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THREE COMPUTER PROGRAMS FOR N-BODY TRAJECTORIES AND INTERPLANETARY TRAJECTORIES

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MECHANICS SECTION

THREE COMPUTER PROGRAMS FOR N-BODY TRAJECTORIES AND INTERPLANETARY TRAJECTORIES*

By

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*Prepared under contract no. NAS2-1706
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GENERAL ELECTRIC

FOREWORD

This report was prepared by the Space Sciences Laboratory, Missile and Space Division, General Electric Company, King of Prussia, Pennsylvania, under NASA contract no. NAS2-1706 for the NASA Ames Research Center, Moffett Field, California.

This report is a description of three computer programs delivered to NASA-Ames according to the terms of the above mentioned contract. The preparation of the report was performed under the supervision of Dr. F. Wendt, Acting Manager-Mechanics Section, by Messrs. J. P. deVries, W. M. Pauson, T. Coffin and F. T. Nicholson.

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ABSTRACT

This report contains complete input instructions, operating instructions and sample problems with computer output for three IBM-7094 Computer Programs. The Programs and their essential features are:

I) Interplanetary Trajectory Program. This program determines the burnout velocity for a trajectory from Earth to any other planet or from any other planet to Earth.

II) N-Body Trajectory Program. This is an N-body program with fourth order Runge-Kutta integration with optional doubling and halving procedure and according to the Cowell or the Encke method. It also can print out central angle, ground trace and azimuth and elevation of the velocity vector.

III) N-Body Program with Sensitivity Coefficients and Differential Correction. This is in purpose similar to the Interplanetary Trajectory Program, but a first estimate of the trajectory must be available from an external source. It computes a 6 x 6 sensitivity matrix by integration along the trajectory; it also makes use of an improved differential correction procedure.

A full description of Program I is available in reference 1 and 2. This report presents an outline of the pertinent sections of those references, to facilitate their use. Full descriptions and analyses are presented for those features that distinguish Programs II and III from Program I.

TABLE OF CONTENTS

1. GENERAL DESCRIPTIONS OF THE THREE PROGRAMS

1.1 Introduction

This section is a general description of three computer programs. Complete operating instructions and the in-and output formats are given in section 4. Sample problems are presented in section 5. The three computer programs described in this report are:

I. Interplanetary Trajectory Program

This program can be used in three ways:

mode 1) To determine the burnout velocity and the trajectory between Earth and any other planet except Pluto (or vice versa) when departure date and trip time are given.

mode 2) To determine the initial configuration angle between departure and destination planets for a given departure date.

mode 3) To compute an N-body trajectory when initial position and velocity components are given; this includes as an option the use of six (or less) midcourse velocity corrections.

II. N-body Program with Options

This is the same program as (I, 3) above, but the units of the output may be miles and feet-per-second (or any other units, depending on input) whereas the units of (I, 3) are always A. U. and A. U. per hour. This program also contains options to compute ground trace information, azimuth and elevation of velocity vector and central angle. It does not have the midcourse correction option.

III. N-body Program with Sensitivity Coefficients and Differential Corrections

This program determines the burnout velocity of interplanetary trajectories (similarly to I, 1) by differential correction of a first estimate. It differs from (I, 1) in three aspects:

a) a first estimate of the initial conditions obtained from another source, must be used as input (the program (I, 1) computes its own estimate)

b) the sensitivity coefficients are computed by integration along the trajectory, whereas (I, 1) computes them by perturbing the initial velocity components.

c) it has an option to print out the 6 x 6 sensitivity coefficient matrices along a nominal trajectory.

The content and operation of program I are reported in G. E. TIS reports R60SD465 (Ref. 1) and R61SD047 (Ref. 2). Ref. 1 deals with the N-body program which is the common basis for all the programs mentioned above. Ref. 2 is a complete description and operation manual for program I, the Interplanetary Trajectory Program, in its three modes of operation. Ref. 2 also contains flow charts.

Programs II and III grew out of program I by a process of adding and modifying. The additions and modifications are described in this report. To facilitate the use of references 1 and 2, this report contains an outline of their content.

The chart on page 3 is a summary of this introduction.

1.2 Program I, The Interplanetary Trajectory Program

This program is fully described in Ref. 2. It may be used in three ways.

1.2.1 Mode 1, Interplanetary Trajectory

In this mode the program computes an interplanetary trajectory from Earth to any other planet (except Pluto) or from any planet to Earth. The initial condition is some altitude above a given launch site at some time during a given departure date. The end condition is intersection of a sphere of given radius (which may or may not be the actual radius from the center of the destination planet after a given trip time. The time of burnout on the departure date is determined to take the best advantage of the planet's rotation. (For those planets of which the rotation is not known this feature is dispensed with.) The trip time is satisfied within a tolerance specified in the input.

The input is principally the identification of departure and arrival planets, the departure date and the trip time; other input consists of physical

SUMMARY OF RELATION BETWEEN THE THREE PROGRAMS

Program I Interplanetary Trajectory

Three modes of operation

mode 1 Interplanetary trajectory

Heliocentric transfer analysis
Geocentric departure analysis
First estimate of trajectory
N-body trajectory
Sensitivity coefficients (by perturbation)
Iteration by differential correction

→ Program III
Interplanetary trajectory
Needs first estimate from other source
N-body trajectory
Iteration by modified differential correction
Sensitivity coefficients by integration
Option to print out 6 x 6 sensitivity matrix

mode 2 Computation of Initial Configuration Angle

mode 3 N-body Trajectory

Doubling and halving, Runge-Kutta
Cowell or Encke, with optional switch
Option to transfer coordinate center
Option for 6 midcourse corrections

Described in references 1 and 2

→ Program II
N-body trajectory
With options: output in mi, ft/sec
ground trace
azimuth and elevation of
velocity vector
central angle
(no option for midcourse corrections)

Described in this report

constants, indicators to make the N-body subroutine compute according to the desired mode and conditions to end the program.

The program begins with the computation of an estimated burnout velocity on the basis of a two body analysis (the "heliocentric transfer analysis") in which the planets are massless. This estimate is refined by introducing the orbital eccentricities of departure and arrival planets (the "geocentric departure analysis"); this part of the program also determines the time of burnout during the departure day to take the best advantage of the planet's rotation. The refined estimate is run as an N-body trajectory; sensitivity coefficients are computed by perturbing the initial velocity components. The estimate is improved iteratively by a differential correction procedure that is based on the sensitivity coefficients.

The heliocentric transfer analysis computes the eccentricity and semi-major axis of the transfer ellipse (or hyperbola, if desired) for given departure date and trip time. This is done by a rather simple iteration procedure, but the formulas to be used depend on the type of transfer to be made. There are four types of elliptic transfers: direct, indirect, perihelion, aphelion; these terms are defined in Ref. 2. To get the iteration started properly, the particular kind of transfer must be indicated in the input. Ref. 2 contains graphs of Earth-planet and planet-Earth transfers; in each graph the trip time -- initial configuration angle space is divided in several areas each of which is associated with a particular type of transfer. Thus, for given trip time and departure date an estimate can be made about which type of transfer is required, since the knowledge of departure date is equivalent to that of initial configuration angle. Since the plots are based on coplanar, circular planetary orbits, their information may be in error especially when a close-to-Hohmann transfer is desired. In that case a number of possible types of transfer may be indicated in the input; the program will then try each type in turn until the correct one is found.

If one does not want to determine the initial configuration angle by referring to the ephemeris and computing the scalar vector of the position vectors, one may use mode 2 of the program to obtain that angle.

1.2.2 Mode 2, Initial Configuration Angle

In this mode the program computes the angle between the departure and destination planets on the departure date by looking up their positions in the ephemeris and computing the scalar product of the position vectors. Input consists of identification of planets, departure date and trip time (but trip time is not used in the computation, it is a "dummy" input).

1.2.3 Mode 3, N-Body Trajectory

The N-body trajectory program is characterized by:

1) Fourth order Runge-Kutta integration in double precision, with optional doubling and halving procedure;

2) Position data of Sun, Moon and all planets except Pluto given at 12 hour intervals, referred to the mean equinox and mean equator of 1950.0;

3) Cowell or Encke solution method with optional switching from one to the other;

4) Optional switching of coordinate center.

The input consists of a) identification of reference planet, b) initial position (in A.U.) and velocity (in A.U./hour) components with respect to the center of the origin planet, c) physical constants, including the identification of planets to be included in the computation, d) indicators for the particular mode of running the program and e) conditions on which to terminate the computation.

Output lists position, velocity and acceleration components and magnitude with respect to origin planet; position components and magnitude with respect to earth, sun and target planet; all this at each integration step or intermittently at any desired number of steps.

This mode of the Interplanetary Trajectory Program also has an option to include the effects on the trajectory of up to six midcourse corrections. The corrections must be defined in the input by the specific impulse and mass flow (the same for all corrections) and the time of initiation, the burning time and the direction as referred to local velocity vector and trajectory plane.

1.3 Program II, the N-Body Program with Options

This is the same program as that described in section 1.2.3, except for the following.

1) It does not have the option for midcourse corrections.

2) It has an option to print out in miles, feet per second and feet per $(\text{second})^2$, or any other units, depending on the input conversion factors.

3) It has an option to print out the ground traces over the Earth's surface in latitude and longitude.

4) It has an option to print out the central angle, i.e. the angle between the instantaneous and the initial position vectors.

5) It has an option to print out aximuth and elevation of the velocity vector.

1.4 Program III, N-Body Program with Sensitivity Coefficients and Differential Correction

This program may take the place of Program I if a first estimate of initial conditions of an interplanetary trajectory is available from some other source. Such a source can be, for instance, the JPL Heliocentric Conic Program. Progam III cannot compute the first estimate (as Program I can), but it does contain improvements in the computation of sensitivity coefficients and the differential correction. These improvements are fully described in section 3 of this report; the following is a general description.

The complete (6 x 6) matrix of sensitivity coefficients is computed along the trajectory by integration of the equations of variations. There is an option to print out the sensitivity matrix at each integration interval.

The differential correction procedure has been improved (or, at least, made more flexible) by permitting the use of only a percentage (determined by the "K-factor" which is supplied as input) of the velocity correction determined by the sensitivity matrix's inversion.

Because of the way in which the sensitivity coefficients are computed, this program must be run with the Cowell solution method and must use the doubling and halving option.

2. OUTLINE OF REFERENCES 1 AND 2

2.1 Introduction

The purpose of this section is to facilitate the use of references 1 and 2. Together, they give a full description of program I (the Interplanetary Trajectory Program). Reference 1 concentrates on the N-body part of the program and discusses two demonstration problems; Reference 2 describes the Interplanetary Trajectory Program and repeats some parts of Reference 1. Reference 2 is thus a complete report in itself. The following is an outline of the relevant materials in the two reports, or an elaborate table of contents (Introductions and abstracts are not mentioned). Some of the contents are merely mentioned, some other matters are discussed in some detail wherever it was felt that further explanation was necessary. Reference is by section number. The title of the reports, "Generalized Interplanetary Trajectory Study," was the title of the WADD contract under which the work was performed. Section IV of Reference 1, "Interplanetary Weather," was the result of a part of that work; it will not be further discussed here.

2.2 Reference 1

("General Interplanetary Trajectory Study," J. P. deVries, coordinator, General Electric TIS R60SD465, August 1960)

Section 2.1 Definition of coordinate system. Reference is the mean equator and mean equinox of 1950.0.

Section 2.2 Equations of motion for the N-body problem are derived for one body with respect to another. The accelerations due to the first earth oblateness term are also indicated.

Section 2.3 Definition of the "Cowell" and "Encke" solution techniques. The program uses either technique (according to input instructions).

Section 2.4 Description of the integration method. The method used is the fourth order Runge-Kutta; the selection of the time step is made by the doubling and halving method. The criterion by which the integration proceeds with the same interval or with half the interval is that both integrations should agree within a certain tolerance. (In our use of the program it has become customary to use a tolerance of 4×10^{-10} A.U.).

A possible criticism is that the tolerance is an absolute distance, whereas a certain percentage of the instantaneous distance to the center of the coordinate system may be more satisfactory. Be this as it may, the program has performed well, and the automatic time step selection provides a great time saving for interplanetary trajectories since small steps are taken near a center of attraction and long steps far away from it. The doubling and halving procedure is optional.

Section 2.5 It may be argued that for trajectories between two planets it is advantageous (for accuracy and speed) to translate the coordinate system from the departure planet to the sun, and later to the arrival planet. Such translations may be executed automatically (by option, according to input instructions). When using the Cowell method the coordinates are translated to the closest body; when using the Encke method the translation is to that body of which the gravitational force is greater than 25% of the total force. Whenever a translation is needed, the relative velocity between the old and the new coordinate system is computed by numerical differentiation. Usually this does not produce a sufficiently accurate velocity; the velocity is corrected in an iteration procedure which is based on running the trajectory in the old as well as in the new coordinate system for a few integration steps.

The program also has an option to switch solution techniques from Cowell to Encke, or from Encke to Cowell. The criterion is (more or less arbitrarily) that the Encke technique is used when the body at the coordinate center is responsible for more than 75% of the total force; otherwise, the Cowell technique is used.

Section 2.6 The sources for the planetary tapes are indicated. Positions of all planets (except Pluto), Moon and Sun are stored on tape at 12 hour intervals. Interpolation was by nine-point Lagrange interpolation of the original ephemerides. Time is Ephemeris Time. Appendix I gives the formulas which were used to convert the Naval Observatory data for the Moon and Mercury.

Section 2.7 All physical constants are input quantities, but this section suggests a set of numbers which is consistent with standards adopted by the International Astronomical Union.

Sections 2.8 and 2.9 Computer input and output are more fully defined in Reference 2 and also in this report, section 4.

Section 3 Two demonstration problems are discussed in detail. The first is the computation of the Lunik III trajectory. The second is the computation of an Earth-Venus trajectory for given departure date and trip time. The method by which a first estimate was obtained is discussed in detail in Appendix II. The method consists of two parts: the heliocentric transfer analysis ("Vertregt") and the geocentric departure analysis ("Moeckel"). These analyses were later generalized and programmed and are now part of the Interplanetary Trajectory Program. The use of differential correction to refine the estimate is also discussed (see also Appendix III); this was also programmed later to be part of Program I.

2.3 Reference 2

("Generalized Interplanetary Trajectory Study," Part II and Supplement I, J. P. deVries, Coordinator, General Electric TIS R61SD047, Jan. 1961)

Section 2 Description of the N-body program. This repeats section 2 of reference 1. The list of astronomical constants contains suggested values for several physical properties of the planets.

Section 3.1 Description of the Interplanetary Trajectory Program. The problem to be solved by this program is briefly: to determine the burnout velocity for a trajectory from Earth to any other planet (or vice versa) when departure date and trip time are known. The program operates in three phases: 1) first estimate, 2) N-body program and 3) differential correction.

Section 3.2 The heliocentric transfer analysis determines the semimajor axis and eccentricity of the transfer ellipse, assuming that the planets have no mass. For elliptic trajectories four kinds of transfer ("routes") are defined (direct, indirect, aphelion and perihelion); two routes are defined for hyperbolic transfers. No retrograde transfers are considered. In plots having trip time as ordinate and initial configuration angle (or departure date) as abscissa each route is confined to a certain region (i.e combination of trip times and departure dates). The four regions belonging to the elliptic routes come together in a point which corresponds to the single trip time-departure date combination for a Hohmann (least energy) transfer. There are fourteen such plots, seven for earth-planet and seven for planet-earth transfers. The plots are used to determine which route will be taken for the desired trip time-configuration angle combination. This information is needed in the input data, because the computation of semi-major axis and eccentricity requires different formulas for different routes. Since the plots are based on an idealized model there is some uncertainty about the route that is required, especially in the neighborhood of the Hohmann point. One may therefore specify a number of routes; the computer will try each one in succession.

This section gives all the formulas for the computation of semi-major axis and eccentricity (for each of the different routes) and the iteration procedure which is used in the program.

Section 3.3 The purpose of the departure analysis is to obtain a first estimate of the burnout velocity. The burnout point is taken to be some given distance vertically above the launch site and the burnout time during the given departure date is determined to take the maximum advantage of the earth's (or other planet's) rotation. (Departures from Mercury and Venus are dealt with slightly differently because not enough is known about their rotations and direction of their axes.) The escape from the planet is taken to be along a hyperbola which is "patched" to the transfer ellipse that was the result of the heliocentric transfer analysis.

Pages 55-57 of this section have a more general significance. A step-by-step procedure is given for the determination of the Universal

Time that corresponds to local mean sidereal time; this is a "translation" of the instructions in the American Ephemeris and Nautical Almanac (as well as the Explanatory Supplement).

Section 3.4 The differential correction by which the first estimate is corrected to produce an impacting trajectory is based on the sensitivity matrix between positions at arrival time and velocities at departure time. The sensitivity coefficients are computed by making small changes in each of the burnout velocity components in turn and computing three new trajectories. The inversion of the sensitivity matrix produces the burnout velocity correction and a new trajectory is run. With the same sensitivity matrix up to ten iterations may be performed; if impact is still not obtained, a new sensitivity matrix is computed along the last trajectory and another 10 iterations may be performed. If after three computations of sensitivity matrices (or a total of 30 tries) impact is still not achieved, the run is given up as a hopeless case. The end condition of the trajectory is impact on a sphere centered at the arrival planet's center with arbitrary (i.e. input data) radius at the arrival time to within a given tolerance (also input data).

Section 3.5 This is a complete discussion of the computation of a Mars-Earth trajectory.

Section 3.7 This is a summary of 7 Earth-Venus trajectories with departure dates at 12-day intervals and a trip time of 100 days.

Sections 4.1 and 4.2 It is argued that to use the Encke method advantageously the coordinate center must be translated from one body to another when the second becomes prominent. To perform this translation relative velocity of the old and the new coordinate center must be determined by differentiation of the ephemeris data. Even when using the iteration procedure described in section 2.5 of reference 1 it has been found that the results are not always consistent. The reasons for this are quite complex, maybe a combination of some small interval inconsistency in the ephemeris with the fact that light significant figures in the initial velocity does not give enough accuracy. Section 4.2 presents an improved method for the velocity determination; this method has been applied with some success, but it has not been incorporated into the present program. (Our practice has become to use the Cowell method always, without translating coordinate centers).

Section 5 This section describes how the program handles the midcourse correction option. Up to six midcourse corrections may be specified. Specific impulse and mass flow must be the same for each correction. The corrections are further specified by giving the starting time (hours from beginning of trajectory), the burning time and two angles which define the thrust direction. The thrust direction is to be specified in a coordinate system that is directed along the local velocity and the trajectory plane.

Appendix I. This reports the two-body formulas that are used in the Encke method.

2.4 Reference 2, Supplement

Section 2 The complete input instructions for the three modes of the Interplanetary Trajectory Program.

Section 3 Description of computer output.

Section 4 Operating instructions.

Note: The contents of sections 2, 3 and 4 of this Supplement are repated in section 4 of this report.

Section 5 Description of the binary tapes of planetary tables.

Section 6 Block diagrams of the most important parts of the Interplanetary Trajectory Program: tape data to angular parameters (fig. 1), heliocentric analysis (fig. 2), departure analysis (fig. 3), Encke integration program (fig. 4), Cowell integration program (fig. 4 cont.) and differential correction analysis (fig. 5).

Appendix I Description of the modified DBC FORTRAN input routine.

Appendix II Complete input and computer output sheets of two examples:

1) Earth-Venus trajectory, 110 days trip time, departure date Dec. 12, 1960 or 346 days from beginning of tape (Jan. 1, 1960) and

2) Venus-Earth trajectory, 110 days trip time, departure date April 1, 1961, or 456 days from beginning of tape.

Appendix III List of Julian dates, calendar dates and table days.

3. ADDITIONS AND MODIFICATIONS OF PROGRAM I

3.1 Introduction

Since the completion of Program I several additions and modifications have been made. Programs II and III are the results. This section describes

1) the computation of ground trace parameters, azimuth and elevation of velocity vector and central angle (for Program II)
2) Computation of 6 x 6 sensitivity matrix (for Program III)
3) Modified differential correction procedure (for Program III).

The use of these additional features is fully explained in the section on input instructions and operating procedures later in this report.

3.2 Computation of Ground Trace, Central Angle and Azimuth and Elevation of Velocity Vector

Program II offers the option of printing out ground trace information, the central angle (or travel angle and aximuth and elevation of the velocity vector. This section describes how this information is obtained. The program can also print out distances in miles, velocities in ft/sec and accelerations in ft/sec^2, or any other units depending on the conversion constants (from A. U. and A. U. /hour) which are supplied as input.

3.2.1 GroundTrace

The parameters computed are the latitude of the vehicle referenced to the equatorial plane and the longitude referenced to the Greenwich meridian. Negative longitudes indicate angles west of Greenwich.

The additional input quantities required for this option are:

H = Greenwich Hour Angle for starting day of trajectory (deg.)
t_o = Table hours for 0^h U.T. on starting day of trajectory
ω_e = Angular rate of Earth's rotation (ω_e = .262516 rad/hour)

The coordinates (X, Y, Z) of the vehicle with respect to the Earth referenced to the vernal equinox of 1950.0, found by the n-body program are transformed to coordinates referenced to the Greenwich meridian at t_o(X', Y', Z'). This involves a rotation about the Z axis through the angle H as shown in Fig. 1.

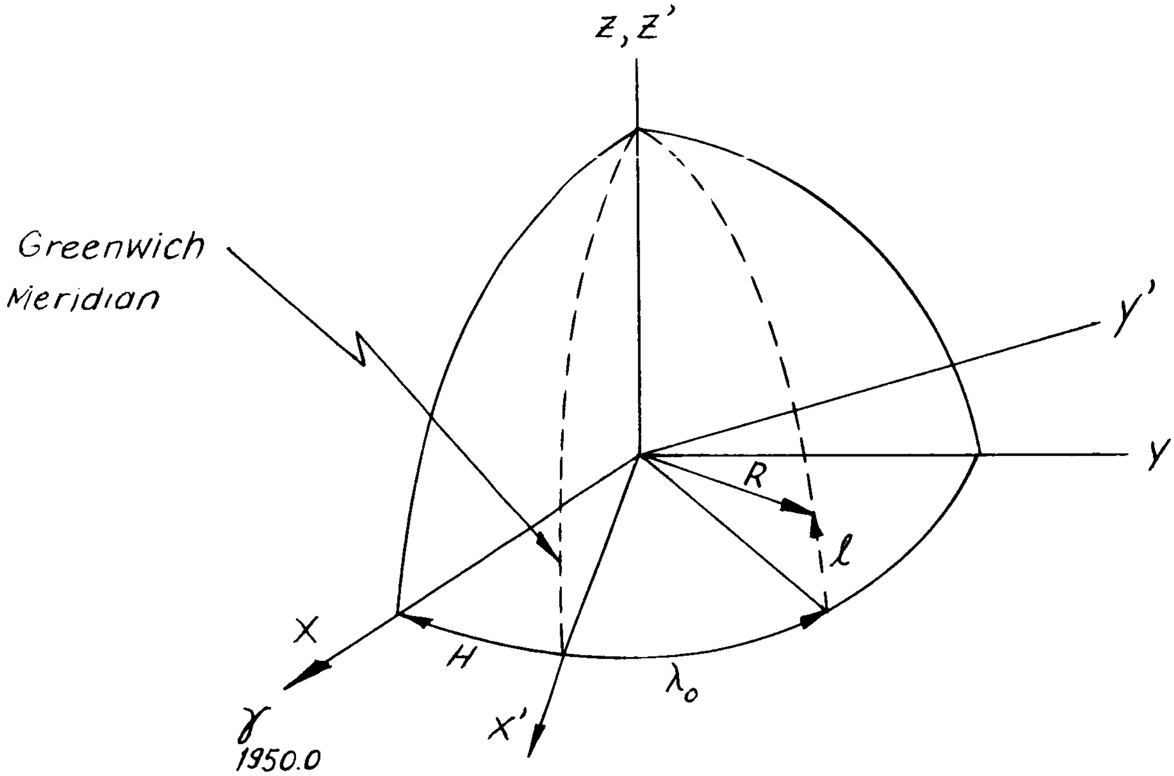


Fig. 1 Computation of Ground Trace

$$X' = X \cos H + Y \sin H$$

$$Y' = -X \sin H + Y \cos H$$

$$Z' = Z$$

Now, if R is the geocentric distance to the vehicle, the latitude ℓ is given by

$$\ell = \sin^{-1}(Z/R)$$

where $-90^\circ \leq \ell \leq 90^\circ$. Negative values of ℓ indicate latitudes in the southern hemisphere.

The longitude with respect to the Greenwich meridian at t_o is

$$\lambda_o = \sin^{-1}(Y'/R \cos \ell)$$

$$\lambda_o = \cos^{-1}(X'/R \cos \ell)$$

To find the longitude, λ, with respect to the Greenwich meridian at the time of the printout step, t, a correction for the Earth's rotation is applied

$$\lambda = \lambda_o - \omega_e (t - t_o)$$

3.2.2 Central Angle and Azimuth and Elevation of Velocity Vector

The desired parameters are shown in Fig. 2

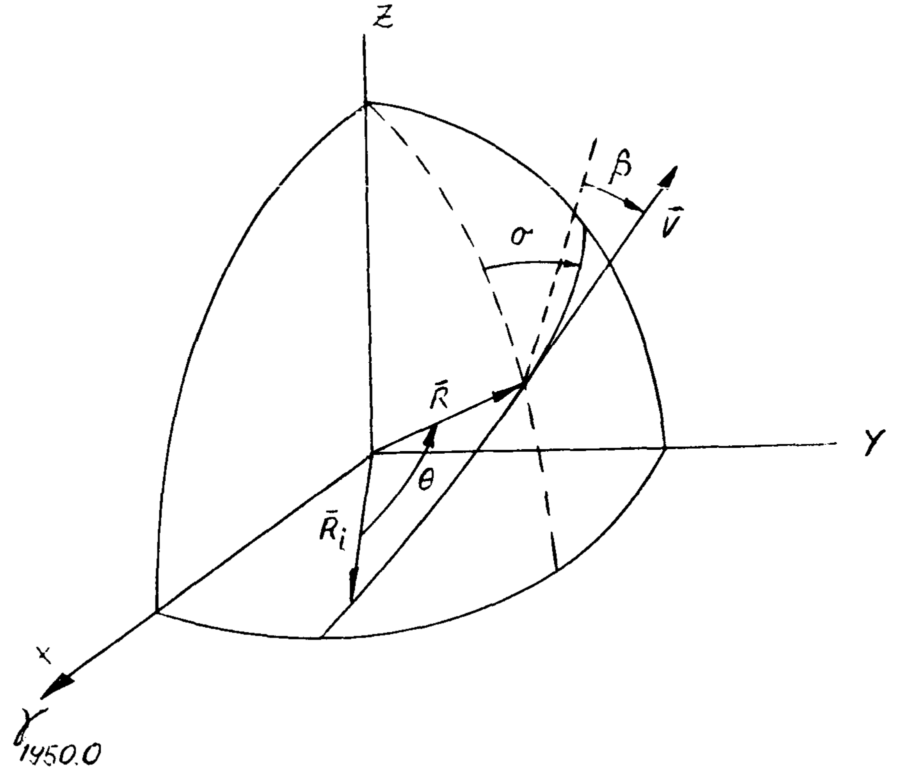


Fig. 2 Central Angle and Direction of Velocity Vector

θ = central angle, or angle traveled from injection, $\bar{R}_i$, to $\bar{R}$ at any time t.

β = elevation of velocity vector, $\bar{V}$, measured positive above local horizontal plane

σ = azimuth of $\bar{V}$, measured positive clockwise from north in local

$X, Y, Z; \dot{X}, \dot{Y}, \dot{Z}$ = components of geocentric position and velocity vectors at any time t subscript i refers to injection time

The central angle is given by

$$\theta = \cos^{-1}\left(\frac{XX_i + YY_i + ZZ_i}{RR_i}\right)$$

where $0^\circ \leq \theta \leq 180^\circ$.

The elevation of $\overline{V}$ is

$$\beta = \sin^{-1}\left(\frac{X\dot{X} + Y\dot{Y} + Z\dot{Z}}{RV}\right)$$

where $-90^{\circ} \leq \beta \leq 90^{\circ}$.

The declination of the vehicle, δ, is the latitude found previously as part of the ground trace computation. The right ascension is found from

$$\alpha = \sin^{-1} (Y/R \cos \delta)$$

$$\alpha = \cos^{-1} (X/R \cos \delta)$$

The azimuth of $\overline{V}$ is thus given by

$$\sigma = \cos^{-1} (\dot{X} \sin \delta \cos \alpha - \dot{Y} \sin \delta \sin \alpha \sin + \dot{Z} \cos \delta)/V \cos\beta$$

where

$$\sigma = \sigma \text{ if } (-\dot{X} \sin\alpha + \dot{Y} \cos\alpha) \geq 0$$

$$\sigma = 2\pi - \sigma \text{ if } (-\dot{X} \sin\alpha + \dot{Y} \cos\alpha) < 0$$

3.2.3 Discussion

It must be noted that some simplifications have been used in the computations described in this section. The ground trace is computed using the mean equator and mean equinox of 1950.0; it should thus be corrected by introducing the nutation and precession to get the ground trace properly in the coordinates of date. The error amounts to about 50 secs of arc per year, counted from 1950; most of the error is in the direction of longitude. (When the option was introduced in the program the purpose warranted neglecting this correction.)

Also in defining the direction of the velocity vector the coordinate system is that of the 1950.0 mean equator and mean equinox.

3.2.4 Modification of the Differential Correction Procedure

In determining the burnout velocity for an interplanetary trajectory with the Interplanetary Trajectory Program (Program I), the linear differential correction procedure has sometimes shown a slowly converging or even diverging behavior. It is typical in such cases that the miss distance will oscillate from one side of the target planet to the other in successive iterations. The modification of the differential correction procedure (described in this section) has improved the computation of such difficult cases considerably.

Since the completion of the Interplanetary Trajectory Program other means of getting a first estimate have become available; in some cases such estimates produce a smaller miss distance (when run in an N-body program) then the estimate determined by the Interplanetary Trajectory Program. An example of such other means is the JPL Heliocentric Patched Conic Program, used directly or with its own differential correction.

The improvement of the Interplanetary Trajectory Program thus consisted of three parts:

1) Make it possible to begin with an estimated burnout velocity which is computed externally.

2) Modify the differential correction procedure to improve convergence.

3) Improve the computation of the sensitivity coefficients by getting them through integration of the equations of variation along the estimated trajectory. (This takes only 10% more computer time than integrating the trajectory alone, whereas the finite perturbation method requires 300% more; the results have consistently agreed within 1%. Also, in this manner the complete 6 x 6 matrix becomes available for printout at any integration interval.

It was therefore decided to let the Interplanetary Trajectory Program stand unchanged and to add the new sensitivity matrix computations and the modified differential correction procedure to the N-body program. The first estimate of initial conditions must now come from an external source. One such source may of course still be the old Interplanetary Trajectory Program (Program I) when it is stopped just before it goes into the N-body subroutine.

The modification of the differential correction procedure consists essentially of limiting the magnitude of the velocity correction that is predicted by the inversion of the sensitivity matrix. A "limiting factor" k is defined below; to date values of .1 and .2 have been used successfully al although not enough experience has been gained to determine a "best" (if there is one!). Low values of k tend to increase the number of iterations required to find an impacting trajectory, larger values may not sufficiently reduce the oscillatory behavior of the differential correction. As shown in Figure 3, only the magnitude of the predicted velocity correction is changed, the direction is maintained.

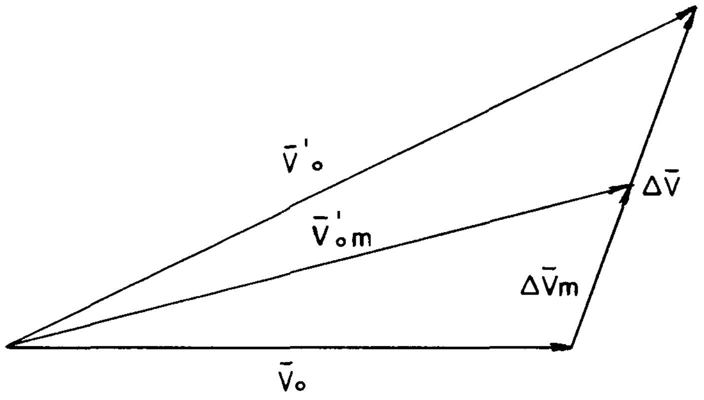


Figure 3. Velocity Diagram

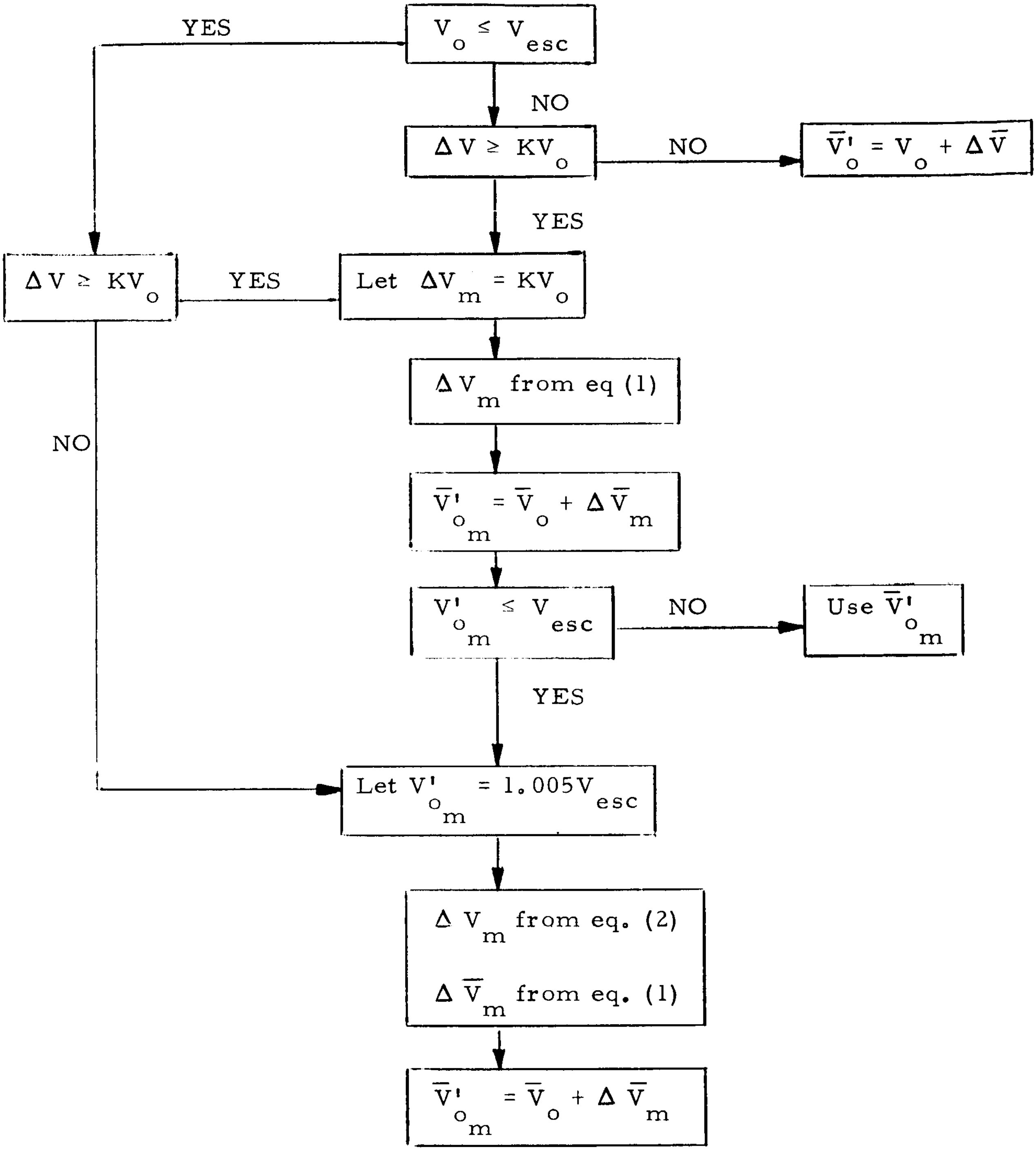
The modification also limits the initial velocity to values greater than the escape velocity of the departure planet.

The modification is described below by a list of symbols, a logic diagram and the pertinent equations.

List of Symbols

r_o	radial distance from origin at injection
$\bar{V}_o$	velocity at injection, from two-body estimate or previous n-body run, to be corrected
$(\dot{X}, \dot{Y}, \dot{Z})$	rectangular components of velocity
$\Delta \bar{V}$	velocity correction predicted by linear differential correction procedure

LOGIC DIAGRAM



3.3 Computations of Sensitivity Coefficients

In Program I (the Interplanetary Trajectory Program) the sensitivity coefficients are computed by perturbation of the initial conditions; a 3 x 3 sensitivity matrix is computed between position components at the end of trajectory and velocity components at the beginning. In Program III the sensitivity coefficients are obtained by integrating the equations of variation along the trajectory. The sensitivity coefficients denote the rate of change of position and velocity components at some time with respect to changes in position and velocity components at the initial time. A 3 x 3 submatrix of the 6 x 6 matrix at the final time is used in the differential correction procedure; the complete 6 x 6 matrix may also be printed out at the integration intervals of the nominal trajectory.

In the computation of the sensitivity coefficients advantage is taken of the doubling and halving procedure of integration. Because of this, the gravity gradients are available at the beginning, the midpoint and the end of the integration interval. This results in a computation of the sensitivity coefficients with errors proportional to the fifth power of the integration interval.

Let the equations of motion be represented by

$$\ddot{\bar{r}} = -\nabla\phi\,(\bar{r},\ t) \tag{1}$$

where ϕ is the gravitational potential of N bodies. Let $\bar{r}_n$ be the position vector on the nominal trajectory, $\bar{r}$ the position vector of a variation trajectory. The equations of variation are then

$$\delta\,\ddot{\bar{r}} = \ddot{\bar{r}} - \ddot{\bar{r}}_n = -\left[\nabla\phi\,(\bar{r}_n, t) + \frac{\partial}{\partial x}\nabla\phi\,(\bar{r}_n, t)\,\delta x + \frac{\partial}{\partial y}\nabla\phi\,\delta y + \frac{\partial}{\partial z}\nabla\phi\,\delta z\right] + \nabla\phi(\bar{r}_n, t)$$

or, expanded,

$$\begin{aligned} \delta\ddot{x} &= G_{xx}\,\delta_x + G_{xy}\,\delta_y + G_{xz}\,\delta_z \\ \delta\ddot{y} &= G_{yx}\,\delta_x + G_{yy}\,\delta_y + G_{yz}\,\delta_z \\ \delta\ddot{z} &= G_{zx}\,\delta_x + G_{zy}\,\delta_y + G_{zz}\,\delta_z \end{aligned} \tag{2}$$

with $G_{xx} = \dfrac{-\partial^2 \phi(\bar{r}_n, t)}{\partial x^2}$, $G_{xy} = \dfrac{-\partial^2 \phi(\bar{r}_n, t)}{\partial y \, \partial x}$, etc.

This is a set of linear second order equations

$$\delta \ddot{\bar{r}} = G(t)\, \delta \bar{r} \tag{3}$$

The G_{ij} are the components of the gravity gradient tensor; they are evaluated on the nominal trajectory as follows.

The gravitational potential is given by

$$\phi = \frac{\mu_k}{r_k}\left[1 + J \frac{r_e^2}{r_k^2}\left(\frac{1}{3} - \frac{z^2}{r_k^2}\right)\right] + \sum_{\substack{j=1 \\ j \neq k}}^{N} \frac{\mu_j}{r_j} \tag{4}$$

where the first term represents the earth potential (including the second harmonic) and the second term the N-body potential. The notation is

r_k = distance from earth center

r_e = earth radius

r_j = distance to center of body j

$\mu_k = k^2 M$, M is mass of earth

$\mu_j = k^2 M_j$, M_j is mass of body j

J = first oblateness constant

z = position coordinate parallel to earth's axis of rotation.

By differentiation of (4) the components of the gravity gradient are obtained

$$G_{xx} = -\sum_j \frac{\mu_j}{r_j^3}\left(1 - 3\frac{x_j^2}{r_j^2}\right) - J\frac{\mu_k r_e^2}{r_k^5}\left(1 - 5\frac{(x_k^2 + z_k^2)}{r_k^2} + 35\frac{x_k^2 z_k^2}{r_k^4}\right)$$

$$G_{yy} = -\sum_j \frac{\mu_j}{r_j^3}\left(1 - 3\frac{y_j^2}{r_j^2}\right) - J\frac{\mu_k r_e^2}{r_k^5}\left(1 - 5\frac{(y_k^2 + z_k^2)}{r_k^2} + 35\frac{y_k^2 z_k^2}{r_k^4}\right)$$

$$G_{zz} = - \sum_j \frac{\mu_j}{r_j^3}\left(1-3\,\frac{z_j^2}{r_j^2}\right) - J\,\frac{\mu_k r_e^2}{r_k^5}\left(3-30\,\frac{z_k^2}{r_k^2} + 35\,\frac{z_k^4}{r_k^4}\right)$$

$$G_{xy} = G_{xy} = \sum_j \mu_j\,\frac{3x_j y_j}{r_j^5} + J\,\frac{\mu_k r_e^2 x_k y_k}{r_k^7}\left(5-35\cdot\frac{z_k^2}{r_k^2}\right)$$

$$G_{xz} = G_{zx} = \sum_j \mu_j\,\frac{3x_j z_j}{r_j^5} + J\,\frac{\mu_k r_e^2 x_k z_k}{r_k^7}\left(15-35\,\frac{z_k^2}{r_k^2}\right)$$

$$G_{yz} = G_{zy} = \sum_j \mu_j\,\frac{3y_j z_j}{r_j^5} + J\,\frac{\mu_k r_e y_k z_k}{r_k^7}\left(15-35\,\frac{z_k^2}{r_k^2}\right)$$

The summation signs $\sum_j$ stand for $\sum_{j=1}^{N}$

Since the positions on the nominal trajectory are available at three points of the integration interval (because of the doubling and halving procedure), the gravity gradient is also available at three points.

If $\bar{X}$ is the vector $\left\{ \delta\,\bar{r},\ \delta\,\dot{\bar{r}} \right\}$, equation (3) can be written as

$$\dot{\bar{X}} = F(t)\,\bar{X} \tag{5}$$

with

$$F = \begin{vmatrix} O & I \\ G & O \end{vmatrix} \tag{6}$$

Here I is the 3 x 3 unit matrix, O is the 3 x 3 null matrix and G is the gravity gradient matrix.

If six solutions of equ. (4) are obtained at some time t_1 with the initial conditions taken as

$$\overline{X}(t_o) = \left|\begin{matrix}1\\0\\0\\0\\0\\0\end{matrix}\right| , \left|\begin{matrix}0\\1\\0\\0\\0\\0\end{matrix}\right| , \left|\begin{matrix}0\\0\\1\\0\\0\\0\end{matrix}\right| , \text{ etc.}$$

and column wise arranged in a matrix, the resulting matrix is the sensitivity matrix from t_o to t_1. Let this matrix be $H_{1,0}$; the following equation then represents the relation between position and velocity deviations at t_1 and t_o:

$$\overline{X}(t_1) = H_{1,0} \overline{X}(t_o) \tag{7}$$

where

$$H_{1,0} = \left|\begin{matrix} \frac{\partial x_1}{\partial x_o} & \frac{\partial x_1}{\partial y_o} & --- \\ \frac{\partial y_1}{\partial x_o} & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & & \cdot \\ \cdot & --- & \frac{\partial \dot{z}_1}{\partial \dot{z}_o} \end{matrix}\right|$$

For a time t_2, equation (7) gives

$$\overline{X}(t_2) = H_{2,1} \overline{X}(t_1)$$

But $\overline{X}(t_1) = H_{1,0} \overline{X}(t_o)$

thus $\overline{X}(t_2) = H_{2,1} H_{1,0} \overline{X}(t_o)$

and $H_{2,0} = H_{2,1} H_{1,0}$

By continuing this procedure, the chain rule for sensitivity matrices follows

$$H_{n,o} = H_{n,n-1} H_{n-1,n-2} --- H_{2,1} H_{1,0} = \prod_{i=n}^{1} H_{i,i-1} \tag{8}$$

This suggests that the sensitivity matrix for an arbitrary time interval can be computed by multiplying the sensitivity matrices for a number of subintervals. As convenient subintervals, the integration intervals of the nominal trajectory may be chosen. If a constant value of G is assumed during an integration interval (for instance the value at the midpoint), simple expressions follow for the H covering that interval, but the errors are proportional to the third power of the interval. Better expressions (resulting in errors proportional to the fifth power of the interval) are found by using the fourth order Runge-Kutta formula. For its use three values of G are needed, at the beginning, the midpoint and the end of the interval. If the doubling and halving procedure is used for the integration of the trajectory, these three values of G are available. Applying this to equation (5), that equation is first written as a matrix equation

$$[\dot{X}] = F\,[X] \qquad (9)$$

To integrate this equation for a time step $\Delta t = t_{n+1} - t_n$, the Runge-Kutta formula is applied as follows.

Let $[X]_{t=t_n}\ [X]_n$,

let also $F_o = F(t_n)$, $F_{1/2} = F(t_n + \frac{1}{2}\Delta t)$, $F_1 = F(t_n + \Delta t)$,

then $K_1 = F_o\,[X]_n \Delta t$

$$K_2 = F_{1/2}\,[I + \tfrac{1}{2} K_1]\,[X]_n\,\Delta t = F_{1/2}\,[I + F_o,\ \tfrac{1}{2}\Delta t]\,[X]_n \Delta t$$

$$K_3 = F_{1/2}\,[I + \tfrac{1}{2} K_2]\,[X]_n\,\Delta t =$$

$$= F_{1/2}[I + F_{1/2},\ \tfrac{1}{2}\Delta t + F_{1/2}\,F_o\ \tfrac{1}{2}\Delta t]\,[X]_n \Delta t$$

and $K_4 = F_1\,[I + K_3]\,[X]_n\,\Delta t =$

$$= F_i\,[I + F_{1/2}\,\Delta t + F^2_{1/2} \cdot \tfrac{1}{2}\Delta t^2 + F^2_{1/2}\,F_o \circ \tfrac{1}{2}\Delta t^3]\,[X]_n\,\Delta t\,.$$

The result of the fourth-order Runge-Kutta integration is thus

$$[X]_{n+1} = [X]_n + [\Delta X] = \Big[I + (F_o + 4F_{1/2} + F_1)\frac{\Delta t}{6} + (F_{1/2}F_o + F_{1/2}^2 + F_1F_{1/2})\frac{\Delta t^2}{6} + (F_{1/2}^2F_o + F_1F_{1/2}^2)\frac{\Delta t^3}{12} + F_1F_{1/2}^2F_o\frac{\Delta t^4}{24}\Big][X_n] \tag{10}$$

or
$$[X]_{n+1} = H_{n+1,n}\,[X]_n \tag{11}$$

where $H_{n+1,n}$ is the "incremental" sensitivity matrix from t_n to t_{n+1}. If equation (6) is substituted in (10), the following expressions result for the submatrices of the partitioned sensitivity matrix $H_{n+1,n}$.

Let

$$H_{n+1,n} = \begin{vmatrix} H_1 & H_2 \\ H_3 & H_4 \end{vmatrix} .$$

Then

$$\left.\begin{aligned}
H_1 &= I + (G_o + 2G_{1/2})\frac{\Delta t^2}{6} + G_{1/2}G_o\frac{\Delta t^4}{24} \\
H_2 &= I\,\Delta t + G_{1/2}\frac{\Delta t^3}{6} \\
H_3 &= (G_o + 4G_{1/2} + G_1)\frac{\Delta t}{6} + (G_{1/2}G_o + G_1G_{1/2})\frac{\Delta t^3}{12} \\
H_4 &= I + (2G_{1/2} + G_1)\frac{\Delta t^2}{6} + G_1G_{1/2}\frac{\Delta t^4}{24} \\
&\text{where } G_o = G(t_n),\ G_{1/2} = G(t_n + \tfrac{1}{2}\Delta t),\ G_1\ G(t_n + \Delta t).
\end{aligned}\right\} \tag{12}$$

In the computation of the sensitivity matrix $H_{n,o}$ by continued product, as in equ. (8), the integration for the i^{th} interval is carried out with $[X]_i = I$, the unit matrix. This method is used in the computer program described in this report. If $H_{n,o}$ were to be computed by continuous integration, the initial condition for the i^{th} interval would be $[X]_i$, the result of all previous integrations. Because of the linearity of the equations of variation the two processes are precisely equivalent with respect to the truncation error. In the build up of round off error they behave differently, although it is not immediately obvious that either one is inferior to the other. To reduce the influence of roundoff error all computations are done in double precision.

Expressions for the truncation errors of a single integration step may be derived by carrying out a few steps of the Peano-Baker method, using a Taylor series expansion for the gravity gradient matrix G. The result may be compared with the expressions obtained by substituting Taylor expansions for $G_{1/2}$ and G_1 in equ. (12). If ϵ is defined as the exact solution minus the computed solution, the truncation errors for the sub-matrices of $H_{n+1,n}$ are

$$\epsilon_1 = \left(\frac{1}{720} \dddot{G}_o + \frac{1}{240} \dot{G}_o G_o + \frac{1}{120} G_o \dot{G}_o \right) \Delta t^5$$

$$\epsilon_2 = \left(\frac{1}{240} \dddot{G}_o + \frac{1}{120} G_o^2 \right) \Delta t^5$$

$$\epsilon_3 = -\left[\frac{1}{2880} \ddddot{G}_o + \frac{1}{480} (G_o \ddot{G}_o + \ddot{G}_o G_o) + \frac{1}{120} \dot{G}_o^2 - \frac{1}{120} G_o^3 \right] \Delta t^5$$

$$\epsilon_4 = -\left(\frac{1}{720} \dddot{G}_o + \frac{1}{120} \dot{G}_o G_o + \frac{1}{240} G_o \dot{G}_o \right) \Delta t^5$$

The program can print out the incremental as well as the total sensitivity matrix at every trajectory print out. The final sensitivity matrix is also printed out, together with its "theoretical" inverse, i.e., the matrix obtained by rearranging the components of the sensitivity matrix according to

$$H^{-1} = \begin{bmatrix} H_4 & -H_2 \\ -H_3 & H_1 \end{bmatrix}^T .$$

4. OPERATING INSTRUCTIONS

This section outlines the operational procedures for the three decks. The Interplanetary Trajectory Program is Program I, the N-Body Program with the ground trace feature and optional output conversion is Program II, and the N-Body Program with sensitivity coefficients is Program III. Each program is completely described in each section so that there need be no cross referencing from one part of the report to another when setting up a job. Included in each section are the FORTRAN statements for the 3 decks showing the main programs as they request input and call for subroutines.

The following listings of input data show certain values for the constants (gravitational constant, radius and mass of the earth, oblateness constant). The values have been chosen to conform with those which are presently in use at J.P.L; they differ therefore from the constants which were recommended in references 1 and 2. Also, the sample problems listed in this report have been run with different values. The J.P.L. constants which are the basis for the values in the input listings are:

$1\text{AU} = 1.49599 \times 10^8 \text{km}$

$k^2 M_E = 398603.2 \text{ km}^3/\text{sec}^2$

$R_E = 6378.165$ km (earth equatorial radius)

$J = 1.62345 \times 10^{-3}$ (oblateness constant)

General Operating Procedure

A. Input Format

The format for input is as follows. Card columns (cc) 1 through 72 may be used. The first character on a data card must be an F or an X. The F denotes floating point numbers and the X, fixed point numbers (Integers). The numbers on each card are to be separated by commas and the last character on each card must be an asterisk. At the end of each program description in this section, there is a sample case with input data included in the proper format. A sample input data card might have the following set of numbers:

e.g. `F1.75,3.4E-06,X20,F.008*`

B. Deck Make-up (Monitor Control Cards)

(1) Card 1 (optional)

This is an ID card. Columns 1 and 2 contain asterisks and columns 3-16 are available for the programmer's name.

(2) Card 2 (optional)

The next card(s) are used to notify the operator if any additional tapes are to be mounted for the run. In our case, an ephemeris tape must be mounted on B-6. CC1 and 2 contain dollar signs ($$), CC5 contains the number 1; (if two tapes had to be used, the second card would have the number 2 in CC5).

CC7-16 contain the library number of the tape(s) to be used. CC20-22 are used to indicate the number of the particular tape drive. CC78-79 is for a sequence number and are the same as CC5.

(3) Card 3 (must be included)

Card #3 has a $ sign in CC1 and the word EXECUTE in CC2-8. CC16-17 contain F2 (indicating FORTRAN II).

(4) Card 4 (must be included)

Card #4 has an asterisk in CC1 and in CC7-9, the letters XEQ.

(5) Card 5 (must be included)

Card #5 is placed directly behind the binary deck and directly in front of the input data. C1 has an asterisk and CC7-10 have the word DATA.

(6) Card 6 (must be included)

This is an end of file card and is the last card on the job deck. It has a multiple 7, 8 punch in CC1.

Thus, the deck make up is as follows:

```
                 CARD COLUMN NUMBERS
        1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 ---- 78 79 80

Card
Number

I       * * T . C O F F I N

II      $ $     1   A 2 9 6  0  7                               0  1

III     $ E X E C U T E                      F  2

IV      *           X E Q

                     +--------+
                     | BINARY |
                     |  DECK  |
                     +--------+

V       *           D A T A

                     +-------+
                     | INPUT |
                     | DATA  |
                     +-------+

VI      7
        8
```

GENERAL OPERATING NOTES FOR THE 3 PROGRAMS

1. Tapes Used

   All tapes used in normal FORTRAN System. Tape B-6, Planet Position Tables Tape.

2. Sense Switches

   Sense Switch 1 is the only switch tested. In the down position, output will be printed on-line as well as on tape A-3. The sense switch may be repositioned at any time. Printing on line uses the Share No. 2 printer board.

3. Card Deck

   Normal FORTRAN System deck set-up with input cases following. Operation is initiated by the FORTRAN System.

4. Stops

   a. All FORTRAN stops.

   b. Stops of form HTR*. These are double precision subroutine stops caused by overflow. Experience has shown these to be due to machine error or input error.

   c. HPR 77777 - Error in two-body solution.

4.1 PROGRAM I

Three types of runs may be made with the Interplanetary Trajectory Program:

1. Interplanetary trajectory
2. Determination of initial configuration angle,
3. N-body trajectory.

The required information is listed for each type of run, along with the corresponding computer output. Also included is a description of the FORTRAN II input-routine to be used with the program.

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*To begin processing another case when machine has stopped, manually transfer to 171_8.

4.1.1 COMPUTER INPUT

This section lists the computer input required for the three types of problems which may be run with the Interplanetary Trajectory Program. These are:

Type 1. Interplanetary trajectory - Computation of trajectory with differential correction procedure from trip time, departure date, and route.

Type 2. Determination of initial configuration angle - Computation of from trip time and departure date. This problem is run prior to run type 1 if the angle ϕ has not been determined by hand computation in order to find the required route.

Type 3. N-body trajectory - Computation of trajectory from initial position and velocity, including midcourse correction capability.

All decimal input is read by a modified DBC FORTRAN subroutine which accepts variable length fields. An (x) following the field number in the following listings indicates a fixed point variable. All other inputs are floating point variables.

TYPE 1.

INTERPLANETARY TRAJECTORY

	FORTRAN Symbols		
Read Statement 1			
Field 1 (x)	NTYPE		code digit for type of run = 1
Read Statement 2			
Field 1	TT	T	trip time, hours
2	DJD	t_s	date of departure, days from beginning of tape (table-days)
3 (x)	ND		code digit for departure planet
4 (x)	NT		code digit for destination planet

Read Statement 3

Field 1 (x)	NR	NR	number of routes to be designated (see note 1)
			NR sets of data, each set composed of the next two fields
2 (x)	NJ(I)		code digit of the route
3 (x)	EC(I)	e_i	initial estimate of the eccentricity (see note 2)
4	GEE	q_i	initial estimate of the ratio a/r_1
5 (x)	KIT	K	number of iterations before attempting next route
6	DT	ΔT	allowable tolerance on trip time, hours
7	DPHI	$\Delta \phi$	allowable tolerance on vehicle travel angle, degrees

Read Statement 4

Field 1	BLE	λ_E	launch site latitude, degrees (0 for non-Earth departure)
2	GEL	L	launch site longitude, degrees (0 for non-Earth departure)
3	RO	r_o	burnout radius, statute miles
4	G(1)	k^2	gravitational constant = 2.9591221×10^{-4}, (radians/day)^2
5	WTE	$M_\oplus$	mass of the Earth = 3.0034424×10^{-6}, solar mass units
6	WTV	m_v	mass of vehicle, solar mass units
7	RADE	$R_{\oplus e}$	equatorial radius of the Earth = $.42635078 \times 10^{-4}$, A.U.

Note 1: One or more of the six possible routes may be designated as input in preferential order. If after K iterations for each route a successful trajectory is not attained with any of these routes, the remaining routes will automatically be attempted by the computer.

Note 2: The initial eccentricities, e_i, used for routes which are not designated as input are 0.5 for elliptic routes and 1.5 for hyperbolic routes.

8	OBJ	J'	oblateness constant for the Earth = $JM_{\oplus}R_{\oplus}^2 = 8.8632361 \times 10^{-18}$, (solar mass units, A.U.^{2})

Read Statement 5

Field 1	EMAX	$D_{1\,max}$	maximum distance from Earth, A. U.
2	GMAX	$D_{2\,max}$	maximum distance from target, A. U.
3	SMAX	$D_{3\,max}$	maximum distance from Sun, A. U.

Read Statement 6

Field 1 (x)	NCKE		control digit indicating computing scheme, 0 = Cowell, 1 = Encke
2 (x)	MSDT		control digit indicating selection of integration time step interval, 0 = doubling and halving procedure 1 = three fixed Δt's
3 (x)	MET		control digit indicating whether computing scheme (Cowell, Encke) is to be switched, 0 = retain original scheme, 1 = switch schemes on test
4 (x)	NOUT	n	print digit n indicates print every n integrations
5 (x)	NOSW		control digit, 0 = retain initial origin, 1 = switch origins on test

Read Statement 7 (used only if Field 2 of Read Statement 6 is 0)

Field 1	DTOR	Δt_i	initial Δt, hours
2	DTMAX	Δt_{max}	maximum Δt, hours
3	EP1	ϵ	value to test accuracy of integration, A. U.

Read Statement 8 (used only if Field 2 of Read Statement 6 is 1)

Field 1	DTA	Δt_1	Δt within 3 radii of origin, hours
2	DTB	Δt_2	Δt within 100 radii of origin, hours
3	DTC	Δt_3	Δt 100 radii from origin, hours

Read Statment 9

Field 1 (x)	NBD	N_b	the number of bodies other than the Earth to be included in the study (the Earth is always included)

Read Statement 10

N_b sets of data, each including

Field 1(x)	NB (I)		code digit of body
2	WT (I)		mass of body, solar mass units
3	RAD (I)		radius of body, A. U.

TYPE 2.

DETERMINATION OF INITIAL CONFIGURATION ANGLE

Read Statement 1

Field 1(x)	NTYPE	2	code digit for type of run

Read Statement 2

Field 1	DJD	t_s	date of departure, days from beginning of tape (table-day)
2	TT	T	trip time, hours
3 (x)	ND		code digit for departure planet
4 (x)	NT		code digit for destination planet

TYPE 3.

N-BODY TRAJECTORY

Read Statement 1

Field 1(x)	NTYPE	3	code digit for type of run

Read Statement 2

Field 1	G91)	k^2	gravitational constant = 2.9591221 x 10^{-4}, $(\text{radians/day})^2$
2	WTE	$M_{\oplus}$	mass of Earth = 3.0034424 x 10^{-6}, solar mass units

3	RADE	$R_{\oplus e}$	equatorial radius of the Earth = $.42635078 \times 10^{-4}$, A. U.
4	WTV	m_v	mass of the vehicle, solar mass units
5	OBJ	J'	oblateness constant for the Earth = $JM_{\oplus} R_{\oplus}^2 = 8.8632361 \times 10^{-18}$, (solar mass units) (A. U. $)^2$

Read Statement 3

Field 1	EMAX	$D_{1\,max}$	maximum distance from Earth, A. U.
2	GMAX	$D_{2\,max}$	maximum distance from target, A. U.
3	SMAX	$D_{3\,max}$	maximum distance from Sun, A. U.
4	TMAX	T	trip time, hours
5	TO	t_s	date of departure, hours from Jan. 1960, 0^h UT

Read Statement 4

Field 1 (x)	NCKE		control digit indicating computing scheme, 0 = Cowell, 1 = Encke
2 (x)	MSDT		control digit indicating selection of integration time step interval, 0 = doubling and halving procedure, 1 = three fixed Δt's
3 (x)	MET		control digit indicating whether computing scheme (Cowell, Encke) is to be switched 0 = retain original scheme 1 = switch schemes on test
4 (x)	NOUT	n	print digit n indicates print every n integrations
5 (x)	NOSW		control digit, 0 = retain initial origin, 1 = switch origins on test
6 (x)	NMC	N_M	number of midcourse corrections

Read Statement 5 (used only if Field 2 of Read Statement 4 is 0)

Field 1	DT	Δt_i	initial Δt, hours
2	DTMAX	Δt_{max}	maximum Δt, hours
3	DP1	ϵ	value to test accuracy of integration, A. U.

Read Statement 6 (used only if Field 2 of Read Statement 4 is 1)

Field 1	DTA	Δt_1	Δt within 3 radii of origin, hours
2	DTB	Δt_2	$\Delta t > 100$ radii from origin, hours

Read Statement 7

Field 1 (x)	N	N_b	number of bodies other than the Earth to be included in the study (Earth is always included)
2 (x)	NEWORG		code digit for origin planet
3 (x)	NTARG		code digit for destination planet

Read Statement 8

N_b sets of data, each including

Field 1 (x)	NB(I)	code digit of body
2	WT(I)	mass of body, solar mass units
3	RAD(I)	radius of body, A. U.

Read Statement 9

Field 1	PM(1)	x_o	position coordinates with respect to origin at burnout, A. U.
2	PM(2)	y_o	
3	PM(3)	z_o	
4	PM(4)	$\dot{x}_o$	velocity components with respect to origin at burnout, A. U. /hour
5	PM(5)	$\dot{y}_o$	
6	PM(6)	$\dot{z}_o$	

Read Statement 10 (used only if Field 6 of Read Statement 4 is not 0)

Field 1	SIMC	I_{sp}	specific impulse, seconds
2	FMC	$\dot{m}$	mass flow, lb_m/sec
3*	WVMC	m_v	mass of vehicle, lb_m

N_M sets of data, each including

4	PPM(I)	T_M	time of midcourse correction application, hours from start of trip

---

*Not necessarilly equal to Field 4 of Read Statement 2.

5	PPL(I)	Δt	burning time, seconds
6	QM(I)	α	component of angle between ΔV and velocity vector in trajectory plane degrees
7	QL(I)	β	component of angle between ΔV and velocity vector normal to trajectory plane, degrees

The code digits for the various planets and routes are as follows:

Planetary Code Digits:

0	Earth
1	Sun
2	Moon
3	Mercury
4	Venus
5	Mars
6	Jupiter
7	Saturn
8	Uranus
9	Neptune

Route Code Digits:

1	Route	D	(direct)
2	Route	P	(perihelion)
3	Route	A	(aphelion)
4	Route	I	(indirect)
5	Route	D_H	(direct hyperbolic)
6	Route	P_H	(perihelion hyperbolic)

4.1.2 COMPUTER OUTPUT

TYPE 1.

INTERPLANETARY TRAJECTORY

Normally, output will be obtained from each major section of the program as follows:

A. Heliocentric Transfer Analysis

1. The eccentricity of the transfer orbit
2. q; the ratio a/r_1

B. Departure Analysis

1. Exact burnout time, hours from beginning of tape

2. Position coordinates at burnout; A. U.
3. Velocity components at burnout; A. U. /hour

C. N-body Integration Program

The following information is printed-out after every n integration steps, where n is an input-control parameter

1. Flight time, hours from start of trajectory
2. Table time, hours from beginning of tape
3. Time increment of integration step, hours
4. Planetary code digit of body at the origin
5. Acceleration components of the vehicle with respect to the origin, A. U. /hr^2
6. Velocity components of the vehicle with respect to the origin, A. U. /hr
7. Position coordinates of the vehicle with respect to the origin, A. U.
8. Position coordinates of the vehicle with respect to the Earth, A. U.
9. Position coordinates of the vehicle with respect to the target, A. U.
10. Position coordinates of the vehicle with respect to the Sun, A. U.

D. Differential Correction Analysis

1. Modified initial velocity components at burnout, A. U. /hr

All times are expressed in hours and distances in astronomical units. Position, Velocity, and acceleration components are expressed in a rectangular coordinate system, referenced to the equatorial plane and mean equinox of 1950. 0

TYPE 2.

DETERMINATION OF INITIAL CONFIGURATION ANGLE

1. Trip time, days
2. Initial configuration angle, degrees

TYPE 3.

N-BODY TRAJECTORY

The following information is printed out after every n integration steps, where n is an input-control parameter.

1. Flight time, hours from start of trajectory
2. Table time, hours from beginning of tape
3. Time increment of integration step, hours
4. Planetary code digit of body at the origin
5. Acceleration components of the vehicle with respect to the origin, A. U. /hr^2
6. Velocity components of the vehicle with respect to the origin, A. U. /hr
7. Position coordinates of the vehicle with respect to the origin, A. U.
8. Position coordinates of the vehicle with respect to the Earth, A. U.
9. Position coordinates of the vehicle with respect to the target, A. U.
10. Position coordinates of the vehicle with respect to the Sun, A. U.

The following information is printed out after every midcourse correction.

1. Mass of the vehicle before correction, lb_m
2. Mass of the vehicle after correction, lb_m
3. Change in velocity components with respect to origin due to correction, A. U. /hr
4. Velocity components of vehicle with respect to origin after correction, A. U. /hr

4.1.3 SAMPLE PROBLEM

PROGRAM I

(Interplanetary Trajectory Program)

Mars to Earth Trajectory

CARD #	
1	X1*
2	F5760.0, 1776.0, X5, 0*
3	X1, 1, F0.2300, 0.8500, X30, F0.10, .10*
4	F0.0, 0.0, 2500.0, 2.959122083E-04, 2.9991123E-06, 0.0*
5	F.426636E-04, 8.8609392E-18*
6	F6.0, 6.0, 8.0*
7	X0, 0, 0, 25, 0*
8	F0.25, 32.0, 4.0E-09*

```
9       X2*
10      X5,F3.2325845E-07,2.26E-05*
11      X1,F1.0,4.655E-03*
```

```
N BODY TRAJECTORY

     GENERAL ELECTRIC CO.

     M.S.V.D.


             STARTING TABLE TIME   = 42636.000

             ORIGIN IS BODY 5   MASS =   3.232584E-07

             DESTINATION IS BODY 0   MASS =   2.999112E-06

             MASS OF VEHICLE =  0.

             OTHER BODIES ARE-

                 BODY  1  MASS =  1.000000E 00

             GRAVITATIONAL CONSTANT =  2.959122E-04

             EPSILON OF INTEGRATION = 4.00E-09   MAXIMUM DELTA T =  32.0

             COWELL METHOD IS USED

             THE ORIGIN IS FIXED
```

```
             T =  240.000     PHI = 170.310     E = 0.23000     Q =  0.85000
ROUTE  1     T =  240.002     PHI = 170.312     E = 0.23231     Q =  0.81167
```

NOTE:

The above indicates the route selection for the nominal trajectory.
The next page is only the nominal trajectory for Route 1. The perturbations on $\dot{x}$, $\dot{y}$, $\dot{z}$ are not included.

```
TIME=   0.        TAPE TIME= 42636.000    DELTA T=    0.250    CENTER IS NO.  5

POSIT FROM   EARTH             TARGET            SUN             CENTER          VELOCITY        ACCELERATION
  X    -1.3737219E 00   -1.3737219E 00   -7.2355542E-01   4.4206819E-06   1.6540647E-04   0.
  Y     6.3417706E-01    6.3417706E-01    1.3188813E 00  -2.1900907E-05  -1.2523806E-04   0.
  Z     3.2789295E-01    3.2789295E-01    6.2481337E-01  -1.5001045E-06   2.3149003E-04   0.
  R     1.5481622E 00    1.5481622E 00    1.6289175E 00   2.6911400E-05   3.1085608E-04   0.

 TIME= 682.6250    TAPE TIME= 43318.625    DELTA T=  32.000    CENTER IS NO.  5

POSIT FROM   EARTH             TARGET            SUN             CENTER          VELOCITY        ACCELERATION
  X    -1.1485864E 00   -1.1485864E 00   -9.3900977E-01   1.0273368E-01   1.4994055E-04  -1.2355676E-09
  Y     1.9889798E-01    1.9889798E-01    1.0816466E 00  -6.9017357E-02  -1.0203517E-04  -4.2305837E-09
  Z     3.2619244E-01    3.2619244E-01    7.0899584E-01   1.5284888E-01   2.1819659E-04  -2.5480977E-08
  R     1.2104597E 00    1.2104597E 00    1.5982408E 00   1.9667329E-01   2.8373067E-04   2.5859324E-08

 TIME=2282.6250    TAPE TIME= 44918.625    DELTA T=  32.000    CENTER IS NO.  5

POSIT FROM   EARTH             TARGET            SUN             CENTER          VELOCITY        ACCELERATION
  X    -3.8443463E-01   -3.8443463E-01   -1.1984684E 00   3.5054766E-01   1.6972536E-04   3.5961078E-08
  Y    -2.1192316E-01   -2.1192316E-01    3.0129476E-01  -2.3839656E-01  -1.0700147E-04   7.6655930E-09
  Z     5.0779058E-01    5.0779058E-01    7.3034565E-01   4.4102425E-01   1.2349472E-04  -9.4694060E-08
  R     6.7123221E-01    6.7123221E-01    1.4354476E 00   6.1173440E-01   2.3559915E-04   1.0158211E-07

 TIME=3882.6250    TAPE TIME= 46518.625    DELTA T=  32.000    CENTER IS NO.  5

POSIT FROM   EARTH             TARGET            SUN             CENTER          VELOCITY        ACCELERATION
  X    -6.7477815E-02   -6.7477815E-02   -9.2271364E-01   7.0019661E-01   2.8698893E-04   1.0452622E-07
  Y    -9.6953183E-02   -9.6953183E-02   -5.8182618E-01  -3.5989172E-01  -5.4421216E-06   1.5941334E-07
  Z     6.4093685E-01    6.4093685E-01    4.3067682E-01   4.8896804E-01  -7.7681046E-05  -1.4397013E-07
  R     6.5173094E-01    6.5173094E-01    1.1727765E 00   9.2676160E-01   2.9736611E-04   2.3888437E-07

 TIME=5482.6250    TAPE TIME= 48118.625    DELTA T=  32.000    CENTER IS NO.  5

POSIT FROM   EARTH             TARGET            SUN             CENTER          VELOCITY        ACCELERATION
  X    -7.6876581E-02   -7.6876581E-02    2.6277050E-02   1.2479724E 00   3.2474670E-04  -1.7631929E-07
  Y    -8.8607190E-03   -8.8607190E-03   -9.3675809E-01  -2.1456329E-02   5.1191758E-04   4.3775526E-07
  Z     2.3591340E-01    2.3591340E-01   -1.6646618E-01   2.2086303E-01  -2.0587374E-04   4.8218697E-08
  R     2.4828140E-01    2.4828140E-01    9.5179681E-01   1.2675472E 00   6.4023747E-04   4.7438718E-07

 TIME=5780.0000    TAPE TIME= 48396.000    DELTA T=    0.062    CENTER IS NO.  5

POSIT FROM   EARTH             TARGET            SUN             CENTER          VELOCITY        ACCELERATION
  X    -7.2022244E-02   -7.2022244E-02    2.2256519E-01   1.3298510E 00   2.6051197E-04  -2.8682716E-07
  Y     1.8778570E-02    1.8778570E-02   -8.7391177E-01   1.3707905E-01   6.2957385E-04   4.0265995E-07
  Z     1.1917100E-01    1.1917100E-01   -2.6794199E-01   1.6638200E-01  -1.8425173E-04   1.0659849E-07
  R     1.4050468E-01    1.4050468E-01    9.4077094E-01   1.3472110E 00   7.0581756E-04   5.0573519E-07

     DESIRED TIME
```

4.1.4 LISTING

This listing is the main program for PROG #1. No subroutines are included since it is here included to assist in setting up job decks.

```
          INTERPLANETARY  PROGRAM  - SHERVIN                        1
 EQUIVALENCE(WM(1),XIM),(WM(2),YI  ),(WM(3),ZIM),(WM(4),XIDM),
1(WM(6),ZIDM),(WM(5),YIDM),(WL(1),XIL),(WL(2),YIL),(WL(3),ZIL),
2(WL(4),XIDL),(WL(5),YIDL),(WL(6),ZIDL)
 DAC PX(9,67),PY(9,67),PZ(9,67),TL(67),RKM(4,6),RKL(4,6),NB(9),
1WT(9),RAD(9),X(9),Y(9),Z(9),DIS(9),DIST(9),PM(6),PL(6),PPM(6),
2PPL(6),QM(6),QL(6),WM(6),WL(6),XX(9),YY(9),ZZ(9),TI(6),CL(6)
 COMMON NEWORG,ZER,FMX,FLX,FMY,FLY,FMZ,FLZ,AM,AL,BM,BL,CM,CL
 COMMON NORG,NTARG,NSUN,N,NN,NNN,T,TO,TMAX,DTMAX,HAFDT,WTE,WTV,MGM,
1XTARG,YTARG,ZTARG,SCEN,VCEN,SMAX,GMAX,EMAX,RAD1,RAD2,RADE,WTA,
2NBE,NBT,NBS,RADORG,SAM,SBM,SCM, SAL,SBL,SCL,XIM,XIL,YIM,YIL,ZIM,
3ZIL,XIDM,XIDL,YIDM,YIDL,ZIDM,ZIDL,RM,RL,RR,RO,SS,RRM,RRL,TEM,TEL,
4GMX,GLX,GMY,GLY,GMZ,GLZ,SUMX,SULX,SUMY,SULY,SUMZ,SULZ,FM,FL,GM,GL,
5HM,HL,RRR,SSS,XM,YM,ZM,A,BX,BY,BZ,WRM,WRL,RCM,RCL,ROM,ROL,RPPM,
6XPM,XPL,YPM,YPL,ZPM,ZPL,
7KOB,VNZ,MEDT,MSDT,MDT,NOT,MET,NOUT,NCKE,TSAV,DTA,DTB,DTC,EP1
 DAC G(2),WTS(2)
 DAC      SOGOOD(6),XDOT(2),YDOT(2),ZDOT(2),WRONG(6),QX(2)
1,QY(2),QZ(2),XP(2),YP(2),ZP(2),XDOTP(2),YDOTP(2),ZDOTP(2),PI(2),
2PI2(2),E(2),XMJ(2),CCON(2),XN(2)
 COMMON A1,A2,FJ1,FJ2,SFJ1,SFJ2,CFJ1,CFJ2,S1,S2,EPS1
 COMMON DT,DTME,RPM
 DAC SM(6),SL(6),XMJP(2)
 COMMON OBJ,TSAVE,REA,RTA,RSN,HPI,THPI,KK,NOSW,PQM(6),PQL(6),ERR,
1SIPO,COPO,SIECC,COECC,SIGVE,COGVE,COZ,GEGEH,DJD,GEL,GE,DL,UO,COPN,
2SIPN,COEE,SIEE,COH,SIH,SIDL,CODL,COAL,SIAL,VH,H,SIBLEH,COBLEH,GEH,
3SIBLE,COBLE,RGEO,BLE,COGE,SIGE,ACOGEH,SCGEH,TAGEH,GBX,GBY,GBZ,GHX,
4GHY,GHZ,COTH3,SITH3,COTH2,SITH2,HX,HY,HZ,CETA,VDP,RDP,SIP1,COP1,
5PHI,QQ,NJ(6),ND,NT,EC(6),RBAR(7),TD,PHID,DPHI,GEE,KIT,TT,RAPA,RDP,
6VOX,VOY,VOZ,NMC,TMC(6),DTMC(6),ALMC(6),BEMC(6),SIMC,FMC,WVMC,VDP,
```

```
     7PN,EE,NR,PSI
      DIMENSION TAB(10,3),TIMV(10)
      RBAR(1) = .387099
      RBAR(2) = .723332
      RBAR(3) = 1.523691
      RBAR(4) = 5.202803
      RBAR(5) = 9.538843
      RBAR(6) = 19.181945
      RBAR(7) = 30.057767
      EPS1 = 1.0E-11
      MGM = 512
B     PI = 202622077325
B     PI(2) = 147042055061
B     PI2 = 203622077325
B     PI2(2) = 150042055061
B     HPI = 201622077325
B     THPI = 203455457437
    2 REWIND 26
      ERASE NOT,NORG,PL,SL(4),SL(5),SL(6)
      RIT 2, 3, NTYPE
    3 FORMAT (3I2)
      IF (NTYPE-2) 4, 40, 80
    4 RIT 2, 3, TT, DJD, ND, NT
      RIT 2, 3, NR, (NJ(I),EC(I),I=1,NR),GEE,KIT,DT,DPHI
      RIT 2,3, BLE,GEL,RO,G(1),WTE,WTV,RADE,OBJ
    5 G(2)=G/576.
      G=-G(2)
      TME = DJD*24.
      TTME = TT
      RIT 2,3, EMAX,GMAT,SMAX
```

```
   RIT 2,3, NCKE,MSDT,MET,NOUT,NOSW
   IF (MSDT) 6, 7, 6
6  RIT 2,3,DTA,DTB,DTC
   ERASE DT
   GO TO 8
7  RIT 2,3, DTOR,DTMAX,EP1
8  RIT 2,3, NBD
   RIT 2,3, (NB(I),WT(I),RAD(I),I=1,NBD)
   ERASE ERR
   CALL PHIS
   REWIND 26
   IF(ERR) 2,10,2
10 IF (NJ-4) 12,12,11
11 NJ = NJ-4
12 CALL DEPART
   GMAX = GMA1
   DT=DTOR
   HAFDT = DT/2.
   NEWORG =ND
   NTARG =NT
   N = NBD
   TO = TME+TO
25 TMAX = TTME
   CALL DIFCOR
   GO TO 199
40 RIT  2,3, DJD,TT,ND,NT
   ERASE NJ(1)
   CALL PHIS
   NDJD = DJD
   PSI = 57.29578*PSI
```

1

```
      WOT 10,42,ND,NT,NDJD,PSI
   42 FORMAT (1H115X,20HFOR DEPARTURE PLANET I2,1X, 22HAND DESTINATION P
     1LANET I2,1X,11HON TAPE DAY I5/1H025X,26HTHE CONFIGURATION ANGLE IS
8    2F7.1,1X,4HDEG.)
      GO TO 2
   80 RIT 2,3,G(1),WTE,RADE,WTV,OBJ
      G(2) = G/576.
      G = -G(2)
   84 RIT 2,3,EMAX,GMAX,SMAX,TMAX,TO
   85 RIT 2,3,NCKE,MSDT,MET,NOUT,NOSW,NMC
      IF (MSDT) 86,87,86
   86 RIT 2,3,DTA,DTB,DTC
      ERASE DT
      GO TO 88
   87 RIT 2,3,DT,DTMAX,EP1
      HAFDT = DT/2.
   88 RIT 2,3,N,NEWORG,NTARG
      RIT 2,3,(NB(I),WT(I),RAD(I),I = 1,N)
      RIT 2,3,(PM(I),I = 1,6)
      ERR = 1.
      TMAXE = TMAX
   92 IF (NMC) 93, 94,93
   93 RIT 2,3,SIMC,FMC,WVMC,(PPM(I),PPL(I),QM(I),QL(I),I = 1,NMC)
      DO 95 I = 1,NMC
      J = NMC+1-I
      TMC(I) = PPM(J)
      DTMC(I) = PPL(J)
      ALMC(I) = QM(J)
   95 BEMC(I) = QL(J)
   94 CALL TITLE
```

```
      CALL ORGN
      T = 0.
      KK = 67
      CALL SETI
      IF (TEM) 97,2,97
   97 IF (NMC) 96,101,96
   96 TMAX = TMC(NMC)
   98 CALL NBODY
      IF (ERR) 99,2,99
   99 CALL MCC
      DT = 1.
      NMC = NMC-1
      IF (NMC) 96,101,96
  101 TMAX = TMAXE
      CALL NBODY
C          PUNCH CARDS FOR MCDONNALD
  199 IF (NORG-NTARG) 195,220,195
  195 IF (NTARG) 196,197,196
  196 NT = NTARG
      GO TO 200
  197 NT = NORG
  200 IF (NNN-6) 201,201,203
  201 NAV = 1
      GO TO 206
  203 IF (NNN-64) 205,205,204
  204 NAV = 59
      GO TO 206
  205 NAV = NNN-6
  206 NBV = 1
      NCV = 5
```

```
207 DO 214 I = NBV,NCV
208 NAV = NAV+1
    IF (TL(NAV)-T) 211,208,211
211 TAB(I,1) = PX(NT,NAV)
    TAB(I,2) = PY(NT,NAV)
    TAB(I,3) = PZ(NT,NAV)
214 TIMV(I) = TL(NAV)
    IF (NBV-7) 209,210,209
209 NBV = 7
    NCV = 10
    GO TO 207
210 IF (NORG) 212,213,212
212 TAB(6,1) = -X
    TAB(6,2) = -Y
    TAB(6,3) = -Z
    GO TO 215
213 NBT = NBT
    TAB(6,1) = X(NBT)
    TAB(6,2) = Y(NBT)
    TAB(6,3) = Z(NBT)
215 TIMV(6) = T
    CALL VELOC(TIMV,TAB,QM)
    IF (NORG) 218,219,218
218 QM(1) = -QM(1)
    QM(2) = -QM(2)
    QM(3) = -QM(3)
219 PM(1) = XTARG
    PM(2) = YTARG
    PM(3) = ZTARG
    PM(4) = PM(4)-QM(1)
```

```
      PM(5) = PM(5)-QM(2)
      PM(6) = PM(6)-QM(3)
      WOT 10,279,(PM(I),I=4,6)
  279 FORMAT (1H015X,27HVEHICLE VELOCITY WRT TARGET/1H02X,3HX =E14.7,2X,
     13HY =E14.7,2X,3HZ =E14.7)
C     ALPHA+DELTA
  220 IF (NTARG-4) 221,221,222
  221 AL = 0.
      DL = 0.
      GO TO 223
  222 N = NTARG
      DJD = (T+TO)/24.+2436934.5
      CALL PLANET
      GO TO 230
  223 IF (NTARG) 230,224,230
  224 DJD = (T+TO)/24.+2436934.5
      DT = (DJD-2433281.5)/365.25
      HDT = DT/2.
      EM =(22.34945E-5)+.13526E-8*HDT
      EN = 9.7169267E-5-.41E-9*HDT
      DO 229 I=1,10
      A = AL
      D = DL
      DAT = EM+EN*SINF(A)*TANF(D)
      DDT = EN*COSF(A)
      AL = -DT*DDT
      DL= -DT*DDT
      AL = (AL+A)/2.
      DL = (DL+D)/2.
      IF (ABSF(A-AL)-5E-8) 228,228,229
```

```
228 IF (ABSF(D-DL)-5E-8) 230,230,229
229 CONTINUE
    AL = (AL+6.2831853)
    DL = (DL+1.5707963)
230 AL = AL*57.29578
    DL = DL*57.29578
    T = T+TO
    WOT 10,280,AL,DL
280 FORMAT (1H04X,38HRIGHT ASCENSION OF TARGET NORTH POLE =1PE15.7/1H0
   14X,34HDECLINATION OF TARGET NORTH POLE =E15.7)
    IF (PM(1)) 231,232,232
231 PUNCH 260,PM(1)
    GO TO 233
232 PUNCH 261,PM(1)
233 IF (PM(2)) 234,235,235
234 PUNCH 262,PM(2)
    GO TO 236
235 PUNCH 263,PM(2)
236 IF (PM(3)) 237,238,238
237 PUNCH 264,PM(3)
    GO TO 239
238 PUNCH 265,PM(3)
239 IF (PM(4)) 240,241,241
240 PUNCH 266,PM(4)
    GO TO 242
241 PUNCH 267,PM(4)
242 IF (PM(5)) 243,244,244
243 PUNCH 268,PM(5)
    GO TO 245
244 PUNCH 269,PM(5)
```

```
245 IF (PM(6)) 246,247,247
246 PUNCH 270,PM(6)
    GO TO 248
247 PUNCH 271,PM(6)
248 PUNCH 272,T
    IF (AL) 249,250,250
249 PUNCH 273,AL
    GO TO 251
250 PUNCH 274,AL
251 IF (DL) 252,253,253
252 PUNCH 275,DL
    GO TO 2
253 PUNCH 276,DL
    GO TO 2
260 FORMAT (5X,6H1UPSIF12X,1PE14.7)
261 FORMAT (5X,6H1UPSIF12X,1PE13.7)
262 FORMAT (5X,6H1LAMIF12X,1PE14.7)
263 FORMAT (5X,6H1LAMIF12X,1PE13.7)
264 FORMAT (5X,6H1GAMIF12X,1PE14.7)
265 FORMAT (5X,6H1GAMIF12X,1PE13.7)
266 FORMAT (5X,7H1UPSIF111X,1PE14.7)
267 FORMAT (5X,7H1UPSIF111X,1PE13.7)
268 FORMAT (5X,7H1LAMIF111X,1PE14.7)
269 FORMAT (5X,7H1LAMIF111X,1PE13.7)
270 FORMAT (5X,7H1GAMIF111X,1PE14.7)
271 FORMAT (5X,7H1GAMIF111X,1PE13.7)
272 FORMAT (5X,6H1TIUPH12X,1PE13.7)
273 FORMAT (5X,6H1ALPND12X,1PE14.7)
274 FORMAT (5X,6H1ALPND12X,1PE13.7)
275 FORMAT (5X,6H1DELND12X,1PE14.7)
```

```
276 FORMAT (5X,6H1DELND12X,1PE13.7)
    END (1,1,0,0,0,1,1,1)
```

4.2 PROGRAM II

This program is a modification of the N-body subroutine in Deck #1. Vehicle ground traces (i.e. longitude, latitude), the central angle, and the elevation and azimuth of the velocity vector are given. Also the option is available for having positions, velocities and acceleration in units of AU, AU/HR, AU/HR^2 or MILES, FT/SEC, FT/SEC^2.

4.2.1 COMPUTER INPUT

N-BODY TRAJECTORY

Read Statement 1

	FORTRAN Symbols	
Field 1 (x)	KGT	code digit to determine whether or not ground traces are to be computed. (1 = yes; 0 = no)*
2 (x)	KLMS	code digit to determine units of output (1 → output in miles, ft/sec and ft/sec^2; 0 → output in AU, AU/hr, AU/hr^2)

Read Statement 2 (used only if field 2 of read statement 1 is = 1)

Field 1	CONV1	conversion factor from AU to miles = 9.2956509E+07
2	CONV2	conversion factor from AU/hr to ft/sec = 1.3633621E+08
3	CONV3	conversion factor from AU/hr^2 to ft/sec^2 = 3.7871170E+04

These conversion factors can of course be changed to suit anyone's particular needs. If one is interested in obtaining position coordinates in kilometers rather than miles or AU, the proper conversion factor could be read in as CONV1 but since KLMS would then = 1, the alphanumeric printout of the beginning of the output would state DISTANCE IS IN MILES, VELOCITY IS IN FT PER SEC, ETC even though this is not the case. This is due to the format of the output statement.

---

* 0 = zero

Read Statement 3 (used only if field 1 of read statement 1 is =1)

Field 1	GHRA		Greenwich hour angle at O^hET on the day of departure in degrees and decimals of a degree
2	OMGE	$\omega_\oplus$	angular velocity of the earth = .262516 rad/hour
3	TTTO		tape hours from O^hET Jan. 1, 1960 of the Greenwich hour angle at O^hET on the day of departure

Read Statement 4

Field 1	G(2)	k^2	Gravitational constant = 5.1373647E-07 (rad/hr)^2
2	WTE	$M_\oplus$	the mass of the earth = 3.0034424E-06 (solar mass units) SMU
3	RADE	$R_\oplus$	$R_\oplus$; the radius of the earth = .426636E-04 (AU)
4	WTV	m_v	mass of the vehicle (usually zero) solar mass units
5	OBJ	J'	oblateness constant for the earth = $JM_\oplus R_\oplus^2 = 8.8609392E-18$ SMU(AU)^2

Read Statement 5

Field 1	EMAX	$D_{1_{max}}$	maximum distance from Earth, A.U.
2	GMAX	$D_{2_{max}}$	maximum distance from target, A.U.
3	SMAX	$D_{3_{max}}$	maximum distance from Sun, A.U.
4	TMAX	T	trip time, hours
5	T0	t_s	date of departure, hours from Jan. 1960, O^hUT

Read Statement 6

Field 1 (x)	NCKE		control digit indicating computing scheme, 0 = Cowell, 1 = Encke
2 (x)	MSDT		control digit indicating selection of integration time step interval, 0 = doubling and halving procedure, 1 = three fixed Δt's

3 (x)	MET		control digit indicating whether computing scheme (Cowell, Encke) is to be switched, 0 = retain original scheme, 1 = switch schemes on test
4 (x)	NOUT	n	print digit n indicates print every n integrations
5 (x)	NOSW		control digit, 0 = retain initial origin, 1 = switch origins on test

Read Statement 7 (used only if Field 2 of read statement 6 is one (1))

Field 1	DTA	Δt_1	Δt within 3 radii of origin, hours
2	DTB	Δt_2	Δt within 100 radii of origin, hours
3	DTC	Δt_3	$\Delta t > 100$ radii from origin, hours

Read Statement 8 (used only if Field 2 of read statement 6 is zero (0))

Field 1	DT	t_i	initial Δt, hours
2	DTMAX	t_{max}	maximum Δt, hours
3	EP1	ϵ	value to test accuracy of integration, A. U.

Read Statement 9

Field 1 (x)	N	N_b	number of bodies other than the Earth to be included in the study (Earth is always included)
2 (x)	NEWORG		code digit for origin planet
3 (x)	NTARG		code digit for destination planet

Read Statement 10

N_b sets of data, each including

Field 1 (x)	NB(I)		code digit of body
2	WT(I)		mass of body, solar mass units
3	RAD(I)		radius of body, A. U.

Read Statement 11

Field 1	PM(1)	x_o	position coordinates with respect to origin at burnout, A. U.
2	PM(2)	y_o	
3	PM(3)	z_o	
4	PM(4)	$\dot{x}_o$	velocity components with respect to origin at burnout, A. U. /hour
5	PM(5)	$\dot{y}_o$	
6	PM(6)	$\dot{z}_o$	

The code digits for the planets are as follows:

0	Earth	5	Mars
1	Sun	6	Jupiter
2	Moon	7	Saturn
3	Mercury	8	Uranus
4	Venus	9	Neptune

4.2.2 COMPUTER OUTPUT

Initially (dependent on the input parameters KMLS and CONV1, CONV2 and CONV3) the units of position velocity and acceleration will be stated.

Then the following information is printed out after every n integration steps, where n is an input control parameter

1. Flight time, hours from start of trajectory
2. Table time, hours from beginning of tape
3. Time increment of integration step, hours
4. Planetary code digit of body at the origin
5. Acceleration components of the vehicle with respect to the origin, in desired units
6. Velocity components of the vehicle with respect to the origin, in desired units
7. Position coordinates of the vehicle with respect to the origin, in desired units
8. Position coordinates of the vehicle with respect to the Earth, in desired units
9. Position coordinates of the vehicle with respect to the target, in desired units
10. Position coordinates of the vehicle with respect to the Sun, in desired units

Then, if KGT = 1

11. Ground trace longitude and latitude (degrees)
12. Control angle (Degrees)
13. Elevation and aximuth of velocity vector (degrees)

4.2.3 SAMPLE PROBLEM

PROGRAM II (N-Body Program with Ground Traces)

Earth to Moon Trajectory

CARD #	
1	X1, 1*
2	F9.2956509E+7, 1.3633621E+8,.378711694E+5*
3	F336.2766,.262516, 75888.0*
4	F5.13736490E-7, 3.00344220E-6, .4266E-4, 0.0, 8.8609392E-18*
5	F2.0, 2.0, 80.0, 75902.422*
6	X0, 0, 0, 1, 0*
7	F.25, 2.0, 4.0E-10*
8	X2, 0, 2*
9	X1, F1.0, 4.655E-3*
10	X2, F3.6942027E-8, 1.1625090E-05*
11	F-2.9325021E-06, 3.8733136E-05, 2.0975379E-05*
12	F-2.6227989E-04, -2.3734089E-06, -3.3926081E-06*

```
N BODY TRAJECTORY

     GENERAL ELECTRIC CO.

     M.S.V.D.


               STARTING TABLE TIME  = 75902.422

               ORIGIN IS BODY 0  MASS =  3.003442E-06

               DESTINATION IS BODY 2  MASS =  3.694203E-08

               MASS OF VEHICLE =  0.

               OTHER BODIES ARE-

                    BODY  1  MASS =  1.000000E 00

               GRAVITATIONAL CONSTANT =  5.137365E-07

               EPSILON OF INTEGRATION = 4.00E-10   MAXIMUM DELTA T =   2.0

               COWELL METHOD IS USED

               THE ORIGIN IS FIXED

GREENWICH  HOUR  ANGLE= 336.2765999    TIME= 75888.000

DISTANCE  IS  IN  MILES

VELOCITY  IS  IN  FT  PER  SEC

ACCELERATION  IS  IN  FT  PER  SEC  PER  SEC

CONVERSION  FACTORS     0.9295651E 08    0.1363362E 09    0.3787117E 05
```

The next page is only the first part of the run to show the ground trace print, etc.

```
   TIME=    0.          TAPE TIME= 75902.422     DELTA T=     0.250     CENTER IS NO.   0

  POSIT FROM    EARTH                TARGET                SUN                 CENTER               VELOCITY              ACCELERATION
       X     -2.7259516E 02      1.9171934E 05      8.5129504E 07     -2.7259516E 02      -3.5758246E 04       0.
       Y      3.6004971E 03      1.1768294E 05     -3.6299660E 07      3.6004971E 03      -3.2358157E 02       0.
       Z      1.9497980E 03      6.0719026E 04     -1.5740930E 07      1.9497980E 03      -4.6253533E 02       0.
       R      4.1036080E 03      2.3300725E 05      9.3874781E 07      4.1036080E 03       3.5762701E 04       0.

  GROUND    TRACE        LATITUDE=   28.3686197     LONGITUDE= -98.8672438    CENTRAL   ANGLE=   -0.
 VELOCITY VECTOR        ELEVATION=     2.9999993        AZIMUTH=  92.4655409

   TIME=    0.0313      TAPE TIME= 75902.453     DELTA T=     0.016     CENTER IS NO.   0

  POSIT FROM    EARTH                TARGET                SUN                 CENTER               VELOCITY              ACCELERATION
       X     -1.0299648E 03      1.9092002E 05      8.5129591E 07     -1.0299648E 03      -3.5236909E 04       7.1320995E 00
       Y      3.5625724E 03      1.1769815E 05     -3.6297972E 07      3.5625724E 03      -3.2055691E 03      -2.4672750E 01
       Z      1.9231061E 03      6.0722002E 04     -1.5740208E 07      1.9231061E 03      -2.0254362E 03      -1.3357479E 01
       R      4.1774497E 03      2.3235849E 05      9.3874086E 07      4.1774497E 03       3.5440342E 04       2.8948811E 01

  GROUND    TRACE        LATITUDE=   27.4099581     LONGITUDE= -87.5419502    CENTR'L   ANGLE=   10.4650738
 VELOCITY VECTOR        ELEVATION=     8.1457831        AZIMUTH=  98.0050592

   TIME=    0.0625      TAPE TIME= 75902.484     DELTA T=     0.016     CENTER IS NO.   0

  POSIT FROM    EARTH                TARGET                SUN                 CENTER               VELOCITY              ACCELERATION
       X     -1.7704708E 03      1.9013755E 05      8.5129694E 07     -1.7704708E 03      -3.4196995E 04       1.1123988E 01
       Y      3.4657611E 03      1.1765446E 05     -3.6296342E 07      3.4657611E 03      -5.8272898E 03      -2.1777266E 01
       Z      1.8645327E 03      6.0693089E 04     -1.5739518E 07      1.8645327E 03      -3.4423600E 03      -1.1747956E 01
       R      4.3153851E 03      2.3168623E 05      9.3873433E 07      4.3153851E 03       3.4860315E 04       2.7129447E 01

  GROUND    TRACE        LATITUDE=   25.5987666     LONGITUDE= -77.0768948    CENTRAL   ANGLE=   20.4134693
 VELOCITY VECTOR        ELEVATION=    13.0350755        AZIMUTH= 102.9033747

   TIME=    0.0938      TAPE TIME= 75902.516     DELTA T=     0.016     CENTER IS NO.   0

  POSIT FROM    EARTH                TARGET                SUN                 CENTER               VELOCITY              ACCELERATION
       X     -2.4846004E 03      1.8938143E 05      8.5129821E 07     -2.4846004E 03      -3.2788030E 04       1.3683321E 01
       Y      3.3168705E 03      1.1755867E 05     -3.6294764E 07      3.3168705E 03      -8.0825922E 03      -1.8267772E 01
       Z      1.7778628E 03      6.0636074E 04     -1.5738855E 07      1.7778628E 03      -4.6567338E 03      -9.8161864E 00
       R      4.5095083E 03      2.3100238E 05      9.3872827E 07      4.5095083E 03       3.4089123E 04       2.4845569E 01

  GROUND    TRACE        LATITUDE=   23.2190573     LONGITUDE= -67.7708359    CENTRAL   ANGLE=   29.6255589
 VELOCITY VECTOR        ELEVATION=    17.5590794        AZIMUTH= 106.9571533

   TIME=    0.1563      TAPE TIME= 75902.578     DELTA T=     0.031     CENTER IS NO.   0

  POSIT FROM    EARTH                TARGET                SUN                 CENTER               VELOCITY              ACCELERATION
       X     -3.8122293E 03      1.8796976E 05      8.5130181E 07     -3.8122293E 03      -2.9469001E 04       1.5140319E 01
       Y      2.8961671E 03      1.1724415E 05     -3.6291733E 07      2.8961671E 03      -1.1407630E 04      -1.1505080E 01
       Z      1.5385471E 03      6.0456053E 04     -1.5737597E 07      1.5385471E 03      -6.4360687E 03      -6.1242596E 00
       R      5.0287178E 03      2.2963832E 05      9.3871771E 07      5.0287178E 03       3.2248705E 04       1.9977554E 01

  GROUND    TRACE        LATITUDE=   17.8154576     LONGITUDE= -52.7711344    CENTRAL   ANGLE=   45.4890680
 VELOCITY VECTOR        ELEVATION=    25.3381155        AZIMUTH= 112.5885353
```

4.2.4 LISTING

This listing is the input routine for Program II. N-body MAIN is called at the end.

```
      N-BODY WITH OPTIONS FOR GROUND TRACE. OUTPUT IN MILES AND FT/SEC.

C     KGT=1, GROUND TRACES, 0, NONE    KMLS=1, OUTPUT IN MILBS, 0, A.U.,S
C     READ CONVERSION FROM A.U.S FOR DISTANCE ,VELOCITY,ACCELERATION
      EQUIVALENCE(WM(1),XIM),(WM(2),YIM),(WM(3),ZIM),(WM(4),XIDM),
     1(WM(6),ZIDM),(WM(5),YIDM),(WL(1),XIL),(WL(2),YIL),(WL(3),ZIL),
     2(WL(4),XIDL),(WL(5),YIDL),(WL(6),ZIDL)
      DAC PX(9,67),PY(9,67),PZ(9,67),TL(67),RKM(4,6),RKL(4,6),NB(9),
     1WT(9),RAD(9),X(9),Y(9),Z(9),DIS(9),DIST(9),PM(6),PL(6),PPM(6),
     2PPL(6),QM(6),QL(6),WM(6),WL(6),XX(9),YY(9),ZZ(9),TI(6),CL(6)
      COMMON NEWORG,ZER,FMX,FLX,FMY,FLY,FMZ,FLZ,AM,AL,BM,BL,CM,CL
      COMMON NORG,NTARG,NSUN,N,NN,NNN,T,TO,TMAX,DTMAX,HAFDT,WTE,WTV,MGM,
     1XTARG,YTARG,ZTARG,SCEN,VCEN,SMAX,GMAX,EMAX,RAD1,RAD2,RADE,WTA,
     2NBE,NBT,NBS,RADORG,SAM,SBM,SCM, SAL,SBL,SCL,XIM,XIL,YIM,YIL,ZIM,
     3ZIL,XIDM,XIDL,YIDM,YIDL,ZIDM,ZIDL,RM,RL,RR,RO,SS,RRM,RRL,TEM,TEL,
     4GMX,GLX,GMY,GLY,GMZ,GLZ,SUMX,SULX,SUMY,SULY,SUMZ,SULZ,FM,FL,GM,GL,
     5HM,HL,RRR,SSS,XM,YM,ZM,A,BX,BY,BZ,WRM,WRL,RCM,RCL,ROM,ROL,RPPM,
     6XPM,XPL,YPM,YPL,ZPM,ZPL,
     7KOB,VNZ,MEDT,MSDT,MDT,NQT,MET,NOUT,NCKE,TSAV,DTA,DTB,DTC,EP1
      DAC G(2),WTS(2)
      DAC      SOGOOD(6),XDOT(2),YDOT(2),ZDOT(2),WRONG(6),QX(2)
     1,QY(2),QZ(2),XP(2),YP(2),ZP(2),XDOTP(2),YDOTP(2),ZDOTP(2),PI(2),
     2PI2(2),E(2),XMJ(2),CCON(2),XN(2)
      COMMON A1,A2,FJ1,FJ2,SFJ1,SFJ2,CFJ1,CFJ2,S1,S2,EPS1
      COMMON DT,DTME,RPM
      DAC SM(6),SL(6),XMJP(2)
      COMMON OBJ,TSAVE,REA,RTA,RSN,HPI,THPI,KK,NOSW,PQM(6),PQL(6),ERR
      COMMON RPA,GHRA,OMGE,TTTO,KGT,KMLS
      COMMON CONV1,CONV2,CONV3
      MGM = 512
      EPS1 = 1.0E-11
B     PI = 202622077325
B     PI(2) = 147042055061
B     PI2 = 203622077325
B     PI2(2) = 150042055061
B     HPI = 201622077325
B     THPI = 203455457437
    2 REWIND 26
1)    RIT2,3, KGT,KMLS
      IF(KMLS)112,113,112
2)112 RIT2,3,CONV1,CONV2,CONV3
  113 IF(KGT)102,103,102
3)102 RIT 2,3,GHRA,OMGE,TTTO  3
4)103 RIT 2,3, G(2),WTE,RADE,WTV,OBJ  4
    3 FORMAT (E9.5)
5)  4 RIT 2,3,EMAX,GMAX,SMAX,TMAX,TO
6)  5 RIT 2,3,NCKE,MSDT,MET,NOUT,NOSW
      IF (MSDT) 6, 7, 6
    6 RIT 2,3,DTA,DTB,DTC
      ERASE DT
      GO TO 8
    7 RIT 2,3,DT,DTMAX,EP1
      HAFDT = DT/2.
    8 RIT 2,3,N,NEWORG,NTARG
      RIT 2,3,(NB(I),WT(I),RAD(I),I = 1,N)
    9 RIT 2,3, (PM(I),I=1,6)
      ERASE T
```

```
    CALL SETI
    G = -G(2)
    ERASE NOT,NORG,PL,SL(4),SL(5),SL(6)
    CALL TITLE
    IF(KGT)104,111,104
104 WOT 10,101,GHRA,TTTO
101 FORMAT(27H0   GREENWICH  HOUR  ANGLE=F12.7,9H    TIME=F10.3/)
    GHRA=GHRA*.017453293
111 IF(KMLS)106,11,106
106 WOT 10,107
107 FORMAT(27H0   DISTANCE  IS  IN  MILES/34H0   VELOCITY  IS  IN  FT
   1 PER  SEC/48H0   ACCELERATION  IS  IN  FT  PER  SEC  PER  SEC)
    WOT 10, 108,CONV1,CONV2,CONV3
108 FORMAT(26H0   CONVERSION  FACTORS   E14.7,3H   E14.7,3H   E14.7)
 11 CALL ORGN
    KK = 67
    NBT = NBT
    IF (TEM) 12,2,12
 12 CALL NBODY
    GO TO 2
    END(1,1,0,0,0,1,1,1,0,1,0,0,0,0,0)
```

4.3 PROGRAM III

This program is a modification of the N-body subroutine of Program I. A complete 6 x 6 differential correction matrix is computed by integration and may be printed out with each integration step. There is also a differential correction procedure (improved over that of Program I) for the computation of interplanetary trajectories.

There is an option to print out at each n^{th} integration step the incremental sensitivity matrix $H_{t,t-\Delta t}$ and/or the sensitivity matrix $H_{t,o}$.

The final sensitivity matrix $H_{t\,final,0}$ is always printed out as well as its inverse. This inverse is the "theoretical" inverse, obtained by rearranging the components of H as follows:

$$H^{-1}\begin{pmatrix} H_4 & -H_2 \\ -H_3 & H_1 \end{pmatrix}^T$$

4.3.1 COMPUTER INPUT

Read Statement 1

Field 1 (x)	KWC	k	control digit for sensitivity coefficients. If k = 1, coefficients are computed. If k = 0, no coefficients are computed. (i.e. same as option 2 in deck #1).
2 (x)	KNOP		control digit for intermediate print of incremental sensitivity coefficients. The program prints them after every KNOP integration steps.
3 (x)	KNOPP		control digit for intermediate print-out of total sensitivity coefficient matrix. Program prints after every KNOPPth integration step.
4 (x)	NOOFT		This is a dummy number with no meaning. Simple set it equal to one (1).

Read Statement 2

Field 1	AK	the limiting ratio of $\Delta V/Vo$. Section 3.4 of the report explains this in detail.

Read Statement 3

Field 1	G(2)	k^2	gravitational constant = 5.1373647E-07 $(\text{rad/hr})^2$
2	WTE	$M_{\oplus}$	mass of Earth = 3.0034424×10^{-6}, solar mass units
3	RADE	$R_{\oplus_e}$	equatorial radius of the Earth = $.42635078 \times 10^{-4}$, A. U.
4	WTV	m_v	mass of the vehicle, solar mass units
5	OBJ	J'	oblateness constant for the Earth $= JM_{\oplus}R_{\oplus}$ $= 8.8632361 \times 10^{-18}$, (solar mass units) $(\text{A. U.})^2$

Read Statement 4

Field 1	EMAX	$D_{1_{max}}$	maximum distance from Earth, A. U.
2	GMAX	$D_{2_{max}}$	maximum distance from target, A. U.
3	SMAX	$D_{3_{max}}$	maximum distance from Sun, A. U.
4	TMAX	T0	trip time, hours
5	T0	t_s	date of departure, hours from Jan. 1960, 0^hUT

Read Statement 5

Field 1 (x)	NCKE		control digit indicating computing scheme, 0 = Cowell, 1 = Encke; This must be = 0 if k = 1
2 (x)	MSDT		control digit indicating selection of integration time step interval, 0 = doubling and halving procedure, 1 = three fixed Δt's
3 (x)	MET		control digit indicating whether computing scheme (Cowell, Encke) is to be switched, 0 + retain original scheme, 1 = switch schemes on test
4 (x)	NOUT	n	print digit n indicates print every n integrations
5 (x)	NOSW		control digit, 0 = retain initial origin, 1 = switch origins on test

Read Statement 6 (If field 2 of read statement 5 = 1)

Field 1	DTA	Δt_1	Δt within 3 radii of origin, hours
2	DTB	Δt_2	Δt within 100 radii of origin, hours
3	DTC	Δt_3	Δt 100 radii from origin, hours

Read Statement 7 (If field 2 of read statement 5 = 0)

Field 1	DT	Δt_i	initial Δt, hours
2	DTMAX	Δt_{max}	maximum Δt, hours
3	EP1	ϵ	value to test accuracy of integration, A. U.

Read Statement 8

Field 1 (x)	N	N_b	number of bodies other than the Earth to be included in the study (Earth is always included)
2 (x)	NEWORG		code digit for origin planet
3 (x)	NTARG		code digit for destination planey

Read Statement 9

N_b sets of data, each including

Field 1 (x)	NB(I)	code digit of body
2	WT(I)	mass of body, solar mass units
3	RAD(I)	radius of body, A. U.

Read Statement 10

Field 1	PM(1)	x_o	position coordinates with respect to origin at burnout, A. U.
2	PM(2)	y_o	
3	PM(3)	z_o	
4	PM(4)	$\dot{x}_o$	velocity components with respect to origin at burnout, A. U. /hour
5	PM(5)	$\dot{y}_o$	
6	PM(6)	$\dot{z}_o$	

The planetary code digits are as follows:

0	Earth	5	Mars
1	Sun	6	Jupiter
2	Moon	7	Saturn
3	Mercury	8	Uranus
4	Venus	9	Neptune

4.3.2 COMPUTER OUTPUT

The following information is printed out after every n integration steps where n is an input control parameter.

1. Flight time, hours from start of trajectory
2. Table time, hours from beginning of tape
3. Time increment of integration step, hours
4. Planetary code digit of body at the origin
5. Acceleration components of the vehicle with respect to the origin, A.U./hr^2
6. Velocity components of the vehicle with respect to the origin, A.U./hr
7. Position coordinates of the vehicle with respect to the origin, A.U.
8. Position coordinates of the vehicle with respect to the Earth, A.U.
9. Position coordinates of the vehicle with respect to the target, A.U.
10. Position coordinates of the vehicle with respect to the Sun, A.U.

The intermediate printout of the incremental sensitivity coefficients (the matrix $H_{t,t-\Delta t}$) is determined by Field 2 of read statement #1, KNOP. (This printout is not shown in the sample output).

The printout of the sensitivity coefficients over the entire trajectory is controlled by Field 3 of read statement #1 KNOPP. On the sample output KNOPP = 1 since we get the matrix at each step. The form is standard; e.g.

$$\begin{vmatrix} \frac{\partial x}{\partial x_o} & \frac{\partial y}{\partial x_o} & \cdots & \frac{\partial \dot{z}}{\partial x_o} \\ \frac{\partial x}{\partial y_o} & & & \vdots \\ \vdots & & & \vdots \\ \frac{\partial x}{\partial \dot{z}_o} & \cdots & \cdots & \frac{\partial \dot{z}}{\partial \dot{z}_o} \end{vmatrix}$$

4.3.3 SAMPLE INPUT

PROGRAM III (N-Body Program with Sensitivity Coefficients)
Earth to Moon Trajectory

CARD #	
1	X1, 500, 1, 1*
2	F1.*
3	F5.137365E-07, 2.9991126E-6,.4266E-4,0,8.8609392E-18*
4	F1.0,1.0,1.1,56.6816,62163.147*
5	X0,0,0,1,0*
6	F.25,2.0,4.00E-10*
7	X2,0,2
8	X1,F1.0,4.655E-3*
9	X2,F3.6821513E-8,1.1625E-5*
10	-1.4158093E-05,3.7906955E-05,1.7647465E-05
11	-2.5124908E-04,-4.8551250E-05,-6.2786291E-05

This run is not shown in its entirety. Only the first few lines of output and the last few lines are here included for the sake of brevity.

```
N BODY TRAJECTORY

     GENERAL ELECTRIC CO.

     M.S.V.D.


          STARTING TABLE TIME   = 62163.147

          ORIGIN IS BODY 0  MASS =  2.999112E-06

          DESTINATION IS BODY 2  MASS =  3.682151E-08

          MASS OF VEHICLE =  0.

          OTHER BODIES ARE-

               BODY  1  MASS =  1.000000E 00

          GRAVITATIONAL CONSTANT =  5.137365E-07

          EPSILON OF INTEGRATION = 4.00E-10   MAXIMUM DELTA T =   2.0

          COWELL METHOD IS USED

          THE ORIGIN IS FIXED
```

```
TIME=    0.       TAPE TIME= 62163.147     DELTA T=    0.250     CENTER IS NO.  0

POSIT FROM    EARTH             TARGET             SUN              CENTER           VELOCITY         ACCELERATION
   X    -1.4158093E-05     1.3166374E-03    -6.7672139E-01    -1.4158093E-05    -2.5124908E-04    0.
   Y     3.7906955E-05     1.9807961E-03     6.5750413E-01     3.7906955E-05    -4.8551250E-05    0.
   Z     1.7647465E-05     9.3058257E-04     2.8512670E-01     1.7647465E-05    -6.2786291E-05    0.
   R     4.4145463E-05     2.5540304E-03     9.8567780E-01     4.4145463E-05     2.6348708E-04

        TIME=       0.0313      DELTA T=       0.0313

    9.9511271E-01      -8.2220472E-03     -3.7484464E-03      3.1204879E-02     -9.0601506E-05     -4.0847932E-05
   -8.2129490E-03       1.0097650E 00      8.3972808E-03     -9.0601506E-05      3.1346810E-02      8.3881941E-05
   -3.7436818E-03       8.3959109E-03      9.9519646E-01     -4.0847932E-05      8.3881941E-05      3.1198310E-02
   -2.7272934E-01      -5.5378598E-01     -2.4917760E-01      9.9632887E-01     -9.0505303E-03     -4.0226670E-03
   -5.5265160E-01       5.9659289E-01      5.1553691E-01     -9.0420599E-03      1.0088042E 00      7.6894564E-03
   -2.4858870E-01       5.1536752E-01     -3.1461631E-01     -4.0182299E-03      7.6881796E-03      9.9493721E-01
  TIME=    0.0313   TAPE TIME= 62163.178     DELTA T=    0.016     CENTER IS NO.  0

POSIT FROM    EARTH             TARGET             SUN              CENTER           VELOCITY         ACCELERATION
   X    -2.1865273E-05     1.3082857E-03    -6.7674575E-01    -2.1865273E-05    -2.4139702E-04    3.7115622E-04
   Y     3.6067478E-05     1.9793299E-03     6.5748811E-01     3.6067478E-05    -6.8825986E-05   -6.1227905E-04
   Z     1.5538511E-05     9.2870773E-04     2.8511844E-01     1.5538511E-05    -7.1912190E-05   -2.6455582E-04
   R     4.4948842E-05     2.5479122E-03     9.8568145E-01     4.4948842E-05     2.6111472E-04

        TIME=       0.0625      DELTA T=       0.0313

    9.8568630E-01      -3.5798635E-02     -1.5861890E-02      6.2304120E-02     -7.9674957E-04     -3.4383577E-04
   -3.5587001E-02       1.0353447E 00      3.0776298E-02     -7.9459535E-04      6.3145143E-02      5.7818004E-04
   -1.5751028E-02       3.0744398E-02      9.8005217E-01     -3.4270717E-04      5.7785523E-04      6.2063202E-02
   -2.8670641E-01      -1.2215165E 00     -5.2647002E-01      9.9585277E-01     -3.9847055E-02     -1.6704380E-02
   -1.2056819E 00       1.0088245E 00      8.9378510E-01     -3.9650556E-02      1.0267136E 00      2.4770575E-02
   -5.1817459E-01       8.9139770E-01     -6.5642517E-01     -1.6601427E-02      2.4740942E-02      9.7844802E-01
  TIME=    0.0625   TAPE TIME= 62163.209     DELTA T=    0.016     CENTER IS NO.  0

POSIT FROM    EARTH             TARGET             SUN              CENTER           VELOCITY         ACCELERATION
   X    -2.9213232E-05     1.3002930E-03    -6.7676975E-01    -2.9213232E-05    -2.2846519E-04    4.4931533E-04
   Y     3.3632378E-05     1.9772679E-03     6.5747150E-01     3.3632378E-05    -8.6524248E-05   -5.1731261E-04
   Z     1.3171923E-05     9.2657518E-04     2.8510992E-01     1.3171923E-05    -7.9228799E-05   -2.0316047E-04
   R     4.6454808E-05     2.5414350E-03     9.8568439E-01     4.6454808E-05     2.5682678E-04

        TIME=       0.0938      DELTA T=       0.0313

    9.7974215E-01      -8.4751801E-02     -3.6475552E-02      9.3625498E-02     -2.7885087E-03     -1.1492530E-03
   -8.3360339E-02       1.0709154E 00      6.2911824E-02     -2.7675297E-03      9.5480096E-02      1.6433104E-03
   -3.5746535E-02       6.2702003E-02      9.5422200E-01     -1.1382602E-03      1.6401462E-03      9.2232393E-02
   -5.8238000E-02      -1.9081708E 00     -7.8759773E-01      1.0123323E 00     -9.0300287E-02     -3.5479904E-02
   -1.8406076E 00       1.2398184E 00      1.1434323E 00     -8.9076141E-02      1.0413645E 00      4.3081501E-02
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 -7.5219689E-01      1.1332428E 00    -9.9361595E-01    -3.4838417E-02     4.2896857E-02     9.5065448E-01

  TIME=   0.0938   TAPE TIME= 62163.241    DELTA T=   0.016    CENTER IS NO.  0

POSIT FROM   EARTH           TARGET            SUN              CENTER          VELOCITY       ACCELERATION
  X    -3.6125793E-05    1.2927357E-03    -6.7679332E-01    -3.6125793E-05   -2.1374815E-04    4.8604773E-04
  Y     3.0692832E-05    1.9747013E-03     6.5745438E-01     3.0692832E-05   -1.0106199E-04   -4.1296899E-04
  Z     1.0606817E-05    9.2424402E-04     2.8510121E-01     1.0606817E-05   -8.4624488E-05   -1.4307391E-04
  R     4.8575995E-05    2.5347263E-03     9.8568664E-01     4.8575995E-05    2.5112367E-04

       TIME=      0.1563      DELTA T=       0.0625

  1.0020923E 00     -2.4323299E-01    -9.9153382E-02     1.5963816E-01    -1.2312696E-02    -4.5970401E-03

 -2.3027675E-01      1.1526092E 00     1.4289540E-01    -1.2000891E-02     1.6063780E-01     5.1840840E-03

 -9.2364120E-02      1.4094178E-01     8.7331368E-01    -4.4336312E-03     5.1370726E-03     1.4930355E-01

  8.3479197E-01     -3.1127384E 00    -1.1896242E 00     1.1149166E 00    -2.1603715E-01    -7.3140742E-02

 -2.7725676E 00      1.3050466E 00     1.3678850E 00    -2.0617557E-01     1.0350196E 00     6.6754824E-02

 -1.0113598E 00      1.3166050E 00    -1.5683395E 00    -6.7972495E-02     6.5268596E-02     8.7208666E-01

  TIME=   0.1563   TAPE TIME= 62163.303    DELTA T=   0.031    CENTER IS NO.  0

POSIT FROM   EARTH           TARGET            SUN              CENTER          VELOCITY       ACCELERATION
  X    -4.8531545E-05    1.2790399E-03    -6.7683904E-01    -4.8531545E-05   -1.8343106E-04    4.6868089E-04
  Y     2.3698334E-05    1.9684521E-03     6.5741901E-01     2.3698334E-05   -1.2083829E-04   -2.2892480E-04
  Z     5.1061518E-06    9.1921101E-04     2.8508341E-01     5.1061518E-06   -9.0420239E-05   -4.9443892E-05
  R     5.4249375E-05    2.5210505E-03     9.8568929E-01     5.4249375E-05    2.3753876E-04

       TIME=      0.2188      DELTA T=       0.0625

  1.0862437E 00     -4.6701584E-01    -1.8207365E-01     2.3440159E-01    -2.9432717E-02    -9.8879658E-03

 -4.1882846E-01      1.2285352E 00     2.2991745E-01    -2.7899013E-02     2.2389198E-01     9.5902399E-03

 -1.5682190E-01      2.2265597E-01     7.6185311E-01    -9.0842406E-03     9.3592352E-03     2.0107389E-01

  1.8457109E 00     -3.9969683E 00    -1.4440558E 00     1.2845136E 00    -3.2637626E-01    -9.2426989E-02

 -3.1850623E 00      1.0990972E 00     1.3966257E 00    -2.9492689E-01     9.8348801E-01     7.1933687E-02

 -1.0185956E 00      1.2743645E 00    -1.9710157E 00    -7.5947804E-02     6.7200899E-02     7.8490094E-01

  TIME=   0.2188   TAPE TIME= 62163.366    DELTA T=   0.031    CENTER IS NO.  0

POSIT FROM   EARTH           TARGET            SUN              CENTER          VELOCITY       ACCELERATION
  X    -5.9122831E-05    1.2671581E-03    -6.7688296E-01    -5.9122831E-05   -1.5623821E-04    3.9769906E-04
  Y     1.5788746E-05    1.9612870E-03     6.5738276E-01     1.5788746E-05   -1.3099295E-04   -1.0623143E-04
  Z    -6.0101537E-07    9.1397114E-04     2.8506541E-01    -6.0101537E-07   -9.1654633E-05    4.0356363E-06
  R     6.1197670E-05    2.5075245E-03     9.8569007E-01     6.1197670E-05    2.2353993E-04

       TIME=      0.2813      DELTA T=       0.0625

  1.2293080E 00     -7.3738809E-01    -2.7785272E-01     3.2058837E-01    -5.2326981E-02    -1.5713766E-02

 -6.2095968E-01      1.2883639E 00     3.1590221E-01    -4.7819266E-02     2.8314417E-01     1.4002269E-02

 -2.1684305E-01      2.9837126E-01     6.2989929E-01    -1.3351930E-02     1.3324076E-02     2.4759422E-01

  2.6977088E 00     -4.6186562E 00    -1.6103842E 00     1.4736097E 00    -4.0007635E-01    -9.1042049E-02

 -3.2394553E 00      8.1001136E-01     1.3471240E 00    -3.3459190E-01     9.1096845E-01     6.8599030E-02
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 -8.8773286E-01      1.1396807E 00      -2.2320133E 00      -5.6738659E-02      5.8758484E-02      7.0576470E-01

 TIME=   0.2813   TAPE TIME= 62163.428    DELTA T=   0.031    CENTER IS NO.  0

POSIT FROM   EARTH            TARGET            SUN              CENTER            VELOCITY         ACCELERATION
  X    -6.8161374E-05     1.2568285E-03    -6.7692532E-01    -6.8161374E-05    -1.3378810E-04    3.2185598E-04
  Y     7.4473313E-06     1.9536894E-03     6.5734604E-01     7.4473313E-06    -1.3519555E-04   -3.5177096E-05
  Z    -6.3013486E-06     9.0873775E-04     2.8504741E-01    -6.3013486E-06    -9.0489997E-05    2.9786098E-05
  R     6.8855956E-05     2.4944587E-03     9.8568946E-01     6.8855956E-05     2.1063127E-04

       TIME=        0.3438      DELTA T=       0.0625

  1.4190557E 00     -1.0406901E 00     -3.8243797E-01      4.1837436E-01     -7.8755630E-02     -2.0980921E-02

 -8.1992807E-01      1.3296705E 00      3.9770705E-01     -6.8852581E-02      3.3772700E-01      1.8147248E-02

 -2.6676555E-01      3.6449257E-01      4.8481009E-01     -1.5793277E-02      1.6658830E-02      2.8957640E-01

  3.3411009E 00     -5.0633135E 00     -1.7308763E 00      1.6525159E 00     -4.4095457E-01     -7.5801409E-02

 -3.1070003E 00      5.1336232E-01      1.2676750E 00     -3.3268013E-01      8.3627487E-01      6.4212712E-02

 -7.0599151E-01      9.7374555E-01     -2.3984624E 00     -1.9100823E-02      4.7961699E-02      6.3993692E-01
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 TIME=  56.6563   TAPE TIME= 62219.803    DELTA T=    0.031     CENTER IS NO.  0

POSIT FROM   EARTH            TARGET            SUN              CENTER           VELOCITY         ACCELERATION
   X    -1.4902017E-05    4.8799018E-06   -7.0633873E-01   -1.4902017E-05   -3.1966136E-05   -4.2260607E-05
   Y    -2.3059363E-03   -8.3186023E-06    6.2889123E-01   -2.3059363E-03   -2.8756910E-05    7.2229898E-05
   Z    -1.1894106E-03   -8.6810032E-06    2.7252977E-01   -1.1894106E-03    1.2097918E-06    7.5273956E-05
   R     2.5946602E-03    1.2975839E-05    9.8422104E-01    2.5946602E-03    4.3014618E-05

       TIME=      56.6719      DELTA T=       0.0156

   4.8688013E 02      -1.0351425E 03     -4.2530309E 02       3.9224710E 02       4.3041454E 01        7.8587552E 01

   2.4977703E 02      -6.3664722E 02     -2.7634847E 02       2.3535028E 02       4.0774745E 01        5.5316250E 01

   1.9807284E 00      -3.2162763E 01     -5.4980786E 01       1.4688702E 01       8.1674490E 00        8.5253605E 00

   2.1479413E 02      -3.5390219E 02     -5.3116207E 01       1.3149495E 02      -5.4621966E 00        1.1352690E 01

  -6.2709739E 02       1.4732052E 03      5.7325200E 02      -5.4491510E 02      -7.6073621E 01       -1.1531669E 02

  -6.6400387E 02       1.3015897E 03      5.6520330E 02      -5.0426905E 02      -4.2875652E 01       -9.6688808E 01

 TIME=  56.6719   TAPE TIME= 62219.819    DELTA T=    0.008     CENTER IS NO.  0

POSIT FROM   EARTH            TARGET            SUN              CENTER           VELOCITY         ACCELERATION
   X    -1.5406358E-05    3.9934957E-06   -7.0634723E-01   -1.5406358E-05   -3.2570680E-05   -3.5017208E-05
   Y    -2.3063766E-03   -8.7527441E-06    6.2888338E-01   -2.3063766E-03   -2.7590215E-05    7.6961544E-05
   Z    -1.1893824E-03   -8.6249201E-06    2.7252658E-01   -1.1893824E-03    2.3908473E-06    7.5741315E-05
   R     2.5950416E-03    1.2920827E-05    9.8422123E-01    2.5950416E-03    4.2752605E-05

       TIME=      56.6797      DELTA T=       0.0078

   4.8842177E 02      -1.0375957E 03     -4.2558095E 02       3.9315686E 02       4.2982404E 01        7.8650478E 01

   2.4482486E 02      -6.2504684E 02     -2.7184946E 02       2.3105894E 02       4.0180686E 01        5.4411415E 01

  -3.1936958E 00      -2.2063054E 01     -5.0602677E 01       1.0773309E 01       7.8411946E 00        7.7783188E 00

   1.7973280E 02      -2.7391481E 02     -1.7989134E 01       1.0132811E 02      -9.6462337E 00        4.7521189E 00

  -6.4026901E 02       1.4955560E 03      5.7808105E 02      -5.5331773E 02      -7.5950762E 01       -1.1624112E 02

  -6.6027920E 02       1.2831223E 03      5.5524868E 02      -4.9776436E 02      -4.0607410E 01       -9.4489591E 01

 TIME=  56.6797   TAPE TIME= 62219.827    DELTA T=    0.004     CENTER IS NO.  0

POSIT FROM   EARTH            TARGET            SUN              CENTER           VELOCITY         ACCELERATION
   X    -1.5661846E-05    3.5469750E-06   -7.0635151E-01   -1.5661846E-05   -3.2829307E-05   -3.1170436E-05
   Y    -2.3065898E-03   -8.9628155E-06    6.2887947E-01   -2.3065898E-03   -2.6980912E-05    7.8978367E-05
   Z    -1.1893615E-03   -8.5899955E-06    2.7252501E-01   -1.1893615E-03    2.9821968E-06    7.5602340E-05
   R     2.5952230E-03    1.2911279E-05    9.8422137E-01    2.5952230E-03    4.2598433E-05

       TIME=      56.6816      DELTA T=       0.0020

   4.8876414E 02      -1.0381110E 03     -4.2560750E 02       3.9334734E 02       4.2962550E 01        7.8658144E 01

   2.4357145E 02      -6.2212117E 02     -2.7071956E 02       2.2997649E 02       4.0032420E 01        5.4184224E 01

  -4.4820793E 00      -1.9562155E 01     -4.9520928E 01       9.8029618E 00       7.7624689E 00        7.5943601E 00

   1.7085285E 02      -2.5376743E 02     -9.1917830E 00       9.3727722E 01      -1.0683213E 01        3.1004267E 00

  -6.4319358E 02       1.5002601E 03      5.7889857E 02      -5.5508782E 02      -7.5868684E 01       -1.1639709E 02
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   -6.5900266E 02      1.2777480E 03      5.5244088E 02      -4.9585199E 02      -4.0005352E 01      -9.3880231E 01

   TIME=  56.6816   TAPE TIME= 62219.829    DELTA T=    0.001    CENTER IS NO.  0

POSIT FROM   EARTH          TARGET             SUN              CENTER           VELOCITY          ACCELERATION
   X    -1.5726025E-05    3.4350378E-06   -7.0635256E-01   -1.5726025E-05   -3.2889231E-05   -3.0189842E-05
   Y    -2.3066424E-03   -9.0146786E-06    6.2887849E-01   -2.3066424E-03   -2.6826200E-05    7.9442545E-05
   Z    -1.1893555E-03   -8.5805367E-06    2.7252460E-01   -1.1893555E-03    3.1297873E-06    7.5527260E-05
   R     2.5952673E-03    1.2910830E-05    9.8422139E-01    2.5952673E-03    4.2557515E-05

        DESIRED TIME
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                              ****   SENSITIVITY  COEFFICIENT  MATRIX   ****

 4.8876414E 02    -1.0381110E 03    -4.2560750E 02     3.9334734E 02     4.2962550E 01     7.8658144E 01
 2.4357145E 02    -6.2212117E 02    -2.7071956E 02     2.2997649E 02     4.0032420E 01     5.4184224E 01
-4.4820793E 00    -1.9562155E 01    -4.9520928E 01     9.8029618E 00     7.7624689E 00     7.5943601E 00
 1.7085285E 02    -2.5376743E 02    -9.1917830E 00     9.3727722E 01    -1.0683213E 01     3.1004267E 00
-6.4319358E 02     1.5002601E 03     5.7889857E 02    -5.5508782E 02    -7.5868684E 01    -1.1639709E 02
-6.5900266E 02     1.2777480E 03     5.5244088E 02    -4.9585199E 02    -4.0005352E 01    -9.3880231E 01

                              ****   INVERSE  MATRIX   ****

 9.3727722E 01    -5.5508782E 02    -4.9585199E 02    -3.9334734E 02    -2.2997649E 02    -9.8029618E 00
-1.0683213E 01    -7.5868684E 01    -4.0005352E 01    -4.2962550E 01    -4.0032420E 01    -7.7624689E 00
 3.1004267E 00    -1.1639709E 02    -9.3880231E 01    -7.8658144E 01    -5.4184224E 01    -7.5943601E 00
-1.7085285E 02     6.4319358E 02     6.5900266E 02     4.8876414E 02     2.4357145E 02    -4.4820793E 00
 2.5376743E 02    -1.5002601E 03    -1.2777480E 03    -1.0381110E 03    -6.2212117E 02    -1.9562155E 01
 9.1917830E 00    -5.7889857E 02    -5.5244088E 02    -4.2560750E 02    -2.7071956E 02    -4.9520928E 01
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4.3.4 LISTING

This is the listing for Program III with the n-body calling sequences and the differential correction routine.

```
      EQUIVALENCE(WM(1),XIM),(WM(2),YIM),(WM(3),ZIM),(WM(4),XIDM),
     1(WM(6),ZICM),(WM(5),YIDM),(WL(1),XIL),(WL(2),YIL),(WL(3),ZIL),
     2(WL(4),XICL),(WL(5),YIDL),(WL(6),ZIDL)
      DAC PX(9,67),PY(9,67),PZ(9,67),TL(67),RKM(4,6),RKL(4,6),NB(9),
     1WT(9),RAD(9),X(9),Y(9),Z(9),DIS(9),DIST(9),PM(6),PL(6),PPM(6),
     2PPL(6),QM(6),QL(6),WM(6),WL(6),XX(9),YY(9),ZZ(9),TI(6),CL(6)
      COMMON NEWORG,ZER,FMX,FLX,FMY,FLY,FMZ,FLZ,AM,AL,BM,BL,CM,CL
      COMMON NORG,NTARG,NSUN,N,NN,NNN,T,TO,TMAX,DTMAX,HAFDT,WTE,WTV,MGM,
     1XTARG,YTARG,ZTARG,SCEN,VCEN,SMAX,GMAX,EMAX,RAD1,RAD2,RADE,WTA,
     2NBE,NBT,NBS,RADORG,SAM,SBM,SCM,SAL,SBL,SCL,XIM,XIL,YIM,YIL,ZIM,
     3ZIL,XIDM,XIDL,YIDM,YIDL,ZIDM,ZIDL,RM,RL,RR,RO,SS,RRM,RRL,TEM,TEL,
     4GMX,GLX,GMY,GLY,GMZ,GLZ,SUMX,SULX,SUMY,SULY,SUMZ,SULZ,FM,FL,GM,GL,
     5HM,HL,RRR,SSS,XM,YM,ZM,A,BX,BY,BZ,WRM,WRL,RCM,RCL,ROM,ROL,RPPM,
     6XPM,XPL,YPM,YPL,ZPM,ZPL,
     7KOB,VNZ,MEDT,MSDT,MDT,NOT,MET,NOUT,NCKE,TSAV,DTA,DTB,DTC,EP1
      DAC G(2),WTS(2)
      DAC        SOGOOD(6),XDOT(2),YDOT(2),ZDOT(2),WRONG(6),QX(2)
     1,QY(2),QZ(2),XP(2),YP(2),ZP(2),XDOTP(2),YDOTP(2),ZDOTP(2),PI(2),
     2PI2(2),F(2),XMJ(2),CCON(2),XN(2)
      COMMON A1,A2,FJ1,FJ2,SFJ1,SFJ2,CFJ1,CFJ2,S1,S2,EPS1
      COMMON DT,DTME,RPM
      DAC SM(6),SL(6),XMJP(2)
      COMMON OBJ,TSAVE,REA,RTA,RSN,HPI,THPI,KK,NOSW,PQM(6),PQL(6),ERR
      COMMON RPA
      COMMON RARBM(3,3),RARBL(3,3),RBRBM(3,3),RBRBL(3,3),
     1RCRBM(3,3),RCRBL(3,3),RDRBM(3,3),RDRBL(3,3)
      COMMON XHINT(10),YHINT(10),ZHINT(10),XNP(3,3),ALX
      COMMON XPCS(3),XVELX(3),NBX(10),XWTX(10),XRADX(10),HDETT
      COMMON KWC,KNOP,KNOPP,NOOFT,KNOPPC,NOOFK
      DIMENSION TRARI(3,3),TRBRI(3,3),TRCRI(3,3),TRDRI(3,3)
      DIMENSION XYZO(6),FGH(10,10)
      MGM=512
      EPS1=1.0E-11
B     PI=202622077325
B     PI(2)=147042055061
B     PI2=203622077325
B     PI2(2)=150042055061
B     HPI=201622077325
B     THPI=203455457437
    2 REWIND 26
      RIT 2,3,KWC,KNOP,KNOPP,NOOFT
      RIT 2,3, AK
      RIT 2,3,G(2),WTE,RADE,WTV,OBJ
    3 FORMAT(E9.5)
    4 RIT 2,3, EMAX,GMAX,SMAX,TMAX,TO
    5 RIT 2,3, NCKE,MSDT,MET,NOUT,NOSW
      IF(MSDT)6,7,6
    6 RIT2,3, DTA,DTB,DTC
      ERASE DT
      GO TO 8
    7 RIT 2,3, CT,DTMAX,EP1
      HAFDT=DT/2.
    8 RIT 2,3, N,NEWORG,NTARG
      RIT2,3,(NB(I),WT(I),RAD(I),I=1,N)
      RIT 2,3,(PM(I),I=1,6)
C     *****SAVE INITIAL CONDITIONS****
```

```
   61 DO 601 I=1,6
  601 XYZO(I)=PM(I)
      VO=SQRTF(PM(4)*PM(4)+PM(5)*PM(5)+PM(6)*PM(6))
      DO 604 I=1,N
      JK=I
      IF(NEWORG-NB(I))604,612,604
  604 WW=WTE
      GO TO 613
  612 WW=WT(JK)
  613 VESC=SQRTF(2.*G(2)*WW/SQRTF(PM(1)**2+PM(2)**2+PM(3)**2))
      IF(KWC)50,51,50
   50 CONTINUE
      ERASE KNOPC,KNOPPC
      ALX=-G(2)*OBJ
      NOOFK=NOOFT
      DO 9 I=1.N
      NBX(I)=NB(I)
      XWTX(I)=WT(I)
      XRADX(I)=RAD(I)
      KTT=I+1
    9 CONTINUE
      NBX(KTT)=0
      XWTX(KTT)=WTE
      XRADX(KTT)=RADE
      ERASE RARBM,RARBL,RBRBM,RBRBL,RCRBM,RCRBL,RDRBM,RDRBL
      DO 24 I=1,3
      RARBM(I,I)=1.0
      RDRBM(I,I)=1.0
   24 CONTINUE
   51 CONTINUE
      ERASE T
      CALL SETI
      G=-G(2)
      ERASE NOT,NORG,PL,SL(4),SL(5),SL(6)
      CALL TITLE
   11 CALL ORGN
      KK=67
      NBT=NBT
      IF(TEM)12,2,12
   12 CALL NBODY
      IF(ERR)55,2,55     → test for impact
   55 IF(KWC)52,53,52
   52 CONTINUE
C     SET UP INVERSE MATRIX
      DO 60 I=1,3
      DO 59 J=1,3
      TRARI(I,J)=RDRBM(J,I)
      TRBRI(I,J)=-RBRBM(J,I)
      TRCRI(I,J)=-RCRBM(J,I)
      TRDRI(I,J)=RARBM(J,I)
   59 CONTINUE
   60 CONTINUE
      WOT 10,230
      WOT 10,231,(RARBM(1,I),I=1,3),(RBRBM(1,J),J=1,3)
      WOT 10,231,(RARBM(2,I),I=1,3),(RBRBM(2,J),J=1,3)
      WOT 10,231,(RARBM(3,I),I=1,3),(RBRBM(3,J),J=1,3)
```

```
      WOT 10,231,(RCRBM(1,I),I=1,3),(RDRBM(1,J),J=1,3)
      WOT 10,231,(RCRBM(2,I),I=1,3),(RDRBM(2,J),J=1,3)
      WOT 10,231,(RCRBM(3,I),I=1,3),(RDRBM(3,J),J=1,3)
      WOT 10,232
      WOT 10,231,(TRARI(1,I),I=1,3),(TRBRI(1,J),J=1,3)
      WOT 10,231,(TRARI(2,I),I=1,3),(TRBRI(2,J),J=1,3)
      WOT 10,231,(TRARI(3,I),I=1,3),(TRBRI(3,J),J=1,3)
      WOT 10,231,(TRCRI(1,I),I=1,3),(TRDRI(1,J),J=1,3)
      WOT 10,231,(TRCRI(2,I),I=1,3),(TRDRI(2,J),J=1,3)
      WOT 10,231,(TRCRI(3,I),I=1,3),(TRDRI(3,J),J=1,3)
   53 CONTINUE
C     ***** DIFFERENTIAL CORRECTIONS=====
      NR=3
      NC=4
      REWIND 26
      FGH(1,4)=-XTARG
      FGH(2,4)=-YTARG
      FGH(3,4)=-ZTARG
      DO 600 I=1,3
      DO 600 J=1,3
  600 FGH(I,J)=RBRBM(I,J)
      CALL MATINV(FGH,NR,NC)
      DO 602 J=1,3
  602 PM(J)=XYZO(J)
      DO 603 J=4,6
  603 PM(J)=XYZO(J)+FGH(J-3,4)
C     ***** TEST CHANGE IN VELOCITY *****
      V1=SQRTF(PM(4)**2+PM(5)**2+PM(6)**2)
      DV1=SQRTF(FGH(1,4)**2+FGH(2,4)**2+FGH(3,4)**2)
      IF(V1-VESC)606,606,605
  605 IF(ABSF(DV1)-AK*VO)61,607,607
  607 DVIP=AK*VO
      J=1
  609 CONST=DVIP/DVI
      DXP=CONST*FGH(1,4)
      DYP=CONST*FGH(2,4)
      DZP=CONST*FGH(3,4)
      PM(4)=XYZC(4)+DXP
      PM(5)=XYZO(5)+DYP
      PM(6)=XYZO(6)+DZP
      V1=SQRTF(PM(4)**2+PM(5)**2+PM(6)**2)
      GO TO (61C,61),J
  610 IF(V1-VESC)608,608,61
  608 VIP=1.005*VESC
      B=2.*(XYZO(4)*DX+XYZO(5)*DY+XYZO(6)*DZ)/DV1
      C=VO*VO-VIP*VIP
      DV1P=(-B-SQRTF(B*B-4.*C))/2.
      J=2
      GO TO 609
  606 IF(ABSF(DVI)-AK*VO)607,608,608
  230 FORMAT(1H1,32X,46H****   SENSITIVITY  COEFFICIENT  MATRIX   ****)
  231 FORMAT(1HC,1P6E19.7)
  232 FORMAT(1H0,32X,29H****   INVERSE  MATRIX   ****)
      END(1,1,0,0,0,1,1,1,0,1,0,0,0,0,0)
```

5. REFERENCES

1. "Generalized Interplanetary Trajectory Study," by J. P. deVries, coordinator, General Electric, Technical Information Series R60SD465, Class III, August 1960. (Air Force Contract AF 33(616)-6296, Project 1431, Task 14014.)

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TECHNICAL INFORMATION SERIES

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SUMMARY

This report contains complete input instructions, operating instructions and sample problems with computer output for three IBM-7094 Computer Programs. The Programs and their essential features are:

I) Interplanetary Trajectory Program. This program determines the burnout velocity for a trajectory from Earth to any other planet or from any other planet to Earth.
II) N-Body Trajectory Program.
III) N-Body Program with Sensitivity Coefficients and Differential Correction.

A full description of Program I is available in reference 1 and 2. This report presents an outline of the pertinent sections of those references, to facilitate their use. Full descriptions and analyses are presented for those features that distinguish Programs II and III from Program I.

KEY WORDS

N-Body Program, interplanetary trajectories, differential correction, sensitivity coefficients

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AUTHOR

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