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THE CONTRACTION OF MOLECULAR HYDROGEN PROTOSTARS

UNPUBLISHED PRELIMINARY DATA

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ABSTRACT

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It is shown that molecular hydrogen protostars larger than about a solar mass suffer a rapid free fall collapse when they are still quite cool ($\sim 50^{\circ}K$). The collapse results from the high luminosity of a transparent protostar for 28μ line radiation resulting from the $J = 2$ to $J = 0$ rotational quadrupole transition in parahydrogen. The possibility of detecting galactic 28μ radiation and star formation is considered.

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I. INTRODUCTION

It is well established that stars form out of condensations of the interstellar gas and that star formation is still taking place in the spiral arms of our galaxy. The quantitative details of this condensation process are poorly understood, however, and the general phenomenon of star formation has remained one of the biggest unsolved problems of modern astrophysics. Two principal difficulties are: (1) how do the dilute gas clouds reach the stage where their gravitational self energy is large enough to allow them to contract against the pressure from the kinetic energy (temperature) of their constituent atoms and molecules, and (2) how is the initial angular momentum of the gas cloud lost as the cloud contracts? The answers to both of these questions are unknown although there have been a number of suggestions. Several useful surveys of the general problem may be found in the collection of prize essays by Burbidge, Kahn, Ebert, von Hoerner, and Temesváry (1960). A more recent review has been given by Spitzer (1963).

Here we shall consider only one particular phase of star formation, a phase which all stars are likely to go through, however. This is the very early stage of stellar evolution in which the protostar is still cool but has reached a density where gravitational interactions are strong enough so that a quasi-equilibrium state exists and the virial theorem holds. We shall not consider the effects of initial rotation of the protostar or of magnetic fields. The assumption is made that either the rotation is small or that there is some efficient process whereby the protostar loses its angular momentum as it contracts. Moreover, it is assumed that either the magnetic field is small or that the degree of ionization is very small. The purpose of this paper is to point out that under these conditions a protostar composed of molecular hydrogen will suffer a rapid collapse when it is still quite cool ($T \sim 50^{\circ}K$). The collapse comes about because the hydrogen molecule is very efficient at radiating away energy (predominantly by de-excitation of the J = 2 rotational level of para-hydrogen) at such low temperatures. Actually, even if the protostar had only a small amount (say, 1%) of molecular hydrogen, the radiation from J = 2 de-excitations would be sufficient to cause a rapid contraction. Because of the difficulty of cooling interstellar matter to temperatures below about 20° - 30°K (cf. Gould and Salpeter 1963, Gould, Gold and Salpeter 1963), the thermal kinetic energy E_k of a gas mass of stellar size is so large that high densities (see Table 1) are required for the magnitude of the gravitational potential energy V to be large enough to satisfy the virial theorem $2E_k + V = 0$. At high densities it is very likely that an appreciable fraction of the hydrogen is in molecular form, since even if recombination on the surfaces of the interstellar grains does not occur, molecules can form by the associative detachment process $H^- + H \rightarrow H_2 + e$ (Gould and Salpeter, loc cit.).

After discussing protostar energy losses in Section II the contraction and collapse is treated in Sections III and IV. The overall phenomenon is summarized in Section V. The possibility of observationally verifying the ideas presented here is considered in the last section.

II. ENERGY LOSSES IN A PROTOSTAR

a) Population of the J = 2 State of Parahydrogen

Consider a gas of molecular hydrogen at a fairly low temperature T (~ 50°K) so that the molecules are predominantly in the ground (J = 0) rotational state. At such low temperatures para-ortho collisional conversion is very slow. The population in the J = 2 state results essentially from collisional excitation from the ground state (cross section: σ_{02}), collisional de-excitation from the J = 2 state (cross section: σ_{20}), and radiative de-excitation from the J = 2 state (transition probability per unit time[1]:

[1] This is the quadrupole transition probability as calculated by Spitzer (1949). Recently it had been thought (cf. Osterbrock 1962) that the J = 2 → 1 → 0 para-ortho-para radiative decay was more likely. However, a more recent investigation by Raich and Good (1963) indicates a negligible probability for para-ortho radiative transitions.

$A_2 = 2.4 \times 10^{-11}$ sec^{-1}). Under steady state conditions the number density n_2 of molecules in the excited state would be

$$\frac{n_2}{n} \approx \frac{n \langle v\, \sigma_{02}\rangle}{n \langle v\, \sigma_{20}\rangle + A_2}, \tag{1}$$

where n is the total density (essentially the density of molecules in the ground state), v is the relative velocity, and the angular brackets denote a thermal mean. From detailed balance we have $\langle v\, \sigma_{20}\rangle = \langle v\, \sigma_{02}\rangle\, (g_0/g_2)\, e^{E_2/kT}$, where E_2 is the excitation energy (7.068×10^{-14} erg) of the J = 2 level, and the g's are the degeneracies ($g_0/g_2 = 1/5$). The de-excitation cross section $\sigma_{20} \equiv \sigma_d$ has been measured indirectly and is 4.0×10^{-18} cm^2 (see Spitzer 1949) approximately independent of energy (temperature). The de-excitation rate constant is then given by $\bar{v}_r \sigma_d$, with $\bar{v}_r = 4\, (kT/\pi m)^{\frac{1}{2}}$ the mean relative velocity (m = mass of H_2). We then have

$$\frac{n_2}{n} = \frac{(g_2/g_0)\, e^{-E_2/kT}}{1 + q}, \tag{2}$$

$$q = A_2/n\, \bar{v}_r \sigma_d;$$

q measures the deviation from thermal equilibrium. For $T \sim 50°$K $q \sim 1$ for $n \sim 60$ cm^{-3}.

b) Energy Losses by Various Processes

Cooling in H I regions has been considered by Spitzer (1949, 1954) and others. Here we give a brief summary (see Fig. 1) of the results. The energy loss (in ergs/cm^3/sec) in a molecular hydrogen gas at low density ($q >> 1$) resulting from $J = 2$ excitations and 28μ emission is (see part a of this section)

$$\Lambda_{H_2H_2} = 4\, E_2\, \sigma_d\; n^2 (kT/\pi m)^{\frac{1}{2}}\, (g_2/g_0)\, e^{-E_2/kT} \qquad (3)$$

For $n \geq 60$ cm^{-3} this expression must be divided by $1 + q^{-1}$ to take account of collisional de-excitations. The cooling by collisions with interstellar grains was also considered by Spitzer. Assuming that in a molecule-grain collision all the kinetic energy of the molecule is transferred to the grain before the molecule leaves the surface with a mean energy $\langle E \rangle \approx kT_g$, where T_g is the grain temperature, we have

$${}_{H_2g} = n n_g\, \pi\, r_g^{\,2}\, (8\, kT/\pi m)^{\frac{1}{2}}\, k\, (T - T_g); \qquad (4)$$

here n_g is the number density of grains and r_g is their radius.

The cooling rates $\Lambda_{H_2H_2}$ and Λ_{H_2g} are plotted in Figure 1 for $r_g = 2 \times 10^{-5}$ cm, $T_g = 10^\circ$K, and $n_g/n = 10^{-13}$ (n = molecule density). Also shown in the figure is the cooling rate Λ_{HH_2} resulting from 28μ excitations resulting from H - H_2 collisions and the effective cooling rate $\Lambda_{ei} - \Gamma_{ei}$ resulting from electron collisions with the ions C^+, Si^+, and Fe^+ and the photoionization of these ions (cf. Gould and Salpeter, op. cit.). The curve $(\Lambda_{ei} - \Gamma_{ei})/n^2$ corresponds to a gas in which the hydrogen is completely molecular and is thus simply four times the curve given in the paper by Gould and Salpeter.

In this paper we shall assume that elements like C, Si, and Fe have condensed into the grains so that there is no $\Lambda_{ei} - \Gamma_{ei}$ cooling rate. Moreover we consider a hydrogen gas which is essentially completely molecular so that the cooling Λ_{HH_2} (which has essentially the same temperature dependence as $\Lambda_{H_2H_2}$)

is inoperative. It will turn out that for the temperatures of interest cooling by molecule-molecule collisions dominates cooling by grain collisions. Thus we shall consider only cooling according to the rate $\Lambda_{H_2H_2}/(1 + q^{-1})$. Since energy loss in protostar contraction is a central problem in star formation, it is clear that the conditions assumed here are not optimum. Specifically, as is clear from Figure 1, if the elements C, Si, and Fe (especially C, since C^+ produces most of the cooling at low temperature) exist in the gas phase, the energy loss at low temperatures is much greater and contraction proceeds more readily. Protostar contraction is considered in the following section.

III. LOW TEMPERATURE CONTRACTION AND COLLAPSE OF A TRANSPARENT PROTOSTAR

Because the cooling by molecular hydrogen increases rapidly with temperature, a protostar composed of hydrogen molecules would be nearly isothermal. The mass distribution of an isothermal gravitational gas sphere follows the Boltzmann law $e^{-m\varphi/KT}$, where φ is the local gravitational potential, and extends to infinity. I shall actually consider a finite gas sphere of radius R, assuming external pressures (for example, radiation pressure on the interstellar grains contained in the protostar) cut off the mass distribution. However, it is assumed that such pressures do not contribute significantly to the value of the total energy of the gas sphere.

Neglecting rotational and vibrational energies (since we are considering low temperatures), the total energy of the molecular protostar can be written, with the help of the virial theorem, in terms of the gravitational potential energy: $E = \frac{1}{2}V$. The potential energy is approximately[2] $-GM^2/R$ so that the

[2] The numerical factor in this expression for V depends on the detailed mass distribution, but is of order unity.

relation between luminosity and contraction rate is

$$\frac{dE}{dt} = \frac{\zeta}{R^2} \frac{dR}{dt} , \tag{5}$$

with $\zeta = \frac{1}{2} GM^2$. By the virial theorem the relation between temperature and radius is

$$T = GmM/3kR. \tag{6}$$

If the energy of the protostar is lost by radiation of the $J = 2 \rightarrow 0$ (28μ) line, and the protostar is transparent to the 28μ line, the luminosity comes from the entire volume and we have, from the relations (2) for n_2/n and (6) for T,

$$dE/dt = - \eta \, e^{-\kappa R}, \tag{7}$$

where

$$\eta = \eta e/(1 + q), \; \kappa = 3E_2/GmM,$$

$$\eta e = (M/m) \quad (g_2/g_0) \; A_2 \; E_2, \tag{8}$$

$$q = q(R) = \frac{\pi^{3/2} A_2 \; m \; R^{7/2}}{\sigma_d (3G)^{1/2} \; M^{3/2}}$$

(the mean density $\bar{n}$ was used to compute q). By equations (5) and (7) the contraction rate is

$$- \frac{1}{R} \frac{dR}{dt} = \gamma_c = \frac{\eta}{\zeta} \; R \; e^{-\kappa R} \tag{9}$$

Because of the exponential dependence, the contraction increases rapidly as the radius decreases (and the temperature rises). An upper limit to the contraction rate is given by the free fall rate

$$\gamma_{ff} = t_{ff}^{-1} \approx (\bar{\rho} \; G)^{\frac{1}{2}} = \xi \; R^{-3/2}, \tag{10}$$

with $\xi = (3MG/4\pi)^{\frac{1}{2}}$.

By comparing the contraction rates γ_c [eq. (9)] and γ_{ff} [eq. (10)] one finds that the contraction caused by 28μ radiation approaches the free fall rate already at fairly large radii R and low temperatures T. The critical radii R_c, temperatures T_c, and mean density $\bar{n}_c$ where $\gamma_c \rightarrow \gamma_{ff}$ are given in Table 1 for various protostar masses. The free fall time is also given for $R = R_c$. The critical temperature determined in this manner does not depend strongly on the exact mass distribution of the protostar since it appears in the exponent $\kappa R_c = E_2/kT_c$, which is fairly large (≥ 10). The reversal in the dependence of T_c on protostar mass at $M/M_\odot \sim 300$ is a result of the deviation from thermal equilibrium in the rotational levels at low density, that is, q is no longer $\ll 1$.

The contraction would begin to slow down when the protostar became opaque to the 28μ radiation or when $\lambda \sim R$, where λ is the mean free path for absorption of the radiation. For $\lambda \ll R$, radiation escapes only from the shell of thickness λ instead of from the whole volume of the protostar, and the luminosity is reduced. This situation is considered in the following section.

IV. CONTRACTION OF AN OPAQUE PROTOSTAR AND DISSOCIATIVE COLLAPSE

a) Absorption Cross Section for the 28μ Line

The cross section for 28μ absorption by interstellar grains composed of ice is $\sigma_g \sim 3 \times 10^{-11}$ cm^2 (cf. Gaustad 1963) and the ratio of grain density to molecule density is $n_g/n \sim 10^{-13}$. With this ratio constant the mean free path λ_g for grain absorption is proportional to $\bar{n}^{-1} \propto R^3 M^{-1}$. The cross section for self absorption of the line may be derived from the transition probability per unit time A_2 for the reverse process with the help of the principal of detailed balance:

$$\sigma_s = \frac{A_2}{\Delta\nu} \frac{\lambda^2}{8\pi} \frac{g_2}{g_0} , \tag{11}$$

where λ is the wavelength and $\Delta\nu$ the frequency width of the line. Taking a Doppler width $\Delta\nu/\nu \approx (kT/mc^2)^{\frac{1}{2}}$ with T given by equation (6), we have the effective cross section

$$\sigma_s \approx (8\pi)^{-1} A_2 \lambda^3 (g_2/g_0) (3R/GM)^{\frac{1}{2}} . \tag{12}$$

The mean free path λ_s for self absorption is then proportional to $R^{5/2} M^{-1/2}$. The effective mean free path is $\lambda = (\lambda_g^{-1} + \lambda_s^{-1})^{-1}$. One finds that at the radius where $\lambda \sim R$, $\lambda_s < \lambda_g$ for $M/M_\odot \leq 100$. However, for $M/M_\odot \geq 10$, by the time the protostar has contracted to the radius where opacity is important, the temperature of the gas will be raised above $\sim 100^\circ$K and the grains will have evaporated (Gaustad loc. cit.). The radius (R_s) and temperature (T_s) of the protostar when $\lambda_s = R$ are given in Table 1 for various stellar masses. The temperature T_s given in the table will not be meaningful if the protostar has reached an opaque condition via a free fall. The radius R_s, which is proportional to $T^{1/8}$, is not in serious error.

b) Contraction of an Opaque Protostar

When the mean free path for absorption of the 28μ line is much less than the radius of the protostar, radiation escapes only from its outer edges. Neglecting stimulated emission one finds, in a straightforward manner, the following expression for the luminosity:

$$dE/dt = -\pi R^2 \sigma^{-1} E_2 A_2 (g_2/g_0) e^{-\kappa R}, \tag{13}$$

where σ is the absorption cross section per molecule; the result is independent of the mass distribution at the edge of the protostar. The effective cross section σ is $\sigma_s + (n_g/n)\,\sigma_g$. Neglecting grain absorption, and employing the expression (12) for σ we get

$$dE/dt = -\eta' R^{3/2} e^{-\varkappa R} ,$$

$$\eta' \equiv 8\,\pi^2 E_2\,\lambda^{-3}\,(GM/3)^{\frac{1}{2}}, \qquad (14)$$

essentially the black body luminosity within the width of the 28μ line. When $\varkappa R$ is no longer appreciably greater than unity, the specific Planck function must be summed over all appropriate frequencies corresponding to quadrupole rotational transitions. By equations (5) and (14) the contraction rate for a cool, opaque protostar is

$$-\frac{1}{R}\,\frac{dR}{dt} = \frac{\eta'}{\zeta}\,R^{5/2}\,e^{-\varkappa R} \qquad (15)$$

By comparing the contraction rate (15) with the free fall rate (10) one finds that for $M = M_\odot$ free fall is attained when the temperature of the protostar reaches about 76°K. However, for smaller mass protostars, the free fall rate is not attained and the contraction proceeds at a slower rate. The rate (15) reaches a maximum when $\varkappa R = E_2/kT$, or when $T = 205°K$.

c) Dissociative Collapse

Cameron (1962) has considered the protostar collapse which results when the temperature is raised to about 1800°K, whereupon the hydrogen is dissociated. The dissociation energy of the gas is balanced by an increase in the (negative) gravitational energy of the gas and an increase in the temperature. As Cameron showed, the temperature of the gas after the dissociative collapse is large enough to ionize the hydrogen and the collapse continues until the hydrogen (and the helium, if present) is fully ionized.

The dissociation energy E_d (4.48 eV) and ionization energy E_i (2 Ry) of the M/m molecules thus goes into increasing the thermal energy of the 4 M/m ions (protons and electrons). The resulting mean temperature after this final collapse is, by the virial theorem, $\overline{T}_f \simeq (E_d + E_i)/6k = 6.1 \times 10^{4}$ °K.

A non-radiating collapse could also ensue if the ratio of specific heats γ approached 4/3. For a gas of molecular hydrogen $\gamma \rightarrow 4/3$ when vibrational levels begin to be populated. As Cameron (loc. cit.) has remarked, γ reaches this value at a temperature of about 1800°K, that is, roughly the same temperature at which the hydrogen begins to dissociate.

V. SUMMARY

First we consider the early evolutionary steps through which a protostar proceeds if there is no fragmentation. At the end of this section we shall discuss the modifications in the condensation process required by fragmentation phenomena. The evolutionary steps may be summarized as follows: (1) A cloud condenses to a fairly high density state where it is stable against disruption (virial theorem holds); it is assumed that molecules also form in the process. (2) The cool gas radiates the 28μ line and the protostar contracts slowly; as it contracts the temperature rises and the rate of 28μ emission increases. (3) The contraction rate increases until finally it reaches the free fall rate. (4) The free fall begins to slow as the protostar becomes opaque to 28μ radiation; a slower contraction follows until the temperature reaches about 1800°K. (5) When the protostar reaches this temperature, the hydrogen dissociates and a free fall collapse ensues in which the temperature rises. (6) The collapse continues until all the hydrogen is ionized; the mean temperature of the protostar is then about 6.1×10^{4} °K. (7) The normal slow Helmholtz contraction to the main sequence begins.

The above describes the evolution of protostars with $M > M_{\odot}$. For smaller masses the first collapse [step (3)] does not occur since the protostar has become opaque to 28μ radiation and its luminosity does not reach such a high value to allow the first free fall collapse. Moreover, for the more massive protostars the first collapse is not halted by an opaque condition, since for the more massive stars the 28μ opacity does not become significant until the temperature of the gas is fairly high (see Table 1). One can obtain a rough value for the radius where the collapse is halted by neglecting the emission in other lines (for example, the $J = 4 \rightarrow 2$ transition and also orthohydrogen transitions) and equating the expression for the opaque contraction rate (15) with the proper Planck factor to the free fall rate (10). The minimum free fall radius R_m is then found from $R_m^4 \sim (\xi\zeta/\eta')(e^{\varkappa R_m} - 1)$. One can then compute the corresponding temperature from equation (6) and one finds that this temperature reaches the dissociation value 1800°K for $M/M_{\odot} \sim 40$. Thus for protostar masses greater than this the first free fall collapse (which is caused by the high 28μ luminosity) continues on through the collapse induced by dissociation and ionization. This is, step (5) above does not occur.

It is evident from Table 1 that the critical densities $\bar{n}_c$ for collapse are much higher than the normal densities in interstellar clouds, except for the more massive protostars. Actually, it is very likely that all stars are formed out of massive condensations (say, $M \sim 10^3 M_{\odot}$) which fragment as they contract. Moreover, it appears that the most likely stage for fragmentation to occur is during the initial collapse from low densities induced by 28μ emission. During this free fall the virial theorem is no longer applicable and the temperature, instead of increasing as $1/R$, will only increase slightly, although the density in the protostar increases as $1/R^3$. Subcondensations will then form and the cloud will fragment.

VI. POSSIBLE OBSERVATION OF 28μ RADIATION AND STAR FORMATION

On the assumption that the collapse process described here occurs in all star formation one can estimate the rate of production of 28μ quanta in the galaxy from the rate of conversion of galactic gas into stars. The number of 28μ quanta that a protostar of mass M emits in contracting to reach the temperature $T_c' = 1800°K$ is $\sim \frac{3}{2}$ (M/m) $(k\ T_c'/E_2)$. If galactic interstellar matter is converted into stars at a rate dM/dt, the number of 28μ quanta produced in the galaxy per second would then be

$$\frac{dN}{dt} \sim \frac{3k\ T_c'}{2m\ E_2}\ \frac{dM}{dt}\ . \qquad (16)$$

Taking $dM/dt = M_g/T$, where M_g ($\sim 10^{43}$ gm) is the galactic mass of interstellar gas and $T \sim 10^9$ yr (see Salpeter 1959), we get $dN/dt \sim 5 \times 10^{50}$ photons/sec. One can get a rough idea of the flux expected from the galactic plane by supposing that this photon source is placed at a distance $\sim$ 10 kpc. The energy flux is then about 3×10^{-16} watt/cm^2. Since to observe 28μ radiation one must carry out (balloon, rocket, or satellite) experiments above the atmosphere, detection of the background flux 3×10^{-16} watt/cm^2 must be considered beyond present day capabilities.

Although the 28μ background radiation from the galactic disk is rather small, one might consider the possibility of detecting this radiation from nearby regions of the galaxy where star formation happens to be taking place rapidly. A protostar of mass 100 $M_\odot$ attains a 28μ luminosity of $L \sim \eta \sim 100\ L_\odot$ for a short time ($\sim 10^6$ yr). If such a collapsing protostar were at a distance of about 500 pc (roughly the distance to the Orion Nebula), the flux which would be received at the earth would be $\sim 10^{-13}$ watt/cm^2. This flux is on the borderline of detectability. Of course, one would have to be fortunate enough to observe the protostar during its maximum 28μ luminosity.

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TABLE 1

CRITICAL PARAMETERS OF A CONTRACTING PROTOSTAR

$M/M_\odot$	R_c (cm)	T_c (°K)	$\bar{n}_c$ (cm^{-3})	$(t_{ff})_c$ (yr)	R_s (cm)	T_s (°K)
0.1					2.0×10^{16}	5.3
0.3					2.9×10^{16}	11
1.0					4.4×10^{16}	25
3.0	5.8×10^{16}	55	2.1×10^{6}	4.6×10^{5}	6.3×10^{16}	51
10	2.3×10^{17}	47	1.2×10^{5}	1.9×10^{6}	9.4×10^{16}	110
30	7.7×10^{17}	42	9.3×10^{3}	7.0×10^{6}	1.4×10^{17}	240
100	2.9×10^{18}	37	6.0×10^{2}	2.7×10^{7}	2.0×10^{17}	530
300	8.9×10^{18}	36	61	8.6×10^{7}	2.9×10^{17}	1100
1000	2.8×10^{19}	38	6.2	2.7×10^{8}	4.4×10^{17}	(2500)

REFERENCES

Burbidge, G. R., Kahn, F. D., Ebert, R., von Hoerner, S., and Temesváry, St. 1960, Die Entstehung von Sternen, (Berlin: Springer-Verlag).

Cameron, A. G. W. 1962, Icarus, **1**, 13.

Gaustad, J. E. 1963, Ap. J., **138**, 1050.

Gould, R. J. and Salpeter, E. E. 1963, Ap. J., **138**, 393.

Gould, R. J., Gold, T., and Salpeter, E. E. 1963, Ap. J., **138**, 408.

Osterbrock, D. E. 1962, Ap. J., **136**, 359.

Raich, J. C. and Good, R. H., Jr. 1963, Ortho-Para Transition in Molecular Hydrogen (preprint).

Salpeter, E. E. 1959, Ap. J., **129**, 608.

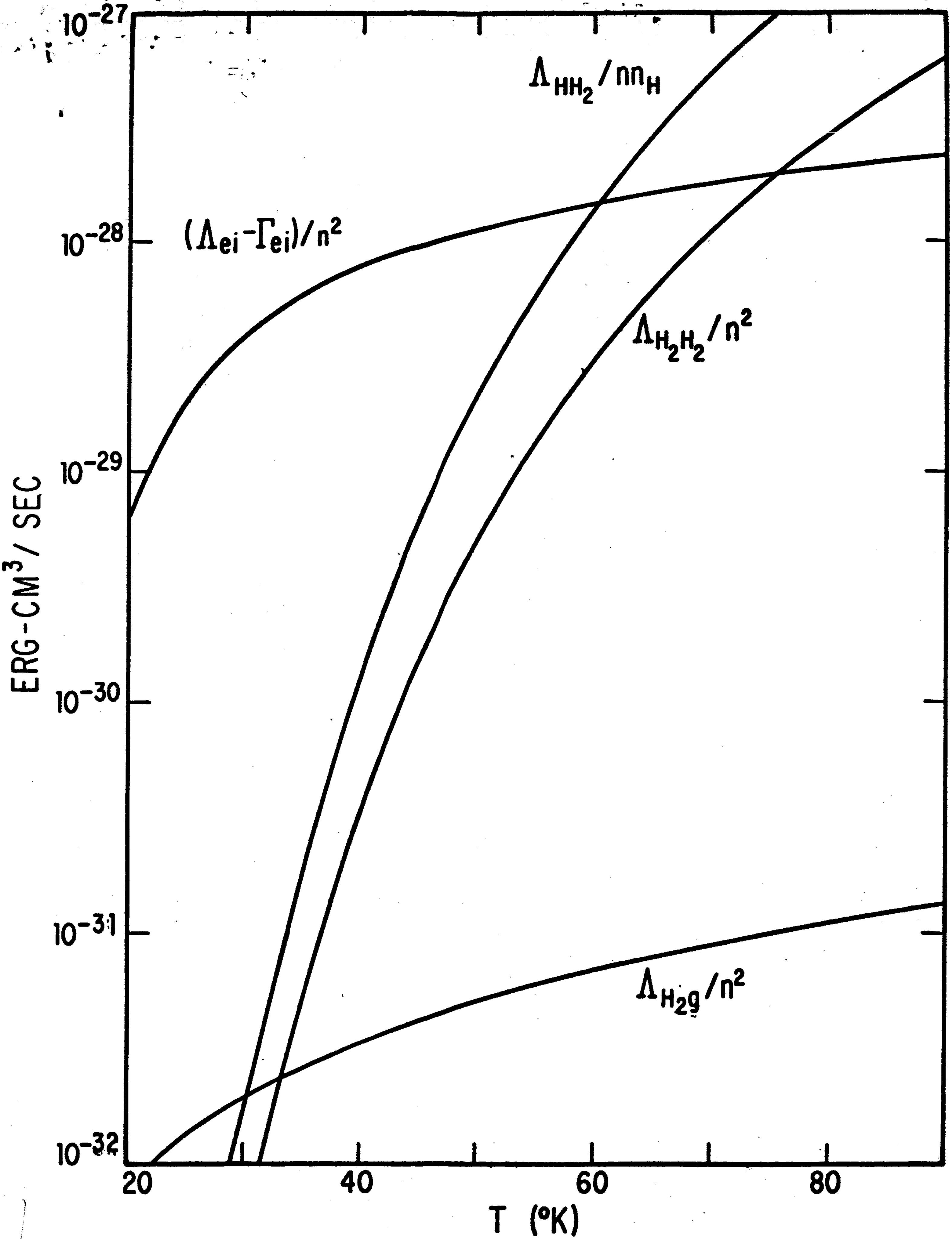
Spitzer, L., Jr. 1949, Ap. J., **109**, 337.

____________ 1954, Ap. J., **120**, 1.

____________ 1963, Origin of the Solar System, R. Jastrow and A. G. W. Cameron, eds. (New York: Academic Press).

FIGURE CAPTION

Fig. 1 Cooling rates in H I regions by various processes in the low density limit (no collisional de-excitation).



10^{-27}
10^{-28}
10^{-29}
10^{-30}
10^{-31}
10^{-32}
ERG-CM^3/ SEC
Λ_{HH_2}/nn_H
$(\Lambda_{ei}-\Gamma_{ei})/n^2$
$\Lambda_{H_2H_2}/n^2$
Λ_{H_2g}/n^2
20
40
60
80
T (°K)