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APPARENT MOVEMENT PHENOMENA ON CRT DISPLAYS:

THRESHOLD DETERMINATIONS OF APPARENT MOVEMENTS OF PULSED LIGHT SOURCES

by H. M. Bowen, L. L. Vallerie, F. J. Affinito, and J. G. Wohl

Prepared under Contract No. NASw-954 *by*
DUNLAP AND ASSOCIATES, INC.
Darien, Conn.
for

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Abstract

The "jump" phenomenon is due to stroboscopic interaction between saccadic eye movements (rapid movement from one fixation point to another) and pulsed light sources. The threshold for perception of a jumped image is primarily a function of the brightness of the pulse and the brightness of the background against which the jumped image is seen. In order to maintain high brightness of display, with a pulse brightness sufficiently low to avoid the jump phenomenon, PRF should be increased as far as possible.

The "shift" phenomenon is believed to be due to stroboscopic interaction between small, involuntary eye movements and pulsed light sources. Experiments with the present equipment indicate that these eifects are very small and, under the particular circumstances of the experiment, could not be seen by some observers. It is reasoned that special conditions (such as those existing on certain CRT displays) are required for the production of the shift phenomenon. Adequate investigation of the shift phenomenon therefore requires simulation of these conditions.

Continuation studies, to investigate a range of conditions wider than that treated in this initial, limited study, are suggested.

Table of Contents

List of Figures and Illustrations

Apparent Movement Phenomena on CRT Displays:

Threshold Determinations of Apparent Movement of Pulsed Light Sources

I. Introduction

Various apparent movement phenomenon which may appear on CRT displays were described in an earlier report*. The present study was concerned with determining the fundamental conditions which are conducive to the production of the two dominant types of movement, which we have termed "jump" and "shift". Both phenomena are a consequence of using pulsed light to write display content. The phenomena first came to attention when apparent movements were seen on CRTs using the P31 phosphor, and when symbols appeared on the screen for a period between 40 and 50 μs, and, typically, were refreshed at intervals of 20 ms.

The jump phenomenon appears as a rapid displacement of display content when the viewer's eyes are moved. The display content appears to jump along the line of eye movement and then return to its normal position; depending on conditions, the jump effect may be seen as a single displaced image or as a string of displaced images.

The shift phenomenon appears as a relatively small and slow wandering or jittering of one part of the display content with respect to another; some part of the display appears to shift around in space. The phenomenon is apparently associated with an appreciable time gap between successive writings of items on the display. It is seen when the eyes are steadily regarding the display.

When operational displays are subject to these apparent movement phenomena, it is probable that the proficiency with which they can be read deteriorates. A previous study** has indicated that observers will tend to commit more errors of reading on such displays; this error tendency probably becomes more pronounced when the displays have to be viewed for long periods.

It is hypothesized that both phenomena are due to stroboscopic effects. The jump phenomenon is apparently produced by successive pulses striking the moving retina at different locations as the eye displaces from one fixation position to another in saccadic movements. The observer interprets the induced perception as a jump of the image. When the pulse rate of the image is high, more than one repetition of the image occurs during the saccadic movement; thus, two or more displaced replications of the original image are seen.

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*Bowen, H., and Guinness, V. Apparent movement phenomena on CRT displays. Report No. 510-TM-2 (DRD-64-131), October, 1964.

**Ibid.

The shift phenomenon is believed to be a consequence of the interaction between involuntary eye movements and pulsed light. Involuntary eye movements are of several forms. Eye tremor is a small, rapid, continuous oscillation of the eye; its frequency is about 140 cps, and its amplitude is about $.05^{\circ}$. The eye tends to drift away from a fixation target; the error so produced is corrected every one or two seconds. The drift generally has to be in the order of $.25^{\circ}$ away from the point of fixation before the error is corrected. The corrective action is in the form of a "flick" which takes about 50 μs to complete. Similar forms of very rapid "flick" motions occur when gaining a new fixation and when a new item is presented to central regard. It is hypothesized that some or all of these motions interact with the pulsed light so that successive images may be perceived in slightly different locations. The small amplitude of the shift phenomenon perceived is probably correlated with the amplitude of the drifting of the eye, which is normally not more than $.25^{\circ}$, corresponding to about 1.5 millimeters at normal reading distances, or about the width of the letters on this page. It is the impression of one author of this report (Bowen), who has seen the shift phenomenon on operational CRT equipment and on laboratory equipment, that the amplitude of a shift is of the right order of magnitude for the above explanation to be tenable.

The experimental purpose was to establish the conditions of display which serve to produce the jump and shift phenomena. Establishment of these threshold conditions is the necessary basis for specifying the electronic conditions which should obtain on CRT displays in order to avoid the apparent movement effects. The scope of the work, however, has been limited up to this point, and only certain of the display conditions which may influence the presence and the degree of the phenomena have been investigated.

II. The Experiments*

A. Introduction

The experiments were planned as a study of the interaction of eye movements and pulsed light sources; more specifically, they were planned as psychophysical determinations of the threshold conditions which are required in order to produce the appearance of movement. In the interests of experimental purity, and in order to minimize equipment complexity and cost, the experiments were conducted on the simplest possible visual stimuli, observed against a neutral, featureless background. Essentially, the experiments consisted of the presentation, in the case of the jump phenomenon, of a single pulsating light, and required the subject to judge whether he saw any jumping of the light when he moved his eyes in a given manner; the characteristics of the pulsating light were varied to determine the threshold conditions. In the case of the shift phenomenon, two lights were used; the observer maintained steady gaze and reported any shifting he saw for different conditions of the lights.

B. Experimental Considerations

The single most important consideration in designing the experiment was that the pulsed image should be stationary in space so that any real movement of the light would be impossible. Such an image was produced by means of a flash tube which fed into an optical sequence and exited through a pupil of desired diameter. Every precaution was taken to render the image physically still.

The image appeared 39" away from the observer, at the other side of a fully enclosed box. The inside of the box was painted a neutral gray, and was illuminated internally by lights whose brightness was controlled and variable.

The variables which are thought to affect the appearance of the jump phenomenon are:

- . pulse peak amplitude
- . pulse mean amplitude
- . pulse length
- . pulse repetition frequency
- . image size (area)
- . ambient illumination
- . eye movement (saccade) extent.

The same variables seem to affect the appearance of the shift phenomenon, with the addition of:

- . separation distance (between the two images)
- . phase difference between the two pulsing images.

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*A detailed account of the experiments is provided in Appendix I.

The experiments did not consider the effects of varying all these variables. Instead, certain variables were pegged at values which were considered to be most conducive to the appearance of the phenomena (as derived by prior analysis and pilot observations), provided that such values could occur in operational conditions (i. e., when CRT displays are used in an operational room to display symbolic data). The basic reason for this approach was the wish to determine the outer threshold boundary for the appearance of the phenomena, rather than some intermediate boundary, while recognizing that the determinations made should be applicable to operational as opposed to hypothetical conditions. In addition, certain equipment limitations were encountered. Most important of these limitations was the inability to vary pulse length over 50 μs. This and other limitations constrained the present experiment to certain values of the variables. The experimental conditions for the experiment were thus established as follows:

1. <u>Pulse Characteristics</u>

The pulse produced by the flash tube took the form of a rise to peak amplitude in about .1 μs, followed by an exponential decay which decreased to a 10% value in 50 μs. The shape and duration of the pulse was essentially the same as that produced on a P31 phosphor when used in typical operations. The amplitude of this pulse could be changed at will so that the pulse appeared brighter or dimmer.

2. <u>Pulse Repetition Frequency</u>

The pulse repetition frequency was variable between 20 and 100 cps.

3. <u>Image Size</u>

The single image was a $\frac{1}{4}$" diameter spot. It was thought that a $\frac{1}{4}$" diameter spot was representative of the largest visual item likely to be produced on operational equipment. Preliminary observations had indicated that larger spots seemed to have a greater tendency to exhibit the jump phenomenon, possibly because larger spots appear brighter (due to spatial summation by the eye).

4. <u>Ambient Brightness</u>

The brightness of the surround of the stimulus scene was pegged at 40 foot candles. 40 foot candles was the measured average brightness of ambient illumination falling on CRT's in the operational checkout areas at Cape Kennedy.

5. <u>Eye Movement</u>

Eye movements were induced by requiring the observer to fixate alternately on two small red lights placed 25^{0} on either side of the stimulus light. The eye-sweep of 50^{0} obtained thereby is probably commensurate with the largest eye-sweeps required in practice. As the angular extent of the eye movement increases, so does the peak angular velocity of the eye; thus, larger eye-sweeps are associated with more pronounced impressions of jump.

The experimental conditions for the study of the shift phenomenon were the same as those described above in terms of pulse characteristics, pulse repetition frequency and surround brightness. The observer focused directly on the stimulus lights in this experiment, so there was no need to induce eye movements.

The experiments conducted to date on the shift phenomenon have been only marginally successful, due to the inability of the present equipment to realize the desired conditions. Specifically, the present equipment was incapable of firing two flash tubes asynchronously to obtain the desired 180° phase shift between them (which would maximize the time differences between successive images appearing at two adjacent locations). Hence, the work to date has been confined to viewing one steady light and one pulsed light spaced $\frac{9}{16}''$ apart.

C. Experimental Procedures and Results

1. Jump Phenomenon

Five observers, drawn from members of the Dunlap and Associates staff, all having normal eyesight without glasses, were used as subjects. Each subject viewed the pulsating light at five pulse-repetition frequencies: 30, 40, 60, 80, and 100 cps. Each frequency was observed ten times, with each observation consisting of a judgment of when the jump phenomenon just appeared or just disappeared as the brightness of the image was raised or lowered in incremental steps centered about a previously estimated threshold point. Arithmetic averages of these data are the reported threshold values.

The results of the experiment are summarized in Figures 1 and 2. The data points in Figure 1 represent threshold values for the appearance of the jump in terms of the apparent steady brightness of the pulsating light as a function of PRF. It is seen that as PRF increases, the apparent steady brightness that the image may have and still be seen not to jump also increases. The data points in Figure 2 represent threshold values for the appearance of jump in terms of the (calculated) brightness of a single pulse of light as a function of PRF. It is seen that as PRF increases, there is an apparent tendency for the threshold pulse brightness to decrease. While the trend of these data points is apparently systematic in the form of a downward-sloped straight line (excepting the data point for a PRF of 30 cps), statistical analysis indicated that the data points did not depart significantly from equality; thus this trend cannot be claimed as an established result.

The threshold point for the 30 cps condition appears to depart from the trend of the other data points. This departure may be attributed to the fact that at 30 cps, the pulsed light was below fusion frequency (which averaged at about

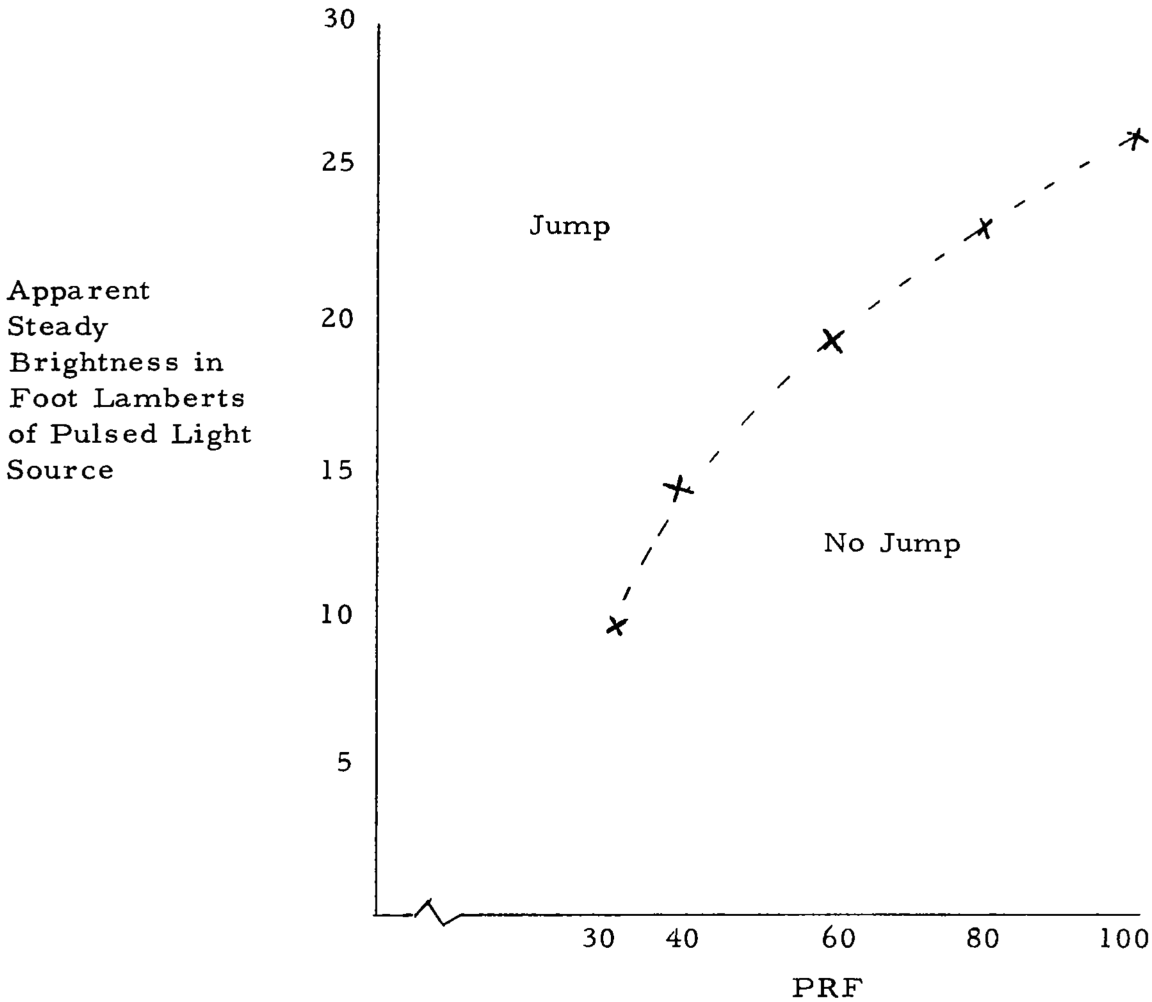


Figure 1. Threshold for the Presence of Jump as a Function of PRF Plotted in Terms of Apparent Steady Brightness of Pulsed Light Source

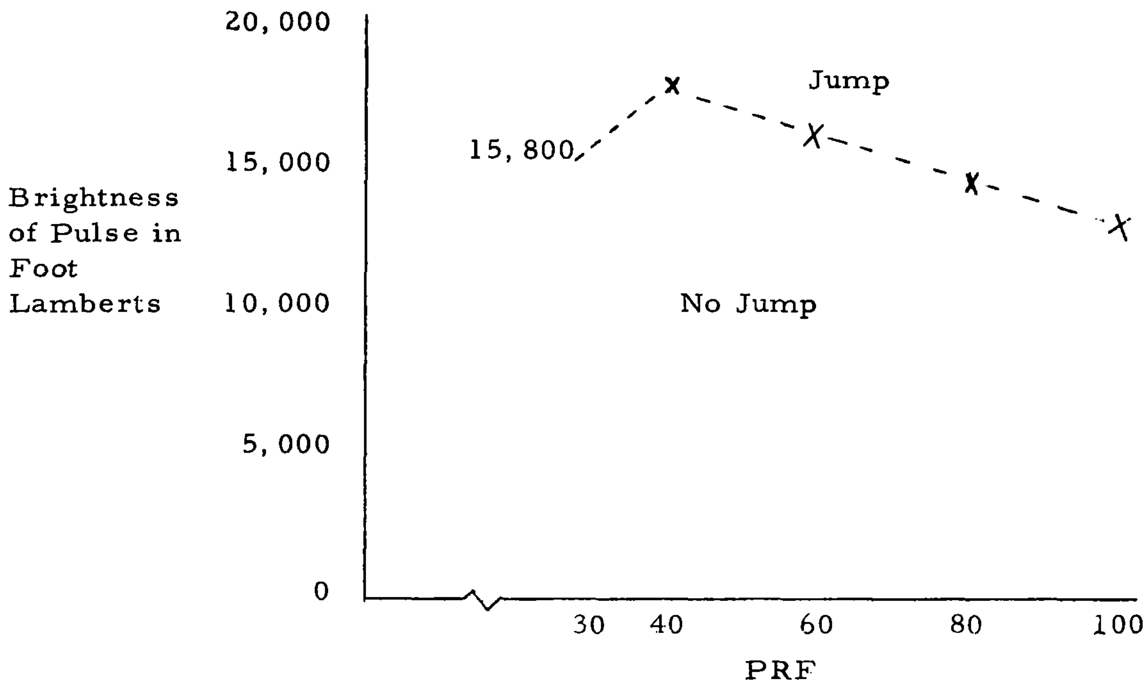


Figure 2. Threshold for the Presence of Jump as A Function of PRF Plotted in Terms of Brightness of Pulse

35 cps for the observers at the brightness involved). The light source appeared therefore qualitatively different, and there were some reported difficulties of separating the sensation of flicker from the sensation of jumping. This data point, additionally, has littler operational significance, for it will always be desirable to operate CRT's displaying symbolic data above the flicker fusion frequency. Hence, on these grounds, this data point has been discounted from the general interpretation of the results.

For understanding the significance of the data, it should be recalled that the jumped image is seen displaced from the true position of the image; the jumped image is seen against the surround of the actual image position. The perception of the jumped image may be expected, therefore, to depend upon the brightness contrast of the jumped image against the surround. Based upon this reasoning, PRF should not affect the visibility of a jumped image because the brightness of a single pulse is independent of PRF. Thus, PRF should not be a determining variable for the perception of jump.

Figure 2 broadly supports the above interpretation. Within the confines of the experiment, the supposition is supported precisely because of the finding that the data points cannot be distinguished from equality (see statistical analysis in Appendix I). However, the strong suggestion of a trend in the data makes this conclusion appear as a fortuitous consequence of the relatively small-scale nature of the experiment. A possible explanation of the trend, if it really exists, is that as PRF increases, the jumped images tend to increase in clarity, appear more frequently near central regard, and are often multiple. They seem to be, in sum, more conspicuous, though not brighter. If this conspicuity is truly a significant determinant in the perception of jumped images, then it appears that a less bright jumped image can be seen at higher PRF's which produce such greater conspicuity. At this time, we prefer to believe that the downward trend in the data is systematic and that the "factor" of conspicuity increases approximately linearly as PRF increases. It can be observed in Figure 2 that the data points for 40, 60, 80 and 100 cps pulse-repetition frequencies conform very closely to a straight line. The data, as reported in Figure 1, may now be interpreted more meaningfully.

At the PRF frequencies and brightnesses involved in this experiment, it can be assumed that the eye acts as an ideal integrator (Talbot's Law), and that the apparent steady brightness (B_a) of a flashing light is represented by the equation:

$$B_a = \frac{1}{T} \int_0^T B(t)\, dt \qquad \text{Equation I.}$$

where B represents values of the instantaneous brightness of the light over time, T. Thus, if the threshold data in terms of pulse brightness as a function of PRF can be represented by a straight line, as in Figure 2, then the threshold data, in terms of average brightness as a function of PRF, should conform to a curve described by an equation of the form of:

$$Y = ax^2 + bx + c \qquad \text{Equation II.}$$

Figure 3 shows the fit obtained (by an approximate curve-fitting technique) by the equation:

$$B = -.0012\,F^2 + .37\,F + 1.3 \qquad \text{Equation III.}$$

where B is Apparent Steady Brightness of the Light Source in Foot Lamberts and F is PRF in cps.

The operational significance of these results is that

a. PRF should be increased as far as possible when a high brightness of display is desired.

b. The apparent steady brightness of the image should be measured, by any standard integrating photometer, and the apparent steady brightness should not appreciably exceed the values given in Figure 1 when pulse length is at or near 50 μs. If the brightness of the surround is appreciably different from 40 foot candles, the data do not apply; however, if the surround brightness is higher than 40 foot candles, higher apparent steady brightnesses will be acceptable and vice versa. At surround brightness values of 40 foot candles, the acceptable apparent steady brightness of the image to just not produce jump may be estimated from Equation III.

2. Shift Phenomenon

Four observers, drawn from Dunlap and Associates' staff, all having normal eyesight without glasses, were used as subjects. The observers viewed two lights, $\frac{1}{16}$" in diameter, separated by $\frac{9}{16}$". Preliminary trials had indicated that smaller images tended to evidence more shift movement. The separation distance of $\frac{9}{16}$" was as small as the equipment would allow. Due to experimental equipment limitations, only one of these lights pulsated; the other (top) light was continuous. Each subject viewed the pulsating light at PRF values between 20 and 100 cps.

The experiment was planned as a psychophysical determination of the appearance of shift as a function, primarily, of PRF. The experiment was a partial failure in achieving its goal, due to the fact that only two observers saw the shift phenomenon with any consistency. Thus the experiment became a form of pilot study of the phenomenon which has suggested some of the conditions that should be present in order to produce the phenomenon.

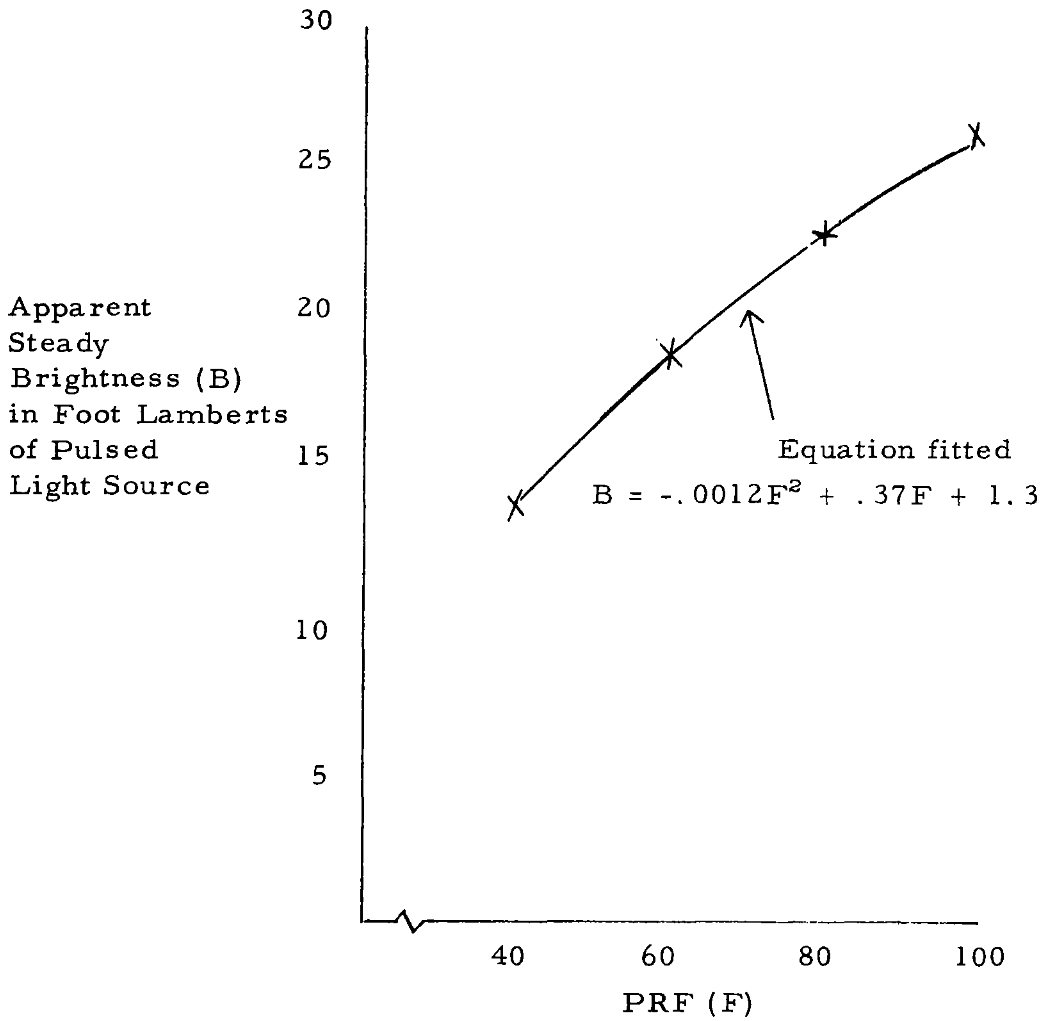


Figure 3. Demonstration of Compatibility of Observed Threshold Points (in terms of apparent steady brightness of pulsed light source as a function of PRF) to an Equation of the Form of $Y = ax^2 + bx + c$. (Data point for 30 cps PRF excluded.)

Studies, with the observer who saw the shift phenomenon most clearly, indicated that the brightness of the sources made little or no difference to the presence of the phenomenon; as brightness was reduced, the movements became more difficult to see, but were still present. Hence, average brightness of the pulsed light was pegged at a bright value (about 50 foot lamberts) which seemed to provide for good conspicuity of the effects. The observer made the following summary of the amount of shift movement that he saw on a final run, after about two hours of observation.

PRF	Response
25	4
30	5
35	4
40	4
45	3
50	1
55	1
60	1
70	1
80	0
90	0
100	0

(Response Values indicate the amount of movement seen; 5 = maximum, 0 = none.)

The other observer, who saw the phenomenon with some consistency, gave results as follows:

PRF	Response
20	No
25	Yes - "I think"
30	No
35	No
40	No
45	Yes
50	Yes - "A little"
55	Yes - "A little"
60	No
65	Yes - "Very slight"
70	No
75	No
80	Yes
85	Yes - "A little"
90	Yes - "Just barely"
95	No
100	Possibly - "A slight one"

The third subject failed to see any movement, while the fourth subject saw a great deal of movement. However, the reports of this fourth subject indicated that she was subject to a large amount of autokinetic phenomena; that is, the perception of motion where there is none. She saw, for instance, the physically motionless continuous light source as being in motion. Hence, her results are discounted.

In summary, it is clear that the shift phenomenon was present to only a marginal degree. To the extent that two observers did see it, there is a suggestion that it is stronger at lower frequencies than at higher frequencies, but that some movement can be seen up to, and perhaps beyond, 100 cps.

The relative potency of the shift phenomenon, as it may occur on CRT displays, suggests that the experimental display was different in important respects from CRT displays. It is hypothesized that the three most important differences are the small extent of the sources, the proximity of the sources and the absence of two pulsating sources.

When the shift phenomenon was observed on CRT displays, it was seen as a relative displacement of one line of symbols with respect to another line of symbols. Each line was painted periodically with a 7 to 15 ms. time interval between the successive paintings, and the lines were $\frac{1}{4}$" to $\frac{1}{2}$" apart. These circumstances may have produced the shift phenomenon by making more obvious the quite small movements of the eye and hence the images. It will be recalled that the amplitude of the movements believed to be responsible for the shift phenomenon is derived from rotations of the eye of $\frac{1}{4}^{\circ}$ or less. Such small movements become more obvious when a large number of elements in a scene undergo displacement, and it is thus reasonable to believe that the relative movements of two lines of symbols would be more obvious than the relative movement of two single light sources. Relative movements of small amplitude are most obvious when the elements are close together; thus, the shift phenomenon would be more likely to be seen should the light sources be moved closer than was possible with the present equipment. Two pulsating sources, rather than the one that was possible with the present equipment, should increase the frequency of movements and perhaps their amplitude (when the eye movements are rotational rather than translatory). These various considerations suggest that a study of the shift phenomenon requires a more elaborate production of pulsed images.

III. Study Continuation

The work to date has demonstrated that the occurrence of the jump phenomenon is systematically related, primarily, to the brightness of the pulse, and, to some extent, to PRF. Continued work should extend the scope of the study to determine whether the lawfulness observed over a very restricted range of variables applies across a wide range of variables. Such work will also establish specific values of the determining variables and will thus provide the basis for drawing up engineering specifications to avoid the jump phenomenon.

Specifically, the continued studies on the jump phenomenon should cover a wide range of pulse lengths, varying between 10 μs to 5 ms., vary pulse shape, and vary ambient illumination between 30 and 60 foot candles. With the constraint that the studies will consider only a single symbol, the combination of a fast phosphor oscilloscope, with adequate programming circuitry, and the present viewing chamber and accessory apparatus, will provide the necessary capability.

The shift phenomenon is also recommended as the subject of continued study. It is understood that certain operational equipment, on which this phenomenon has been seen to a pronounced degree, will continue in important use. The shift phenomenon is almost certainly produced by stroboscopic interaction between small-amplitude, involuntary eye movements and pulsed light sources; thus, its basis is the same as that of the jump phenomenon. Proper understanding of one will illuminate the other, allowing for a general understanding of the response of the visual apparatus to pulsed light sources.

An investigation of the shift phenomenon may be conducted with a double-gun fast phosphor oscilloscope, having adequate programming circuitry, combined with the present viewing chamber and accessory apparatus. The study would utilize two extended lines of dots, varying the following factors:

. time interval between painting the two lines
. spatial proximity of the two lines
. pulse length
. pulse shape
. pulse amplitude
. PRF
. ambient illumination.

Appendix I. Detailed Account of Experiments

A. Equipment Description

A block diagram of the experimental equipment is shown in Figure I. 1.

The pulsating light source was produced by an FX-6A Xenon Flash Tube. The tube was operated by discharging a capacitor across an RC circuit. The characteristics of this circuit controlled the shape of the generated light pulse. The shape of the light pulse consisted of a rapid rise to a peak value followed by an exponential decay over the pulse length. The peak brightness of the pulse was further controlled by means of neutral density filters and an iris diaphragm.

A system of optics was used to collect, diffuse, and focus the light for presentation to the observers. The optical path is shown in Figure I. 2. The light from the flash tube was first collimated through a condensing system, passed through a variable aperature diaphragm, focused on the end of a incoherent fiber optic tube and finally fed through a mask which formed the circular image seen by the observer. This image was $\frac{1}{4}$'' or $\frac{1}{16}$'' in diameter. Since the fiber optic tube was of the incoherent type, it served to diffuse and evenly distribute the light at the exit end of the tube. Since the tube is flexible, it eliminated the problem usually associated with alignment of optical equipment.

The circular image of the pulsating light appeared inside a viewing chamber. The chamber served to protect the image against stray light from other sources in the laboratory. Figure I. 3 contains a sketch of the chamber. The interior of the chamber was coated with a neutral gray paint and was essentially featureless except for the two red fixation lights, the stimulus image itself, and a light sensor. The fixation lights were located in a horizontal plane, 25^{o} (visual arc) either side of the light image. Ambient illumination within the light chamber was supplied by four cool white flourescent lamps which were located where they could not be seen. Since these lamps flicker on AC, they were run on DC. Provisions were made to switch polarity of the lamps to avoid building up deposits on the end acting as the cathode, and thereby eliminating uneven "blackening" of the lamps and an uneven distribution of ambient illumination within the chamber. The spectral characteristic of the lamps was chosen to be as close as possible to the spectral characteristic of the flash tube. The flash tube emits energy in the range of 2,000 to 20,000 angstroms with a primary peak at 8,000 angstroms and a secondary peak at 5,000 angstroms. The flourescent lamps emit in the range of 3,000 to 7,500 angstroms with a primary peak at 5,800 angstroms and a

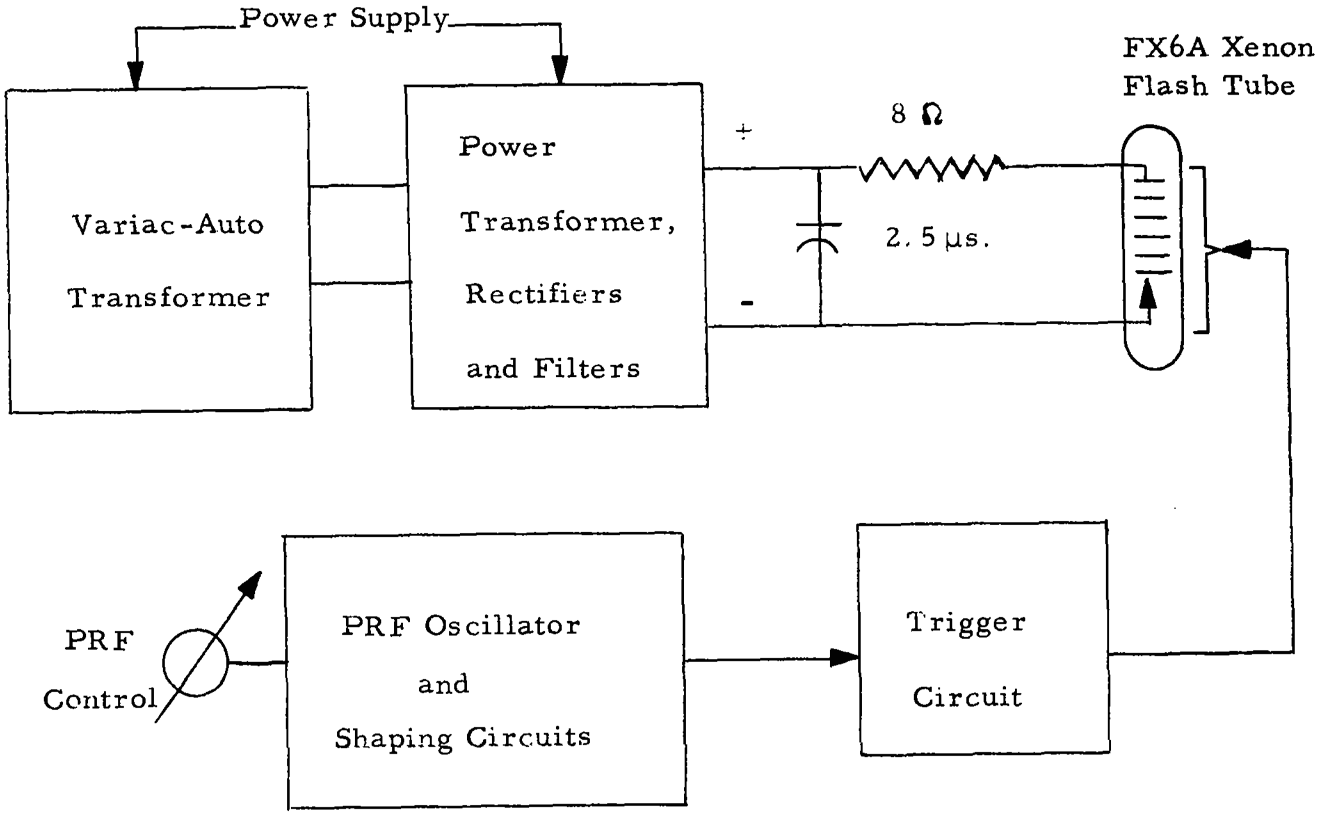


Figure I. 1. Block Diagram of Single Light Source System.

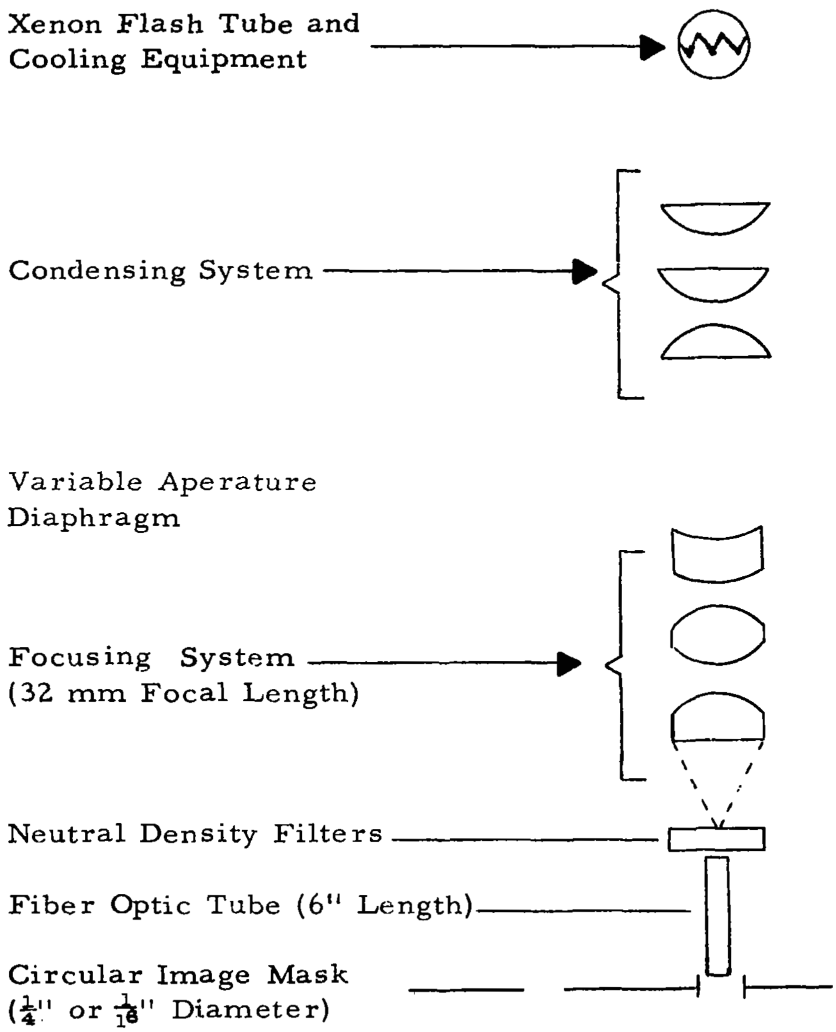
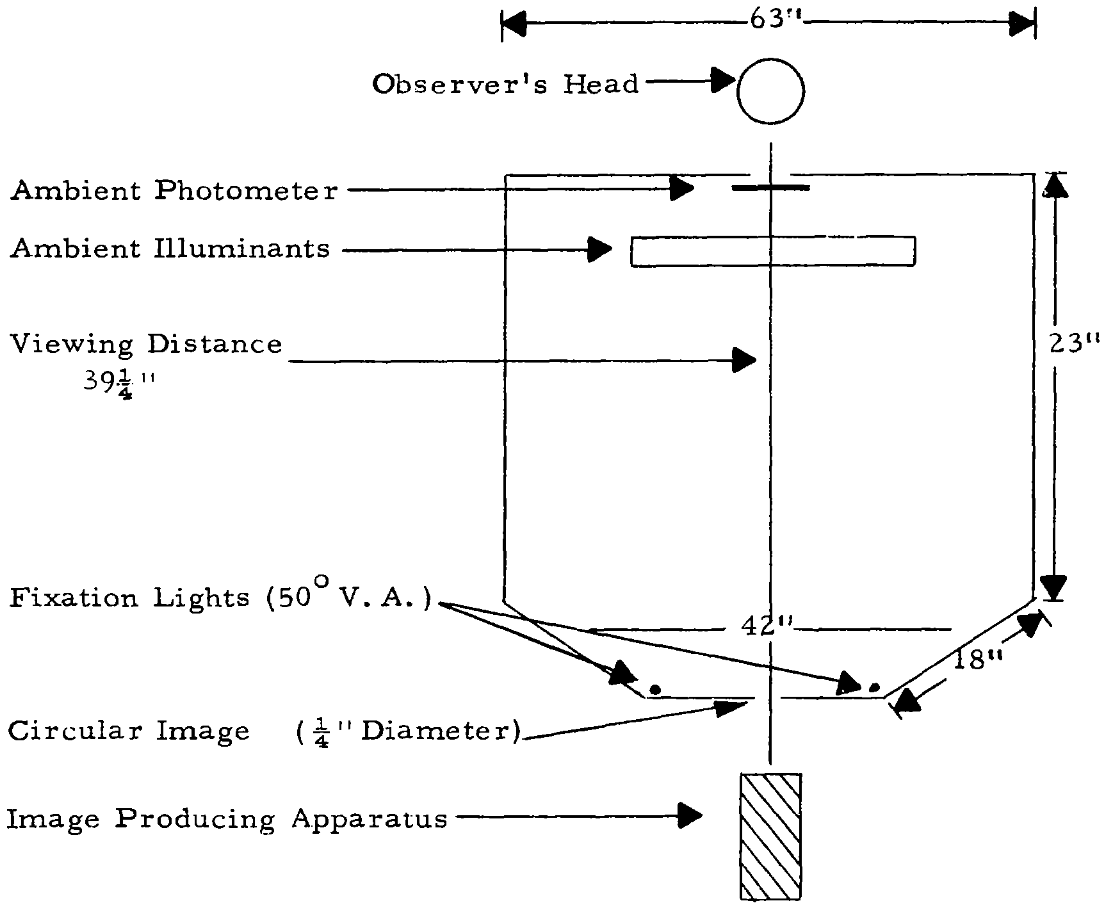


Figure I. 2. Optical Path.



Height of Chamber - 42"

Figure I. 3. Plan View of the Viewing Chamber.

secondary peak at 4,700 angstroms. The flash tube was, therefore, a little more red than the ambient. However, direct comparison between the two light sources showed that the difference is barely perceptible.

Ambient illumination was maintained so that background brightness was 40 foot-candles by means of a voltage control and a photometer mounted inside the light chamber.

A rapid response planar photo sensor, no larger than the head of a match, was used to monitor the peak brightness of the pulsating light. The sensor was situated inside the chamber and directed at the light image, but not in the line of sight of the observer. The output of the sensor was fed into an oscilloscope. In this way, the peak brightness of an individual light pulse could be controlled by varying the aperture diaphragm in the optical path and noting the height of the pulse in millimeters on the scope.

The scope and the photo sensor were calibrated in absolute terms (foot lamberts) using a spot photometer. Since the spot photometer measures the apparent steady brightness of a pulsating light, it was necessary to calculate the brightness of an individual pulse. Pulse brightness, B_p may be calculated from the equation

$$B_p = \frac{B_a}{PRF \times 20\mu s.}$$

where B_a is the apparent steady brightness of the train of pulses. (Derivation of this formula is presented in Appendix II.)

B. Experimental Procedures

The general procedure used to conduct the study on the jump phenomenon was to obtain a report from the observer as to the presence or absence of the "jump" phenomenon at each pulse repetition frequency as he moved his eyes quickly between two fixation points. At each frequency, five threshold estimates were made starting with a pulse peak amplitude well above threshold and then descending in discrete levels to a point where a "no" report was received from the observer. Five additional threshold estimates were made starting below threshold and then increasing pulse amplitude until a "yes" report was made by the observer. In this way, a total of ten threshold estimates were obtained from each observer at each PRF used in the study.

The experimental design is presented in Figure I.4.

Figure I. 4

Experimental Design for
The Jump Phenomenon Study

Subjects	30	40	60	80	100
1	*				
2					
3					
4					
5					

Constant Conditions: Ambient Illumination: 40FL
Pulse Length: 50μs
Eye Sweep: 50°

*Ten threshold estimates per cell

C. Experimental Results and Analysis

Tables I. 1 and I. 2 provide tabulations of the observations. Table I. 1 reports the threshold determinations of each subject in terms of the apparent steady brightness of the light source as a function of PRF. Table I. 2 reports the same data, this time in terms of pulse brightness.

Table I. 3 reports the analysis of variance summary of the analysis performed on the data recorded in Table I. 2 and appearing in Figure 2 of the main text. The table shows that neither PRF nor subjects occasioned, by themselves, a significant departure from equality; however, their interaction is significant, indicating that different subjects had different patterns of threshold values across the PRF variable. The variance estimate for the residual term (within cells) is 653214 foot lamberts; hence, the standard deviation of the residual term is 810 foot lamberts. Inspection of Figure 2 shows that a standard deviation of 810 foot lamberts represents a highly consistent performance on the part of the subjects when the effects of subject mean difference and the effects due to RFP are taken into account. Inspection of the mean squares column in Table I. 3 indicates that considerable variation is attributable to differences between subjects. From the present limited data, it appears that there may be some real differences between the threshold levels for the perception of the jump phenomenon for different observers and that different observers respond differentially to changes in PRF. However, it is considered that the present data are too scant to make any firm statement, at this time, regarding differences between observers; further interpretation of individual differences should await the collection of additional data.

Table I. 1

Threshold Estimates in Terms of Average Brightness (FL)

PRF (cps)

Subjects	30		40		60		80		100	
	8.0	8.0	12.2	12.2	19.5	18.0	22.0	22.0	23.5	23.5
	8.0	8.0	12.2	12.2	18.0	18.0	22.0	22.0	23.5	23.5
1	8.0	8.0	12.2	12.2	18.0	18.0	22.0	22.0	23.5	23.5
	8.0	8.9	12.2	12.2	18.0	18.0	22.0	22.0	23.5	23.5
	8.0	8.9	12.2	12.2	18.0	18.0	22.0	22.0	23.5	23.5
	7.6	7.6	15.0	16.7	19.5	19.6	22.0	24.8	28.3	30.5
	7.6	7.6	16.9	17.1	19.5	20.7	24.8	22.0	28.3	28.3
2	7.6	7.6	16.3	17.2	19.5	21.8	26.7	24.8	28.5	28.3
	7.6	7.6	17.0	17.1	20.7	19.5	24.8	24.8	30.5	28.3
	7.6	7.6	16.9	17.2	20.7	19.5	24.8	24.8	30.3	28.3
	9.8	11.5	13.4	14.3	19.5	19.5	22.0	24.8	28.3	23.5
	12.6	9.8	13.4	14.3	18.0	19.5	22.0	22.0	23.5	28.3
3	9.8	9.8	13.4	13.4	19.5	18.0	22.0	24.8	23.5	28.3
	9.8	9.8	13.4	14.3	19.5	18.0	22.0	22.0	23.5	28.3
	9.8	8.9	14.3	13.4	19.5	19.5	22.0	24.8	28.3	28.3
	8.9	8.0	12.2	12.2	19.5	19.5	24.8	22.0	23.5	23.5
	8.0	8.9	12.2	13.4	18.0	18.0	22.0	22.0	23.5	23.5
4	8.9	8.0	12.2	12.2	18.0	18.0	22.0	24.8	23.5	23.5
	8.0	8.0	12.2	12.2	18.0	18.0	22.0	24.8	23.5	23.5
	8.9	8.9	13.4	12.2	19.5	18.0	24.8	22.0	28.3	23.5
	13.0	13.0	15.7	16.3	19.5	22.7	22.0	24.8	30.5	28.3
	13.0	12.9	17.1	14.3	19.5	20.7	24.8	22.0	28.3	28.3
5	12.9	13.0	16.7	16.9	18.0	22.7	22.0	24.8	30.5	28.3
	13.0	13.0	17.0	14.3	19.5	19.5	24.8	24.8	28.3	28.3
	13.0	13.1	17.1	16.3	21.8	21.8	22.0	24.8	28.3	28.3
$\Sigma x =$	473.8		713.1		963.1		1160.7		1316.0	
$\frac{\Sigma x}{n} =$	9.5		14.3		19.3		23.4		26.3	

Table I. 2

Threshold Estimates in Terms of Pulse Brightness (FL)

PRF (cps)

Subjects	30		40		60		80		100	
	13333	13333	15250	15250	16250	15000	13750	13750	11750	11750
	13333	13333	15250	15250	15000	15000	13750	13750	11750	11750
1	13333	13333	15250	15250	15000	15000	13750	13750	11750	11750
	13333	14833	15250	15250	15000	15000	13750	13750	11750	11750
	13333	14833	15250	15250	15000	15000	13750	13750	11750	11750
	12667	12667	18750	20875	16250	16250	13750	15500	14150	15250
	12667	12667	21125	21375	16250	16250	15500	13750	14150	14150
2	12667	12667	20375	21500	16250	18167	16688	15500	14150	14150
	12667	12667	21250	21375	17250	16250	15500	15500	15250	14150
	12667	12667	21125	21500	17250	16250	15500	15500	15250	14150
	16333	19167	16750	17875	16250	16250	13750	15500	14150	11750
	21000	16333	16750	17875	15000	16250	13750	13750	11750	14150
3	16333	16333	16750	16750	16250	15000	13750	15500	11750	14150
	16333	16333	16750	17875	16250	15000	13750	13750	11750	14150
	16333	14833	17875	16750	16250	16250	13750	15500	14150	14150
	14833	13333	15250	15250	16250	16250	15500	13750	11750	11750
	13333	14833	15250	16750	15000	15000	13750	13750	11750	11750
4	14833	13333	15250	15250	15000	15000	13750	15500	11750	11750
	13333	13333	15250	15250	15000	15000	13750	15500	11750	11750
	14833	14833	16750	15250	16250	15000	15500	13750	14150	11750
	21667	21667	19625	20375	16250	18917	13750	15500	15250	14150
	21667	21500	21375	17875	16250	17250	15500	13750	14150	14150
5	21500	21667	20875	21125	15000	18917	13750	15500	15250	14150
	21667	21667	21250	17875	16250	16250	15500	15500	14150	14150
	21667	21833	21375	20375	18167	18167	13750	15500	14150	14150
$\Sigma x =$	789833		891375		802585		725438		657800	
$\frac{\Sigma x}{n} =$	15796.66		17827.50		16051.70		14508.76		13156.00	

Table I. 3

Analysis of Variance of Threshold Scores*

Source	df	Mean Squares	F	P
PRF (P)	4	153518108	5.21	
Subjects (S)	4	110594240	3.75	
(Cells)	(24)			
P x S	16	29456259	45.09	$<.001$
Within Cells	225	653214		

*Scores expressed in FL of pulse brightness

Appendix II. Calculation of Pulse Brightness

The FX-6A Zenon Flash Tube was operated in a stretched-pulse mode. The generated pulse of light varied as a function of time as shown in Figure II. 1.

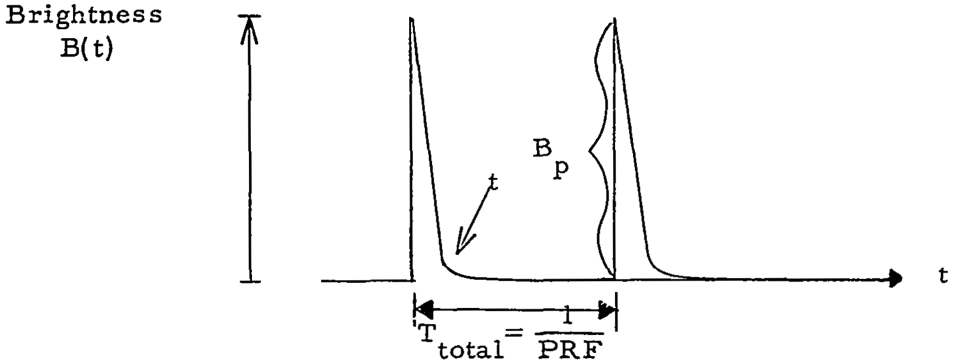


Figure II. 1. Representation of pulse train emitted from flash tube.

Knowing PRF, the decay time constant (τ), and the apparent steady brightness (B_a) of the train of pulses as measured by a spot photometer, it is possible to calculate the pulse brightness (B_p) of an individual light pulse. First if we consider the analogous variables for a train of rectangular light pulses, we obtain the following relationship:

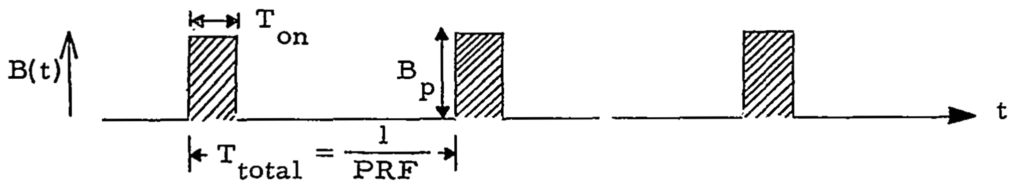


Figure II. 2. Pulse train represented as rectangular pulses.

$$B_a = \text{duty cycle} \times B_p \qquad \text{Equation II. 1}$$

$$= \frac{T_{on}}{T_{total}} \times B_p \qquad \text{Equation II. 2}$$

$$= PRF \times T_{on} \times B_p \qquad \text{Equation II. 3}$$

We notice that $T_{on} \times B_p$ is equal to the area under the (rectangular) light pulse, i.e.,

$$A_{rect.} = T_{on} \times B_p \qquad \text{Equation II. 4}$$

For an <u>exponential</u> light pulse, the area under a single pulse is:

$$A_{exp} = \int_0^T B_p \, e^{-t/\tau} \, dt, \qquad \text{Equation II. 5}$$

where τ = decay time constant. Integrating,

$$A_{exp} = B_p \int_0^T e^{-t/\tau} \, dt$$

$$= B_p \times \tau \left[-e^{-t/\tau} \right]_{t=0}^{t=T}$$

or

$$A_{exp} = B_p \times \tau, \text{ since } T >> \tau. \qquad \text{Equation II. 6}$$

The significance of Equation II. 6 can be seen by examining Figure II. 3.

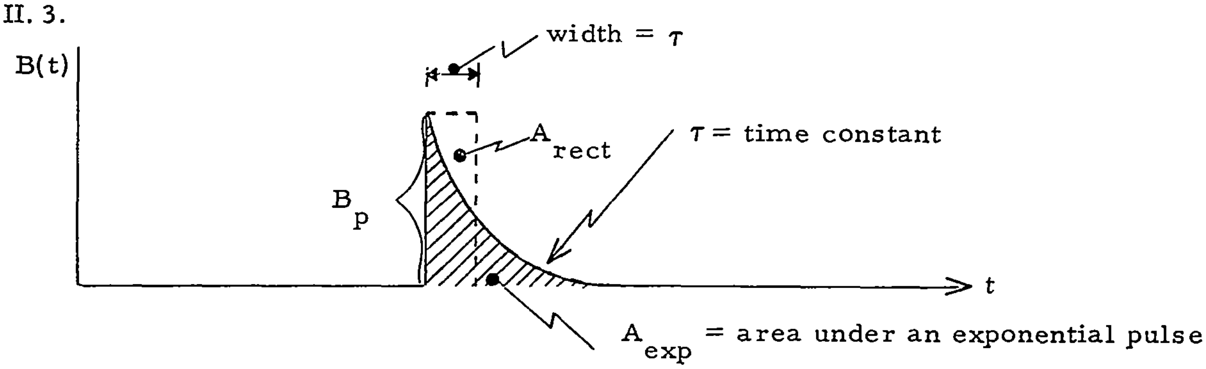


Figure II. 3. Exponentially decaying pulse and its rectangular equivalent.

The area under a rectangular pulse of width τ and peak value B_p is equal to the total area under the exponential pulse. Thus, by appeal to Talbot's Law (brightness is a function of total flux emitted in a pulse irrespective of how the energy is distributed in the pulse, provided the pulse is not greater than 30 ms.), we state that the exponential pulse obtained from the FX-6A flash tube is equivalent to a rectangular pulse of the same pulse brightness B_p, and of width τ.

We are now in a position to determine B_p for an exponential pulse in terms of τ, PRF, and B_a, all of which were measured. The first step is to replace the exponential pulse by its equivalent rectangular pulse. Equation II. 3 states that for a rectangular pulse, the average brightness is equal to the PRF multiplied on T_{on}, and the pulse brightness, B_p. For the equivalent rectangular pulse, T_{on} is equal to τ. Thus, Equation II. 3 may be rewritten for the equivalent pulse as:

$$B_a = PRF \times \tau \times B_p \qquad \text{Equation II. 7}$$

which may now be solved for B_p :

$$B_p = \frac{B_a}{PRF \times \tau} \qquad \text{Equation II. 8}$$

The value of B_a was measured with a spot photometer; PRF, by means of the calibrated time base of a Tektronix 561A oscilloscope, and the value of τ is determined by the value of the discharge resistor and the energy storage capacitor:

$$\tau = R \times C = 8\,(\text{ohms}) \times 2.5 \times 10^{-6} \text{ farads}$$

$$\tau = 20\ \mu s,$$

hence,

$$B_p = \frac{B_a}{PRF \times 20\,\mu s} .$$