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A Postulated Law for the Masses of the Elementary Particles*

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Abstract:

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The formula previously postulated for the masses of the elementary particles is stated in a revised form and shown to be in good agreement with the empirical masses. The existence of a neutral shift is shown. Implications of these facts are discussed. Possible application of the formula to resonances is considered.

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I. Introduction:

A possible rule for the masses of the elementary particles has long been known.[1] In this paper this rule is revised, greatly improving its agreement with experiment. Possible consequences and implications of the rule are discussed.

In this paper the speed of light, Planck's constant, and the mass of the electron are taken as unity. This seems to be the most natural system of units for all physical quantities appearing in elementary particle physics.

In section II, we shall state the rule for the masses of the elementary particles, which was first discovered by Nambu, and discuss its implications. The question of the applicability of the law to the resonances appearing in the interaction of strongly interacting particles is discussed in section III. Section IV covers the application of the law to nuclei. It seems that the system of units used in this paper is the natural one to use in elementary particle physics. In section V we list for reference the values of certain constants which are important when this system of units is used, as well as the values of certain other constants which have been used throughout this paper.

1. Nambu; Prog. of Theoretical Physics 7, 595 (52)
Reulos; Arch. Sci. (Geneva) 11, 112 (58)
Frohlich; Nuclear Physics, 7, 148 (58)
See also: Levitt, Curr. Sci. 27, 131 (58)
Darling; Phys. Rev. 80, 460 (50)

11. The Postulated Mass Law:

It is postulated that the masses of the elementary particles are given by the equation . II-1. $M = n\left(\frac{1}{\alpha} + a\right)$ to within about one electron mass. The Nambu number n is either integral or half integral. The term "a" is zero or ± 1. $\frac{1}{\alpha} = 137.0373$

The formula differs from the previously stated one[1] by the inclusion of the "a" term.

The formula does not hold for the electron, but the electron is the basic particle of the system.

In table 11-1 the computed and experimental values of the masses are given.

(Insert table 11-1)

It is seen that the rule gives the masses of the charged particles to within one unit except for the Σ where the discrepancy is about two units and the Ξ, whose mass is not yet known with complete accuracy, and for which the discrepancy may be about two or three units.

A complete statistical analysis of the numbers has not been carried out. However in order to give a rough idea of how well the rule agrees with experiment we have included the last two columns of the table which give the ratios of the discrepancy to the average discrepancy that would be expected from purely statistical considerations in two limiting cases. For the first table the "statistical error" was computed by taking the difference between mass value corresponding to the nambu number for the particle with "a" equal to one, and the mass number corresponding to the next nambu number.

with "a" equal to minus one. This was then divided by four to get the statistical discrepancy. In the last column the mass levels were taken as evenly spaced and their spacing was again divided by four to get the statistical discrepancy. The next to the last column gives a better measure of the agreement than does the last, especially for the lower mass particles.

It is seen that the agreement is rather good. Moreover the formula is quite simple and further more the deviations from it all follow a simple pattern in that all the neutral particles (with the exception of the zero mass particles) are shifted. For these reasons it seems quite likely that the formula is approximately correct. It shall be assumed so for the rest of the paper and the consequences of it will be explored.

It may be that the formula is correct for lower mass particles but not for the hyperons, as the support for it is much less in this case. Nevertheless it seems quite likely that it is correct here also and it shall be taken as correct in this paper. A more precise determination of the masses of the hyperons would help to clarify this situation.

While the charged particles are in very good agreement with Nambu's rule there is a consistent discrepancy between the rule and the masses of the neutral particles. This "neutral shift" applies even to the Λ^0 which has no charged counterpart and whose "a" value cannot thus be determined, (the discrepancy for the two possible "a" values are both less than the discrepancy for the π^0), and to the Σ^- whose mass is shifted even though it is charged.

than others (except for the electron) seems likely to be correct[3].

Furthermore it should be noted that the muon fits into the formula. It seems thus incorrect to regard the muon as in any way mysterious[4] or as being in some way identical to the electron except for some different interaction which causes it to have a large mass. The mass is given by the formula and needs no explanation (except in the sense that all the masses do).

It is a remarkable fact that the nambu numbers and the "a" value seem to be completely independent of the properties of the particles. They are, for example, completely independent of the spin and the isotopic spin. This seems to indicate difficulty for those theories which treat integral and half integral spin on a different basis[5].

Equation 11-1 is a simple formula for masses of the elementary particles. These masses are apparently eigenvalues of some sort. We would thus expect that a theory of elementary particles would produce a formula for these masses. Theories in which the masses are calculated individually from complicated approximate formulae are thus not likely to be correct.[5]

It should be noted that all the discrepancies are positive with the exception of the K which could be positive within experimental error. While the number of cases is too small to have much statistical significance it does suggest that a better determination of the K mass may be of interest.

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3. See discussion by Feynman; Proc. Ann. Rochester Conf. High Energy Phy. 10, 501 (60)
Ohnuki; Rochester Conf. High Energy Phy 10, 844 (60)
Fermi and Yang Phy. Rev. 76, 1739 (43)
4. Taylor; Phys. Rev. 110, 1216 (58)
5. Heisenberg, Proc. Rochester Conf. High Energy Phy. 10, 851 (60)
Nambu, Rochester Conf. High Energy Phys. 10, 858 (60)

Table II-1: The masses of the elementary particles:

Particle:	Experimental Mass:[a]	Computed Mass	Nambu No.	a	Discrep-ancy		
G	0	0	0	?	0		
γ	0	0	0	?	0		
ν	0	0	0	?	0		
e	1	-	-	-	-		
μ	206.77±0.03	207.0595	1.5	1	.29	1/56	1/19
π^0	264.20±0.1				9.87		
$\pi^{\pm}$	273.18±0.1	274.0746	2	0	.89	1/18	1/6
$K^{\pm}$	966.6±0.4	966.2611	7	1	-.3	1/45	1/18
K^0	974.2±1.2				-7.9		
p	1836.12±0.02	1836.5036	13.5	-1	.38	1/27	1/14.5
n^0	1838.65±0.02				-2.19		
Λ^0	2182.80±0.3	2176.5968 2192.5968	16	-1 0	-6.20 9.80		
Σ^+	2327.7±0.4	2329.6341	17	0	1.9	1/4.5	1/3
Σ^0	2331.8±1.0				-2.2		
Σ^-	2340.6±0.6				-11.0		
Ξ^0	2566±16				19.0		
Ξ^-	2580.2±2.5	2584.7087	19	-1	4.5	1/1.7	8/10

a/ Snow and Shapiro; Reviews of Modern Physics 33, 231 (61)

111. Resonances

A resonance is sometimes regarded as a particle of very short lifetime.[6] There are now at least seven resonances known in elementary particles physics. It would be of interest therefore to see whether the masses of the "particles" corresponding to these resonances obey Nambu's rule. In the following table we list the masses of the resonances as well as the closest masses allowed by the formula.

(Insert table 111-1).

We can immediately see from the table that the positions of the resonances are not yet known well enough to allow us to draw definite conclusions. However, it seems quite possible that the masses will be in agreement with the rule.

It is certainly hoped that the positions of these resonances will soon be determined with greater accuracy.

It would also be of interest to look for a "neutral shift" in these resonances. Consider for example the second pion-nucleon resonance. Ordinarily, the reaction $\pi^- + p$ is studied, with the result that the "particle causing the resonance" is neutral. Would the position of the resonance be the same if the more difficult reaction $\pi^+ + n$ were studied? This also has an isotopic spin $\frac{1}{2}$ component, but the "particle" is positive.

It is interesting to note that in the first pion-nucleon resonance which seems in excellent agreement with the formula the "particle" has a charge of two for the reaction $\pi^+ + p$. However, the position of the resonance is not yet known with enough accuracy to allow us to make definite conclusions. It would be of interest to see whether the position of the resonance is the same for the doubly charged $\pi^+ + p$ as for singly

6. Barut; Phys. Rev., 122, 1340 (61)

charged $\pi^{+} + n$, the uncharged $\pi^{-} - p$, and the negativly charged $\pi^{-} + n$. that is whether a neutral shift hold here also.

It should be noted that these "particles" have high spin.[7] Thus if the formula should prove to hold for the resonances we would be able to conclude that this factor is irrelevant to the determination of the mass.

7. Peierls; Phys. Rev. 118, 325 (60)

Table III-1: Resonances

Resonance	Experimental Mass		Computed Mass	Nambu Number	a
πN 33	2400	(a)	2,398.2	17.5	0
πN Second	2980	(a)	2,992.8	22	-1
πN Third	3295	(a)	3,288.8	24	0
Y_1^*(b,c)	2710±10	(c)	2,691.83	19.5	1
Y_0^*(d)	2727±12	(d)	2,720.75	20.	-1
Λ^*(b)	3555	(b)	3,562.97	26.	0
K^*(b)	1735±6	(e)	1,725.47	12.5	1
$W^0(\pi\pi)$(b,f)	603	(b)	612.17	4.5	-1

a. Sternheimer and Lindenbaum, Phys. Rev. 123, 333 (61)

b. Salam, Review Mod. Phys. 33, 426 (61)

c. Alston and Ferro-Luzzi, Review Mod. Phys. 33, 416 (61)

d. Alston et al.; Phys. Rev. Lett. 6, 698 (61)

e. Alston et al.; Phys. Rev. Lett. 6, 300 (61)

f. Booth et al.; Phys. Rev. Lett. 1, 35 (61)

IV. The masses of nuclei:

The masses of the elementary particles obey a simple rule. But what particles are elementary? Does obedience to Nambu's rule determine the elementarity of particles? In this connection the masses of the nuclei are of some interest.

A comparison of the masses of the nuclei with the rule is not simple because the nambu numbers become of the order of 137 resulting in an overlapping of levels. It might be reasonable to assume that the discrepancy would also increase. Thus a complete statistical analysis would have to be carried out. It is hoped to do this statistical analysis and report later on the question of the elementarity of nuclei.

V. Units:

It is quite clear that the mass of the electron is fundamental to Nambu's rule. Thus this constant now takes its place alongside the speed of light and Planck's constant as one of the fundamental constants of physics. It is hoped that all numbers in elementary particle physics will be expressed in terms of it in the future.

For convenience we list the values of some other constants[8] which can be seen to be important in this system of units which we will call the natural system.

[illegible] system.

(V-1) Mass of the electron: $m_e = 0.510976 \pm 0.000007$ mev

(V-2) $= 9.1085 \times 10^{-28}$ gm

(V-3) Compton wavelength of the electron: $\lambda = \frac{\hbar}{m_e c} = 2.42627 \times 10^{-10}$ cm

(V-4) $\alpha = 7.29729 \pm 0.00003 \times 10^{-3}$

(V-5) $\frac{1}{\alpha} = 137.0373 \pm 0.0006$

(V-6) $1 \text{ mev} = 1.60207 \pm 0.00007 \times 10^{-6}$ erg [9]

It would also be of interest to look at the weak interaction coupling constant[10]

(V-7) $G_V = 1.416 \pm 0.003 \times 10^{-49}$ erg-cm^3

and the gravitational coupling constant[8]

8. DuMond and Cohen: Fundamental Constants of Physics (Interscience)
9. Kaplan. Nuclear Physics (Addison-Wesley)
10. Bardin: Physics Rev. Lett., 5, 323 (60)

(V-8) $G = 6.670 \pm 0.005 \times 10^{-11}$ new.-m^2/kgms^2.

In the natural number system these become pure numbers. These are respectively:

(V-9) $G_v = 1.206 \times 10^{-14}$

and

(V-10) $G = 2.735 \times 10^{-46}$.

We believe that these numbers are the proper ones to compare with each other and with other coupling constants in order to find the relative orders of magnitude.

Vl: Conclusions:

The masses of the elementary particles have been shown to be represented accurately by a simple law. The implications of this result have been discussed. The possible application of the law to resonances and the implications of such application have been discussed.

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