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DETERMINATION OF NEWTON'S GRAVITATIONAL
CONSTANT, G, WITH IMPROVED PRECISION

Status Report
covering the period

October 1, 1964 - March 31, 1965
under NASA Grant NGR-47-005-022

Principal Investigators:

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A. R. Kuhlthau
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SECTION I INTRODUCTION

This is the first semi-annual status report under this grant, the object of which is to apply new techniques to the laboratory determination of the Newtonian Gravitation Constant, G , with the expectation of improvement of the accuracy by about two orders of magnitude. The improvement is attributed to both a new approach to the measurement, which should eliminate or bring under better control most of the difficulties encountered in earlier methods, and to improvements in materials, metrology, fabrication procedures, etc. which should circumvent many problems and limitations previously encountered.

With reference to the schematic diagram of figure 1, two small masses, m , are connected by a rigid horizontal rod. These are then placed in proximity to another pair of masses, M , and the gravitational interaction between the two sets of masses results in a pure torque acting on the m -system. A measure of this torque plus a knowledge of the separation distance, d , enables G to be determined. Previous experimenters either measured the torque directly by observing the static deflection of a torsion fiber suspension or introduced variations on this method such as inducing resonant torsional oscillations in the m -system by suitably driving the large M system.

In the present proposed method the gravitational torque is maintained constant by detecting changes in the relative angular position β of the two mass systems and correcting the relative position by a servo mechanism to maintain β constant to the necessary accuracy. Thus the nearly constant torque being applied to the m -system results in a nearly constant angular acceleration of this system which can be permitted to act for an extended period of time. The ultimate measurement, then, is one of determining the angular speed of the m -system at the end of this extended period of time, i. e. of measuring large displacements and long times, both of which can be done accurately.

In designing the experiment, three methods for suspending the m -system have been considered. Two of these involve the use of magnetic suspensions and for future reference these will be designated Models A-1 and A-2. The other

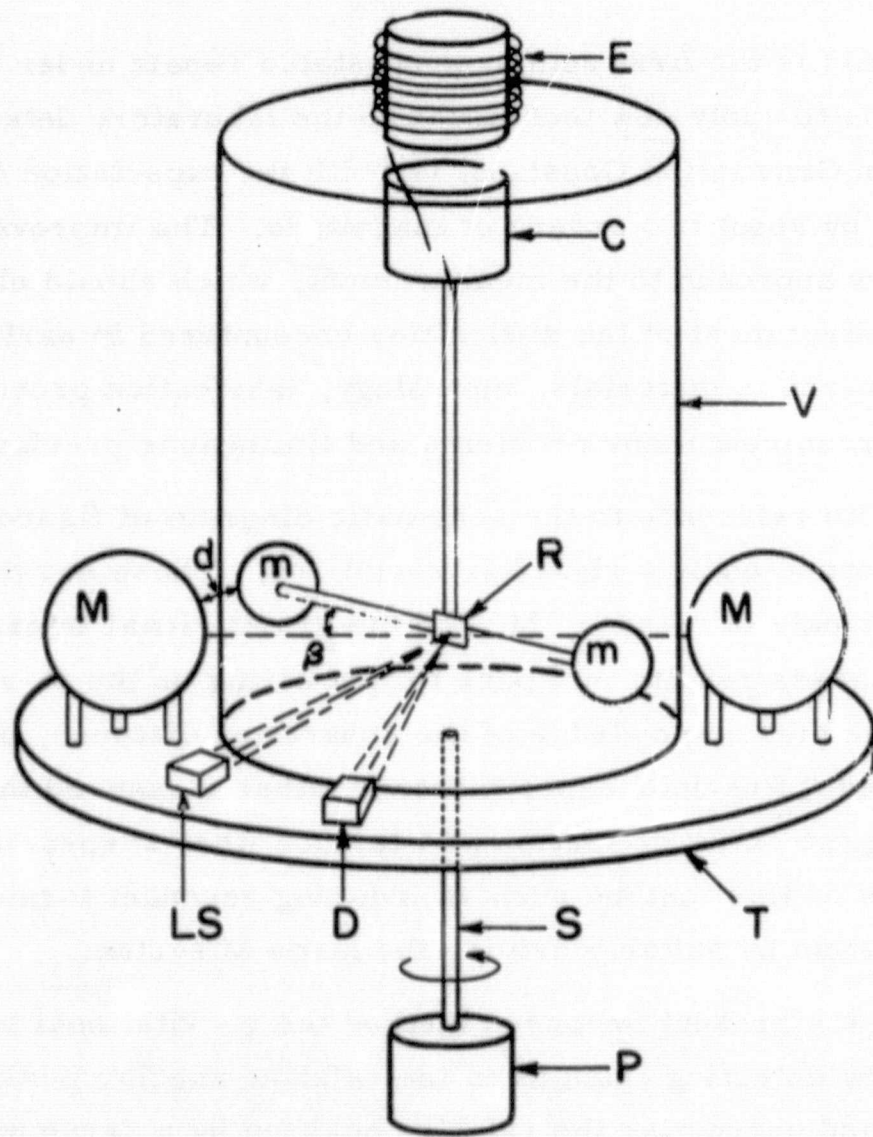


FIGURE-1

SCHEMATIC OF APPARATUS

involves the use of a more conventional torsion fiber] and this will be designated as model B. Each system has many advantages and disadvantages in comparison with the others, and in fact it is at this stage impossible to make the final selection. They are all being studied more carefully as part of the initial phases of the program and it is quite probable that preliminary working models of each design will be constructed.

For the orientation of the reader, the principal differences are:

Model A-1 and A-2-Magnetic Support

Again referring to figure 1, the center of the rod connecting the small masses, m , is attached to a rigid vertical rod containing at its upper end, a small cylinder of magnetic material, C . This is freely suspended by the electromagnet E [1]. The major advantage of the electromagnetic suspension models is that the precision to which the angle β must be held constant in order to get the desired precision in the determination of G is only 10^{-3} radians, an easy prospect. The major disadvantage is the rather elaborate magnetic shielding required.

In Model A-1 the coil of the electromagnet, E , is attached to the rotating table.

In Model A-2 the coil of the electromagnet is free of the rotating table and is fixed in the laboratory frame of reference.

The advantages and disadvantages between these two versions are more subtle. They are discussed in more detail in reference [2], and for the present purposes it need only be said that they do exist, are significant, and can only be evaluated by further experimentation.

Model B-Torsion Fiber

Model B replaces the electromagnetic suspension with a delicate torsion fiber mounted from the rotating table. As the m -system begins to rotate as a result of its interaction with the M system, the fiber support is rotated, due to the table rotation, and the balance always operates at essentially constant deflection (which will be as close as possible to the null point). The major disadvantage here is that this deflection angle and β , must be held constant to 10^{-6} radians in order

to get accuracy in the G measurement comparable to what can be obtained by knowing β to 10^{-3} radians in the case of the magnetic support. The big advantage, of course, is the elimination of the magnetic field, and hence a minimization of magnetic effects.

At the present time the program is divided into three principle tasks, the status of each of which is summarized in the following sections. These tasks are:

1. Theoretical Analysis
2. Apparatus Design - Details and Procurement of Components
3. Assembly of Apparatus and Development of Techniques

SECTION II
TASK I - THEORETICAL ANALYSIS

A principal effort here has been the study of the various sources of errors with the objective of reducing them to an analytical form suitable for experimental verification. If this can be successfully done then corrections can often be made in cases where the errors cannot be avoided through proper design.

A second major problem has been to develop the equation of motion for the experiment into a form suitable for data reduction.

A. Equation of Motion

With regard to the latter, the differential equation describing the rotational motion of the m-system can be written as:

$$\ddot{\theta} = a + A \sin (2\theta + \phi)$$

where

a represents the angular acceleration due to the m-M interaction and the interactions of the m-system with other masses fixed on the rotating table.

$A \sin (2\theta + \phi)$ represents a typical term due to the interaction of the m-system with masses fixed in the laboratory.

In general one will expect a large number of terms of this type such that the second term will in fact be $\sum_i A_i (\sin 2\theta + \phi_i)$. For the sake of simplicity in presenting this summary we limit our consideration to one such term.

The first integral with $\dot{\theta} = 0$ at $t = 0$ is

$$\dot{\theta}^2 - \dot{\theta}_0^2 = 2 a \theta - A [\cos (2\theta + \phi) - \cos \phi]$$

Solving for $\dot{\theta}$ and then dt and integrating from $t = 0$ to $t = t_m$ and $\theta = 0$ to $\theta = 2\pi m$ one obtains an expression for the time requested to make m revolutions.

$$t_n = \int_0^{2\pi n} \frac{d\theta}{\sqrt{\theta^2 - 2a\theta}} \left\{ 1 + \frac{A}{2} \frac{\cos(2\theta + \phi) - \cos \theta}{\theta^2 + 2a\theta} + \frac{3A^2}{8} \frac{[\cos(2\theta + \phi) - \cos \phi]^2}{(\theta^2 + 2a\theta)^2} + \dots \right\}$$

Even in the case where the perturbation term (2nd term) in the original differential equation is an infinite series, the result may be readily put in the form.

$$t_n = \frac{\theta_0}{a} \left[\sqrt{1 + \frac{4\pi na}{\theta_0^2}} - 1 \right] + K_1 n + K_2 n^2 + \dots + K_i n^i + \dots$$

where the K_i are functions of a , A_i , ϕ_i , θ_0

In a typical case the K 's decrease rather rapidly with increasing i . This form of the solution seems quite suitable for at least squares reduction of data.

B. Errors in Geometry

A coplanar analysis of the following kind has been made. With reference to figure 2, and with the assumption of perfect isotropic spherical masses, the following "errors" were assumed.

$$\begin{array}{ll} r_1 = r + \delta & ; r_2 = r - \delta \\ l_1 = l + \alpha & ; r_2 = l - \alpha \\ m_1 = m + \mu & m_2 = m - \mu \\ M_1 = M + \nu & M_2 = M - \nu \end{array}$$

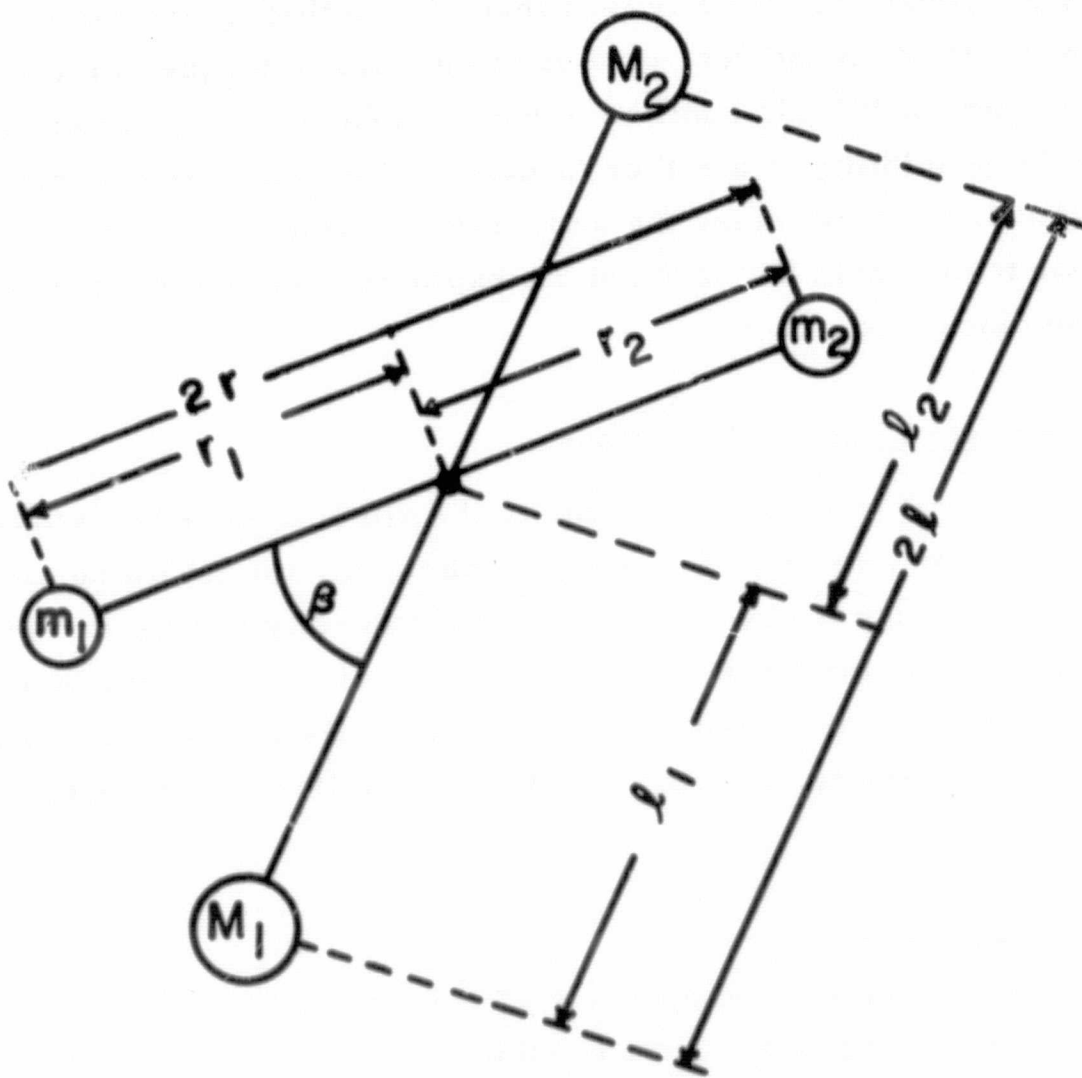


FIGURE-2

GEOMETRY OF INTERACTING SYSTEMS

The torque on the m-system was calculated as a function of $\frac{\delta}{r}$, $\frac{\alpha}{l}$, $\frac{\mu}{m}$, $\frac{\nu}{M}$ to second order in these quantities. As may be seen from symmetry, the first order contribution to the torque vanishes. The coefficients of the second order terms in the expression for the fractional change in torque are typically less than 10, so that fractional errors of, say, 10^{-3} in the above quantities, give rise to errors in torque of the order less than 1 part in 10^5 . In addition to being an encouraging result, the analysis offers the possibility of a rather fundamental check on the precision of the experiment. For example, the deliberate introduction of an error ($\frac{\delta}{r}$ is an easy one to do) could be made and its experimental effect could be compared with the calculated effect.

C. Effect of Density Variations

An effort was made to estimate the effect of density variation in the small and large masses. For a small mass, m, the following model was chosen: Imagine m to be a perfect constant density sphere except that it contains a much smaller spherical volume whose center may be located at a variable point within m such that its surface always touches the surface of m, and whose density, although constant and uniform is different from the rest of m, i. e.

density of m = ρ

density of inscribed sphere = $\rho'' = 1 + \alpha\rho$

average density assumed constant.

The analysis then calculates the value of α , as a function of where the inner sphere is centered, so that the ratio of the torque to moment of inertia changes by less than 1 part in 10^6 from its value when $\alpha = 0$. The result showed that α must be larger than about 2×10^{-4} . For the small masses the changes in the moment of inertia and in the torque tend to cancel each other.

For the large masses, a different model seemed more appropriate. A particle was assumed to be on the surface of the sphere, and a calculation was made to estimate the effect of this particle, as a function of its position, on the torque. The result was that in the most unfavorable position, the particle could be no larger than about 0.4 mm in diameter for a 10 cm diameter sphere in order to affect the torque by less than 1 part in 10^6 . Experimentally, one should be able to determine the dependence of the results on the orientation of the large masses, and if it proves to be significant a more detailed analysis of the problem must be considered.

SECTION III
TASK II - APPARATUS DESIGN
DETAILS AND PROCUREMENT OF EQUIPMENT

A. Large Mass System

Considerable attention has been given to the design of both mass systems. It was discovered that the Union Carbide Corporation, Nuclear Division, at Oak Ridge has unique capabilities in machining components to extremely high tolerance. Although final details have not yet been settled, some tentative possibilities evolved as a result of frequent consultation with the UCCND staff.

It appears as though tungsten will be the optimum material to use. It is a compromise between cost, heavy mass, workability, resistance to corrosion, etc. As presently envisioned the large masses will be about 4 inches in diameter, weighing about 22 lbs. each. It is hoped that the sphericity can be held to 10^{-5} inch or better, the diameter determined to 10^{-5} inch, and the mass to within 1 part in 10^6 . Calculations indicate that the center of mass and the geometrical center should coincide to about 10 microinches. However this does not assure uniform density; and if the density variations are not to produce significant errors in the experiment, then the density averaged over any volume element 1/4 inch in diameter should vary less than 1 part in 10^4 .

This last requirement is the only one that appears difficult to meet, and at the present time the UCCND group is studying various batches of tungsten in an attempt to evaluate this possibility.

B. Small Mass System

The small mass system will also contain tungsten spheres, and a very satisfactory design has evolved from discussions with the UCCND staff. It appears as though they can proceed with the fabrication of this system with good expectation of achieving the desired tolerances as soon as they receive a request for service to other government agencies which must be filed by NASA. A letter to NASA requesting that they file the necessary forms on our behalf is in preparation at this writing.

The tolerances on the small spheres are a little more rigid than those on the large masses, but it is easier to hold them on the smaller sizes. The small masses will be of the order of 6-8 mm in diameter. An important feature is that most of the mass of this dumbbell arrangement be in the spheres. To accomplish this, it is proposed that the spheres be joined by pressing them into each end of a beryllium tube which will be fabricated from 2-4 mil thick foil. An arrangement for attaching a vertical shaft to this dumbbell at the center of mass must still be devised.

C. Rotary Table

The rotary table which supports the large mass system and whose rotational speed is servoed in order to maintain the desired value of β must be a precision piece of equipment. After considerable theoretical study the following specifications were chosen.

1. Speed: To be controlled by a servoed motor over the range of 0.1 to 10 rpm. Servo and motor will be supplied separately.
2. Direction of Rotation: Both directions
3. Access Hole: An axial access hole is required for electrical leads. Its dimension is not critical and may be 1/2" to 1" diameter.
4. Diameter of Table: 9" - 14"
5. Weight Capability: 100 lbs. (axis vertical)
6. Maximum Allowable Axial and Radial T. I. R. of Spindle of Table: 0.000015"
7. Maximum Wobble: 0.4 sec. of arc.
8. Surface Finish of Table Top: 4μ inches.
9. Materials: It is desired that all materials in spindle and rotary table be non magnetic.
10. Mounting: Spindle and rotary table are to be mounted in a granite surface plate with the axis of the spindle perpendicular to the surface plate to within 1 sec. The spindle will be operated with its axis vertical.

These specifications were sent to a large number of vendors. Several quoted and the Micrometrical Division of the Bendix Corporation was able to supply what was considered an acceptable spindle. The only real compromise was the necessity of accepting a steel spindle. An order has been placed with this company with delivery anticipated in June.

As in the case of the mass systems we are grateful to the staff of the UCCND for their advice and council based on their experience with various designs of precision rotary tables.

D. Servo-Motor Drive for Rotary Table

An Inland Motor Corp. A-C torque motor with a solid core rotor has been chosen to drive the table. Delivery is expected about the end of May. This type of motor produces an extremely smooth output torque. An analysis of the system has been performed, and from the root-locus and Bode plots, the compensation has tentatively been determined.

The transistor drive for the two phase motor has been designed, and most of the components are on hand. A Sorenson 60 volt, 25 amp power supply has been ordered with delivery expected by mid-June.

The splendid cooperation of the engineering staff of the Specialty Controls Department of the General Electric Corporation in Waynesboro, Virginia, is acknowledged. Discussions with this group laid the ground work for our final selections.

E. Measurement of Period of Rotation of Rotary Table

As indicated earlier, the ultimate experimental measurement from which G will be calculated will be one of determining the total angle through which the apparatus has turned in a time Δt . The approach has been to utilize the experience obtained by members of the staff of the Johns Hopkins University, Applied Physics Laboratory, in their work with determining the periods of satellites to high precision. Thus with their helpful assistance the method shown in figure 3 has been devised.

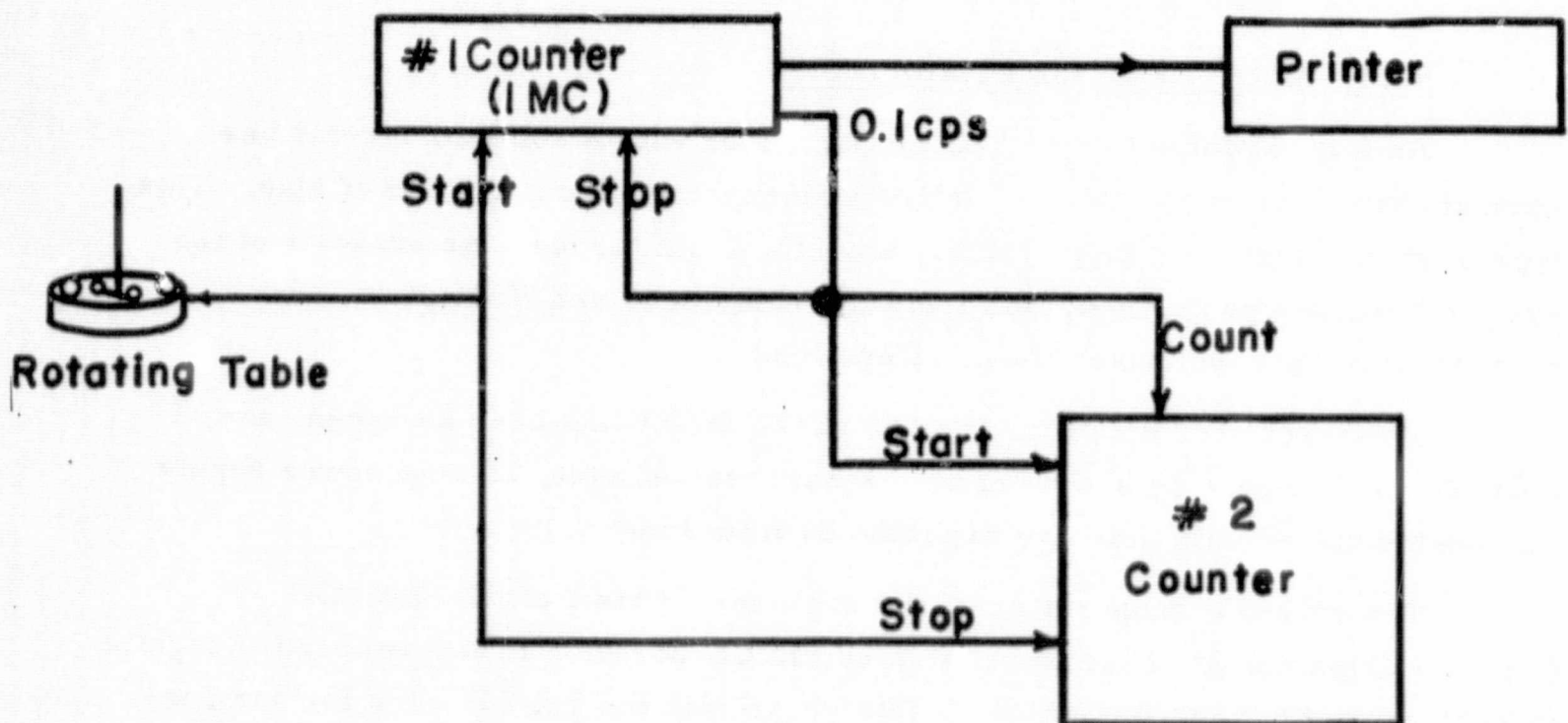


FIGURE -3

DETERMINATION OF PERIOD OF ROTARY
TABLE SCHEMATIC

Counter No. 1 is a very stable (≤ 3 parts in 10^9 per 24 hours) 1 MC counter operating in the time interval mode. It also has output frequencies in decade steps down to 0.1 cps. This counter starts to count on signal from some fiducial mark on the table and stops counting at the next 0.1 cps pulse from its own time base. This count is then printed on the printer. This same time pulse also starts the Number 2 counter which then counts the number of 0.1 cps pulses coming from the No. 1 counter time base until it is turned off by the pulse from the rotary table. This pulse also starts the No. 1 counter which then proceeds to count cycles until turned off by its next 0.1 cps pulse, and so on. The arrangement is diagramed in figure 4. With reference to this figure it is easily seen that the period can be obtained from the relations.

The time base drift and absolute setting of the No. 1 counter can be readily checked using Loran-C techniques.

All equipment is either on hand or on order. We are grateful to APL/JHU for the loan of the printer.

F. Suspension - Model B

This is the torsion fiber suspension. The design of this model has been essentially completed and the device will be constructed in our own shops. It will be a copy of a similar device that we have used successfully in connection with another NASA project (NASA-2538)[3] .

G. Suspension - Model A

Design studies have begun on the electromagnetic suspension apparatus for the small m-system. No difficulties are anticipated as the group has had considerable experience in the design of such systems, and the present application seems straightforward. The major problems, if they do arise, will probably be with the magnetic homogeneity of the active material attached to the small m-system.

H. Dust Free Facility

It is absolutely essential that this experiment be carried out in a dust-free facility. Various commercial clean rooms have been evaluated and the

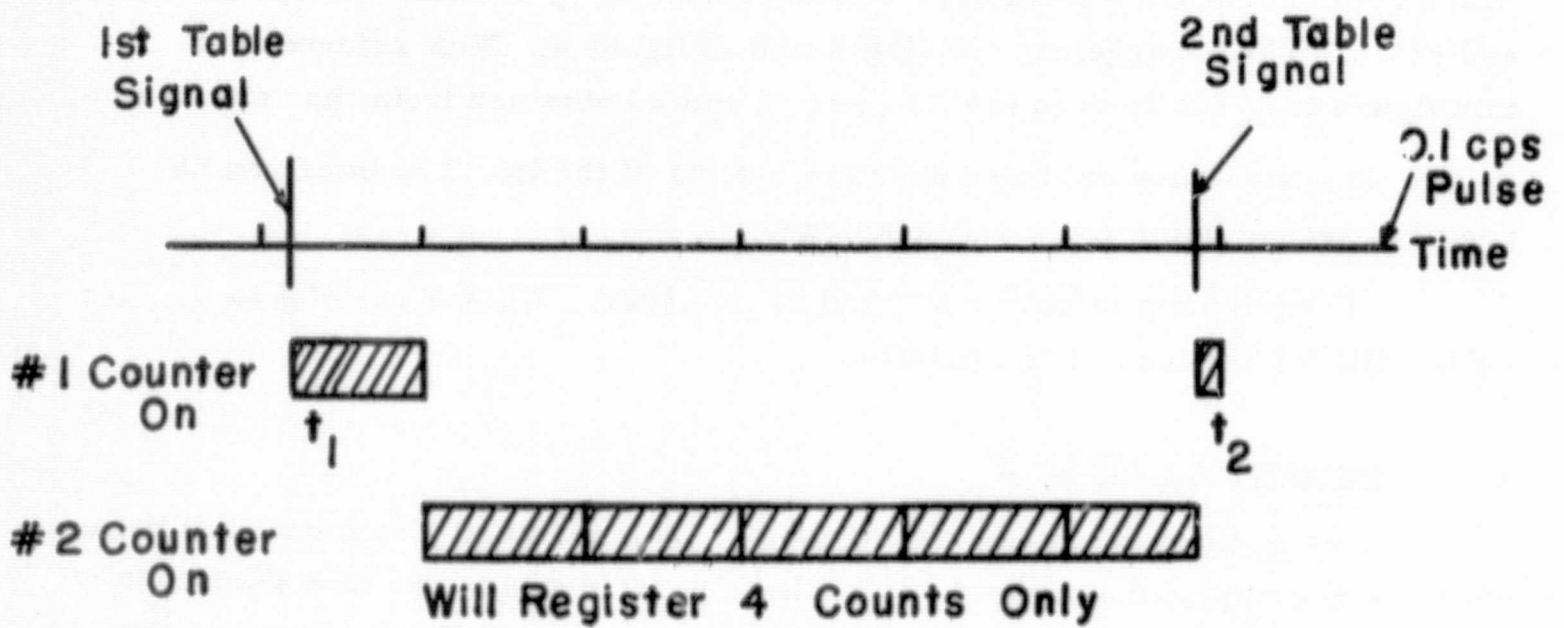


FIGURE - 4

DIAGRAM OF COUNTER OPERATION

optimum from the point of view of meeting requirements for the minimum expense was a modular kit unit manufactured by Agnew Higgins, Inc. An order has been placed for one of these units with delivery expected in July.

At the moment no plans have been made for temperature control for this unit. The air input will be taken from a room that is air-conditioned, and it is hopeful that this may suffice. However the temperature problem is being given very careful study. The most important item of concern at this moment is the realization of temperature equilibrium in the entire apparatus, so that the dimensions at the time of a run may be known precisely.

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SECTION IV
TASK III - ASSEMBLY OF APPARATUS
AND DEVELOPMENT OF TECHNIQUES

A. Space

An experimental area has been provided in a corner of the basement of the Aero-Mechanical Building of the School of Engineering and Applied Science. This location was selected because it will be reasonably easy to isolate the area during experimental observations. A pier, isolated from the rest of the building, was sunk into the earth, and this will be located within the dust-free facility when that is erected.

B. Prototype Apparatus

In order to gain experience with the performance of the various units of the apparatus, a prototype mass system has been designed and is currently nearing completion. The large and small mass systems will be made of brass, and will not possess the desired accuracy, but otherwise the prototype will use the components of the final device.

Present thinking will have only the small mass-system within a vacuum and this vacuum system has been designed and is about 50% completed. This will use equipment at hand for the first try, but may require refinements involving new equipment.

C. Metrology

The techniques for the determination of the various fixed constants of the apparatus are receiving serious attention. Some preliminary conversations with members of the staff of the National Bureau of Standards have been held and they have graciously agreed to work with us on these problems. Arrangements are being made for a visit to the Bureau for consultation regarding the optimum exact sizes for various components and the proper measurement procedures.

D. Density Determinations

One of the most serious possible problems is the determination of the local density throughout both the large and the small mass systems. Several methods are under consideration, and one that shows good promise and is being most actively pursued at the moment is based upon electromagnetic weighing techniques.

SECTION V
STAFF

In addition to the principal investigators, the assistance of the following colleagues is gratefully acknowledged.

Drs. John Boring and Robert Humphris, members of the senior research staff of the Department of Aerospace Engineering, who have assumed responsibilities for various phases of the development.

Mr. James Dickerson, Design Engineer, who has developed the detailed specifications for many of the components to be purchased commercially and is supervising the design of the first prototype.

Mr. William Towler, Electronic Engineer, who is developing the magnetic suspension systems.

Miss Jo Anne Soderguist, a graduate student in Aerospace Engineering, who has assisted in several of the calculations.

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