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APP E

APPENDIX E

LIMITED PHASE DEVIATION FREQUENCY MULTIPLIER PHASE MODULATOR

Submitted as part of the Final Report

for RF Test Console on JPL

Contract No. 950144

NAS 7-100

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DATE: December 15, 1964

WESTINGHOUSE DEFENSE AND SPACE CENTER

SURFACE DIVISION

ADVANCED DEVELOPMENT ENGINEERING

Appendix 1.

PM TRANSMITTER:

DEVIATION MULTIPLICATION PHASE MODULATOR

I. Purpose of this Memorandum:

This Memorandum is prepared for the purpose of reviewing the more conventional phase modulator circuits, those generally requiring deviation multiplication and also some specialized devices. The application of such techniques to the specific requirements of the RF Test Console is treated. Identification of problem areas is made and some candidate systems are analyzed. The adaptability of the systems to dual PM-FM modes of operation is considered.

Finally a summary comparison to the phase-locked modulator technique, previously reported, is made with conclusions and recommendations for RF Test Console equipment design.

II. General Discussions

A. Specifications and Objectives

The specific requirements for the RF Test Console phase modulator are given in the JPL Spec No. GPG-15062-DSN. A summary of the pertinent requirements was included in a previous report, dated August 25, 1964, covering a phase locked oscillator phase modulator concept. Rather than repeat the requirements again reference to these documents is suggested.

In the interest of simplification and economy of circuits an additional objective not specifically stated is the desirability of dual mode PM-FM operational capability of the modulators and systems described below.

B. A Review of Typical Modulators for Analog Input Signals

The generation of candidate systems for a PM transmitter depends to considerable extent upon the physical circuits available to produce phase modulation of a carrier frequency by an analog input or command signal. In this section discussions of some typical methods are given, but no claim is offered that this review is all-inclusive. Broad classifications of phase modulator circuits include the following major concepts:

1. The Armstrong Phase Modulator: This method was used in the early FM systems*1 where the input signal was operated upon by an integrator prior to application to the phase modulator. If the integrator is omitted, phase modulation is produced. A crystal controlled oscillator source is used to provide two carrier signals in quadrature phase relationship, one of which is amplitude modulated by the signal. The output of the balanced modulator is vectorially summed with the unmodulated carrier thereby producing a direct phase modulation of the resultant signal. For acceptable fidelity the peak deviation of the modulator must be limited to about 0.2 radians.

*1 Armstrong, E.H., 'A Method of reducing Disturbances in Radio Signaling by a System of Frequency Modulation', Proc. IRE, Vol. 24, pp 689-740, May 1936.

2. Phase Shifters, and Variable Tuning of Resonant Circuits: This is a broad classification appropriate for the most generally used circuits for both PM and FM. Countless variations of the exact circuit configuration are found in the literature. Typically most of these circuits require a buffer stage or amplifier to provide isolation of the basic modulator from the reference crystal oscillator to avoid any degradation of carrier stability. For this class of modulators the maximum phase deviations achievable with "acceptable" linearity is limited to less than 0.1 to 0.2 radians.

In the early development[*2] of PM and FM radio systems the reactance tube modulator was common. More recently the use of transistors, voltage-variable capacitors, and varactors has become widespread.[*3,4,5,6]

*2 Crosby, M.G. "Communication by Phase Modulation", Proc. IRE, vol. 27, pp. 126-136, Feb. 1939.

*3 Todd, A.C., Schuck, R.P., and Sachs, H.M., "Using Voltage-Variable Capacitors in Modulator Design", Electronics, Vol. 34, pp 56-59, Jan. 20,1961.

*4 Thomas, L. D. and Brotherton, C.E., "Modulator Investigation" Westinghouse Electric Corp., Electronics Rept. EE-4416, 40 pps, Dec. 12, 1963.

*5 Fairley, D.O., "One Circuit: Phase Modulation, Frequency Multiplication", Electronics, Vol. 37, pp 71-74, Nov. 2, 1964.

*6 ______________ Multiplier-Modulator - X3 and Phase, 66.0 MC/S Schematic Diagram, Jet Propulsion Laboratory drawing J9330970 (also Design Specification DSIF S-Band and RF Subsystem X3 Frequency Multiplier and Modulator, 66 MC, JPL Spec DOR-9469-1-DSN-C, 11 May 1964.)

3. Variable Delay Lines: If several variable phase shift circuits are connected in cascade to form an artificial delay line, the phase deviations of each circuit due to the modulating voltages will be additive so that a large composite deviation may be expected. The modulating voltage may be applied to a Varicap or Varactor in each section to produce phase shift. Delay compensation is required for the modulating signal so that the carrier and modulating signal signals arrive at each section of the line at the same time. An analysis included below does not however confirm that the expected large phase deviation is in fact attainable.

Small phase deviation may be obtained with the transmission line terminated in an impedance which is varied by the modulating signal.*2 The output is taken from the transmission line at a point where the incident and reflected waves combine to produce a resultant which varies in phase.

*2 Crosby, M.G. ibid.

4. <u>Pulse Position (and Serrascid) Modulators:</u>*7, 8 These circuits are capable of wide deviation phase modulation without further multiplication, at least over a modest frequency spectrum of the modulating signal. The concept utilizes a carrier signal modulated in frequency by a periodic saw-tooth waveform. This signal is applied to a gate circuit which is controlled in time by a pulse generated from the desired modulating signal. The output is then filtered to remove harmonics, the resultant signal being phase modulated with comparatively wide deviations. Basic limitations of the system would appear to be a limited frequency response imposed by the FM ramp generator and difficulty in achieving adequate carrier stability.

*7 Day, J. R., "Serrasoid FM Modulator", Electronics, P 72, Oct. 1948.

*8 Columbia Univ., "The Cascade Phase Shift Modulator Employing Negative Feedback", Tech. Rept. T-4/164, July 1961 RADC-TN-61-187.

LS-335
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5. Special Devices: References found in the literature to several special devices which are capable of direct wide deviation phase modulation. For example *9, an Archimedian spiral target may be used with a CRT to obtain several hundred degrees of phase deviation of the Shelby modulator where an amplitude modulated carrier is applied through quadrature phase shifters to the orthogonal deflection plates of the CRT.

Another special device for direct wide deviation phase modulation is the Phasitron tube *10. The tube generates a rotating beam, shaped much like the spokes of a wheel. The rate of rotation is advanced or retarded by the modulating signal acting through an external magnetic field. The author claimed phase deviations 12 to 16 times conventional modulators.

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*9 Hund, A., "Frequency Modulation" pp 190-193, 1st. Ed. McGraw Hill, 1942.

*10 Adler, R., "A New System of Frequency Modulation", Proc. IRE, Vol. 35, pp 25-31, Jan. 1947.

65-335
APP E

6. Differentiator-Frequency Modulator Cascade: - In theory phase modulation may be produced by applying the modulating signal through a perfect differentiator (having a +6 db per octave slope over the signal spectrum thence to any linear frequency modulator. Reference is made to a previous report*11 for the detailed treatment of the phase-locked phase modulator which is a specialized concept properly classified under this heading.

The more conventional frequency modulators permit a very small linear phase deviation, however the phase-locked phase modulator is capable of direct wide phase deviations with minimum degradation of carrier stability.

Another potential method*12 for direct wide phase deviation lies in the use of the dual klystron-mixer system where a free running klystron is frequency modulated to the full desired deviation while the secondary klystron is controlled by a precision AFC loop to stabilize the difference carrier frequency obtained by mixing the two klystron outputs. This system is widely used in continental microwave FM communications and has also been used for intercontinental TV via satellite. In addition to the requirement for a differentiating preemphasis network required to generate PM, the technique presently used for AFC does not give sufficient carrier stability for phase coherent applications.

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*11 Vaughan, G.R., Osborne, E.F., and Entwistle, J.S., "Phase Locked Oscillator Phase Modulator Analysis", Westinghouse Electric Corp., Preliminary Rept., August 25, 1964 on Contract: JPL 950144.

*12 Giger, A.J., Pardee, S. Jr. and Wickliffe, P.R., Jr., "The Ground Transmitter and Receiver", Bell Syst. Tech. Jour., Vol. 42, Part 1, pp 1063-1107, July 1963.

C. Identification of General Problem Areas Peculiar to the RF Test Console

Prospective systems for the PM Transmitter must be evaluated to determine performance and comparative advantages in the RF Test Console application. In addition to the requirements for phase coherence, stability, and fidelity, the requirement for a high deviation on a 50 mc carrier results in a large percentage bandwidth to carrier ratio that is peculiar to the RF Test Console.

General problem areas encountered in making a decision as to the most desirable mechanization include the following:

1) Assuming that the more orthodox modulation techniques of section B (1) and (2) above are employed, their linear modulation capability of the order of 0.1 to 0.2 radians will require large frequency multiplications in the subsequent circuitry in order to achieve the desired 4.0 radians peak requirement. For example a multiplication factor of 32 would require a linear modulation capability of 0.125 radians peak.

2) In the frequency multiplication chain the percentage bandwidth requirements do not remain constant but become increasingly difficult (wider) as one recedes from the output through each multiplier stage toward the phase modulator. From the standard Bessel functions one may estimate that for a phase deviation of 4 radians and the transmission of all components of energy level greater than 1% of the unmodulated carrier, a bandwidth is required adequate for at least the 6th order of sidebands. It is important to remember that all sidebands are created in the initial modulation, and that frequency multiplication creates no new sidebands but only redistributes the energy among those sidebands already existent. A distortion free system must therefore have adequate absolute bandwidth at all points in the system. Thus for a top modulating frequency of 1.5 mcs the required two sides (rf) bandwidth is 9 mcs

for a deviation of 1 radian, accommodating at least the third order sideband at the lower deviation of the wideband channel.

3) As a consequence of 1) and 2) it is obvious that a direct multiplication of the phase modulator spectrum, by the large ratio required to obtain the specified output deviation, is incompatible with a 50 mc output center frequency. Modulation methods requiring large frequency multiplications must therefore provide one or more frequency translations by heterodyne mixing.

4) A phase modulator, frequency multiplier and mixer chain requires careful spectral analysis to avoid the production of harmonics and image spectra that may fall within the desired bands. In addition to the careful selection of nominal center frequencies the insertion of filters to remove undesired components cannot be made indiscriminately but design procedures must be employed to ensure a linear phase response over the full modulation bandwidth at all center frequencies if delay distortion is to be avoided.

D. Candidate Systems for the RF Test Console

In the selection of candidate systems for the FM transmitter portion of the RF Test Console, practical factors as well as potential performance capability must be considered. It is desirable to utilize concepts inherently reliable and maintenance free. Preferably the equipment should be of modern solid state design in order to minimize size and weight, and power supply requirements. Since the RF Test Console has a dual mode capability of operation with either PM or FM signals some economies might be achieved if consideration is given to circuit utilization in both modes.

The following candidate systems were chosen for comparison and further examination in satisfying the objectives of the PM transmitter. They are:

1) The phase-locked oscillator modulator (described in seperate report, reference 11 above).

2) A delay line of cascaded phase shift networks as illustrated by the Diagram Figure 1.

3) A dual heterodyne mixer-multiplier chain as shown in the block diagram of Figure 2.

4) A direct multiplier chain with final output heterodyne translation to the desired 50 mc center frequency, shown in Figure 3.

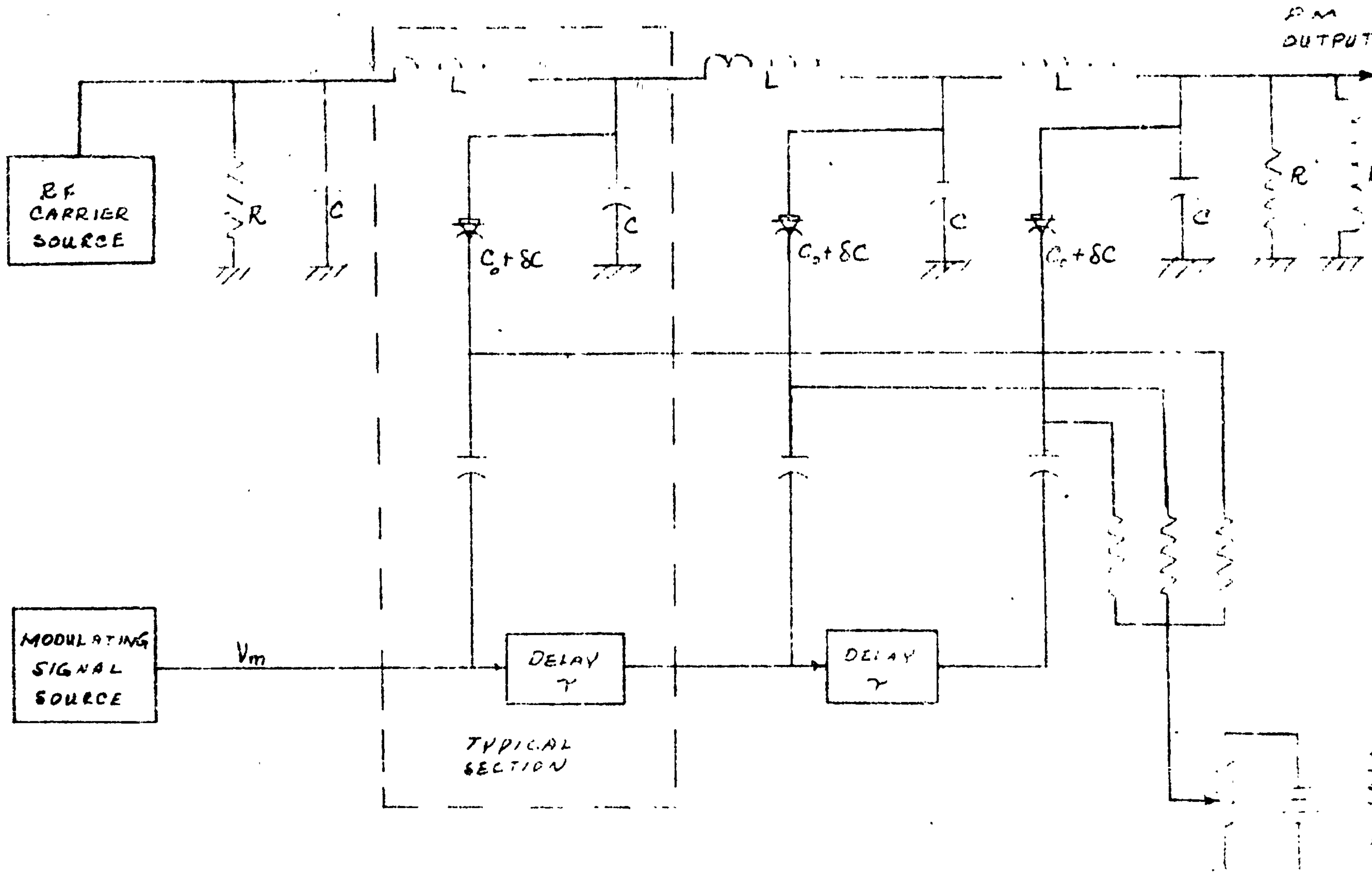


FIG. 1 DELAY LINE PHASE MODULATOR

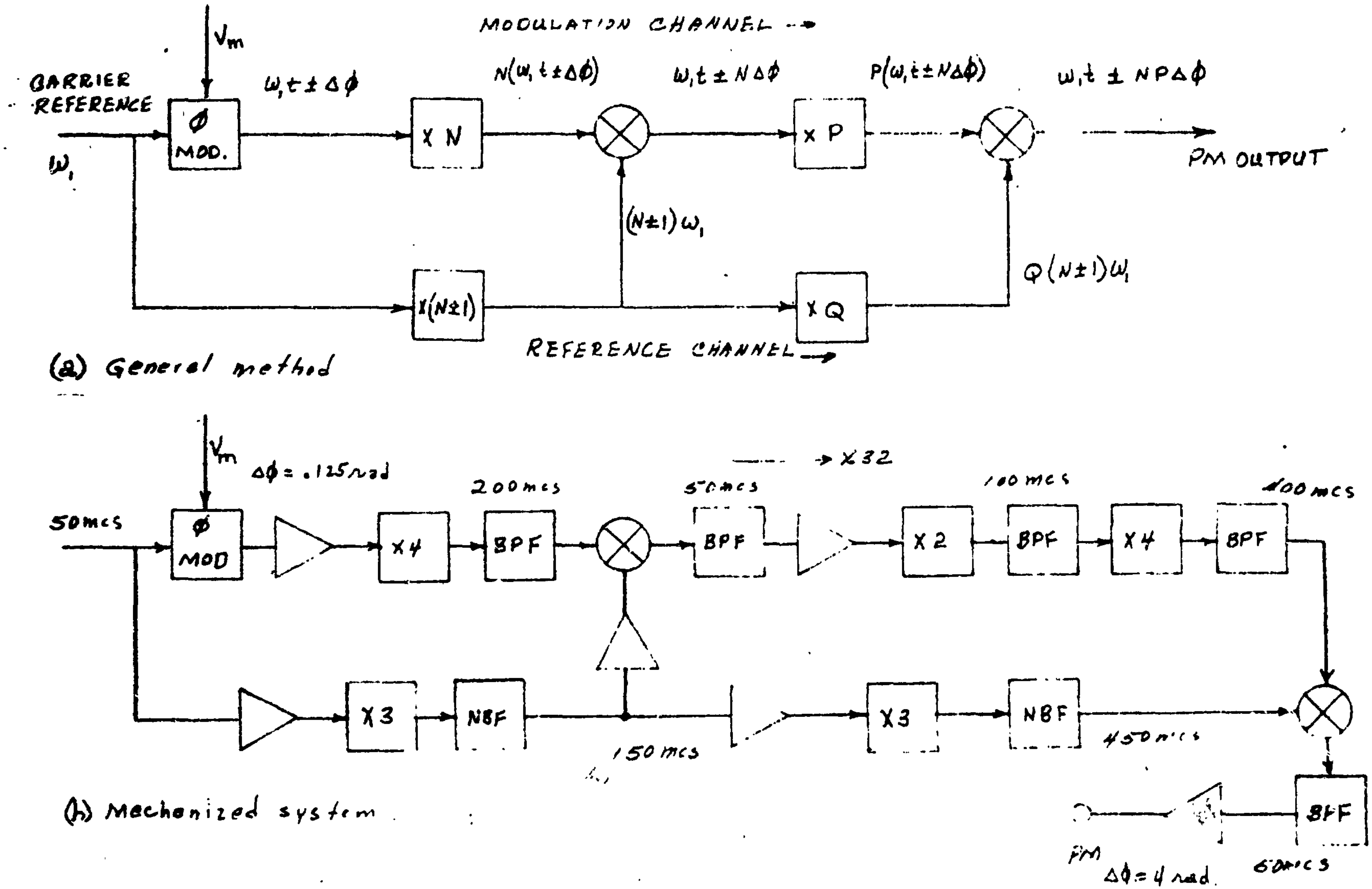
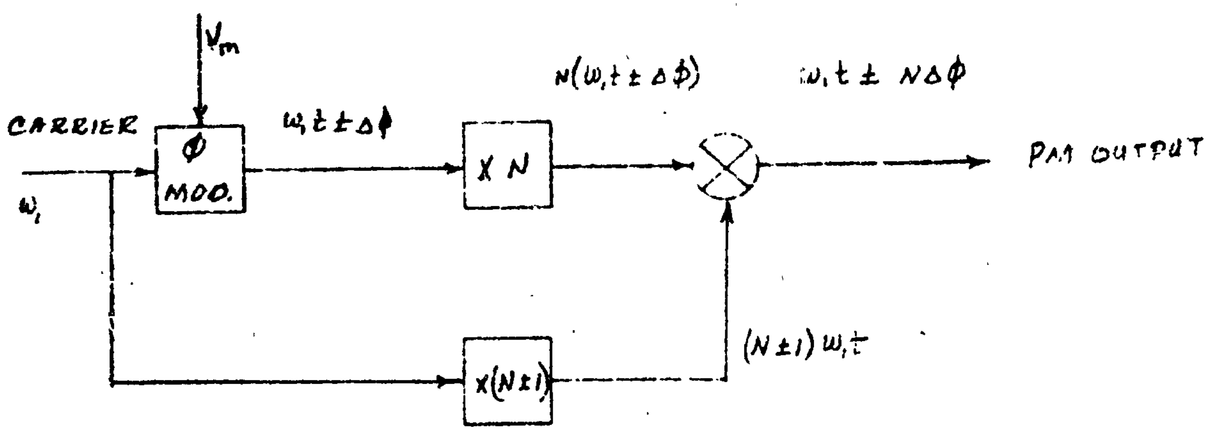
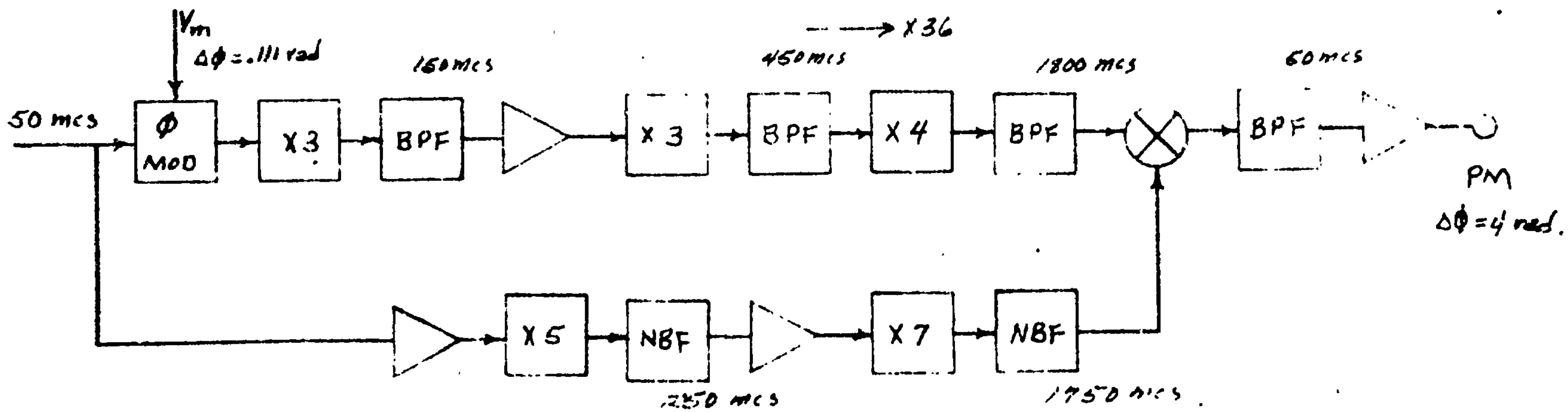


FIG. 2 DUAL MIXER-MULTIPLIER PM TRANSMITTER



(a) General method.



(b) Mechanization of X36 Multiplier System *

FIG 3 DIRECT MULTIPLIER PM TRANSMITTER

III. ANALYSIS OF MODULATOR EFFICIENCY

Before proceeding with a comparative evaluation of the candidate systems an examination of deviation vs. fidelity is made for the basic modulator circuit to show more factually the multiplication factor required. The vectorial relationships of the Armstrong technique are chosen as being typical of most small deviation modulators. This is followed by a similar examination of the phase shift network taken as an element in the delay line candidate modulator.

A. Linearity Computations for the Armstrong Modulator:

First the modulator is examined for phase, then for amplitude distortion. Reference to the diagram of Fig. 4a is suggested.

(1) PM Analysis of Armstrong Modulator: - In the diagrams the modulating signal, e_m, is given by:

$$e_m = k_m \sin W_m t \qquad (1)$$

and the carrier, or reference signal, e_c, is:

$$e_c = E_c \sin W_c t \qquad (2)$$

assuming $2Ke_c = 1.0$

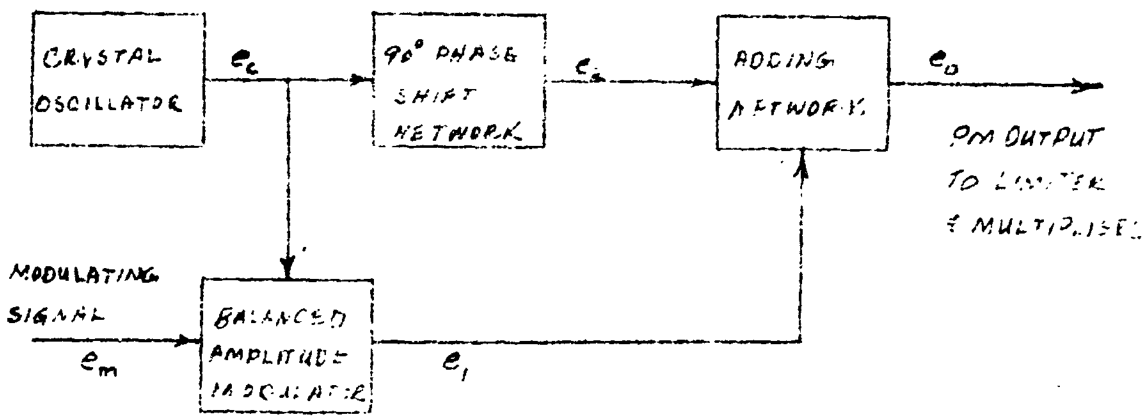
The output of the balanced modulator is

$$e_1 = E_m \sin (W_c + W_m)t + E_m \sin (W_c - W_m)t \qquad (3)$$

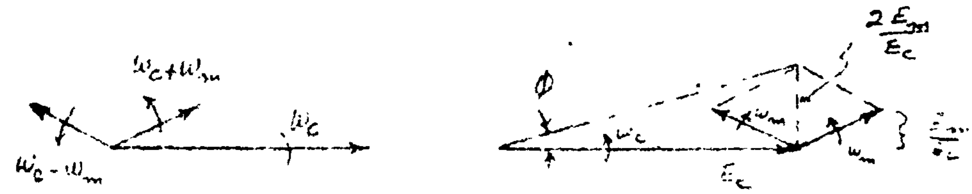
After summing with the quadrature carrier, e_2, the instantaneous modulated signal is obtained, being:

$$e_o = E_c \cos W_c t + E_m \sin (W_c + W_m)t + E_m \sin (W_c - W_m)t \qquad (4)$$

Now it is desired to examine this output signal more fully in terms of its phase and amplitude variations. The phase angle, ϕ, of the instantaneous vector, Fig. 4b, is:



(a). PHASE MODULATOR



(b) VECTOR REPRESENTATIONS

FIG. 11 ARMSTRONG TYPE PHASE MODULATOR

65-335
APP. E

$$\phi(t) = \omega_c t + \Delta\phi(t) = W_c t \pm \arctan \frac{2E_m}{E_c} \sin W_m t \qquad (5)$$

Under the constraint that:

$$\frac{2E_m}{E_c} \ll 1.0 \qquad (6)$$

the instantaneous phase is

$$\phi(t) = W_c t \pm \frac{2E_m}{E_c} \sin W_m t \qquad (7)$$

consequently the instantaneous vector may be described by amplitude and phase functions, namely,

$$e_c = \sqrt{E_c^2 + (2E_m \sin W_m t)^2} \cos\left(W_c t + \frac{2E_m}{E_c} \sin \omega_m t\right) \qquad (8)$$

having a peak phase deviation

$$\Delta\phi = \frac{2E_m}{E_c} \qquad (9)$$

The peak deviation that may be obtained with linearity is limited by the condition that

$$\Delta\phi = \arctan \Delta\phi. \qquad (10)$$

The generation of intermodulation distortion can be estimated by examining a two-tone modulation having the peak deviation of eq (9) and an instantaneous deviation described by

$$\Delta\phi = \arctan \frac{E_m}{E_c} (\sin pt + \sin qt), \qquad (11)$$

where p and q represent modulating frequencies. This function is expanded in the power series for the arctangent

$$\arctan x = X - \frac{1}{3}X^3 + \frac{1}{5}X^5 - \frac{1}{7}X^7 + \cdots \qquad (12)$$

65-335
APP. E

Upon substitution:

$$\Delta \phi = \frac{E_m}{E_c}(\sin pt + \sin qt) - \frac{1}{3}\left(\frac{E_m}{E_c}\right)^3(\sin pt + \sin qt)^3$$

$$+ \frac{1}{5}\left(\frac{E_m}{E_c}\right)^5(\sin pt + \sin qt)^5 - \frac{1}{7}\left(\frac{E_m}{E_c}\right)^7(\sin pt + \sin qt)^7 + \cdots \quad (13)$$

Obtaining a solution for the first three terms of the power series, relative signal levels for the various frequency terms are as shown in Table I.

TABLE I. FM SPECTRA FOR ARMSTRONG MODULATION

Frequency Terms	Magnitude of Signal	Comparative Level in db	
		$E_m/E_c = .2$	$E_m/E_c = .1$
p, q	$1 - \frac{3}{4}\left(\frac{E_m}{E_c}\right)^2 - \frac{5}{12}\left(\frac{E_m}{E_c}\right)^4$	0	0
3p, 3q	$\left(\frac{E_m}{E_c}\right)^2\left\{\frac{1}{12} + \frac{5}{48}\left(\frac{E_m}{E_c}\right)^2\right\}$	-68.5	-101
5p, 5q	$\left(\frac{E_m}{E_c}\right)^4 \frac{1}{240}$	-123	-147.6
$(p \pm 2q)$, $(q \pm 2p)$	$\left(\frac{E_m}{E_c}\right)^2\left\{\frac{1}{4} + \frac{7}{40}\left(\frac{E_m}{E_c}\right)^2\right\}$	-59.3	-71.4
$(3p \pm 2q)$, $(3q + 2p)$	$\left(\frac{E_m}{E_c}\right)^4 \frac{1}{24}$	-103	-128
$(p \pm 4q)$, $(q \pm 4p)$	$\left(\frac{E_m}{E_c}\right)^4 \frac{1}{80}$	-113	-138
$(2p \pm q)$, $(2q \pm p)$	$\left(\frac{E_m}{E_c}\right)^4 \frac{1}{30}$	-105	-129.6
$(2p \pm 3q)$, $(2q \pm 3p)$	$\left(\frac{E_m}{E_c}\right)^4 \frac{1}{40}$	-107	-132
$(4p \pm 2)$, $(4q \pm p)$	$\left(\frac{E_m}{E_c}\right)^4 \frac{1}{120}$	-117.5	-142.5

65-335
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In the data of Table I evaluation at $\frac{E_m}{E_c}$ of 0.2 and 0.1 represents peak phase deviations of 0.4 and 0.2 radians respectively as will be obvious from eq. (11). On the basis of phase linearity alone quite acceptable values are obtained, the most severe intermodulation being the third order component. The performance of the Armstrong Modulator must be examined further to determine the effect of the amplitude function contained within the radical of eq (8).

(2) AM Analysis of Armstrong Modulator: - Continuing with the equal two-tone analysis we have for the amplitude function of eq. (8)

$$A = \sqrt{E_c^2 + E_m^2 (\sin pt + \sin qt)^2} \qquad (14)$$

$$= E_c \sqrt{1 + \left(\frac{E_m}{E_c}\right)^2 (\sin pt + \sin qt)^2}$$

Using the binomial series,

$$(1 + x)^{1/2} = 1 + \frac{X}{2} - \frac{X^2}{8} + \frac{X^3}{16} \text{ ---------} \qquad (15)$$

Obtain

$$\frac{A}{E_c} = 1 + \frac{1}{2}\left(\frac{E_m}{E_c}\right)^2 (\sin pt + \sin qt)^2 - \frac{1}{8}\left(\frac{E_m}{E_c}\right)^4 (\sin pt + \sin qt)^4$$

$$+ \frac{1}{16}\left(\frac{E_m}{E_c}\right)^6 (\sin pt + \sin qt)^6 \text{-----} \qquad (16)$$

After expansion and collection of coefficients relative signal levels for the various frequencies generated because of the amplitude function are as shown in Table II. The data indicates the presence of relatively large 2nd harmonics (of each modulating tone) and 2nd order intermodulation components.

TABLE II - AM SPECTRA FOR ARMSTRONG MODULATOR

Frequency Terms	Magnitude of Signal	Comparative Level in db $E_m/E_c = .2$	$E_m/E_c = .1$
0	$1 + \frac{1}{2}\left(\frac{E_m}{E_c}\right)^2 - \frac{7}{32}\left(\frac{E_m}{E_c}\right)^4$	0	0
2p, 2q	$\left(\frac{E_m}{E_c}\right)^2\left[-\frac{1}{4} + \frac{3}{16}\left(\frac{E_m}{E_c}\right)^2\right]$	-40	-72
4p, 4q	$\frac{1}{64}\left(\frac{E_m}{E_c}\right)^4$	-112	-136
(p + q), (p-q)	$\left(\frac{E_m}{E_c}\right)^2\left[+\frac{1}{2} - \frac{1}{4}\left(\frac{E_m}{E_c}\right)^2\right]$	-54.2	-66
(3p + q), (3p - q)	$\frac{1}{16}\left(\frac{E_m}{E_c}\right)^4$	-80	-124
(p + 3q), (p-3q)	$\frac{1}{16}\left(\frac{E_m}{E_c}\right)^4$	-80	-124
(2p - 2q, (2p + 2q)	$\frac{1}{16}\left(\frac{E_m}{E_c}\right)^4$	-80	-124

From the data of Tables I and II a composite spectrum is produced having odd order components due to phase distortion plus even order terms due to amplitude effect. Since the desired 1st order sideband is at a level 23 db below the carrier for $\frac{E_m}{E_c} = .1$ then a peak phase deviation of about .2 radians is possible within the fidelity requirements of the specification. Thus assuming no other degradation within the transmitter it is seen that for a peak output deviation of 4.0 radians, the multiplier chain must provide a factor of at least 20. Admitting that further degradation will occur within the transmitter it is advisable to provide a larger multiplication factor which is selected for circuit convenience, as for example a doubler followed by two quadruplers giving a total multiplication of 32 with the basic modulator operating at 0.125 radians peak.

B. Linearity Computations for Delay Lines Phase Shift Section:

(1) General Discussion: A distributed phase modulator, which takes the form of an artificial delay line was shown in Figure 1. The voltage variable capacitors have a constant d-c bias applied to obtain operation on the linear portion of the voltage-capacitance curve. The modulating voltage, applied through the appropriate capacitors, is used to bias the VARICAPS and causes a small change in time and phase. If each section of the delay line is the same, then the phase deviation attributed to one section is multiplied by the number of sections. If the modulating signal delay for each section is the same as the delay experienced by the carrier, then a greater linear phase deviation can be achieved than by a conventional phase modulator before multiplication.

The coil L1 is a RF choke to ground the line to the modulator and to D-C, to prevent undesired interaction between the diodes (VARICAPS). The difficulties encountered with this modulator are listed as follows:

(1) losses in the lines

(2) reflections due to improper terminations and change of line impedance with modulation

(3) distortion due to non-linearity in the VARICAPS and inherent distortion

(4) errors in switching delays and variation in delay over the modulating frequency band.

Difficulty (1) may be overcome by amplifiers, (2) and (3) may be minimized by the small degree of modulation in each stage, and (4) may be minimized by the use of a wide band delay device for the modulating signal such as a coaxial cable. An analysis of the distributed phase modulator follows.

(2) <u>Analysis of Delay Line Modulation:</u> The circuit paramters are defined as:

- δC - change in line cap. due to Varicap
- δT - change in line delay due to δC
- $\delta \phi$ - change in phase of line signal due to δC
- C_T - total shunt capacitance = $C + Co$
- Co - constant component of the Varicap
- C - fixed shunt capacitor
- V_m - modulating signal

$$\delta C = f(V_m) \qquad (17)$$

$$\delta T = \sqrt{L}\left(\sqrt{C+\delta C} - \sqrt{C}\right) \qquad (18)$$

$$\phi = 2\pi f_o \sqrt{L}\left(\sqrt{C+\delta C} - \sqrt{C}\right). \qquad (19)$$

$$= K\left(\sqrt{1+\frac{\delta C}{C}} - 1\right) \qquad (20)$$

where $K = 2\pi f_o (T)$ and $T = \sqrt{LC}$

expanding $\sqrt{1+\delta C/C}$ in a Binominal series

$$\delta\phi = K\left[1/2\left(\frac{\delta C}{C}\right) - 1/8\left(\frac{\delta C}{C}\right)^2 + 1/16\left(\frac{\delta C}{C}\right)^3 - \dots\right] \qquad (21)$$

when $\frac{\delta C}{C}$ is expressed as a function of the modulating signal, distortion may be determined

for f_o = 50 Mc

L = 1 μh

C = 5 μμf (22)

Co = 5 μμf

C_T = 10 μμf

so that $X_L = X_{C_T}$ = 315 ohms

A varicap was chosen such that

C_o (at 16 v d-c bias) = 5 μμf (23)

and with a ±4 volt swing of the modulating signal

Varicap sensitivity = -0.164 μμf/volt (24)

$$\therefore \left(\frac{\delta C}{C_T}\right)_{max.} = \frac{0.164 \quad (4)}{10} = 0.0656 \qquad (25)$$

and $K = 2\pi fo \sqrt{LC_T}$

$$= 2\pi\,(50 \times 10^6) \sqrt{10 \times 10^{-18}}$$

$$= 0.993 \qquad (26)$$

Using only the first term of the equation (21), the peak phase deviation for one section is given by

$$\delta \phi_{peak} = 0.993 \; \frac{(0.0656)}{2} \qquad (27)$$

= 0.0325 radians per delay section. To compare with the deviation expected, using the Armstrong modulator, above, an output deviation of ±0.125 radians will require .

$$\frac{0.125}{0.0325} \approx 4 \text{ sections} \qquad (28)$$

The distortion is now calculated using $\left(\frac{\delta C}{C}\right)$ peak = 2x with three terms of the expansion. Two modulating tones are used:

65-335 MPP.E

$$\delta\phi \approx K\left[\frac{1}{2}\ \frac{\delta c}{c} - \frac{1}{8}\left(\frac{\delta c}{c}\right)^2 + \frac{1}{16}\left(\frac{\delta c}{c}\right)^3 - \cdots\right] \tag{29}$$

letting $\frac{\delta c}{c} = x\left[\sin pt + \sin qt\right]$

$$\frac{\delta\phi(t)}{2} \approx K\left[\frac{x}{2}(\sin pt + \sin qt) - \frac{x^2}{8}(\sin^2 pt + 2\sin pt \sin qt + \sin^2 qt) + \frac{x^3}{16}(\sin^3 pt + 3\sin^2 pt \sin qt + 3\sin pt \sin^2 qt + \sin^3 qt) - \cdots\right] \tag{30}$$

As in eq. (25), letting $x = 6.56 \times 10^{-2}$, evaluation of (30) indicates the spectrum of Table III.

TABLE III - SPECTRUM FOR A SINGLE DELAY LINE SECTION

Frequency	Amplitude	db
p, q	3.32×10^{-2}	0
d-c	5.38×10^{-4}	-35.8 db
2p, 2q	2.69×10^{-4}	-41.8 db
p + q, p - q	5.38×10^{-4}	-35.8 db
q + 2p, q - 2p	1.325×10^{-5}	-68.0 db
p + 2q, p - 2 q	1.325×10^{-5}	-68.0 db
3p, 3q	4.42×10^{-6}	-78.5 db

The maximum allowable value of x may be now determined (the maximum undesired component must be $\leq$ - 50 db)

$$20 \log \frac{\left(\frac{x}{2} + \frac{9x^3}{64}\right)}{\frac{x^2}{8}} = 50$$

$$\frac{\frac{x}{2} + \frac{9x^3}{64}}{\frac{x^2}{8}} = 316$$

$$9x^2 - 2528x + 32 = 0$$

$$x^2 - 280.89x + 3.55 = 0 \qquad (31)$$

using the quadratic formula and log tables the maximum value is found to be

$$x_{max} \approx 0.02 \qquad (32)$$

if we again choose only the first term of the series

$$\frac{\delta \phi}{2} \max = 0.993 \left(\frac{x_{max}}{2} \right) \qquad (33)$$

$$\delta \phi_{max} = 0.01986 \text{ radians per section.} \qquad (34)$$

The number of sections required for an output deviation of 0.125 radians is therefore:

$$\frac{0.125}{0.0199} = 6.3, \text{ or at least 7 sections.} \qquad (35)$$

65-325
APP. E

The advantage cited initially for the delay line modulator was its potential capability of generating large phase deviations directly without a necessity for subsequent frequency multiplication. To achieve the full desired deviation apparently requires:

$$\frac{4.0}{.0199} \approx 200 \text{ sections!} \tag{36}$$

This result requires an extremely long delay line of such complexity as to negate its practicality from a mechanization viewpoint.

IV. EVALUATION OF CANDIDATE SYSTEMS

A. The Delay Line Modulator:

The computations presented in the analysis above, based upon the use of a conventional Varicap, have shown that a prohibitive number of sections are required to achieve even moderate phase deviations with the desired linearity of modulation. The analysis was predicated upon the non-linearity of one section and no account was taken of mismatch among sections or imperfections in time delay matching of the modulating signals to the carrier channel.

While it is possible that improvement may be effected by a choice of linearized characteristics for each Varicap of the delay line, nevertheless the complexity of the equipment and the difficulties cited in paragraph III B (1) above require a conclusion that this concept is inadequate for the RF Test Console.

B. Dual Mixer - Multiplier FM Transmitter

The dual mixer-multiplier transmitter is illustrated by the general block diagram of Figure 2a. In the first multiplication the modulated signal is multiplied by the factor N while the reference oscillator for the mixer is multiplied by a factor N $\pm$ 1 so that the phase deviation is multiplied by the factor N but the mixer output is returned to a carrier frequency of f_1. In the final multiplication the modulation channel is further multiplied by a factor P, while the reference channel is multiplied by an additional factor Q. The resultant output is taken at the 50 mc output with multiplication of the phase deviation by the full product of NP.

The output center frequency is specified at 50 mcs. The intial carrier frequency is selected sufficiently high to be compatible with the modulating spectrum, deviation, and consequent required bandwidth. Since the bandwidth required at the output frequency is also required at any multiple or sub-multiple frequency, there will be a lower limit on the frequency at which phase modulation is initially obtained. From a standpoint of circuit design, it is desired that the percentage information bandwidth be kept low (i.e. ≈ 20%) where percentage information bandwidth is defined as

$$\% \text{ BW} = \frac{\text{Required Bandwidth}}{\text{Output Center Frequency}} \times 100\% \qquad (37)$$

In the wide bandwidth case, a 9 mc information bandwidth is required. This limits the system carrier frequency to a minimum of approximately 50 Mc. Although some other frequency of this order would be perfectly acceptable, for convenience and compatibility of the mechanization it is logical to operate at the phase modulator with an initial carrier frequency of 50 mc, this being easily obtained by simple synthesis from the frequency standard oscillator.

The actual mechanization of the transmitter is much more complicated than indicated by the simple diagram of Fig. 2a. Major complication required for an adequate system lies in the necessity in providing selective filtering particularly in the modulation multiplication chain where constant group delays are required if distortion is to be avoided in the filtering. The filter designs must therefore be of the Bessel type with parameters selected so that the total bandwidth of all such filters taken in cascade within the modulation channel possess a flat delay response over the 9 mc information bandwidth. In addition to the basic filtering problem, amplification will be required for isolation and for establishing practical operating levels.

With these requirements in mind the more complicated diagram of Fig. 2b is representative of the RF Test Console requirements where the phase deviation is limited to 1/8 radian at the modulator. A total multiplication factor of 32 provides the desired 4.0 radians deviation at the output 50 mcs terminal. The frequency multipliers are quite feasible being selected as doublers, triplers, and quadruplers as shown.

On the modulation-multiplication channel the most critical filter requirement which appears upon brief study of the system appears to be that of the bandpass filter at 100 mcs following the first mixer and frequency doubler. For example, from the universal data for a six pole Bessel filter included in Table IV it is estimated that for symmetrical attenuation of 50 db at the 50 mcs input center frequency a group delay variation over the 9 mcs bandwidth centered at 100 mcs would be less than 3.5 nanoseconds.

In the diagram Fig. 2b it is assumed that a pure harmonic-free reference signal is available at 50 mcs. Should this not be the case then additional filtering would be required prior to phase modulation and prior to multiplication in the reference channel. Otherwise degradation will occur. For example the carrier frequency harmonics which could give cross-products falling in the information passband are indicated by the simple data of Table IV.

Normally, it may be expected that the higher harmonics of the carrier will converge to relatively small levels. Therefore, as a rule of thumb, the unwanted component may be neglected if

$$M + n \leq 10 \qquad (38)$$

where M and n are the numbers of the harmonics. Therefore, only two components of Table IV require attention.

TABLE IV - CARRIER HARMONICS AND MIXER CROSS-PRODUCTS

Carrier Harmonic	Frequency	1st Mixer L.O. X3	Modulation Channel X4	
1	50	150 →	200	→ Desired passband
2	100	300	400	Unwanted products
3	150	450	600	
4	200	600	800	
5	250	750	1000	
6	300	900	1200	
7	350	1050	1400	
8	400	1200	1600	
9	450	1350	1800	
10	500	1500	2000	

C. Direct Multiplier Chain (with single output mixer):

As an alternative to the dual mixer-multiplier transmitter just discussed, a simple direct multiplication to S band with a final output mixer to translate the fully deviated signal to the required 50 mcs is considered. The basic block diagram is given in Fig. 3a. Except for the output mixer typical mechanizations for such systems are seen in references 5 and 6. (Page 3).

In the expansion of the elementary diagram of Fig. 3a into a practical model difficulties are encountered in the choice of satisfactory multiplication factors. For example for a peak deviation of 4 radians the systems tabulated in Table V might be considered.

TABLE V - MULTIPLIER FACTORS

Modulator Deviation	Multiplier N	Reference Channel N +1	Reference Channel N - 1
.1425	28 = 4 x 7	29	27 = 3x3x3
.125	32 = 2 x 4 x 4	33 = 3 x 11	31
.111	36 = 3 x 3 x 4	37	35 = 5 x 7
.0835	48 = 3 x 4 x 4	49 = 7 x 7	47

Recalling that the peak phase deviation for the desired fidelity is limited to about 0.125 radians, study of the systems of Table V suggests that a suitable compromise would utilize a multiplication factor N = 36. This requires two triplers and a quadrupler in the modulation channel. Fig. 3b. While the times 5 and times 7 multipliers in the reference channel are fairly large no modulation need be sustained and very narrow band filtering is permitted.

In the direct multiplication method the problem of spurious products of mixing due to harmonics of the reference carrier are avoided since no combination of harmonics and multiplier ratios produce cross-products falling within the output passband.

In either the dual mixer-multiplier or direct multiplier mechanizations some distortion will be added by the multipliers and mixers in the system. The distortion and bandwidth problems increase with large values of multiplication, but the total multiplication factor should be small enough that adequate bandwidth and negligible distortion can be maintained.

The multiplication process increases phase error in the manner shown below:

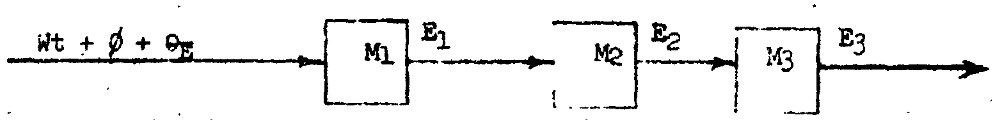

Figure 5. Multiplication of Errors in a multiplier chain.

$$E_1 = M_1 (Wt + \phi + \theta_E) + \theta_1 \tag{39}$$

$$E_2 = M_1 M_2 (Wt + \phi + \theta_E) + M_2 \theta_1 + \theta_2 \tag{40}$$

$$E_3 = M_1 M_2 M_3 (Wt + \phi + \theta_E) + M_2 M_3 (\theta_1) + M_3 \theta_2 + \theta_3 \tag{41}$$

where θ_E is the original phase error and θ_1, θ_2, θ_3 are the phase errors attributed to M_1, M_2, and M_3. θ_1, θ_2, and θ_3 can probably be assumed negligible. If θ_E is small then the degrading effects of the multipliers can be ignored.

The distortion due to the mixers is assumed negligible when compared to the other sources of error.

GC-324 APP. E

V. COMPAI ;ON OF PHASE LOCKED MODULATOR AND SMALL DEVIATION MULTIPLIER SYSTEMS

The choice of a technique for developing the required PM output signal for the RF Test Console has been narrowed to candidate systems including the phase locked modulator (ref. 11)* and the small deviation multiplier chains described above. The basic characteristics of these systems are summarized as follows:

Small Deviations Systems:

	Dual Mixer-Multiplier	Direct Multiplier
Deviation at modulator	— .125 rad peak	.111 rad peak
Multiplication factor	— 32	36
Multipliers	— X4, X2, X4	X3, X3, X4
Reference Channel Multipliers	— X3, X3	X5, X7
Number of Bessel Bandpass Filters	— 5	4
Highest Modulation Channel Frequency	— 400 mcs	1800 mcs
Highest Reference Frequency	— 450 mcs	1750 mcs
Oscillators	— Reference Crystal only	
System	— Bandpass beyond signal input	

Phase Locked Mo ator:

Deviation at Modulator	— 4.0 rad peak (direct deviation at 50 mcs)
Frequency multiplication	— None Required.
Bandpass filtering	— None Required.
Bandpass Characteristic	— Shaped by distribution of low frequency poles of the loop transfer function
Feedback Loop	— Type I phase locked with (-6 db) per octave over bandwidth of modulation spectrum.

* Page 7.

Reference frequency — 12.5 mcs

Oscillators — VCO plus reference crystal

System — Low Pass feedback loop.

A. Ease of Mechanization:

Of the candidate systems under discussion the more conventional approach certainly follows the small deviation-frequency multiplication schemes. During the course of this present investigation, however, the survey has not revealed such conventional systems that have demonstrated performance comparable to RF Test Console requirements. Although the systems appear simple on superficial examination, the multiplier, mixer, and filter designs are quite severe and once fabricated are most difficult to modify or change.

On the other hand the application of phase lock principles to modulation systems is a novel approach. This however should not be a deterrent to its application, for advances in this technology have accelerated at exponential rates within the last decade, particularly in receiver systems where performance is complicated by the presence of noise. It has been shown by the study, reported in reference 11 above,* that the full peak deviation can be achieved with the desired frequency response and fidelity. The design and mechanization, while not easy, is clearly practical with present technology. It was demonstrated that the phase locked modulator is readily adaptable to various design objectives such as maximally flat (amplitude of modulation) frequency response, or maximally flat group delay by simple adjustments of the loop gain and time constants. The necessity for design and fabrication of complex filters at carrier frequencies is avoided.

* Page 7.

Considering the unique requirements of the RF Test Console this study effort has shown that a satisfactory system may be provided with least effort utilizing the phase locked modulator approach.

B. Dual Mode PM/FM Utilization:

The RF Test Console requires a FM as well as the PM capability. It is therefore importance to consider what economy or advantage may be obtained by dual mode operation of the transmitters or its subcircuits.

A phase modulator may of course be used for FM provided the input signal is first applied to an integrator, or conversely, a frequency modulator may generate PM if proceded by a differentiator in the signal path.

The Test Console specification for FM operation requires extremely high modulation indices to accommodate extremely low signal frequencies such as 3 cps. True FM for such a low modulation frequency and for a deviation of ±500 KC would require a phase deviation of 1.67×10^5 radians which is clearly beyond the capability of the contemplated small deviation multiplier (X32 or X96) chains.

Similar problems are encountered with direct operation with the phase locked modulator because of the limited dynamic phase range of its detector. However the VCO is inherently a frequency modulation device, consequently the phase locked loop bandwidth may be constrained to a very small value to provide an AFC for the FM carrier. The details of this concept are covered in a separate report. Thus the phase locked modulator components are more adaptable to dual mode PM/FM operation.

VI. CONCLUSIONS AND RECOMMENDATIONS

During the course of this present investigation a review and classification of six basic types of phase modulators was made from which the small deviation dual mixer-multiplier and direct multiplier methods are serious candidate systems for the RF Test Console along with the phase locked modulator previously reported.

Upon consideration of the design and mechanization problems, potential performance, complexity, and adaptability to dual mode PM/FM operation it is concluded that the phase locked modulator approach offers greater economy and ease of mechanization for the RF Test Console Application. It is recommended therefore that the phase locked modulator concept be chosen for the RF Test Console equipment fabrication.

TABLE IV

MAXIMALLY FLAT DELAY NETWORKS   N = 1 THRU N = 11

INPUT DATA

FREQUENCY RANGE			NUMBER OF POINTS
0.	TO	2.00000E 01	101
0.	TO	0.	0
0.	TO	0.	0

FREQUENCY IN RADIANS PER SECOND
OUTPUT POINTS IN ARITHMETIC SERIES

SIGMA(J) IS REAL COMPONENT OF POLE OR ZERO.
RHO(J) IS IMAGINARY COMPONENT OF POLE OR ZERO.
N(J) IS ORDER OF THE POLE OR ZERO AND HAS + SIGN
FOR A POLE AND - SIGN FOR A ZERO.

J	SIGMA(J)	RHO(J)	N(J)
1	-4.24836E 00	8.67510E-01	1
2	-4.24836E 00	-8.67510E-01	1
3	-3.73571E 00	2.62627E 00	1
4	-3.73571E 00	-2.62627E 00	1
5	-2.51593E 00	4.49267E 00	1
6	-2.51593E 00	-4.49267E 00	1

TOTAL NUMBER OF POLES AND ZEROS K= 6 ,CONSTANT= 1.03950E 04

OUTPUT DATA

FREQ(I) = FREQUENCY IN RADIANS PER SECOND
ALPHA(I) IS VALUE OF THE GAIN IN DECIBELS
D1GAIN(I) IS FIRST DERIVATIVE OF GAIN (IN DECIBELS)
D2GAIN(I) IS SECOND DERIVATIVE OF GAIN
THETA(I) IS VALUE OF THE PHASE IN RADIANS
D1PHASE(I) IS FIRST DERIVATIVE OF PHASE (IN RADIANS)
D2PHASE(I) IS SECOND DERIVATIVE OF PHASE

I	FREQ(I)	ALPHA(I)	THETA(I)	D1GAIN(I)	D1PHASE(I)	D2GAIN(I)	D2PHASE(I)
1	0.	5.17719E-07	-0.	-0.	-10.00000E-01	-7.89626E-01	0.
2	2.00000E-01	-1.57956E-02	-2.00000E-01	-1.57989E-01	-10.00000E-01	-7.90585E-01	6.98492E-10
3	4.00000E-01	-6.32210E-02	-4.00000E-01	-3.16363E-01	-10.00000E-01	-7.93482E-01	-9.31323E-10
4	6.00000E-01	-1.42393E-01	-6.00000E-01	-4.75515E-01	-10.00000E-01	-7.98378E-01	3.25963E-09
5	8.00000E-01	-2.53506E-01	-8.00000E-01	-6.35855E-01	-10.00000E-01	-8.05382E-01	1.02445E-08
6	10.00000E-01	-3.96843E-01	-10.00000E-01	-7.97819E-01	-10.00000E-01	-8.14661E-01	1.01048E-07
7	1.20000E 00	-5.72774E-01	-1.20000E 00	-9.61886E-01	-10.00000E-01	-8.26452E-01	7.07572E-07
8	1.40000E 00	-7.81772E-01	-1.40000E 00	-1.12859E 00	-10.00000E-01	-8.41096E-01	3.64357E-06
9	1.60000E 00	-1.02443E 00	-1.60000E 00	-1.29855E 00	-9.99998E-01	-8.59075E-01	1.48171E-05
10	1.80000E 00	-1.30146E 00	-1.80000E 00	-1.47249E 00	-9.99992E-01	-8.81062E-01	5.02039E-05
11	2.00000E 00	-1.61374E 00	-2.00000E 00	-1.65130E 00	-9.99974E-01	-9.07970E-01	1.46913E-04
12	2.20000E 00	-1.96237E 00	-2.19999E 00	-1.83608E 00	-9.99924E-01	-9.40971E-01	3.80982E-04
13	2.40000E 00	-2.34853E 00	-2.39996E 00	-2.02819E 00	-9.99803E-01	-9.81449E-01	8.92079E-04
14	2.60000E 00	-2.77424E 00	-2.59990E 00	-2.22925E 00	-9.99534E-01	-1.03083E 00	1.91254E-03
15	2.80000E 00	-3.24109E 00	-2.79976E 00	-2.44119E 00	-9.98981E-01	-1.09026E 00	3.79404E-03
16	3.00000E 00	-3.75158E 00	-2.99946E 00	-2.66605E 00	-9.97927E-01	-1.16004E 00	7.02020E-03
17	3.20000E 00	-4.30851E 00	-3.19887E 00	-2.90582E 00	-9.96044E-01	-1.23894E 00	1.21893E-02
18	3.40000E 00	-4.91501E 00	-3.39779E 00	-3.16201E 00	-9.92878E-01	-1.32349E 00	1.99508E-02
19	3.60000E 00	-5.57445E 00	-3.59590E 00	-3.43518E 00	-9.87851E-01	-1.40742E 00	3.08867E-02
20	3.80000E 00	-6.29015E 00	-3.79276E 00	-3.72434E 00	-9.80288E-01	-1.48163E 00	4.53450E-02
21	4.00000E 00	-7.06598E 00	-3.98780E 00	-4.02645E 00	-9.69481E-01	-1.53503E 00	6.32614E-02
22	4.20000E 00	-7.90124E 00	-4.18029E 00	-4.33620E 00	-9.54792E-01	-1.55627E 00	8.40284E-02
23	4.40000E 00	-8.79955E 00	-4.36942E 00	-4.64618E 00	-9.35757E-01	-1.53607E 00	1.06480E-01
24	4.60000E 00	-9.75914E 00	-4.55429E 00	-4.94753E 00	-9.12193E-01	-1.46954E 00	1.29032E-01
25	4.80000E 00	-1.07774E 01	-4.73401E 00	-5.23096E 00	-8.84253E-01	-1.35760E 00	1.49953E-01

26	5.00000E 00	-1.18498E 01	-4.90773E 00	-5.48798E 00	-8.52424E-01	-1.20697E 00	1.67697E-01
27	5.20000E 00	-1.29703E 01	-5.07477E 00	-5.71190E 00	-8.17461E-01	-1.02865E 00	1.81167E-01
28	5.40000E 00	-1.41320E 01	-5.23457E 00	-5.89846E 00	-7.80278E-01	8.35574E-01	1.89852E-01
29	5.60000E 00	-1.53271E 01	-5.38680E 00	-6.04598E 00	-7.41834E-01	-6.40266E-01	1.93808E-01
30	5.80000E 00	-1.65478E 01	-5.53128E 00	-6.15511E 00	-7.03037E-01	-4.53087E-01	1.93530E-01
31	6.00000E 00	-1.77868E 01	-5.66804E 00	-6.22826E 00	-6.64656E-01	-2.81398E-01	1.89769E-01
32	6.20000E 00	-1.90370E 01	-5.79721E 00	-6.26900E 00	-6.27306E-01	-1.29511E-01	1.83363E-01
33	6.40000E 00	-2.02925E 01	-5.91906E 00	-6.28150E 00	-5.91433E-01	8.51824E-04	1.75125E-01
34	6.60000E 00	-2.15480E 01	-6.03391E 00	-6.27008E 00	-5.57332E-01	1.09878E-01	1.65754E-01
35	6.80000E 00	-2.27997E 01	-6.14212E 00	-6.23888E 00	-5.25169E-01	1.98991E-01	1.55816E-01
36	7.00000E 00	-2.40425E 01	-6.24411E 00	-6.19167E 00	-4.95014E-01	2.70286E-01	1.45739E-01
37	7.20000E 00	-2.52750E 01	-6.34026E 00	-6.13179E 00	-4.66862E-01	3.26124E-01	1.35827E-01
38	7.40000E 00	-2.64945E 01	-6.43098E 00	-6.06210E 00	-4.40658E-01	3.68856E-01	1.26284E-01
39	7.60000E 00	-2.76994E 01	-6.51665E 00	-5.98499E 00	-4.16315E-01	4.00674E-01	1.17236E-01
40	7.80000E 00	-2.88882E 01	-6.59762E 00	-5.90242E 00	-3.93726E-01	4.23526E-01	1.08752E-01
41	8.00000E 00	-3.00601E 01	-6.67425E 00	-5.81605E 00	-3.72775E-01	4.39099E-01	1.00861E-01
42	8.20000E 00	-3.12144E 01	-6.74684E 00	-5.72717E 00	-3.53342E-01	4.48515E-01	9.35612E-02
43	8.40000E 00	-3.23508E 01	-6.81568E 00	-5.63684E 00	-3.35312E-01	4.53361E-01	8.68361E-02
44	8.60000E 00	-3.34691E 01	-6.88105E 00	-5.54588E 00	-3.18572E-01	4.55212E-01	8.06566E-02
45	8.80000E 00	-3.45692E 01	-6.94319E 00	-5.45494E 00	-3.03015E-01	4.53666E-01	7.49879E-02
46	9.00000E 00	-3.56511E 01	-7.00233E 00	-5.36456E 00	-2.88545E-01	4.49868E-01	6.97925E-02
47	9.20000E 00	-3.67151E 01	-7.05867E 00	-5.27511E 00	-2.75069E-01	4.44341E-01	6.50324E-02
48	9.40000E 00	-3.77613E 01	-7.11241E 00	-5.18691E 00	-2.62505E-01	4.37504E-01	6.06706E-02
49	9.60000E 00	-3.87899E 01	-7.16373E 00	-5.10018E 00	-2.50777E-01	4.29692E-01	5.66721E-02
50	9.80000E 00	-3.98014E 01	-7.21278E 00	-5.01508E 00	-2.39815E-01	4.21176E-01	5.30041E-02
51	10.00000E 00	-4.07961E 01	-7.25970E 00	-4.93174E 00	-2.29555E-01	4.12168E-01	4.96362E-02
52	1.02000E 01	-4.17743E 01	-7.30464E 00	-4.85023E 00	-2.19942E-01	4.02839E-01	4.65407E-02
53	1.04000E 01	-4.27363E 01	-7.34772E 00	-4.77062E 00	-2.10923E-01	3.93323E-01	4.36924E-02
54	1.06000E 01	-4.36826E 01	-7.38905E 00	-4.69291E 00	-2.02450E-01	3.83727E-01	4.10684E-02
55	1.08000E 01	-4.46136E 01	-7.42873E 00	-4.61712E 00	-1.94492E-01	3.74133E-01	3.86480E-02
56	1.10000E 01	-4.55296E 01	-7.46687E 00	-4.54325E 00	-1.86979E-01	3.64606E-01	3.64126E-02
57	1.12000E 01	-4.64310E 01	-7.50355E 00	-4.47127E 00	-1.79905E-01	3.55196E-01	3.43454E-02
58	1.14000E 01	-4.73182E 01	-7.53886E 00	-4.40116E 00	-1.73230E-01	3.45941E-01	3.24312E-02
59	1.16000E 01	-4.81916E 01	-7.57287E 00	-4.33289E 00	-1.66924E-01	3.36870E-01	3.06565E-02

46-335 APP E

39.

60	1.18000E 01	-4.90515E 01	-7.60565E 00	-4.26640E 00	-1.60959E-01	3.28003E-01	2.90090E-02
61	1.20000E 01	-4.98983E 01	-7.63727E 00	-4.20167E 00	-1.55312E-01	3.19356E-01	2.74777E-02
62	1.22000E 01	-5.07323E 01	-7.66780E 00	-4.13864E 00	-1.49961E-01	3.10937E-01	2.60526E-02
63	1.24000E 01	-5.15539E 01	-7.69728E 00	-4.07726E 00	-1.44885E-01	3.02754E-01	2.47247E-02
64	1.26000E 01	-5.23633E 01	-7.72577E 00	-4.01753E 00	-1.40065E-01	2.94809E-01	2.34860E-02
65	1.28000E 01	-5.31610E 01	-7.75332E 00	-3.95934E 00	-1.35485E-01	2.87103E-01	2.23291E-02
66	1.30000E 01	-5.39471E 01	-7.77998E 00	-3.90267E 00	-1.31129E-01	2.79634E-01	2.12475E-02
67	1.32000E 01	-5.47221E 01	-7.80578E 00	-3.84747E 00	-1.26981E-01	2.72393E-01	2.02350E-02
68	1.34000E 01	-5.54862E 01	-7.83078E 00	-3.79370E 00	-1.23030E-01	2.65396E-01	1.92863E-02
69	1.36000E 01	-5.62397E 01	-7.85501E 00	-3.74130E 00	-1.19263E-01	2.58618E-01	1.83965E-02
70	1.38000E 01	-5.69828E 01	-7.87850E 00	-3.69023E 00	-1.15668E-01	2.52061E-01	1.75609E-02
71	1.40000E 01	-5.77159E 01	-7.90129E 00	-3.64046E 00	-1.12235E-01	2.45719E-01	1.[illegible]7756E-02
72	1.42000E 01	-5.84391E 01	-7.92340E 00	-3.59193E 00	-1.08955E-01	2.39585E-01	1.60363E-02
73	1.44000E 01	-5.91527E 01	-7.94488E 00	-3.54461E 00	-1.05818E-01	2.33653E-01	1.53411E-02
74	1.46000E 01	-5.98570E 01	-7.96574E 00	-3.49946E 00	-1.02816E-01	2.27916E-01	1.46854E-02
75	1.48000E 01	-6.05522E 01	-7.98601E 00	-3.45343E 00	-9.99410E-02	2.22369E-01	1.40669E-02
76	1.50000E 01	-6.12385E 01	-8.00572E 00	-3.40950E 00	-9.71865E-02	2.17004E-01	1.34329E-02
77	1.52000E 01	-6.19161E 01	-8.02489E 00	-3.36662E 00	-9.45457E-02	2.11816E-01	1.29[illegible]10E-02
78	1.54000E 01	-6.25852E 01	-8.04355E 00	-3.32476E 00	-9.20121E-02	2.06797E-01	1.24091E-02
79	1.56000E 01	-6.32460E 01	-8.06171E 00	-3.28389E 00	-8.95802E-02	2.01942E-01	1.19152E-02
80	1.58000E 01	-6.38988E 01	-8.07939E 00	-3.24397E 00	-8.72443E-02	1.97245E-01	1.14473E-02
81	1.60000E 01	-6.45437E 01	-8.09661E 00	-3.20498E 00	-8.49996E-02	1.92700E-01	1.10039E-02
82	1.62000E 01	-6.51809E 01	-8.11339E 00	-3.16688E 00	-8.28413E-02	1.88300E-01	1.05832E-02
83	1.64000E 01	-6.58105E 01	-8.12975E 00	-3.12965E 00	-8.07649E-02	1.[illegible]4041E-01	1.01839E-02
84	1.66000E 01	-6.64328E 01	-8.14570E 00	-3.09326E 00	-7.87664E-02	1.79713E-01	9.80455E-03
85	1.68000E 01	-6.70479E 01	-8.16126E 00	[illegible]05768E 00	-7.68418E-02	1.75925E-01	9.44398E-03
86	1.70000E 01	-6.76559E 01	-8.17644E 00	-3.02288E 00	-7.49876E-02	1.72057E-01	9.10102E-03
87	1.72000E 01	-6.82571E 01	-8.19126E 00	-2.98884E 00	-7.32003E-02	1.68309E-01	8.77460E-03
88	1.74000E 01	-6.88515E 01	-8.20573E 00	-2.95555E 00	-7.14767E-02	1.64678E-01	8.46372E-03
89	1.76000E 01	-6.94393E 01	-8.21956E 00	-2.92297E 00	-6.98139E-02	1.61158E-01	8.16747E-03
90	1.78000E 01	-7.00207E 01	-8.23366E 00	-2.89108E 00	-6.82088E-02	1.57745E-01	7.88499E-03
91	1.80000E 01	-7.05958E 01	-8.24714E 00	-2.85986E 00	-6.66590E-02	1.54436E-01	7.61548E-03
92	1.82000E 01	-7.11647E 01	-8.26032E 00	-2.82930E 00	-6.51618E-02	1.51226E-01	7.35821E-03

4

93	1.84000E 01	-7.17276E 01	-8.27321E 00	-2.79937E 00	-6.37150E-02	1.48112E-01	7.11248E-03
94	1.86000E 01	-7.22845E 01	-8.28581E 00	-2.77005E 00	-6.23161E-02	1.45090E-01	6.87765E-03
95	1.88000E 01	-7.28356E 01	-8.29814E 00	-2.74132E 00	-6.09632E-02	1.42156E-01	6.65312E-03
96	1.90000E 01	-7.33810E 01	-8.31020E 00	-2.71318E 00	-5.96542E-02	1.39309E-01	6.43833E-03
97	1.92000E 01	-7.39209E 01	-8.32201E 00	-2.68559E 00	-5.83873E-02	1.36543E-01	6.23275E-03
98	1.94000E 01	-7.44553E 01	-8.33356E 00	-2.65856E 00	-5.71605E-02	1.33857E-01	6.03588E-03
99	1.96000E 01	-7.49844E 01	-8.34487E 00	-2.63205E 00	-5.59724E-02	1.31247E-01	5.84728E-03
100	1.98000E 01	-7.55082E 01	-8.35595E 00	-2.60605E 00	-5.48211E-02	1.28711E-01	5.66650E-03
101	2.00000E 01	-7.60268E 01	-8.36680E 00	-2.58056E 00	-5.37053E-02	1.26246E-01	5.49315E-03

THE END.