FACILITY FORM 602

N65-36430	
(ACCESSION NUMBER)	(THRU)
70	1
(PAGES)	(CODE)
CR67550	30
(NASA CR OR TMX OR AD NUMBER)	(CATEGORY)

CENTER FOR SPACE RESEARCH

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

GPO PRICE $ ____________

CSFTI PRICE(S) $ ____________

Hard copy (HC) 3.00

Microfiche (MF) .75

ff 653 July 65

DYNAMICS OF A
SOLAR PRESSURE STABILIZED SATELLITE

by
John R. Carroll
and
Robert C. Limburg

T-65-1 June 1965

DYNAMICS OF A SOLAR PRESSURE STABILIZED SATELLITE

by

John R. Carroll

Robert C. Limburg

36430

ABSTRACT

It is the objective of this thesis to investigate the body dynamics of a solar pressure stabilized satellite in a heliocentric orbit of non-zero eccentricity. The equations of motion for an unsymmetrical satellite are simplified by finding satellite equilibrium position and making small angle assumptions about this position to linearize the equations. The motions predicted by the simplified model are compared with digital computer solutions of the general equations of motion. Passive damping and injection of the satellite into orbit are investigated briefly.

In its equilibrium position, the satellite has one principal axis along the radial line. Of the two remaining principal axes, the one with the greater moment of inertia is perpendicular to the orbital plane. For the simplified model, motion about the three principal axes can be uncoupled for small disturbances from equilibrium. Motion about the radial line is characterized by oscillations at approximately orbital frequency rate.

Author

This report was submitted to the Department of Aeronautics and Astronautics on May 21, 1965 in partial fulfillment of the requirements for the degree of Master of Science.

ACKNOWLEDGEMENTS

The authors wish to express their appreciation to their thesis supervisor, Professor John V. Harrington, for suggesting the problem as part of the Sunblazer study, and for his continued help and encouragement. Thanks are also due to the staff of the Center for Space Research, Mrs. Joan Limburg, and Miss Suzanne Boyd who aided in the preparation and typing of the manuscript.

Acknowledgement is made to the Department of the Air Force and the Air Force Institute of Technology who made the work possible.

Acknowledgement is also made to the MIT Computation Center for the use of the IBM 7094 Computer. The work was done as problem number M4114.

This report was printed under National Aeronautics and Space Administration Contract Number NASr-249.

TABLE OF CONTENTS

SYMBOLS

1,2,3	principal body axes
A,B,C	moment of inertia ratios
a	semi-major axis
D	damping-moment of inertia ratio
d	viscous damping coefficient
E	eccentric anomoly
ΔE	energy dissipated
EI	flexural rigidity
e	eccentricity
F	solar pressure force
f	true anomoly
g	earth gravity
I_i	moment of inertia about i-axis
K	solar pressure constant
L	distance between satellite center of mass and center of pressure
L_1	tube distance from satellite center of mass
L_2	antenna length
M	mean anomoly
M_i	moment about i-axis
m	mass
N	number of collisions
r	orbital radius

$(r_{cm})_i$	distance between center of mass of component i and satellite center of mass
s	Laplace operator
t	time
U	potential energy
V	satellite velocity
W_B	ball weight
X,Y,Z	attitude reference axes
α	structural damping factor
β	general angular displacement
γ	phase angle
Δ	determinant
ζ	damping ratio
$\Theta, \Phi, \Psi,$	Laplace transformed variables
θ	pitch angle
$\theta(0), \phi(0), \psi(0)$	initial conditions
θ_0, ϕ_0, ψ_0	equilibrium offset angles
μ	gravitational constant times the mass of the sun
ρ	density
τ	time of perehelion passage
ϕ	roll angle
ψ	yaw angle
ω	orbital angular velocity
$\dot{\omega}$	orbital angular acceleration
ω_i	angular velocity about axis i

CHAPTER 1

INTRODUCTION

A satellite in heliocentric orbit can be made sun-pointing through the relatively simple and economical use of solar pressure stabilization. Use of such a method, however, causes long period oscillations making conventional damping schemes difficult. For a satellite of this type, the orbit is usually of non-zero eccentricity, further complicating the analysis. It will be the purpose of this thesis to investigate the dynamic behavior of such a satellite in order to establish its physical requirements.

In order to keep the problem as general as possible, it is assumed that the satellite does not possess axial symmetry. The equations of motion for such a case are quite complex due to nonlinearities, and system behavior is extremely difficult to predict. For this reason, a body centered coordinate system, rotating at orbital angular velocity, will be chosen as a reference frame, and the satellite equilibrium position as a function of orbital parameters will be defined with respect to this frame. Perturbation methods will then be utilized in order to linearize the equations. The validity of these simplifications will be verified using a digital computer.

This analysis is contingent on whether or not the satellite will damp to its equilibrium position following injection into orbit. For this purpose, the feasibility of using simple passive damping techniques will be looked into, and the injection lock-in process will be studied using the computer.

For purposes of computation, the satellite will be placed in an orbit with the following characteristics[1]:

1. an eccentricity of 0.31
2. a sidereal period of eight months
3. an aphelion of one astronomical unit

To satisfy different communication requirements, two satellite configurations will be considered, one with long antennas and the other with short.

CHAPTER 2

THE SIMPLIFIED MODEL

2.1 General Considerations

The motion of the satellite center of mass is described by the orbital parameters, and is independent of motion about the center of mass. The converse does not hold true, however, since the motion about the center of mass is influenced by the direction of solar radiation pressure which changes with the body's orbital position.

Choose an attitude reference frame, X, Y, Z, with origin at the center of mass of the satellite. The Z-axis of the coordinate frame is maintained along the radius vector from the body center of mass to the sun. (See Figure 1a). The X - Z plane forms the plane of the orbit, and the Y-axis is along the angular velocity vector, thus forming an orthogonal triad. The 1, 2, 3 axis system is coincident with the principal body axes and body motion relative to the reference frame is described by Euler angles. In this case, the Euler angles are the roll angle, ϕ, pitch angle, θ, and yaw angle, ψ. Roll motion is about the X-axis. When roll is zero, pitch is about the Y-axis and describes motion in the orbital plane. If roll and pitch are both zero, then yaw is about the Z axis or radial line. The order of rotation is defined as ϕ, θ, ψ.

Euler's equations in principal body axes (see Figure 2) can be used to describe the motion about the center of mass and can be written as,

$$M_1 = I_1\dot{\omega}_1 + (I_3 - I_2)\omega_3\omega_2 \tag{2.1}$$

$$M_2 = I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 \tag{2.2}$$

$$M_3 = I_3\dot{\omega}_3 + (I_2 - I_1)\omega_2\omega_1 \tag{2.3}$$

where M is the applied moment due to solar pressure restoring torque and viscous damping torque. For the chosen coordinate system and the given satellite configuration,

$$M_1 = -FL(\sin\phi\ \cos\psi + \cos\phi\ \sin\theta\ \sin\psi) - d_1\omega_1 \tag{2.4}$$

$$M_2 = -FL(\cos\phi\ \sin\theta\ \cos\psi - \sin\phi\ \sin\psi) - d_2\omega_2 \tag{2.5}$$

$$M_3 = -d_3\omega_3 \tag{2.6}$$

$$\omega_1 = \dot{\phi}\ \cos\theta\ \cos\psi + \dot{\theta}\ \sin\psi + \omega\ \cos\phi\ \sin\psi + +\omega\ \sin\phi\ \sin\theta\ \cos\psi \tag{2.7}$$

$$\omega_2 = \dot{\theta}\ \cos\psi - \dot{\phi}\ \cos\theta\ \sin\psi + \omega\ \cos\phi\ \cos\psi + -\omega\ \sin\phi\ \sin\theta\ \sin\psi \tag{2.8}$$

$$\omega_3 = \dot{\psi} + \dot{\phi}\ \sin\theta - \omega\ \sin\phi\ \cos\theta \tag{2.9}$$

where

F = solar pressure force

L = Distance between satellite center of mass and center of pressure

and

d_i = viscous damping coefficient about the i-axis

The complete equations of motion are derived in detail and presented in Appendix A.

2.2 Equilibrium Position and Linearized Equations of Motion

To simplify the equations of motion, small angle assumptions will be made on ϕ and θ. The assumptions would appear to be valid, since these angles are constrained by the solar pressure torque. However, since the body does not have axial symmetry there will be a preferred orientation about the radial line. Substitute $(\psi_o + \psi)$ for the yaw angle, where ψ_o is the preferred angle and ψ is a small angle perturbation about ψ_o. Using trigonometric identities and making small angle assumptions on ψ,

$$\sin(\psi_o + \psi) \cong \sin\psi_o + \psi\cos\psi_o \tag{2.10}$$

$$\cos(\psi_o + \psi) \cong \cos\psi_o - \psi\sin\psi_o \tag{2.11}$$

Further assume that time derivatives of these angles are also small, and neglect the product of two small quantities. Using these assumptions, the equations of motion become linear with time varying coefficients and are given by,

$$\begin{aligned}(\cos\psi_o)\ddot{\phi} &+ (D_1\cos\psi_o)\dot{\phi} + \left(\left[\frac{FL}{I_1} - C\omega^2\right]\cos\psi_o\right)\phi + (\sin\psi_o)\ddot{\theta} + \\ &+ (D_1\sin\psi_o)\dot{\theta} + \left(\frac{FL}{I_1}\sin\psi_o\right)\theta + ([C+1]\omega\cos\psi_o)\dot{\psi} + \\ &+ (\dot{\omega}\cos\psi_o + D_1\omega\cos\psi_o)\psi = -\dot{\omega}\sin\psi_o - \omega D_1\sin\psi_o\end{aligned} \tag{2.12}$$

where

$$C = \frac{I_3 - I_2}{I_1} \quad \text{and} \quad D_1 = \frac{d_1}{I_1}$$

$$(-\sin\psi_o)\ddot{\phi} + (-D_2\sin\psi_o)\dot{\phi} + ([B\omega^2 - \frac{FL}{I_2}]\sin\psi_o)\phi +$$

$$+ (\cos\psi_o)\ddot{\theta} + (D_2\cos\psi_o)\dot{\theta} + (\frac{FL}{I_2}\cos\psi_o)\theta +$$

$$- ([1 + B]\omega\sin\psi_o)\dot{\psi} + (-\dot{\omega}\sin\psi_o - \omega D_2\sin\psi_o)\psi =$$

$$-\dot{\omega}\cos\psi_o - \omega D_2\cos\psi_o \tag{2.13}$$

where

$$B = \frac{I_3 - I_1}{I_2} \quad \text{and} \quad D_2 = \frac{d_2}{I_2}$$

and

$$([A\{\cos^2\psi_o - \sin^2\psi_o\} - 1]\omega)\dot{\phi} + (-\dot{\omega} - D_3\omega)\phi +$$

$$+ (2A\omega\sin\psi_o\cos\psi_o)\dot{\theta} + \ddot{\psi} + D_3\dot{\psi} +$$

$$+ (A\omega^2[\cos^2\psi_o - \sin^2\psi_o])\psi = -A\omega^2\sin\psi_o\cos\psi_o \tag{2.14}$$

where

$$A = \frac{I_2 - I_1}{I_3} \quad \text{and} \quad D_3 = \frac{d_3}{I_3}$$

If the oscillations of the satellite are damped, the yaw, pitch, and roll angles will assume a steady-state value, which is defined as the equilibrium position. Denote this position by ϕ_o, θ_o, and ψ_o. To find these angles, let $\ddot{\theta}$, $\dot{\theta}$, $\ddot{\phi}$, $\dot{\phi}$, $\ddot{\psi}$, $\dot{\psi}$, and the perturbation angle ψ go to zero, thereby obtaining

$$[(\frac{FL}{I_1} - C\omega^2)\cos\psi_o]\phi_o + [\frac{FL}{I_1}\sin\psi_o]\theta_o =$$

$$-\dot{\omega}\sin\psi_o - \omega D_1\sin\psi_o \tag{2.15}$$

$$\left[\left(B\omega^2 - \frac{FL}{I_2}\right) \sin\psi_o\right]\phi_o + \left[\frac{FL}{I_2} \cos\psi_o\right]\theta_o =$$

$$-(\dot{\omega} + D_2\omega) \cos\psi_o \tag{2.16}$$

$$(\dot{\omega} + D_3\omega)\ \phi_o = A\omega^2 \sin\psi_o \cos\psi_o \tag{2.17}$$

Solutions to these equations are (see Appendix B),

$$\psi_o = 0$$

$$\phi_o = 0$$

$$\theta_o = \frac{-\ I_2\ (\dot{\omega} + \omega D_2)}{FL} \tag{2.18}$$

or

$$\psi_o = 90°$$

$$\phi_o = 0$$

$$\theta_o = \frac{-\ I_1\ (\dot{\omega} + \omega D_1)}{FL} \tag{2.19}$$

Note that the equilibrium angle θ_o is a function of position in the orbit due to the non-zero eccentricity.

With the value of ψ_o established, the equations can be further simplified. If $\psi_o = 0$, the equations are,

$$\ddot{\phi} + D_1\dot{\phi} + \left(\frac{FL}{I_1} - C\omega^2\right)\phi + (C + 1)\omega\dot{\psi} + (D_1\omega + \dot{\omega})\psi = 0 \tag{2.20}$$

$$\ddot{\theta} + D_2\dot{\theta} + \frac{FL}{I_2}\ \theta = -D_2\omega - \dot{\omega} \tag{2.21}$$

$$(A - 1)\omega\dot{\phi} - (D_3\omega + \dot{\omega})\phi + \ddot{\psi} + D_3\dot{\psi} + A\omega^2\ \psi = 0 \tag{2.22}$$

If ψ_o is chosen as 90° then

$$\ddot{\phi} + D_2\dot{\phi} + \left(\frac{FL}{I_2} - B\omega^2\right)\phi + (B + 1)\omega\dot{\psi} + (D_2\omega + \dot{\omega})\psi = 0 \qquad (2.23)$$

$$\ddot{\theta} + D_1\dot{\theta} + \frac{FL}{I_1}\theta = -D_1\omega - \dot{\omega} \qquad (2.24)$$

$$(-A - 1)\omega\dot{\phi} - (D_3\omega + \dot{\omega})\phi + \ddot{\psi} + D_3\dot{\psi} - A\omega^2\psi = 0 \qquad (2.25)$$

To investigate the stability of these equilibrium positions, choose $\psi_o = 0$ and solve (2.20), (2.21), and (2.22) for the characteristic equation. With the assumptions

$$\frac{FL}{I_1} = \omega_1^2 >> \omega^2$$

and

$$\omega_1^2 >> \dot{\omega}$$

Lin's Method of Approximation[2] can be employed to obtain the characteristic equation. (See Appendix C.) For $\psi_o = 0$, it is

$$(s^2 + D_1 s + \omega_1^2)(s^2 + D_2 s + \omega_2^2)(s^2 + D_3 s + A\omega^2) = 0 \qquad (2.26)$$

Looking at equations (2.20) through (2.25), it is obvious that the analysis for $\psi_o = 90°$ can be carried out simply by interchanging subscripts 1 and 2 in the $\psi_o = 0$ equations. Since A is defined as $\frac{I_2 - I_1}{I_3}$, this operation causes the coefficient of ω^2 to become $\frac{I_1 - I_2}{I_3} = -A$. This gives a characteristic equation for $\psi_o = 90°$,

$$(s^2 + D_2 s + \omega_2^2)(s^2 + D_1 s + \omega_1^2)(s^2 + D_3 s - A\omega^2) = 0 \qquad (2.27)$$

For the particular satellite in question, $I_2 > I_1$ and A is a positive quantity (see Appendix E). Using Routh's criterion, $\psi_o = 0$ is stable and $\psi_o = 90°$ is unstable. This means that for the stable case, the axis having the greater moment of inertia will be perpendicular to the plane of the orbit.

2.3 System Response to Disturbances

In order to use Laplace transforms, the angular variables must have constant coefficients. Since, the coefficients in equations (2.20), (2.21) and (2.22) change at orbital frequency, they can be assumed constant over a small portion of the orbit.

Expressing system disturbances as non-zero initial conditions, the transformed equations are written as

$$\begin{bmatrix} (C+1)\omega s + \dot{\omega} + \omega D_1 & s^2 + D_1 s + \frac{FL}{I_1} - C\omega^2 & 0 \\ 0 & 0 & s^2 + D_2 s + \frac{FL}{I_2} \\ s^2 + D_3 s + A\omega^2 & (A-1)\omega s - \dot{\omega} - \omega D_3 & 0 \end{bmatrix} \begin{bmatrix} \Psi \\ \Phi \\ \Theta \end{bmatrix} =$$

$$\begin{bmatrix} (s+D_3)\psi(o) + (A-1)\omega\phi(o) + \dot{\phi}(o) \\ -\frac{\dot{\omega}}{s} - \frac{D_2\omega}{s} + (s+D_2)\theta(o) + \dot{\theta}(o) \\ (s+D_1)\phi(o) + (C+1)\omega\,\psi(o) + \dot{\psi}(o) \end{bmatrix} \tag{2.28}$$

Since small angle assumptions have been made, let the initial rotation rates be zero. Solving equation (2.28) gives

$$\Psi(s) = \frac{\phi(o)}{\Delta_1}\left\{(s + D_1)[(1 - A)\omega\, s + \dot{\omega} + D_3\omega]\right\} +$$

$$+ \frac{\phi(o)(A - 1)\omega}{s^2 + D_3 s + A\omega^2} + \frac{\psi(o)(s + D_3)}{s^2 + D_3 s + A\omega^2} +$$

$$+ \frac{\psi(o)}{\Delta_1}\left\{(C + 1)\omega[(1 - A)\omega\, s + \dot{\omega} + D_3\omega]\right\} \qquad (2.29)$$

$$\Theta(s) = \frac{1}{\Delta}[(1 - A)\omega\, s + \dot{\omega} + D_3\omega]\cdot\left[\frac{-\dot{\omega} - D_2\omega}{s} + (s + D_2)\theta(o)\right]\cdot$$

$$\cdot[(C + 1)\omega\, s + D_1\omega + \dot{\omega}] + \frac{1}{s^2 + D_2 s + \omega_2{}^2}\left[\frac{-\dot{\omega} - D_2\omega}{s} + (s + D_2)\theta(o)\right]$$

$$(2.30)$$

$$\Phi(s) = -\frac{\psi(o)}{\Delta_1}\left\{[s + D_3]\cdot[(C + 1)\omega s + D_1\omega + \dot{\omega}]\right\} +$$

$$+ \frac{\psi(o)(C + 1)\omega}{s^2 + D_1 s + \omega_1{}^2} + \frac{\phi(o)(s + D_1)}{s^2 + D_1 s + \omega_1{}^2} - \frac{\phi(o)}{\Delta_1}\left\{[(A - 1)\omega][(C + 1)\omega s +\right.$$

$$\left.+ \omega D_1 + \dot{\omega}]\right\} \qquad (2.31)$$

where

$$\Delta = (s^2 + D_1 s + \omega_1{}^2)(s^2 + D_2 s + \omega_2{}^2)(s^2 + D_3 s + A\omega^2) \qquad (2.32)$$

$$\Delta_1 = (s^2 + D_1 s + \omega_1{}^2)(s^2 + D_3 s + A\omega^2) \qquad (2.33)$$

$$\omega_1{}^2 = \frac{FL}{I_1} \qquad (2.34)$$

and

$$\omega_2{}^2 = \frac{FL}{I_2} \qquad (2.35)$$

Inverse transforming[3] and neglecting comparatively small terms gives

$$\psi(t) = \frac{\psi(o)\ e^{-\xi_3\omega_3 t}}{\sqrt{1-\xi_3^2}} \sin[\omega_3\sqrt{1-\xi_3^2}\ t + \gamma_1] +$$

$$+ \phi(o)(1 - A)\ \{\frac{\omega}{\omega_1}\ e^{-\xi_1\omega_1 t}\sin[\omega_1\sqrt{1-\xi_1^2}\ t] +$$

$$+ e^{-\xi_3\omega_3 t}\sin[\omega_3\sqrt{1-\xi_3^2}\ t]\} \tag{2.36}$$

where

$$\omega_3^2 = A\omega^2 \tag{2.36a}$$

and

$$\gamma_1 = \tan^{-1}\frac{\sqrt{1-\xi_3^2}}{\xi_3} \tag{2.36b}$$

$$\theta(t) = \frac{\theta(o)\ e^{-\xi_2\omega_2 t}}{\sqrt{1-\xi_2^2}} \sin[\omega_2\sqrt{1-\xi_2^2}\ t + \gamma_2] +$$

$$-\left\{\frac{\dot{\omega} + D_2\omega}{\omega_2^2}\right\}\left\{1 + \frac{e^{-\xi_2\omega_2 t}}{\sqrt{1-\xi_2^2}}\sin[\omega_2\sqrt{1-\xi_2^2}\ t - \gamma_3]\right\} \tag{2.37}$$

where

$$\gamma_2 = \tan^{-1}\frac{\sqrt{1-\xi_2^2}}{\xi_2} \tag{2.37a}$$

and

$$\gamma_3 = \tan^{-1}\frac{\sqrt{1-\xi_2^2}}{-\xi_2} \tag{2.37b}$$

$$\phi(t) = \frac{\phi(o)\ e^{-\xi_1\omega_1 t}}{\sqrt{1 - \xi_1^2}} \sin[\omega_1\sqrt{1 - \xi_1^2}\ t + \gamma_4] +$$

$$+ \left\{\frac{\psi(o)(C + 1)\omega}{\omega_1}\right\} \left\{\frac{e^{-\xi_1\omega_1 t}}{\sqrt{1 - \xi_1^2}} \sin[\omega_1\sqrt{1 - \xi_1^2}\ t]\right\} \qquad (2.38)$$

where

$$\gamma_4 = \tan^{-1} \frac{\sqrt{1 - \xi_1^2}}{\xi_1} \qquad (2.38a)$$

For first order calculations, only one term need be computed in each of the above equations, since the primary terms are several orders of magnitude greater than the cross-coupling terms. However, when primary terms are not present, cross-coupling terms should be considered.

CHAPTER 3

USE OF THE DIGITAL COMPUTER

To check the validity of the simplified equations of motion, the IBM 7094 Digital Computer was used. Equations (A.13), (A.14), and (A.15) were solved iteratively and the angles ϕ, ψ, and θ were printed as a function of time.

Orbital quantities had to be inserted as a function of time or true anomoly. Thus, for a satellite in an elliptical orbit, the velocity of the body is given by

$$V^2 = \mu \left[\frac{2}{r} - \frac{1}{a}\right]$$

$$= \frac{\mu}{a\ (1 - e^2)} \quad [1 + 2e \cos f + e^2] \tag{3.1}$$

where f is the true anomoly.

The angular velocity is then

$$\omega = V/r$$

$$= \sqrt{\frac{\mu}{a^3\ (1 - e^2)^3}\ [1 + e(4 \cos f) + e^2\ (1 + 5 \cos^2 f) + e^3(2 \cos f + 2 \cos^3 f) + e^4(\cos^2 f)]^{1/2}} \tag{3.2}$$

Differentiating (3.2) gives an angular acceleration of

$$\dot{\omega} = -\frac{1}{2}\sqrt{\frac{\mu}{a^3(1-e^2)^3}}\ [\omega e \sin f]\ \cdot$$

$$\cdot \left\{\frac{[4 + 10e \cos f + 2e^2 + 6e^2\cos^2 f]}{[1 + e(4 \cos f) + e^2(1 + 5 \cos^2 f) + e^3(2 \cos f + 2 \cos^3 f)]}\right\}^{1/2} \tag{3.3}$$

The solar pressure force acting on the satellite is given by

$$F = K/r^2$$

$$= \frac{K\ [1 + 2e \cos f + e^2\cos^2 f]}{[a(1-e^2)]^2} \tag{3.4}$$

where K = constant. At a distance of one astronomical unit, the solar pressure[4] is 9.40×10^{-8} lb./ft^2. For the satellite considered, the effective sail area is 1.11 ft^2. Thus the constant in equation (3.4) is

$$K = 0.252 \times 10^{17}\ \text{lb.} - \text{ft}^2. \tag{3.5}$$

In using the computer, it is easier to have time as the independent variable. Consequently, it is necessary to solve Kepler's equation. Rather than solve a transcendental equation for the true anomoly, f, a series solution involving mean anomoly, M, and eccentric anomoly, E, was computed. As given in Battin[5],

$$M = \sqrt{\frac{\mu}{a^3}}\,(t - \tau) \tag{3.6}$$

where τ = time of perehelion passage

$$E = M + [e\left(1 - \frac{e^2}{8}\right) + \frac{e^4}{194} \sin M + \frac{e^2}{2}\left(1 - \frac{e^2}{3}\right) \sin 2M +$$

$$+ \frac{3e^3}{8}\left(1 - \frac{9e^2}{16}\right) \sin 3M] \tag{3.7}$$

$$f = 2 \tan^{-1}\left[\sqrt{\frac{1 + e}{1 - e}} \tan\left(\frac{E}{2}\right)\right] \tag{3.8}$$

The library of the computer contains a subprogram called INDV, DPNV which enables the programmer to obtain the numerical solution of a system of N^{th} order, non-linear, simultaneous ordinary differential equations. The program requires that the equations be written with the second derivatives as a function of first and zero order terms. (See Appendix D). This requirement causes the equations to have an artificial singularity when $\theta = 90°$. For this reason, care was taken to limit θ to values between $-85°$ and $+85°$.

The iterative process is very sensitive to the size of the time increment used for integration. It was found that an increment of 0.00125 times the smallest period of oscillation worked very well.

CHAPTER 4

PASSIVE DAMPING

4.1 General

The equations of motion derived in Chapter 2 included viscous damping torques. For other forms of damping, difficulty is encountered expressing the friction torque in the equations of motion with the proper direction over a complete cycle of oscillation. For this reason an equivalent viscous damping coefficient will be used and will be determined on the basis of equal energy dissipation.

Considering the oscillation to be nearly harmonic, the energy dissipated per cycle by viscous damping can be written as[6]

$$\Delta E = \pi d \omega \beta^2 \tag{4.1}$$

where

$$\beta = \beta_o e^{j\omega t}$$

4.2 Ball-in-tube Damper

If a small metallic ball with low coefficient of restitution is placed in a tube perpendicular to an axis of satellite rotation and a distance L_1 from the center of mass, then the ball will make inelastic collisions with the ends of the tube, thus dissipating energy. Since angular velocities are small, the friction force on the ball due to the tube must be nearly zero, in order to provide any ball motion at all (i.e., the ball must overcome stiction). For this reason, the coefficient of friction must be very small and the tube must have a radius of curvature L_1 in order to minimize the force normal to the tube.

Assume that the tube is frictionless, and that the ball makes a perfectly non-elastic collision with the end of the tube. Over one cycle the ball will make two collisions and the energy dissipated by the ball over this cycle is given by

$$\Delta E = 2\left(\frac{1}{2} m_B V^2\right)$$

$$= \frac{W_B}{g} L_1{}^2 \dot{\beta}^2 \qquad (4.2)$$

where $\dot{\beta}$ is the angular velocity of the satellite about the axis of rotation and is given by

$$\dot{\beta} = \beta_o \omega e^{j\omega t} \qquad (4.3)$$

therefore

$$\Delta E = \frac{W_B L_1{}^2 \beta_o{}^2 \omega^2 e^{2j\omega t}}{g} \qquad (4.4)$$

Equating this to equation (4.1) gives

$$d = \frac{W_B}{\pi g} L_1{}^2 \omega \qquad (4.5)$$

If I is the moment of inertia about the axis of rotation, then the equivalent viscous damping coefficient is also given by

$$d = 2\xi\omega I \qquad (4.6)$$

where ξ is the damping ratio. Combining (4.5) and (4.6) gives a ball weight of

$$W_B = \frac{2g\pi\xi I}{L_1{}^2} \tag{4.7}$$

To reduce the disturbance to 95% of its value within N periods,

$$e^{-\xi\omega t} \triangleq e^{-3} \tag{4.8}$$

$$t = N\left(\frac{2\pi}{\omega}\right) = \frac{3}{\xi\omega} \tag{4.9}$$

$$\xi = \frac{3}{2\pi N} \tag{4.10}$$

and

$$W_B = \frac{3g\ I}{N\ L_1{}^2} \tag{4.11}$$

For the satellite in question, there are two configurations. With long antennas, the principal moments of inertia are

$$I_1 = 0.294 \text{ slug-ft}^2$$

$$I_2 = 6.30 \text{ slug-ft}^2$$

$$I_3 = 6.55 \text{ slug-ft}^2 \tag{4.12}$$

The periods of oscillation at aphelion are then

$$P_1 = 2.44 \text{ hours}$$

$$P_2 = 11.42 \text{ hours}$$

$$P_3 = 7.8 \times 10^3 \text{ hours} \tag{4.13}$$

To damp within five days from the initial disturbance

$$N_1 \cong 49$$

$$N_2 \cong 11 \tag{4.14}$$

It is obvious that the third mode cannot be damped in this period. Using a distance L_1 of 1.75 feet, the weights of the balls are

$$W_{B1} = 0.190 \text{ pounds}$$

$$W_{B2} = 18.1 \text{ pounds} \tag{4.15}$$

Neither of these weights seem small enough to be useful.

For the configuration with short antennas,

$$I_1 = 0.054 \text{ slug-ft}^2$$

$$I_2 = 0.111 \text{ slug-ft}^2$$

$$I_3 = 0.123 \text{ slug-ft}^2 \tag{4.16}$$

Then

$$P_1 = 1.05 \text{ hours}$$

$$P_2 = 1.49 \text{ hours}$$

$$P_3 = 5.56 \times 10^3 \text{ hours} \tag{4.17}$$

Again using a five day damping time

$$N_1 \cong 114$$

$$N_2 \cong 81 \tag{4.18}$$

and

$$W_{B1} = 0.015 \text{ pounds}$$

$$W_{B2} = 0.043 \text{ pounds} \qquad (4.19)$$

Both of these balls appear to be feasible and their radii are given by

$$r_B = \sqrt[3]{\frac{3W_B}{4\pi\rho}} \qquad (4.20)$$

where ρ is the ball density. For lead,

$$r_{B1} = 0.208 \text{ inches}$$

$$r_{B2} = 0.247 \text{ inches} \qquad (4.21)$$

If the maximum displacement from equilibrium is defined as β_o and the tube has an arc length of $2L_1\beta_1$, centered at $\beta = 0$, then a collision will occur provided $\beta_1 < \beta_o$. To determine the optimum tube length, assume

$$\beta(t) = \beta_o \cos\omega t \qquad (4.22)$$

then

$$\dot{\beta}(t) = -\beta_o\omega \sin\omega t \qquad (4.23)$$

If the ball starts at one end of the tube, it must travel an angular distance $2\beta_1$ before making a collision. When the collision is made, it is desirable to have $\dot{\beta}$ at its maximum value in order to increase the energy dissipated. Thus

$$\ddot{\beta}(t) = 0 = -\beta_o\omega^2\cos\omega t$$

or

$$t = \frac{\pi}{2\omega} \tag{4.24}$$

When the collision occurs,

$$\beta(t) = \beta_o - 2\beta_1 = 0$$

or

$$\beta_1 = \frac{\beta_o}{2} \tag{4.25}$$

Thus, to maximize the velocity at collision, β_1 should be approximately one half of β_o. For the satellite in question, it is imperative that the large angular displacements at injection be damped quickly. However, the tube length must be such that it can fit within the sail. Thus, β_1 is limited to twenty-six degrees, giving best damping for angular displacements in the area of fifty degrees. Such a tube will provide damping until the displacement from equilibrium becomes less than twenty-five degrees. If a second tube with an arc length of six inches is placed parallel to the first, then damping to small angles should be assured.

4.3 Structural Damping in Antenna

The antennas can be considered as cantilever beams excited by the satellite acceleration, and thus energy will be dissipated due to structural damping. The amplitude of vibration at any point along the length of the antenna can be approximated by the static deflection curve of a weightless beam with a concentrated load at the end. The energy dissipated by structural damping is given by[7]

$$\Delta E = 2\pi\alpha U_{max} \tag{4.26}$$

where α = structural damping factor

U_{max} = the maximum potential energy

For the assumed beam the maximum potential energy is given by[6]

$$U_{max} = \frac{P^2 L_2{}^3}{6EI} \tag{4.27}$$

where P = force due to satellite acceleration

L_2 = length of antenna

and EI = flexural rigidity of antenna

If the satellite is assumed to have harmonic motion the force P, due to angular acceleration, is

$$P = mL_2\ddot{\beta}$$

$$= - mL_2{}^2\omega^2\beta_o e^{j\omega t} \tag{4.28}$$

The energy dissipated by structural damping is then

$$\Delta E = \frac{\pi\alpha m^2 L_2{}^5\omega^4}{3EI} \quad \beta_o{}^2 e^{2j\omega t} \tag{4.29}$$

Equating (4.1) and (4.29) gives

$$d = \frac{\alpha m^2 L_2{}^5 \omega^3}{3EI} \tag{4.30}$$

For the satellite with long antennas, the antennas are considered as two twenty-one foot and two seven foot 0.25 in. O.D. aluminum tubes. The flexural rigidity is then approximately 100 lb. ft^2. The weight of each antenna is 0.69 lbs. and 0.24 lbs. respectively. The damping factor, α, is assumed to be 0.01.

The equivalent damping coefficients in the vicinity of aphelion are then

$$d_1 = 1.07 \times 10^{-14} \text{ ft lb/rad/sec}$$

$$d_2 = 3.76 \times 10^{-13} \text{ ft lb/rad/sec}$$

$$d_3 = 1.31 \times 10^{-23} \text{ ft lb/rad/sec} \qquad (4.31)$$

For the short antenna configuration the values obtained are even less. For comparison, it is noted that the values of damping coefficient required for a ξ of 0.01 are

$$d_1 = 4.20 \times 10^{-6} \text{ ft lb/rad/sec}$$

$$d_2 = 1.93 \times 10^{-5} \text{ ft lb/rad/sec} \qquad (4.23)$$

Thus, negligible damping is provided due to the very small magnitude of the angular acceleration.

In general, structural damping in the antennas is not considered a feasible means of damping satellite oscillations unless very large angular accelerations are present.

CHAPTER 5

DISCUSSION OF RESULTS

The ability of the simplified model to predict system response to disturbances was tested using five combinations of angular initial conditions. The system disturbances were approximated by small angular displacements and zero angular rates. The general equations of motion (A.13), (A.14), and (A.15) for the long antenna configuration were solved on a digital computer. These results, along with the corresponding simplified equations (2.36), (2.37) and (2.38), were plotted in Figures 3 through 6.

The five cases investigated were (all angles in radians):

1. $\phi(o) = 0, \quad \theta(o) = 0, \quad \psi(o) = 0$ (Figure 3a)

2. $\phi(o) = 0, \quad \theta(o) = 0.1, \quad \psi(o) = 0$ (Figure 3b)

3. $\phi(o) = 0.1, \quad \theta(o) = 0.216 \times 10^{-3}, \quad \psi(o) = 0$ (Figure 4a)

4. $\phi(o) = 0, \quad \theta(o) = 0.216 \times 10^{-3}, \quad \psi(o) = 0.1$ (Figure 5)

5. $\phi(o) = 0.1, \quad \theta(o) = 0.1, \quad \psi(o) = 0.1$ (Figure 6)

In each of the cases, the satellite motion was investigated near aphelion, and all modes used a predicted damping ratio of 0.1.

Except for the discrepancies noted below, predicted values of frequency, phase, and damping time were within five percent of the computer results. This is considered to be within the accuracy of the simplified model.

The purpose of case 1 was to determine the validity of predicting the equilibrium offset angle due to the forcing terms in the θ equation. The offset by computer solution was determined to be 0.216 milliradians, while the simplified model predicted 0.308 milliradians. This is a difference of about 50 percent in the amplitudes of the two curves (see Figure 3a), using the computer solution as a reference. No reason could be found to explain this difference. However, it should be remembered that this equilibrium angle has a maximum of about 0.55 milliradians at perihelion and can be neglected entirely, without causing extensive errors.

For the $\phi(o)$ input (case 3, Figure 4a), the simplified model does not predict the transient oscillation in the response. Since it is rapidly damped, however, it leaves a dominant mode of much lower frequency. The slope of this slower mode is matched quite accurately by the predicted slope, but the two differ slightly in phase.

When a $\psi(o)$ input is present, the θ and ϕ oscillations will be coupled by $\sin\psi$. The effect of this cross-coupling is not predicted by the simplified model since products of small quantities were neglected. This effect is evident, however, in Figures 5 and 6, and causes errors of approximately ten percent for $\psi(o) = 0.1$ radians.

The stability of the equilibrium position defined by $\psi_o = 90°$ for the satellite considered was not verified due to the excessive computer time required.

Several computer solutions of the equations of motion were obtained for the injection of the satellite into orbit. A typical computer run is plotted in Figure 7. For this example the long antenna configuration was used, the damping ratio was 0.1 for all modes, and the initial conditions were $\phi(o) = 0.1$, $\theta(o) = 1.485$, and $\psi(o) = 0.785$ radians. The ϕ and θ responses damped out in a reasonable amount of time, but the ψ motion was characterized by a residual spin. When the satellite was torqued about a principal axis ($\theta(o) = 1.485$, $\phi(o) = 0$, and $\psi(o) = 0$ or 1.5708 radians), however, the residual spin no longer appeared.

CHAPTER 6

CONCLUSIONS AND RECOMMENDED FURTHER STUDY

6.1 Conclusions

The results of this investigation of a solar-pressure stabilized satellite can be summarized as follows:

1. Motion about either principal axis perpendicular to the radial line is characterized by a frequency which is directly proportional to the square root of both the effective sail area and the sail distance from the center of mass, and inversely proportional to the square root of the respective moment of inertia. Motion about the radial line, for small disturbances, is an oscillation at orbital frequency times the square root of the moment of inertia ratio, A.
2. Solution of the equations of motion gives a stable equilibrium position about the radial line when the principal axis having the greater moment of inertia is perpendicular to the plane of the orbit. An unstable equilibrium position was predicted for the case when the axis with the smaller moment of inertia is perpendicular to the plane of the orbit, but this position was not verified on a computer. The steady state lag angle, θ_o, is extremely small, thus the equilibrium position can be re-

garded as along the axes of the attitude reference frame. (One body axis along the radial line, one along the orbital angular velocity vector, and the third forming a right handed orthogonal set.)

3. For small angular displacements from equilibrium, the motion of the satellite can be described by the simplified equations for small portions of the orbit. Furthermore, these equations can be uncoupled completely since cross-coupling terms are several orders of magnitude smaller than the uncoupled terms.
4. It is virtually impossible to damp oscillations about the radial line within a reasonable portion of the orbit unless an active damping system is used.
5. For the short antenna configuration, ball-in-tube damping about principal axes perpendicular to the radial line is feasible, provided two different size tubes are used per axis. Unless sail area is increased considerably, this type of damping is impractical for the long antenna model. Structural damping for both configurations is negligible, thus passive damping seems impractical for the satellite with large antennas.
6. At injection, torquing the satellite into alignment with the radial line about a non-principal axis causes a spin to develop about the yaw axis (i.e., causes $\dot{\psi}$). The other two angles are damped within a reasonable amount of time.

6.2 Further Study Areas

Two major areas stand out for further investigation; passive damping and motion about the solar pointing axis of the satellite.

For ball-in-tube type damping, the stiction problem should be examined in detail, as should non-zero coefficients of restitution. Along this same line, the effect on damping parameters due to an increase in sail area should be looked into. The two methods of passive damping covered in this report were very elementary types, and a more ingeneous system might give better results.

Lack of ability to damp about the radial line, coupled with the residual spin from injection, seems to indicate that an active damping mechanism is needed to align a principal axis with the plane of the orbit. The feasibility of using a gyroscope for an inertial reference and coupling it to the satellite with a viscous damper is one of many systems that could be studied.

APPENDIX A

DERIVATION OF THE EQUATIONS OF MOTION

The attitude reference frame and the body axes are initially aligned. Three rotations are then made in the order ϕ, θ, and ψ. Referring to Figure 1, the angular velocity components are obtained as follows:

Step 1. Rotate about the x-axis by ϕ. Then,

$$\omega 1_{\phi} = \dot{\phi}$$

$$\omega 2_{\phi} = (\cos\phi)\omega$$

$$\omega 3_{\phi} = -\ \omega \sin\phi$$

Step 2. Rotate about the $\omega 2_{\phi}$ axis by θ.

$$\omega 1_{\theta} = \omega 1_{\phi}\cos\theta - \omega 3_{\phi}\sin\theta$$

$$= \dot{\phi}\ \cos\theta + \omega\ \sin\phi\ \sin\theta$$

$$\omega 2_{\theta} = \omega 2_{\phi} + \dot{\theta}$$

$$= \omega\ \cos\phi + \dot{\theta}$$

$$\omega 3_{\theta} = \omega 1_{\phi}\sin\theta + \omega 3_{\phi}\cos\theta$$

$$= \dot{\phi}\ \sin\theta - \omega\ \sin\phi\ \cos\theta$$

Step 3. Rotate about axis ω_{3_θ} by ψ to get the final position.

$$\omega_1 = \omega_{1_\theta}\cos\psi + \omega_{2_\theta}\sin\psi$$

$$= \dot{\phi}\cos\theta\cos\psi + \omega\sin\phi\sin\theta\cos\psi + \omega\cos\phi\sin\psi + \dot{\theta}\sin\psi \tag{A.1}$$

$$\omega_2 = -\omega_{1_\theta}\sin\psi + \omega_{2_\theta}\cos\psi$$

$$= -\dot{\phi}\cos\theta\sin\psi - \omega\sin\phi\sin\theta\sin\psi + \omega\cos\phi\cos\psi + \dot{\theta}\cos\psi \tag{A.2}$$

$$\omega_3 = \omega_{3_\theta} + \dot{\psi}$$

$$= \dot{\phi}\sin\theta - \omega\sin\phi\cos\theta + \dot{\psi} \tag{A.3}$$

The solar pressure is always directed along the negative Z axis of the attitude reference coordinates. The solar pressure force, F, is equal to the solar pressure times the projected sail area. Using the same rotations given in steps 1, 2, and 3 above, the force can be resolved into body axes to give:

Step 1.

$$F_{1_\phi} = 0$$

$$F_{2_\phi} = -F\sin\phi$$

$$F_{3_\phi} = -F\cos\phi$$

Step 2.

$$F_{1_\theta} = - F_{3_\phi} \sin\theta$$

$$= + F \cos\phi \sin\theta$$

$$F_{2_\theta} = F_{2_\phi} = - F \sin\phi$$

$$F_{3_\theta} = F_{3_\phi} \cos\theta$$

$$= - F \cos\phi \cos\theta$$

Step 3.

$$F_1 = F_{1_\theta} \cos\psi + F_{2_\theta} \sin\psi$$

$$= F \cos\phi \sin\theta \cos\psi - F \sin\phi \sin\psi \qquad (A.4)$$

$$F_2 = - F_{1_\theta} \sin\psi + F_{2_\theta} \cos\psi$$

$$= - F \cos\phi \sin\theta \sin\psi - F \sin\phi \cos\psi \qquad (A.5)$$

$$F_3 = F_{3_\theta} = - F \cos\phi \cos\theta \qquad (A.6)$$

The distance from the center of mass to the center of pressure is the moment arm for the solar pressure force, and is given by the constant L about axes 1 and 2, and given by zero about axis 3. If a damping moment proportional to the angular velocity in body axes is also included, the total moment applied is given by,

$$M_1 = F_2 L - d_1 \omega_1 \qquad (A.7)$$

$$M_2 = - F_1 L - d_2 \omega_2 \qquad (A.8)$$

$$M_3 = - d_3 \omega_3 \qquad (A.9)$$

where d is a viscous damping coefficient.

Euler's equations of motion in body axes are given by,

$$M_1 = I_1\dot{\omega}_1 + (I_3 - I_2)\omega_3\omega_2 \tag{A.10}$$

$$M_2 = I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 \tag{A.11}$$

$$M_3 = I_3\dot{\omega}_3 + (I_2 - I_1)\omega_2\omega_1 \tag{A.12}$$

Differentiating ω_1, ω_2, and ω_3 and substituting in equations (A.10), (A.11) and (A.12) gives,

$$\begin{aligned}
&- FL(\sin\phi\ \cos\psi + \cos\phi\ \sin\theta\ \sin\psi) = I_1(\ddot{\phi}\ \cos\theta\ \cos\psi + \\
&- \dot{\phi}\ \dot{\theta}\ \sin\theta\ \cos\psi - \dot{\phi}\ \dot{\psi}\ \cos\theta\ \sin\psi + \ddot{\theta}\ \sin\psi + \dot{\theta}\ \dot{\psi}\ \cos\psi + \\
&+ \dot{\omega}\ \sin\phi\ \sin\theta\ \cos\psi + \omega\ \dot{\phi}\ \cos\phi\ \sin\theta\ \cos\psi + \\
&+ \dot{\omega}\ \dot{\theta}\ \sin\phi\ \cos\theta\ \cos\psi - \omega\ \dot{\psi}\ \sin\phi\ \sin\theta\ \sin\psi + \dot{\omega}\ \cos\phi\ \sin\psi + \\
&- \omega\ \dot{\phi}\ \sin\phi\ \sin\psi + \omega\ \dot{\psi}\ \cos\phi\ \cos\psi) + (I_3 - I_2)(\dot{\psi} + \dot{\phi}\ \sin\theta + \\
&- \omega\ \sin\phi\ \cos\theta)(\dot{\theta}\ \cos\psi - \dot{\phi}\ \cos\theta\ \sin\psi + \omega\ \cos\phi\ \cos\psi + \\
&- \omega\ \sin\phi\ \sin\theta\ \sin\psi) + d_1(\dot{\phi}\ \cos\theta\ \cos\psi + \dot{\theta}\ \sin\psi + \\
&+ \omega\ \cos\phi\ \sin\psi + \omega\ \sin\phi\ \sin\theta\ \cos\psi)
\end{aligned} \tag{A.13}$$

$$- FL(\cos\phi \sin\theta \cos\psi - \sin\phi \sin\psi) = I_2(\ddot{\theta} \cos\psi +$$
$$- \dot{\theta} \dot{\psi} \sin\psi - \ddot{\phi} \cos\theta \sin\psi + \dot{\phi} \dot{\theta} \sin\theta \sin\psi - \dot{\phi} \dot{\psi} \cos\theta \cos\psi +$$
$$+ \dot{\omega} \cos\phi \cos\psi - \omega \dot{\phi} \sin\phi \cos\psi - \omega \dot{\psi} \cos\phi \sin\psi +$$
$$- \dot{\omega} \sin\phi \sin\theta \sin\psi - \omega \dot{\phi} \cos\phi \sin\theta \sin\psi +$$
$$- \omega \dot{\theta} \sin\phi \cos\theta \sin\psi - \omega \dot{\psi} \sin\phi \sin\theta \cos\psi) + (I_1 - I_3) \cdot$$
$$\cdot (\dot{\phi} \cos\theta \cos\psi + \dot{\theta} \sin\psi + \omega \cos\phi \sin\psi +$$
$$+ \omega \sin\phi \sin\theta \cos\psi)(\dot{\psi} + \dot{\phi} \sin\theta - \omega \sin\phi \cos\theta) + d_2 \cdot$$
$$\cdot (\dot{\theta} \cos\psi - \dot{\phi} \cos\theta \sin\psi + \omega \cos\phi \cos\psi - \omega \sin\phi \sin\theta \sin\psi) \tag{A.14}$$

$$0 = I_3(\ddot{\psi} + \ddot{\phi} \sin\theta + \dot{\phi} \dot{\theta} \cos\theta - \dot{\omega} \sin\phi \cos\theta +$$
$$- \omega \dot{\phi} \cos\phi \cos\theta + \omega \dot{\theta} \sin\phi \sin\theta) + (I_2 - I_1)(\dot{\theta} \cos\psi +$$
$$- \dot{\phi} \cos\theta \sin\psi + \omega \cos\phi \cos\psi - \omega \sin\phi \sin\theta \sin\psi) \cdot$$
$$\cdot (\dot{\phi} \cos\theta \cos\psi + \dot{\theta} \sin\psi + \omega \cos\phi \sin\psi +$$
$$+ \omega \sin\phi \sin\theta \cos\psi) + d_3(\dot{\psi} + \dot{\phi} \sin\theta - \omega \sin\phi \cos\theta) \tag{A.15}$$

Assume ϕ, $\dot{\phi}$, $\ddot{\phi}$, θ, $\dot{\theta}$, $\ddot{\theta}$, $\dot{\psi}$ and $\ddot{\psi}$ are small quantities and neglect the products of two of these quantities. (Note: Angle ψ is not small). Also let

$$\sin\theta \cong \theta, \ \cos\theta \cong 1, \text{ etc.}$$

so that the equations of motion simplify to

$$\frac{-FL}{I_1}(\phi \cos\psi + \theta \sin\psi) = \ddot{\phi} \cos\psi + \ddot{\theta} \sin\psi + \dot{\omega} \sin\psi +$$
$$+ (C + 1)\omega \dot{\psi} \cos\psi - C \phi \omega^2 \cos\psi + D_1(\dot{\phi} \cos\psi + \dot{\theta} \sin\psi + \omega \sin\psi) \tag{A.16}$$

where

$$C = \frac{I_3 - I_2}{I_1} \qquad \text{and} \qquad D_1 = \frac{d_1}{I_1} \tag{A.16a}$$

$$\frac{FL}{I_2}(\phi \sin\psi - \theta \cos\psi) = \ddot{\theta} \cos\psi - \ddot{\phi} \sin\psi + \dot{\omega} \cos\psi +$$
$$- \omega \dot{\psi} \sin\psi - B \omega \dot{\psi} \sin\psi + B \phi \omega^2 \sin\psi + D_2(\dot{\theta} \cos\psi +$$
$$- \dot{\phi} \sin\psi + \omega \cos\psi) \tag{A.17}$$

where

$$B = \frac{I_3 - I_1}{I_2} \qquad \text{and} \qquad D_2 = \frac{d_2}{I_2} \tag{A.17a}$$

$$\ddot{\psi} - \dot{\omega} \phi - \omega \dot{\phi} + 2A \omega \dot{\theta} \sin\psi \cos\psi + A \omega \dot{\phi}(\cos^2\psi +$$
$$- \sin^2\psi) + A \omega^2 \sin\psi \cos\psi + D_3(\dot{\psi} - \omega \phi) = 0 \tag{A.18}$$

where

$$A = \frac{I_2 - I_1}{I_3} \qquad \text{and} \qquad D_3 = \frac{d_3}{I_3} \tag{A.18a}$$

Let ψ_o be the equilibrium offset angle at any point in the orbit, and let the body oscillate about this point by a small angle, ψ. Replacing ψ by $\psi_o + \psi$ in the above equations now makes it possible to make small angle assumptions on ψ. That is,

$$\sin(\psi + \psi_o) = \sin\psi_o \cos\psi + \cos\psi_o \sin\psi$$

$$\cong \sin\psi_o + \psi \cos\psi_o$$

and

$$\cos(\psi + \psi_o) = \cos\psi_o \cos\psi - \sin\psi_o \sin\psi$$

$$\cong \cos\psi_o - \psi \sin\psi_o$$

Making these substitutions in equations (A.16), (A.17) and (A.18) gives the linear equations of motion with time varying coefficients.

$$(\cos\psi_o)\ddot{\phi} + (D_1\cos\psi_o)\dot{\phi} + \left(\left[\frac{FL}{I_1} - C\omega^2\right]\cos\psi_o\right)\phi + (\sin\psi_o)\ddot{\theta} +$$

$$+ (D_1\sin\psi_o)\dot{\theta} + \left(\frac{FL}{I_1}\sin\psi_o\right)\theta + ([C + 1]\omega\cos\psi_o)\dot{\psi} +$$

$$+ (\dot{\omega}\cos\psi_o + D_1\omega\cos\psi_o)\psi = -\dot{\omega}\sin\psi_o - \omega D_1\sin\psi_o \tag{A.19}$$

$$(-\sin\psi_o)\ddot{\phi} + (-D_2\sin\psi_o)\dot{\phi} + \left(\left[B\omega^2 - \frac{FL}{I_2}\right]\sin\psi_o\right)\phi +$$

$$+ (\cos\psi_o)\ddot{\theta} + (D_2\cos\psi_o)\dot{\theta} + \left(\frac{FL}{I_2}\cos\psi_o\right)\theta +$$

$$- ([1 + B]\omega\sin\psi_o)\dot{\psi} + (-\dot{\omega}\sin\psi_o - \omega D_2\sin\psi_o)\psi =$$

$$- \dot{\omega}\cos\psi_o - \omega D_2\cos\psi_o \qquad (A.20)$$

and

$$\left(\left[A\{\cos^2\psi_o - \sin^2\psi_o\} - 1\right]\omega\right)\dot{\phi} + (-\dot{\omega} - D_3\omega)\phi +$$

$$+ (2A\ \omega\sin\psi_o\cos\psi_o)\dot{\theta} + \ddot{\psi} + D_3\dot{\psi} +$$

$$+ \left(A\ \omega^2\left[\cos^2\psi_o - \sin^2\psi_o\right]\right)\psi = -A\ \omega^2\sin\psi_o\cos\psi_o \qquad (A.21)$$

APPENDIX B

SATELLITE EQUILIBRIUM POSITION

Using equations (19), (20) and (21) of Appendix A, let $\ddot{\phi}$, $\dot{\phi}$, $\ddot{\theta}$, $\dot{\theta}$, $\ddot{\psi}$, $\dot{\psi}$ and ψ go to zero. Substitute $\phi = \phi_o$, and $\theta = \theta_o$, and solve for ϕ_o, θ_o and ψ_o to define the equilibrium position as a function of ω, $\dot{\omega}$ and F. The equations reduce to,

$$\left[\left(\frac{FL}{I_1} - C\,\omega^2 \right) \cos\psi_o \right] \phi_o + \left[\frac{FL}{I_1} \sin\psi_o \right] \theta_o =$$

$$- \dot{\omega} \sin\psi_o - \omega D_1 \sin\psi_o \tag{B.1}$$

$$\left[\left(B\,\omega^2 - \frac{FL}{I_2} \right) \sin\psi_o \right] \phi_o + \left[\frac{FL}{I_2} \cos\psi_o \right] \theta_o =$$

$$- (\dot{\omega} + \omega D_2) \cos\psi_o \tag{B.2}$$

$$(\dot{\omega} + D_3\omega)\phi_o = A\,\omega^2 \sin\psi_o \cos\psi_o \tag{B.3}$$

Solving equations (B.1) and (B.2) for θ_o and equating gives,

$$\frac{\left[\left(C\omega^2 - \frac{FL}{I_1} \right) \cos\psi_o \right] \phi_o - (\dot{\omega} + \omega\, D_1) \sin\psi_o}{\frac{FL}{I_1} \sin\psi_o} =$$

$$\frac{\left[\left(\frac{FL}{I_2} - B\omega^2\right)\sin\psi_o\right]\phi_o - (\dot{\omega} + \omega\, D_2)\cos\psi_o}{\frac{FL}{I_2}\cos\psi_o} \tag{B.4}$$

Solving (B.3) for ϕ_o and substituting gives,

$$\frac{\cos\psi_o\,\sin\psi_o}{FL}\left\{I_1\left(C\omega^2 - \frac{FL}{I_1}\right)\cos^2\psi_o\,\frac{A\omega^2}{\dot{\omega} + D_3\omega} + \right.$$

$$- I_1(\dot{\omega} + \omega D_2) + I_2\left(B\omega^2 - \frac{FL}{I_2}\right)\sin^2\psi_o\,\frac{A\omega^2}{\dot{\omega} + D_3\omega} +$$

$$\left. + I_2(\dot{\omega} + D_1\omega)\right\} = 0 \tag{B.5}$$

The solution of equation (B.5) is either $\psi_o = 0°$, $\psi_o = 90°$, or the bracketed term is zero. Investigating the latter and substituting

$$\sin^2\psi_o = 1 - \cos^2\psi_o$$

gives

$$\cos^2\psi_o = \frac{AFL\omega^2 + I_1(\dot{\omega} + \omega\, D_1)(\dot{\omega} + D_3\omega)}{\omega^4(I_1AC - I_2AB)} +$$

$$- \frac{I_2AB\omega^4 + I_2(\dot{\omega} + \omega\, D_2)(\dot{\omega} + D_3\omega)}{\omega^4(I_1AC - I_2AB)} \tag{B.6}$$

The first term is much larger than any other term in the equation and can be approximated by

$$\text{term} \cong \frac{FL\ \omega^2}{I_1 C\omega^4} = \frac{{\omega_1}^2}{C\omega^2} >> 1$$

Since $\cos^2\psi_o$ cannot be greater than one, there are no real roots in this expression. Therefore, the only real roots are $\psi_o = 0$ and $\psi_o = 90°$.

If $\sin\psi_o = 0$, then

$$\psi_o = 0$$

$$\phi_o = 0$$

$$\theta_o = \frac{-\ I_2(\dot{\omega} + \omega D_2)}{FL} \tag{B.7}$$

If $\cos\psi_o = 0$, then

$$\psi_o = 90°$$

$$\phi_o = 0$$

$$\theta_o = \frac{-\ I_1(\dot{\omega} + D_1\omega)}{FL} \tag{B.8}$$

Stability of these equilibrium positions is investigated in Chapter 2.

APPENDIX C

FACTORIZATION OF THE CHARACTERISTIC EQUATION USING LIN'S METHOD OF APPROXIMATION

If $\sin\psi_o \triangleq 0$ and damping is included, then the Laplace transform of the equations of motion can be given as,

$$\begin{bmatrix} (C+1)\omega s + \dot{\omega} + \omega D_1 & s^2 + D_1 s + \frac{FL}{I_1} - C\omega^2 & 0 \\ 0 & 0 & s^2 + D_2 s + \frac{FL}{I_2} \\ s^2 + D_3 s + \omega^2 A & (A-1)\omega s - \dot{\omega} - \omega D_3 & 0 \end{bmatrix} \begin{bmatrix} \psi \\ \phi \\ \theta \end{bmatrix} =$$

$$\begin{bmatrix} (s+D_3)\psi(o) + (A-1)\omega\,\phi(o) \\ -\frac{\dot{\omega}}{s} - \frac{D_2\omega}{s} + (s+D_2)\theta\,(o) \\ (s+D_1)\phi(o) + (C+1)\omega\,\psi(o) \end{bmatrix} \qquad (C.1)$$

The θ equation can be decoupled from the ψ and ϕ equations leaving

$$\begin{bmatrix} (C+1)\omega\, s + \dot{\omega} + \omega D_1 & s^2 + D_1 s + \frac{FL}{I_1} - C\omega^2 \\ s^2 + D_3 s + A\,\omega^2 & (A-1)\omega\, s - \dot{\omega} - \omega D_3 \end{bmatrix} \begin{bmatrix} \Psi \\ \Phi \end{bmatrix} =$$

$$\begin{bmatrix} (s + D_3)\psi(o) + (A-1)\omega\,\phi(o) \\ (s + D_1)\phi(o) + (C+1)\omega\,\psi(o) \end{bmatrix} \tag{C.2}$$

The determinant is then given by

$$- \Delta = s^4 + [D_1 + D_3]s^3 + [(1 - AC)\omega^2 + \frac{FL}{I_1} + D_1 D_3]s^2 +$$
$$+ [C\omega\dot{\omega} + 2\omega\dot{\omega} + \omega^2 D_3 - A\omega\dot{\omega} + D_1\omega^2 + D_3\frac{FL}{I_1}]s +$$
$$+ [\dot{\omega}^2 + \omega\dot{\omega}(D_3 + D_1) + \omega^2 D_1 D_3 + \frac{AFL}{I_1}\omega^2 - AC\omega^4] \tag{C.3}$$

As a <u>first</u> step toward finding the characteristic frequencies, let $D_1 = D_2 = D_3 = 0$ which simplifies the equation to

$$- \Delta = s^4 + [(1 - AC)\omega^2 + \frac{FL}{I_1}]s^2 + [(C - A + 2)\omega\dot{\omega}]s +$$
$$+ [\dot{\omega}^2 + \frac{FL}{I_1}A\omega^2 - AC\omega^4] \tag{C.4}$$

Furthermore, assume that the coefficient of s is small and can be neglected. It is also known that the libration frequency, $\sqrt{\frac{FL}{I_1}}$, is much greater than the orbital frequency and the orbital angular acceleration. The determinant then reduces to

$$- \Delta = s^4 + \omega_1{}^2 s^2 + A\omega^2 \omega_1{}^2 \tag{C.5}$$

where

$$\omega_1{}^2 = \frac{FL}{I_1} \tag{C.6}$$

Factoring gives,

$$s_{1,2}{}^2 = \frac{- \omega_1{}^2 \pm \sqrt{\omega_1{}^4 - 4A\omega^2\omega_1{}^2}}{2} \tag{C.7}$$

Using the binomial expansion,

$$s_{1,2}{}^2 \cong \frac{- \omega_1{}^2 \pm \omega_1{}^2 \left(1 - \frac{2A\omega^2}{\omega_1{}^2} \right)}{2} \tag{C.8}$$

$$s_1{}^2 = - A\omega^2$$

$$s_2{}^2 = - \omega_1{}^2 + A\omega^2 \tag{C.9}$$

Furthermore, $\omega^2 << \omega_1{}^2$ and the determinant is approximated by

$$- \Delta = (s^2 + A\omega^2)(s^2 + \omega_1{}^2) \tag{C.10}$$

This simplification shows that the two modes of the characteristic equation are widely separated and that equation (C.3) can be factored using Lin's approximation[2].

Given a fourth order differential equation composed of two widely separated second order equations, Lin's method can be applied as follows:

$$\Delta = s^4 + Es^3 + Bs^2 + Fs + D$$

Take the terms $(Bs^2 + Fs + D)$ and change to $\left(s^2 + \frac{F}{B} s + \frac{D}{B}\right)$. Divide this second order factor into the determinant. If the remainder is small, then the divisor and the quotient are the factors; if not, then a more complicated process must be used.

The factors are then

$$\Delta = \left[s^2 + \frac{F}{B} s + \frac{D}{B}\right]\left[s^2 + \left(E - \frac{F}{B}\right) s + B - \frac{D}{B} - \left(E - \frac{F}{B}\right)\frac{F}{B}\right] \tag{C.11}$$

with a remainder of

$$Re = \left[F - \left(E - \frac{F}{B}\right)\frac{D}{B} - \left(B - \frac{D}{B}\right)\frac{F}{B} + \left(E - \frac{F}{B}\right)\left(\frac{F}{B}\right)^2\right] s + $$

$$+ D - \frac{D}{B}\left(B - \frac{D}{B}\right) + \left(E - \frac{F}{B}\right)\frac{FD}{B^2} \tag{C.12}$$

Assume $\frac{FL}{I_1} = \omega_1{}^2 >> \omega^2$

and $\omega_1{}^2 >> \dot{\omega}$

Utilizing these two assumptions, equations (C.3) and (C.4) reduce to

$$- \Delta = s^4 + [D_1 + D_3]s^3 + [\omega_1{}^2 + D_1D_3]s^2 + [D_3\omega_1{}^2]s +$$

$$+ \omega\dot{\omega}[D_1 + D_3] + \omega^2 D_1 D_3 + A\omega^2\omega_1{}^2 \tag{C.3a}$$

and

$$-\Delta = s^4 + \omega_1{}^2 s^2 + (C - A + 2)\omega\dot{\omega} s + A\omega^2\omega_1{}^2 \qquad \text{(C.4a)}$$

Looking at (C.4a), the simpler of the two equations,

$$E = 0$$

$$B = \omega_1{}^2$$

$$F = (C - A + 2)\omega\dot{\omega}$$

$$D = A\omega_1{}^2\omega^2$$

Substituting into equation (C.11) and neglecting small terms gives

$$-\Delta = \left(s^2 + \frac{(C - A + 2)}{\omega_1{}^2}\omega\dot{\omega} s + A\omega^2\right)\left(s^2 - \frac{(C - A + 2)}{\omega_1{}^2}\omega\dot{\omega} s + \omega_1{}^2\right) \qquad \text{(C.13)}$$

with a remainder which is small compared to the original equation.

When damping is included,

$$E = D_1 + D_3$$

$$B = \omega_1{}^2 + D_1 D_3$$

$$F = D_3\omega_1{}^2$$

$$D = \omega\dot{\omega}(D_1 + D_3) + \omega^2 D_1 D_3 + A\omega^2\omega_1{}^2$$

and substitution yields,

$$- \Delta = \left[s^2 + \left(\frac{D_3 \omega_1{}^2}{\omega_1{}^2 + D_1 D_3} \right) s + \frac{\omega \dot{\omega} (D_1 + D_3) + \omega^2 D_1 D_3 + A \omega^2 \omega_1{}^2}{\omega_1{}^2 + D_1 D_3} \right] \cdot$$

$$\left[s^2 + \left(1 + \frac{D_3{}^2}{\omega_1{}^2 + D_1 D_3} \right) D_1 s + \omega_1{}^2 + D_1 D_3 + \right.$$

$$\left. \left(\frac{D_1 D_3 \omega_1{}^2}{\omega_1{}^2 + D_1 D_3} \right) \left(\frac{- D_3{}^2}{\omega_1{}^2 + D_1 D_3} - 1 \right) \right] \qquad \text{(C.14)}$$

The remainder, fortunately, is small again and can be neglected. Since D_1 and D_3 are expected to be much less than ω_1, (C.14) can be further simplified to

$$- \Delta \cong [s^2 + D_3 s + A\omega^2] [s^2 + D_1 s + \omega_1{}^2] \qquad \text{(C.15)}$$

APPENDIX D

COMPUTER PROGRAM

In order to solve the equations of motion on the digital computer, it was necessary to solve for the second derivative in terms of first order quantities. Thus,

$$M_1 = I_1\dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3 + d_1\omega_1$$

$$\dot{\omega}_1 = \frac{M_1}{I_1} - \frac{(I_3 - I_2)}{I_1}\,\omega_2\omega_3 - \frac{d_1\omega_1}{I_1} = \text{TENT}$$

From the derivation, $\dot{\omega}_1$ is also given by

$$\dot{\omega}_1 = \ddot{\phi}\cos\theta\,\cos\psi + \ddot{\theta}\sin\psi + \text{EN}$$

where

$$\text{EN} = -\dot{\phi}\,\dot{\theta}\sin\theta\,\cos\psi - \dot{\phi}\,\dot{\psi}\cos\theta\,\sin\psi + \dot{\theta}\,\dot{\psi}\cos\psi +$$

$$+\,\dot{\omega}\sin\phi\,\sin\theta\,\cos\psi + \omega\,\dot{\phi}\cos\phi\,\sin\theta\,\cos\psi +$$

$$+\,\omega\,\dot{\theta}\sin\phi\,\cos\theta\,\cos\psi - \omega\,\dot{\psi}\sin\phi\,\sin\theta\,\sin\psi +$$

$$+\,\dot{\omega}\cos\phi\,\sin\psi - \omega\,\dot{\phi}\sin\phi\,\sin\psi + \omega\,\dot{\psi}\cos\phi\,\cos\psi$$

Thus

$$\ddot{\phi}\cos\theta\cos\psi + \ddot{\theta}\sin\psi + EN = TENT$$

$$\ddot{\phi}\cos\theta\cos\psi + \ddot{\theta}\sin\psi = TENT - EN = Q \qquad (D.1)$$

Similarly

$$M_2 = I_2\dot{\omega}_2 + (I_1 - I_3)\omega_1\omega_3 + d_2\omega_2$$

$$\dot{\omega}_2 = \frac{M_2}{I_2} - \frac{(I_1 - I_3)}{I_2}\omega_1\omega_3 - \frac{d_2}{I_2}\omega_2 = ZETA$$

From the derivation

$$\dot{\omega}_2 = \ddot{\theta}\cos\psi - \ddot{\phi}\cos\theta\sin\psi + GAMMA$$

where

$$GAMMA = -\dot{\theta}\dot{\psi}\sin\psi + \dot{\phi}\dot{\theta}\sin\theta\sin\psi - \dot{\phi}\dot{\psi}\cos\theta\cos\psi +$$

$$+\dot{\omega}\cos\phi\cos\psi - \omega\dot{\phi}\sin\phi\cos\psi - \omega\dot{\psi}\cos\phi\sin\psi +$$

$$-\dot{\omega}\sin\phi\sin\theta\sin\psi + \omega\dot{\phi}\cos\phi\sin\theta\sin\psi +$$

$$-\omega\dot{\theta}\sin\phi\cos\theta\sin\psi - \omega\dot{\psi}\sin\phi\sin\theta\cos\psi$$

So that

$$\ddot{\theta}\cos\psi - \ddot{\phi}\cos\theta\sin\psi = ZETA - GAMMA = R \qquad (D.2)$$

The third equation of motion is given by

$$0 = I_3\dot{\omega}_3 + (I_2 - I_1)\omega_2\omega_1 + d_3\omega_3$$

$$\dot{\omega}_3 = -\frac{(I_2 - I_1)}{I_3}\omega_2\omega_1 - \frac{d_3}{I_3}\omega_3 = YOU$$

And

$$\dot{\omega}_3 = \ddot{\psi} + \ddot{\phi} \sin\theta + \text{TAMB}$$

where

$$\text{TAMB} = \dot{\phi}\, \dot{\theta} \cos\theta - \dot{\omega} \sin\phi \cos\theta - \omega\, \dot{\phi} \cos\phi \cos\theta +$$

$$+ \omega\, \dot{\theta} \sin\phi \sin\theta$$

Thus,

$$\ddot{\psi} + \ddot{\phi} \sin\theta = \text{YOU} - \text{TAMB} = S \tag{D.3}$$

Putting equations (D.1), (D.2), and (D.3) in matrix form gives,

$$\begin{bmatrix} \cos\theta \cos\psi & \sin\psi & 0 \\ -\cos\theta \sin\psi & \cos\psi & 0 \\ \sin\theta & 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} Q \\ R \\ S \end{bmatrix}$$

The determinant is then given by

$$\Delta = \cos\theta \cos^2\psi + \cos\theta \sin^2\psi$$

$$= \cos\theta$$

$$\ddot{\phi} = \frac{Q \cos\psi - R \sin\psi}{\cos\theta} \tag{D.4}$$

$$\ddot{\theta} = R \cos\psi + Q \sin\psi \tag{D.5}$$

$$\ddot{\psi} = S + \frac{R \sin\psi \sin\theta}{\cos\theta} - \frac{Q \cos\psi \sin\theta}{\cos\theta} \tag{D.6}$$

A copy of the program is also contained in this appendix.

```
*       M4114-3627,FMS,DEBUG,1,1,500,0
*       XEQ
*       LIST
*       LABEL
C           SOLUTION OF THE GENERAL EQUATIONS OF MOTION
        COMMON D1,D2,D3, DELTPR,TOT,DELT,T,TH,DTH,PH,
   2     DPH,PS,DPS,TPRINT,A,B,C,AL,CON,ERT1,ERT2,ERT3
        READ8,D1,D2,D3
        READ8,TH,PH,PS
        READ8,DTH,DPH,DPS
        READ8,A,B,C
        READ8,ERT1,ERT2,ERT3
        READ8,TOT,T,TPRINT
        READ9,AL,CON
        READ9,DELT,DELTPR
  8     FORMAT(3E20.5)
  9     FORMAT(2E20.5)
  1     FORMAT(8E15.6)
C       CONSTANTS
        E=.31
        UM=0.469E22
        SMA=0.37E12
        TAU=0.103244E8
        SENSE LIGHT 1
C       ALGEBRAIC EQUATIONS (MEAN ANOMOLY= AM, MU= UM)
  2     AM= ((UM/(SMA)**3.)**(.5))*(T-TAU)
        EA=AM+E*(1.-(E**2.)*(.125)+(E**4.)*(.00515))*SINF(AM)
   2    +(E**2.)*(.5)*(1.-(E**2.)*(.333))*SINF(2.*AM) + (.375)*
   3    (E**3.)*(1.-(9./16.)*(E**2.))* SINF (3.*AM)
        F=2.* ATANF (    (((1.+E)/(1.-E))**(.5))* TANF (.5*EA))
        RA=(SMA)*(1.-(E**2.))/(1.+E* COSF(F))
        V=(UM*((2./RA)-(1./SMA)))**(.5)
        W=V/RA
        FP=(CON)/( RA**2.)
C        WW IS THE DERIVATIVE OF W, THE ANGULAR VEL.
        WW=-(((  UM/(((SMA)*(1.-(E**2.)))**3.))**(.5))*W*E*
   2    SINF(F)* (4. + 10.*E*COSF(F)+(.1922)+(.5766)*(COSF(F)**2.))
   3    /((1.+E*4.*COSF(F)+(.0961)*(1.+5.*(COSF(F)**2.))+(E**3.)*
   4    (2.*COSF(F)+2.*(COSF(F)**3.)))**(.5)))*(.5)
C       EQUATIONS OF MOTION
        D2TH=R*COSF(PS)+Q*SINF(PS)
        R=ZETA-GAMMA
        ZETA=((TWOMO)/ERT2)+B*W1*W3-D2*W2
        W1=DPH*COSF(TH)*COSF(PS)+DTH*SINF(PS)+W*COSF(PH)
   2    *SINF(PS)+W*SINF(PH)*SINF(TH)*COSF(PS)
        W2=DTH*COSF(PS)-DPH*COSF(TH)*SINF(PS)+
   2    W*COSF(PH)*COSF(PS)-W*SINF(PH)*SINF(TH)*SINF(PS)
        W3=DPS+DPH*SINF(TH)-W*SINF(PH)*COSF(TH)
        GAMMA=-DTH*DPS*SINF(PS)+DPH*DTH*SINF(TH)*SINF(PS)
   2    -DPH*DPS*COSF(TH)*COSF(PS)+WW*COSF(PH)*COSF(PS)-
   3    W*DPH*SINF(PH)*COSF(PS)-W*DPS*COSF(PH)*SINF(PS)-
   4    WW*SINF(PH)*SINF(TH)*SINF(PS)-W*DPH*COSF(PH)*
   5    SINF(TH)*SINF(PS)-W*DTH*SINF(PH)*COSF(TH)*SINF(PS)
   6    -W*DPS*SINF(PH)*SINF(TH)*COSF(PS)
        TWOMO=-FP*AL*(-SINF(PH)*SINF(PS)+COSF(PH)*SINF(TH)*
```

```
  2 COSF(PS))
    Q=TENT-EN
    TENT=(ONEMO)/(ERT1)-C*W2*W3-D1*W1
    ONEMO=-(FP*AL)*(SINF(PH)*COSF(PS)+COSF(PH)*
  2 SINF(TH)*SINF(PS))
    EN=-DPH*DTH*SINF(TH)*COSF(PS)-DPH*DPS*COSF(TH)
  2 *SINF(PS)+DTH*DPS*COSF(PS)+WW*SINF(PH)*SINF(TH)
  3 * COSF(PS)+W*DPH*COSF(PH)*SINF(TH)*COSF(PS)+W*DTH*
  4 SINF(PH)*COSF(TH)*COSF(PS)-W*DPH*SINF(PH)*SINF(PS)
  5 +W*DPS*COSF(PH)*COSF(PS)-W*DPS*SINF(PH)*SINF(TH)*SINF(PS)
  6 +WW*COSF(PH)*SINF(PS)
    D2PH=(1./COSF(TH))*(Q*COSF(PS)-R*SINF(PS))
    D2PS=S                +(R*SINF(PS)*SINF(TH))*(1./
  2 COSF(TH))-(Q*SINF(TH)*COSF(PS))*(1./COSF(TH))
    S=YOU-TAMB
    YOU=-A*W1*W2-D3*W3
    TAMB=DPH*DTH*COSF(TH)-WW*SINF(PH)*COSF(TH)
  2 -W*DPH*COSF(PH)*COSF(TH)+W*DTH*SINF(PH)*SINF(TH)
    IF (T-TPRINT) 4,3,3
 3  PRINT1,T,TH,PH,PS,DTH,DPH,DPS,F
    TPRINT=TPRINT+DELTPR
 4  IF(T-TOT) 5,6,6
 5  T=INDVF(T,DELT)
    TH=DPNVF(TH,DTH)
    PH=DPNVF(PH,DPH)
    PS=DPNVF(PS,DPS)
    DTH=DPNVF(DTH,D2TH)
    DPH=DPNVF(DPH,D2PH)
    DPS=DPNVF(DPS,D2PS)
    GO TO 2
 6  CALL EXIT M
    END
*   DATA
```

APPENDIX E

CALCULATION OF THE MOMENTS OF INERTIA

Two possible antenna arrangements were considered for this satellite. Since the antenna size markedly effects the moments of inertia and the frequencies of oscillation, calculations for both body configurations are given below.

E.1 General Satellite Configuration

The satellite is composed of a six inch wide hollow disk, of radius ten inches, which houses electronic equipment and solar cells. Three capacitors are placed in line and attached to the disk, as are four antennas. The solar sail is circular with center of mass twenty inches from the front of the disk, with a width of eight inches and a radius of ten inches. (See Figure 2)

Approximate:

(a.) the hollow disk, electronics, solar cells and assorted equipment by a solid disk of weight 7.51 pounds, with a thickness of three inches and a radius of nine inches.

(b.) the three capacitors by a rectangular bar of weight 6.29 pounds, with dimensions 3 in. x 3 in. x 16 in.

(c.) the sail by a circular cylindrical shell with a radius of nine inches, negligible thickness, and a weight of 0.25 pounds.

E.2 Long Antenna Configuration

Approximate the two twenty foot antennas by a forty-two foot rod weighing 1.37 pounds, and the two six foot antennas by a fourteen foot rod weighing 0.462 pounds.

The moments of inertia are then given by (neglecting small quantities),

$$I_3 = I_{3\ \text{disk}} + I_{3\ \text{bar}} + I_{3\ \text{antenna}}$$

$$= 6.55 \text{ slug-ft}^2 \tag{E.1}$$

$$I_2 = (I_2 + mr^2_{cm})_{\text{disk}} + I_{2\ \text{long anten.}} + (mr^2_{cm})_{\text{sail}} +$$

$$+ (I_2 + mr^2_{cm})_{\text{bar}}$$

$$= 6.30 \text{ slug-ft}^2 \tag{E.2}$$

$$I_1 = (I_1 + mr^2_{cm})_{\text{disk}} + I_{1\ \text{short ant.}} + (mr_{cm}^2)_{\text{sail}} +$$

$$+ (I_1 + mr_{cm}^2)_{\text{bar}}$$

$$= 0.294 \text{ slug-ft}^2 \tag{E.3}$$

Thus,

$$A \triangleq \frac{I_2 - I_1}{I_3} = 0.919$$

$$B \triangleq \frac{I_3 - I_1}{I_2} = 0.990$$

$$C \triangleq \frac{I_3 - I_2}{I_1} = 0.855$$

E.3 Short Antenna Configuration

Approximate the two 2.46 foot antennas by a rod of 6.55 feet and weight of 0.217 pounds, and the two 0.82 foot antennas by a rod of 3.28 feet and 0.108 pounds.

Using equations (E.1, 2, and 3), the moments become

$$I_3 = 0.123 \text{ slug-ft}^2$$

$$I_2 = 0.111 \text{ slug-ft}^2$$

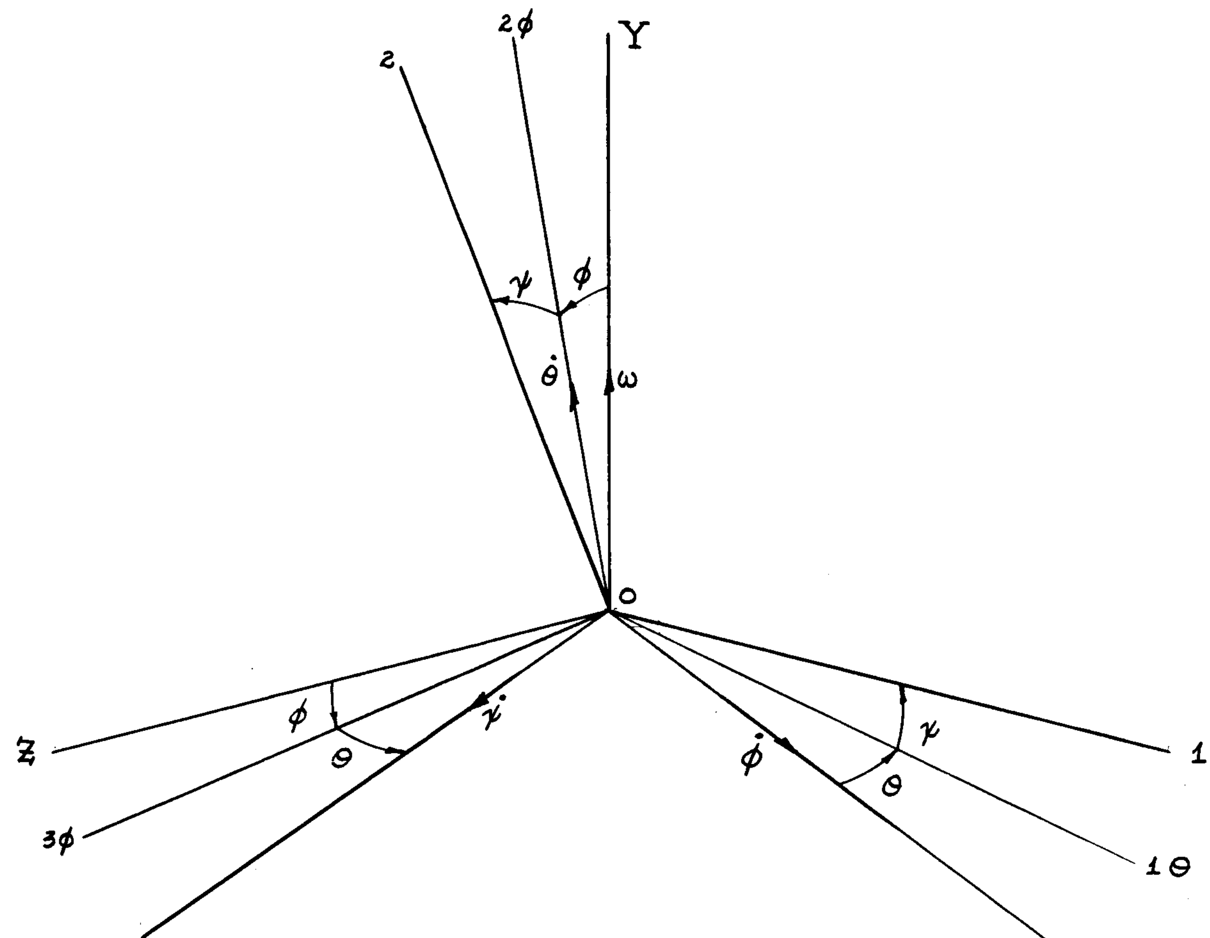
$$I_1 = 0.054 \text{ slug-ft}^2$$

The parameters A, B and C then become

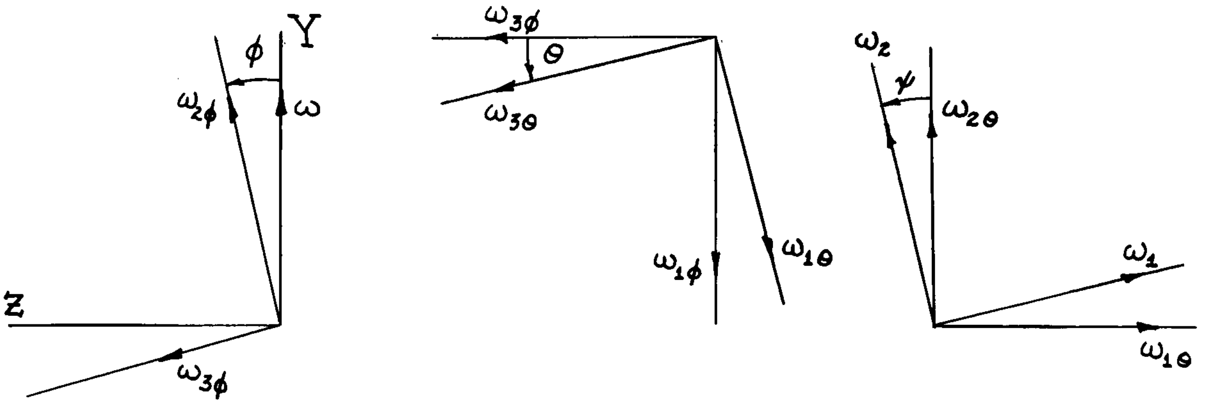
$$A = 0.460$$

$$B = 0.625$$

$$C = 0.232$$



a. *Angular Rotations*



b. *φ Rotation* c. *θ Rotation* d. *ψ Rotation*

Figure 1. Coordinate System

Antenna Length

Long: $L_1 = 20$ ft. $L_2 = 6$ ft.

Short: $L_1 = 2.46$ ft. $L_2 = 0.82$ ft.

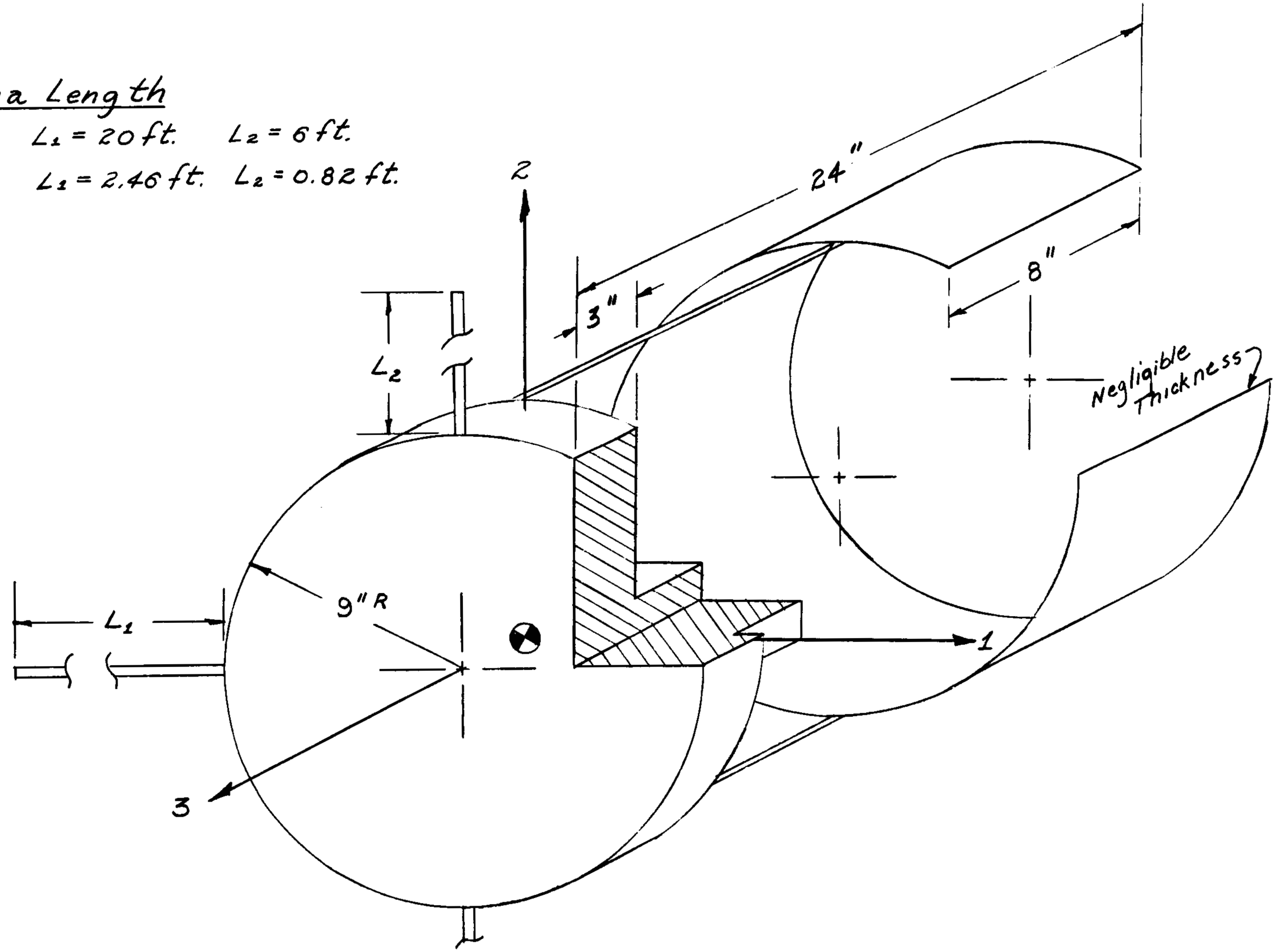


Figure 2. Satellite Model

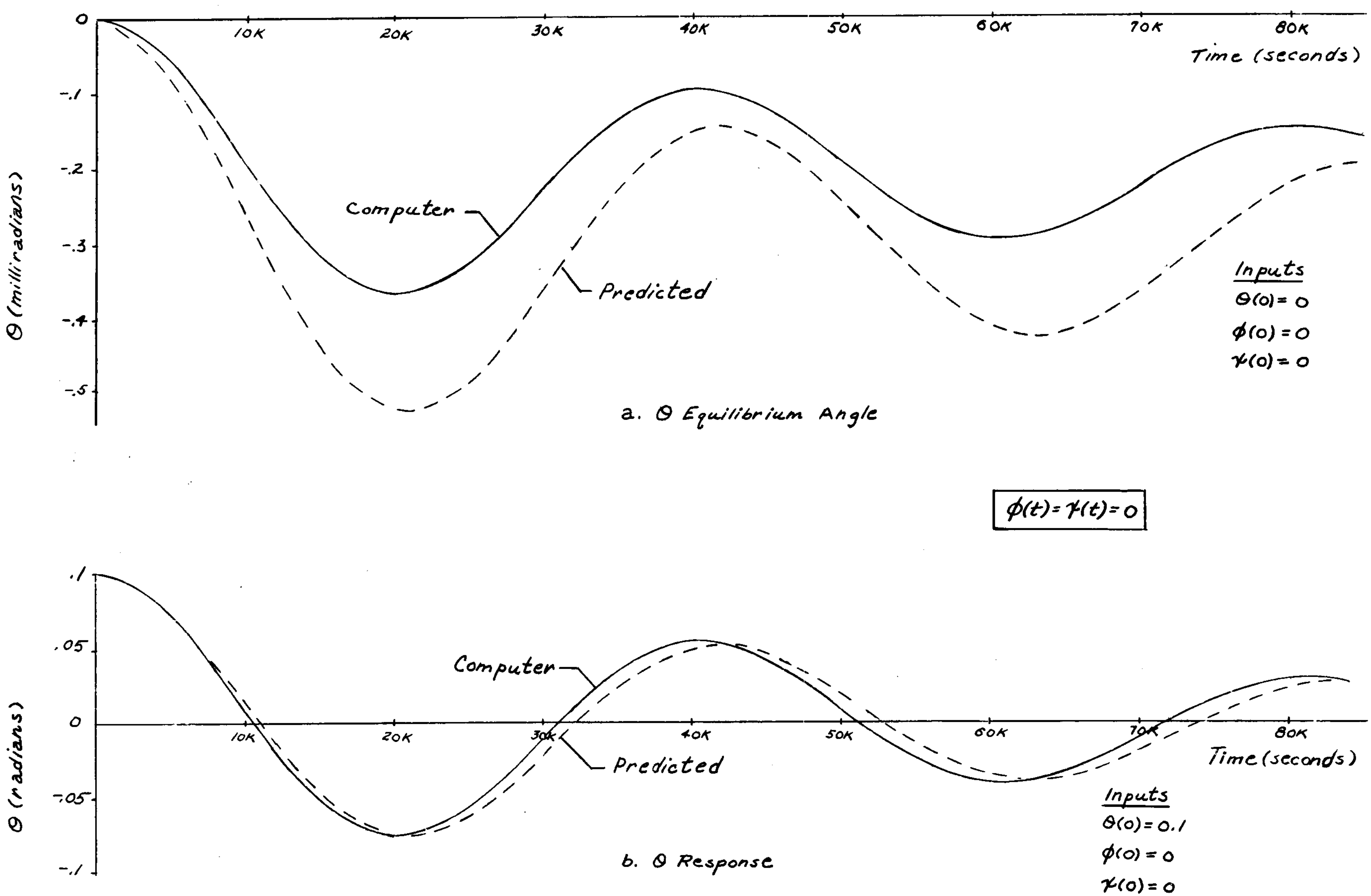
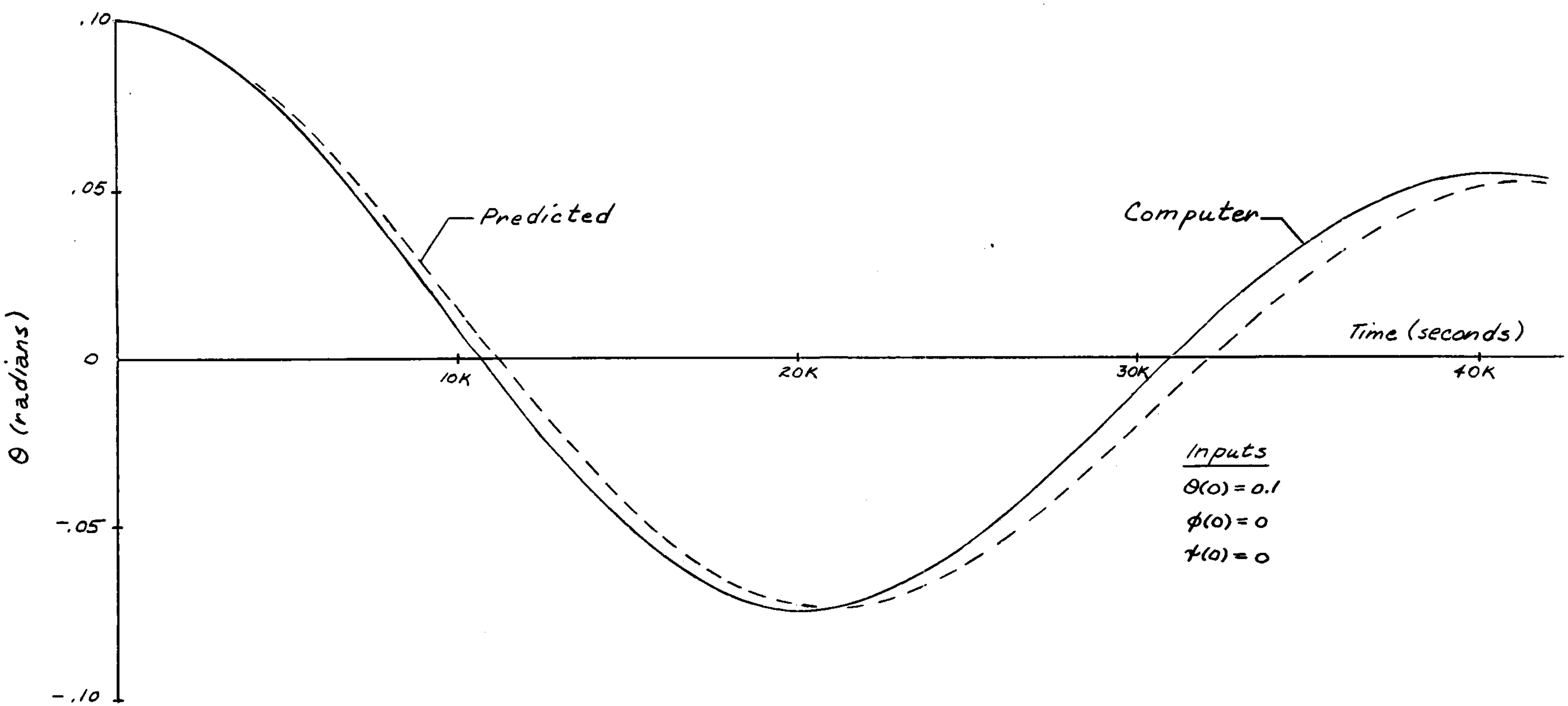


Figure 3. System Response to Θ Input



c. Enlarged View of Θ Response

Figure 3. System Response to Θ Input (continued)

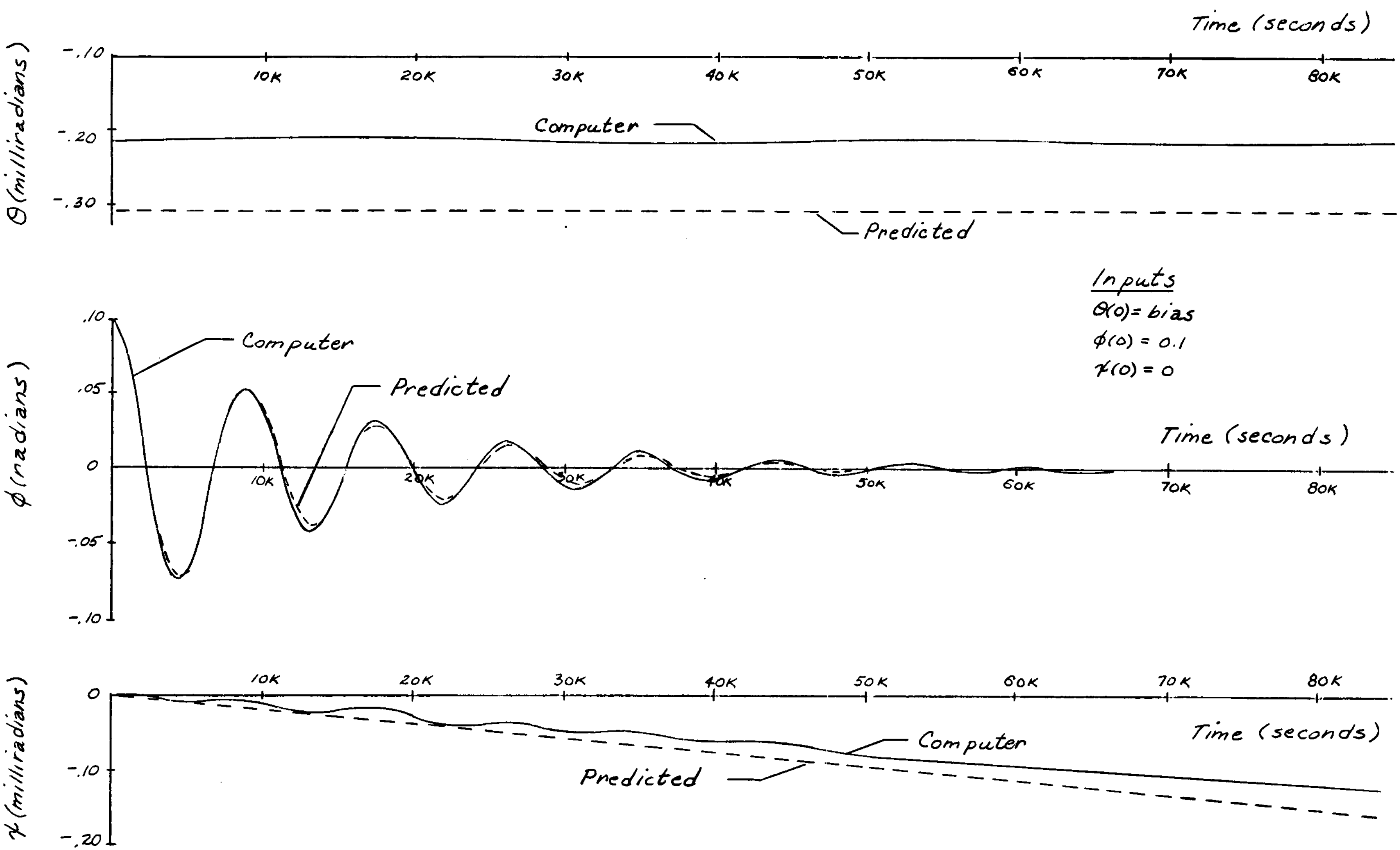
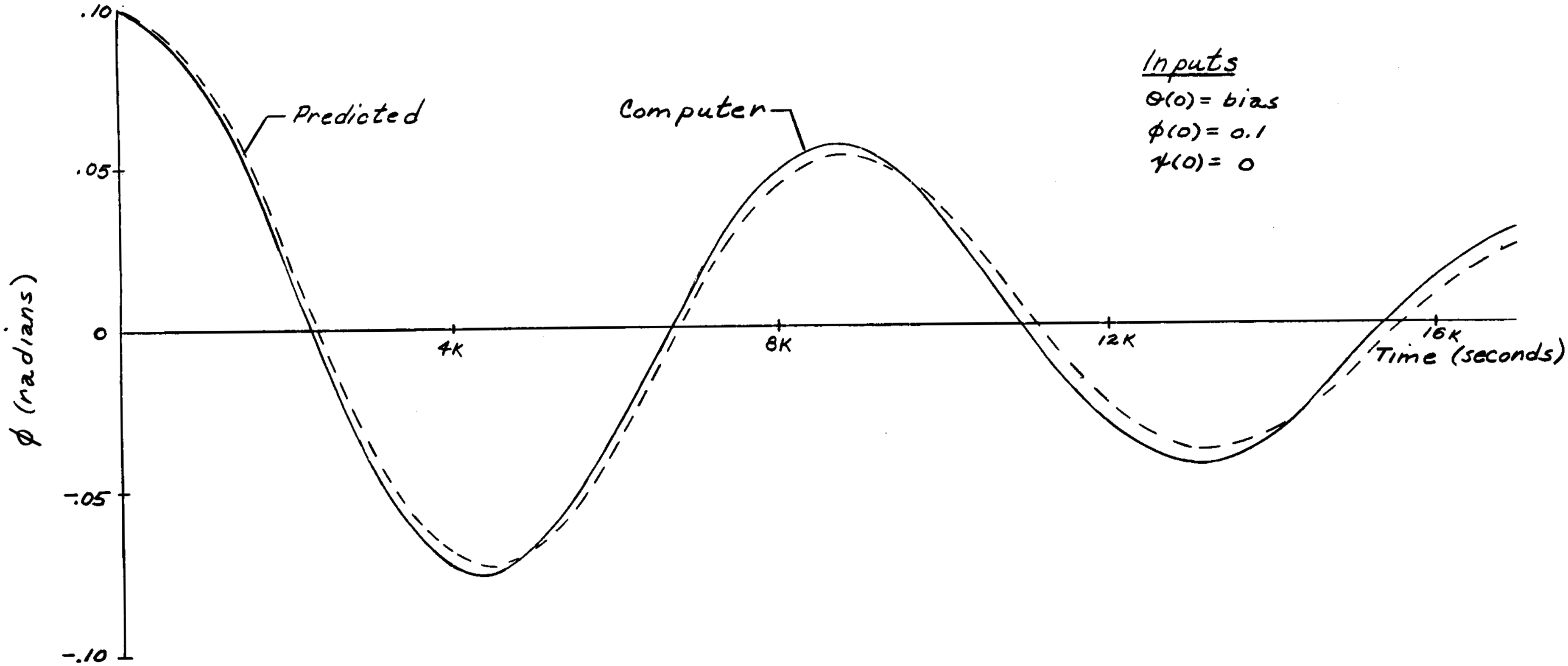


Figure 4. System Response to ϕ Input



a. Enlarged View of ϕ Response

Figure 4. System Response to ϕ Input (continued)

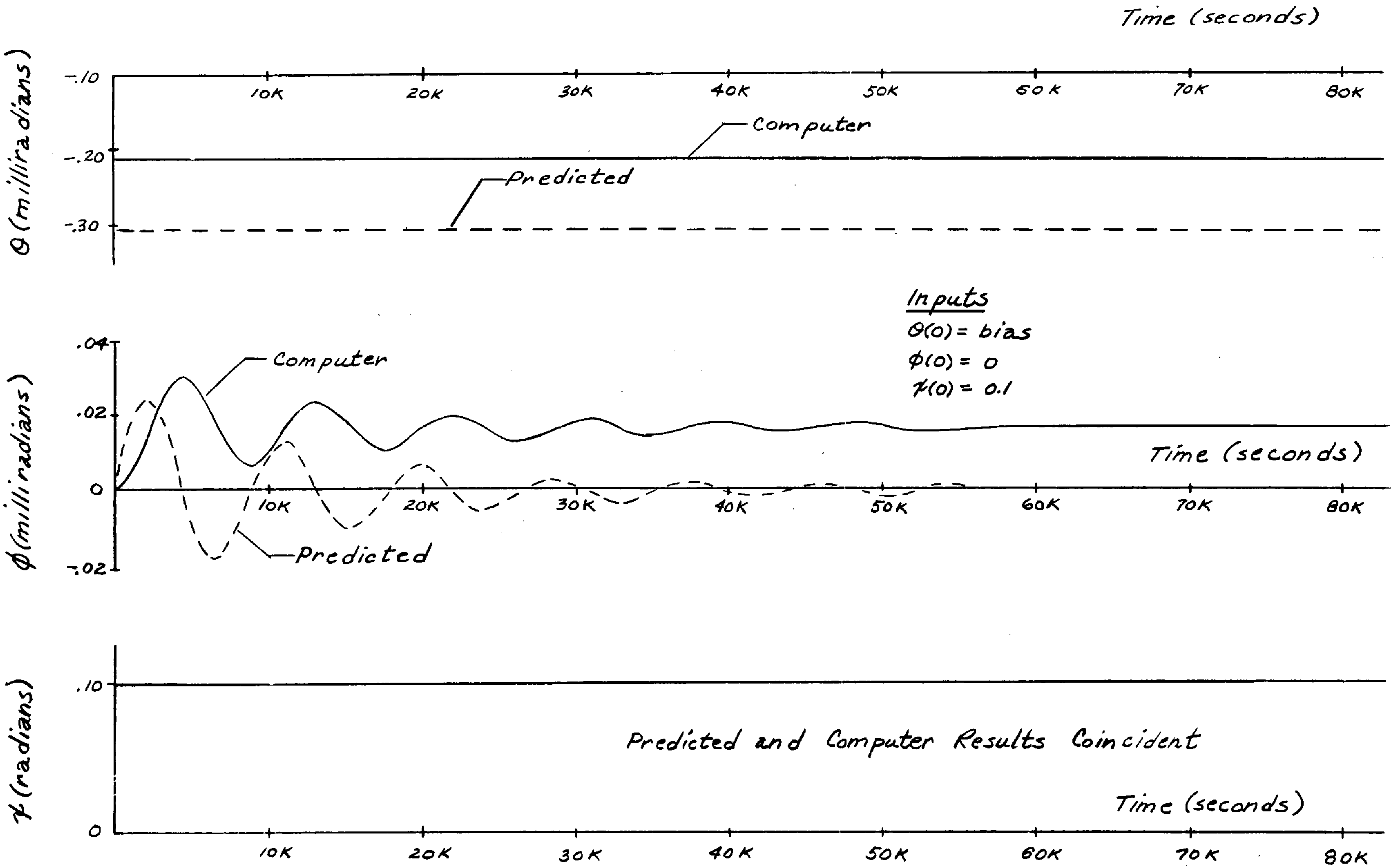


Figure 5. System Response to ψ Input

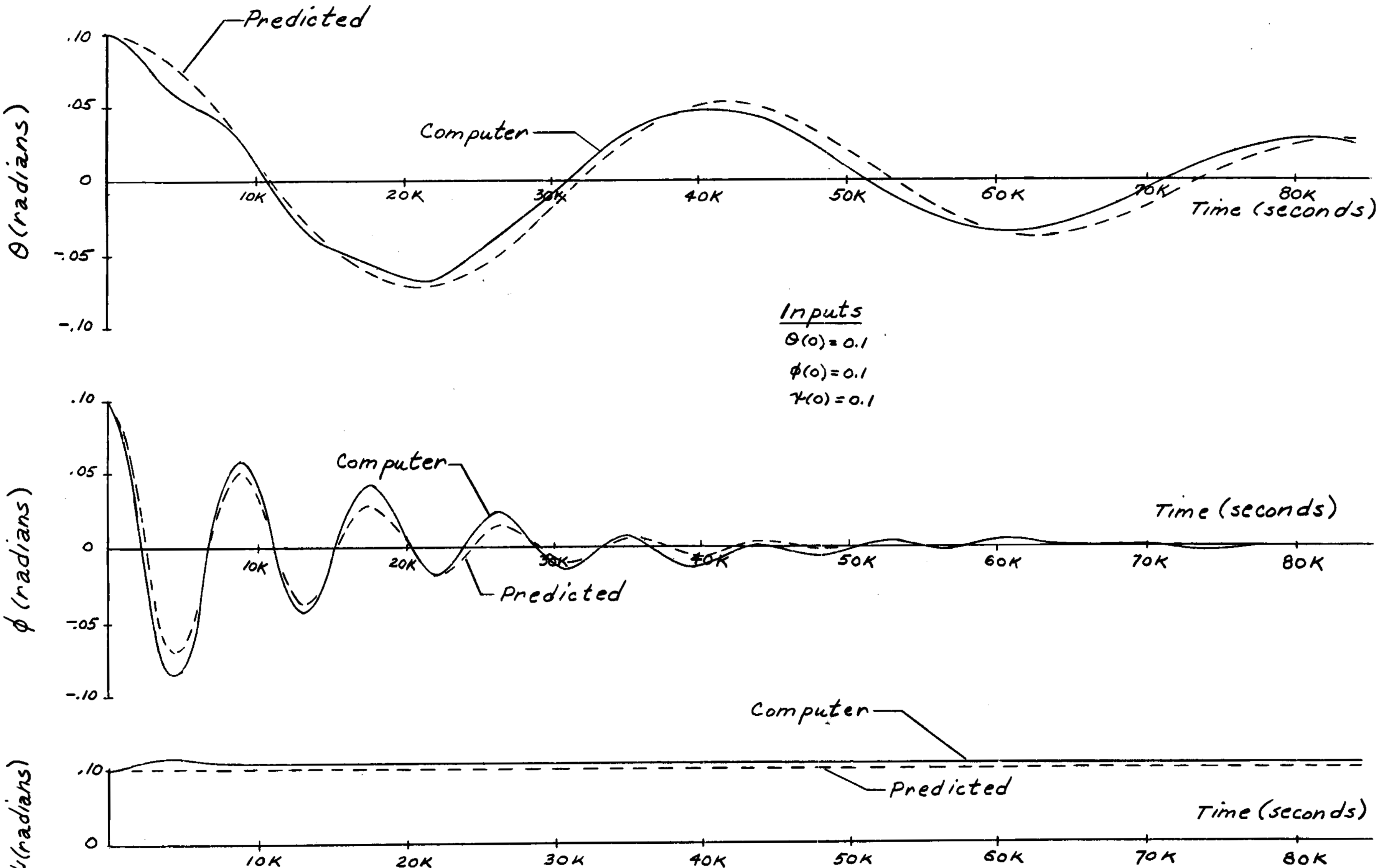


Figure 6. System Response to Combined θ, φ, and ψ Input

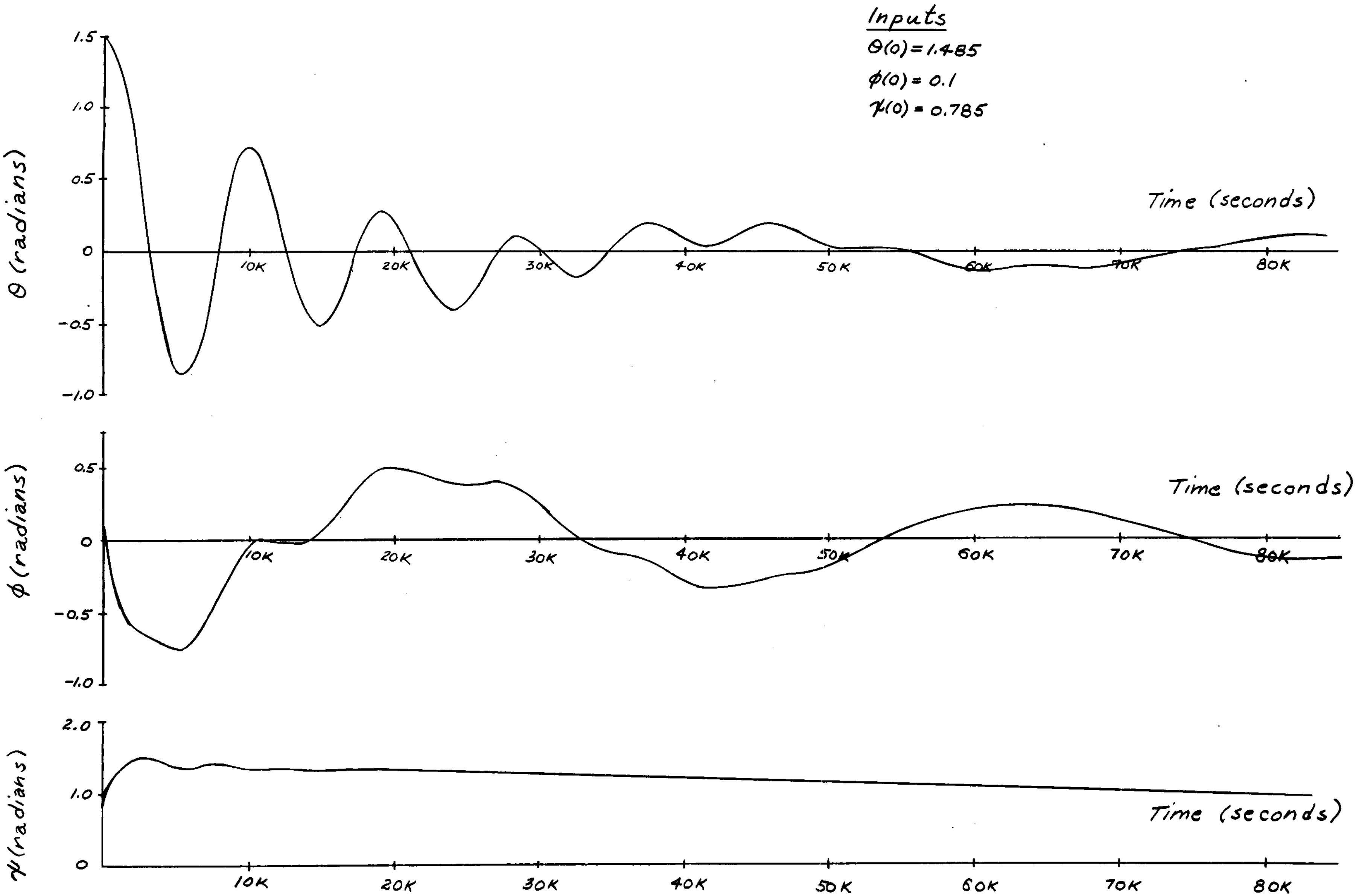


Figure 7. System Response for Orbit Injection

REFERENCES

1. Harrington, J. V., "First Order Analysis of Sunblazer Orbits," Massachusetts Institute of Technology, Center for Space Research Memorandum, July 20, 1964

2. Lecture Notes, Course 16.11, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, 1964

3. Nixon, F. E., Handbook of Laplace Transformations: Tables and Examples, Prentice-Hall, Inc., Englewood Cliffs, N.J., 1960

4. Accord, J. D., and Nicklas, J. C., "Theoretical and Practical Aspects of Solar Pressure Attitude Control for Interplanetary Spacecraft," Jet Propulsion Laboratory Technical Report, No. 32-467, May 15, 1964

5. Battin, R. H., Astronautical Guidance, McGraw-Hill Co., N.Y., 1964

6. Thomson, W. T., Mechanical Vibrations, 2nd Ed., Prentice-Hall, Inc., N.Y., 1953

7. Thomson, W. T., Introduction To Space Dynamics, John Wiley and Sons Inc., N.Y., 1961