Revised

HIGH FREQUENCY SCATTERING FROM A COATED CYLINDER+

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ABSTRACT

The scattered field produced by a plane electromagnetic wave incident on an infinitely long imperfectly conducting cylinder coated with a layer of material with complex index of refraction is considered.

The geometric optics and the creeping wave contributions to the back scattered field are obtained, for normal incidence and small wavelengths.

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1. INTRODUCTION

The determination of the high frequency radar cross section of a smooth convex conducting body covered with one or more thin absorbing layers of materials with large complex indexes of refraction (e.g. ferrites) can be greatly simplified by observing that under certain general assumptions the total electric and magnetic fields satisfy an impedance boundary condition on the outer surface of the outer coating layer [Weston, 1963].

In certain practical applications, such a scatterer is in turn covered by another layer of material whose index of refraction is no longer large compared to unity. It is then of great practical importance to investigate the influence that this outer layer has on the magnitude of the far back scattered field, and therefore on the value of the monostatic radar cross section.

The analysis is complicated by the fact that the exact boundary conditions (i.e. the continuity of the tangential components of the total electric and magnetic fields) must be imposed at the outer surface of the outer layer, while an impedance boundary condition may still be assumed on its inner surface, as we shall see in the following.

In this paper, the investigation is carried out for the case of an infinitely long circular coated cylinder. It is supposed that the material of the outer coating layer has a complex refractive index whose absolute value has a lower bound that is only moderately large compared to unity (e.g. 1.5 or 2), and whose argument is bounded away from both zero and $\pi/2$. An asymptotic evaluation of the far back scattered field is obtained in terms of the geometric optics and of the creeping

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wave contributions, for small wavelengths and normal incidence.

The problem of scattering of plane electromagnetic waves by concentric infinite cylinders has been considered by many authors. The first calculated results for the case of a metal cylinder surrounded by a dielectric sleeve have been published by Adey [1956], who also gave a survey of the previous work on this subject. This case has been recently reconsidered by Kodis [1959, 1961, 1963] and by Helstrom [1963], among others. The boundary value problem for an arbitrary number of concentric cylinders has been solved by Kerker and Matijevic [1961].

As far as the author knows, the case in which an impedance boundary condition holds on the surface of the cylindrical core has not been previously considered.

The rationalized MKS system of units is used, and the time dependence factor $e^{-i\omega t}$ is omitted throughout the paper.

2. THE INFINITE SERIES SOLUTION

Consider an infinitely long cylinder of radius b, coated with a layer of constant thickness d and surrounded by free space; the radius a of the outer surface is then equal to $(b+d)$. The geometry of the scatterer is illustrated in Fig. 1, which also shows the two systems of Cartesian (x, y, z) and cylindrical (r, ϕ, z) coordinates.

Let ϵ_o, μ_o and $Z = \sqrt{\mu_o/\epsilon_o}$ be respectively the permittivity, the permeability and the intrinsic impedance of free space, let ϵ and μ be the relative permittivity and permeability of the material of the layer, and suppose that on the surface $r = b$ of the cylinder the following impedance boundary condition holds:

$$\vec{E}_1 - (\vec{E}_1 \cdot \hat{r})\hat{r} = \eta Z \hat{r} \times \vec{H}_1 , \tag{1}$$

where $\hat{r}$ is a unit vector directed radially from the axis z of the cylinder, $\vec{E}_1$ and $\vec{H}_1$ are the total electric and magnetic fields, and η is the relative surface impedance. The parameters ϵ, μ and η are supposed constant in space and time.

Consider the plane incident electromagnetic wave:

$$E_z^{(i)} = -ZH_y^{(i)} = e^{ikx} \tag{2}$$

where $k = \omega \sqrt{\epsilon_o \mu_o} = 2\pi/\lambda$ is the free space wave number.

The wave number k_1 of the coating is related to the index of refraction $N = \sqrt{\epsilon\mu}$ by the expression

$$k_1 = Nk .$$

The incident wave is propagating in the positive x direction, perpendicularly to the axis z of the cylinder, and is polarized in the (x, z) plane. The results for the other polarization ($\vec{E}^{(i)}$ parallel to the y axis) may be easily obtained by replacing ϵ and ϵ_o with μ and μ_o and vice versa, E with H, H with -E, and η with η^{-1}, throughout the paper [Senior, 1962].

The scattered electric field is given by

$$\begin{aligned} E_z^{(s)} &= \sum_{n=0}^{\infty} h_n i^n a_n H_n^{(1)}(kr) \cos n\phi , \\ E_x^{(s)} &= E_y^{(s)} = 0 , \end{aligned} \tag{3}$$

where $h_o = 1$ and $h_n = 2$ for $n = 1, 2,$ etc.

The constants a_n are determined by imposing the boundary conditions, i.e. the continuity of the tangential components of the total electric and magnetic fields across the outer surface $r = a$, and the impedance boundary condition (1) at the inner surface $r = b$. One finds:

$$a_n = - \frac{J_n'(ka) - A_n J_n(ka)}{H_n^{(1)'}(ka) - A_n H_n^{(1)}(ka)} , \tag{4}$$

where

$$A_n = \frac{N}{\mu} \frac{\partial(\ln C_n)}{\partial(k_1 a)} \; \frac{1 - i\eta \dfrac{N}{\mu} \dfrac{\partial}{\partial(k_1 b)} \left[\ln \dfrac{\partial C_n}{\partial(k_1 a)} \right]}{1 - i\eta \dfrac{N}{\mu} \dfrac{\partial(\ln C_n)}{\partial(k_1 b)}} , \tag{5}$$

with

$$C_n = J_n(k_1 a) H_n^{(1)}(k_1 b) - J_n(k_1 b) H_n^{(1)}(k_1 a) . \tag{6}$$

By making use of the results of Leontovich as discussed by Weston [1963], one finds that the impedance boundary condition (1) at $r = b$ is a very good approximation provided that the index of refraction of the absorber is very large and has a large imaginary part, and that the radius b is large compared to the wavelength inside the absorber.

For an absorbing layer of thickness Δ, relative permittivity ϵ' and relative permeability μ' backed by a metal core, the relative impedance η on its outer surface $r = b$ is given by the expression:

$$\eta \sim -i \sqrt{\frac{\mu'}{\epsilon'}} \tan(k \Delta \sqrt{\epsilon' \mu'}) . \tag{7}$$

A rigorous derivation of (7) and of similar expressions for the case of several absorbing layers may be obtained by considering the exact solution of the corresponding boundary value problem, and by applying to this solution a procedure similar to the one used by Weston and Hemenger [1962] for the coated sphere.

No particular expression for η will be assumed in this paper; then η could not only represent the effect of absorbing layers, but could account for the finite conductivity of the core, or for the roughness of its surface [Senior, 1960].

In the following it will be assumed that

$$\Phi < \arg N < \frac{\pi}{2} - \Phi' \tag{8}$$

with

$$\Phi >> |k_1 d|^{-2/3}, \qquad \Phi' >> |k_1 b|^{-1}, \tag{9}$$

and that $|k_1 d|$ is not large compared to unity.

3. HIGH FREQUENCY BACKSCATTERED FIELD: GEOMETRIC OPTICS CONTRIBUTION

The intensity of the scattered electric field is given by relation (3), which in the case of the backscattered far field becomes

$$E_z^{(b.s)} \sim \sqrt{\frac{2}{\pi kr}}\, e^{ikr - i\frac{\pi}{4}} \left[a_o + 2 \sum_{n=1}^{\infty} (-1)^n a_n \right]. \tag{10}$$

Treating the summation over n as a residue series, the summation is replaced by a contour integral C in the complex ν plane taken in the clockwise direction around the poles at $\nu = 1, 2, \ldots,$ giving:

$$(11) \qquad E_z^{(b.s.)} \sim \sqrt{\frac{2}{\pi kr}}\, e^{ikr - i\frac{\pi}{4}} \left[a_o + \int_C \frac{ia_\nu}{\sin \pi\nu}\, d\nu \right].$$

Following a Watson-type transform technique, the contour C is deformed to include the poles of the integrand which lie in the first quadrant.

The asymptotic evaluation of the line integral and of the term containing a_o for large ka gives the geometric optics contribution to the far backscattered field:

$$(12) \qquad \left[E_z^{(b.s.)}\right]_{g.o.} \sim \frac{\tilde{\eta}-1}{\tilde{\eta}+1} \sqrt{\frac{a}{2r}}\, e^{ikr - i2ka} \times \times \left[1 + \frac{i}{2ka} \left\{ \frac{5}{8} + \frac{\tilde{\eta}}{\tilde{\eta}^2 - 1} \left[1 - 2\tilde{\eta}\left(1 + \frac{ip}{N^2}\right)\right]\right\} + \ldots \right].$$

where:

$$(13) \qquad \tilde{\eta} = -i\frac{\mu}{N} \tan\left[k_1 d + \arctan\left(i\eta \frac{N}{\mu}\right)\right],$$

$$(14) \qquad p = \frac{\frac{N}{\mu}\, \mathrm{cotg}\, k_1 d}{1 + i\eta \frac{N}{\mu}\, \mathrm{cotg}\, k_1 d} \left[\frac{1}{2} \left\{ 1 + \left(\eta\frac{N}{\mu}\right)^2 \right\} - \frac{k_1 d}{\sin 2k_1 d} \left\{ 1 - \left(\eta\frac{N}{\mu}\right)^2 \right\} - i\eta\frac{N}{\mu} \tan k_1 d \right],$$

If we let a go to infinity, we find the field that would be reflected by a plane of surface impedance η coated with a layer of thickness d and refractive index N, for normal incidence.

In the case of a perfectly conducting cylinder ($d=0$, $\eta=0$), relation (12) becomes

$$(15) \qquad \left[E_z^{(b.s.)}\right]_{\substack{g.o.\\ d=0,\ \eta=0}} \sim -\sqrt{\frac{a}{2r}}\, e^{ikr - i2ka} \left(1 + \frac{5i}{16ka} + \ldots\right).$$

This formula checks with the result obtained by Imai [1954].

If the core is perfectly conducting and the material of the coating is a (lossy) dielectric (η 0, $\mu = 1$),

$$\left[E_z^{(s)}\right]_{g.o.} \sim -\sqrt{\frac{a}{2r}}\, e^{ikr - i2ka} \left[\frac{N + i\tan k_1 d}{N - i\tan k_1 d} + O\left(\frac{1}{ka}\right)\right] . \tag{16}$$

In order to compare (16) with the geometric optics term of Kodis [1963]:

$$\left[E_z^{(s)}\right]_{\substack{g.o.\\ \text{Kodis}}} \sim \sqrt{\frac{a}{2r}}\, e^{ikr - i2ka} \left[\frac{1-N}{1+N} + \frac{4N}{N^2-1} \sum_{s=1}^{\infty} \left(1 + \frac{sd}{Nb}\right)^{-\frac{1}{2}} \left\{\frac{1-N}{1+N}\, e^{i2k_1 d}\right\}^{s}\right] , \tag{17}$$

observe that since Im k_1 is positive, only the lowest values of s are of importance, and therefore

$$\left(1 + \frac{sd}{Nb}\right)^{-\frac{1}{2}} \sim 1 + O\left(\frac{1}{kb}\right) , \tag{18}$$

provided that s_{max} is not large compared to $(2N)/(kd)$. Relation (16) is then easily obtained from (17) and (18).

4. HIGH FREQUENCY BACKSCATTERED FIELD: CREEPING WAVE CONTRIBUTION

The creeping wave contribution to the **backscattering** far field is given by the residue series:

$$\left[E_z^{(b.s.)}\right]_{cr.w.} \sim -\frac{4i}{ka}\sqrt{\frac{2}{\pi kr}}\, e^{ikr - i\frac{\pi}{4}} \times \sum_s \left[\sin(\pi\nu)\, H_\nu^{(1)}(ka) \frac{\partial}{\partial\nu} \left\{H_\nu^{(1)'}(ka) - A_\nu H_\nu^{(1)}(ka)\right\}\right]^{-1}_{\nu = \nu_s} \tag{19}$$

where ν_s are the roots of

$$H_{\nu}^{(1)'}(ka) - A_{\nu} H_{\nu}^{(1)}(ka) = 0 \tag{20}$$

which have positive imaginary parts.

Since the main contribution arises from the roots of (20) which are close to ka, we introduce the Fock asymptotic approximation

$$\begin{aligned} H_{\nu}^{(1)}(ka) &\sim -\frac{i}{\sqrt{\pi}} m^{-1} w_1(t), \\ H_{\nu}^{(1)'}(ka) &\sim \frac{i}{\sqrt{\pi}} m^{-2} w_1'(t), \end{aligned} \tag{21}$$

where:

$$\nu = ka + mt, \qquad m = (ka/2)^{1/3} \tag{22}$$

and $w_1(t)$ is the Airy integral in the Fock notation.

If we assume that the absolute value of N is sufficiently large compared to unity, so that the inequalities

$$|\nu - k_1 a| > |\nu|^{1/3}, \qquad |\nu - k_1 b| > |\nu|^{1/3} \tag{23}$$

are satisfied by the first few roots of equation (20) and if

$$\begin{aligned} &|t| << m^2, \\ &\left| \frac{kd}{2\sqrt{N^2-1}} \frac{t}{m^2} \right| << 1, \end{aligned} \tag{24}$$

then

$$A_{\nu} \sim -\frac{i}{\eta_1} + p_1 \frac{t}{m^2}, \tag{25}$$

$$\eta_1 = -i \frac{\mu}{\sqrt{N^2-1}} \tan\left[kd \sqrt{N^2-1} + \arctan\left(i\eta \frac{\sqrt{N^2-1}}{\mu} \right) \right], \tag{26}$$

$$p_1 = \frac{1}{2\mu\sqrt{N^2-1}}\left(1 + i\eta\,\frac{\beta\,\mathrm{cotg}\,\beta}{\mu kd}\right)^{-2}\left[\frac{\beta}{\sin^2\beta} - \mathrm{cotg}\,\beta + \right.$$

(27)

$$\left. + \, i\,\frac{2\eta\beta}{\mu kd}\left(1 + i\eta\,\frac{\beta\,\mathrm{cotg}\,\beta}{\mu kd}\right)\right],$$

and

(28) $$\beta = kd\sqrt{N^2-1}\ .$$

With the approximations (21) and (25), the creeping wave contribution becomes:

$$\left[E_z^{(b.s.)}\right]_{cr.w.} \sim \frac{2\sqrt{2\pi}}{m}\,e^{-i\frac{3\pi}{4}}\,\frac{e^{ikr}}{\sqrt{kr}}\sum_s\left[\sin(\pi\nu_s)\,w_1^2(t_s)\left\{\eta_1^{-2} + \right.\right.$$

(29)

$$\left.\left. + \frac{p_1}{m^3} + \frac{t_s}{m^2}\left(1 + i\,\frac{2p_1}{\eta_1}\right)\right\}\right]^{-1},$$

where t_s are the roots of

(30) $$\frac{w_1'(t)}{w_1(t)} = \frac{im}{\eta_1} - p_1\,\frac{t}{m}\ .$$

An approximate evaluation of t_s gives:

(31) $$t_s \sim t_{os}\left(1 + \frac{p_1}{mt_{os} + m^3\eta_1^{-2}}\right)^{-1},$$

where t_{os} are the roots of the equation

(32) $$\frac{w_1'(t)}{w_1(t)} = \frac{im}{\eta_1}\ ,$$

and may be obtained from the values of $w_1'(t)/w_1(t)$ which were computed by

Logan and Yee [1962] when t lies in the first quadrant.

A detailed discussion of the creeping wave contribution to the scattered field for the case of a perfectly conducting core has been given by Helstrom [1963], who made use of Olver's asymptotic expansion for the Hankel function.

The total backscattered field is obtained by adding together the contributions (12) and (29).

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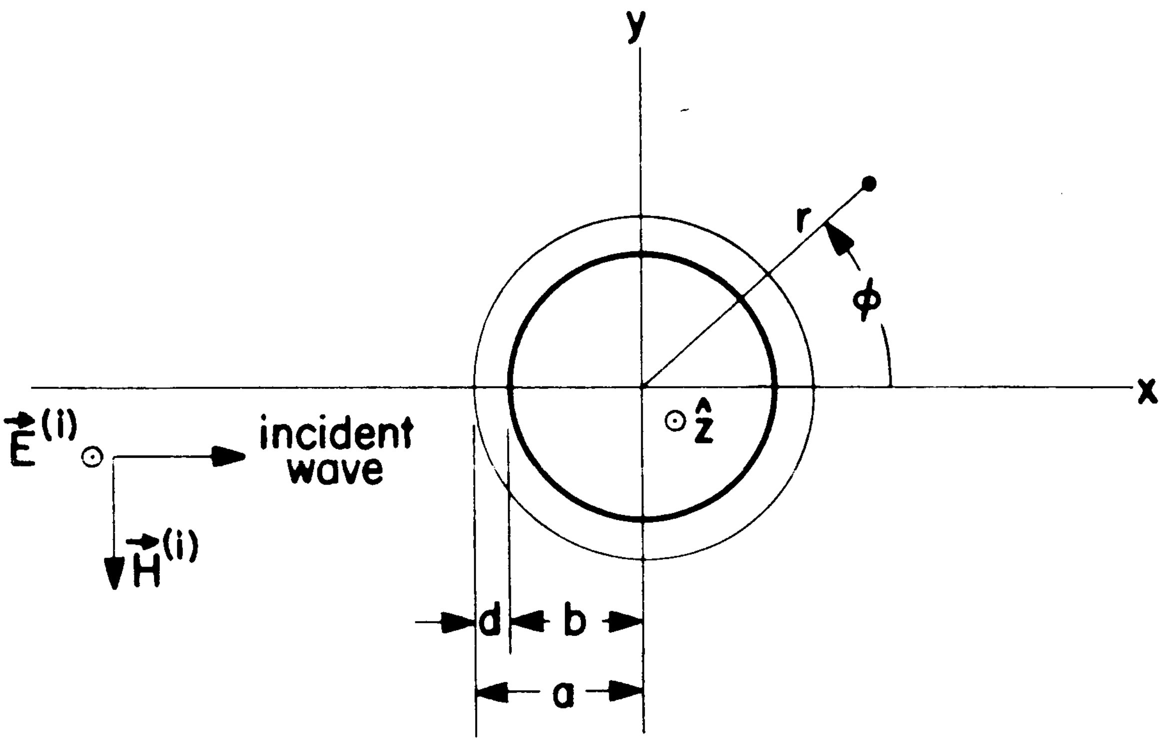

Fig. 1. Basic geometry for the scattering problem.