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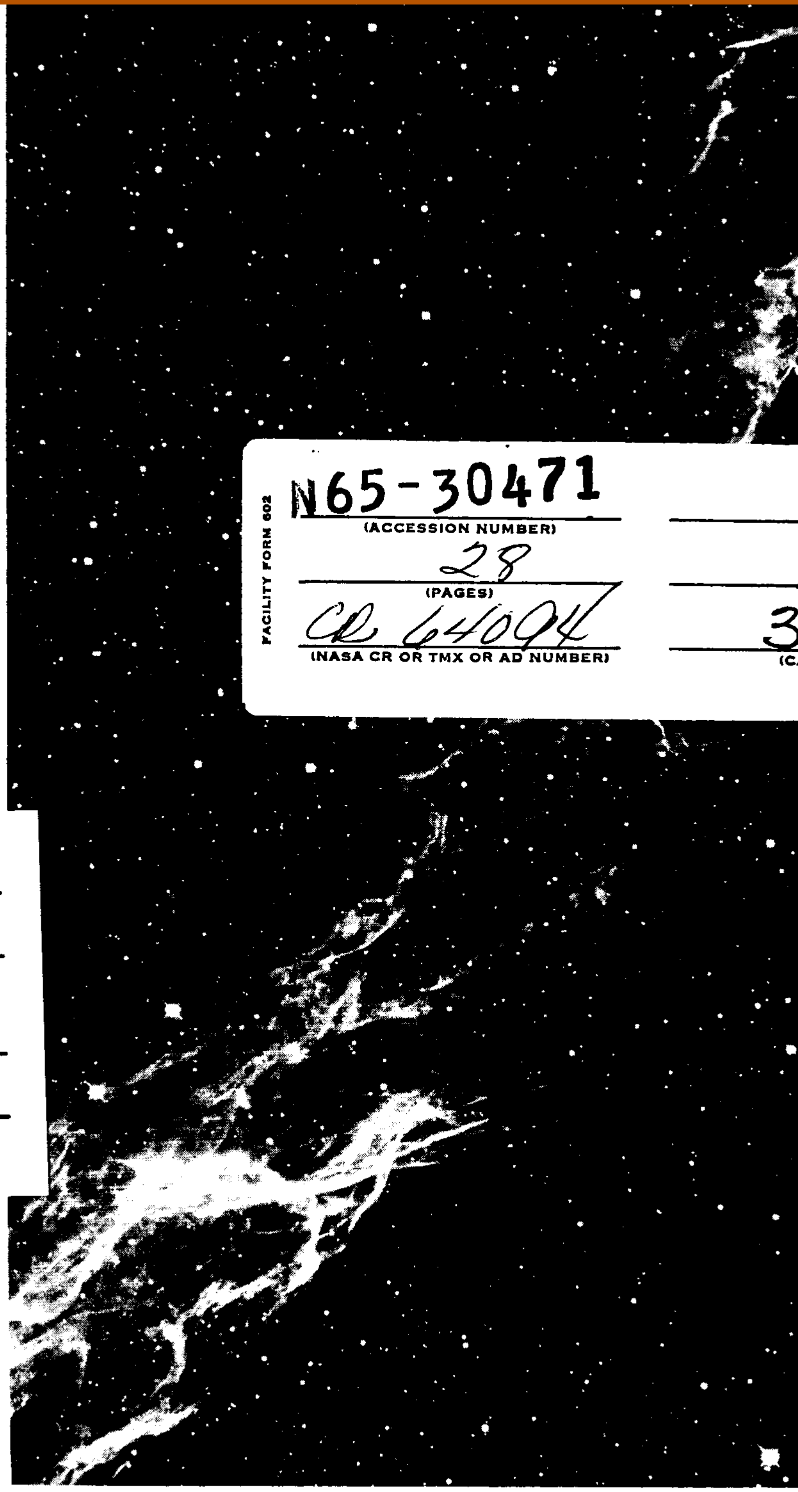

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SPATIAL DISTRIBUTION OF THE KNOWN ASTEROIDS

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by

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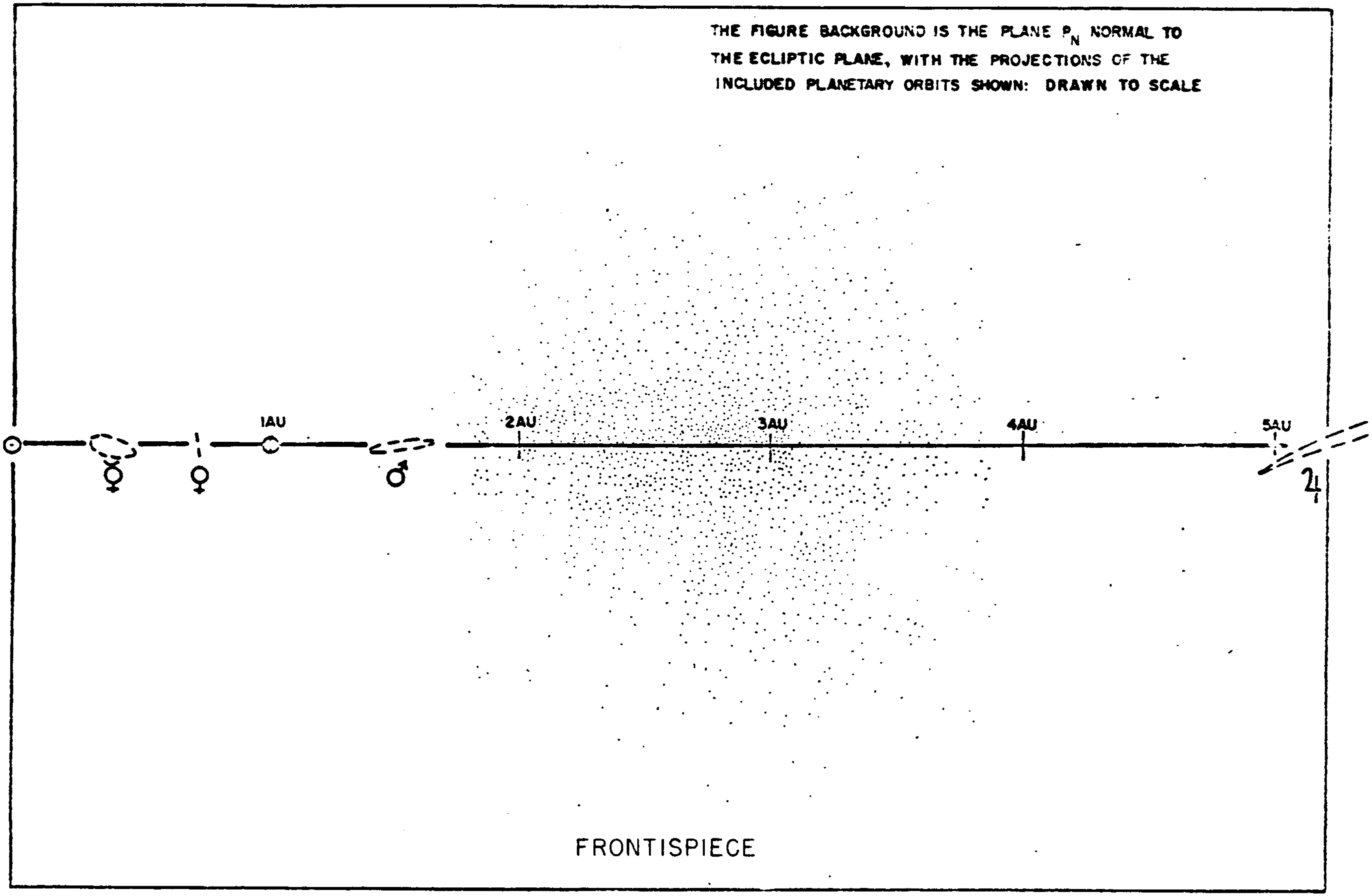
FRONTISPIECE

The distribution of numbered asteroids normal to the ecliptic plane on 4/19/73. In any given latitude range $\Delta\beta$ and radius range ΔR are shown all the numbered asteroids with β within $\Delta\beta$, R with ΔR, and any longitude λ.



FRONTISPIECE

DISTRIBUTION OF 1563 NUMBERED ASTEROIDS IN β AND R ON 4/19/73

ABSTRACT

SPATIAL DISTRIBUTION OF THE KNOWN ASTEROIDS

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The question of the distribution of asteroids in the solar system has significance in planning space flights to either avoid asteroid encounters or to maximize asteroid encounters. The orbital elements of the asteroids have been studied extensively; many regularities have been found in these orbital elements. This report differs from much of the previous work in that here the actual positions of the asteroids at specific times are analyzed, rather than their orbital elements.

In order to answer the question "Are there, at a given time, significant nonuniformities in the spatial distribution of the asteroids within the asteroid belt?" the positions of 1563 numbered asteroids and 445 unnumbered asteroids were analyzed for the 1950 to 1995 time period. The total number of asteroid position-time points used in the study was 135,000. The main conclusion of this study is that the asteroid spatial distribution does appear to contain a slight non-statistical clustering of the asteroids. The asteroids also show a slight tendency to be at latitudes β below the ecliptic plane. The primary conclusions are:

author

1) The numbered asteroid distribution in λ, the heliocentric longitude, shows clusters both because of the limited number (2008) of asteroids studied (random events cluster) and also because the asteroids are not quite homogeneous within the belt. If all of the asteroids have orbital elements similar to those of the known asteroids, the distribution in λ would not quite be uniform.

2) The numbered asteroid distribution in β, the heliocentric latitude, peaks below the ecliptic plane at a latitude of -0.1 ± 0.03 degrees.

3) The numbered asteroid distribution in R, distance from the Sun, peaks at slightly below 3.0 AU. The average R is about 2.8 AU.

4) The unnumbered asteroids exhibit the same phenomena as the numbered asteroids.

5) Since there is a statistical clustering of the known asteroids, judicious choice of trajectories could alter the chance of encounters with the known asteroids by as much as $\pm$ 30 percent. However, if all the asteroids were known, their distribution would probably be almost uniform. If all the asteroids tend to be below the ecliptic plane, as do the known asteroids, a slight decrease in asteroid encounter probability could be obtained by traversing the asteroid belt above the ecliptic plane.

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SPATIAL DISTRIBUTION OF THE KNOWN ASTEROIDS

1. INTRODUCTION

During various discussions of possible exploratory missions to the asteroid belt or beyond the question was raised whether there are, at a given time, significant non-uniformities in the spatial distribution of the asteroids within the asteroid belt. This question has significance for planning exploratory missions to either minimize or maximize asteroid encounters, and is possibly of theoretical interest in relation to the problem of the origin of the asteroid belt.

In the millions of years since the asteroid belt was formed, many factors such as planetary perturbations and collisions within the belt, have affected the positions of the asteroids. This report considers the problem of whether these factors have effected the asteroid positions in such a way as to homogenize the distribution of the asteroids within the belt, or alternatively whether the asteroids are clustered (inhomogeneous) within the belt. Even if the asteroids are effectively homogenized within the belt any calculations of asteroid distribution using a finite number of asteroids would show some clustering (statistically random events cluster). Thus the question of homogeneity or clustering in the belt is ultimately one of whether the clusters are due to the statistics of the sampling process or due to genuine physical effects.

In the work reported here the positions of 2008 asteroids were calculated between 1950 and 1995, and the number of asteroids in various volumes of space analyzed for non-statistical deviations from uniformity. Of course, since the positions of the asteroids can be calculated, the phenomena under analysis are not truly random; nevertheless the question of deviation from homogeneity may be investigated using the statistical formalism. Since there are a number of regularities in the asteroid orbital parameters (such as the tendency for asteroid perihelion longitudes to be similar to the perihelion longitude of Jupiter and the Kirkwood gaps of the asteroid periods at orbital periods which are commensurable with the orbital period of Jupiter), it is reasonable to suspect that there would be some clustering of the asteroids.

The ASC/IITRI Conic Section Trajectory System (Pierce and Narin 1964) has on its orbital elements tape the orbital elements of approximately 1600 numbered asteroids (Cincinnati Observatory 1961). This study considered the first 1563 numbered asteroids, and the 445 unnumbered asteroids for which elliptical orbital elements are on the tape, and excluded the unnumbered asteroids for which only (the very approximate) circular elements are known. For each time considered, the actual positions of the asteroids were calculated using the standard 2-body, conic section equations (Ehricke 1960).

It should be mentioned here that the relationships between asteroid orbital parameters and asteroid positions are non-linear, involving trigonometric functions and their inverses. As a result it is not an easy matter to draw conclusions about the average positions of the asteroids from the average orbital parameters.

2. DISCUSSION OF RESULTS

2.1 Clustering in Longitude λ

Figure 1 shows the heliocentric, ecliptic coordinate system. Figure 2 shows the number of asteroids in a given $\Delta\lambda$ range for 7 dates in 1951 to 1953. For ease of interpretation the areas of the graphs above average = 43.4 asteroids/10 degrees $\Delta\lambda$ are darkened. Thus the longitude clustering is shown by the darkened areas on the graphs. The motion, to the right, of the darkened areas in Figure 2 as time advances allows one to observe the formation and disintegration of the clusters as the asteroids move in and out of a given (moving) segment of space; a given cluster tends to move in the orbit at a speed quite similar to a typical asteroid speed (4.5 years per 360°). Figure 3 shows the average and median λ's for the asteroids over the 1965 to 1995 time period. The lower part of Figure 3, for the numbered asteroids, appears more smooth than the unnumbered asteroids for statistical reasons, since 3-1/2 times as many numbered as unnumbered asteroids were used in the study. Note that the curves, particularly for the numbered asteroids, seem to be symmetric about 180 degrees.

A χ^2 test has been applied to data on the λ clusters to ascertain if these clusters have a non-statistical origin. This was done in the following manner. First, the asteroids were assumed to be uniformly distributed, 43.4/10 degree $\Delta\lambda$ and

$$\chi^2 = \sum_{i=1}^{36} \frac{(N/L_i - 43.4)^2}{43.4}$$

was calculated, where N/L_i is the number of asteroids in the i_{th} $\Delta\lambda$ range. Next, using standard tables (Crow, Davis, Maxfield 1960) the probability P (χ^2) of occurrence of the given χ^2 deviation from uniformity was calculated. If the P (χ^2) were due to statistical effects, one would find that ten percent of the time P $(\chi^2) < 10$ percent, 20 percent of the time P $(\chi^2) < 20$ percent etc. with a linear distribution of

P (χ^2) in time. Consideration of Figure 4, P (χ^2) vs. time for 1950 through 1975, shows that there is a tendency for P (χ^2) to have extreme values, that is, for the asteroids to be too clustered or too uniform. From this it is concluded that some non-statistical clustering does occur in the asteroid belt.

2.2 Clustering in Latitude β

Figure 5 shows the asteroid clustering characteristics with respect to β on 1/1/65. Figure 6 shows the average and median β's for the asteroids over the 1965 to 1995 time period. As in Figure 3 the lower part of Figure 6, for the numbered asteroids, appears more smooth than the upper part of the figure for the unnumbered asteroids. Note that for both the numbered and unnumbered asteroids the curves are not symmetric about 0; rather there is a definite tendency for the average and median β's to be negative, and for the asteroids to be below the ecliptic plane.

2.3 Clustering in Radius R

Figure 7 shows the asteroid lumping characteristics with respect to R on 1/1/65. The distribution is somewhat asymmetric with a maximum asteroid density in the 2.9 to 3.0 AU range, and an $\overline{R}$ of 2.82 AU.

2.4 Positions of the Asteroids in Space

Figures 8 through 10 present, in a different manner, the spatial distributions of the asteroids. Since the distribution in λ appears to be almost uniform except for statistical clustering, these figures group all the asteroids in a given $\Delta\beta$, ΔR range together, and ignore the λ distribution.

The figure background for Figures 8 through 10 show the traces of planetary orbits, i.e., the paths the planets would trace on a plane moving (in longitude) with the planets. On any one day the planets would appear as a single dot. The horizontal widths of the traces of the planetary orbits are a measure of the eccentricity of the orbit and the vertical heights are a measure of the inclination of the orbits (Narin 1964).

Figure 8 shows, for 3/20/73, the number of asteroids in each $\Delta\beta$, ΔR segment of space. Figure 9 is similar to Figure 8, except that discrete dots representing individual asteroids have been filled uniformly in the $\Delta\beta$, ΔR segments instead of writing the numbers on the figure. The spread of the known asteroids in space is immediately apparent from Figure 9. Figure 10 for 4/19/73 is similar to Figure 8; the figure corresponding to Figure 9 for 4/19/73 is included as the frontispiece.

2.5 Average Asteroid Positions

Table 1 shows the grand averages of the β, λ and R average and median values for all of the times covered. For example, in 1965 to 1979 the average β's for the numbered asteroids were found for 37 different dates (and plotted on Figure 6). The grand average $\bar{\bar{\beta}}$ of -0.14 is the average of these 37 averages. The final average $\bar{\bar{\beta}}$ of the grand average $\bar{\beta}$'s for all asteroids for all times is -0.10 ± 0.03, where the standard deviation is taken as $\sqrt{\bar{\bar{\beta}}}$. The other grand and final averages are found in an analogous manner. It is clear from these averages, as well as the previous figures, that the asteroids tend to be about 1/10 degree below the ecliptic plane.

3. <u>CONCLUSIONS</u>

1) The numbered asteroid distribution in λ, the heliocentric longitude, shows clusters both because of the limited number (2008) of asteroids studied (random events cluster) and also because the asteroids are not quite homogeneous within the belt. If all of the asteroids have orbital elements similar to those of the known asteroids, the distribution in λ would not quite be uniform.

2) The numbered asteroid distribution in β, the heliocentric latitude, peaks below the ecliptic plane at a latitude of -0.1 ± 0.03 degrees.

3) The numbered asteroid distribution in R, distance from the Sun, peaks at slightly below 3.0 AU. The average R is about 2.8 AU.

4) The unnumbered asteroids exhibit the same phenomena as the numbered asteroids.

5) Since there is a statistical clustering of the known asteroids, judicious choice of trajectories could alter the chance of encounters with the known asteroids by as much as ± 30 percent. However, if all the asteroids

were known, their distribution would probably be almost uniform. If all the asteroids tend to be below the ecliptic plane, as do the known asteroids, a slight decrease in asteroid encounter probability could be obtained by traversing the asteroid belt above the ecliptic plane.

REFERENCES

Cincinnati Observatory "Minor Planets for 1947: Elements and Opposition Ephemerides", published under the auspices of the International Astronomical Union (1948).

Cincinnati Observatory "Elements of Unnumbered Minor Planets", prepared by the Minor Planet Center of the International Astronomical Union at the Cincinnati Observatory (1961).

Pierce, P. M. and Narin, F. "Accuracy and Capabilities of the ASC/IITRI Conic Section Trajectory System", ASC/IITRI Report No. T-5, Contract No. NASr-65(06), 1964.

Narin, F. "The Accessible Regions Method of Energy and Flight Time Analysis for One-Way Ballistic Interplanetary Missions", ASC/IITRI Report No. T-6, Contract No. NASr-65(06), 1964.

Ehricke, K. E. "Space Flight I. Environment and Celestial Mechanics", D. Van Nostrand Co. Inc., 1960.

Crow, E. L., Davis, F. A. and Maxfield, M. W. "Statistics Manual", Dover Publications, Inc. New York, 1960.

NOMENCLATURE

Ω = LONGITUDE OF THE ASCENDING NODE
$\bar{\omega}$ = LONGITUDE OF PERIHELION
ω = ARGUMENT OF PERIHELION
u = ARGUMENT OF LATITUDE
η = TRUE ANOMALY
β = HELIOCENTRIC LATITUDE
λ = HELIOCENTRIC LONGITUDE
i = INCLINATION

USEFUL RELATIONS

$$\lambda = \Omega + \mathrm{TAN}^{-1}(\mathrm{COS}\, i\ \mathrm{TAN}\, u)$$
$$\mathrm{SIN}\,\beta = \mathrm{SIN}\,u\ \mathrm{SIN}\,i$$

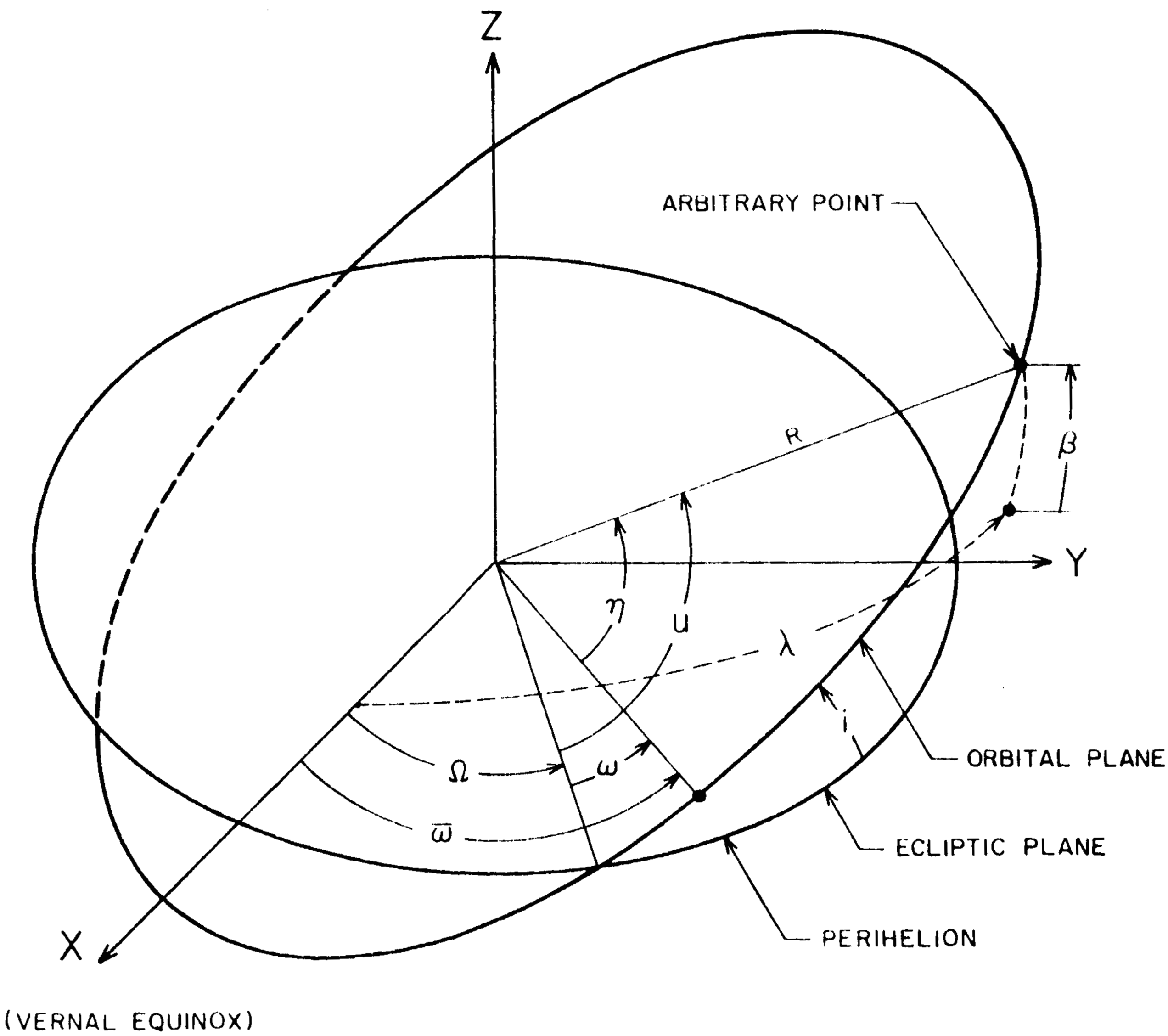

FIG. 1 HELIOCENTRIC, ECLIPTIC COORDINATE SYSTEM

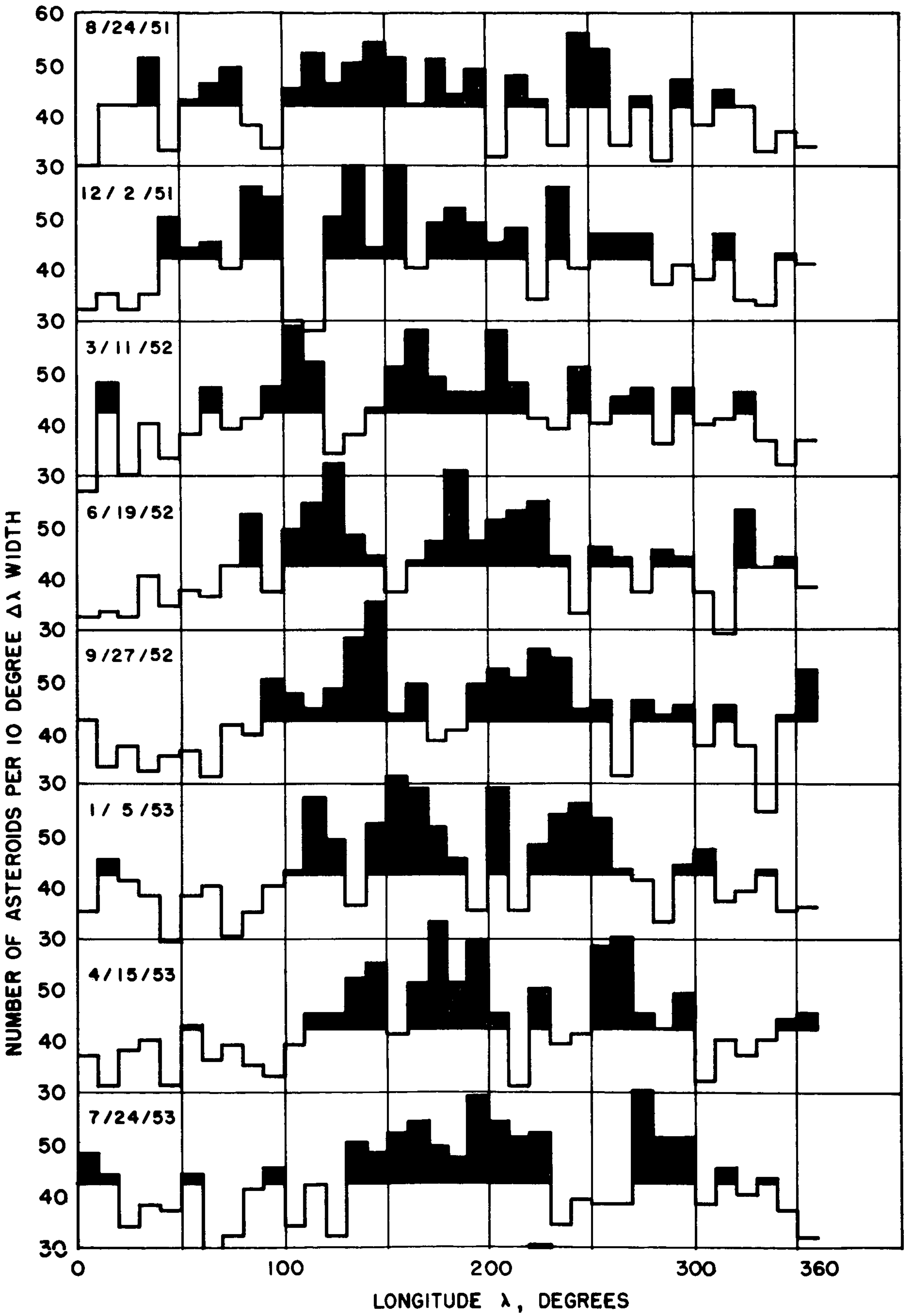

FIG. 2 LONGITUDE DISTRIBUTIONS FOR 1563 NUMBERED ASTEROIDS (ABOVE AVERAGE AREAS DARKENED)

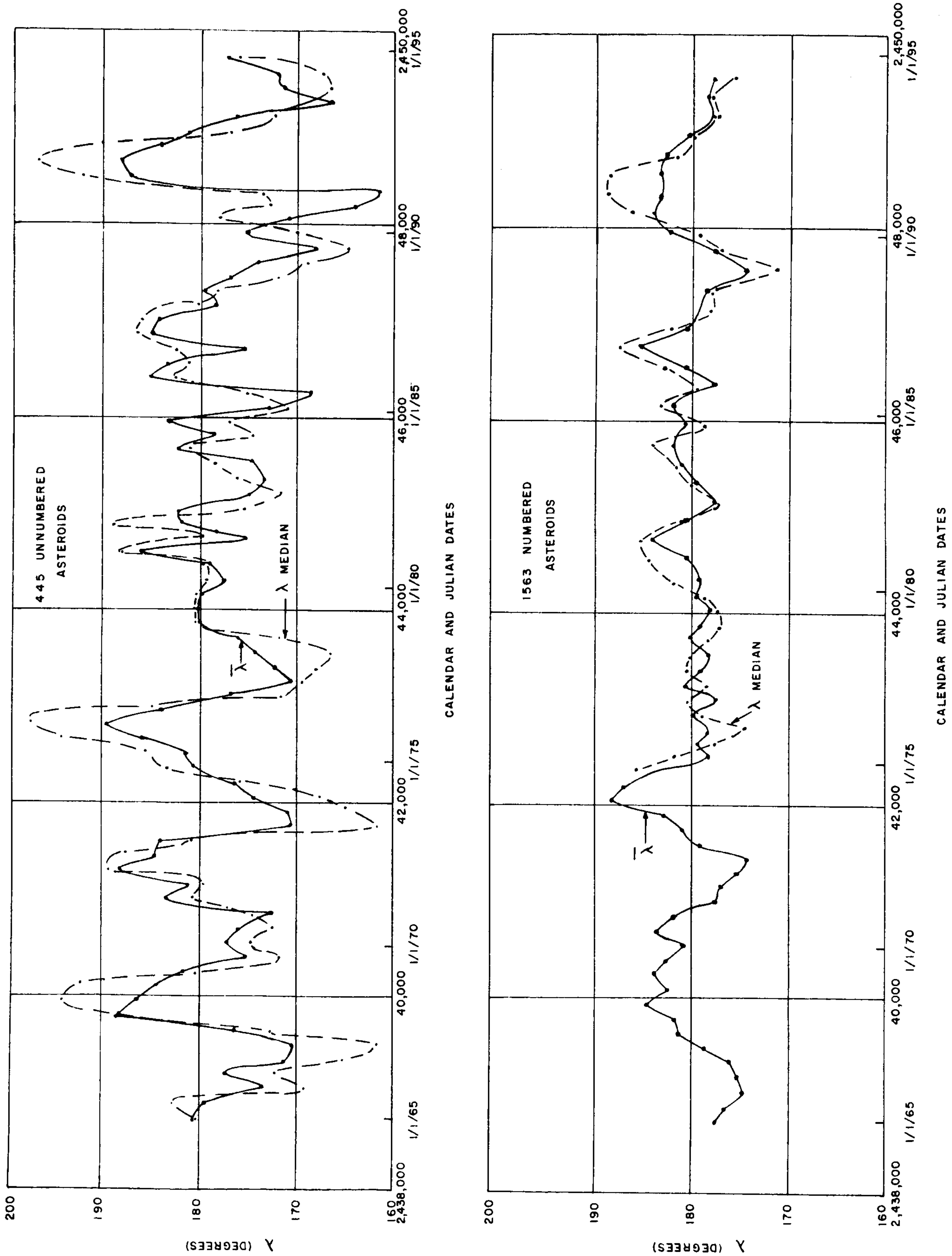

FIGURE 3 AVERAGE AND MEDIAN LONGITUDES FOR THE ASTEROIDS

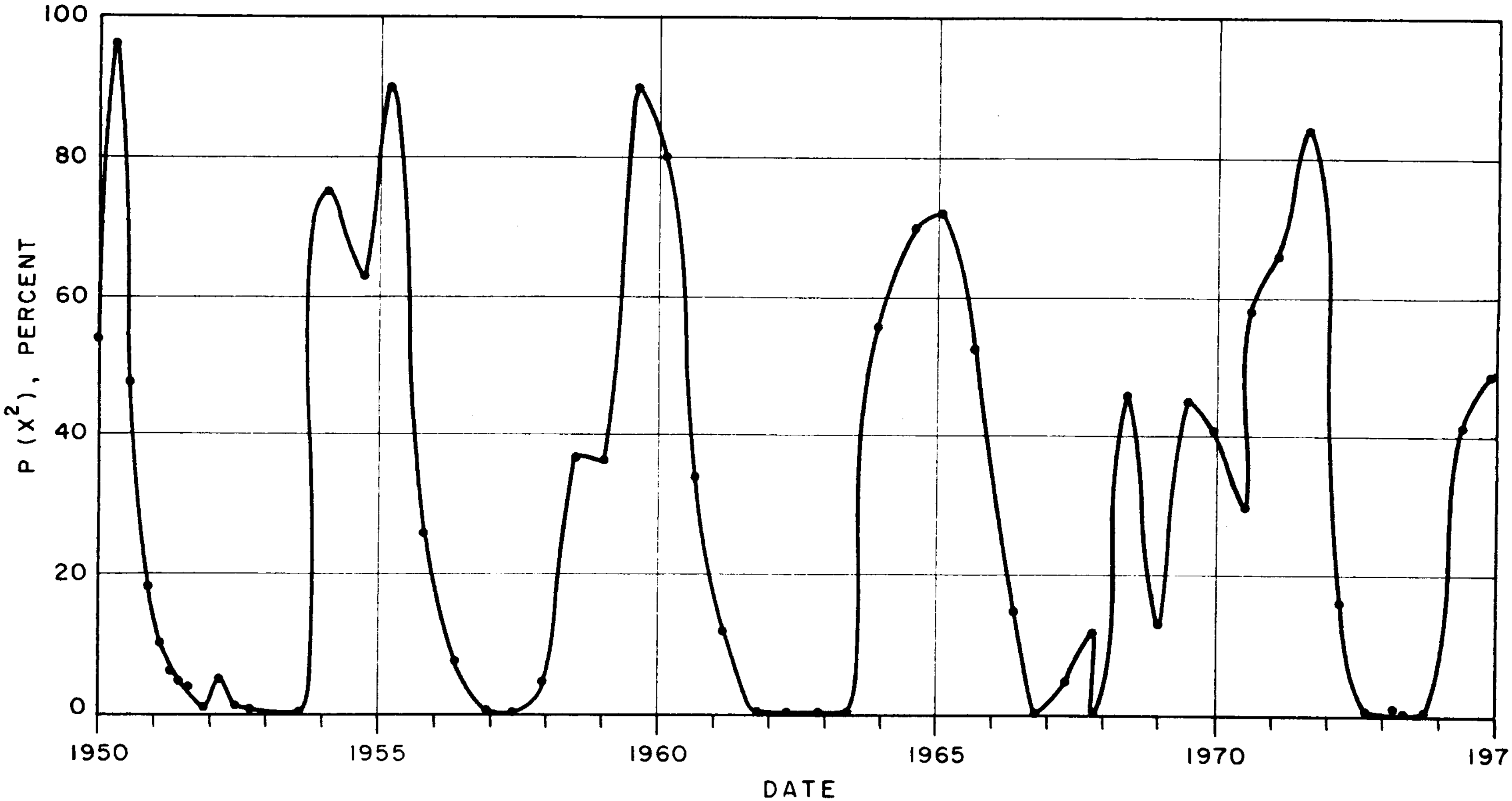

FIGURE 4 OBSERVED $P(X^2)$ DEVIATION FROM UNIFORM OF ASTEROID LONGITUDES

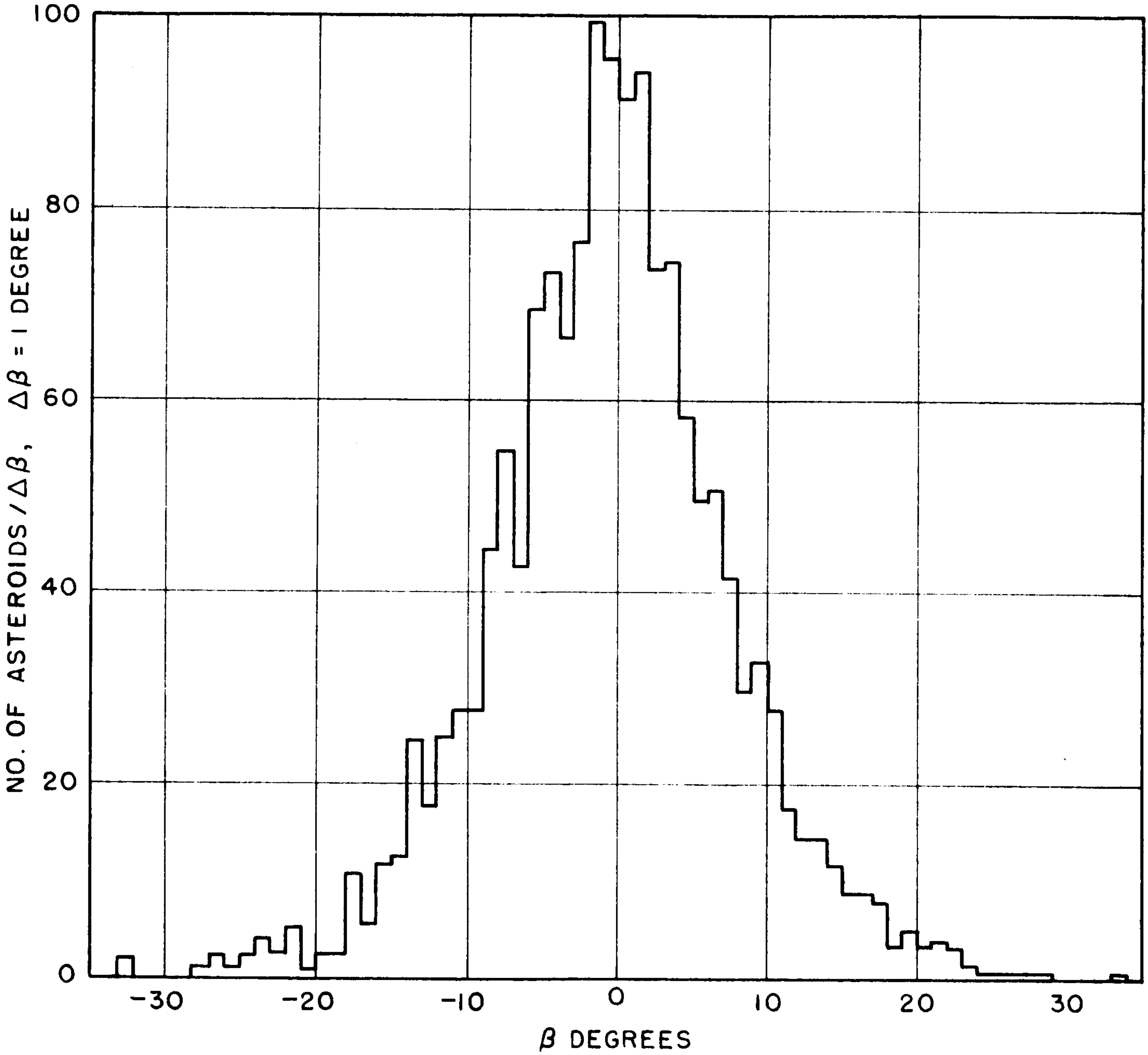

FIG. 5 LATITUDE DISTRIBUTION FOR 1563 NUMBERED ASTEROIDS ON 1/1/65

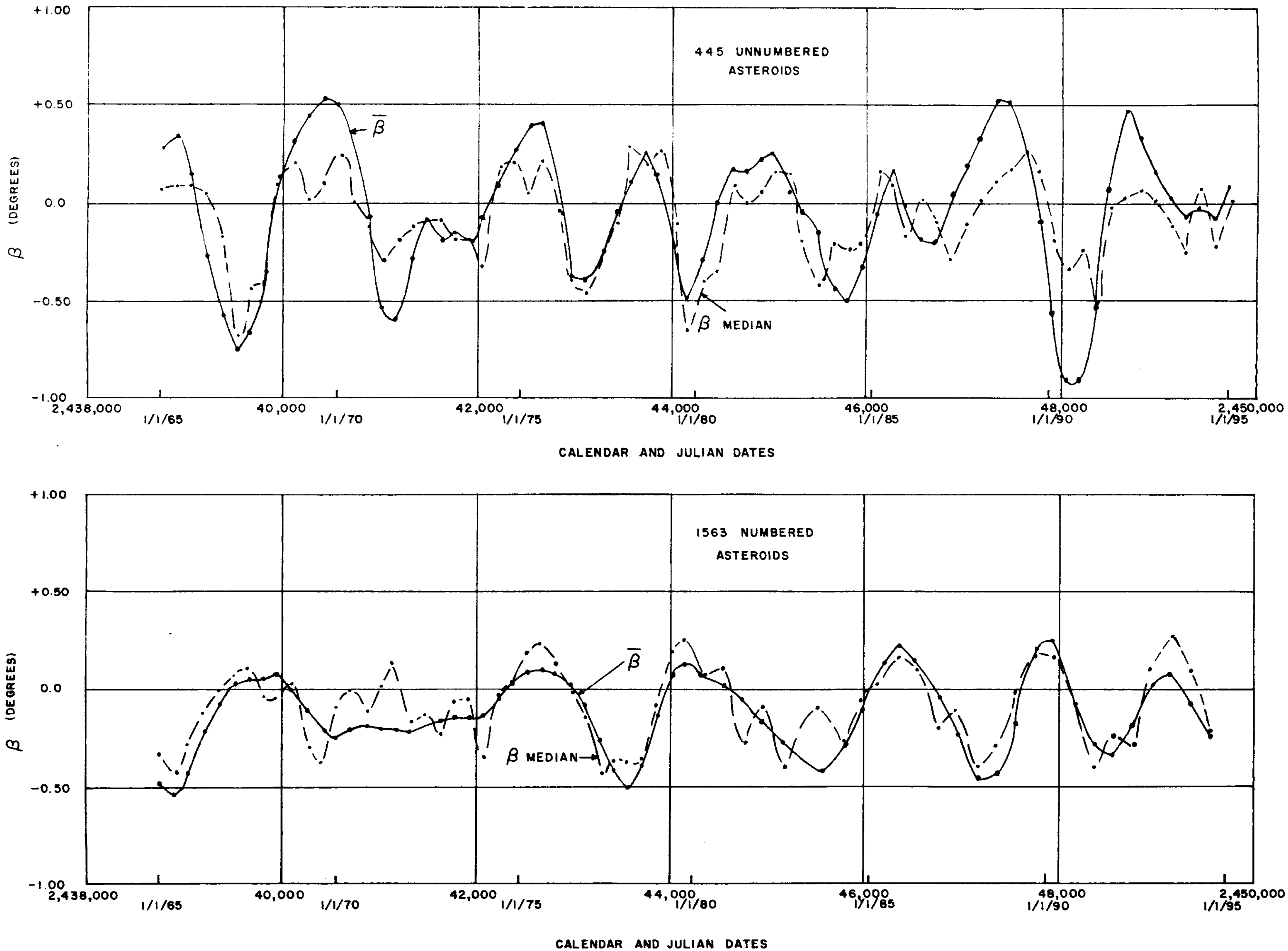

FIGURE 6 AVERAGE AND MEDIAN LATITUDES FOR THE ASTEROIDS

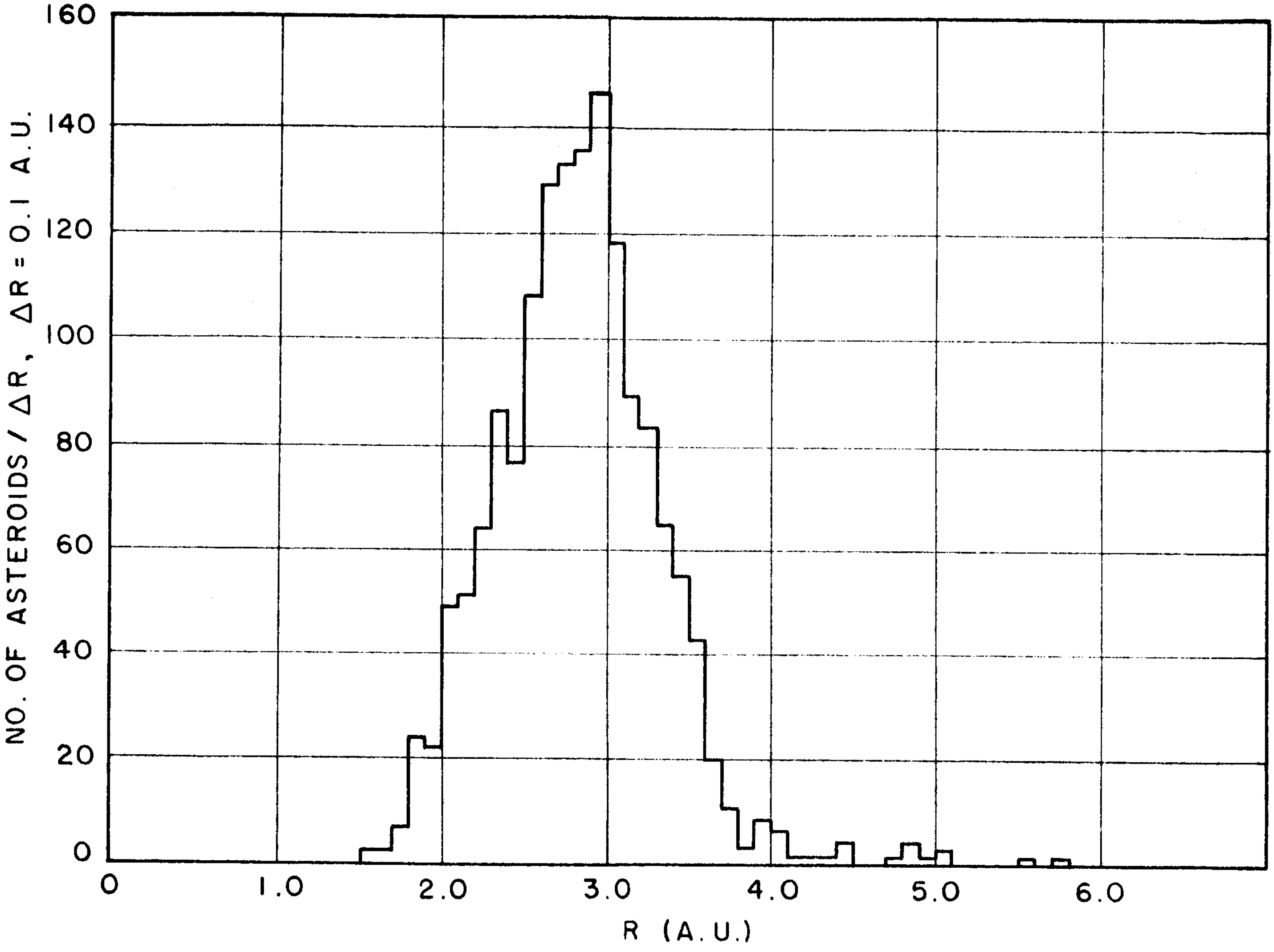

FIG. 7 RADIAL DISTRIBUTION FOR 1563 NUMBERED ASTEROIDS ON 1 / 1 / 65

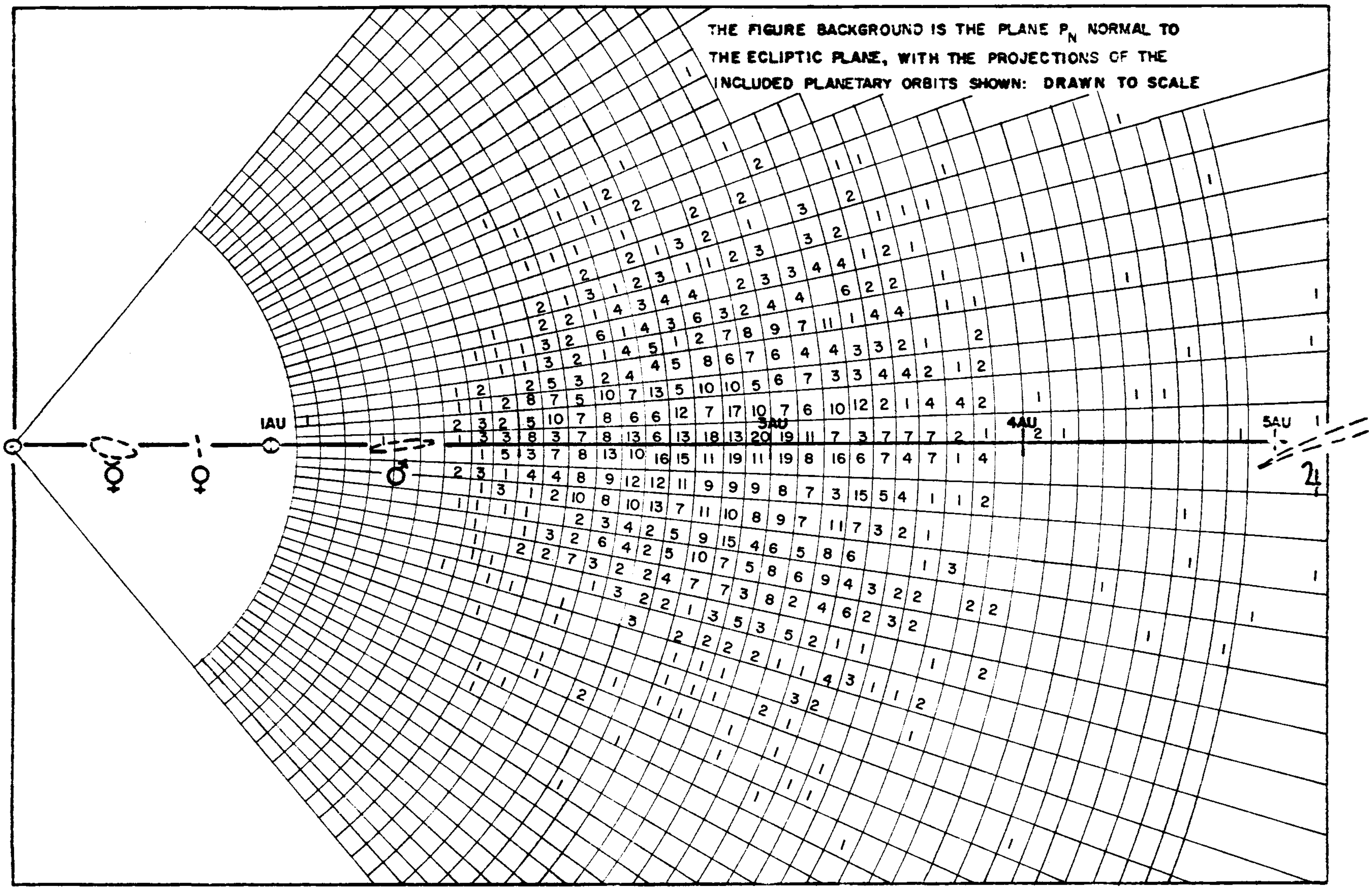

FIG. 8 DISTRIBUTION OF 1563 NUMBERED ASTEROIDS BY $\Delta\beta$, ΔR SEGMENTS ON 3/20/73

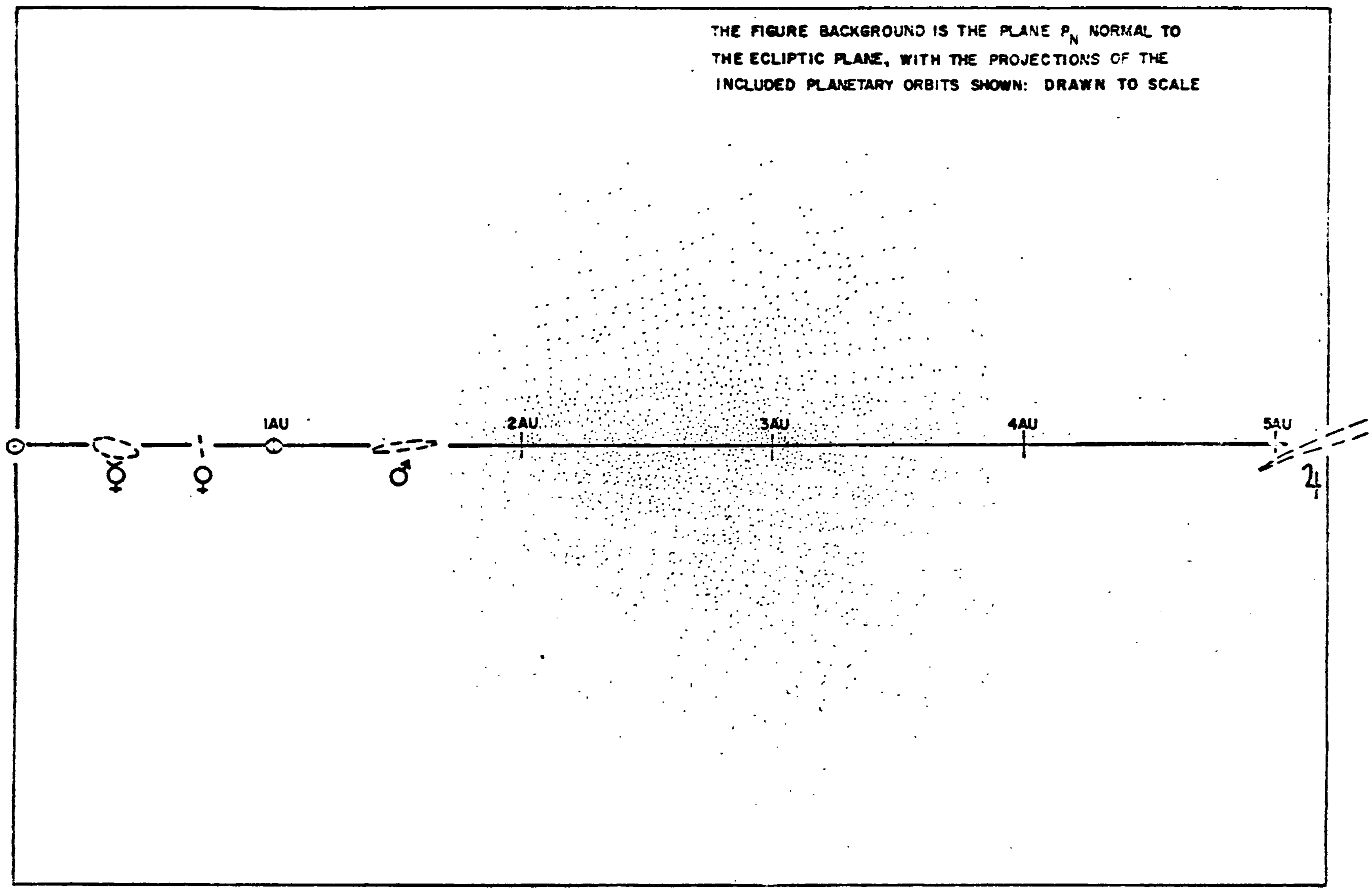

FIG. 9 DISTRIBUTION OF 1563 NUMBERED ASTEROIDS IN β AND R ON 3/20/73

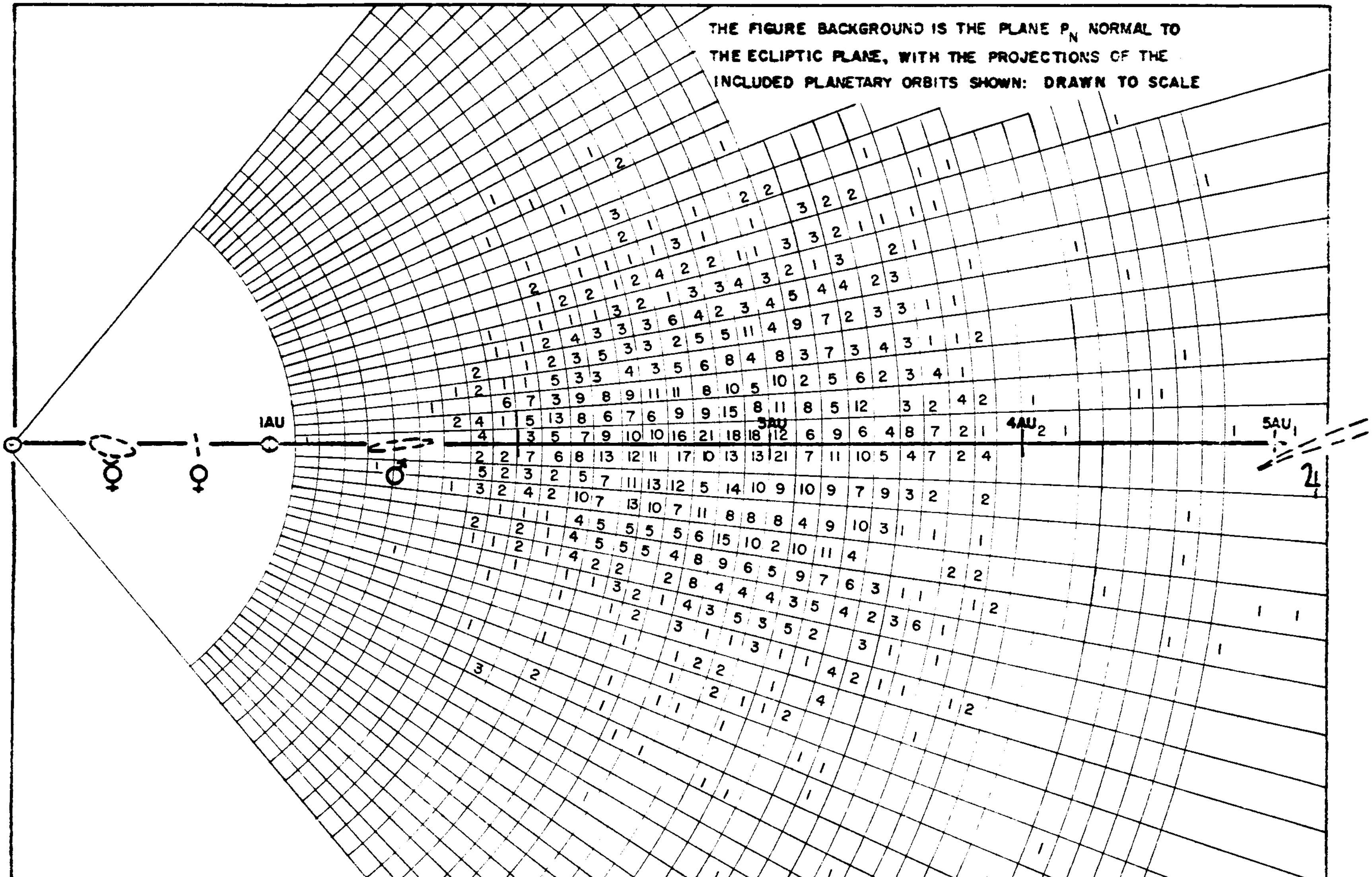

FIG. 10 DISTRIBUTION OF 1563 NUMBERED ASTEROIDS BY $\Delta\beta$, ΔR SEGMENTS ON 4/19/73

Table 1

AVERAGES OF THE ASTEROID β, λ AND R AVERAGES AND MEDIANS

Objects	Sampling Rates and Dates	Number of Times	Grand Averages					
			$\bar{\beta}$ (deg)	β med (deg)	$\bar{\lambda}$ (deg)	λ med (deg)	$\bar{R}$ (AU)	R med (AU)
1563 Numbered Asteroids	1965-1979 every 150 days	37	-0.14	-0.10	179.9	179.4	2.84	2.82
1563 Numbered Asteroids	1980-1994 every 200 days	28	-0.10	-0.08	180.6	181.2	2.83	2.82
445 Unnumbered Asteroids	1965-1979 every 150 days	37	-0.05	-0.08	178.9	177.7	2.80	2.70
445 Unnumbered Asteroids	1980-1994 every 150 days	38	-0.04	-0.09	178.4	177.9	2.79	2.70
Final Averages for all Asteroids and all Times		134,970	-0.10	-0.09	179.8	179.6	2.83	2.79