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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

Technical Report No. 32-820

Geometric Aspects of Ground Station/ Satellite Communications

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FIGURES (Cont'd)

ABSTRACT

This Report presents a discussion of the geometric aspects of communications between a ground station and a satellite in an elliptic orbit whose line of apsides is fixed. The range to the satellite along a given vector from the ground station is shown to take on, at maximum, two values; a means for calculating these values is presented. In addition, an averaging method is used to develop an expression for the long-term fraction of time a satellite and ground station are covisible. Applications of these results to various communications satellites and ground stations are shown.

I. INTRODUCTION

A. Statement of the Problem

This Report presents a discussion of the geometric aspects of communications between a ground station and a satellite in an elliptic orbit whose line of apsides is fixed. It will be shown that the range to the satellite along any given vector from the ground station takes on, at maximum, two values, and a means for calculating these values will be presented. Also, an expression is developed for the long-term average fraction of time during which a satellite and ground station are covisible; a discussion is presented on the extension to the calculation of the fraction of time the satellite is in view of several stations.

B. Summary of Analysis and Results

An averaging method is used to calculate the satellite/ground station range and viewing characteristics. A principal aid in this analysis is the determination of the locus of all possible satellite positions as seen from a coordinate system rotating with the planet. This locus is a toroidal surface and is termed the surface on which the satellite always lies, or simply the satellite surface. From the satellite surface concept, the set of all possible lander-to-satellite ranges is immediately available.

The long-term average fraction of time in view (viewing fraction) may be found by noting what portion of the satellite surface is visible from the ground station and then integrating the satellite's time density over that portion. The viewing fraction is obtained from a single integral and is shown to be a function of the orbit semi-major axis, eccentricity, inclination, argument of periapsis, the ground station's latitude, and the minimum elevation angle above the local horizontal plane at which sighting can be established.

Numerical results for selected eccentric orbits are given, and the range of the viewing fraction is shown. The communications implications of these results are discussed.

C. Background

Geometric aspects of satellite communications have been studied extensively (Refs. 1 through 5). Karrenberg and Lüders (Ref. 1) review the work done in the field prior to August, 1963, and they thoroughly treat the problem of several satellites in various circular orbits. The lenticular region on the surface of a planet concentric sphere, which is commonly visible to two stations, is treated in their paper. The viewing fraction is assumed to be proportional to the angular arc length of the possible satellite traces across the region of mutual communications. References 1 and 2 establish the validity of an averaging method.

R. Heppe (Ref. 3) has developed a graphical method for calculating the fraction of time that two or more ground stations can see a single satellite in a circular orbit. His method can be used also for eccentric orbits that have periapsis at minimum or maximum latitude; unfortunately, it is somewhat cumbersome for this application.

A Russian article by Stetsevich (Ref. 4) treats viewing fractions for one or more stations and circular orbits.

A report by the Moore School of Electrical Engineering at the University of Pennsylvania (Ref. 5) includes an analysis of satellite-ground station visibility. Essentially, it extends the graphical system of Heppe (Ref. 3) to arbitrary eccentric orbits; it does not go on to a calculation of the viewing fraction. The graphical method involves templates drawn for a particular orbit/ground station combination and is, therefore, relatively inflexible.

The motivation for the effort presented here is interesting in its own right. The *Voyager* Mars Exploration Program may invoke either of two schemes of returning data from a capsule landed on the surface of the planet. The first scheme concerns transmitting data from the capsule directly to a receiving station on Earth. The second involves transmitting the lander information to a Mars orbiting satellite, which then relays the information to the Earth. (This means of recovering data by satellite from a remote ground station is closely analogous to meteorological-data-recovery schemes proposed for the Earth (Ref. 6).) Analysis and design of the relay scheme requires knowledge of range and viewing characteristics of the lander-orbiter combination. These requirements prompted the current study. The results differ from those previously published in that they formulate analytic range and viewing fractions for arbitrary orbits.

II. FORMULATION

A. Satellite Surface and Viewing Cone Concepts

To an observer, in a coordinate system fixed at the planet center but not rotating with it, the set of all possible satellite positions can be thought of as a closed curve, an ellipse. Essentially, this curve is the collection over all time of all satellite locations. Similarly, to an observer on a rotating planet, the set of all possible satellite positions is seen as a surface. This surface is the collection over all time of all satellite orbits. Relative to the rotating planet, the motion of a satellite consists of its elliptical movement in its orbital plane, plus a constant rotation of this plane about the planet axis. Thus, the spatial locus of all satellite positions is generated by rotating the orbit ellipse about the planet axis. This surface generated by the rotation of the orbit ellipse is referred to as the satellite surface.

The generated satellite surface is toroidal in shape, but its details vary widely, depending on the elements of the orbit. Figure 1 shows an isometric representation

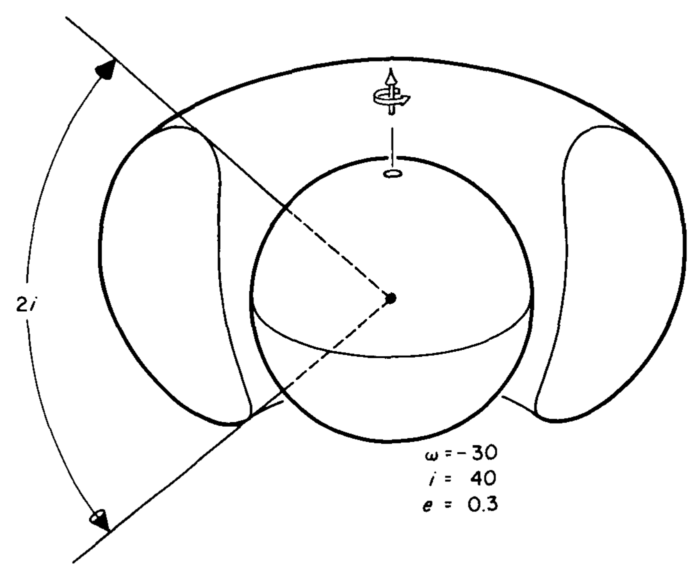

Fig. 1. Satellite surface and planet

of the primary planet and a typical satellite surface. It should be noted that the satellite does not enter the volume of the torus, but lies only on its surface.

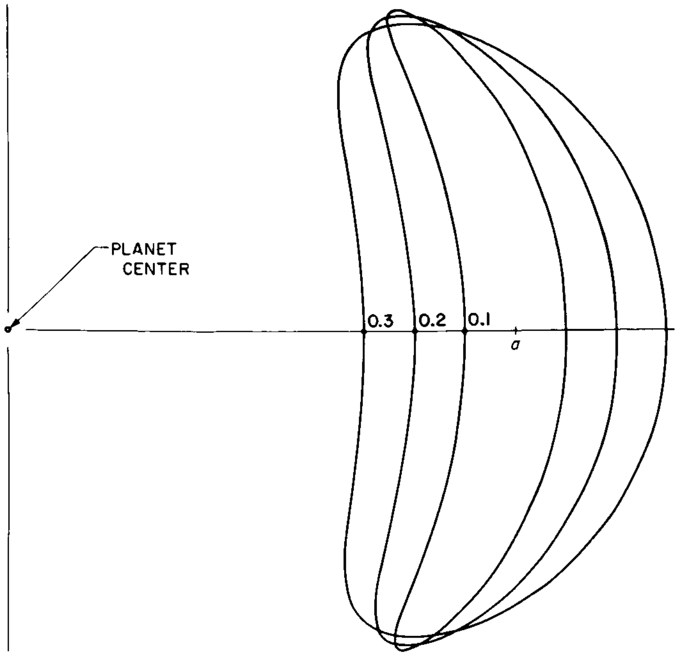

Fig. 2. Satellite surface cross sections for various eccentricities: ω = 0 deg; i = 40 deg

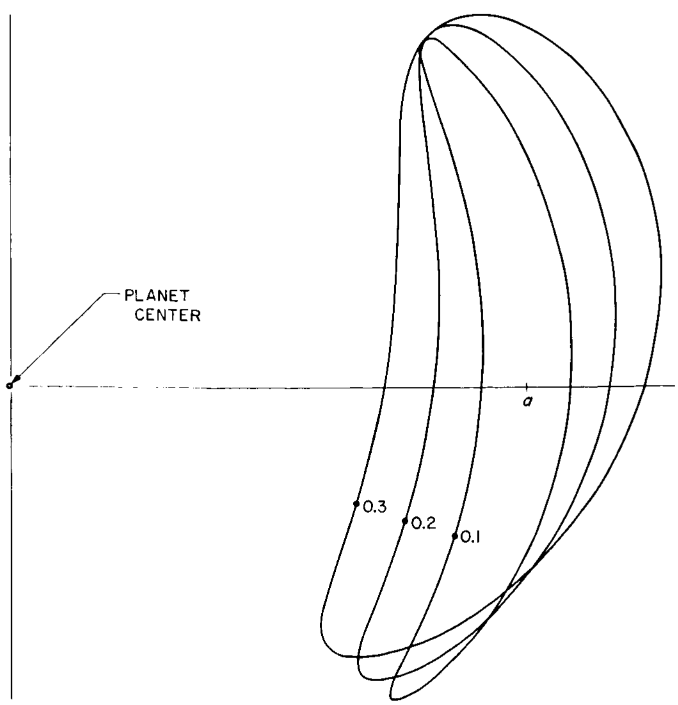

Fig. 3. Satellite surface cross sections for various eccentricities: ω = −30 deg; i = 40 deg

Since the satellite surface is a figure of revolution, a complete description may be obtained by determining its intersection with a plane containing the axis of symmetry; i.e., a cross section. A cross section may be analytically generated by plotting the satellite-planet center distance as a function of its latitude. The range may be expressed in terms of the orbital elements and the true anomaly which, in turn, is related to the satellite latitude through the orbit inclination and argument of periapsis. These generating equations are

$$r = \frac{a(1 - e^2)}{(1 + e \cos f)} \tag{1}$$

$$\sin \phi = \sin i \sin (f + \omega) \tag{2}$$

where

r = planet center/satellite range

a = semi-major axis

e = eccentricity

f = true anomaly

ϕ = latitude

i = inclination

ω = argument of periapsis

Equations 1 and 2 were programmed on an analog computer, and plots of cross sections are shown in Figs. 2 through 8. The value of the eccentricity is shown at the location of periapsis. Figure 3 is a cross section of the surface presented in Fig. 1.

Two special cases occur. When periapsis lies at minimum or maximum latitude, there is only one satellite

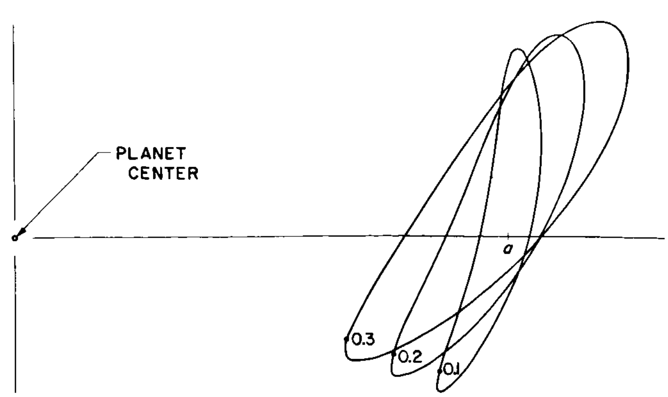

Fig. 4. Satellite surface cross sections for various eccentricities: ω = −60 deg; i = 20 deg

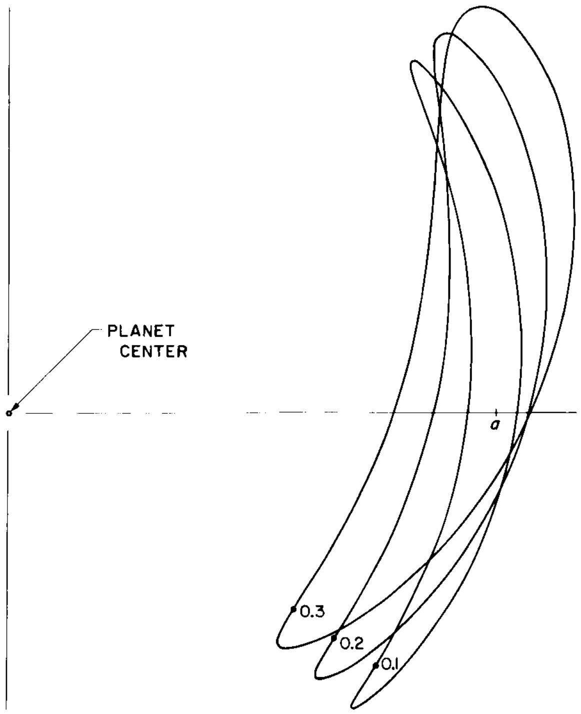

Fig. 5. Satellite surface cross sections for various eccentricities: $\omega = -60$ deg; $i = 40$ deg

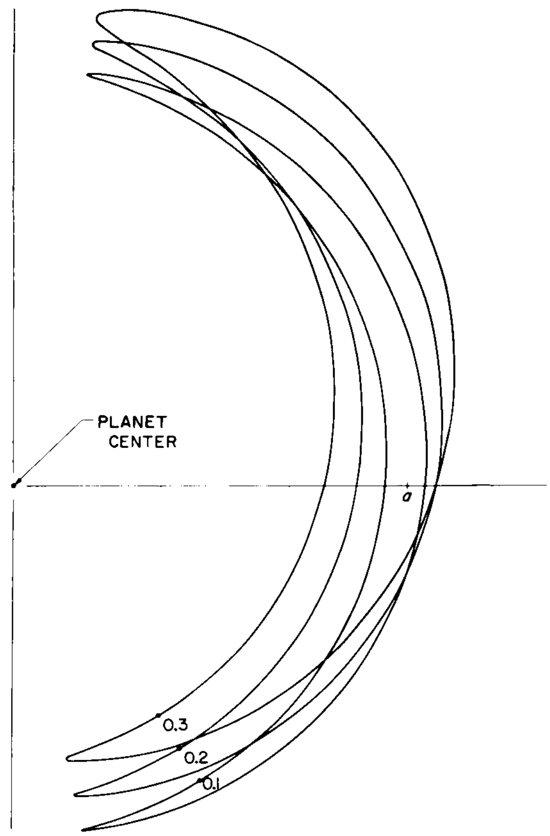

Fig. 7. Satellite surface cross sections for various eccentricities: $\omega = -60$ deg; $i = 80$ deg

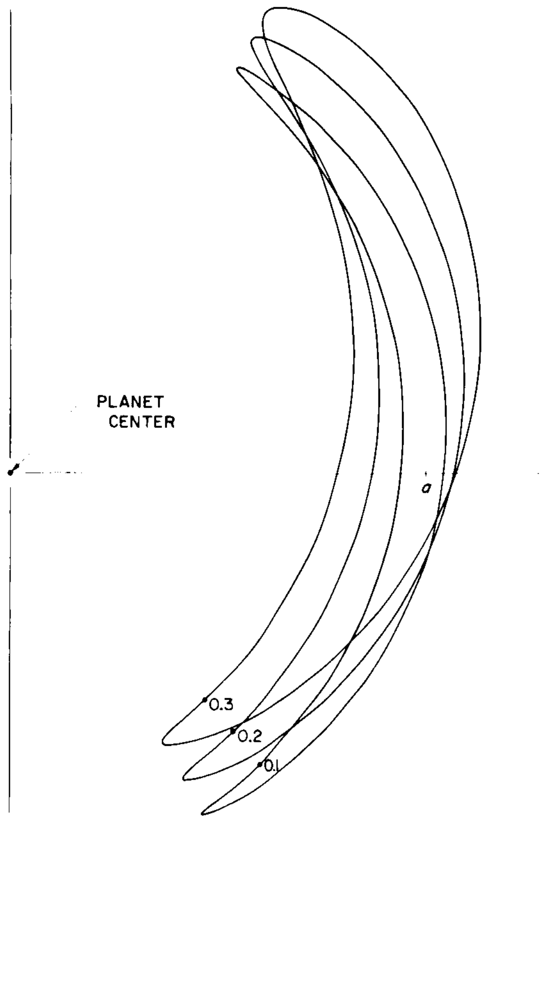

Fig. 6. Satellite surface cross sections for various eccentricities: $\omega = -60$ deg; $i = 60$ deg

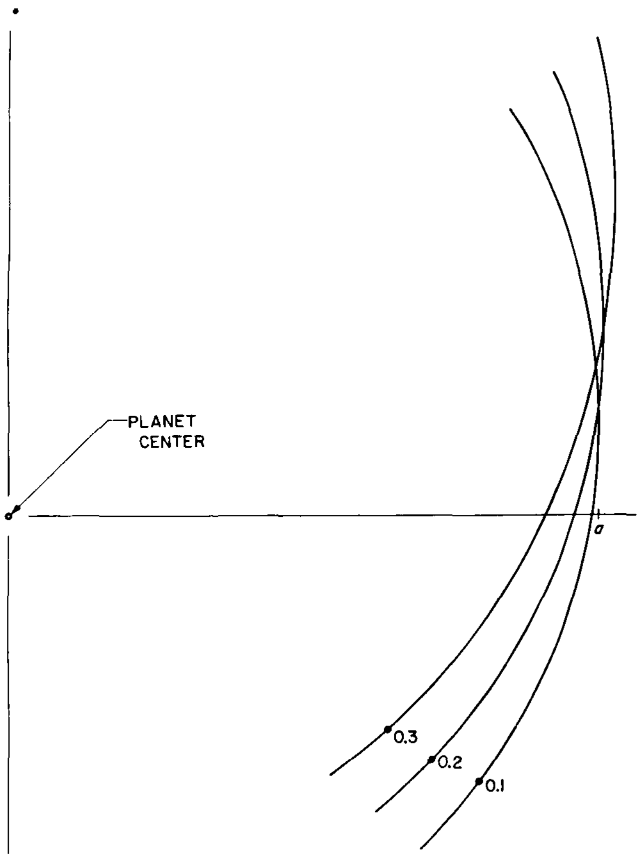

Fig. 8. Satellite surface cross sections for various eccentricities: $\omega = -90$ deg; $i = 40$ deg

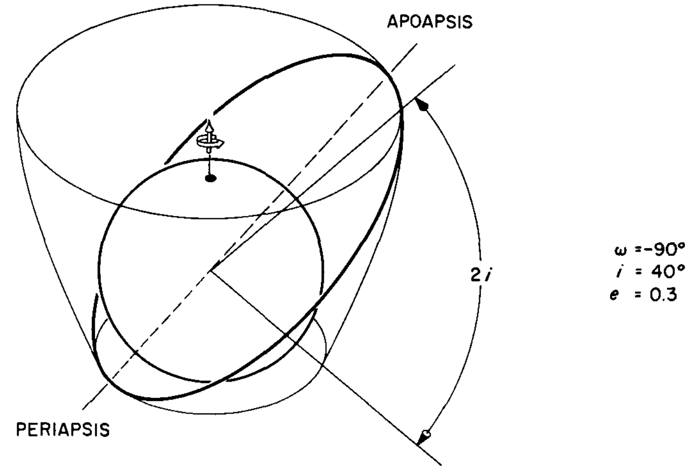

Fig. 9. Degenerate satellite surface; periapsis at minimum latitude

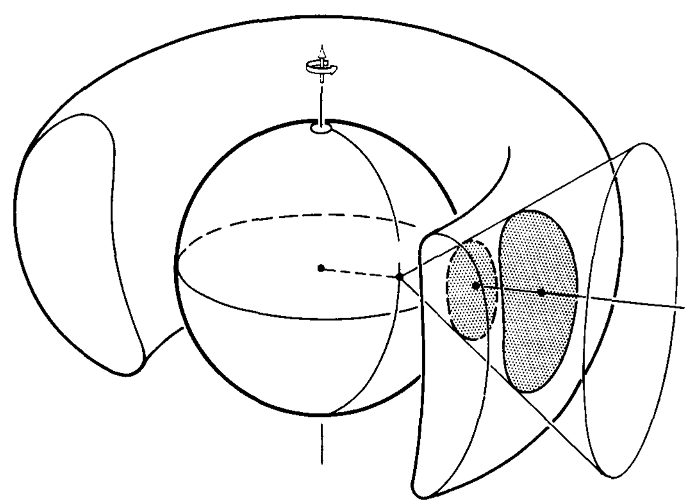

Fig. 10. Satellite surface-viewing cone intersection

radius associated with each latitude; the double satellite surface degenerates to a single one. An isometric representation of this case is shown in Fig. 9. Note that in Fig. 8, where periapsis is at minimum latitude, the satellite surface cross section is simply a single line, as opposed to a closed loop. The second special case occurs when the orbit is circular; the satellite surface is just a spherical zone. This planet concentric sphere has been used by several authors in calculations of viewing fractions. In particular, Ref. 2 refers to it as the *orbit sphere.*

Consider now a station located at latitude β. If the view of the sky from the station is restricted by a minimum elevation angle γ (independent of azimuth), then objects can be seen from the station interior to a cone of half angle $(\pi/2) - \gamma$, whose apex is at the station location and whose axis is along the zenith. Those portions of the satellite surface which are enclosed by the viewing cone represent the locus of all positions the satellite can occupy when in view of the ground station. (In general, because of the toroidal shape of the satellite surface, there is an inner and outer region.) Figure 10 shows a viewing cone intersecting the satellite surface of Fig. 1. Note the distinct inner and outer surfaces. Under some circumstances, the two may be joined at one or both of the latitude extremes of the satellite surface as shown in Fig. 11.

The inner and outer surfaces can be separated analytically by assuming that the satellite is ascending in latitude on the inner surface and descending on the outer. This contrains the argument of periapsis ω to the range $-\pi/2$ to $+\pi/2$, but this constraint is only a matter of computational convenience and does not restrict the generality of the analytic approach.

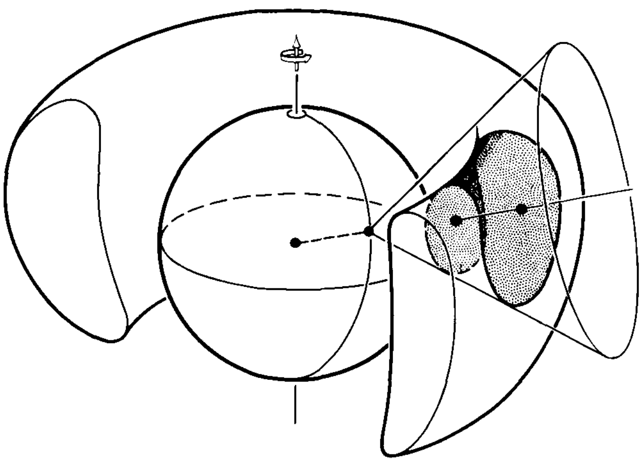

Fig. 11. Satellite surface-viewing cone intersection (inner and outer surfaces joined)

B. Range Characteristics

Next, consider the possible station-to-satellite ranges. Over the lander can be thought to be a distorted umbrella on which the satellite, when in view, always lies. A different umbrella-like pattern is associated with the inner and outer surfaces, but, unlike the circular orbit case where the umbrella is just a spherical cap, the surfaces for the general orbit case are not necessarily symmetrical about the station zenith. The general shape of the umbrella-like surfaces may be seen by noting the two intersections of Fig. 10.

Since the shape of the satellite surface is known analytically from Eqs. (1) and (2), the possible station-to-satellite ranges may be obtained quantitatively.

C. Viewing Fraction

The long-term average fraction of covisibility, (viewing fraction) is defined as the limit of the ratio of the time in view to the total time as the latter approaches infinity. Thus

$$v = \lim_{n \to \infty} \sum_{i=1}^{n} \frac{\tau_i}{t_n} \qquad (3)$$

where

τ_i = duration of the i^{th} view period

t_n = total time elapsed at the end of the n^{th} view period

A straightforward method of determining the fraction of time in which a ground station and satellite are covisible is to analytically generate the ephemeris of the satellite, record the durations of the sightings, and form the partial sum of Eq. 3. This simulation method has the advantage that it gives a goodly amount of information about the particular problem; e.g., the distribution and durations of the sightings are found and the satellite's position and range are available at all times. This method has the following disadvantages: considerable computational time is required, especially for elliptic orbits. The method is inefficient because the satellite is out of sight a majority of the time. General results are difficult to infer from single examples, and only one case at a time may be run on a digital computer. The averaging method presented here circumvents some of the disadvantages of the simulation method.

Consider the intersection of the lander viewing cone and the inner satellite surface (shown in detail in Fig. 12). Unlike the circular orbit case whose satellite surface is a spherical zone, the intersection boundary is not simply a circle; furthermore, α, the lander-planet center-intersection boundary angle (Fig. 12), is a function of the latitude. That portion of the surface lying between latitudes ϕ and $\Delta\phi$ and subtending a longitude difference of 2θ is indicated by a shaded area in Fig. 12. Assuming that the planet's rate of rotation and the satellite's period are not harmonically related, the long-term average fraction of time the satellite lies within the longitude bounds of the shaded strip is $2\theta/2\pi$. This follows from the fact that, in a planet-fixed system, all longitudes of the satellite are equally likely over the continuum of time. Thus, the long-term probability of finding the orbiter between

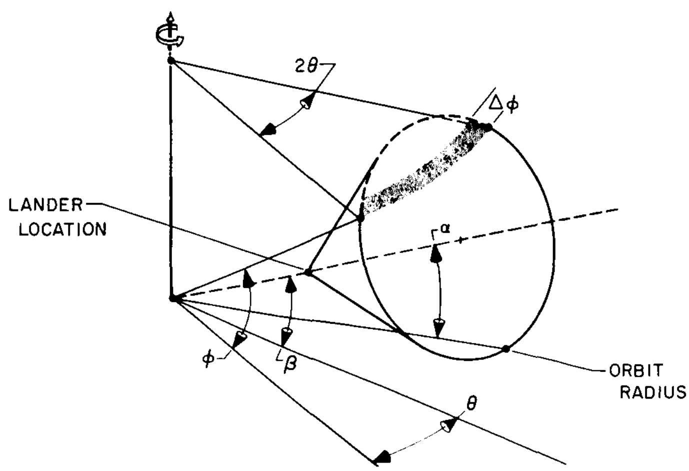

Fig. 12. Detail of surface-viewing cone intersection

any two longitudes is proportional only to their difference, and is independent of their magnitudes. The fraction of time within the strip itself is

$$\frac{\theta}{\pi}\,\frac{t(\phi + \Delta\phi) - t(\phi)}{T}$$

where $T =$ the orbit period. From spherical trigonometry, the angle θ is given by

$$\theta = \cos^{-1}\frac{\cos\alpha(\phi) - \sin\beta\sin\phi}{\cos\beta\cos\phi} \tag{4}$$

where α is obtained from

$$\cos\alpha = [r(\phi)]^{-1}\,[r_p + r_{LO}(\phi)\sin\gamma] = \left[1 - \frac{r_{LO}^2\,(\phi)}{r^2(\phi)}\cos^2\gamma\right]^{1/2}$$

$r(\phi) =$ orbit radius at latitude ϕ

$r_{LO} =$ lander/orbiter range (5)

$= [r^2 - r_p^2\cos^2\gamma]^{1/2} - r_p\sin\gamma$

$r_p =$ planet radius

The time interval dt within the shaded strip is

$$dt = \frac{dt}{df}\,\frac{df}{d\phi}\,d\phi = \frac{T\,r^2}{2\pi\,a^2}\,\frac{1}{[1 - e^2]^{1/2}}\,\frac{df}{d\phi}\,d\phi$$

The derivative $dt/d\phi$ is the time density of the satellite with respect to latitude. The view fraction may be obtained by integrating the product of this density and the factor $\theta/\pi T$ over the latitude range of interest. The total fraction of time the satellite is visible on the inner surface is then

$$v_1 = \frac{1}{2\pi^2\,(1 - e^2)^{1/2}}\int_{\phi_{11}}^{\phi_{12}}\cos^{-1}\left[\frac{\cos\alpha(\phi) - \sin\beta\sin\phi}{\cos\beta\cos\phi}\right]\frac{r^2}{a^2}\,\frac{df}{d\phi}\,d\phi \tag{6}$$

The limits ϕ_{11} and ϕ_{12} are the extremes in latitude of the viewing cone/inner surface intersection. They are given by

$$\phi_{11} = \begin{cases} -\,i; & \beta - \alpha < i \\ \beta - \alpha; & \beta - \alpha > -\,i \end{cases} \qquad \phi_{12} = \begin{cases} +\,i; & \beta + \alpha > i \\ \beta + \alpha; & \beta + \alpha < i \end{cases} \tag{7}$$

Since α is a complicated function of ϕ given by the set of equations (5), the limits on the lower line must be obtained iteratively from the equation $\beta + \alpha(\phi_{12}) = \phi_{12}$.

The expression has been formulated with the variable of integration ϕ for ease in identification of the limits. It is appropriate at this point, however, to change the variable of integration, since the limits on the top line of Eq. 7 are poles of the term $df/d\phi$ and the integrand therefore diverges at these end points. This may be avoided by integrating with respect to the true anomaly f.[1] The integral then becomes

$$v_1 = \frac{(1 - e^2)^{3/2}}{2\pi^2} \int_{f_{11}}^{f_{12}} \cos^{-1} \left\{ \frac{\cos \alpha(f) - \sin \beta \sin i \sin (f + \omega)}{\cos \beta \left[1 - \sin^2 i \sin^2 (f + \omega)\right]^{1/2}} \right\} \frac{df}{(1 + e \cos f)^2} \tag{6'}$$

and the limits are

$$f_{11} = \begin{cases} -\dfrac{\pi}{2} - \omega; & \beta - \alpha < -i \\ \sin^{-1}\left(\dfrac{\sin \phi_{11}}{\sin i}\right) - \omega; & \beta - \alpha > -i \end{cases} \qquad f_{12} = \begin{cases} \dfrac{\pi}{2} - \omega; & \beta + \alpha > i \\ \sin^{-1}\left(\dfrac{\sin \phi_{12}}{\sin i}\right) - \omega; & \beta + \alpha < i \end{cases} \tag{7'}$$

Since v_1 is for the inner surface, f is restricted to the interval

$$-\frac{\pi}{2} \leq f + \omega \leq \frac{\pi}{2}$$

The outer surface viewing fraction v_2 is given by the same integral with limits f_{21} and f_{22}.

$$f_{21} = \begin{cases} \dfrac{3\pi}{2} - \omega; & \beta - \alpha < -i \\ \sin^{-1}\left(\dfrac{\sin \phi_{21}}{\sin i}\right) - \omega; & \beta - \alpha > -i \end{cases} \qquad f_{22} = \begin{cases} \dfrac{\pi}{2} - \omega; & \beta + \alpha > i \\ \sin^{-1}\left(\dfrac{\sin \phi_{22}}{\sin i}\right) - \omega; & \beta + \alpha < i \end{cases} \tag{8}$$

[1]With this change of variable, the integration is carried out along the arc of an orbit passing through the viewing cone rather than along a longitude line, a technique similar to that used in Refs. 1 and 2.

Note that the upper limit is actually the smaller because it is associated with the maximum extreme in latitude; since the satellite is descending on the outer surface, this extreme corresponds to a smaller value of the true anomaly. The integration on the outer surface uses f in the interval

$$\frac{\pi}{2} \leq f + \omega \leq \frac{3\pi}{2}$$

The total viewing fraction v is given by the sum of the integrals for v_1 and v_2.

The limits of integration take on the values on the upper lines of Eqs. 7′ and 8 when the viewing cone encompasses one or both of the surface's extremes in latitude. In this case, the inner and outer surfaces are joined as in Fig. 11.

When $\omega = \pm\,\pi/2$, the inner and outer surfaces merge (Fig. 9) and $v = 2v_1 = 2v_2$. In this case, the satellite is at a single altitude over a given latitude, and the graphical method of Ref. 3 may be used.

For a circular orbit of radius R, the expression for v simplifies to

$$v = \int_{f_1}^{f_2} \cos^{-1} \frac{\cos\alpha - \sin\beta \sin i \sin f}{\cos\beta\,(1 - \sin^2 i \sin^2 f)^{1/2}}\, df \qquad (9)$$

where

$$\cos\alpha = \left(1 - \frac{r_{LO}^2}{R^2} \cos^2\gamma\right)^{1/2}$$

$$r_{LO} = (R^2 - r_p^2 \cos^2\gamma)^{1/2} - r_p \sin\gamma$$

Stetsevich (Ref. 4) calculates a quantity equivalent to the viewing fraction, but he neglects the fact that the satellite spends more time over the extremes in latitude. Stetsevich assumes v is proportional to the area of the satellite-surface viewing-cone intersection regardless of where this intersection lies. The results obtained by this simplified method do not coincide with those of Eq. 9, and it is the opinion of this author that the method presented here is the correct one.

The preceding analysis has relied heavily on the assumption that no longitude is preferred; i.e., the satellite and planet periods are noncommensurable. Reference 2 treats this assumption in detail and shows that it is valid. The proof is an application of the ergodic theorem of Birkoff and leads to the conclusion that the average viewing fraction of any system may be calculated by the method given here, but that the short-term value of v (Eq. 3, with the limit removed) may fluctuate markedly from the average.

The perturbations of the orbital elements affect this analysis to some degree. To an observer on the planet, the effect of the regression of the nodes appears simply as a change in the planet's rotation rate. Since the derivation of the viewing fractions assumes that the satellite's longitude has been equally distributed by the rotation of the planet, the secular perturbation of the longitude of the ascending node is of no consequence in this analysis.

The precession of the line of apsides is a result of the secular perturbation of the argument of periapsis, and the change of the satellite surface due to this rotation is pronounced. The distortion caused by ω changing from 0 to 90 deg can be seen by noting consecutively Figs. 2, 3, 5, and 8. For long-duration missions, at inclinations for which the rate of apsidal precession is significant, the viewing fractions change considerably. In addition, the locus of possible ground station-to-satellite ranges changes markedly with the satellite surface distortion.

D. Limitations of the Approach

The method presented correctly evaluates v as defined in Eq. 1. No insight is given, however, into the nature of the sequence

$$v_n = \sum_{i=1}^{n} \frac{\tau_i}{t_n} \qquad (10)$$

and how this converges to v. From a practical standpoint, knowledge of v is not meaningful without an accompanying statement about its convergence. For those few cases where the satellite and planet periods are nearly harmonic (i.e., the sub-satellite trace is almost periodic), the sequence of v_n's may vary widely; these cases must be investigated individually.

This analysis provides no information regarding the distribution and duration of the view periods. If the average view-period duration is short and the time necessary for signal acquisition occupies a significant fraction of the total view period, then the calculated value of v is high. This problem has been treated to some extent in Ref. 5.

The secular perturbation of the argument of periapsis and its effect on this analysis have been discussed. Sizeable rotation rates of the line of apsides invalidate the satellite surface concept. Those orbits whose inclinations are near 63.4 deg (e.g., the many USSR satellites at 65 deg) do not suffer this limitation.

E. Applications

The most obvious application for this analysis is the calculation of the view fraction for a particular station-satellite combination. For instance, the recently launched Soviet communications satellite, *Molniya I*, had the following orbital elements on May 26, 1965 (Ref. 7)

$$a = 26{,}624 \text{ km}$$

$$e = 0.7405$$

$$i = 65.19 \text{ deg}$$

$$\omega = 323.5 \text{ deg}$$

The average fraction of time this satellite can be viewed is plotted vs minimum elevation angle (Fig. 13) for a station at 43.1° N latitude (*viz.*, Vladivostok). Note the distinct inner and outer viewing fractions.

It is interesting to note the effect that a sizeable precession rate has on the view fraction calculation. The initial elements of the *Telstar II* orbit were:

$$a = 12{,}266 \text{ km}$$

$$e = 0.401$$

$$i = 42.74 \text{ deg}$$

$$\omega = 171.8 \text{ deg}$$

and the line of apsides rotated approximately 1.22 deg per day. The viewing fraction for Andover, Maine (latitude 44.6° N) is plotted as a function of time for the actual, observed, orbital elements (Ref. 8) in Fig. 14. Note that the viewing fractions vary markedly, due primarily to the apsidal precession. The peaks in July of 1963 and May of 1964 correspond to perigee at its most southerly location. Similarly, the intervening dips correspond to perigee at its most northerly location. A minimum elevation angle of 7.5 deg was taken for the Andover station. For this orbit/station pair, the satellite surface and view cone are similar to those of Fig. 11.

For communications between two separated ground stations (e.g., Andover, Maine, and Goonhilly Downs, England), that portion of the satellite surface which is common to the interior of both viewing cones must be considered. Integration of the time density must then be performed over the lenticular region of interest. An integral similar to that of Eq. (6′) will be encountered, but the angle θ will not be given by the simple arc-cosine.

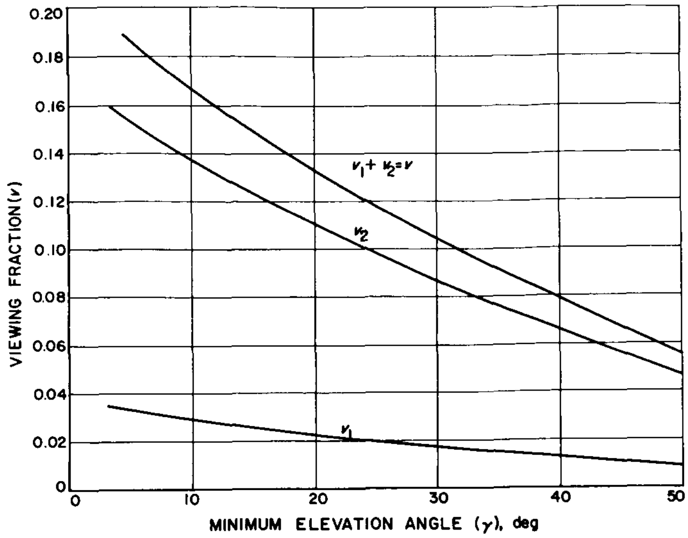

Fig. 13. Viewing fraction for *Molniya I* and station at 43.1° N latitude

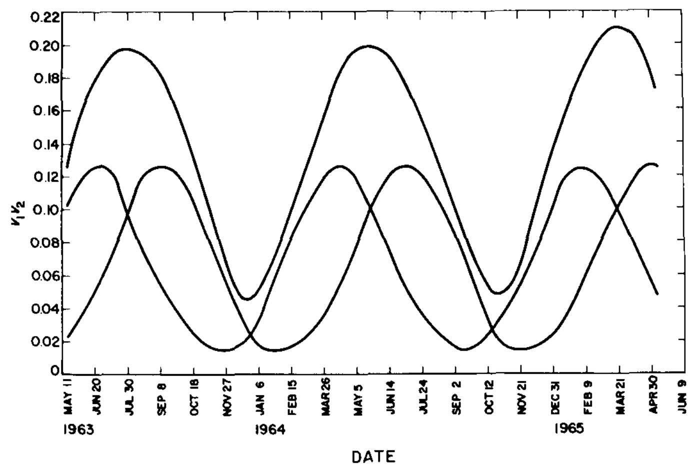

Fig. 14. Viewing fraction for *Telstar II*

The satellite surface concept lends insight into the problem of designing non-steerable ground antennas for satellite communications. For many of the satellites now in use or being designed, the orbiter-antenna radiation pattern is Earth-oriented rather than inertially oriented. This feature allows calculation of the gain-product of the transmitting and receiving antennas for a particular ground station and a given satellite surface. For the most efficient utilization of radiated power, the signal-to-noise ratio at the receiver should be a constant during the entire period of transmission. Thus, the antenna gain product should be proportional to the range squared, and the range is available for the satellite surface. The ground station/satellite antenna pair can take on a wide range of beam shapes, as long as the product meets this requirement. The two beam shapes can be thought to fit together and, for example, one could be moderately directive when the other is conical. Although it may be impossible to realize the desired gain-product exactly, the range-squared criterion can be used as a guideline in the design.

The combined view fraction and satellite surface results are also valuable for calculating optimum antenna-utilization policies. Consider a digital-satellite communications system whose maximum data rate is a monotonically decreasing function of the signal-to-noise ratio for a given bit-error probability. Assume that a constant data rate is used throughout the period of transmission and that the data rate is governed by the minimum signal-to-noise ratio during the transmission; there is a question as to whether communications should be started when the satellite is low on the horizon to maximize the duration of the pass or high above the horizon to minimize the inverse-square attenuation. If ψ is the angle off the ground station zenith, then the maximum amount of information is transmitted (over the long term) if the following product is maximized

$$v(\psi) \min_{\psi' < \psi} [\dot{I}(\psi')] \tag{11}$$

where $\dot{I}$ is the information rate and is a function of the satellite/ground station range, the antenna gains, the atmospheric attenuation, and several other angular dependent quantities. The angle ψ is the complement of the minimum elevation angle γ, so that both the necessary geometric quantities are available from the analysis of this Report. The value of ψ which maximizes expression (11) is the angle off the zenith at which transmission should begin, and, thus, it specifies the range, antenna gains, etc. These quantities, in turn, determine the design signal-to-noise ratio and bit rate.

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