
TA 20
MODE 3
f = 36.063
M.M. = .35522×10^3

SM 46646


5.8169
2
0
200
400
600
STATION
2
4
6
8
10
TA 21
MODE 1
f = 10.694
M.M. = .10916×10^5

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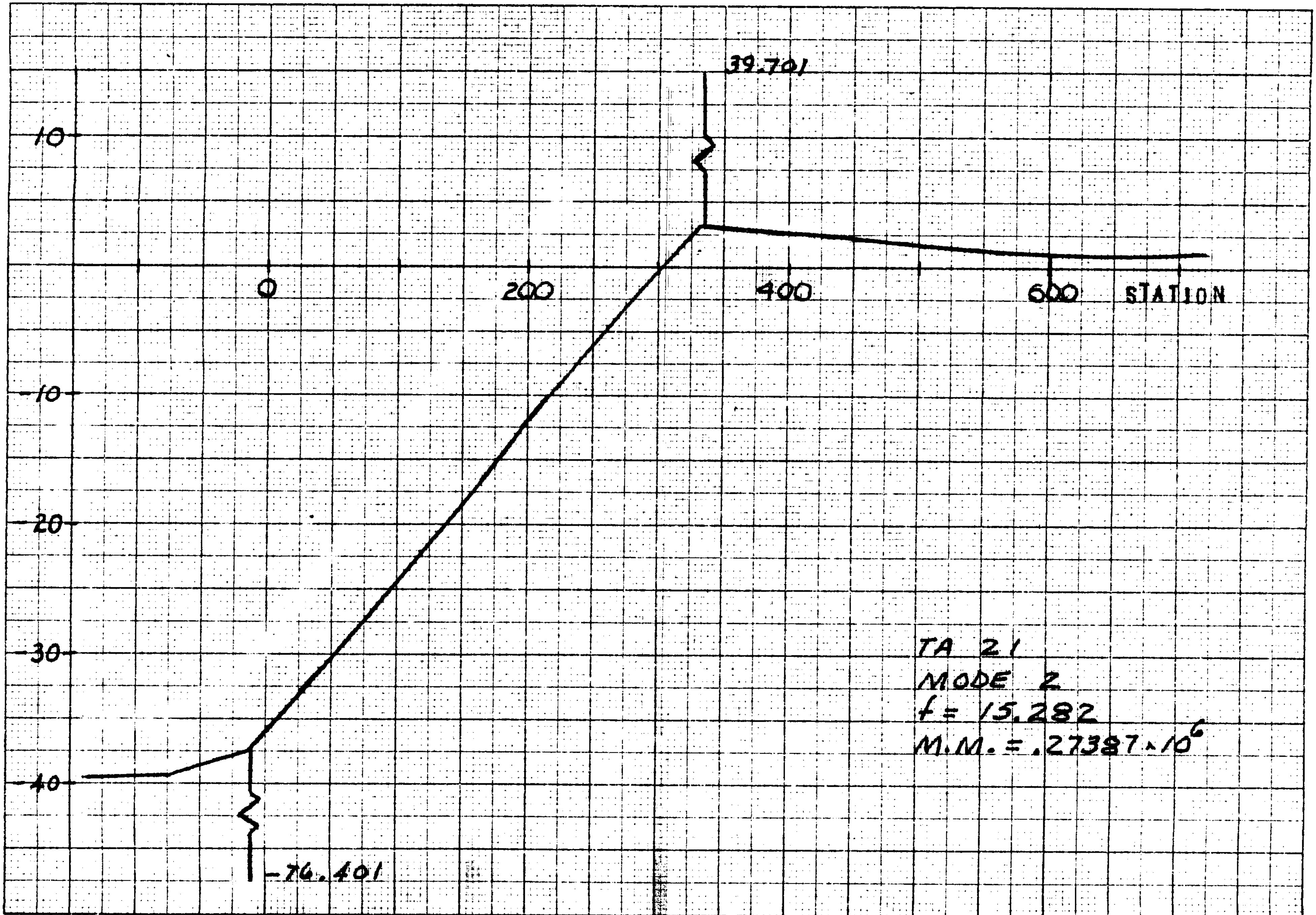
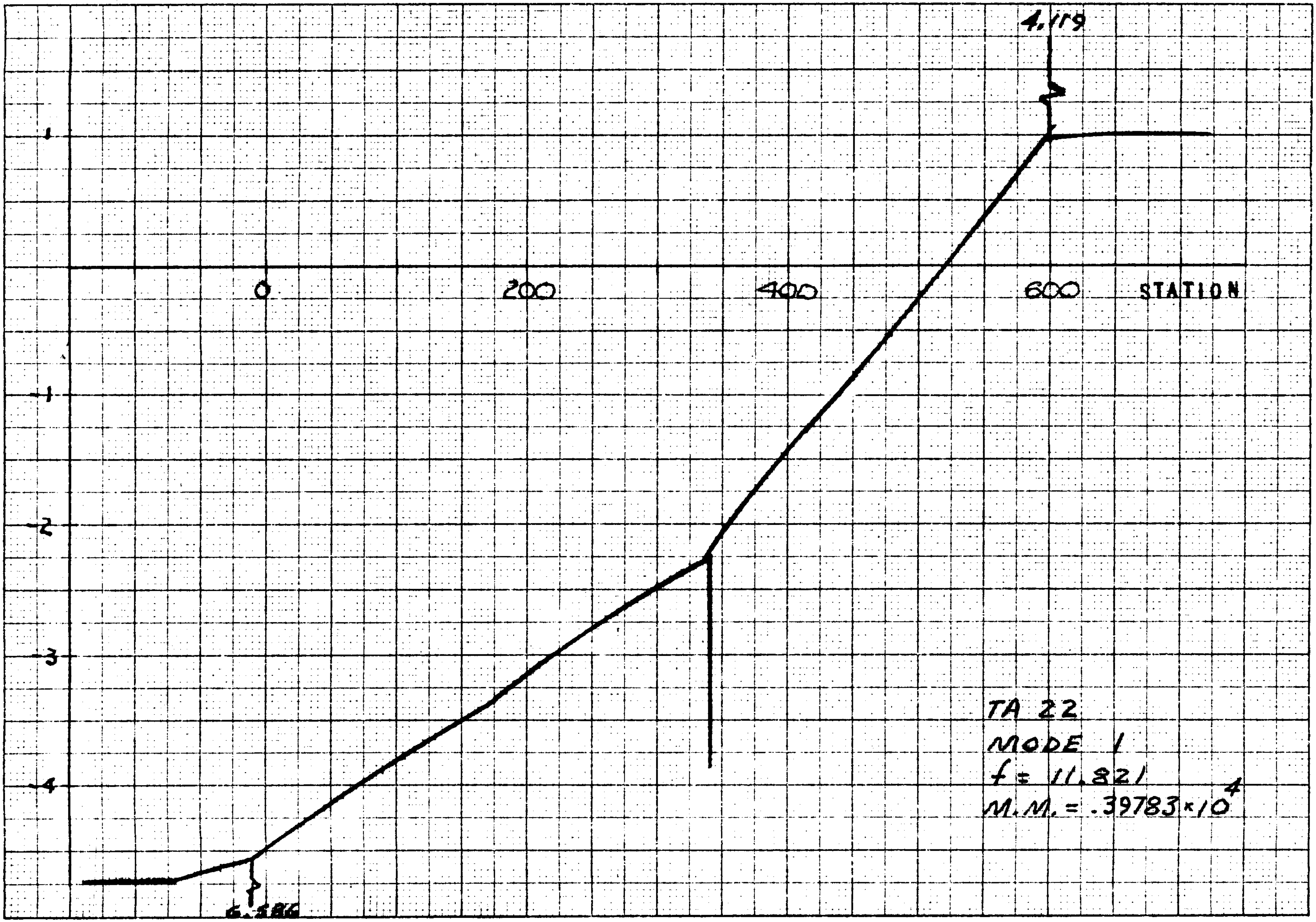
SM 46346



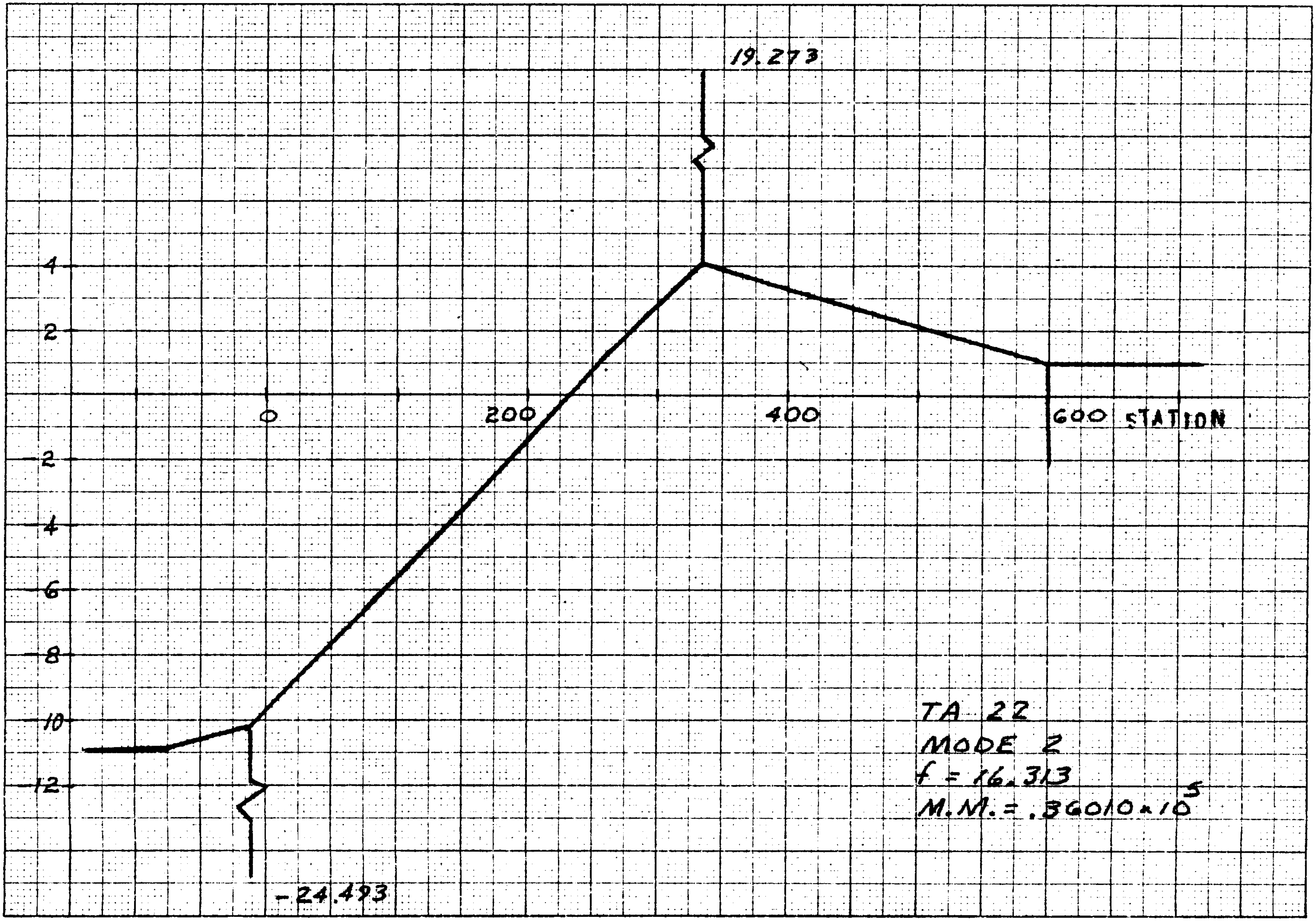
SM 46346


5
4
3
2
1
0
200
400
600
STATION
-1.7466
TA 21
MODE 3
f = 36.145
M.M. = .38906 × 10^3

SM 46346


4.119
1
-1
-2
-3
-4
0
200
400
600
STATION
6.586
TA 22
MODE 1
f = 11.821
M.M. = .39783×10^4

SM 46346


19.273
4
2
0
200
400
600 STATION
2
4
6
8
10
12
-24.493
TA 22
MODE 2
f = 16.313
M.M. = .36010 x 10^5

K+E 10 X 10 TO 1/2 INCH 46 1327
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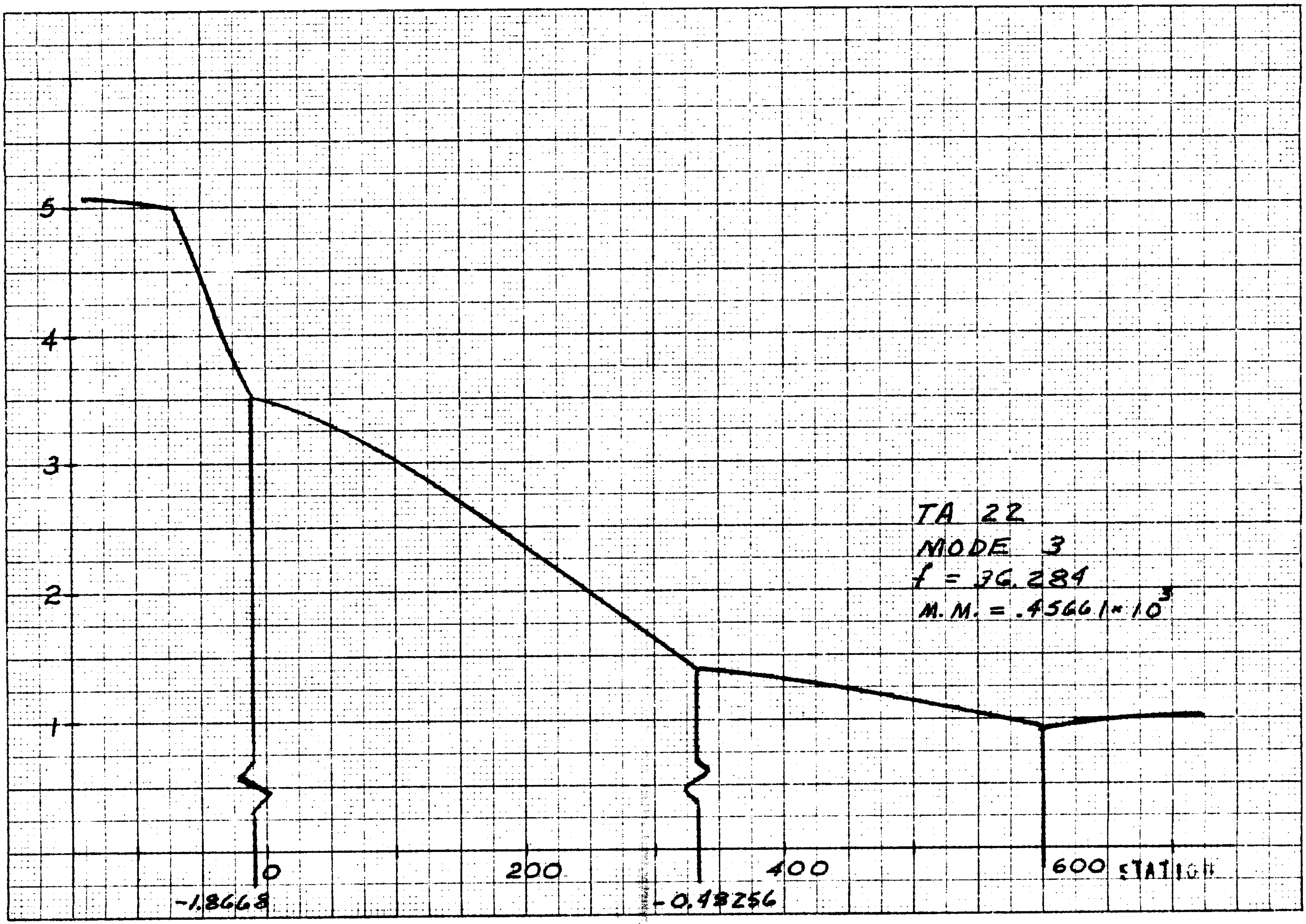
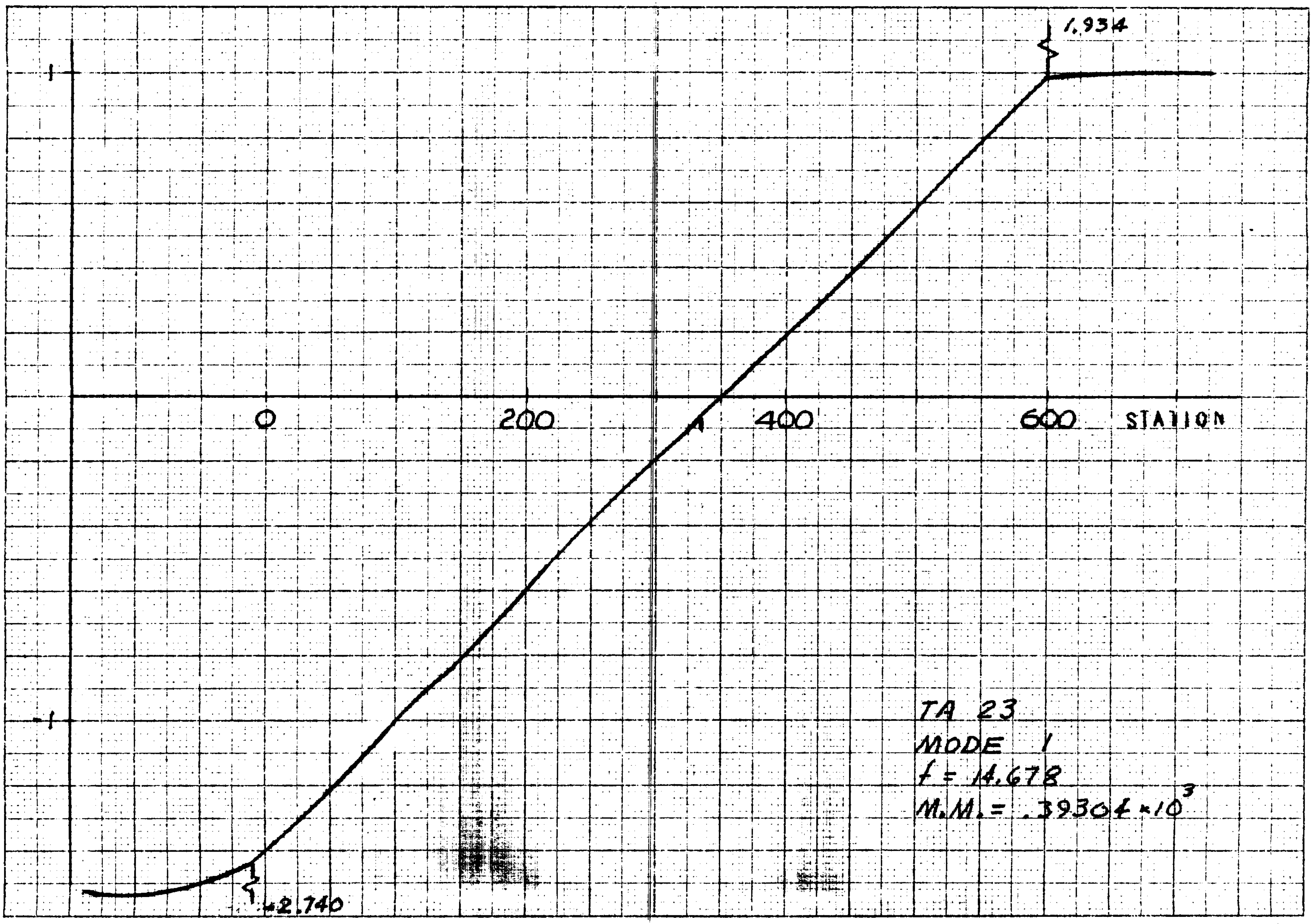
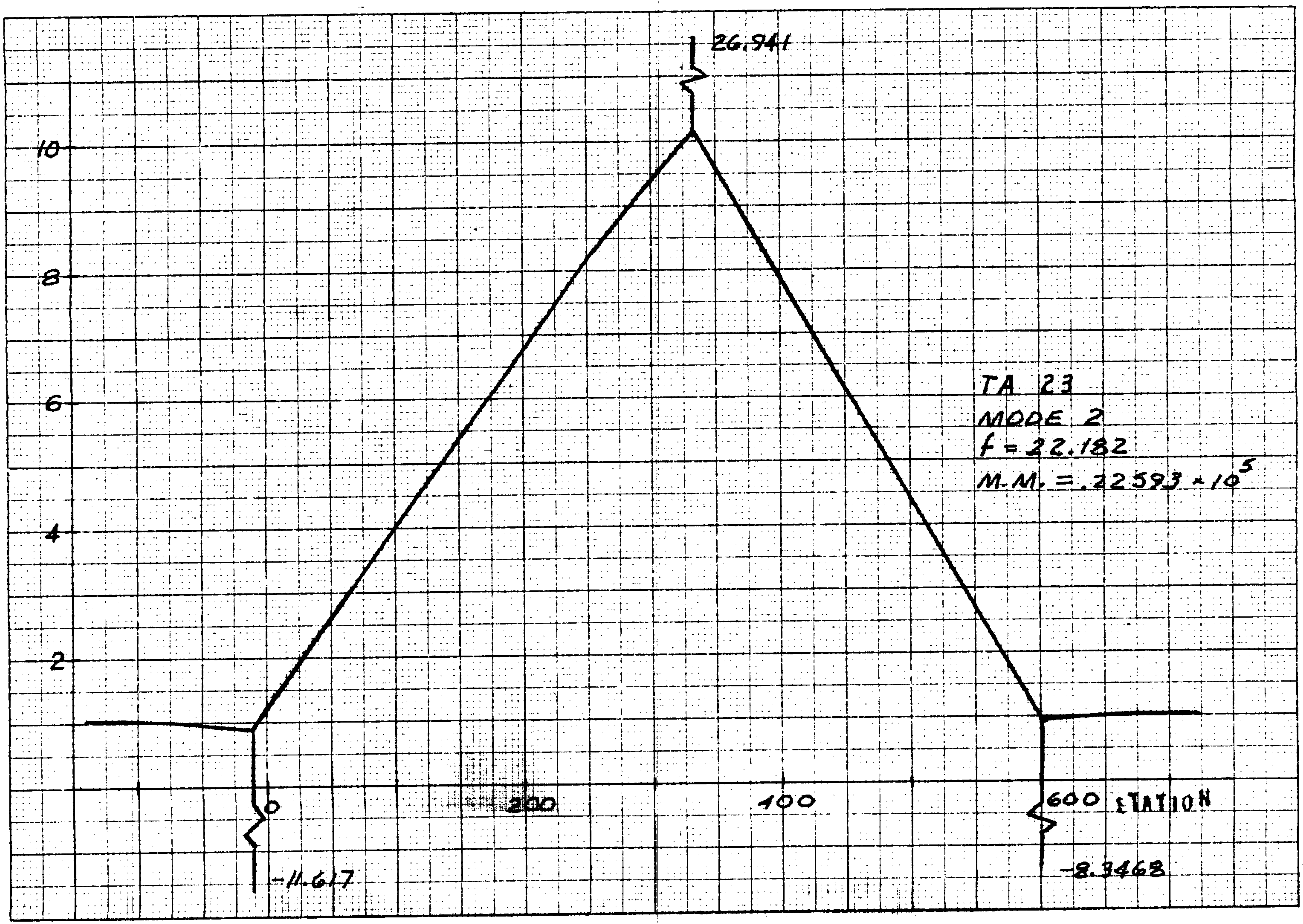
SM 46346



SM 46346

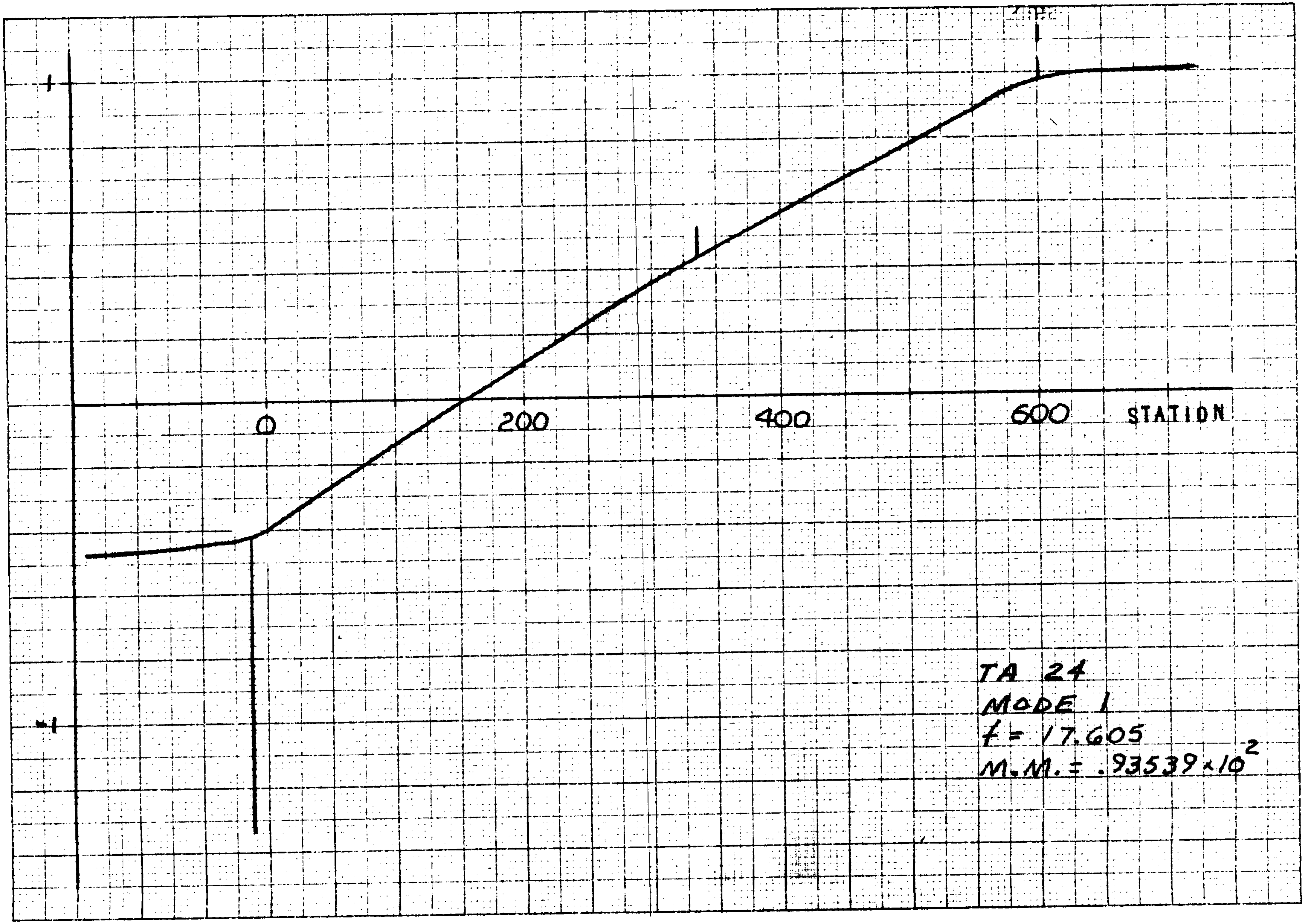

1.934
1
-1
0
200
400
600
STATION
-2.740
TA 23
MODE 1
f = 14.678
M.M. = .39304 × 10^3

SM 46346

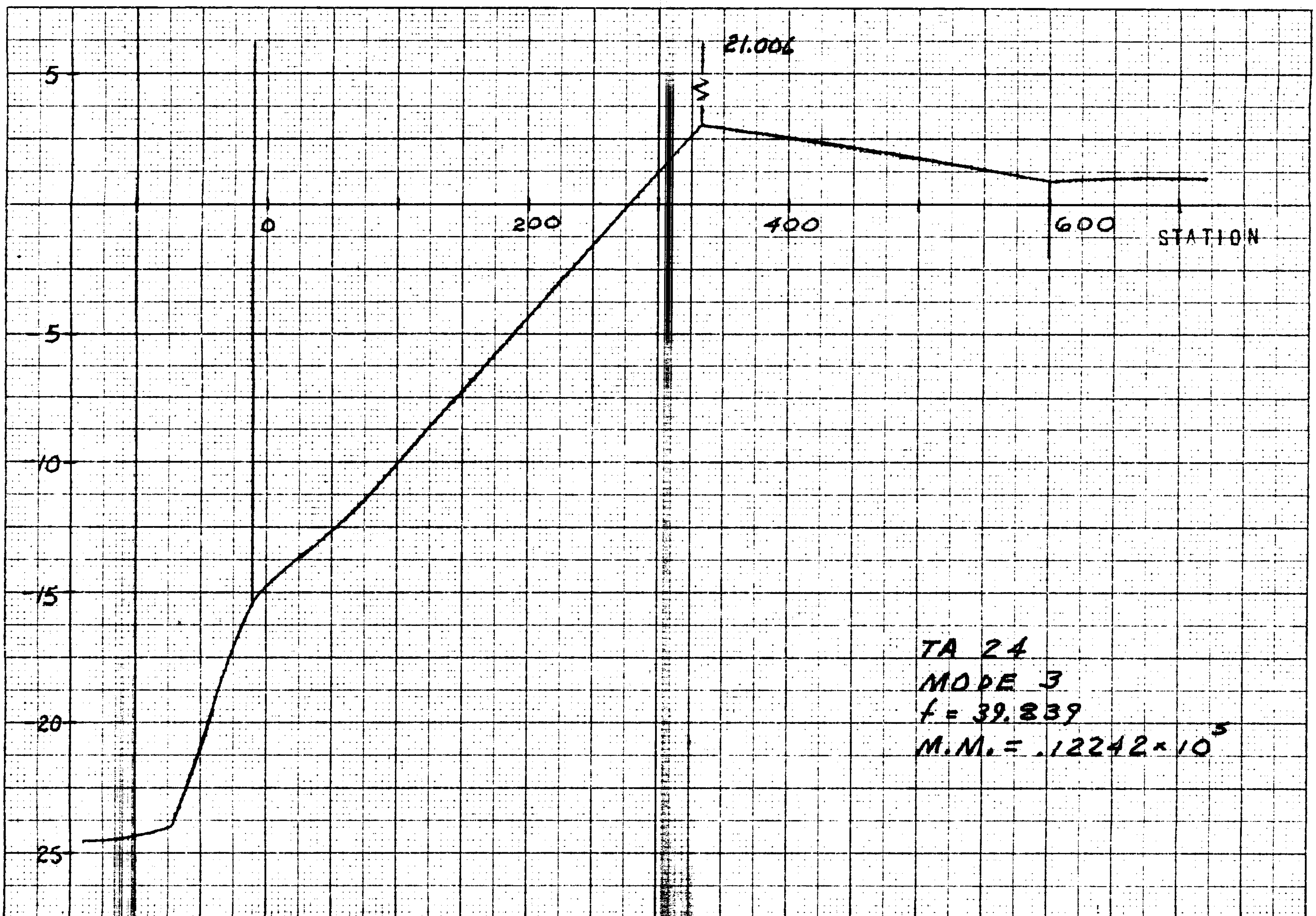

26.941
10
8
6
4
2
TA 23
MODE 2
f = 22.182
M.M. = .22593 × 10^5
0
200
400
600 STATION
-11.617
-8.3468


10
8
6
4
2
2
0
200
400
600 STATION
-3.0459
TA 23
MODE 3
f = 37.087
M.M. = .14220 × 10^4

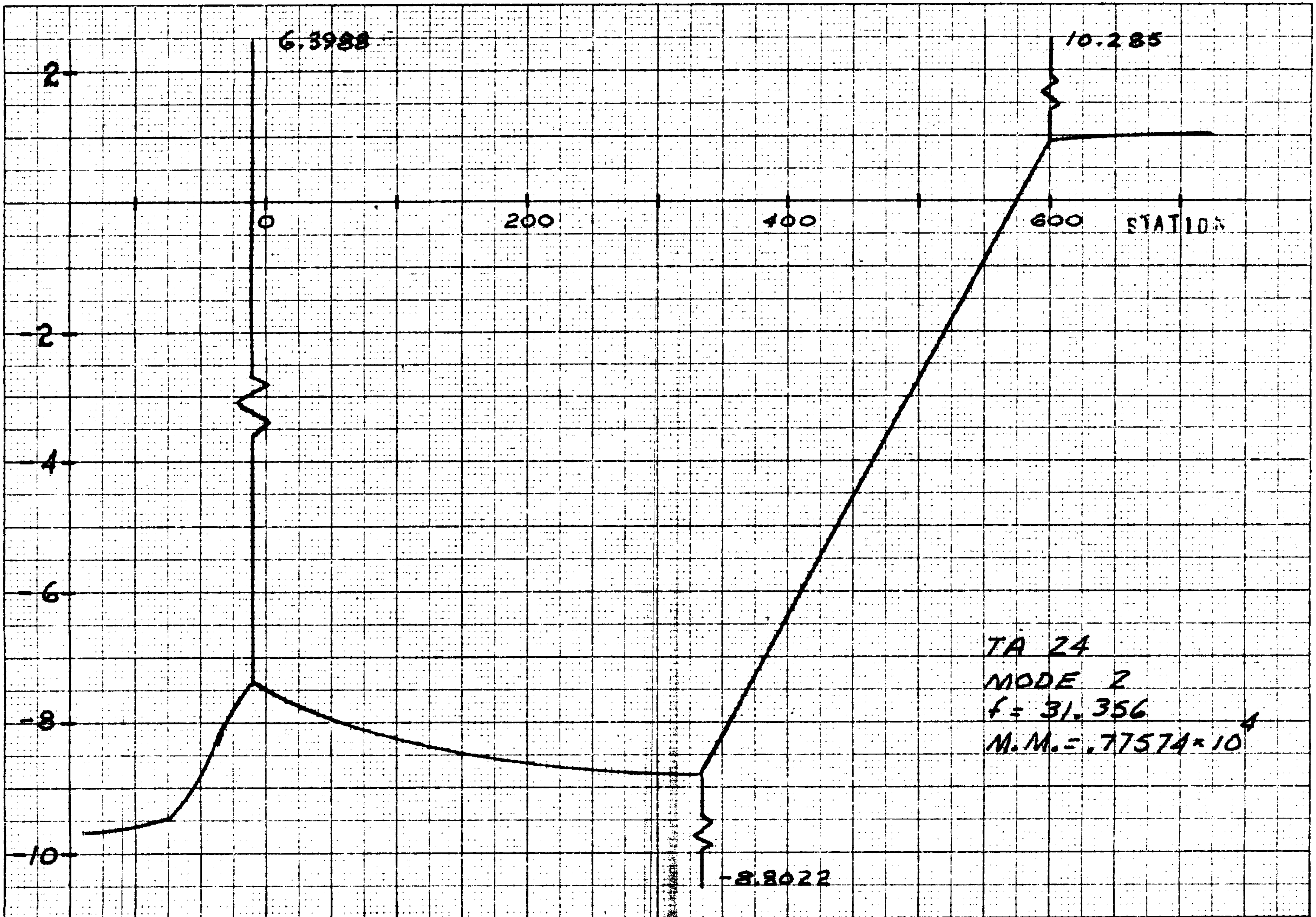
SM 46346



SM 46346

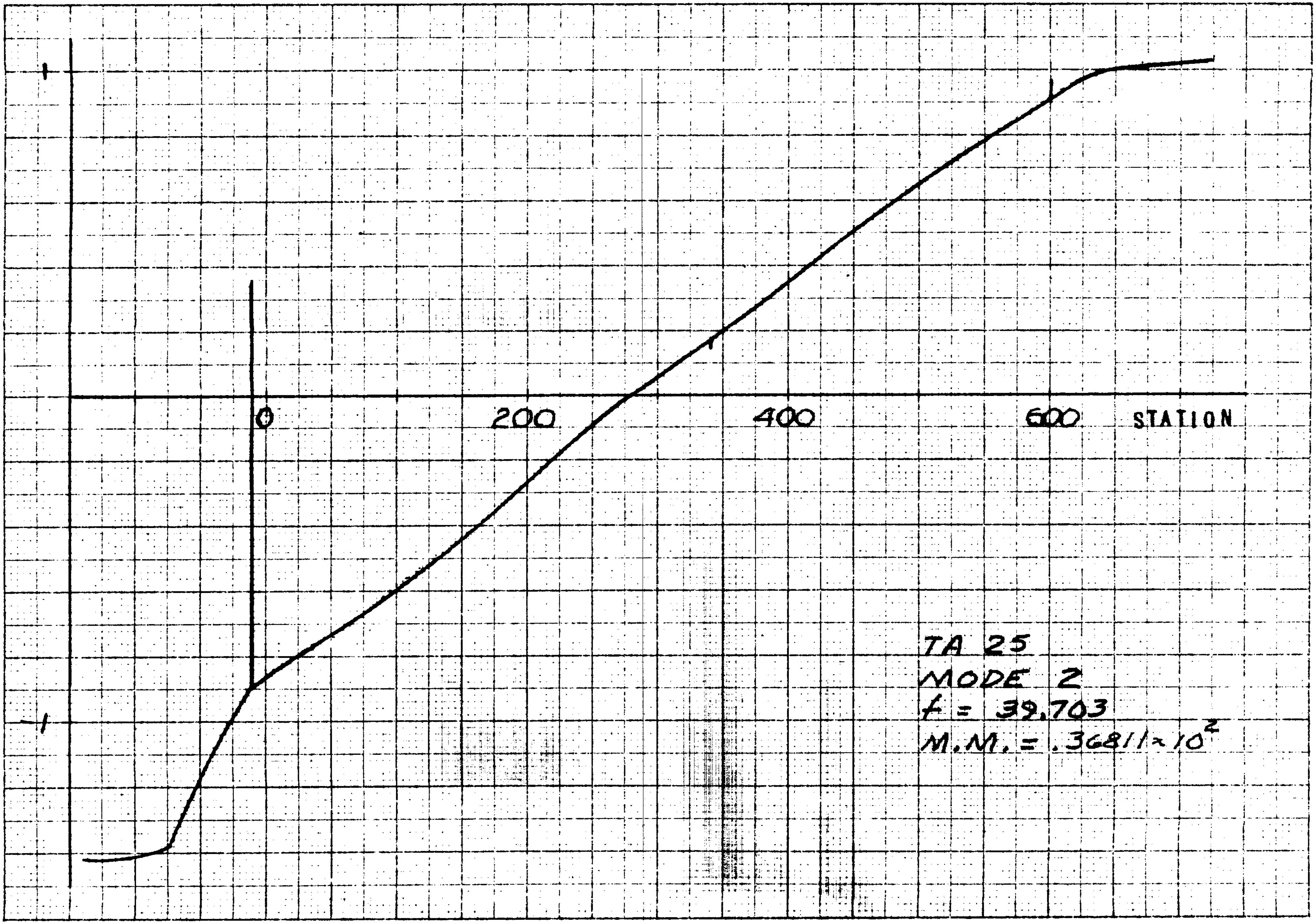

21.006
5
-5
-10
-15
-20
-25
0
200
400
600
STATION
TA 24
MODE 3
f = 39.839
M.M. = .12242 × 10^5

SM 46346

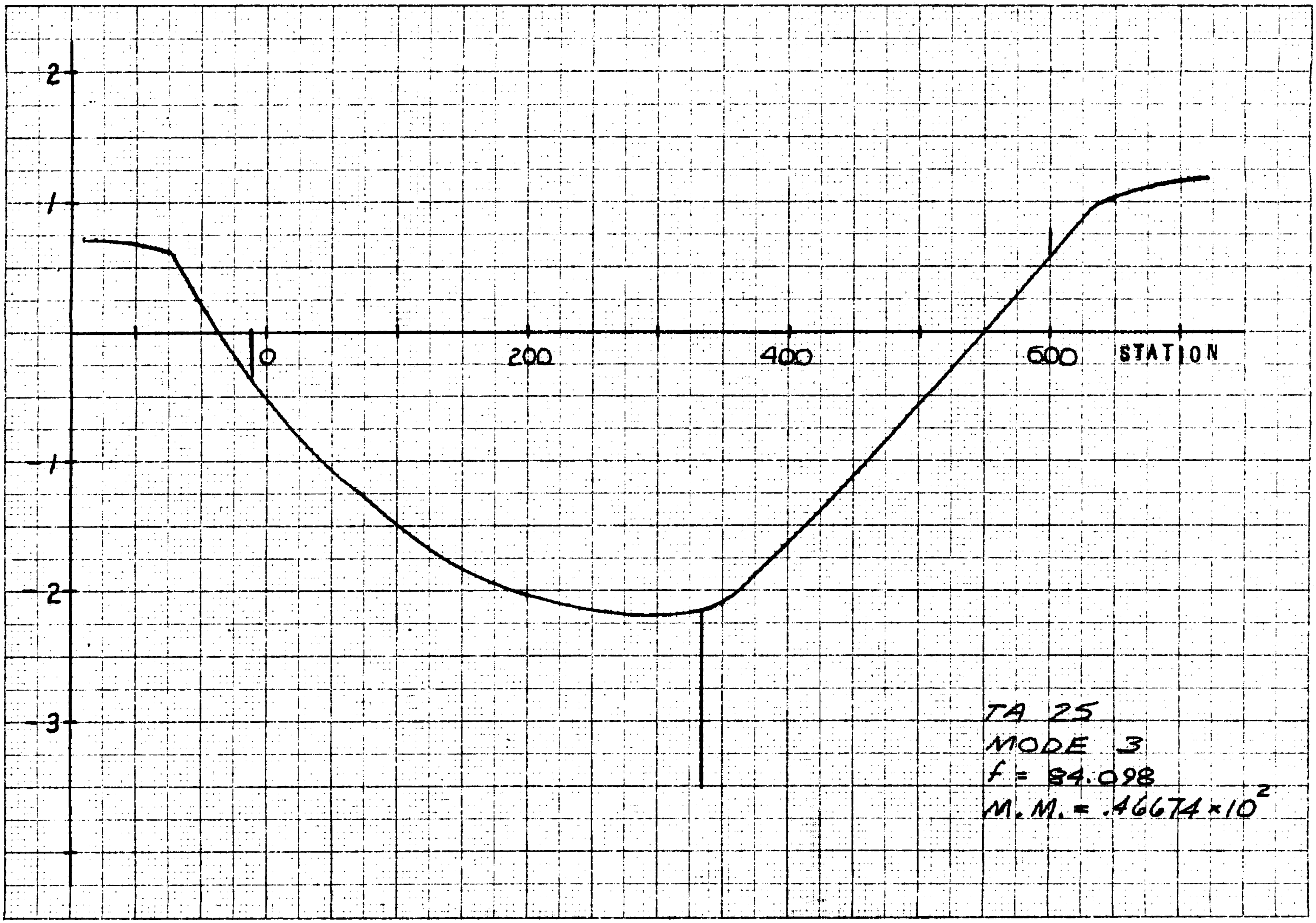

6.3988
10.285
2
-2
-4
-6
-8
-10
0
200
400
600
STATION
-8.8022
TA 24
MODE 2
f = 31.356
M.M. = .77574 × 10^4

SM 46346

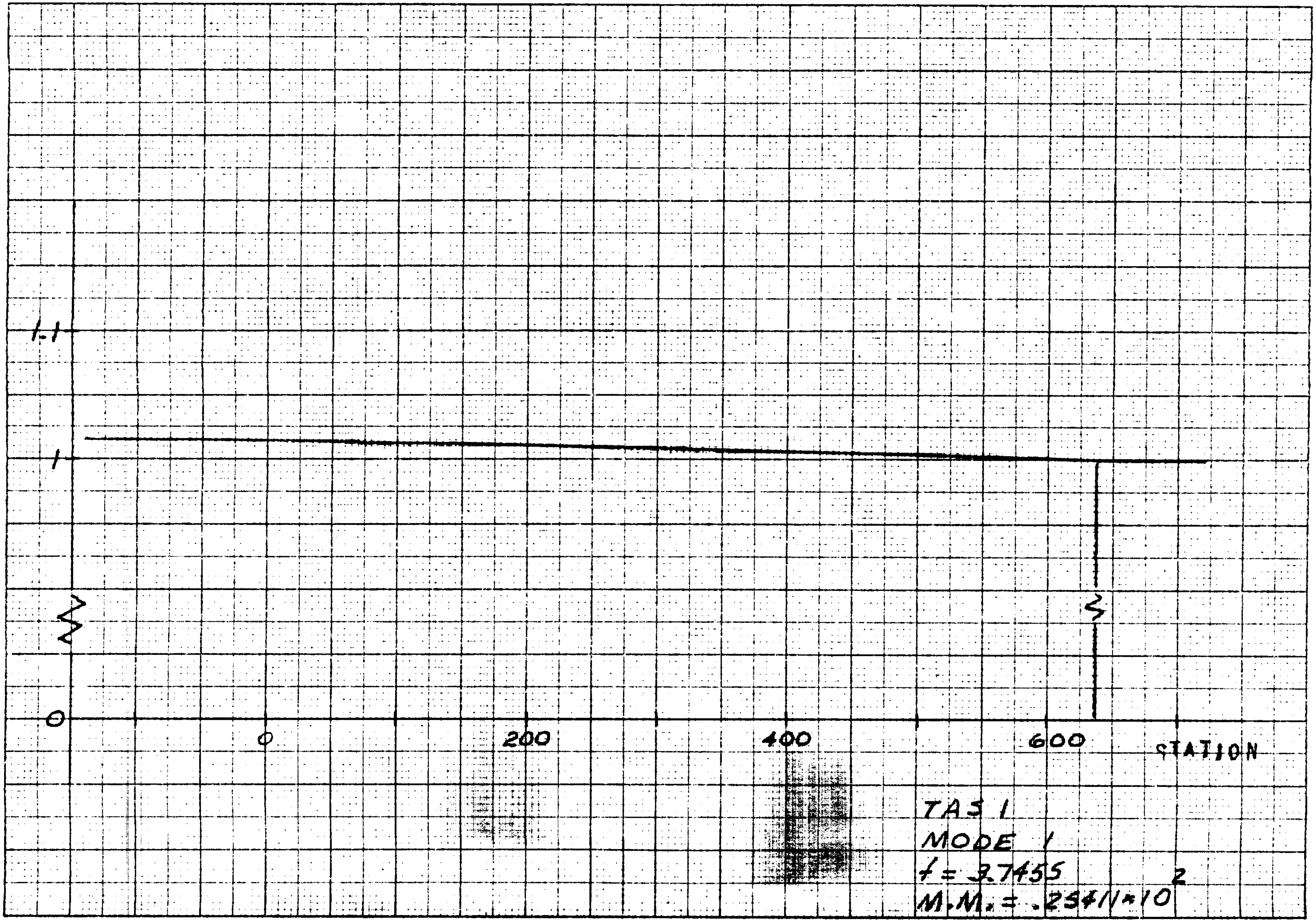
SM 46346



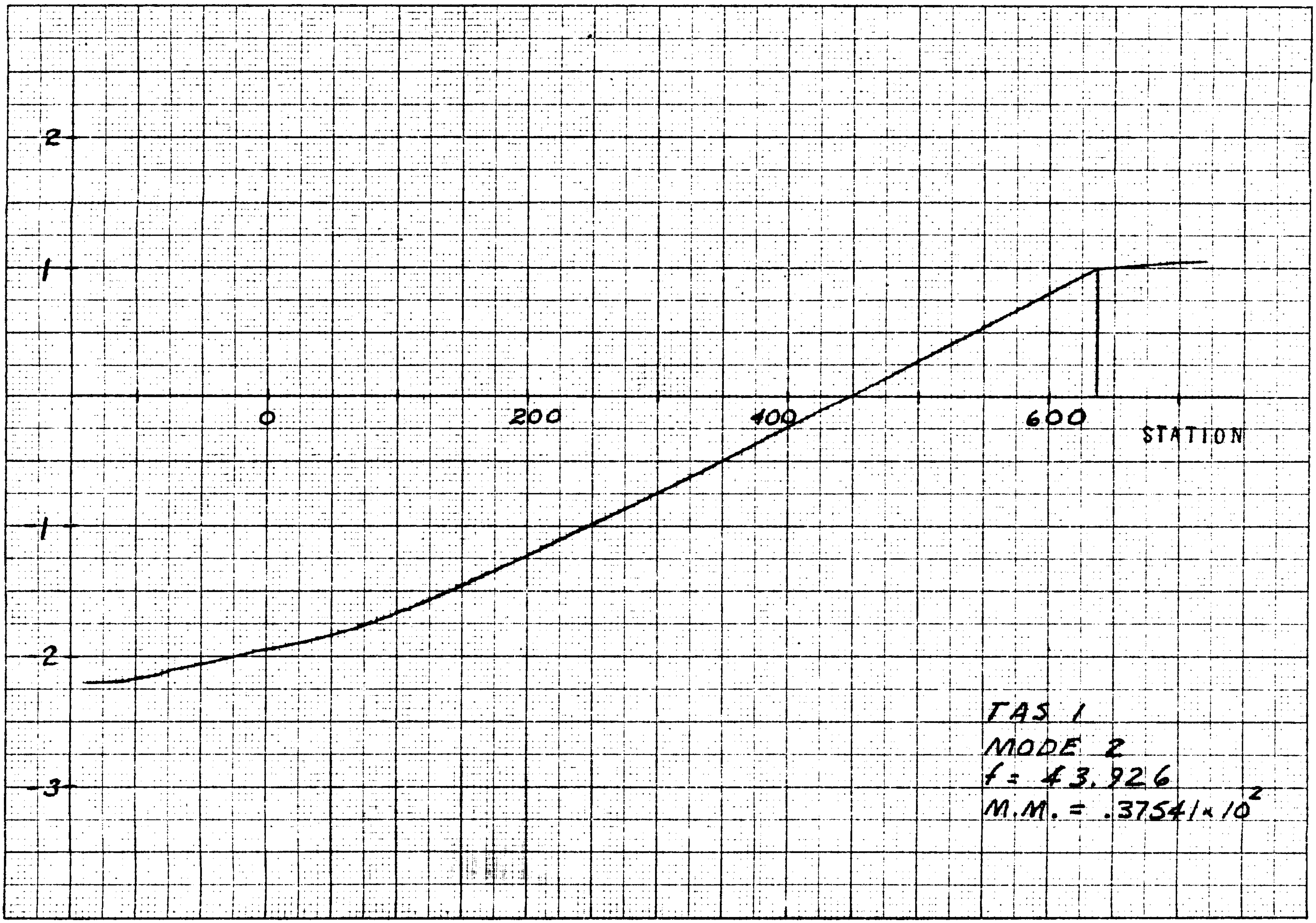
SM 46346



SM 46346

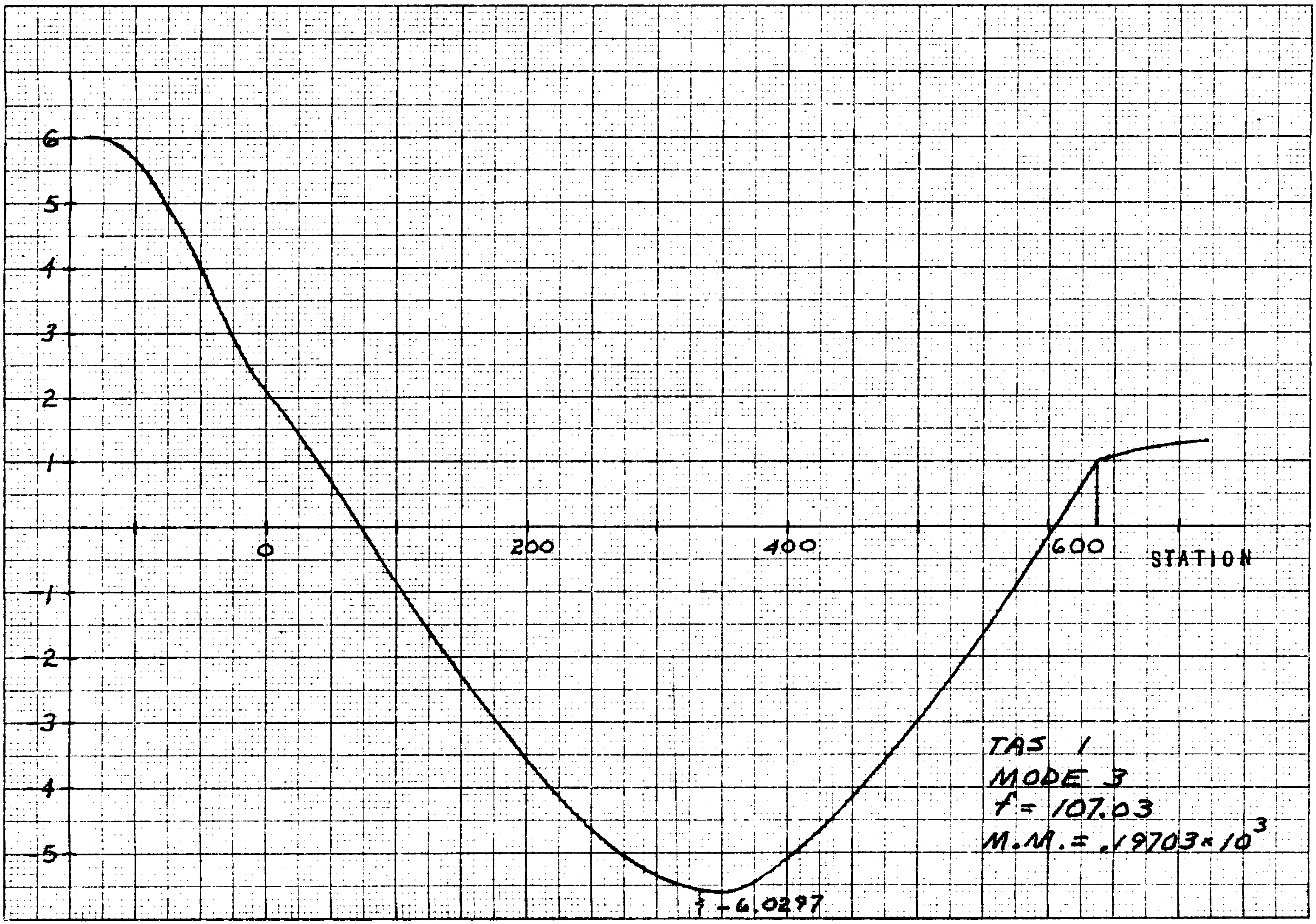


SM 40340

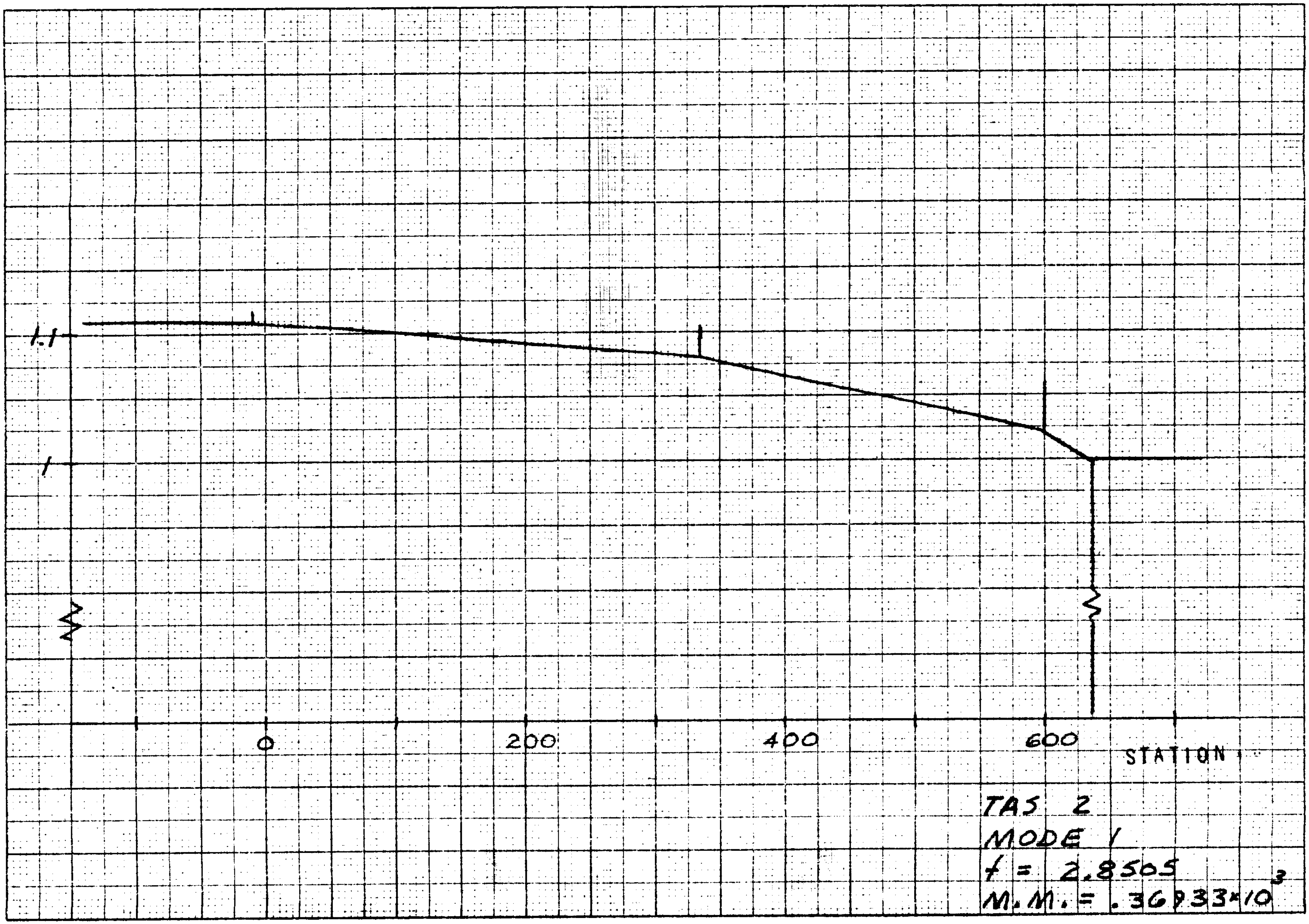


SM 46346

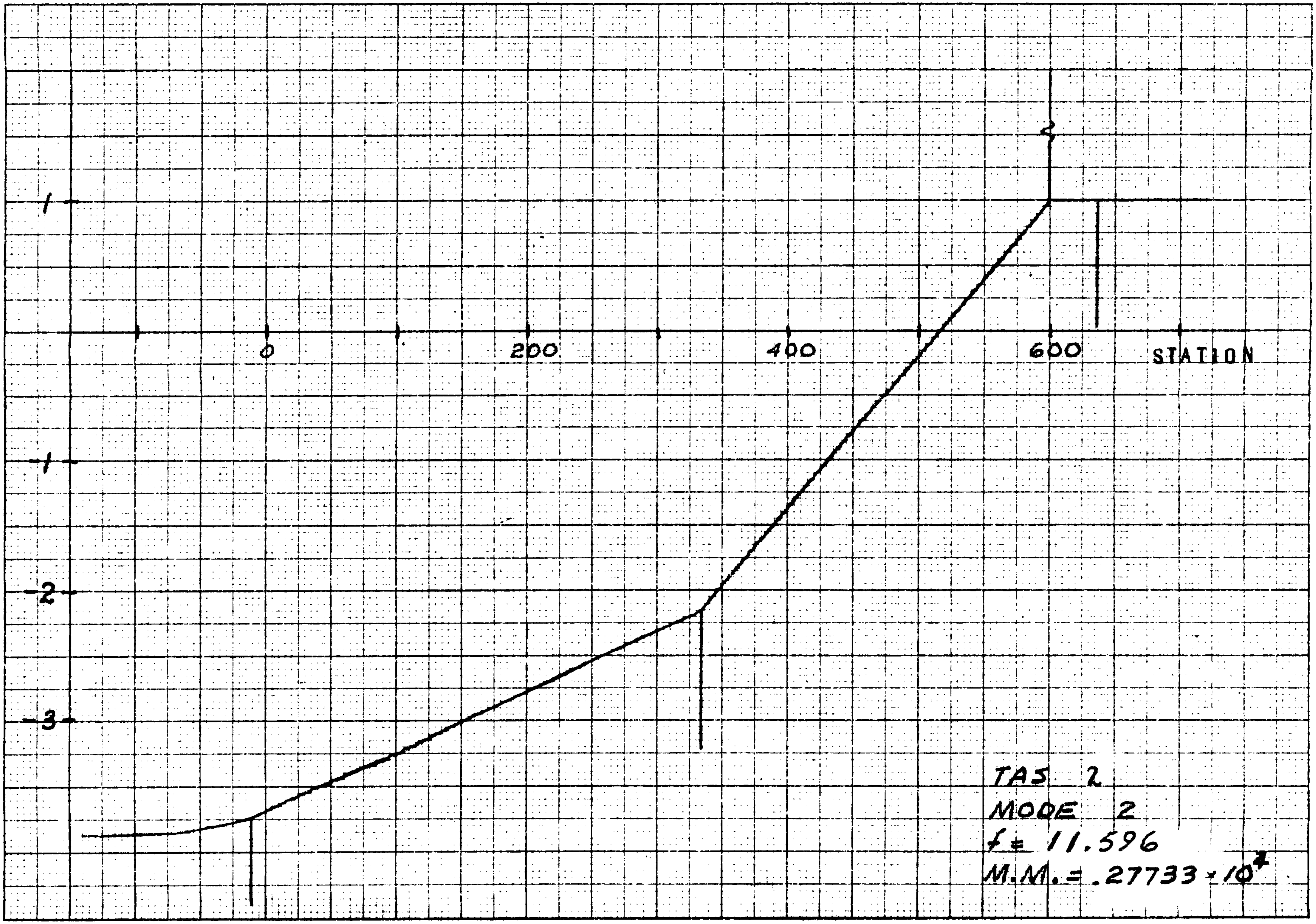
K+E 10 X 10 TO ½ INCH 46 1327
KEUFFEL & ESSER CO



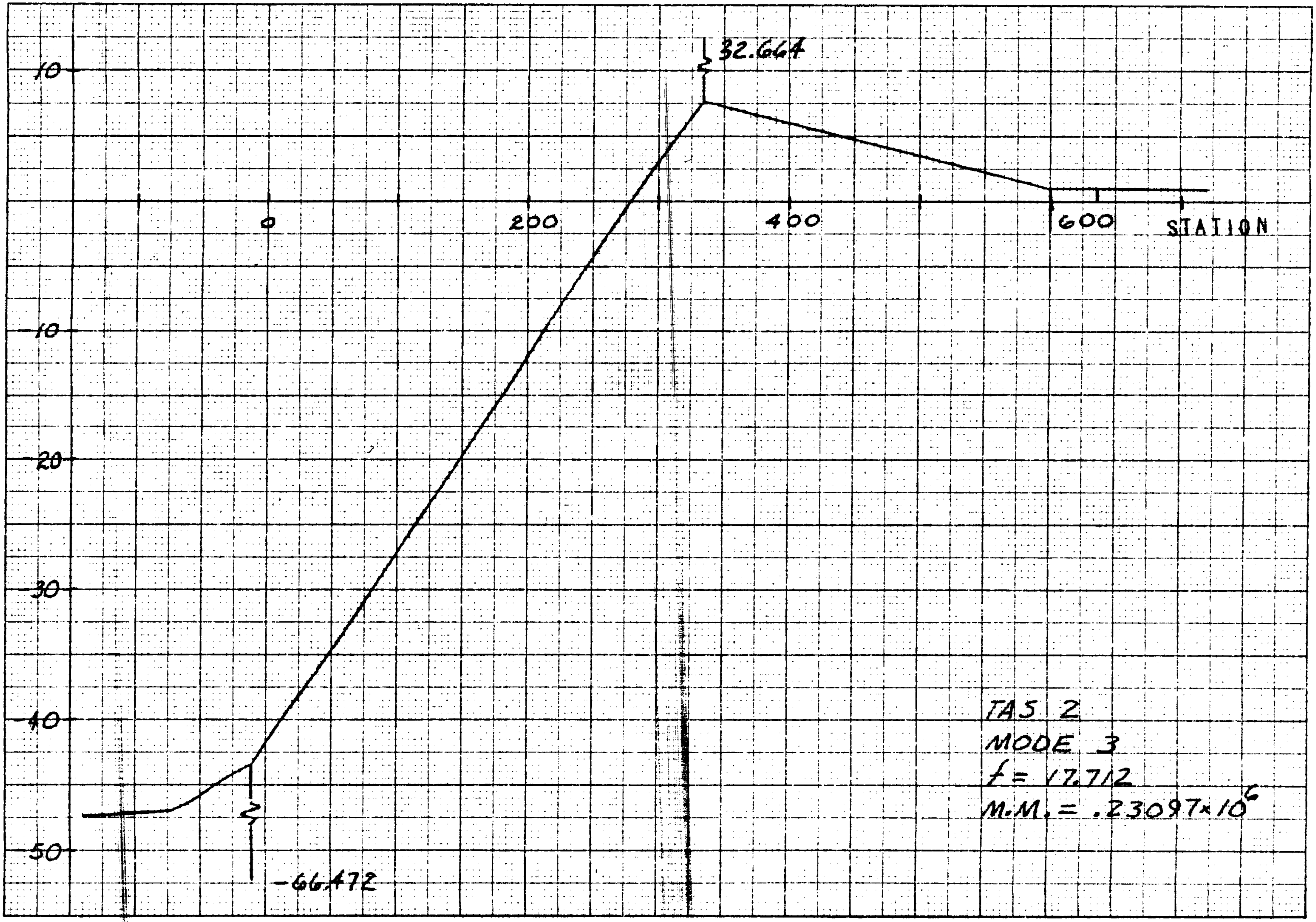
SM 4634



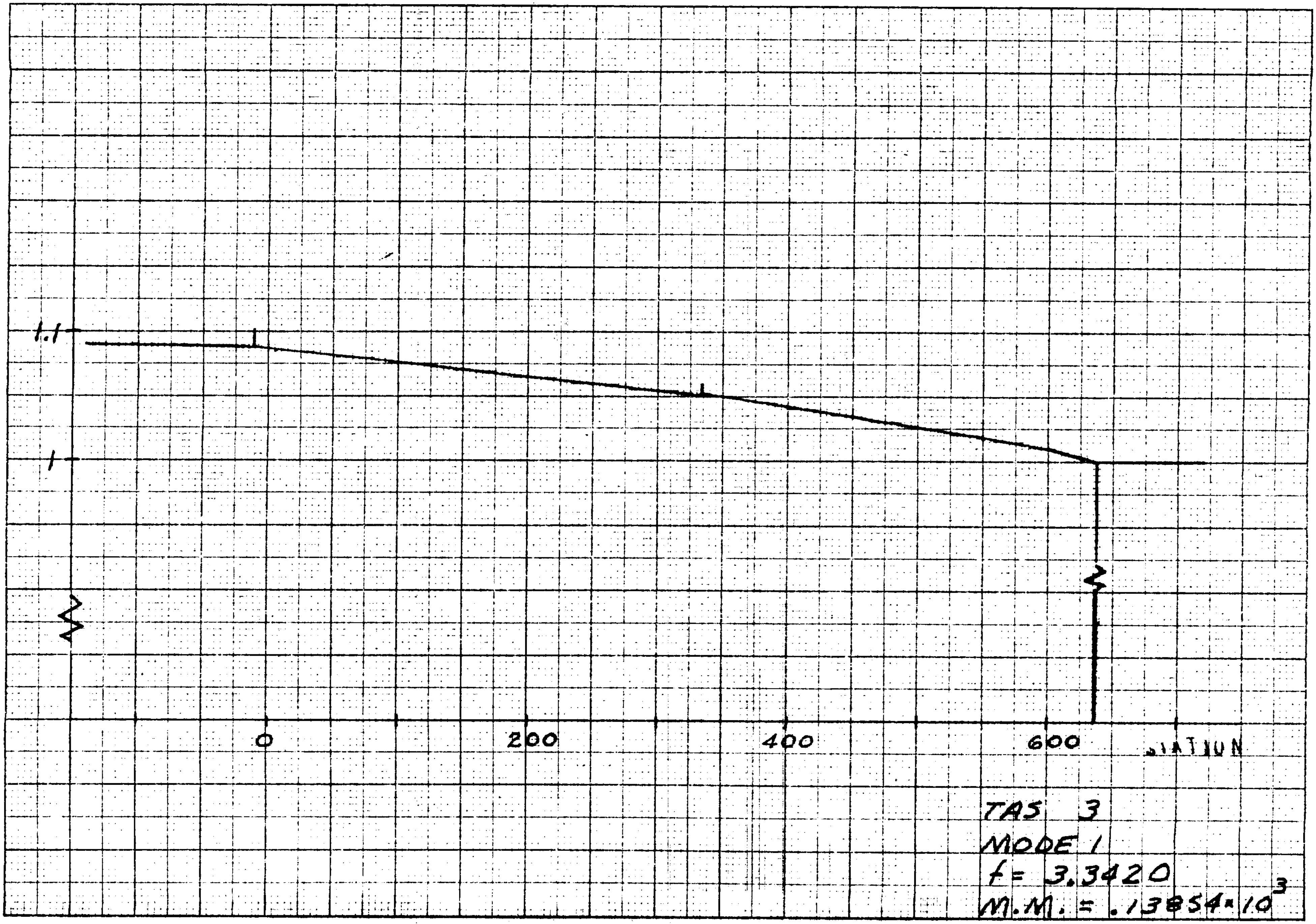
SM 40346

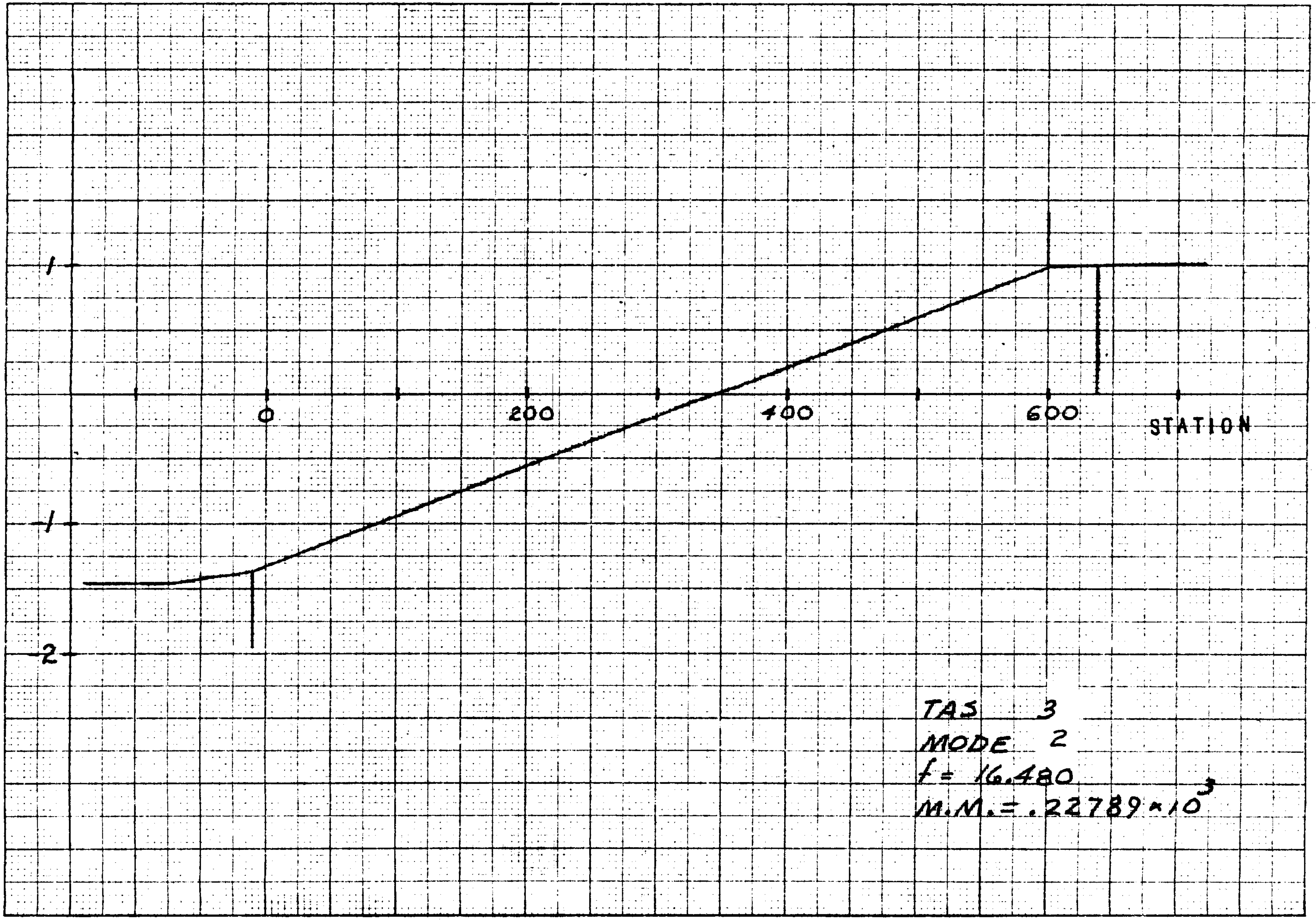


SM 46346

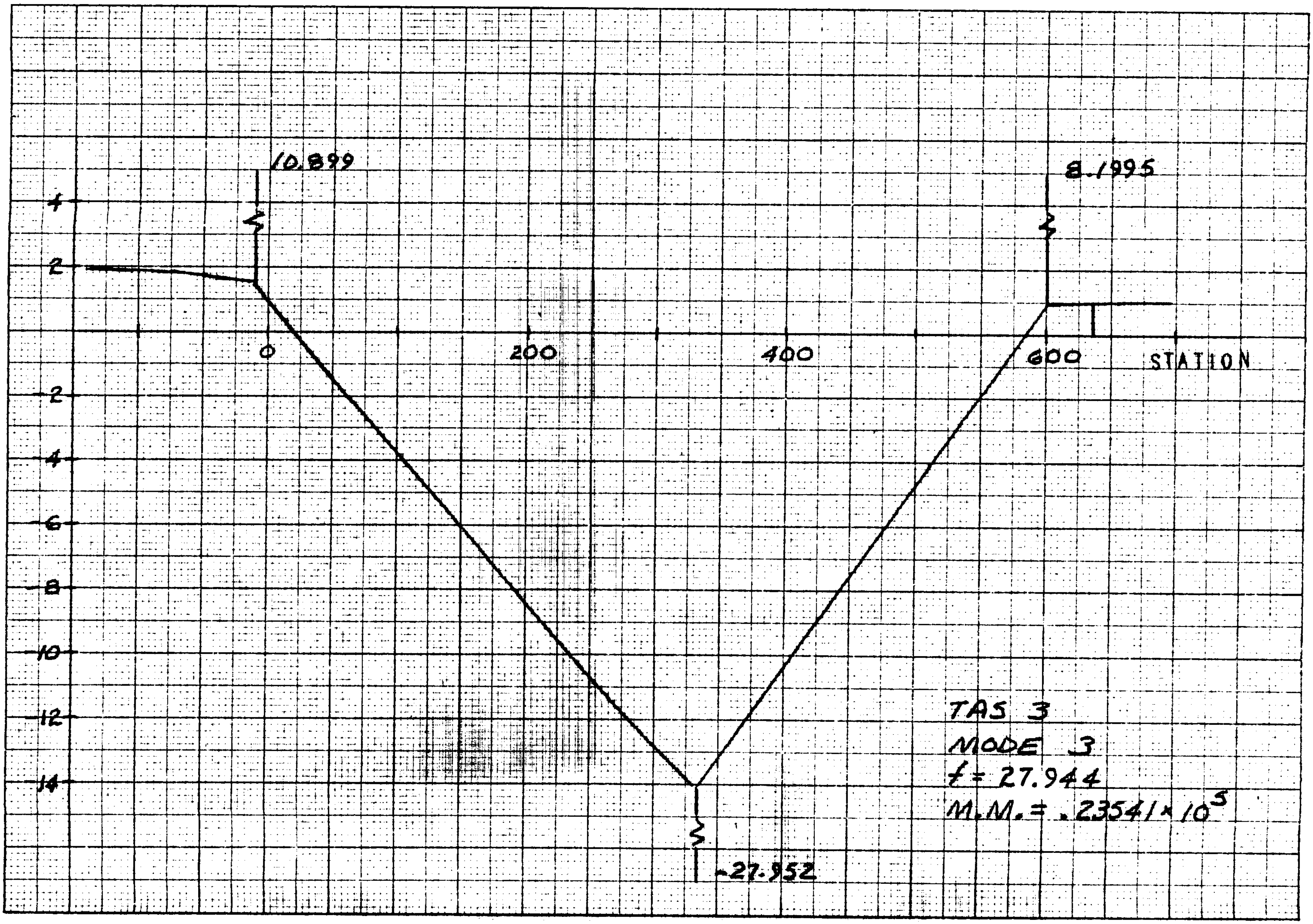

32.664
10
-10
-20
-30
-40
-50
0
200
400
600
STATION
-66.472
TAS 2
MODE 3
f = 17.712
M.M. = .23097×10^6

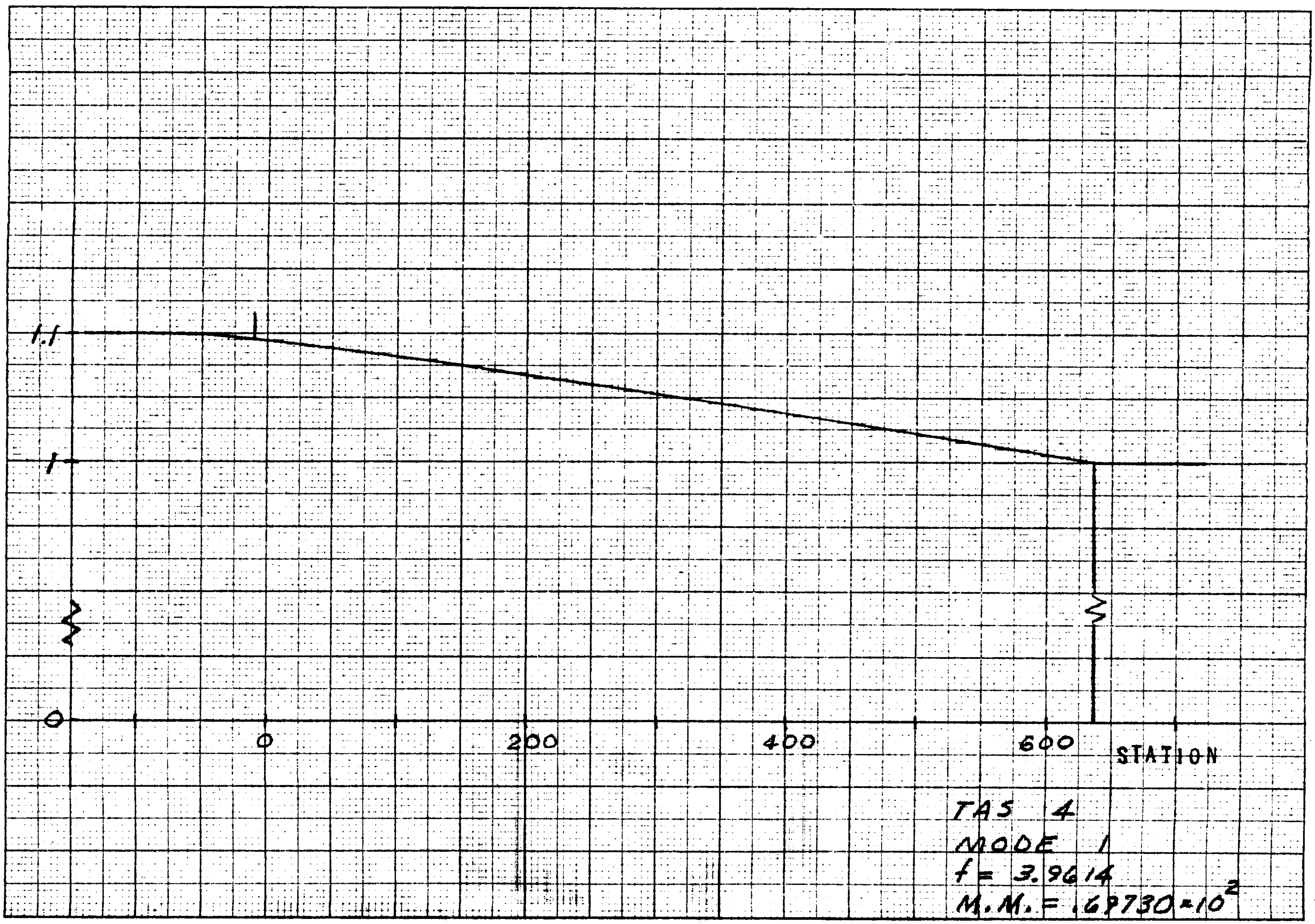
SM 46340



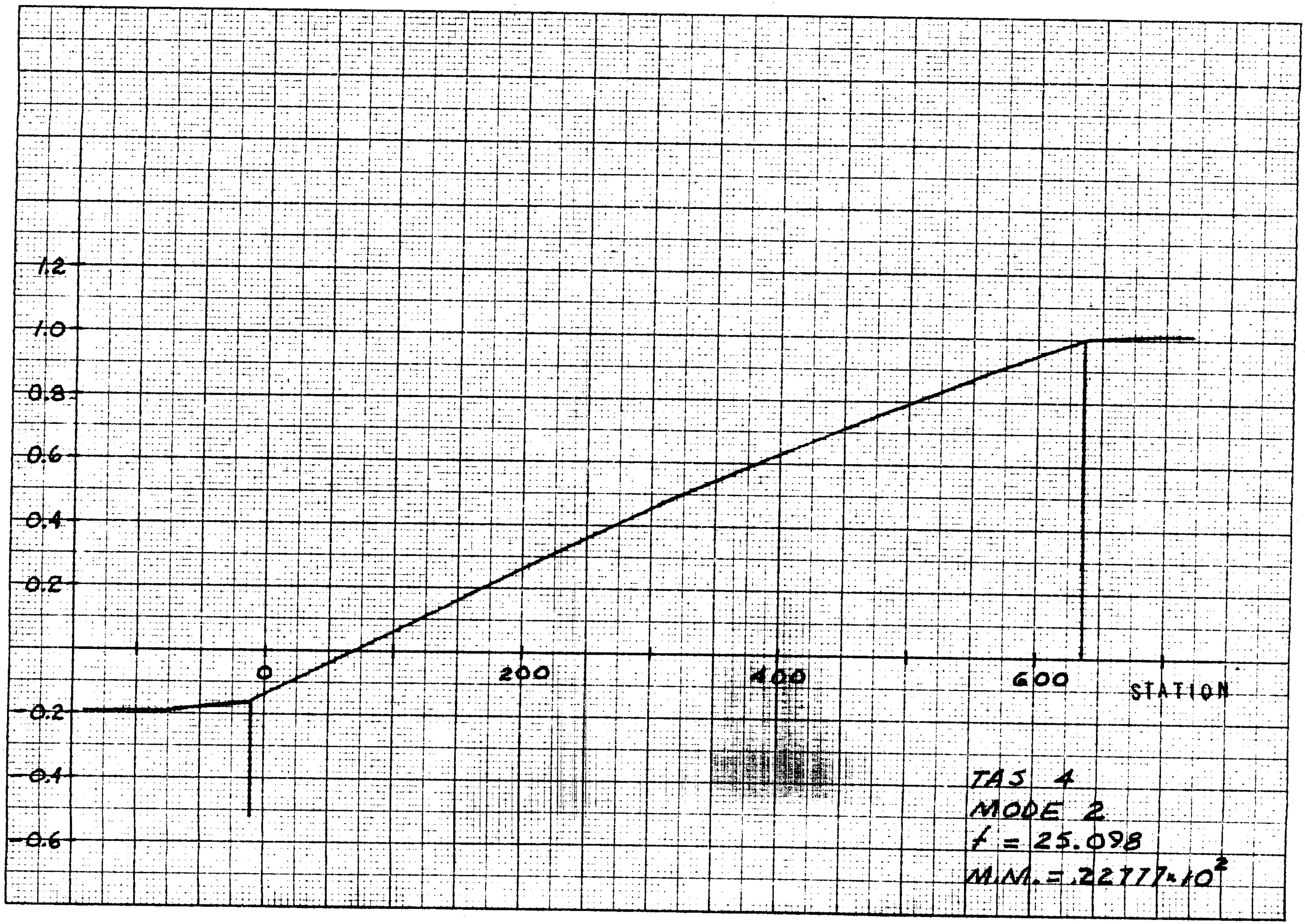

1
-1
-2
0
200
400
600
STATION
TAS 3
MODE 2
f = 16.480
M.M. = .22789 × 10³

SM 46346

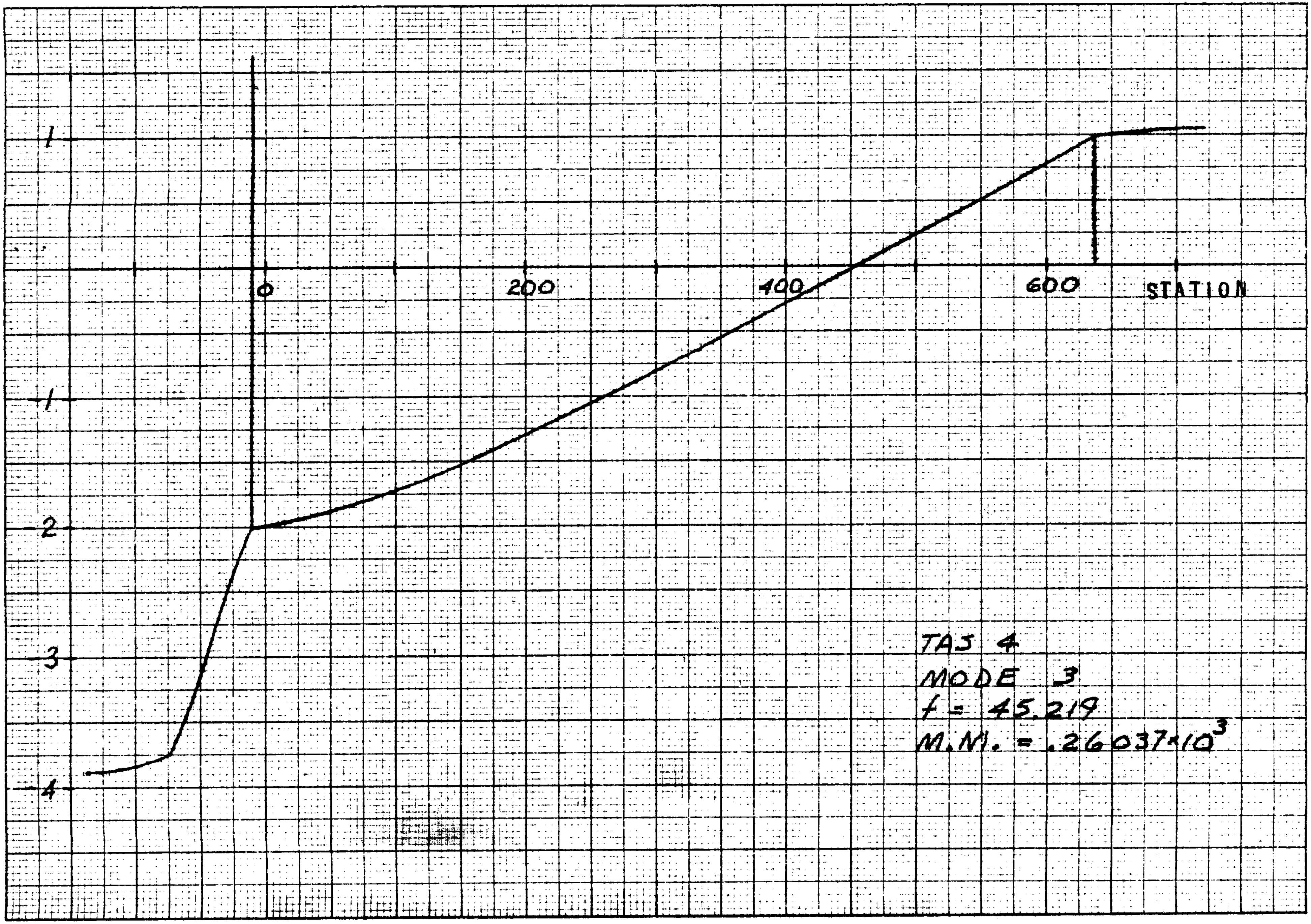

10.899
8.1995
4
2
0
200
400
600
STATION
-2
-4
-6
-8
-10
-12
-14
-27.952
TAS 3
MODE 3
f = 27.944
M.M. = .23541 × 10^5


1.1
1
0
0
200
400
600
STATION
TAS 4
MODE 1
f = 3.9614
M.M. = .69730×10²

SM 46346


1.2
1.0
0.8
0.6
0.4
0.2
-0.2
-0.4
-0.6
0
200
400
600
STATION
TAS 4
MODE 2
f = 25.098
M.M. = .22777×10²

SM 46346



APPENDIX

D. Table of modal masses

THOR MODAL MASSES

Case	Mode 1	Mode 2	Mode 3
T 1	$.16312 \times 10^2$	$.96763 \times 10^2$	$.23313 \times 10^3$
T 2	$.22935 \times 10^2$	$.51263 \times 10^3$	$.13878 \times 10^3$
T 3	$.54822 \times 10^3$	$.10265 \times 10^2$	$.13453 \times 10^5$
T 4	$.41047 \times 10^3$	$.10457 \times 10^2$	$.85255 \times 10^4$
T 5	$.27451 \times 10^3$	$.10849 \times 10^2$	$.16425 \times 10^4$
T 6	$.14069 \times 10^3$	$.12080 \times 10^2$	$.13642 \times 10^3$
T 7	$.20328 \times 10^2$	$.74905 \times 10^3$	$.76676 \times 10^2$
T 8	$.20399 \times 10^4$	$.90809 \times 10^1$	$.18010 \times 10^3$
T 9	$.14824 \times 10^4$	$.91754 \times 10^1$	$.18676 \times 10^3$
T 10	$.94338 \times 10^3$	$.93642 \times 10^1$	$.20095 \times 10^3$
T 11	$.42320 \times 10^3$	$.99314 \times 10^1$	$.24980 \times 10^3$
T 12	$.22181 \times 10^2$	$.39722 \times 10^3$	$.21733 \times 10^6$
T 13	$.74349 \times 10^4$	$.98284 \times 10^1$	$.65865 \times 10^2$
T 14	$.52444 \times 10^4$	$.99196 \times 10^1$	$.67025 \times 10^2$
T 15	$.32030 \times 10^4$	$.10098 \times 10^2$	$.69440 \times 10^2$
T 16	$.12970 \times 10^4$	$.10604 \times 10^2$	$.77283 \times 10^2$
T 17	$.22402 \times 10^2$	$.39033 \times 10^4$	$.21563 \times 10^3$

AGENA MODAL MASSES

Case	Mode 1	Mode 2	Mode 3
A 1	$.24464 \times 10$	$.16512 \times 10^{2}$	$.42036 \times 10$
A 2	$.13359 \times 10^{2}$	$.11759 \times 10^{4}$	$.12160 \times 10$
A 3	$.74353 \times 10$	$.11371 \times 10^{3}$	$.13391 \times 10$
A 4	$.17399 \times 10^{2}$	$.75239 \times 10^{2}$	$.12176 \times 10$
A 5	$.11209 \times 10^{2}$	$.26058 \times 10^{2}$	$.14028 \times 10$
A 6	$.20061 \times 10^{2}$	$.49056 \times 10^{2}$	$.12234 \times 10$
A 7	$.14382 \times 10^{2}$	$.17746 \times 10^{2}$	$.14778 \times 10$

SM 46346

THOR-AGENA MODAL MASSES

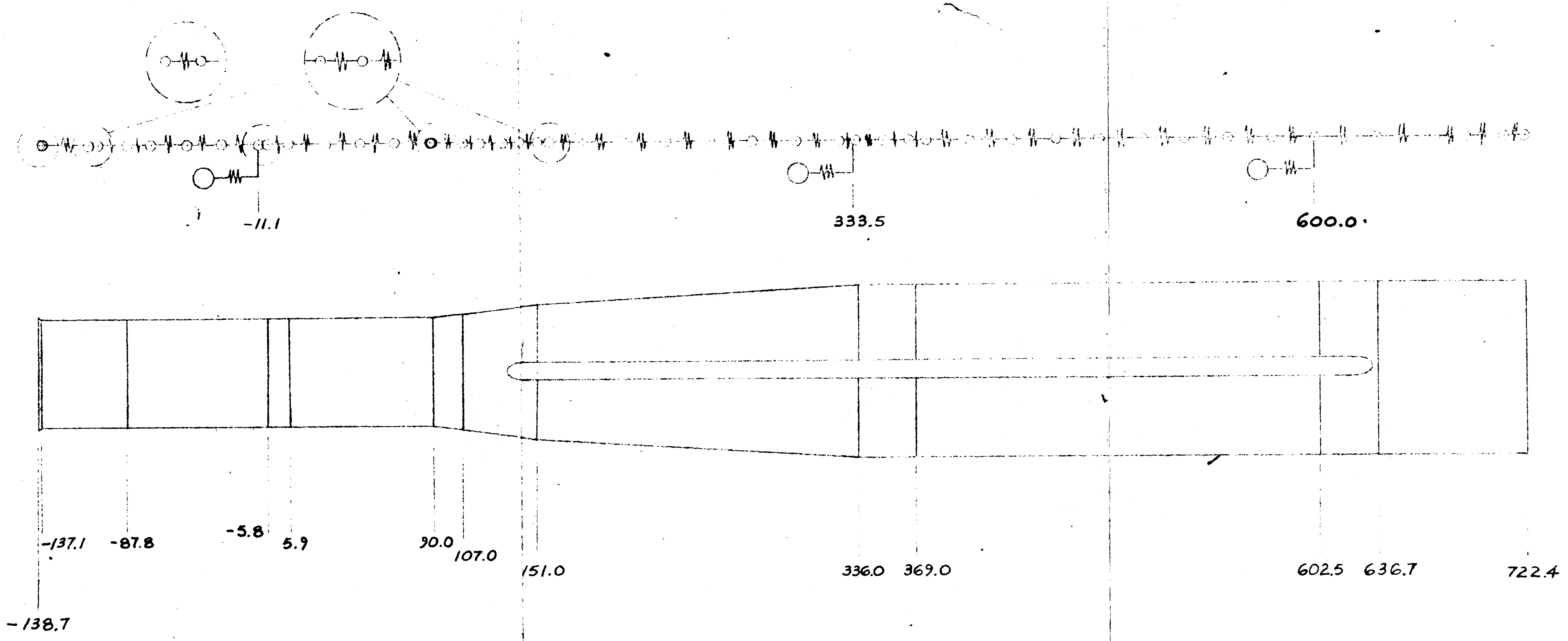
Case	Mode 1	Mode 2	Mode 3
TA 1	$.19202 \times 10^2$	$.76222 \times 10^2$	$.39149 \times 10^3$
TA 2	$.49079 \times 10^3$	$.38007 \times 10^4$	$.43055 \times 10^4$
TA 3	$.39343 \times 10^3$	$.22432 \times 10^4$	$.19476 \times 10^4$
TA 4	$.29463 \times 10^3$	$.12576 \times 10^4$	$.66165 \times 10^3$
TA 5	$.11753 \times 10^3$	$.34212 \times 10^3$	$.24363 \times 10^4$
TA 6	$.48940 \times 10^2$	$.12980 \times 10^3$	$.35566 \times 10^4$
TA 7	$.13377 \times 10^2$	$.19474 \times 10^3$	$.54450 \times 10^2$
TA 8	$.11214 \times 10^4$	$.25484 \times 10^7$	$.74659 \times 10^3$
TA 9	$.60675 \times 10^3$	$.16555 \times 10^6$	$.96936 \times 10^3$
TA 10	$.53252 \times 10^3$	$.29064 \times 10^5$	$.15771 \times 10^4$
TA 11	$.16221 \times 10^3$	$.21870 \times 10^4$	$.86164 \times 10^4$
TA 12	$.57897 \times 10^2$	$.41621 \times 10^3$	$.31505 \times 10^5$
TA 13	$.14525 \times 10^2$	$.48534 \times 10^3$	$.43492 \times 10^2$
TA 14	$.27405 \times 10^4$	$.22220 \times 10^6$	$.30021 \times 10^4$
TA 15	$.17533 \times 10^4$	$.15408 \times 10^6$	$.35113 \times 10^4$
TA 16	$.99556 \times 10^3$	$.26518 \times 10^6$	$.46491 \times 10^4$
TA 17	$.22191 \times 10^3$	$.24315 \times 10^5$	$.49210 \times 10^5$
TA 18	$.68627 \times 10^2$	$.11680 \times 10^4$	$.30197 \times 10^4$
TA 19	$.16313 \times 10^2$	$.10343 \times 10^3$	$.44681 \times 10^2$
TA 20	$.24730 \times 10^5$	$.74558 \times 10^6$	$.35522 \times 10^3$
TA 21	$.10916 \times 10^5$	$.27387 \times 10^6$	$.38906 \times 10^3$
TA 22	$.39783 \times 10^4$	$.36010 \times 10^5$	$.45661 \times 10^3$
TA 23	$.39304 \times 10^3$	$.22593 \times 10^5$	$.14220 \times 10^4$
TA 24	$.93539 \times 10^2$	$.77574 \times 10^4$	$.12242 \times 10^5$
TA 25	$.22870 \times 10^2$	$.36811 \times 10^2$	$.46674 \times 10^2$

THOR-AGENA WITH SHAKER

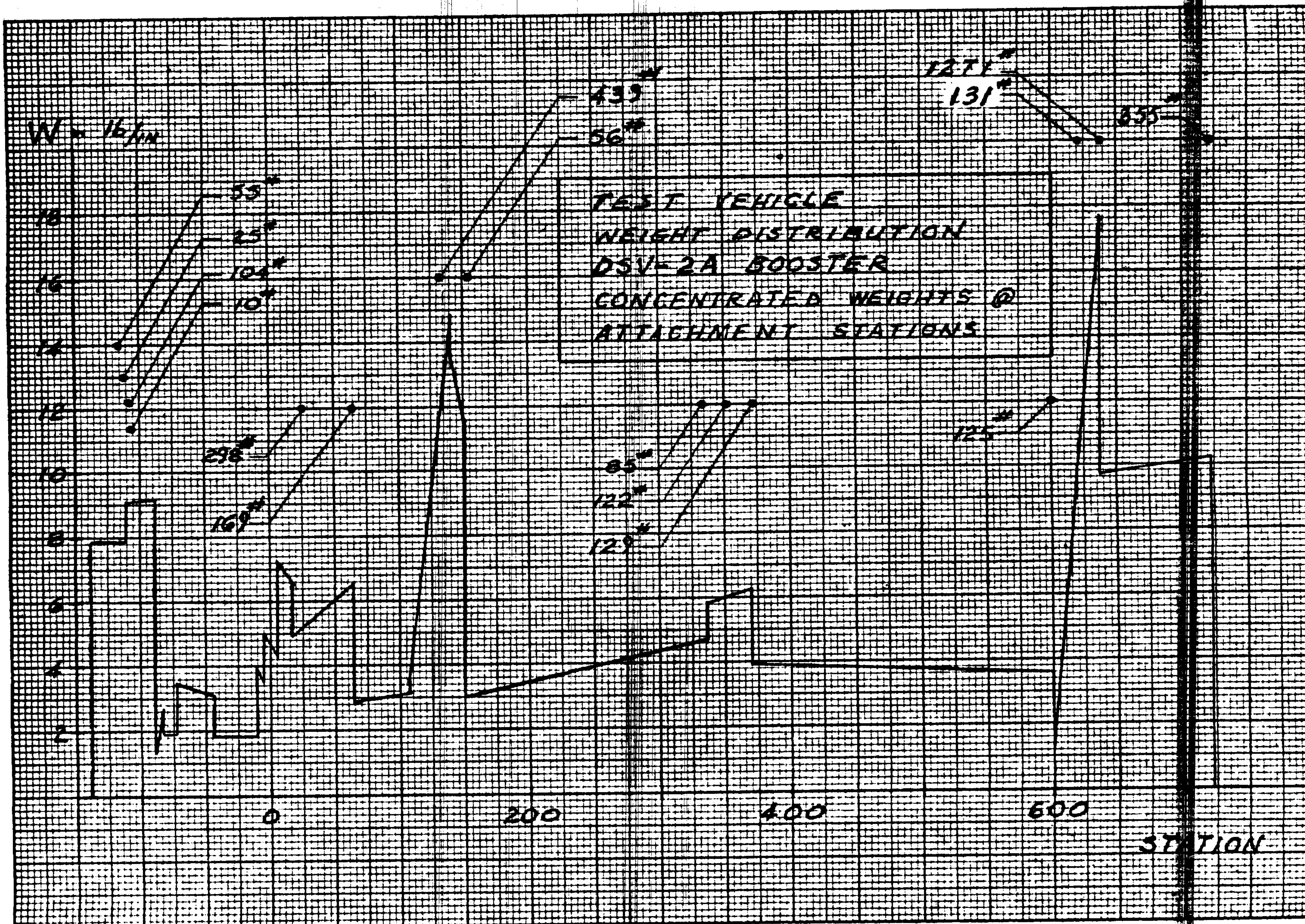
Case	Mode 1	Mode 2	Mode 3
TAS 1	$.25411 \times 10^2$	$.37541 \times 10^2$	$.19703 \times 10^3$
TAS 2	$.36933 \times 10^3$	$.27733 \times 10^4$	$.23097 \times 10^6$
TAS 3	$.13854 \times 10^3$	$.22789 \times 10^3$	$.23541 \times 10^5$
TAS 4	$.69730 \times 10^2$	$.22777 \times 10^2$	$.26037 \times 10^3$

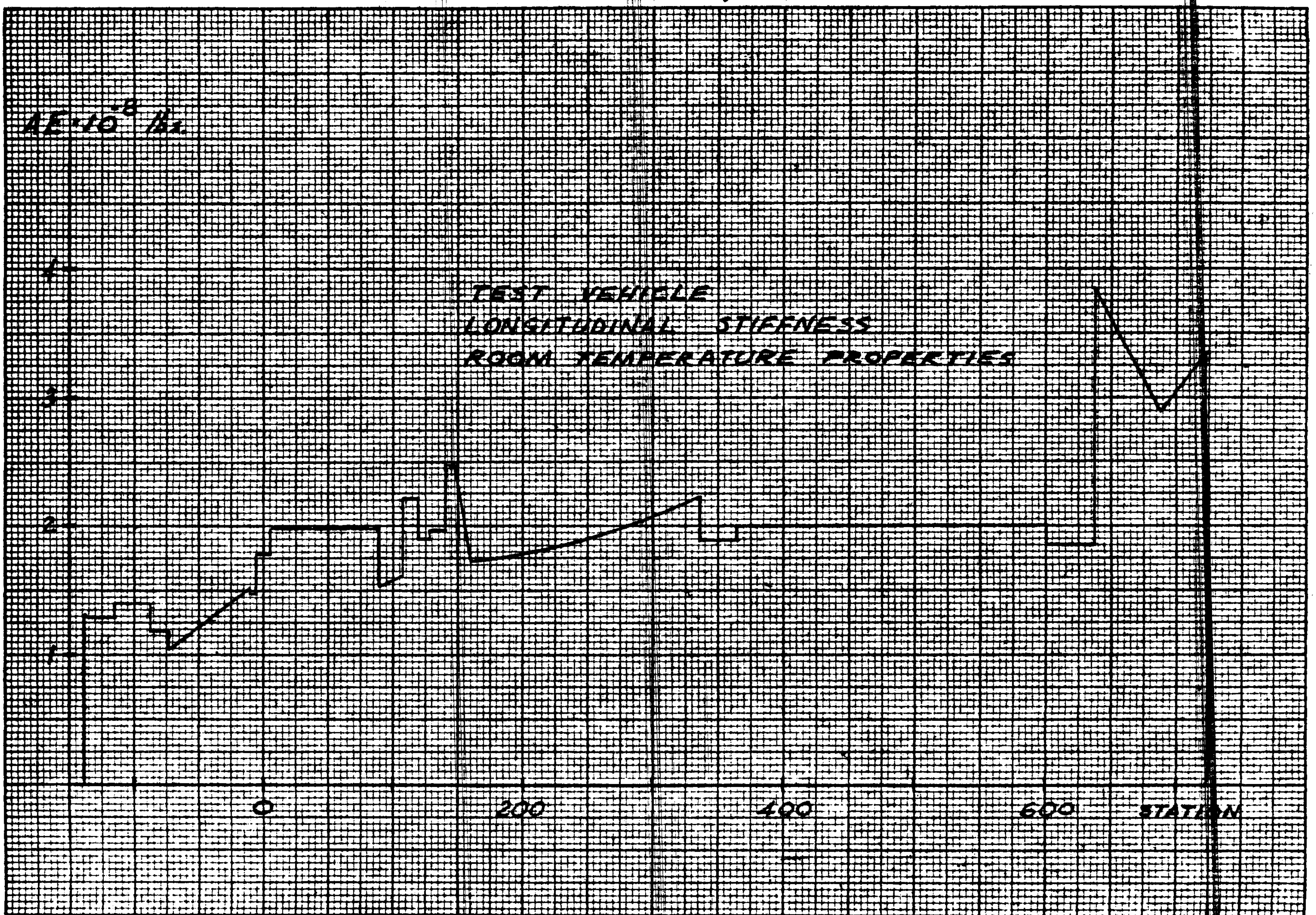
APPENDIX

E. Weight, stiffness and configuration graphs



THOR-AGENA TEST VEHICLE ℄ IDEALIZATION


TEST VEHICLE
WEIGHT DISTRIBUTION
DSV-2A BOOSTER
CONCENTRATED WEIGHTS @
ATTACHMENT STATIONS
W - lb/in
STATION
0
200
400
600
2
4
6
8
10
12
14
16
18
53#
25#
104#
10#
298#
169#
433#
56#
85#
122#
129#
1277#
131#
835#
125#


AE·10⁻⁸ lb/in.
TEST VEHICLE
LONGITUDINAL STIFFNESS
ROOM TEMPERATURE PROPERTIES
1
2
3
4
0
200
400
600
STATION