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MANNED FLYING SYSTEMS (MFS) CRYOGENIC ENGINE

Prepared Under Contract No. NAS8-20082

F. B. Tatom and L. M. Bhalla

NORTHROP SPACE LABORATORIES
Huntsville, Alabama

For

NASA - GEORGE C. MARSHALL SPACE FLIGHT CENTER
Huntsville, Alabama

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ABSTRACT

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The results are presented of a preliminary investigation to compare the performance of a cryogenic (liquid oxygen and liquid hydrogen) propulsion system with an earth storable (nitrogen tetroxide and a 50:50 mixture of hydrazine and unsymmetrical dimethylhydrazine) propulsion system for the Manned Flying System (MFS). The mass of the propulsion system necessary to meet the mission requirements of the MFS, as outlined in the Apollo Extension System Studies, is used as the basis for comparison. A semi-empirical mathematical model is developed to predict the mass of the propulsion systems. The results, which are presented in both tabular and graphical form, indicate that the cryogenic system offers no apparent advantage over the earth storable system under the present state-of-the-art.

FOREWORD

The investigation described in this report was requested by Mr. Lynn L. Bradford of the Systems Concepts Planning Office, Aero-Astrodynamics Laboratory, George C. Marshall Space Flight Center. The study was carried out by Mr. F. B. Tatom and Mr. L. M. Bhalla of the Huntsville Department of Northrop Space Laboratories, under Contract No. NAS8-20082, Appendix F-1, Schedule Order 4, Technical Directive No. 2. Work commenced on May 3, 1965 and ended on July 19, 1965, with a total of five man weeks expended.

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DEFINITION OF SYMBOLS

A. English Letters

Symbol	Definition	Units
C	General structure factor	
C_{fw}	Thrust-to-weight ratio	
C_{pf}	Structure factor for propellant fuel	
C_{po}	Structure factor for propellant oxidizer	
C_{sf}	Structure factor for storage of propellant fuel	
C_{so}	Structure factor for storage of propellant oxidizer	
c_p	Heat capacity	Btu/lb_m°F
F	Thrust produced by one engine	lb_f
g	Acceleration of gravity on earth's surface	ft/sec^2
h_c	Latent heat generated by change of phase	Btu/lb_m
I_{sp}	Specific impulse of propellant	sec
k_{in}	Thermal conductivity of insulation	Btu/ft°F hr
m_a	Mass of accessory equipment	lb_m
m_c	Mass of fuel or oxidizer subjected to a change of phase	lb_m
m_e	Mass of the rocket engines	lb_m
m_f	Mass of the fuel	lb_m
m_{in}	Mass of insulation	lb_m
m_o	Mass of oxidizer	lb_m
m_{ps}	Mass of propellant system (aboard MFS)	lb_m
m_{ss}	Mass of the storage system (including propellant)	lb_m

DEFINITION OF SYMBOLS

A. English Letters (Cont'd)

Symbol	Definition	Units
m_s	Mass of the shell	lb_m
m_t	Total mass of system	lb_m
m_x	Total mass of fuel or oxidizer within container	lb_m
$\dot{m}_f$	Mass flow rate of the fuel for each engine	lb_m/sec
$\dot{m}_o$	Mass flow rate of the oxidizer for each engine	lb_m/sec
$\dot{m}_p$	Mass flow rate of propellant for each engine	lb_m/sec
n_e	Number of engines	
n_m	Number of missions	
P_p	Design pressure for a propellant container	psia
P_{pmax}	Maximum design MFS propellant tank pressure	psia
P_{pmin}	Minimum design MFS propellant tank pressure	psia
P_s	Propellant storage pressure	psia
Q_s	Total heat flow from the shell	Btu
q	Rate of heat flow from the shell	Btu/hr
r_i	Inner radius of insulation	ft
r_o	Outer radius of insulation	ft
r_s	Radius of the shell	ft
T_c	Temperature for change of phase	°R
T_{fr}	Fusion temperature of propellant	°R
T_i	Temperature of propellant or inside insulation	°R
T_{io}	Initial storage temperature	°R

DEFINITION OF SYMBOLS

A. English Letters (Cont'd)

Symbol	Definition	Units
T_o	Outside temperature of insulation	°R
T_{omax}	Maximum outer surface temperature of propellant container	°R
T_{omin}	Minimum outer surface temperature of propellant container	°R
T_{sat}	Saturation temperature of propellant	°R
t	Shell thickness	ft
x	Mixture ratio of oxidizer to fuel	

B. Greek Letters

Symbol	Definition	Units
ΔT	Temperature difference across insulation	°F
$\overline{\Delta T}$	Mean temperature difference across insulation as defined by Eq. (B-8)	°F
η	Safety factor in pressure vessel design	
λ	Thermodynamic parameter defined by Eq. (C-7)	hr^{-1}
ρ_{in}	Density of the insulation	lb_m/ft^3
ρ_s	Density of shell material	lb_m/ft^3
ρ_x	Density of fuel or oxidizer	lb_m/ft^3
σ_s	Tensile strength of shell material	psia
τ	Time	(days, hr or sec)
τ_b	Engine burning time during a single mission	sec
τ_m	Mission time	hr
τ_s	General storage time	hr
τ_{ss}	Propellant long-term storage time	days

SUMMARY

An analytical investigation has been conducted to determine whether or not the use of a cryogenic (liquid oxygen and liquid hydrogen) propulsion system for the Manned Flying System (MFS) offers any advantages over the earth storable (nitrogen tetroxide and a 50:50 mixture of hydrazine and unsymmetrical dimethylhydrazine) system presently under consideration. The mass of the necessary equipment associated with each propulsion system is used as a basis for comparison. This mass includes not only that equipment aboard the MFS but also the propellant storage system which is contained within the LEM/A, LEM/D, and/or LEM/S stages.

A semi-empirical mathematical model is developed to permit calculation or prediction of the total mass of the propulsion systems. This model utilized certain dimensionless parameters including thrust-to-weight ratios and structure factors. The latter are mass parameters which express the ratio of structural mass to propellant mass. A method of evaluating these structure factors is provided.

By means of the mathematical model developed, the mass of the cryogenic propulsion systems is calculated along with the mass of the earth storable system. Such calculations reveal that the cryogenic system appears to be heavier than the earth storable system under the same mission requirements. The conclusion is reached that there is no apparent justification for using the cryogenic system in place of the earth storable system.

1.0 INTRODUCTION

As part of the Apollo Extension System (AES) program, a Manned Flying System (MFS) will be used to provide rapid transportation from point to point on the lunar surface. The operational characteristics of this craft have been described in previous research efforts (refs. 1, 2, and 3). As indicated in these references, present plans call for a propulsion system consisting of five 100 lb_f thrust, throttable rocket engines. The propellants for these engines would be a 50:50 mixture by weight of hydrazine (N_2H_4) and unsymmetrical dimethylhydrazine (UDMH) for fuel, and nitrogen tetroxide (N_2O_4) for oxidizer. These propellants, commonly referred to as "earth storables", are also used in the descent and ascent stages of the LEM vehicle. Considerable thought has been given to possible arrangements whereby the propellants required for the MFS might be supplied by using the residual propellant from the LEM/D (descent stage of LEM) and/or by using a portion of the propellants from the LEM/A (ascent stage of LEM).

Because of their greater specific impulse, liquid hydrogen and liquid oxygen (cryogenic propellants) appear to be promising alternatives to the earth storable propellants. There is the possibility that the auxiliary power supply for the LEM/A and the LEM/S (shelter stage of LEM) will involve fuel cells using liquid hydrogen and liquid oxygen (refs. 4, 5, 6, and 7). By means of a common storage system, these cryogenic propellants might be used both to provide auxiliary power for the LEM/S and LEM/A, and to propel the MFS. Because of the cryogenic temperature involved, however, such an arrangement would appear to require more elaborate facilities than would be necessary for the earth storables. This report presents the results of a preliminary study by the Huntsville Department of Northrop Space Laboratories concerning the relative merits of the two propulsion systems.

2.0 TECHNICAL DISCUSSION

2.1 Background

In order to obtain a more up-to-date understanding of the two propulsion systems under consideration, a short literature search was conducted. The performance and overall characteristics of both cryogenic and earth storable propulsion systems for thrusts ranging from 100,000 to 1,000,000 lb_f are relatively well established. In the low thrust range from 100 to 500 lb_f , however, especially for cryogenic systems, a scarcity of published material exists. For this reason, letters were written to seven aerospace companies, prominent in cryogenic propulsion, requesting information regarding their most recent work in the area of low-thrust cryogenic propulsion. A sample of these letters is provided in Appendix A, along with the responses obtained. Although limited information was available concerning low-thrust earth storable propulsion systems, several reports (refs. 2 and 3) were obtained which were both detailed and pertinent. Thus, written inquiries to the appropriate aerospace firm for this case were unnecessary.

Because of the relatively short time period involved in the study, the investigation was carried out for the most part before the answers to the written inquiries were received. Thus, certain assumptions which were made initially out of necessity, regarding propulsion system performance, may differ to some extent from the actual data provided in the responses contained in Appendix A. Such differences, however, do not significantly affect the overall validity of the analysis.

2.2 Basis for Comparison

There are numerous parameters upon which the performance of a propulsion system can be judged. In the problem under consideration, a number of these parameters have already been fixed either by the mission requirements or by the system interface requirements as discussed in reference 1. These include:

Propellant long-term storage time (τ_{ss}) = 180 days

Engine thrust (F) = 100 lb_f

Number of engines (n_e) = 5.

Certain other quantities while not truly constant can or must be treated as such for simplicity. These quantities are presented in Table 2-1. In drawing conclusions from this study, consideration must be given to the assignment of constant values to the parameters indicated in this table.

The most logical parameter upon which to compare the performance would appear to be the total mass involved in the operation of the propulsion system. This mass would take into account, not only the mass of the rocket engines and of the propellant storage and feed system aboard the MFS, but also the mass of the MFS propellant storage system located within the LEM/A, LEM/D, and/or LEM/S stages. That system with the least mass which can meet the operational requirements

TABLE 2-1

Values of Important Propulsion Parameters
(Assumed to be constants)

Parameter	Value
Number of missions (n_m)	3
Mission time (τ_m)	3 hours
Minimum design MFS propellant tank pressure (P_{pmin})	150 psia
Maximum design MFS propellant tank pressure (P_{pmax})	1000 psia
Cryogenic propellant storage pressure (P_s)	50 psia
Earth storable propellant storage pressure (P_s)	14.7 psia
Structure safety factor (η)	1.67
Maximum outer surface temperature of propellant container (T_{omax})	600°R
Minimum outer surface temperature of propellant container (T_{omin})	200°R
Earth storable propellant specific impulse (I_{sp})	300 sec
Cryogenic propellant specific impulse (I_{sp})	400 sec
Earth storable propellant mixture ratio (x)	2.0
Cryogenic propellant mixture ratio (x)	5.0

specified by the constant parameters already listed, represents the optimum propulsion system.

2.3 Mathematical Model Development

Based on the discussion presented in the preceding subsections, the need arises for developing a mathematical relationship which will permit calculation of the total system mass. In the most primitive form, the total system mass, m_t, can be expressed as

$$m_t = m_e + m_{ps} + m_{ss} \tag{1}$$

where

m_e = mass of the rocket engines (lb_m)

m_{ps} = mass of the propellant system (aboard MFS) (lb_m)

m_{ss} = mass of the storage system (including the propellant) (lb_m).

Notice should be taken that the mass described by Eq. (1) does not include certain equipment, such as the pressurization system and the attitude rocket assembly. These items were not included because, for low thrust levels, the associated masses would remain essentially constant regardless of the type of propellant.

An exact analytical prediction of m_e, the mass of the rocket engines, would be exceedingly complex due to the nature of the technologies involved. A reasonably accurate value based on the ratio of thrust-to-weight (C_{fw}) can be obtained empirically. Based on the definition of this ratio,

$$m_e = n_e \frac{F}{g\, C_{fw}} \tag{2}$$

where

n_e = number of rocket engines

F = engine thrust (lb_f)

g = acceleration of gravity on earth's surface (ft/sec^2).

The factor C_{fw} is primarily a function of the nature of the propellants and the magnitude of the thrust. Figure 2-1 represents the approximate variation of C_{fw} with thrust for both cryogenic and earth storable propellants. For a thrust of 100 lb_f Figure 2-1 indicates that the appropriate thrust-to-weight ratios are

$C_{fw} = 14$ (earth storables)

$C_{fw} = 42$ (cryogenics).

Notice should be taken that, due to the lack of existing data for low-thrust cryogenic systems, considerable extrapolation is necessary in order to obtain a value of C_{fw} for cryogenic systems with thrusts in the range of 100 to 500 lb_f.

The mass of the propellant systems, m_{ps}, includes all equipment aboard the MFS which is associated with the propulsion system other than the rocket engines. The mass consists primarily of the propellant feed tanks and does not include the mass of the propellants themselves. As already noted, the pressurization system mass and attitude rocket

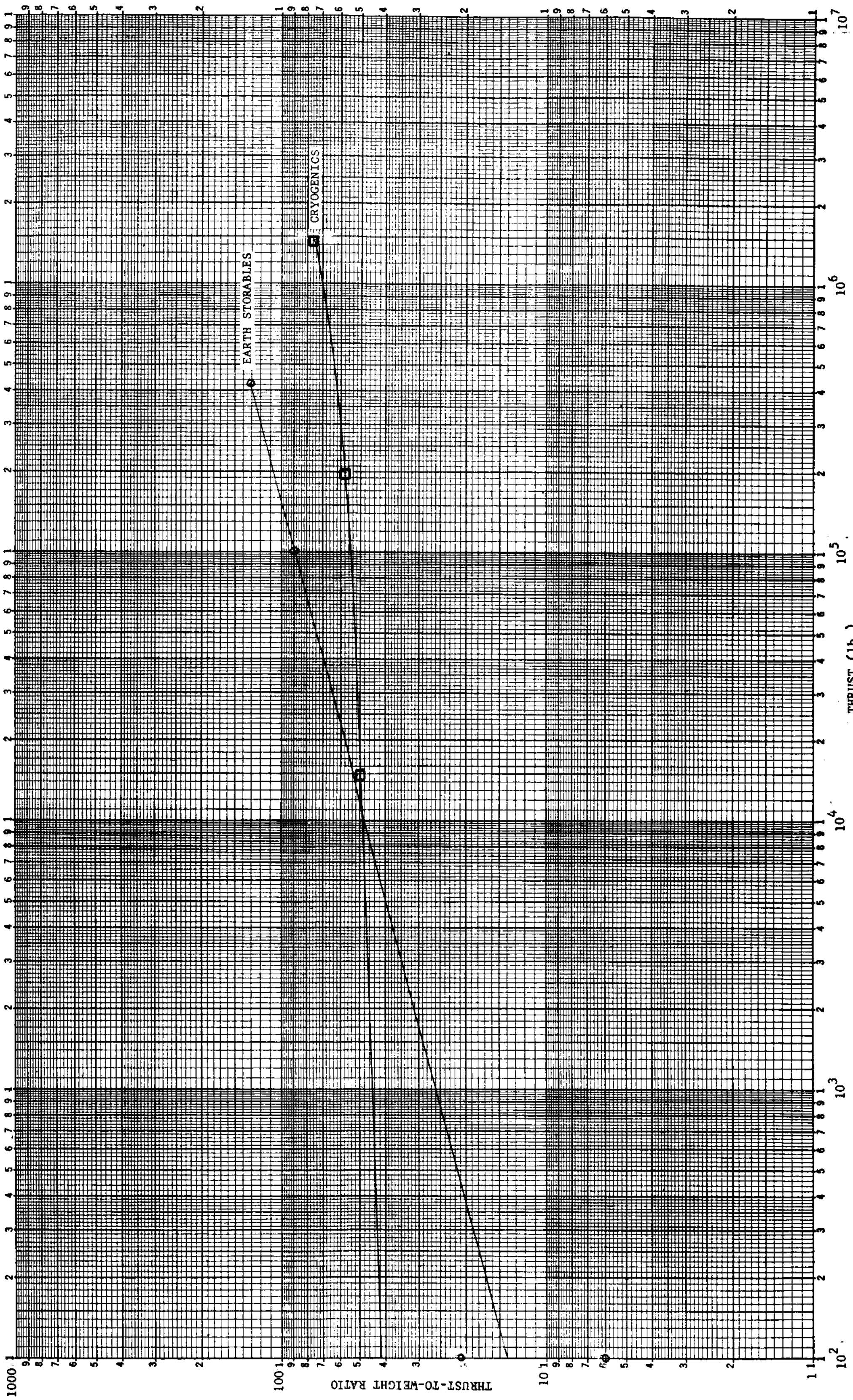

FIGURE 2-1 VARIATION OF THRUST-TO-WEIGHT RATIO WITH THRUST

assembly mass are constants which can be neglected in the present analysis. For convenience, the mass m_{ps} can be expressed in the form

$$m_{ps} = (C_{pf}-1)\, m_f + (C_{po}-1)\, m_o \tag{3}$$

where

C_{pf} = Propellant system fuel structure factor (m_{pf}/m_f)

m_f = Mass of fuel required for a single mission (lb_m)

m_{pf} = Mass associated with the fuel system including fuel (lb_m)

C_{po} = Propellant system oxidizer structure factor (m_{po}/m_o)

m_o = Mass of oxidizer required for a single mission (lb_m)

m_{po} = Mass associated with the oxidizer system including the oxidizer (lb_m).

Now the masses m_f and m_o can be expressed as

$$m_f = n_e \int_0^{\tau_b} \dot{m}_f \, d\tau \tag{4}$$

$$m_o = n_e \int_0^{\tau_b} \dot{m}_o \, d\tau \tag{5}$$

where

τ = time (sec)

τ_b = burning time for a single mission (sec)

$\dot{m}_f$ = mass flow rate of fuel for each engine (lb_m/sec)

$\dot{m}_o$ = mass flow rate of oxidizer for each engine (lb_m/sec).

A combination of Eqs. (3), (4), and (5) yields

$$m_{ps} = n_e \{ (C_{pf}-1) \int_o^{b} \dot{m}_f d + (C_{po}-1) \int_o^{b} \dot{m}_o d \} \tag{6}$$

By definition

$$\dot{m}_p = \dot{m}_o + \dot{m}_f \tag{7}$$

and

$$x = \dot{m}_o / \dot{m}_f \tag{8}$$

where

$\dot{m}_p$ = mass flow rate of propellant (lb_m/sec)

x = mixture ratio.

Then

$$\dot{m}_o = \dot{m}_p \frac{x}{x+1} \tag{9}$$

and

$$\dot{m}_f = \dot{m}_p \frac{1}{x+1} \tag{10}$$

Thus by substitution,

$$m_{ps} = n_e \left[(C_{pf}-1) \int_o^{b} \frac{\dot{m}_p}{x+1} d + x(C_{po}-1) \int_o^{b} \frac{\dot{m}_p d}{x+1} \right]$$

$$= \frac{n_e}{x+1} \left[(C_{pf}-1) + x(C_{po}-1) \right] \int_o^{b} \dot{m}_p d \tag{11}$$

The mass of the storage system m_{ss}, includes the mass of all fuel and oxidizer necessary to perform "n_m" missions, where for the case under consideration, as shown in Table 2-1,

$$n_m = 3.$$

This mass can be expressed in a manner similar to that developed for m_{ps}. The resulting expression is

$$m_{ss} = n_m \cdot n_e \left(C_{sf} \int_o^{\tau_b} \dot{m}_f d\tau + C_{so} \int_o^{\tau_b} \dot{m}_o d\tau\right)$$

$$= \frac{n_m \cdot n_e}{x+1} \left(C_{sf} \int_o^{\tau_b} \dot{m}_p d\tau + x\, C_{so} \int_o^{\tau_b} \dot{m}_p d\tau\right)$$

$$= \frac{n_m \cdot n_e}{x+1} \left(C_{sf} + x\, C_{so}\right) \int_o^{\tau_b} \dot{m}_p d\tau \qquad (12)$$

where

C_{sf} = storage system fuel structure factor $\dfrac{m_{sf}}{n_m m_f}$

m_{sf} = mass of fuel storage system including mass of fuel itself (lb_m)

C_{so} = storage system oxidizer structure factor $m_{so}/n_m m_o$

m_{so} = mass of oxidizer storage system including mass of oxidizer itself (lb_m).

The total mass, m_t, can be expressed as a combination of Eqs. (1), (2), (11), and (12), yielding,

$$m_t = \frac{n_e F}{g\, C_{fw}} + \frac{n_e}{x+1} \left[(C_{pf}-1) + x(C_{po}-1)\right] \int_o^{\tau_b} \dot{m}_p d\tau$$

$$+ \frac{n_m \cdot n_e}{x+1} \left(C_{sf} + x\, C_{so}\right) \int_o^{\tau_b} \dot{m}_p d\tau$$

$$= n_e \left\{ \frac{F}{g\, C_{fw}} + \left(\frac{1}{x+1}\right) \left[(C_{pf} - 1 + n_m C_{sf}) + x(C_{po} - 1 + n_m C_{so})\right] \int_o^{\tau_b} \dot{m}_p d\tau \right\} \qquad (13)$$

Eq. (13) represents the mathematical model developed and used in the analysis presented in this report. In using this equation, the integral term, $n_e \int_o^{\tau_b} \dot{m}_p d\tau$, must be carefully evaluated, and appropriate values of the structure factors, C_{pf}, C_{po}, C_{sf}, and C_{so}, must be selected.

2.4 Evaluation of Propellant Consumption

The integral, $n_e \int_o^{\tau_b} \dot{m}_p d\tau$, represents the amount of propellant burned during a single mission which requires the engines to be operating for a burning time of τ_b. A previous study (ref. 1) has shown that for the MFS a trajectory consisting of a vertical ascent phase, a horizontal leg, and a vertical descent phase is more desirable than a ballistic trajectory. Accordingly, the former type of trajectory has been used as a basis for comparison in the present investigation. Based on Table 2-1 from reference 1, for an altitude of 1,300 feet and a coast velocity of 900 ft/sec, the propellant consumption for the earth storable system is as follows:

$$\left[n_e \int_o^{\tau_b} \dot{m}_p d\tau \right]_{\text{earth storable}} = 484.74 \text{ lb}_m .$$

The mixture ratio for earth storables from Table 2-1 is

$$x = 2.$$

Thus by the definition of mixture ratio,

$$\left. \begin{aligned} m_f &= 161.58 \text{ lb}_m \\ m_o &= 323.16 \text{ lb}_m \end{aligned} \right\} \quad \text{for earth storables.}$$

Exact evaluation of cryogenic propellant consumption can only be achieved by performing a dynamic analysis of the MFS (along the trajectory already described) in the same manner as that carried out in reference 1. Due to the shortage of time such an analysis was not feasible. Instead, as indicated in Section 2.2 and Table 2-1 of this report, the thrust and burning time for each leg of the trajectory were assumed to be the same for the cryogenics as for the earth storables. With this assumption,

$$\left[\int_o^{\tau_b} F d\tau\right]_{\substack{\text{earth}\\\text{storables}}} = \left[\int_o^{\tau_b} F d\tau\right]_{\text{cryogenics}} \tag{14}$$

Now by definition of specific impulse

$$\frac{1}{gI_{sp}} \int_o^{\tau_b} F d\tau = \int_o^{\tau_b} \dot{m}_p d\tau \tag{15}$$

Thus

$$\left[n_e \int_o^{\tau_b} \dot{m}_p d\tau\right]_{\text{cryogenics}} = \left[n_e \int_o^{\tau_b} \dot{m}_p d\tau\right]_{\substack{\text{earth}\\\text{storables}}} \left[\frac{I_{sp}\ (\text{earth storables})}{I_{sp}\ (\text{cryogenics})}\right] \tag{16}$$

Then

$$\left[n_e \int_o^{\tau_b} \dot{m}_p d\tau\right]_{\text{cryogenic}} = 484.74 \cdot \frac{300}{400}$$

$$= 363.54\ lb_m\ .$$

The mixture ratio for liquid oxygen and liquid hydrogen from Table 2-1 is

$$x = 5.$$

Then as before,

$$\left.\begin{aligned} m_f &= 60.59 \ lb_m \\ m_o &= 302.95 \ lb_m \end{aligned}\right\} \quad \text{for cryogenics.}$$

It is important to note that the assumption of the cryogenic burning time and thrust being equal to their counterparts in the earth storable system permits a comparison on the basis of burning time. However, with its greater specific impulse, the cryogenic system should travel further than the earth storable system during the same time period.

2.5 Evaluation of Structure Factors

The definitions of the structure factors are deceptively simple. These factors are not constants but are functions of a number of variables including:

- o storage time
- o storage environment
- o propellant mass
- o propellant density
- o propellant specific heat
- o propellant latent heat of fusion
- o propellant latent heat of vaporization
- o propellant storage temperature
- o propellant storage pressure
- o propellant tank shape

- o propellant tank material density
- o propellant tank material tensile strength
- o insulation density
- o insulation thermal conductivity.

For spherical tanks the general relationship for a structure factor, C, is

$$C = \left(1.0 + \frac{m_c}{m_x}\right)\left\{1.0 + \frac{\rho_s \eta P_p}{2\rho_x \sigma_s}\left[3 + \frac{3\eta P_p}{2\sigma_s} + \left(\frac{\eta P_p}{2\sigma_s}\right)^2\right] + \frac{\rho_{in}}{\rho_x}\left(1 + \frac{\eta P_p}{2\sigma_s}\right)^3 \left[\frac{Q^3}{\left[Q - 4\pi\left(\frac{3}{4\pi}\right)^{1/3}\left(\frac{m_x + m_c}{\rho_x}\right)^{1/3}\left(1 + \frac{\eta P_p}{2\sigma_s}\right) k_{in}\overline{\Delta T}\,\tau_s\right]^3} - 1\right]\right\} \quad \text{(B-13)}$$

with

$$Q = (m_x + m_c)\, c_p\, (T_{io} - T_i) + m_c h_c \quad (T_{fr} \le T_i \le T_{sat}) \quad \text{(B-12)}$$

where

m_c - mass of fuel or oxidizer subjected to a change of phase (lb_m)

m_x - total mass of fuel or oxidizer (lb_m)

ρ_x - density of fuel or oxidizer (lb_m/ft^3)

ρ_s - density of shell material (lb_m/ft^3)

η - safety factor

P_p - design pressure for a propellant container (psia)

σ_s - tensile strength of shell material (psi)

ρ_{in} - density of the insulation (lb_m/ft^3)

q - rate of heat flow from the shell (Btu/hr)

k_{in} - thermal conductivity of the insulation (Btu / ft°F hr)

$\overline{\Delta T}$ - mean temperature difference between inner and outer insulation temperature (°F) as defined by Eq. (B-8)

τ_s - storage time (hr)

c_p - heat capacity (Btu/lb_m °F)

T_{io} - initial storage temperature (°R)

T_i - temperature of propellant or inner surface of insulation (°R)

T_c - temperature for change of phase (°R)

h_c - latent heat generated by change of phase (Btu/lb_m)

T_{fr} - fusion temperature of propellant (°R)

T_{sat} - saturation temperature of propellant (°R).

The development of Eqs. (B-12) and (B-13) is provided in Appendix B.

The manner in which the structure factors are defined obviously requires that the minimum possible value for any such factor is 1.0. Previous studies (refs. 7 and 8) have indicated that for an 180-day period the storage system structure factor for cryogenics would be

$$2.2 \leq C_{sf} \leq 5.35 \text{ (for liquid hydrogen)} \tag{17}$$

$$C_{so} = 1.11 \text{ (for liquid oxygen)} \tag{18}$$

Because of the variation of the lunar environment during a period of 180 days, actual calculation of the factors C_{so} and C_{sf} for the earth storables using Eq. (B-13) is quite involved. Due to the physical characteristics of the earth storables, however, as shown in Table 2-2, the associated structure factors should be no greater than that for

TABLE 2-2

Physical Characteristics of Earth Storable and Cryogenic Propellants

Earth Storable Propellants

Propellant	Fusion Temperature (°R)	Saturation Temperature at 14.7 psia (°R)	Density (lb_m/ft^3)	Heat Capacity (Btu/lb_m °F)
Nitrogen tetroxide (N_2O_4)	472	530	93	0.084
50:50 mixture of Hydrazine + UDMH	480	618	56.2	0.4503
Hydrazine (N_2H_4)	495	696	63.0	0.3552
UDMH	289	606	49.18	0.6285

Cryogenic Propellants

Propellant	Saturation Temperature at 50 psia (°R)	Density (lb_m/ft^3)	Latent heat of Vaporization (Btu/lb_m)
Liquid Hydrogen	46	4.0	180
Liquid Oxygen	188	68.0	430

liquid oxygen. The assignment of a value of 1.11 for the C_{so} and C_{sf} for the earth storables thus appears reasonable.

The propellant system structure factors C_{po} and C_{ps} involve only a short-term storage period of three hours, which is the duration of one mission. During this time period, variation of the lunar environment is negligible. Thus actual calculations of these structure factors by means of Eq. (B-13) in Appendix B are feasible for both the earth storables and the cryogenics. Because both a maximum and minimum design MFS propellant tank pressure must be considered as given in Table 2-1, two separate values of each structure factor, C_{po} and C_{pf}, will result.

In carrying out the calculations for C_{po} and C_{pf}, the necessity arises to select specific materials for use in the storage facilities under consideration. Accordingly, an analysis was made of the most desirable materials for such use. Based on this analysis, titanium was selected for the shell material and SI-91 was chosen for the insulation. The pertinent physical characteristics of these materials are found in Table 2-3.

Notice should be taken that cryogenic propellants are normally stored at their saturation temperature. Thus boil-off must be taken into account. Because of the form of Eq. (B-13), an optimization process is involved when boil-off occurs. An example of the necessary calculations is given in Appendix B. Obviously the minimum value of

TABLE 2-3

Physical Characteristics of Titanium
and SI-91 Insulation

Titanium

density (ρ_s) - 281 lb_m/ft^3

tensile strength (σ_s) - 80,000 psi

SI-91 Insulation

density (ρ_{in}) - 7.8 lb_m/ft^3

thermal conductivity (k_{in}) - 1×10^{-5} Btu/hr ft °R the structure factor is the desired value. Figures 2-2 and 2-3 present the manner in which the values of C_{po} and C_{pf} for the cryogenic propellant vary with boil-off. As indicated by these figures

$$\left.\begin{array}{lll} C_{pf} = 3.373 & (P_{pf} = 1{,}000 \text{ psia}) \\ C_{pf} = 1.373 & (P_{pf} = 150 \text{ psia}) \\ C_{po} = 1.136 & (P_{po} = 1{,}000 \text{ psia}) \\ C_{po} = 1.020 & (P_{po} = 150 \text{ psia}) \end{array}\right\} \begin{array}{l}\text{Cryogenic} \\ \text{Propellant}\end{array}$$

When boil-off does not occur, as is generally the case with earth storables, evaluation of the propellant system structure factors involves selection of the optimum storage temperature in the propellant tanks. A comparison of the maximum and minimum lunar temperatures provided in Table 2-1, with the boiling and freezing points of the earth storable propellants, as presented in Table 2-2, indicates that the possibility of these propellants freezing as well as boiling must be guarded against. The true thermal problem involved is transient in nature and its exact solution is too complex to attempt in this preliminary investigation. Instead, the approximate analysis presented in Appendix C may be used. Based on this analysis the optimum propellant temperatures, T_{io} , for 3-hour time periods are

$$T_{io} \ (N_2O_4) = 518°R$$

$$T_{io} \ (N_2H_4\text{+UDMH}) = 618°R \quad \text{(Saturation temperature).}$$

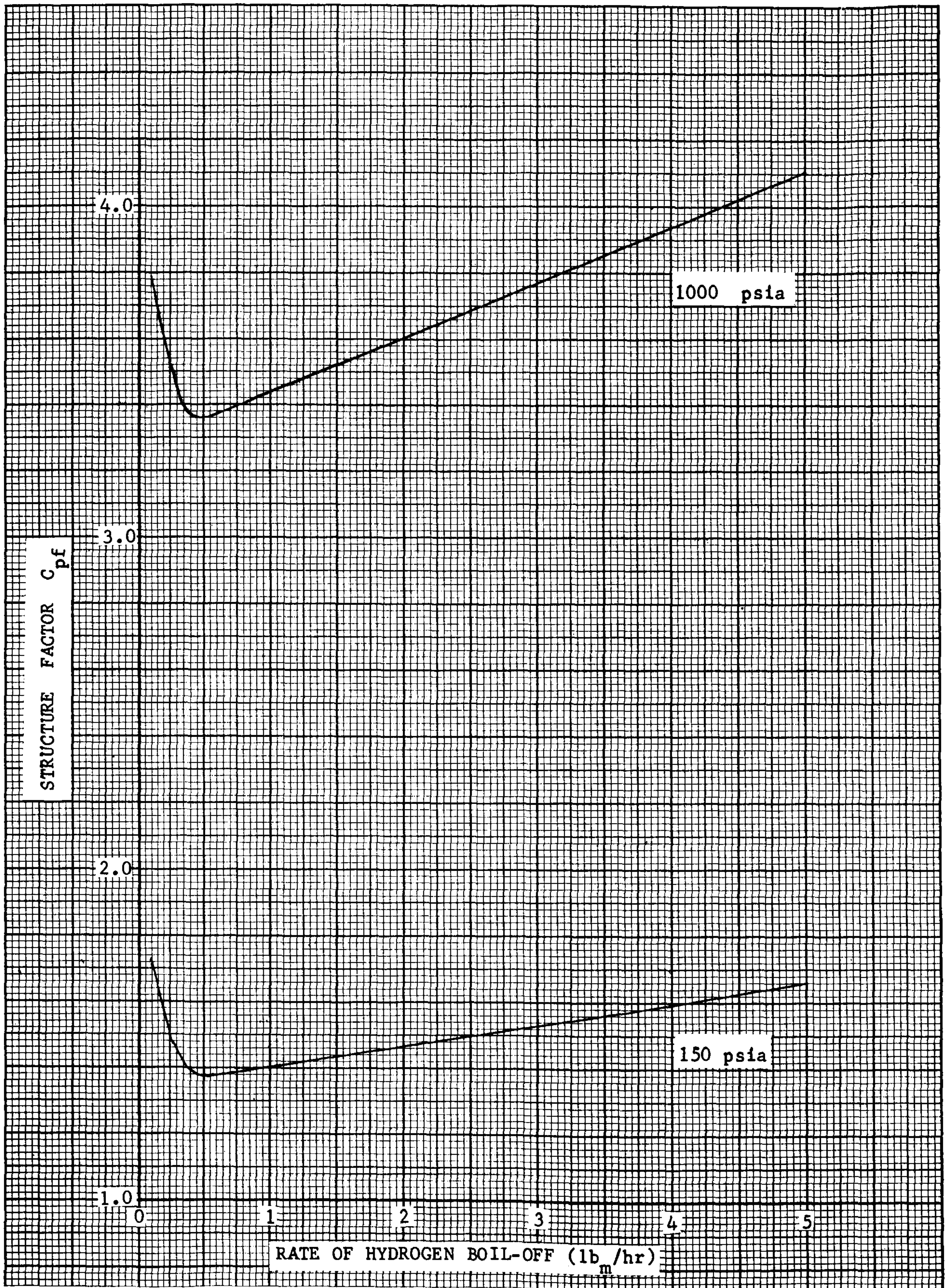

Figure 2-2 VARIATION OF STRUCTURE FACTOR, C_{pf} , WITH HYDROGEN BOIL-OFF RATE

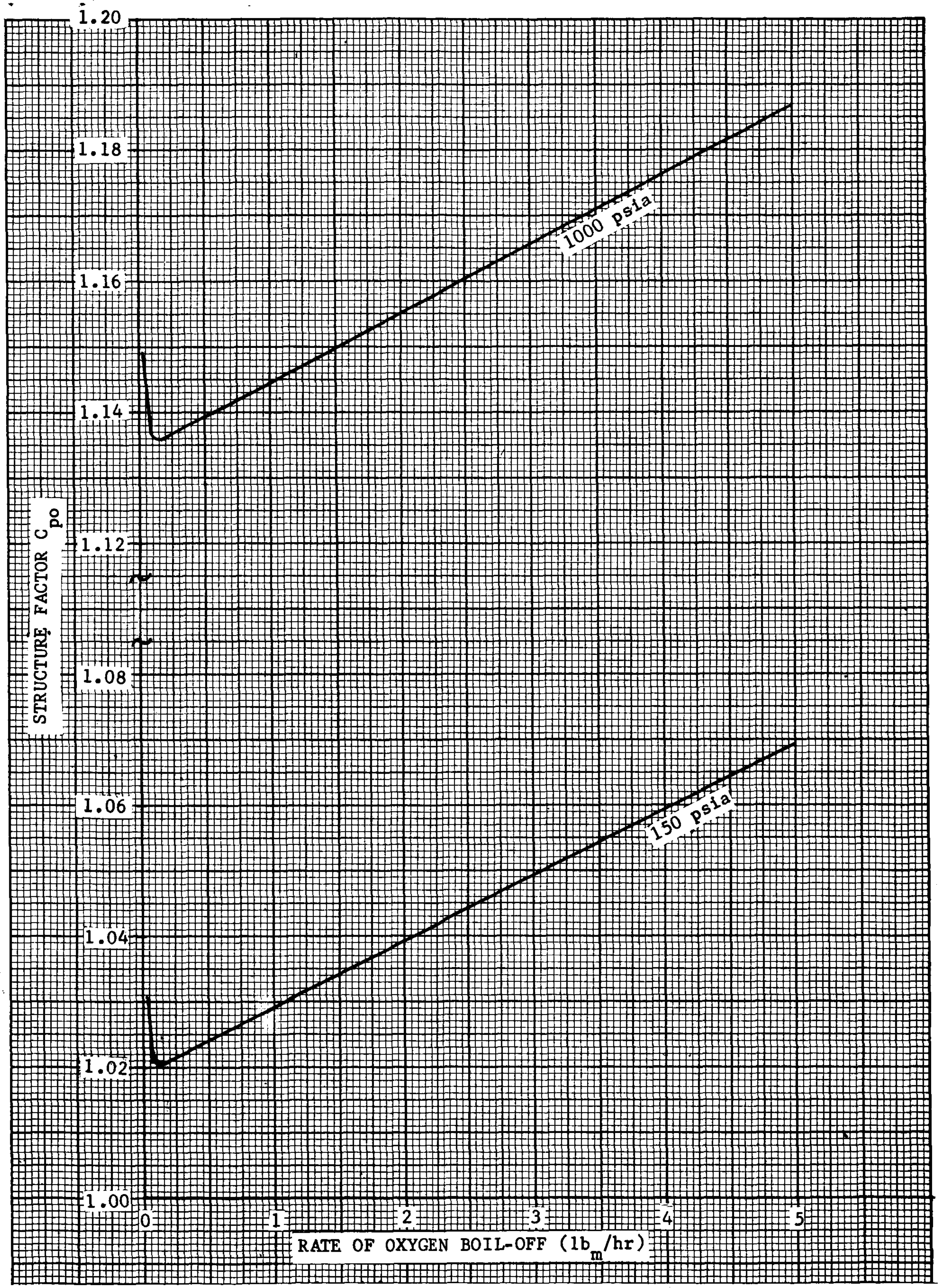

Figure 2-3 VARIATION OF STRUCTURE FACTOR, C_{po}, WITH OXYGEN BOIL-OFF RATE

With these values, the propellant system structure factors are as follows:

$$\left.\begin{array}{ll} C_{pf} = 1.162 & (P_{pf} = 1{,}000 \text{ psia}) \\ C_{pf} = 1.028 & (P_{pf} = 150 \text{ psia}) \\ C_{po} = 1.100 & (P_{po} = 1{,}000 \text{ psia}) \\ C_{po} = 1.016 & (P_{po} = 150 \text{ psia}) \end{array}\right\} \text{Earth Storables.}$$

Notice should be taken that the calculated values of C_{po} and C_{pf} for earth storables are less than the corresponding calculated values of C_{po} and C_{pf} for cryogenics at the same propellant tank pressure.

2.6 Calculation of Total System Mass

Based on the procedure described in subsections 2.4 and 2.5, values of $n_e \int_0^{\tau_b} \dot{m}_p d\tau$, C_{sf}, C_{so}, C_{pf}, and C_{po} were obtained for use in Eq. (13), for both cryogenic and earth storable propellants. For the cryogenic C_{sf}, a value of 4.68 was assigned based on reference 7. For the thrust-to-weight ratios, those values listed in subsection 2.3, corresponding to 100 lb_f thrust engines, were used. By means of Eq. (13), the total system mass was then calculated. The results are presented in Table 2-4. A sample of the calculations for the total systems mass is provided in Appendix D. For each propellant tank pressure, the calculated mass of the earth storable propulsion system was less than the calculated mass of the cryogenic propulsion system.

TABLE 2-4

Calculated Total Mass of Earth Storable and Cryogenic Propulsion Systems

a.	$m_t = 2056.31\ lb_m$	$(P_p = 1000$ psia)	cryogenic
b.	$m_t = 1900.0\ lb_m$	$(P_p = 150$ psia)	
c.	$m_t = 1708.35\ lb_m$	$(P_p = 1000$ psia)	earth storable
d.	$m_t = 1663.22\ lb_m$	$(P_p = 150$ psia)	

2.7 Variation of Propulsion Parameter

As noted previously, the value of the structure factor C_{sf} for cryogenic propellants used in the mass calculations was 4.68. This value is the primary reason for the cryogenic system being heavier than the corresponding earth storable system. Because of the range in possible values of C_{sf} for liquid hydrogen as indicated by Eq. (17), it is of interest to consider what effect variation of this factor has upon total system mass for cryogenics. Figure 2-4 represents this variation for a propellant tank pressure of 1,000 psia, while Figure 2-5 represents such variation for a pressure of 150 psia. In each figure, the value of the cryogenic C_{so} was 1.11. Also shown in each figure is the variation of system mass with earth storable C_{sf} for several different values of C_{so}. The points with letter designators correspond to the calculated values given in Table 2-4.

Examination of Figures 2-4 and 2-5 reveals that with a propellant tank pressure of 1,000 psia, if the cryogenic C_{sf} is less than 2.80 (with the cryogenic C_{so} held at 1.11), the cryogenic system mass will be less than the earth storable system mass given by Eq. (13), with the earth storable C_{sf} and C_{so} both equal to 1.11. Likewise, with a propellant tank pressure of 150 psia, if the cryogenic C_{sf} is less than 3.37, the cryogenic system mass will be less than the earth storable system mass. As before, this condition applies only when the cryogenic C_{so} and the earth storable C_{sf} and C_{so} are all equal to 1.11.

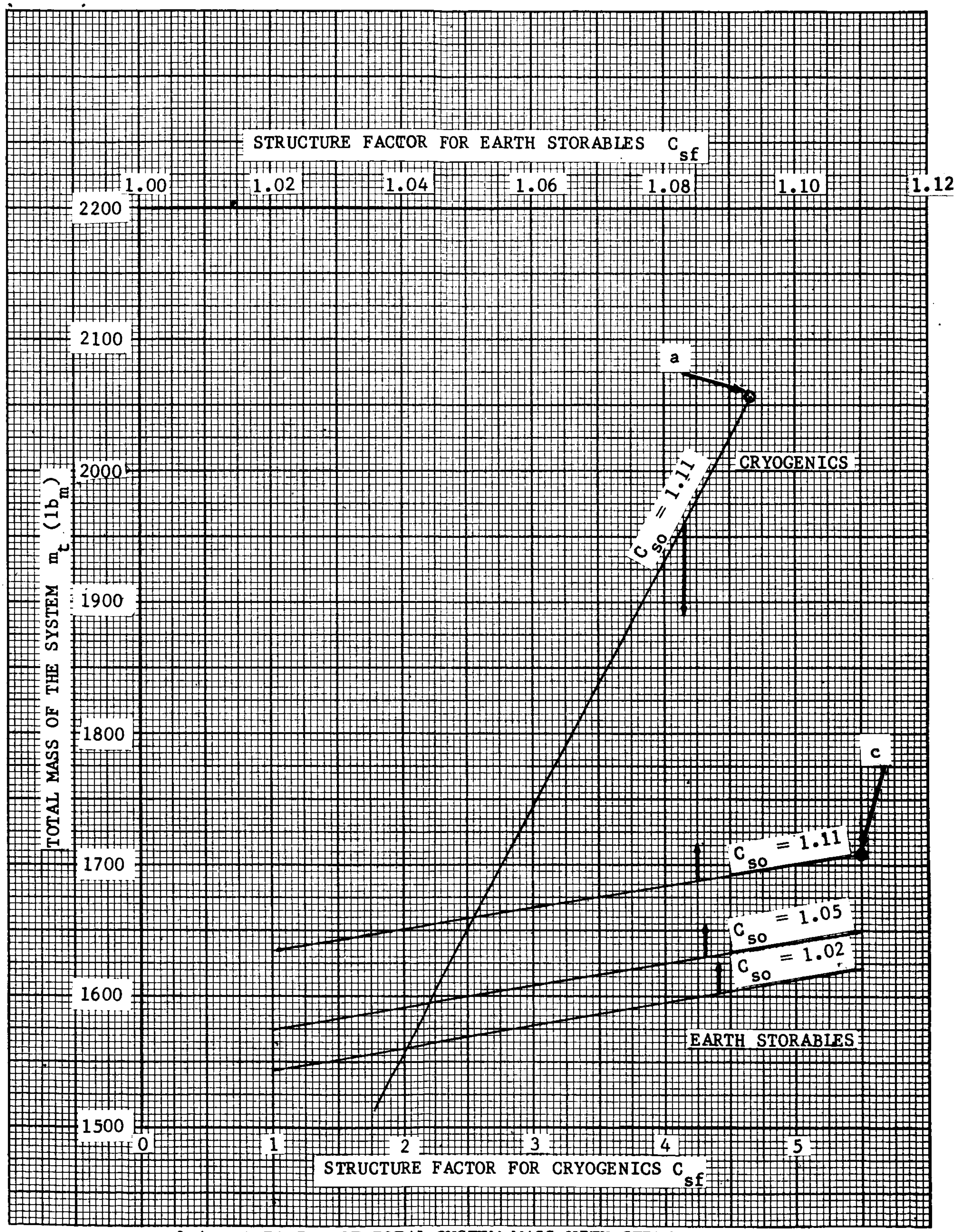

Figure 2-4 VARIATION OF TOTAL SYSTEM MASS WITH STRUCTURE FACTORS FOR A PROPELLANT TANK PRESSURE OF 1000 PSIA

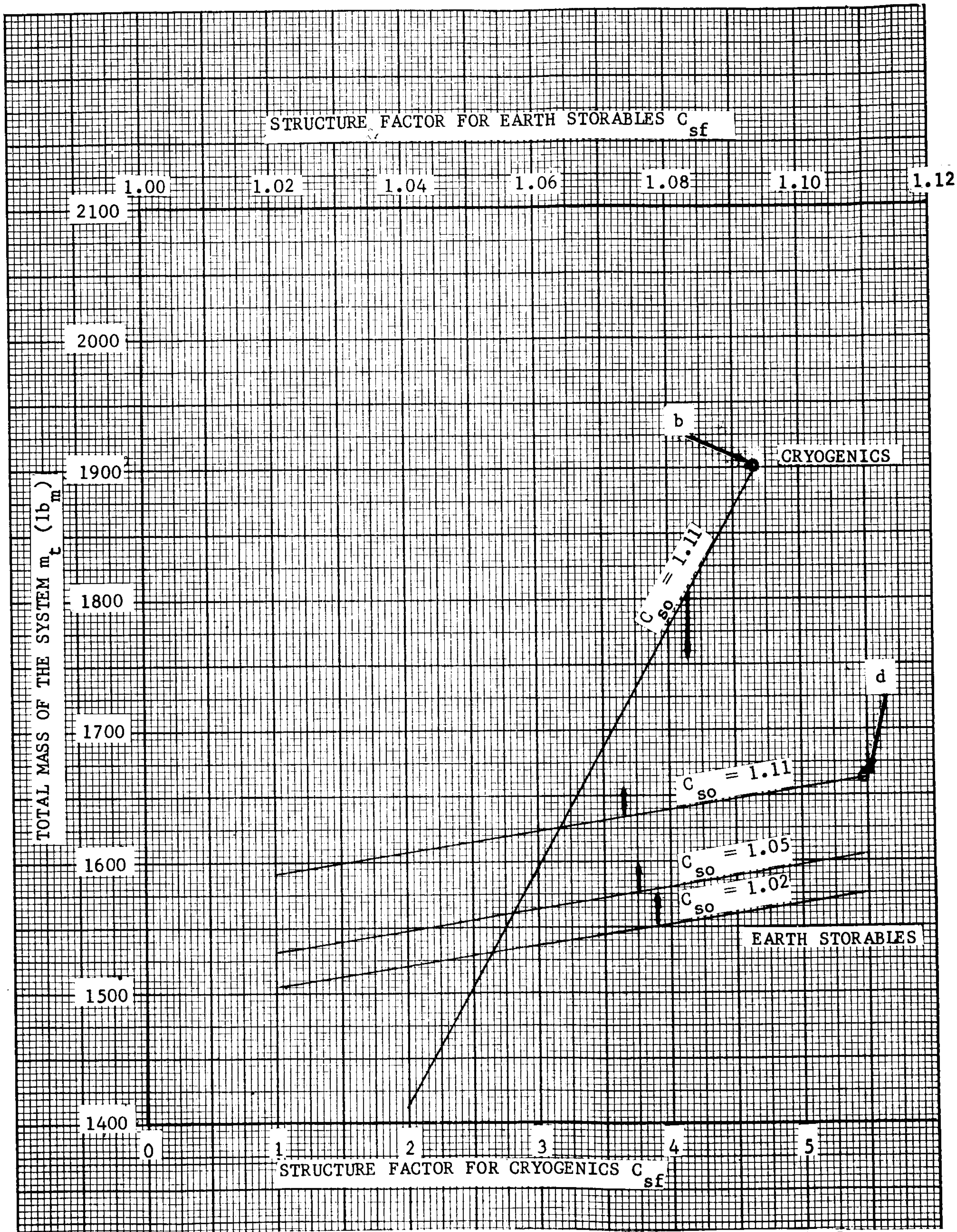

Figure 2-5 VARIATION OF TOTAL SYSTEM MASS WITH STRUCTURE FACTORS FOR A PROPELLANT TANK PRESSURE OF 150 PSIA

Such reductions in the cryogenic structure factor C_{sf}, as noted in reference 7, would represent considerable advances in the present state-of-the-art for cryogenic storage.

Time did not permit an extensive analysis of the effect of the variation of other important parameters on total system mass. An examination of the mathematical model as given by Eq. (13) does indicate, however, that as the number of missions increases, the mass of the earth storable systems probably increases more slowly than the mass of the cryogenic system. Also, for the cryogenic system, as the mixture ratio decreases, the mass of the system will increase. A decrease in the mixture ratio for the earth storable system, however, does not appreciably change the total system mass.

3.0 CONCLUSIONS

Based on the results presented in Section 2.0, the use of cryogenic propulsion systems for the Manned Flying System does not appear to offer any improvement with regard to weight savings over the earth storable propulsion system. The basic reason for this fact is the heavier storage apparatus required for long-term storage of the cryogenic propellants. Liquid hydrogen, with its low density and low saturation temperature, is especially noticeable in this respect. The development of improved insulation might conceivably result in a cryogenic system which would be as light as the earth storable system. Such a development, however, would require considerable advances in the present state-of-the-art.

The most general conclusion that can be drawn from this preliminary investigation is that there appears to be no justification for using a cryogenic propulsion system in place of an earth storable system for the Manned Flying System. The possibility exists that a more detailed investigation might reveal advantages offered by the cryogenic system which were not uncovered in this study.

4.0 REFERENCES CITED

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APPENDIX A

Survey of the Present State-of-the-Art for Low Thrust Cryogenic Propulsion Systems

Because of the scarcity of information concerning liquid oxygen and liquid hydrogen propulsion systems with thrusts ranging from 100 lb_f to 500 lb_f, written inquiries were sent to the following aerospace companies:

Aerojet-General Corporation
Sacremento, California

Jet Propulsion Laboratory
Pasadena, California

Martin Marietta Corporation
Denver, Colorado

Pratt and Whitney
East Hartford, Connecticut

Reaction Motor Division
Thiokol Chemical Corporation
Denville, New Jersey

Rocketdyne
Canoga Park, California

Space Technology Laboratories, Inc.
Redondo Beach, California

The remainder of this appendix consists of a sample of the written inquiry, along with the responses received.

S A M P L E

17 May 1965

Company "X"

Dear Sirs:

Northrop Space Laboratories under Contract NAS8-20082 is presently providing support to the Aero-Astrodynamics Laboratory of NASA's George C. Marshall Space Flight Center, Huntsville, Alabama. As part of this contract, Northrop has been asked to determine the present state-of-the-art with regard to low thrust (100-500 lbs.), cryogenic fueled (liquid hydrogen and liquid oxygen) rocket engines. Any information or data with regard to such engines, which your company can provide without compromising your own proprietary interests, is requested.

Thank you,

Frank B. Tatom
Senior Engineer
Northrop Space Laboratories

FBT/lb

AEROJET GENERAL TIRE GENERAL

AEROJET-GENERAL CORPORATION

P. O. BOX 1947 • SACRAMENTO, CALIFORNIA

TL No. 5-1042

DATE 7 June 1965
DHM:ecs

REPLY TO TECHNICAL INFORMATION REQUEST

TO:

Northrop Space Laboratories
Northrop Corporation
Attn: Frank B. Tatom, Senior Engineer
6025 Technology Drive
Huntsville, Alabama

REFERENCE(S):

(a). Your Ltr., dtd. 17 May 1965, requesting information on the present state-of-the-art with regard to low thrust (100-500lbs.) cryogenic fueld (Liquid hydrogen and Liquid oxygen) rocket engines.

_____ Information requested is CLASSIFIED. Please resubmit your request through the agency monitoring the contract on which it is needed.

_____ Information requested was prepared under government contract. Please resubmit your request to:

_____ Report(s) requested out-of-print, and our supply has been exhausted.

_____ Available from the Defense Documentation Center, as AD_____________.

_____ Available from NASA Scientific and Technical Information Facility, as,

N ____________

X ____________

_____ Information requested cannot be identified.

__X__ Other - This is to advise you that we have not published any research on this subject.

Signed:

D. T. Bedsole, Manager for
Technical Library
Sacramento Plant

AGC 2-1053

Jet Propulsion Laboratory

(No reply received)

Martin Marietta Corporation

(No reply received)

Pratt & Whitney Aircraft DIVISION OF UNITED AIRCRAFT CORPORATION

July 21, 1965

Mr. Frank B. Tatom
Northrop Space Laboratories
6025 Technology Drive
Huntsville, Alabama

Dear Mr. Tatom:

Attached is a copy of the estimated design data on low thrust O_2/H_2 engines which you requested in your letter to Pratt & Whitney Aircraft, dated 17 May 1965.

Because of the general nature of this request providing no specific application, the engines presented were not optimized but designed with representative values of chamber pressure, area ratio, and mixture ratio for low thrust engines. The design data presented utilizes a fixed bed catalyst for the ignition source.

It also includes two mixtures ratios. This is because the catalyst is limited to the temperature corresponding to a maximum mixture ratio of approximately 1.2. The upper limit varies according to the temperature of the inlet propellants. A too-high temperature will result in fusion of the catalyst pellets. If a higher mixture ratio should be required, it can be obtained by adding additional oxygen downstream of the catalyst chamber where it will mix and ignite with the catalytic combustion gases. In this manner, the overall mixture ratio can be increased; however, it would require additional controls and a cooled thrust chamber.

The estimated engine weights include the solenoid operated inlet valves, injector, combustion chamber including the catalyst bed, and nozzles.

Suite 4, Holiday Office Center
Huntsville, Alabama

We hope this information will be helpful to Northrop in determining state-of-the-art of low thrust oxygen/hydrogen engines. If there are any further questions, please do not hesitate in calling this office.

Very truly yours,

John G. Campbell /W C Keathley
John G. Campbell

Encl

JGC/ah

cc: Mr. L. M. Bhalla

DESIGN DATA FOR LOW THRUST OXYGEN-HYDROGEN ENGINES

Thrust, Lb.	100	100	500	500	100	100	500	500
Chamber Pressure, psia	50	50	50	50	50	50	50	50
Vacuum Specific Impulse (95% Theoretical Shifting), Sec.	344	363	344	363	400	427	400	427
Mixture Ratio	1	1	1	1	5	5	5	5
Thrust Chamber Cooling	.... Radiation........				........Regenerative........			
Fuel Flow, lbs/sec	.146	.138	.720	.690	.0415	.039	.208	.195
Oxidizer Flow, lbs/sec	.146	.138	.720	.690	.2085	.195	1.04	.975
Engine Weight, lbs	5.5	6.5	18	23	7	9	21	31
Area Ratio	10	30	10	30	10	30	10	30
Length, Max, in.	11	16	22	40	10	15	20	31
Dia, Max, in.	5	7	11	15.5	4	7	9	15

Space Technology Laboratories, Inc.

(No reply received)

ROCKETDYNE
A DIVISION OF NORTH AMERICAN AVIATION, INC.
6633 CANOGA AVENUE, CANOGA PARK, CALIFORNIA 91304

1 June 1965

IN REPLY REFER TO:
65RC8975

Northrop Space Laboratories
Northrop Corporation
6025 Technology Drive
Huntsville, Alabama

Attention: Mr. Frank B. Tatom
Senior Engineer

Subject: Low Thrust Cryogenic Fueled Rocket Engines

Reference: (a) Northrop Request for Information dated 17 May 1965

Gentlemen:

Your reference (a) request for information concerning the present state-of-the-art with regard to low thrust (100-500 pounds) rocket engines utilizing liquid hydrogen and liquid oxygen as propellants has been received. Although we have extensive experience on engines using cryogenic propellants we regret to inform you that we do not have information or data on engines in thrust ranges as low as 100 to 500 pounds.

If additional information is desired, please contact Mr. W. P. O'Dea, 347-5651, extension 6064.

Very truly yours,

NORTH AMERICAN AVIATION, INC.
Rocketdyne Division

A F Pietrowski

A. F. Pietrowski
Chief, Program Administration
Spacecraft Engines

WPO'D:fml
RC65-1265
08724RC

Thiokol

CHEMICAL CORPORATION

REACTION MOTORS DIVISION

DENVILLE, NEW JERSEY 07834
Phone Area Code 201, 627--7000
TWX 201, 627--3913

28 June 1965

Northrop Space Laboratories
Northrop Corporation
6025 Technology Drive
Huntsville, Alabama

Attention: Mr. F. B. Tatom

Subject: Cryogenic Low Thrust Engines

Reference: Northrop letter dated 17 May 1965, same subject

Enclosure: (1) PI 2-65, Attitude Control System Technology

Gentlemen:

Thiokol Chemical Corporation, Reaction Motors Division, regrets that we are not able to furnish you information at this time on low thrust, cryogenic attitude control engines. We are presently engaged in the earth storable, bi-propellant attitude control engine area. We have several contracts for attitude control engines, however, these are with N_2O_4 and monomethyl.

I am forwarding you our latest report on attitude control engine and system work that is presently being conducted at RMD. I trust that you will find this report helpful in the general area of attitude control systems - unfortunately it does not cover cryogenics. If after review of this report there is additional information that we may be able to supply please do not hesitate to contact either Mr. D. Culbertson at our Southern District Office, Huntsville, Alabama, Area Code 205-881-2661, or Mr. Paul O'Dea, Denville, New Jersey, Area Code 201-627-7000.

Very truly yours,

THIOKOL CHEMICAL CORPORATION
Reaction Motors Division

Warren P. Turner
Director of Applications Engineering

APPENDIX B

Analytical Expression for Structure Factors

A. General Development

The general structure factor, C, by definition is

$$C = \frac{m_x + m_s + m_{in} + m_c + m_a}{m_x} \qquad \text{(B-1)}$$

where

m_x = mass of useful fuel or oxidizer within container (lb_m)

m_s = mass of the shell (lb_m)

m_{in} = mass of insulation (lb_m)

m_c = mass of fuel or oxidizer subjected to a change of phase (lb_m)

m_a = mass of accessory equipment (lb_m).

The mass, m_x , is fixed by the mission requirements. Thus the problem is to express the masses m_s, m_{in}, and m_a in terms of m_x and/or other known quantities.

For spherical containers the development of such an expression for m_s proceeds as follows:

$$m_x + m_c = \frac{4}{3} \pi r_s^3 \rho_x \qquad \text{(B-2)}$$

where

r_s = radius of the shell (ft)

ρ_x = density of the fuel or oxidizer (lb_m/ft^3).

Then

$$r_s = \left(\frac{3}{4\pi}\right)^{1/3} \left(\frac{m_x + m_c}{\rho_x}\right)^{1/3} \tag{B-3}$$

For thin-shelled spheres, the thickness t is

$$t = \eta \frac{P_p r_s}{2\sigma_s} \tag{B-4}$$

where

η = safety factor (1.67)

P_p = design pressure for the shell (psia)

σ_s = tensile strength of shell material (psi)

Thus the mass of the shell can be expressed as

$$\begin{aligned} m_s &= \frac{4}{3}\pi \rho_s \left[\left[r_s + \frac{\eta P_p r_s}{2\sigma_s}\right]^3 - r_s^3\right] \\ &= \frac{4}{3}\pi \rho_s \left[\frac{3}{4\pi} \frac{m_x + m_c}{\rho_x}\right]\left[\left[1 + \frac{\eta P_p}{2\sigma_s}\right]^3 - 1\right] \\ &= \frac{\rho_s (m_x + m_c)}{\rho_x} \frac{\eta P_p}{2\sigma_s}\left[3 + \frac{3\eta P_p}{2\sigma_s} + \left(\frac{\eta P_p}{2\sigma_s}\right)^2\right] \end{aligned} \tag{B-5}$$

The mass of the insulation can be expressed as follows:

From Fourier's Law for steady-state radial heat transfer in a sphere

$$q = 4\pi \frac{r_i r_o}{r_o - r_i} k_{in} \Delta T \tag{B-6}$$

or

$$r_o = \frac{r_i\ q}{q - 4\pi\ k_{in}\ \Delta T\ r_i} \tag{B-7}$$

where

q = rate of heat flow from the shell (Btu/hr)

r_o = outer radius of insulation (ft)

r_i = inner radius of insulation (ft)

k_{in} = thermal conductivity of the insulation (Btu/hr ft °F)

ΔT = temperature difference across insulation (°F)

For the present case, the temperature difference is not always a constant. For simplicity, however, a mean temperature difference, $\overline{\Delta T}$ can be defined such that,

$$Q_s = \int_0^{\tau_s} q d\tau = 4\pi\ \frac{r_o r_i}{r_o - r_i}\ k_{in}\ \overline{\Delta T}\ \tau_s \tag{B-8}$$

where

Q_s = total heat flow from the shell (Btu)

τ_s = general storage time (hr).

Then for the transient case under consideration,

$$r_o = \frac{r_i\ Q}{Q - 4\pi\ k_{in}\ \overline{\Delta T}\ r_i \tau_s} \tag{B-9}$$

Now

$$r_i = r_s \left[1 + \frac{\eta\ P}{2\sigma_s}\right] \tag{B-10}$$

Then

$$m_{in} = \frac{4}{3}\pi\rho_{in}\left(r_o^3 - r_i^3\right)$$

$$= \frac{4}{3}\pi\rho_{in}\left[\frac{r_i^3 Q^3}{\left(Q-4\pi k_{in}\overline{\Delta T}\, r_i \tau_s\right)^3} - r_i^3\right]$$

$$= \frac{4}{3}\pi\rho_{in} r_s^3\left(1+\frac{\eta P_p}{2\sigma_s}\right)^3\left\{\frac{Q^3}{\left[Q-4\pi r_s\left(1+\frac{\eta P_p}{2\sigma_s}\right)k_{in}\overline{\Delta T}\,\tau_s\right]^3} - 1\right\}$$

$$= \frac{4}{3}\pi\rho_{in}\left[\frac{3}{4\pi}\frac{(m_x+m_c)}{\rho_x}\right]\left[1+\frac{\eta P_p}{2\sigma_s}\right]^3\left\{\frac{Q^3}{\left[Q-4\pi\left(\frac{3}{4\pi}\right)^{1/3}\left(\frac{m_x+m_c}{\rho_x}\right)^{1/3}\left(1+\frac{\eta P_p}{2\sigma_s}\right)k_{in}\overline{\Delta T}\,\tau_s\right]^3} - 1\right\}$$

$$= \frac{\rho_{in}(m_x+m_c)}{\rho_x}\left(1+\frac{\eta P_p}{2\sigma_s}\right)^3\left\{\frac{Q^3}{\left[Q-4\pi\left(\frac{3}{4\pi}\right)^{1/3}\left(\frac{m_x+m_c}{\rho_x}\right)^{1/3}\left(1+\frac{\eta P_p}{2\sigma_s}\right)k_{in}\overline{\Delta T}\,\tau_s\right]^3} - 1\right\}$$

(B-11)

From thermodynamics

$$Q = (m_x+m_c)\, c_p (T_{io}-T_i) + m_c h_c \quad (T_{fr} \le T_i \le T_{sat}) \qquad \text{(B-12)}$$

where

c_p = heat capacity (Btu/lb_m °F)

T_{io} = initial storage temperature (°R)

T_i = temperature of propellant or inner surface of insulation (°R)

T_c = temperature for change of phase (°R)

m_c = mass subjected to a change of phase (lb_m)

h_c = heat generated by a change of phase (Btu/lb_m)

T_{fv} = fusion temperature (°R)

T_{sat} = saturation temperature (°R).

The mass of accessory equipment, m_a, is difficult to express analytically. Furthermore, this mass is relatively small and does not appear to have an appreciable effect on the value of the structure factor. For these reasons m_a is neglected in this analysis.

By means of a combination of Eqs. (B-1), (B-5), and (B-11), the structure factor can be written

$$C = 1 + \frac{\rho_s (1+m_c/m_x)\eta P_p}{2\rho_x \sigma_s} \left[3 + \frac{3\eta P_p}{2\sigma_s} + \left(\frac{\eta P_p}{2\sigma_s}\right)^2 \right]$$

$$+ \frac{\rho_{in}(1+m_c/m_x)}{\rho_x} \left(1 + \frac{\eta P_p}{2\sigma_s}\right)^3 \left\{ \frac{Q^3}{\left[Q - 4\pi\left(\frac{3}{4\pi}\right)^{1/3} \left(\frac{m_x + m_c}{\rho_x}\right)^{1/3} \left(1 + \frac{\eta P_p}{2\sigma_s}\right) k_{in} \overline{\Delta T}\, \tau_s\right]^3} - 1 \right\}$$

$$+ m_c/m_x$$

$$= (1 + m_c/m_x) \left\{ 1 + \frac{\rho_s \eta P_p}{2\rho_x \sigma_s} \left[3 + \frac{3\eta P_p}{2\sigma_s} + \left(\frac{\eta P_p}{2\sigma_s}\right)^2 \right] \right.$$

$$\left. + \frac{\rho_{in}}{\rho_x} \left(1 + \frac{\eta P_p}{2\sigma_s}\right)^3 \left[\frac{Q^3}{\left[Q - 4\pi\left(\frac{3}{4\pi}\right)^{1/3} \left(\frac{m_x + m_c}{\rho_x}\right)^{1/3} \left(1 + \frac{\eta P_p}{2\sigma_s}\right) k_{in} \overline{\Delta T}\, \tau_s\right]^3} - 1 \right] \right\} \qquad \text{(B-13)}$$

To demonstrate the procedure for using Eq. (B-13), the remaining portion of this appendix consists of sample calculations.

B. Sample Calculations of C_{po} for Oxygen

$$P_p = 1000 \text{ psia}$$
$$T_{io} = T_i = 188\,°R$$
$$T_c = 188\,°R$$
$$\rho_x = 68 \text{ lb}_m/\text{ft}^3$$
$$m_x = m_o = 302.95 \text{ lb}_m$$
$$\tau_s = \tau_m = 3 \text{ hr}$$
$$\overline{\Delta T} = T_{omax} - T_{io} = 600 - 188 = 412\,°R$$
$$k_{in} = 10^{-5} \text{ Btu/hr ft}\,°F$$
$$\eta = 1.67$$
$$\sigma_s = 80{,}000 \text{ psi}$$

Assume

$$m_c = 15 \text{ lb}_m.$$

Now from Eq. (B-13), for the case of the structure factor C_{po} for oxygen,

$$C_{po} = (1+m_c/m_x)\left\{1+\frac{\rho_s \eta P_p}{2\rho_x \sigma_s}\left[3+\frac{3\eta P_p}{2\sigma_s}+\left(\frac{\eta P_p}{2\sigma_s}\right)^2\right]\right.$$
$$\left.+\frac{\rho_{in}}{\rho_x}\left(1+\frac{\eta P_p}{2\sigma_s}\right)^3\left[\frac{Q^3}{\left[Q-4\pi\left(\frac{3}{4\pi}\right)^{1/3}\left(\frac{m_x+m_c}{\rho_x}\right)^{1/3}\left(1+\frac{\eta P_p}{2\sigma_s}\right)k_{in}\,\overline{\Delta T}\,\tau_m\right]^3}-1\right]\right\} \tag{B-13a}$$

where from Eq. (B-12)

$$Q = (m_x + m_c)\ c_p\ (T_{io} - T_i) + m_c h_c\ (T_{fr} \leq T_i \leq T_{sat}) \tag{B-12}$$

Then

$$Q = \ 5.430 = 6450 \text{ Btu}$$

$$c_{po} = (1 + \frac{15}{302.95}) \left\{ 1 + \frac{281.7 \cdot 1.67 \cdot 1000}{2 \cdot 68 \cdot 80,000} \left[3 + \frac{3 \cdot 1.67 \cdot 1000}{2 \cdot 80,000} + \left(\frac{1.67 \cdot 1000}{2 \cdot 80,000} \right)^2 \right] + \frac{7.8}{68} \left(1 + \frac{1.67 \cdot 1000}{2 \cdot 80,000} \right)^3 \left[\frac{6450^3}{\left[6450 - 4\pi (\frac{3}{4\pi})^{1/3} (\frac{317.95}{68})^{1/3} (1 + \frac{1.67 \cdot 1000}{2 \cdot 80,000}) \cdot 10^{-5} \cdot 412 \cdot 3 \right]^3} - 1 \right] \right\}$$

$$= 1.04952 \left[1 + .04324\ [3 + .031313 + .00011] + \frac{7.8}{68} \cdot 1.03164 \left[\frac{268 \cdot 336125 \cdot 10^9}{[6450 - (4\pi)(.62)(1.67)(1.014)(1.236)(10^{-2})]^3} - 1 \right] \right]$$

$$= 1.04952 (1 + .13108 + .00000895)$$

$$= 1.187$$

or

$$T_i = T_{io} - \int_o^\tau \frac{qd\tau}{m_x c_p} \tag{C-3}$$

Then if the thermal capacitance of the insulation is neglected,

$$\int_o^\tau qd\tau = \int_o^\tau \frac{4\pi r_o r_i \, k_{in}}{r_o - r_i} \left[(T_{io} - \int_o^\tau \frac{qd\tau}{m_x c_p}) - T_o \right] d\tau \tag{C-4}$$

or

$$q = \frac{-4\pi r_o r_i \, k_{in}}{r_o - r_i} \left[\int_o^\tau \frac{qd\tau}{m_x c_p} + T_o - T_{io} \right] \tag{C-5}$$

or

$$\frac{dq}{d\tau} = \frac{-4\pi r_o r_i \, k_{in}}{(r_o - r_i) \, m_x c_p} \, q \tag{C-6}$$

Let

$$\lambda \equiv \frac{4\pi r_o r_i \, k_{in}}{(r_o - r_i) \, m_x c_p} \tag{C-7}$$

Then

$$q = C_1 \, e^{-\lambda\tau} \tag{C-8}$$

In order to satisfy the boundary conditions,

$$C_1 = m_x c_p \, \lambda \, (T_{io} - T_o) \tag{C-9}$$

Thus

$$q = m_x c_p \, \lambda \, (T_{io} - T_o) \, e^{-\lambda\tau} \tag{C-10}$$

APPENDIX C

Calculation of Optimum Storage Temperature and Insulation Thickness for Earth Storable Propellants

The actual heat transfer problem involved in the flow of heat from the propellant containers is quite complex due to its transient nature. In the analysis which follows, the thermal capacitance of the insulation is ignored. This simplification permits a closed-form solution but the accuracy of the solution is reduced. For purposed of this preliminary investigation, however, the technique appears sufficient.

A. General Development

As already noted

$$Q_s = \int_o^{\tau_s} q d\tau \qquad \text{(B-8)}$$

Now, for steady radial heat transfer

$$q = \frac{4\pi \; r_o r_i \; k_{in}}{r_o - r_i} \qquad (T_i - T_o)$$

where

T_o = outside temperature of insulation (°R)

T_i = temperature of propellant or inner surface of insulation (°R).

Also, if no change of phase occurs,

$$(T_{io} - T_i) \; m_x c_p = \int_o^{\tau} q d\tau \qquad \text{(C-2)}$$

Now

$$T_i = T_{io} - \int_o^{\tau} \frac{m_x c_p \; \lambda(T_{io}-T_o)}{m_x c_p} \; e^{-\lambda\tau} d\tau$$

$$= T_{io} + (T_{io}-T_o)\;(e^{-\lambda\tau} -1)$$

$$= T_o + (T_{io}-T_o)\; e^{-\lambda\tau} \qquad \text{(C-11)}$$

In order to avoid both freezing and boiling for a time period of τ_m,

$$T_{sat} \geq T_{omax} + (T_{io}-T_{omax})\; e^{-\lambda\tau_m} \qquad \text{(C-12)}$$

and

$$T_{fr} \geq T_{omin} + (T_{io}-T_{omin})\; e^{-\lambda\tau_m} \qquad \text{(C-13)}$$

where

T_{sat} = saturation temperature of propellant (°R)

T_{omax} = maximum outer surface temperature of propellant container (°R)

T_{fr} = fusion temperature of propellant (°R)

T_{omin} = minimum outer surface temperature of propellant container (°R)

B. Optimum Storage Temperature for N_2O_4

For N_2O_4, as noted in Tables 2-1 and 2-2,

$$T_{sat} < T_{omax}$$

and

$$T_{fr} > T_{omin}.$$

Under these conditions logic reveals that the most desirable condition involves

$$T_{sat} = T_{omax} + (T_{io}-T_{omax})e^{-\lambda\tau_m} \qquad \text{(C-12a)}$$

and

$$T_{fr} = T_{omin} + (T_{io}-T_{omin})e^{-\lambda\tau_m} \qquad \text{(C-13a)}$$

These equations can be arranged to yield

$$T_{io} = (T_{sat}-T_{omax})e^{\lambda\tau_m} + T_{omax} \qquad \text{(C-14)}$$

and

$$T_{io} = (T_{fr}-T_{omin})e^{\lambda\tau_m} + T_{omin} \qquad \text{(C-15)}$$

or

$$(T_{sat}-T_{omax})e^{\lambda\tau_m} + T_{omax} - (T_{fr}-T_{omin})e^{\lambda\tau_m} - T_{omin} = 0 \qquad \text{(C-16)}$$

Thus

$$\lambda = \frac{1}{\tau_m} \ln\left[\frac{T_{omax} - T_{omin}}{T_{omax} - T_{sat} + T_{fr} - T_{omin}}\right] \qquad \text{(C-17)}$$

By means of the definition of λ, as given in Eq. (C-7), r_o can be expressed as

$$r_o = \frac{r_i\, m_o\, c_p \ln\left[\dfrac{T_{omax} - T_{omin}}{T_{omax}-T_{sat}+T_{fr}-T_{omin}}\right]}{m_o\, c_p \ln\left[\dfrac{T_{omax} - T_{omin}}{T_{omax}-T_{sat}+T_{fr}-T_{omin}}\right] - 4\pi\, r_i\, k_{in}\, \tau_m} \qquad \text{(C-18)}$$

If SI-91 insulation is used, for N_2O_4

$$r_o = 1.0001\ r_i$$

$$\lambda = .05217\ hr^{-1}$$

and

$$T_{io} = 518\ ^{o}R.$$

C. Optimum Storage Temperature for a 50:50 mixture of N_2H_4 and UDMH

For this fuel, as shown in Tables 2-1 and 2-2,

$$T_{sat} > T_{omax}$$

and

$$T_{fr} > T_{omin}$$

Under these conditions, from basic thermodynamics, it is desirable to have

$$T_{io} = T_{sat} \qquad \text{(C-19)}$$

By means of Eq. (C-13)

$$T_{fr} = T_{omin} + (T_{sat} - T_{omin})e^{-\lambda\tau_m} \qquad \text{(C-20)}$$

or

$$\lambda = \frac{1}{\tau_m}\ \ln\left[\frac{T_{sat} - T_{omin}}{T_{fr} - T_{omin}}\right] \qquad \text{(C-21)}$$

Then by the definition of λ,

$$r_o = \frac{r_i \; m_f \; C_p \quad \ln\left[\frac{T_{sat} - T_{omin}}{T_{fr} - T_{omin}}\right]}{m_f \; C_p \; \ln\left[\frac{T_{sat} - T_{omin}}{T_{fr} - T_{omin}}\right] - 4\pi r_i \; k_{in} \; \tau_m} \tag{C-22}$$

With SI-91 insulation,

$$r_o = 1.0000115 \; r_i$$

and

$$\lambda = 0.1321 \text{ hr}^{-1} .$$

Based on Eq. (C-19) and Table 2-2,

$$T_{io} = 618 \text{ °R.}$$

APPENDIX D

Sample Calculations for Total System Mass

The calculations which follow are provided to demonstrate the procedure for calculation of total system mass as expressed by Eq. (13).

Propellants: N_2O_4 and 50:50 mixture of N_2H_4 and UDMH

$$n_e = 5$$

$$F = 100 \text{ lb}_f$$

$$n_m = 3$$

$$C_{fw} = 14$$

$$x = 2$$

$$n_e \int_0^{\tau_b} \dot{m} d\tau = 484.74 \text{ lb}_m$$

$$P_p = 1{,}000 \text{ psia}$$

$$C_{pf} = 1.162$$

$$C_{po} = 1.100$$

$$C_{sf} = 1.11$$

$$C_{so} = 1.11$$

Then

$$m_t = n_e \left[\frac{F}{g\, C_{fw}} + \left(\frac{1}{x+1}\right) \left[(C_{pf} - 1 + n_m C_{sf}) + x(C_{po} - 1 + n_m C_{so}) \right] \int_0^{\tau_b} \dot{m}_p d\tau \right] \tag{13}$$

$$= 5 \cdot \frac{100}{14} + \frac{1}{2+1} \left[(1.162 - 1 + 3 \cdot 1.11) + 2(1.100 - 1 + 3 \cdot 1.11) \right] \; 484.74$$

$$= 35.68 + \frac{1}{3}(3.492+6.86)\ 484.74$$

$$= 35.68 + 1672.67$$

$$= 1708.35 \quad lb_m.$$

E20-130

MANNED FLYING SYSTEMS (MFS)

CRYOGENIC ENGINE

Research & Analysis Section Tech. Memo. #113

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R-AERO-DIR
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  -SP (23)
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R-P&VE-DIR
  -A
  -AB (15)
  -AL (5)
R-RP-DIR
  -J (5)
R-FP-DIR
R-FP (2)
R-QUAL-DIR
  -J (3)
R-COMP-DIR
  -RSP
R-ME-DIR
  -X
R-TEST-DIR
  -I
I-DIR
MS-IP
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