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BETWEEN THE SOLAR WIND AND THE EXOSPHERE"

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## Reprint Enclosed:

A MODEL FOR A BROAD DISORDERED TRANSITION  
BETWEEN THE SOLAR WIND AND THE MAGNETOSPHERE,  
J. Geophysical Research 69, No. 7, April 1, 1964

I. PROGRESS OF WORK (March 15 through June 14)\*

During this period, F. L. Scarf attended the Solar Wind Conference (J.P.L. April 1-4) and the Spring Meeting of the American Geophysical Union (Washington, April 20-24). Many discussions were held with Explorer 18 experimentalists concerning the latest measurements of the wind-magnetosphere interface. At the Solar Wind Conference the Explorer 18 data for pass No. 6 (outbound) were shown together and Scarf discussed the relation between the observations and the model developed in the present study. His remarks will be published in the conference proceedings and a copy ("On the Role of Ion Acoustic Waves in the Transition Region") appears as Appendix B .

A more comprehensive discussion of the role of the ion wave instability in the transition region and its relation to the production of keV and MeV electrons has been prepared and will be submitted to the Journal of Geophysical Research. A copy of this work ("Electron Acceleration and Plasma Instabilities in the Transition Region") appears in this report as Appendix C . At the time of writing only scattered Explorer 18 data had been released. If more complete results are available in the near future, they will be incorporated in the version to be published.

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\* An extension of this contract is presently being negotiated and the final report for the first year is also a progress report for work now being performed.

During the last few weeks of this work period, numerical investigations were initiated in an attempt to analyze quantitatively the electron acceleration associated with the ion wave-electron cyclotron resonance. The preliminary numerical results strongly support the conjecture that this mechanism can yield the observed spikes of  $E > 30$  keV electrons. Final evaluation is now being carried out, and a comprehensive report on this work is to be prepared soon.

## II. KEY PERSONNEL

During this period F. L. Scarf, R. W. Fredricks, L. M. Noble, S. Altshuler, and A. Peskoff worked on this problem; L. M. Noble terminated on May 15.

## III. REPORTABLE ITEMS

None

## IV. SUMMARY OF WORK (June 14, 1963 to June 14, 1964)

### Publications

1. Plasma Instabilities in the Magnetopause, F. L. Scarf, W. Bernstein, and R. W. Fredricks, Trans. A.G.U. 44, No. 4, December 1963, p. 880. (See Appendix A)
2. A Model for a Broad Disordered Transition between the Solar Wind and the Magnetosphere, W. Bernstein, R. W. Fredricks, and F. L. Scarf, J. Geophys. Res. 69, No. 7, April 1, 1964. (Submitted with Status Report No. 2)\*
3. On the Role of Ion Acoustic Waves, F. L. Scarf, to appear in Proceedings Solar Wind Conference, J.P.L., April 1 - 4, 1964. (See Appendix B)

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\* Previously submitted "Preprints." ~~Twenty reprint copies are enclosed herewith.~~

4. Electron Acceleration and Plasma Instabilities in the Transition Region, F. L. Scarf, W. Bernstein, and R. W. Fredricks, to be submitted to J. Geophys. Res. (Preprint, see Appendix C).
5. Numerical Study of Electron Acceleration Mechanism, R. W. Fredricks, F. L. Scarf, and W. Bernstein, in preparation.

(Although W. Bernstein worked on this problem, his support was derived from another source.)

## Plasma Instabilities in the Magnetopause\*

F. L. SCARF, W. BERNSTEIN and R. W. FREDRICKS

(Space Technology Laboratories, Inc., Redondo Beach, Calif.)

It is known (Piddington, 1960) that charge separation electric fields in the Chapman-Ferraro sheath generate currents which are large enough to trigger the two-stream plasma instability. We argue that as the electrostatic and drift energy are lowered and the electron temperature rises, the instability is generally not quenched. Instead, it can change form so that growing ion waves appear (Fried and Gould, 1961). The equilibrium state then involves fluctuating electromagnetic fields which allow "fast" plasma diffusion across the main magnetic field (Spitzer, 1960). In this case, the exospheric thermal plasma, together with that part of the solar wind plasma which has attained energy equipartition, can form current systems leading to a broad, disordered, self-consistent transition region between the magnetosphere and the solar wind.

---

\* Trans. A.G.U. 44, No. 4, December 1963, p. 880.

ON THE ROLE OF ION ACOUSTIC WAVES IN THE TRANSITION REGION

F. L. Scarf\*

(to appear in Proc. Solar Wind Conf., Held at J.P.L. on April 1-4, 1964)

The comparison of plasma probe and magnetometer data for the sixth outbound pass of Explorer 18 shows that the transition region can be remarkably broad and complex. I would like to return to Dr. Axford's physical explanation of the meaning of supersonic flow, and comment briefly on the possible origin of these complications.

In order to justify the use of continuum flow for a collisionless plasma-field interaction, one examines the waves which can be produced in the transition region, and if  $u_0$  (the wind speed) is greater than any reasonable wave speed, then the flow is "supersonic"; since  $u_0$  is considerably greater than the local Alfvén speed it has become customary to expect the highly super-Alfvénic flow to be associated with a distinct shock front. However, as Dr. Axford pointed out other kinds of waves can be generated in the interface, and one type which has not been discussed here is a longitudinal ion acoustic wave; the speed is  $\sqrt{\gamma kT_e/m_p}$  where  $kT_e$  is electron thermal energy,  $m_p$  is the ion mass and  $\gamma \geq 1$  depends on the shape of the electron distribution. For  $\gamma = 1$ ,  $T_e = T_i$  the ion wave speed is somewhat less than the Alfvén speed and the incident flow is supersonic in terms of sound waves as well as in terms of Alfvén waves. Actually, the ion waves are heavily damped for  $T_e = T_i$  so that they need not be considered in this case.

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\* These comments were made following the presentation of Explorer 18 data by E. F. Lyon and N. F. Ness.



However, if nonlinear effects (such as those associated with charge separation electric fields) locally increase  $T_e/T_i$  then the damping goes down and only a very small current or electron-proton drift speed is needed to generate instability.<sup>1</sup> In the outer transition region the MIT plasma probe shows that the electrons do, in fact, undergo greater thermalization than the ions. The maximum value of  $kT_e$  possible from equipartition is  $m_p u_o^2/4$ , and this must reduce the local ion drift speed to  $u^2 \approx u_o^2/2$ . When this happens the ion wave velocity increases to  $\sqrt{\gamma} u_o/2$  and thus the wave speed approaches or exceeds the local ion speed,  $u_o/\sqrt{2}$ .

I want to suggest that the ion wave instability is relevant in explaining the broadening and variability of the transition region, the appearance of upstream precursors, and superthermal electron peaks, and the occasional disappearance of a distinct outer boundary. The ion wave frequencies overlap the local electron gyrofrequency ( $\nu_{\max} = \sqrt{\gamma kT_e/m_p} / \lambda_D$ , where  $\lambda_D$  is the Debye length) and the wave-particle interaction can distort the electron velocity distribution at low energies (0.5 - 3 keV), and produce a small non-Maxwellian tail ( $E \approx 30$  keV). [ In some experiments at Oak Ridge,<sup>2</sup> 100 keV electrons were easily produced by a related beam plasma instability; Stix<sup>3</sup> showed that in this case the electron plasma waves interact via the electron cyclotron resonance. ] This distortion produces high energy peaks, particularly in regions where the local magnetic disorder is

small, and it tends to raise  $\chi$  and broaden the ion wave source region by the fast diffusion process. The incident wind becomes less supersonic and small fluctuations allow isolated precursors to travel upstream and dump ion wave energy in isolated electron spikes. When the distortion of the velocity distribution is high enough, the incident wind becomes subsonic with respect to ion acoustic waves and the entire sheath relaxes, until the next solar wind enhancement initiates a new transient.

#### References

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Electron Acceleration and Plasma Instabilities  
in the Transition Region  
(June 1964)

F. L. Scarf, W. Bernstein, and R. W. Fredricks  
TRW Space Technology Laboratories  
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1. Introduction

In a recent article (Bernstein, et al., 1964; henceforth referred to as BFS) a theory for a broad disordered transition between the solar wind and the magnetosphere was constructed. The basic feature of the model is the assertion that an electron-proton plasma does not behave as a collection of equal mass particles with  $m \approx \frac{1}{2}(m_e + m_p)$  when it flows into an inhomogeneous magnetic field. In particular it was assumed that in some region the incident ions remain relatively cool while local charge separation electric fields induced by magnetic fields transfer a portion of the ion streaming energy to the electrons. Ultimately, this energy becomes randomized and  $kT_e(t) \rightarrow c(t)m_p u_o^2$  where  $u_o$  is the solar wind speed and  $c(t_o) < c(t) \leq \frac{1}{4}$ , with the upper bound determined by a form of energy equipartition. The ion acoustic wave speed,  $V_i = (\gamma kT_e/m_p)^{1/2}$  then tends toward  $\sqrt{c} u_o$  and it becomes comparable to the wind speed. It was shown in BFS that under these conditions (1) the magnetopause is generally unstable with respect to generation of ion waves; (2) the high frequency electric field fluctuations associated with the waves allow fast diffusion of plasma across the magnetic field. Since exospheric plasma also diffuses out, local currents produce local magnetic fields, and a Chapman-Ferraro (CF) magnetopause sheath relaxes into a broad disordered transition region which contains higher electron fluxes than the solar wind; (3) the ion waves also interact with electrons via the cyclotron resonance ("betatron" mechanism) and drastically distort the electron velocity distribution.

In this note we wish to extend the original theory by starting from a stage in the development of a hypothetical collisionless shock, rather than a C-F sheath. An interplanetary field ( $B_I$ ) is always present and the transverse component is generally reasonably strong and steady. This permits information to be propagated upstream from the magnetopause with the Alfvén speed,  $V_A = (B_I^2/4\pi Nm)^{1/2}$ , and the Newtonian C-F theory gives way to a continuum description. However for  $B_I \approx 5$  gamma,  $N \approx 2-3$  protons/cm<sup>3</sup>, and  $u_0 \approx 600$  km/sec, one finds  $u_0 \approx (8-9)V_A$  and the incident flow is "super-Alfvénic". Thus for periods when the solar wind and interplanetary fields are quiet, it has become customary to modify the C-F sheath in two ways: continuum equations are used and a shock discontinuity [based on the deHoffman-Teller (1950) jump relations for equal mass particles,  $m = \frac{1}{2}(m_e + m_p)$ ] with quiet conditions upstream is postulated.

We argue here that before this idealized hydromagnetic shock can be set up, the instabilities discussed in BFS become important. At an early stage in the shock formation, a region with  $T_e/T_i \gg 1$  develops and becomes unstable with respect to ion acoustic wave generation. The nonlinear wave-particle interaction of (3) then becomes extremely important. Although the mean electron energy is still limited by  $kT_e \leq \frac{1}{4}m_p u_0^2$ , the betatron mechanism broadens and distorts the distribution at low energies and a significant very high energy tail is also produced. The ion wave speed becomes  $V_i = (\gamma kT_e/m_p)^{1/2}$  where  $\gamma \geq 1$  depends on the shape of the distribution for nearly thermal electrons. Thus  $V_i$  increases and since the ions lose speed in the thermalization region,  $c = [V_i(\gamma, u)/u]^2$  should exceed the bound suggested by equipartition with a Maxwell distribution. (Note that  $c$  is now redefined in terms of a local velocity ratio.) Generally  $c(r, t) < 1$ ,  $V_i < u_0$  and the solar wind is still supersonic with respect to ion wave speeds.

During this period the ion waves primarily propagate into the magnetosphere, and solar plasma diffuses inside; fluctuations in the wind parameters also occasionally produce temporary subsonic conditions and isolated precursors then travel upstream.

At a later stage of the breakup,  $c$  exceeds unity and  $V_1$  exceeds  $u$ , so that the incident wind becomes subsonic with respect to ion acoustic waves and the concept of a distinct shock front then becomes meaningless. Ion waves are copiously produced in the hot electron region, they are driven upstream and into the outer magnetosphere, and the situation becomes similar to that discussed in BFS; the presence of a steady interplanetary field is then unimportant except for the following effects.

The nonlinear wave-particle betatron interaction which "heats" the electrons past  $c = \frac{1}{4}$  must actually be of limited effectiveness wherever the local magnetic field is highly disordered, and the experiments show (Sect. 4) that  $|\Delta B/B| \approx 1$  can frequently be found in the region where the ions and electrons undergo the original thermalization. It is likely that these large amplitude magnetic field fluctuations are associated with low frequency hydromagnetic waves or quasi-static current distributions, and not with the high frequency ion acoustic waves which should produce small amplitude magnetic perturbations. Nevertheless, in this disordered transition region, the electron heating must be controlled and limited by the coherence properties of the local fields. The ion waves which are driven out of this region begin to dissipate as they encounter lower electron temperatures, but the magnetic fields also become quieter within the magnetosphere and in interplanetary space, so that the betatron heating is less limited. When  $B$  becomes sufficiently steady, it is possible that all of the ion wave energy can be given

to the electrons, and thus a high energy electron spike can separate the unstable and stable regions. A distinct outer boundary may be apparent, but it is not a collisionless shock front, since at this point the wind is sonic or subsonic and the boundary is (1) upstream from the region with  $c \geq \frac{1}{4}$ ; (2) unaccompanied by a discontinuity in  $B$  large enough (10-15 gammas) to be associated with a shock front; (3) related to local spikes of very high energy electrons. Since ion acoustic waves do not propagate freely across a magnetic field, it is expected that the relaxation mechanism outlined here proceeds upstream most rapidly in the direction of the interplanetary field, or approximately  $45^\circ$  west of the earth-sun line.

The ion waves also enter the magnetosphere freely in regions where they need not propagate across the magnetic field at wide angles. This suggests that the magnetopause may be relatively well defined near the sub-solar point but that ion waves can enter the outer magnetosphere more readily away from the noon-meridian and the equator. In these outer regions we expect considerable magnetopause broadening, high frequency ( $0 < \omega < 2\pi \sqrt{6kT_e/m_p} / \lambda_0 \approx 10^3 - 10^4$  rad/sec) electromagnetic fluctuations, appearance of solar plasma which has diffused into the magnetosphere and significant production of high energy (keV and MeV) electrons via the ion wave-particle interaction. The outer magnetosphere may be so disordered by these effects that it is difficult to distinguish trapped or injected particles and scattered "free" particles, since they differ only in trapping lifetime. We conjecture that the outer magnetosphere contains particles which are well trapped with respect to times on the order of ion wave periods (milliseconds) but that this

population responds rapidly to changes in the wind parameters, and to the state of relaxation of the transition region. If these conditions are met, the ion waves can supply the bulk of the energy needed to accelerate the outer magnetosphere electrons, and the population can be severely depleted by a sudden enhancement in the wind.

## 2. Temporal Variations

The Mariner plasma probe (Snyder, Neugebauer and Rao, 1963) and magnetometer (Coleman, Davis, Smith and Sonett, 1962) data show that interplanetary conditions are rarely constant for periods longer than several hours, even during non-storm days. Moreover, the gross changes in wind velocity do not appear to be correlated with immediate surface geomagnetic activity; instead the geomagnetic "noise" level appears to be simply related to the magnitude of the wind velocity. These facts can support a theory that the interface is always unstable, and that the gross fluctuations merely change the nature and state of the instability by delivering varying amounts of energy to the interface, and by initiating a new decaying transient.

With these comments in mind, it is useful to describe the wind-magnetosphere interaction as a series of transients and to contemplate the development of the interface after a sudden enhancement in solar wind flux has compressed the magnetosphere and swept away the flux of quasi-trapped particles. At  $t = 0$ , the subsolar field and flux profiles should resemble those predicted by the C-F theory (Fig. 1a). Collisions are unimportant and very soon after the enhancement occurs, waves produced near the interface cannot significantly distort the Newtonian flow by interacting with particles which have not yet arrived.

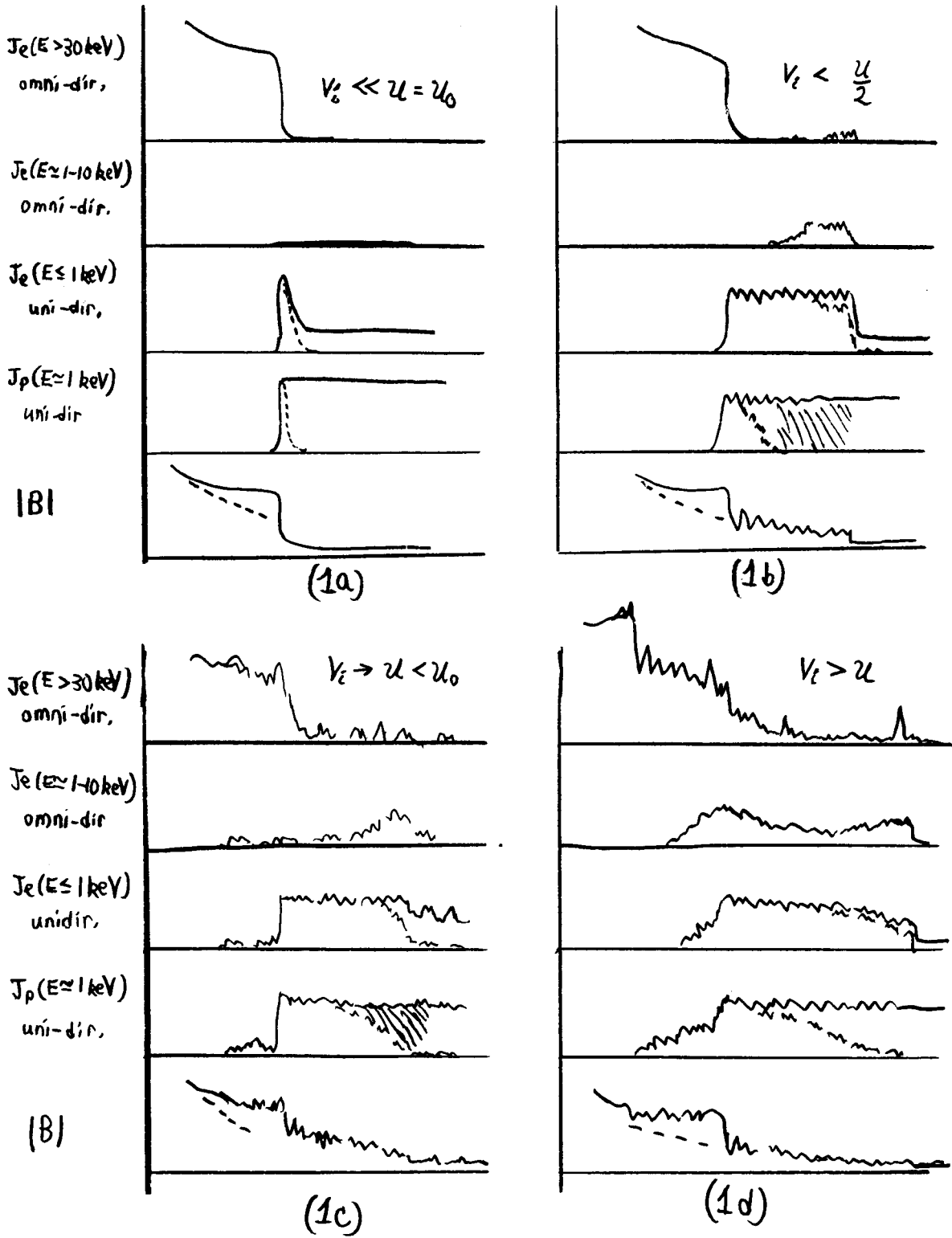


Fig. 1



At later times one must recognize at least two limiting possibilities. If the interplanetary conditions remain reasonably constant after  $t = 0$ , then Alfvén waves generated in the sheath of Fig. (1a) can begin to travel upstream, scatter and heat the incoming particles, and ultimately produce a momentary quasi-shock front such as that shown in Fig. (1b); the term "quasi-shock" describes the case in which the electrons are heated and thermalized throughout a specific transition region while the ion thermalization is primarily completed near the inner boundary. It takes some time for the interface to develop from (1a) to (1b), and the growth time must certainly exceed  $t_1 = |r_2 - r_1| / V_A$ , the time needed to propagate an Alfvén wave from a magnetopause ( $r_1$ ) to a shock front ( $r_2$ ) [for  $|r_2 - r_1| \approx 4 R_e$ ,  $V_A \approx 70$  km/sec,  $t_1 \approx 6$  minutes]. However, the true growth time should be considerably longer than  $t_1$ , since the electrons are to be completely thermalized, and appropriate currents must be set up. Since the magnetic field is also disordered beyond the magnetopause (see Sect. 4), it is likely that the effective Alfvén speed is considerably smaller than  $\sqrt{B^2/4\pi Nm}$ , where  $B$  is the average field in the transition region.

We anticipate that the configuration shown in Fig. (1b) develops in a time on the order of hours after a sudden enhancement and that at this stage the electrons are thermalized and heated as soon as they penetrate into a specific transition region but that the ions merely lose drift energy near the outer boundary. Nearer to  $r_1$  ion heating does occur, and here both  $kT_e$  and  $kT_i$  initially tend toward  $\frac{1}{4} m_p u_o^2$ . However, in the outer cross-hatched region  $T_e/T_i \gg 1$ , and thus almost any small current can trigger the ion wave instability.

If a significant steady transverse magnetic field component is locally present, charge separation electric fields are strong enough to induce the threshold current (see BFS) and stimulate the waves.

In Fig. (1c) the beginning of the quasi-shock breakup is depicted. Several complicating effects are included here: (1) ion waves interact with electrons via the betatron mechanism and raise  $c(t)$  toward unity so that the cross-hatched source region becomes bigger; (2) the high frequency field fluctuations are driven past the conventional magnetopause and they accelerate local bunches of electrons to keV and MeV energies when the resonance conditions are met; as the field becomes more regular with decreasing range, the wave energy is absorbed more efficiently and the instability quenching is marked by a buildup of high energy electrons; (3) the fluctuating electric fields allow "fast" diffusion of plasma across the magnetopause so that some solar directed plasma is found in the outer magnetosphere and whistler plasma populates the transition region; (4) As  $c \rightarrow 1$ , the ion wave speed tends toward  $u_0$ , and small fluctuations in  $u_0$  then allow isolated ion waves to propagate upstream, producing local bursts of magnetic disordering and superthermal electrons.

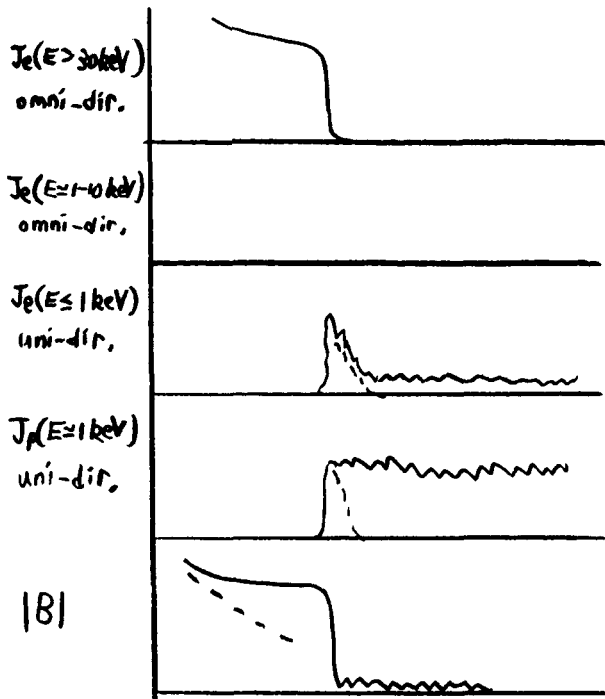
The completely subsonic interaction is shown in Fig. (1d). For  $c > 1$  the ion waves are copiously produced and they travel far upstream. The wave energy can be dumped into the electrons in some small region yielding a spike of superthermal electrons with quiet conditions upstream. In this case the outer magnetosphere could be highly disordered, with a gradual transition to the interface region marked by a gradual softening with range of the quasi-trapped particle distributions; the inner limit of this region may be associated with a spike of superthermal particles, or moderate heating of the more dense whistler plasma.

The development in the second limiting case which corresponds to noisy interplanetary conditions is shown in Fig. 2. Here the C-F sheath is again set up by an enhancement at  $t = 0$ , but the incoming wind and field are assumed to be so disordered that (low frequency) Alfvén waves cannot propagate over large distances. This is the case discussed in (BFS) and the C-F sheath relaxes as shown in (2b) and (2c).

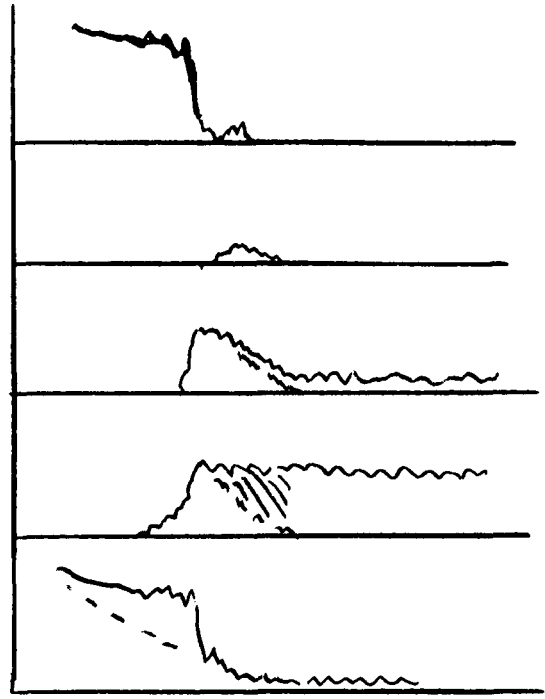
The temporal variability of the wind-magnetosphere interface is an essential point of our argument. However, although many different configurations are predicted, these cases have some fundamental features in common. In particular, it is expected that with the exclusion of a short time period following a wind enhancement, a region with  $T_e/T_i \gg 1$  exists because the massive protons cannot respond to perturbations as rapidly as the electrons. If this region also contains a finite magnetic field perpendicular to the ion streaming speed and steady for a time long compared to ion wave periods ( $\approx 10^{-3}$  secs), then currents produced by the charge separation electric fields are large enough to generate the ion wave instability and associated electric field fluctuations induce superthermal heating of some electrons.

### 3. Electron Acceleration Mechanisms

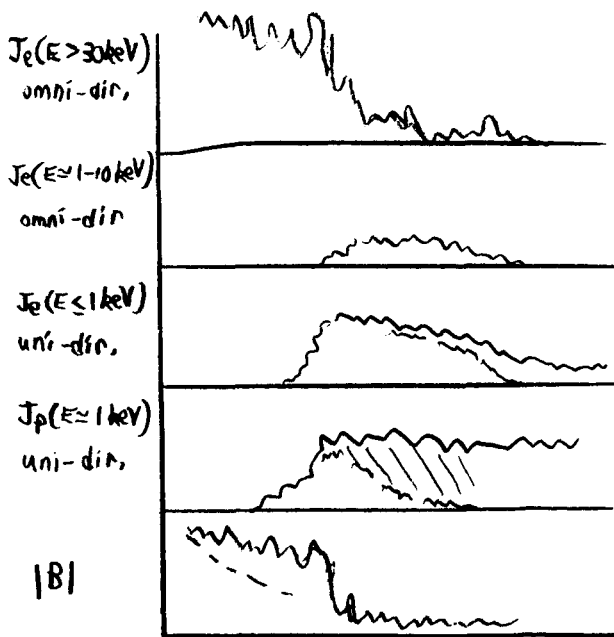
The charge separation electric fields can raise the electron temperature only to  $kT_e \approx m_p u_o^2 / 4$ , and this is generally sufficiently high to trigger the current or ion wave instability. In this section we wish to consider in more detail the subsequent superthermal acceleration of a small fraction of the electron population by the ion acoustic waves.



(2a)



(2b)



(2c)

|||||  $T_e/T_i \gg 1$

Fig. 2

The longitudinal waves may be described by a potential function

$$\phi = \phi_0 \cos(\mathbf{k} \cdot \mathbf{r} - \omega t) \quad (1)$$

where  $\omega = \sqrt{kT_e/m_p} k$  and  $0 < k < k_D$ , with  $k_D$  the Debye wave number. In the presence of an external magnetic field,  $\mathbf{B}_0$ , the equation of motion for an electron is

$$m_e \frac{d\mathbf{v}}{dt} = + e \mathbf{k} \phi_0 \sin(\mathbf{k} \cdot \mathbf{r} - \omega t) - e \mathbf{v} \times \frac{\mathbf{B}_0}{c} \quad (2)$$

For simplicity, let us first examine an idealized case with  $\mathbf{B}_0 = B_0 \mathbf{i}_z$ ,  $\mathbf{k} = k \mathbf{i}_x$ . Equation (2) yields

$$\ddot{y} = \dot{x} \omega_c \quad (3)$$

$$\ddot{x} = \frac{ek\phi_0}{m_e} \sin(kx - \omega t) - \dot{y} \omega_c \quad (4)$$

with  $\omega_c = eB_0/m_e c$ . Equation (3) gives  $\dot{y} - x \omega_c = \text{constant}$ , and for those electrons for which the constant vanishes,

$$\ddot{x} + \omega_c^2 x = \frac{ek\phi_0}{m_e} [\sin kx \cos \omega t - \cos kx \sin \omega t] \quad (5)$$

For small excursions ( $kx \ll \pi/2$ ), Eq. (5) becomes

$$\ddot{x} + \omega_c^2 x - \frac{ek^2\phi_0}{m_e} x \cos \omega t \approx \frac{-ek\phi_0}{m_e} \sin \omega t \quad (6)$$

and the right hand side may be regarded as the driving term; in this case, it is of interest to examine the solutions to the homogeneous equation,

$$\ddot{x} + \left[ \omega_c^2 - \frac{ek^2\phi_0}{m_e} \cos \omega t \right] x = 0 \quad (7)$$

Equation (7) is the Mathieu equation, and it is known that for certain ranges of the parameters, unstable growing solutions are found. We define  $2\tau = \omega t$  and rewrite Eq. (7) in the standard form

$$\frac{d^2 x}{d\tau^2} + (b - h^2 \cos^2 \tau) x = 0$$

with  $b = 4 \omega_c^2 / \omega^2$ ,  $h^2/2 = 4 e k^2 \Phi_0 / m_e \omega^2$ ; the unstable regions in the  $b$ - $h$  plane are given for instance on page 563 of Morse and Feshbach, vol. 1. For  $h \ll 1$  (very small ion wave electric fields), the unstable regions are quite narrow and they are centered around  $b = n^2$ ,  $n = 1, 2, \dots$  so that growth occurs if  $\omega = 2 \omega_c / n$ ; since the inhomogeneous driving term also oscillates with frequency  $\omega$ , one can expect extremely rapid acceleration when the resonance conditions are met.

Of course, this exercise is extremely crude and simplified, and it is merely presented to illustrate the basic "betatron" or wave-particle interaction of interest. Aside from the unrealistic limitation  $kx \ll \pi/2$ , we have ignored the disorder in  $B_0$ , and the fact that ion waves propagate most freely in directions near  $\underline{B}_0/B_0$ . More realistic calculations are now being carried out on an analog computer; Eq. (2) is solved for cases with  $\underline{k} \cdot \underline{B}_0 \neq 0$ , and a long coherence time for  $\underline{B}_0$  is introduced. Preliminary analysis indicates that moderate ion wave fields which have  $\frac{1}{2} \omega$  near a suitable doppler shifted  $\omega_c$  are quite effective in producing keV and even MeV particles in short times even when  $\underline{k} \cdot \underline{B}_0 \neq 0$ , although small  $k_{\parallel}/k_{\perp}$  still produces the greatest acceleration. When  $k_{\parallel} \neq 0$ , the mean particle drift velocity along the

field must be related to the speed with which the ion wave propagates along  $\underline{B}_0$ , and this requirement limits the energy transfer. Near the subsolar point  $k_{\parallel}$  is indeed small, but the ion waves are highly reflected in a fairly narrow "magnetopause" so that small fluxes of high energy electrons should be produced. Away from this region  $k_{\parallel}/k_{\perp}$  is not so small and one might therefore expect production of softer electrons. Actually, the ion wave spectrum is quite broad and in terms of an effective monochromatic wave source it may be appropriate to discuss an effective value for  $k_{\parallel}/k_{\perp}$  which is quite small (the electron resonance can involve  $\omega = \omega(t)$ ) so that superthermal electrons can be generated in the entire outer magnetosphere. [The phenomena discussed here were suggested by a description of an electron cyclotron-electron plasma wave interaction recently considered by Stix (1964) in an attempt to account for the production of 100 keV electrons in laboratory beam-plasma experiments (Alexoff, et al., 1963)].

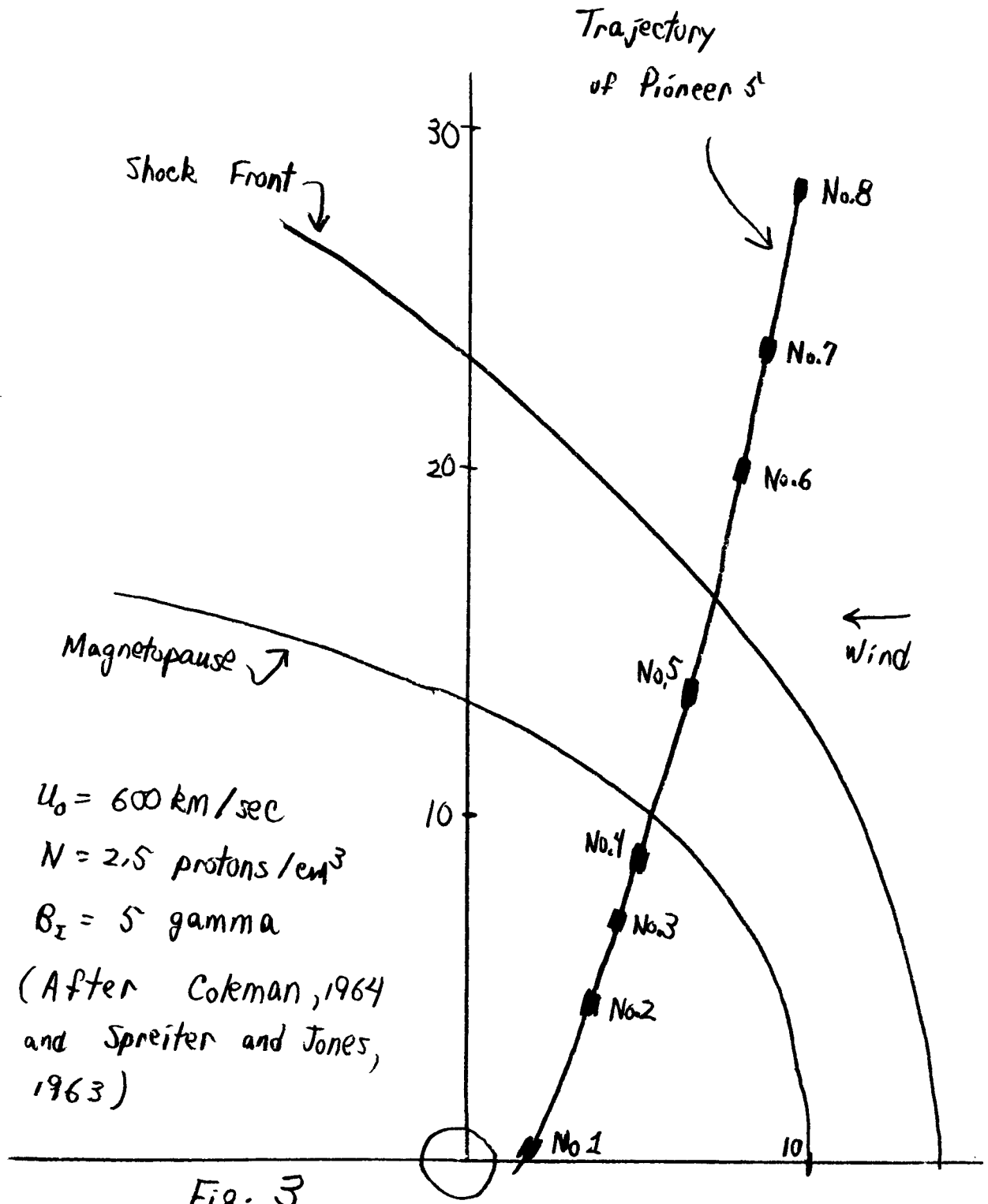
It is again assumed here that the large amplitude  $\underline{B}_0$  fluctuations of Figs. (1), (2) have coherence times long compared with ion wave periods (milliseconds) so that reasonably small electric fields have enough time to produce high energy electrons. If the  $|\Delta \underline{B}_0/\underline{B}_0| \approx 1$  fluctuation periods are also on the order of milliseconds, then little superthermal acceleration would be expected, but the  $\underline{B}_0$  fluctuations with kilocycle frequencies produced by the ion waves should generally have very small amplitudes.

#### 4. Comparison with Experiment

The most complete measurements of the outer magnetosphere and the transition region were made using the sensitive, high resolution detectors on Explorer 18, but only scattered and preliminary discussions of the results are presently available. However, important characteristics of this region were determined on earlier flights, and it is now possible to make some comparison between our theory and experiment for the sunlit hemisphere.

The earliest direct indication of an extremely broad, disordered and complex transition region is derived from the Pioneer 5 magnetometer data (Coleman, 1964). In Fig. (3), the trajectory and the locations of the first eight periods of data transmission are shown and compared with theoretical magnetopause and shock front boundaries derived from work of Spreiter and Jones (1963). The experimental values for  $B_{\perp}$  are shown in Fig. (4) for six transmissions of interest and the instantaneous values are compared with the extrapolated dipolar values. It is possible to make a fairly unambiguous identification of the  $8.5 R_e$  results with a geomagnetic field which is slightly compressed, and at  $30 R_e$  the moderately quiet field has a value which agrees with the interplanetary value of Mariner 2 (Coleman, et al., 1962). However, the entire region between at least  $10.5$  and  $25.4 R_e$  appears to be a highly disordered transition region. In particular, transmission No. 4, which is supposed to occur within the hypothetical magnetopause, exhibits rapid large field fluctuations, and very similar effects are seen through transmissions No. 6 and No. 7, which are supposed to be beyond the shock front. There is no real evidence for a discontinuity in  $B_{\perp}$  between transmissions No. 5 and No. 6 (the average field is however somewhat closer to





$U_0 = 600 \text{ km/sec}$   
 $N = 2.5 \text{ protons/cm}^3$   
 $B_z = 5 \text{ gamma}$   
 (After Coleman, 1964  
 and Spreiter and Jones,  
 1963)

Portions of Pioneer 5 Transmissions (After Coleman, 1963)

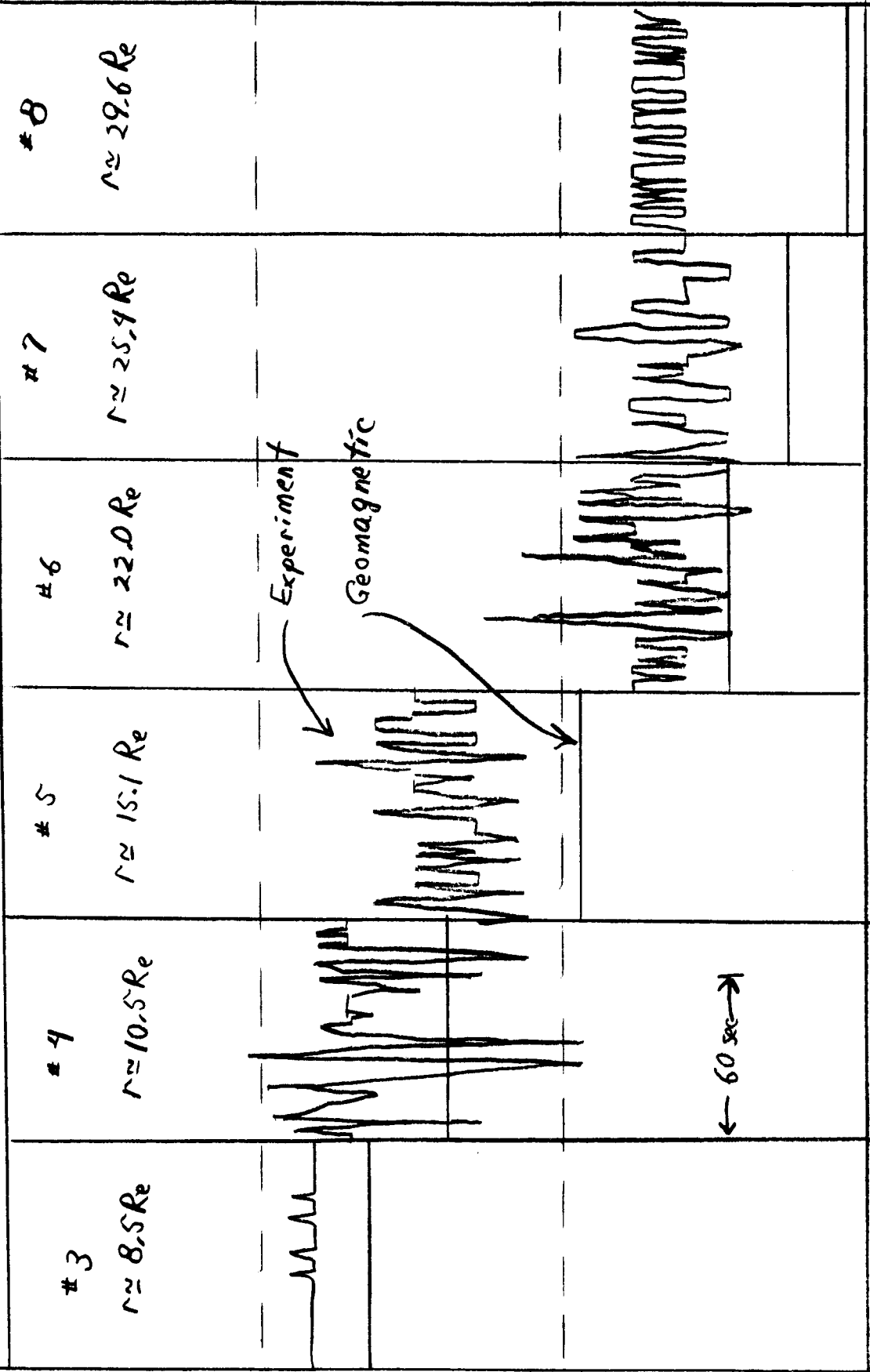


Fig. 4

geomagnetic for No. 6) and there is no evidence for quiet conditions upstream of a shock at about  $18 R_e$ .

The most notable result of the Explorer 12 program was the CdSTE detector measurement (Freeman, Van Allen and Cahill, 1963; Freeman, 1964) of an extremely broad and variable region ( $r_1 \approx 5.2$  to  $6.4 \times 10^4$  km,  $r_2 \approx 6.5$  to  $> 8.4 \times 10^4$  km,  $0 < L_{\text{sep}} < 70^\circ$ ) populated by large fluxes ( $J \approx 10^8 - 10^{10}/\text{cm}^2\text{sec}$ ) of electrons with  $E \approx 0.5$  to 2-3 keV. The inner boundary of this thermalization region appeared to be associated with termination of magnetospheric electron flux ( $40 \text{ keV} < E < \text{MeV}$ ) and with the onset of large amplitude magnetic fluctuations. However,  $r_1 = r_1(\theta, \phi, u_o, t)$  was not always well defined (particularly away from  $L_{\text{sep}} = 0^\circ$ ) and  $r_2$  was generally (i.e., on 25 out of 38 inbound passes) beyond the apogee of Explorer 12 (83,600 km).

In this belt, the electrons undoubtedly acquire the maximum thermal energy allowed by equipartition ( $kT_e = \frac{1}{4} m_p u_o^2$ ) so that one must discuss an enormously broadened ( $\delta \approx 10,000$  km to  $> 25,000$  km) wind-magnetosphere interface. However, not enough information is available from the Explorer 12 detectors to confirm or deny the predictions of Figs. (1), (2). The plasma probe which could yield  $T_e/T_i$  was apparently not operative, the magnetometer resolution was not sufficient to detect a specific shock transition at  $r_2$  ( $V_i < u$ ) or upstream precursors ( $V_i \geq u$ ), and the 40-50 keV detector was not sufficiently sensitive to measure the presence of a superthermal high energy tail in the transition region.

Nevertheless, other evidence does suggest that more sensitive instruments would rarely (if ever) measure a sharp boundary for thermalized electrons. The retarding potential analyzer on Explorer 18 (Serbu, 1964) found (see Appendix )  $J(E > 100 \text{ eV}) \approx 10^7 / \text{cm}^2 \text{ sec}$  at  $7 R_e$  and  $J(E > 100 \text{ eV}) \approx 2 \times 10^8 / \text{cm}^2 \text{ sec}$  for  $8 R_e < r < 16 R_e$  during an outbound pass (Nov. 27, 1963) for which the magnetometer and high energy particle detectors indicated a magnetopause near  $11 R_e$ . The inner "boundary" discrepancy of 18,000 km to 25,000 km suggests significant inward diffusion of hot electrons from a source beyond  $11 R_e$ , and fast diffusion across a magnetic field is consistent with the presence of an ion wave instability (BFS). [Night side measurements may reveal similar broadening; Explorer 12 and Lunik 2 (Gringauz, et al., 1961) detected a belt of thermalized electrons past  $7-9 R_e$ , while whistler analysis (Liemohn and Scarf, 1964) indicates a small but significant flux of  $200 \text{ eV} < E < 3 \text{ keV}$  electrons between  $2.5$  and  $4.5 R_e$ .]

The superthermal  $E > 30 \text{ keV}$  distribution near and beyond the conventional magnetopause has recently been explored using more sensitive instruments on Explorer 14 (Frank, Van Allen and Macagno, 1963) and Explorer 18 (Fan, Gloecker and Simpson, 1964). The main results appear to be the following:

(1) For  $L_{\text{sep}}$  near  $0^\circ$ , a moderately distinct boundary for high  $E > 30 \text{ keV}$  electron fluxes defines the limit to the magnetosphere. The boundary is usually characterized by the appearance of a sharp electron spike, but as  $L_{\text{sep}}$  increases toward  $70^\circ$ , the boundary becomes jagged and less distinct, and beyond  $L_{\text{sep}} = 70^\circ$ , no sharp boundary appears to be definable. Even for  $L_{\text{sep}} < 70^\circ$ , the boundary location is highly variable, and it may appear between  $7 R_e$  and  $13 R_e$ .

(2) Large spikes of  $E > 30$  keV electrons are usually, but not always found beyond the boundary. For  $L_{\text{sep}} < 70^\circ$ ,  $r_2 - r_1 \approx (4-10)R_e$  and for  $L_{\text{sep}} > 70^\circ$ ,  $r_2 \approx 22 R_e$  to  $30 R_e$ . Single spikes are generally not seen at larger values of  $L_{\text{sep}}$ . For example, on the Explorer 18 November 27 outbound pass ( $L_{\text{sep}} \approx 30^\circ$ ), the 30 keV magnetosphere boundary was at 74,000 - 75,000 km, with many spikes seen out to 90,000 km and a small isolated burst at 94,000 km. However, in this case the outer boundary for  $J(E > 100 \text{ eV})$  occurred at 103,000 km (Serbu, 1964) and it therefore seems that the superthermal electrons were distributed throughout the thermalization region; there is some evidence that when isolated spikes are found they are located near or beyond the outer boundary of the thermalized electron region.

(3) The entire outer magnetosphere region ( $r > 6 R_e$ ) is one in which rapid ( $\sim$  day) temporal variations (in  $J(E > 30 \text{ keV})$ ) of a factor of ten or larger occur.

None of the results quoted directly confirm the basic assumption of our theory, that a region with  $T_e/T_i \gg 1$  exists. However, certain general conclusions can be drawn from the data already at hand, and we might summarize as follows.

On any given pass the location of a "magnetopause" or a "shock front" appears to depend greatly on the type of detector used to measure it, as well as on the range and sensitivity of the instrument. As an extreme example, we refer again to the retarding potential analyzer data and the University of Chicago results on the November 27, 1963 outbound pass of Explorer 18. The large discrepancies are consistent with an ion wave induced fast diffusion mechanism.

In fact, if the physical processes which operate in the interface involve the complex phenomena discussed here (ion wave instabilities, unequal temperatures, fast diffusion, nonlinear wave-particle interactions, non-Maxwellian distributions, superthermal accelerations) there is no reason to expect various measurements to be at all correlated in space or in time. For instance, low frequency magnetic fluctuation analysis ( $T \geq$  minutes) should not reveal the boundaries of the ion wave region ( $T \approx 10^{-3}$  sec) and so any limits established by such a study will generally be unrelated to those suggested by examination of high energy electron fluxes. By the same token, the extent of the ion thermalization region should not generally indicate where wind electrons are thermalized, or be related to the limit of magnetospheric trapping, and the separate limit of the whistler plasma. Indeed, if the wind-magnetosphere interface is broad, time dependent (i.e. unstable) and governed by gross fluctuations in the wind, as conjectured here, then the concept of a fixed and well-defined transition region is not a useful one.

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APPENDIX 1Comments on the Explorer 18 Retarding Potential Analyzer

The IMP-1 retarding potential analyzer (RPA) was only operative on the first outbound pass, and since spacecraft outgassing effects could possibly have complicated these observations, it is necessary to comment further on the general relation between the RPA results and other measurements.

Below  $r = 8 R_e$ , the  $0 < E < 5$  eV flux was in general agreement with that expected if the plasma density has the general form [ say,  $N(r) \approx \bar{N}_3 = 14,100 (R_e/r)^3$  ] suggested by night side whistler analysis (Liemohn and Scarf, 1964) with  $T_e(r) \approx 1 - 4 \times 10^3$  K for  $3 R_e < r < 5 R_e$  and  $T_e(r) \rightarrow 50,000$  K as  $r \rightarrow 8 R_e$ . There are some moderate discrepancies but the uncompressed night side extrapolation is somewhat questionable and so are the absolute values of the RPA flux readings. Nevertheless, we can conclude that even out to  $8 R_e$  there is no reason to distrust the RPA results; the whistler plasma appears to extend out to at least  $8 R_e$ , with a gradual rise in temperature and the development of a more sizeable non-Maxwellian tail ( $0 < E < 5$  eV and  $E > 100$  eV) with increasing range.

However, beyond  $8 R_e$  the flux in all three RPA channels remained relatively constant out to  $16 R_e$ . [ It is true that  $J(E > 100$  eV) was at least an order of magnitude less than the thermalized flux determined two years earlier on Explorer 12, but this discrepancy is not necessarily significant because of the difference in time and solar activity. ] We wish to suggest that the constancy of the outer region flux could be related to the presence of local large amplitude

electric oscillations associated with ion acoustic waves. At the high wave frequencies expected (kilocycles) the spacecraft potential would not follow the oscillations in plasma potential, and the RPA would average over a range of retarding potentials. The interpretation of the electron flux measurements, the effective vehicle-plasma potential difference, and the effective velocity distribution, under such conditions is very complex.



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