Downtown

2.38

[redacted]

9p.

Dispersion Strengthening Models

J. S. Hirschhorn

G. S. Ansell

Materials Engineering Department
Rensselaer Polytechnic Institute
Troy, New York

[redacted]

Dispersion Strengthening Models

In recent years a number of theoretical models [1-3] have been proposed to explain the observed strengthening behavior of crystalline solids containing a uniform dispersion of fine particles. A cursory examination of the structure-property relationships derived from these models shows them apparently to be quite dissimilar. A more detailed examination, however, reveals a basic similarity of approach utilized in each model. It is therefore appropriate to consider why these basically similar models predict such differing structure-strength relationships.

The dislocation model in each case ascribes yielding to occur when the applied stress on the bulk alloy permits dislocations to freely move through the lattice by cutting through the distributed second phase. In calculating the yield strength Ansell [1] assumed that the stress required to cut the distributed phase arose from the magnification of the applied stress due to a piled-up array of curved dislocations. Kelly and Nicholson [2] and Marcinkowski and Wriedt [3], on the other hand, related the work done during deformation, due to the applied stress, to the energy required to fracture the particles. The magnification of the applied stress due to pile-up was originally derived on the basis of such a work consideration [4]. Therefore all three models use essentially

the same dislocation-particle interaction model and similar criteria for yielding. Where these models widely differ, however, lies in the authors' treatments of the description of these interactions in relation to the distribution of the particles within the solid.

Each of the models considers the interaction of dislocations with the particles contained within a slip plane. Kelly and Nicholson and Marcinkowski and Wriedt assume a particular distribution function for the arrangement of the particles in the slip plane and then extrapolate this distribution function to the distribution of particles contained within the solid. Ansell [1], however, first considers the nature of the distribution in three dimensions and from this determines the second phase distribution within a slip palne. On the surfaces these two approaches would appear to be the same; however, the authors' use of these approaches (i.e., planar and volumetric distribution) are not compatible and lead to their divergent results.

The planar approach invokes the assumption that the dispersed particles may be imagined to be arranged on a square-mesh in the slip plane. In reality there are a multiplicity of such slip planes, especially in cubic structures. Thus, according to this approach the particles would not be uniformly distributed within the matrix lattice.

Instead, one must imagine a rather peculiar spatial distribution such that many random planes intersect the particles in such a manner that on these planes the particles are arranged on a square-mesh. In addition, one must consider an average effective diameter of the particles in such planes. Indeed, there can be no such three dimensional distribution of spheres in a solid which would yield these results.

The volumetric approach starts with the assumption that the particles are uniformly distributed in space; and that this distribution may be represented by a cubic arrangement of the particles. Thus any random plane within the volume of the alloy possesses an equally probable planar distribution of particles.

It is then possible to derive two equations for each of the proposed models; that is, one representing a planar discription and the other representing a volumetric description of the dispersion. To do this we must make note of some basic equations appropriate for each approach. In the planar method the center to center spacing between particles is given by [2]

$$L = (\pi/6)^{\frac{1}{2}} d/F^{\frac{1}{2}} \qquad (1)$$

where d is the true diameter of the spherical particles and F is the volume fraction of dispersed particles. The effective intercept diameter (di) of the particles on the planes is given by

$$di = (2/3)^{\frac{1}{2}} d \qquad (2)$$

or

$$di = (\pi d/2)\, 6^{-\frac{1}{2}} \qquad (3)$$

In the volumetric method L is given by [1]

$$L = \left(\frac{\pi}{6}\right)^{1/3} d \,/\, F^{1/3} \qquad (4)$$

and the true diameter of the particles is considered.

The first model proposed was that of Ansell [1]. Using the volumetric approach he derives

$$\tau = \tau_m + \frac{\mu^* F^{1/3}}{4C\left[\left(\frac{\pi}{6}\right)^{1/3} - F^{1/3}\right]} \qquad (5)$$

where τ is the yield stress of the alloy, τ_m is the strength of the matrix, μ^* is the shear modulus of the dispersed phase, and C is a constant equal to about 30. With the use of eqns. (1) and (2) we obtain from this model, for the planar case

$$\tau = \tau_m + \frac{\mu^* (3)^{\frac{1}{2}} F^{\frac{1}{2}}}{4C (2)^{\frac{1}{2}} \left[\left(\frac{\pi}{2}\right)^{\frac{1}{2}} - F^{\frac{1}{2}}\right]} \qquad (6)$$

The model of Kelly and Nicholson in which the planar approach was used gave

$$\tau = \frac{\gamma (r)^{\frac{1}{3}} F^{\frac{1}{2}}}{2b} \qquad (7)$$

where γ is the energy of the additional matrix-particle interface produced by the shear and the energy of the dislocations produced at the interface and b is the Burgers vector of the dislocation cutting the particle. With the use of eqn, (4) we obtain from their model, for the volumetric case

$$\tau = \left(\frac{6}{\pi}\right)^{1/3} \frac{\gamma F^{1/3}}{b} \qquad (8)$$

Marcinkowski and Wriedt also used the planar approach and found

$$\tau = \tau_m + 2 (\tau_p - \tau_m) F^{\frac{1}{2}} / (\pi)^{\frac{1}{2}} \qquad (9)$$

where τ_p is the strength of the particle. Using eqn. (4) we obtain from their model, for the volumetric case

$$\tau = \tau_m + (6/\pi)^{1/3} (\tau_p - \tau_m) F^{1/3} \qquad (10)$$

The model of Dew-Hughes [5] may also be similarly treated; and this would yield equations similar to eqns. (5) and (6). This model involves the cutting of dislocation loops by other dislocations; and did not use either the planar or volumetric type of description, but left the final equation in terms of general parameters.

We now can conclude that if we use a similar method of describing the geometrical distribution of the dispersion, then all three models yield the same basic relationship; that is, if the planar description is used, then the yield strength is a function of $F^{\frac{1}{2}}$, and if the volumetric description is used, then the yield strength is a function of $F^{1/3}$.

The crucial question to now consider is whether the planar or volumetric type of description is more realistic. We have already noted that it is impossible to extrapolate a three dimensional distribution which is compatible with the planar description employed. We conclude that the volumetric type of description represents more faithfully the actual physical distribution of dispersed phase particles. Thus, regardless of the model one believes correctly treats the dislocation-particle interaction, we believe that the yield strength is a function of $F^{1/3}$. Mathematically one finds that equally good agreement may be obtained if one plots $\hat{\tau}$ vs. $F^{1/3}$ or $\hat{\tau}$ vs. $F^{\frac{1}{2}}$; thus, the data in the literature found to agree with the $F^{\frac{1}{2}}$ dependence may be found to be in equally good agreement with $F^{1/3}$.

The work reported in this paper was supported by the National Aeronautics and Space Administration at the NASA Interdisciplinary Materials Research Center, Rensselaer Polytechnic Institute.

J. S. Hirschhorn
G. S. Ansell

Materials Engineering Department

Rensselaer Polytechnic Institute

Troy, New York

References

1. G. S. Ansell, Acta Met., 9, 518 (1961).

2. A. Kelly and R. B. Nicholson, Progr. Materials Science, 10, 148 (1963).

3. M. J. Marcinkowski and D. F. Wriedt, J. Electrochemical Soc., 111, 92 (1964).

4. A. H. Cottrell, Dislocations and Plastic Flow in Crystals, Oxford, London (1953).

5. D. Dew-Hughes, Acta Met., 8, 816 (1960).