FAILURE CRITERIA FOR VISCOELASTIC MATERIALS

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In the last semi-annual report[1] we presented an outline for the investigation of cumulative damage in viscoelastic materials. We need not repeat here the details of that outline, nor the reasons for following certain lines of investigation, but content ourselves with reporting the progress made in the various aspects of failure investigations. In particular, we shall report on results related to the following areas of fracture

a) Specimen preparation.
b) Monotonic loading histories.
c) Cyclic loading histories.
d) Analytical work.
e) Newly started investigations.

Specimen Preparation

The preparation of tensile specimens is very important in fracture studies, because surface defects introduced in the production process may lead to erroneous results. The most common method of specimen production is the cutting of rings from sheet stock. Because of the cutting operation the circular surfaces often appear uneven and the question arises whether the possible existence of small cracks may cause the material to fail prematurely.

In order to check the effect of surface finish, it is, therefore, necessary to prepare specimens in such a manner that immaculate surfaces are obtained. Specimens were therefore cast against polished surfaces in the mold shown in Figure 1. A cast specimen is shown in Figure 2. The surface finish on the cast specimens is very much smoother than a machine cut surface and free of defects.

For comparison purposes, ring specimens were cut on a milling machine from 0.1 inch thick sheets with a four blade cutter developed at the Jet Propulsion Laboratory. The ring dimensions were: OD = 0.75 inch, ID = 0.62 inch. Each ring was measured individually for data reduction purposes.

Figure 3 shows the comparison of failure data obtained from the ring specimens and from the cast specimens. Note that there is very little difference between the two sets of data. The cast specimens tended to have a slightly lower Young's modulus than the ring specimens and their failure stresses tend to be slightly higher than those for the ring specimens. However, all the data points fall well within the σ_1-limit which was calculated from failure data on the cast specimens*. It should be noted, however, that the variance of the cast specimens appeared larger than for the mechanical specimens. From this comparison it should be quite clear that properly cut ring specimens can give the same information regarding failure behavior as the carefully cast tensile specimens.

Although the cast specimens were significantly more economical than the previously used tensile samples of similar shape, though machined, the ring specimens are most cheaply produced and are now used exclusively and with confidence.

Monotonic Loading Histories

Uniaxial stress state. Drawing on earlier calculations of failure behavior the meaning and uniqueness of the failure envelope has come under closer scrutiny during the past reporting period. As reported in detail in reference 3 it appears that strain rate histories for which the second time derivative of the strain, $d\epsilon^2/dt^2$, is always smaller than or equal to zero should lead to failure as predicted by the envelope. However, if $d^2\epsilon/dt^2$ is greater than zero then failure may not be predicted by the envelope. The reason for this possible behavior is given in detail in reference 3 and reference 4. For the present purpose it suffices to indicate briefly the difference in stress-strain behavior for the two types of monotonic loading histories.

Consider the strain histories shown in Figure 4A, which lead to the same failure strains ϵ_b. If one adjusts the load magnitudes and strain rates such that the stress-strain curves pass all through the same point (σ_b, ϵ_b)** then one obtains the stress-strain curves shown in Figure 4B. Since fracture is presumed to accumulate more strongly the closer fracture is approached,

* Note that only a fraction of the data for the cast specimens has been plotted.

** The details of this calculation are given in references 4 and 5.

we are concerned with the stress-strain behavior near the end point. From Figure 4B it is obvious that in this region the stress-strain behavior is very similar for the strain histories for which $d^2\epsilon/dt^2 < 0$. Only in those cases exemplified by curve (4) and for which $d^2\epsilon/dt^2 > 0$ should one expect different failure behavior. Tests are presently in progress to check the implications of these calculations. The results of these tests should cast light on the uniqueness question regarding the failure envelope.

Fracture energy measurements. The concept of the fracture energy has been used by several investigators in fracture work of polymers[2, 6, 7]. A relatively easy way of measuring the fracture energy is to determine the rate of crack propagation in a biaxially strained strip[2]. There exists, however, the question of whether the quantity measured in this fashion is independent of the sheet geometry. In particular, it needs to be determined whether the measured fracture energy is independent of the sheet thickness. Tests are therefore in progress to measure the fracture energy in sheets of 0.25 and 0.03 inch thickness and compare these measurements with those obtained from 0.1 inch thick sheets. The fracture energy for the latter sheet thickness is given in Figure 5.

As indicated in the figure the data were obtained at four different temperatures. If one attempts to shift the data to account for the rate-temperature correspondence the limited amount of data seem to indicate that the shift factor does not agree too well with that obtained from relaxation data. Such a difference in thermorheological behavior has been previously reported by Mullins[7]. However, more, and more careful tests are indicated to check this possibility. Inasmuch as uniaxial failure data obeys the same thermorheological law as relaxation data a discrepancy for crack propagation in this respect would be surprising but could shed new light on the fracture process.

The problem of fracture initiation has been examined in some detail. The result of this research has been summarized in reference 8.

Cyclic Loading Histories

Temperature effects. In order to arrive at a meaningful comparison of failure data in monotonic and cyclic load histories it is necessary to understand the effect of heat generation. In dealing with the temperature rise during testing one might either choose to correct for it through proper data reduction or to test under a situation which assures nearly constant temperature conditions throughout the test specimen.

The temperature in a heat generating specimen depends on the ratio of the volume to the surface, because the heat generated is proportional to the volume and the heat conducted away is proportional to the surface area. A conservative estimate of the temperature rise may be obtained by calculating the change in temperature in an infinite sheet of thickness h. It can be easily shown[4, 5] that the steady state temperature distribution is

$$\Delta T = \frac{\overline{\dot{D}}}{2k} (h^2 - x^2)$$

where $\overline{\dot{D}}$ = the dissipation rate averaged over one cycle

k = thermal conductivity

x = thickness coordinate.

The maximum temperature rise is therefore

$$\Delta T_{max} = \frac{\overline{\dot{D}} h^2}{2k} .$$

Alternately one may ask, what specimen thickness is required to assure that the maximum heat buildup does not exceed a certain value. It turns out that unless the deformations occur in the near rubbery state the specimen thickness has to be impractically thin[4, 5]. For testing in the transition range one must therefore expect to encounter sizable temperature variations.

An estimate of the temperature change may be obtained by calculating the adiabatic temperature rise. An example of the temperature history is shown in Figure 6 for an initial temperature of -20°C in Solithane 113 (equivoluminal composition) and for a frequency of 60 radians per second. Note, that due to the strong initial dissipation, the temperature rises first rapidly and then as the material heats up and the dissipation decreases, the rate of temperature increase decreases also.

Testing program. At present equipment is near completion which will be used to test rubber under cyclic loading conditions. The equipment will be used in conjunction with the Instron tester and consists primarily of a motor driven cam. The immediate objective is to examine what effect cyclic load histories have on the failure envelope. Testing conditions will first be chosen such as to leave the specimen at a constant temperature.

Analytical Work

One phase of research in fracture mechanics relates to the analytical determination of stress distribution around cracks in various sheet geometries. As pointed out earlier[9] this type of information is important to estimate the effect of cracks in structures weakened by such defects, even if the geometry is not identical to the one considered in the analytical problem.

Following the completion of the analysis for the cracked strip geometry with clamped boundaries[10] one of the remaining problems, namely that of prescribed shear stresses or shear displacements at the strip boundaries, has been solved and partially evaluated numerically. With the shear problems the stresses for the whole class of cracked strip geometries under in-plane loading have been determined.

Similarly, the solution for the penny-shaped crack given by Sneddon[11] has been evaluated and the evaluation of the same geometry for shear loading has been completed. Comparison of these stress fields with their two dimensional approximations has also been made.[12]

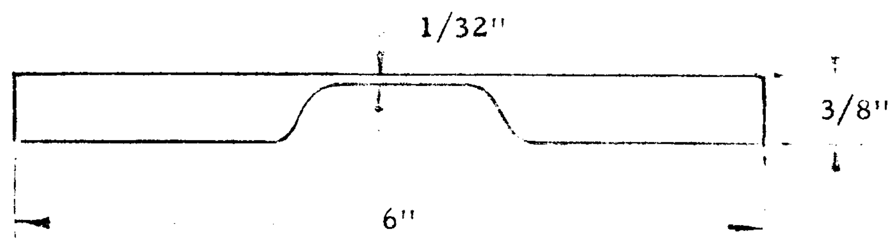
Newly Started Investigations

In an effort to better understand the influence of viscoelasticity on the fracture process a series of experiments are being started which consider fracture properties of polymers in the swollen state. By swelling the rubber it is hoped to internally lubricate the material and thus reduce the effect on internal viscosity of the material.

Initial efforts will be spent on crack propagation studies in the swollen state. The determination of the mechanical properties will follow to the extent necessary. Equipment to strain swollen rubber sheets has been partially constructed and should soon be in operation.

A second effort is directed at the investigation of time dependent fracture in multiaxial stress states. The major difficulty with multiaxial tests has been that failure could seldom be induced at the point where the stress field was known to be uniform. Thus, failure stresses could seldom be determined accurately.

Equipment has been constructed to make failure measurements on equi-biaxially stretched membranes. The test is the well-known bubble test but with a major modification of the test sample. Previously, investigators employed sheet-stock which was clamped along a circular boundary and pressurized from one side. Failure occurred almost invariably along the clamped boundary as a consequence of the high, local bending stresses. In contrast we shall be using a sheet specimen of 3/8 inch thickness, 6 x 6 inches in lateral dimensions and cast (from Solithane) with a central, thin section. A cross-section of the specimen is shown below.



The diameter of the indentation can be made either one or two inches. In preliminary tests the specimen has withstood twice the pressure load sustained by the conventional sheet specimens without rupturing. The experimental set-up for a two-inch indented specimen is shown in Figure 7.

It is intended to perform these tests at various pressurization rates and temperatures to attempt delineating a failure envelope for biaxial failure. A later stage of multiaxial failure would then consider the failure envelope obtained from the triaxial tension test, the so-called poker-chip test.

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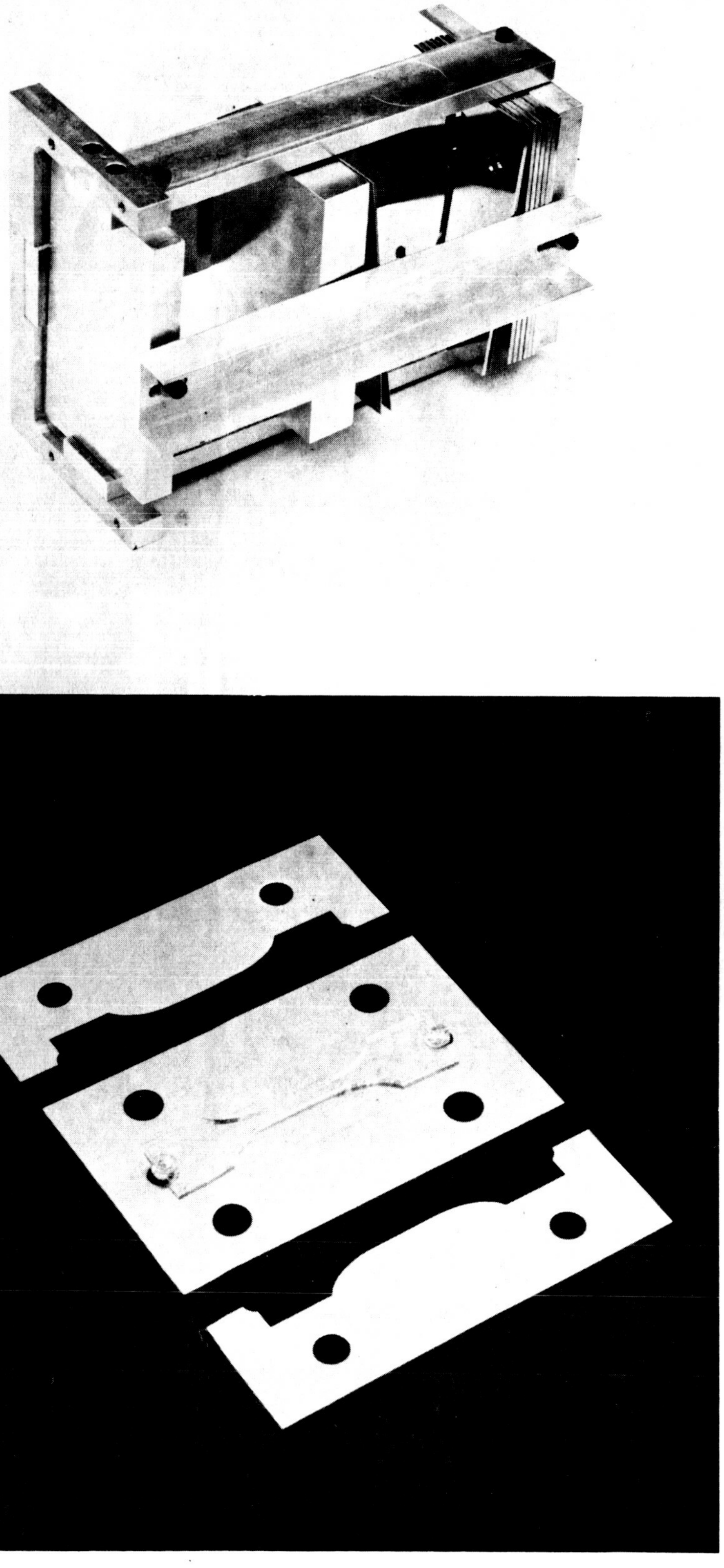


FIG. 1. COMPOSITE VIEW OF SPECIMEN MOLD AND INDIVIDUAL MOLD FORMS

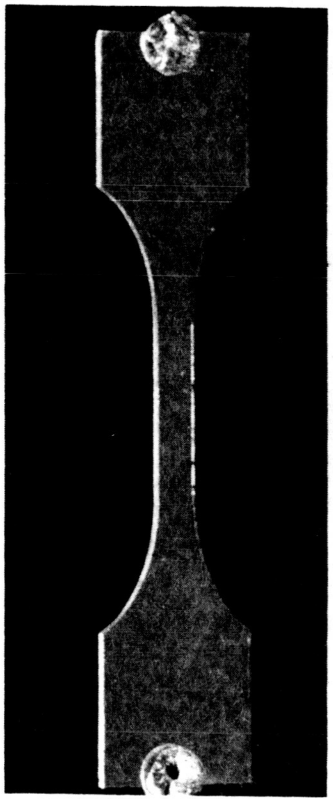


FIG. 2. CAST TENSILE SPECIMEN APPROXIMATELY FULL SCALE

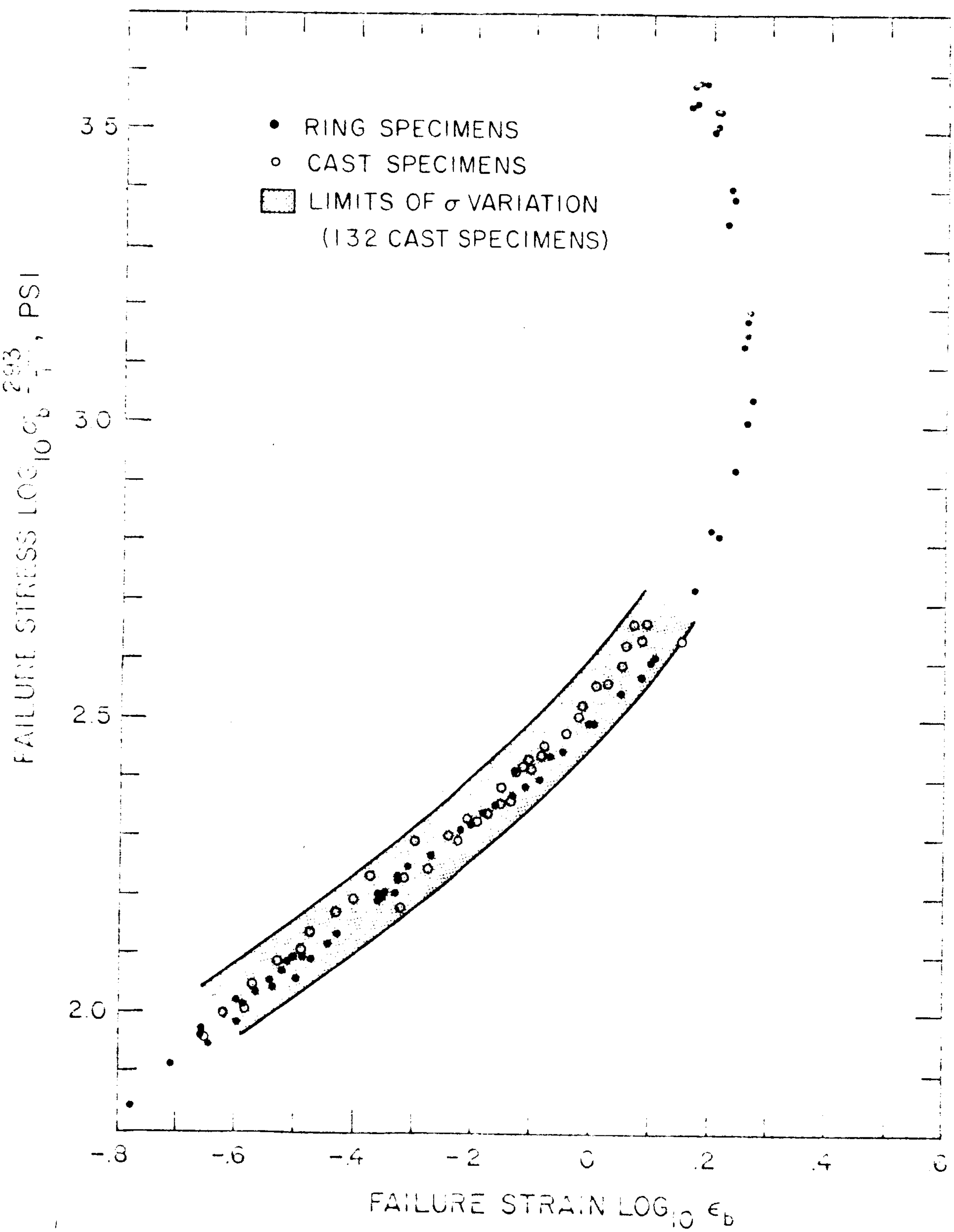
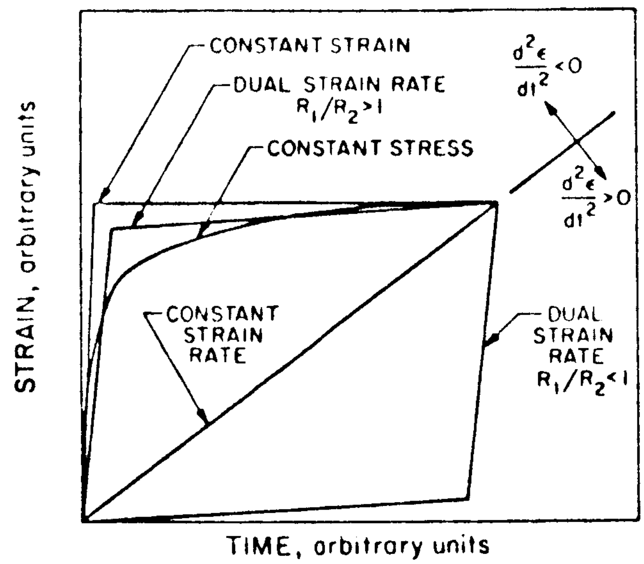
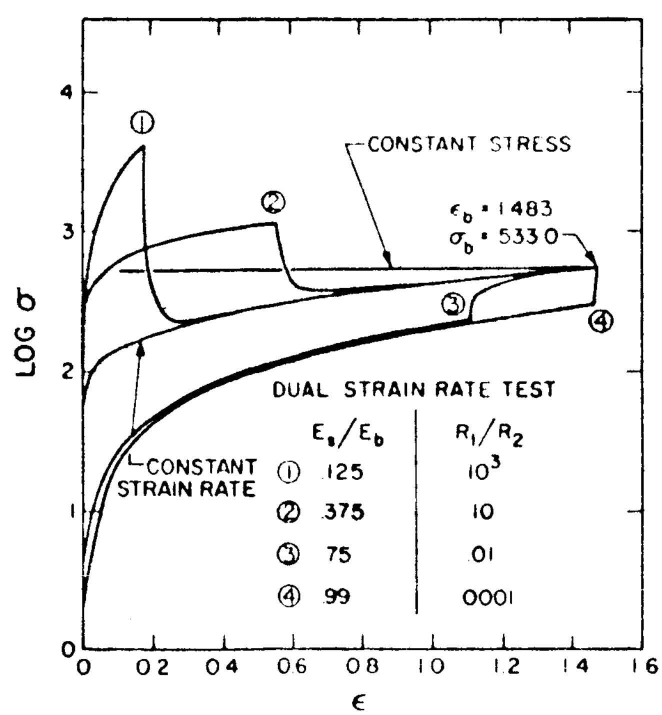


FIG 3 COMPARISON OF FAILURE DATA OBTAINED FROM MACHINED RING AND CAST SPECIMENS



A. STRAIN HISTORIES LEADING TO THE SAME STRAIN



B. STRAIN HISTORIES LEADING TO THE SAME STRESS AND STRAIN ON THE FAILURE ENVELOPE

FIG. 4 SEVERAL STRAIN HISTORIES LEADING TO IDENTICAL VALUES OF STRESS AND STRAIN AT FAILURE (LINEARLY VISCOELASTIC BEHAVIOR OF A POLYURETHANE RUBBER)

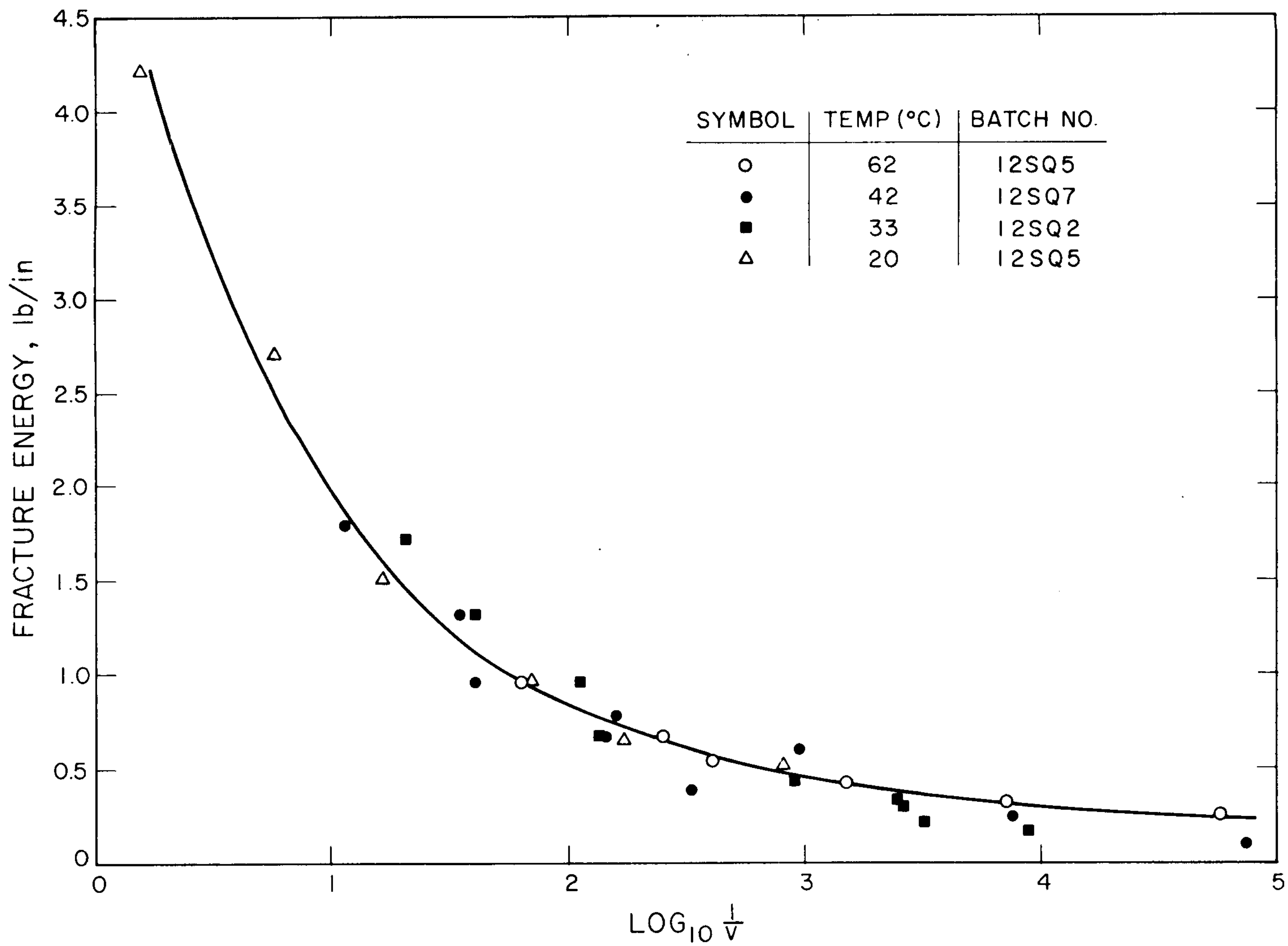


FIG. 5 FRACTURE ENERGY vs CRACK PROPAGATION VELOCITY
(SHEET THICKNESS 0.1 INCH)

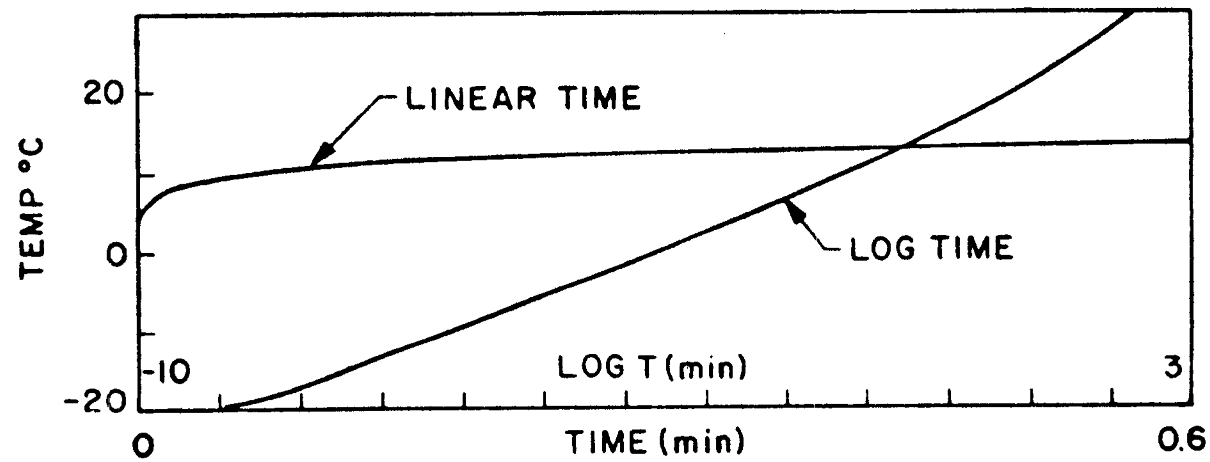


FIG. 6 ADIABATIC TEMP RISE $\epsilon = 0.10 \sin 60t$
$T_0 = -20°C$ $C = 1.8$ in-lb/in^3 °C

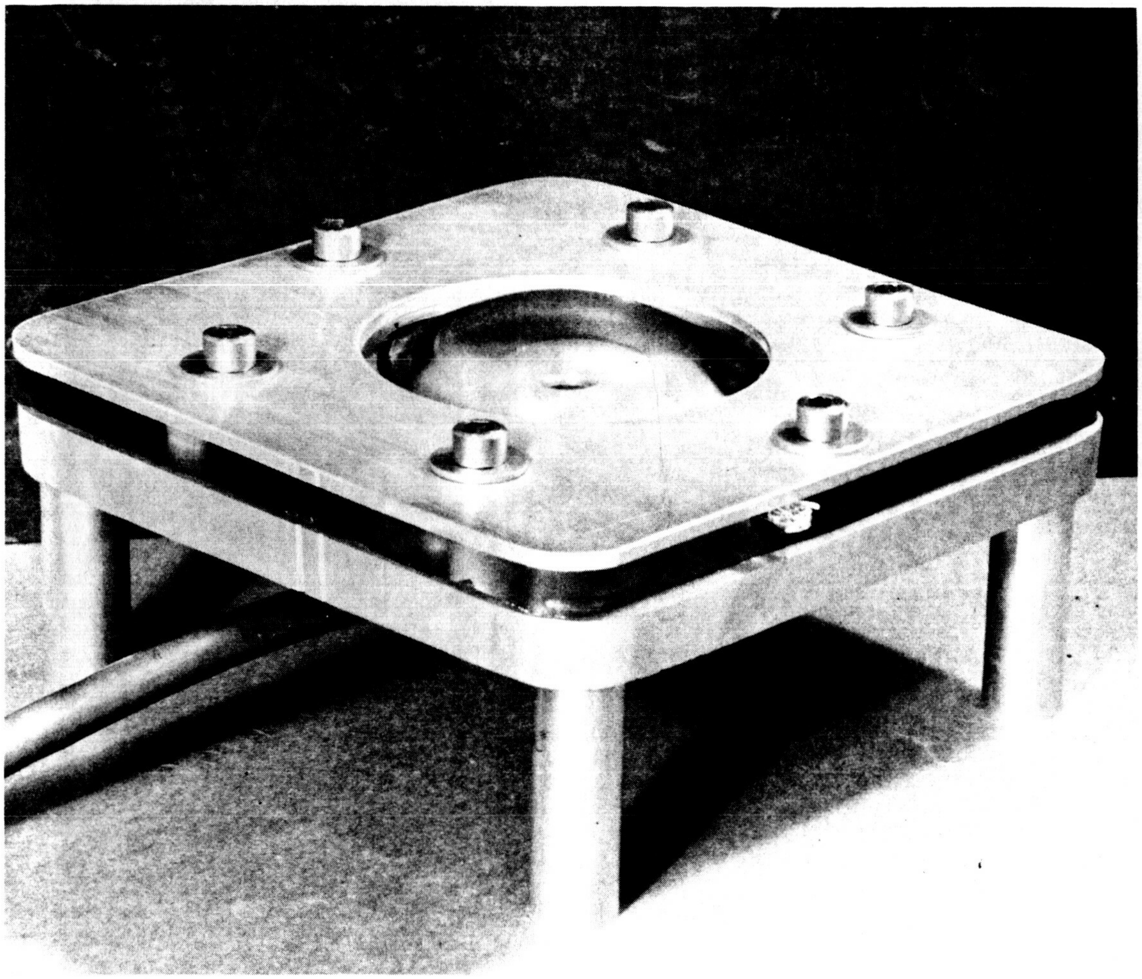


FIG. 7. EXPERIMENTAL SET-UP OF EQUI-BIAXIAL TENSION TEST EMPLOYING A SPECIAL TEST SPECIMEN