NASA CONTRACTOR REPORT

NASA CR-327

NASA CR-327

COMMUNICATION PROBABILITIES FOR ORDERLY-SPACED SATELLITES

by John C. Houbolt

Prepared under Contract No. NAS 1-4585 *by*
AERONAUTICAL RESEARCH ASSOCIATES OF PRINCETON, INC.
Princeton, N. J.
for Langley Research Center

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION - WASHINGTON, D. C. - NOVEMBER 1965

NASA CR-327

COMMUNICATION PROBABILITIES FOR ORDERLY-SPACED SATELLITES

By John C. Houbolt

Prepared under Contract No. NAS 1-4585 by
AERONAUTICAL RESEARCH ASSOCIATES OF PRINCETON, INC.
Princeton, N. J.

for Langley Research Center

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

For sale by the Clearinghouse for Federal Scientific and Technical Information
Springfield, Virginia 22151 – Price $2.00

COMMUNICATION PROBABILITIES FOR ORDERLY-SPACED SATELLITES

by

John C. Houbolt

SUMMARY

Probability equations to determine the amount of time communication is available for various combinations of equal and random spacing of communication satellites are given. Application is made to selected communication links including Boston-London, and Los Angeles-Hawaii, to bring out the effects of using equal spacing. Improvements are noted, but for randomly-spaced planes and equally spaced satellites per plane, the improvements are only small over the case of completely random distribution. Equally-spaced planes, and equal spacing of the satellites per plane, the most sophisticated ordering, gives the most improvement; one example: for Boston-London at 2,000 miles altitude, 24 satellites with four equally-spaced planes, six equally-spaced satellites per plane yields a communication probability equal to .98, contrasted to .8 for random spacing. Optimum conditions are indicated when six equally-spaced satellites per plane are used, whether the planes are randomly or equally spaced.

INTRODUCTION

In the study of communication satellites, whether of the passive or active type, one of the chief considerations is the determination of the number required for given probabilities of communication time between fixed ground locations. This problem is considered in Reference 1 for the situation of a random distribution of satellites (at fixed altitudes). Reference 2 gives further applications of the techniques developed in Reference 1 to a large number of North-South and East-West communication links.

The purpose of the present report is to establish communication probabilities for the case of orderly or equally-spaced satellites, or for mixed distributions of orderly and random spacings; the report is thus a companion volume to Reference 1. Mixed distributions can arise because the satellites may be equally spaced in orbital planes that have random spacing, or they may be randomly distributed in equally-spaced orbital planes. Besides the spacing factor, the influence of altitude, orbital plane inclination, elevation angle for communication, and station separation distance are studied.

The theory is first developed for various combinations of distributions--eight cases in all. Application is then made by way of illustration to three possible communication links of interest: Boston-London, Newfoundland-Ireland, Los Angeles-Hawaii.

Results are given to show a comparative assessment between the use of random or orderly spacing; aspects such as launching difficulties, launch vehicle requirements, cost, etc., are not considered here.

SYMBOLS

h	satellite orbit altitude
i	orbit inclination angle with respect to equator
M	number of orbital planes
N	number of satellites in each orbital plane

$P_c(MN)$	probability of communicating between two ground stations with M • N satellites in orbit
$P_{nc}(MN)$	probability of not communicating between two ground stations with M • N satellites in orbit
R	mean radius of earth, 3,960 U. S. statute miles
$\Delta\alpha$	geocentric angle outside of region of mutual communication for each orbital pass
β	minimum station elevation angle for communicating with satellite

DEVELOPMENT OF PROBABILITY FORMULAE

The governing equations for determining the percentage of time that communications are possible (or not possible) between two ground stations are developed in this section.

In order to keep the development from being overly complicated, and in order to make the results more meaningful from a general point of view, the following restrictions or assumptions are adopted. The parameters, orbital altitude h, orbital plane inclination i, and minimum station elevation angle for communication β, while variables, are assumed essentially the same for all satellites in a given case; that is, cases involving a mixture of satellites at different altitudes, or a mixture of different orbital plane inclinations, are not considered. (This assumption is of course consistent with the establishment of a realistic system.)

Central in this theoretical development is the establishment of the proportion of time that a given satellite is not mutually visible during each orbital path. The basic theory for this determination is given in Reference 1; as a reminder of some of the basic geometrical aspects involved, see Figure 1, which is reproduced from this reference. For present purposes, these times were found as follows. The succession of orbital passes that a given satellite makes about the earth is represented by the equivalent array of equally-spaced passes as shown in Figure 2--effectively as though a ratchet mechanism advanced the earth by the rotational increment $\Delta\phi$ after each revolution of the satellite. It follows that for any pass i, the probability that the satellite is not mutually visible is simply

$$p_i = \frac{\Delta\alpha^0}{360}, \qquad i = 1, 2, 3, \ldots m \tag{1}$$

where $m = \frac{360}{\Delta\phi^0}$

It is to be noted that the p_i 's are recurring, that is

$$p_{i+m} = p_i$$

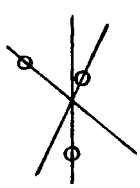
These p_i 's, which form the basic ingredients in the equations to follow, can be established readily in practice through use of a globe. A compass is used to establish the region of mutual communication, where the radius is established from the given values of altitude h, minimum elevation angle for communication β, and chosen transmission and reception stations through use of the equations given in Reference 1. If not already on the globe, a latitude circle drawn with the North Pole as the center and at a latitude equal to the given orbital inclination is also helpful. The arc of each pass over the mutual region is then measured quickly and easily by means of a simply constructed protractor, set in great circle fashion on the globe and tangent to the latitude circle; this arc subtracted from 360 gives $\Delta\alpha$. (Note, the National Geographic Globe was found ideal for this purpose.) A value of $m = 36$

was found convenient and satisfactory for all purposes.

The use of the p_i's so developed, greatly simplifies the probability notation and more particularly leads to rather easy numerical evaluation of the appropriate probability equations, as will be seen.

In the consideration of the effects of various satellite spacing, at least eight basic cases may be formulated. These cases are treated individually in the remainder of this section. The sketches that are included in some of the cases are schematic in nature and are intended to depict the case being treated; they are drawn for the case of 90° orbital plane inclination, for satellite passage over the Northern Hemisphere and as if the observer were far above the North Pole.

<u>Case I:</u> 1 satellite per orbital plane
M randomly-spaced planes
(This is the case of Reference 1)



(M = 3)

From the p_i's as defined by Equation 1, the probability of not communicating with any one of the M satellites is given through use of the mutually exclusive events theorem as

$$\frac{p_1 + p_2 + p_3 + \ldots. + p_m}{m}$$

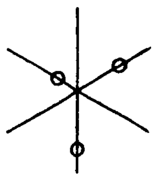
Then, by the independent events probability law, the probability of not communicating with any of the satellites is

$$P_{nc}(M) = \left(\frac{p_1+p_2+p_3+\ldots+p_m}{m}\right)_1 \left(\frac{p_1+p_2+p_3+\ldots+p_m}{m}\right)_2 \ldots \left(\frac{p_1+p_2+p_3+\ldots+p_m}{m}\right)_M$$

This equation actually applies for the case of different satellite altitudes (and inclinations). For the case of equal altitudes, the equation is simply

$$P_{nc}(M) = \left(\frac{p_1 + p_2 + p_3 + \dots + p_m}{m}\right)^M \qquad (2)$$

Case II: 1 satellite per orbital plane
M equally-spaced planes



(M = 3)

The probability of not communicating with any of the satellites for a single pass combination is

$$p_i \cdot p_{i+k} \cdot p_{i+2k} \cdots p_{i+(M-1)k}$$

where $k = \frac{m}{M}$

The probability of not communicating with any of the satellites for all the pass combinations is then (by the mutually exclusive law)

$$P_{nc}(M) = \frac{1}{m}\Big[p_1 \cdot p_{1+k} \cdot p_{1+2k} \cdots p_{1+(M-1)k} + p_2 \cdot p_{2+k} \cdot p_{2+2k} \cdots p_{2+(M-1)k} + \dots + p_m \cdot p_{m+k} \cdots p_{m+(M-1)k}\Big] \qquad (3a)$$

Since

$$p_i \cdot p_{i+k} \cdot p_{i+2k} \cdots p_{i+(M-1)k} = p_{i+k} + p_{i+k+k} \cdot p_{i+k+2k} \cdots p_{i+k+(M-1)k}$$

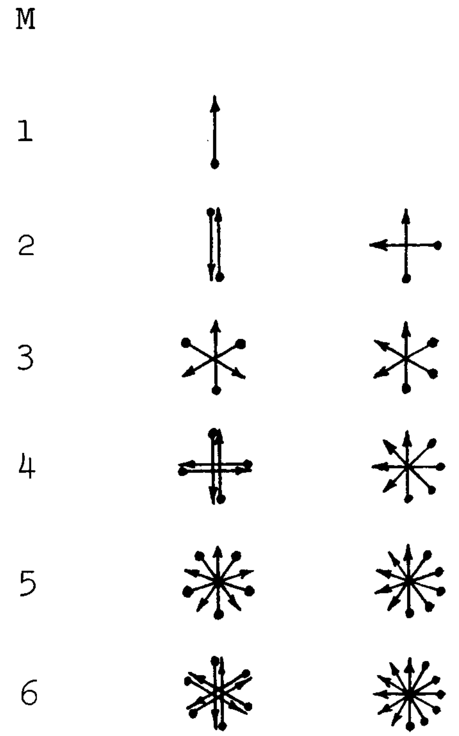
this equation may be written in the abbreviated alternate form

$$P_{nc}(M) = \frac{1}{k}\Big[p_1 \cdot p_{1+k} \cdot p_{1+2k} \cdots p_{1+(M-1)k}$$

$$+ p_2 \cdot p_{2+k} \cdot p_{2+2k} \cdots p_{2+(M-1)k} + \cdots \qquad (3b)$$

$$+ p_k \cdot p_{k+k} \cdot p_{k+2k} \cdots p_{k+(M-1)k}\Big]$$

When $i = 90$ for this case, two solutions occur, depending on the interpretation of what is meant by equally-spaced planes. This anomaly of differing possibilities may be indicated by the following sketches



The cases on the right are to be preferred in general for the polar type orbits shown (although there is no difference in the results for odd values of M), while the cases on the left apply otherwise. The solution for the cases on the right is simply Equation 3 with k redefined as $k = \frac{m}{2M}$.

Case III: N randomly-spaced satellites in each orbit
1 orbital plane

The probability of not communicating with the N satellites during one orbital pass is p_i^N. The probability of not communicating with any of the satellites for all the passes is then

$$P_{nc}(N) = \frac{p_1^N + p_2^N + p_3^N + \dots + p_m^N}{m} \tag{4}$$

Case IV: N equally-spaced satellites per orbital plane
1 orbital plane

For a given orbital pass, the probability of communicating with one satellite is $1 - p_i$; the probability of communicating with the N satellites in the orbit is then $N(1 - p_i)$, from whence it follows that the probability of not communicating with any of the N satellites during the entire single pass is

$$\begin{aligned} q_i &= 1 - N(1 - p_i) \\ &\geq 0 \end{aligned} \tag{5}$$

The proviso that q_i must be greater or equal to zero is added because if N becomes large enough, one of the satellites, because of their equal spacing, will always be visible during a full orbital pass over the mutual region (q_i negative has no meaning). The probability of not seeing any of the satellites for all the passes follows as

$$P_{nc}(N) = \frac{q_1 + q_2 + q_3 + \dots + q_m}{m} \tag{6}$$

Case V: N randomly-spaced satellites per orbital plane
M randomly-spaced planes

This case follows directly from Case III; the probability of not communicating with the $M \cdot N$ satellites is

$$P_{nc}(MN) = \left[P_{nc}(N)\right]^{M} \tag{7}$$

where $P_{nc}(N)$ is defined by Equation 4, Case III.

Case VI: N randomly-spaced satellites per orbital plane
M equally-spaced planes

The probability of not seeing any of the N satellites during one full orbital pass is p_1^N; the probability of not seeing any of the $M \cdot N$ satellites for one pass combination is (see Case II)

$$p_1^N \cdot p_{1+k}^N \cdot p_{1+2k}^N \cdots p_{1+(M-1)k}^N$$

leading hence to the probability of not seeing any of the $M \cdot N$ satellites for all pass combinations as

$$P_{nc}(MN) = \frac{1}{m}\Big[p_1^N \cdot p_{1+k}^N \cdot p_{1+2k}^N \cdots p_{1+(M-1)k}^N + p_2^N \cdot p_{2+k}^N \cdot p_{2+2k}^N \cdots p_{2+(M-1)k}^N + \ldots + p_m^N \cdot p_{m+k}^N \cdots p_{m+(M-1)k}^N\Big] \tag{8a}$$

where $k = \frac{m}{M}$

As with Equation 3b, this equation may also be written in the alternate form

$$P_{nc}(MN) = \frac{1}{k}\Big[p_1^N \cdot p_{1+k}^N \cdot p_{1+2k}^N \cdots p_{1+(M-1)k}^N$$

$$+ p_2^N \cdot p_{2+k}^N \cdot p_{2+2k}^N \cdots p_{2+(M-1)k}^N \qquad (8b)$$

$$+ \dots + p_k^N \cdot p_{k+k}^N \cdots p_{k+(M-1)k}^N\Big]$$

As in Case II, an anomaly occurs when $i = 90°$; thus, solutions are also possible by Equations 8 with k defined as $k = \frac{m}{2M}$. With M even, the use of $k = \frac{m}{2M}$ will usually lead to higher communication probabilities than with $k = \frac{m}{M}$, but sometimes this is not the case (for N small in particular).

Case VII: N equally-spaced satellites per orbital plane
M randomly-spaced planes

This case is an extension of Case IV; the probability of not seeing any of the $M \cdot N$ satellites becomes

$$P_{nc}(MN) = \left[P_{nc}(N)\right]^M \qquad (9)$$

where $P_{nc}(N)$ is defined by Equation 6, Case IV.

Case VIII: N equally-spaced satellites per orbital plane
M equally-spaced planes
(The most orderly case of all)

The q_i 's of Case IV, Equation 5, are used here in a manner similar to the p_i 's in Case II. Thus, the probability of not communicating with any of the $M \cdot N$

satellites is

$$P_{nc}(MN) = \frac{1}{m}\Big[q_1 \cdot q_{1+k} \cdot q_{1+2k} \cdots q_{1+(M-1)k}$$

$$+ q_2 \cdot q_{2+k} \cdot q_{2+2k} \cdots q_{2+(M-1)k} \qquad (10)$$

$$+ \ldots + q_m \cdot q_{m+k} \cdots q_{m+(M-1)k}\Big]$$

where $k = \frac{m}{M}$ and q_i is defined by Equation 5. Again as in Cases II and VI, another solution is possible for $i = 90^\circ$ with k defined by $k = \frac{m}{2M}$. The comments made at the end of Case VI apply here also.

RESULTS OF EXAMPLE APPLICATIONS

As a means for bringing out the effects of using orderly spacing as compared to random spacing, the equations of the previous section were applied to the following selected communication links

a) Boston-London, 3250 miles; Lat. from 42° to 51°
b) Los Angeles-Hawaii, 2440 miles; Lat. 33° to 19°
c) Newfoundland-Ireland, 2000 miles (Limited results; this tract falls essentially along the Boston-London track and is intended primarily to show influence of shorter separation distances between stations)

Parameters investigated include altitudes of 1500, 2000, 3000 statute miles, orbital inclination of 45, 60, 90°, and the influence of minimum elevation angle of reception.

Number Required. Figure 3 gives the p_i 's that apply to each link. As mentioned earlier, these are of fundamental importance in determining communication probabilities.

Although the probability equations of the previous section were developed in terms of discrete p_i values, the p_i curves could be used in a continuous function sense as well. For example, Equation 7 for Case V would appear in the form

$$P_{nc}(MN) = \left[\frac{1}{2\pi}\int_0^{2\pi} p^N d\phi\right]^M$$

for p treated continuously, where ϕ is longitude. In other cases, such as Case II, correlation type integrals would be involved. In any case, the probabilities are seen to be related to an averaging process of the p_i curves. Thus, the relative behavior expected between two links for differring situations, such as different orbital plane inclinations or trading off altitude for inclinations, can usually be established by visual evaluation of the p_i curves; in general, the lower the average value of p_i, the better the communication probabilities.

The probability $P_c(MN)$ for communication, equal to $1 - P_{nc}(MN)$, is shown for the three links in Figures 4, 5, and 6, for h, i, and β as indicated on the figures. These figures not only show probability levels, but are a good means for evaluating the effectiveness of using ordered spacing. To illustrate, consider that the number of satellites to be launched is limited to 24; the question is, what distributions of satellites lead to the highest probability for communication. The answer to this question for the Boston-London link is shown by the dotted lines labeled $M \cdot N = 24$ in Figure 4. At least four observations may be made:

For randomly-spaced orbital planes (top of figure),

a) For random spacing in each orbital plane, it is best to launch all the satellites completely at random, that is, $N = 1$.

b) For equal spacing in each orbital plane, an optimum distribution does occur at $M = 4$, $N = 6$; the peak is only slightly higher than for complete random distribution however; $P_c = .84$ as compared to .78. The question thus arises, is this slight increase in probability worth the added difficulty of achieving equal spacing in each orbital plane?

For equal spacing of the orbital planes (bottom of figure),

c) For random distribution in each orbital plane, negligible difference in communication probability is found for $M > 3$; the probability for 24 equally spaced planes is only slightly better than 24 random planes, $P_c = .8$ compared to .78.

d) For equal spacing in each plane, a pronounced improvement develops, with a maximum probability of near .98 at $M = 3$, $N = 8$ or $M = 4$, $N = 6$. This case is the most sophisticated ordering, and the price of obtaining it (and maintaining it) must be evaluated against the benefits derived.

Some of the results shown in Figure 4 are presented in tabular form in Table I for comparative purposes. As indicated by this table, two ways of evaluating the problem should be kept in mind; one is to determine the number of satellites for stipulated communication time, the other is to determine the amount of communication time that is available for a given number of satellites. This difference in interpretation becomes more evident for probabilities near unity. For example, other results indicate that 50 satellites are required to give a $P_c = .9993$; on the other hand, only 25 are needed to yield a $P_c = .98$, which represents a significant savings in number required for only a modest drop off in communication time.

For the Los Angeles-Hawaii link, Figure 5, $i = 45^{\circ}$ was found to be better than $i = 90^{\circ}$. The results show trends similar to that of Figure 4, but over-all probability levels are higher due primarily to the shorter distances between stations.

The limited results shown in Figure 6 for the Newfoundland-Ireland link are primarily for comparison with the results shown in Figure 4. The shorter distance between station is seen to increase markedly the communication probabilities.

<u>Effect of Altitude</u>. The effect of satellite altitude on communication probabilities is shown in Figure 7. To keep the presentation in simple form, only selected results are included, as shown. The following is indicated. Below a certain altitude, no communication is possible since there is no region of mutual visibility. The variation of communication probability with altitude depends on the spacing combination and may be: (a) gradually increasing with altitude (b) a marked rise at first with only a slow rise thereafter so that there is not much point in going beyond a certain altitude because only modest gains are realized or (c) an altitude may be reached which yields 100% communication time.

The results for the Boston-London link also bring out the effect of using both $k = \frac{m}{M}$ and $k = \frac{m}{2M}$ for $i = 90^{\circ}$ for equal spacing on M. Specifically, the comparison of curve a with the dotted curves shows that in this instance the use of $k = \frac{m}{2M}$ leads to appreciably better probabilities.

<u>Effect of Orbital Plane Inclination</u>. Figure 8 shows the effect of orbital plane inclination on communication probabilities for the same cases shown in Figure 7 for altitude. The figure indicates that there may be an inclination below which no communication is possible. Further, depending on the spacing combination, $i = 90$, may be best, there may

be an optimum i below 90, or, because the curve becomes fairly flat, there is a range of i within which essentially the same results are obtained.

The results also show the effect of using $k = \frac{m}{M}$ and $k = \frac{m}{2M}$ for combination (a); it is seen that as $i = 90^{\circ}$ is approached, $k = \frac{m}{2M}$ leads to the best equal spacing for M.

With respect to an optimum i, a rule of thumb may be used in the absence of specific results; that is, optimum i should occur at an i a few degrees less (perhaps 10°) than the maximum latitude value of the region of mutual communication. These maximum latitude values are indicated by the longer tick marks along the abscissa.

For the Los Angeles-Hawaii link, the following observation is also pertinent. For $i = 0$, none of the combinations shown lead really to Case IV, the situation of 24 equally-spaced satellites around the equator. If 24 satellites were equally spaced around the equator, a 100% communication would result; actually, for this situation with $i > 0$, P_c would hold at $P_c = 1$ for a range of i between 0 and some finite value of i before dropping off.

Effect of Minimum Communication Angle. An interesting fact is that changes in the minimum angle relative to the horizon for which communication is possible may be likened unto changes in orbital altitude. A given altitude h, and given minimum angle β, fixes a geocentric cone angle θ_0' according to the following equation

$$\frac{\sin\,(\theta_0' - \beta)}{\cos\,\beta} = \frac{R}{R + h}$$

which is Equation A7 of Reference 1, written in different form; R is the radius of the earth. A plot of this relation is given in Figure 9, which also shows the definition of θ_0'.

For a given communications link, fixing θ_0' has the effect of defining similar regions of mutual communication, and hence a fixed set of p_i 's, regardless of the combinations of h and β. Thus, if communication probabilities are known for various altitudes, the effect of changing β can be determined readily by simply moving vertically along the appropriate θ_0' line. The illustrative path shown on the figure, for example, indicates that if the minimum β is increased from 5 to 10°, with $h = 2000$, then the communication probabilities would decrease to what they are for $\beta = 5$ and $h = 1500$; alternatively, to retain the same communication probabilities in increasing β from 5 to 10°, the altitude would have to be increased from 2000 to 2600. The figure thus serves to eliminate specific probability computations to establish the effect of varying β (or to establish the effect of varying h if results for various β 's are known).

CONCLUDING REMARKS

The probability equations developed herein allow an assessment to be made of the use of orderly spaced communication satellites for improving communication probabilities. Based on the results for two communication links, Boston-London, and Los Angeles to Hawaii, the following observations are made. Spacing combinations in order of increasing improvements appear to be

1. Randomly-spaced orbital planes, random spacing in each plane.
2. Equally-spaced orbital planes, random spacing in each plane.
3. Randomly-spaced orbital planes, equal spacing in each plane.

4. Equally-spaced orbital planes, equal spacing in each plane.

For condition 1, it is best to have completely random spacing, that is, all the planes should be randomly spaced with only one satellite per plane. For condition 2, and a given total number of satellites, M • N, probabilities are essentially independent of the number of equally spaced planes, M, as long as M is greater than 3. For conditions 3 and 4, definite optimum combinations of M and N occur in ranges defined by the following numbers

MN	M No. of Planes	N Satellites Per Plane
6	1	6
12	2	6
24	4	6
36	6	6

Thus six equally-spaced satellites per plane appears to lead to the best communication probabilities, in general; the peaks of the curves leading to these combinations are fairly flat, however, and so nearby combinations give about the same results. For MN = 24, for example, M = 3 and N = 8 gives equal or even slightly better results than M = 4, N = 6. It is to be noted that equal spacing in only one plane is to be avoided in general. For example, 24 equally-spaced satellites in one plane gives results which are appreciably lower than a complete random distribution of the 24 satellites (i.e., a bad ordered spacing can be worse than wholly random spacing).

The improvement of conditions 2 and 3 over 1 is only small, and it is doubtful, whether the improvements are enough to offset the added problems brought about in achieving equal spacing, namely, in the case of 3, the pinpoint rendezvous operation required and the station keeping problem that results thereafter. Whether condition 4 is sufficiently better can only be adjudged by considering the tradeoffs that are involved in each application.

With respect to altitude, increasing altitudes lead in general to better communication probabilities. On the other hand, from a system point of view, increasing altitude leads to greater transmission distances and requires much greater power requirements (as evidenced by the fourth power distance law in the radar equation). With respect to orbital plane inclination, each link of course has an optimum value. The value to be used depends on the number of links to be serviced. When the stations are more towards either pole than the equator, such as the Boston-London link, North-South rather than East-West type orbits tend to be favored. The consequence, then, from a launching point of view, is a reduced orbital weight.

The effects of minimum elevation angle for communication is shown to be equivalent to altitude effects; increasing the minimum angle corresponds to decreasing the altitude.

REFERENCES

1. Bennett, F. V., Coleman, T. L., and Houbolt, J. C. Determination of the Required Number of Randomly-Spaced Communication Satellites. NASA TN D-619, January 1961.

2. Bennett, F. V. Further Developments on the Required Number of Randomly-Spaced Communication and Navigation Satellites. NASA TN D-1020, February 1962.

TABLE I. CONDENSED LISTING OF COMMUNICATION PROBABILITIES

Boston-London; h = 2000 s.m., i = 90°, $\beta = 5^{\circ}$

Number of Satellites Required:

% Communication	Randomly-Spaced Planes		Equally-Spaced Planes	
	Randomly-Spaced Satellites	Equally-Spaced Satellites	Randomly-Spaced Satellites	Equally-Spaced Satellites
70	*1 × 19 = 19	4 × 4 = 16	6 × 3 3 × 6 = 18 1 × 18	7 × 2 = 14
80	1 × 25 = 25	6 × 4 = 24	8 × 3 6 × 4 4 × 6 = 24 1 × 24	5 × 3 = 15
90	1 × 36 = 36	7 × 5 = 35	7 × 5 5 × 7 = 35	6 × 3 = 15

*First number is number of satellites in a given plane
Second number is number of planes, i.e.

NM = Total Number
\\— No. of Planes
\\— No. of Satellites Per Plane

Presented Another Way (Percent communication obtained for a given number of satellites):

Total No. of Satellites	Randomly-Spaced Planes		Equally-Spaced Planes	
	Randomly-Spaced Satellites	Equally-Spaced Satellites	Randomly-Spaced Satellites	Equally-Spaced Satellites
12	1 × 12 54%	6 × 2 59%	4 × 3 3 × 4 1 × 12 55%	6 × 2 67%
24	1 × 24 78%	6 × 4 84%	8 × 3 6 × 4 4 × 6 1 × 24 80%	8 × 3 98%
36	1 × 36 90%	6 × 6 92%	9 × 4 6 × 6 4 × 9 2 × 18 1 × 36 91%	12 × 3 100%

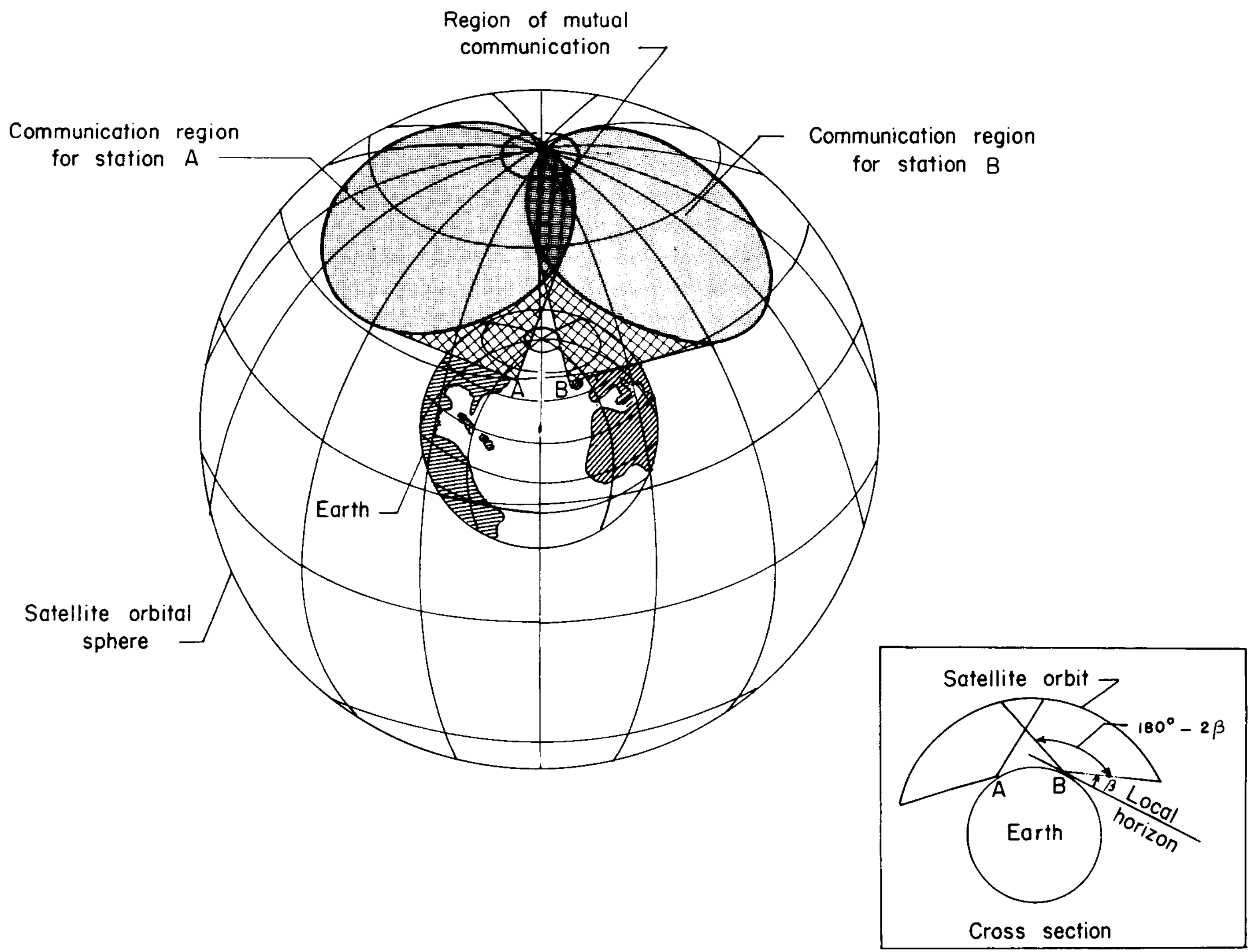


Figure 1.- Problem geometry.

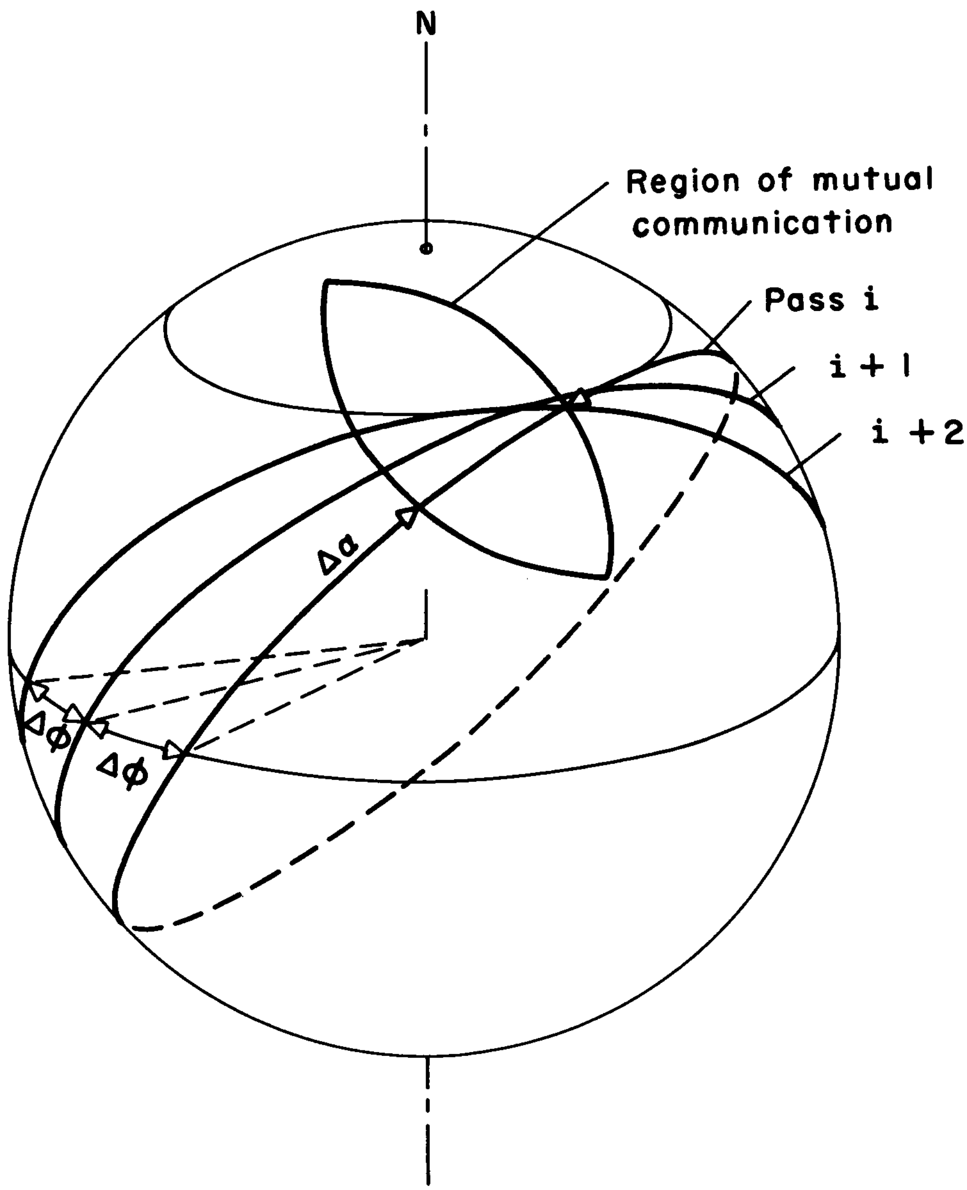
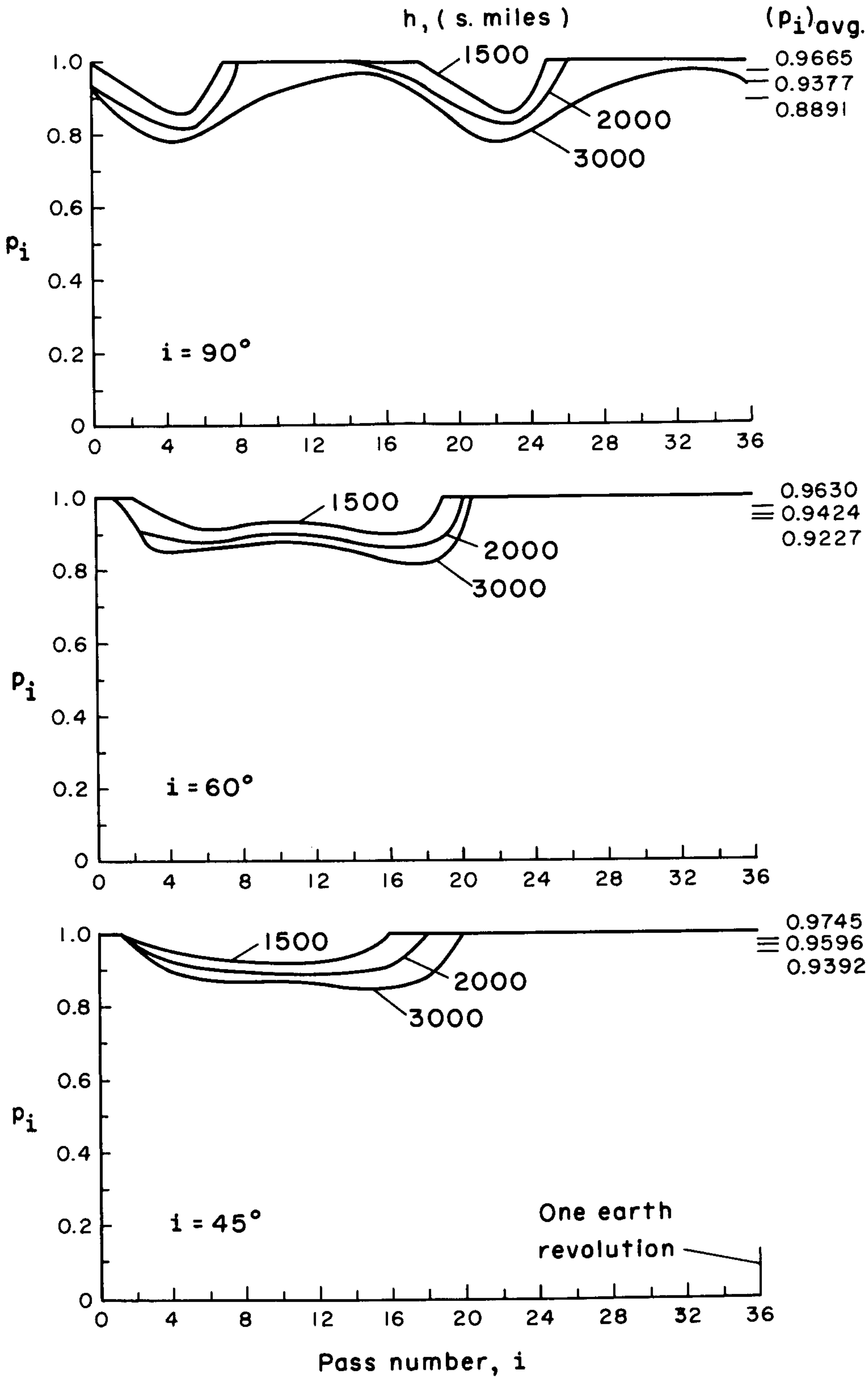
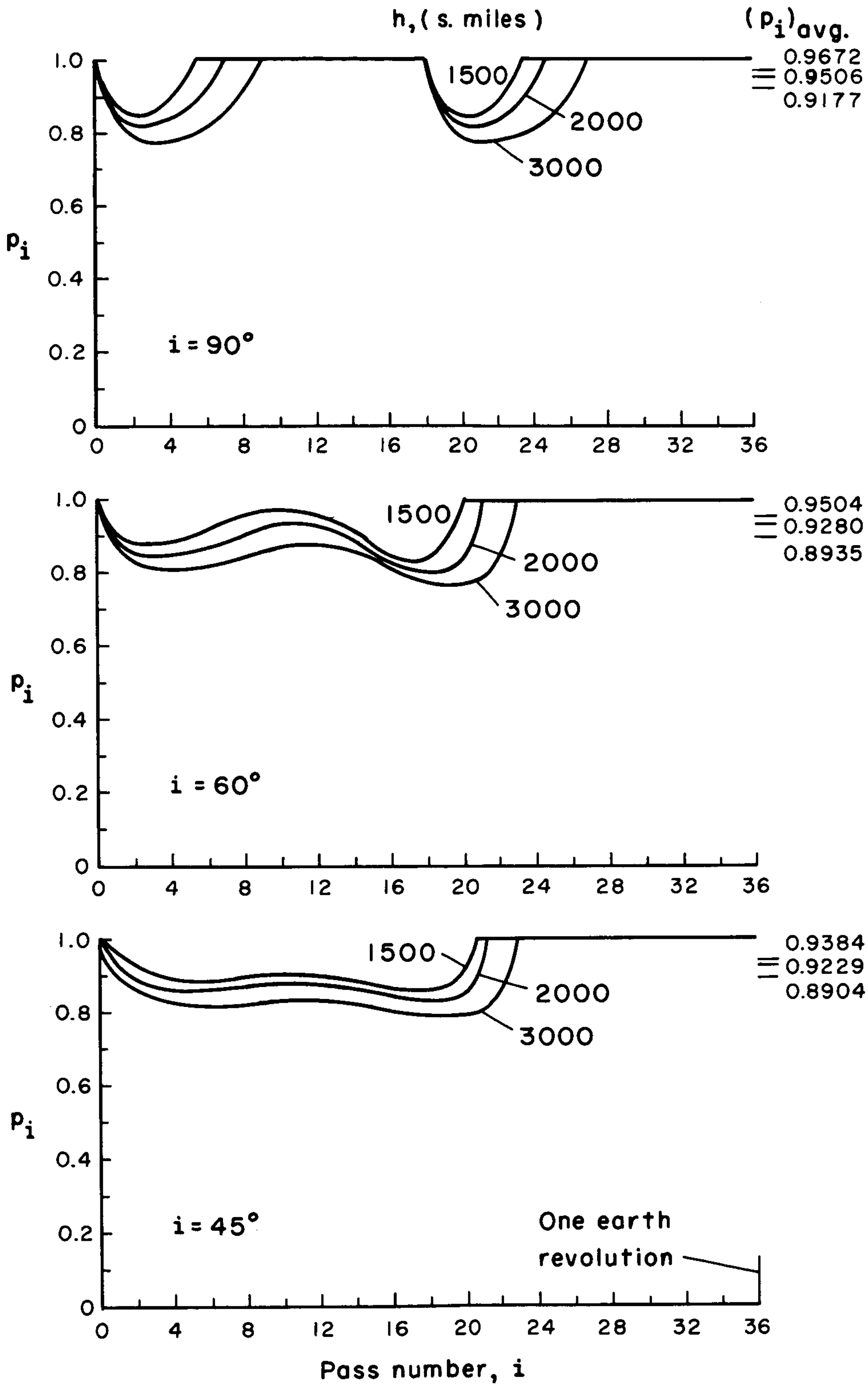


Figure 2.- Equivalent array of equally-spaced passes.



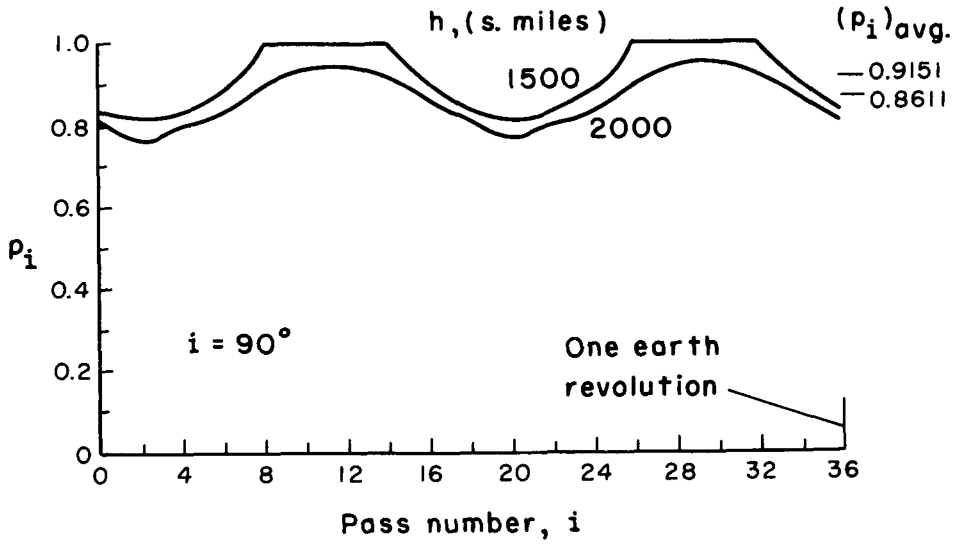
(a) Boston-London

Figure 3.- Probabilities for non-communication for individual passes.



(b) Los Angeles-Hawaii

Figure 3.- Continued.



(c) Newfoundland-Ireland

Figure 3.- Concluded.

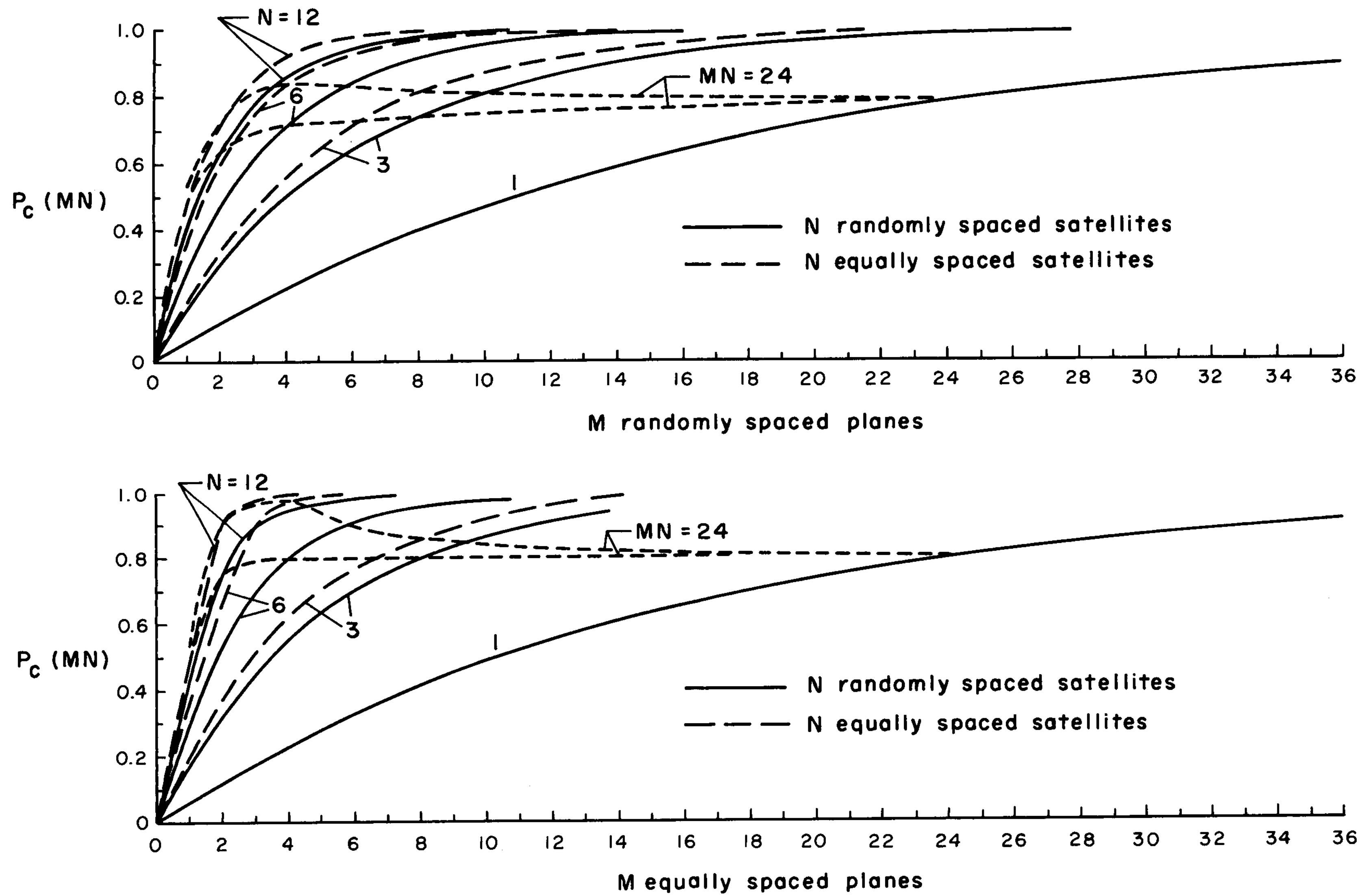


Figure 4.- Communication probabilities for Boston-London link,
h = 2,000 s.m., $i = 90^{\circ}$, $\beta = 5^{\circ}$.

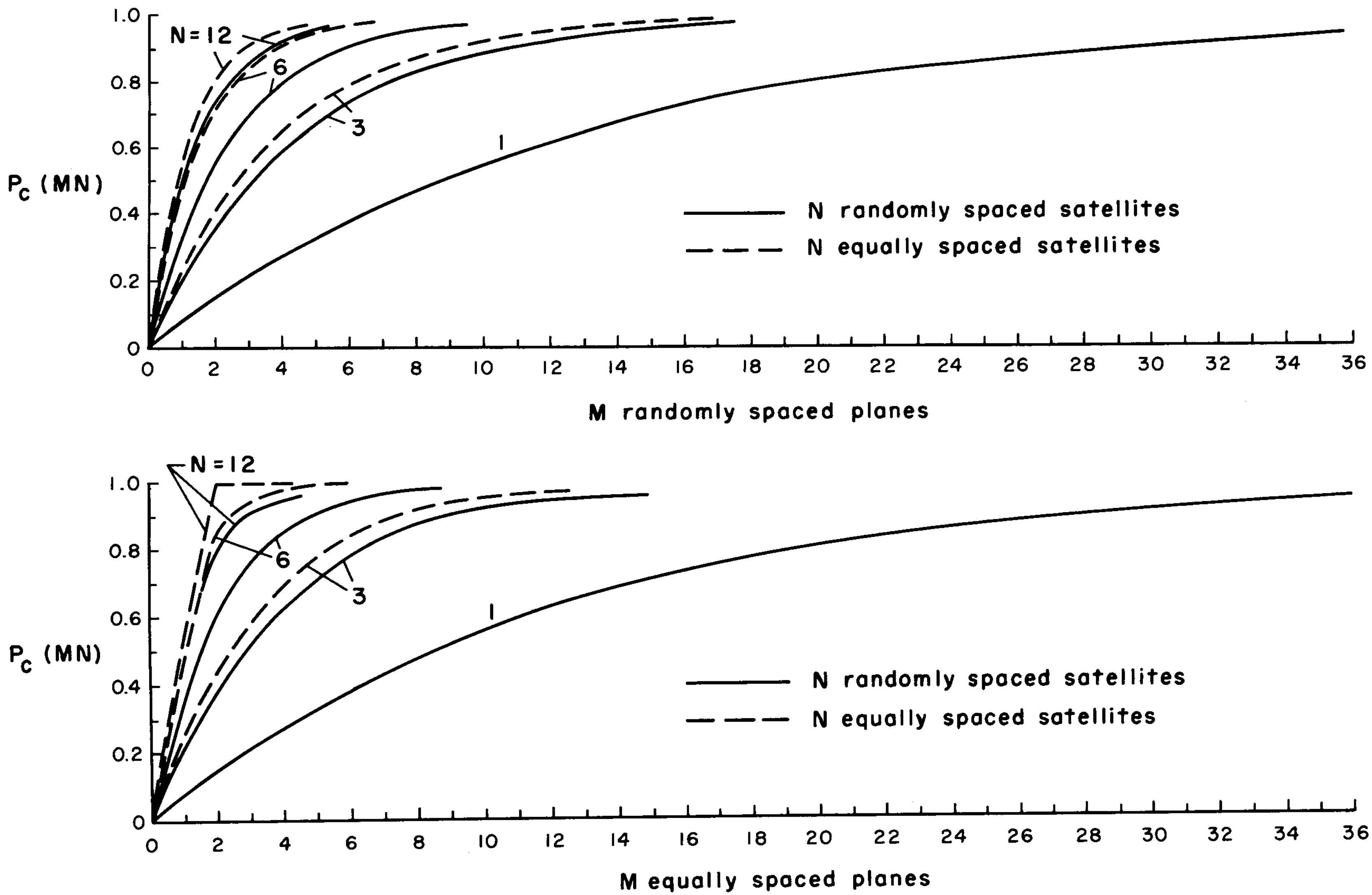


Figure 5.- Communication probabilities for Los Angeles-Hawaii link, $h = 2{,}000$ s.m., $i = 45^{\circ}$, $\beta = 5^{\circ}$.

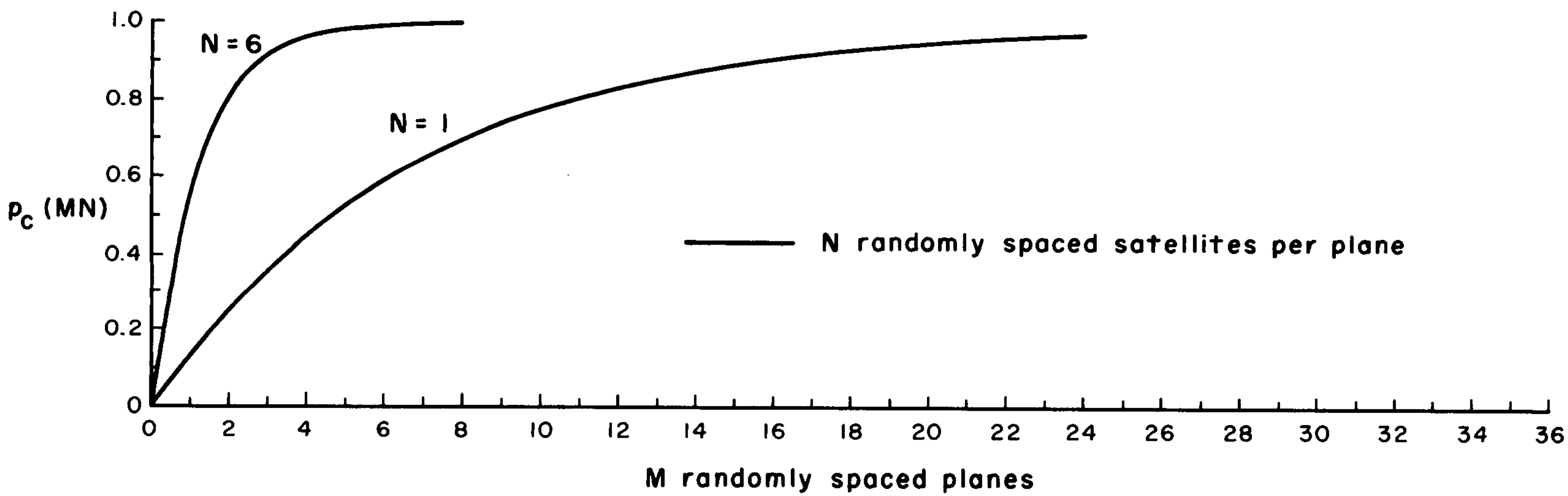
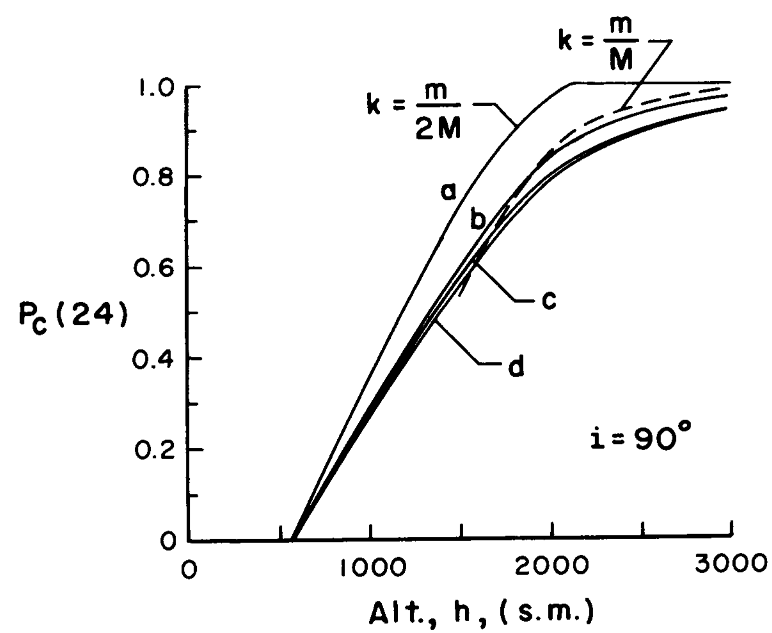
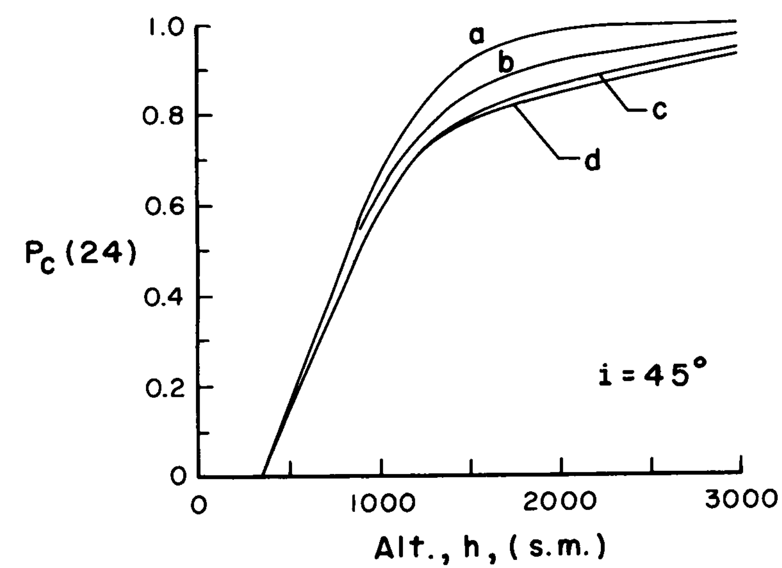


Figure 6.- Communication probabilities for Newfoundland-Ireland link, $h = 2{,}000$ s.m., $i = 90^{\circ}$, $\beta = 5^{\circ}$.

	M	N
(a)	4 Equal	6 Equal
(b)	4 Random	6 Equal
(c)	24 Equal	1
(d)	24 Random	1



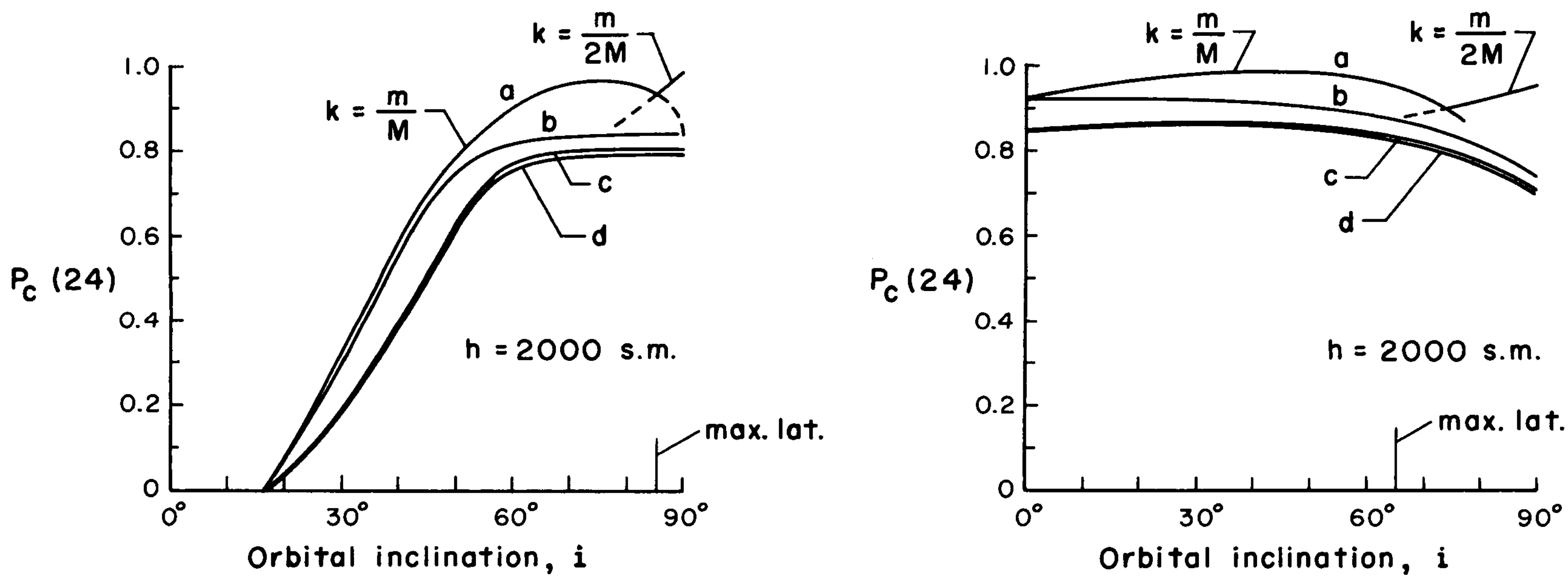
(a) Boston-London



(b) Los Angeles-Hawaii

Figure 7.- Effect of altitude on communication probabilities.

	M	N
(a)	4 Equal	6 Equal
(b)	4 Random	6 Equal
(c)	24 Equal	1
(d)	24 Random	1



(a) Boston-London

(b) Los Angeles-Hawaii

Figure 8.- Effect of orbital inclination on communication probabilities.

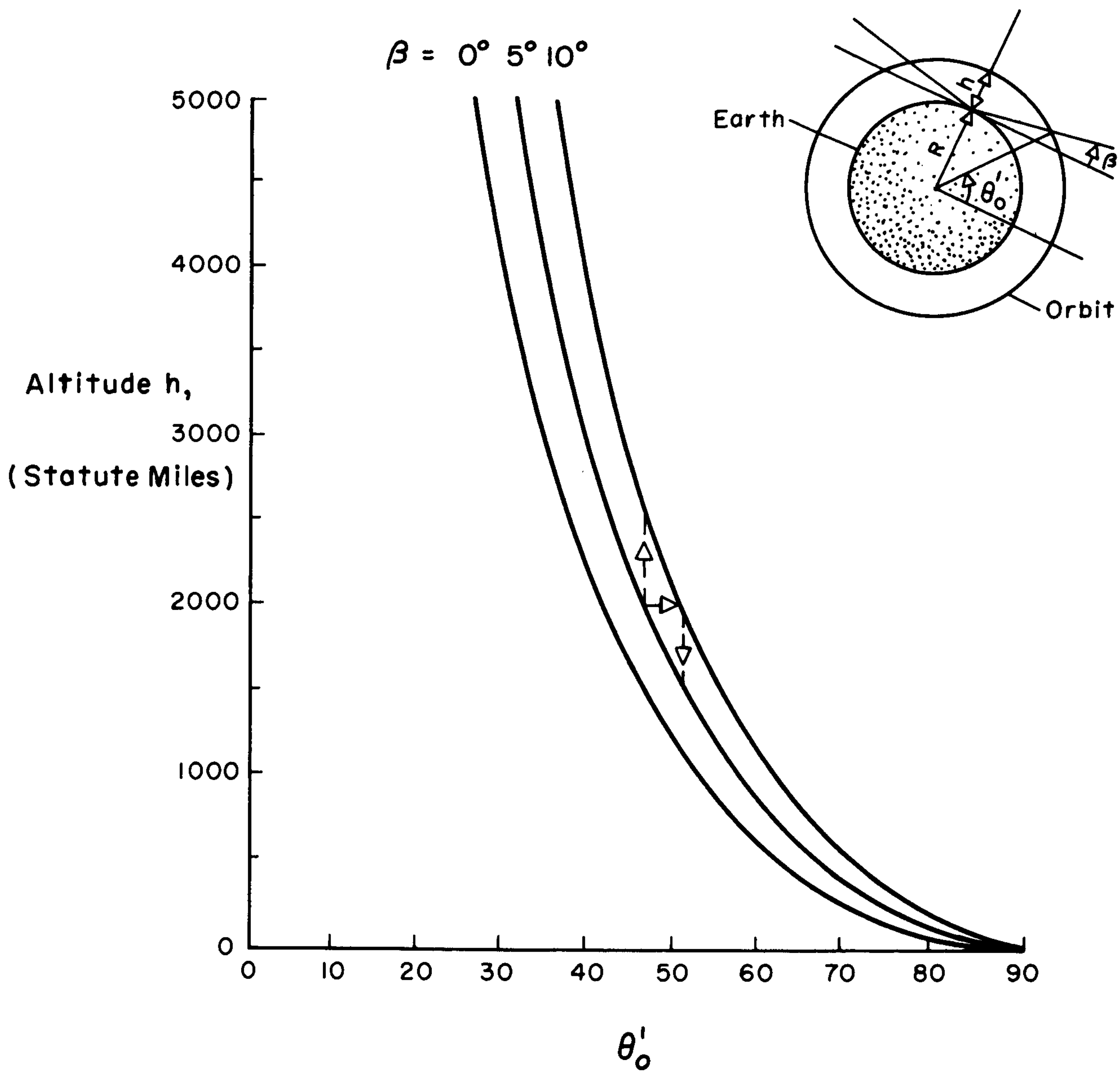


Figure 9.- Relation between h and β.

NASA-Langley, 1965 CR-327