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An experimental–theoretical approach based on continuous media theory has been suggested as a means of characterizing the multiaxial mechanical behavior of solid propellants (allowing for compressibility). The applicability of this approach to solid propellants has been the subject of one research program at the Jet Propulsion Laboratory. Two multiaxial experiments that have been developed in this program are the inflated cylinder test and the biaxial sheet test. This Report deals with characterizing the multiaxial data from these tests in terms of strain energy; only the elastic portion of the viscoelastic response is considered. However, the ultimate aims of the study are to characterize materials with memory.

Many of the observations previously reported by the author for compressible propellant were largely repeated upon examination of an unfilled polyurethane binder, which was essentially incompressible. The compressibility theory suggested by the author experimentally converges to the theory as used by Rivlin for an incompressible material.

author

I. INTRODUCTION

Previous work with the inflated cylinder test has shown that polyurethane solid propellant behaves as a compressible material (Refs. 1, 2, and 3). Hence, the third strain invariant (I_3), as well as the partial derivative of strain energy with respect to the third strain invariant ($\partial W/\partial I_3$), must be considered if the strain energy function (W) is ever to be explicitly defined. Another observation was that the three partial derivatives of strain energy with respect to the three strain invariants ($\partial W/\partial I_k$) as a function of the invariants implied that the strain energy function could not be described in the relatively simple form of a Mooney material, in the small-strain realm.

The stored energy function W, as proposed by G. Green, has been argued to exist by Lord Kelvin, according to the first and second laws of thermodynamics. Essentially, the function W represents the elastically stored energy in the deformed state per unit of volume measured in the undeformed state. The following constitutive equation, which incorporates the stored energy concept, has been widely used in the literature of applied mechanics:

$$\tau^{ij} = \left(\frac{g}{G}\right)^{1/2} \frac{\partial W}{\partial \epsilon_{ij}} = \frac{1}{\sqrt{I_3}} \frac{\partial W}{\partial \epsilon_{ij}} \tag{1a}$$

where

τ^{ij} = contravariant components of stress tensor in the deformed state.

$\epsilon_{ij} = \epsilon_{ji} = \frac{1}{2}(G_{ij} - g_{ij})$ = strain tensor associated with the general deformation gradients of the volume element enclosing the stress tensor.

g_{ij}, G_{ij} = metric tensors of the undeformed and deformed state.

$\left(\frac{g}{G}\right)^{1/2} = \frac{1}{\sqrt{I_3}}$ = ratio of the undeformed to deformed determinants of the metric tensors g_{ij} *and* G_{ij}.

It has been assumed that the stored energy function is dependent only upon the three strain invariants, I_i, for a body that is homogeneous and isotropic, hence

$$W = W(I_1, I_2, I_3) \tag{1b}$$

where

I_1, I_2, I_3 = strain invariants; for the deformation in which a cube strains into a cuboid and the material and space coordinates are cartesian:

$$I_1 = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$$

$$I_2 = \lambda_1^2\lambda_2^2 + \lambda_1^2\lambda_3^2 + \lambda_2^2\lambda_3^2$$

$$I_3 = \lambda_1^2\lambda_2^2\lambda_3^2$$

$$\lambda_i = 1 + e_i = \text{stretches} \tag{1c}$$

The relationship between the extensions e_i and the strain tensor ϵ_{ii} is

$$e_i = (1 + \epsilon_{ii})^{1/2} - 1 \tag{1d}$$

From Eqs. (1a) and (1b) it can readily be shown that the constitutive equations of a compressible rubber-like continuum that is homogeneous and isotropic may reasonably be assumed to have the following form (Ref. 4):

$$\tau^{ij} = \frac{1}{\sqrt{I_3}}\left(\frac{\partial W}{\partial I_1}\frac{\partial I_1}{\partial \epsilon_{ij}} + \frac{\partial W}{\partial I_2}\frac{\partial I_2}{\partial \epsilon_{ij}} + \frac{\partial W}{\partial I_3}\frac{\partial I_3}{\partial \epsilon_{ij}}\right) \tag{1e}$$

Equation (1e) can readily be reduced to physical principal stress (t_{ii}) and extensions ($e_i = \partial u_i/\partial x_i$). For the experiment in which an infinitesimal cube deforms into a cuboid of dimensions $(1 + e_i)$, Eq. (1e) reduces to

$$t_{ii} = \frac{\lambda_i}{\sqrt{I_3}}\frac{\partial W}{\partial I_k}\frac{\partial I_k}{\partial e_i} \tag{1f}$$

Reference 1 shows that $\partial I_k/\partial e_i$ can be experimentally obtained for any given equilibrium state by independently measuring e_1, e_2, and e_3 at a point region, and employing the following equations:

$$\frac{\partial I_1}{\partial e_1} = 2 + 2e_1 \quad \frac{\partial I_1}{\partial e_2} = 2 + 2e_2 \quad \frac{\partial I_1}{\partial e_3} = 2 + 2e_3$$

$$\frac{\partial I_2}{\partial e_1} = 4 + 4e_1 + 4e_2 + 4e_3 + 2e_1e_2^2 + 2e_1e_3^2 + 4e_1e_2 + 4e_3e_1 + 2e_2^2 + 2e_3^2$$

$$\frac{\partial I_2}{\partial e_2} = 4 + 4e_1 + 4e_2 + 4e_3 + 4e_1e_2 + 4e_2e_3 + 2e_1^2e_2 + 2e_2e_3^2 + 2e_1^2 + 2e_3^2$$

$$\frac{\partial I_2}{\partial e_3} = 4 + 4e_1 + 4e_2 + 4e_3 + 4e_2e_3 + 4e_1e_3 + 2e_3e_2^2 + 2e_1^2 e_3 + 2e_2^2 + 2e_1^2$$

$$\frac{\partial I_3}{\partial e_1} = 2 + 4e_2 + 2e_2^2 + 2e_1 + 4e_2e_1 + 2e_1e_2^2 + 4e_3 + 8e_2e_3 + 4e_3e_2^2 + 4e_1e_3 + 8e_2e_3e_1 + 4e_1e_3e_2^2 + 2e_3^2 + 4e_2e_3^2 + 2e_2^2 e_3^2 + 2e_1e_3^2 + 4e_2e_1e_3^2 + 2e_1e_2^2 e_3^2$$

$$\frac{\partial I_3}{\partial e_2} = 2 + 2e_2 + 4e_1 + 4e_1e_2 + 2e_1^2 + 2e_1^2 e_2 + 4e_3 + 4e_2e_3 + 8e_1e_3 + 8e_1e_3e_2 + 4e_3e_1^2 + 4e_2e_3e_1^2 + 2e_3^2 + 2e_2e_3^2 + 4e_1e_3^2 + 4e_1e_2e_3^2 + 2e_1^2 e_3^2 + 2e_1^2 e_2e_3^2$$

$$\frac{\partial I_3}{\partial e_3} = 2 + 4e_2 + 2e_2^2 + 4e_1 + 8e_1e_2 + 4e_1e_2^2 + 2e_1^2 + 4e_2e_1^2 + 2e_1^2e_2^2 + 2e_3 + 4e_2e_3 + 2e_3e_2^2 + 4e_1e_3 + 8e_1e_2e_3 + 4e_1e_3e_2^2 + 2e_1^2e_3 + 4e_2e_3e_1^2 + 2\,e_1^2\,e_2^2\,e_3 \tag{2}$$

As a consequence of experimental investigations, various explicit forms of the stored energy function have been proposed as being representative of rubber-like materials, e.g., by Mooney (Ref. 5), Treloar (Ref. 6), Rivlin and Saunders (Ref. 7), Thomas (Ref. 8), Blatz and Ko (Ref. 9), Landel (Ref. 10), and San Miguel (Ref. 1). However, an explicit form of the stored energy function that is truly representative of rubber-like material response and that would cover the range between molecular interpretation and physical property response has not been discovered. The literature records the many inadequacies of the explicitly proposed forms of the universal stored energy function. Notable discrepancies have been discussed by Treloar (Refs. 5 and 11), Gumbrel et al. (Ref. 12), Gent and Rivlin (Ref. 13), Thomas (Ref. 8), Gent and Thomas (Ref. 14), Ko (Ref. 15) and San Miguel and Duran (Ref. 16). Hence it is recognized that the stored energy concept is in a transition state and may be subject to drastic reinterpretation.

II. EXPERIMENTAL PROCEDURE

The form of Eq. (1f) suggests that inferences regarding an explicit form of the stored energy function W may be arrived at simply by measuring the geometric changes, $\partial I_i/\partial e_j$, associated with the boundary tractions t_{ii}. The value of $\partial I_i/\partial e_j$ can be explicitly found for a homogeneous strain state by measuring the three principal extensions e_j. Associated with this strain state will be a stress state t_{jj}, which, presumably, can be approximated by resorting to analytical techniques that are independent of the material properties to be examined. For the experiments in which one of the principal stresses is zero ($t_{33} = 0$), Eq. (1f) reduces to

$$\begin{aligned}
t_{11} = \lambda_1 \tau^{11} &= \frac{\lambda_1}{\sqrt{I_3}}\left(\frac{\partial W}{\partial I_1}\frac{\partial I_1}{\partial e_1} + \frac{\partial W}{\partial I_2}\frac{\partial I_2}{\partial e_1} + \frac{\partial W}{\partial I_3}\frac{\partial I_3}{\partial e_1}\right)\\
t_{22} = \lambda_2 \tau^{22} &= \frac{\lambda_2}{\sqrt{I_3}}\left(\frac{\partial W}{\partial I_1}\frac{\partial I_1}{\partial e_2} + \frac{\partial W}{\partial I_2}\frac{\partial I_2}{\partial e_2} + \frac{\partial W}{\partial I_3}\frac{\partial I_3}{\partial e_2}\right)\\
0 &= \frac{\lambda_3}{\sqrt{I_3}}\left(\frac{\partial W}{\partial I_1}\frac{\partial I_1}{\partial e_3} + \frac{\partial W}{\partial I_2}\frac{\partial I_2}{\partial e_3} + \frac{\partial W}{\partial I_3}\frac{\partial I_3}{\partial e_3}\right)
\end{aligned} \tag{3}$$

The $\partial W/\partial I_i$ can be directly computed for each stress–strain state by measuring e_j and t_{jj}. Equations (3) are considerably different in form from those used by Rivlin (Ref. 17) in that not only are $\partial I_3/\partial e_j$ and $\partial W/\partial I_3$ explicitly retained, but also a measurement of e_3 is required in the homogeneous strain state being investigated.

Multiaxial stress experiments must be performed where the measurement of at least two simultaneous stresses in conjunction with the three principal strains at a point region are independently obtained, if characterization of the stored energy function is contemplated. A requirement normally associated with multiaxial experiments is that they be "homogeneous," i.e., that the stress-extension state of a test structure be the same at all points in the test structure and that curved or straight lines deform into curved or straight lines. Two experiments used in this Report are the biaxial sheet test and the inflated cylinder test.

A. *Biaxial Sheet Test*

A biaxial tensile sheet experiment (Ref. 3) is reasonably "homogeneous" (in the center realm of the sheet) and consists of applying two principal tension loadings to a sheet of material. This multiaxial test has been used by Treloar (Ref. 18) in observing photoelasticity stress-optic phenomena and by Rivlin and Saunders (Ref. 7), in their experimental studies of the stored energy function of rubber. Blatz and Ko (Ref. 9) have conducted finite-strain experimental studies with polyurethane rubber and foam.

The parameters that are measured are the two principal loads [from which the two principal stresses t_{11} and t_{22} ($t_{33} = 0$) are estimated] and the three principal extensions e_1, e_2, and e_3 in the central region (where a reasonably homogeneous stress-extension state exists) of a biaxially loaded sheet.

The biaxial sheet test data were obtained at arbitrarily chosen extension ratios of the order of (a) 1:1, (b) 1:4, (c) 1:5, (d) 1:6, (e) 1:9. This approach differs from that of Rivlin and Saunders (Ref. 7) in that no attempt was made to obtain specific data where two of the strain invariants remained essentially constant per experiment.

The consequence of obtaining data in a semirandom fashion is that the partial derivatives ($\partial W/\partial I_i$) measured at different extensions at the same extension ratio are not strictly comparable, since they have been obtained at varying I_j. However, since the main purpose is to examine the trends of these gradients as the invariants increase, and not to obtain an analytic form for W, this complication is not of grave concern at the present.

Data measurements were made in a near-equilibrium state (static). Sheet relaxation or creep was not significant near room temperature.

B. An Inflated Cylinder Test

A number of multiaxial experiments must be performed in order to eventually determine one explicit form for the stored energy function W. One experiment that may be of interest to solid-propellant engineers is the inflated cylinder test that has been described by San Miguel and Silver (Ref. 2). This experiment consists of pressurizing a greased rubber bag within the cavity of an unrestricted thick-walled cylinder. The bag is longitudinally retained within the cavity by two self-adjusting end plates, which are monitored by a servo system that ensures bag containment yet does not allow any consequential longitudinal load to exist in the thick-walled cylinder.

The point principal extensions and their corresponding directions are measured in a low-reinforcing surface coating that exhibits birefringence upon deformation (Ref. 19). The coating technique measures the two-dimensional surface extensions and their corresponding directions during loading. The third principal extension, which is normal to the free surface, is obtained from the longitudinal contraction of the circular cylinder.

The point principal stresses are computed directly from the Lamé–Maxwell form of the equilibrium equations. The validity of the magnitude obtained from these equations for the principal stresses t_{11} and t_{22} ($t_{33} = 0$) has been experimentally shown by San Miguel and Silver (Ref. 20) to be reasonable, at least for strain fields of the order of 20%.

The inflated cylinder test data, like the biaxial sheet test data, were obtained by arbitrarily examining various strain states; i.e., no attempt was made to examine specific points where two of the strain invariants remained constant. The cylinder was pressurized in 5-psi intervals, and the stress–strain state of some five specific neighboring points was examined. Data measurements were again made in a near-equilibrium state (static).

III. MATERIAL

The material examined in this investigation was Solithane 113. This material was selected because of its linear elastic properties, its similarity to polyurethane solid-propellant binder with respect to the stress-extension trade-off characteristics, and its photoelastic properties (Ref. 19).

The preparation of the material used was as follows: Castor oil (Baker Caster Oil Co., Los Angeles, Calif., D. B. Grade) is dried under a vacuum for 24 hr and preheated to 130°F. Solithane 113 (Thiokol Chemical Corp., Trenton, N. J.) is also preheated to 130°F. Next, 100.0 wt % D. B. castor oil (70% trifunctional, 30% difunctional) is added to 75.0 wt % Solithane 113, a polyurethane prepolymer made from tolylene diisocyanate and castor oil, which contains about 10.6% free isocyanate groups. This temperature (130°F) is maintained during the mixing and degassing phase (approximately 20 min). The cure is in a sealed (closed) mold for 2 hr as the temperature rises from 130 to 230°F. Finally, the material is post-cured for 30 days at room temperature in a desiccator.

IV. DISCUSSION

It was noted earlier in this Report that the form of the stored energy function has been subject to controversy. However, the main object of this Report is to present one method to characterize multiaxial data in terms of a strain energy concept. Hence, the interpretation of the characterized data in terms of theoretical or physical concepts is left to others. Any discrepancies between theoretical stress and experimentally obtained stress for the same strain invariant path must be due to: (1) a poor constitutive equation and theory, (2) error in manipulating the theory, (3) error in assumptions that simplify the theory, e.g., the incompressibility assumption, or (4) error in obtaining data.

The data obtained from the biaxial sheet test are shown in Fig. 1 for extension ratios of 1:1, 1:4, 1:5, 1:6, and 1:9, using Eq. (3). The pertinent observations are: (1) $\partial W/\partial I_2$ is negative, (2) $\partial W/\partial I_i$ decrease with increasing I_j, (3) $-\partial W/\partial I_2 \approx \partial W/\partial I_3 \ll \partial W/\partial I_1$, (4) $\partial W/\partial I_3$ changes sign from negative to positive, and (5) all $\partial W/\partial I_i$ have significant values for small strain invariants.

The various curves in Fig. 1 are due to the different $\partial W/\partial I_i$ vs I_j paths taken by each of the five polyurethane sheets. The various values of $\partial W/\partial I_i$ vs I_i show by their relative smoothness that the stored energy function is itself of a smooth character. It should be noted that the biaxial sheet data approached those of the stress–strain failure state of the polyurethane, for it was the tearing of the sheet that limited the magnitude of the data obtained. However, the biaxial sheet data were sufficiently accurate to preclude any of the unaccountable sign oscillation associated with almost a quarter of Rivlin and Saunders' raw data (Ref. 7).

A question now arises concerning the experimental error inherent in the biaxial sheet test in general. As noted in Ref. 3, the greatest experimental error deals with assigning some effective length to the sheet to obtain the stressed area to which a prescribed load is assigned. One way to detour this obstacle is to perform a different biaxial stress experiment in which this area assumption is not a prerequisite.

The inflated cylinder test does not make any assumptions with regard to stressed area. However, it makes assumptions that are intrinsically different from the biaxial sheet test, e.g., those of the stress–optic law, the Lamé cylinder equations, etc. If the data obtained from this experiment corroborate the multiaxial sheet observations, then it would be reasonable to assume that the observations in this Report are truly an indication of the material response and not the experimental error associated with technique.

The data obtained from the inflated cylinder test are shown in Fig. 2, using Eq. (3). The observations from Fig. 2 are very similar to those in Fig. 1. It should be noted that although the magnitudes of the strain invariants are small, the stretch magnitudes are of the same order of magnitude as those of the sheet experiment. The limitation of the stretch magnitudes was due to failure of the unrestricted polyurethane cylinder. The numerous points in Fig. 2 represent some seven different homogeneous strain states (located at different radial distances) occurring for one pressurization experiment. Hence, numerous arbitrary $\partial W/\partial I_i$ vs I_j paths are represented in Fig. 2. Finally, it is interesting to compare Fig. 2 with Fig. 3, which was obtained by San Miguel (Ref. 1) for a compressible, filled-polyurethane cylinder of similar dimensions using Eqs. (3). The data used in Fig. 3 were not near the fracture realm of the unrestricted cylinder.

Perhaps the most distressing observations resulting from characterizing multiaxial data as suggested in this Report is that $\partial W/\partial I_2$ is negative. The second most disturbing observation is that all $\partial W/\partial I_k$ appear to be significant in the realm of strains normally experienced by solid propellant motors. Hence, the material assumptions of Mooney–Rivlin and others may be unrealistic for practical propellant stress analysis.

The question whether the $\partial W/\partial I_2$ can be negative could be disturbing. On the other hand, Truesdell (Ref. 21) and Eringen (Ref. 22) have shown theoretically that $\partial W/\partial I_2 \approx -\mu/2$ upon approaching a state of zero extensions, where μ is the shear modulus. The tensile modulus E of the sheet was approximately 400 psi. This would suggest that μ was about 130 psi, and consequently $\partial W/\partial I_2$ should approach -65 psi. It should be recognized that the stress–strain curve of a rubber is difficult to measure for extremely small deformations; hence an initial modulus (Ref. 16) is somewhat indeterminate. On the other hand, the predicted value for $\partial W/\partial I_2$, at zero deformation, is reasonably close to that measured. It should be recognized that $\partial W/\partial I_2$ is extremely sensitive

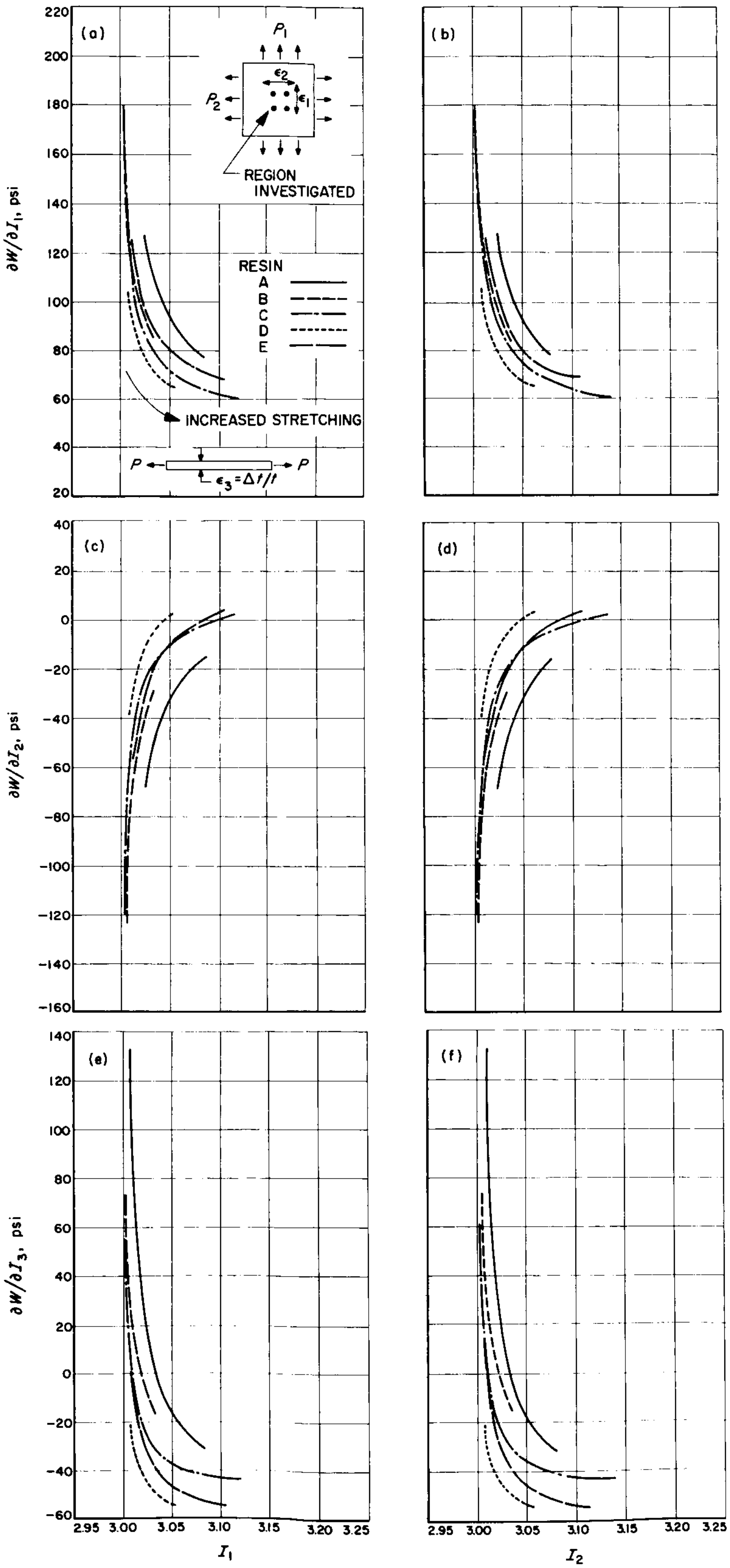

Fig. 1. The $\partial W/\partial I_k$ vs I_j for an unfilled resin sheet

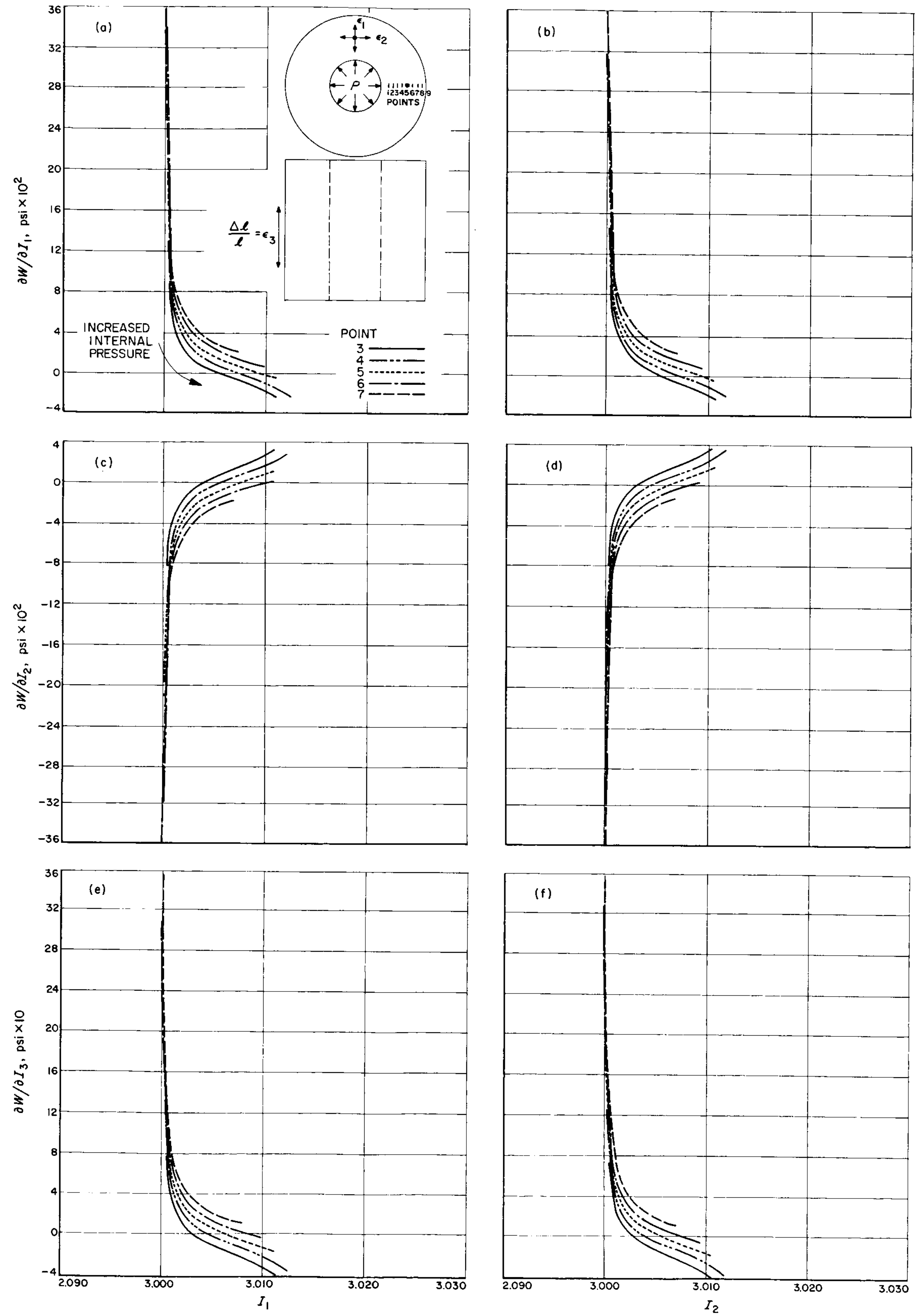

Fig. 2. The $\partial W/\partial I_k$ vs I_j for an unfilled resin cylinder

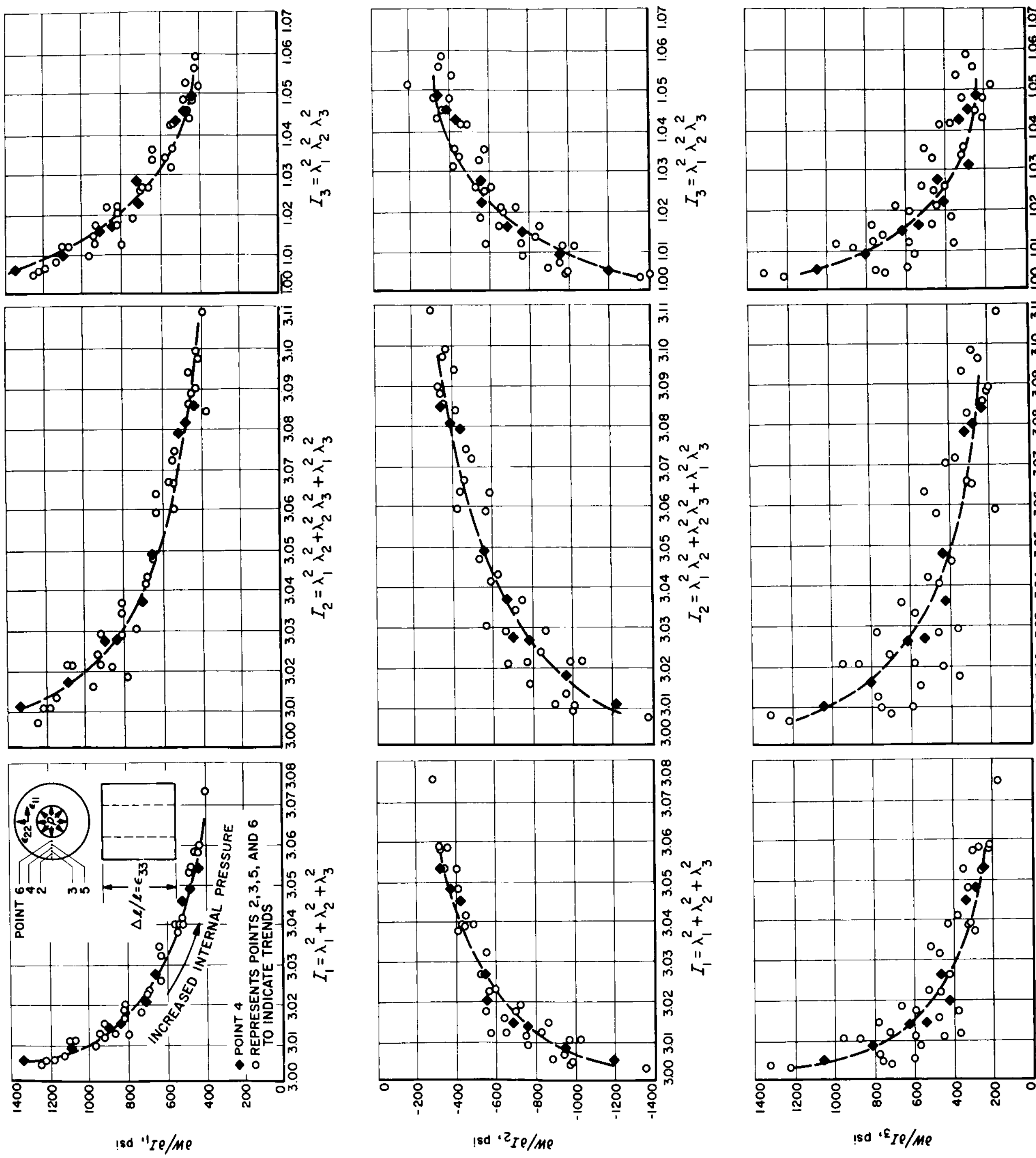

Fig. 3. The $\partial W/\partial I_k$ vs I_j for a filled resin cylinder

to the input data required in Eq. (3). Thus, the smoothness of the experimental results is considered rather satisfactory. An analogous argument for the solid-propellant results (Fig. 3) can also be presented.

With regard to the second disturbing observation, it should be noted that $\partial W/\partial I_k$ is shown only for I_j and I_i, which are "approximately" constant. Hence, the curves shown in Figs. 1, 2, and 3 are not exactly correct since I_j and I_k did vary by a small amount. The significance of these variations remains to be seen. On the other hand, one might assume that the curves shown in the Figures are representative in mode and magnitude of the true $\partial W/\partial I_k$ curves when holding I_i and I_j constant. In this context one would surmise that such materials as those assumed by Mooney–Rivlin and others may have to be reinterpreted in the realm of small strains.

A final note is that the results of Rivlin and Saunders (Ref. 7) are identically obtained if their raw data are used in Eqs. (3) and if e_3, which they did not give, is calculated on the basis of incompressibility. Thus, it is inferred that Eqs. (3) are correct. (Note that Eqs. (3) are more general than those employed by Rivlin (Ref. 7); Eqs. (3) are applicable to either a compressible or an incompressible material.)

V. CONCLUSION

A format has been suggested for characterizing multiaxial data in terms of a strain energy function. The precise interpretation of such data characterization is not known, but it is suspected that new modifications will have to be introduced into the strain energy concept from the continuous media theory, together with experimental observations at finite strains (greater than 20%).

REFERENCES

1. San Miguel, A., *An Experimental Method to Measure the Strain Energy Function (W)*, Technical Report No. 32-365, Jet Propulsion Laboratory, Pasadena, Calif., December 1962. *Transactions of the Society of Rheology*, Vol. VII, 1963, p. 422.
2. San Miguel, A., and Silver, R. H., *A Normal-Incidence Reflective Polariscope for Viscoelasticity Measurements*, Technical Report No. 32-573, Jet Propulsion Laboratory, Pasadena, Calif., June 1964. (To be published in *Experimental Mechanics*).
3. San Miguel, A., "A Biaxial Tester," *Bulletin of the Third Meeting of ICRPG Working Group on Mechanical Behavior*, November 15, 1964, SPIA, The Johns Hopkins University, Applied Physics Laboratory, Silver Spring, Maryland.
4. Sneddon, I. N., and Berry, D. S., "The Classical Theory of Elasticity," *Handbuch der Physik*, Vol. VI, S. Flugge, ed., Springer-Verlag, Berlin, Vol. 3/1, 1958, p. 17.
5. Mooney, M., "A Theory of Large Elastic Deformation," *Journal of Applied Physics*, Vol. 11, 1940, p. 582.

REFERENCES (Cont'd)

6. Treloar, L. R. G., "The Elasticity of a Network of Long Chain Molecules-II," *Transactions of the Faraday Society,* Vol. 39, 1943, p. 241.

7. Rivlin, R. S., and Saunders, D. W., "Large Elastic Deformations of Isotropic Materials, VII. Experiments on the Deformation of Rubber," *Philosophical Transactions of the Royal Society,* Series A, Vol. 243, p. 251.

8. Thomas, A. G., "The Departures from the Statistical Theory of Rubber Elasticity," *Transactions of the Faraday Society,* Vol. 51, 1955, p. 569.

9. Blatz, P. J., and Ko, W. L., "Application of Finite Elastic Theory to the Deformation of Rubber Materials," *Transactions of the Society of Rheology,* Vol. VI, 1962, p. 223.

10. Landel, R. F., "An Approximate Stored Energy Function for Amorphous Elastomers," presented before the West Coast Regional Meeting of the Society of Rheology, February 1–2, 1962, Pasadena, Calif.

11. Treloar, L. R. G., "Stress–Strain Data for Vulcanized Rubber Under Various Types of Deformation," *Transactions of the Faraday Society,* Vol. 40, 1944, p. 59.

12. Gumbrell, S. M., Mullins, L., and Rivlin, R. S., "Departures of the Elastic Behavior of Rubbers in Simple Extension from the Kinetic Theory," *Transactions of the Faraday Society,* Vol. 49, 1953, p. 1495.

13. Gent, A. N., and Rivlin, R. S., "Experiments on the Mechanics of Rubber, III: Small Torsions of Stretched Prisms," *Proceedings of the Physical Society,* B, Vol. 65, 1952, p. 645.

14. Gent, A. N., and Thomas, A. G., "Forms for the Stored (Strain) Energy Function for Vulcanized Rubber," *Journal of Polymer Science,* Vol. 28, Issue No. 118, 1958, p. 625.

15. Ko, W. L., Personal communication.

16. San Miguel, A., and Duran, E. N., *Some Experimental and Theoretical Significances Associated with Irradiated Propellant,* Technical Report No. 32-518, Jet Propulsion Laboratory, Pasadena, Calif., November 1963.

17. Rivlin, R. S., "Large Elastic Deformations of Isotropic Materials, IV. Further Developments of the General Theory," *Philosophical Transactions of the Royal Society,* Series A, Vol. 241, 1948, p. 387.

18. Treloar, L. R. G., "Stresses and Birefringence in Rubber Subjected to General Homogeneous Strain," *Proceedings of the Physical Society,* Vol. 60, 1948, p. 135.

19. San Miguel, A., and Duran, E. N., "Some Low-Modulus Birefringent Resins," *Experimental Mechanics,* Vol. 4, No. 3, 1964.

20. San Miguel, A., and Silver, R. H., "Radial Stress Measurements in Propellant Grains," *Bulletin of the Second Meeting of ICRPG Working Groups on Mechnical Behavior,* Applied Physics Laboratory, Johns Hopkins University, November 1964.

21. Truesdell, C., "Elasticity and Fluid Dynamics," *Journal of Rational Mechanics and Analysis,* Vol. 1, Eq. 41.7, 1952, p. 180.

22. Eringen, A. C., *Nonlinear Theory of Continuous Media,* McGraw-Hill Book Co., New York, 1962, p. 173.

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