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# OPTIMUM CORRECTION OF THRUST TRANSIENT MEASUREMENTS

*by J. E. Irby and J. C. Hung*

Prepared under Grant No. NsG-351 *by*
UNIVERSITY OF TENNESSEE
Knoxville, Tenn.
*for*

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## SUMMARY

A study has been made of the methods of recovering the distorted thrust transient measurement. The distortion is caused mostly by the heavy mass of the measuring thrust stand.

Two known methods are reviewed and discussed. The first method makes use of Fourier series approximation and numerical computation. This method, in general, requires the use of a digital computer. The second method employs a closed-loop network connected in cascade with the thrust stand output. The network is realized by analog computer simulation.

A third method is proposed, which makes use of an open-loop network in cascade with the thrust stand output. The overall system is adjusted to achieve a maximally flat frequency response. Experimental results indicate that this method is the best.

## INTRODUCTION

The measurement of transient thrust characteristics is an important problem. In aerospace technology the study of thrust transients requires accurate measurement of the thrust build-up and tail-off produced by rocket engines. This measurement is often complicated by the following facts:

(1) Noise in the measuring instrument corrupts the input signal.

(2) The bandwidth of the measuring instrument is much smaller than the bandwidth of the signal.

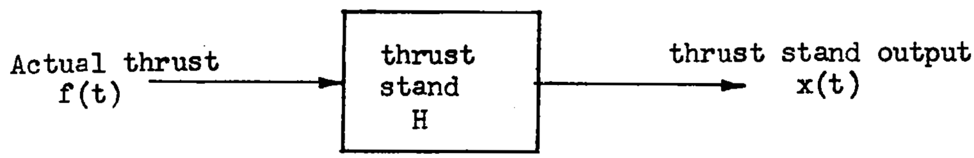
(3) The dynamic properties of the measuring instrument are time varying.

The subject of this paper is to investigate methods to cope with the difficulty caused by the second fact.

Because of the high thrust level, the supporting frame of the thrust stand must be relatively heavy, but the spring constant of the entire system cannot satisfactorily be increased. Since the thrust transient is very fast at build-up and tail-off, this leads to the situation that the bandwidth of the measuring device is narrower than the bandwidth of thrust transient signal. When this situation exists the output of the measuring device is no longer a replica of its input. Fig. 1 illustrates this phenomena.

Two methods have been developed for recovery of the actual thrust transients from the distorted measurement results. The first method makes use of Fourier series approximation and numerical computation. This method, in general, requires the use of a digital computer. The second method employs a closed-loop network connected in cascade with the thrust stand output[2]. The closed-loop network is realized by analog computer simulation.

These two methods are reviewed and discussed in this paper. In addition, a third method is proposed. This new method makes use of an open-loop network in cascade with the thrust stand output. The network can be realized either by a simple RLC network or by analog computer simulation. Comparison of the experimental results using all three methods



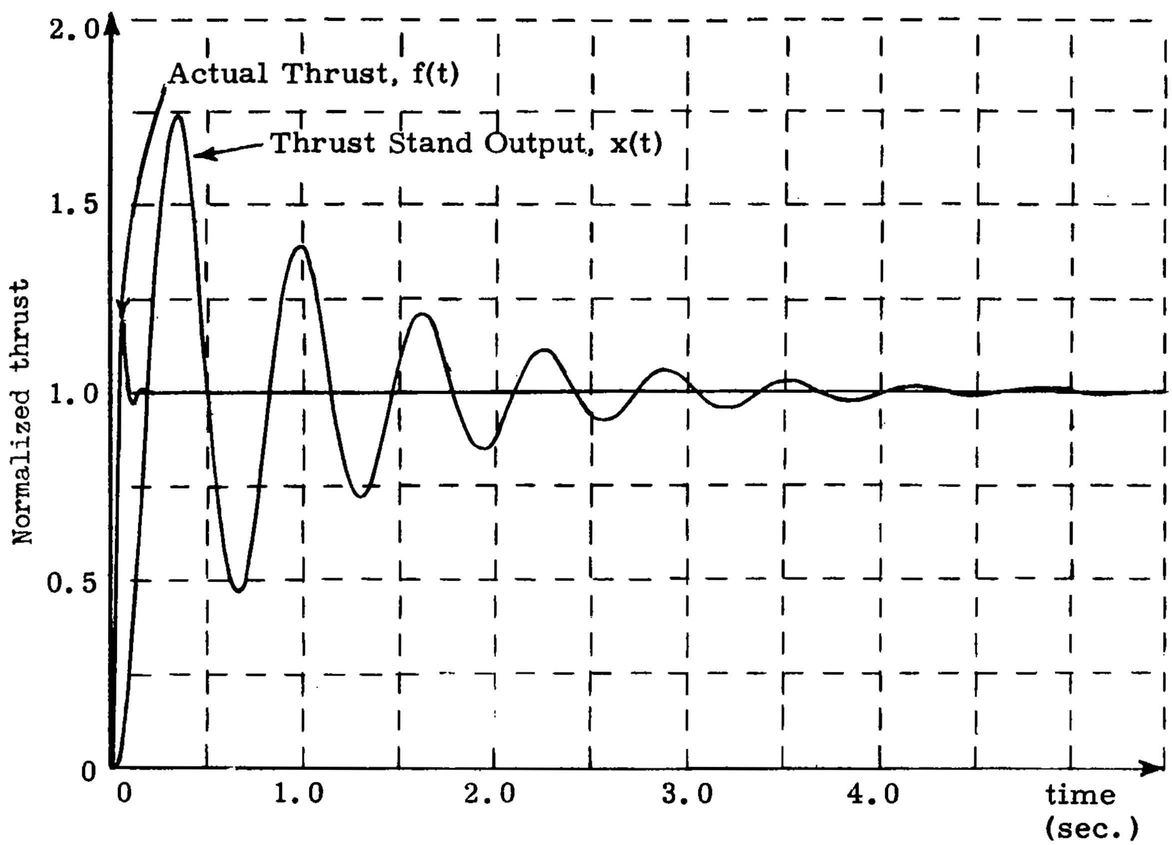




Fig. 1. Actual thrust signal and thrust stand output

indicate that the proposed method is superior.

## THRUST STAND CHARACTERISTICS

A thrust stand is normally considered to be composed of simple masses, springs, and linear dampings. Its dynamic characteristics can be determined analytically or experimentally. In many cases the dynamic equation of a thrust stand can be described satisfactorily by a second-order differential equation of the form

$$M\ddot{x}(t) + D\dot{x}(t) + Kx(t) = f(t) \qquad (1)$$

where x(t) is thrust stand displacement; M, D, and K are the mass, damping constant, and spring constant of thrust stand, respectively, and f(t) is the rocket thrust. An alternative form of (1) is

$$\frac{K}{\omega_0^2}\left[\ddot{x}(t) + 2\xi\omega_0\dot{x}(t) + \omega_0^2 x(t)\right] = f(t) \qquad (2)$$

where

$$\omega_0 = \sqrt{\frac{K}{M}} = \text{undamped natural frequency} \qquad (3)$$

$$\xi = \frac{D}{2\sqrt{MK}} = \left(\frac{D}{2K}\right)\omega_n = \text{damping ratio} \qquad (4)$$

Thus, the parameters K, $\omega_0$, and $\xi$ completely specify the characteristics of the thrust stand. These 3 parameters are considered available in the following development.

Note that, if the mathematical expression for x(t) is known explicitly, f(t) can be obtained analytically. However, in our problem, only the measured time record of x(t) is available.

## SERIES APPROXIMATION METHOD

The function x(t) can be represented in terms of sine series over the interval of interest. This is done by extending x(t) into an odd periodic function beyond the interval of interest as shown in Fig. 2. The period

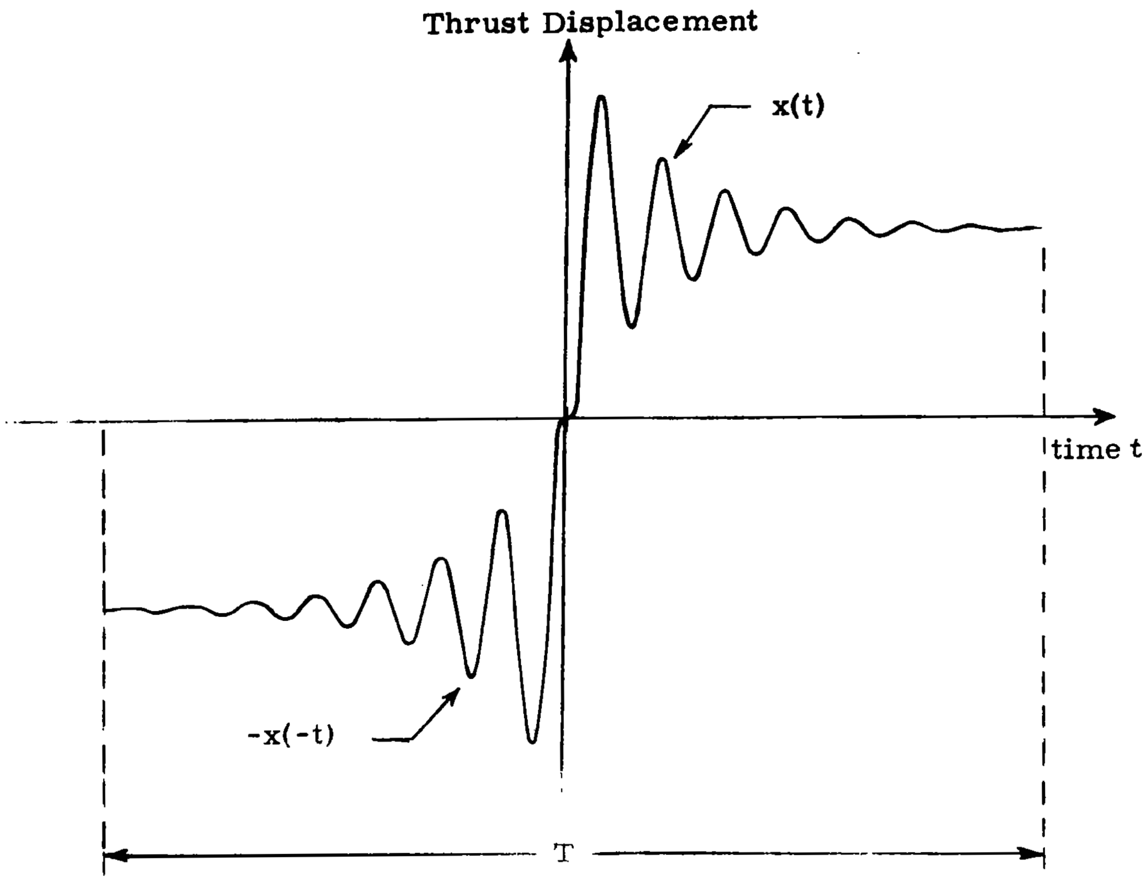



Fig. 2. Thrust stand output as an odd function.

T must be large enough so that the important portion of x(t) is contained in $\frac{T}{2}$. Analytically,

$$x(t) = \sum_{n=1}^{\infty} b_n \sin n\omega t \tag{5}$$

with

$$b_n = \frac{4}{T} \int_0^{\frac{T}{2}} x(t) \sin n\omega t \, dt \tag{6}$$

and

$$\omega = \frac{2\pi}{T} \, . \tag{7}$$

Taking the Laplace transforms of (2) and (3) and noting that $x(0) = \dot{x}(0) = 0$, since the motion of the thrust stand starts from rest, one has

$$\frac{K}{\omega_0^2} \left(s^2 + 2 \xi \omega_0 s + \omega_0^2\right) X(s) = F(s) \tag{8}$$

and

$$X(s) = \sum_{n=1} b_n \frac{n\omega}{s^2 + n^2\omega^2} \, . \tag{9}$$

Substituting (9) into (8), the n-th term for F(s) is

$$F_n(s) = \frac{n\omega b_n K \left(s^2 + 2 \xi \omega_0 s + \omega_0^2\right)}{\omega_0^2 \left(s^2 + n^2\omega^2\right)}$$

$$= \frac{n\omega b_n K}{\omega_0^2} + \frac{n\omega b_n K \left\{ \frac{2\xi}{\omega_0} + \left[1 - \left(\frac{n\omega}{\omega_0}\right)^2\right]\right\}}{s^2 + n^2\omega^2} \tag{10}$$

Taking the inverse Laplace transform of (10)

$$f_n(t) = \frac{n\omega b_n K}{\omega_0^2} \delta(t) + b_n K \left\{ \frac{2n\omega\xi}{\omega_0} \cos n\omega t + \left[1 - \left(\frac{n\omega}{\omega_0}\right)^2\right] \sin n\omega t \right\} \tag{11}$$

$$= \frac{n\omega b_n K}{\omega_0^2} \delta(t) + \frac{2n\omega \xi b_n K}{\omega_0 \sin \phi_n} \sin \left(n\omega t + \phi_n\right) \tag{12}$$

where

$$\phi_n = \tan^{-1} \frac{\frac{2n\xi\omega}{\omega_0}}{1 - \left(\frac{n\omega}{\omega_0}\right)^2} \tag{13}$$

Summing $f_n(t)$, for all n, to give f(t),

$$f(t) = \frac{\omega K}{\omega_0^2} \delta(t) \sum_{n=1}^{\infty} nb_n + \frac{2\omega\xi K}{\omega_0} \sum_{n=1}^{\infty} nb_n \sin(n\omega t + \phi_n) \tag{14}$$

The initial thrust, f(0), of a physical rocket engine can never be infinite. Therefore the impulse function term must vanish, giving the final expression.

$$f(t) = \frac{2\omega\xi K}{\omega_0} \sum_{n=0}^{\infty} nb_n \sin(n\omega t + \phi_n) \tag{15}$$

Given the measured thrust stand output x(t), the thrust stand input f(t) is obtained by using (6) and (15).

The first source of error is in the evaluation of the Fourier coefficients $b_n$, Eq. (6). The error exists because x(t) is of an irregular shape and cannot be described by a simple equation, therefore numerical integration must be used. In the numerical integration process, the expression for $b_n$ has the following form

$$b_n = \frac{4}{T} \sum_{i=1}^{q} x(t_i) \sin n\omega t_i \, \Delta t \tag{16}$$

where $\Delta t = \frac{T}{2q}$. The error can be made small by using a large q, but can never be completely eliminated.

The running index n in (15) can never go to infinity, but must stop at some finite value N. Thus another error source is due to the use of a finite series representation. This error can be made small by choosing N large. Therefore

$$f(t) = \frac{2\omega\xi K}{\omega_0} \sum_{n=0}^{\infty} nb_n \sin(n\omega t + \phi_n) \tag{17}$$

CLOSED-LOOP NETWORK METHOD

This method, developed by McGregor and Spouse[2], involves the use of a simulated closed-loop network connected in cascade with the thrust stand, as shown in Fig. 3. In this figure F(s), X(s), and G(s) are the thrust stand input, thrust stand output, and corrected output respectively. T(s), the transfer function of the thrust stand, is given by

$$T(s) = \frac{X(s)}{F(s)} = \frac{\omega_0^2/K}{s^2 + 2\xi\omega_0 s + \omega_0^2} \tag{18}$$

The corrected output can easily be obtained as

$$G(s) = \frac{kA\ T(s)}{1 + k(A + Ds)\ T(s)}\ F(s) \tag{19}$$

where k, A, and D are all pure gain constants. Substituting (16) to (17) and simplifying the expression, gives

$$G(s) = \frac{\omega_0^2\ kA}{s^2 + 2\xi_c\omega_c s + \omega_c^2}\ F(s) \tag{20}$$

with

$$\omega_c = \omega_0\sqrt{1 + kA} = \text{undamped natural frequency} \tag{21}$$

$$\xi_c = \frac{2\xi + \omega_0 D}{\sqrt{1 + kA}} = \text{damping ratio} \tag{22}$$

Comparing (18) and (20) one sees that both are second order systems, but with different undamped natural frequencies and different damping ratios.

Eq. (19) reveals that G(s) can be made as close to F(s) as one pleases by choosing sufficiently large k and A. This approach, however, would not work in a practical system. In the first place, very large values of k and A result in a very wide system bandwidth, which is highly susceptible to noise contamination. Secondly, use of amplifiers with very high gains may not be convenient.

McGregor and Sprouse chose the constants k, A, and D in such a way that

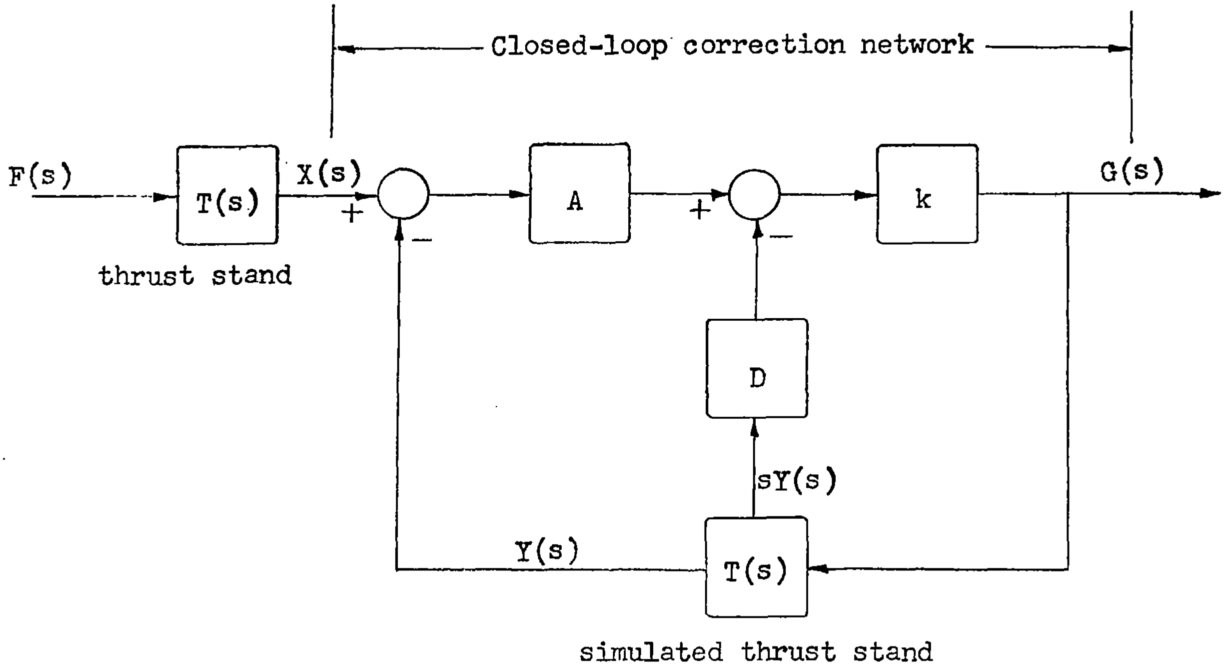



Fig. 3. Block diagram of closed-loop network method

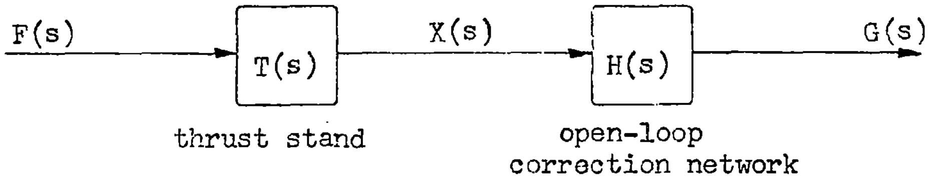



Fig. 4. Block diagram of open-loop network method

$$\left.\begin{aligned} \omega_c &= 10\omega_0 \\ \xi_c &= 0.65 \end{aligned}\right\} \tag{23}$$

The justification of this choice is not obvious. In general, the desired value of $\omega_c$ depends on the relative magnitudes of the bandwidth of the thrust signal and $\omega_0$. Eq. (23) may be optimum for a particular type of thrust signal. However, (23) is certainly not the best, as will be discussed in the next section.

It should be pointed out that the use of a closed-loop system is always troublesome. The high frequency phase shifts associated with pure gain amplifiers and the phase shift of simulated transfer function may easily cause oscillations, especially at high frequencies. Further, when the feedback loop is more complex than the forward path the advantage of a closed-loop system vanishes. The noise generated in the simulated thrust stand would not be reduced by the use of feedback.

OPEN-LOOP NETWORK METHOD

Fig. 4 is the block diagram of the open-loop network method, where the cascade network H(s) is connected in cascade with the thrust stand T(s). In general, H(s) has the form

$$H(s) = \frac{Q(s)}{P(s)} \tag{24}$$

where P(s) and Q(s) are polynomials in s.

Mathematically, the ideal correction network should have a transfer function $H(s) = T(s)^{-1}$ which is the reciprocal of thrust stand transfer function T(s). Then, the corrected output is

$$G(s) = T(s)^{-1} X(s) = T(s)^{-1} T(s) F(s) = F(s) \tag{25}$$

In view of the form of T(s), Eq. (18), $T(s)^{-1}$ is a generalized differentiator which has increasing gain with increasing frequencies. Due to the ever present amplifier noise, it is almost impossible to simulate $T(s)^{-1}$ on a real analog computer.

In practice, a satisfactory cascade correction network must either have constant gain at high frequencies or possess a low-pass characteristic. This requires that the denominator order of H(s) be equal or greater than the numerator order.

It is proposed that the numerator Q(s) of H(s) be made equal to $T^{-1}(s)$. Then the corrected output is

$$G(s) = \frac{T^{-1}(s)}{P(s)} X(s) = \frac{T^{-1}(s)}{P(s)} T(s)F(s) = \frac{1}{P(s)} F(s) \tag{26}$$

The denominator polynomial P(s) is chosen to give an optimum G(s). In general, one tries to make the order of P(s) as low as possible so that the correction network be as simple as possible. When the noise problem is not very severe, it is often satisfactory to make P(s) of the same order as Q(s). For our present problem, P(s) has the form

$$P(s) = \frac{s^2 + 2\xi_p \omega_p s + \omega_p^2}{\omega_p^2} . \tag{27}$$

Then the ratio of the corrected output to the thrust stand input is

$$M(s) = \frac{G(s)}{F(s)} = \frac{\omega_p^2}{s^2 + 2\xi_p \omega_p s + \omega_p^2} \tag{28}$$

As mentioned in the last section, the choice of $\omega_p$ depends upon the relative magnitudes of the undamped natural frequency of the thrust stand and the frequency content of the thrust signal. But for a chosen $\omega_p$, the damping ratio $\xi_p$ is selected so that M(s) has a "maximally flat" property

The amplitude square of M(s), at any frequency $\omega$, is

$$|M(\omega)|^2 = \frac{\omega_p^4}{(\omega_p^2 - \omega^2)^2 + (2\xi_p \omega_p \omega)^2}$$

$$= \frac{\omega_p^4}{\omega^4 + 2\omega_p^2 (2\xi_p^2 - 1)\omega^2 + \omega_p^4} \tag{29}$$

which is a function of $\omega^2$. The term "maximal flatness" means that the amplitude response of M(s), as a function of frequency, is made as flat as possible[3]. Mathematically, the requirement is that the derivative of $|M|^2$, with respect to $\omega^2$, be zero at $\omega = 0$. That is,

$$\left.\frac{d|M|^2}{d\omega^2}\right|_{\omega=0} = -\omega_p^4 \left.\frac{2\omega^2 + 2\omega_p^2(2\xi_p^2 - 1)}{\omega^4 + (4\xi_p^2\omega_p^2 - 2\omega_p^2)\omega^2 + \omega_p^4}\right|_{\omega=0} = 0 \quad (30)$$

Eq. (30) is satisfied when

$$2\xi_p^2 - 1 = 0$$

giving the desired damping ratio

$$\xi_p = \frac{1}{\sqrt{2}} = 0.707 \quad (31)$$

Therefore, the optimum H(s) is

$$H(s) = \frac{T^{-1}(s)}{P(s)} = K\left(\frac{\omega_p}{\omega_0}\right)^2 \frac{s^2 + 2\xi\omega_0 s + \omega_0^2}{s^2 + 1.414\omega_p s + \omega_p^2} \quad (32)$$

In general[3], if P(s) is of n-th order and of the form

$$P(s) = \frac{s^n + a_1 s^{n-1} + a_2 s^{n-2} + - - - - + a_n}{a_n} \quad (33)$$

then

$$|M|^2 = \left|\frac{1}{P(s)}\right|^2 = \frac{A_n}{\omega^{2n} + A_1\omega^{2n-2} + A_2\omega^{2n-4} + --- + A_n} \quad (34)$$

where $A_1$, $A_2$, ----, $A_n$ are functions of $a_1$, $a_2$, ----, $a_n$. For maximal flatness, $a_1$, $a_2$, ---, $a_n$ are so adjusted that

$$A_1 = A_2 = - - - - - - = A_{n-1} = 0 \quad (35)$$

The network H(s) can easily be simulated on an analog computer, or be realized by a RLC circuit and a pure gain amplifier. The use of an open-loop correction network avoids the oscillations caused by feedback. Furthermore, the computer simulation of an open-loop network is usually simpler than that of a closed-loop network.

EXAMPLE

Considering a hypothetical problem where the thrust stand dynamic is given by

$$T(s) = \frac{100}{s^2 + 2s + 100} \tag{36}$$

with

$$\omega_0 = 10 \text{ radians/second}$$
$$\xi = 0.1 \tag{37}$$
$$k = 1,$$

and where the hypothetical thrust build-up, which is the thrust stand input, is given in Fig. 5. Three different methods will be used to correct the thrust stand output.

Series Approximation Method

The period T is selected to be 12 seconds, which is, by judgement, much larger than the settling time of the thrust build-up. Therefore, $\omega = \frac{2\pi}{T} = \frac{2\pi}{12} = 0.5233$ radians per second.

Substituting the values of $\omega_0$, $\xi$, K and $\omega$ into (17) and (13) gives

$$f(t) = \sum_{n=1}^{N} \frac{0.01056nb_n}{\sin \phi_n} \sin (0.5233nt + \phi_n) \tag{38}$$

$$\phi_n = \tan^{-1} \frac{0.01040n}{1-0.00274n^2} \tag{39}$$

The Fourier coefficients, $b_n$, are determined by using (16),

$$b_n = \frac{1}{3} \sum_{i=1}^{q} x(t_i) \text{ Sin } 0.5233nt_i \, \Delta t \tag{40}$$

where the number q is chosen to be 120. The computation of (40) is done on an IBM 1620 computer.

f(t) is first calculated for N = 20. The calculated thrust signal along with the actual thrust signal are shown in Fig. 6. That twenty are insufficient is evident from the figure. The value of N is then increased to 100, and F(t) is calculated once more. The resulting curve is also shown in Fig. 6, showing a great improvement.

Closed-Loop Network Method

Using (23) and (37) in (21) and (22), gives

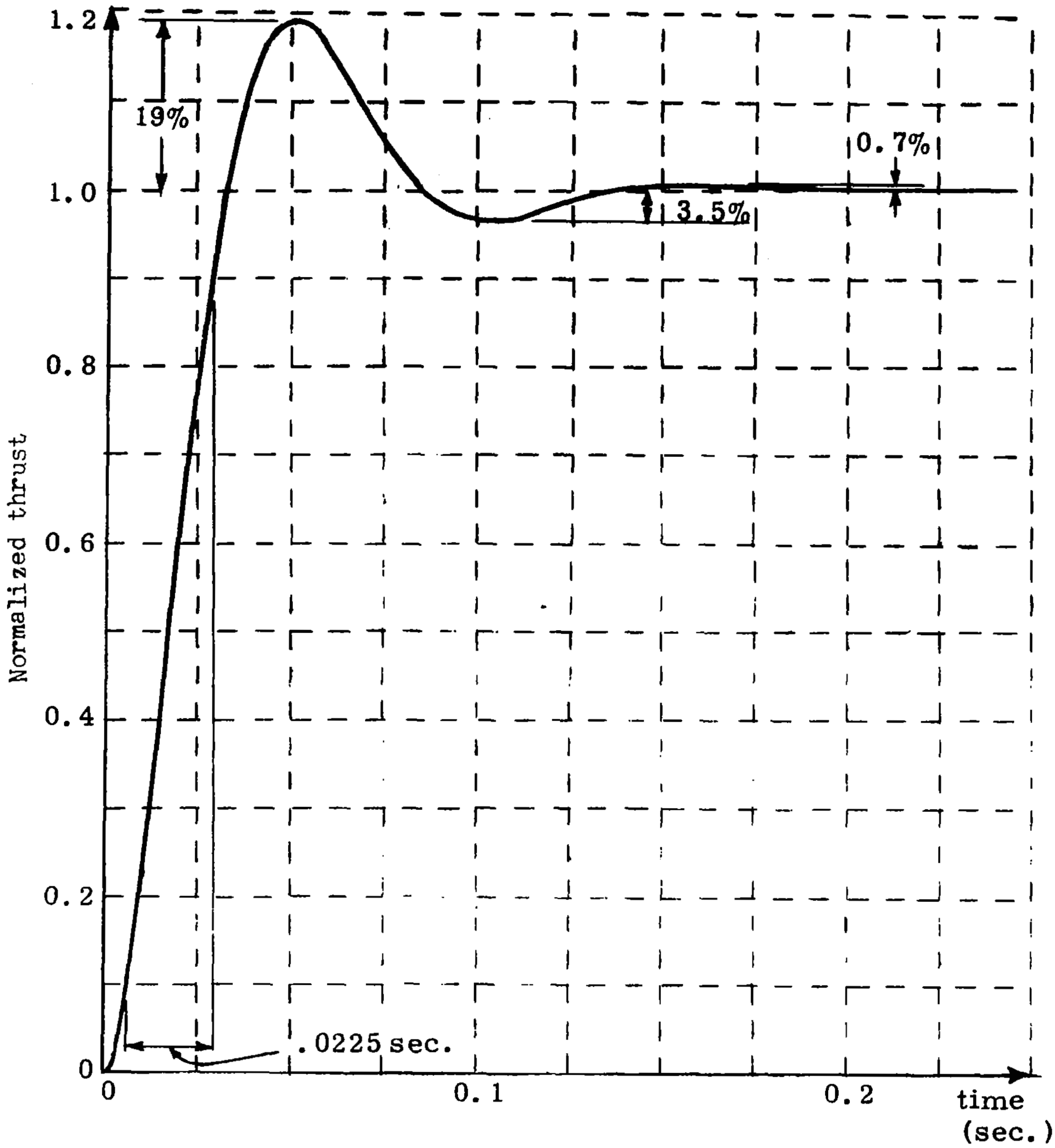



Fig. 5. The hypothetical thrust build-up

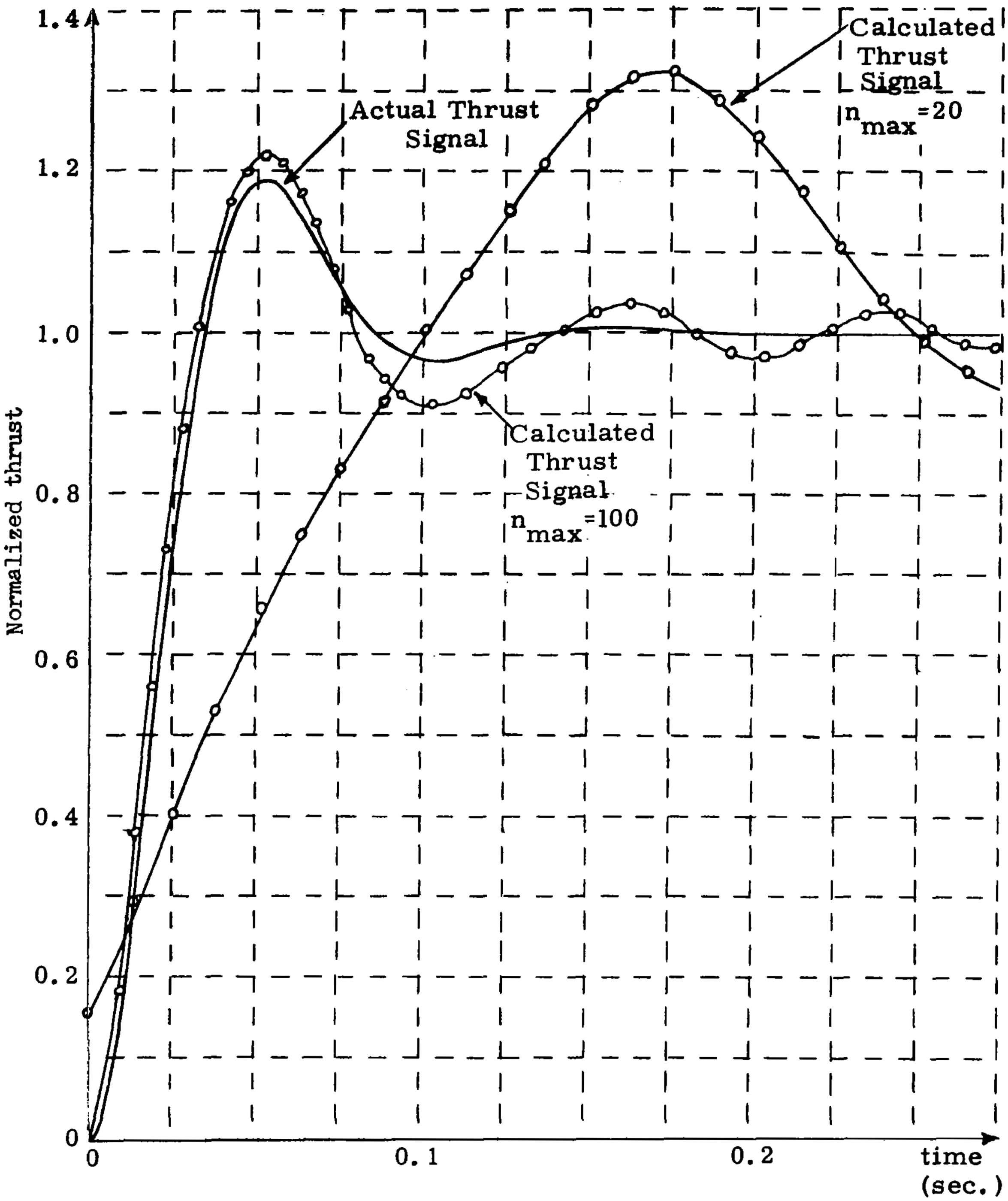



Fig. 6. Recovered thrust build-up using series approximation method

$$\sqrt{1 + A} = 10$$

$$\frac{0.2 + 10D}{\sqrt{1 + A}} = 0.65 \qquad (41)$$

Solving for A and D

$$A = 99$$
$$D = 12.8 \qquad (42)$$

The closed-loop network, Fig. 3, is then simulated on an analog computer.

The recovered thrust signal is recorded in Fig. 7. Comparing Figs. (5) and (7), one sees that an appreciable discrepancy occurs at the first peak. The recovered overshot is 24% as compared to 19% for the actual thrust. It is also seen that a heavy noise is riding on the recovered thrust curve. This noise may be due to amplifier noise or high frequency oscillation, or both.

Open-Loop Network Method

For the purpose of comparison with the preceding methods, choose P(s) to be second order having

$$\omega_p = 10\ \omega_0 \qquad (43)$$

Substituting (39) and (43) into (32), the open-loop correction network is

$$H(s) = 100\ \frac{s^2 + 2s + 100}{s^2 + 141.4s + 10{,}000} \qquad (44)$$

and the overall system transfer function is

$$\frac{G(s)}{F(s)} = T(s)\ H(s) = \frac{10{,}000}{s^2 + 141.4s + 10{,}000} \qquad (45)$$

The network (44) is simulated on an analog computer and then used to correct the thrust stand output. The result is extremely good as shown in Fig. 8. Comparing Figs. (5) and (8), no appreciable discrepancy is noted and no noise is seen.

COMPARISON AND CONCLUSION

In the series approximation method, it may be assumed that if a sufficient number of divisions were taken for the computation of the Fourier coefficients by numerical integration, and if a sufficient number

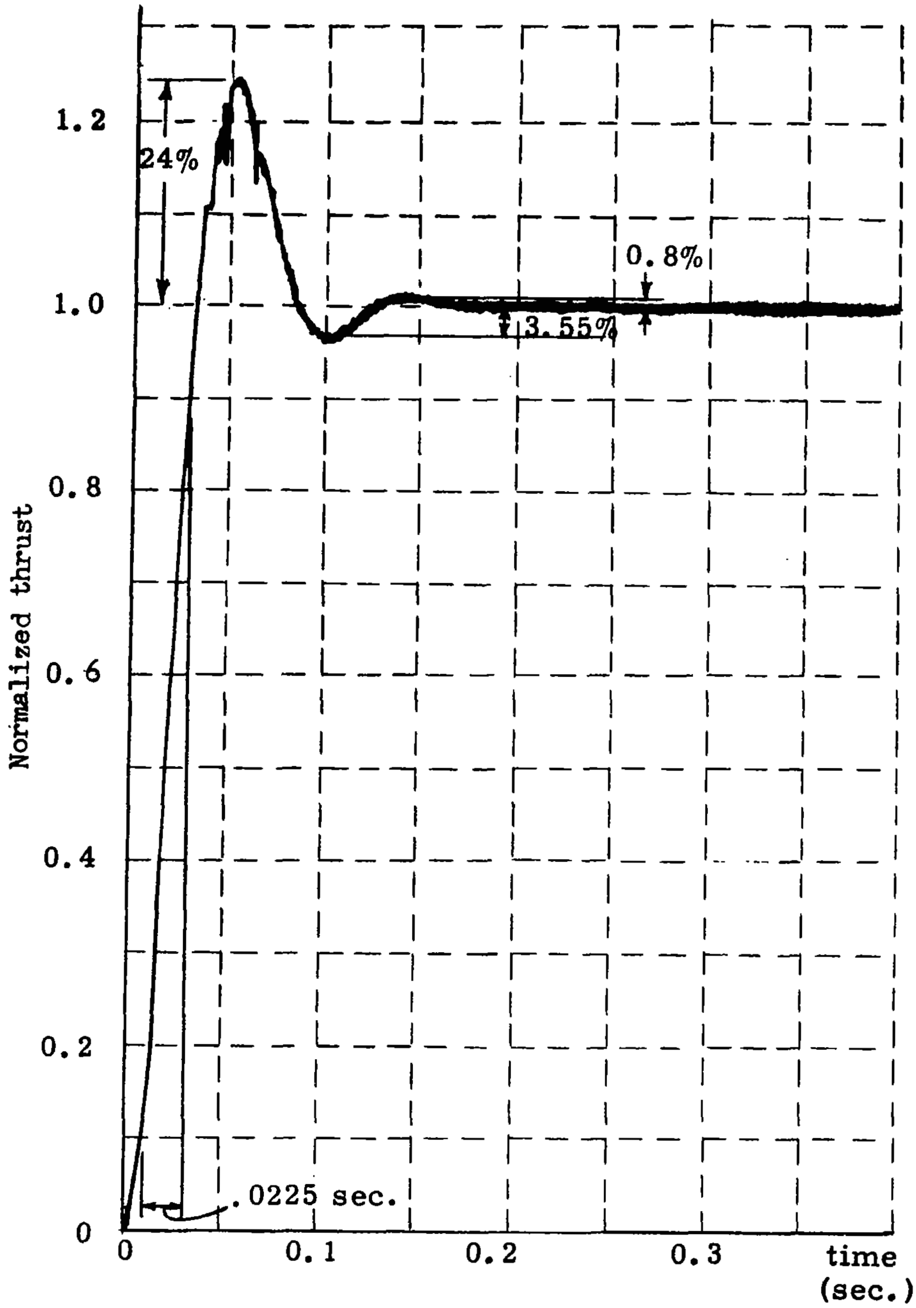



Fig. 7. Recovered thrust build-up using closed-loop correction network

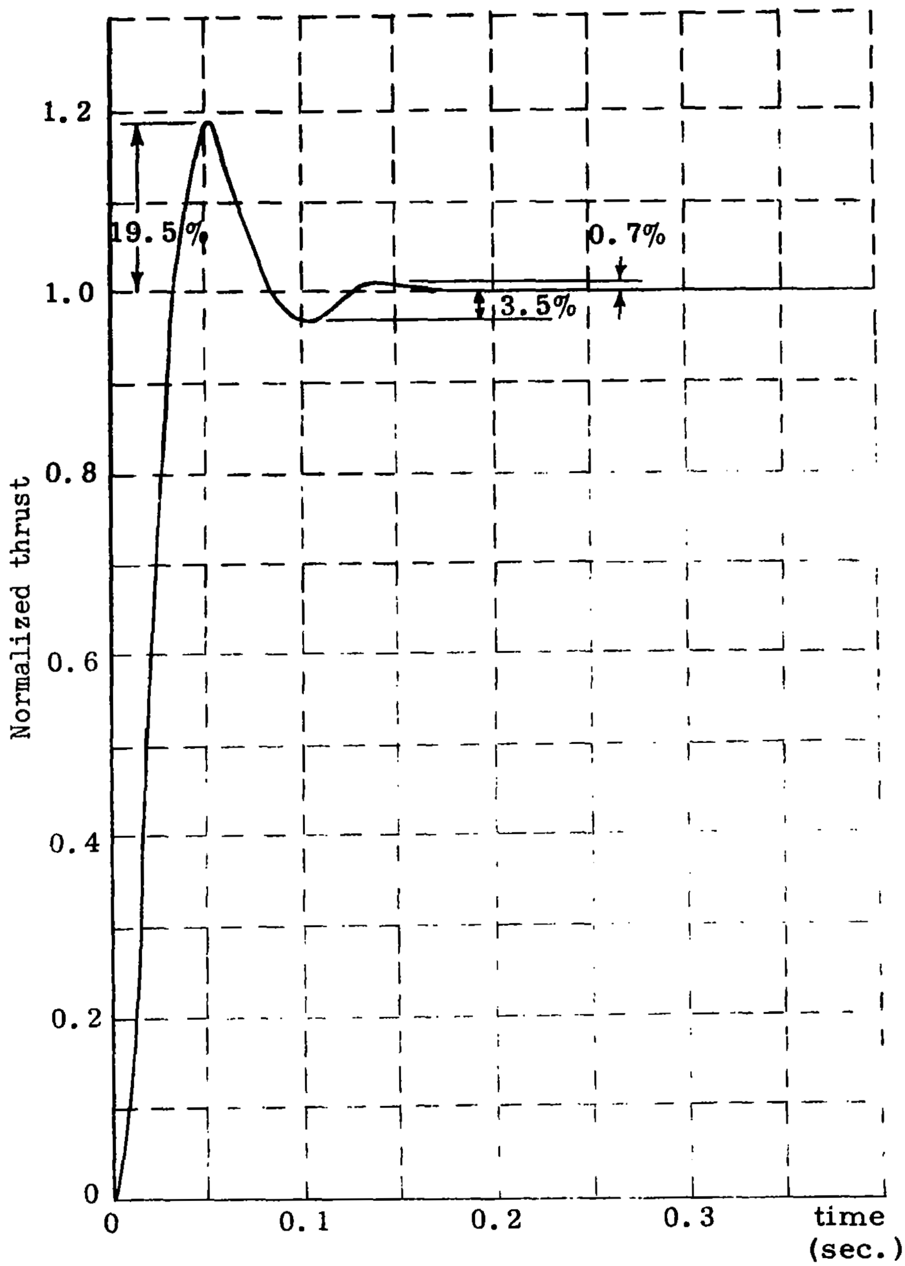



Fig. 8. Recovered thrust build-up using open-loop correction network.

of terms are utilized in the summation expression, then this method will reproduce exactly the actual rocket thrust. However, the number of divisions used in the numerical integration and the maximum value of n in the preceding example were not large enough to produce a calculated f(t) that accurately reproduces the actual f(t), and yet it took four hours from the thrust stand displacement curve to the IBM 1620 computer print-out of the time expression coordinates. Possibly this length of time could be cut in half by using a machine to record the data instantaneously for computing the Fourier coefficients. However, this is the only improvement in the time required that would be possible, due to the tremendous number of calculations necessary for each instant of time. For N = 100, 84,000 operations were necessary to compute $b_n$ and 1600 were necessary to compute f(t) at each instant of time. If 40 instants of time were computed, then 64,000 + 84,000, i.e., 148,000 mathematical operations were required. However, N = 100 is too small. N should probably be in the neighborhood of 1000 in order to produce the desired accuracy. An N of this magnitude would require the number of mathematical operations to exceed the million mark, and the time required on a digital computer would be several hours.

Therefore, it is concluded that, although the mathematical method is potentially exact, the time required to produce the accuracy desired makes this method too cumbersome to be practical.

Both the closed-loop network and open-loop network methods have the advantage that on-line correction is readily obtained and no further time is required for computation.

Comparing Figs. (7) and (8), one sees that the open-loop network method is superior to the closed-loop network method in that the former gives a more exact signal recovery and has much less noise content.

Another aspect which must be considered when discussing the advantages and disadvantages of each method is the equipment involved in the operation of each method With the numerical method, a digital computer is a necessity. With the open loop and closed loop methods, an analog computer is obviously necessary. The open loop method also demonstrates superiority in that it requires about half as many summing amplifiers and integrators as the closed loop method.

Therefore, of the three methods, the open loop method is the best.

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