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NUMERICAL RESIDUAL PERTURBATION SOLUTIONS APPLIED TO THE PROBLEM OF A CLOSE SATELLITE OF THE SMALLER BODY IN THE RESTRICTED THREE-BODY PROBLEM

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ABSTRACT

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The purpose of this report is to demonstrate the new method of numerical residual perturbation solution as applied to the problem of a close satellite of the smaller body in the restricted three-body problem. Cowell demonstrated a method of numerically solving the total differential equations of motion of an orbiting object. The variation of parameters and Encke's methods take advantage of the known analytic solution to the two-body problem and numerically handle only the perturbations to the orbit. This report demonstrates the use of an analytic series perturbation solution as a reference orbit (rather then using conics as a reference) and the numerical solution of the residual (i.e., not accounted for by the analytic solution) perturbation equations of motion. The new method shows advantages over the full region of validity of the analytic reference orbit. The analytic theory of motion used is such as to limit the application of the results to orbits of small inclination and eccentricity with special initial conditions. The program is thus just for demonstration.
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Section 1
INTRODUCTION

The purpose of this paper is to show the relative merits for prediction of future position of a satellite of the smaller body in the restricted three-body problem, of the analytic solution of reference 1, of numerical solution of the residual perturbation differential equations not accounted for by the theory of motion in reference 1, of the perturbation differential equations from a Keplerian ellipse, and of the total equations of motion.

In a numerical solution, the time interval is normally chosen so as to make the error due to series truncation in the approximating function approximately equal to the error due to truncation of numbers in the computation of the equations of motion and the approximating function. The balancing of these errors assures maximum accuracy from the method. (Actually, to determine mathematically the time interval, it was necessary in this program to make the series truncation error slightly larger than number truncation error.) In a numerical solution of the total equations, if δY is added to Y_i at each point, then the error builds up as:

```
      Yi = xxxxxxxx
                    ┌──── digit affected by series truncation
                   /┌─── digit affected by number truncation
                   ↓↓
  +   δY =  xxxxxxxx
  ─────────────────
  = Yi+1 = xxxxxxxx
                  ↑
                  └──── digit where error accumulates
```

If it is possible to write Y as $Y_a + \Delta Y$ where Y_a is obtained by an analytic approximation and only ΔY is obtained by numerical solution of the differential equations, then the error accumulates as:

ΔY_i = xxxxxxxx

digit affected by series truncation

digit affected by number truncation

+ δY_i = xxxxxxxx

= ΔY_{i+1} = xxxxxxxx

digit where error accumulates

+ Y_{ai+1} = xxxxxxxx

= Y_{i+1} = xxxxxxxx(xx)

digit where error accumulates

digit where number truncation occurs but is not accumulated.

It is, of course, obvious that the better the approximation Y_a, the smaller ΔY will be and the slower the accumulation of number truncation.

This approach in this paper differs from the work of Encke primarily in that an approximate analytic solution of the perturbed motion is used as Y_a rather than Y_a being an unperturbed Kepler ellipse. The procedure assumes a differential equation of the form:

$$\ddot{Y} = f(X, Y, \dot{Y}) + g(X, Y, \dot{Y}) \tag{I-1}$$

It is approximated by:

$$\ddot{Y} = f(X, Y, \dot{Y}) \tag{I-2}$$

and equation (I-2) is solved analytically to yield the approximation Y_a in the form of a finite asymptotic series. The exact solution, Y, of equation (I-1) is written in the form:

$$Y = Y_a + Z \tag{I-3}$$

Equation (I-3) is differentiated twice and the result substituted for the left-hand side of equation (I-1). The value of Z is computed by a numerical solution of this differential equation. The solution, Y, involves two approximations in this case. First, it is derived from equation (I-2), not equation (I-1); second, the truncation of the series for Y_a represents an error. It may be seen that the numerical procedure for Z corrects for both approximations.

The above discussion is independent of whether the machine computations are done single-precision, double-precision, or n-precision since numerical solution of the total equations of motion, of the perturbations from a conic, and of the residual perturbation from a theory may all be done to n-precision without changing the relative advantages.

Reference 1 was an early work in the series of papers on celestial mechanics described in the list of references. Besides being limited to small eccentricities and inclinations, it does not generalize initial conditions, but always begins with the perigee and node lined up along the Earth-Moon line. Therefore, rectification such as Encke used is not possible. In using it as a basis for the numerical solution, there results a demonstration for the new numeric techniques but not a useful analytic tool. For demonstration purposes, it was not deemed worth-while to attempt the cancellation of all oscillatory terms of the highest order carried by the approximation in order to provide the minimum residual perturbations. What has been done in this paper is to use the precessing elliptic orbit which provides the zero order portion of reference 1 as the reference orbit. This is an appreciable improvement over a fixed initial osculating ellipse and serves to demonstrate the usefulness of the technique.

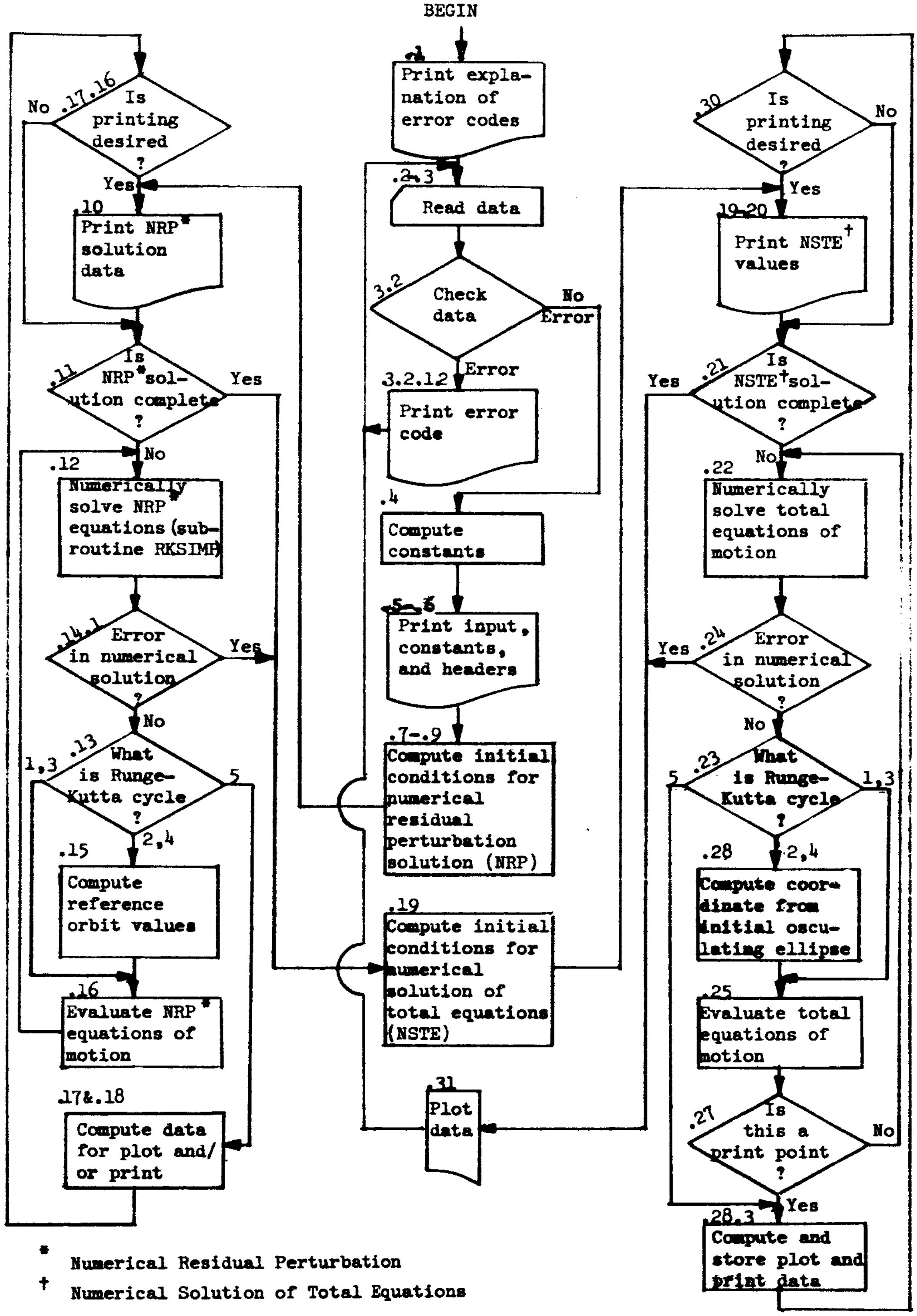
In this example, the equations of motion are those of the restricted three-body problem. No other perturbations are considered. However, no real increase in complexity or decrease in accuracy result from adding other perturbations. This is being demonstrated in a companion report for a satellite of an oblate planet. In the companion report, an analytic approximation is used for J_2 and J_4 perturbations; J_3 and higher-order Earth potential terms as well as Sun and Moon perturbations are treated numerically.

This report consists of a description of the program, a discussion of the results, and plotted results for a variety of mass ratios, semi-major axes, eccentricities, and inclinations. The general flow of the program is illustrated in figure 1. The subsection numbers on the top of each block refer to Section 4 where the equations are detailed.

In this report, the equal sign is often used in the FORTRAN sense, i.e.,

$$x = x + 1$$

means replace x by x + 1. Similarly, symbols such as A(N) are used to refer to an array of N numbers.



* Numerical Residual Perturbation

† Numerical Solution of Total Equations

Figure 1

Section 2

DEFINITION OF SYMBOLS

a	Semi-major axis
a	Subscript for analytic solution from reference 1
a_1	Semi-major axis in analytic solution = A1
a_2	a^2
a_3	a^3 = A3
a_{12}	$\sqrt{a}$
a_{32}	$a^{3/2}$
a_{92}	$a^{9/2}$ = A92
a_e	$a(1-e^2)$ = AEC
$a_{e1/2}$	$\sqrt{a(1-e^2)}$
A_p	Plotting array of dimension 13,000 with elements

$$\left.\begin{array}{ll} ap_{1+13i} = (\Delta C/C)_{ai} & ap_{7+13i} = (\Delta\rho/a)_{pki} \\ ap_{2+13i} = (\Delta C/C)_{pki} & ap_{8+13i} = (\Delta\rho/a)_{ci} \\ ap_{3+13i} = (\Delta C/C)_{ci} & ap_{9+13i} = (\Delta\rho/a)_{ki} \\ ap_{4+13i} = (\Delta C/C)_{ki} & ap_{10+13i} = (\Delta\dot{\rho}a^{1/2})_{ai} \\ ap_{5+13i} = (\Delta C/C)_{pi} & ap_{11+13i} = (\Delta\dot{\rho}a^{1/2})_{pki} \\ ap_{6+13i} = (\Delta\rho/a)_{ai} & ap_{12+13i} = (\Delta\dot{\rho}a^{1/2})_{ci} \\ \multicolumn{2}{c}{ap_{13+13i} = (\Delta\dot{\rho}a^{1/2})_{ki}} \end{array}\right\} i = 0, 1, \ldots, 999$$

a_s	Semi-major axis of precessing ellipse

a_{s32}	$a_s^{3/2}$
A1	FORTRAN symbol for semi-major axis, a
A2	FORTRAN symbol for a^2
A3	FORTRAN symbol for $a_3 = a^3$
A12	FORTRAN symbol for $\sqrt{a}$
A32	FORTRAN symbol for $a^{3/2} = a_{32}$
A92	FORTRAN symbol for $a^{9/2} = a_{92}$
ACOBI	FORTRAN symbol for the Jacobi integral $- 1/\varepsilon^2 r$ (Computed at initial time using numerical residual perturbation solution values)
AEC	FORTRAN symbol for $a(1-e^2)$
AESQ	FORTRAN symbol for $\sqrt{a_s(1-e_s^2)}$
ALPHA1	FORTRAN symbol for α_1
ALPHA2	FORTRAN symbol for α_2
AP	FORTRAN symbol for A_p
AP(I)	FORTRAN symbol for a_{pi}
ARNODE	FORTRAN symbol for argument of node, Ω
AS	FORTRAN symbol for a_s
AS32	FORTRAN symbol for $a_s^{3/2}$

B1	FORTRAN symbol for $C_{15} + \rho_s C_{16}$
B2	FORTRAN symbol for $C_{17} + \rho_s C_{18}$
BETA	FORTRAN symbol for β
C	FORTRAN symbol for the Jacobi constant
c	Subscript for values obtained from numerical solution of total equations of motion
C_1	$\frac{15}{4} \varepsilon a^3 e$
C_2	$\frac{15}{4} \varepsilon a^{1/2} e$
C_3	$\dfrac{\frac{3}{8} \varepsilon i a^{5/2}}{1-\varepsilon a^{3/2}}$
C_6	$\dot{\Omega}$
C_7	$\varepsilon^4 \lambda_1^2 = \dot{\Omega}^2$
C_8	$1-\varepsilon^4$
C_9	$a^{3/2}(1-e^2)^{3/2}$
C_{10}	$\dfrac{\varepsilon^2 \omega_1}{1 + \varepsilon^2 \omega_1} + \varepsilon_1 a^{3/2}(1 + \varepsilon_1 m_1)$
C_{11}	$2P_1 - 1$
C_{12}	$2\dot{\Omega} \;/\; \sqrt{a(1-e^2)} = 2\dot{\Omega}/a_{e1/2}$
C_{13}	$2\dot{\omega} \;/\; \sqrt{a(1-e^2)} = 2\dot{\omega}/a_{e1/2}$

C_{14}	$2C_{\dot{\omega}\dot{\Omega}} = 2\dot{\omega}\dot{\Omega}$
C_{15}	$-\frac{2}{ae^{1/2}}(\dot{\Omega} + \dot{\omega}\cos i)$
C_{16}	$-2\dot{\omega}\dot{\Omega} - (\dot{\omega}^2 + \dot{\Omega}^2)\cos i$
C_{17}	$\frac{2}{ae^{1/2}}(\dot{\Omega}\cos i + \dot{\omega})$
C_{18}	$2\dot{\omega}\dot{\Omega}\cos i + \dot{\omega}^2 + \dot{\Omega}^2$
C_{19}	$\dot{\omega}^2 \sin i$
C_{20}	$\frac{2e\sin i}{ae^{1/2}}\dot{\omega}$
C_{21}	$\frac{2\sin i}{ae^{1/2}}\dot{\omega}$
C_{22}	$-\frac{2e}{ae^{1/2}}(\dot{\Omega} + \dot{\omega}\cos i)$
C_{23}	$\frac{2e}{ae^{1/2}}(\dot{\Omega}\cos i + \dot{\omega})$
C_{24}	$\sqrt{1-e_s^2}$
C_E	cos E
C_i	cos i
C_{init}	Initial value of Jacobi constant

C_{max}	Maximum of absolute values of the components of the velocity increment over the last two Runge-Kutta cycles = CMAX
$C_{P1\psi}$	$\cos (2p_1 - 1)\psi$
C_{part}	Part of Jacobi constant (see Section 3, equation (68))
C_v	$\cos v$
$C_{\varepsilon t}$	$\cos \varepsilon t$
C_{θ_1}	$\cos \theta_1$
C_{ϕ_s}	$\cos \phi_s = \cos (v + \omega)$
C_{ψ}	$\cos \psi$
C_{Ω}	$\cos \Omega$
C_{ω}	$\cos \omega$
$C_{\dot{\omega}\dot{\Omega}}$	$\dot{\omega}\dot{\Omega}$
$C_{\dot{\omega}^2\dot{\Omega}^2}$	$\dot{\omega}^2 + \dot{\Omega}^2$
C1 through C3	FORTRAN symbol for C_1 through C_3
C6 through C21	FORTRAN symbol for C_6 through C_{21}
CALPHA	FORTRAN symbol for $\cos \alpha_1$
CC11PS	FORTRAN symbol for $\cos (C_{11}\psi)$

CEC	FORTRAN symbol for $\cos E$
CEPT	FORTRAN symbol for $\cos \epsilon t$
CI	FORTRAN symbol for cosine inclination, $\cos i$
CIS	FORTRAN symbol for $\cos i_s$
CMAX	FORTRAN symbol for the maximum of absolute values of the components of the velocity increment over the last two Runge-Kutta cycles = C
CNO	FORTRAN symbol for $\cos \Omega$
COPER	FORTRAN symbol for $\cos \omega_s$
COV	FORTRAN symbol for $\cos v_s$
CP1	FORTRAN symbol for $\cos (2p_1 - 1)\psi$
CP_3	FORTRAN symbol for $\cos p_3\psi$
CP1PS	FORTRAN symbol for $\cos (p_1\psi + \alpha_1)$
CP1PST	FORTRAN symbol for $\cos (p_1\psi)$
CPS	FORTRAN symbol for $\cos \psi$
CSNO	FORTRAN symbol for $\cos \Omega_s$
CTHETA	FORTRAN symbol for $\cos \theta_1$
CTRL	FORTRAN symbol for $\cos (v + \omega_s)$

D_n	$1 + e \cos E$
DEGI	FORTRAN symbol for i (degrees)
DELM	FORTRAN symbol for the error in mean anomaly when solving Kepler's equation
DELXD, DELYD, DELZD	In the Runge-Kutta subroutine, FORTRAN symbols for the increment of the components of velocity accumulated for the time interval computation
DEN	FORTRAN symbol for D_n
DNOD	FORTRAN symbol for $\dot{\Omega} = C_6$
DPDN	FORTRAN symbol for $\dot{\omega}\dot{\Omega} = C_{\dot{\omega}\dot{\Omega}}$
DPER	FORTRAN symbol for $\dot{\omega}$
D2PHDT	FORTRAN symbol for $d^2\phi/dt^2$
DPHIDT	FORTRAN symbol for $d\phi/dt$
DPNSQ	FORTRAN symbol for $\dot{\omega}^2 + \dot{\Omega}^2 = C_{\dot{\omega}^2\dot{\Omega}^2}$
D2PSDT	FORTRAN symbol for $d^2\psi/dt^2$
DPSIDT	FORTRAN symbol for $d\psi/dt$
DR	FORTRAN symbol for $(\Delta x^2 + \Delta y^2 + \Delta z)^2$
DRC	FORTRAN symbol for the magnitude of the vector position difference between the numerical residual perturbation solution and the fixed initial osculating ellipse solution at a given time

DRDC	FORTRAN symbol for the magnitude of the vector velocity difference between the numerical residual perturbation solution and the numerical solution of the total equation's of motion at a given time
DRDF	FORTRAN symbol for the magnitude of the vector velocity difference between the numerical residual perturbation solution and the initial osculating ellipse value at a given time
DRF	FORTRAN symbol for the magnitude of the vector position difference between the numerical residual perturbation solution and the initial osculating ellipse value at a given time
DSDPSI	FORTRAN symbol for $ds/d\psi$
DSDT	FORTRAN symbol for ds/dt
DT	FORTRAN symbol for Δt
DT2	FORTRAN symbol for $\Delta t/2$
DT3	FORTRAN symbol for $\Delta t/3$
DT4	FORTRAN symbol for $\Delta t/4$
DT6	FORTRAN symbol for $\Delta t/6$
DT1DPS	FORTRAN symbol for $\varepsilon^2(\frac{dt}{d\psi})_1$
DTDPSI	FORTRAN symbol for $dt/d\psi$
DTM	FORTRAN symbol for multiplier on Δt if error is less than the minimum allowable = 1.5 if input $W_9 \leq 1$
DTODPS	FORTRAN symbol for $(\frac{dt}{d\psi})_o$

DX, DY, DZ,	FORTRAN symbol for Δx, Δy, Δz
DXD, DYD, DZD,	$\Delta\dot{x}$, $\Delta\dot{y}$, $\Delta\dot{z}$
DX2D, DY2D, DZ2D	$\Delta\ddot{x}$, $\Delta\ddot{y}$, $\Delta\ddot{z}$
	position, velocity and acceleration obtained from residual perturbation equations. Coordinate system references to plane of planetary motion.
E	Eccentric anomaly in main program (measured from apogee). In subroutine RKSIMP, the maximum component of the difference between the Runge-Kutta and Simpson rule solutions for velocity
e	Eccentricity (initial)
e_2	$\sqrt{1-e^2}$
E_{min}	Minimum error in Runge-Kutta routine
E_n	Tenative guess at eccentric anomaly
E_{r_1}	$(1-\epsilon^4)\epsilon^2/r^3$
E_{r_2}	$(1-\epsilon^4)\epsilon^2[2(x' \cos \epsilon t + y' \sin \epsilon t) + \epsilon^2\rho^2] \cdot \frac{[1+r(1+r)]}{r^3(1+r)}$
EN	FORTRAN symbol for E_n
e_s	Eccentricity of precessing ellipse
E_{2x}	$\epsilon^2 x$
e_2	$\sqrt{1-e^2}$ = EC2; (also temporary value of $1-e^2$)
e_3	$1 + e$

EALL	FORTRAN symbol for maximum allowable error in Runge-Kutta = $\lvert W_8 \rvert$ set = 10^{-7} if input $W_8 = 0$
EC1	FORTRAN symbol for e
EC2	FORTRAN symbol for $e_2 = \sqrt{1-e^2}$; (also temporary value of $1-e^2$ in Sections 4.3.22 - 4.4)
EC3	FORTRAN symbol for $1+e$
ECAN	FORTRAN symbol for eccentric anomaly (measured from apogee) in analytic solution
ELMB	FORTRAN symbol for λ_1
EM	FORTRAN symbol; in subroutine PLOT, the time at the beginning of each series of $\Delta C/C$, $\Delta\rho/a$, $\Delta\dot{\rho}a^{1/2}$ plots
EN	FORTRAN symbol; in subroutine PLOT, the time at the end of each series of $\Delta C/C$, $\Delta\rho/a$, $\Delta\dot{\rho}a^{1/2}$ plots
EP1	FORTRAN symbol for ε
EP2	FORTRAN symbol for ε^2
EP4	FORTRAN symbol for ε^4
EPA	FORTRAN symbol for $\varepsilon a^{3/2}$
EPR1	FORTRAN symbol for $(1-\varepsilon^4)\varepsilon^2/r^3$
EPR2	FORTRAN symbol for $(1-\varepsilon^4)(1 - 1/r^3)$
EP2X	FORTRAN symbol for $\varepsilon^2 x = E_{2X}$
ERMIN	FORTRAN symbol for the minimum allowable error = $\lvert W_{12} \rvert$

ERROR	FORTRAN symbol for $t(\psi) - t$ (should equal zero)
ES	FORTRAN symbol for e_s
ESA	FORTRAN symbol for the eccentric anomaly of two-body solution
ESTER	FORTRAN symbol for estimated error in Runge-Kutta subroutine
f	$\frac{1 - (1 + 2q)^{-3/2}}{q}$ = Encke's f series
FAC(40)	FORTRAN symbol for the factor in polynomial expression for f
FTD	FORTRAN symbol for a factor (< 1) by which the calculated computing interval is multiplied before use to protect against round-off error
FPS(I)	FORTRAN symbol for $F_{\psi i}$
FQ	FORTRAN symbol for $fq = 1 - (1 + 2q)^{-3/2}$ See Section 3, equation (102)
FRA(40)	FORTRAN symbol for the fraction by which a term in series for f must be multiplied to give succeeding term
$\dot{G}$	$\frac{-(\frac{dt_o}{d\psi} \varepsilon^2 \omega_1 + \varepsilon^2 \frac{dt_1}{d\psi} (1 + \varepsilon^2 \omega_1)}{\frac{dt}{d\psi} \frac{dt_o}{d\psi} (1 + \varepsilon^2 \omega_1)} - \varepsilon^2 \omega_1 - \dot{\omega}$
GAMMA	FORTRAN symbol for εt modulo 2π

GNU1	FORTRAN symbol for ν_1
HX, HY, HZ HXD, HYD, HZD HX2D, HY2D, HZ2D	FORTRAN symbols for dummy variables of position, velocity, and acceleration in Runge-Kutta subroutine. Equivalent to: $\Delta\vec{x}$, $\Delta\dot{\vec{x}}$, $\Delta\ddot{\vec{x}}$ for numerical residual perturbation solution $\vec{x}$, $\dot{\vec{x}}$, $\ddot{\vec{x}}$ for numerical solution of total equations of motion
HXDA, HYDA, HZDA	FORTRAN symbol for Simpson's rule; integrated values of $\dot{x}$, $\dot{y}$, $\dot{z}$, respectively
I	FORTRAN symbol, main program cards 220-230 -- index for zeroing input array W Main program cards 563-564 -- index for zeroing first 13 elements of plotting array AP Main program card 1030 -- index for printing input array W Main program cards 2510-2660 -- index for Encke's f series Main program card 5380 -- index used to print plot-table data at computing point ITD Runge-Kutta subroutine cards 6670-6680 -- index for restoring initial condition array S from array SP after print point, which is not a normal computing point while integrating total equations of motion Runge-Kutta subroutine cards 7140-7150, 7670-7680, and 7850-7860 -- index for restoring initial condition array S from array SS when computing interval fails

i	Inclination (initial) held constant in analytic solution (radians)
i^{o}	Inclination (degrees)
i_s	Inclination of precessing ellipse
ICH	FORTRAN symbol for the flag to signal which equations are being numerically solved Residual perturbation equations if ICH = 1 Total equations of motion if ICH = 2
IERR	FORTRAN symbol; in subroutine PLOT for number of off-scale points
II	FORTRAN symbol; in subroutine PLOT, an index giving the plot point number of the first point of each series of $\Delta C/C$, $\Delta\rho/a$, $\Delta\dot{\rho}a^{1/2}$ plots
III	FORTRAN symbol; in subroutine PLOT, an index giving the plot point number of the last point of each series of $\Delta C/C$, $\Delta\rho/a$, $\Delta\dot{\rho}a^{1/2}$ plots
IIK	FORTRAN symbol; in subroutine PLOT, an index for successive sets of $\Delta C/C$, $\Delta\rho/a$, $\Delta\dot{\rho}a^{1/2}$ plots
IP	FORTRAN symbol for the initial point flag IP = 1 on first point IP = 2 on all others Set = 1 by main program Set = 2 by Runge-Kutta subroutine

IPRINT	FORTRAN symbol for print flag IPRINT = 1 to call for print IPRINT = 2 to supress print IPRINT = 3 in integration of total equations of motion, designates a print point that is also a regular computed point
IS	FORTRAN symbol for i_s (a floating point number)
IT	FORTRAN symbol for the index for first element in plot array AP for computing point ITP IT = 13ITP - 12
ITP	FORTRAN symbol for the index for plotting array
ITQ	FORTRAN symbol for the index for last element in plot array AP for computing point ITP ITQ = IT + 12
J	FORTRAN symbol for main program cards 2570-2590 -- index used to compute coefficients in Encke's f series
JACOBI	FORTRAN symbol for the initial value of Jacobi constant (a floating point variable)
K	FORTRAN symbol for the index used to compute coefficients of Encke's f series = I - J
K	FORTRAN symbol; in subroutine PLOT, index to locate proper values in AP array for each plot. When K = - 12 + 13I $\Delta C/C$ values are plotted K = - 7 + 13I $\Delta \rho/a$ values are plotted K = - 3 + 13I $\Delta\dot{\rho}a^{1/2}$ values are plotted

K	In subroutine PLOT, subscript for initial osculating ellipse values
KC	FORTRAN symbol for flag to signal Simpson's rule computations No if KC = 1 Yes if KC = 2 Under control of Runge-Kutta subroutine
KF	FORTRAN symbol for intermediate failure counter in Runge-Kutta subroutine
KFAIL	FORTRAN symbol for failure counter in Runge-Kutta subroutine
KHALT	FORTRAN symbol for halt flag If = 1, run continued If = 2, run halted, next case called If = 3, perturbed trajectory completed, numerical solution of total equations of motion to be initiated
KR	FORTRAN symbol for Runge-Kutta flag to signal which pass of Runge-Kutta. Initially set = 1 in main program; thereafter under control of Runge-Kutta subroutine
KT	FORTRAN symbol for computing interval counter
L	FORTRAN symbol for index used in computing coefficients of Encke's f series = I - 1

L FORTRAN symbol; in subroutine PLOT, an index used to establish for which plot the grid and common labeling are being done

$L = 1 \quad \Delta C/C$

$L = 2 \quad \Delta \rho / a$

$L = 3 \quad \Delta \dot{\rho} a^{1/2}$

M Mean anomoly

m_1 $\dfrac{-a_{3/2}(1 + 12e)}{6}$

M1 FORTRAN symbol for m_1

MFAIL FORTRAN symbol for the maximum number of failures allowed in Runge-Kutta subroutine = $|W_{11}|$

MRKPT FORTRAN symbol; in subroutine PLOT, an array to identify the plotting symbols to be used for subroutine APLOTV

Term No.	Value	Symbol Indicated	Type of Solution
1	38	o	Analytic Solution
2	55	×	Precessing Ellipse
3	63	□	Numerical Solution of Total Equations of Motion
4	44	*	Initial Osculating Ellipse
5	42	.	Numerical Residual Perturbation Solution

n Point number index in explanation of Runge-Kutta method in Section 3.4

NDIV	FORTRAN symbol for the error code

NDIV	Reason for Halt
1	$a \leq 0$
2	$1 - e^2 \leq 0$
7	$S = 0$
8	$dt/d\psi = 0$

NX	FORTRAN symbol; in subroutine PLOT, the number of characters in horizontal labels
NY	FORTRAN symbol; in subroutine PLOT, the number of characters in vertical labels
OMEG1	FORTRAN symbol for ω_1
p	Subscript for numerical residual perturbation solution results
P_{ss}	Period of mean precessing ellipse
P_t	A 700-element array of plot times
$P_{t(ITP)}$	ITPth element of array P_t
p_1	$\frac{1}{1 + \varepsilon_2\omega_1} - \varepsilon_1 a^{3/2} (1 + \varepsilon_1 m_1)$
p_3	$\frac{1}{1 + \varepsilon_2\omega_1} - \varepsilon_a [2 + \varepsilon(\nu_1 - 2\lambda_1)]$
P_Ω	Period of node
P_ω	Period of perigee

P1	FORTRAN symbol for p_1
P3	FORTRAN symbol for p_3
PHI	FORTRAN symbol for ϕ
pk	Subscript for precessing ellipse values
PNOD	FORTRAN symbol for the period of the node
PPER	FORTRAN symbol for the period of the perigee
P1PS	FORTRAN symbol for $2p_1\psi$ where $0 \leq \psi < 360$ approximately
PSI	FORTRAN symbol for ψ
PS(I)	ith guess on ψ in solution of Kepler's equation = array of 25 elements
PSS	FORTRAN symbol for the two-body period of the precessing ellipse which is the reference orbit of the numerical residual perturbation solution
PTIME	FORTRAN symbol for the 700-element array P_t
Q	FORTRAN symbol for q
q	$= [(x_s' + \frac{\Delta x'}{2})\Delta x' + (y_s' + \frac{\Delta y'}{2})\Delta y' + (z_s' + \frac{\Delta z'}{2})\Delta z']/\rho_s^2$
r	Distance from larger body
r_a	$\sqrt{1 + 2\varepsilon^2(x'\cos\gamma + y'\sin\gamma) + \varepsilon^2\rho'^2}$ where $\rho'^2 = x'^2 + y'^2 + z'^2$

r_f	Distance from larger body, according to fixed ellipse
r_o	Initial distance from larger body
r	$\sqrt{1 + \epsilon^4\rho^2 + 2\epsilon^2(x \cos \epsilon t + y \sin \epsilon t)}$
RA	FORTRAN symbol for r_a
RADI	FORTRAN symbol for inclination, i (radians)
RCU	FORTRAN symbol for r^3
RF	FORTRAN symbol for r_f
RHO	FORTRAN symbol for ρ
RHOASQ	FORTRAN symbol for ρ_a^2
RHOCU	FORTRAN symbol for ρ^3
RHOD	FORTRAN symbol for $\dot{\rho} = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$
RHOF	FORTRAN symbol for ρ_f
RHOFSQ	FORTRAN symbol for ρ_f^2
RHOS	FORTRAN symbol for ρ_s = two-body orbit radius
RHOSCU	FORTRAN symbol for ρ_s^3
RHOSQ	FORTRAN symbol for ρ^2
RKSIMP	FORTRAN symbol for the name of Runge-Kutta subroutine

RK1X, RK1Y, RK1Z RK2X, RK2Y, RK2Z RK3X, RK3Y, RK3Z	FORTRAN symbol for the Runge-Kutta values of Δt times the acceleration on successive passes of Runge-Kutta
RNAUT	FORTRAN symbol for r_o
RSQ	FORTRAN symbol for r^2
S	FORTRAN symbol for s
s	Reciprocal of radius vector, ρ
S_E	$\sin E$
S_i	$\sin i$
S_{P1}	$\sin (2p_1 - 1)\psi$
$S_{P_1\psi}$	$\sin (p_1\psi)$
S_v	$\sin v$
S_{θ_1}	$\sin \theta_1$
S_{ϕ_s}	$\sin (v + \omega)$
$S_{\epsilon t}$	$\sin (\epsilon t)$
S_ψ	$\sin \psi$
S_ω	$\sin \omega$
S_Ω	$\sin \Omega$

S(10)	FORTRAN symbol for the 10-element array in which values of time, position, velocity, and acceleration are saved for ordinary Runge-Kutta use
SALPHA	FORTRAN symbol for $\sin \alpha_1$
SC11PS	FORTRAN symbol for $\sin (C_{11}\psi)$
SEC	FORTRAN symbol for $\sin E$
SEPT	FORTRAN symbol for $\sin \gamma = S_{\epsilon t}$
SI	FORTRAN symbol for $\sin i$
SIPER	FORTRAN symbol for $\sin \omega_s$
SIS	FORTRAN symbol for $\sin i_s$
SIV	FORTRAN symbol for $\sin v_s$
SNO	FORTRAN symbol for $\sin \Omega$
SNODE	FORTRAN symbol for Ω_s
SP	FORTRAN symbol for the 10-element array in which values of time, position, velocity, and acceleration are saved when print or plot occurs at other than a regular compute point during the numerical solution of the total equations of motion
SP1	FORTRAN symbol for $\sin (2p_1 - 1)\psi$
SP1PS	FORTRAN symbol for $\sin (p_1\psi + \alpha)$

SP1PST	FORTRAN symbol for $\sin(p_1\psi)$
SP3	FORTRAN symbol for $\sin(p_3\psi)$
SPDT	FORTRAN symbol for the saved value of computing interval when print or plot occurs at other than a regular compute point during the numerical solution of the total equations of motion
SPER	FORTRAN symbol for ω_s
SPS	FORTRAN symbol for $\sin\psi$

SS — FORTRAN symbol in subroutine RKSIMP for the 17-element array in which values of time, position, velocity, and acceleration are saved for

1. in case computing interval selection fails
2. Simpson's rule integration

$ss_1 = t$	$ss_7 = \dot{z}_h$	$ss_{13} = t_\Omega$
$ss_2 = x_h$	$ss_8 = \ddot{x}_h$	$ss_{14} = p_\omega$
$ss_3 = y_h$	$ss_9 = \ddot{y}_h$	$ss_{15} = \theta_1$
$ss_4 = z_h$	$ss_{10} = \ddot{z}_h$	$ss_{16} = \sin\theta_1$
$ss_5 = \dot{x}_h$	$ss_{11} = t_s$	$ss_{17} = \gamma$
$ss_6 = \dot{y}_h$	$ss_{12} = t_\psi$	

SSNO	FORTRAN symbol for $\sin\Omega_s$
STHET1	FORTRAN symbol for $\sin\theta_1$. Old value not removed to avoid unnecessary change in code

STHETA	FORTRAN symbol for $\sin \theta_1$
STRL	FORTRAN symbol for $\sin (\nu + \omega_s)$
T	FORTRAN symbol for time, t
t	Time
t_{a32}	$t \cdot a^{3/2}$
$T_{den\ i}$	ith term in Encke's series $f = 3[1 - \frac{5}{2} q + \frac{5\cdot 7}{2\cdot 3} q^2 - \frac{5\cdot 7\cdot 9}{2\cdot 3\cdot 4} q^3 + \ldots]$
T_f	Final time
t_i	Integer part of t_R
$T_{num\ i}$	Numerator by which ith term in series for f must be multiplied
$T_{num\ (i-1)}$	Numerator by which the $(i-1)$th term in series for f must be multiplied
t_p	Print and plot time during numerical residual perturbation solution
t_{pT}	Print time during numerical solution of total equations of motion
t_R	Time interval plotted on each page
t_s	Time, modulo the period of precessing ellipse

t_o — Initial time

T_1 — Period of t expression defined in

$$t(2\pi) = T_1 = a^{3/2}\, 2\pi + \frac{15}{4}\, \varepsilon a^3 e \sin\,[(2p_1 - 1)2\pi]$$
$$- \varepsilon^2 a^{9/2}\, \frac{11}{8} \sin 4p_1\pi$$

t_ψ — Time since last passage of apocenter by analytic solution

t_Ω — Time since the last passage of the node through zero

t_ω — Time since the last time the perigee was zero

TAS32 — FORTRAN symbol for $t_s/a_s^{3/2}$

TDEN — FORTRAN symbol for the 40-element array of the denominators by which the terms in series for f must be divided

TERM — FORTRAN symbol for the 40-element array of succeeding terms in series for f

TF — FORTRAN symbol for the total flight time; input as a run stop condition

THETA1 — FORTRAN symbol for θ_1

THETA2 — FORTRAN symbol for θ_2

TN — FORTRAN symbol for "next time" = $t + \Delta t$

TNOD — FORTRAN symbol for t_Ω

TNUM	FORTRAN symbol for the 40-element array of numerators by which the terms in the series for f must be multiplied
TP1	FORTRAN symbol for the print or plot interval
TPER	FORTRAN symbol for t_ω
TPI	FORTRAN symbol for the print or plot time
TPRINT	FORTRAN symbol for t_p and t_{pt}
TPSI	FORTRAN symbol for t_ψ
TR	FORTRAN symbol for t_R
TRUL	FORTRAN symbol for $v + \omega_s$
TS	FORTRAN symbol for t_s
V	FORTRAN symbol for true anomaly in two-body equations, v
v	true anomaly, V
VSQ	FORTRAN symbol for $\dot{\rho}^2$
VSSQ	FORTRAN symbol for V_s^2 at initial point
W	FORTRAN symbol for the 30-element input array consisting of W_1 through W_{30}
W_1	Input a

W_2 Input e

W_3 Input i (degrees)

W_4 Input $\epsilon^4 (\equiv \mu)$

W_5 Input FTD modifier of Δt

W_6 Not used

W_7 Input run stop time

W_8 Input maximum allowable error in Runge-Kutta

W_9 Input multiplier on computing interval if error $<$ minimum allowable. Must be > 1. If it is not, it is set = 1.5

W_{10} Input initial computing interval. If zero, it is set = 0.005

W_{11} Input maximum failures permitted by program in selection of computing interval

W_{12} Input minimum allowable error in Runge-Kutta

W_{13} Input flag for secular rates

If $W_{13} \leq 0$, only initial and final points printed

If $W_{13} > 0$, approximately every 1/4 radian of ψ printed

WR FORTRAN symbol for the 30-element input reference run array

X, Y, Z XD, YD, ZD X2D, Y2D, Z2D	FORTRAN symbol for x, y, z $\dot{x}, \dot{y}, \dot{z}$ $\ddot{x}, \ddot{y}, \ddot{z}$ Total position, velocity, and acceleration components. Coordinate system referenced to plane of planetary motion.
x'	Inertial coordinate in plane of massive bodies, from smaller body away from larger
x'_a	Inertial coordinate from analytic solution
x''_a	Coordinate in orbital plane direction to the node
x'_f	Inertial coordinate from fixed ellipse
x_h, y_h, z_h $\dot{x}_h, \dot{y}_h, \dot{z}_h$ $\ddot{x}_h, \ddot{y}_h, \ddot{z}_h$	In subroutine RKSIMP, dummy variables of position, velocity, and acceleration, respectively
X_L	Time for the left edge of each plot (in subroutine PLOT) = EM
x'_s	Inertial coordinate from precessing ellipse
X_T	Time at right edge of each plot = EN
x'_o	Initial inertial coordinate
XD	See X, Y, Z, for definition
X2D	See X, Y, Z, for definition

XF	FORTRAN symbol for the x' coordinate of position determined from the initial osculating ellipse
XL	FORTRAN symbol in subroutine PLOT for left-most value of abscissa
XNAUT	FORTRAN symbol for x'_o
XP, YP, ZP XDP, YDP, ZDP X2DP, Y2DP, Z2DP	FORTRAN symbol for x', y', z' $\dot{x}', \dot{y}', \dot{z}'$ $\ddot{x}', \ddot{y}', \ddot{z}'$ Approximate position, velocity and acceleration components. Coordinate system referenced to plane of planetary motion.
X2P, Y2P, Z2P XD2P, YD2P, ZD2P X2D2P, Y2D2P, Z2D2P	FORTRAN symbol for x'', y'', z'' $\dot{x}'', \dot{y}'', \dot{z}''$ $\ddot{x}'', \ddot{y}'', \ddot{z}''$ Approximate position, velocity and acceleration components. Coordinate system referenced to orbit plane of satellite
XS, YS, ZS	FORTRAN symbol for x'_s, y'_s, z'_s
XSD, YSD, ZSD	FORTRAN symbol for $\dot{x}'_s, \dot{y}'_s, \dot{z}'_s$
y'	Inertial coordinate in plane of massive bodies, perpendicular to x' in direction of motion
y'_a	Inertial position component from analytic solution
y''_a	Coordinate in satellite orbital plane from analytic solution

y'_f	Inertial coordinate from initial osculating ellipse
y'_s	Inertial coordinate from precessing ellipse
YACAF	FORTRAN symbol for ΔC_f and $\Delta C_f/C_{init}$
YACAPP	FORTRAN symbol for ΔC_a and $\Delta C_a/C_{init}$
YACAS	FORTRAN symbol for $\Delta C_s/C_{init}$
YACOBI	FORTRAN symbol for Jacobi constant on first point ΔC_E. Thereafter equals difference between instantaneous value and initial value $= \Delta C_E/C_{init}$
YB	FORTRAN symbol in subroutine PLOT for the bottom value of the scale
YF	FORTRAN symbol for y' coordinate of position as derived from the initial osculating ellipse
YFD	FORTRAN symbol for y' coordinate of the velocity derived from the initial osculating ellipse
YT	Top of ordinate in plots
z'	Inertial coordinate normal to plane of massive bodies, positive north
z'_a	Inertial coordinate from analytic solution
z''_a	Coordinate perpendicular to orbital plane from analytic solution
z'_f	Inertial coordinate from fixed ellipse

z_s'	Inertial coordinate from precessing ellipse
ZF	FORTRAN symbol for z' coordinate of the position as derived from the initial osculating ellipse
ZFD	FORTRAN symbol for z' coordinate of the velocity derived from the initial osculating ellipse
α_1	Part of $2p_1 \psi_{tot}$ corresponding to an integral number of revolutions of $\psi = 2n\pi \cdot 2p_1$, modulo 2π
α_2	$\alpha_1 + 2\pi \cdot 2p_1$, modulo $2\pi = 2(n+1)\pi \cdot 2p_1$, modulo 2π
β	Part of $p_3 \cdot \psi_{tot}$ corresponding to an integral number of revolutions of $\psi = 2p_3 n\pi$, modulo 2π
γ	εt, modulo 2π
ΔC	Error in Jacobi constant
ΔC_a	Error in Jacobi constant calculated from analytic solution
ΔC_c	Error in Jacobi constant calculated from numerical solution of total equations of motion
ΔC_E	Error in Jacobi constant calculated from the residual perturbation numerical solution
ΔC_F	Error in Jacobi constant calculated from initial osculating ellipse
ΔC_s	Error in Jacobi constant calculated from precessing ellipse

$(\Delta C/C)_{ai}$	Fractional error of Jacobi constant of analytic solution
$(\Delta C/C)_{ci}$	Fractional error of Jacobi constant for numerical solution of total equations of motion
$(\Delta C/C)_{ki}$	Fractional error of Jacobi constant from initial osculating ellipse
$(\Delta C/C)_{pi}$	Fractional error in Jacobi constant from numerical residual perturbation solution
$(\Delta C/C)_{pki}$	Fractional error in Jacobi constant from precessing Kepler ellipse
ΔE	Difference between successive guesses at E in iterative solution of Kepler's equation
ΔE_p	Difference between successive guesses at E in iterative solution of perturbed Kepler's equation
Δt	Computing interval
Δt_p	Time interval for print
Δt_{sp}	In numerical solution of total differential equations, the saved normal computing interval when a special print point is being computed
$\Delta x'$, $\Delta y'$, $\Delta z'$	Increments in position, velocity, and acceleration obtained numerically from the residual perturbation equations. Coordinate system is referenced to plane of planetary motion

$\Delta\dot{x}$, $\Delta\dot{y}$, $\Delta\dot{z}$	In subroutine RKSIMP, the accumulated change in $\dot{x}$, $\dot{y}$, and $\dot{z}$ over two computing intervals
$\Delta\rho_a$	Distance between analytic solution and numerical residual perturbation solution
$\Delta\dot{\rho}_a$	Magnitude of vector velocity difference between the analytic solution and the numerical residual perturbation solution
$\Delta\rho_c$	Distance between numerical solution of total differential equations and the numerical residual perturbation solution
$\Delta\dot{\rho}_c$	Magnitude of vector velocity difference between the numerical solution of the total equations of motion and the numerical residual perturbation solution
$\Delta\rho_F$	Distance between initial osculating ellipse solution and the numerical residual perturbation solution
$\Delta\dot{\rho}_F$	Magnitude of vector velocity difference between the initial osculating ellipse solution and the numerical residual perturbation solution
$\Delta\rho_s$	Distance between the precessing ellipse solution and the numerical residual perturbation solution
$\Delta\dot{\rho}_s$	Magnitude of vector velocity difference between the precessing ellipse and the numerical residual perturbation solution
ε	$\mu^{1/4}$

ε_a	$\varepsilon \cdot a^{3/2} = \varepsilon a_{3/2}$
ε_1	$\varepsilon = \mu^{1/4}$
ε_2	$\varepsilon^2 = \mu^{1/2}$
ε_4	$\varepsilon^4 = \mu$
ε_4	Mass ratio of sun to sun + planet = μ
θ_1	Part of $(2p_1-1)\ \psi_{tot}$ resulting from integer n rotations of $\psi = 2\pi n\ (2p_1-1)$, modulo 2π
θ_2	$\theta_1 + 2\pi\ (2p_1-1)$, modulo 2π $= 2(n+1)\pi(2p_1-1)$, modulo 2π
λ_1	Angular rate of change of Ω
μ	Mass ratio of smaller body to sum of smaller and larger bodies
ν_1	$a^{3/2}\ (11 + 24e)/12 = \dot{\bar{\phi}}/\varepsilon^2$
ρ	Radial distance from smaller body
$\dot{\rho}$	Total velocity in blown up coordinates
ρ_a	Radial distance obtained from analytic solution
ρ_s	Radial distance obtained from precessing ellipse
ϕ	Angle from node to satellite

ϕ_s	Angular distance of the satellite from the node as obtained from precessing ellipse; $\phi_s = v + \omega$
ϕ_1	Integer part of ϕ modulo $2\pi = \left[\frac{2n\pi}{1+\varepsilon^2\omega_1} - \varepsilon^2\nu_1 \, t(2\pi n)\right]$, modulo 2π
ψ	True anomaly measured from apogee
Ω	Argument of the node in analytic solution
ω	Argument of pericenter
ω_1	$\frac{a^3(24e - 7)}{12}$

Section 3
DEVELOPMENT OF EQUATIONS

3.0 GENERAL

The purpose of this section is to describe the sources of the equations used in the program and to outline the derivation of the exact forms programmed. When an equation from a reference is first introduced, the equation number in the source is shown to the left of the equation. The equation numbers of the present document are to the right of the equations. The equations are not derived in the exact order they are used; however, the numbers of the subsections in the Equations in Order of Solution, Section 4, where the equation is used, are stated after each derivation. Section 4 presents the equation numbers of this section so that the cross reference is complete.

3.1 EQUATIONS OF MOTION

The motion considered is that of a satellite of negligible mass in close proximity to the smaller of the two massive bodies in the restricted three-body problem, which may serve as a simplified model for an actual case like a lunar satellite in the earth-moon system.

The coordinate system used is centered at the smaller mass and rotates with the constant angular velocity of the two massive bodies around their center of mass. With the notation as in reference 1, the x^*, y^*, and z^* rectangular system is oriented such that the x^*, y^* plane coincides with the plane of motion of the two massive bodies, the x^*-axis pointing away from the larger body, the y^*-axis 90° ahead in the direction of motion, and z^*-axis parallel to the rotation axis as to complete a right hand system.

In order to nondimensionalize the equations of motion, the following scales are used:

1. Unit of length = Distance between the two massive bodies
2. Unit of time = Period of rotation of the x^*, y^*, z^* system in inertial space, divided by 2π

3. Unit of mass = Total mass of the system.

The equations of motion of the satellite in these coordinates are given in reference 1 by:

$$(1) \quad \frac{d^2\vec{x}^*}{dt^{*2}} = \text{grad}^*\left[\frac{1-\mu}{\vec{x}^* + \hat{i}} + \frac{\mu}{\vec{x}^*}\right] - 2\hat{k} \times \frac{d\vec{x}^*}{dt^*} - \hat{k} \times \{\hat{k} \times [\vec{x}^* + (1-\mu)\hat{i}]\} \qquad (1)$$

where $\vec{x}^* = (x^*, y^*, z^*)$ and $\hat{i}$, $\hat{j}$, and $\hat{k}$ are the unit vectors in the x^*, y^*, and z^* directions.

Since the motion of the satellite takes place in a close vicinity of the smaller body, the following "blown-up" coordinates are introduced:

$$x = \frac{x^*}{\mu^{1/2}}, \quad y = \frac{y^*}{\mu^{1/2}}, \quad z = \frac{z^*}{\mu^{1/2}}, \quad t = \frac{t^*}{\mu^{1/4}} \qquad (2)$$

(Reference 1, page 205, equation above (6), there in vector notation) where μ is the mass ratio of the smaller to the total mass.

In order to avoid terms of order $\mu^{1/4} = \varepsilon$ in the equations of motion, another transformation is made to inertially oriented coordinates by multiplying the vector $\vec{x} = (x, y, z)$ by the matrix $\{R\}$ with:

$$(7c) \quad \{R\} = \begin{bmatrix} \cos \varepsilon t & -\sin \varepsilon t & 0 \\ \sin \varepsilon t & \cos \varepsilon t & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad (3)$$

[Reference 1, page 206, equation (7c)]

then

$$(7b) \quad \{\vec{x}\}' = \{R\}\{\vec{x}\} \qquad (4)$$

The primed system is therefore inertially oriented, but centered at the smaller body.

When the transformations (2) and (4) are performed on the equations of motion (1), the following equations result:

$$\frac{d^2\vec{x}'}{dt^2} = \frac{\vec{x}'}{\rho^3} - \frac{(1 - \varepsilon^4)\varepsilon^2\vec{x}'}{r^3} + (1 - \varepsilon^4)(1 - \frac{1}{r^3}) \begin{bmatrix} \cos \varepsilon t \\ \sin \varepsilon t \\ 0 \end{bmatrix} \tag{5}$$

where

$$\vec{x}' = (x', y', z') \tag{6}$$

$$\rho^2 = x'^2 + y'^2 + z'^2 \tag{7}$$

$$r^2 = 1 + \varepsilon^4\rho^2 + 2\varepsilon^2(x' \cos \varepsilon t + y' \sin \varepsilon t) \tag{8}$$

$$\varepsilon = \mu^{1/4} \tag{9}$$

Equations (5), (6), and (7) are used in Section 4.25. Equations (8) and (9) are used in Sections 4.25, 4.19.2, 4.16.2, and 4.8.3.

Geometrically, ρ is the distance of the satellite from the smaller body and r the distance between satellite and larger body, both in blown-up coordinates.

3.2 THE JACOBI CONSTANT

In order to check the accuracy of the solutions, the Jacobian integral will be used. From reference 2, page 281, equation (7), one obtains:

$$(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2 + (\frac{dz}{dt})^2 = x^2 + y^2 + \frac{2(1 - \mu)}{r_1} + \frac{2\mu}{r_2} + C \tag{10}$$

This is in rotating coordinates, centered at the center of mass; r_1 and r_2 are the distances of the satellite from the larger and the smaller primary, respectively. To transform equation (10) to blown-up coordinates, centered at the smaller body, the above variables are to be replaced as follows:

$$
\begin{array}{lll}
x \to 1 - \mu + \varepsilon^2 x & \frac{dx}{dt} \to \varepsilon\dot{x} & r_1 \to r \\
y \to \varepsilon^2 y & \frac{dy}{dt} \to \varepsilon\dot{y} & r_2 \to \varepsilon^2 \rho \\
z \to \varepsilon^2 z & \frac{dz}{dt} \to \varepsilon\dot{z} & \mu \to \varepsilon^4
\end{array}
\tag{11}
$$

Then after applying the transformation (4), the Jacobi integral takes on the form:

$$
\dot{x}'^2 + \dot{y}'^2 + \dot{z}'^2 + 2\varepsilon(\dot{x}'y' - \dot{y}'x') - 2(1 - \varepsilon^4)(x' \cos \varepsilon t + y' \sin \varepsilon t)
$$

$$
- \frac{2}{\rho} - \frac{2(1 - \varepsilon^4)}{\varepsilon^2 r} + \frac{(1 - \varepsilon^4)^2}{\varepsilon^2} = C' \tag{12}
$$

The last terms on the left side are numerically large. To avoid the loss of significant figures in the computation of the error in the Jacobi constant, the initial value of the Jacobi constant is subtracted in the following way:

$$
\dot{x}'^2 + \dot{y}'^2 + z'^2 + 2\varepsilon(\dot{x}'y' - \dot{y}'x') - 2(1 - \varepsilon^4)(x'\cos \varepsilon t + y'\sin \varepsilon t)
$$

$$
- \frac{2}{\rho} - \frac{2(1 - \varepsilon^4)}{\varepsilon^2 r} + \frac{(1 - \varepsilon^4)^2}{\varepsilon^2} - C_{init} = \Delta C \tag{13a}
$$

where $C_{init} = - \dot{x}'^2(0) - \dot{y}'^2(0) - \dot{z}'^2(0) - 2\varepsilon[\dot{x}'(0)y'(0) - \dot{y}'(0)x'(0)]$

$$
+ 2(1 - \varepsilon^4)x'(0) + \frac{2}{\rho(0)} + \frac{2(1 - \varepsilon^4)}{\varepsilon^2 r(0)} - \frac{(1 - \varepsilon^4)^2}{\varepsilon^2} \tag{13b}
$$

The quantity ΔC is zero if the solution is exact. For an approximate solution, ΔC serves as a test quantity on the accuracy of the approximation. Define:

$$C_{part} = +\dot{x}'^2(0) + \dot{y}'^2(0) + \dot{z}'^2(0) + 2\varepsilon[\dot{x}'(0)y'(0) - \dot{y}'(0)x'(0)]$$

$$- 2(1 - \varepsilon^4)x'(0) + \frac{2}{\rho(0)} \tag{14}$$

Using equation (8) and noting that with apocenter as the initial condition, then the difference of the $\frac{1}{\varepsilon^2}$ terms of (13a) is written as:

$$-\frac{2(1-\varepsilon^4)}{\varepsilon^2 r} + \frac{2(1-\varepsilon^4)}{\varepsilon^2 r_o} = \frac{2(1-\varepsilon^4)}{\varepsilon^2} \frac{(r^2 - r_o^2)}{rr_o(r + r_o)}$$

$$= 2\,\frac{1-\varepsilon^4}{\varepsilon^2}\,\frac{1 + \varepsilon^4\rho^2 + 2\varepsilon^2(x' \cos \varepsilon t + y' \sin \varepsilon t) - 1 - \varepsilon^4 x_o'^2 - 2\varepsilon^2 x'}{rr_o(r + r_o)}$$

$$= 2(1 - \varepsilon^4)\,\frac{2(x'\cos \varepsilon t + y'\sin \varepsilon t - x') + \varepsilon^2(\rho^2 - x'^2)}{rr_o(r + r_o)} \tag{15}$$

where $r(0) = 1 + \varepsilon^2 x_o' = r_o$ (*)

Then the error in the Jacobi constant becomes:

$$\Delta C' = (\dot{x}'^2 + \dot{y}'^2 + \dot{z}'^2) + 2[\varepsilon(\dot{x}'y' - \dot{y}'x') - \frac{1}{\rho}$$

$$- (1 - \varepsilon^4)(x'\cos \varepsilon t + y'\sin \varepsilon t)] - C_{part}$$

$$+ 2(1 - \varepsilon^4)\,\frac{[2(x'\cos \varepsilon t + y'\sin \varepsilon t - x_o) + \varepsilon^2(\rho^2 - x_o^2)]}{rr_o(r + r_o)} \tag{16}$$

Equation (16) is used in Sections 4.17.15, 4.28.1, and 4.28.2.

3.3 SOLUTIONS

In the following, three different methods for the solution of equation (5) will be outlined:

(*) See equations (8), (42), and (61)

1. An approximate, analytic solution as given in reference 1.

2. A numerical residual perturbation method which uses the Keplerian part plus the secular perturbations to order ε^2 of the analytic solution as a reference orbit.

3. Direct application of numerical integration to the total exact equations of motion.

3.3.1 Analytic Solution

The developments as given in reference 1 start from an approximate set of equations that result from equation (5) when terms having ε^3 or a higher power of ε as a factor are neglected.

The approximate equations of motion are:

$$\frac{d^2x'}{dt^2} = -\frac{x'}{\rho^3} + \frac{\varepsilon^2 x'}{2} + \frac{3}{2}\varepsilon^2(x'\cos 2\varepsilon t + y'\sin 2\varepsilon t)$$

(8a)
$$\frac{d^2y'}{dt^2} = -\frac{y'}{\rho^3} + \frac{\varepsilon^2 y'}{2} + \frac{3}{2}\varepsilon^2(x'\sin 2\varepsilon t - y'\cos 2\varepsilon t) \qquad (17)$$

$$\frac{d^2z'}{dt^2} = -\frac{z'}{\rho^3} - \varepsilon^2 z'$$

These equations are identical with the vector equation (8a) of reference 1.

The solution of equation (17) is given in reference 1 in the following form:

(18a)
$$s(\psi) = s_o(\psi) + \varepsilon^2 s_1(\psi,\varepsilon) + O(\varepsilon^3)$$

(18b)
$$t(\psi) = t_o(\psi) + \varepsilon^2 t_1(\psi,\varepsilon) + O(\varepsilon^3) \qquad (18)$$

(18c) $$z''_a(\psi) = z_o(\psi) + \varepsilon^2 z_1(\psi,\varepsilon) + O(\varepsilon^3)$$

where

(25a) $$s_o(\psi) = \frac{1 - e \cos \psi}{a(1 - e^2)} \qquad (19)$$

(25b) $$t_o(\psi) = a^{3/2} \left[\frac{e(1 - e^2)^{1/2} \sin \psi}{1 - e \cos \psi} + \cos^{-1} \frac{\cos \psi - e}{1 - e \cos \psi}\right] \qquad (20)$$

$$z_o(\psi) = 0 \qquad (21)$$

(38) $$s_1/a^2 = \frac{2}{3}(1 + 5e) - \frac{1}{3}[5 + (95e/8)]\cos \psi + \cos 2p_1\psi$$

$$+ (5/8)e \cos (2p_1 + 1)\psi - 15Qe \sin p_1\psi$$

$$\times \sin (p_1 - 1)\psi + O(e^2)$$

(39) $$t_1/a^{9/2} = [(10/3) + (83e/12)]\sin \psi - (11/8)\sin 2p_1\psi$$

$$+ (3e/4)\sin (2p_1 - 1)\psi - 2e \sin (2p_1 + 1)\psi$$

$$+ (5e/2)\sin 2\psi - 30Qe \cos p_1\psi$$

$$\times \sin (p_1 - 1)\psi + O(e^2)$$

(42) $$z_1 = (3ia^{4/64}) [\varepsilon a^{3/2})^{-1} \cos \frac{1}{2}(p_3 + 1)\psi \sin \frac{1}{2}(p_3 - 1)\psi$$

$$+ \sin \frac{1}{2}(p_3 + 1)\psi \cos \frac{1}{2}(p_3 - 1)\psi]$$

$$Q = -\left[\frac{1}{4\varepsilon a^{3/2}} + \frac{67}{80} + O(\varepsilon)\right]$$

Equation (19) is used in Section 4.17.3.

The geometric interpretation of the variables in equation (18) is as follows:

- s = reciprocal radius vector from the smaller body to the satellite, in the blown-up scale.
- t = time, in the blown-up scale.
- z''_a = distance of the satellite from a plane that passes through the smaller body and is oriented with an inclination i and a node Ω against the x', y' plane. The node is varying with time according to:

(12)
$$\Omega = \Omega_o + \varepsilon^2 \lambda_1 t \qquad (39)$$

In order to restrict Ω to values smaller than 2π, equation (39) is written as:

$$\Omega = \Omega_o + \varepsilon^2 \lambda_1 T_\Omega \qquad (39a)$$

used in Section 4.17.5, where T_Ω is reduced in Section 4.17.1 by:

$$P_\Omega = \frac{2\pi}{\varepsilon^2 \lambda} \qquad (39b)$$

whenever it exceeds P_Ω. Ω_o is zero by the special choice of initial conditions in reference 1.

The constants a, e, and i are the semi-major axis, eccentricity, and inclination of a Keplerian ellipse represented by the leading terms s_o, t_o, and z_o. The angular variable, ψ, represents the true anomaly, but counted from the apocenter. All the terms of order $\varepsilon^2 e$ were dropped from s_1, t_1, and z_1 in this program for consistency since ε, e, and i were all considered small variables in reference 1 and the paper omitted ε^3 and εei terms. Therefore, the equations for the above quantities become:

$$s_1(\psi,\varepsilon) = a^2[\frac{2}{3} - \frac{5}{3}\cos\psi + \cos 2p_1\psi$$

$$+ \frac{15}{8}\ \frac{e}{\varepsilon a^{3/2}}(\cos\psi - \cos(2p_1 - 1)\psi)] \tag{22}$$

$$t_1(\psi,\varepsilon) = a^{9/2}\ [\frac{10}{3}\sin\psi - \frac{11}{8}\sin 2p_1\psi$$

$$- \frac{15}{4}\ \frac{e}{\varepsilon a^{3/2}}(\sin\psi - \sin(2p_1 - 1)\psi)] \tag{23}$$

$$z_1(\psi,\varepsilon) = \frac{3}{8}\, i\ \frac{a^{5/2}}{\varepsilon(1 - \varepsilon a^{3/2})}[\sin p_3\psi - p_3\sin\psi] \qquad (24)^*$$

Equation (22) is used in Section 4.17.3; equation (24) in Section 4.17.4. The eccentric anomaly:

$$E = \cos^{-1}(\frac{\cos\psi - e}{1 - e\cos\psi}) = \sin^{-1}(\frac{\sqrt{1 - e^2}\sin\psi}{1 - e\cos\psi}) \tag{25}$$

is defined such that

$$2n\pi \leqq E \leqq 2(n + 1)\pi$$

for

$$2n\pi \leqq \psi \leqq 2(n + 1)\pi$$

The time t and the eccentric anomaly are therefore single valued and increasing function of ψ . However, in order to avoid large arguments of

*Equation (24) was taken from equation (57) of reference 7 instead of being taken from equation (42) of reference 1.

the trigonometric functions for large times, a time, t_ψ, which is the time since the last passage of the apocenter, is introduced. If the total ψ and the total E are denoted by ψ_{tot} and E_{tot}, and the symbols ψ and E are used for the principal values only, such that:

$$\psi_{tot} = 2\pi n + \psi$$

$$E_{tot} = 2\pi n + E$$

Then the equation for the time is written [c.f. equations (20), (23), and (25)]:

$$t(\psi_{tot}) = a^{3/2}[E_{tot} + e \sin E_{tot}] - \frac{15}{4} \varepsilon a^3 e[\sin \psi_{tot} - \sin (2p_1 - 1)\psi_{tot}]$$

$$+ \varepsilon^2 a^{9/2}[\frac{10}{3} \sin \psi_{tot} - \frac{11}{8} \sin 2p_1\psi_{tot}] \qquad (26)$$

The time after n revolutions is:

$$t(2n\pi) = a^{3/2} \cdot 2n\pi + \frac{15}{4} \varepsilon a^3 e \sin [(2p_1 - 1) \cdot 2n\pi]$$

$$- \varepsilon^2 a^{9/2} \cdot \frac{11}{8} \sin (2p_1 \cdot 2n\pi) \qquad (27)$$

Then, by subtraction of equation (27) from equation (26) and rearranging:

$$t (\psi_{tot}) - t(2n\pi) - t_\psi = a^{3/2}[E_{tot} - 2\pi n + e \sin E_{tot}]$$

$$- \frac{15}{4} \varepsilon a^3 e\{\sin \psi_{tot} - \sin[(2p_1-1)(\psi_{tot}-2\pi n)$$

$$+ 2\pi n(2p_1-1)] + \sin [(2p_1-1) \cdot 2n\pi]\}$$

$$+ \varepsilon^2 a^{9/2}\{\frac{10}{3} \sin \psi_{tot} - \frac{11}{8} \sin[2p_1 (\psi_{tot} - 2\pi n)]$$

$$+ 4p_1\pi n] + \frac{11}{8} \sin [4p_1 n\pi]\}$$

or

$$t_\psi = a^{3/2}[E + e \sin E] - \frac{15}{4} \varepsilon a^3 e\{\sin \psi - \sin (2p_1 - 1)\psi \cos \theta_1$$

$$+ [1 - \cos (2p_1 - 1)\psi] \sin \theta_1 + \varepsilon^2 a^{9/2}\{\frac{10}{3} \sin \psi$$

$$- \frac{11}{8} \sin 2p_1\psi \cos \alpha_1 + \frac{11}{8} (1 - \cos 2p_1\psi) \sin \alpha_1\} \qquad (28)$$

where

$$\theta_1 = [2n\pi(2p_1 - 1)] \text{ modulo } 2\pi$$

and

$$\alpha_1 = [2n\pi p_1] \text{ modulo } 2\pi$$

When n becomes large, forming the angle and then computing the modulo with respect to 2π does not improve the accuracy. It is necessary to form the θ_1 and α_1 without developing large angles. For this purpose, initially θ_1 and α_1 are set to zero. The period of the first revolution T_{1_o} is computed by substituting $\psi = 2\pi$ in equation (26) yielding:

$$t(2\pi) = T_1 = a^{3/2}\, 2\pi + \frac{15}{4} \varepsilon a^3 e \sin [(2p_1 - 1)2\pi] - \varepsilon^2 a^{9/2} \frac{11}{8} \sin 4p_1\pi \qquad (28a)$$

When t_ψ exceeds T_1, we set:

$$t_\psi = t_\psi - T_1 \qquad (28b)$$

$$\theta_1 = [\theta_1 + 2\pi(2p_1 - 1] \text{ modulo } 2\pi \qquad (28c)$$

$$\alpha_1 = \{[(\alpha_1 + 2\pi p_1) \text{ modulo } 2\pi] + 2\pi p_1\} \text{ modulo } 2\pi \qquad (28d)$$

The period for the next revolution is computed by applying equation (26) to obtain:

$$T_1 = t\big(2(n + 1)\pi\big) - t(2n\pi)$$

or

$$T_1 = 2\pi a^{3/2} + \frac{15}{4}\,\varepsilon a^3 e\{\sin\theta_2 - \sin\theta_1\} - \frac{11}{8}\,\varepsilon^2 a^{9/2}\,\{\sin\alpha_2 - \sin\alpha_1\} \qquad (28e)$$

where θ_2 is computed from the θ_1 just obtained by equation (28c) by:

$$\theta_2 = [\theta_1 + 2\pi(2p_1 - 1)] \text{ modulo } 2\pi \qquad (28f)$$

and α_2 from the α_1 of equation (28d) by:

$$\alpha_2 = \{[(\alpha_1 + 2\pi p_1) \text{ modulo } 2\pi] + 2\pi p_1\} \text{ modulo } 2\pi \qquad (28g)$$

Similarly the argument $p_3\psi$ in z''_a [equation (24)] is taken as $p_3\psi + \beta$. β is set equal to zero initially and incremented each time $t_\psi > T_1$ by:

$$\beta = (\beta + 2p_3\pi) \text{ modulo } 2\pi \qquad (29)$$

Equations 28b through 29 are used in Section 4.17.1.2.

Now expression (28) may be solved for ψ when n and t_ψ are known. The resulting value is between 0 and 2π. It is solved by Newton's method using $M = \psi$ as a first guess in Section 4.17.2.4 with E, defined by equation (20), computed in Section 4.17.2.3.

The constants p_1 and p_3 are given by equations (27a) and (27c) of reference 1. However, these quantities can be obtained with more accuracy from (26a) and (26c) of reference 1,

by requiring:

$$\varepsilon^2 = \sin 2 (\bar{\phi} - qt) = \varepsilon^2 \sin 2 (p_1 \psi) + 0 (\varepsilon^3)$$

$$\varepsilon^2 \cos 2 (\bar{\phi} - qt) = \varepsilon^2 \cos 2 (p_1 \psi) + 0 (\varepsilon^3) \tag{29a}$$

$$\varepsilon^2 \sin(\bar{\phi} - \ell t) = \varepsilon^2 \sin (p_3 \psi) + 0 (\varepsilon^3) \tag{29b}$$

$$\bar{\phi} - qt = p_1\psi \tag{30}$$

$$\bar{\phi} - \ell t = p_3\psi \tag{31}$$

where

(16a) $$q = \varepsilon[1 + \varepsilon m_1] \tag{32}$$

(16b) $$\ell = \varepsilon[2 + \varepsilon(\nu_1 - 2\lambda_1)] \tag{33}$$

(16c) $$m_1 = \nu_1 - \lambda_1 \tag{34}$$

past (42) $$\nu_1 = - \frac{1}{12} a^{3/2}(11 + 24e) \tag{35}$$

past (41) $$\lambda_1 = - \frac{3}{4} a^{3/2} \tag{36}$$

With the relations between ψ and $\bar{\phi}$ given in equation (40) below and observing only the secular part of the relation between t and ψ in equations (20) and (23) (see also equation (71) below), which gives $t = a^{3/2}$, the following values for p_1 and p_3 are derived without expansion:

$$p_1 = \frac{1}{1 + \varepsilon^2 \omega_1} - \varepsilon a^{3/2}(1 + \varepsilon m_1) \tag{37}$$

$$p_3 = \frac{1}{1 + \varepsilon^2 \omega_1} - \varepsilon a^{3/2}[2 + \varepsilon(\nu_1 - 2\lambda_1)] \tag{38}$$

Equations (34) through (38) are used in Section 4.4. The relation between s, t, z'' and the primed coordinates is given by the following transformations of variables:

(17) $$\psi = (1 + \varepsilon^2 \omega_1)\bar{\Phi} \tag{40}$$

(14) $$\bar{\Phi} = \phi + \varepsilon^2 \nu_1 t \tag{41}$$

(13a) $$s = \frac{1}{\rho} \tag{42}$$

(13b) $$x''_a = \rho \cos \phi \tag{43}$$

(13c) $$y''_a = \rho \sin \phi \tag{44}$$

(11a–11c) $$\begin{bmatrix} x''_a \\ y''_a \\ z''_a \end{bmatrix} = \begin{bmatrix} \cos \Omega & \sin \Omega & 0 \\ -\sin \Omega \cos i & \cos \Omega \cos i & \sin i \\ \sin \Omega \sin i & -\cos \Omega \sin i & \cos i \end{bmatrix} \begin{bmatrix} x'_a \\ y'_a \\ z'_a \end{bmatrix} \tag{45}$$

past (32) $$\omega = \omega_1 a^{-3} \tag{46a}$$

(36) $$\omega = (-7 + 24e)/12 \tag{46b}$$

The initial conditions for any of the variables follow simply by putting $\psi = 0$ for $t = 0$, which means that the motion starts at apocenter, which, in turn, coincides with the node for $\Omega_0 = 0$.

Equations (40) and (41) may be combined to give:

$$\phi = \frac{\psi}{1 + \varepsilon^2 \omega_1} - \varepsilon^2 \nu_1 t_\psi \qquad (47)$$

which is used in Section 4.17.1.2.

To avoid large arguments of trigonometric functions in equations (43) and (44), these equations are written as:

$$x_a'' = \frac{1}{s} \cos (\phi + \phi_1) \qquad (48)$$

$$y_a'' = \frac{1}{s} \sin (\phi + \phi_1)$$

where ϕ_1 is the value of ϕ at n revolutions, modulo 2π:

$$\phi_1 = (\frac{2n}{1 + \varepsilon^2 \omega_1} - \varepsilon^2 \nu_1 \; t(2n\pi)) \text{ modulo } 2\pi \qquad (49)$$

Equation (48) is used in Section 4.17.4. As in the case of α_1, θ_1, and β, ϕ_1 is formed in Section 4.17.1.2 at appropriate times by adding increments to the initial value $\phi_1 = 0$ formed by evaluating equation (49) for $n = 1$. This avoids the angle ever becoming large, which would result in loss of accuracy.

Solving the linear system (45) for x', y', and z' yields:

$$\left.\begin{aligned} x'_a &= x''_a \cos \Omega - y''_a \sin \Omega \cos i + z''_a \sin \Omega \sin i \\ y'_a &= x''_a \sin \Omega + y''_a \cos \Omega \cos i - z''_a \cos \Omega \sin i \\ z'_a &= y''_a \sin i + z''_a \cos i \end{aligned}\right\} \quad (50)$$

which are used in Section 4.17.6.

Differentiation of equations (18) through (24) with respect to ψ yields velocity components:

$$\frac{ds}{d\psi} = \frac{e \sin \psi}{a(1 - e^2)} + \varepsilon^2 a^2 \left(\frac{5}{3} \sin \psi - 2 \sin 2p_1\psi\right.$$

$$- \frac{15}{8} \varepsilon a^{1/2} e[\sin \psi - \sin (2p_1 - 1)\psi] \quad (51)$$

$$\frac{dt}{d\psi} = \frac{a^{3/2}(1 - e^2)^{3/2}}{(1 - e \cos \psi)^2} - \frac{15}{4} \varepsilon e a^3[\cos \psi - \cos (2p_1 - 1)\psi \cos \theta$$

$$+ \sin (2p_1 - 1)\psi \sin \theta] + \varepsilon^2 a^{9/2}\left[\frac{10}{3} \cos \psi - \frac{11}{4} \cos 2p_1\psi \cos \alpha\right.$$

$$+ \frac{11}{4} \sin 2p_1\psi \sin \alpha] \quad (52)$$

$$\frac{dz''_a}{d\psi} = \frac{3}{8} \frac{i\varepsilon a^{5/2}}{(1 - \varepsilon a^{3/2}} [\cos p_3\psi - \cos \psi] \quad (53)$$

Terms of order ε^3, $\varepsilon^2 e$, $\varepsilon^3 i$, and $\varepsilon e i$, are neglected in equation (51) through (53). Equation (51) is used in Section 4.17.9 and equation (52) is used in Sections 4.17.7 and 4.17.2.5. Differentiation of (48) with respect to t yields

$$\dot{x}''_a = - \frac{1}{s^2} \frac{ds}{dt} \cos (\phi - \phi_1) - \frac{1}{s} \frac{d\phi}{dt} \sin (\phi - \phi_1) \quad (53a)$$

$$\dot{y}''_a = - \frac{1}{s^2} \frac{ds}{dt} \sin (\phi - \phi_1) + \frac{1}{s} \frac{d\phi}{dt} \cos (\phi - \phi_1)$$

or

$$\dot{x}''_a = - y''_a \frac{d\Phi}{dt} - \frac{x''_a}{s} \quad \frac{ds}{dt} \tag{54a}$$

$$\dot{y}''_a = + x''_a \frac{d\phi}{dt} - \frac{y''_a}{s} \quad \frac{ds}{dt} \tag{54b}$$

and from equation (53):

$$\dot{z}''_a = \frac{3}{8} \frac{i\varepsilon a^{5/2}}{(1-\varepsilon a^{3/2})} [\cos p_3\psi - \cos \psi]\frac{d\psi}{dt} \tag{54c}$$

where, from equation (47):

$$\frac{d\phi}{dt} = \frac{1}{1 + \varepsilon^2\omega_1} \frac{d\psi}{dt} - \varepsilon^2\nu_1 \tag{55}$$

and

$$\frac{d\psi}{dt} = \frac{1}{\frac{dt}{d\psi}} , \frac{ds}{dt} = \frac{ds}{d\psi} \cdot \frac{d\psi}{dt} \tag{56}$$

Equations (54a), (54b) are used in Section 4.17.10. Equations (55) and (56) are used in Section 4.17.9.

Differentiation of (50) with respect to time yields the velocity components **in the inertial frame:**

$$\left.\begin{aligned}
\dot{x}'_a &= (-x''_a \sin \Omega - y''_a \cos \Omega \cos i + z''_a \cos \Omega \sin i)\dot{\Omega} \\
&\quad + \dot{x}''_a \cos \Omega - \dot{y}''_a \sin \Omega \cos i + \dot{z}''_a \sin \Omega \sin i \\
\dot{y}'_a &= (x''_a \cos \Omega - y''_a \sin \Omega \cos i + z''_a \sin \Omega \sin i)\dot{\Omega} \\
&\quad + \dot{x}''_a \sin \Omega + \dot{y}''_a \cos \Omega \cos i - \dot{z}''_a \cos \Omega \sin i \\
\dot{z}'_a &= \dot{y}''_a \sin i + \dot{z}''_a \cos i
\end{aligned}\right\} \tag{57}$$

which are computed in Section 4.17.12.

3.3.1.1 Initial Conditions for the Analytic Solution

The initial conditions are obtained by setting $\psi = t = \Omega = 0$. From equations (19) through (24):

$$s_o(0) = \frac{1}{a(1 + e)} \qquad s(0) = 0$$

$$t_o(0) = 0 \qquad t_1(0) = 0 \tag{58}$$

$$z_o(0) = 0 \qquad z_1(0) = 0$$

From equations (51) through (53):

$$\frac{ds(0)}{d\psi} = 0 \tag{59a}$$

$$\frac{dt(0)}{d\psi} = \frac{a^{3/2}(1 + e)^{3/2}}{(1 - e^2)^{1/2}} + \frac{7}{12} \varepsilon^2 a^{9/2} \tag{59b}$$

$$\frac{dz_a''(0)}{d\psi} = 0 \tag{59c}$$

Equation (59b) is used in Sections 4.8 and 4.19.2.

From equations (47), (48), and (98):

$$\phi(0) = 0 \tag{60}$$

$$x_a''(0) = \frac{1}{s(0)} = a(1 + e) \tag{61}$$

$$y_a''(0) = 0 \tag{62}$$

$$z_a''(0) = 0 \tag{62a}$$

By (58) equations (61) and (62) are used in Section 4.8.3.

From equations (50) and (61):

$$\left.\begin{aligned} x_a'(0) = x_a''(0) &= a(1 + e) \\ y_a'(0) &= 0 \\ z_a'(0) &= 0 \end{aligned}\right\} \tag{63}$$

which are used in Section 4.8.3.

From (54a), (54b), (54c), (59a), (59c), and (62):

$$\left.\begin{aligned} \dot{x}_a''(0) &= 0 \\ \dot{y}_a''(0) &= x_a''(0) \ . \ \frac{d\phi(0)}{dt} \\ \dot{z}_a''(0) &= 0 \end{aligned}\right\} \tag{64}$$

where by equation (55):

$$\frac{d\phi(0)}{dt} = [(1 + \varepsilon^2\omega_1) \frac{dt(0)}{d\psi}]^{-1} - \varepsilon^2\nu_1 \tag{65}$$

Equations (64) are computed in Sections 4.8.4 and equation (65) is used in Section 4.8.2 and 4.19.2.

And finally from (57), (61), (63), (62a), and (62):

$$\left.\begin{aligned} \dot{x}_a'(0) &= 0 \\ \dot{y}_a'(0) &= x_a''(0) \cdot \dot{\Omega} + \dot{y}_a''(0) \cos i \\ \dot{z}_a'(0) &= \dot{y}_a''(0) \sin i \end{aligned}\right\} \qquad (66)$$

which are computed in Sections 4.8.4 and 4.19.2.

3.3.1.2 Solution for Perturbed Kepler's Equation

Since the original equations of motion (5) are written with t being the independent variable, equation (28) has to be inserted to give ψ as a function of t. To accomplish this, the following iteration scheme is used to solve the perturbed Kepler's equation for E.

First guess of E for a given t_ψ:

$$E_1 = a^{-3/2} t_\psi - e \sin (t_{\psi/a32}) (1 - e \cos t_{\psi/a32}) \qquad (67)$$

i.e., true anomaly = mean anomaly

Then use the formula

$$E_i = E_{i-1} + \frac{t_\psi - t_\psi(E_{i-1})}{\frac{dt(E_{i-1})}{dE}} \qquad i = 2, 3, \ldots \qquad (68)$$

to obtain a better value for E, and so on. On the right side of equation (68), the solution (28) with the argument E_{i-1} is used. From equation (25) ψ is obtained as a function of E as:

$$\sin \psi = \frac{\sqrt{1 - e^2} \sin E}{1 + e \cos E} \qquad (68aa)$$

$$\cos \psi = \frac{\cos E + e}{1 + e \cos E} \qquad (68ab)$$

In the limit, the second term of equation (68) is to go to zero. However, t_ψ as defined by equation (28) is a very complex function. Round off may limit the smallness of this quantity. Taking the derivative of equation (28) in the form:

$$\frac{dt}{dE} = a^{3/2} (1 + e \cos E) + \varepsilon^2 \frac{dt_1}{d\psi} \frac{d\psi}{dE} \tag{68ac}$$

where

$$\varepsilon^2 \frac{dt_1}{d\psi} = - \frac{15}{4} \varepsilon a^3 e \{\cos \psi - \cos [(2p - 1)(\psi) + \theta_1]\}$$

$$+ \varepsilon^2 a^{9/2} \{\frac{10}{3} \cos \psi - \frac{11}{4} \cos (2p_1\psi + \alpha_1)\} \tag{68ad}$$

and with equation (68ab):

$$\varepsilon^2 \frac{dt_1}{dE} = \varepsilon^2 \frac{dt_1}{d\psi} \frac{d\psi}{dE} = \frac{\sqrt{1 - e^2}}{1 + e \cos E} \varepsilon^2 \frac{dt_1}{d\psi} \tag{68ae}$$

and substituting in equation (68) gives after cancellation and rearrangement:

$$E_i = \frac{a^{3/2} e(E_{i-1} \cos E_{i-1} - \sin E_{i-1}) + E_{i-1}(\varepsilon^2 \frac{dt_1}{dE})_{i-1} + t_\psi - t_{1(i-1)}}{a^{3/2}(1 + e \cos E) + \varepsilon^2 \frac{dt_1}{dE}} \tag{68af}$$

where

$$t_1 = - \frac{15}{4} \varepsilon a^3 e \{\sin \psi - \sin [(2p_1 - 1)\psi + \theta_1] + \sin \theta_1\}$$

$$+ \varepsilon^2 a^{9/2} \{\frac{10}{3} \sin \psi - \frac{11}{8} [\sin (2p_1\psi + \alpha_1) - \sin \alpha_1]\} \tag{68ag}$$

Equations (68aa) and (68ab) are computed in Section 4.17.2.2; equations (68ad), (68ae), (68af), and (68ag), in Section 4.17.2.4.

When ψ is determined by arc tangent function, it is placed in the correct one of the first four quadrants by comparing the signs of the sine and cosine. However, when E is near 0 or 2π, an intermediate step of the iteration may result in a value of E in the fifth or minus first quadrant. To assure that ψ and E be in the same quadrant when cos $\psi > 0$ the equation is then:

$$\psi = \text{sign } E[\psi + \pi(1 - 1 \text{ sign } \psi + 2 \text{ integer part of } \frac{E}{2\pi})] \quad (68ah)$$

3.3.1.3 Jacobi Integral

The test quantity, ΔC_a, for the analytic solution is simply obtained by subscripting the coordinates in equation (16) with the subscript a as shown in Section 4.17.15.

The initial value of the Jacobi constant is given by [see equations (13b) and (14)]:

$$C_{init} = C_{part} - \frac{2(1 - \varepsilon^4)}{\varepsilon^2 r_o} \quad (68a)$$

Substitution of the initial conditions (63) and (66) in equation (14) yields:

$$C_{part} = x_a''(0)^2\dot{\Omega}^2 + 2x_a''(0)\dot{y}_a''(0)\dot{\Omega}\cos i + \dot{y}_a'(0)^2\cos^2 i + \dot{y}_a''(0)^2\sin^2 i$$

$$- 2\,x_a''(0)[x_a''(0)\dot{\Omega} + \dot{y}_a''(0)\cos i] - 2x_a''(0)(1 - \varepsilon^4) - \frac{2}{x_a''(0)}$$

or

$$C_{part} = x_a'(0)^2 \{\dot{\Omega}^2 + 2\dot{\Omega} \frac{d\phi(0)}{dt} \cos i + (\frac{d\phi(0)}{dt})^2$$

$$- 2\varepsilon[\dot{\Omega} + \frac{d\phi(0)}{dt} \cos i]\} - 2(1 - \varepsilon^4)x_a'(0) - \frac{2}{x_a'(0)} \qquad (68b)$$

C_{init} is computed in Section 4.8.6. The fractional error in the Jacobi constant is then given by $\frac{\Delta C_a}{C_{init}}$

3.3.2 Numerical Residual Perturbation Method

The numerical residual perturbation method to be described below uses as a reference orbit the two-body ellipse as given by s_o, t_o, but accounts for the secular perturbations, which are reflected by the rotation of this ellipse with respect to the x', y', z' frame. This rotation is composed of the recession of the node as defined by equation (39) and the precession of the pericenter, which is comprised in equations (30) and (31). However, equations (30) and (31) contain some oscillatory terms due to the elliptic motion of the satellite. In order to separate the secular part, the argument of pericenter, ω, is introduced as follows:

$$\phi = \psi + \omega \qquad (69)$$

Substituting equations (52) and (41) in equation (40), one obtains:

$$\psi = \psi + \omega + \varepsilon^2\omega_1\psi + \varepsilon^2\omega_1\omega + \varepsilon^2\nu_1 t + \varepsilon^4\omega_1\nu_1 t \qquad (70)$$

Now from equation (26) or equation (27), it can be seen that the secular part of the relation between t and ψ is given by:

$$\psi = a^{-3/2}t \qquad (71)$$

If equations (71), (46), and (35) are substituted in equation (70), one obtains for ω:

$$\omega = \varepsilon^2 \frac{\frac{3}{2} a^{3/2} - \nu_1 \varepsilon^2 \omega_1}{1 + \varepsilon^2 \omega_1} t_\omega = \dot{\omega}\, t_\omega \tag{72}$$

To keep ω smaller than 2π, t is replaced by:

$$t_\omega = (t \text{ modulo } 2\pi)/\dot{\omega} \tag{72a}$$

The quantity $\dot{\omega}$ is computed in Section 4.4, t_ω is incremented in the RKSIMP subroutine (Section 7), and the modulo is formed in Section 4.17.1. The rotating ellipse is then described by:

$$\begin{aligned} \xi_s &= \rho_s \cos v \\ \eta_s &= \rho_s \sin v \\ \zeta_s &= 0 \end{aligned} \tag{73}$$

where the origin of ξ, η, ζ is the smaller body, ξ points towards apocenter, η points 90° ahead of ξ in the direction of motion, ζ is normal to the orbit plane, and

$$\begin{bmatrix} \xi_s \\ \eta_s \\ \zeta_s \end{bmatrix} = \{M\} \begin{bmatrix} x'_s \\ y'_s \\ z'_s \end{bmatrix} \tag{74}$$

The subscript s indicates approximate coordinates as derived from the reference orbit; ρ_s and v are radial distance and true anomaly, the latter measured from apocenter.

$\{M\}$ is a matrix composed of three rotation matrices: A rotation around the z' axis by the angle Ω, described by (see page 41 ff. reference 3):

$$\{\Omega\} = \begin{bmatrix} \cos\Omega & \sin\Omega & 0 \\ -\sin\Omega & \cos\Omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{75}$$

A rotation around the line of nodes by the angle i, described by:

$$\{i\} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos i & \sin i \\ 0 & -\sin i & \cos i \end{bmatrix} \tag{76}$$

A rotation around the polar axis of the orbital plane by the angle ω, described by:

$$\{\omega\} = \begin{bmatrix} \cos\omega & \sin\omega & 0 \\ -\sin\omega & \cos\omega & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{77}$$

Then

$$\{M\} = \{\omega\}\cdot\{i\}\{\Omega\} \tag{78}$$

$$\{M\} = \begin{bmatrix} \cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i & \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i & \sin\omega\sin i \\ -\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i & -\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i & \cos\omega\sin i \\ +\sin\Omega\sin i & -\cos\Omega\sin i & \cos i \end{bmatrix} \tag{79}$$

Now, using the vector notation:

$$\vec{\xi}_s = \begin{bmatrix} \xi_s \\ \eta_s \\ \zeta_s \end{bmatrix} \qquad \vec{x}_s' = \begin{bmatrix} x_s' \\ y_s' \\ z_s' \end{bmatrix} \tag{80}$$

in inertial coordinates the two-body solution may be written as:

$$\vec{x}_s' = \{M\}^T \xi_s \tag{81}$$

or, using equation (73) and the transposed matrix of matrix (79)

$$\left.\begin{aligned} x_s' &= \rho_s[\cos(v+\omega)\cos\Omega - \sin(v+\omega)\sin\Omega\cos i] \\ y_s' &= \rho_s[\cos(v+\omega)\sin\Omega + \sin(v+\omega)\cos\Omega\cos i] \\ z_s' &= \rho_s\sin(v+\omega)\sin i \end{aligned}\right\} \tag{82}$$

Equation (82) is computed in Section 4.15.6.

The velocity components are obtained by differentiation of equation (81) with respect to time:

$$\dot{\vec{x}}_s' = \frac{d}{dt}\{M\}^T \cdot \vec{\xi}_s + \{M\}^T \dot{\xi}_s \tag{83}$$

Differentiating again yields the accelerations:

$$\ddot{\vec{x}}_s' = \{M\}^T \ddot{\vec{\xi}}_s + 2\,\frac{d\{M\}^T}{dt}\,\dot{\vec{\xi}}_s + \frac{d^2\{M\}^T}{dt^2}\,\vec{\xi}_s \tag{84}$$

Evaluating the matric derivatives yields:

$$\frac{d\{M\}^T}{dt} = -\dot{\Omega}\begin{bmatrix} \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i & -\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i & -\cos\Omega\sin i \\ -\cos\omega\cos\Omega + \sin\omega\sin\Omega\cos i & \sin\omega\cos\Omega + \cos\omega\sin\Omega\cos i & -\sin\Omega\sin i \\ 0 & 0 & 0 \end{bmatrix}$$

$$-\dot{\omega}\begin{bmatrix} \sin\omega\cos\Omega + \cos\omega\sin\Omega\cos i & \cos\omega\cos\Omega - \sin\omega\cos\Omega\cos i & 0 \\ \sin\omega\sin\Omega - \cos\omega\cos\Omega\cos i & \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i & 0 \\ -\cos\omega\sin i & \sin\omega\sin i & 0 \end{bmatrix} \quad (85)$$

$$\frac{d^2\{M\}^T}{dt^2} = -\dot{\Omega}^2\begin{bmatrix} \cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i & -\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i & +\sin\Omega\sin i \\ \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i & -\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i & -\cos\Omega\sin i \\ 0 & 0 & 0 \end{bmatrix}$$

$$+ 2\dot{\Omega}\dot{\omega}\begin{bmatrix} \sin\omega\sin\Omega - \cos\omega\cos\Omega\cos i & \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i & 0 \\ -\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i & -\cos\omega\cos\Omega + \sin\omega\sin\Omega\cos i & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$-\dot{\omega}^2\begin{bmatrix} \cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i & -\sin\omega\cos\Omega - \cos\omega\sin\Omega\cos i & 0 \\ \cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i & -\sin\omega\sin\Omega + \cos\omega\cos\Omega\cos i & 0 \\ \sin\omega\sin i & \cos\omega\sin i & 0 \end{bmatrix} \quad (86)$$

The rates of change of node and pericenter, $\dot{\Omega}$ and $\dot{\omega}$, are obtained by differentiating equations (39) and (72) with respect to t:

$$\dot{\Omega} = -\varepsilon^2 \lambda_1 \qquad (87)$$

$$\dot{\omega} = \varepsilon^2 \frac{\frac{3}{2} a^{3/2} - \varepsilon^2 \nu_1 \omega_1}{1 + \varepsilon^2 \omega_1} \qquad (88)$$

Equations (87) and (88) are computed in Section 4.4.

The vector $\dot{\vec{\xi}}_s$ in Keplerian motion is given by:

$$\dot{\xi}_s = -\frac{\sin v}{a^{1/2}(1 - e^2)^{1/2}}$$

$$\dot{\eta}_s = \frac{\cos v - e}{a^{1/2}(1 - e^2)^{1/2}} \qquad (89)$$

$$\dot{\zeta}_s = 0$$

Equations (89) can be derived from equation (73) by differentiation with respect to time, making use of the area-integral $\rho_s^2 \dot{v} = a^{1/2}(1 - e^2)^{1/2}$ and the elliptic formula

$$\rho_s = \frac{a(1 - e^2)}{1 - e \cos v} \qquad (90)$$

Substituting equations (73), (89), (79), and (85) in equation (83) and rearranging yields the following expression for the approximate velocity vector:

$$\begin{aligned} \dot{x}'_s = \frac{-1}{\sqrt{a(1 - e^2)}} & \{+ \sin (v + \omega) \cos \Omega + \cos (v + \omega) \sin \Omega \cos i \\ & - e(\sin \omega \cos \Omega + \cos \omega \sin \Omega \cos i)\} \\ & - \rho_s\{\cos (v + \omega) \sin \Omega \, [\dot{\Omega} + \dot{\omega} \cos i] \\ & + \sin (v + \omega) \cos \Omega \, [\dot{\omega} + \dot{\Omega} \cos i]\} \end{aligned} \qquad (91)$$

$$\dot{y}'_s = \frac{1}{\sqrt{a(1 - e^2)}} \{- \sin \Omega \sin (v + \omega) + \cos \Omega \cos (v + \omega) \cos i$$

$$+ e[\sin \omega \sin \Omega - \cos \omega \cos \Omega \cos i]\}$$

$$+ \rho_s\{\cos (v + \omega) \cos \Omega (\dot{\Omega} + \dot{\omega} \cos i)$$

$$- \sin (v + \omega) \sin \Omega (\dot{\omega} + \dot{\Omega} \cos i)\} \qquad (92)$$

$$\dot{z}'_s = \frac{\sin i}{\sqrt{a(1 - e^2)}} [\cos (v + \omega) - e \cos \omega] + \rho_s \dot{\omega} \sin i \cos (v + \omega) \qquad (93)$$

Equations (91) through (93) are used in Section 4.17.13.

The residual perturbation equations of motion are obtained by substitution of the two-body equations of motion, i.e.:

$$\ddot{\vec{\xi}}_s = - \frac{\vec{\xi}_s}{\rho_s^3} \qquad (94)$$

in equation (84), subtitution of equation (74) in the result, and then subtraction of this from the exact restricted three-body equations (5), yielding:

$$\ddot{\vec{x}}' - \ddot{\vec{x}}'_s = \Delta\ddot{\vec{x}}' = - \frac{\vec{x}'}{\rho^3} + \frac{\vec{x}'_s}{\rho_s^3} - \frac{(1 - \varepsilon^4)\varepsilon^2 x'}{r^3} + (1 - \varepsilon^4)(1 - \frac{1}{r^3}) \begin{bmatrix} \cos \varepsilon t \\ \sin \varepsilon t \\ 0 \end{bmatrix}$$

$$- 2 \frac{d\{M\}^T}{dt} \dot{\vec{\xi}}_s - \frac{d^2\{M\}^T}{dt^2} \vec{\xi}_s \qquad (95)$$

The vector $\Delta\vec{x}'$ is the correction vector which is to be added to the solution of the reference orbit in order to obtain the correct coordinates, i.e.,

$$\vec{x}' = \vec{x}'_s + \Delta\vec{x}' \quad (96)$$

(This addition is accomplished in Section 4.16.2.)

Equations (95) are to be integrated numerically. The first two terms on the right side of equation (95) represent a small difference of two large numbers. In order to avoid the loss of significant figures, the Encke transformation as described in detail in reference 3, starting at the bottom of page 176, will be applied.

Substitution of equations (85), (86), (89), and (73) in equation (95) and rearranging yields:

$$\begin{aligned}
\Delta\ddot{x}' = {} & \frac{1}{\rho_s^3}\left[fq(x'_s + \Delta x') - x'\right] - \frac{(1-\varepsilon^4)\varepsilon^2 x'}{r^3} + (1-\varepsilon^4)\left(1-\frac{1}{r^3}\right)\cos\varepsilon t \\
& + \frac{2\dot{\Omega}}{\sqrt{a(1-e^2)}}\{-\sin(v+\omega)\sin\Omega + \cos(v+\omega)\cos\Omega\cos i \\
& + e(\sin\omega\sin\Omega - \cos\omega\cos\Omega\cos i)\} \\
& + \frac{2\dot{\omega}}{\sqrt{a(1-e^2)}}\{+\cos(v+\omega)\cos\Omega - \sin(v+\omega)\sin\Omega\cos i \\
& - e(\cos\omega\cos\Omega - \sin\omega\sin\Omega\cos i)\} \\
& - 2\rho_s\dot{\Omega}\dot{\omega}\{\sin(v+\omega)\sin\Omega - \cos(v+\omega)\cos\Omega\cos i\} \\
& + \rho_s(\dot{\omega}^2 + \dot{\Omega}^2)\{\cos(v+\omega)\cos\Omega - \sin(v+\omega)\sin\Omega\cos i\}
\end{aligned} \quad (98)$$

$$\begin{aligned}
\Delta\ddot{y}' = {} & \frac{1}{\rho_s^3}\left[fq(y'_s + \Delta y') - y'\right] - \frac{(1-\varepsilon^4)\varepsilon^2(y'_s + \Delta y')}{r^3} + (1-\varepsilon^4)\left(1-\frac{1}{r^3}\right)\sin\varepsilon t \\
& + \frac{2\dot{\Omega}}{\sqrt{a(1-e^2)}}\{\sin(v+\omega)\cos\Omega + \cos(v+\omega)\sin\Omega\cos i
\end{aligned}$$

$$- e\,(\sin\omega\cos\Omega + \cos\omega\sin\Omega\cos i)\}$$

$$+ \frac{2\dot{\omega}}{\sqrt{a(1-e^2)}}\{\cos(v+\omega)\sin\Omega + \sin(v+\omega)\cos\Omega\cos i$$

$$- e\,(\cos\omega\sin\Omega + \sin\omega\cos\Omega\cos i)\}$$

$$- 2\rho_s\dot{\Omega}\dot{\omega}\{-\sin(v+\omega)\cos\Omega - \cos(v+\omega)\sin\Omega\cos i\}$$

$$+ \rho_s(\dot{\Omega}^2 + \dot{\omega}^2)\{\cos(v+\omega)\sin\Omega + \sin(v+\omega)\cos\Omega\cos i\} \qquad (99)$$

$$\Delta\ddot{z}' = \frac{1}{\rho_s^3}[fq(z_s' + \Delta z'] - \frac{(1-\varepsilon^4)\varepsilon^2(z_s' + \Delta z']}{r^3}$$

$$+ \dot{\omega}\sin i\{[\frac{2}{\sqrt{a(1-e^2)}} + \rho_s\dot{\omega}]\sin(v+\omega) - \frac{2e}{\sqrt{a(1-e^2)}}\sin\omega\}$$

(100)

Equations (98) through (100) are used in Section 4.16.2 where the quantities f and q are defined by:

$$q = \frac{1}{\rho_s^2}[(x_s' + \frac{1}{2}\Delta x')\Delta x' + (y_s' + \frac{1}{2}\Delta y')\Delta y' + (z_s' + \frac{1}{2}\Delta z')\Delta z'] \qquad (101)$$

$$f = \frac{1 - (1+2q)^{-3/2}}{9} \qquad (102)$$

((7,6))

$$= 3[1 - \frac{5}{2}q + \frac{5\cdot7}{2\cdot3}q^2 - \frac{5\cdot7\cdot9}{2\cdot3\cdot4}q^3 + \ldots]$$

$$= 3\{1 - \frac{5}{2}q(1 - \frac{7}{3}q(1 - \frac{9}{4}q(1 - \frac{11}{5}q(\ldots))))\}$$

The above expansion can be found in reference 4, page 98, equation ((7,6)). They are evaluated in Section 4.16.1.

Now it is still necessary to define ρ_s and v as functions of time. Introducing the eccentric anomaly E, the theory of elliptic motion (e.g., reference 2, page 164, equations (48) and (49), with $k(\sqrt{1 + m})$ being 1 in this case) gives:

$$t = a^{-3/2}[E + e \sin E] \tag{103}$$

$$\cos v = \frac{\cos E + e}{1 + e \cos E} \tag{104}$$

$$\sin v = \frac{\sqrt{1 - e^2} \sin E}{1 + e \cos E} \tag{105}$$

$$\rho_s = a(1 + e \cos E) \tag{106}$$

There are plus signs instead of minus signs in the above equations because all the anomalies are measured from apocenter. Equations (104) and (105) are evaluated in Section 4.15.2. Equations (106) is evaluated in Section 4.15.3. Kepler's equation (103) is solved for E in Section 4.15.1 by iteration as follows:

$$E_{i+1} = E_i + \frac{a^{-3/2}[t_s - t_s(E_i)]}{\frac{dt_s(E_i)}{dE}} = E_i + \frac{a^{-3/2}t_s - E_i - e_s \sin E_i}{1 - e_s \cos E_i}$$

$$= \frac{a^{-3/2}t_s - e_s(E_i \cos E_i - \sin E_i)}{1 - e_s \cos E_i} \tag{107}$$

with a first estimate:

$$E_1 = a^{-3/2}t_s - e_s \sin (a^{-3/2}t_s) + \frac{1}{2} e_s^2 \sin 2(a^{-3/2}t_s) \qquad (108)$$

Equation (108) is taken from reference 2, page 169, equation (59); M being $a^{-3/2}t_s$. The quantity t_s is the input time, modulo $2\pi a^{3/2}$ (see Section 17.1).

3.3.2.1 Initial Conditions for the Residual Perturbation Solution

The initial conditions on the residual perturbation equations of motion (98) through (100) are chosen such that for t = 0 the total values of x', y', and z' agree with the values x'_a, y'_a, and z'_a of the analytic solution.

$$\vec{x}'(0) = \vec{x}'_s(0) + \Delta\vec{x}'(0) = \vec{x}'_a(0) \qquad (109)$$

They are set equal in Section 4.8.3. The initial conditions for the reference orbit are:

from equations (103) through (106), (73), (39), (72), and (82):

$$E(0) = v(0) = 0,\ \rho_s(0) = a(1 + e) \qquad (110)$$

$$\xi_s(0) = \rho_s(0),\ \eta_s(0) = \zeta_s(0) = 0 \qquad (111)$$

$$\Omega(0) = 0 \qquad (112)$$

$$\omega(0) = 0$$

$$x'_s(0) = \rho_s(0)$$

$$y_s'(0) = z_s'(0) = 0 \tag{113}$$

The initial conditions from equations (113) are established in Section 4.8.3

From equations (91) through (93):

$$\dot{x}_s'(0) = 0 \tag{114}$$

$$\dot{y}_s'(0) = [a(1 - e^2)]^{-1/2}\cos i\ (1 - e) + \rho_s(0)(\dot{\Omega} + \dot{\omega}\cos i)$$

$$= x_s'(0)[\dot{\Omega} + (\frac{1}{\frac{dt_o(0)}{d\psi}} + \dot{\omega})\cos i] \tag{115}$$

$$\dot{z}'(0) = \{[a(1 - e^2)]^{-1/2}(1 - e) + \rho_s(0)\ \dot{\omega}\}\sin i$$

$$= x_s'(0)[\frac{1}{\frac{dt_o(0)}{d\psi}} + \dot{\omega}]\sin i \tag{116}$$

Equation (114) is computed in Section 4.8.6. Equations (115) and (116) are computed in Section 4.8.4.

Now from equation (109):

$$\Delta\vec{x}'(0) = \vec{x}_a'(0) - \vec{x}_s'(0)$$

$$\Delta\dot{\vec{x}}'(0) = \dot{\vec{x}}_a'(0) - \dot{\vec{x}}_s'(0) \tag{117}$$

Substituting equations (113) through (116), (63), and (66) in equation (117) yields:

$$\left.\begin{aligned} \Delta x'(0) &= 0 \\ \Delta y'(0) &= 0 \\ \Delta z'(0) &= 0 \\ \Delta\dot{x}'(0) &= 0 \end{aligned}\right\} \tag{118}$$

$$\Delta \dot{y}'(0) = x'\dot{G} \cos i$$

$$\Delta \dot{z}'(0) = x'\dot{G} \sin i \tag{119}$$

where

$$\dot{G} = [(1 + \varepsilon^2\omega_1) \frac{dt(0)}{d\psi}]^{-1} - \varepsilon^2 v_1 - (\frac{dt_o(0)}{d\psi})^{-1} - \dot{\omega}$$

$$= \frac{\frac{dt_o(0)}{d\psi} (\frac{dt_o(0)}{d\psi} + \varepsilon^2 \frac{dt_1(0)}{d\psi}) (1 + \varepsilon^2\omega_1)}{\frac{dt(0)}{d\psi} (1 + \varepsilon^2\omega_1) \frac{dt_o(0)}{d\psi}} - \varepsilon^2 v_1 - \dot{\omega}$$

$$= \frac{- \varepsilon^2\omega_1 \frac{dt_o(0)}{d\psi} - \varepsilon^2 \frac{dt_1(0)}{d\psi} (1 + \varepsilon^2\omega_1)}{\frac{dt(0)}{d\psi} \frac{dt_o(0)}{d\psi} (1 + \varepsilon^2\omega_1)} - \varepsilon^2 v_1 - \dot{\omega} \tag{120}$$

The Δ velocities are not zero at the initial time because the precessing ellipse has been chosen to be a mean ellipse rather than an initial osculating ellipse. This was necessary in order that the period of the ellipse did not vary in a secular manner from the period of the actual motion. Equations (117) through (120) are computed in Section 4.8.4.

Substitution of the initial conditions into equations (98) through (100) yields the initial residual accelerations:

$$\Delta \ddot{x}'(0) = \frac{1 - \varepsilon^4}{r_o^3} [2\varepsilon^2 x'(0) + 3\varepsilon^4 x'(0)^2 + \varepsilon^6 x'(0)^3]$$

$$+ \frac{2(1 - e)}{\sqrt{a(1 - e^2)}} (\dot{\Omega} \cos i + \dot{\omega})$$

$$+ 2x'(0)\dot{\Omega}\dot{\omega} \cos i + x'(0)(\dot{\omega}^2 + \dot{\Omega}^2) \tag{121}$$

$$\Delta \ddot{y}'(0) = \Delta \ddot{z}'(0) = 0 \tag{122}$$

which are computed in Section 4.8.5.

3.3.3 Fixed Reference Orbit

For comparison, a fixed, osculating Kepler ellipse is also calculated from the initial conditions. The node and the argument of apocenter are zero at $t = 0$, and the inclination has the same value as in the previous solutions. However, the osculating values for a and e are different from those used previously because the perturbations in the velocities do not vanish initially in the analytic solution.

To obtain the osculating values for a, the formula in reference 3, page 48, equation (88) is used:

$$\frac{1}{a} = \frac{2}{r} - V^2 \qquad (123)$$

or, in our notation:

$$a = \frac{1}{\frac{2}{\rho(0)} - \dot{\rho}(0)^2} \qquad (124)$$

On the same page, further below, the following equation is found:

$$e \cos u = 1 - \frac{r}{a} \qquad (125)$$

Since the eccentric anomaly, u, is 180° initially, this becomes in our notation:

$$e = \frac{\rho(0)}{a} - 1 = \frac{x'(0)}{a} - 1 \qquad (126)$$

Equations (124) and (126) are used in Section 4.19.3. The coordinates and velocity components are then obtained as in equations (82) and (91) through (93), but setting $\omega = \Omega = \dot{\omega} = \dot{\Omega} = 0$:

$$x_f' = \rho_s \cos v$$

$$y_f' = \rho_s \sin v \cos i$$

$$z_f' = \rho_s \sin v \sin i \qquad (127)$$

$$\dot{x}_f' = - [a(1 - e^2)]^{-1/2} \sin v$$

$$\dot{y}_f' = + [a(1 - e^2)]^{-1/2} (\cos v - e) \cos i$$

$$\dot{z}_f' = + [a(1 - e^2)]^{-1/2} (\cos v - e) \sin i$$

Equations (127) are computed in Section 4.28.2.

Again, the Jacobi constant is calculated according to equation (16), subscripting all the variables with f, indicating that they are computed from the fixed orbit. The radius vector and the total velocity are also compared with the corresponding values obtained by the numerical residual perturbation method:

$$\Delta\rho_f = \frac{1}{a}[(x_f' - x')^2 + (y_f' - y')^2 + (z_f' - z')^2]^{1/2}$$

$$\Delta\dot{\rho}_f = a^{1/2}[(\dot{x}_f' - \dot{x}')^2 + (\dot{y}_f' - \dot{y}')^2 + (\dot{z}_f' - \dot{z}')^2]^{1/2} \qquad (128)$$

Equations (128) are used in Section 4.28.2

The quantities $\Delta\rho_f$ and $\Delta\dot{\rho}_f$ are normalized with the semi-major axis and the circular velocity $a^{-1/2}$.

3.3.4 Numerical Integration of Total Equations

For the integration of the total equations the same initial conditions as in the analytic solution are used, but the exact equations of motion (5) are integrated numerically. The results are checked by means of the Jacobi integral equation (16). The radius vector and the total velocity are also compared with the values obtained from the numerical residual perturbation method by computing the differences, normalized with a and $a^{-1/2}$, respectively:

$$\Delta\rho_c = \frac{1}{a}[(x'_c - x')^2 + (y'_c - y')^2 + (z'_c - z')^2]^{1/2}$$
$$\Delta\dot{\rho}_c = a^{1/2}[(\dot{x}'_c - \dot{x}')^2 + (\dot{y}'_c - \dot{y}')^2 + (\dot{z}'_c - \dot{z}')^2]^{1/2} \qquad (129)$$

where x'_c, y'_c, z'_c, $\dot{x}'_c$, $\dot{y}'_c$, and $\dot{z}'_c$ stand for the values obtained by numerical integration of the exact equations of motion (5). Equations (129) were used in Section 4.28.2.

3.4 METHOD FOR NUMERICAL SOLUTION OF DIFFERENTIAL EQUATIONS

The Runge-Kutta method is used for the numerical solution of the differential equations. The method is a simple extension of the methods for the second order and simultaneous equations given by Hildebrand (reference 5, page 237) which are:

1. Given the simultaneous first order equations:

(6.16.7)
$$\frac{dy}{dx} = F(x,y,u),$$
$$\frac{du}{dx} = G(x,y,u) \qquad (130)$$

the solution may be written as:

(6.16.8)
$$y_{n+1} = y_n + \frac{1}{6}(k_0 + 2k_1 + 2k_2 + k_3) + 0(h^5)$$
$$u_{n+1} = u_n + \frac{1}{6}(m_0 + 2m_1 + 2m_2 + m_3) + 0(h^5) \qquad (131)$$

where

(6.16.9)

$$k_0 = hF(x_n, y_n, u_n),$$

$$k_1 = hF(x_n + \frac{1}{2}h,\ y_n + \frac{1}{2}k_0,\ u_n + \frac{1}{2}m_0),$$

$$k_2 = hF(x_n + \frac{1}{2}h,\ y_n + \frac{1}{2}k_1,\ u_n + \frac{1}{2}m_1), \tag{132}$$

$$k_3 = hF(x_n + h,\ y_n + k_2,\ u_n, + m_2)$$

and

(6.16.10)

$$m_0 = hG(x_n, y_n, u_n),$$

$$m_1 = hG(x_n + \frac{1}{2}h,\ y_n + \frac{1}{2}k_0,\ u_n + \frac{1}{2}m_0),$$

$$m_2 = hG(x_n + \frac{1}{2}h,\ y_n + \frac{1}{2}k_1,\ u_n + \frac{1}{2}m_1), \tag{133}$$

$$m_3 = hG(x_n + h,\ y_n + k_2,\ u_n + m_2).$$

2. The second order equation

(6.16.11)

$$\frac{d^2y}{dx^2} = G(x,y,y'), \tag{134}$$

can be written as the two simultaneous first order differential equations

$$\frac{dy}{dx} = u$$

and

$$\frac{du}{dx} = G(x,y,u)$$

Then equation (6.16.9) gives:

$$k_0 = hy'_n, \quad k_1 = hy'_n + \frac{h}{2} m_0, \quad k_2 = hy'_n + \frac{h}{2} m_1, \quad k_3 = hy'_n + hm_2,$$

and hence equations (6.16.8) and (6.16.10) give:

(6.16.12)

$$\left.\begin{aligned} y_{n+1} &= y_n + hy'_n + \frac{h}{6}(m_0 + m_1 + m_2) + O(h^5), \\ y'_{n+1} &= y'_n + \frac{1}{6}(m_0 + 2m_1 + 2m_2 + m_3) + O(h^5), \end{aligned}\right\} \quad (135)$$

where

(6.16.13)

$$\left.\begin{aligned} m_0 &= hG(x_n, y_n, y'_n), \\ m_1 &= hG(x_n + \frac{1}{2}h,\ y_n + \frac{1}{2}hy'_n,\ y'_n + \frac{1}{2}m_0), \\ m_2 &= hG(x_n + \frac{1}{2}h,\ y_n + \frac{1}{2}hy'_n + \frac{1}{2}hm_0,\ y'_n + \frac{1}{2}m_1), \\ m_3 &= hG(x_n + h,\ y_n + hy'_n + \frac{1}{2}hm_1,\ y'_n + m_2). \end{aligned}\right\} \quad (136)$$

The extension of the above technique to three second-order differential equations:

$$\frac{d^2x_i}{dt^2} = f_i(t, x_1, x_2, x_3) \text{ where } i = 1, 2, 3 \quad (137)$$

yields

$$\frac{dx_i}{dt} = g_i(t, x_1, x_2, x_3) \quad \text{for } i = 1, 2, 3 \quad (138)$$

$$\frac{dg_i}{dt} = f_i(t, x_1, x_2, x_3) \quad (139)$$

$$m_{oi} = hf_i(t_n, x_{1n}, x_{2n}, x_{3n}) \quad (140)$$

$$m_{1i} = hf_i(t_n + \frac{h}{2}, x_{1n} + \frac{h}{2}\dot{x}_{1n}, x_{2n} + \frac{h}{2}\dot{x}_{2n}, x_{3n} + \frac{h}{2}\dot{x}_{3n}) \quad (141)$$

$$m_{2i} = hf_i(t_n + \frac{h}{2}, x_{1n} + \frac{h}{2}\dot{x}_{1n} + \frac{h}{4}m_{o1}, x_{2n} + \frac{h}{2}\dot{x}_{2n} + \frac{h}{4}m_{o2},$$

$$x_{3n} + \frac{h}{2}\dot{x}_{3n} + \frac{h}{4}m_{o3}) \quad (142)$$

$$m_{3i} = hf_i(t_n + h, x_{1n} + h\dot{x}_{1n} + \frac{h}{2}m_{11}, x_{2n} + h\dot{x}_{2n} + \frac{h}{2}m_{12},$$

$$x_{3n} + h\dot{x}_{3n} + \frac{h}{2}m_{13}) \quad (143)$$

$$x_{i(n+1)} = x_{in} + h\dot{x}_{in} + \frac{h}{6}(m_{oi} + m_{1i} + m_{2i}) + 0(h^5) \quad (144)$$

$$\dot{x}_{i(n+1)} = \dot{x}_{in} + \frac{1}{6}[m_{oi} + 2(m_{1i} + m_{2i}) + m_{3i}] + 0(h^5) \quad (145)$$

With the substitution of x, y, z for x_1, x_2, x_3, equation (140) and the parameter for equation (141) are computed in Section 7.8; equation (141) and the parameter for equation (142) are computed in Section 7.26; equation (142) and the parameters for equation (143) are computed in Section 7.27; and equation (143) through (145) are computed in Section 7.28.

After integration over two intervals of equal size, the results for the velocity components are compared with an integration over the same intervals using Simpson's rule which is also of fourth order accuracy. Simpson's rule is given on page 73 of reference 5 as:

$$\int_{x_o}^{x_2} f(x)\,dx = \frac{h}{3}(f_o + 4f_1 + f_2) - \frac{h^5 f^{IV}(\xi)}{90} \quad (146)$$

where $x_o < \xi < x_2$

or in the notation of this problem:

$$\left.\begin{aligned}\dot{x}_{n+2} &= \dot{x}_n + \frac{\Delta t}{3}[\ddot{x}_n + 4\ddot{x}_{n+1} + \ddot{x}_{n+2}]\\ \dot{y}_{n+2} &= \dot{y}_n + \frac{\Delta t}{3}[\ddot{y}_n + 4\ddot{y}_{n+1} + \ddot{y}_{n+2}]\\ \dot{z}_{n+2} &= \dot{z}_n + \frac{\Delta t}{3}[\ddot{z}_n + 4\ddot{z}_{n+1} + \ddot{z}_{n+2}]\end{aligned}\right\} \qquad (147)$$

where the subscript n means evaluated at time t_n, the subscript $n+1$ means evaluated at $t_n + \Delta t$, and the subscript $n+2$ means evaluated at $t_n + 2\Delta t$.

By virtue of the comparison between the two integrated results, decisions, are made by the program concerning the accuracy of the integration, and the computing interval for the next two intervals is chosen. The logic underlying these program decisions will now be explained using one first order differential equation as an example.

Let the differential equation to be solved be of the form:

$$\dot{x} = \dot{x}(t, x) \qquad (148)$$

If this equation is integrated over an interval, h, by Runge-Kutta methods of fourth order, then the numerical value of that function corresponds to a Taylor series expansion with an error term of $O(h^5)$, i.e.:

$$x_{n+1} = x_n + \dot{x}_n h + \ddot{x}_n \frac{h^2}{2!} + \frac{\dddot{x}h^3}{3!} + \frac{x^{IV}h^4}{4!} + O(h^5) \qquad (149)$$

The complete functional form of the coefficient of the error term is unknown but it is known to contain x^V. For the purposes of this program the coefficient of the fifth order term is assumed to be the next term in the Taylor series $\frac{x^V}{5!}$ and x^V is assumed to be a slowly varying function. The coefficient of the fifth order term in Simpson's rule is known to be $-\frac{x^V}{90}$. Thus if we let x_c

be the correct value of x at the end of the two equal intervals, and let x_{RK} and x_{SR} be the Runge-Kutta and Simpson's rule integrated values respectively, we may write:

$$x_c = x_{RK} + 2\left(\frac{x^V h^5}{5!}\right) \tag{150}$$

$$x_c = x_{SR} - \frac{x^V h^5}{90} \tag{151}$$

Eliminating x_c between these two equations and solving for x^V results in:

$$x^V = \frac{36(x_{SR} - x_{RK})}{h^5} \tag{152}$$

From equations (150) through (152) the error in the Runge-Kutta solution is estimated to be:

$$\delta x = \frac{3}{5}(x_{SR} - x_{RK}) \tag{153}$$

A factor of $\frac{3}{5}$ is dropped in the use of this equation in Section 7.14 because an arbitrary constant is introduced at this point.

Letting $\Delta\dot{x}$, $\Delta\dot{y}$, $\Delta\dot{z}$ be the changes in the $\dot{x}$, $\dot{y}$, $\dot{z}$ values over the double interval, then what is required in the program is that:

$$E = \text{maximum}\left(|\delta\dot{x}|, |\delta\dot{y}|, |\delta\dot{z}|\right) < E_{all} = \text{maximum}\left(|W_8| C_{max}\right), \tag{154}$$
$$10^{-9}\ \text{maximum}\left(|\dot{x}|, |\dot{y}|, |\dot{z}|\right)$$

where $C_{max} = \text{maximum}\left(|\Delta\dot{x}|, |\Delta\dot{y}|, |\Delta\dot{z}|\right)$ (155)

and W_8 is an input number designed to require a series truncation greater than number truncation but as small as possible. An error which is less than

10^{-9} of the maximum of the absolute values of $\dot{x}$, $\dot{y}$, and $\dot{z}$ is always acceptable, since it will be lost in the first addition anyway because of the limits of machine word length.

If $E \leq E_{all}$, the computation proceeds. If $E > E_{all}$, the last two steps are done over.

If E is greater than an input minimum error $E_{min} \cdot C_{max}$, then Δt is computed by:

$$\Delta t_{new} = FDT * \Delta t_{old} \left(\frac{E_{all}}{E}\right)^{.25} \quad (156)$$

If it assumed that $E_{all} = W_8 C_{max} = K\Delta t$, where K is some constant (since x is roughly proportional to Δt and c_{max} is normally proportional to Δx) and FDT = 1, then by equations (150), (153), and (154) Δt_{new} would result in an error of exactly E_{all}. FDT is an input number < 1 to prrvent Δt_{new} from resulting in an error $E > E_{all}$ due to number truncation or change of x^V over the two new intervals as compared to the x^V of the previous two intervals.

If $E < E_{min} \cdot C_{max}$, then it is assumed that the error in x_{RK} is primarily due to number truncation in the computations. In this case equation (150) does not apply. The new computing interval is then computed by:

$$\Delta t_{new} = \Delta t_{min} \cdot \Delta t_{old} \quad (157)$$

where Δt_{min} is an input quantity > 1. The checks against E_{all} are made in Sections 7.14 and 7.15 and checks against E_{min} are made in Section 7.22. The computation of the time interval by formula (156) is accomplished in Section 7.19. the computation of Δt by equation (156) is done in Section 7.24.

Section 4
EQUATIONS IN ORDER OF SOLUTION

4.1 PRINT EXPLANATION OF ERROR CODE

Error Code	Reason for Halt of Run
1	Semi-major axis = 0
2	Eccentricity equals or exceeds 1
7	S (Inverse of radius vector) = 0
8	Derivative of time with respect to ψ = 0

4.1.1 Set Input Arrays = 0

Set $W_I = 0$ for $I = 1, 2, \ldots, 30$

4.2 PROGRAM INPUT

Up to 30 quantities can be input to this program. Those specified thus far are given below.

4.2.1 Osculating Orbital Elements

W_1 = a, semi-major axis in units of $\mu^{1/2}$ D

W_2 = e, eccentricity

W_3 = i, inclination (degrees)

4.2.2 Planetary Mass Ratio

$W_4 = \mu \equiv \varepsilon^4$

4.2.3 Time and Accuracy Specifications

W_5 = FTD, modifier of estimated computing interval for the next pass

W_6 = Not used

W_7 = Run stop time

W_8 = Maximum allowable velocity error

W_9 = Modifier of previous computing interval if velocity error is less than the minimum allowable

W_{10} = Initial value of computing interval

W_{11} = Maximum number of failures permitted in computing interval determination

W_{12} = Minimum allowable velocity error

W_{13} = Flag to call for print

If $W_{13} \leq 0$, print will be suppressed.

If $W_{13} > 0$, print will be called for.

4.3 MOVE INPUT TO WORKING STORAGE AND CHECK INPUT

4.3.1 Move Input to Working Storage and Form Necessary Constants

$$a = |\text{input } a|$$

$$a_2 = a^2$$

$$a_s = a$$

$$e = |\text{input } e|$$

$$e_s = e$$

$$i^\circ = \text{input } i$$

$$\varepsilon_4 = \text{input } \mu$$

$$T_f = W_7$$

4.3.2 Check on Constant Divisors, Halt Run if Any Are Zero

4.3.2.1 Test Semi-Major Axis

If $a \leq 0$, go to Step 4.3.2.1.1.

If $a > 0$, go to Step 4.3.2.2.

4.3.2.1.1 Set Error Code = 1

4.3.2.1.2 Print Error Code and go to Step 4.2

4.3.2.2 Compute and Test e_2

$$e_2 = 1 - e^2$$

If $e_2 \leq 0$, go to Step 4.3.2.2.1.

If $e_2 > 0$, go to Step 4.4.

4.3.2.2.1 Set Error Code to 2 and go to Step 4.3.2.1.2

4.4 COMPUTE CONSTANTS

$$a_e = ae_2$$

$$a_e 1/2 = \sqrt{a_e}$$

$$e_2 = \sqrt{e_2}$$

$$a_3 = a^3$$

$$\varepsilon_2 = \sqrt{\varepsilon_4}$$

$$T_{num\ 1} = 3$$

$$T_{den\ 1} = 1$$

$$\left. \begin{array}{l} T_{num\ i} = T_{num\ i-1} + 2 \\ T_{den\ i} = T_{den\ i-1} + 1 \end{array} \right\} \begin{array}{l} \text{Form for} \\ i = 2, 3, \ldots, 40 \end{array}$$

$$ITP = 2$$

$$P_{t1} = 0$$

$$ap_i = 0 \quad i = 1, \ldots, 13$$

$$a_{32} = \sqrt{a^3}$$

$$a_{92} = (a^{3/2})^3$$

$$a_{S32} = a_{32}$$

$$\varepsilon = \sqrt{\varepsilon_2}$$

$$C_1 = 3.75 \varepsilon a_3 e$$

$$\nu_1 = \frac{- a_{32} (11 + 24e)}{12} \qquad (35)$$

$$\varepsilon_a = \varepsilon a_{32}$$

$$\lambda_1 = - .75 \cdot a_{32} \qquad (36)$$

$$m_1 = \frac{- a_{32} (1 + 12e)}{6} \qquad (34), (35), (36)$$

$$\omega_1 = \frac{a_3 (- 7 + 24e)}{12} \qquad (46)$$

$$p_3 = \frac{1}{1 + \varepsilon_2\omega_1} - \varepsilon_a[2 + \varepsilon(\nu_1 - 2\lambda_1)] \quad (38)$$

$$i = i^\circ/57.2957795$$

$$C_i = \cos i$$

$$S_i = \sin i$$

$$i_s = i$$

$$C_{is} = \cos i_s$$

$$S_{is} = \sin i_s$$

$$\dot{\omega} = (\tfrac{3}{2}\,\varepsilon_2 a_{32} - \varepsilon_4\nu_1\omega_1)/(1 + \varepsilon_2\omega_1) \quad (88)$$

$$\dot{\Omega} = -\,\frac{3}{4}\,\varepsilon \cdot \varepsilon_a \quad (87),\ (36)$$

$$C_{\dot{\omega}\dot{\Omega}} = \dot{\omega}\dot{\Omega}$$

$$C_{\dot{\omega}^2\dot{\Omega}^2} = \dot{\omega}^2 + \dot{\Omega}^2$$

$$a_{12} = a^{1/2}$$

$$p_1 = \frac{1}{1 + \varepsilon_2\omega_1} - \varepsilon_a\,(1 + \varepsilon_1 m_1) \quad (37)$$

$$C_2 = 1.875\,\varepsilon_1 a_{12} e$$

$$C_3 = \frac{.375\,\varepsilon_1 i\ a\ a_{32}}{1 - \varepsilon_1 a_{32}}$$

$$C_6 = \dot{\Omega}$$

$$C_7 = \dot{\Omega}^2$$

$$C_8 = 1 - \varepsilon_4$$

$$C_9 = a_e a_{12} e_2$$

$$C_{10} = \frac{\varepsilon_2 \omega_1}{1 + \varepsilon_2 \omega_1} + \varepsilon_1 a_{32} (1 + \varepsilon_1 m_1)$$

$$C_{11} = 2p_1 - 1$$

$$C_{12} = \frac{2\dot{\Omega}}{a_e^{1/2}}$$

$$C_{13} = \frac{2\dot{\omega}}{a_e^{1/2}}$$

$$C_{14} = 2C_{\dot{\omega}\dot{\Omega}}$$

Compute $- \frac{2}{a_e^{1/2}} (\dot{\Omega} + \dot{\omega} \cos i)$

$$C_{15} = -C_{12} - C_{13} C_i$$

Compute $-2\dot{\omega}\dot{\Omega} - (\dot{\omega}^2 + \dot{\Omega}^2) \cos i$

$$C_{16} = -C_{14} - C_{\dot{\omega}^2 \dot{\Omega}^2} \cdot C_i$$

Compute $\frac{2}{a_e^{1/2}} (\dot{\Omega} \cos i + \dot{\omega})$

$$C_{17} = C_{12} C_i + C_{13}$$

Compute $2\dot{\omega}\dot{\Omega} \cos i + \dot{\omega}^2 + \dot{\Omega}^2$

$$C_{18} = C_{14}\ C_i + C_{\dot{\omega}^2\dot{\Omega}^2}$$

Compute $\dot{\omega}^2 \sin i$

$$C_{19} = \dot{\omega}^2 S_i$$

Compute $\frac{2\ e \sin i}{a_e 1/2}\ \dot{\omega}$

$$C_{20} = eS_i C_{13}$$

Compute $\frac{2 \sin i}{a_e 1/2}\ \dot{\omega}$

$$C_{21} = S_i C_{13}$$

Compute $- \frac{2e}{a_e 1/2} (\dot{\Omega} + \dot{\omega} \cos i)$

$$C_{22} = eC_{15}$$

Compute $\frac{2e}{a_e 1/2} (\dot{\Omega} \cos i + \dot{\omega})$

$$C_{23} = eC_{17}$$

$$C_{24} = \sqrt{1 - e_s^2}$$

$$T_1 = 2\pi a_{32} + C_1 \sin 2\pi\ C_{11} \tag{28a}$$

$$- 1.375\ \varepsilon_2\ a_{92} \sin 4\pi p_1$$

$$P_{ss} = 2\pi a_{32}$$

$$P_{\Omega} = 2\pi/\dot{\Omega} \tag{39b}$$

$$P_{\omega} = 2\pi/\dot{\omega} \tag{72a}$$

$$\Delta E_{max} = 0$$

$$\Delta E_{pmax} = 0$$

$$\alpha_1 = 0$$

$$\theta_1 = 0$$

$$\phi_1 = 0$$

$$C_{\alpha} = 1$$

$$S_{\alpha} = 0$$

$$C_{\theta} = 1$$

$$S_{\theta} = 0$$

4.5 PRINT CONSTANTS

W_1	W_2	W_3	W_4	W_5
W_6	W_7	W_8	W_9	W_{10}
W_{11}	W_{12}	W_{13}	W_{14}	W_{15}

ω_1	ν_1	λ_1	p_1	p_3	
c_1	c_2	c_3	c_4	c_5	c_6
c_7	c_8	c_9	c_{10}	c_{11}	$\sqrt{1 - e^2}$
$\dot{\omega}$	$\dot{\Omega}$	$\dot{\omega}\dot{\Omega}$	$\dot{\Omega}^2 + \dot{\Omega}^2$		

4.6 PRINT HEADER FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION LISTING THE INPUT VALUES OF a, e, i, and μ

4.7 SET PERTURBATION-TOTAL FLAG

ICH = 1; the signal that residual perturbation equations are to be numerically solved and that the analytic solution is to be computed.

4.8 COMPUTE INITIAL VALUES OF POSITION, VELOCITY, AND ACCELERATION

$$\text{Set } e_3 = 1 + e$$

$$\frac{dt_o}{d\psi} = \frac{a_{32}(e_3)^{1.5}}{(1 - e)^{1/2}} \tag{59}$$

$$\varepsilon^2 \frac{dt_1}{d\psi} = \frac{7}{12} \varepsilon_2 a_{92}$$

$$\frac{dt}{d\psi} = \frac{dt_o}{d\psi} + \varepsilon_2 \frac{dt_1}{d\psi} \tag{59}$$

4.8.1 <u>Test Value of</u> $\frac{dt}{d\psi}$

If $\frac{dt}{d\psi} = 0$, go to Step 4.17.8.1.

If $\frac{dt}{d\psi} \neq 0$, continue below.

4.8.2 Compute $\frac{d\phi}{dt}$ and Set Initial Values of t and ψ

$$\frac{d\psi}{dt} = 1/\frac{dt}{d\psi}$$

$$\frac{d\phi}{dt} = \frac{\frac{d\psi}{dt}}{(1 + \varepsilon_2\omega_1)} - \varepsilon_2\nu_1 \qquad (65)$$

$$t = 0$$

$$\psi = 0$$

4.8.3 Position Coordinates

$$x_a'' = ae_3, \text{ where } e_3 = 1 + e \qquad (61)$$

$$y_a'' = z_a'' = 0 \qquad (62)$$

$$x_a' = x_a''$$

$$y_a' = z_a' = 0 \qquad (63)$$

$$x' = x_a'$$

$$y' = z' = 0 \qquad (109)$$

$$\Delta x' = \Delta y' = \Delta z' = 0 \qquad (118)$$

$$\rho = x'$$

$$r = 1 + \varepsilon_2 x' \qquad (t = 0) \qquad (8)$$

$$r^3 = (1 + \varepsilon_2 x')^3$$

$$x'_s = x'_a$$

$$y'_s = z'_s = 0 \tag{113}$$

4.8.4 Velocity Components

$$\dot{x}''_a = 0 \tag{64}$$

$$\dot{y}''_a = x'_a \frac{d\phi}{dt}$$

$$\dot{z}''_a = 0$$

$$\dot{y}'_a = \dot{y}''_a C_i + x''_a \dot{\Omega} \tag{66}$$

$$\dot{z}'_a = \dot{y}''_a S_i$$

$$\dot{x}' = 0$$

$$\dot{y}' = \dot{y}'_a \tag{117}$$

$$\dot{z}' = \dot{z}'_a$$

$$\Delta\dot{x} = 0$$

$$\dot{G} = \frac{-\left(\frac{dt_o}{d\psi} \varepsilon_2\omega_1 + \varepsilon_2 \frac{dt_1}{d\psi} (1 + \varepsilon_2\omega_1)\right)}{\frac{dt}{d\psi} \frac{dt_o}{d\psi} (1 + \varepsilon_2\omega_1)} - \varepsilon_2\omega_1 - \dot{\omega}$$

$$\Delta \dot{y}' = x' \dot{G} \, C_i$$

$$\Delta \dot{z}' = x' \dot{G} \, S_i \tag{119}$$

$$\dot{\rho} = (\dot{y}'^2 + \dot{z}'^2)^{1/2}$$

$$\dot{x}_s' = 0 \tag{114}$$

$$\dot{y}_s' = x' \left[\dot{\Omega} + \left(\frac{1}{\frac{dt_o}{d\psi}} + \dot{\omega}\right) C_i\right] \tag{115}$$

$$\dot{z}_s' = x' \left(\frac{1}{\frac{dt_o}{d\psi}} + \dot{\omega}\right) S_i \tag{116}$$

4.8.5 Acceleration Components

$$E_{2x} = \varepsilon_2 x$$

$$\begin{aligned} \Delta \ddot{x}' = {} & C_8 \, E_{2x} \, [2 + E_{2x} \, (3 + E_{2x})]/r^3 \\ & + 2(\dot{\omega} + \dot{\Omega} \, C_i)\sqrt{\frac{(1 - e)}{a(1 + e)}} \\ & + x_s' \, (2 \, C_{\dot{\omega}\dot{\Omega}} \, C_i + C_{\dot{\omega}^2\dot{\Omega}^2}) \end{aligned} \tag{121}$$

$$\Delta \ddot{y}' = \Delta \ddot{z}' = 0 \tag{122}$$

4.8.6 Initial Value of Jacobi Constant

$$C_{part} = x'^2 \left\{\left(\frac{d\phi}{dt}\right)^2 + \dot{\Omega}^2 - 2\left[\varepsilon\dot{\Omega} - \frac{d\phi}{dt} (\dot{\Omega} - \varepsilon) \cos i\right]\right\} - 2\left[\frac{1}{x'} + C_8 \, x'\right] \tag{68b}$$

$$C_{init} = C_{part} - \frac{2\ C_8}{\varepsilon_2 r} \tag{68a}$$

$$x'_o = x'$$

$$r_o = 1 + \varepsilon_2 x'$$

$$(\Delta c/c)_{approx} = (\Delta c/c)_{exact} = 0$$

Print Jacobi integral, C_{init}.

4.9 SET RUN START FLAGS

IP = 1

IPRINT = 2

KHALT = 1

KR = 1

4.9.1 Test Perturbation-Total

If ICH = 1, go to Step 4.10.

If ICH = 2, go to Step 4.19.4.

4.10 OUTPUT OF NUMERICAL RESIDUAL PERTURBATION SOLUTION

The format will be:

t	$\Delta x'$	$\Delta \dot{x}'$	$\Delta \ddot{x}'$	ρ
	$\Delta y'$	$\Delta \dot{y}'$	$\Delta \ddot{y}'$	$\dot{\rho}$
$\frac{\Delta C_a}{C_{init}}$	$\Delta z'$	$\Delta \dot{z}'$	$\Delta \ddot{z}'$	

	x'	$\dot{x}'$	x'_a	$\dot{x}'_a$
	y'	$\dot{y}'$	y'_a	$\dot{y}'_a$
$\frac{\Delta C_E}{C_{init}}$	z'	$\dot{z}'$	z'_a	$\dot{z}'_a$

If this is the initial point (IP = 1), then go to Step 4.12; otherwise, go to Step 4.18.1.

4.11 TEST HALT FLAG, KHALT

If KHALT = 1, continue numerical residual perturbation at Step 4.12.

If KHALT = 2, go to Section 4.14.1 to discontinue the solution.

If KHALT = 3, initiate numerical solution of total equations of motion at Step 4.19.

4.12 CALL NUMERICAL SOLUTION SUBROUTINE FOR RESIDUAL PERTURBATION EQUATIONS

The arguments for subroutine RKSIMP are:

KR	Runge-Kutta flag
IP	Initial point flag
KC	Simpson's rule flag
t	Time
Δt	Computing interval
ICH	Perturbation-total flag
$\Delta x'$, $\Delta y'$, $\Delta z'$ $\Delta\dot{x}'$, $\Delta\dot{y}'$, $\Delta\dot{z}'$ $\Delta\ddot{x}'$, $\Delta\ddot{y}'$, $\Delta\ddot{z}'$	Position, velocity, acceleration components

KHALT	Halt flag
t_p	Print time
T_f	Run stop time

4.13 TEST RUNGE-KUTTA FLAG, KR

If KR = 1, go to Step 4.14.

If KR = 2 or 4, go to Step 4.15.

If KR = 3, go to Step 4.16.

If KR = 5, go to Step 4.17.

4.14 TEST HALT FLAG, KHALT

If KHALT = 1 or 3, go to Step 4.16.

If KHALT = 2, go to Step 4.14.1.

4.14.1 Print Computing Interval Selection Failed

Set T_f = last print time.

If ICH = 1, initiate numerical solution of total equations of motion at Step 4.19.

If ICH = 2, begin plotting results (Step 4.31).

4.15 COMPUTE TWO-BODY EQUATIONS

4.15.1 Solve Kepler's Equations

4.15.1.1 Form Constant

$$t_{a32} = t_s/a_{32}$$

4.15.1.2 Compute a First Estimate for Eccentric Anomaly

$$E = t_{a32} - e_s \sin (t_{a32})[1 + e_s \cos (t_{a32})] \qquad (108)$$

$$\Delta E = 10{,}000$$

4.15.1.3 Compute New Guess at E

$$S_E = \sin E$$

$$C_E = \cos E$$

$$D_n = 1 + e_s C_E$$

$$E_n = \frac{t_{a32} + e_s (EC_E - S_E)}{D_n}$$

If $E_n = E$, go to Step 4.15.2.

If $|E_n - E| \geq \Delta E$, go to Step 4.15.1.4; otherwise set $E = E_n$, $\Delta E = |E_n - E|$ and repeat Step 4.15.1.3.

4.15.1.4 Store Maximum ΔE

$$\Delta E_{max} = \max (|\Delta E|, \Delta E_{max})$$

4.15.2 <u>Compute Sine and Cosine of the True Anomaly</u>

$$\sin v = \frac{\sqrt{1 - e_s^2} \sin E}{1 + e_s \cos E} = \frac{C_{24} S_E}{D_n} \qquad (105)$$

$$\cos v = \frac{\cos E + e_s}{1 + e_s \cos E} = \frac{C_E + e_S}{D_n} \qquad (104)$$

4.15.3 Compute $\rho_s = a_s(1 + e\ C_E)$ (106)

4.15.4 Test Perturbation-Total Flag

If ICH = 1, continue below.

If ICH = 2, go to Step 4.28.2.

4.15.5 Compute Arguments of Node and Perigee and Their Sines and Cosines

$$\Omega = \dot{\Omega} t_\Omega\ ; \qquad C_\Omega = \cos\ \Omega; \qquad S_\Omega = \sin\ \Omega$$

$$\omega = \dot{\omega} t_\omega\ ; \qquad C_\omega = \cos\ \omega; \qquad S_\omega = \sin\ \omega$$

$$C_{\phi_s} = \cos\ v\ \cos\ \omega - \sin\ v\ \sin\ \omega$$

$$S_{\phi_s} = \sin\ v\ \cos\ \omega + \cos\ v\ \sin\ \omega$$

4.15.6 Compute the Precessing Two-Body Position Coordinates in the Inertial Coordinate System

$$x'_s = \rho_s(C_{\phi_s}\ C_\Omega - S_{\phi_s}\ S_\Omega\ C_i)$$

$$y'_s = \rho_s(C_{\phi_s}\ S_\Omega + S_{\phi_s}\ C_\Omega\ C_i)$$

$$z'_s = \rho_s\ S_{\phi_s}\ S_i$$

$$C_{\epsilon t} = \cos\ \gamma$$

$$S_{\epsilon t} = \sin\ \gamma$$

4.16 ACCELERATION COMPONENTS FOR NUMERICAL RESIDUAL PERTURBATION EQUATIONS OF MOTION

4.16.1 Computation of the Small Differences Between the Two-Body Terms

$$\text{Set } q = \frac{(x_s' + \frac{\Delta x'}{2})\Delta x' + (y_s' + \frac{\Delta y'}{2})\Delta y' + (z_s' + \frac{\Delta z'}{2})\Delta z'}{\rho_s^2} \qquad (101)$$

Determine the term of lowest order in the following series which is less than 2×10^{-9}

$$-\frac{5}{2!} q + \frac{5 \cdot 7}{3!} q^2 - \frac{5 \cdot 7 \cdot 9}{4!} q^3 + \ldots$$

up to a limit of 40 terms. If the fortieth term still exceeds 2×10^{-9}, print its value, set final time $t_f = P_{t(ITP - 1)}$, and begin the solution of the total equations of motion at Step 4.19. If the absolute value of the nth term is $< 2 \times 10^{-9}$ and $n \leq 40$, then compute

$$fq = 3q \{1 - \frac{5}{2} q (1 - \frac{7}{3} q (1 - \frac{9}{4} q (1 - \frac{11}{5} q \ldots))))\} \qquad (102)$$

etc., to the nth term

4.16.2 Compute Numerical Residual Perturbation Acceleration Components

$$x' = x_s' + \Delta x'$$

$$y' = y_s' + \Delta y' \qquad (96)$$

$$z' = z_s' + \Delta z'$$

$$\rho^2 = x'^2 + y'^2 + z'^2$$

$$r^2 = 1 + \epsilon_2[2(x'C_{\epsilon t} + y'S_{\epsilon t}) + \epsilon_2\rho^2] \qquad (8)$$

$$r = \sqrt{r^2}$$

$$r^3 = r^2 r$$

$$E_{r_1} = C_8\ \epsilon_2/r^3$$

$$E_{r_2} = C_8\ \epsilon_2[2(x'\ C_{\epsilon t} + y' S_{\epsilon t}) + \epsilon_2 \rho^2][1 + r(1 + r)]/r^3(1 + r)$$

$$\rho_s^3 = (\rho_s)^3$$

Compute $B_1 = - \{\frac{2(\dot{\Omega} + \dot{\omega} \cos i)}{\sqrt{a(1 - e^2)}} + \rho_s[2\dot{\omega}\ \dot{\Omega} + (\dot{\omega}^2 + \dot{\Omega}^2) \cos i]\}$

$= C_{15} + \rho_s\ C_{16}$

Compute $B_2 = \frac{2(\dot{\Omega} \cos i + \dot{\omega})}{\sqrt{a(1 - e^2)}} + \rho_s[2\dot{\omega}\dot{\Omega} \cos i + (\dot{\omega}^2 + \dot{\Omega}^2)]$

$= C_{17} + \rho_s\ C_{18}$

Following the preliminary computations, the acceleration components may be computed as:

$$\Delta\ddot{x}' = (f_q x' - \Delta x')/\rho_s^3 - E_{r_1}\ x' + E_{r_2}\ C_{\epsilon t}$$

$$+ S_{\phi_s}\ S_\Omega\ B_1 + C_{\phi_s}\ C_\Omega\ B_2 - (S_\omega\ S_\Omega\ C_{22} + C_\omega\ C_\Omega\ C_{23}) \quad (98)$$

$$\Delta\ddot{y}' = (f_q y' - \Delta y')/\rho_s^3 - E_{r_1} y' + E_{r_2}\ S_{\epsilon t}$$

$$- S_{\phi_s}\ C_\Omega\ B_1 + C_{\phi_s}\ S_\Omega\ B_2 + (S_\omega\ C_\Omega\ C_{22} + C_\omega\ S_\Omega\ C_{23}) \quad (99)$$

$$\Delta\ddot{z}' = (f_q z' - \Delta z')/\rho_s^3 - E_{r_1} z' + S_{\phi_s} s_i(C_{13} + \rho_s \omega^2) - e\, s_i S_\omega C_{13}$$

$$= (f_q z' - \Delta z')/\rho_s^3 - E_{r_1} z' + S_{\phi_s} (C_{21} + \rho_s C_{19}) - S_\omega C_{20} \qquad (100)$$

where C_Ω, S_Ω, C_ω, S_ω, C_{ϕ_s}, and S_{ϕ_s} are defined in Section 4.15.5, $C_{\epsilon t}$ and $S_{\epsilon t}$ in Section 4.15.6, and C_{13} and C_{19} through C_{23} in Section 4.4.

4.16.3 Test Runge-Kutta Flag, KR

If KR = 1, go to Step 4.16.4.
If KR > 1, go to Step 4.12.

4.16.4 Test Print Flag, IPRINT

If IPRINT = 2, go to Step 4.12.
If IPRINT ≠ 2, go to Step 4.17.

4.17 COMPUTATIONS FOR PRINT ONLY

4.17.1 Reduce All Angles that Increase with Time by Integer Multiples of 2π

$$T_s = T_s, \text{ modulo } P_{ss} \qquad (108)$$

$$T_\Omega = T_\Omega, \text{ modulo } P_\Omega \qquad (39a, b)$$

$$T_\omega = T_\omega, \text{ modulo } P_\omega \qquad (72a)$$

$$\gamma = \gamma, \text{ modulo } 2\pi$$

4.17.1.1 Test Value of T_ψ

If $t_\psi \geq T_1$, go to Step 4.17.1.2.

If $t_\psi > T_1$, go to Step 4.17.2.1.

(28b)

4.17.1.2 Set $T_\psi = T_\psi - T_1$

Recompute T_1

$$\phi_1 = [\phi_1 + \frac{2\pi}{1 + \varepsilon_2 \omega_1} - \varepsilon_2 \nu_1 T_1] \text{ modulo } 2\pi \quad (49)$$

$$\alpha_1 = \{ [(\alpha_1 + 2\pi p_1) \text{ modulo } 2\pi] + 2\pi p_1 \} \text{ modulo } 2\pi \quad (28d)$$

$$S_{\alpha_1} = \sin \alpha_1$$

$$C_{\alpha_1} = \cos \alpha_1$$

$$\alpha_2 = \{ [(\alpha_1 + 2\pi p_1) \text{ modulo } 2\pi] + 2\pi p_1 \} \text{ modulo } 2\pi \quad (28g)$$

$$\theta_1 = (\theta_1 + 2\pi C_{11}) \text{ modulo } 2\pi \quad (28c)$$

where C_{11} is defined in Section 4.4.

$$S_{\theta_1} = \sin \theta_1$$

$$C_{\theta_1} = \cos \theta_1$$

$$\theta_2 = (\theta_1 + 2\pi C_{11}) \text{ modulo } 2\pi \quad (28f)$$

$$T_1 = 2\pi a_{32} + C_1(\sin\theta_2 - S_{\theta_1}) - 1.375\ \varepsilon_2 a_{92}\ (\sin\alpha_2 - S_{\alpha_1}) \qquad (28e)$$

where C_1 is defined in Section 4.4.

$$\beta = (\beta + 2\pi p_3) \text{ modulo } 2\pi \qquad (29)$$

4.17.2 <u>Solution of the Perturbed Kepler Equation</u>

As in the unperturbed case, ψ (true anomaly) is an implicit function of time and $\psi(t)$ must be found by iteration.

4.17.2.1 Compute Initial Guess at E

$$E = t_\psi/a_{32} - e\sin(t_\psi/a_{32})(1 - e\cos t_\psi/a_{32})$$

$$\Delta E_p = 10{,}000$$

4.17.2.2 Compute Sine and Cosine of ψ

$$S_E = \sin E$$

$$C_E = \cos E$$

$$D_n = 1 + e\,C_E$$

$$S_\psi = \sin\psi = \frac{e_2 S_E}{D_n} \qquad (68aa)$$

where $e_2 = \sqrt{1 - e^2}$ is computed in Section 4.4

$$C_\psi = \cos\psi = \frac{C_E + e}{D_n} \qquad (68ab)$$

4.17.2.3 Compute ψ

4.17.2.3.1 Test $\sin\psi$

If $\sin\psi = 0$, go to Step 4.17.2.3.2; otherwise go to Step 4.17.2.3.3.

4.17.2.3.2 Set $\psi = 0$ or π

$$\psi = \frac{\pi}{2}(1 - 1 \text{ sign } C_\psi)$$

Go to Step 4.17.3.

4.17.2.3.3 Compute principal value of ψ $(-\pi/2 < \psi < \pi/2)$

$$\psi = \tan^{-1}\left(\frac{\sin\psi}{\cos\psi}\right)$$

4.17.2.3.4 Test $\cos\psi$

If $\cos\psi < 0$, then set $\psi = \psi + \pi$ and go to Step 4.17.2.4.

If $\cos\psi = 0$, go to Step 4.17.2.3.5.

If $\cos\psi > 0$, go to Step 4.17.2.3.6.

4.17.2.3.5 Set $\psi = \pi/2$ or $3\pi/2$

$$\psi = \frac{\pi}{2}(2 - 1 \text{ sign } S_\psi)$$

Go to Step 4.17.2.4.

4.17.2.3.6 Place ψ in first or third quadrant

$$\psi = \text{sign } E\left[\psi + \pi\left(1 - 1 \text{ sign } S_\psi + 2 \text{ integer part of } \frac{E}{2\pi}\right)\right] \qquad (68ah)$$

4.17.2.4 Compute New Guess at E

Compute $(2p_1 - 1)\psi$

$$C_{11\psi} = C_{11} \cdot \psi$$

Compute $\sin (2p_1 - 1)\psi$

$$S_{C11\psi} = \sin C_{11\psi}$$

Compute $\cos (2p_1 - 1)\psi$

$$C_{C11\psi} = \cos C_{11\psi}$$

Compute $\sin (2p_1 - 1)(\psi + \theta_1)$

$$S_{P1} = S_{C11\psi}C_{\theta_1} + C_{C11\psi}S_{\theta_1}$$

Compute $\cos (2p_1 - 1)(\psi + \theta_1)$

$$C_{P1} = C_{C11\psi}C_{\theta_1} - S_{C11\psi}S_{\theta_1}$$

Compute $\cos 2p_1\psi$

$$C_{2P1\psi} = C_{C11\psi}C_{\psi} - S_{C11\psi}S_{\psi}$$

Compute $\sin 2p_1\psi$

$$S_{2P1\psi} = S_{C11\psi}C_{\psi} + C_{C11\psi}S_{\psi}$$

Compute $\cos 2p_1(\psi + \alpha_1)$

$$C_{P1\psi} = C_{2P1\psi}C_{\alpha_1} - S_{2P1\psi}S_{\alpha_1}$$

Compute $\sin 2p_1(\psi + \alpha_1)$

$$S_{P1\psi} = S_{2P1\psi}C_{\alpha_1} + C_{2P1\psi}S_{\alpha_1}$$

$$\varepsilon^2 \frac{dt_1}{d\psi} = - C_1(C_\psi - C_{P1}) + \varepsilon_2 a^{9/2}[\frac{10}{3} C_\psi - 2.75 C_{P1\psi}] \qquad (68ad)$$

$$\varepsilon^2 \frac{dt_1}{dE} = \frac{e_2}{D_n} \varepsilon^2 \frac{dt_1}{d\psi} \qquad (68ae)$$

$$\varepsilon^2 t_1 = - C_1(S_\psi - S_{P1} + S_{\theta_1}) + \varepsilon_2 a_{92}[\frac{10}{3} S_\psi - 1.375(S_{P1\psi} - S_{\alpha_1})] \qquad (68af)$$

$$E_n = \frac{a_{32} e(E \cos E - \sin E) + E \frac{dt_1}{dE} + t_\psi - \varepsilon^2 t_1}{a_{32}(1 + e \cos E) + \varepsilon^2 \frac{dt_1}{dE}} \qquad (68ag)$$

If $E_n = E$, go to Step 4.17.3.

If $|E_n - E| \geq \Delta E_P$, go to Step 4.17.2.5.

Otherwise set

$$\Delta E_P = |E_n - E|$$

and

$$E = E_n$$

and go to Step 4.17.2.2.

4.17.2.5 Store Maximum Final Value of ΔE_P

$$\Delta E_{Pmax} = \max(\Delta E_{Pmax}, |\Delta E_P|)$$

4.17.3 Compute Auxiliary Functions of ψ and t

$$C_{P_3} = \cos p_3\psi \cos \beta - \sin p_3\psi \sin \beta \qquad (29)$$

$$S_{P_3} = \sin p_3\psi \cos \beta + \cos p_3\psi \sin \beta$$

$$s = \frac{1 - eC_\psi}{a_e} + C_2(C_\psi - C_{P1}) + \varepsilon_2 a_2[(2 - 5C_\psi)/3 + C_{P1\psi}] \qquad (18, 19, 22)$$

where C_2 is defined in Section 4.4, C_ψ and S_ψ in Section 4.17.2.2, and C_{P1} and $C_{P1\psi}$ in Section 4.17.2.4.

Test the value of s.

If $s \leq 0$, set and print the error code = 7 and go to Step 4.3.2.1.2.

If $s > 0$, continue below.

4.17.4 Compute Double Prime Coordinates

$$\phi = \frac{\psi}{1 + \varepsilon_2\omega_1} - \varepsilon_2\nu_1 t_\psi \qquad (49)$$

$$x_a'' = (\cos \phi \cos \phi_1 - \sin \phi \sin \phi_1)/s \qquad (48)$$

$$y_a'' = (\sin \phi \cos \phi_1 + \cos \phi \sin \phi_1)/s$$

$$z_a'' = C_3(S_{P_3} - p_3 S_\psi) \qquad (24)$$

where C_3 is defined in Section 4.4, S_{P_3} in Section 4.17.3, and S_ψ in Section 4.17.2.2.

4.17.5 Compute Node

$$\Omega = C_6 T_\Omega \qquad \text{where } C_6 = \dot{\Omega} \qquad (39)$$

$$C_\Omega = \cos \Omega$$

$$S_\Omega = \sin \Omega$$

4.17.6 Compute Approximate Coordinates in Plane of Planets

$$x'_a = x''_a C_\Omega - y''_a S_\Omega C_i + z''_a S_\Omega S_i$$

$$y'_a = x''_a S_\Omega + y''_a C_\Omega C_i - z''_a C_\Omega S_i \qquad (50)$$

$$z'_a = y''_a S_i + z''_a C_i$$

where $S_i = \sin i$ and $C_i = \cos i$

4.17.7 Compute Time Derivatives of ψ, ϕ, and s

$$\frac{dt}{d\psi} = C_9/(1 - eC_\psi)^2 - C_1(C_\psi - C_{P1}) + \varepsilon_2 a_{92}\left(\frac{10}{3} C_\psi - 2.75\, C_{P1\psi}\right) \qquad (52)$$

where C_9 is defined in Section 4.4 and C_ψ, C_{P1}, and $C_{P1\psi}$ are defined in Section 4.17.2.

4.17.8 Test Value of $\frac{dt}{d\psi}$

If $\frac{dt}{d\psi} = 0$, go to Step 4.17.8.1.

If $\frac{dt}{d\psi} \neq 0$, go to Step 4.17.9.

4.17.8.1 Set the Error Code = 8 and Go to Step 4.3.2.1.2

4.17.9 Compute Time Derivatives of ϕ and s

$$\frac{d\psi}{dt} = \frac{1}{\frac{dt}{d\psi}} \qquad (56)$$

$$\frac{d\phi}{dt} = \frac{d\psi}{dt\ (1 + \varepsilon_2\omega_1)} - \varepsilon_2\nu_1 \qquad (55)$$

$$\frac{ds}{d\psi} = \frac{eS_\psi}{a_e} - C_2\ (S_{P1} - S_\psi) + \varepsilon_2 a_2\ (\frac{5}{3}\ S_\psi - 2\ S_{P1\psi}) \qquad (51)$$

where S_{P1}, S_ψ, and $S_{P1\psi}$ are defined in Section 4.17.2 and C_2 is defined in Section 4.4.

$$\frac{ds}{dt} = \frac{ds}{d\psi}\frac{d\psi}{dt}$$

4.17.10 <u>Compute Approximate Velocity Components in Orbit Plane</u>

$$\dot{x}''_a = y''_a \frac{d\phi}{dt} - \frac{x''_a}{s}\frac{ds}{dt}$$

$$\dot{y}''_a = x''_a \frac{d\phi}{dt} - \frac{y''_a}{s}\frac{ds}{dt} \qquad (54)$$

$$\dot{z}''_a = C_3\ (C_{P_3} - C_\psi)\ \frac{d\psi}{dt}$$

where C_3 is defined in Section 4.4, C_{P_3} in Section 4.17.3, and C_ψ in Section 4.17.2.2.

4.17.11 <u>Compute ρ</u>

$$\rho = \sqrt{\rho^2}$$

4.17.11.1 <u>Test Perturbation-Total Flag</u>

If ICH = 1, go to Step 4.17.12.

If ICH = 2, go to Step 4.17.14.1.

4.17.12 Compute Approximate Velocity Components in Inertial System

$$\dot{x}'_a = C_6 \left(- x''_a S_\Omega - y''_a C_\Omega C_i + z''_a C_\Omega S_i\right) + \dot{x}''_a C_\Omega - \dot{y}''_a S_\Omega C_i + \dot{z}''_a S_\Omega S_i$$

$$\dot{y}'_a = C_6 \left(x''_a C_\Omega - y''_a S_\Omega C_i + z''_a S_\Omega S_i\right) + \dot{x}''_a S_\Omega + \dot{y}''_a C_\Omega C_i - \dot{z}''_a C_\Omega S_i$$

$$\dot{z}'_a = \dot{y}''_a S_i + \dot{z}''_a C_i \qquad (57)$$

where $C_6 = \dot{\Omega}$, $C_\Omega = \cos \Omega$, $S_\Omega = \sin \Omega$, $C_i = \cos i$, and $S_i = \sin i$ as computed previously.

4.17.13 Compute Precessing Keplerian Velocity Components in Inertial System

$$\dot{x}'_s = - \left[S_{\phi_s} C_\Omega + C_{\phi_s} S_\Omega C_i - e(S_\omega C_\Omega + C_\omega S_\Omega C_i)\right]/a_e{}^{1/2}$$
$$- \rho_s \left[\dot{\Omega}(C_{\phi_s} S_\Omega + S_{\phi_s} C_\Omega C_i) + \dot{\omega}(S_{\phi_s} C_\Omega + C_{\phi_s} S_\Omega C_i)\right] \qquad (91)$$

$$\dot{y}'_s = - \left[S_{\phi_s} S_\Omega - C_{\phi_s} C_\Omega C_i - e(S_\omega S_\Omega - C_\omega C_\Omega C_i)\right]/a_e{}^{1/2}$$
$$+ \rho_s \left[\dot{\Omega}(C_{\phi_s} C_\Omega - S_{\phi_s} S_\Omega C_i) - \dot{\omega}(S_{\phi_s} S_\Omega - C_{\phi_s} C_\Omega C_i)\right] \qquad (92)$$

$$\dot{z}'_s = \left[(C_{\phi_s} - eC_\omega)/a_e{}^{1/2} + \rho_s \dot{\omega} C_{\phi_s}\right] S_i \qquad (93)$$

where C_Ω, S_Ω, C_ω, S_ω, and C_{ϕ_s} are defined in Section 4.15.5 and $a_e{}^{1/2}$, C_i, and S_i are computed in Section 4.4.

4.17.14 <u>Compute Total Inertial Velocity Components and Vector Magnitudes</u>

$$\dot{x}' = \dot{x}'_s + \Delta\dot{x}'$$

$$\dot{y}' = \dot{y}'_s + \Delta\dot{y}' \qquad (117)$$

$$\dot{z}' = \dot{z}'_s + \Delta\dot{z}'$$

4.17.14.1 Compute Inertial Velocity

$$V^2 = \dot{z}'^2 + \dot{y}'^2 + \dot{z}'^2$$

$$\dot{\rho} = \sqrt{V^2}$$

4.17.15 <u>Compute Jacobi Constant Fractional Errors</u>

$$r = \sqrt{r^2}$$

$$\Delta C_E = V^2 + 2[\varepsilon_1(\dot{x}'y' - \dot{y}'x') - 1/\rho - C_8(x'C_{\varepsilon t} + y'S_{\varepsilon t})] \qquad (16)$$

$$-C_{part} + 2C_8 \frac{[2(x'C_{\varepsilon t} + y'S_{\varepsilon t} - x_o) + \varepsilon_2(\rho^2 - x_o^2)]}{rr_o(r_o + r)}$$

where V is defined in Section 4.17.14.1, C_8 in Section 4.4, and $C_{\varepsilon t}$ and $S_{\varepsilon t}$ in Section 4.15.6.

$$\frac{\Delta C_E}{C} = \Delta C_E/C_{init}$$

$$\rho_a^2 = x_a'^2 + y_a'^2 + z_a'^2$$

$$r_a = \{1 + \epsilon_2[2(x_a'C_{\epsilon t} + y_a S_{\epsilon t}) + \epsilon_2 \rho_a^2]\}^{1/2}$$

$$\Delta C_a = \dot{x}_a'^2 + \dot{y}_a'^2 + \dot{z}_a'^2 + 2[\epsilon_1(\dot{x}_a' y_a' - \dot{y}_a' x_a') - \frac{1}{\sqrt{\rho a^2}}$$

$$-C_8(x_a' C_{\epsilon t} + y_a' S_{\epsilon t})] - C_{part} \qquad (16)$$

$$+ 2C_8 \frac{[2(x_a'C_{\epsilon t} + y_a'S_{\epsilon t} - x_o) + \epsilon_2(\rho_a^2 - x_o^2)]}{r_a r_o (r_o + r_a)}$$

$$\frac{\Delta C_a}{C} = \frac{\Delta C_a}{C_{init}}$$

$$\rho_s^2 = x_s^2 + Y_s^2 + z_s^2$$

$$\rho_s = \sqrt{\rho_s^2}$$

$$r_s = \{1 + \epsilon_2[2(x_s'C_{\epsilon t} + y_s'S_{\epsilon t}) + \epsilon_2 \rho_s^2]\}^{1/2}$$

$$\frac{\Delta C_s}{C} = \{\dot{x}_s'^2 + \dot{y}_s'^2 + \dot{z}_s'^2 + 2[\epsilon_1(\dot{x}_s' y_s - \dot{y}_s' x_s) - 1/\rho_s$$

$$- C_8(x_s'C_{\epsilon t} + y_s'S_{\epsilon t})] - C_{part}$$

$$+ 2C_8 \frac{[2(x_s'C_{\epsilon t} + y_s'S_{\epsilon t} - x_o) + \epsilon_2(\rho_s^2 - x_o^2)]}{r_s r_o (r_o + r_s)} \} / C_{init}$$

If KHALT = 1, go to Step 4.17.16.

If KHALT = 2, go to Step 4.14.1

If KHALT = 3, go to Step 4.10.

4.17.16 Test if Printing is Desired

If $W_{13} \leq 0$, go to Step 4.18.1.

If $W_{13} > 0$, go to Step 4.10.

4.18 SAVE PLOT VALUES AND TEST IF PLOTTING ARRAY IS FULL

4.18.1 Save Values from Numerical Solution of Total Equations of Motion

$$P_{t(ITP)} = t$$

$$P_{t(ITP + 1)} = T_f$$

$$IT = 13\ ITP - 12$$

$$a_{p(IT)} = \left|\frac{\Delta C_a}{C}\right|$$

$$a_{p(IT + 1)} = \left|\frac{\Delta C_s}{C}\right|$$

$$a_{p(IT + 2)} = x'$$

$$a_{p(IT + 3)} = y'$$

$$a_{p(IT + 4)} = \left|\frac{\Delta C_E}{C}\right|$$

$$a_{p(IT + 5)} = [(x'_a - x')^2 + (y'_a - y')^2 + (z'_a - z')^2]^{1/2} / a$$

$$a_{p(IT + 6)} + [(x'_s - x')^2 + (y'_s - y')^2 + (z'_s - z')^2]^{1/2} / a$$

$$a_{p(IT + 7)} = z'$$

$$a_{p(IT + 8)} = \dot{x}'$$

$$a_{p(IT+9)} = a_{12}[(\dot{x}'_a - \dot{x}')^2 + (\dot{y}'_a - \dot{y}')^2 + (\dot{z}'_a - \dot{z}')^2]^{1/2}$$

$$a_{p(IT+10)} = a_{12}[(\dot{x}'_s - \dot{x}')^2 + (\dot{y}'_s - \dot{y}')^2 + (\dot{z}'_s - \dot{z}')^2]^{1/2}$$

$$a_{p(IT+11)} = \dot{y}'$$

$$a_{p(IT+12)} = \dot{z}'$$

Increment index of plot arrays.

4.18.2 Test Value of ITP

If ITP > 700, set $T_f = P_{t(ITP)}$, and go to Step 4.19.

If ITP $\leq$ 700, set ITP = ITP + 1, set $P_{t(ITP)} = T_f$, and to to Step 4.11.

4.19 INITIATE NUMERICAL SOLUTION OF TOTAL EQUATIONS OF MOTION

4.19.1 Set Perturbation-Total Flag

ICH = 2

KT = 0

4.19.2 Compute Initial Conditions for Numerical Solution of Total Equations of Motion

$x' = ae_3$ where e_3 is computed in Section 4.8.

$$y' = 0$$

$$z' = 0$$

$$\gamma = 0$$

$$\rho = x' \qquad (59)$$

$$t = 0$$

$$\frac{dt}{d\psi} = \frac{a_{32}e_3^{1.5}}{\sqrt{1 - e}} + 7\ \varepsilon_2 a_{92}/12$$

where a_{32}, a_{92}, and ε_2 are computed in Section 4.4.

$$\frac{d\psi}{dt} = \frac{1}{\frac{dt}{d\psi}}$$

$$\frac{d\phi}{dt} = \frac{d\psi}{dt(1 + \varepsilon^2\omega_1)} = \varepsilon_2\nu_1 \qquad (65)$$

$$\dot{x}' = 0$$

$$\dot{y}' = x'(\frac{d\phi}{dt} C_i + C_6) \qquad (66), (64)$$

$$\dot{z}' = x' \frac{d\phi}{dt} S_i \qquad (66), (64)$$

$$\dot{\rho} = (\dot{y}'^2 + \dot{z}'^2)^{1/2}$$

$$r^3 = (1 + \varepsilon_2 x')^3$$

$$E_{2x} = \varepsilon_2 x'$$

$$\ddot{x}' = -\frac{1}{x'^2} + C_8\ E_{2x}(2 + E_{2x}(3 + E_{2x}))/r^3$$

where C_8 is computed in Section 4.4.

$$\ddot{y}' = \ddot{z}' = \frac{\Delta C_c}{C} = 0 \qquad (5)$$

4.19.3 <u>Compute Semi-Major Axis and Eccentricity of Initial Osculating Ellipse</u>

$$a_s = \frac{1}{\frac{2}{\rho} - \dot{\rho}^2} \tag{124}$$

$$e_s = \frac{x'}{a_s} - 1 \tag{125}$$

$$a_{s32} = (a_s)^{1.5}$$

$$P_{ss} = 2\pi\, a_{s32}$$

$$a_{e1/2} = \sqrt{a_s(1 - e_s^2)}$$

Go to Step 4.9.

4.19.4 <u>Print Header for Numerical Solution of Total Equations of Motion</u>

4.20 PRINT NUMERICAL SOLUTION OF TOTAL EQUATIONS OF MOTIONS

$$t' \quad x' \quad \dot{x}' \quad \ddot{x}' \quad \rho$$

$$y' \quad \dot{y}' \quad \ddot{y}' \quad \dot{\rho}$$

$$\frac{\Delta C_c}{C} \quad z' \quad \dot{z}' \quad \ddot{z}'$$

$$\frac{\Delta C_a}{C} \quad \frac{\Delta C_s}{C} \quad \frac{\Delta C_c}{C} \quad \frac{\Delta C_F}{C} \quad \frac{\Delta C_E}{C}$$

$$\Delta\rho_a \quad \Delta\rho_s \quad \Delta\rho_c \quad \Delta\rho_F$$

$$\Delta\dot{\rho}_a \quad \Delta\dot{\rho}_s \quad \Delta\dot{\rho}_c \quad \Delta\dot{\rho}_F$$

4.21 TEST HALT FLAG, KHALT

If KHALT = 1, continue below.

If KHALT = 2, go to Step 4.14.1.

If KHALT = 3, go to Step 4.31.

4.22 CALL NUMERICAL SOLUTION SUBROUTINE FOR TOTAL EQUATIONS OF MOTIONS

The arguments for subroutine RKSIMP are:

KR

IP

KC

t

Δt

ICH

x', y', z'

$\dot{x}', \dot{y}', \dot{z}'$

$\ddot{x}', \ddot{y}', \ddot{z}'$

KHALT

t_{PT}

T_p

4.23 TEST RUNGE-KUTTA FLAG, KR

If KR = 1, continue below.

If KR = 2, 3, or 4, go to Step 4.25.

If KR = 5, go to Step 4.28.1.

4.24 TEST HALT FLAG, KHALT

If KHALT = 2, set $t_f = P_{t(ITP - 1)}$ and go to Step 4.14.1.

If KHALT = 1 or 3, continue below.

4.25 COMPUTE TOTAL EQUATIONS OF MOTION

$$\rho^2 = x'^2 + y'^2 + z'^2 \tag{7}$$

$$\rho^3 = (\rho^2)^{1.5}$$

$\gamma = \gamma$ modulo 2π $\gamma = \varepsilon t$ is developed in subroutine RKSIMP

$$C_{\varepsilon t} = \cos \gamma$$

$$S_{\varepsilon t} = \sin \gamma$$

$$r^2 = 1 + \varepsilon_4 \rho^2 + 2\varepsilon_2(x'C_{\varepsilon t} + y'S_{\varepsilon t}) \tag{8}$$

$$r = (r^2)^{.5}$$

$$r^3 = r^2 r$$

$E_{r_1} = C_8 \varepsilon_2 / r^3$ C_8 is defined in Section 4.4.

$$E_{r_2} = C_8 \varepsilon_2 [2(x'C_{\varepsilon t} + y'S_{\varepsilon t}) + \varepsilon_2 \rho^2] \frac{[1 + r(1 + r)]}{r^3(1 + r)} \tag{5}$$

$$
\left.
\begin{aligned}
\ddot{x}' &= -\frac{x'}{\rho^3} - E_{r_1} x' + E_{r_2} C_{\epsilon t} \\
\ddot{y}' &= -\frac{y'}{\rho^3} - E_{r_1} y' + E_{r_2} S_{\epsilon t} \\
\ddot{z}' &= -\frac{z'}{\rho^3} - E_{r_1} z'
\end{aligned}
\right\} \qquad (5)
$$

When the expression for E_{r_1} and E_{r_2} are substituted in the equations above, one arrives at a set of differential equations identical to (5) except that the last term on the right side of (5) has been put over the common demoninator $r^3(1 + r)$ in order to cancel the large terms in the numerator.

4.26 TEST RUNGE-KUTTA FLAG, KR

If KR = 1, continue below.

If KR = 2, 3, 4, or 5, go to Step 4.22.

4.27 TEST PRINT FLAG, IPRINT

If IPRINT = 1, continue below.

If IPRINT = 2 or 3, go to Step 4.22.

4.28 COMPUTE DATA FOR PRINT AND FOR PLOT ROUTINE

4.28.1 Compute Error in Jacobi Integral and Limit t_s for Numerical Solution of Total Equations

$$\rho = .(\rho^2)$$

$$r = (r^2)^{.5}$$

$$V^2 = \dot{x}'^2 + \dot{y}'^2 + \dot{z}'^2$$

$$\dot{\rho} = (V^2)^{.5}$$

$$\Delta C_c = V^2 + 2[\varepsilon_1(\dot{x}'y' - \dot{y}'x') - \frac{1}{\rho} - C_8(x'C_{\varepsilon t} + y'S_{\varepsilon t}) - C_{part}$$

$$+ 2C_8 \frac{[2(x'C_{\varepsilon t} + y'S_{\varepsilon t} - x_o) + \varepsilon_2(\rho^2 - x_o^2)]}{rr_o(r_o + r)} \tag{16}$$

where C_8 is computed in Section 4.4 and $C_{\varepsilon t}$ and $S_{\varepsilon t}$ are computed in Section 4.25.

$$\frac{\Delta C_c}{C} = \frac{\Delta C_c}{C_{init}}$$

$$T_s = T_s \text{ modulo } P_{ss}$$

Go to Step 4.15 for solution of Kepler's equation.

4.28.2 Compute the Fixed Orbit Values

$$x'_f = \rho_s C_V$$

$$y'_f = \rho_s S_v C_i$$

$$z'_f = \rho_s S_v S_i$$

$$\dot{x}'_f = - S_v/a_e 1/2 \tag{127}$$

$$\dot{y}'_f = \frac{(C_v - e_s)}{a_{e_s} 1/2} C_i$$

$$\dot{z}'_f = \frac{(C_v - e_s)}{a_{e_s} 1/2} S_i$$

$$\rho_f^2 = x_f'^2 + y_f'^2 + z_f'^2$$

$$\rho_f = (\rho_f^2)^{.5}$$

$$r_f = \{ 1 + \varepsilon_2[2(x_f'C_{\varepsilon t} + y_f'S_{\varepsilon t}) + \varepsilon_2\rho_f^2] \}^{.5}$$

$$\Delta C_f = \dot{x}_f'^2 + \dot{y}_f'^2 + \dot{z}_f'^2 + 2[\varepsilon_1(\dot{x}_f'y_f' - \dot{y}_f'x_f') - \frac{1}{\rho_f}$$

$$- C_8(x_f'C_{\varepsilon t} + y_f'S_{\varepsilon t})] - C_{part}$$

$$+ 2C_8 \frac{[2(x_f'C_{\varepsilon t} + y_f'S_{\varepsilon t} - x_o) + \varepsilon_2(\rho_f^2 - x_o^2)]}{r_f r_o(r_o + r_f)} \qquad (16)$$

where C_8 is computed in Section 4.4 and $C_{\varepsilon t}$ and $S_{\varepsilon t}$ are computed in Section 4.25.

$$IT = 13\ ITP - 12$$

$$\frac{\Delta C_f}{C} = \frac{\Delta C_f}{C_{init}}$$

$$\Delta\rho_f = \frac{1}{a}[(x_f' - a_{p(IT+2)})^2 + (y_f' - a_{p(IT+3)})^2 + (z_f' - a_{p(IT+7)})^2]^{1/2}$$

$$\Delta\dot{\rho}_f = a^{1/2}[(\dot{x}_f' - a_{p(IT+8)})^2 + (\dot{y}_f' - a_{p(IT+11)})^2$$

$$+ (\dot{z}_f' - a_{p(IT+12)})^2]^{1/2}$$

$$\Delta\rho_c = [(x' - a_{p(IT+2)})^2 + (y' - a_{p(IT+3)})^2$$

$$+ (z' - a_{p(IT+7)})^2]^{1/2}/a$$

$$\Delta\dot{\rho}_c = a^{1/2}[(\dot{x}' - a_{p(IT+8)})^2 + (\dot{y}' - a_{p(IT+11)})^2$$

$$+ (\dot{z}' - a_{p(IT+12)})^2]^{1/2}$$

4.28.3 <u>Set Quantities Just Computed Equal to Values in Plot Array</u>

$$a_{p(IT+2)} = |\Delta C_c/C|$$

$$a_{p(IT+3)} = |\Delta C_f/C|$$

$$a_{p(IT+7)} = \Delta\rho_c$$

$$a_{p(IT+8)} = \Delta\rho_f$$

$$a_{p(IT+11)} = \Delta\dot{\rho}_c$$

$$a_{p(IT+12)} = \Delta\dot{\rho}_f$$

4.29 TEST VALUE OF HALT FLAG, KHALT

If KHALT = 3, go to Step 4.20.

If KHALT = 2, go to Step 4.14.1.

If KHALT = 1, continue below.

4.30 TEST, IF PRINTING IS DESIRED

If $W_{13} \leq 0$, go to Step 4.22.

If $W_{13} > 0$, go to Step 4.20.

4.31 CALL THE PLOT SUBROUTINE

The arguments for subroutine PLOT are:

$$a^{3/2}$$

$$T_f$$

a

e

i°

μ

C_{init}

P_{ss}

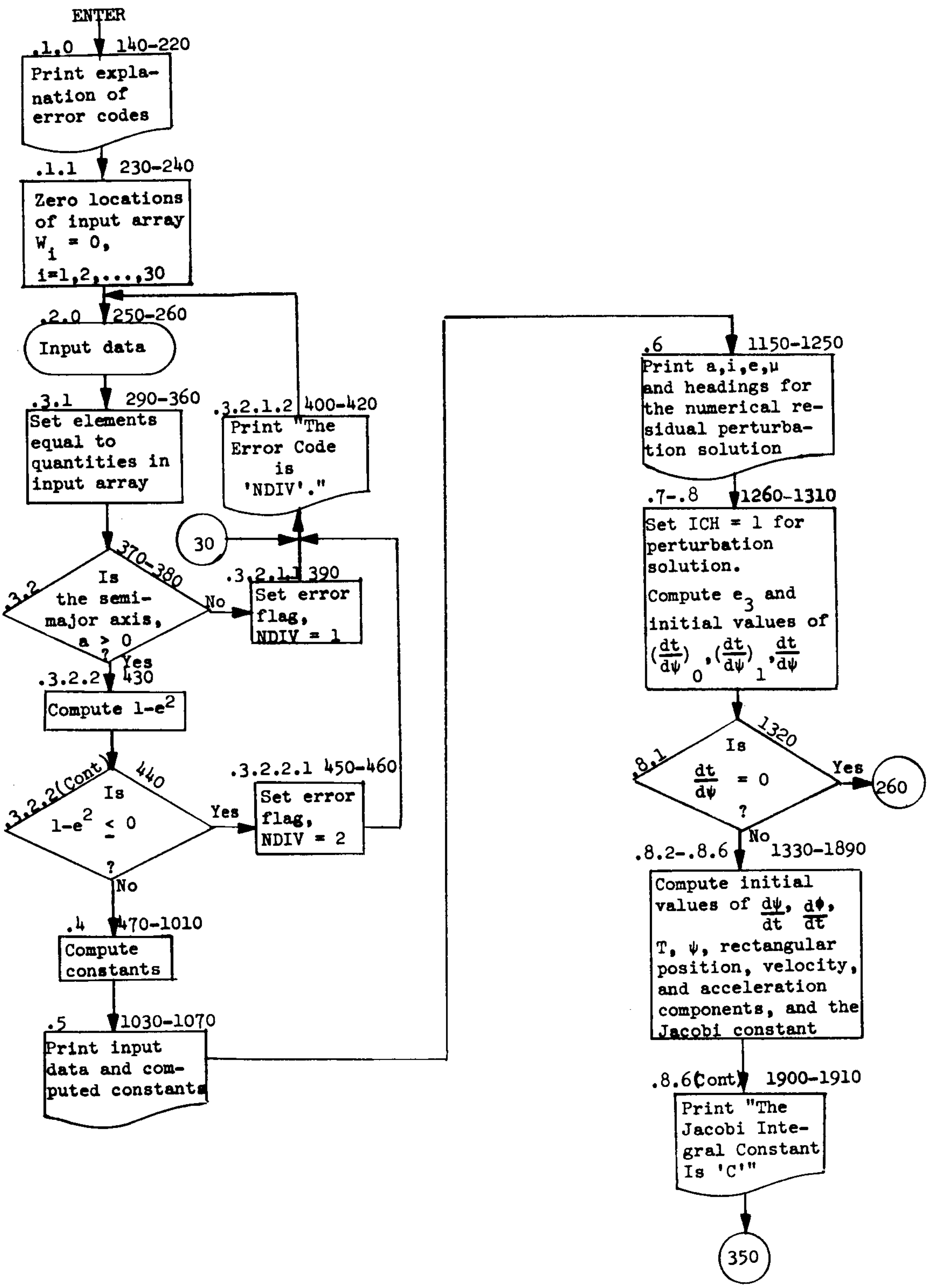
4.32 RUN IS COMPLETED, CALL THE NEXT CASE AT STEP 4.2

Section 5

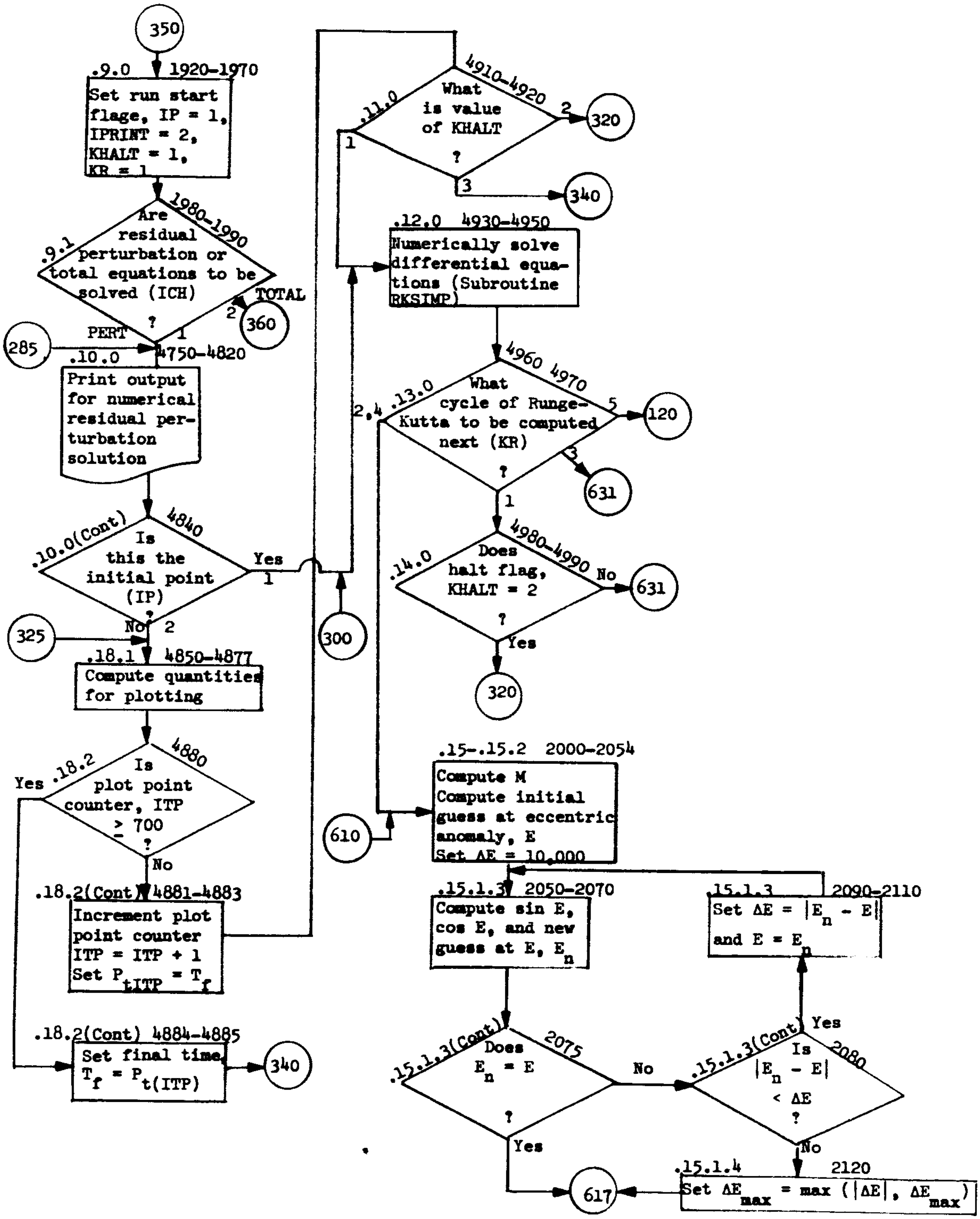
MAIN PROGRAM DETAIL FLOW CHARTS

5.1 EXPLANATION

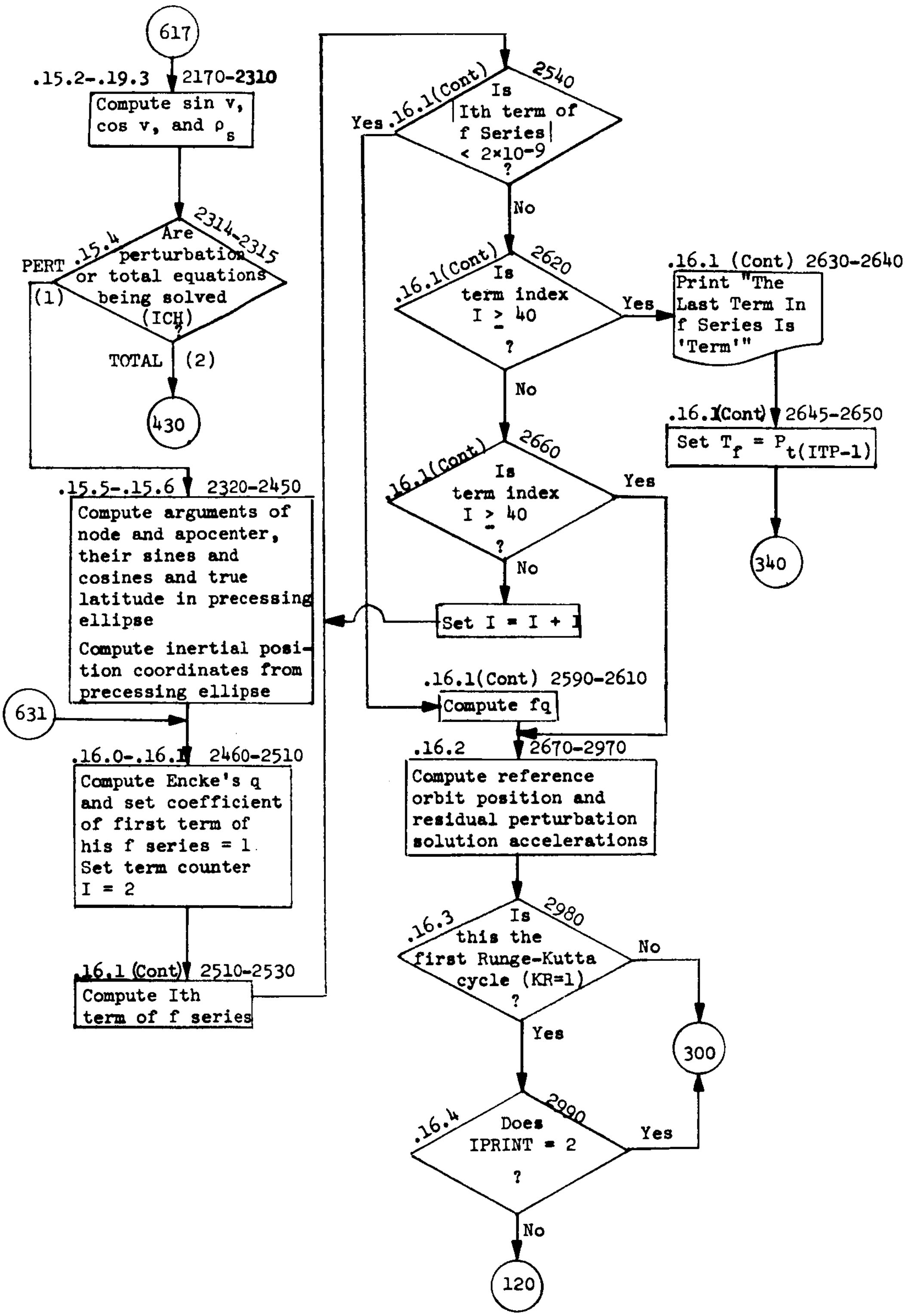
The flow charts of this section describe the logic of the main program completely, i.e., all conditional transfers are shown. The numbers to the top left at each box are the subsection numbers of Section 4 where the operations mentioned in the box are detailed. The numbers on the top right are the card numbers of the program listing, Section 6.



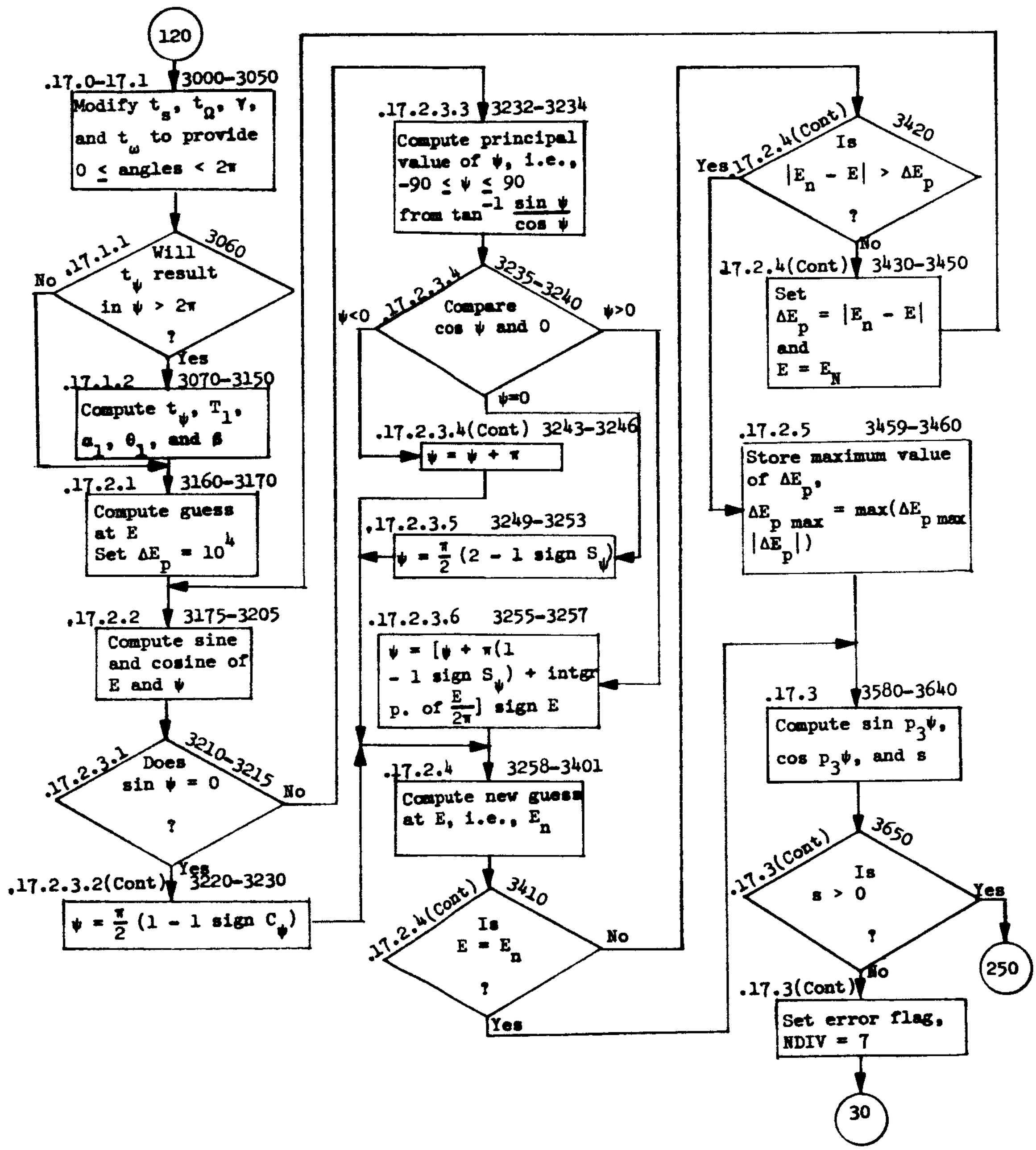
ENTER
.1.0 140-220
Print explanation of error codes
.1.1 230-240
Zero locations of input array W_i = 0, i=1,2,...,30
.2.0 250-260
Input data
.3.1 290-360
Set elements equal to quantities in input array
.3.2 370-380
Is the semi-major axis, a > 0 ?
No
Yes
.3.2.1.1 390
Set error flag, NDIV = 1
.3.2.1.2 400-420
Print "The Error Code is 'NDIV'."
30
.3.2.2 430
Compute 1-e^2
.3.2.2(Cont) 440
Is 1-e^2 ≤ 0 ?
Yes
No
.3.2.2.1 450-460
Set error flag, NDIV = 2
.4 470-1010
Compute constants
.5 1030-1070
Print input data and computed constants
.6 1150-1250
Print a,i,e,μ and headings for the numerical residual perturbation solution
.7-.8 1260-1310
Set ICH = 1 for perturbation solution.
Compute e_3 and initial values of (dt/dψ)_0, (dt/dψ)_1, dt/dψ
.8.1 1320
Is dt/dψ = 0 ?
Yes
260
No
.8.2-.8.6 1330-1890
Compute initial values of dψ/dt, dϕ/dt, T, ψ, rectangular position, velocity, and acceleration components, and the Jacobi constant
.8.6(Cont) 1900-1910
Print "The Jacobi Integral Constant Is 'C'"
350

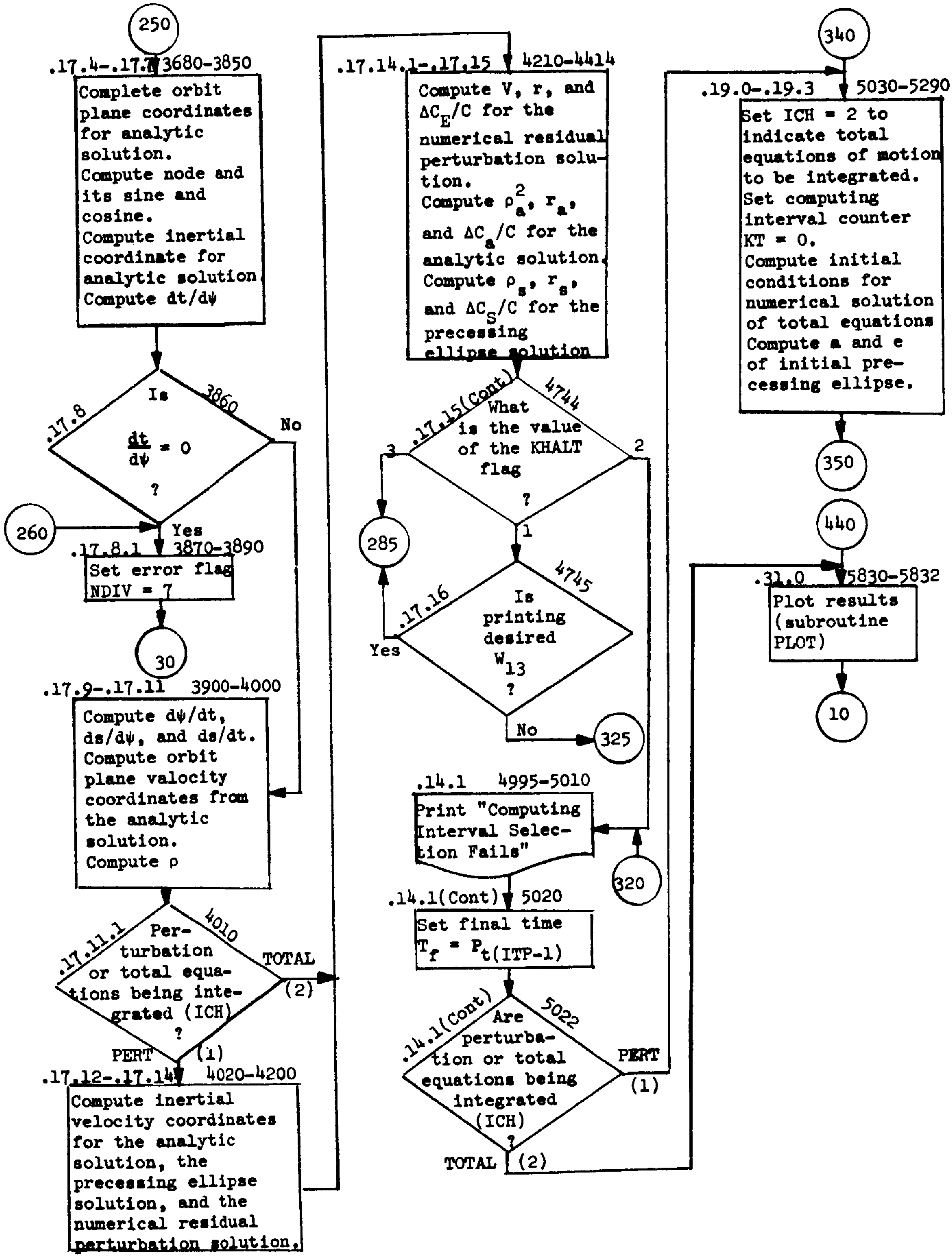


350
.9.0 1920-1970
Set run start flage, IP = 1, IPRINT = 2, KHALT = 1, KR = 1
9.1 1980-1990
Are residual perturbation or total equations to be solved (ICH) ?
TOTAL 2
360
PERT 1
285
.10.0 4750-4820
Print output for numerical residual perturbation solution
.10.0(Cont) 4840
Is this the initial point (IP) ?
Yes 1
No 2
325
.18.1 4850-4877
Compute quantities for plotting
.18.2 4880
Is plot point counter, ITP ≥ 700 ?
Yes
No
.18.2(Cont) 4881-4883
Increment plot point counter ITP = ITP + 1 Set P tITP = Tf
.18.2(Cont) 4884-4885
Set final time, Tf = Pt(ITP)
340
300
.11.0 4910-4920
What is value of KHALT ?
1
2
320
3
340
.12.0 4930-4950
Numerically solve differential equations (Subroutine RKSIMP)
.13.0 4960 4970
What cycle of Runge-Kutta to be computed next (KR) ?
2,4
5
120
3
631
1
.14.0 4980-4990
Does halt flag, KHALT = 2 ?
No
631
Yes
320
.15-.15.2 2000-2054
Compute M Compute initial guess at eccentric anomaly, E Set ΔE = 10,000
610
.15.1.3 2050-2070
Compute sin E, cos E, and new guess at E, En
.15.1.3 2090-2110
Set ΔE = |En - E| and E = En
.15.1.3(Cont) 2075
Does En = E ?
No
Yes
.15.1.3(Cont) 2080
Is |En - E| < ΔE ?
Yes
No
.15.1.4 2120
Set ΔEmax = max (|ΔE|, ΔEmax)
617

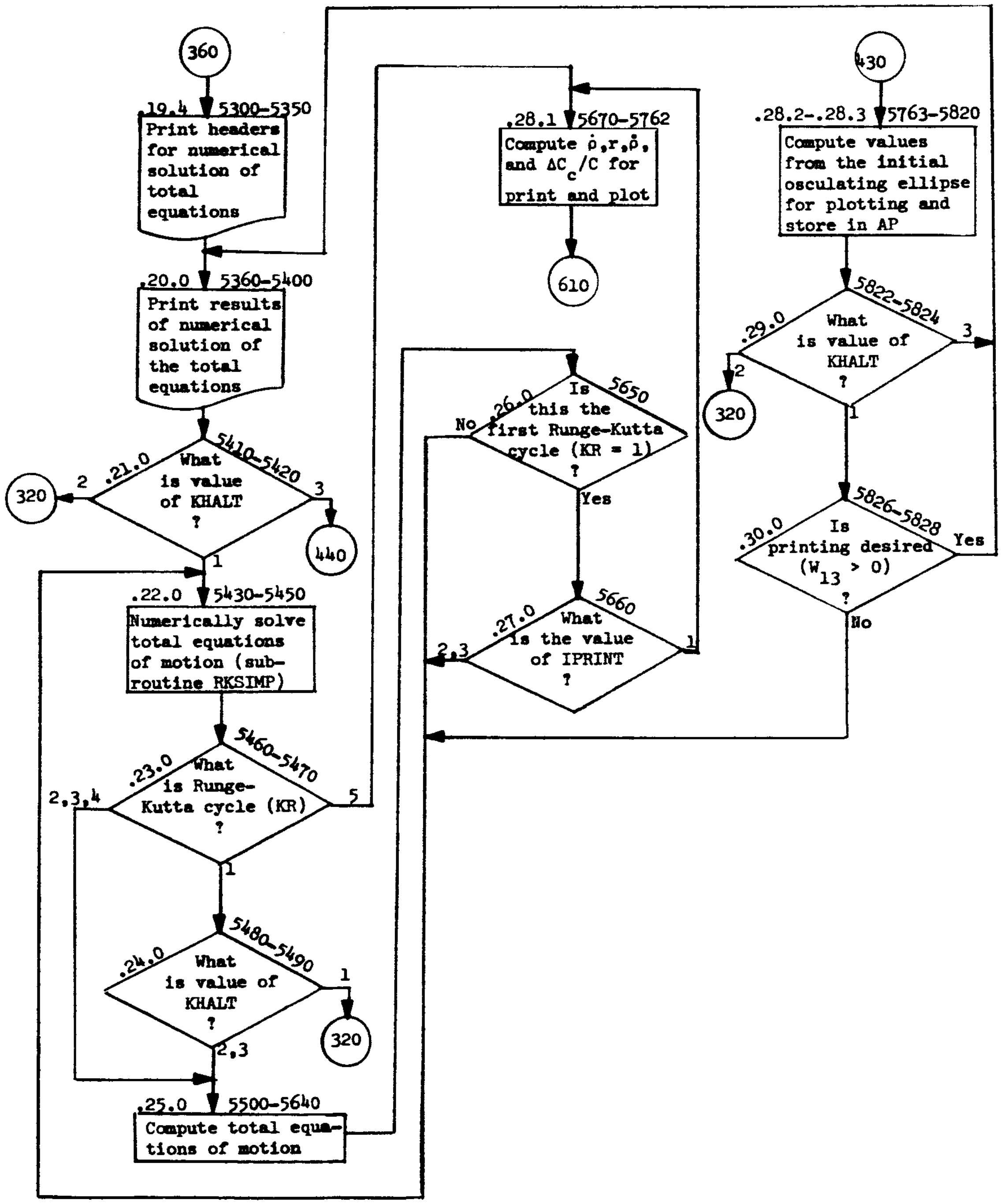


617
.15.2-.19.3 2170-2310
Compute sin v, cos v, and ρs
.15.4 2314-2315
Are perturbation or total equations being solved (ICH)?
PERT (1)
TOTAL (2)
430
.15.5-.15.6 2320-2450
Compute arguments of node and apocenter, their sines and cosines and true latitude in precessing ellipse
Compute inertial position coordinates from precessing ellipse
631
.16.0-.16.1 2460-2510
Compute Encke's q and set coefficient of first term of his f series = 1.
Set term counter I = 2
.16.1 (Cont) 2510-2530
Compute Ith term of f series
.16.1(Cont) 2540
Is |Ith term of f Series| < 2×10-9 ?
Yes
No
.16.1(Cont) 2620
Is term index I ≥ 40 ?
Yes
No
.16.1 (Cont) 2630-2640
Print "The Last Term In f Series Is 'Term'"
.16.1(Cont) 2645-2650
Set Tf = Pt(ITP-1)
340
.16.1(Cont) 2660
Is term index I ≥ 40 ?
Yes
No
Set I = I + 1
.16.1(Cont) 2590-2610
Compute fq
.16.2 2670-2970
Compute reference orbit position and residual perturbation solution accelerations
.16.3 2980
Is this the first Runge-Kutta cycle (KR=1) ?
No
Yes
300
.16.4 2990
Does IPRINT = 2 ?
Yes
No
120


120
.17.0-17.1 3000-3050
Modify t_s, t_Ω, γ, and t_ω to provide 0 ≤ angles < 2π
.17.1.1 3060
Will t_ψ result in ψ > 2π ?
No
Yes
.17.1.2 3070-3150
Compute t_ψ, T_1, a_1, θ_1, and β
.17.2.1 3160-3170
Compute guess at E
Set $\Delta E_p = 10^4$
.17.2.2 3175-3205
Compute sine and cosine of E and ψ
.17.2.3.1 3210-3215
Does sin ψ = 0 ?
No
Yes
.17.2.3.2(Cont) 3220-3230
$\psi = \frac{\pi}{2}(1 - 1 \text{ sign } C_\psi)$
.17.2.3.3 3232-3234
Compute principal value of ψ, i.e., -90 ≤ ψ ≤ 90 from $\tan^{-1} \frac{\sin \psi}{\cos \psi}$
.17.2.3.4 3235-3240
Compare cos ψ and 0
ψ<0
ψ>0
ψ=0
.17.2.3.4(Cont) 3243-3246
ψ = ψ + π
.17.2.3.5 3249-3253
$\psi = \frac{\pi}{2}(2 - 1 \text{ sign } S_\psi)$
.17.2.3.6 3255-3257
$\psi = [\psi + \pi(1 - 1 \text{ sign } S_\psi) + \text{intgr p. of } \frac{E}{2\pi}] \text{ sign } E$
.17.2.4 3258-3401
Compute new guess at E, i.e., E_n
.17.2.4(Cont) 3410
Is $E = E_n$?
No
Yes
.17.2.4(Cont) 3420
Is $|E_n - E| > \Delta E_p$?
Yes
No
.17.2.4(Cont) 3430-3450
Set $\Delta E_p = |E_n - E|$ and $E = E_N$
.17.2.5 3459-3460
Store maximum value of ΔE_p, $\Delta E_{p\,max} = \max(\Delta E_{p\,max}, |\Delta E_p|)$
.17.3 3580-3640
Compute sin $p_3\psi$, cos $p_3\psi$, and s
.17.3(Cont) 3650
Is s > 0 ?
Yes
250
No
.17.3(Cont)
Set error flag, NDIV = 7
30



250
.17.4-.17.7 3680-3850
Complete orbit plane coordinates for analytic solution.
Compute node and its sine and cosine.
Compute inertial coordinate for analytic solution
Compute dt/dψ
.17.8 3860
Is dt/dψ = 0 ?
No
Yes
260
.17.8.1 3870-3890
Set error flag NDIV = 7
30
.17.9-.17.11 3900-4000
Compute dψ/dt, ds/dψ, and ds/dt. Compute orbit plane velocity coordinates from the analytic solution. Compute ρ
.17.11.1 4010
Perturbation or total equations being integrated (ICH) ?
TOTAL (2)
PERT (1)
.17.12-.17.14 4020-4200
Compute inertial velocity coordinates for the analytic solution, the precessing ellipse solution, and the numerical residual perturbation solution.
.17.14.1-.17.15 4210-4414
Compute V, r, and ΔC_E/C for the numerical residual perturbation solution.
Compute ρ_a^2, r_a, and ΔC_a/C for the analytic solution.
Compute ρ_s, r_s, and ΔC_S/C for the precessing ellipse solution
.17.15(Cont) 4744
What is the value of the KHALT flag ?
3
2
1
285
.17.16 4745
Is printing desired W13 ?
Yes
No
325
.14.1 4995-5010
Print "Computing Interval Selection Fails"
320
.14.1(Cont) 5020
Set final time T_f = P_t(ITP-1)
.14.1(Cont) 5022
Are perturbation or total equations being integrated (ICH) ?
PERT (1)
TOTAL (2)
340
.19.0-.19.3 5030-5290
Set ICH = 2 to indicate total equations of motion to be integrated.
Set computing interval counter KT = 0.
Compute initial conditions for numerical solution of total equations
Compute a and e of initial precessing ellipse.
350
440
.31.0 5830-5832
Plot results (subroutine PLOT)
10


360
.19.4 5300-5350
Print headers for numerical solution of total equations
.20.0 5360-5400
Print results of numerical solution of the total equations
.21.0 5410-5420
What is value of KHALT ?
2
320
3
440
1
.22.0 5430-5450
Numerically solve total equations of motion (subroutine RKSIMP)
.23.0 5460-5470
What is Runge-Kutta cycle (KR) ?
2,3,4
5
1
.24.0 5480-5490
What is value of KHALT ?
1
320
2,3
.25.0 5500-5640
Compute total equations of motion
.28.1 5670-5762
Compute $\dot{\rho}$, r, $\ddot{\rho}$, and $\Delta C_c/C$ for print and plot
610
.26.0 5650
Is this the first Runge-Kutta cycle (KR = 1) ?
No
Yes
.27.0 5660
What is the value of IPRINT ?
2,3
1
430
.28.2-.28.3 5763-5820
Compute values from the initial osculating ellipse for plotting and store in AP
.29.0 5822-5824
What is value of KHALT ?
3
2
320
1
.30.0 5826-5828
Is printing desired (W_{13} > 0) ?
Yes
No

Section 6

MAIN PROGRAM LISTING

This section contains the listing of the main program. The subsection numbers on the comment cards refer to the subsections of Section 4, Equations in Order of Solution, wherein the code is explained.

```
    H144   JERRY                                                        09/01/65                          PAGE  1
        EXTERNAL FORMULA NUMBER   -   SOURCE STATEMENT   -   INTERNAL FORMULA NUMBER(S)

C       NUMERICAL RESIDUAL PERTURBATION SOLUTICNS APPLIED TO THE PROBLEM  0070H144
C       OF A CLOSE SATELLITE OF THE SMALLER BODY IN THE RESTRICTED THREE- 0080H144
C       BODY PROBLEM                                                      0085H144
        DIMENSION  W(30), WR(30), PS(25), FPS(25)                         0090H144
        DIMENSION ECS(25), TERM (40), TNUM(40), TDEN(40),                 0100H144
       1 FRA(40), FAC(40), PTIME(700), AP( 9100)                          0110H144
        COMMON W, IPRINT, TS, TPSI, TNOD, TPER, GAMMA, EP1,               0120H144
       1 ITP, PTIME, AP                                                   0130H144
C1.0    PRINT EXPLANATION OF ERROR CODE                                   0140H144
        WRITE (6, 901)                                                    0150H144    .1     .2
  901 FORMAT(1H1,30H ERROR CODE    REASON FOR HALT//4X2H 1,8X20H SEMI-MA0160H144
       1JOR AXIS = 0/4X2H 2,8X33H ECCENTRICITY EQUALS OR EXCEEDS 1/       0170H144
       5        4X2H 7,8X33H S (THE APPROX RADIUS VECTOR) = 0/4X2H 8,8X31H0210H144
       6 DERIVATIVE OF TIME WRT PSI = 0)                                  0220H144
C1.1    SET INPUT ARRAYS = 0                                              0225H144
        DO 7 I = 1, 30                                                    0230H144    .3
    7 W(I) = 0.                                                           0240H144    .4     .5
        KKKK = 1                                                          0245H144
C 2.0   PROGRAM INPUT                                                     0250H144    .6
   10 CALL INPUT1 (W(1), W(30), WR(1))                                    0260H144    .7
        KFAIL = 0                                                         0280H144
C3.0    MOVE INPUT TO WORKING STORAGE AND CHECK INPUT                     0285H144
C3.1    MOVE INPUT TO WORKING STORAGE AND FORM NECESSARY CONSTANTS        0290H144    .8
        A1 = ABS (W(1))                                                   0300H144    .9
        A2 = A1 ** 2                                                      0310H144    .10
        AS = A1                                                           0320H144    .11
        EC1 = ABS (W(2))                                                  0330H144    .12
        ES = EC1                                                          0340H144    .13
        DEGI = W(3)                                                       0350H144    .14
        TF = W (7)                                                        0355H144    .15
        EP4 = ABS (W(4))                                                  0360H144
C3.2    CHECK ON CONSTANT DIVISORS. HALT RUN IF ANY ARE ZERO              0370H144
C3.2.1 TEST SEMI MAJOR AXIS                                               0375H144    .16
        IF (A1) 20, 20, 50                                                0380H144
C3.2.1.1 SET ERROR CODE = 1                                               0385H144    .17
   20 NDIV = 1                                                            0390H144
C3.2.1.2 PRINT ERROR CODE AND GO TO STEP 2.0                              0395H144    .18
   30 WRITE (6, 40) NDIV                                                  0400H144    .19    .20    .21
   40 FORMAT(1H ,18H THE ERROR CODE IS I5)                                0410H144
        GO TO 10                                                          0420H144
C3.2.2 COMPUTE AND TEST E SUB 2                                           0425H144    .22
   50 EC2 = 1. - EC1 ** 2                                                 0430H144    .23
        IF (EC2) 60, 60, 70                                               0440H144
C3.2.2.1 SET ERROR CODE TO 2 AND GO TO STEP 3.2.1.2                       0445H144    .24
   60 NDIV = 2                                                            0450H144    .25
        GO TO 30                                                          0460H144
C4.0    COMPUTE CONSTANTS                                                 0465H144    .26
   70 AEC = A1 * EC2                                                      0470H144    .27
        AESQ = SQRT (AEC)                                                 0480H144    .28
        EC2 = SQRT (EC2)                                                  0490H144    .29
        A3 = A1 * A1 * A1                                                 0500H144    .30
        EP2 = SQRT (EP4)                                                  0510H144    .31
        TNUM(1) = 3.                                                      0520H144    .32
        TDEN(1) = 1.                                                      0530H144    .33
        DO 80 I = 2, 40                                                   0540H144    .34
```

```
     EXTERNAL FORMULA NUMBER  -  SOURCE STATEMENT  -  INTERNAL FORMULA NUMBER(S)

    TNUM(I) = TNUM(I-1) + 2.                                 0550H144   ,35
 80 TDEN(I) = TDEN(I-1) + 1.                                 0560H144   ,36   ,37
    ITP = 2                                                  0561H144   ,38
    PTIME(1) = 0.                                            0562H144   ,39
    DO 91 I = 1, 13                                          0563H144   ,40
 91  AP(I) = 0.                                              0564H144   ,41   ,42
 90 A32 = SQRT (A3)                                          0570H144   ,43
    A92 = A32 ** 3                                           0580H144   ,44
    AS32 = A32                                               0590H144   ,45
    EP1 = SQRT (EP2)                                         0600H144   ,46
    C1 = 3.75 * EP1 * A3 * EC1                               0610H144   ,47
    EPA = EP1 * A32                                          0620H144   ,48
    GNU1 = (-A32 ) * ( 24. * EC1 + 11.)/12.                  0630H144   ,49
    ELMB = (-.75) * A32                                      0640H144   ,50
    REAL M1                                                  0650H144
    M1 = - A32 * (1. + 12. * EC1) / 6.                       0660H144   ,51
    OMEG1 = A3 *( -7. + 24. * EC1)/12.                       0670H144   ,52
    P3 = 1. / ( 1. + EP2 * OMEG1  )- EPA * ( 2. + EP1 * ( GNU1 -  0680H144
   1 2. * ELMB))                                             0690H144   ,53
110 RAD I = DEG I / 57.2957795                               0710H144   ,54
    CI = COS (RAD I)                                         0720H144   ,55
    SI = SIN (RAD I)                                         0730H144   ,56
    REAL IS                                                  0740H144
    IS = RADI                                                0750H144   ,57
    CIS = COS (IS)                                           0760H144   ,58
    SIS = SIN (IS)                                           0770H144   ,59
602 DPER = (1.5 * EP2 * A32 - EP4 * OMEG1 * GNU1) /          0790H144
   1 (1. + EP2 * OMEG1)                                      0800H144   ,60
    DNOD =-.75 * EP1 * EPA                                   0810H144   ,61
    DPDN = DNOD * DPER                                       0820H144   ,62
    DPNSQ = DNOD ** 2 + DPER ** 2                            0830H144   ,63
    A12 = SQRT (A1)                                          0840H144   ,64
    P1 = 1. / (1. + EP2 * OMEG1) - EPA * (1. + EP1 * M1)     0850H144   ,65
    C2 = 1.875 * EP1 * A12 * EC1                             0860H144   ,66
    C3 = .375 * EP1 * RADI * A1 * A32 / (1. - EP1 * A32)     0870H144   ,67
    C6 = DNOD                                                0880H144   ,68
    C7 = C6 * C6                                             0890H144   ,69
    C8  = 1. - EP4                                           0900H144   ,70
    C9  = AEC * A12 * EC2                                    0910H144   ,71
    C10 = EP2 + OMEG1 / (1. + EP2 * OMEG1) + EPA *           0920H144
   1        (1. + EP1 * M1)                                  0930H144   ,72
    C11 = 2. * P1 - 1.                                       0940H144   ,73
    C12 = 2.* DNOD / AESQ                                    0941H144   ,74
    C13 = 2.* DPER / AESQ                                    0942H144   ,75
    C14 = 2.* DPDN                                           0943H144   ,76
    C15 =(-C12 - C13 * CI)                                   0944H144   ,77
    C16 = -C14 - DPNSQ * CI                                  0945H144   ,78
    C17 =(C12 * CI  + C13)                                   0946H144   ,79
    C18 = C14 * CI + DPNSQ                                   0947H144   ,80
    C19 = DPER * DPER * SI                                   0948H144   ,81
    C20 = EC1 * SI * C13                                     0949H144   ,82
    T1 = 6.2831853 * A32 + C1 * SIN (6.2831853 * C11)        0950H144
   1 - AS2 * 1.375 * SIN (6.2831853 * 2. * P1) *EP2          0960H144   ,83
    C21 = SI * C13                                           0961H144   ,84
    C22 = EC1 * C15                                          0962H144   ,85
    C23 = EC1 * C17                                          0963H144   ,86
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H144    JERRY                                                     09/01/65
     EXTERNAL FORMULA NUMBER   -   SOURCE STATEMENT   -   INTERNAL FORMULA NUMBER(S)

      C24 = SQRT (1. - ES ** 2)                              0964H144    ,87
      PSS= 6.2831853 * A32                                   0970H144    ,88
      PNOD = 6.2831853 / DNOD                                0980H144    ,89
      PPER = 6.2831853 / DPER                                0990H144    ,90
      DEM = 0.                                               0993H144    ,91
      DEPM = 0.                                              0996H144    ,92
      ALPHA1= 0.                                             1000H144    ,93
      THETA1 = 0.                                            1010H144    ,94
      PHI1 = 0                                               1020H144    ,95
      CALPHA = 1.                                            1021H144    ,96
      SALPHA = 0.                                            1022H144    ,97
      STHETA = 0.                                            1023H144    ,98
      CTHETA = 1.                                            1024H144
C5.0  PRINT CONSTANTS                                        1025H144    ,99
      WRITE (6, 1) (W(I), I = 1,15), OMEG1, GNU1, ELMB,  P1,  1030H144
     1 P3, C1, C2, C3, C4, C5, C6, C7, C8, C9, C10, C11, EC2  1040H144
     2 , DPER, DNOD, DPDN, DPNSQ                             1050H144    ,100  ,101  ,102  ,103  ,104  ,105

    1 FORMAT(1H0,5E18.8/5E18.8/5E18.8//5E18.8/6E18.8/6E18.8/  1060H144
     1 /4F20.8)                                              1070H144
C6.0  PRINT HEADER FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION LISTING 1145H144
C     THE INPUT VALUES OF A,E,I, AND MU                      1146H144
  117 WRITE (6, 902) A1, DEG I, EC1, EP4                     1150H144    ,106  ,107  ,108
  902 FORMAT(1H1,48H THE INITIAL CONDITIONS ARE      SEMI-MAJOR AXIS E17.1160H144
     18.4X17H INCLINATION,DEG E17.8/31X14H ECCENTRICITY E20.8,4X12H MASS1170H144
     2 RATIO E22.8)                                          1180H144
      WRITE (6, 903)                                         1190H144    ,109  ,110
  903 FORMAT(1H0/61H TIME       DELTA X       DELTA X DOT      DELTA X DBL DO1200H144
     1T      RHO/65H          DELTA Y      DELTA Y DOT      DELTA Y DBL DOT1210H144
     2      RHO DOT/53H C APPROX DELTA Z      DELTA Z DOT      DELTA Z DBL 1220H144
     3DOT/51H             X       X DOT      X APPROX      X DOT APPROX/51H   1230H144
     4        Y       Y DOT      Y APPROX      Y DOT APPROX/51H C EXACT  Z    1240H144
     5 Z DOT       Z APPROX      Z DOT APPROX)               1250H144
C7.0  SET PERTURBATION-TOTAL FLAG                            1255H144
      ICH = 1                                                1260H144
C8.0  COMPUTE INITIAL VALUES OF POSITION, VELOCITY, AND ACCELERATION   1270H144    ,111
      EC3 = 1. + EC1                                         1280H144    ,112
      DTODPS = A32 * EC3 ** 1.5 / SQRT (1. - EC1)            1290H144    ,113
      DT1DPS = 7./12. * EP2 * A92                            1300H144    ,114
      DTDPS1 = DTODPS + DT1DPS                               1310H144
C8.1  TEST THE VALUE OF THE DERIVATIVE OF TIME WITH RESPECT TO PSI     1315H144    ,115
      IF (DTDPSI) 119, 260, 119                              1320H144
C8.2  COMPUTE THE DERIVATIVE OF PHI WITH RESPECT TO T AND SET INITIAL   1325H144
C     VALUES OF T AND PSI                                    1326H144    ,116
  119 DPSIDT = 1. / DTDPSI                                   1330H144    ,117
      DPHIDT = DPSIDT / (1.+EP2 * OMEG1)  - EP2 * GNU1       1340H144    ,118
      T = C.                                                 1350H144    ,119
      PSI = 0.                                               1360H144
C 8.3  POSITION COORDINATES                                  1370H144    ,120
      X2P = A1 * EC3                                         1380H144    ,121
      Y2P = 0.                                               1390H144    ,122
      Z2P = 0.                                               1400H144    ,123
      XP = X2P                                               1410H144    ,124
      YP = 0.                                                1420H144    ,125
      ZP = 0.                                                1430H144    ,126
      X = XP                                                 1440H144    ,127
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       EXTERNAL FORMULA NUMBER  -  SOURCE STATEMENT  -  INTERNAL FORMULA NUMBER(S)
       Y = C.                                                       1450H144  .128
       Z = C.                                                       1460H144  .129
       DX = 0.                                                      1470H144  .130
       DY = 0.                                                      1480H144  .131
       DZ = 0.                                                      1490H144  .132
       RHO = X                                                      1500H144  .133
       R = 1. + EP2 * X                                             1510H144  .134
       RCU = (1. + EP2 * X) ** 3                                    1520H144  .135
       XS = XP                                                      1530H144  .136
       YS = 0.                                                      1540H144  .137
       ZS = 0.                                                      1550H144  .138
       XSD = 0.                                                     1560H144
C8.4   VELOCITY COMPONENTS                                          1570H144  .139
       XD2P = 0.                                                    1580H144  .140
       YD2P = XP * DPHIDT                                           1590H144  .141
       ZD2P = 0.                                                    1600H144  .142
       XDP = 0                                                      1610H144  .143
       YDP = YD2P * CI + X2P * C6                                   1620H144  .144
       ZDP = YD2P * SI                                              1630H144  .145
       XD = 0.                                                      1640H144  .146
       YD = YDP                                                     1650H144  .147
       ZD = ZDP                                                     1660H144  .148
       DXD = 0.                                                     1670H144  .149
       GDOT = -(DTODPS * EP2 * OMEG1  + DT1DPS * ( 1. + EP2 * OMEG1))  1675H144
      1 / (DTOPSI * ( 1. + EP2 * OMEG1) * DTODPS) - EP2 * GNU1-DPER  1676H144  .150
       DYD =  X * GDOT * CI                                         1680H144  .151
       DZD =  X * GDOT * SI                                         1690H144  .152
       RHOD = SQRT (YD ** 2 + ZD ** 2)                              1700H144  .153
       XSD = 0.                                                     1710H144  .154
       YSD = X * (DNOD + (1. / DTODPS + DPER) * CI)                 1720H144  .155
       ZSD = X * (1. / DTODPS + DPER) * SI                          1730H144
C 8.5   ACCELERATION COMPONENTS                                     1740H144  .156
       EP2X = EP2 * X                                               1750H144  .157
       DX2D = C8 * EP2X * (2. + EP2X * (3. + EP2X)) / RCU           1760H144
      1 + 2. * (DPER + DNOD * CI)* SQRT((1. - ES) / AS / (1.        1770H144
      2 + ES)) + XS * (2. * DPDN * CI + DPNSQ)                      1780H144  .158
       DY2D = 0.                                                    1790H144  .159
       DZ2D = 0.                                                    1800H144
C8.6   INITIAL VALUE OF JACOBI CONSTANT                             1810H144  .160
       ACOBI = X ** 2 * (DPHIDT ** 2 + C7 -2. * (C6 * EP1 -         1820H144
      1 DPHIDT * (C6 - EP1) * CI)) - 2. * (1. / X + C8 * X)         1830H144  .161
       REAL JACOBI                                                  1840H144
       JACOBI = ACOBI - 2. * C8 / EP2 / (1. + EP2 * X)              1850H144  .162
       XNAUT = X                                                    1860H144  .163
       RNAUT = 1. + EP2 * X                                         1870H144  .164
       YACOBI = 0.                                                  1880H144  .165
       YACAPP = 0.                                                  1890H144  .166
       WRITE (6, 351) JACOBI                                        1900H144  .167  .168  .169
   351 FORMAT(1H0,32H THE JACOBI INTEGRAL CONSTANT IS E17.8)        1910H144
C9.0   SET RUN START FLAGS                                          1920H144
   350 IP = 1                                                       1930H144  .170
       IPRINT = 2                                                   1940H144  .171
       KHALT = 1                                                    1950H144  .172
       KR  = 1                                                      1970H144
C9.1   TEST PERTURBATION-TOTAL FLAG                                 1980H144  .173
       GO TO (285, 360), ICH                                        1990H144
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      H144    JFRRY                                                       09/01/65                            PAGE  5
           EXTERNAL FORMULA NUMBER    -    SOURCE STATEMENT    -    INTERNAL FORMULA NUMBER(S)

C15.0 COMPUTE TWO-BODY EQUATIONS                                          2000H144
C15.1 SOLVE KEPLER'S EQUATION                                             2010H144
C15.1.1 FORM CONSTANTS                                                    2015H144     .174
  610 TAS32 = TS / AS32                                                   2020H144
C15.1.2 COMPUTE A FIRST ESTIMATE FOR ECCENTRIC ANOMALY                    2025H144     .175
      ESA = TAS32 - ES *  SIN( TAS32)*(1.+ ES*COS (TAS32))                2030H144     .176
      DLTE = 10000.                                                       2040H144
C15.1.3 COMPUTE NEW GUESS AT E                                            2050H144     .177
  616 SESA = SIN (ESA)                                                    2057H144     .178
      CESA = COS (ESA)                                                    2058H144     .179
      DEN = 1. + ES * CESA                                                2060H144     .180
      ESAN = (TAS32 + ES * (ESA * CESA - SESA)) / DEN                     2070H144     .181
      IF (ESA - ESAN) 612,617,612                                         2075H144     .182
  612 IF ( ABS(ESA -ESAN) - DLTE)  613,220,220                            2080H144     .183
  613 DLTE = ABS (ESA - ESAN)                                             2090H144     .184
      ESA = ESAN                                                          2100H144     .185
      GO TO 616                                                           2110H144     .186
  220 DEM = AMAX1 ( DEM, ABS (DLTE))                                      2120H144
C15.2 COMPUTE SINE AND COSINE OF TRUE ANOMOLY                             2170H144     .187
  617 SIV = C24 * SESA / DEN                                              2190H144     .188
      COV =(CESA + ES) / DEN                                              2200H144
C15.3 COMPUTE RHO SUB S                                                   2305H144     .189
  623 RHOS = AS * DEN                                                     2310H144
C15.4 TEST PERTURBATION-TOTAL FLAG                                        2314H144     .190
      GO TO (431, 430), ICH                                               2315H144
C15.5 COMPUTE ARGUMENTS OF NODE AND PERIGEE AND THEIR SINES AND COSINES  2320H144     .191
  431 SNODE = DNOD * TNOD                                                 2330H144     .192
      SPER  = DPER * TPER                                                 2340H144     .193
      CSNO =  COS (SNODE)                                                 2350H144     .194
      SSNO = SIN (SNODE)                                                  2360H144     .195
      COPER =  COS (SPER)                                                 2363H144     .196
      SIPER =  SIN (SPER)                                                 2373H144     .197
      STRL = SIV * COPER + COV * SIPER                                    2380H144     .198
      CTRL = COV * COPER - SIV * SIPER                                    2390H144
C15.6 COMPUTE THE PRECESSING TWO BODY POSITION COORDINATE IN THE          2420H144
C     INERTIAL COORDINATE SYSTEM                                          2421H144     .199
      XS = RHOS * (CTRL * CSNO - STRL * SSNO * CIS)                       2430H144     .200
      YS = RHOS * (CTRL * SSNO + STRL * CSNO * CIS)                       2440H144     .201
      ZS = RHOS * STRL * SIS                                              2450H144     .202
      SEPT  =  SIN (GAMMA)                                                2453H144     .203
      CEPT  =  COS (GAMMA)                                                2456H144
C16.0 ACCELERATION COMPONENTS FOR NUMERICAL RESIDUAL PERTURBATION          2460H144
C     EQUATIONS OF MOTION                                                 2465H144
C16.1 COMPUTATION OF THE SMALL DIFFRENCES BETWEEN THE TWO BODY TERMS       2470H144     .204
  631 Q = ( (XS + DX / 2.) * DX  + (YS + DY / 2.) * DY  +                 2480H144
     1      (ZS + DZ / 2.) * DZ ) / RHOS ** 2                             2490H144     .205
      TERM(1) = 1.                                                        2500H144     .206
      DO 625 I = 2, 40                                                    2510H144     .207
      FRA (I) = Q * TNUM(I) / TDEN(I)                                     2520H144     .208
      TERM(I) = TERM(I - 1) * FRA(I)                                      2530H144     .209
      IF (ABS(TERM(I))-2.E-9)624,624, 626                                 2540H144     .210
  624 FAC(I) = 1. - FRA(I)                                                2550H144     .211
      L = I - 1                                                           2560H144     .212
      DO 625 J = 2, L                                                     2570H144     .213
      K=I-J                                                               2580H144     .214
  625 FAC(J)=1.-FRA(K+1)*FAC(J-1)                                         2590H144     .215   .216
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    EXTERNAL FORMULA NUMBER   -   SOURCE STATEMENT   -   INTERNAL FORMULA NUMBER(S)

      FQ = 3.0 * Q * FAC(L)                                          2600H144   .217
      GO TO 630                                                      2610H144   .218
  626 IF (I - 40) 629, 627, 627                                      2620H144   .219
  627 WRITE (6, 628) TERM(I)                                         2630H144   .220  .221  .222
  628 FORMAT(1H0,29H THE LAST TERM IN F SERIES IS E17.8)             2640H144
      TF = PTIMF (ITP - 1)                                           2645H144   .223
      ITP = 1                                                        2647H144   .224
      GO TO 340                                                      2650H144   .225
  629 CONTINUE                                                       2660H144
C16.2 COMPUTE NUMERICAL RESIDUAL PERTURBATION ACCELERATION COMPONENTS  2670H144   .226  .227
  630 X = XS + DX                                                    2680H144   .228
      Y = YS + DY                                                    2690H144   .229
      Z = ZS + DZ                                                    2700H144   .230
      RHOSC = X*X + Y*Y + Z*Z                                        2710H144   .231
      RSQ = 1. + EP2 * (2. * (X * CEPT + Y * SEPT) + EP2 * RHOSQ)    2740H144   .232
      R = SQRT (RSQ)                                                 2750H144   .233
      RCU = RSQ * R                                                  2760H144   .234
      FPR1 = C8 * EP2 / RCU                                          2770H144   .235
      FPR2 = C8 * EP2 * (2. * (X * CEPT + Y * SEPT) + EP2 *          2780H144
     1 RHOSQ) * (1. + R * (1. + R)) / RCU / (1. + R)                 2790H144   .236
      RHOSCU = RHOS ** 3                                             2800H144   .237
      B1 =   C15 + C16 * RHOS                                        2803H144   .238
      B2 =   C17 + C18 * RHOS                                        2804H144   .239
      DX2D = (FQ * X - DX) / RHOSCU - EPR1* X + EPR2 * CEPT          2810H144
     1+STRL * SSNO * B1 + CTRL * CSNO * B2 - SIPER * SSNO * C22 - COPER 2811H144
     2 * CSNO  * C23                                                 2812H144   .240
      DY2D = (FQ * Y - DY) / RHOSCU - EPR1 * Y + EPR2 * SEPT         2880H144
     1 - STRL * CSNO * B1 + CTRL * SSNO * B2 + SIPER * CSNO * C22 -COPER2881H144
     2 * SSNO * C23                                                  2882H144   .241
      DZ2D = (FQ * Z - DZ) / RHOSCU - EPR1 * Z                       2950H144
     1 +  STRL *(C21 + RHOS * C19 ) -SIPER * C20                     2951H144
C16.3 TEST THE RUNGE-KUTTA FLAG, KR                                  2975H144   .242
      GO TO (121, 300, 300, 300, 300),KR                             2980H144
C16.4 TEST THE PRINT FLAG, IPRINT                                    2985H144   .243
  121 GO TO (120, 300, 120), IPRINT                                  2990H144
C17.0 COMPUTATIONS FOR PRINT ONLY                                    3000H144
C17.1 REDUCE ALL ANGLES THAT INCREASE WITH TIME BY INTEGER MULTIPLES OF 3010H144
C     2 PI                                                           3011H144   .244
  120 TS = AMOD (TS, PSS)                                            3020H144   .245
      TNOD = AMOD (TNOD, PNOD)                                       3030H144   .246
      TPER = AMOD (TPER, PPER)                                       3040H144   .247
      GAMMA = AMOD (GAMMA, 6.2831853)                                3050H144
C17.1.1 TEST VALUE OF T SUB PSI                                      3055H144   .248
      IF (TPSI - T1)1007, 632, 632                                   3060H144
C17.1.2 SET T SUB PSI                                                3065H144   .249
  632 TPSI = TPSI - T1                                               3070H144   .250
      PHI1=AMOD(PHI1+6.2831853 / (1.+ EP2 * OMEG1) - EP2 * GNU1 * T1, 3075H144
     1 6.2831853)                                                    3076H144   .251
      ALPHA1 = AMOD( AMOD ( ALPHA1 + 6.2831853 * P1, 6.2831853)      3090H144
     1 + 6.2831853 * P1  , 6.2831853 )                               3091H144   .252
      CALPHA = COS (ALPHA1)                                          3094H144   .253
      SALPHA = SIN (ALPHA1)                                          3097H144   .254
      ALPHA2 = AMOD( AMOD ( ALPHA1 + 6.2831853 * P1, 6.2831853)      3100H144
     1 + 6.2831853 * P1 ,  6.2831853)                                3101H144   .255
      THETA1 = AMOD (THETA1 + 6.2831853 * C11, 6.2831853)            3110H144   .256
      STHETA = SIN (THETA1)                                          3114H144   .257
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   H144    JERRY                                                     09/01/65
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      CTHETA = COS (THETA1)                                            3117H144    ,258
      THETA2 = AMOD (THETA1 + 6.2831853 * C11, 6.2831853)             3120H144    ,259
      T1 = 6.2831853 * A32 + C1 * (SIN(THETA2) -SIN(THETA1))           3130H144
     1 - 1.375 * A92 * EP2     * (SIN (ALPHA2) - SIN (ALPHA1))         3140H144    ,260
      BETA = AMOD (BETA + 6.2831853 * P3, 6.2831853)                   3150H144
C17.2 SOLUTION OF THE PERTURBED KEPLER EQUATION                        3155H144
C  17.2.1 COMPUTE INITIAL GUESS AT E                                   3160H144    ,261
 1007 ECAN =(TPSI/A32) - EC1 * SIN(TPSI/A32) * (1. - EC1 * COS(TPSI/ A323165H144
     1 ))                                                              3166H144    ,262
      DLTEP= 10000.                                                    3170H144
C  17.2.2 COMPUTE SINE AND COSINE OF PSI                               3175H144    ,263
    8 SEC = SIN (ECAN)                                                 3180H144    ,264
      CEC = COS (ECAN)                                                 3185H144    ,265
      DEN = 1. + EC1 * CEC                                             3190H144    ,266
      SPS = EC2 * SEC / DEN                                            3200H144    ,267
      CPS =(CEC + EC1) / DEN                                           3205H144
C  17.2.3 COMPUTE PSI                                                  3210H144
C  17.2.3.1 TEST SIN PSI                                               3211H144    ,268
      IF ( SPS)  150,125,150                                           3215H144
C  17.2.3.2  SET PSI = ZERO OR PI                                      3220H144    ,269
  125 PSI = 1.5707963 * (1. - SIGN (1., CPS))                          3225H144    ,270
      GO TO 3                                                          3230H144
C17.2.3.3 COMPUTE PRINCIPLE VALUE OF PSI                               3232H144    ,271
  150 PSI = ATAN (SPS/CPS)                                             3234H144
C17.2.3.4 TEST COS PSI                                                 3235H144    ,272
      IF(CPS) 170,180,2                                                3240H144    ,273
  170  PSI = PSI + 3.1415926                                           3243H144    ,274
       GO TO 3                                                         3246H144
C17.2.3.5 SET PSI = PI/2 OR 3PI/2                                      3249H144    ,275
  180 PSI = 1.5707963 * (2. - SIGN(1.,SPS))                            3251H144    ,276
      GO TO 3                                                          3253H144
C17.2.3.6 PLACE PSI IN THE FIRST OR THIRD QUADRANT                     3255H144    ,277
    2 PSI = SIGN(PSI +3.14159265 * (1.- SIGN(1.,SPS) + 2. *  AINT (ECAN 3257H144
     1 / 6.2831853)), ECAN)                                            3258H144
C  17.2.4 COMPUTE NEW GUESS AT ECENTRIC ANOMOLY                        3259H144    ,278
    3 C11PS = C11 * PSI                                                3280H144    ,279
      SC11PS = SIN(C11PS)                                              3290H144    ,280
      CC11PS = COS(C11PS)                                              3300H144    ,281
      CP1 = CC11PS * CTHETA - STHETA * SC11PS                          3310H144    ,282
      SP1 = SC11PS * CTHETA + STHETA * CC11PS                          3320H144    ,283
      C2P1PS =  CC11PS * CPS    - SC11PS * SPS                         3330H144    ,284
      S2P1PS =  SC11PS * CPS + CC11PS * SPS                            3340H144    ,285
      CP1PS = C2P1PS * CALPHA - S2P1PS * SALPHA                        3350H144    ,286
      SP1PS = S2P1PS * CALPHA + C2P1PS * SALPHA                        3360H144    ,287
      DT1DPS = -C1 * (CPS - CP1) + A92 * (3.3333333 * CPS - 2.75 * CP1PS3370H144
     1 ) * EP2                                                         3371H144    ,288
      DT1DE = EC2 * DT1DPS / DEN                                       3380H144    ,289
      TONE = - C1 *(SPS - SP1 + STHETA)+ EP2 * A92 * (3.3333333 * SPS - 3390H144
     1 1.375 * (SP1PS - SALPHA))                                       3391H144    ,290
      EN = (A32 * EC1 * ( ECAN * CEC - SEC) +ECAN*DT1DE+TPSI-TONE)/(   3400H144
     1A32 * DEN +DT1DE)                                                3401H144    ,291
      IF ( EN - ECAN) 4, 235, 4                                        3410H144    ,292
    4 IF     (ABS(EN - ECAN) - DLTEP) 5, 6, 6                          3420H144    ,293
    5 DLTEP = ABS(EN - ECAN)                                           3430H144    ,294
      ECAN = EN                                                        3440H144    ,295
      GO TO 8                                                          3450H144
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      FXTERNAL FORMULA NUMBER   -   SOURCE STATEMENT   -   INTERNAL FORMULA NUMBER(S)

C17.2.5 STCRE MAXIMUM FINAL VALUE OF DELTA E PERTURBED                3459H144   .296
    6 DEPM = AMAX1 ( DEPM, ABS (DLTEP))                               3460H144
C17.3 COMPUTE AUXILIARY FUNCTIONS OF PSI AND T                        3580H144   .297
  235 CP3 = COS (P3 * PSI) * COS (BETA) - SIN (P3 * PSI) *            3590H144
     1 SIN (BETA)                                                     3600H144   .298
      SP3 = SIN (P3 * PSI) * COS (BETA) + COS (P3 * PSI) *            3610H144
     1 SIN (BETA)                                                     3620H144   .299
      S = (1. - EC1 * CPS) / AEC + C2 * (CPS - CP1)                   3630H144
     1 + A2 * ((2. - 5. * CPS) / 3. + CP1PS) *EP2                     3640H144   .300
      IF (S) 240, 240, 250                                            3650H144   .301
  240 NDIV = 7                                                        3660H144   .302
      GO TC 30                                                        3670H144
C17.4 COMPUTE DOUBLE PRIME COORDINATES                                3680H144   .303
  250 PHI  = PSI / (1. + EP2 * OMEG1) - EP2 * GNU1 * TPSI             3690H144   .304
      X2P  =(COS (PHI) * COS(PHI1) - SIN (PHI) * SIN (PHI1))/ S       3700H144   .305
      Y2P  =( SIN(PHI) * COS(PHI1) + COS (PHI) * SIN (PHI1))/ S       3710H144   .306
      Z2P = C3 * (SP3 - P3 * SPS)                                     3720H144
C17.5 COMPUTE NODE                                                    3730H144   .307
      ARNODE = C6 * TNOD                                              3740H144   .308
      CNO = COS (ARNODE)                                              3750H144   .309
      SNO = SIN (ARNODE)                                              3760H144
C17.6 COMPUTE APPROXIMATE COORDINATES IN PLANE OF PLANETS             3770H144
C     SINGLE PRIME SYSTEM                          6.1.6.2.6.3        3780H144   .310
      XP = X2P * CNO - Y2P * SNO * CI + Z2P * SNO * SI                3790H144   .311
      YP = X2P * SNO + Y2P * CNO * CI - Z2P * CNO * SI                3800H144   .312
      ZP =             Y2P     *      SI + Z2P     *      CI          3810H144
C17.7 COMPUTE TIME DERIVATIVES OF PSI, PHI AND S                      3820H144   .313
      DTDPSI = C9 / ( (1. - EC1 * CPS) ** 2) - C1 *                   3830H144
     1  (CPS - CP1)                                                   3840H144
     2 + AS2 * (10. / 3. * CPS - 2.75 * CP1PS) * EP2                  3850H144
C17.8 TEST THE VALUE OF THE DERIVATIVE OF TIME WITH RESPECT TO PSI    3855H144   .314
      IF (DTDPSI) 270, 260, 270                                       3860H144
C17.8.1 SET THE ERROR CODE = 8 AND GO TO STEP 3.2.1.2                 3865H144   .315
  260 NDIV =8                                                         3870H144   .316
      GO TC 30                                                        3890H144
C17.9 COMPUTE TIME DERIVATIVES OF PHI AND S                           3895H144   .317
  270 DPSIDT = 1. / DTDPSI                                            3900H144   .318
      DPHIDT = DPSIDT /(1. + EP2 * OMEG1) - EP2 * GNU1                3910H144   .319
      DSDPSI = EC1 * SPS / AEC + C2 * (SP1 - SPS)                     3920H144
     1 + A2 * (5. / 3. * SPS - 2. * SP1PS) * EP2                      3930H144   .320
      DSDT = DSDPSI * DPSIDT                                          3940H144
C17.10 COMPUTE APPROXIMATE VELOCITY COMPONENTS IN ORBIT PLANE         3950H144   .321
      XD2P = - Y2P * DPHIDT - X2P * DSDT / S                          3960H144   .322
      YD2P =   X2P * DPHIDT - Y2P * DSDT / S                          3970H144   .323
      ZD2P = C3 * (CP3 - CPS) * DPSIDT                                3980H144
C17.11 COMPUTE RHO                                                    3990H144   .324
      RHO = SQRT (RHUSQ)                                              4000H144
C17.11.1 IF ICH = 1 GO TO 17.12.  IF ICH = 2 GO TO 17.14.1            4005H144   .325
      GO TO (800, 801), ICH                                           4010H144
C17.12 COMPUTE APPROXIMATE VELOCITY COMPONENTS IN INERTIAL SYSTEM     4015H144   .326
  800 XDP = C6 * (-X2P * SNO - Y2P * CNO*CI + Z2P * CNO*SI)           4020H144
     1 +XD2P * CNO -YD2P * SNO * CI +ZD2P * SNO * SI                  4030H144   .327
      YDP = C6 * (X2P * CNO -Y2P * SNO * CI +Z2P * SNO * SI)          4040H144
     1 +XD2P * SNO +YD2P * CNO * CI - ZD2P * CNO * SI                 4050H144   .328
      ZDP =           YD2P     *     SI + ZD2P     *     CI           4060H144
C17.13 COMPUTE PRECESSING KEPLERIAN VELOCITY COMPONENTS IN INERTIAL   4070H144
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  H144    JERRY                                                        09/01/65                                PAGE  9
      EXTERNAL FORMULA NUMBER   -   SOURCE STATEMENT   -   INTERNAL FORMULA NUMBER(S)

C       SYSTEM                                                        4071H144   ,329
        XSD = - (STRL * CSNO + CTRL * SSNO * CIS                      4080H144
       1 - FS * (SIPER * CSNO + COPER * SSNO * CIS) ) / AESQ          4090H144
       2 - RHOS *(DNOD *(CTRL * SSNO + STRL * CSNO * CIS)             4100H144
       3 + DPER * (STRL * CSNO + CTRL * SSNO * CIS))                  4110H144   ,330
        YSD = - (STRL * SSNO - CTRL * CSNO * CIS                      4120H144
       1 - FS * (SIPER * SSNO - COPER * CSNO * CIS) ) / AESQ          4130H144
       2 + RHOS *(DNOD *(CTRL * CSNO - STRL * SSNO * CIS)             4140H144
       3 -        DPER *(STRL * SSNO - CTRL * CSNO * CIS))            4150H144   ,331
        ZSD = ((CTRL  - FS * COPER) / AESQ                            4160H144
       1 + RHOS * DPER * CTRL) * SIS                                  4170H144
C17.14 COMPUTE TOTAL INERTIAL VELOCITY COMPONENTS AND VECTOR MAGNITUDES 4175H144   ,332
        XD = XSD + DXD                                                4180H144   ,333
        YD = YSD + DYD                                                4190H144   ,334
        ZD = ZSD + DZD                                                4200H144
C17.14.1 COMPUTE INERTIAL VELOCITY                                    4205H144   ,335
   801 VSQ = XD * XD + YD * YD + ZD * ZD                              4210H144   ,336
        RHOD = SQRT (VSQ)                                             4220H144
C17.15 COMPUTE JACOBI CONSTANT FRACTIONAL ERRORS                      4225H144   ,337
        R = SQRT (RSQ)                                                4230H144   ,338
        YACOBI = VSQ + 2. * (EP1 * (XD * Y - YD * X) - 1. /RHO        4240H144
       1 - C8 * (X * CEPT + Y * SEPT)) - ACOBI + 2. * C8 * (2.        4250H144
       2 * (X * CEPT + Y * SEPT - XNAUT) + EP2 * (RHOSQ - XNAUT       4260H144
       3 ** 2)) / R / RNAUT / (RNAUT + R)                             4270H144   ,339
        YACOBI = YACOBI / JACOBI                                      4280H144   ,340
        RHOASQ = XP ** 2 + YP ** 2 + ZP ** 2                          4290H144   ,341
        RA = SQRT (1. + EP2 * (2. * (XP * CEPT + YP * SEPT) +         4300H144
       1 EP2 * RHOASQ))                                               4310H144   ,342
        YACAPP = XDP ** 2 + YDP ** 2 + ZDP ** 2 + 2. * (EP1 *         4320H144
       1 (XDP * YP - YDP * XP) - 1. / SQRT (RHOASQ) - C8 * (XP        4330H144
       2 * CEPT + YP * SEPT)) - ACOBI + 2. * C8 * (2. * (XP *         4340H144
       3 CEPT + YP * SEPT - XNAUT) + EP2 * (RHOASQ - XNAUT ** 2       4350H144
       4 )) / RA / RNAUT / (RNAUT + RA)                               4360H144   ,343
        YACAPP = YACAPP / JACOBI                                      4370H144   ,344
        RHOSQ = XS *XS + YS *YS + ZS * ZS                             4380H144   ,345
        RHOS = SQRT ( RHOSQ )                                         4390H144   ,346
        RS = SQRT ( 1. + EP2 * ( 2. * ( XS * CEPT + YS * SEPT )       4400H144
       1 + EP2 * RHOSQ ))                                             4401H144   ,347
        YACAS = ( XSD * XSD + YSD * YSD + ZSD * ZSD + 2.* ( EP1 * ( XSD   4410H144
       1 * YS - YSD * XS ) - 1. / RHOS - C8 * ( XS * CEPT + YS * SEPT ) ) 4411H144
       2 - ACOBI + 2. * C8 * ( 2. * ( XS * CEPT + YS * SEPT - XNAUT )  4412H144
       3 + EP2 * ( RHOSQ - XNAUT * XNAUT ) ) / RS / RNAUT / ( RNAUT + RS))4413H144
       4 / JACOBI                                                     4414H144   ,348
        GO TO (810, 320, 285), KHALT                                  4730H144
C17.16 TEST IF PRINTING IS DESIRED                                    4740H144   ,349
   810 IF (W(13)) 325, 325, 285                                       4745H144
C10.0  OUTPUT OF NUMERICAL RESIDUAL PERTURBATION SOLUTION             4747H144   ,350
   285 WRITE (6, 904) T, DX, DXD, DX2D, RHO, DY, DYD, DY2D,           4750H144
       1 RHOD, YACAPP, DZ, DZD, DZ2D                                  4760H144
       2, X, XD, XP, XDP, Y, YD, YP, YDP, YACOBI, Z, ZD, ZP, ZDP      4770H144   ,351  ,352  ,353
   904 FORMAT(1H0,/5F17.8/E34.8,3E17.8/4E17.8//E34.8,3E17.8/          4810H144
       1E34.8, 3F17.8/5F17.8)                                         4820H144
        GO TO (300, 325), IP                                          4840H144
C18.0  SAVE PLOT VALUES AND TEST IF PLOTTING ARRAY IS FULL            4845H144
C18.1  SAVE VALUES FOR PLOT ROUTINE AND FOR LATER COMPARISON WITH VALUES4850H144
C      FROM NUMERICAL SOLUTION OF TOTAL EQUATIONS OF MOTION           4851H144   ,354
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       EXTERNAL FORMULA NUMBER   -   SOURCE STATEMENT   -   INTERNAL FORMULA NUMBER(S)

  325 PTIME (ITP) = T                                              4859H144  ,355
      IT = 13 * ITP - 12                                           4860H144  ,356
      AP (IT) = ABS (YACAPP)                                       4861H144  ,357
      AP (IT + 1) = ABS (YACAS)                                    4862H144  ,358
      AP (IT + 2) = X                                              4863H144  ,359
      AP (IT + 3) = Y                                              4864H144  ,360
      AP (IT + 4) = ABS ( YACOBI)                                  4865H144  ,361
      AP (IT + 5) = SORT ((XP - X) * * 2 +(YP - Y) * * 2           4866H144
     1 + (ZP - Z) ** 2) / A1                                       4867H144  ,362
      AP (IT + 6) = SORT ((XS - X) * *2  +(YS - Y) * *2            4868H144
     1 + (ZS - Z) ** 2) / A1                                       4869H144  ,363
      AP (IT + 7) =  Z                                             4870H144  ,364
      AP (IT + 8) = XD                                             4871H144  ,365
      AP (IT + 9) = A12 *  SORT((XDP - XD) * * 2 +(YDP -           4872H144
     1 YD) ** 2 + (ZDP - ZD) ** 2)                                 4873H144  ,366
      AP (IT + 10) = A12 * SORT((XSD - XD) * * 2 +(YSD -           4874H144
     1 YD) ** 2 + (ZSD - ZD) ** 2)                                 4875H144  ,367
      AP (IT + 11) = YD                                            4876H144  ,368
      AP (IT + 12) = ZD                                            4877H144
C18.2 TEST THE VALUE OF ITP                                        4879H144  ,369
      IF (ITP -  700) 326, 327, 327                                4880H144  ,370
  326 ITP = ITP + 1                                                4881H144  ,371
       PTIME (ITP) = TF                                            4882H144  ,372
      GO TO 290                                                    4883H144  ,373
  327 TF = PTIME (ITP)                                             4884H144  ,374
      GO TO 340                                                    4885H144
C11.0 TEST HALT FLAG, KHALT                                        4910H144  ,375
  290 GO TO (300, 320, 340), KHALT                                 4920H144
C12.0 CALL NUMERICAL SOLUTION SUBROUTINE FOR RESIDUAL PERTURBATION 4930H144
C     EQUATIONS                                                    4935H144  ,376
  300 CALL RKSIMP (KR, IP, KC, T, DT, ICH, DX, DY, DZ, DXD,        4940H144
     1 DYD, DZD, DX2D, DY2D, DZ2D, KHALT, TPRINT, TF)              4950H144
C 13.0 TEST RUNGE-KUTTA FLAG                                       4960H144  ,377
      GO TO (310, 610, 631, 610, 120), KR                          4970H144
C 14.0 TEST HALT FLAG FOR INTEGRATION ACCURACY ERROR               4980H144  ,378
  310 GO TO (631, 320, 631), KHALT                                 4990H144
C14.1 PRINT COMPUTING INTERVAL SELECTION FAILED                    4995H144  ,379
  320 WRITE (6, 330)                                               5000H144  ,380  ,381
  330 FORMAT(1H0,35H COMPUTING INTERVAL SELECTION FAILS)           5010H144
      TF = PTIME (ITP - 1)                                         5020H144  ,382
      GO TO (340, 440), ICH                                        5022H144
C19.0  INITIATE NUMERICAL SOLUTION OF TOTAL EQUATIONS OF MOTION    5030H144
C19.1  SET PERTURBATION-TOTAL FLAG                                 5035H144  ,383
  340 ICH = 2                                                      5040H144  ,384
      KT = 0                                                       5050H144
C19.2  COMPUTE INITIAL CONDITIONS FOR NUMERICAL SOLUTION OF TOTAL  5060H144
C      EQUATIONS OF MOTION                                         5065H144  ,385
      X = A1 * EC3                                                 5070H144  ,386
      Y = C.                                                       5080H144  ,387
      Z = C.                                                       5090H144  ,388
      GAMMA = 0.                                                   5100H144  ,389
      RHO = X                                                      5110H144  ,390
      T = C.                                                       5120H144  ,391
      DTDPSI = A32 * EC3 ** 1.5 / SQRT (1. - EC1)                  5130H144
     1 + AS2 * EP2 * 7.             / 12.                          5150H144  ,392
      DPSIDT = 1. / DTDPSI                                         5160H144  ,393
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      H144    JERRY                                                   09/01/65                                   PAGE  11
         EXTERNAL FORMULA NUMBER    -    SOURCE STATEMENT    -    INTERNAL FORMULA NUMBER(S)

        DPHIDT = DPSIDT / ( 1. + EP2 * OMEG1 ) - EP2 * GNU1               5170H144    ,394
        XD = 0.                                                           5180H144    ,395
        YD = X *(DPHIDT * CI + C6)                                        5190H144    ,396
        ZD = X * DPHIDT * SI                                              5200H144    ,397
        RHOD = SQRT (YD ** 2 + ZD **2)                                    5210H144    ,398
        RCU = (1. + EP2 * X) ** 3                                         5220H144    ,399
        EP2X = EP2 * X                                                    5230H144    ,400
        X2D = - 1. / X ** 2 + C8 * EP2X * (2. + EP2X * (3. +              5240H144
       1 EP2X)) / RCU                                                     5250H144    ,401
        Y2D = 0.                                                          5260H144    ,402
        Z2D = 0.                                                          5270H144    ,403
        YACOBI = 0.                                                       5280H144
C19.3   COMPUTE SEMI-MAJOR AXIS AND ECCENTRICITY OF INIT. OSC. ELLIPSE    5281H144    ,404
        AS = 1. / (2. / RHO - RHOD ** 2)                                  5282H144    ,405
        ES = X / AS - 1.                                                  5283H144    ,406
        AS32 = AS ** 1.5                                                  5284H144    ,407
        PSS = 6.2831853 * AS32                                            5285H144    ,408
        AESQ = SQRT (AS * (1. - ES ** 2))                                 5286H144    ,409
        GO TO 350                                                         5290H144
C19.4   PRINT HEADER FOR NUMERICAL SOLUTION OF TOTAL EQUATIONS OF MOTION 5300H144
C       PRINT                                                             5305H144    ,410
  360 WRITE (6, 370)                                                      5310H144    ,411  ,412
  370 FORMAT(1H1,37H INTEGRATION OF COWELL EQNS OF MOTION//6X5H TIME,16X5320H144
     12H X,17X6H X DOT,13X10H X DBL DOT,14X4H RHO/27X2H Y,17X6H Y DOT,135330H144
     2X10H Y DBL DOT,12X8H RHO DOT/17H JACOBI CONSTANT,10X2H Z,17X6H Z D5340H144
     3OT,13X10H Z DBL DOT)                                                5350H144
C20.0   PRINT NUMERICAL SOLUTION OF TOTAL EQUATIONS OF MOTION DATA        5360H144
  380 IT = 13 * ITP - 12                                                  5363H144    ,413
        ITO = IT + 12                                                     5366H144    ,414
        WRITE (6, 390) T, X, XD, X2D, RHO, Y, YD, Y2D,RHOD,YACOBI, Z, ZD, 5370H144
       1 Z2D, (AP(I), I = IT,ITO)                                         5380H144    ,415  ,416  ,417  ,418  ,419  ,420

  390 FORMAT(1H0,/E16.8,E19.8,3E21.8/E35.8,3E21.8/E16.8,                  5390H144
       1 E19.8, 2E21.8 /5E20.8/4E20.8)                                    5400H144
C21.0 TEST THE HALT FLAG, KHALT                                           5410H144
  400 GO TO (410, 320, 440), KHALT                                        5420H144
C22.0   CALL NUMERICAL SOLUTION SUBROUTINE FOR TOTAL EQUATIONS OF MOTION 5430H144    ,421
  410 CALL RKSIMP (KR, IP, KC, T, DT, ICH, X, Y, Z, XD, YD,               5440H144
       1 ZD, X2D, Y2D, Z2D, KHALT, TPRINT, TF)                            5450H144
C23.0 TEST THE RUNGE-KUTTA FLAG, KR                                       5460H144    ,422
        GO TO (420, 275, 275, 275, 283), KR                               5470H144
C24.0 TEST THE HALT FLAG, KHALT                                           5480H144    ,423
  420 GO TO (275, 320, 275), KHALT                                        5490H144
C25.0   COMPUTE TOTAL EQUATIONS OF MOTION                                 5500H144    ,424
  275 RHOSQ = X * X + Y * Y + Z * Z                                       5510H144    ,425
        RHOCU = RHOSQ ** 1.5                                              5520H144    ,426
        GAMMA = AMOD (GAMMA, 6.2831853    )                               5530H144    ,427
        CEPT = COS (GAMMA)                                                5540H144    ,428
        SEPT = SIN (GAMMA)                                                5550H144    ,429
        RSQ = 1. + EP4 * RHOSQ + 2. * EP2 * (X *CEPT +Y *SEPT)            5560H144    ,430
        R = SQRT (RSQ)                                                    5570H144    ,431
        RCU = RSQ * R                                                     5580H144    ,432
        EPR1 = C8 * EP2 / RCU                                             5590H144    ,433
        EPR2 = C8 * EP2 * (2. * (X * CEPT + Y * SEPT) + EP2 *             5600H144
       1 RHOSQ) * (1. + R * (1. + R)) / RCU / (1. + R)                    5610H144    ,434
        X2D = - X / RHOCU - EPR1 * X + EPR2 * CEPT                        5620H144    ,435
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    H144    JERRY                                                    09/01/65
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       Y2D = - Y / RHOCU - EPR1 * Y + EPR2 * SEPT                    5630H144    ,436
       Z2D = - Z / RHOCU - EPR1 * Z                                  5640H144
C26.0  TEST THE RUNGE-KUTTA FLAG, KR                                 5645H144    ,437
       GO TO (411, 410, 410, 410, 410), KR                           5650H144
C27.0  TEST THE PRINT FLAG, IPRINT                                   5655H144    ,438
   411 GO TO (283, 410, 410), IPRINT                                 5660H144
C28.0  COMPUTE DATA FOR PRINT AND FOR PLOT ROUTINE                   5670H144
C28.1   COMPUTE ERROR IN JACOBI INTEGRAL AND LIMIT T SUB S FOR NUMERICAL 5675H144
C        SOLUTION OF TOTAL EQUATIONS                                 5676H144    ,439
   283 RHO = SQRT (RHOSQ)                                            5680H144    ,440
       R = SQRT (RSQ)                                                5690H144    ,441
       VSQ = XD **2 + YD ** 2 + ZD ** 2                              5700H144    ,442
       RHOD = SQRT (VSQ)                                             5710H144    ,443
       YACOBI = VSQ + 2. * (EP1 * (XD * Y - YD * X) - 1. /RHO        5720H144
      1 - C8 * (X * CEPT + Y * SEPT)) - ACOBI + 2. * C8 * (2.        5730H144
      2 * (X * CEPT + Y * SEPT - XNAUT) + EP2 * (RHOSQ - XNAUT       5740H144
      3 ** 2)) / R / RNAUT / (RNAUT + R)                             5750H144    ,444
       YACOBI = YACOBI / JACOBI                                      5752H144    ,445
       TS = AMOD (TS, PSS)                                           5755H144    ,446
       GO TO 610                                                     5758H144
C28.2  COMPUTE THE FIXED ORBIT VALUES                                5760H144    ,447
   430 XF = RHOS * COV                                               5763H144    ,448
       YF = RHOS * SIV * CI                                          5764H144    ,449
       ZF = RHOS * SIV * SI                                          5765H144    ,450
       XFD = - SIV / AESQ                                            5766H144    ,451
       YFD = (COV - FS) / AESQ * CI                                  5767H144    ,452
       ZFD = (COV - FS) / AESQ * SI                                  5768H144    ,453
       RHOFSQ = XF ** 2 + YF ** 2 + ZF ** 2                          5769H144    ,454
       RHOF = SQRT (RHOFSQ)                                          5770H144    ,455
       RF = SQRT (1. +EP2 * (2. * (XF * CEPT + YF * SEPT) +          5771H144
      1 EP2 * RHOFSQ))                                               5772H144    ,456
       YACAF = XFD ** 2 + YFD ** 2 + ZFD ** 2 + 2. * (EP1 *          5773H144
      1 (XFD * YF - YFD * XF) - 1. / RHOF - C8 * (XF * CEPT +        5774H144
      2 YF * SEPT)) - ACOBI + 2. * C8 * (2. * (XF * CEPT + YF        5775H144
      3 * SEPT - XNAUT) + EP2 * (RHOFSQ - XNAUT ** 2)) / RF /        5776H144
      4 RNAUT / (RNAUT + RF)                                         5777H144    ,457
       IT = 13 * ITP - 12                                            5778H144    ,458
       YACAF = YACAF / JACOBI                                        5782H144    ,459
       DRF = SQRT ((XF - AP(IT + 2))** 2 + (YF - AP(IT +             5785H144
      1 3)) ** 2 + (ZF - AP(IT + 7)) ** 2) / A1                      5786H144    ,460
       DRDF = A12 * SQRT ((XFD - AP(IT + 8)) ** 2 + (YFD             5790H144
      1 - AP(IT + 11)) ** 2 + (ZFD - AP(IT + 12)) ** 2)              5791H144    ,461
       DRC = SQRT ((X - AP(IT + 2)) ** 2 + (Y - AP(IT +              5795H144
      1 3)) ** 2 + (Z - AP(IT + 7)) ** 2) / A1                       5796H144    ,462
       DRDC = A12 * SQRT ((XD - AP(IT + 8)) ** 2 + (YD -             5800H144
      1 AP(IT + 11)) ** 2 + (ZD - AP(IT + 12)) ** 2 )                5801H144
C28.3  SET QUANTITIES JUST COMPUTED EQUAL TO VALUES IN PLOT ARRAY    5805H144    ,463
        AP(IT + 2) = ABS (YACOBI)                                    5810H144    ,464
        AP(IT + 3) = ABS (YACAF)                                     5812H144    ,465
        AP(IT + 7) = DRC                                             5814H144    ,466
        AP(IT + 8) = DRF                                             5816H144    ,467
        AP(IT + 11) = DRDC                                           5818H144    ,468
        AP(IT + 12) = DRDF                                           5820H144
C29.0  TEST VALUES OF HALT FLAG, KHALT                               5822H144    ,469
       GO TO (815, 320, 380), KHALT                                  5824H144
C30.0   TEST IF PRINTING IS DESIRED                                  5826H144    ,470
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      H144    JERRY                                                                 09/01/65                              PAGE  13
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   815 IF (W(13)) 410, 410, 380                                                     5828H144
 C31.0 CALL THE PLOT SUBROUTINE                                                     5829H144    ,471
   440 CALL PLOT (A32,TF,A1,EC1,DEGI,EP4,JACOBI,PSS)                                5831H144    ,472
       WRITE (6,230) DEM,DEPM                                                       5832H144    ,473  ,474  ,475
   230 FORMAT(1H016HMAXIMUM DELTA E= E17.8,27HMAXIMUM DELTA E PERTURBED =5833H144
     1 F17.8                         )                                              5834H144
 C32.0  RUN IS COMPLETED, CALL THE NEXT CASE AT STEP 4.2                            5835H144
       GO TO 10                                                                     5836H144    ,476
       END                                                                          5838H144    ,477
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Section 7

SUBROUTINE FOR NUMERICAL SOLUTION OF DIFFERENTIAL EQUATIONS, RKSIMP

7.1 EQUATIONS IN ORDER OF SOLUTION

7.1.1 Subroutine Input

7.1.1.1 Calling Sequence

KR	Runge-Kutta flag; takes on five values from 1 to 5 indicating the cycle of Runge-Kutta. Initially set = 1 by the main program and thereafter under the control of the subroutine.
IP	Initial point flag; signals first point calculations by the subroutine. Initially set to 1 by the main program and advanced to 2 by the subroutine.
KC	Simpson's rule computation flag. The computations are bypassed if KC = 1, computed if KC = 2. Wholly under the control of the subroutine.
t	Time; initial value set by main program, thereafter under control of the subroutine.
Δt	Computing interval; initial value is input; if input is left blank, it is set equal to 0.005 by the subroutine.
ICH	A flag that signals which equations are being integrated. ICH = 1 corresponds to numerical residual perturbation solution

	ICH = 2 corresponds to numerical solution of total equations of motion
	Wholly under control of the main program.
x_h, y_h, z_h $\dot{x}_h$, $\dot{y}_h$, $\dot{z}_h$ $\ddot{x}_h$, $\ddot{y}_h$, $\ddot{z}_h$	Dummy variables of position, velocity, and acceleration, respectively.
KHALT	Halt = 2, call next case.
t_p	Print time
T_f	Run stop time

7.1.1.2 Common

W	The input array
IPRINT	Print flag; print if equal to 1. Initially set = 1 by main program for numerical residual perturbation solution Initially set = 2 by main program for numerical solution of total equations of motion Under control of subroutine during numerical solution of total equations of motion
t_s	Time modulo the period of the precessing ellipse
t_ψ	Time since last passage of the apocenter by the analytic solution

t_Ω	Time modulo the period of the node
t_ω	Time modulo the period of the apocenter
γ	$= \varepsilon t$
ε	$\mu^{1/4}$
ITP	Plot point index

7.1.2 <u>Test Runge-Kutta Flag, KR</u>

If KR = 1, continue below (Step 7.1.3).

If KR = 2, go to Step 7.1.26.

If KR = 3, go to Step 7.1.27.

If KR = 4, go to Step 7.1.28.

If KR = 5, go to Step 7.1.30.

7.1.3 <u>Test Initial Flag, IP</u>

If IP = 1, first point computations must be executed; continue below (Step 7.1.4).

If IP = 2, go to Step 7.1.9.

7.1.4 <u>First Point Computations</u>

7.1.4.1 Set Flags and Initial Conditions

Set IP = 2 Advance initial point flag.

LL = 0	Obsolete.
KT = -2	Set computing interval counter = -2.
ITP = 1	Set initial value of index for plotting arrays A_p and P_t.
KC = 1	Indicates next two Runge-Kutta cycles to be at the same Δ_t.
$E_{min} = \lvert W_{12} \rvert$	Set value for the minimum allowable error equal to input value.
$FDT = \lvert W_7 \rvert$	Modifier of allowable computing interval.
KF = 0 KFAIL = 0	Set intermediate and total failure counters to zero.
$MFAIL = \lvert W_{11} \rvert$	Set the maximum number of failures allowed equal to an input value.
$t_{pT} = P_{t1}$	Set first print time for numerical solution of total equations of motion. (Note: Unnecessary on numerical residual perturbation solution, but saves coding if included here.)

$\Delta\dot{X} = 0$
$\Delta\dot{Y} = 0$
$\Delta\dot{Z} = 0$

Sets velocity increments for use in computing interval tests initially equal to zero.

$\gamma = 0$
$t_\psi = 0$
$\beta = 0$
$t_\Omega = 0$
$t_\omega = 0$
$t_s = 0$

Sets initial values of angle and time variables to zero.

$\Delta t_p = a^{3/2}/4$
$t_p = \Delta t_p$

Sets print interval to 1/4 radian of M

7.1.4.2 Set the Maximum Allowable Error Equal to the Input Value or Equal to 10^{-7} if the Input Value is Zero

$$E_{all} = |W_8| \text{ or } E_{all} = 10^{-7} \text{ if } W_8 = 0$$

7.1.4.3 Set the Computing Interval Multiplier (Used if Error is Less Than the Minimum Allowed) Equal to Input Value or Equal to 1.5 if Input Value ≤ 1

$$\Delta t_{min} = W_9 \text{ or } \Delta t_{min} = 1.5 \text{ if } W_9 \leq 1$$

7.1.4.4 Set the Initial Computing Interval Equal to Input Value or Equal to 0.005 if the Input Value is Zero

$$\Delta t = W_{10} \text{ or } \Delta t = 0.005 \text{ if } W_{10} = 0$$

7.1.4.5 Test Perturbation - Total Flag

If ICH = 1, go to Step 7.1.5 (numerical residual perturbation solution).

If ICH = 2, go to Step 7.1.4.6 (numerical solution of total equations of motion).

7.1.4.6 Test the First Print Time in Numerical Solution of Total Equations of Motion Versus Twice the Computing Interval

If $t_{pT} < 2\Delta t$, go to Step 7.1.4.7.

If $t_{pT} = 2\Delta t$, go to Step 7.1.4.8.

If $t_{pT} > 2\Delta t$, go to Step 7.1.5.

7.1.4.7 Set $\Delta t = \frac{1}{2} t_{pT}$

7.1.4.8 Set IPRINT = 3 to Signal Both Print and Testing of Computed Values

7.1.5 Save Quantities at Start of Span for Computing Interval Calculations

Save positions, velocities, accelerations, time, t_s, t_ψ, t_Ω, t_ω, θ_1, and $\sin \theta_1$ for restart in case computing interval selection fails and for use in Simpson's rule calculations.

The array SS(17) is used for this purpose with:

$$ss_1 = \text{time}, t$$

$$ss_2, ss_3, ss_4 = \text{position coordinates}, x_h, y_h, z_h$$

$$ss_5, ss_6, ss_7 = \text{velocity coordinates}, \dot{x}_h, \dot{y}_h, \dot{z}_h$$

$$ss_8, ss_9, ss_{10} = \text{acceleration components}, \ddot{x}_h, \ddot{y}_h, \ddot{z}_h$$

$$ss_{11} = t_s$$

$$ss_{12} = t_\psi$$

$$ss_{13} = t_\Omega$$

$$ss_{14} = t_\omega$$

$$ss_{15} = \theta_1$$

$$ss_{16} = \sin \theta_1$$

$$ss_{17} = \gamma$$

7.1.5.1 Increment Computing Interval Counter

$$KT = KT + 2$$

7.1.6 <u>Save Positions, Velocities, Accelerations, and Time for Ordinary Runge-Kutta Use</u>

The array S(17) is used for this purpose and the indices are ordered as is the SS array in Step 7.1.5.

7.1.7 <u>Compute the Next Value of Time (i.e., At the End of the Next Computing Interval) and Make Special Time Tests</u>

$$TN = s_1 + \Delta t$$

7.1.7.1 Test Perturbation - Total Flag

If ICH = 1 (numerical residual perturbation solutions), go to Step 7.1.7.3.

If ICH = 2 (numerical solutions of total equations of motion), continue below (Step 7.1.7.2).

7.1.7.2 Test Run Stop Time Against the Next Print Time

If $T_f > t_p$, go to Step 7.1.7.6.

If $T_f \leq t_p$, continue below (Step 7.1.7.3).

7.1.7.3 Test the Next Value of Time Against Run Stop Time

If $TN > T_f$, continue below (Step 7.1.7.4).

If $TN = T_f$, go to Step 7.1.7.5.

If $TN < T_f$, go to Step 7.1.8.

7.1.7.4 Set Computing Interval So That Time Will Equal the Run Stop Time at the End of the Next Interval

$$\Delta t = T_f - t$$

7.1.7.5 Set Print and Halt Flags

$$IPRINT = 1$$

$$KHALT = 3$$

Go to Step 7.1.8.

7.1.7.6 Test the Next Value of Time Against the Next Print Time

(Enter here from Step 7.1.7.2 if $T_f > t_p$.)

If $TN > t_p$, continue below (Step 7.1.7.7).

If $TN = t_p$, go to Step 7.1.7.8.

If $TN < t_p$, go to Step 7.1.8.

7.1.7.7 Save Initial Conditions and Compute Interval for Print Only

Save the value of computing interval for use after print, and then set the computing interval so that the "next time" will equal the print time.

$$\Delta t_{sp} = \Delta t$$

$$\Delta t = t_p - t$$

Set the print flag.

$$IPRINT = 1$$

Save the values of positions, velocities, accelerations, and time in the array SP(10) for use after print. The indices correspond to those in Steps 7.1.5 and 7.1.6.

Set $sp_I = s_I$ for $I = 1, 2, \ldots 10$.

Go to Step 7.1.8.

7.1.7.8 Set IPRINT = 3

7.1.8 Complete First Runge-Kutta Pass

Compute intermediate value of time.

$$t = s_1 + \frac{1}{2}\Delta t$$

$$t_\psi = t_\psi + \frac{1}{2}\Delta t$$

$$t_s = t_s + \frac{1}{2}\Delta t$$

$$t_\Omega = t_\Omega + \frac{1}{2}\Delta t$$

$$t_\omega = t_\omega + \frac{1}{2}\Delta t$$

$$\gamma = \gamma + \frac{\varepsilon\Delta t}{2}$$

Compute the intermediate position values.

$$x_h = s_2 + \frac{1}{2}\Delta t \times s_5$$

$$y_h = s_3 + \frac{1}{2}\Delta t \times s_6 \qquad \text{parameters for (141)}$$

$$z_h = s_4 + \frac{1}{2}\Delta t \times s_7$$

Compute the Runge-Kutta parameters.

$$RK1X = \Delta t \times s_8$$

$$RK1Y = \Delta t \times s_9 \text{ computing intervals, etc. (140)}$$

$$RK1Z = \Delta t \times s_{10}$$

Set Runge-Kutta flag, KR = 2 and return to main program.

7.1.9 Test Whether Residual Perturbation of Total Equations are Being Solved

(Enter here from Step 7.1.3 on a first pass of Runga-Kutta which is not the first point of the trajectory.)

If ICH = 1 (residual perturbation), go to Step 7.1.12.

If ICH = 2 (total), continue below (Step 7.1.10).

7.1.10 Test Print Flag

If IPRINT = 1 (print), continue below (Step 7.1.11).

If IPRINT = 2 or 3 (don't print), go to Step 7.1.12.

7.1.11 Advance Print Index and Set the Next Print Time

$$ITP = ITP + 1$$

$$t_{PT} = P_{t(ITP)}$$

If IPRINT = 1, go to Step 7.1.11.1.

If IPRINT = 2, go to Step 7.1.5.

If IPRINT = 3, go to Step 7.1.11.2.

7.1.11.1 Restore Values Saved Previously in Step 7.1.7.7

$$\Delta t = \Delta t_{sp}$$

$$s_I = sp_I \text{ for } I = 1, 2, \ldots \ldots 10$$

Set IPRINT = 2

Return to Step 7.1.7.

7.1.11.2 Set IPRINT = 2 and Go to Step 7.1.5

7.1.12 Test Simpson's Rule Flag

(Enter here from Step 7.1.9 when numerical residual perturbation solution is being computed and from Step 7.1.10 on a nonprint point during the numerical solution of the total equations of motion.)

If KC = 1, continued below (Step 7.1.13).

If KC = 2, go to Step 7.1.14.

7.1.13 Advance Simpson's Rule Flag

KC = 1 signals that the first phase of the two-cycle Runge-Kutta is in progress. KC = 2, and return to Step 7.1.6.

7.1.14 Reset Simpson's Rule Flag and Compute Actual and Allowable Errors

(Enter here from Step 7.1.12 if KC = 2. Signals that two Runga-Kutta cycles have been completed and accuracy tests can now be made.)

$$KC = 1$$

Compute the velocity increments by Simpson's rule:

$$\dot{x}_h = 1/3\ \Delta t\ (ss_8 + 4s_8 + \ddot{x}_h)$$

$$\dot{y}_h = 1/3\ \Delta t\ (ss_9 + 4s_9 + \ddot{y}_h)$$

$$\dot{z}_h = 1/3\ \Delta t\ (ss_{10} + 4s_{10} + \ddot{z}_h)$$

Compute the estimated error (E) as follows:

Set C_{max} = the maximum absolute value of the Runge-Kutta velocity increments over the last two cycles of Runge-Kutta $\Delta\dot{X}$, $\Delta\dot{Y}$, $\Delta\dot{Z}$.

Determine E as the maximum absolute value of the respective differences between the Simpson's rule and Runge-Kutta velocity increments.

Reset the Runga-Kutta increments to zero for proper computation during the next two cycles:

$$\Delta\dot{X} = \Delta\dot{Y} = \Delta\dot{Z} = 0$$

Compute maximum and minimum allowable errors:

$$E_{all} = \max\left(|W_8| C_{max},\ 10^{-9} \max,\ (|\dot{x}_h|,\ |\dot{y}_h|,\ |\dot{z}_h|)\right)$$

$$E_{min} = W_{12}\ C_{max}$$

7.1.15 Test the Estimated Error Against the Maximum Allowable Error

If $E > E_{all}$, continue below (Step 7.1.16).

If $E < E_{all}$, go to Step 7.1.22.

7.1.16 Increment and Test the Total Failure Counter

$$\text{KFAIL} = \text{KFAIL} + 1$$

Test the number of failures against the maximum allowed.

If KFAIL > MFAIL, continue below (Step 7.1.17).

If KFAIL < MFAIL, go to Step 7.1.18.

7.1.17 Set Halt Flag to Stop Trajectory Computations and Compute Final Time

$$\text{KHALT} = 3$$

$$T_f = t$$

Go to step 7.1.29.

7.1.18 Increment the Intermediate Failure Counter

(Enter here from Step 7.1.16 if the number of failures is less than the maximum allowed.)

$$KF = KF + 1$$

$$KHALT = 1$$

7.1.19 Compute a New Δt Based on the Value of Estimated Error, E

$$\Delta t = FDT \cdot \Delta t \; (E_{all}/E)^{1/4}$$

If $\Delta t/t > 10^{-8}$, go to Step 7.1.20; otherwise print the computer interval and go to Step 7.1.17.

7.1.20 Test Intermediate Failure Counter

If $KF \leq 0$, continue below (Step 7.1.20.1).

If $KF > 0$, go to Step 7.1.21.

7.1.20.1 Test Perturbation - Total Flag

If ICH = 1, go to Step 7.1.20.2.

If ICH = 2, go to Step 7.1.20.3.

7.1.20.2 Perturbation Solution Calculations

$$KR = 5$$

7.1.20.2.1 Check time against print time

If $t \geq t_p$, go to Step 7.1.20.2.2.

If $t < t_p$, go to Step 7.1.5.

7.1.20.2.2 Increment print time

$$t_p = t_p + \Delta t_p$$

Go to Step 7.1.29.

7.1.20.3 Test if This is Both a Print and a Regular Compute Point (IPRINT=3)

If IPRINT = 1 or 2, go to Step 7.1.5.

If IPRINT = 3, go to Step 7.1.20.2.

7.1.21 <u>Restore Values Saved in Step 7.1.5 to Those in Step 7.1.6</u>

Set $s_I = ss_I$ for I = 1, 2, . . . , 17

IPRINT = 2

Return to Step 7.1.8.

7.1.22 <u>Test the Estimated Error Against the Minimum Allowable Error, E_{min}</u>

(Enter from Step 7.1.15 if the estimated error < the maximum allowed.)

If $E \leq E_{min}$, continue below (Step 7.1.23).

If $E > E_{min}$, go to Step 7.1.25.

7.1.23 <u>Increment Total Failure Counter</u>

$$KFAIL = KFAIL + 1$$

Test the number of failures against the maximum allowed.

If KFAIL > MFAIL, return to Step 7.1.17.

If KFAIL $\leq$ MFAIL, continued below (Step 7.1.24).

7.1.24 Increment Intermediate Failure Counter, Compute New Δt, and Restore Initial Conditions

$$KF = KF + 1$$

$$KHALT = 1$$

Compute new Δt by multiplying the old by an input factor (>1):

$$\Delta t = \Delta t_{min} \Delta t$$

Restore the values saved in Step 7.1.5 to those in Step 7.1.6.

Set $s_I = ss_I$ for $I = 1, 2, \ldots, 17$

Return to Step 7.1.17.

7.1.25 Set the Intermediate Failure Counter to Zero

(Enter here from Step 7.1.22 with $E_{min} < E < E_{all}$.)

$$KF = 0$$

7.1.26 Second Pass Computations

(Enter here on second pass of Runga-Kutta (KR = 2).)

Advance Runge-Kutta:

$$KR = 3$$

Compute new intermediate position values:

$$x_h = x_h + \frac{1}{4}(\Delta t) \quad (RKIX)$$

$$y_h = y_h + \frac{1}{4}(\Delta t) \quad (RKIY) \qquad \text{parameters for (142)}$$

$$z_h = z_h + \frac{1}{4}(\Delta t) \quad (RKIZ)$$

Compute Runge-Kutta parameters:

$$RK2X = \Delta t \ddot{x}_h$$

$$RK2Y = \Delta t \ddot{y}_h \qquad (141)$$

$$RK2Z = \Delta t \ddot{z}_h$$

Return to main program.

7.1.27 Third Pass Computations

(Enter here on third pass of Runge-Kutta (KR = 3).)

Advance Runge-Kutta flag:

$$KR = 4$$

Compute new time and intermediate position values:

$$t_\psi = t_\psi + \frac{1}{2} \Delta t$$

$$t_s = t_s + \frac{1}{2} \Delta t$$

$$t_\Omega = t_\Omega + \frac{1}{2} \Delta t$$

$$t_\omega = t_\omega + \frac{1}{2} \Delta t$$

$$\gamma = \gamma + \varepsilon \Delta t/2$$

$$T = s_1 + \Delta t$$

Compute Runge-Kutta parameters:

$$x_h = s_2 + \Delta t \times s_5 + \frac{1}{2} \Delta t \times RK2X$$

$$y_h = s_3 + \Delta t \times s_6 + \frac{1}{2} \Delta t \times RK2Y \quad \text{parameters for (143)}$$

$$z_h = s_4 + \Delta t \times s_7 + \frac{1}{2} \Delta t \times RK2Z$$

$$RK3X = \Delta t \ddot{x}_h$$

$$RK3Y = \Delta t \ddot{x}_h \qquad (142)$$

$$RK3Z = \Delta t \ddot{x}_h$$

Return to main program.

7.1.28 Fourth Pass Computations

(Enter here on fourth pass of Runge-Kutta (KR = 4).)

Reset Runge-Kutta flag:

$$KR = 1$$

Compute integrated position, velocities, and velocity increments.

$$x_h = s_2 + \Delta t \times s_5 + \frac{1}{6} \Delta t \quad (RK1X + RK2X + RK3X)$$

$$y_h = s_3 + \Delta t \times s_6 + \frac{1}{6} \Delta t \quad (RK1Y + RK2Y + RK3Y) \qquad (144)$$

$$z_h = s_4 + \Delta t \times s_7 + \frac{1}{6} \Delta t \quad (RK1Z + RK2Z + RK3Z)$$

$$\dot{x}_h = s_5 + \frac{1}{6} \left(RK1X + 2 \left(RK2X + RK3X\right) + \Delta t \ \ddot{x}_h\right) \qquad (143,$$

$$\dot{y}_h = s_6 + \frac{1}{6} \left(RK1Y + 2 \left(RK2Y + RK3Y\right) + \Delta t \ \ddot{y}_h\right) \qquad 145)$$

$$\dot{z}_h = s_7 + \frac{1}{6} \left(RK1Z + 2 \left(RK2Z + RK3Z\right) + \Delta t \ \ddot{z}_h\right)$$

7.1.28.1 Test Whether Residual Perturbation Equations or Total Equations of Motion are Being Integrated

If ICH = 1 (perturbation), go to Step 7.1.28.3.

If ICH = 2 (total), go to Step 7.1.28.2.

7.1.28.2 Test Value of Print Flag

If IPRINT = 1, go to Step 7.1.29.

If IPRINT = 2 or 3, go to Step 7.1.28.3.

7.1.28.3 Compute Two Interval Increments in Velocity

$$\Delta\dot{X} = \Delta\dot{X} + \dot{x}_h - s_5$$

$$\Delta\dot{Y} = \Delta\dot{Y} + \dot{y}_h - s_6$$

$$\Delta\dot{Z} = \Delta\dot{Z} + \dot{z}_h - s_7$$

7.1.29 Return to Main Program

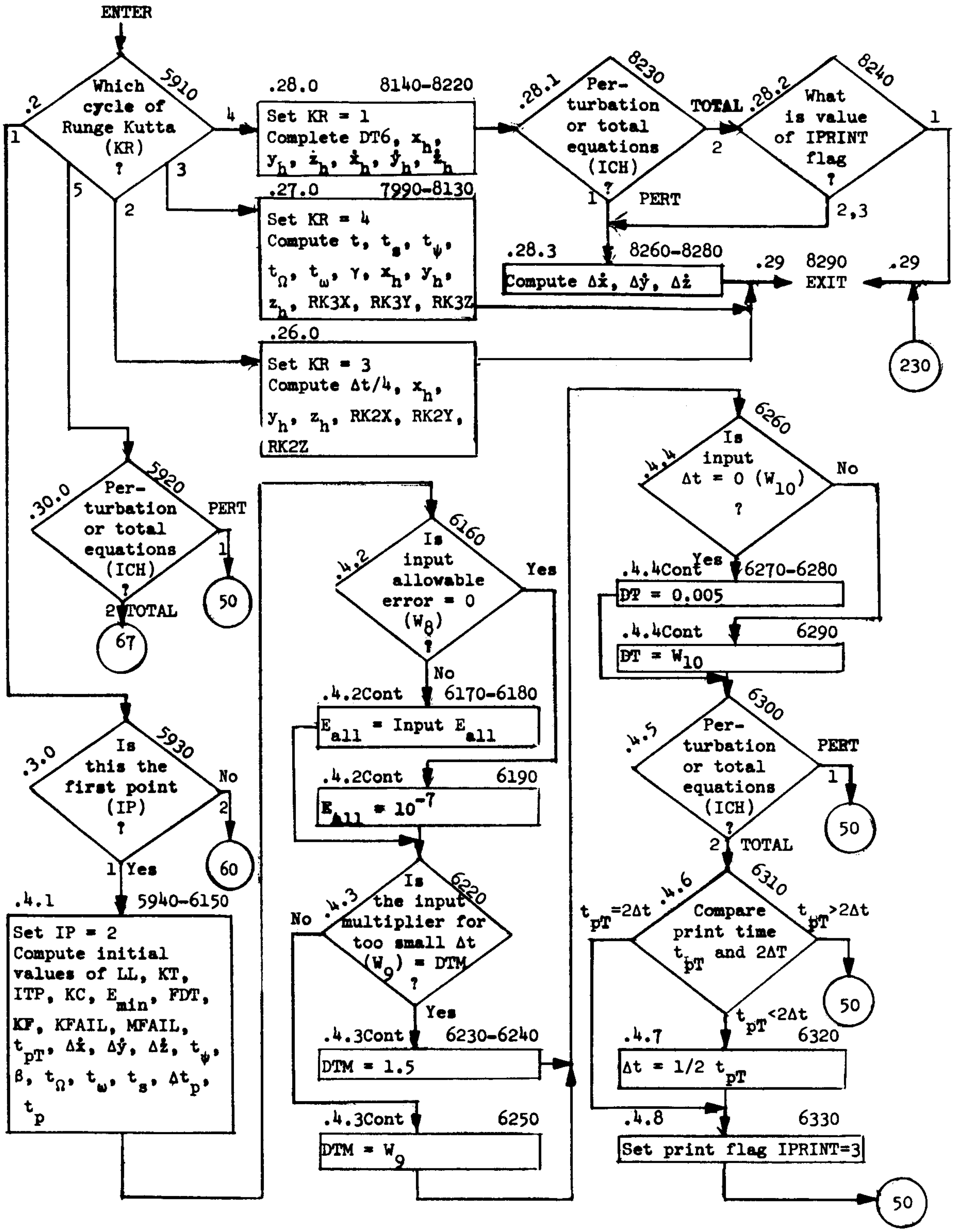
7.1.30 Test Whether Residual Perturbation or Total Equations of Motion are Being Integrated

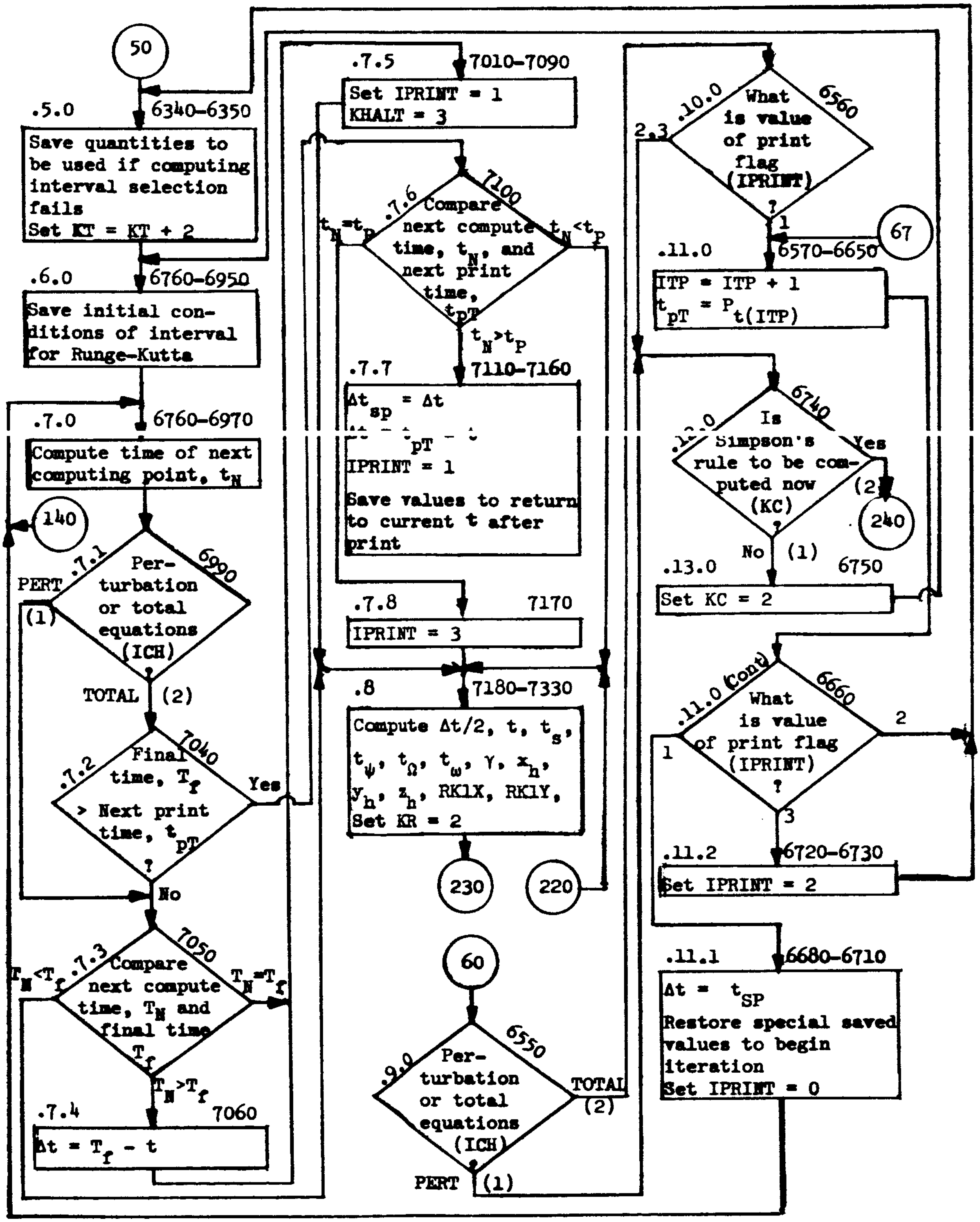
If ICH = 1 (perturbation), go to Step 7.1.5.

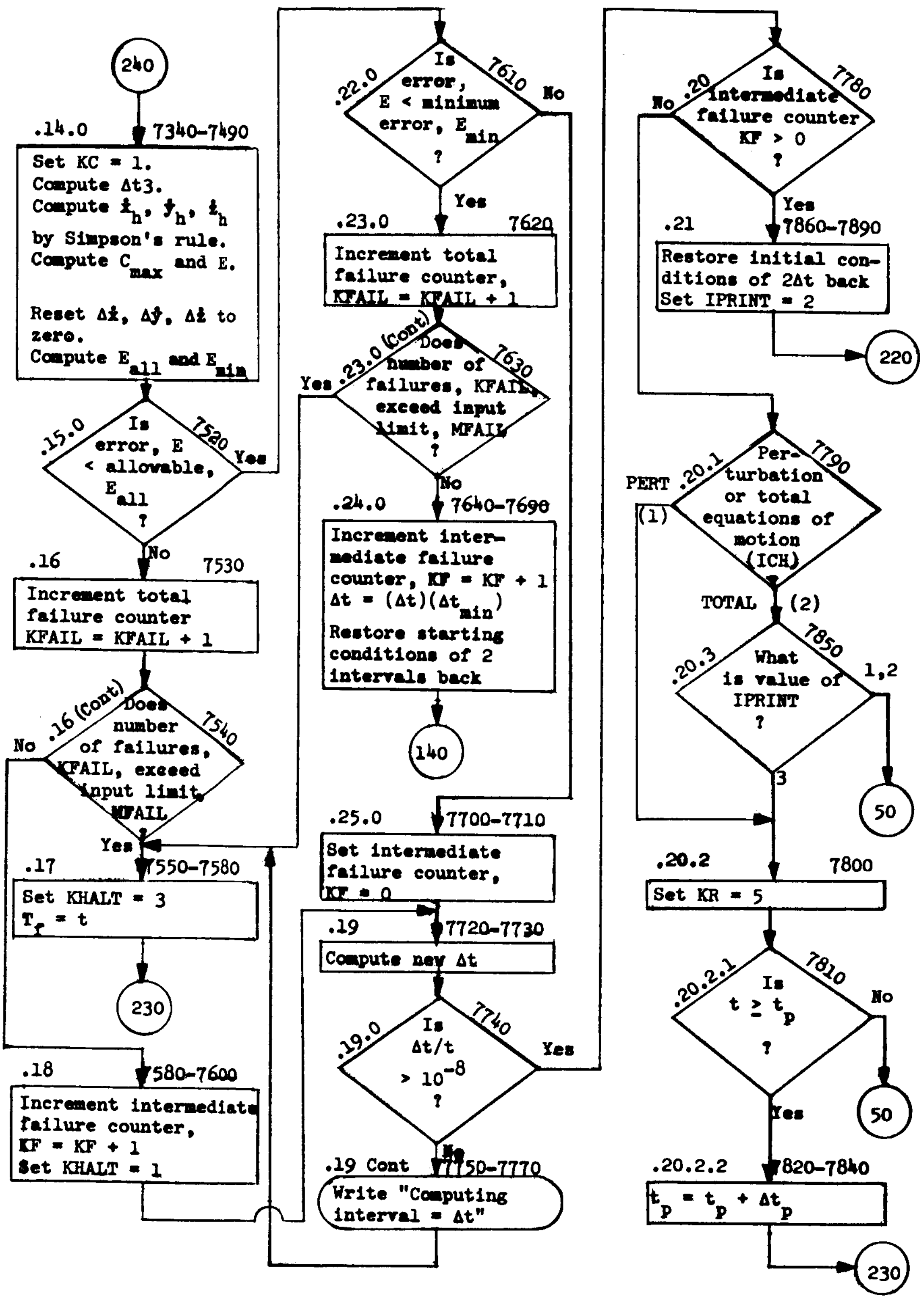
If ICH = 2 (total), go to Step 7.1.11.

7.2 SUBROUTINE RKSIMP DETAIL FLOW CHARTS

The flow charts of this section describe the logic of subroutine RKSIMP completely, i.e., all conditional transfers are shown. The numbers to the top left of each box are the subsection numbers of Section 7.1 where the operations mentioned in the box are detailed. The numbers at the top right are the card numbers of the subroutine listing, Section 7.3.


ENTER
5910 .2 Which cycle of Runge Kutta (KR) ?
1 2 3 4 5
.28.0 8140-8220 Set KR = 1 Complete DT6, x_h, y_h, z_h, $\dot{x}_h$, $\dot{y}_h$, $\dot{z}_h$
.27.0 7990-8130 Set KR = 4 Compute t, t_s, t_ψ, t_Ω, t_ω, γ, x_h, y_h, z_h, RK3X, RK3Y, RK3Z
.26.0 Set KR = 3 Compute $\Delta t/4$, x_h, y_h, z_h, RK2X, RK2Y, RK2Z
8230 .28.1 Perturbation or total equations (ICH) ?
TOTAL 2
1 PERT
8240 .28.2 What is value of IPRINT flag ?
1
2,3
.28.3 8260-8280 Compute $\Delta\dot{x}$, $\Delta\dot{y}$, $\Delta\dot{z}$
.29 8290 EXIT
.29
230
5920 .30.0 Perturbation or total equations (ICH) ?
PERT 1
50
2 TOTAL
67
5930 .3.0 Is this the first point (IP) ?
No 2
60
1 Yes
.4.1 5940-6150 Set IP = 2 Compute initial values of LL, KT, ITP, KC, E_{min}, FDT, KF, KFAIL, MFAIL, t_{pT}, $\Delta\dot{x}$, $\Delta\dot{y}$, $\Delta\dot{z}$, t_ψ, β, t_Ω, t_ω, t_s, Δt_p, t_p
6160 .4.2 Is input allowable error = 0 (W_8) ?
Yes
No
.4.2Cont 6170-6180 E_{all} = Input E_{all}
.4.2Cont 6190 $E_{all} = 10^{-7}$
6220 .4.3 Is the input multiplier for too small Δt (W_9) = DTM ?
No
Yes
.4.3Cont 6230-6240 DTM = 1.5
.4.3Cont 6250 DTM = W_9
6260 .4.4 Is input Δt = 0 (W_{10}) ?
No
Yes
.4.4Cont 6270-6280 DT = 0.005
.4.4Cont 6290 DT = W_{10}
6300 .4.5 Perturbation or total equations (ICH) ?
PERT 1
50
2 TOTAL
6310 .4.6 Compare print time t_{pT} and 2ΔT
$t_{pT}=2\Delta t$
$t_{pT}>2\Delta t$
50
$t_{pT}<2\Delta t$
.4.7 6320 $\Delta t = 1/2\ t_{pT}$
.4.8 6330 Set print flag IPRINT=3
50


50
.5.0
6340-6350
Save quantities to be used if computing interval selection fails
Set KT = KT + 2
.6.0
6760-6950
Save initial conditions of interval for Runge-Kutta
.7.0
6760-6970
Compute time of next computing point, t_N
140
.7.1
6990
Perturbation or total equations (ICH)
PERT (1)
TOTAL (2)
.7.2
7040
Final time, T_f > Next print time, t_{pT} ?
Yes
No
.7.3
7050
Compare next compute time, T_N and final time T_f
$T_N<T_f$
$T_N=T_f$
$T_N>T_f$
.7.4
7060
$\Delta t = T_f - t$
.7.5
7010-7090
Set IPRINT = 1
KHALT = 3
.7.6
7100
Compare next compute time, t_N, and next print time, t_{pT}
$t_N=t_P$
$t_N<t_P$
$t_N>t_P$
.7.7
7110-7160
$\Delta t_{sp} = \Delta t$
IPRINT = 1
Save values to return to current t after print
.7.8
7170
IPRINT = 3
.8
7180-7330
Compute $\Delta t/2$, t, t_s, t_ψ, t_Ω, t_ω, γ, x_h, y_h, z_h, RK1X, RK1Y,
Set KR = 2
230
220
60
.9.0
6550
Perturbation or total equations (ICH)
TOTAL (2)
PERT (1)
.10.0
6560
What is value of print flag (IPRINT) ?
2,3
1
67
.11.0
6570-6650
ITP = ITP + 1
$t_{pT} = P_{t(ITP)}$
.12.0
6740
Is Simpson's rule to be computed now (KC)
Yes (2)
240
No (1)
.13.0
6750
Set KC = 2
.11.0 (Cont)
6660
What is value of print flag (IPRINT) ?
1
2
3
.11.2
6720-6730
Set IPRINT = 2
.11.1
6680-6710
$\Delta t = t_{SP}$
Restore special saved values to begin iteration
Set IPRINT = 0



240
.14.0
7340-7490
Set KC = 1.
Compute Δt3.
Compute ẋh, ẏh, żh by Simpson's rule.
Compute Cmax and E.
Reset Δẋ, Δẏ, Δż to zero.
Compute Eall and Emin
.15.0
7520
Is error, E < allowable, Eall ?
Yes
No
.16
7530
Increment total failure counter KFAIL = KFAIL + 1
.16 (Cont)
7540
Does number of failures, KFAIL, exceed input limit, MFAIL
No
Yes
.17
7550-7580
Set KHALT = 3
Tf = t
230
.18
7580-7600
Increment intermediate failure counter,
KF = KF + 1
Set KHALT = 1
.22.0
7610
Is error, E < minimum error, Emin ?
No
Yes
.23.0
7620
Increment total failure counter,
KFAIL = KFAIL + 1
.23.0 (Cont)
7630
Does number of failures, KFAIL, exceed input limit, MFAIL ?
Yes
No
.24.0
7640-7690
Increment intermediate failure counter, KF = KF + 1
Δt = (Δt)(Δtmin)
Restore starting conditions of 2 intervals back
140
.25.0
7700-7710
Set intermediate failure counter,
KF = 0
.19
7720-7730
Compute new Δt
.19.0
7740
Is Δt/t > 10^-8 ?
Yes
No
.19 Cont
7750-7770
Write "Computing interval = Δt"
.20
7780
Is intermediate failure counter KF > 0 ?
No
Yes
.21
7860-7890
Restore initial conditions of 2Δt back
Set IPRINT = 2
220
.20.1
7790
Perturbation or total equations of motion (ICH)
PERT
(1)
TOTAL
(2)
.20.3
7850
What is value of IPRINT ?
1,2
3
50
.20.2
7800
Set KR = 5
.20.2.1
7810
Is t ≥ tp ?
No
Yes
50
.20.2.2
7820-7840
tp = tp + Δtp
230

7.3 RKSIMP LISTING

This section contains the listing for subroutine RKSIMP. The subsection numbers on the comment cards refer to the subsections in Section 7.1, wherein the code is explained.

```
      SUBROUTINE RKSIMP (KR, IP, KC, T, DT, ICH, HX, HY, HZ,            5850H144
     1 HXD, HYD, HZD, HX2D, HY2D, HZ2D, KHALT, TPRINT, TF)              5860H144
      COMMON W, IPRINT, TS, TPSI, TNOD, TPER, GAMMA, EP1,               5870H144
     1 ITP, PTIME, AP                                                   5880H144
      DIMENSION W(30), S(17), SS(17), SP(17),                           5890H144
     1 PTIME( 700),                           AP( 9100)                 5900H144
C2.0  TEST RUNGE-KUTTA FLAG, KR                                         5905H144
      GO TO (10, 360, 370, 380, 5), KR                                  5910H144
C30.0 TEST WHETHER RESIDUAL PERTURBATION OR TOTAL EQUATIONS OF MOTION   5915H144
C     ARE BEING INTEGRATED                                              5916H144    ,1
    5 GO TO (50, 67), ICH                                               5920H144
C3.0  TEST THE INITIAL POINT FLAG, IP                                   5925H144    ,2
   10 GO TO (20, 60), IP                                                5930H144
C4.0  FIRST POINT COMPUTATIONS                                          5933H144
C4.1  SET FLAGS AND INITIAL CONDITIONS                                  5937H144    ,3
   20 IP = 2                                                            5940H144    ,4
      LL = 0                                                            5950H144    ,5
      KT = - 2                                                          5960H144    ,6
      ITP = 1                                                           5970H144    ,7
      KC = 1                                                            5980H144    ,8
      ER MIN = ABS (W(12))                                              5990H144    ,9
      EDT = ABS (W(5))                                                  6000H144    ,10
      KF = 0                                                            6010H144    ,11
      KFAIL = 0                                                         6020H144    ,12
      MFAIL = ABS (W(11))                                               6030H144    ,13
      TPRINT = PTIME(1)                                                 6040H144    ,14
      DELXD = 0.                                                        6050H144    ,15
      DELYD = 0.                                                        6060H144    ,16
      DELZD = 0.                                                        6070H144    ,17
      GAMMA = 0.                                                        6080H144    ,18
      TPSI = 0.                                                         6090H144    ,19
      BETA = 0.                                                         6100H144    ,20
      TNOD = 0.                                                         6110H144    ,21
      TPER = 0.                                                         6120H144    ,22
      TS = 0.                                                           6130H144    ,23
      TP1 = .25 * (ABS (W(1))) ** 1.5                                   6140H144    ,24
      TPI = TP1                                                         6150H144
C4.2  SET THE MAXIMUM ALLOWABLE ERROR EQUAL TO THE INPUT VALUE OR EQUAL  6155H144
C     TO 10 TO THE -7 IF THE INPUT VALUE IS ZERO                        6156H144    ,25
      IF (W(8)) 21, 22, 21                                              6160H144    ,26
   21 EALL = ABS (W(8))                                                 6170H144    ,27
      GO TO 30                                                          6180H144    ,28
   22 EALL = 1. E-7                                                     6190H144
C4.3  SET THE COMPUTING INTERVAL MULTIPLIER (USED IF ERROR IS LESS THAN  6215H144
C     THE MINIMUM ALLOWED) EQUAL TO INPUT VALUE OR EQUAL TO 1.5 IF INPUT6216H144
C     VALUE IS LESS THAN OR EQUAL TO 1                                  6217H144    ,29
   30 IF (W(9) - 1.) 32, 32, 33                                         6220H144    ,30
   32 DTM = 1.5                                                         6230H144    ,31
      GO TO 40                                                          6240H144    ,32
   33 DTM = W(9)                                                        6250H144
C4.4  SET THE INITIAL COMPUTING INTERVAL EQUAL TO INPUT VALUE OR EQUAL   6255H144
C     TO 0.005 IF INPUT VALUE = 0                                       6256H144    ,33
   40 IF (W(10)) 42, 41, 42                                             6260H144    ,34
   41 DT = 0.005                                                        6270H144    ,35
      GO TO 45                                                          6280H144    ,36
```

```
       EXTERNAL FORMULA NUMBER  -  SOURCE STATEMENT  -  INTERNAL FORMULA NUMBER(S)

   42 DT = W(10)                                                          6290H144
C4.5  TEST PERTURBATION-TOTAL FLAG                                        6295H144   .37
   45 GO TO (50, 43), ICH                                                 6300H144
C4.6  TEST FIRST PRINT TIME IN NUMERICAL SOLUTION OF TOTAL EQUATIONS OF   6305H144
C     MOTION VS. TWICE THE COMPUTING INTERVAL                             6306H144   .38
   43 IF (TPRINT - 2. * DT) 44, 46, 50                                    6310H144
C4.7  SET DT = 1/2 T SUB PT                                                6315H144   .39
   44 DT = 0.5 * TPRINT                                                    6320H144
C4.8  SET IPRINT = 3 TO SIGNAL BOTH PRINT AND TESTING OF COMPUTED VALUES6325H144   .40
   46 IPRINT = 3                                                          6330H144
C5.0  SAVE QUANTITIES AT START OF SPAN FOR COMPUTING INTERVAL             6340H144
C     CALCULATIONS                                                        6350H144   .41
   50 SS(1) = T                                                           6360H144   .42
      SS(2) = HX                                                          6370H144   .43
      SS(3) = HY                                                          6380H144   .44
      SS(4) = HZ                                                          6390H144   .45
      SS(5) = HXD                                                         6400H144   .46
      SS(6) = HYD                                                         6410H144   .47
      SS(7) = HZD                                                         6420H144   .48
      SS(8) = HX2D                                                        6430H144   .49
      SS(9) = HY2D                                                        6440H144   .50
      SS(10) = HZ2D                                                       6450H144   .51
      SS(11) = TS                                                         6460H144   .52
      SS(12) = TPSI                                                       6470H144   .53
      SS(13) = TNCD                                                       6480H144   .54
      SS(14) = TPER                                                       6490H144   .55
      SS(15) = THETA1                                                     6500H144   .56
      SS(16) = STHET1                                                     6510H144   .57
      SS(17) = GAMMA                                                      6520H144
C5.1  INCREMENT COMPUTING INTERVAL COUNTER                                6525H144   .58
      KT = KT + 2                                                         6530H144   .59
      GO TO 110                                                           6540H144
C9.0  TEST WHETHER RESIDUAL PERTURBATION OR TOTAL EQUATIONS ARE BEING     6545H144
C     SOLVED                                                              6546H144   .60
   60 GO TO (90, 63), ICH                                                 6550H144
C10.0 TEST THE PRINT FLAG                                                 6555H144   .61
   63 GO TO (67, 90, 90), IPRINT                                          6560H144
C11.0 ADVANCE PRINT INDEX AND SET NEXT PRINT TIME                         6565H144   .62
   67 ITP = ITP + 1                                                       6570H144   .63
      TPRINT = PTIME (ITP)                                                6650H144   .64
      IF (IPRINT - 2) 75, 50, 85                                          6660H144
C11.1 RESTORE VALUES SAVED PREVIOUSLY IN STEP 7.1.7.7                     6665H144   .65
   75 DT = SPDT                                                           6670H144   .66
      DO 80  I = 1, 17                                                    6680H144   .67
   80 S(I) = SP(I)                                                        6690H144   .68   .69
      IPRINT = 2                                                          6700H144   .70
      GO TO 140                                                           6710H144
C11.2 SET IPRINT = 2 AND GO TO STEP 7.1.5                                 6715H144   .71
   85 IPRINT = 2                                                          6720H144   .72
      GO TO 50                                                            6730H144
C12.0 TEST THE SIMPSON'S RULE FLAG                                        6735H144   .73
   90 GO TO (100, 240), KC                                                6740H144
C13.0 ADVANCE THE SIMPSON'S RULE FLAG                                     6745H144   .74
  100 KC = 2                                                              6750H144
C6.0  SAVE POSITIONS, VELOCITIES, ACCELERATIONS AND TIME FOR              6760H144
C      ORDINARY RUNGE-KUTTA USE                                           6770H144   .75
```

```
     EXTERNAL FORMULA NUMBER   -   SOURCE STATEMENT   -   INTERNAL FORMULA NUMBER(S)

   110 S (1) = T                                                   6780H144   ,76
       S(2) = HX                                                   6790H144   ,77
       S(3) = HY                                                   6800H144   ,78
       S(4) = HZ                                                   6810H144   ,79
       S(5) = HXD                                                  6820H144   ,80
       S(6) = HYD                                                  6830H144   ,81
       S(7) = HZD                                                  6840H144   ,82
       S(8) = HX2D                                                 6850H144   ,83
       S(9) = HY2D                                                 6860H144   ,84
       S(10) = HZ2D                                                6870H144   ,85
       S(11) = TS                                                  6880H144   ,86
       S(12) = TPSI                                                6890H144   ,87
       S(13) = TNOD                                                6900H144   ,88
       S(14) = TPER                                                6910H144   ,89
       S(15) = THETA1                                              6920H144   ,90
       S(16) = STHET1                                              6930H144   ,91
       S(17) = GAMMA                                               6940H144   ,92
       GO TO 140                                                   6950H144
C7.0   COMPUTE THE NEXT VALUE OF TIME AND MAKE SPECIAL PRINT TIME TESTS 6960H144 ,93
   140 TN = S(1) + DT                                              6970H144
C7.1   TEST PERTURBATION-TOTAL FLAG                                6985H144   ,94
   139 GO TO (150, 145), ICH                                       6990H144
C7.2   TEST RUN STOP TIME AGAINST THE NEXT PRINT TIME              7035H144   ,95
   145 IF (TF - TPRINT) 150, 150, 180                              7040H144
C7.3   TEST THE NEXT VALUE OF TIME AGAINST RUN STOP TIME           7045H144   ,96
   150 IF (TN - TF) 220, 170, 160                                  7050H144
C7.4   SET COMPUTING INTERVAL SO THAT TIME WILL EQUAL THE RUN STOP TIME 7055H144
C      AT THE END OF THE NEXT INTERVAL                             7056H144   ,97
   160 CT = TF - T                                                 7060H144
C7.5   SET THE PRINT AND HALT FLAGS                                7065H144   ,98
   170 IPRINT = 1                                                  7070H144   ,99
       KHALT = 3                                                   7080H144   ,100
       GO TO  220                                                  7090H144
C7.6   TEST THE NEXT VALUE OF TIME AGAINST THE NEXT PRINT TIME     7095H144   ,101
   180 IF (TN - TPRINT) 220, 200, 190                              7100H144
C7.7   SAVE INITIAL CONDITIONS AND COMPUTE INTERVAL FOR PRINT ONLY 7105H144   ,102
   190 SPDT = DT                                                   7110H144   ,103
       DT = TPRINT - T                                             7120H144   ,104
       IPRINT = 1                                                  7130H144   ,105
       CO 210 I = 1, 17                                            7140H144   ,106
   210 SP(I) = S(I)                                                7150H144   ,107  ,108
       GO TO 220                                                   7160H144
C7.8   INDICATE COMPUTE POINT = PRINT POINT                        7165H144   ,109
   200 IPRINT = 3                                                  7170H144
C8.0   COMPLETE 1ST R-K PASS, COMPUTE NEW TIME AND POSITIONS       7180H144   ,110
   220 DT2 = DT / 2.                                               7190H144   ,111
       T = S(1) + DT2                                              7200H144   ,112
       TS = S(11) + DT2                                            7210H144   ,113
       TPSI = S(12) + DT2                                          7220H144   ,114
       TNOD = S(13) + DT2                                          7230H144   ,115
       TPER = S(14) + DT2                                          7240H144   ,116
       GAMMA = S(17) + EP1 * DT2                                   7250H144   ,117
       HX = S(2) + DT2 * S(5)                                      7260H144   ,118
       HY = S(3) + DT2 * S(6)                                      7270H144   ,119
       HZ = S(4) + DT2 * S(7)                                      7280H144   ,120
       RK1X = DT * S(8)                                            7290H144   ,121
```

```
     EXTERNAL FORMULA NUMBER   -   SOURCE STATEMENT   -   INTERNAL FORMULA NUMBER(S)

        RK1Y = DT * S(9)                                              7300H144   .122
        RK1Z = DT * S(10)                                             7310H144   .123
        KR = 2                                                        7320H144   .124
    901 GO TO 230                                                     7330H144
  C14.0 RESET SIMPSONS RULE FLAG AND COMPUTE ACTUAL AND ALLOWABLE ERRORS 7340H144   .125
    240 KC = 1                                                        7350H144   .126
        DT3 = DT / 3.                                                 7360H144   .127
        HXDA = DT3 * (SS(8) + 4. * S(8) + HX2D)                       7370H144   .128
        HYDA = DT3 * (SS(9) + 4. * S(9) + HY2D)                       7380H144   .129
        HZDA = DT3 * (SS(10) + 4. * S(10) + HZ2D)                     7390H144
  C     COMPUTE MAXIMUM ESTIMATED ERROR                               7400H144   .130
        CMAX = AMAX1 (ABS(DELXD), ABS(DELYD), ABS(DELZD))             7410H144   .131
        ESTER = AMAX1 (ABS(DELXD - HXDA), ABS(DELYD - HYDA),          7420H144
       1 ABS (DELZD - HZDA))                                          7430H144   .132
        DELXD = 0.                                                    7440H144   .133
        DELYD = 0.                                                    7450H144   .134
        DELZD = 0.                                                    7460H144   .135
        EALL = AMAX1 (ABS (W(8)) * CMAX, 1.E-9 * AMAX1 (ABS (HXD      7470H144
       1 ), ABS (HYD), ABS (HZD)))                                    7480H144   .136
        ERMIN = ABS (W(12)) * CMAX                                    7490H144
  C15.0 TEST THE ESTIMATED ERROR AGAINST THE MAXIMUM ALLOWABLE ERROR  7495H144   .137
        IF (ESTER - EALL) 280, 280, 250                               7520H144
  C16.0 INCREMENT AND TEST THE TOTAL FAILURE COUNTER                  7525H144   .138
    250 KFAIL = KFAIL + 1                                             7530H144   .139
        IF (KFAIL - MFAIL) 270, 270, 260                              7540H144
  C17.0 SET HALT FLAG TO STOP TRAJECTORY COMPUTATIONS AND COMPUTE FINAL 7545H144
  C     TIME                                                          7546H144   .140
    260 KHALT = 3                                                     7550H144   .141
        TF = T                                                        7560H144   .142
        GO TO 230                                                     7570H144
  C18.0 INCREMENT THE INTERMEDIATE FAILURE COUNTER                    7575H144   .143
    270 KF = KF + 1                                                   7580H144   .144
        KHALT = 1                                                     7590H144   .145
        GO TO 330                                                     7600H144
  C22.0 TEST THE ESTIMATED ERROR AGAINST THE MINIMUM ALLOWABLE ERROR  7605H144
  C     (ERMIN)                                                       7606H144   .146
    280 IF (ESTER - ERMIN) 290, 290, 320                              7610H144
  C23.0 INCREMENT THE TOTAL FAILURE COUNTER AND TEST THE NUMBER OF    7615H144
  C     FAILURES AGAINST THE MAXIMUM ALLOWED                          7616H144   .147
    290 KFAIL = KFAIL + 1                                             7620H144   .148
        IF (KFAIL - MFAIL) 300, 300, 260                              7630H144
  C24.0 INCREMENT THE INTERMEDIATE FAILURE COUNTER, COMPUTE NEW DELTA T, 7635H144
  C     AND RESTORE INITIAL CONDITIONS                                7636H144   .149
    300 KF = KF + 1                                                   7640H144   .150
        KHALT = 1                                                     7650H144   .151
        DT = DTM * DT                                                 7660H144   .152
        DO 310 I = 1, 17                                              7670H144   .153
    310 S (I) = SS (I)                                                7680H144   .154   .155
        GO TO 140                                                     7690H144
  C25.0 SET THE INTERMEDIATE FAILURE COUNTER TO ZERO                  7695H144   .156
    320 KF = 0                                                        7700H144   .157
        GO TO 330                                                     7710H144
  C19.0 COMPUTE NEW DT BASED ON THE VALUE OF THE ESTIMATED ERROR, ESTER 7720H144   .158
    330 DT = FDT * DT * (EALL / ESTER) ** 0.25                        7730H144   .159
        IF (DT / T - 1. E-8) 334, 334, 336                            7740H144   .160
    334 WRITE (6,335) DT                                              7750H144   .161   .162   .163
```

```
  335 FORMAT (1H0.16H COMP INTERVAL = E17.8)                7760H144
      GO TO 260                                             7770H144
C20.0 TEST INTERMEDIATE FAILURE COUNTER                     7775H144   ,164
  336 IF (KF) 331, 331, 340                                 7780H144
C20.1 TEST PERTURBATION-TOTAL FLAG                          7785H144   ,165
  331 GO TO (332, 333), ICH                                 7790H144
C20.2 PERTURBATION SOLUTION CALCULATIONS                    7795H144   ,166
  332 KR = 5                                                7800H144
C20.2.1 CHECK TIME AGAINST PRINT TIME                       7805H144   ,167
      IF (T - TPI) 50, 337, 337                             7810H144
C20.2.2 INCREMENT PRINT TIME                                7815H144   ,168
  337 TPI = TPI + TP1                                       7820H144   ,169
      GO TO 230                                             7830H144
C20.3 TEST IF THIS IS BOTH A PRINT AND A REGULAR COMPUTE POINT  7835H144
C     (IPRINT = 3)                                          7836H144   ,170
  333 GO TO ( 50, 50 , 332), IPRINT                         7840H144
C21.0 RESTORE VALUES SAVED IN STEP 7.1.5 TO THOSE IN STEP 7.1.6  7845H144   ,171
  340 DO 350 I = 1, 17                                      7850H144   ,172
  350 S (I) = SS (I)                                        7860H144   ,173   ,174
      IPRINT = 2                                            7870H144   ,175
      GO TO 220                                             7880H144
C26.0 SECOND PASS OF RUNGE-KUTTA                            7890H144   ,176
  360 KR = 3                                                7900H144   ,177
      DT4 = DT / 4.                                         7910H144   ,178
      HX = HX + DT4 * RK1X                                  7920H144   ,179
      HY = HY + DT4 * RK1Y                                  7930H144   ,180
      HZ = HZ + DT4 * RK1Z                                  7940H144   ,181
      RK2X = DT * HX2D                                      7950H144   ,182
      RK2Y = DT * HY2D                                      7960H144   ,183
      RK2Z = DT * HZ2D                                      7970H144   ,184
      GO TO 230                                             7980H144
C27.0 THIRD PASS OF RUNGE-KUTTA                             7990H144   ,185
  370 KR = 4                                                8000H144   ,186
      T = S(1) + DT                                         8010H144   ,187
      TS =    S(11) + DT                                    8020H144   ,188
      TPSI =  S(12) + DT                                    8030H144   ,189
      TNOD =  S(13) + DT                                    8040H144   ,190
      TPER =  S(14) + DT                                    8050H144   ,191
      GAMMA = S(17) + FP1 * DT                              8060H144   ,192
      HX = S(2) + DT * S(5) + DT2 * RK2X                    8070H144   ,193
      HY = S(3) + DT * S(6) + DT2 * RK2Y                    8080H144   ,194
      HZ = S(4) + DT * S(7) + DT2 * RK2Z                    8090H144   ,195
      RK3X = DT * HX2D                                      8100H144   ,196
      RK3Y = DT * HY2D                                      8110H144   ,197
      RK3Z = DT * HZ2D                                      8120H144   ,198
      GO TO 230                                             8130H144
C28.0 FOURTH PASS OF RUNGE-KUTTA                            8140H144   ,199
  380 KR = 1                                                8150H144   ,200
      DT6 = DT / 6.                                         8160H144   ,201
      HX = S(2) + DT * S(5) + DT6 * (RK1X + RK2X + RK3X)    8170H144   ,202
      HY = S(3) + DT * S(6) + DT6 * (RK1Y + RK2Y + RK3Y)    8180H144   ,203
      HZ = S(4) + DT * S(7) + DT6 * (RK1Z + RK2Z + RK3Z)    8190H144   ,204
      HXD = S(5) + (RK1X + 2. * (RK2X + RK3X) + DT *HX2D)/6.  8200H144   ,205
      HYD = S(6) + (RK1Y + 2. * (RK2Y + RK3Y) + DT *HY2D)/6.  8210H144   ,206
      HZD = S(7) + (RK1Z + 2. * (RK2Z + RK3Z) + DT *HZ2D)/6.  8220H144
C28.1 TEST WHETHER RESIDUAL PERTURBATION EQUATIONS OR TOTAL EQUATIONS  8225H144
```

```
     EXTERNAL FORMULA NUMBER  -  SOURCE STATEMENT  -  INTERNAL FORMULA NUMBER(S)

C      OF MOTION ARE BEING INTEGRATED                            8226H144   ,207
       GO TO (382, 381), ICH                                     8230H144
C28.2  TEST VALUE OF PRINT FLAG                                  8235H144   ,208
  381  GO TO (230, 382, 382), IPRINT                             8240H144
C28.3  COMPUTE TWO INTERVAL INCREMENT IN VELOCITY                8245H144   ,209
  382  DELXD = DELXD +(RK1X +2.*(RK2X + RK3X) + DT * HX2D)/6.    8250H144   ,210
       DELYD = DELYD +(RK1Y +2.*(RK2Y + RK3Y) + DT * HY2D)/6.    8260H144   ,211
       DELZD = DELZD +(RK1Z +2.*(RK2Z + RK3Z) + DT * HZ2D)/6.    8270H144
C29.0  RETURN TO MAIN PROGRAM                                    8275H144   ,212
  230  RETURN                                                    8280H144   ,213
       END                                                       8290H144   ,214
```

Section 8

SUBROUTINE PLOT

8.1 DEVELOPMENT OF EQUATIONS

The purpose of this subroutine is to plot the error in the Jacobi constant, $\Delta C/C$, for the analytic solution, for the numerical residual perturbation solution, for the precessing ellipse which forms the basis of the numerical perturbation solution, for a straightforward numerical solution of the total equations of motion, and for the initial osculating ellipse which would form the basis of an ordinary Encke method; and to plot the vector position and velocity differences of each of the other solutions from the numerical residual perturbation solution. The numerical residual perturbation solution is taken as the standard of comparison because it normally holds the Jacobi integral most constant. The routines are taken from reference 8.

The abscissa of the plots is time. The plotting symbols are 6 rasters wide and the length of the scales is 1,024 rasters less space required for labeling and scales. This allows about 152 points to a plot. Hand plots indicate a point about every quarter radian of true anomaly is necessary to describe the motion adequately. In the dimensionless variables, the mean motion is $a^{-3/2}$ (ref 1, page 208). Thus, the time to be plotted per page to produce 152 points (corresponding to $152/4 = 38$ radians) is given by:

$$t_R = (38a^{3/2})$$

To make the plot readable, t_R is rounded to have only one significant figure. If

$1 \times 10^n \leq t_R < 2 \times 10^n$, then grid is drawn at intervals of 0.1×10^n; if $2 \times 10^n \leq t_R < 5 \times 10^n$, then the grid is drawn at intervals of 0.2×10^n; and if $5 \times 10^n \leq t_R < 10^{n+1}$, then the grid is drawn at intervals of 0.5×10^n.

8.2 EQUATIONS IN ORDER OF SOLUTIONS

8.2.1 Input

The input to subroutine PLOT is:

a_{32}	= The semi-major axis to the three-halves power
T_f	= Final time
a	= The semi-major axis
e	= The eccentricity
i°	= The inclination in degrees
μ	= The ratio of the mass of the smaller body to the sum of the masses of the two bodies
C_{init}	= The Jacobi constant
P_{ss}	= The anomalistic period of the precessing ellipse
PTIME	= Up to 700 values of time at which the data are to be plotted
AP	= A 9,100-element array defined as follows, (where $i = 0, 1, 2, \ldots, 699$ is the plot point count index):

$ap_{1+13i} = (\Delta C/C)_{ai}$ The subscripts are defined by:

$ap_{2+13i} = (\Delta C/C)_{pki}$ a = analytic solution from reference 1

$ap_{3+13i} = (\Delta C/C)_{ci}$ pk = precessing ellipse value

$ap_{4+13i} = (\Delta C/C)_{ki}$ c = value obtained from numerical solution of total equations of motion

$$ap_{5+13i} = (\Delta C/C)_{pi}$$

$$ap_{6+13i} = (\Delta\rho/a)_{ai}$$

$$ap_{7+13i} = (\Delta\rho/a)_{pki}$$

$$ap_{8+13i} = (\Delta\rho/a)_{ci}$$

$$ap_{9+13i} = (\Delta\rho/a)_{ki}$$

$$ap_{10+13i} = (\Delta\dot{\rho}\ a^{1/2})_{ai}$$

$$ap_{11+13i} = (\Delta\dot{\rho}\ a^{1/2})_{pki}$$

$$ap_{12+13i} = (\Delta\dot{\rho}\ a^{1/2})_{ci}$$

$$ap_{13+13i} = (\Delta\dot{\rho}\ a^{1/2})_{ki}$$

k = initial osculating ellipse value

p = value from numerical residual perturbation solution

8.2.2 Initial Setup

1. The machine is instructed to provide both 9 in. by 9 in. transparencies and 35mm slides.

2. The plotting symbols are chosen as:

o = Analytic solution

× = Precessing ellipse

□ = Numerical solution of total equations of motion

* = Initial osculating ellipse

• = Numerical residual perturbation solution

3. Compute abscissa scale.

Set initial time for initial plot = 0.

A. Compute unrounded t_R:

$$t_R = 38. + a_{32}$$

B. Set number of characters to be displayed in horizontal labels:

$$NX = 3$$

C. Develop $A = 10^n$ such that $10^n \leq t_R \leq 10^{n+1}$

(1) Set $A = 1$

(2) Compare t_R and 1:

If $t_R < 1$, go to Step 8.2.2-3 C(3).

If $t_R = 1$, go to Step 8.2.2-3 C(6).

If $t_R > 1$, go to Step 8.2.2-3 C(5).

(3) Develop $A < 1$:

Set $A = A/10$ and increment digit count NX:

$$NX = NX+1$$

Compare t_R and A:

If $t_R < A$, repeat Step 8.2.2-3 C(3).

If $t_R \geq A$, go to Step 8.2.2-3 C(6).

(4) Set A = B and increment NX:

$$A = B$$

$$NX = NX+1$$

(5) Develop A > 1:

$$B = 10\ A$$

Compare t_R and B:

If $t_R \leq B$, go to Step 8.2.2-3 C(6).

If $t_R > B$, go to Step 8.2.2-3 C(4).

(6) Compute integer portion of $t_R/A = t_i$:

Compare t_i and 2:

If $t_i < 2$ (i.e., = 1), go to Step 8.2.2-3D.

If $t_i = 2$, go to Step 8.2.2-3E.

If $t_i > 2$, go to Step 8.2.2-3F.

D. Compute t_R and vertical line spacing for $t_i = 1$:

$$\delta X = A/10$$

$$t_R = 10\delta X t_i$$

Go to Step 8.2.2-3H.

E. Compute t_R and vertical line spacing for $t_i = 2, 3,$ or 4:

$$\delta X = A/5$$

$$t_R = 5\delta Xt$$

Go to Step 8.2.2-3H.

F. Compare t_i and 4:

If $t_i \leq 4$ (i.e., = 3 or 4), go to Step 8.2.2-3E.

If $t_i > 4$, go to Step 8.2.2-3G.

G. Compute t_R and vertical line spacing for $4 < t_i < 10$:

$$\delta X = A/2$$

$$t_R = 2\delta Xt_i$$

H. Print scale values:

$$t_R, A, \delta X, t_i, NX$$

4. The machine is instructed to use semi-log scales, log on vertical, and linear on horizontal.

5. The point count index II for beginning of plot point number is set II = 1. The page count IIK is set = 1.

8.2.3 Computation for Each Set of Three Plots

1. Set point count index III for maximum end of plot point number III = II + 152.

2. Compute initial time EM and final time EN for each set of plots:

$$EM = EN$$

$$EN = EM + t_R$$

8.2.4 Set L = 1 to Indicate Plot for ΔC/C

8.2.5 Draw Grid and Do Common Labeling

1. Draw grid (subroutine GRIDV), calling sequence: general (L, XL, XR, YB, YT, DX, DY, N, M, I, J, NX, NY)

   L = 1 indicates film to be advanced, job number, and frame count to be displayed

   XL = EM is first time

   XR = EN is last time

   YB = 10^{-10} is bottom of ordinate

   YT = 1 is top of ordinate

   DX = δX indicates vertical lines drawn at increments of δX

   DY = 1 indicates horizontal lines drawn at increments of 1

   N = 0 causes all vertical lines to be of equal intensity

   M = 2 with logarithmic ordinate, this is a dummy

   I = -1 causes each vertical line to be labeled and labels to be below the scale

   J = -4 with logarithmic ordinate, this is a dummy

NX = NX indicates NX significant places on X scale

NY = 1 indicates 1 significant place in Y scale

2. List and define symbols.

Symbol	Location (Raster Counts)	Definition	Location (Raster Counts)
o	125, 939	ANALYTIC SOLUTION	145-289, 939
×	125, 919	PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PRETURBATION SOLUTION)	145-408, 919 145-432, 899
□	125, 879	NUMERICAL SOLUTION OF TOTAL EQUATIONS	145-416, 879
*	125, 859	INITIAL OSCULATING ELLIPSE	145-336, 859

3. Label horizontal scale.

The work "TIME" goes in raster counts 475-507, 0.

4. List run parameters and time scale.

"SEMI-MAJOR AXIS = 'a' ECCENTRICITY = 'e' INCLINATION (DEGREES) = 'i' MASS RATIO = 'μ'" is printed in raster counts 162-942, 989.

"TIME RANGE FROM 'EN' TO 'EM'" is printed in raster counts 434-658, 969.

5. If L = 1, go to Step 8.2.6.

If L = 2, go to Step 8.2.7.

If L = 3, go to Step 8.2.8.

8.2.6 Label and Plot Error in Jacobi Integral, $\Delta C/C$

1. Label vertical scale and title.

   "ABSOLUTE VALUE OF $\Delta C/C$" is entered in raster counts 0, 800-348.

   Label plot "ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL" in raster counts 302-727, 1012.

2. Add additional symbol definition and parameter values. The symbol '•' is printed at raster locations 125, 839.

   "NUMERICAL RESIDUAL PERTURBATION SOLUTION" is printed in raster positions 145-465, 839.

   "JACOBI CONSTANT, C = 'C_{init}'" is printed at raster positions 132-348, 969.

   "ANOMALISTIC PERIOD = 'P_{ss}'" is printed at raster positions 718-950, 969.

3. Plot $\Delta C/C$.

   A. Set point count index $i = II$.

   B. If $t_i \leq t_f$, go to Step 8.2.6-3C; otherwise go to Step 8.2.7.

   C. Set index K to find location of $(\Delta C/C)_{ai}$ in array AP:

   $$K = -12 + 13i$$

   D. If $t_i \leq EN$, go to Step 8.2.6-3E; otherwise go to Step 8.2.7.

   E. Plot $(\Delta C/C)_{ai}$, $(\Delta C/C)_{pki}$, $(\Delta C/C)_{ci}$, $(\Delta C/C)_{ki}$, and $(\Delta C/C)_{pi}$.

F. If $i < III$, set $i = i+1$ and return to Step 8.2.6-3B; otherwise go to Step 8.2.7.

8.2.7 Label and Plot Position Differences from Numerical Residual Perturbation Value, $\Delta\rho/a$

1. Draw grid and do common labeling.

   Set $L = 2$ and go to Step 8.2.5.

2. Label vertical scale and title.

   "DELTA RHO/A" is printed in raster locations 0, 602-446.

   "NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION" is printed in raster locations 302-822, 1012.

3. Plot $\Delta\rho/a$

   A. Set point count index $i = II$.

   B. If $t_i \leq t_f$, go to Step 8.2.7-3C; otherwise go to Step 8.2.8.

   C. Set index K to find location of $(\Delta\rho/a)_{ai}$:

$$K = -7 + 13i$$

   D. If $t_i \leq EN$, go to Step 8.2.7-3E; otherwise go to Step 8.2.8.

   E. Plot $(\Delta\rho/a)_{ai}$, $(\Delta\rho/a)_{pki}$, $(\Delta\rho/a)_{ci}$, and $(\Delta\rho/a)_{ki}$.

   F. If $i < III$, set $i = i+1$ and return to Step 8.2.7-3B; otherwise go to Step 8.2.8.

8.2.8 <u>Label and Plot Velocity Differences from Numerical Residual Perturbation Value, $\Delta\dot{\rho}\sqrt{a}$</u>

1. Draw grid and do common labeling. Set L = 3 and go to Step 8.2.5.

2. Label vertical scale and title.

   "DELTA RHO DOT TIMES SQ RT A" is printed in rasters 0, 620-324.

   "NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION" is printed in raster count 258-866, 1012.

3. Plot $(\Delta\dot{\rho}\ a^{1/2})_{ai}$, $(\Delta\dot{\rho}\ a^{1/2})_{pki}$, $(\Delta\dot{\rho}\ a^{1/2})_{ci}$, and $(\Delta\dot{\rho}\ a^{1/2})_{ki}$.

   A. Set point count index i = II.

   B. If $t_i \leq t_I$, go to Step 8.2.8-3C; otherwise go to Step 8.2.10.

   C. Set index K to find location of $(\Delta\dot{\rho}\ a^{1/2})_{ai}$ in array AP:

$$K = -3 + 13i$$

   D. If t_i < EN, go to Step 8.2.8-3E.

      If t_i = EN, go to Step 8.2.8-3F.

      If t_i > EN, go to Step 8.2.8-3G.

   E. Plot $(\Delta\dot{\rho}\ a^{1/2})_{ai}$, $(\Delta\dot{\rho}\ a^{1/2})_{pki}$, $(\Delta\dot{\rho}\ a^{1/2})_{ci}$, and $(\Delta\dot{\rho}\ a^{1/2})_{ki}$.

      If i < III, set i = i+1 and return to Step 8.2.8-3B; otherwise set II = III and go to Step 8.2.9.

   F. Plot $(\Delta\dot{\rho}\ a^{1/2})_{ai}$, $(\Delta\dot{\rho}\ a^{1/2})_{pki}$, $(\Delta\dot{\rho}\ a^{1/2})_{ci}$, and $(\Delta\dot{\rho}\ a^{1/2})_{ki}$.

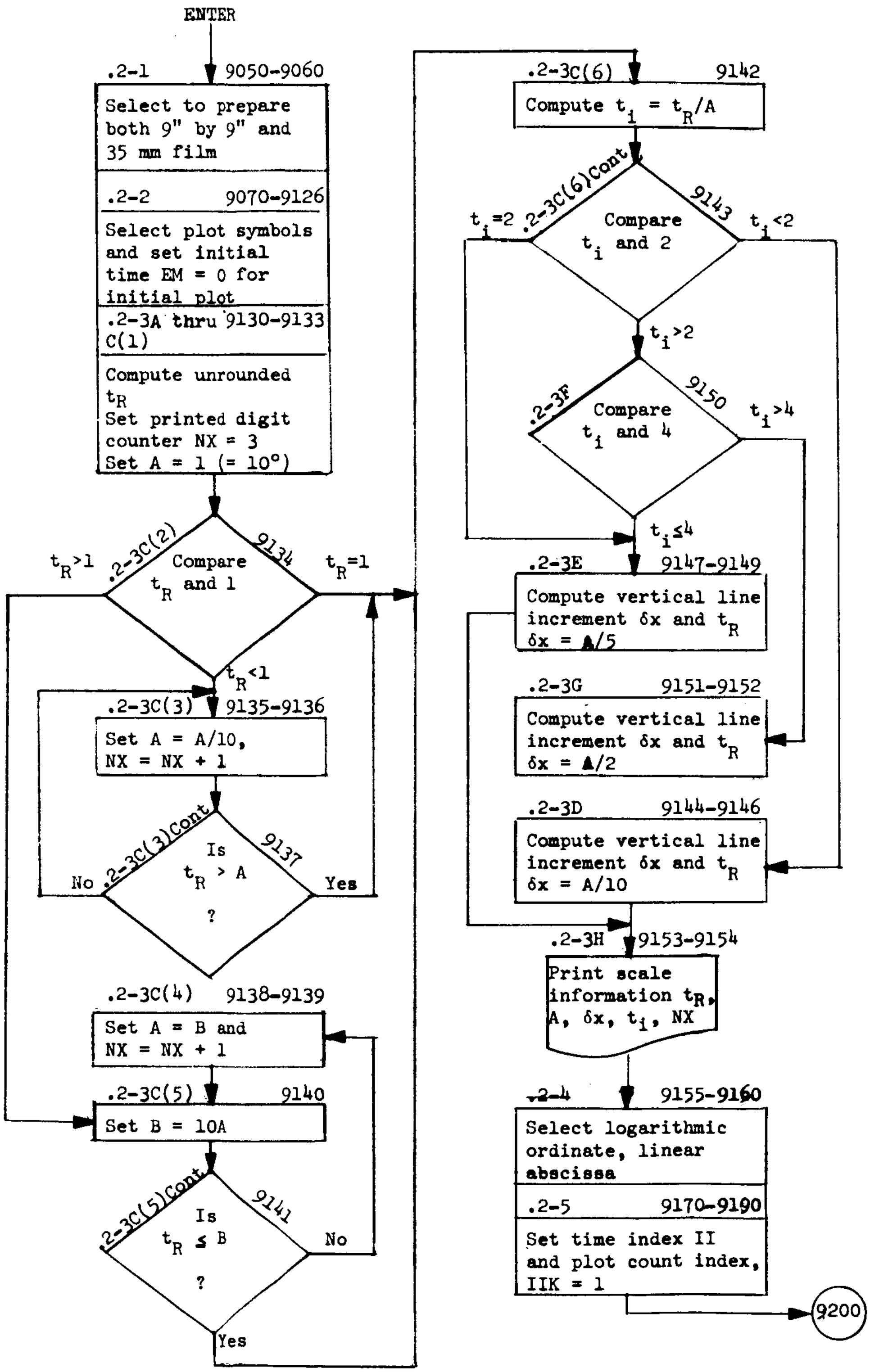
   G. Set II = i.

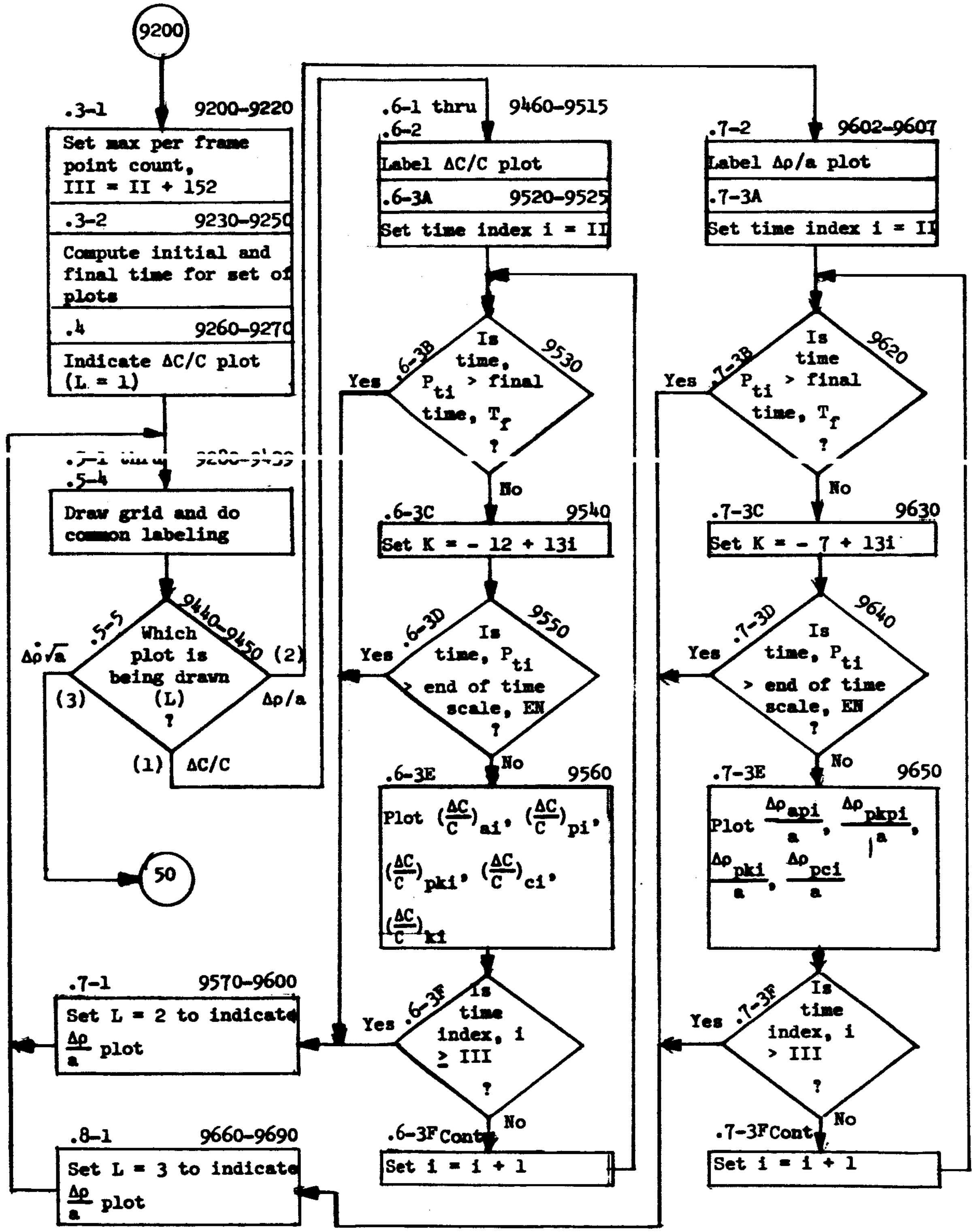
8.2.9 If IIK < 6, set IIK = IIK + 1 and Return to Step 8.2.3-1; Otherwise Go To Step 8.2.10.

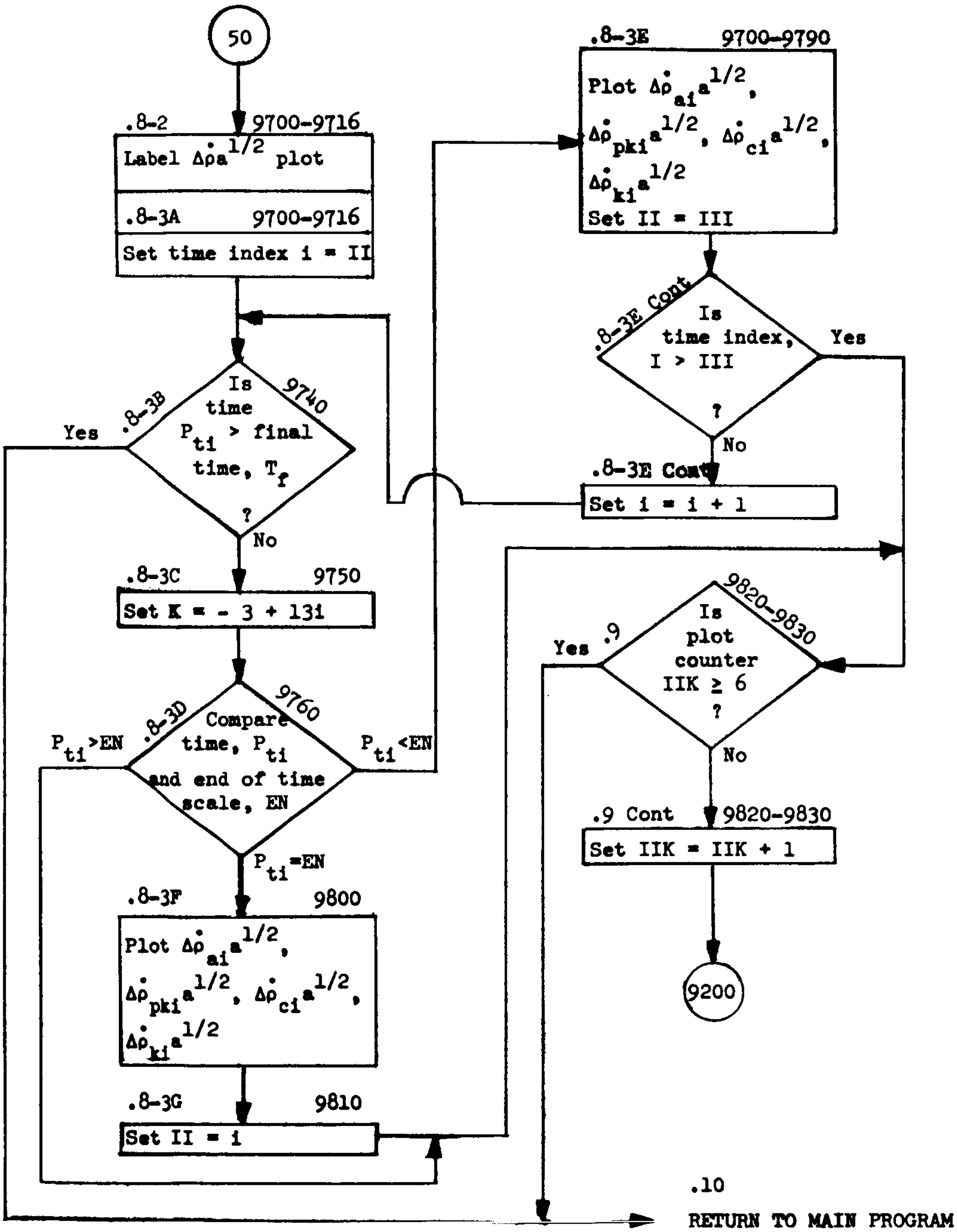
8.2.10 Return to Main Program

8.3 SUBROUTINE PLOT DETAIL FLOW CHARTS

The flow charts of this section describe the logic of subroutine PLOT completely, i.e., all conditional transfers are shown. The numbers to the top left of each box are the subsection numbers of Section 8.2 where the operations mentioned in the box are detailed. The numbers at the top right are the card numbers of the subroutine listing, Section 8.4.


ENTER
.2-1 9050-9060
Select to prepare both 9" by 9" and 35 mm film
.2-2 9070-9126
Select plot symbols and set initial time EM = 0 for initial plot
.2-3A thru C(1) 9130-9133
Compute unrounded tR
Set printed digit counter NX = 3
Set A = 1 (= 10°)
.2-3C(2) 9134
Compare tR and 1
tR>1
tR=1
tR<1
.2-3C(3) 9135-9136
Set A = A/10, NX = NX + 1
.2-3C(3)Cont 9137
Is tR > A ?
No
Yes
.2-3C(4) 9138-9139
Set A = B and NX = NX + 1
.2-3C(5) 9140
Set B = 10A
.2-3C(5)Cont 9141
Is tR ≤ B ?
No
Yes
.2-3C(6) 9142
Compute ti = tR/A
.2-3C(6)Cont 9143
Compare ti and 2
ti=2
ti<2
ti>2
.2-3F 9150
Compare ti and 4
ti>4
ti≤4
.2-3E 9147-9149
Compute vertical line increment δx and tR δx = A/5
.2-3G 9151-9152
Compute vertical line increment δx and tR δx = A/2
.2-3D 9144-9146
Compute vertical line increment δx and tR δx = A/10
.2-3H 9153-9154
Print scale information tR, A, δx, ti, NX
.2-4 9155-9160
Select logarithmic ordinate, linear abscissa
.2-5 9170-9190
Set time index II and plot count index, IIK = 1
9200


9200
.3-1 9200-9220
Set max per frame point count, III = II + 152
.3-2 9230-9250
Compute initial and final time for set of plots
.4 9260-9270
Indicate ΔC/C plot (L = 1)
.5-4
Draw grid and do common labeling
.5-5 9440-9450
Which plot is being drawn (L) ?
Δρ/a (3)
(2) Δρ/a
(1) ΔC/C
50
.6-1 thru 9460-9515
.6-2
Label ΔC/C plot
.6-3A 9520-9525
Set time index i = II
.6-3B 9530
Is time, Pti > final time, Tf ?
Yes
No
.6-3C 9540
Set K = - 12 + 13i
.6-3D 9550
Is time, Pti > end of time scale, EN ?
Yes
No
.6-3E 9560
Plot (ΔC/C)ai, (ΔC/C)pi, (ΔC/C)pki, (ΔC/C)ci, (ΔC/C)ki
.6-3F
Is time index, i ≥ III ?
Yes
No
.6-3F Cont.
Set i = i + 1
.7-1 9570-9600
Set L = 2 to indicate Δρ/a plot
.8-1 9660-9690
Set L = 3 to indicate Δρ/a plot
.7-2 9602-9607
Label Δρ/a plot
.7-3A
Set time index i = II
.7-3B 9620
Is time Pti > final time, Tf ?
Yes
No
.7-3C 9630
Set K = - 7 + 13i
.7-3D 9640
Is time, Pti > end of time scale, EN ?
Yes
No
.7-3E 9650
Plot Δρapi/a, Δρpkpi/a, Δρpki/a, Δρpci/a
.7-3F
Is time index, i > III ?
Yes
No
.7-3F Cont.
Set i = i + 1


50
.8-2 9700-9716
Label Δρ̇a^1/2 plot
.8-3A 9700-9716
Set time index i = II
.8-3B 9740
Is time P_ti > final time, T_f ?
Yes
No
.8-3C 9750
Set K = - 3 + 13i
.8-3D 9760
Compare time, P_ti and end of time scale, EN
P_ti>EN
P_ti<EN
P_ti=EN
.8-3F 9800
Plot Δρ̇_ai a^1/2, Δρ̇_pki a^1/2, Δρ̇_ci a^1/2, Δρ̇_ki a^1/2
.8-3G 9810
Set II = i
.8-3E 9700-9790
Plot Δρ̇_ai a^1/2, Δρ̇_pki a^1/2, Δρ̇_ci a^1/2, Δρ̇_ki a^1/2
Set II = III
.8-3E Cont
Is time index, I > III ?
Yes
No
.8-3E Cont
Set i = i + 1
.9 9820-9830
Is plot counter IIK ≥ 6 ?
Yes
No
.9 Cont 9820-9830
Set IIK = IIK + 1
9200
.10
RETURN TO MAIN PROGRAM

8.4 PLOT LISTING

This section contains the listing for subroutine PLOT. The subsection numbers on the comment cards refer to the subsections in Section 8.2, wherein the code is explained.

```
     H144    PLOT                                                        09/01/65
         EXTERNAL FORMULA NUMBER   -   SOURCE STATEMENT   -   INTERNAL FORMULA NUMBER(S)

      SUBROUTINE PLOT (A32,TF,A1,EC1,DEGI,EP4,JACOBI,PSS)          9010H144
      DIMENSION  W(30), PTIME( 700),AP( 9100), MRKPT(5)            9020H144
      COMMON  W,IPRINT,TS,TPSI,TNOD,TPER,GAMMA,EP1,ITP,PTIME       9030H144
     1,AP                                                          9031H144
C8.2.2 INITIAL SETUP                                               9040H144
C8.2.2-1 SET FOR BOTH 9 IN BY 9 IN AND 35 MM OUTPUT                9050H144
      CALL CAMRAV (935)                                            9060H144
C8.2.2-2 SELECTION OF PLOTTING SYMBOLS                             9070H144   ,1
      MRKPT (1) = 38                                               9080H144   ,2
      MRKPT (2) = 55                                               9090H144   ,3
      MRKPT (3) = 63                                               9100H144   ,4
      MRKPT (4) = 44                                               9110H144   ,5
      MRKPT (5) = 42                                               9120H144
C8.2.2-3 COMPUTE ABSCISSA SCALE                                    9123H144   ,6
      FN = 0.                                                      9126H144   ,7
      TR = 38. * A32                                               9131H144   ,8
      NX = 3                                                       9132H144   ,9
      A = 1.                                                       9133H144   ,10
  210 IF (TR - A)220, 230, 240                                     9134H144   ,11
  220 A = A /10.                                                   9135H144   ,12
      NX =NX + 1                                                   9136H144   ,13
      IF(TR -A) 220,230,230                                        9137H144   ,14
  250 A =B                                                         9138H144   ,15
      NX = NX + 1                                                  9139H144   ,16
  240 B = A * 10.                                                  9140H144   ,17
      IF(TR - B) 230,230,250                                       9141H144   ,18
  230 TI = AINT (TR / A)                                           9142H144   ,19
      IF(TI - 2.)  260,270,280                                     9143H144   ,20
  260 DX = A /10.                                                  9144H144   ,21
      TR = DX * 10. * TI                                           9145H144   ,22
      GO TO 300                                                    9146H144   ,23
  270 DX = A/5.                                                    9147H144   ,24
      TR = DX * 5. * TI                                            9148H144   ,25
      GO TO 300                                                    9149H144   ,26
  280 IF(TI - 4.) 270,270,290                                      9150H144   ,27
  290 DX = A/2.                                                    9151H144   ,28
      TR = DX * 2.* TI                                             9152H144   ,29
  300 WRITE (6,310) TR , A , DX, TI, NX                            9153H144   ,30   ,31   ,32
  310 FORMAT (1H 4E18.8, I8)                                       9154H144
C8.2.2-4 SET FOR LOGARITHIMIC ORDINATE, LINEAR ABSCISSA            9155H144
      CALL SMXYV ( 0, 1)                                           9160H144
C8.2.2-5 SET INITIAL FLAGS                                         9170H144   ,33
      II  =1                                                       9180H144   ,34
      DO 150 IIK=1,6                                               9190H144
C8.2.3 COMPUTATION FOR EACH SET OF 3 PLOTS                         9200H144
C8.2.3-1 SET MAXIMUM PER FRAME POINT COUNT                         9210H144   ,35
      III =II + 152                                                9220H144
C8.2.3-2 COMPUTE INITIAL AND FINAL TIME FOR EACH SET OF PLOTS      9230H144   ,36
      EM = FN                                                      9240H144   ,37
      EN = EM + TR                                                 9250H144
C8.2.4 SET L = 1 TO INDICATE PLOT FOR DELTA C/C                    9260H144   ,38
      L=1                                                          9270H144
C8.2.5 DRAW GRID AND DO COMMON LABELING                            9280H144
C8.2.5-1 DRAW GRID (SUBROUTINE GRIDV)                              9290H144   ,39
   20 CALL GRIDIV (1,EM,EN,1.E-10,1.0,DX,1.0,0.2 ,-1,-4,NX,-1)     9300H144
```

```
      EXTERNAL FORMULA NUMBER   -   SOURCE STATEMENT   -   INTERNAL FORMULA NUMBER(S)

C8.2.5-2 LIST AND DEFINE SYMBOLS                                  9310H144    40
     CALL PLOTV(125,939,38)                                       9320H144    41
     CALL PRINTV (18,18H ANALYTIC SOLUTION,145,939)               9330H144    42
     CALL PLOTV(125,919,55)                                       9340H144    43
     CALL PRINTV(39,39HPRECESSING ELLIPSE (REFERENCE ORBIT FOR,137,919)9350H144    44
     CALL PRINTV(41,41HNUMERICAL RESIDUAL PERTURBATION SOLUTION),145,899360H144
    19)                                                           9361H144    45
     CALL PLOTV(125,879,63)                                       9370H144    46
     CALL PRINTV (38,38H NUMERICAL SOLUTION OF TOTAL EQUATIONS,137,879)9380H144    47
     CALL PLOTV (125,859,44)                                      9390H144    48
     CALL PRINTV (27,27H INITIAL OSCULATING ELLIPSE,137,859)      9400H144
C8.2.5-3 LABEL HORIZONTAL SCALE                                   9410H144    49
     CALL PRINTV ( 4, 4HTIME, 475, 0 )                            9420H144
C8.2.5-4 LIST RUN PARAMETERS AND TIME SCALE                       9422H144    50
     CALL PRINTV ( 17,17HSEMI-MAJOR AXIS =, 132, 989 )            9424H144    51
     CALL LABLV (A1,268,989,7,1,1)                                9426H144    52
     CALL PRINTV ( 14,14HECCENTRICITY =, 340,989)                 9428H144    53
     CALL LABLV (EC1,452,989,7,1,0)                               9430H144    54
     CALL PRINTV ( 23,23HINCLINATION (DEGREES) =, 524,989)        9432H144    55
     CALL LABLV (DEGI,718,989,7,1,3)                              9433H144    56
     CALL PRINTV ( 12,12HMASS RATIO =,   790,989)                 9434H144    57
     CALL LABLV (EP4,886,989,-3,1,1)                              9435H144    58
     CALL PRINTV(15,15HTIME RANGE FROM,434,969)                   9436H144    59
     CALL LABLV (FM,562,969,4,1,3)                                9437H144    60
     CALL PRINTV( 2, 2HTO,602,969)                                9438H144    61
     CALL LABLV (EN,626,969,4,1,3)                                9439H144
C8.2.5-5 DETERMINE WHICH PLOT IS BEING DRAWN                      9440H144    62
     GO TO (30,40,50),L                                           9450H144
C8.2.6 LABEL AND PLOT ERROR IN JACOBI INTEGRAL, DELTA C/C         9460H144
C8.2.6-1 LABEL VERTICAL SCALE AND TITLE                           9470H144    63
  30 CALL APRNTV(C,-12,29,29HABSOLUTE VALUE OF DELTA C / C, 0,800)  9480H144    64
     CALL PRINTV(53,53HABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INT9484H144
    1EGRAL, 302,1012)                                             9485H144
C8.2.6-2 ADD ADDITIONAL SYMBOL DEFINITION AND PARAMETER VALUES    9490H144    65
     CALL PLOTV (125,839,42)                                      9500H144    66
     CALL PRINTV (19,19HJACOBI CONSTANT,C = , 132,969)            9502H144    67
     CALL LABLV (JACOBI,292,969, 7 ,1,4)                          9504H144    68
     CALL PRINTV (21,21H ANOMOLISTIC PERIOD =,718,969)            9506H144    69
     CALL LABLV (PSS,894,969, 7,1, 2)                             9508H144    70
     CALL PRINTV(40,40HNUMERICAL RESIDUAL PERTURBATION SOLUTION,145,8399510H144
    1)                                                            9511H144
C8.2.6-3 PLOT DELTA C/C                                           9520H144    71
     DO90I = I1,II1                                               9525H144    72
     IF(PTIME(I)-TF) 10,10,100                                    9530H144    73
  10 K =-12+13* I                                                 9540H144    74
     IF(PTIME(I) - EN) 90,90,100                                  9550H144    75
  90 CALL APLOTV (5,PTIME(I), AP(K), 0,1,5,MRKPT,IERR)            9560H144
C8.2.7 LABEL AND PLOT POSITION DIFFERENCES FROM NUMERICAL RESIDUAL  9570H144
C      PERUTRBATION VALUE, DELTA RHO/A                            9575H144
C8.2.7-1 DRAW GRID AND DO COMMON LABELING                         9580H144    76   ,77
 100 L=2                                                          9590H144    78
     GO TO 20                                                     9600H144
C8.2.7-2 LABEL VERTICAL SCALE AND TITLE                           9602H144    79
  40 CALL APRNTV (0,-12,   13,13HDELTA RHO / A,0,602)             9604H144    80
     CALL PRINTV (65,65HNORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PER9606H144
    1TURBATION POSITION, 302,1012)                                9607H144
```

```
 H144    PLOT                                                           09/01/65                              PAGE  83
      EXTERNAL FORMULA NUMBER   -   SOURCE STATEMENT   -   INTERNAL FORMULA NUMBER(S)

C8.2.7-3 PLOT DELTA RHO/A                                              9608H144    .81
      DO 60  I = II,III                                                9610H144    .82
      IF(PTIME(I)- TF)130,130  ,110                                    9620H144    .83
  130 K = -7 + 13*I                                                    9630H144    .84
      IF(PTIME - EN)60,60,110                                          9640H144    .85
   60 CALL APLOTV (4,PTIME(I),AP(K) ,0,1,4,MRKPT,IERR)                 9650H144
C8.2.8 LABEL AND PLOT VELOCITY DIFFERENCES FROM NUMERICAL RESIDUAL     9660H144
C      PERTURBATION VALUE, DELTA RHO DOT SQ RT A                       9665H144
C8.2.8-1 DRAW GRID AND DO COMMON LABELING                              9670H144    .86    .87
  110 L=3                                                              9680H144    .88
      GO TO 20                                                         9690H144
C8.2.8-2 LABEL VERTICAL SCALE AND TITLE                                9700H144    .89
   50 CALL APRNTV (0,-12,27,27HDELTA RHO DOT TIMES SQ RT A, 0, 620)    9710H144    .90
      CALL PRINTV (76,76HNORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL R9715H144
     1ESIDUAL PERTURBATION SOLUTION, 258,1012)                         9716H144
C8.2.8-3 PLOT (DELTA RHO DOT * SQ RT A) SUB AI, (DELTA RHO DOT * SQ    9720H144
C        RT A) SUB PKI, (DELTA RHO DOT * SQ RT A) SUB CI, AND (DELTA RHO9722H144
C        DOT * SQ RT A) SUB KI                                         9724H144    .91
      DO 70  I = II,III                                                9730H144    .92
      IF (PTIME(I) - TF)120,120,80                                     9740H144    .93
  120 K =-3 + 13 * I                                                   9750H144    .94
      IF (PTIME(I)-EN)70, 160,140                                      9760H144    .95
   70 CALL APLOTV (4,PTIME(I),AP(K),0,1,4,MRKPT,IERR)                  9770H144    .96    .97
      II = III                                                         9780H144    .98
      GO TO 150                                                        9790H144    .99
  160 CALL APLOTV (4,PTIME(I),AP(K),0,1,4,MRKPT,IERR)                  9800H144    .100
  140 II = I                                                           9810H144
C8.2.9 IF IIK IS LESS THAN 6, SET IIK = IIK + 1 AND RETURN TO STEP     9820H144
C      8.2.3-1 OTHERWISE GO TO STEP 8.2.10                             9825H144    .101
  150 CONTINUE                                                         9830H144
C8.2.10 RETURN TO MAIN PROGRAM                                         9840H144    .102   .103
   80 RETURN                                                           9850H144    .104
      END                                                              9860H144    .105
```

Section 9
INPUT AND OUTPUT

9.1 INPUT

The input to this program is by the Douglas subroutine INPUT 1. A load sheet for this program is shown as figure 2. The card format is:

column 1 - a one
columns 2 through 6 - location number of piece of data
columns 7 through 15 - input number
columns 16 and 17 - location of decimal place from beginning of field
positive if to the right

Three other pieces of data may be entered on the card. The location numbers are punched in columns 18-22, 34-38, and 50-54. The data are punched in columns 23-31, 39-47, and 55-63. The exponents, as explained above, are punched in columns 32-33, 48-49, and 64-65, respectively. The remaining information required is:

columns 66-68, zeros
columns 69-70, reference run number
columns 71-73, case number.

This routine allows identification on the card of each piece of input data by relative location number; only-non-zero numbers need be entered. It has a "Reference Run," "Case" setup. If the case number (card columns 71 to 73) is non-zero, but the reference run number (card columns 69, 70) is zero, then the data on the load sheet are assumed to be sufficient and the case is computed. If the case number is zero and the reference run number is non-zero, the data are stored in array WR and no case is attempted. If the following load sheets with non-zero case numbers have also the reference run number of the stored array, then a case is run using the input of array WR as modified by the new load sheet.

FORTRAN DATA LOAD SHEET

Form 60-720-1 (11-62)

PROBLEM ________

PREPARED BY ________ DATE ________ PAGE ____ OF ____

MUST BE FILLED IN FOR PROPER PROCESSING

69 70	71 73
RR	CASE

ENGINEER NOTE:

LOCATION 00000 <u>MUST NOT</u> BE USED

QUAN	LOC	±	VALUE	±	E	QUAN	LOC	±	VALUE	±	E

KEY PUNCH STANDARD DATA INPUT 1

Figure 2

The order of stacking cases is then:

1. All cases with zero reference run number
2. First reference run (zero case number)
3. All cases with first reference run number and non-zero case number
4. Second reference run (zero case number)

   .
   .
   .

It is one of the peculiarities of this routine that cases must be run in numerical order.

9.2 OUTPUT

The first information that is printed each time the program is run consists of an explanation of the error codes. The program makes certain checks on the input to make sure that the input is reasonable and consistent with the theory being used. Throughout the program there are additional checks made to ascertain if division by zero has occurred which would cause the program to produce nonsense if the case continued. In these cases, an error code is printed and the case terminated. The print is as follows:

ERROR CODE	REASON FOR HALT
1	SEMI-MAJOR AXIS = 0
2	ECCENTRICITY EQUALS OR EXCEEDS 1
7	S (THE APPROX RADIUS VECTOR) = 0
8	DERIVATIVE OF TIME WRT PSI = 0 (i.e., $\frac{dt}{d\psi}$)

The error code is set to 1 in Section 4.3.2.1.1. The error code is set to 2 in Section 4.3.2.2.1. The error code is set to 7 in Section 4.17.3. The error code is set to 8 in Section 4.17.8.1. The error code is printed in Section 4.3.2.1.2.

The program then reads in a case and INPUT 1 prints:

```
REFERENCE RUN NO.    (RR#)    CASE NO.    (CASE #)
```

There follows a floating point print of the input array W and of constants computed by the program as described in Section 4.4.1.

Section 4.6 then prints the headings for the print during the numerical residual perturbation solution as follows:

```
THE INITIAL CONDITIONS ARE    SEMI-MAJOR AXIS  a    INCLINATION  i
                              ECCENTRICITY  e       MASS RATIO  μ

TIME      DELTA X       DELTA X DOT        DELTA X DBL DOT    RHO
          DELTA Y       DELTA Y DOT        DELTA Y DBL DOT    RHO DOT
C APPROX  DELTA Z       DELTA Z DOT        DELTA Z DBL DOT
          X     X DOT       X APPROX        X DOT APPROX
          Y     Y DOT       Y APPROX        Y DOT APPROX
C EXACT   Z     Z DOT       Z APPROX        Z DOT APPROX
```

The algebraic interpretation of these headings is given in Section 4.10.

The initial value of the Jacobi constant, C_{init}, is printed in Section 4.8.6 as:

```
THE JACOBI INTEGRAL CONSTANT IS  (C_init)
```

During computation of the numerical residual perturbation solution, the printing according to the above heading is accomplished in Section 4.10. If the input quantity $W_{13} > 0$, then printing takes place about every 1/4 radian of ψ. Otherwise printing takes place at the first and last point only.

The headers for the identification of the print during the numerical solution of the total equations of motion are printed in Section 4.19.4 and are as follows:

NUMERICAL SOLUTION OF TOTAL EQUATIONS OF MOTION DATA

TIME	X	X DOT	X DBL DOT	RHO
	Y	Y DOT	Y DBL DOT	RHO DOT
JACOBI CONSTANT,	Z	Z DOT	Z DBL DOT	

The printing for the numerical solution of the total equations of motion is done in Section 4.20 and the explanation of the above headers is shown in that section. Additional print for which there are no headers is also shown there. Printing during the solution of the total equations of motion has the same rules with regard to the input quantity W_{13} as does the print during the numerical residual perturbation solution.

The plotted output is described in subroutine PLOT, Section 8.

SAMPLE PRINTED OUTPUT

ERROR CODE	REASON FOR HALT
1	SEMI-MAJOR AXIS = 0
2	ECCENTRICITY EQUALS OR EXCEEDS 1
7	S (THE APPROX RADIUS VECTOR) = 0
8	DERIVATIVE OF TIME WRT PSI = 0

```
REFERENCE RUN NO.     3    CASE NO.    90

  0.1478C000E 01    0.                0.                0.30019999E-05    0.95000000E 00
 0.2CCC0000E-06    0.90000000E 03    0.09999999E-0?    0.15000000E 01    0.09999999E-02
 0.50000000E 03    0.09999999E-08    0.                0.                0.

-0.18833892E 01   -0.16471118E 01   -0.13476369E 01    0.92941266E 00    0.85042351E 00
 0.                0.                0.               -0.00000000E-19   -0.00000000E-19   -0.23349535E-02
 0.9452CC79E-05    0.99999700E 00    0.17968492E 01    0.70587333E-01    0.85882531E 00    0.09999999E 01

   0.46750527E-02     -0.23349539E-02     -0.10917198E-04     0.27315607E-04
```

```
THE INITIAL CONDITIONS ARE     SEMI-MAJOR AXIS    0.14780000E 01     INCLINATION,DEG    0.
                              ECCENTRICITY       0.                 MASS RATIO         0.30019999E-05

TIME      DELTA X      DELTA X DOT      DELTA X DBL DOT      RHO
          DELTA Y      DELTA Y DOT      DELTA Y DBL DOT      RHO DOT
C APPROX  DELTA Z      DELTA Z DOT      DELTA Z DBL DOT
          X       X DOT      X APPROX      X DOT APPROX
          Y       Y DOT      Y APPROX      Y DOT APPROX
C EXACT   Z       Z DOT      Z APPROX      Z DOT APPROX

THE JACOBI INTEGRAL CONSTANT IS  -0.11550964E 04

 0.              0.                0.                0.89611433E-02    0.14780000E 01
                 0.               -0.26841874E-02    0.                0.82332655E 00
 0.              0.               -0.                0.

                 0.14780000E 01    0.                0.14780000E 01    0.
                 0.                0.82332655E 00    0.                0.82332655E 00
 0.              0.                0.                0.                0.
```

```
INTEGRATION OF COWELL EQNS OF MOTION

      TIME                    X                        X DOT                   X DBL DOT                  RHO
                              Y                        Y DOT                   Y DBL DOT                RHO DOT
JACOBI CONSTANT.              Z                        Z DOT                   Z DBL DOT


 0.                    0.14780000E 01        0.                      -0.45267198E-00           0.14780000E 01
                       0.                    0.82332654E 00           0.                       0.82332654E 00
 0.                    0.                    0.                       0.
      0.28676210E-06         0.71000228E-05        0.14498003E 01          0.23612494E-03        0.55294169E-08
      0.27158640E-00         0.27841849E-00        0.                     -0.13303044E-03
      0.26952379E-00         0.28065864E-00        0.81663435E 00          0.


 0.31457543E 03        0.14443220E 01       -0.14989252E-00          -0.45220289E-00           0.14687005E 01
                       0.26648622E-00        0.81386065E 00          -0.80971745E-01           0.82754873E 00
 0.88302965E-07        0.                   -0.                      -0.
      0.28676210E-06         0.71000228E-05        0.88302965E-07          0.89011711E-03        0.55294169E-08
      0.27158640E-00         0.27841849E-00        0.20873854E-01          0.13723116E 01
      0.26952379E-00         0.28065864E-00        0.20775224E-01          0.13778877E 01
    0.55555999E 02      0.09999999E 02       0.49999999E 01      0.59999999E 01          4
ERRNLV  XL =     0.09999999E-09 XU =      0.09999999E 01 ML =                40 MU =         40 DXY =     0.09999999E 01
ERRNLV  XL =     0.09999999E-09 XU =      0.09999999E 01 ML =                40 MU =         40 DXY =     0.09999999E 01
ERRNLV  XL =     0.09999999E-09 XU =      0.09999999E 01 ML =                40 MU =         40 DXY =     0.09999999E 01
ERRNLV  XL =     0.09999999E-09 XU =      0.09999999E 01 ML =                40 MU =         40 DXY =     0.09999999E 01
ERRNLV  XL =     0.09999999E-09 XU =      0.09999999E 01 ML =                40 MU =         40 DXY =     0.09999999E 01
ERRNLV  XL =     0.09999999E-09 XU =      0.09999999E 01 ML =                40 MU =         40 DXY =     0.09999999E 01
ERRNLV  XL =     0.09999999E-09 XU =      0.09999999E 01 ML =                40 MU =         40 DXY =     0.09999999E 01
ERRNLV  XL =     0.09999999E-09 XU =      0.09999999E 01 ML =                40 MU =         40 DXY =     0.09999999E 01
ERRNLV  XL =     0.09999999E-09 XU =      0.09999999E 01 ML =                40 MU =         40 DXY =     0.09999999E 01
ERRNLV  XL =     0.09999999E-09 XU =      0.09999999E 01 ML =                40 MU =         40 DXY =     0.09999999E 01
ERRNLV  XL =     0.09999999E-09 XU =      0.09999999E 01 ML =                40 MU =         40 DXY =     0.09999999E 01
ERRNLV  XL =     0.09999999E-09 XU =      0.09999999E 01 ML =                40 MU =         40 DXY =     0.09999999E 01
ERRNLV  XL =     0.09999999E-09 XU =      0.09999999E 01 ML =                40 MU =         40 DXY =     0.09999999E 01
ERRNLV  XL =     0.09999999E-09 XU =      0.09999999E 01 ML =                40 MU =         40 DXY =     0.09999999E 01
ERRNLV  XL =     0.09999999E-09 XU =      0.09999999E 01 ML =                40 MU =         40 DXY =     0.09999999E 01
ERRNLV  XL =     0.09999999E-09 XU =      0.09999999E 01 ML =                40 MU =         40 DXY =     0.09999999E 01
ERRNLV  XL =     0.09999999E-09 XU =      0.09999999E 01 ML =                40 MU =         40 DXY =     0.09999999E 01
ERRNLV  XL =     0.09999999E-09 XU =      0.09999999E 01 ML =                40 MU =         40 DXY =     0.09999999E 01

MAXIMUM DELTA E=   0.             MAXIMUM DELTA E PERTURBED =   0.11920929E-06
```

Section 10

DISCUSSION OF RESULTS

A summary of numerical results is included in graphical plot form in the Appendix. For the numerical studies, the mass ratios were $\mu = 3 \times 10^{-6}$, which corresponds to the sun-earth (or quite closely to the sun-Venus) systems and $\mu = 0.0123$, which corresponds to the earth-moon system. In reference 1 it was stated that the solutions should be valid for a region of order $\mu^{1/2}$ around the origin in the natural variables, i.e., semi-major axes of order unity in the blown-up variable could be tolerated. This would represent the limit to which the series could be taken. Further approximation is involved in that the actual series are only computed to order ε^2. All $\varepsilon^2 e$ terms were dropped in this program for consistency since the switch-back terms from ε^3 (presumed to be of order $\varepsilon^2 e$) and terms of forms $\varepsilon e i$ and ε^3 are not included in reference 1. For the earth-moon mass ratio, semi-major axes of 0.041 (corresponding to a near grazing lunar satellite), 0.1, and 0.4 were used. For the earth-sun mass ratio, the semi-major axes were chosen to be 0.15 (corresponding to about the synchronous altitude), 0.5, and 1.478 (corresponding to the moon). In reference 1 the values of eccentricity and inclination that could be tolerated consistently with the accuracy with respect to μ were not specified explicitly, but it was merely stated that these elements must be small. We have used eccentricities of 0 and 0.05 and inclinations of 0° and 5°. The plots show the time history of the error in the Jacobi constant, the vector distance from the numerical residual preturbation value, and the vector velocity difference from the numerical residual perturbation value. The summary of the cases computed is given in Table I.

The error in the Jacobi constant is represented as the variation with time along the solution, divided by the initial value. It is presented for the analytic solution of reference 1, for a precessing mean Kepler ellipse which has essentially the same precession rate as the analytic solution but no oscillatory perturbations, for a numerical integration of the total equations of motion, for the fixed initial osculating ellipse, and for the numerical residual perturbation solution. The precessing ellipse is the reference orbit of the numerical residual perturbation solution. The fixed initial osculating ellipse is provided for comparison since it would form the basis of a normal Encke's method solution. The variation of the Jacobi constant should, of course, be zero in the exact solution of the restricted three-body problem. The other two variables were chosen to represent position and velocity errors, and were referenced to the numerical residual perturbation solution since it was anticipated that this solution would, in general, maintain the Jacobi integral most accurately. For each set of parameters, the progress of these results was monitored for approximately 28 revolutions. This limit is set by the program which, by the way it is coded, imposes a storage limit at this level. Since it was desired to show both the short- and long-term performance of the technique, an expanded time scale was used. This causes a plot for 28 revolutions to extend over five to six pages. For the 24-hour circular earth orbit and the moon's orbit, the entire time span is exhibited. For the other cases, only the beginning and ending pages are included.

It will be seen from the error in Jacobi integral plots that for the 24-hour circular earth orbit (or 13,000-mile Venus orbit) and the grazing lunar orbit, the numerical residual perturbation solution approximates the sun's perturbations no better than the analytic solution, which is already at the round off level of the machine throughout the 28 orbits. It will also be noted that for these cases, the initial osculating ellipse maintains the Jacobi integral better than the particular numerical solution of the total equations of motion used. It will be noted that in most cases the initial osculating ellipse holds the Jacobi integral better than the precessing mean-ellipse. However, the position and velocity errors of the initial osculating ellipse are greater than (sometimes orders of magnitude greater) or equal to the precessing ellipse errors. This may be explained as follows. For motion of a close satellite around the smaller primary, the Jacobi integral is sensitive to errors in radius and its derivative. It is almost totally insensitive to errors along the direction of motion. The s, or one-over-radius, expression in the analytic solution contains a constant term in the perturbations of order ε^2 showing that the radius vector oscillates about a value which is not the Keplerian value of the precessing ellipse but is actually less. The $dt/d\psi$ expression contains a perturbation term whose value is not zero at initial time. This term is used in the computation of the elements of the osculating ellipse but not in the computation of the elements of the mean ellipse. The result is that the period of the osculating ellipse is in error, but the perigee is lowered, partially compensating for the neglected term in s . It is thus better in radius but poorer in angle.

At the largest semi-major axes considered (the case of the moon about the earth, and the 10 lunar radii lunar satellite), the analytic solution is very poor. This would be anticipated since de Pontecoulant demonstrated in the last century that this series was, at best, only barely convergent for the moon and only second order terms have been carried in this study. Over half of the precession of perigee and over half the "evection" were lost when the higher order terms were dropped (ref 3, pages 322 and 326). At these semi-major axes, the advantage of the numerical residual perturbation solution without rectification over the numerical solution of the total equations of motion in holding the Jacobi integral drops to a factor of 6 to 7. However, the sharp difference in slope between the two values of error in the Jacobi constant for small times suggests that with rectification, the numerical residual perturbation solution will be much better than the numerical solution of the total equations of motion. But since the $\Delta\rho/a$ and $\Delta\dot{\rho}\sqrt{a}$ plots show that the initial osculating ellipse is nearly as good as the precessing mean ellipse at small times, the advantage of the numerical residual perturbation solution over a numerical perturbation solution is not so clear. This suggests that when frequent rectification is required, as it seems to be with the large orbits, the use of a precessing ellipse whose initial orbital parameters are equal to those of an initial osculating ellipse might be better than using a precessing mean ellipse.

Comparing the results for $e = 0.05$ with the results for $e = 0$, it seems that $e = 0.05$ is a large e for this expansion. However, this program was constructed merely for demonstration of the residual perturbation method and is not deemed to be a practical analysis tool. In reference 6, the theory is developed without the restriction of small eccentricity or small inclination. The accuracy shown here for the case of zero inclination and zero eccentricity then would be the only one applicable to a program based on that theory, and it would be expected that it would hold to the same order of accuracy at other eccentricities and inclinations. From the cases with zero-eccentricity, it appears that the assumption that the precessing ellipse would be nearly as useful a reference orbit as the analytic solution is not a good one. In hind sight it is seen that this would follow from the fact that in an accumulation, as is done in the numerical solution of differential equations, the decimal place where error accumulates is proportional to the maximum value of the solution. Thus, when an oscillation is being accumulated numerically, the error accumulates at a rate roughly proportional to the full magnitude of the oscillation. If the method of reference 6 were programmed, then it is anticipated that the additional algebra required to algebraically cancel all εe terms and ε^2 in the residual perturbation equations of motion would result in appreciably improved numerical results.

Table I. - SUMMARY OF COMPUTED CASES

Mass Ratio μ	Semi-major Axis a	Eccentricity e	Inclination i°	Notes	Frame Identification Serial Number	Frame Identification Page Numbers
3×10^{-6}	0.15	0	0	Synchronous earth or 13,000-mile Venus satellite	281071	145-162
	0.15	0.05	5		281071	127-129, 139-144
	0.5	0.05	5		281071	109-111, 124-126
	1.478	0	0		302074	1-3, 13-18
	1.478	0.0549	5.15	Moon's orbit or 120,000-mile Venus satellite	281071	91-108
1.23×10^{-2}	0.041	0	0	Grazing lunar satellite	281071	55-57, 67-69
	0.041	0.05	5		281071	37-39, 49-51
	0.1	0.05	5		281071	19-21, 34-36
	0.4	0.05	5	10 lunar radii lunar satellite	281071	1-3, 13-15

REFERENCES

1. Kevorkian, J.: A Uniformly Valid Asymptotic Representation for All Times of a Satellite in the Vicinity of the Smaller Body in the Restricted Three-Body Problem. Astron. J., vol. 67, no.4, no. 1299, May 1962 (Revised version of reference 7.)

2. Moulton, Forest Ray: An Introduction to Celestial Mechanics. The Macmillian Company, 1914.

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4. Herget, Paul: The Computation of Orbits. 1948.

5. Hildebrand, F. B.: Introduction to Numerical Analysis. McGraw-Hill Book Co., Inc., 1956.

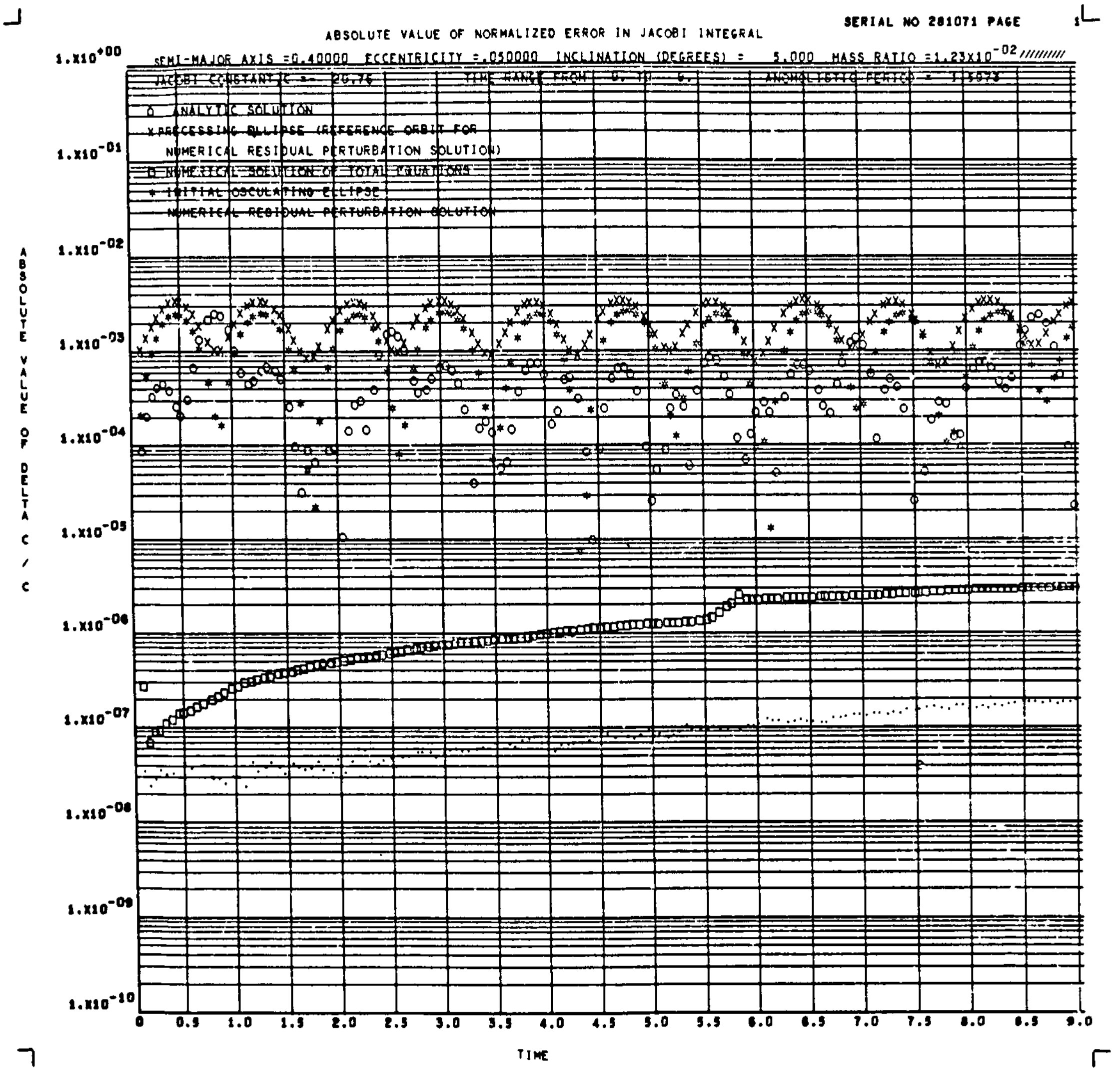
6. Eckstein, M.; Shi, Y; and Kevorkian, J.: Satellite Motion for Arbitrary Eccentricity and Inclination Around the Smaller Body in the Restricted Three-Body Problem. Douglas Paper No. 3489, April 1965.

7. Kevorkian, J.: A Uniformly Valid Asymptotic Representation for All Times of a Satellite in the Vicinity of the Smaller Body in the Restricted Three-Body Problem. Douglas Paper No. 1163, May 1961.

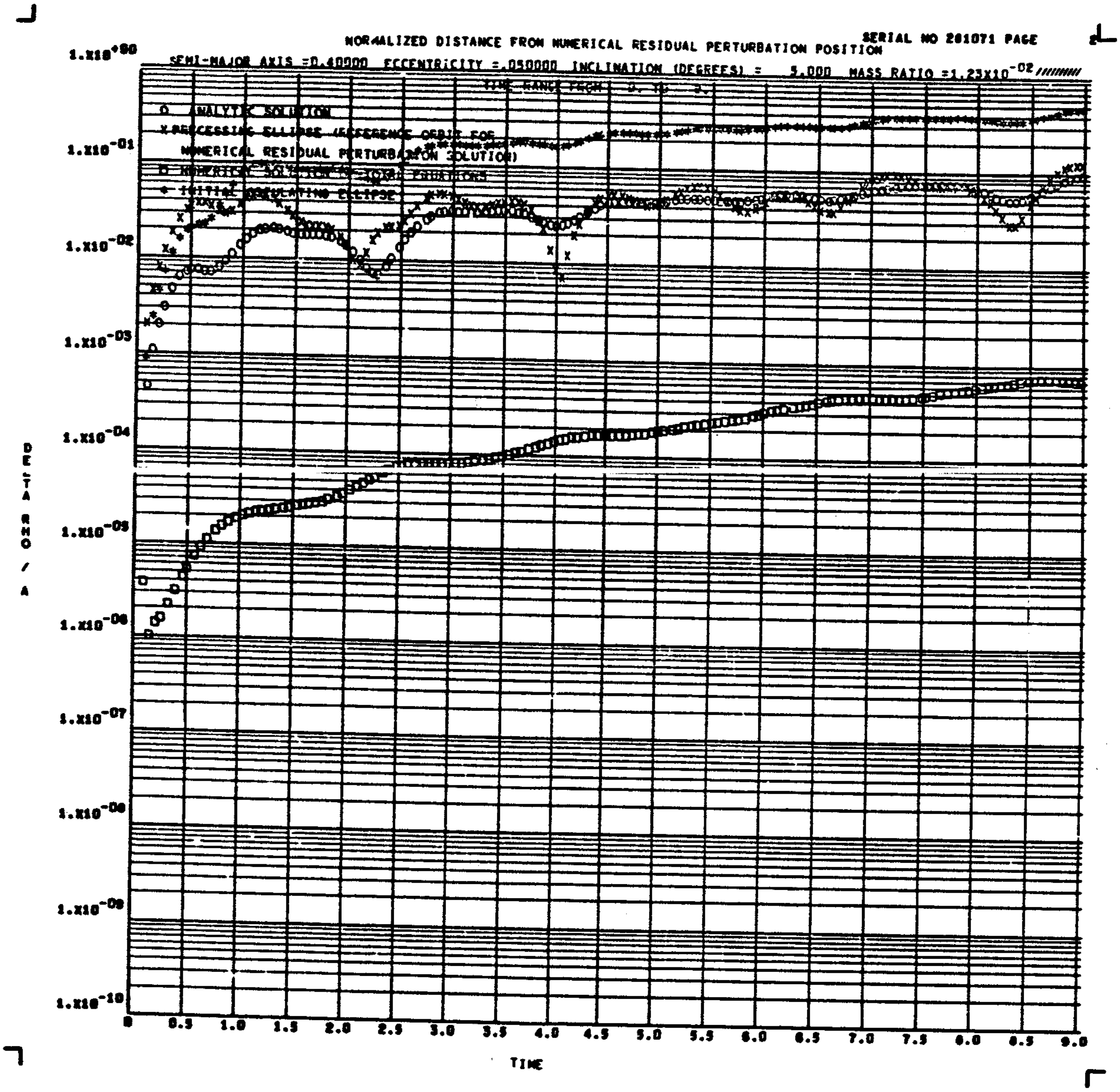
8. Anon.: North American Aviation, Inc., Engineers Computing Manual, Region 74, S-C 4020 Subprograms, Oct. 1963.

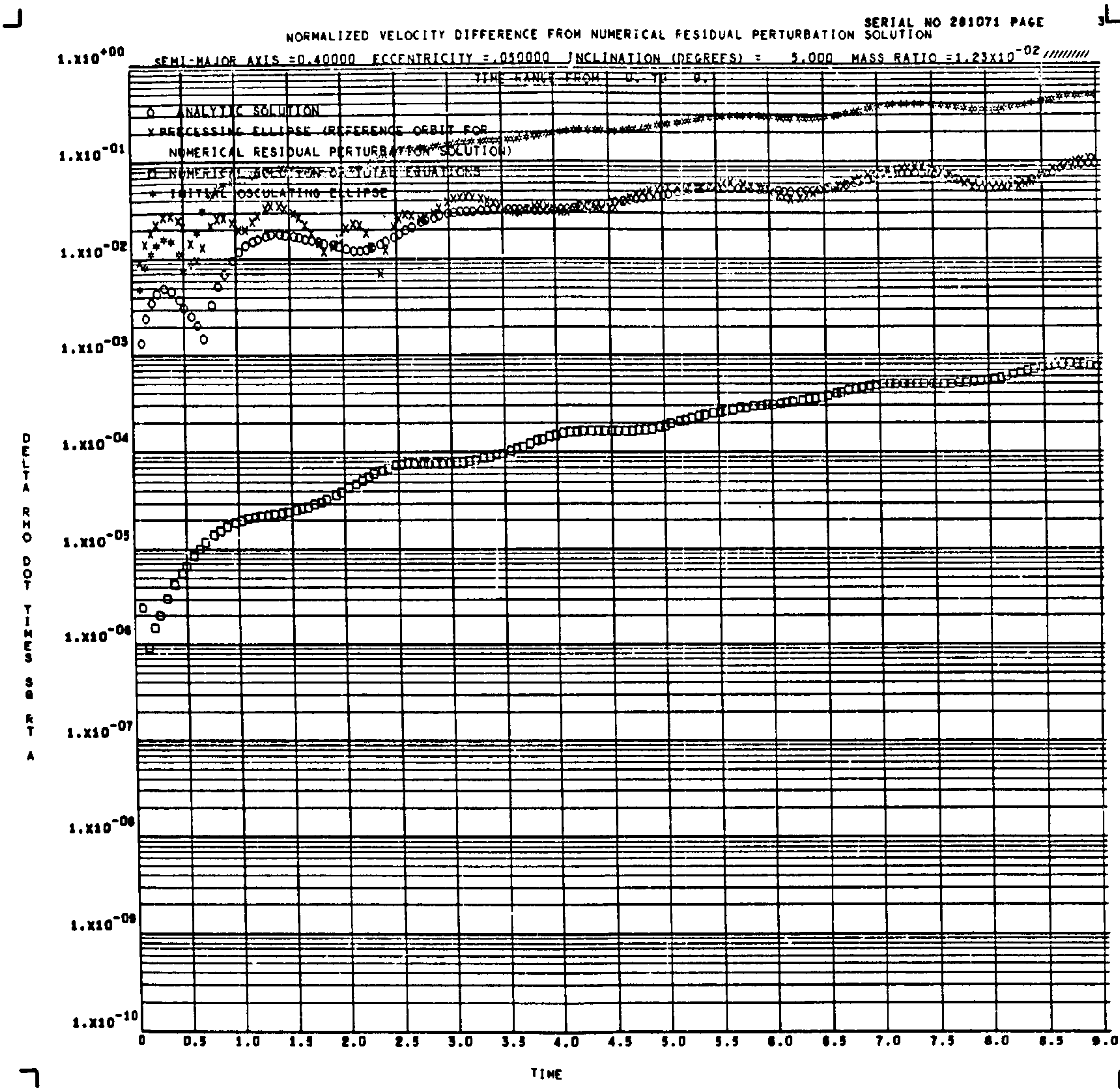
APPENDIX

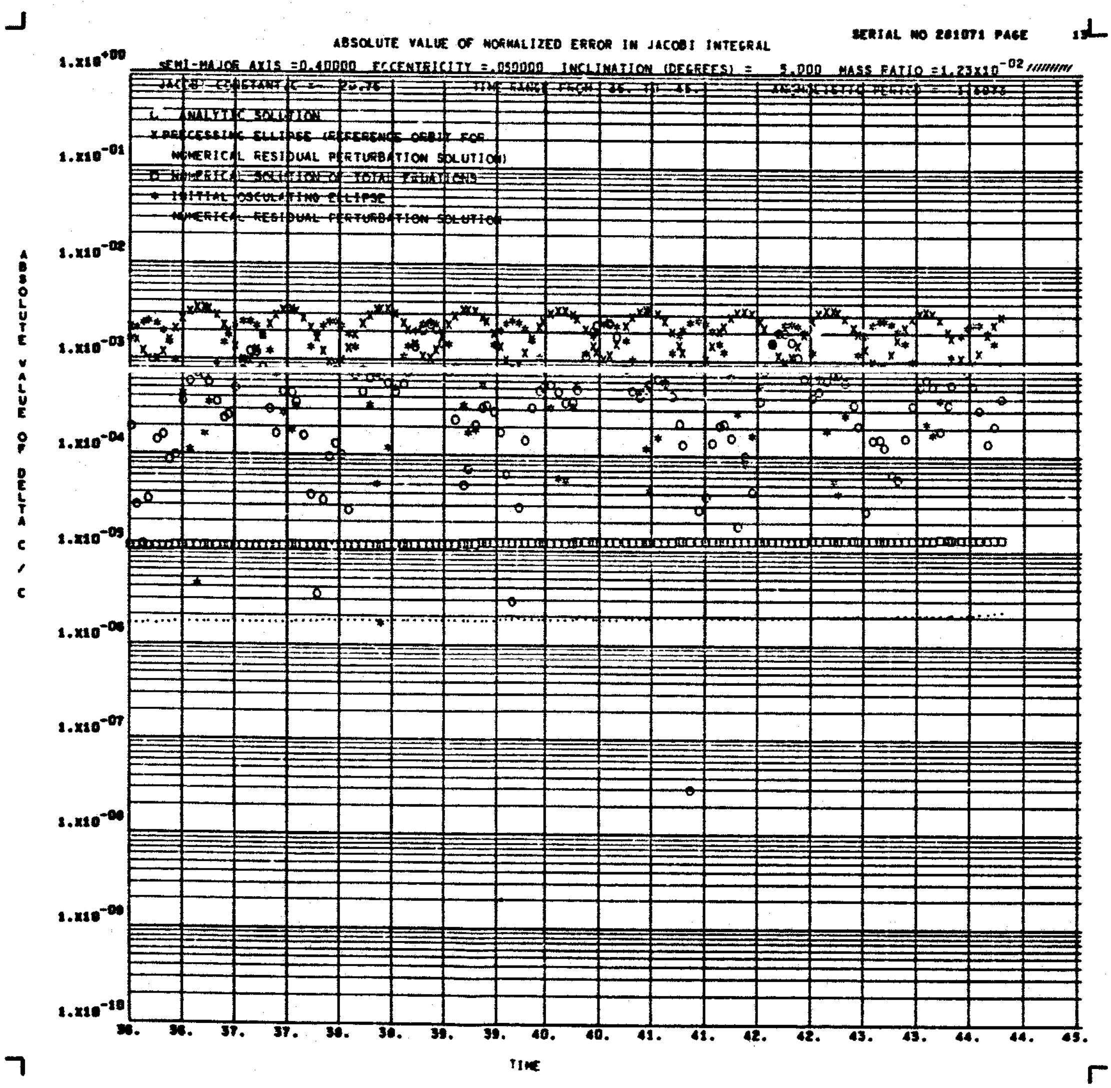
SAMPLE GRAPHICAL RESULTS

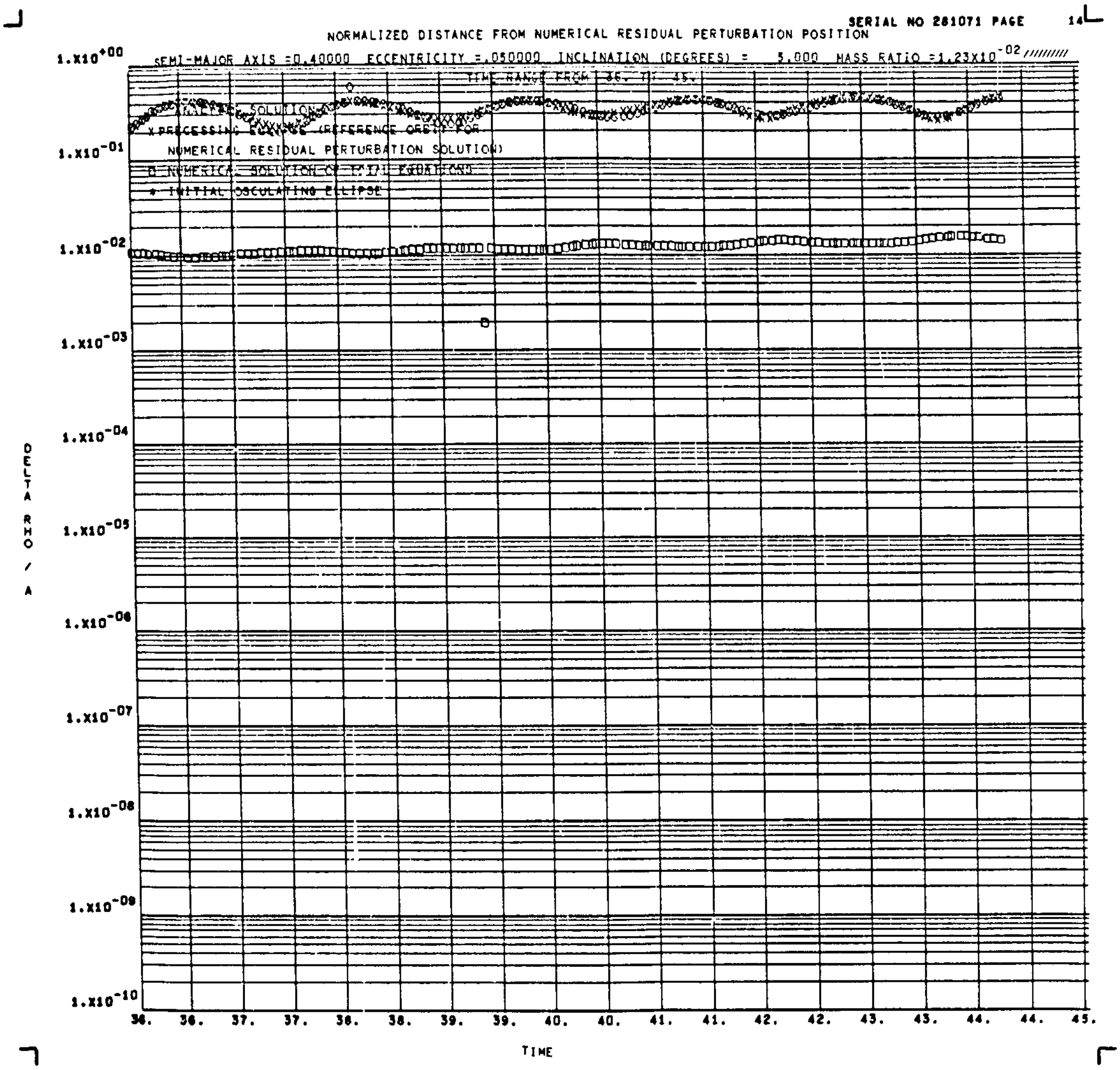


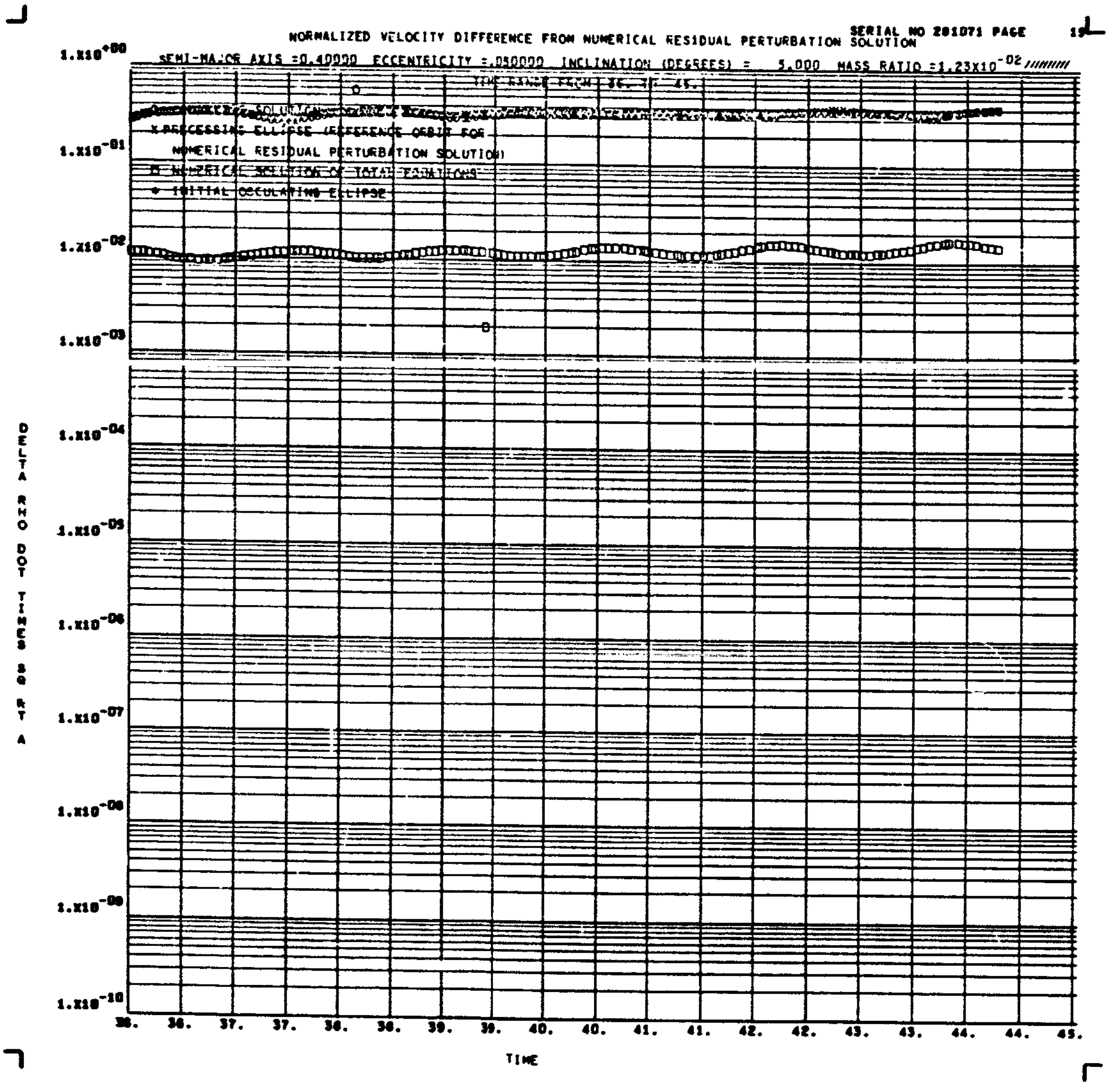
SERIAL NO 281071 PAGE
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =0.40000 ECCENTRICITY =.050000 INCLINATION (DEGREES) = 5.000 MASS RATIO =1.23X10^-02
O ANALYTIC SOLUTION
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
ABSOLUTE VALUE OF DELTA C / C
0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0 7.5 8.0 8.5 9.0
TIME


NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SERIAL NO 281071 PAGE
SEMI-MAJOR AXIS =0.40000 ECCENTRICITY =.050000 INCLINATION (DEGREES) = 5.000 MASS RATIO =1.23X10-02
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION
+ INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO / A
0
0.5
1.0
1.5
2.0
2.5
3.0
3.5
4.0
4.5
5.0
5.5
6.0
6.5
7.0
7.5
8.0
8.5
9.0
TIME

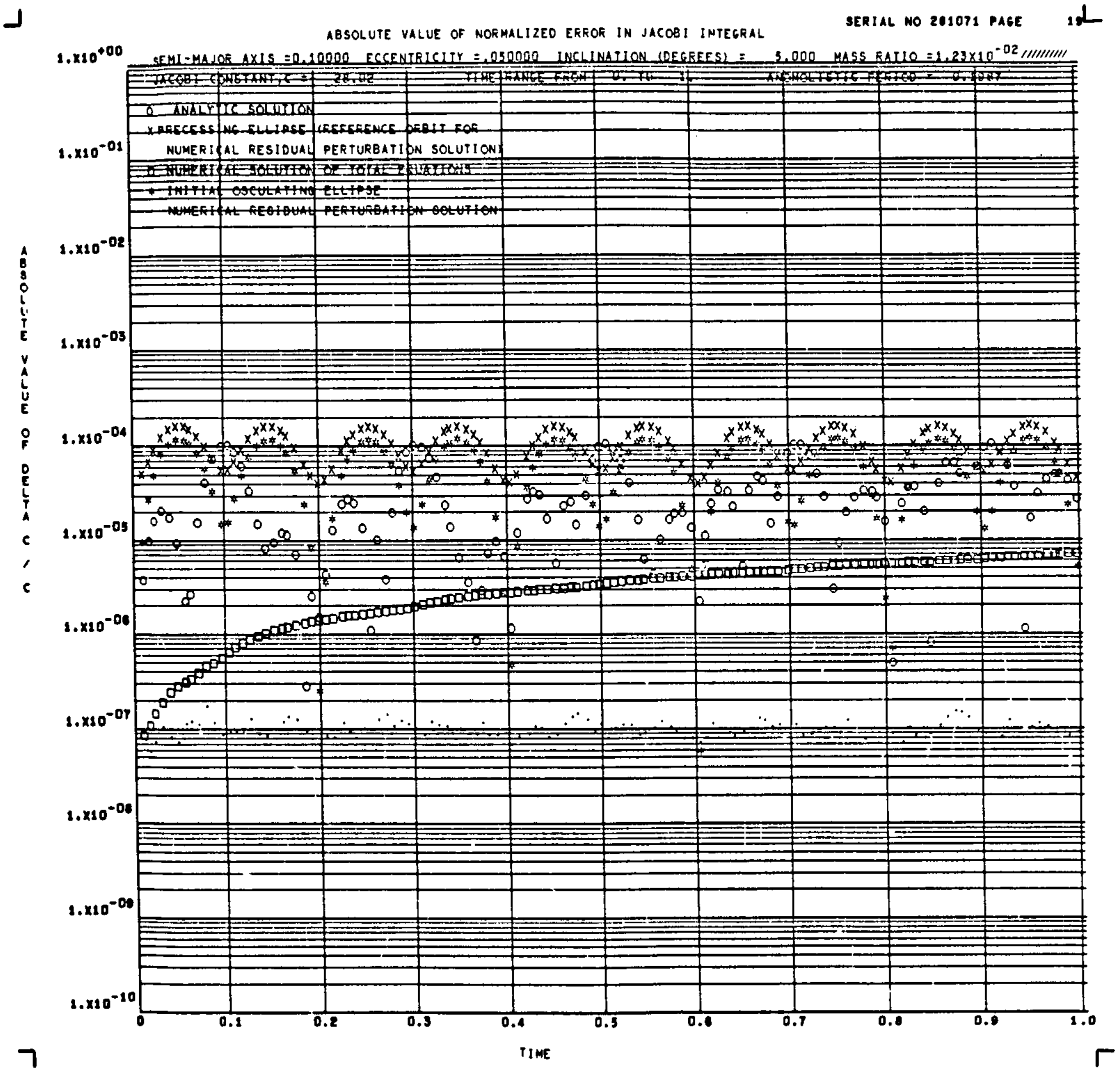

SERIAL NO 281071 PAGE 3
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =0.40000 ECCENTRICITY =.050000 INCLINATION (DEGREES) = 5.000 MASS RATIO =1.23X10-02
TIME RANGE FROM 0. TO 9.
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO DOT TIMES SQ RT A
0 0.5 1.0 1.5 2.0 2.5 3.0 3.5 4.0 4.5 5.0 5.5 6.0 6.5 7.0 7.5 8.0 8.5 9.0
TIME


SERIAL NO 281071 PAGE
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =0.40000 ECCENTRICITY =.050000 INCLINATION (DEGREES) = 5.000 MASS RATIO =1.23x10^-02
ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
NUMERICAL SOLUTION OF TOTAL EQUATIONS
INITIAL OSCULATING ELLIPSE
NUMERICAL RESIDUAL PERTURBATION SOLUTION
ABSOLUTE VALUE OF DELTA C / C
1.x10^+00
1.x10^-01
1.x10^-02
1.x10^-03
1.x10^-04
1.x10^-05
1.x10^-06
1.x10^-07
1.x10^-08
1.x10^-09
1.x10^-10
36. 36. 37. 37. 38. 38. 39. 39. 40. 40. 41. 41. 42. 42. 43. 43. 44. 44. 45.
TIME

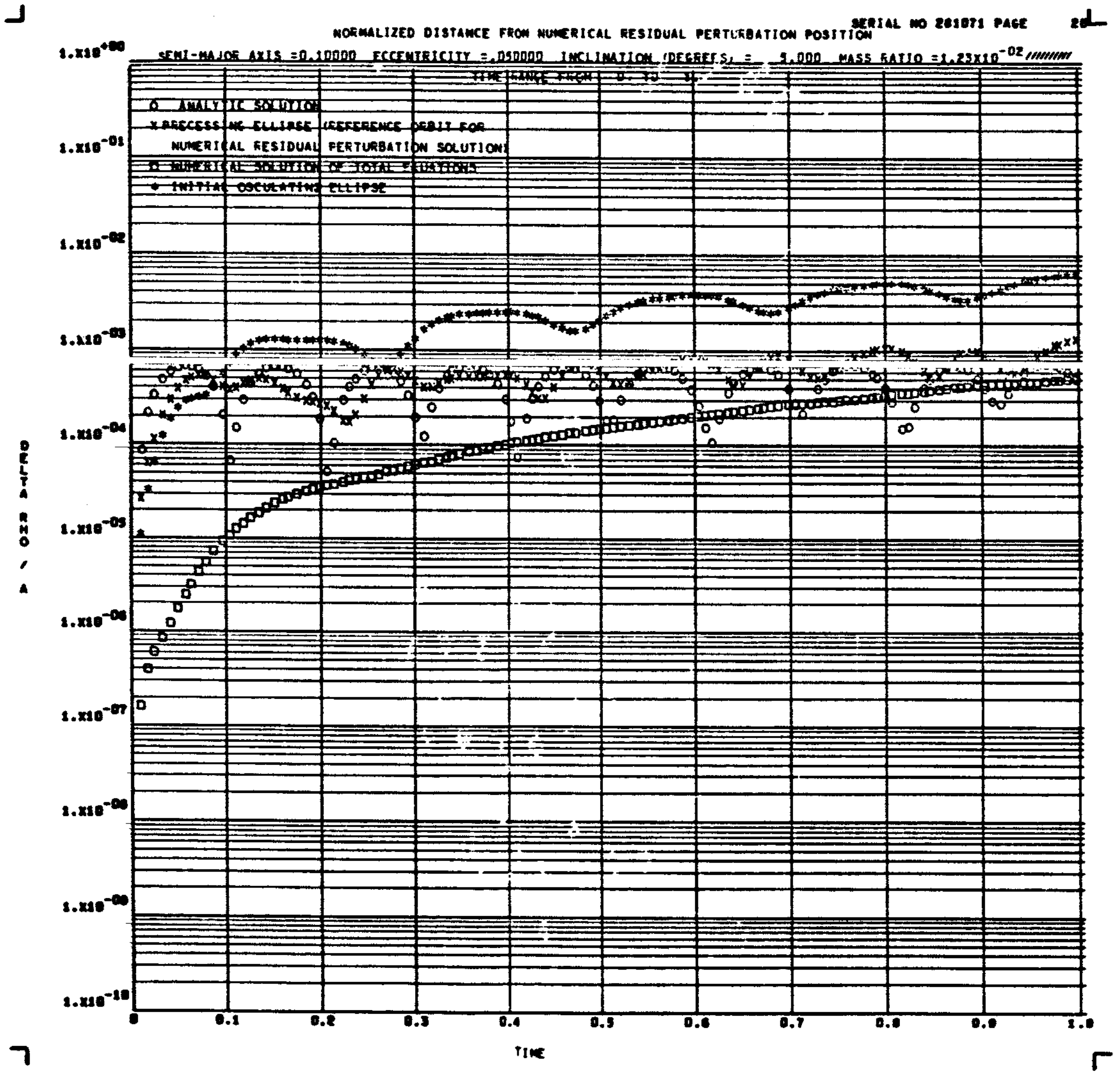

SERIAL NO 281071 PAGE 14
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =0.40000 ECCENTRICITY =.050000 INCLINATION (DEGREES) = 5.000 MASS RATIO =1.23X10-02
SOLUTION
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO / A
36. 36. 37. 37. 38. 38. 39. 39. 40. 40. 41. 41. 42. 42. 43. 43. 44. 44. 45.
TIME

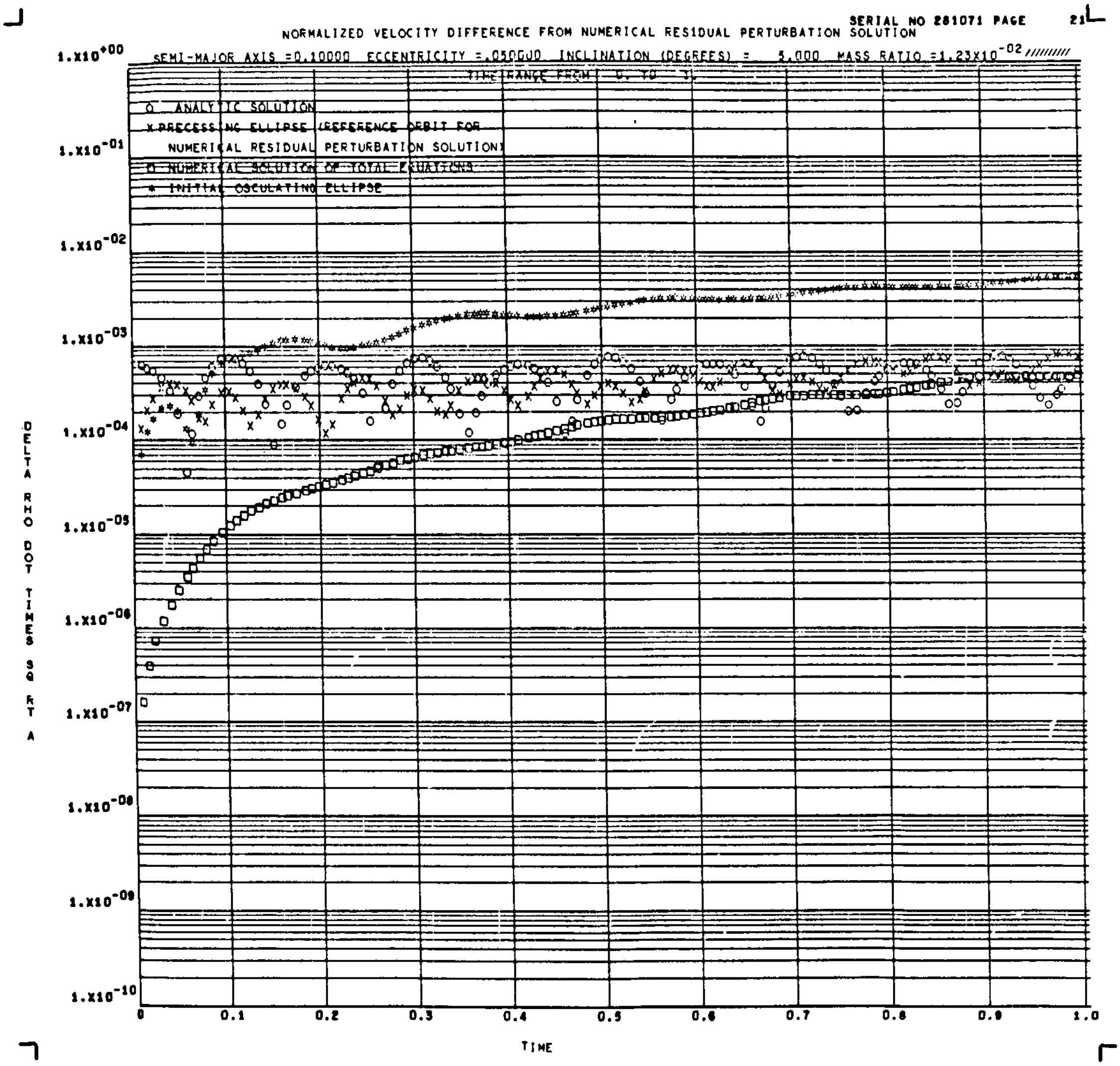


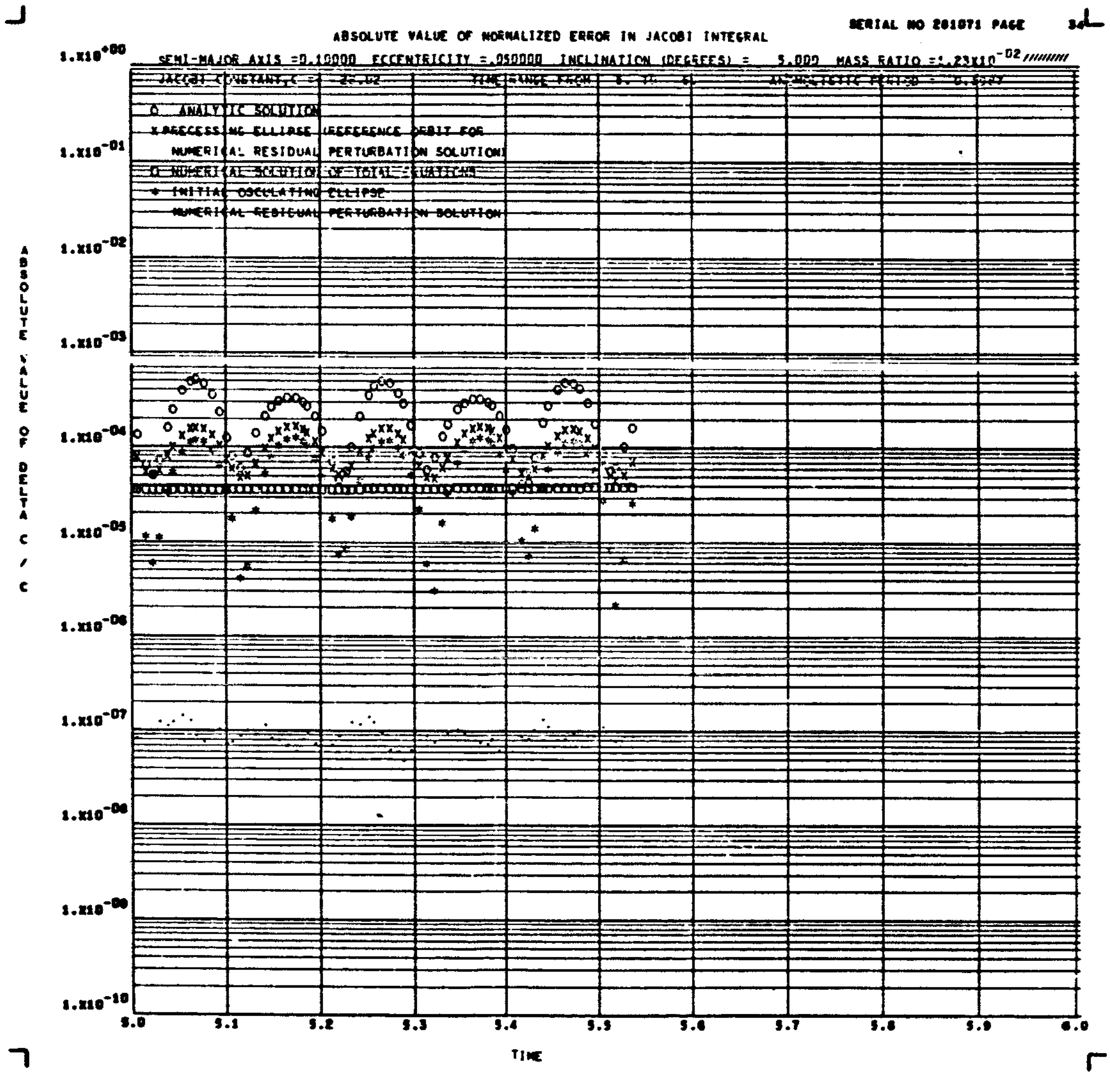
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SERIAL NO 281071 PAGE
SEMI-MAJOR AXIS =0.40000 ECCENTRICITY =.050000 INCLINATION (DEGREES) = 5.000 MASS RATIO =1.23X10-02
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO DOT TIMES SQ RT A
36. 36. 37. 37. 38. 38. 39. 39. 40. 40. 41. 41. 42. 42. 43. 43. 44. 44. 45.
TIME



SERIAL NO 281071 PAGE 19
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =0.10000 ECCENTRICITY =.050000 INCLINATION (DEGREES) = 5.000 MASS RATIO =1.23X10-02
JACOBI CONSTANT,C = 28.02 TIME RANGE FROM 0. TO 1.
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
NUMERICAL RESIDUAL PERTURBATION SOLUTION
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
ABSOLUTE VALUE OF DELTA C / C
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
TIME

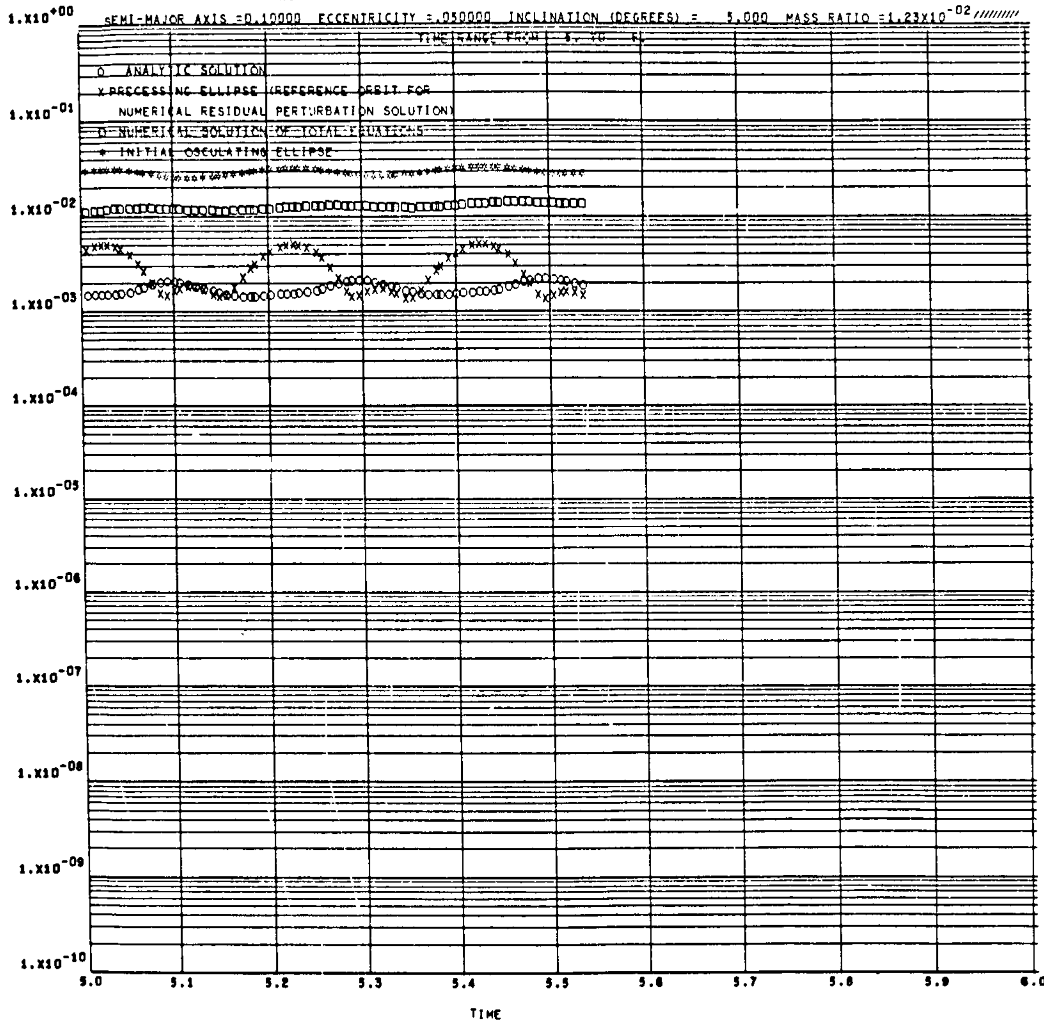

SERIAL NO 281871 PAGE
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =0.10000 ECCENTRICITY =.050000 INCLINATION (DEGREES) = 5.000 MASS RATIO =1.23X10^-02
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
DELTA RHO / A
TIME
1.X10^+00
1.X10^-01
1.X10^-02
1.X10^-03
1.X10^-04
1.X10^-05
1.X10^-06
1.X10^-07
1.X10^-08
1.X10^-09
1.X10^-10
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0

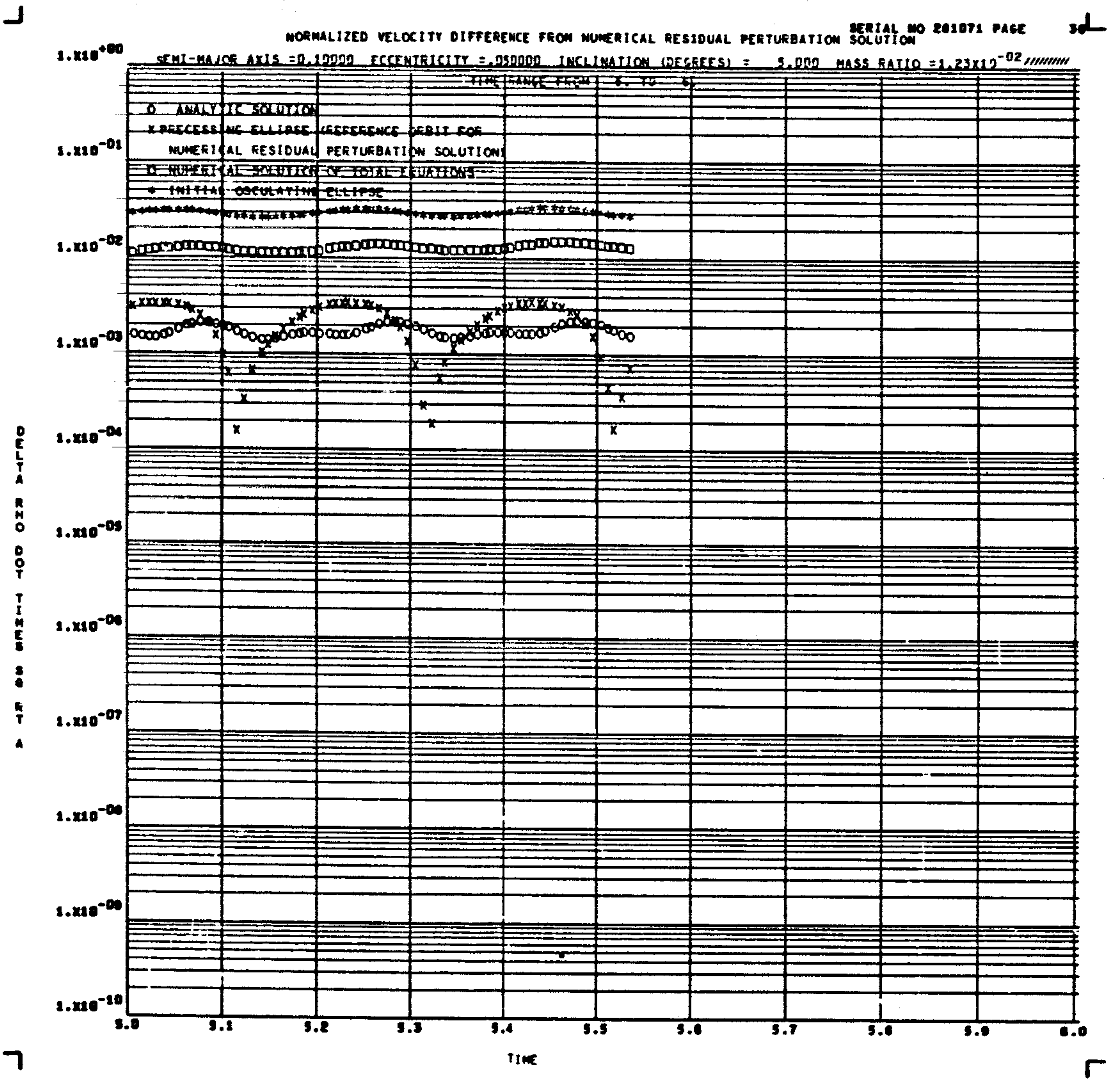

SERIAL NO 281071 PAGE 21
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =0.10000 ECCENTRICITY =.050000 INCLINATION (DEGREES) = 5.000 MASS RATIO =1.23X10-02
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
DELTA RHO DOT TIMES SQ RT A
TIME
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0


SERIAL NO 281071 PAGE
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =0.10000 ECCENTRICITY =.050000 INCLINATION (DEGREES) = 5.000 MASS RATIO =1.23X10-02
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
NUMERICAL RESIDUAL PERTURBATION SOLUTION
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
ABSOLUTE VALUE OF DELTA C / C
5.0
5.1
5.2
5.3
5.4
5.5
5.6
5.7
5.8
5.9
6.0
TIME

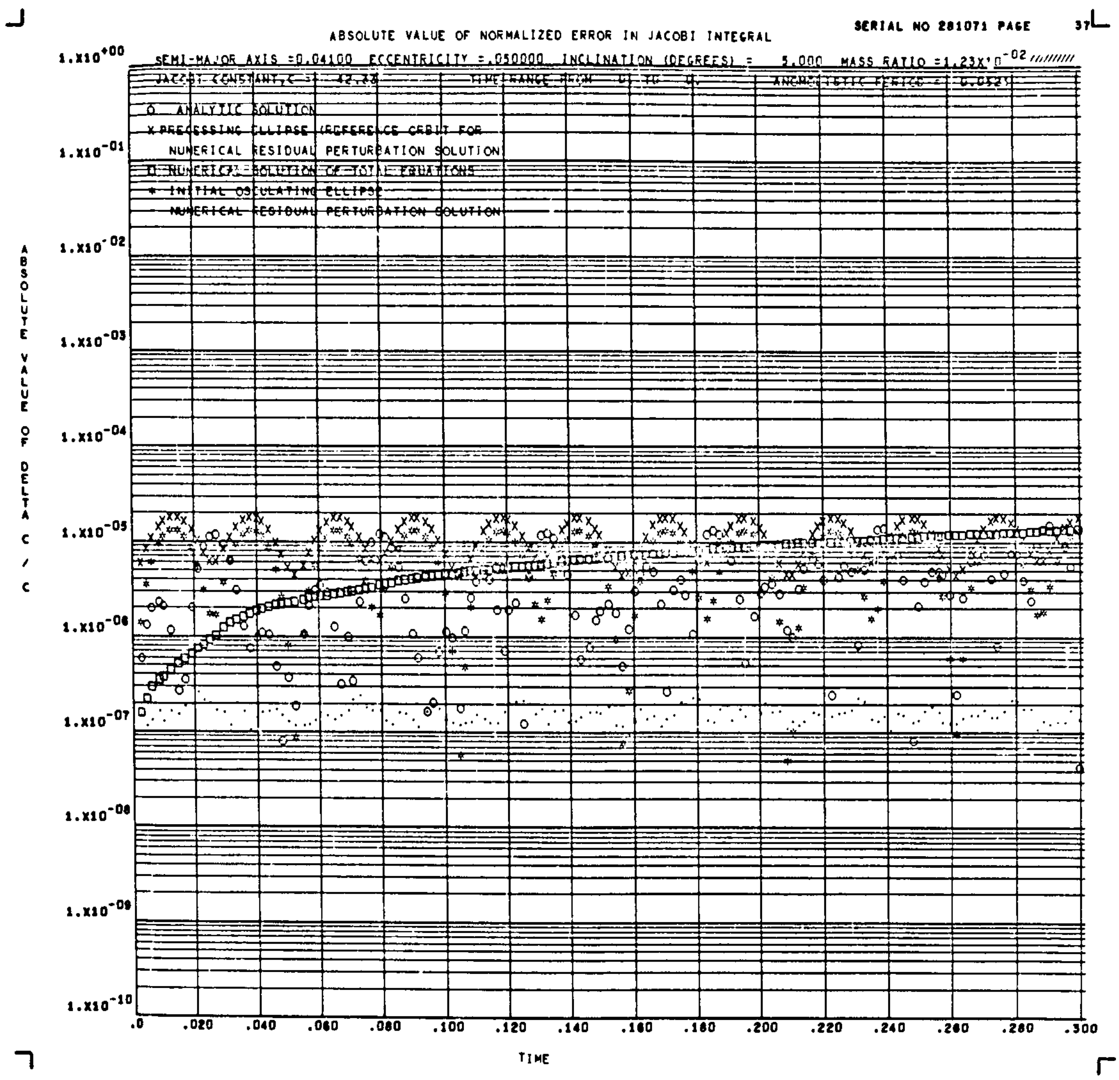
SERIAL NO 281071 PAGE 35

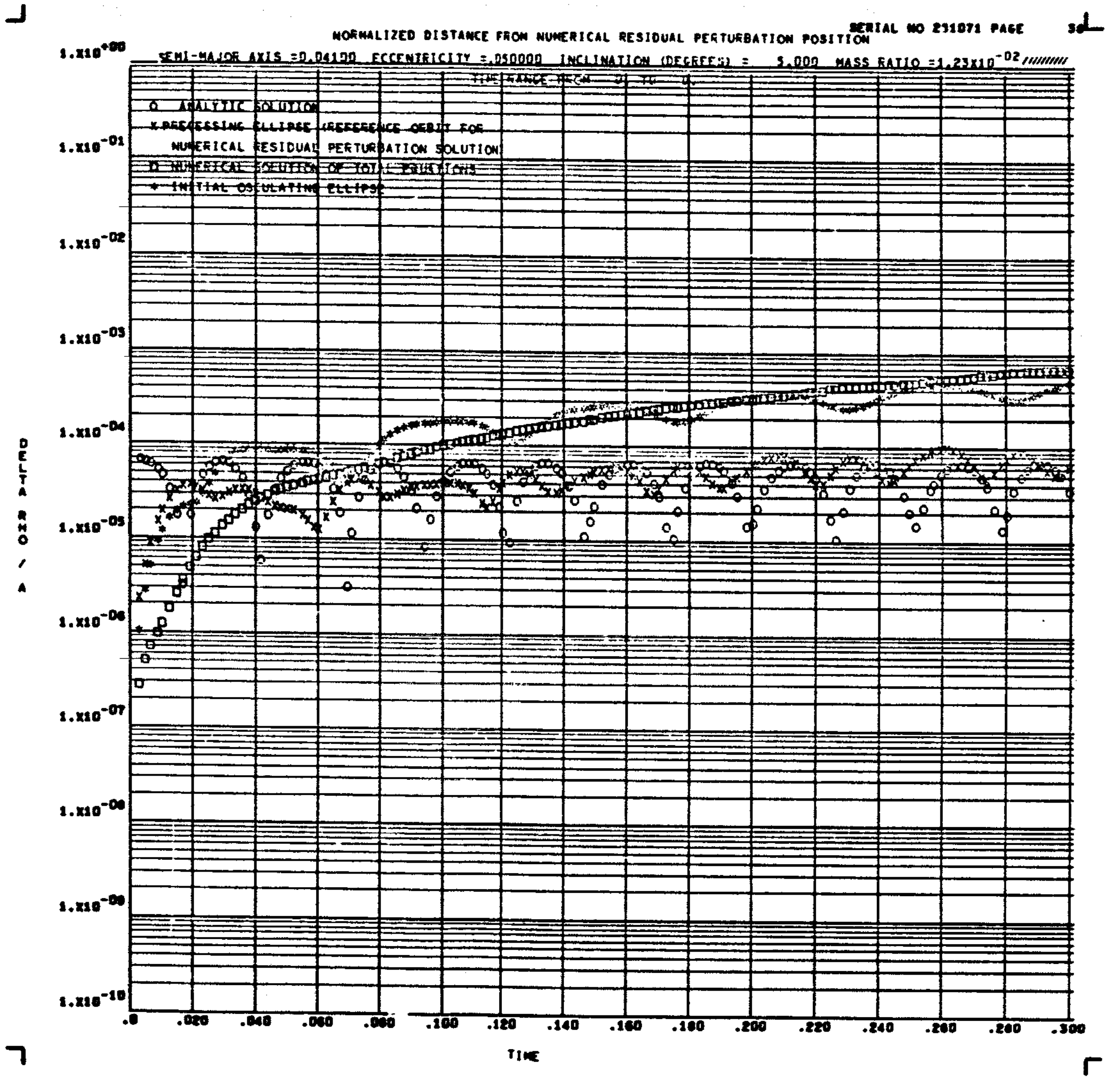
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION



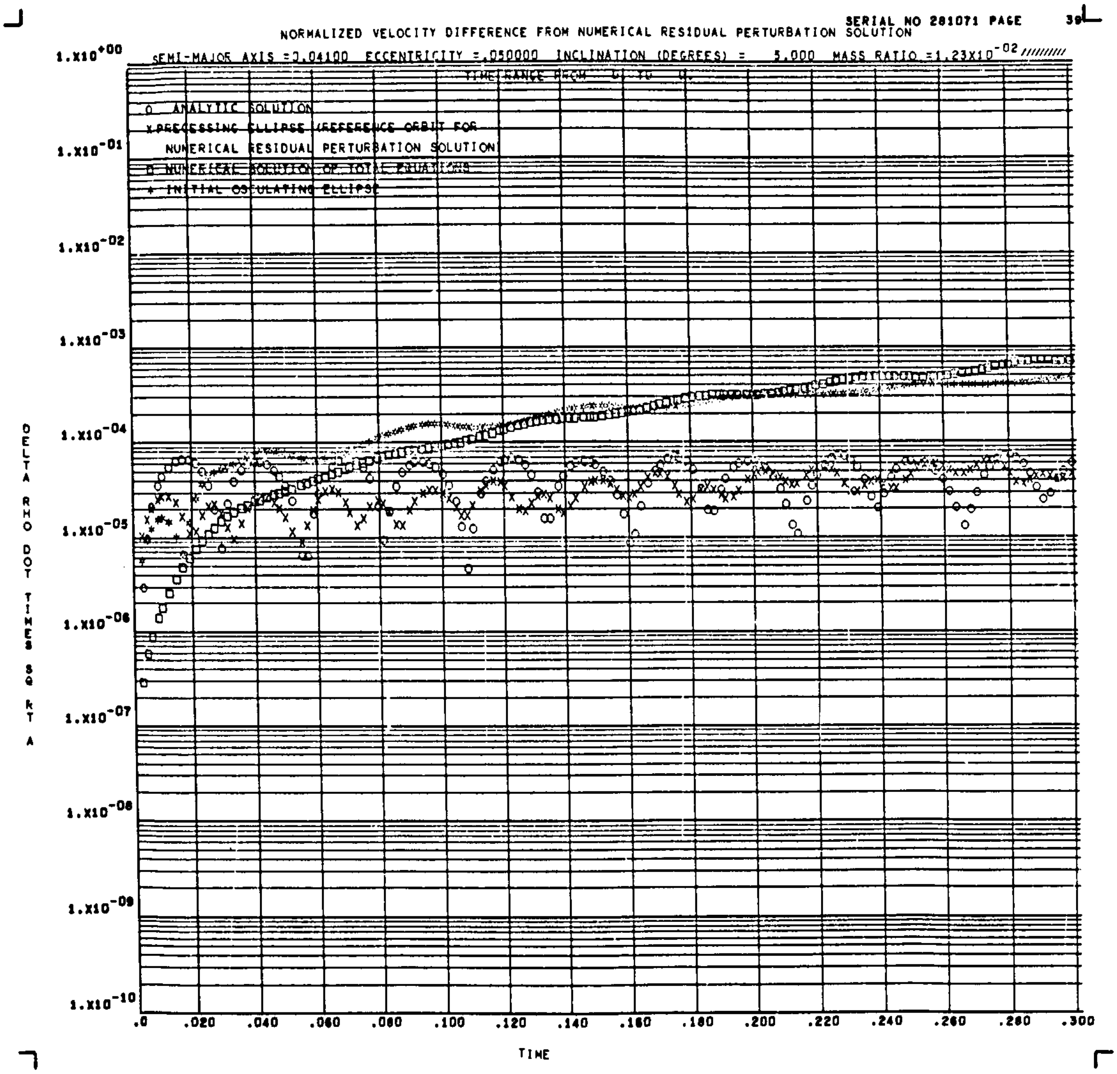


SERIAL NO 281071 PAGE
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =0.10000 ECCENTRICITY =.050000 INCLINATION (DEGREES) = 5.000 MASS RATIO =1.23x10^-02
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
1.x10^+00
1.x10^-01
1.x10^-02
1.x10^-03
1.x10^-04
1.x10^-05
1.x10^-06
1.x10^-07
1.x10^-08
1.x10^-09
1.x10^-10
DELTA RHO DOT TIMES SQRT A
5.0
5.1
5.2
5.3
5.4
5.5
5.6
5.7
5.8
5.9
6.0
TIME

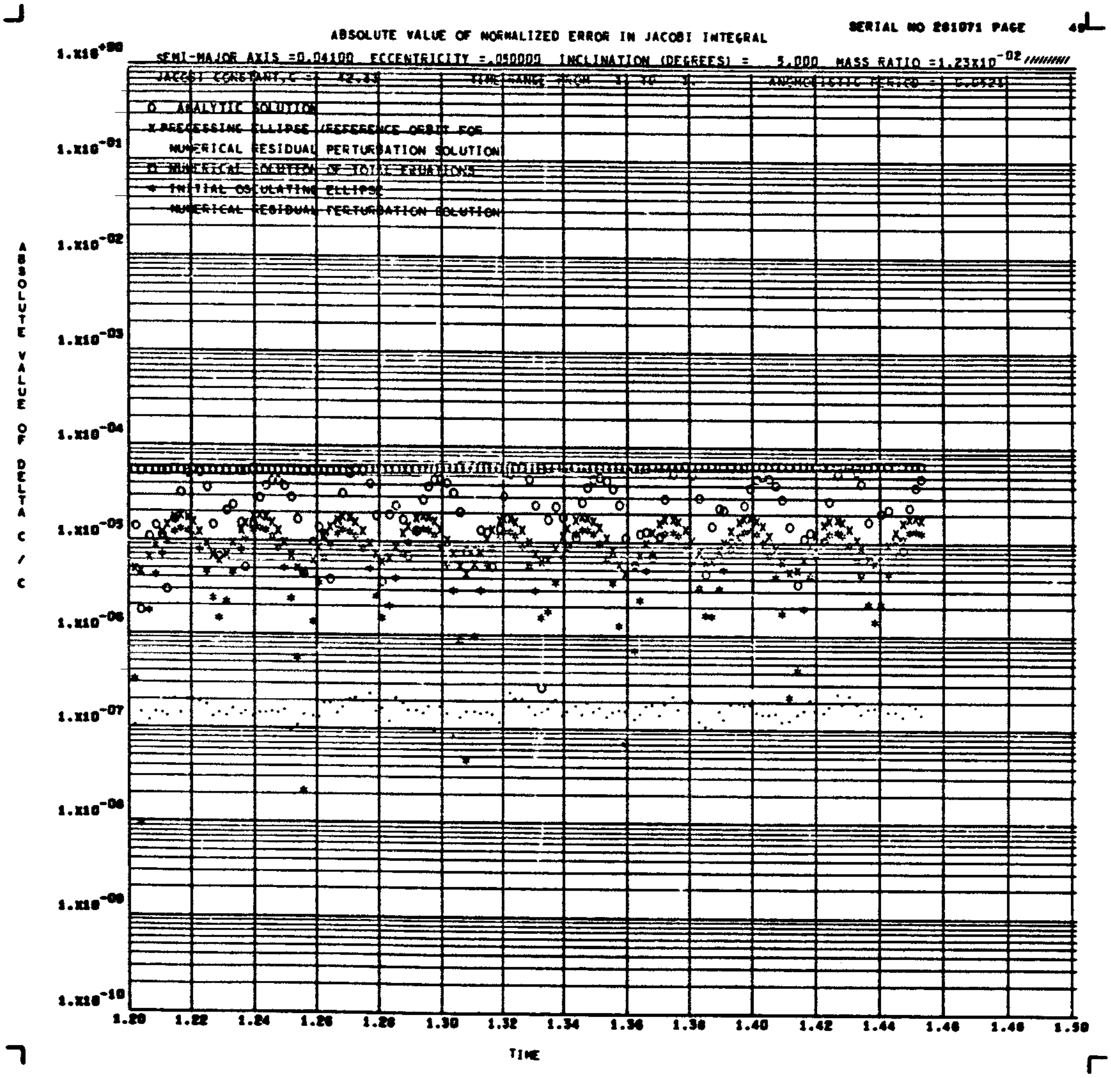

SERIAL NO 281071 PAGE 37
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =0.04100 ECCENTRICITY =.050000 INCLINATION (DEGREES) = 5.000 MASS RATIO =1.23X10-02
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
ABSOLUTE VALUE OF DELTA C / C
.0
.020
.040
.060
.080
.100
.120
.140
.160
.180
.200
.220
.240
.260
.280
.300
TIME



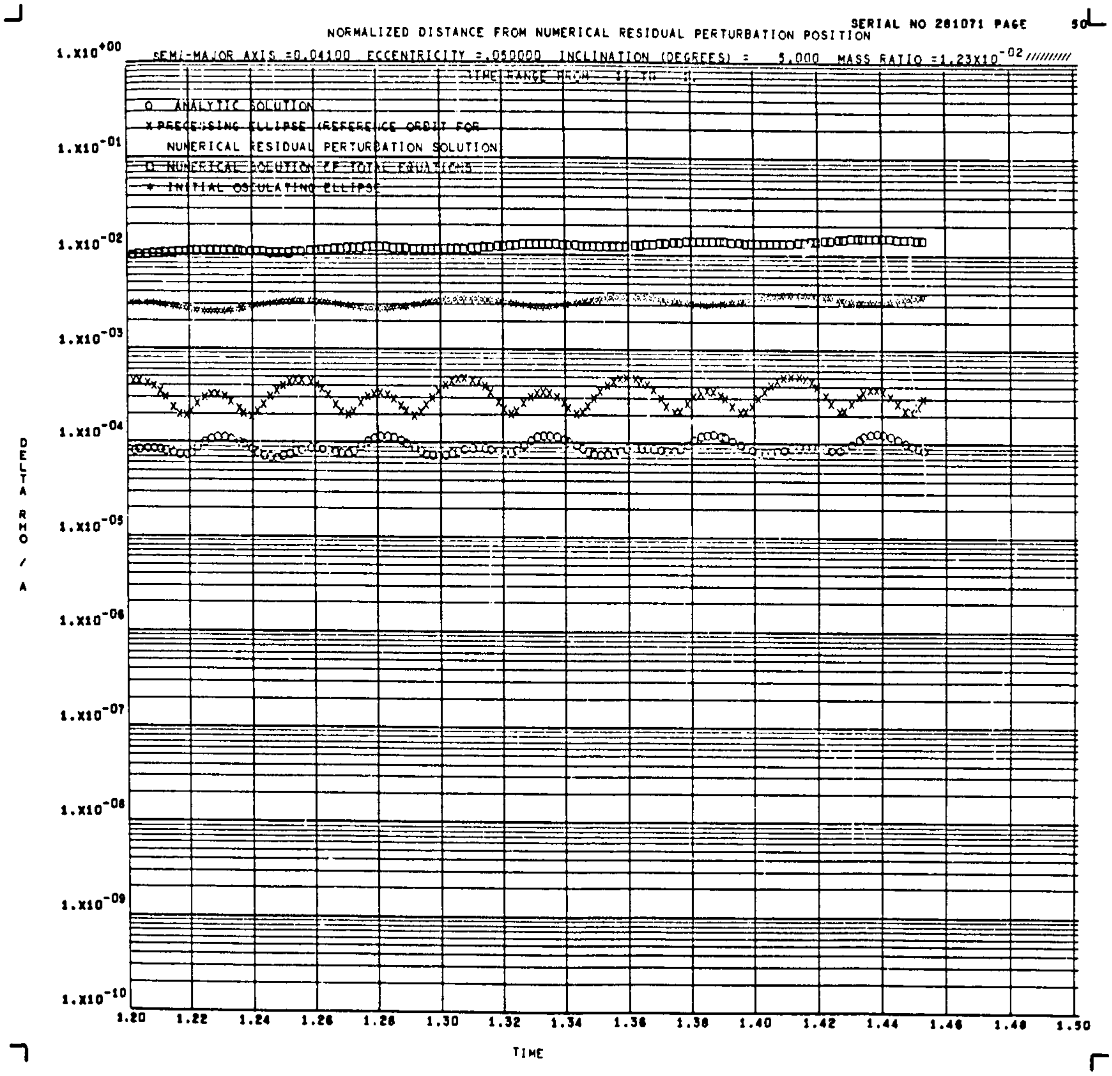
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SERIAL NO 231871 PAGE
SEMI-MAJOR AXIS =0.04100 ECCENTRICITY =.050000 INCLINATION (DEGREES) = 5.000 MASS RATIO =1.23x10-02
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
DELTA RHO / A
TIME



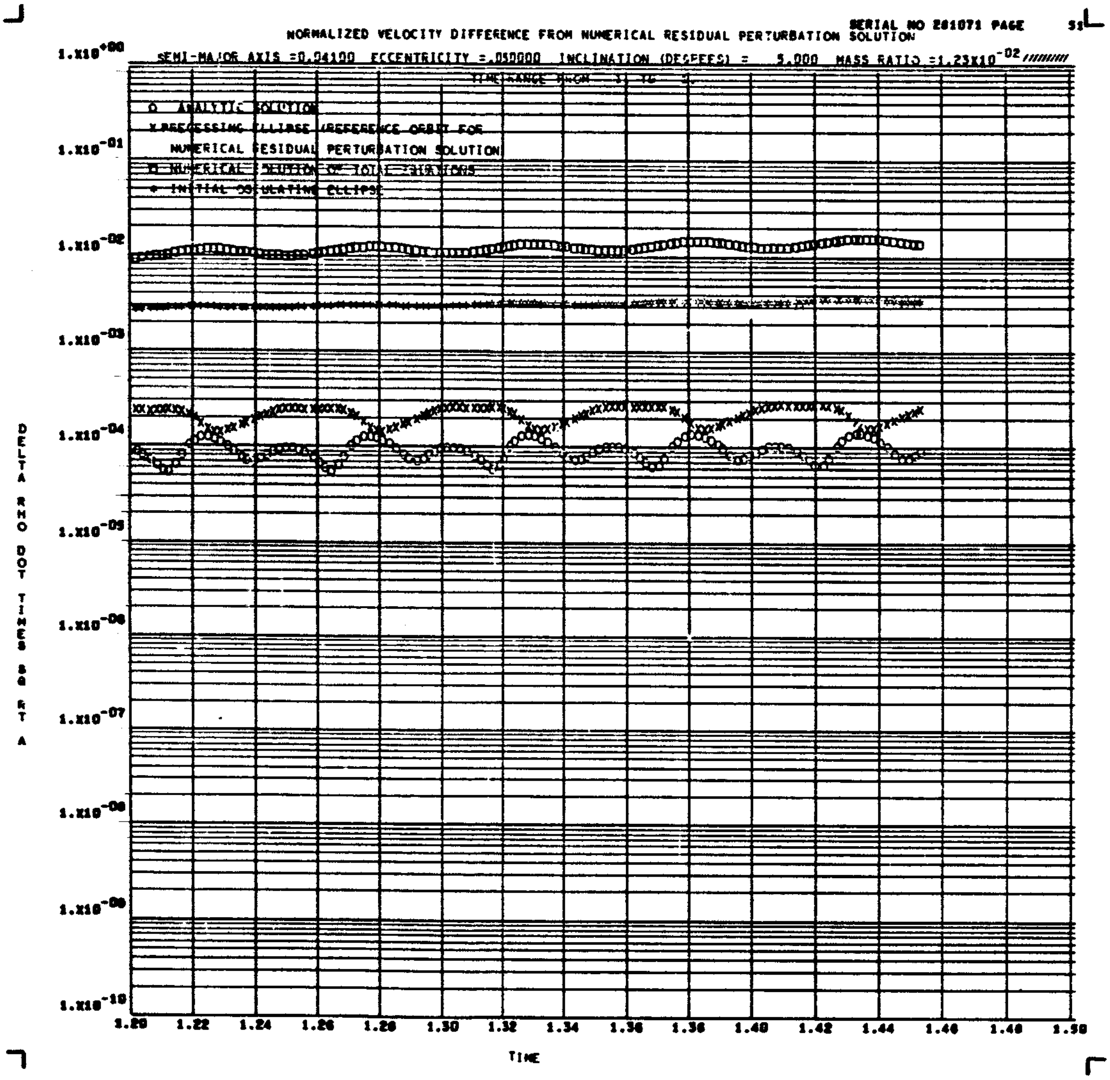
SERIAL NO 281071 PAGE 39
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =0.04100 ECCENTRICITY =.050000 INCLINATION (DEGREES) = 5.000 MASS RATIO =1.23X10-02
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO DOT TIMES SQ RT A
.0
.020
.040
.060
.080
.100
.120
.140
.160
.180
.200
.220
.240
.260
.280
.300
TIME

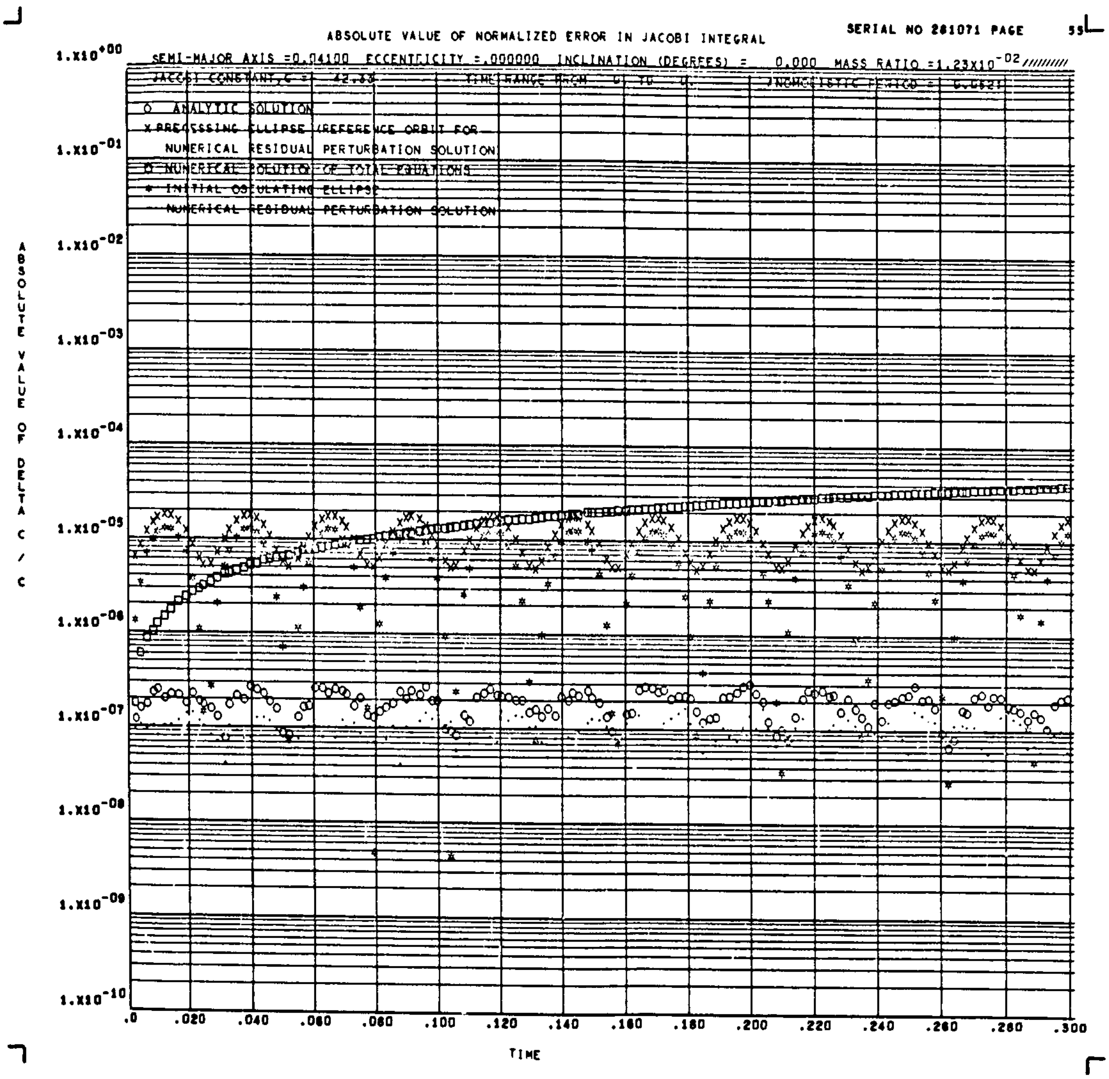


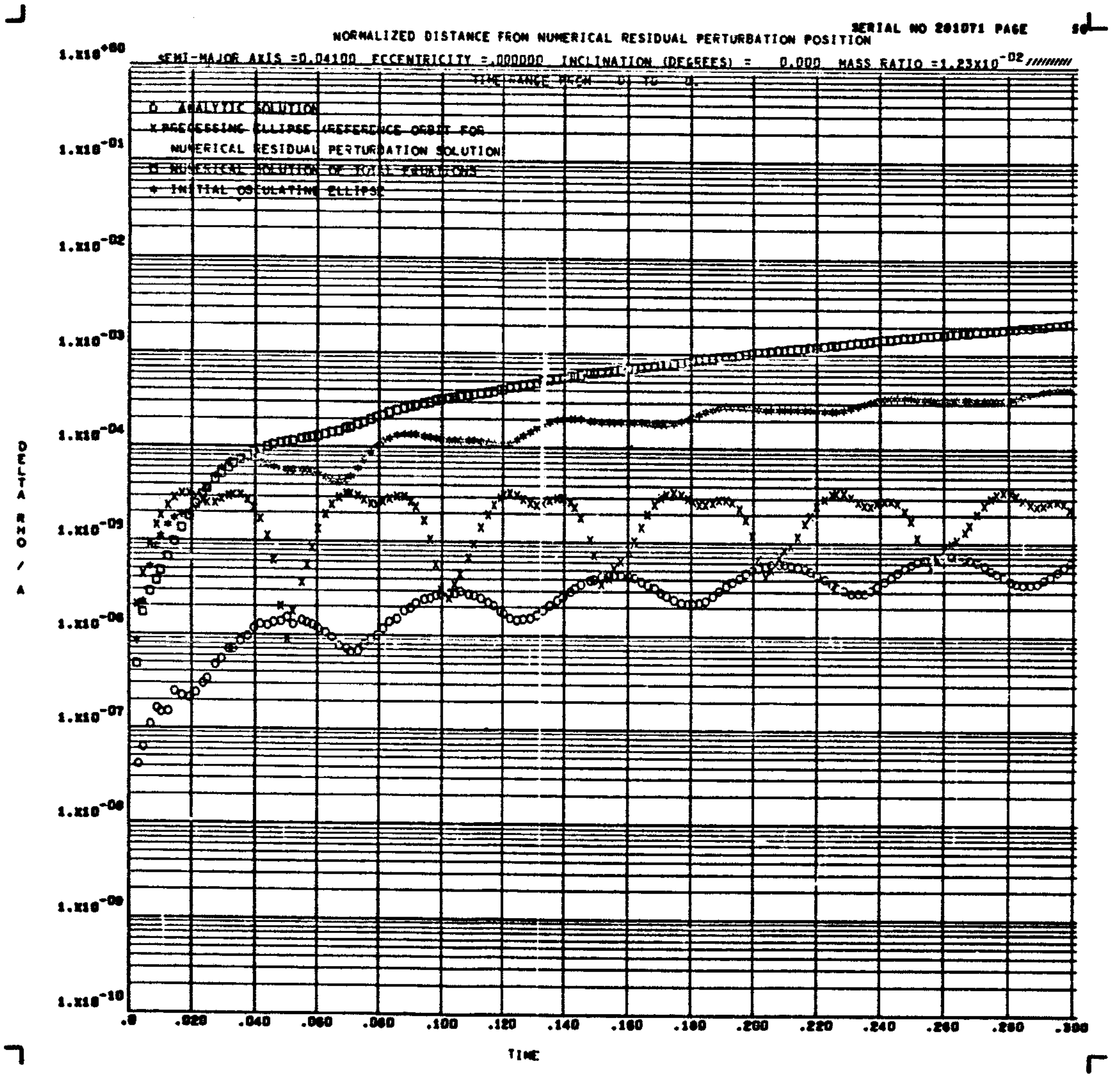
SERIAL NO 281071 PAGE 49
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =0.04100 ECCENTRICITY =.050000 INCLINATION (DEGREES) = 5.000 MASS RATIO =1.23x10^-02
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
O NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
· NUMERICAL RESIDUAL PERTURBATION SOLUTION
ABSOLUTE VALUE OF DELTA C / C
TIME
1.20 1.22 1.24 1.26 1.28 1.30 1.32 1.34 1.36 1.38 1.40 1.42 1.44 1.46 1.48 1.50



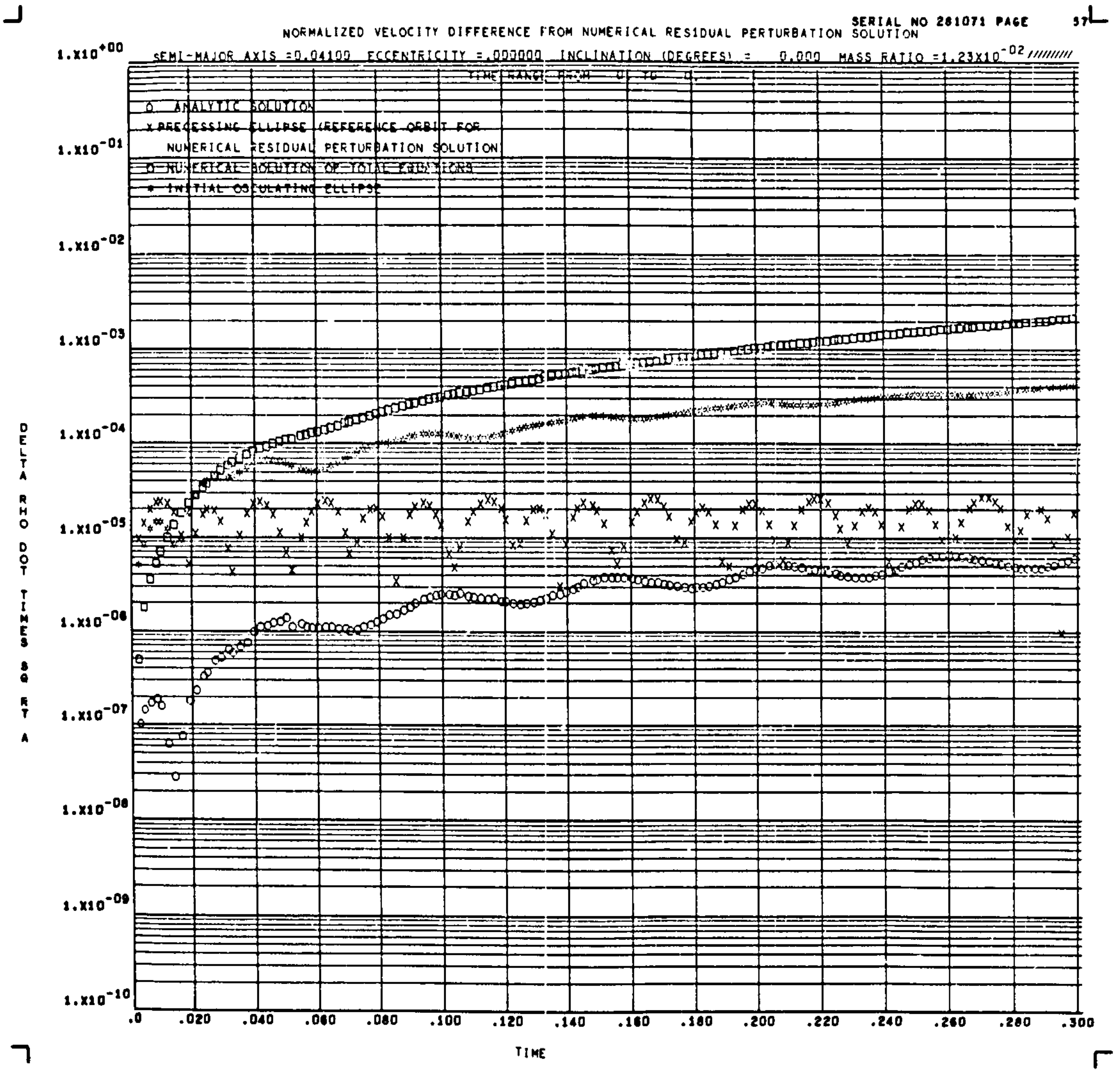
SERIAL NO 281071 PAGE 50
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =0.04100 ECCENTRICITY =.050000 INCLINATION (DEGREES) = 5.000 MASS RATIO =1.23X10^-02
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
NUMERICAL SOLUTION OF TOTAL EQUATIONS
INITIAL OSCULATING ELLIPSE
1.X10^+00
1.X10^-01
1.X10^-02
1.X10^-03
1.X10^-04
1.X10^-05
1.X10^-06
1.X10^-07
1.X10^-08
1.X10^-09
1.X10^-10
DELTA RHO / A
1.20
1.22
1.24
1.26
1.28
1.30
1.32
1.34
1.36
1.38
1.40
1.42
1.44
1.46
1.48
1.50
TIME

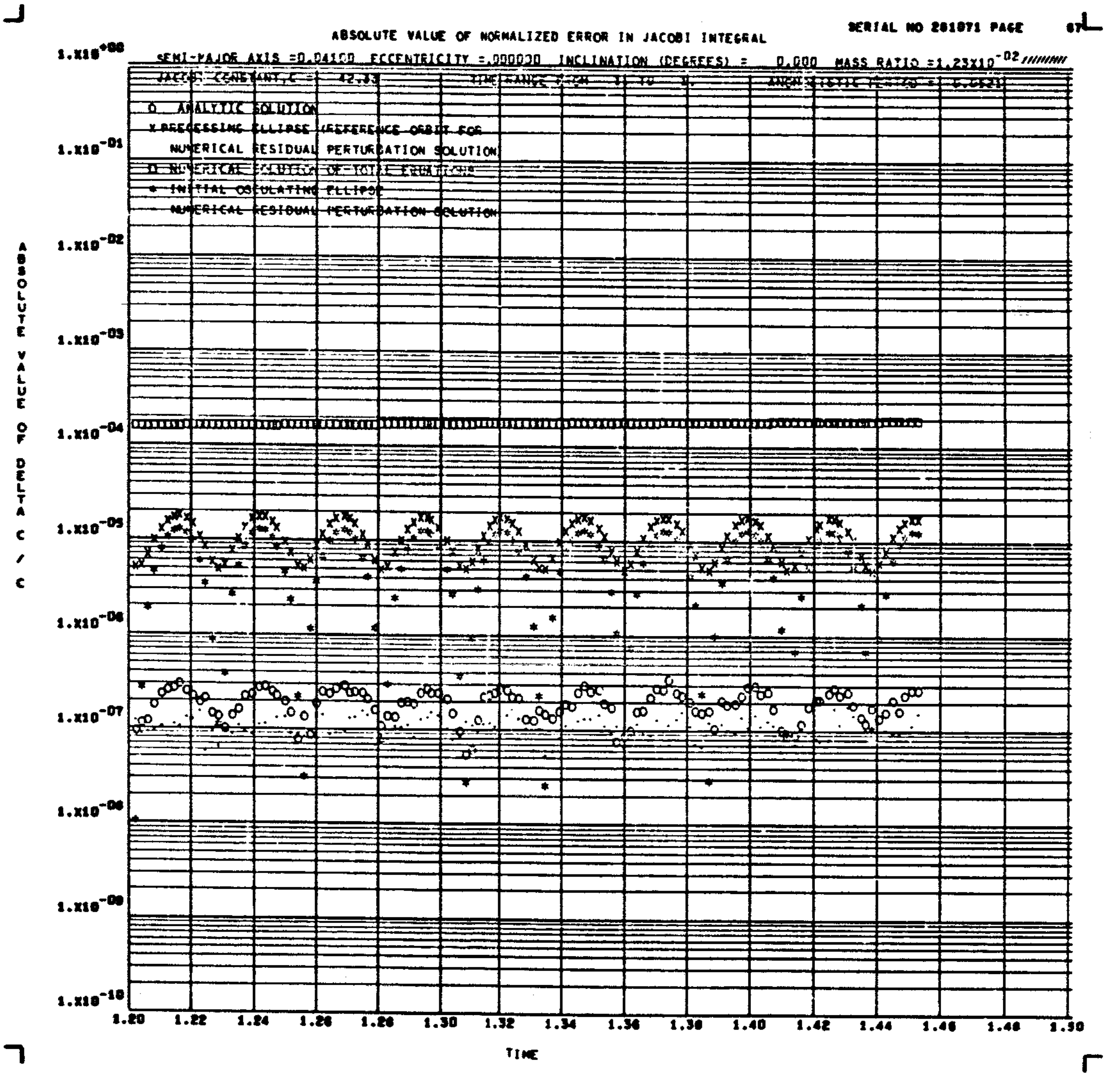

SERIAL NO 281071 PAGE 51
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =0.04100 ECCENTRICITY =.050000 INCLINATION (DEGREES) = 5.000 MASS RATIO =1.23x10-02
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
NUMERICAL SOLUTION OF TOTAL EQUATIONS
INITIAL OSCULATING ELLIPSE
1.x10+00
1.x10-01
1.x10-02
1.x10-03
1.x10-04
1.x10-05
1.x10-06
1.x10-07
1.x10-08
1.x10-09
1.x10-10
DELTA RHO DOT TIMES SQ RT A
1.20
1.22
1.24
1.26
1.28
1.30
1.32
1.34
1.36
1.38
1.40
1.42
1.44
1.46
1.48
1.50
TIME

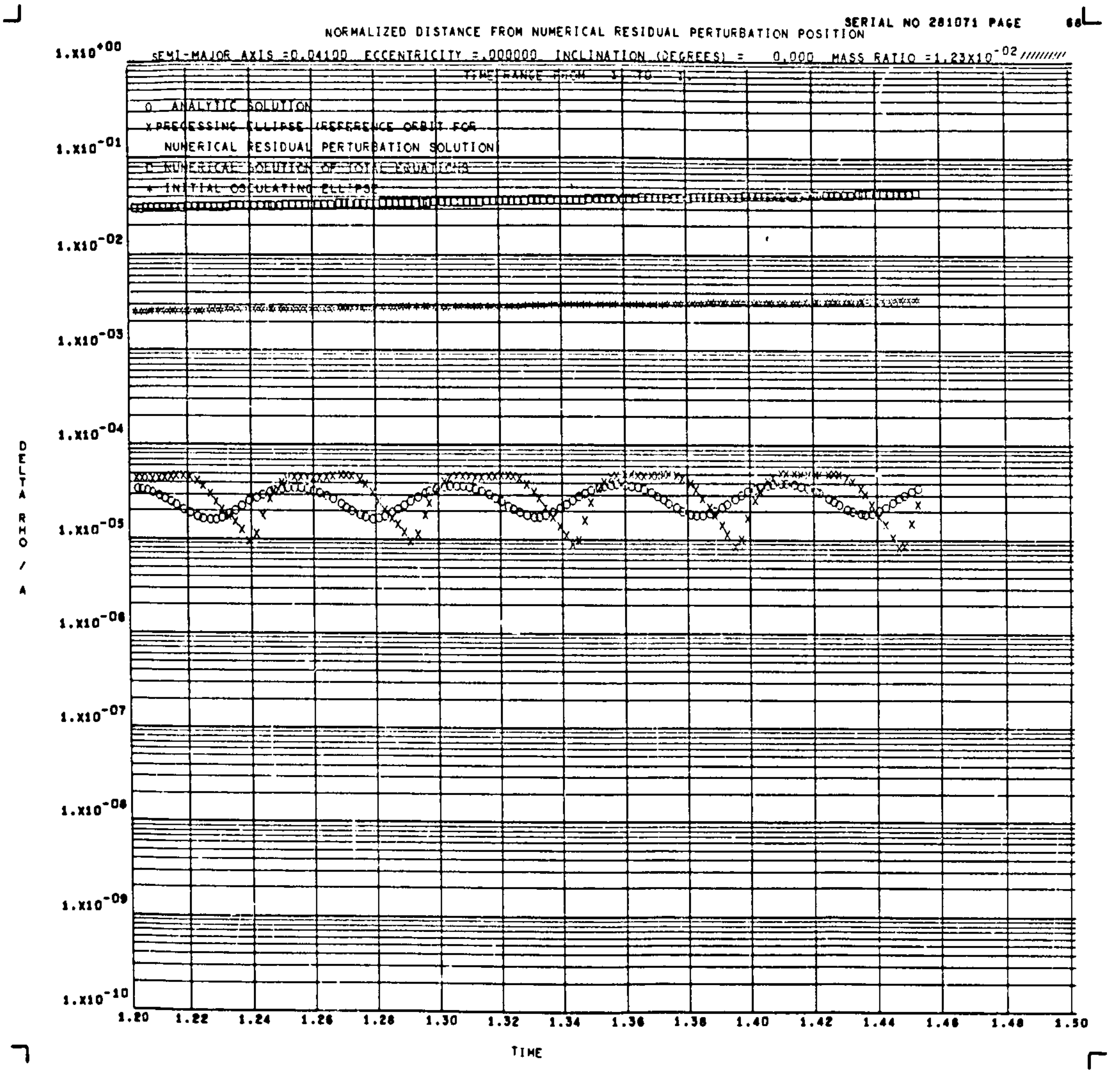

SERIAL NO 281071 PAGE 55
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =0.04100 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =1.23X10-02
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
NUMERICAL RESIDUAL PERTURBATION SOLUTION
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
ABSOLUTE VALUE OF DELTA C / C
.0
.020
.040
.060
.080
.100
.120
.140
.160
.180
.200
.220
.240
.260
.280
.300
TIME



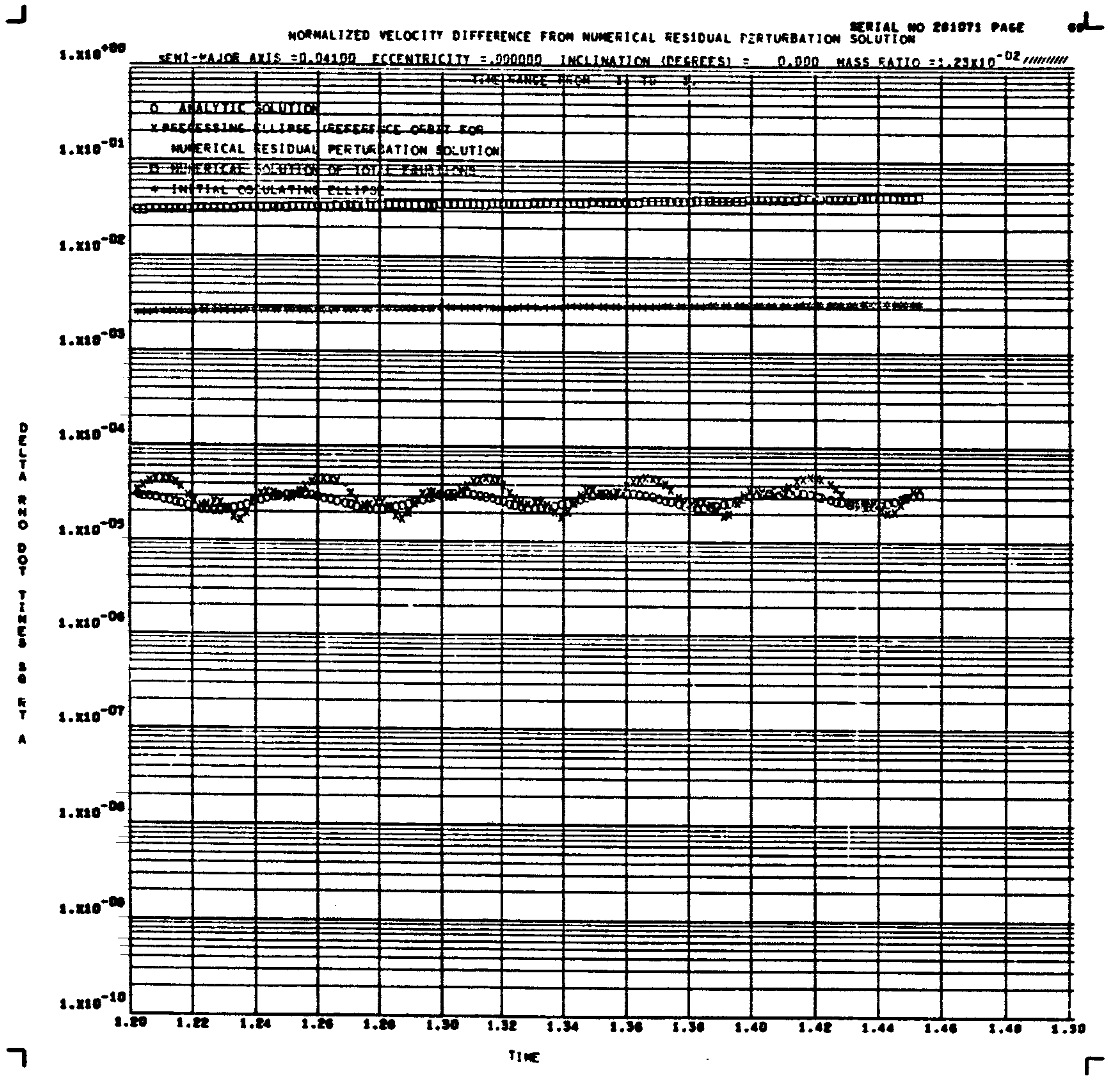
SERIAL NO 201071 PAGE
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =0.04100 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =1.23x10^-02
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
\+ INITIAL OSCULATING ELLIPSE
DELTA RHO / A
1.x10^+00
1.x10^-01
1.x10^-02
1.x10^-03
1.x10^-04
1.x10^-05
1.x10^-06
1.x10^-07
1.x10^-08
1.x10^-09
1.x10^-10
.0
.020
.040
.060
.080
.100
.120
.140
.160
.180
.200
.220
.240
.260
.280
.300
TIME


SERIAL NO 281071 PAGE 57
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =0.04100 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =1.23X10-02
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO DOT TIMES SQ RT A
.0 .020 .040 .060 .080 .100 .120 .140 .160 .180 .200 .220 .240 .260 .280 .300
TIME


ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SERIAL NO 281871 PAGE
SEMI-MAJOR AXIS =0.04100 ECCENTRICITY =.000030 INCLINATION (DEGREES) = 0.000 MASS RATIO =1.23X10-02
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
* INITIAL OSCULATING ELLIPSE
NUMERICAL RESIDUAL PERTURBATION SOLUTION
ABSOLUTE VALUE OF DELTA C / C
TIME
1.20
1.22
1.24
1.26
1.28
1.30
1.32
1.34
1.36
1.38
1.40
1.42
1.44
1.46
1.48
1.50

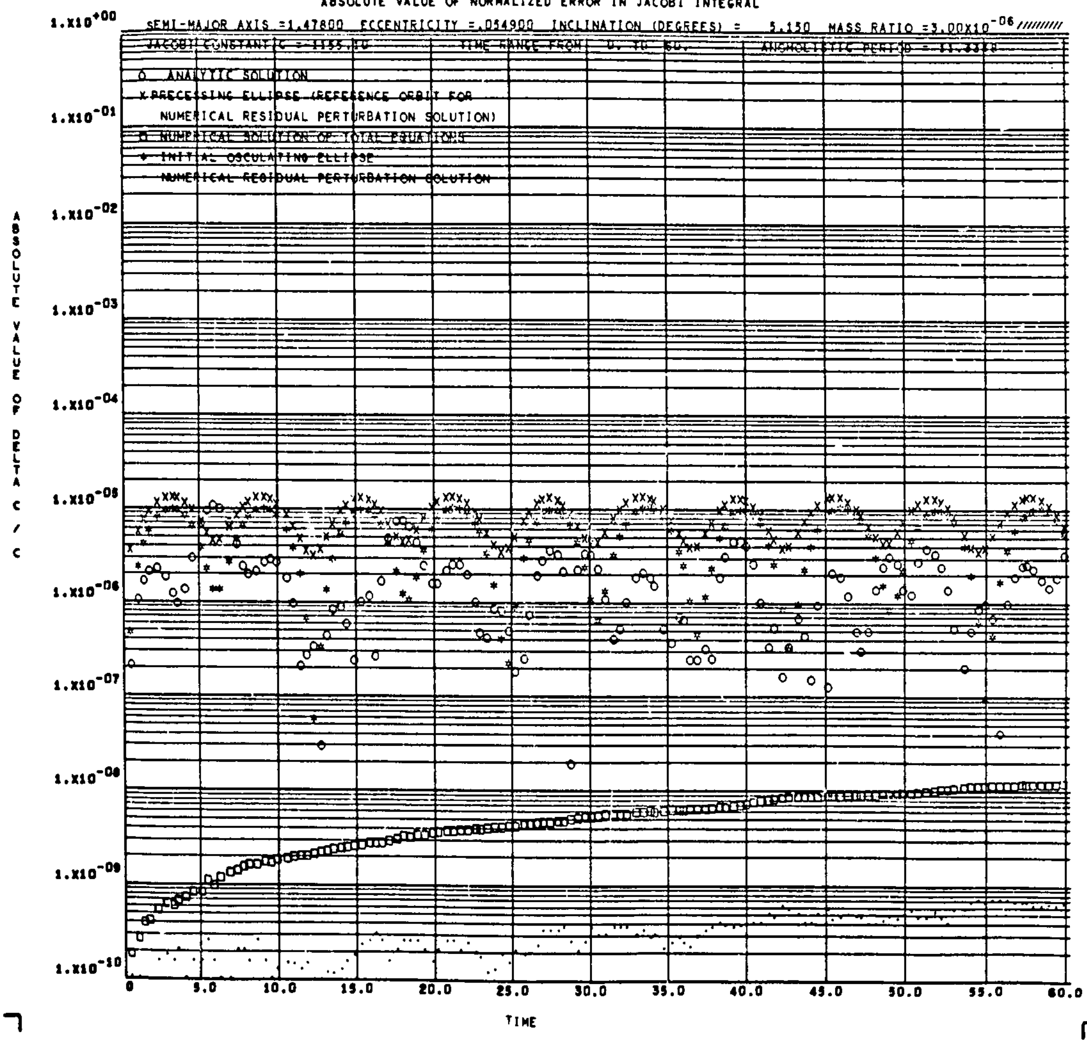


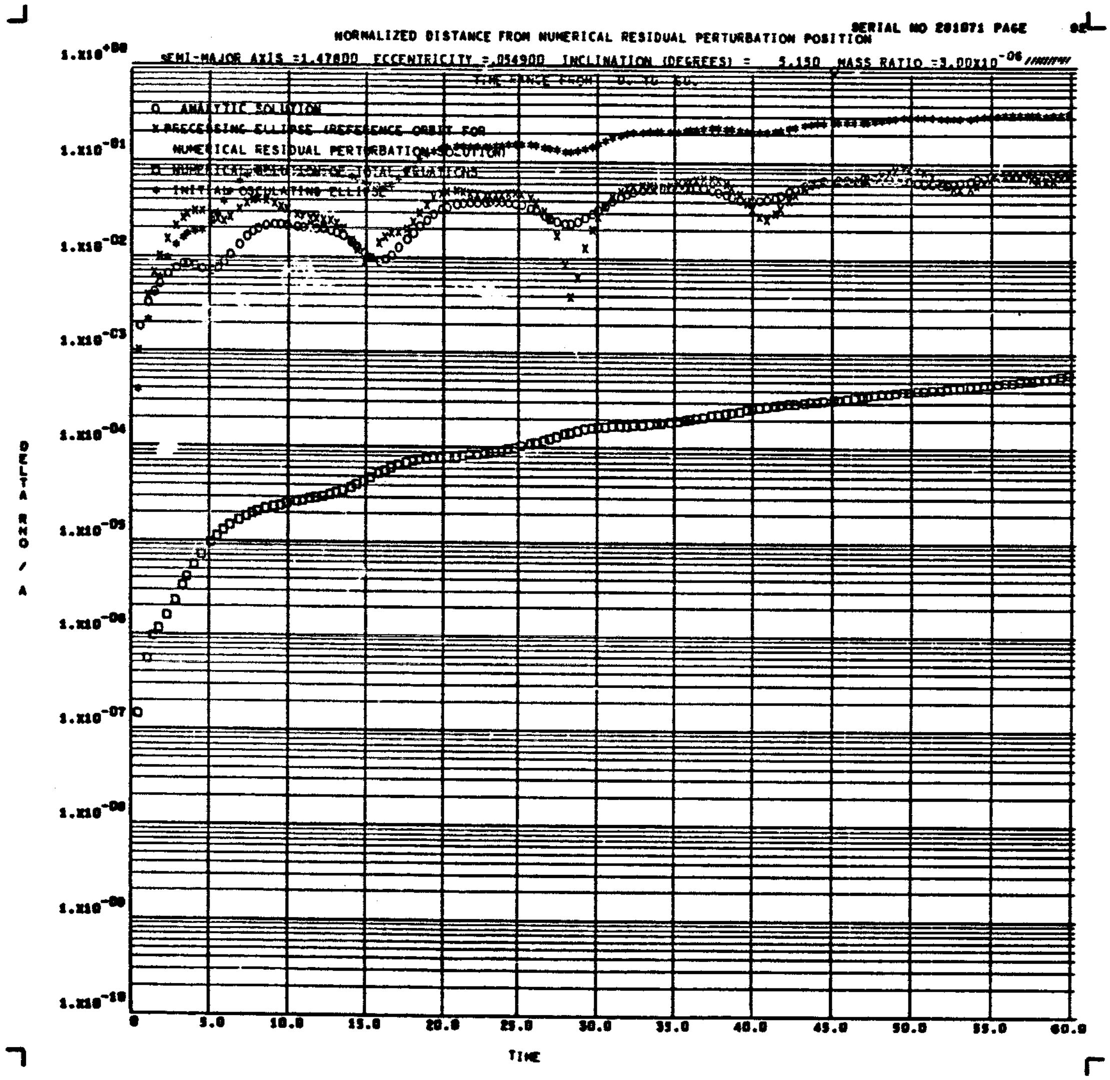
SERIAL NO 281071 PAGE 68
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =0.04100 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =1.23X10-02
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
DELTA RHO / A
TIME

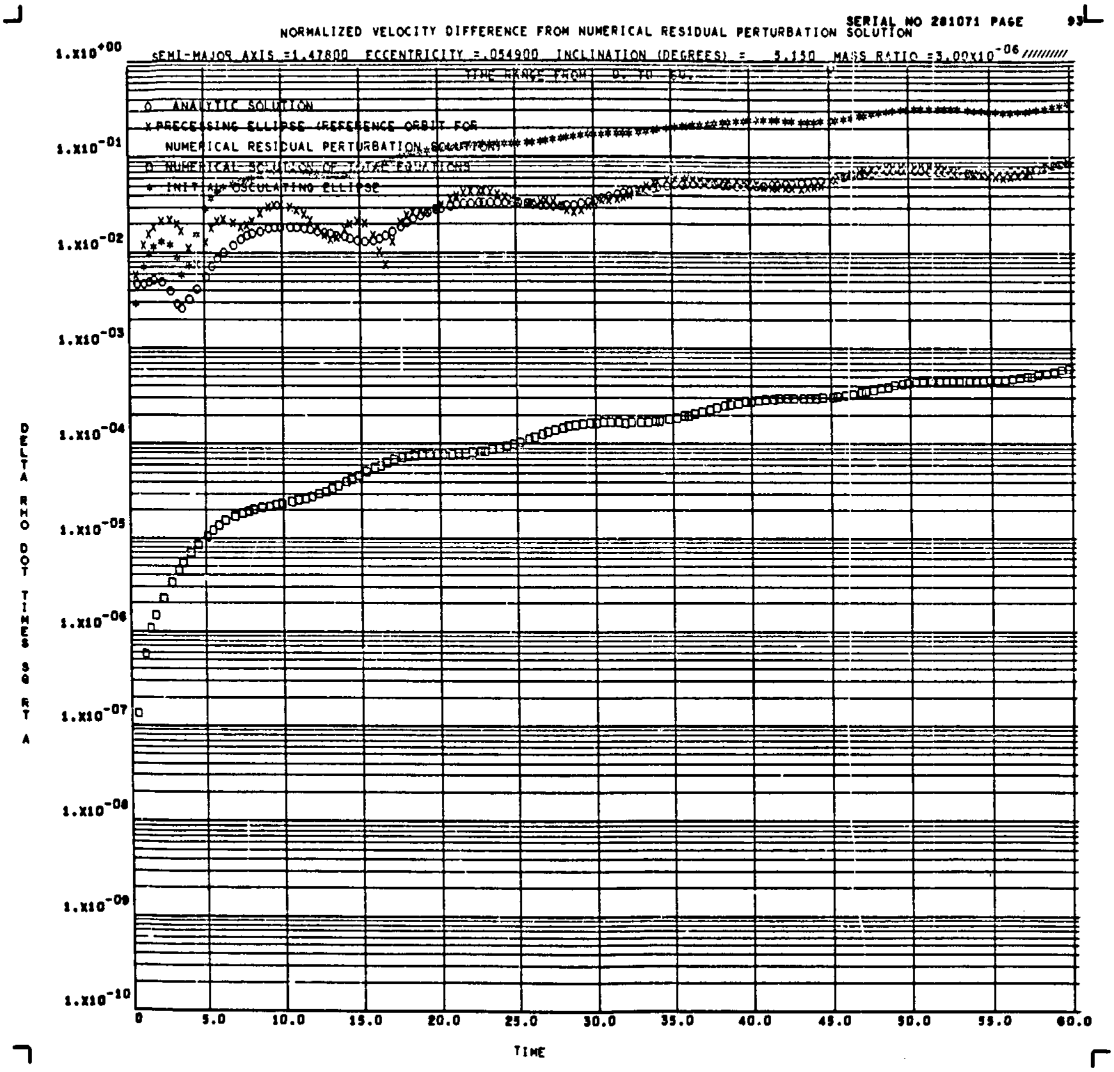

SERIAL NO 281071 PAGE
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =0.04100 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =1.23x10^-02
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
DELTA RHO DOT TIMES SQ RT A
TIME

ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL

SERIAL NO 281071 PAGE 91

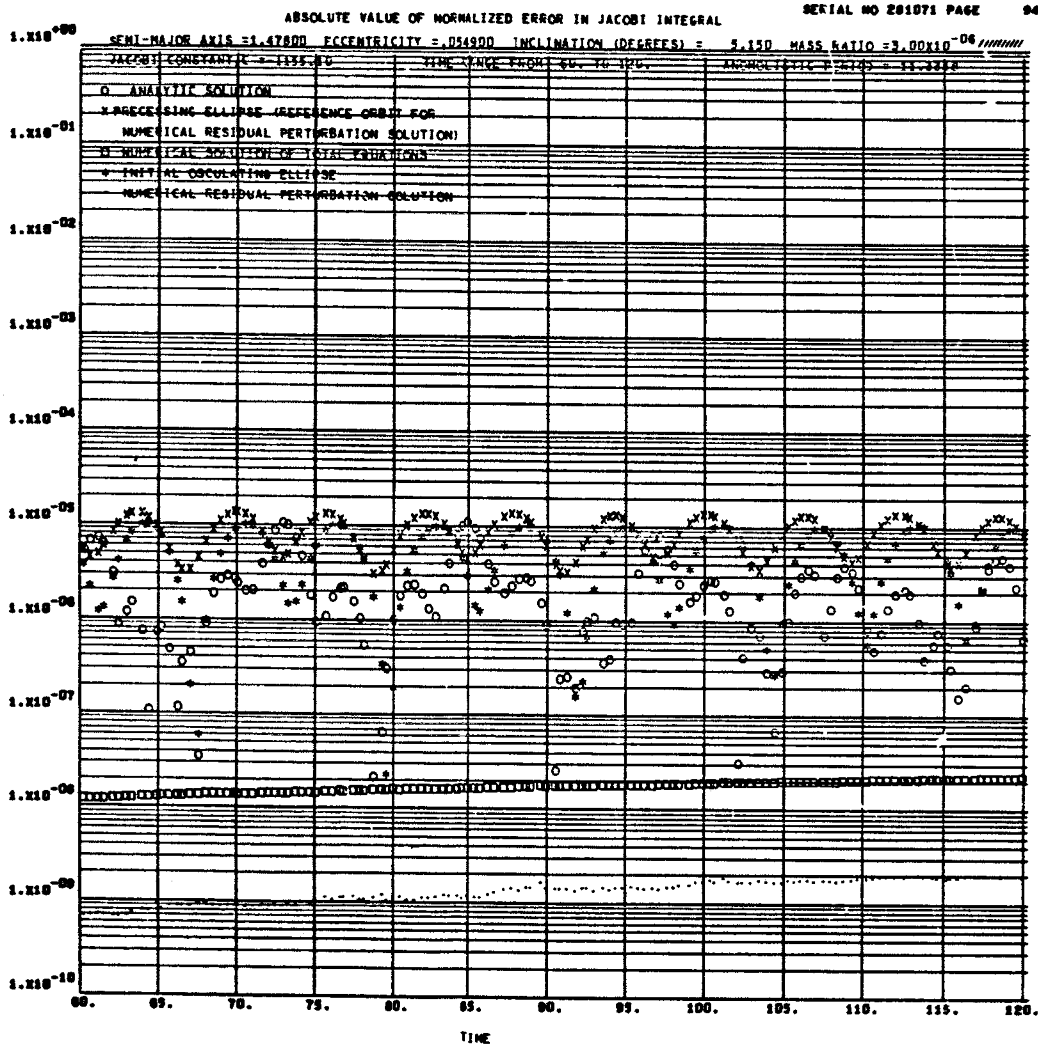


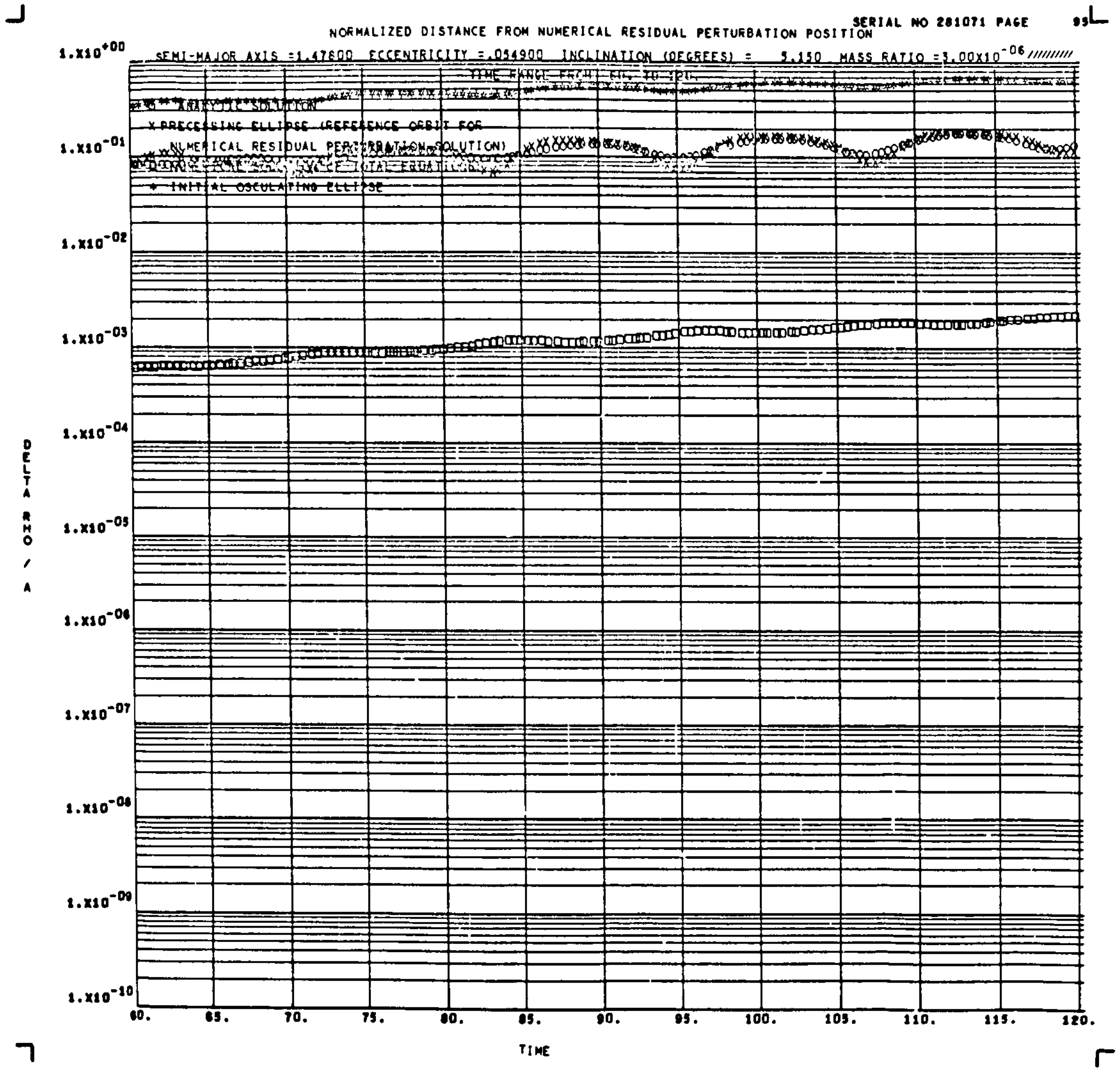

SERIAL NO 201071 PAGE
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00x10-06
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
INITIAL OSCULATING ELLIPSE
1.x10+00
1.x10-01
1.x10-02
1.x10-03
1.x10-04
1.x10-05
1.x10-06
1.x10-07
1.x10-08
1.x10-09
1.x10-10
DELTA RHO / A
0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
55.0
60.0
TIME

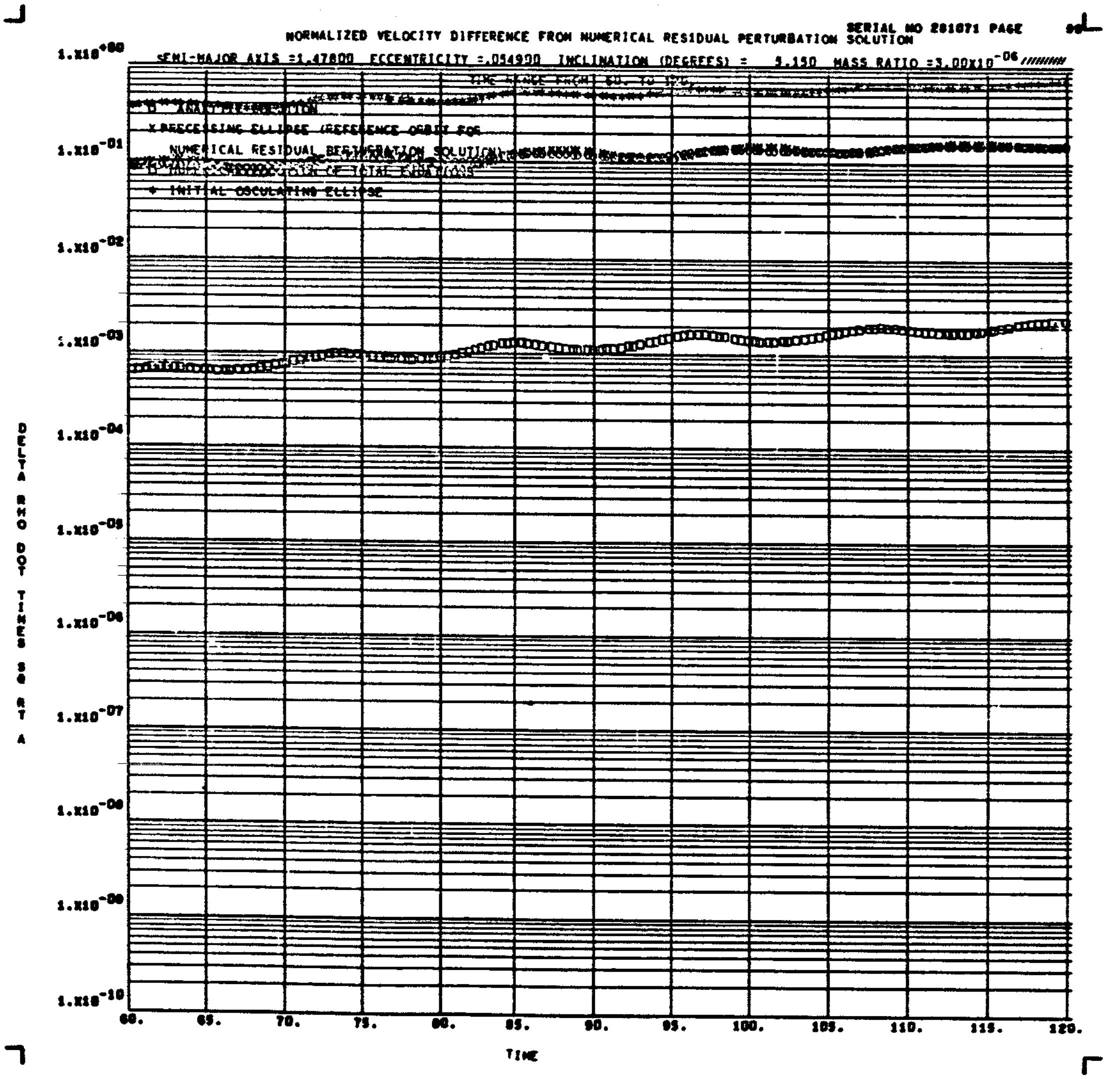

NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00X10-06
TIME RANGE FROM 0. TO 60.
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO DOT TIMES SQ RT A
0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
55.0
60.0
TIME

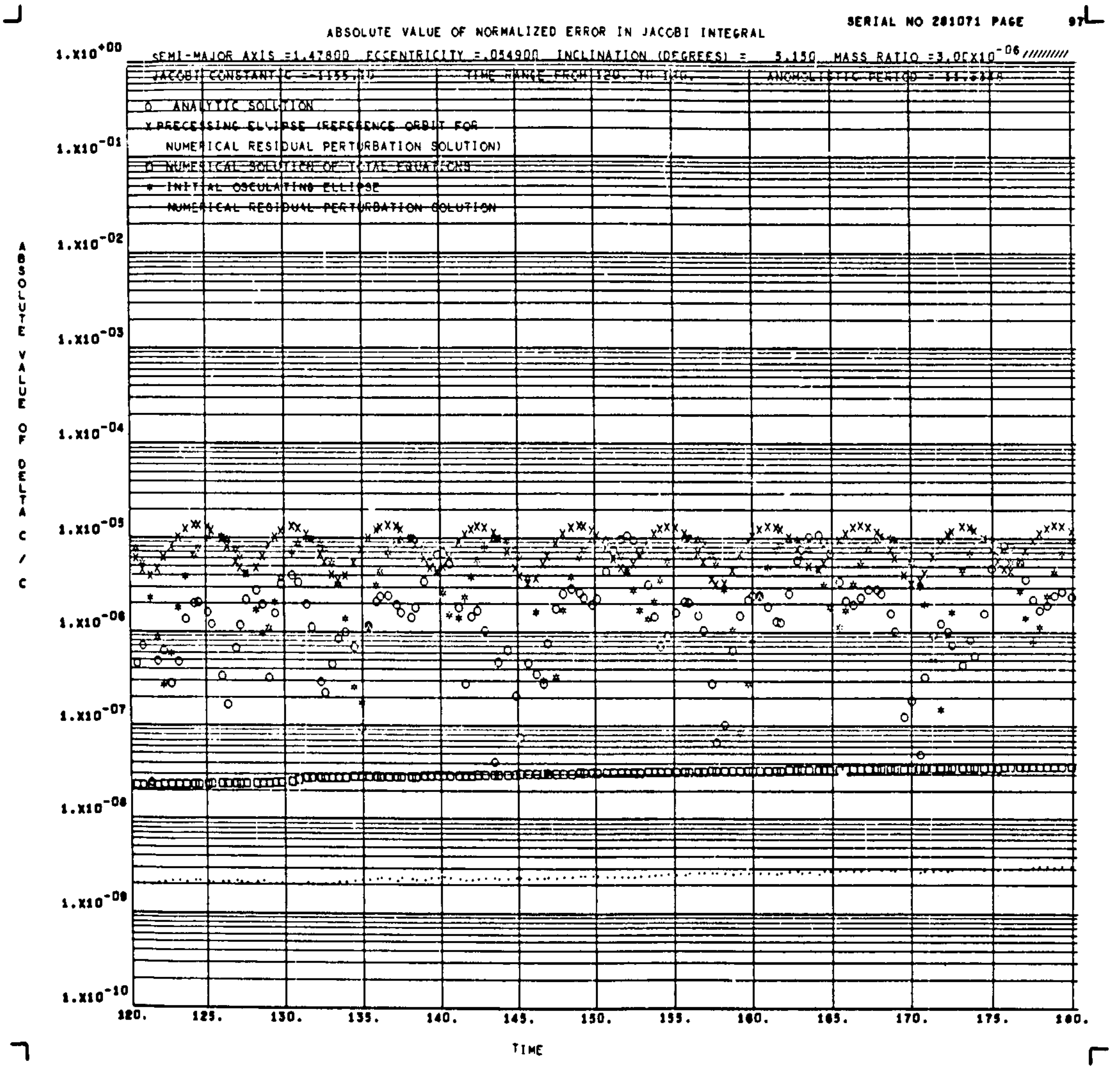
SERIAL NO 281071 PAGE

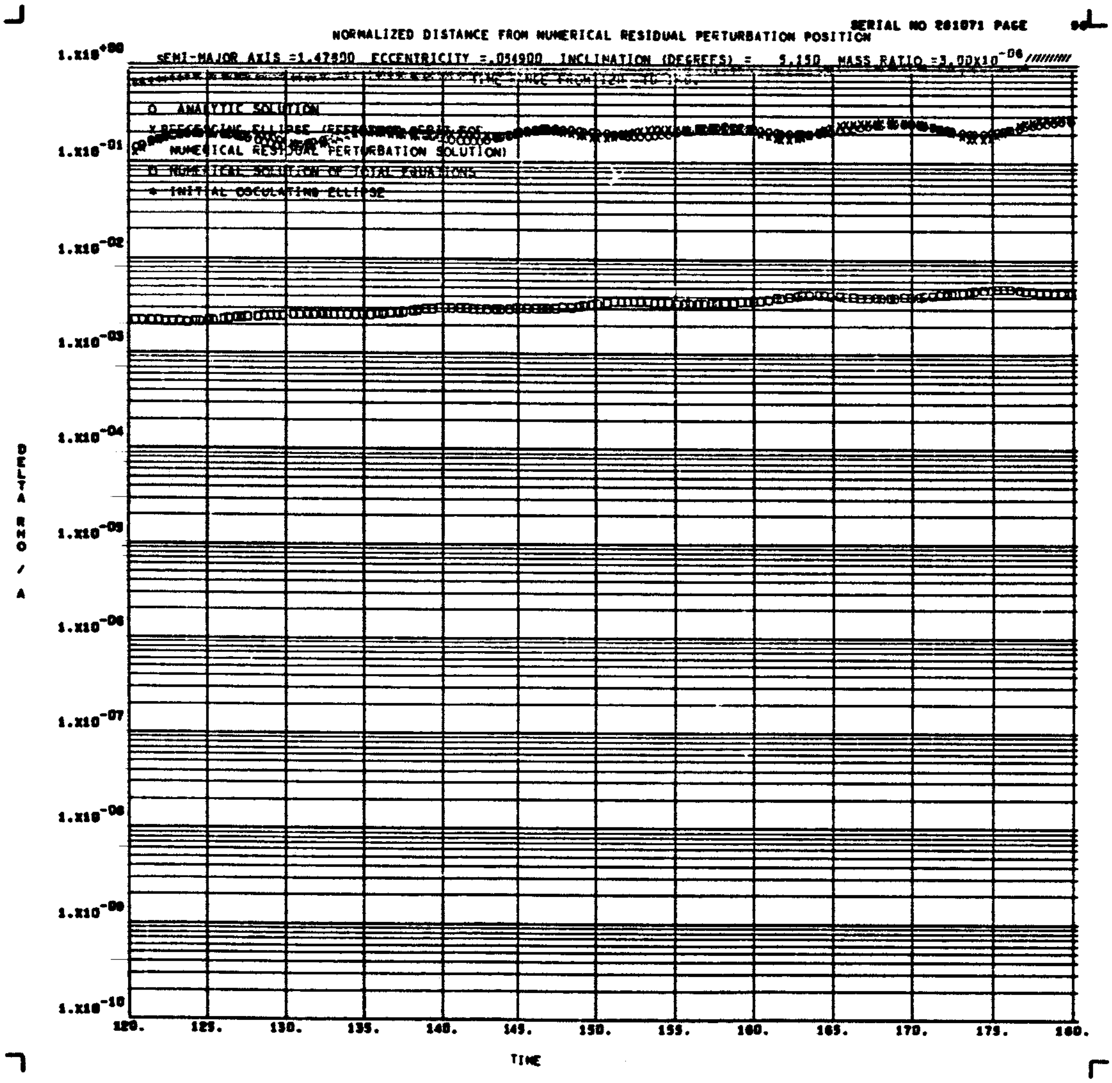
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL

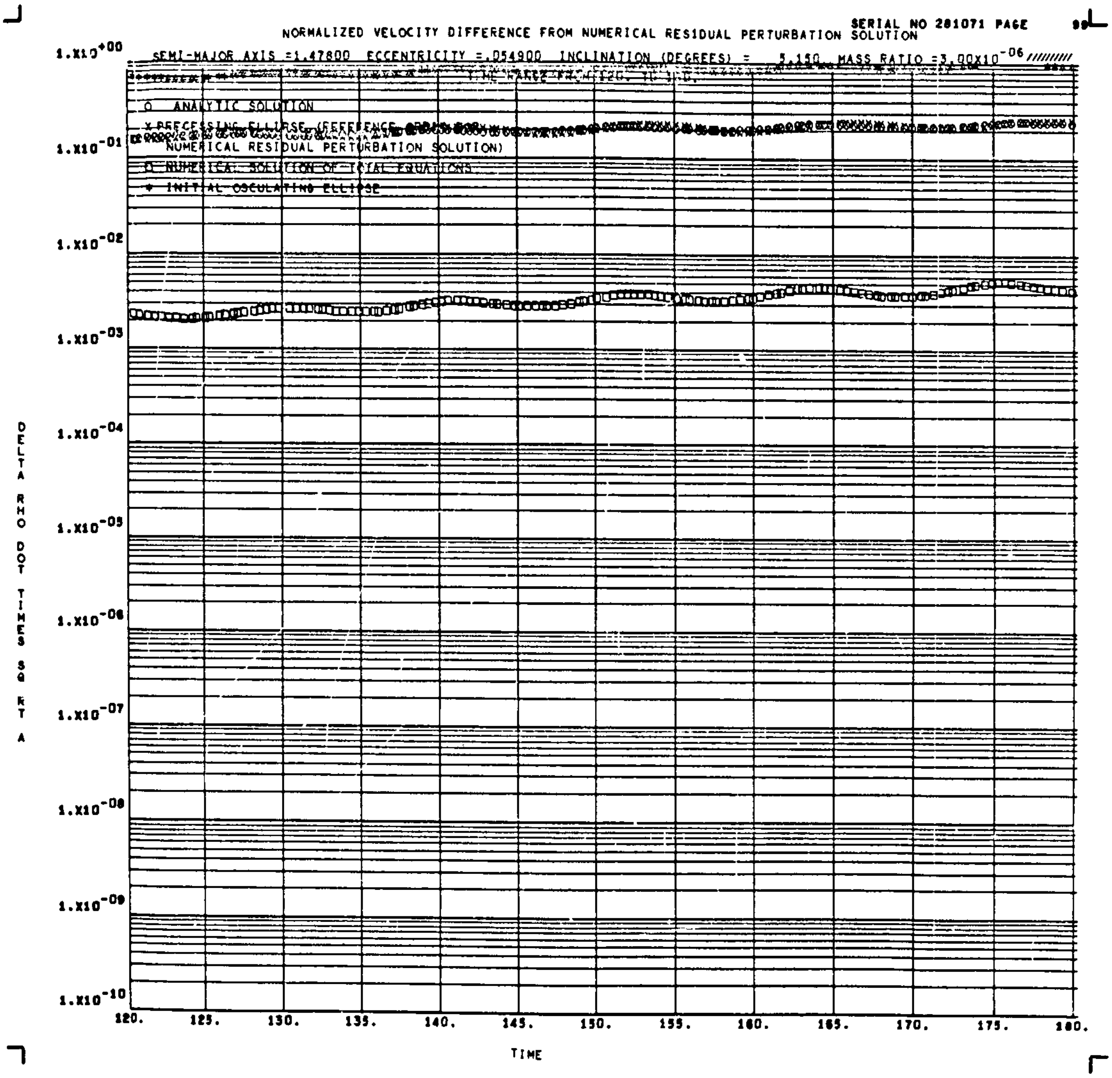


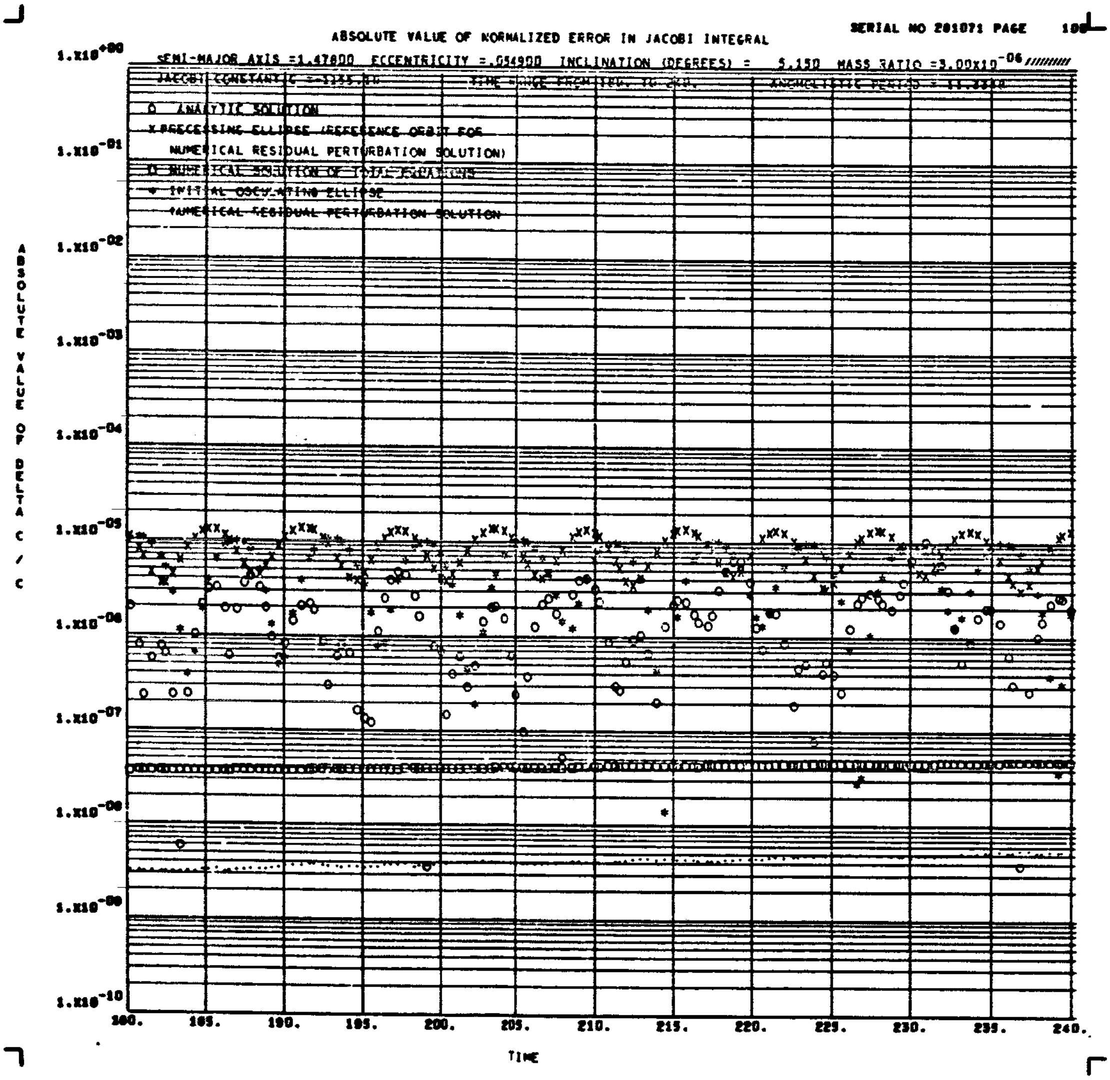

SERIAL NO 281071 PAGE 95
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =1.47600 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00X10-06
ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
+ INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO / A
60. 65. 70. 75. 80. 85. 90. 95. 100. 105. 110. 115. 120.
TIME

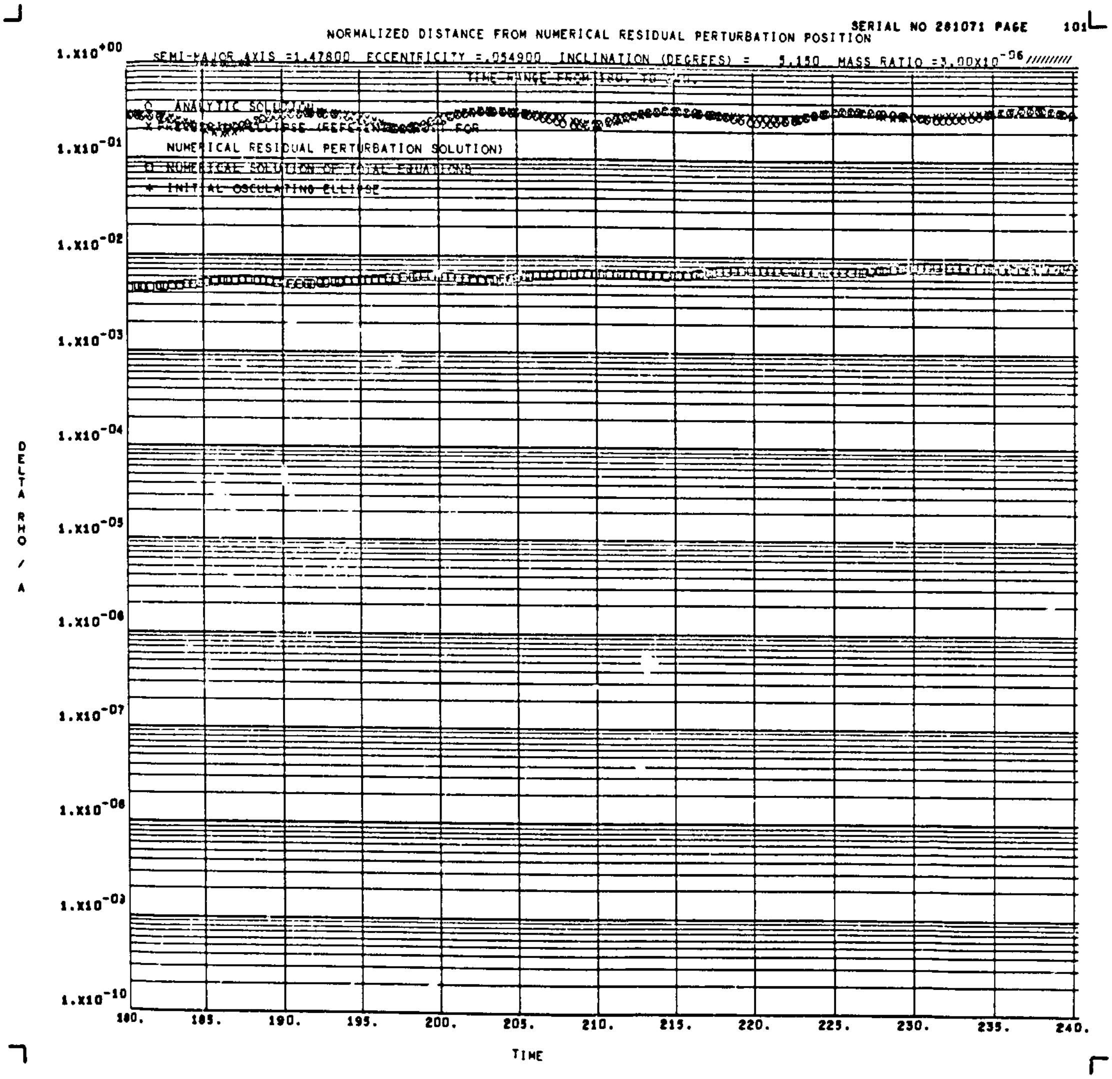

SERIAL NO 281871 PAGE
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00x10^-06
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
+ INITIAL OSCULATING ELLIPSE
1.x10^+00
1.x10^-01
1.x10^-02
1.x10^-03
1.x10^-04
1.x10^-05
1.x10^-06
1.x10^-07
1.x10^-08
1.x10^-09
1.x10^-10
DELTA RHO DOT TIMES SQ RT A
60.
65.
70.
75.
80.
85.
90.
95.
100.
105.
110.
115.
120.
TIME

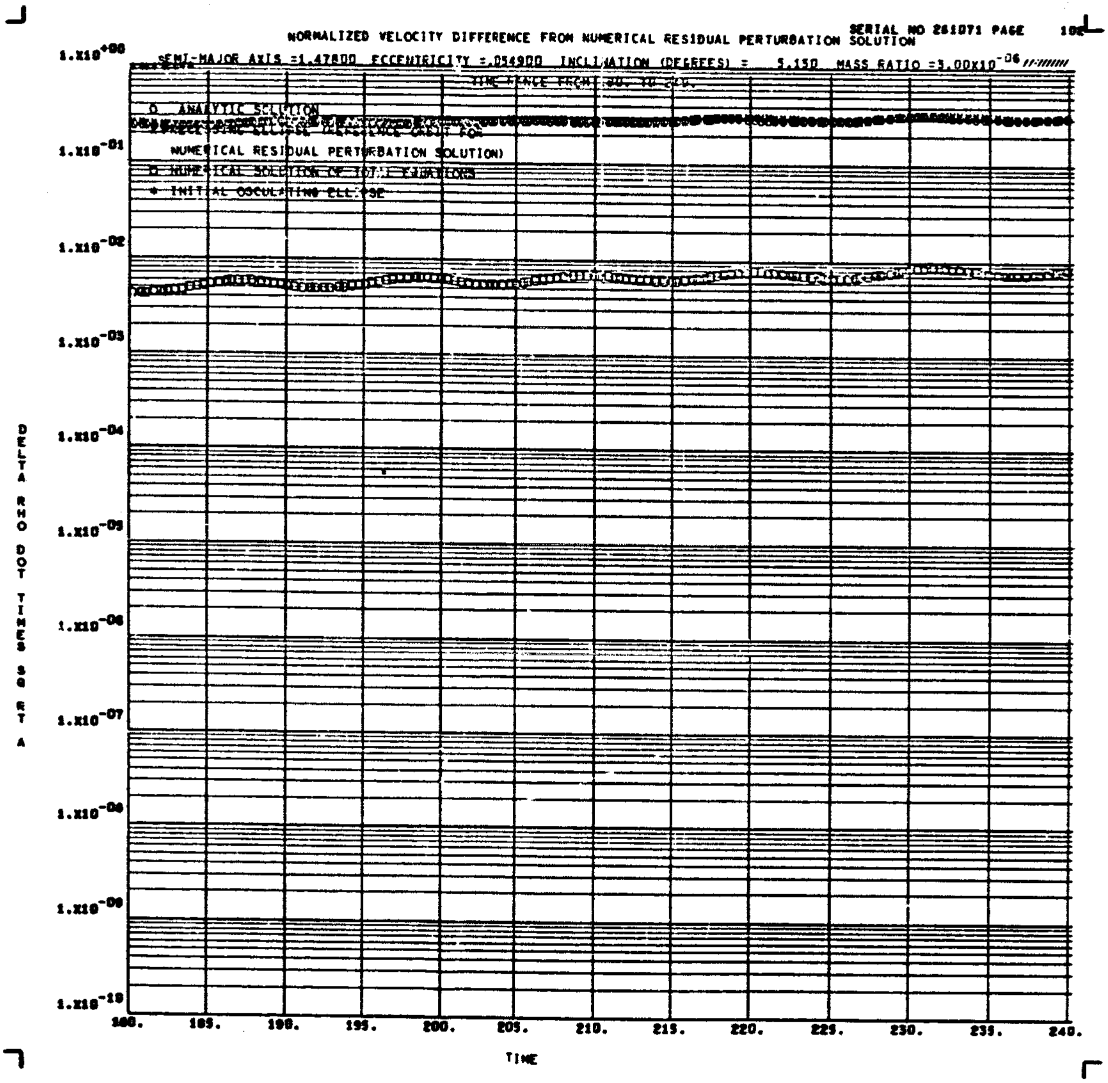

SERIAL NO 281071 PAGE 97
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.0€X10-06
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
NUMERICAL RESIDUAL PERTURBATION SOLUTION
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
ABSOLUTE VALUE OF DELTA C / C
120.
125.
130.
135.
140.
145.
150.
155.
160.
165.
170.
175.
180.
TIME


SERIAL NO 281071 PAGE
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00X10-06
O ANALYTIC SOLUTION
X REFERENCING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO / A
120. 125. 130. 135. 140. 145. 150. 155. 160. 165. 170. 175. 180.
TIME

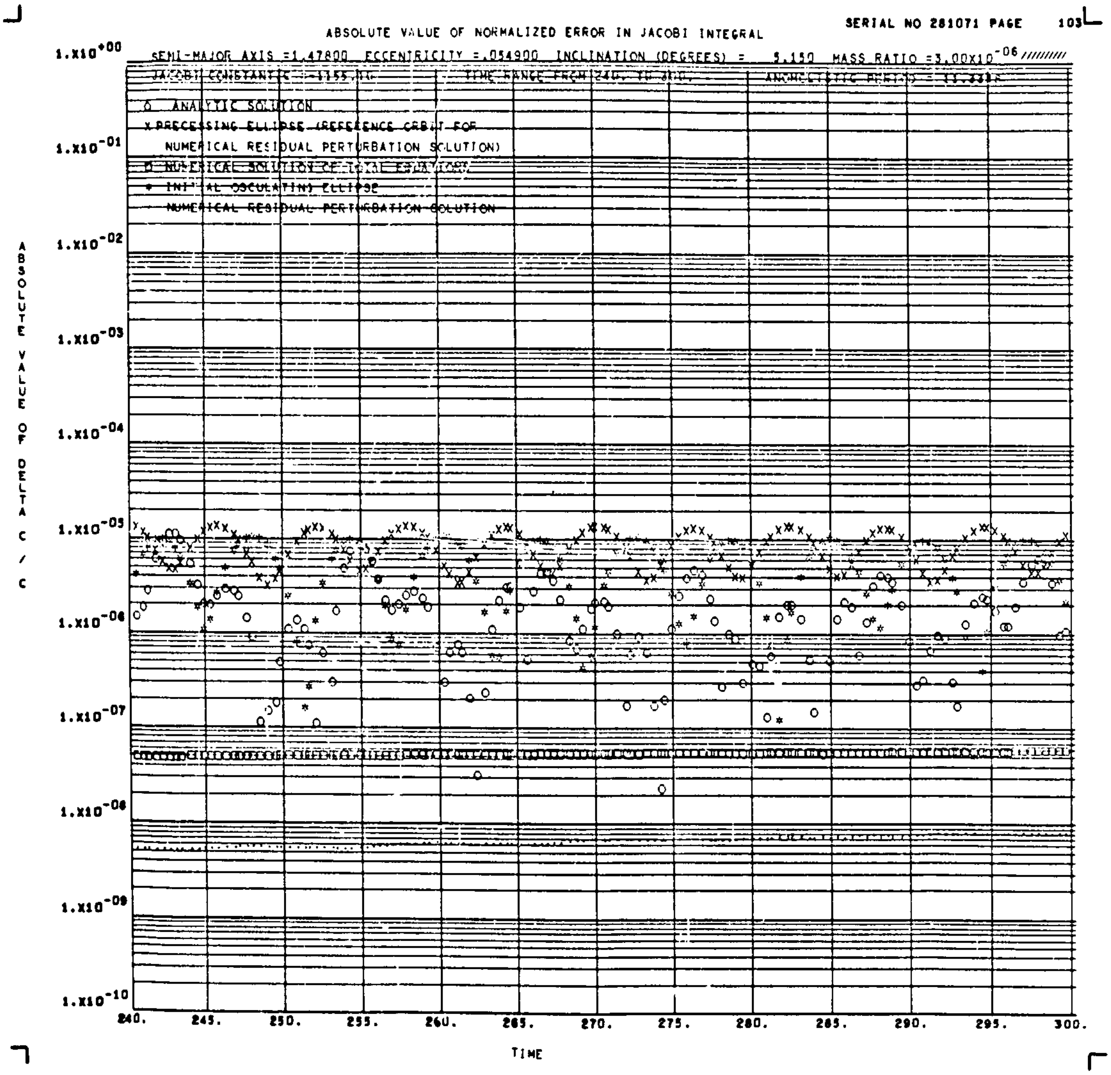

SERIAL NO 281071 PAGE
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00X10^-06
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
DELTA RHO DOT TIMES SQ RT A
1.X10^+00
1.X10^-01
1.X10^-02
1.X10^-03
1.X10^-04
1.X10^-05
1.X10^-06
1.X10^-07
1.X10^-08
1.X10^-09
1.X10^-10
120. 125. 130. 135. 140. 145. 150. 155. 160. 165. 170. 175. 180.
TIME

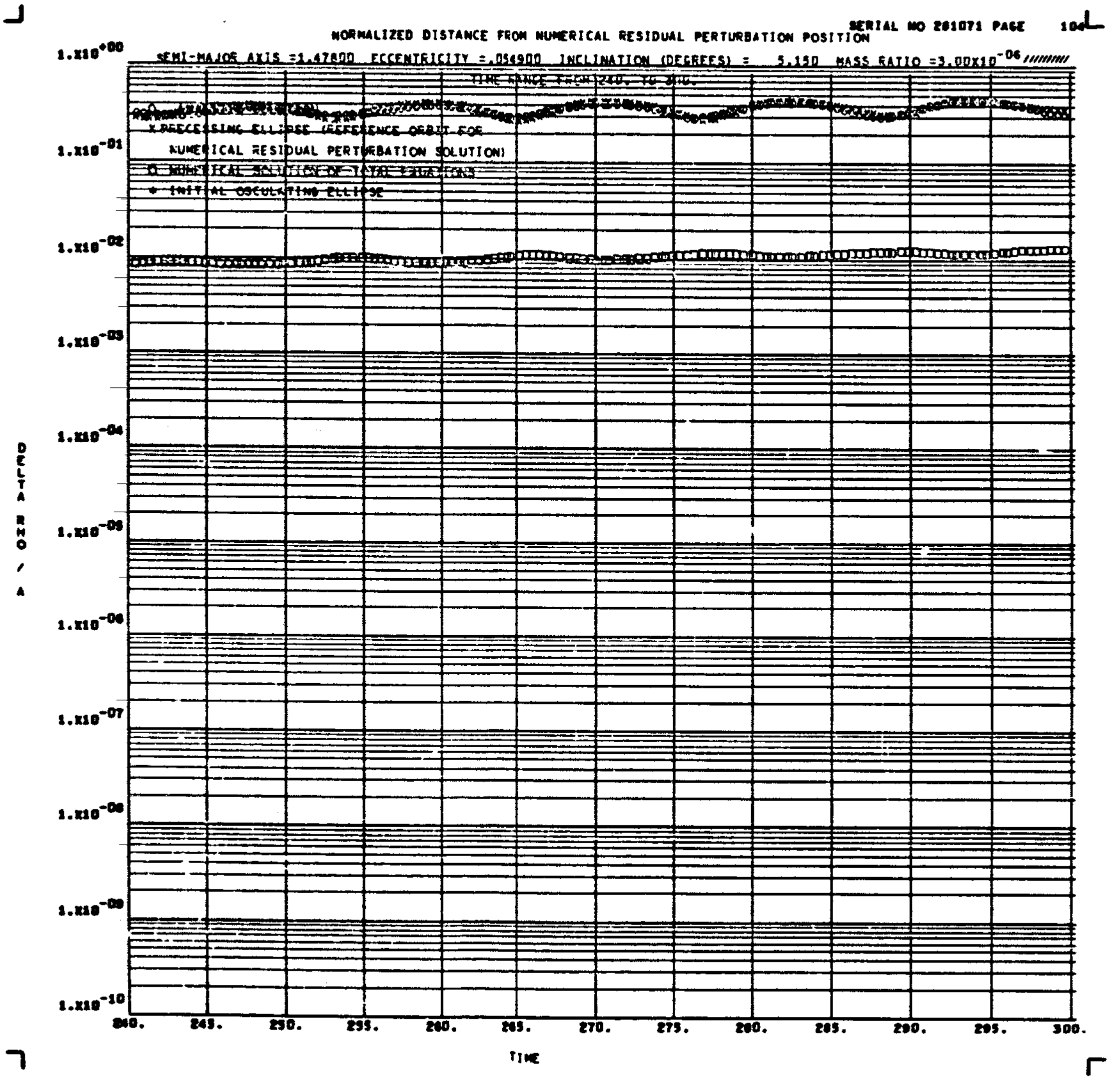

SERIAL NO 201071 PAGE
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00x10^-06
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
* INITIAL OSCULATING ELLIPSE
NUMERICAL RESIDUAL PERTURBATION SOLUTION
ABSOLUTE VALUE OF DELTA C / C
1.x10^+00
1.x10^-01
1.x10^-02
1.x10^-03
1.x10^-04
1.x10^-05
1.x10^-06
1.x10^-07
1.x10^-08
1.x10^-09
1.x10^-10
180. 185. 190. 195. 200. 205. 210. 215. 220. 225. 230. 235. 240.
TIME

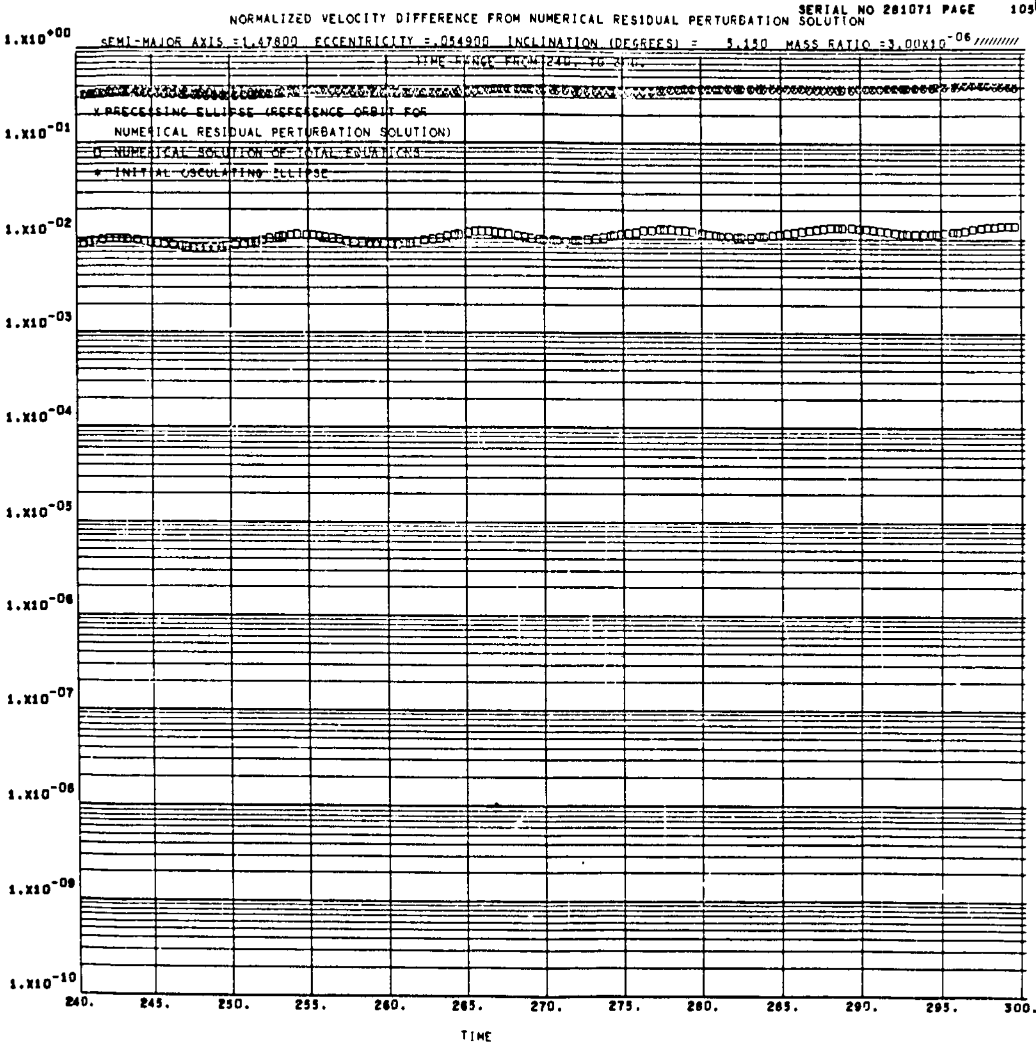

SERIAL NO 281071 PAGE 101
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00x10-06
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
1.x10+00
1.x10-01
1.x10-02
1.x10-03
1.x10-04
1.x10-05
1.x10-06
1.x10-07
1.x10-08
1.x10-09
1.x10-10
DELTA RHO / A
180.
185.
190.
195.
200.
205.
210.
215.
220.
225.
230.
235.
240.
TIME

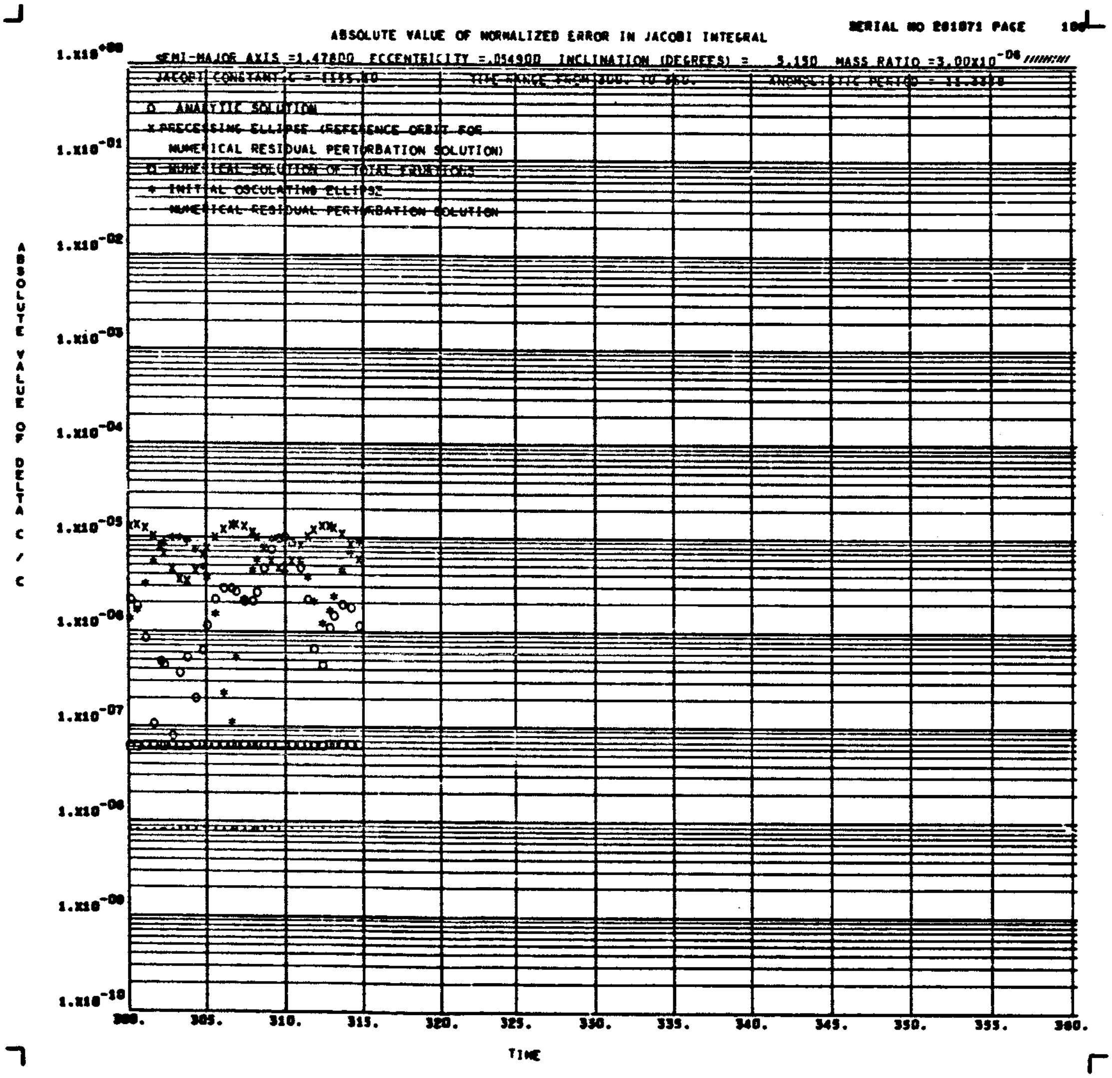


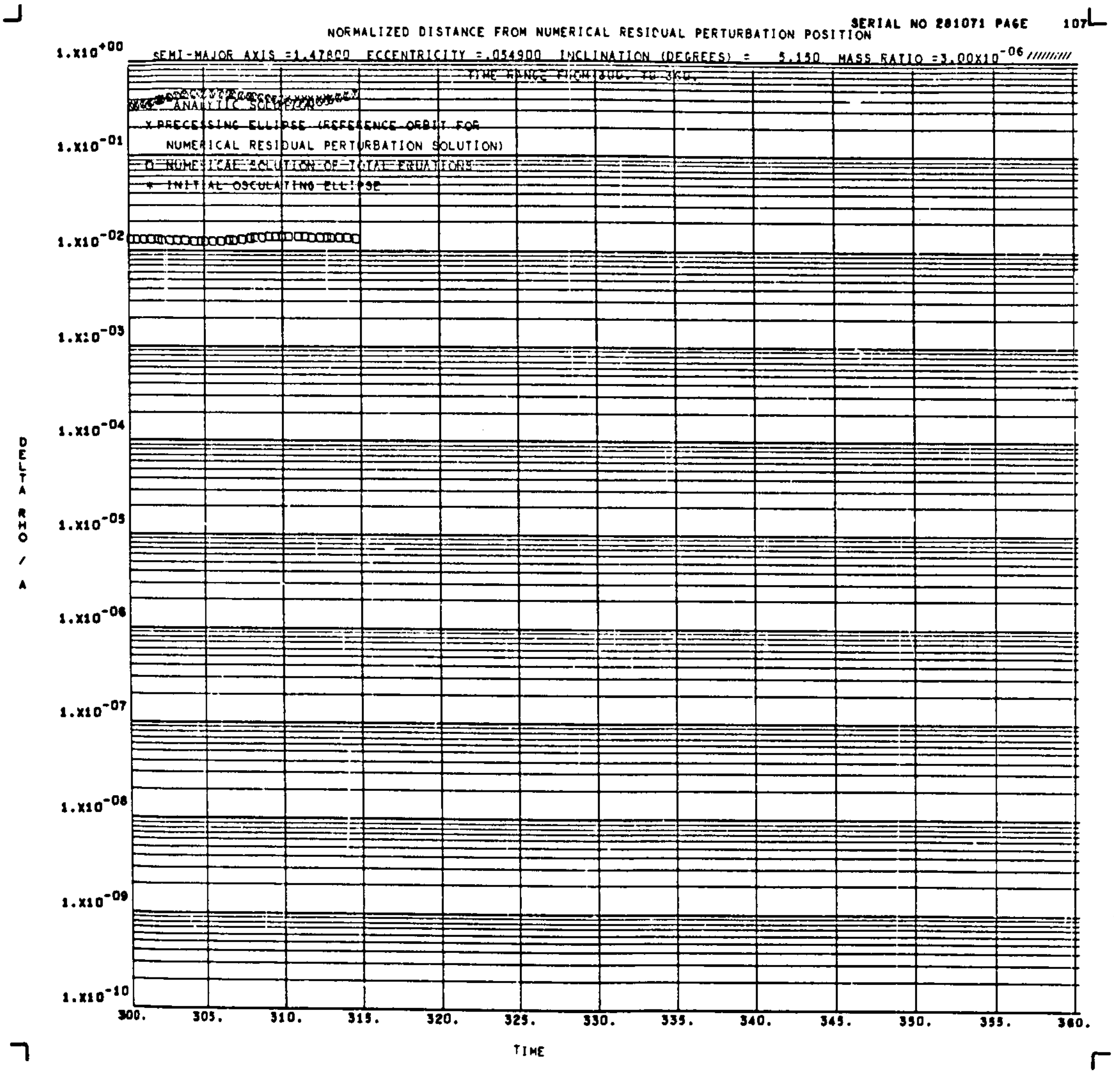
SERIAL NO 261071 PAGE 10
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00X10-06
O ANALYTIC SOLUTION
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO DOT TIMES SQ RT A
180.
185.
190.
195.
200.
205.
210.
215.
220.
225.
230.
235.
240.
TIME

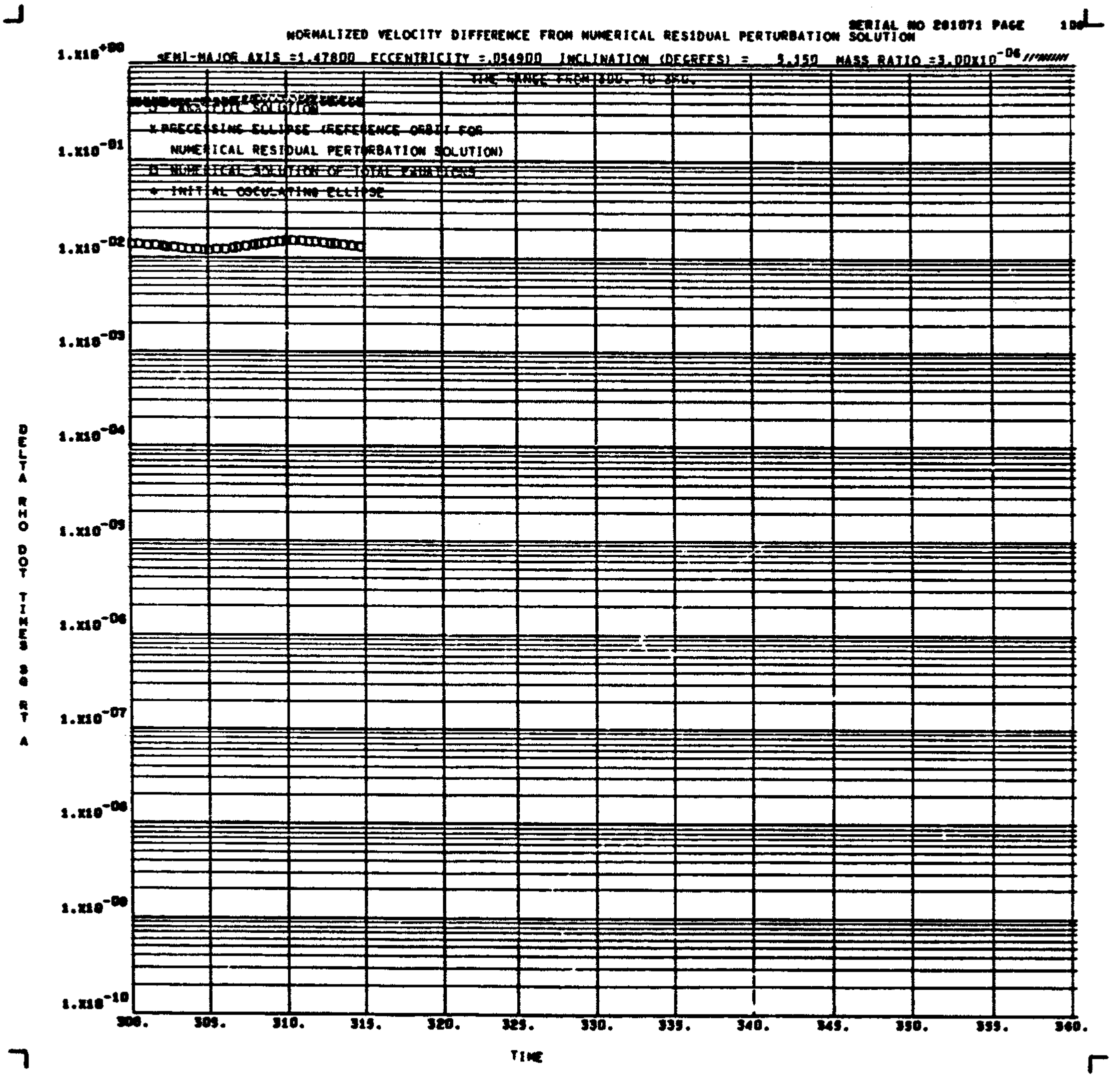

ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00x10^-06
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
NUMERICAL SOLUTION OF TOTAL EQUATIONS
INITIAL OSCULATING ELLIPSE
NUMERICAL RESIDUAL PERTURBATION SOLUTION
ABSOLUTE VALUE OF DELTA C / C
TIME

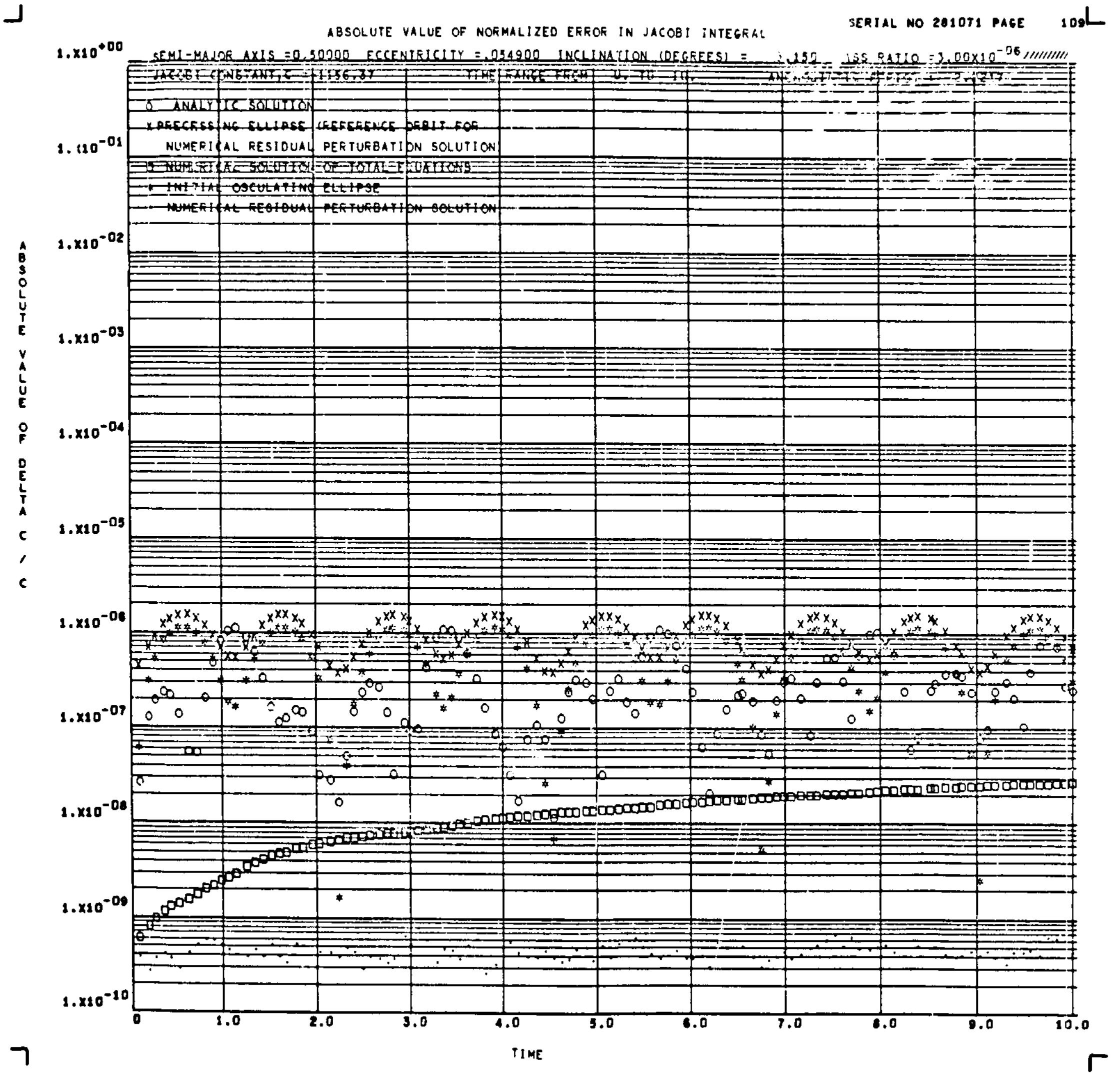

SERIAL NO 281071 PAGE 104
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00x10-06
TIME RANGE FROM 240. TO 300.
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
O NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
DELTA RHO / A
TIME

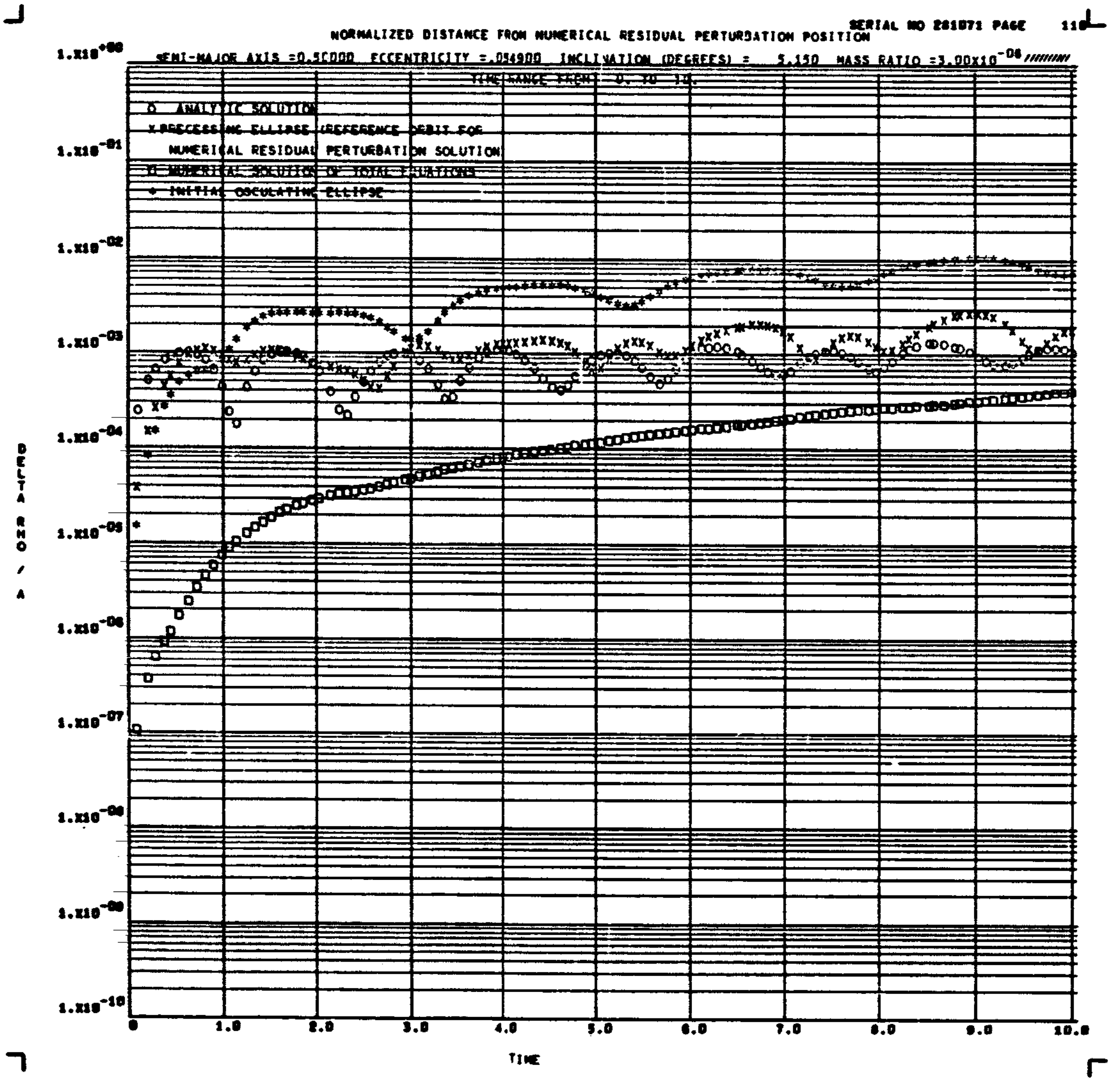

NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00X10^-06
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
1.X10^+00
1.X10^-01
1.X10^-02
1.X10^-03
1.X10^-04
1.X10^-05
1.X10^-06
1.X10^-07
1.X10^-08
1.X10^-09
1.X10^-10
DELTA RHO DOT TIMES SQ RT A
240.
245.
250.
255.
260.
265.
270.
275.
280.
285.
290.
295.
300.
TIME

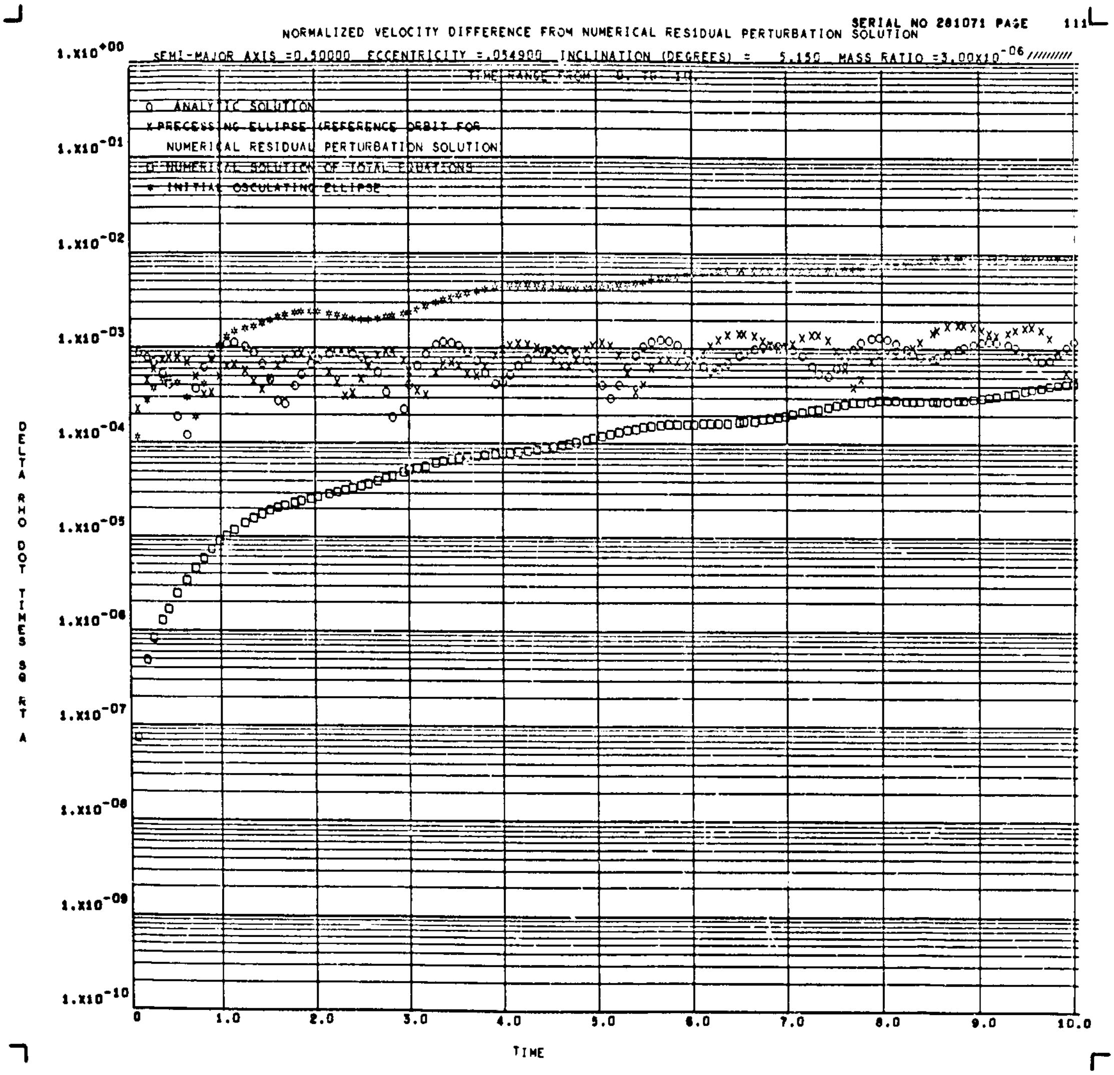

SERIAL NO 201071 PAGE 100
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00x10-06
O ANALYTIC SOLUTION
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
+ INITIAL OSCULATING ELLIPSE
1.x10+00
1.x10-01
1.x10-02
1.x10-03
1.x10-04
1.x10-05
1.x10-06
1.x10-07
1.x10-08
1.x10-09
1.x10-10
ABSOLUTE VALUE OF DELTA C / C
300.
305.
310.
315.
320.
325.
330.
335.
340.
345.
350.
355.
360.
TIME

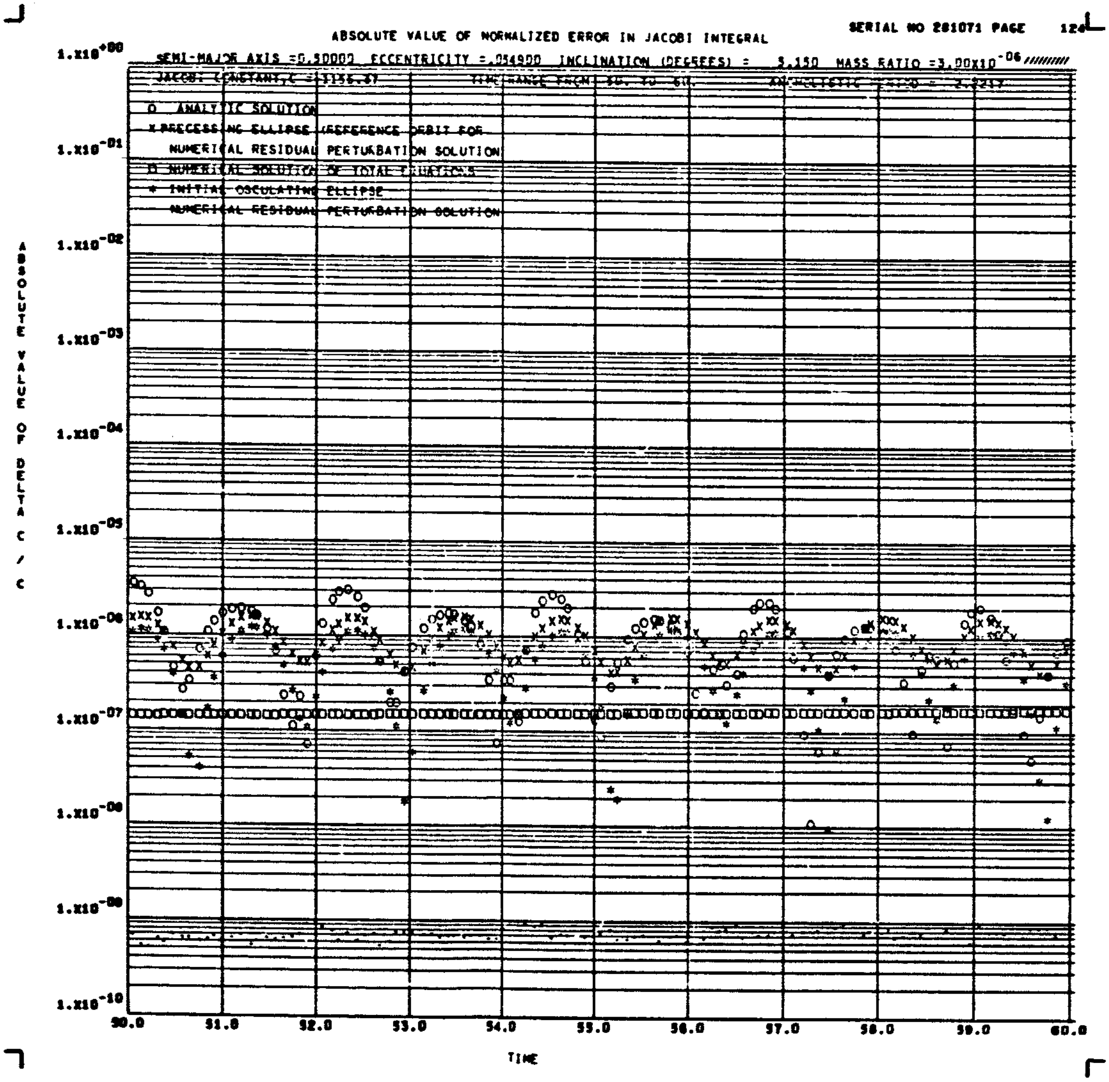

SERIAL NO 281071 PAGE 107
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00X10-06
TIME RANGE FROM 300. TO 360.
ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
O NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO / A
300. 305. 310. 315. 320. 325. 330. 335. 340. 345. 350. 355. 360.
TIME


SERIAL NO 201071 PAGE
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00X10-06
ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
NUMERICAL SOLUTION OF TOTAL EQUATIONS
INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO DOT TIMES SQ RT A
300.
305.
310.
315.
320.
325.
330.
335.
340.
345.
350.
355.
360.
TIME

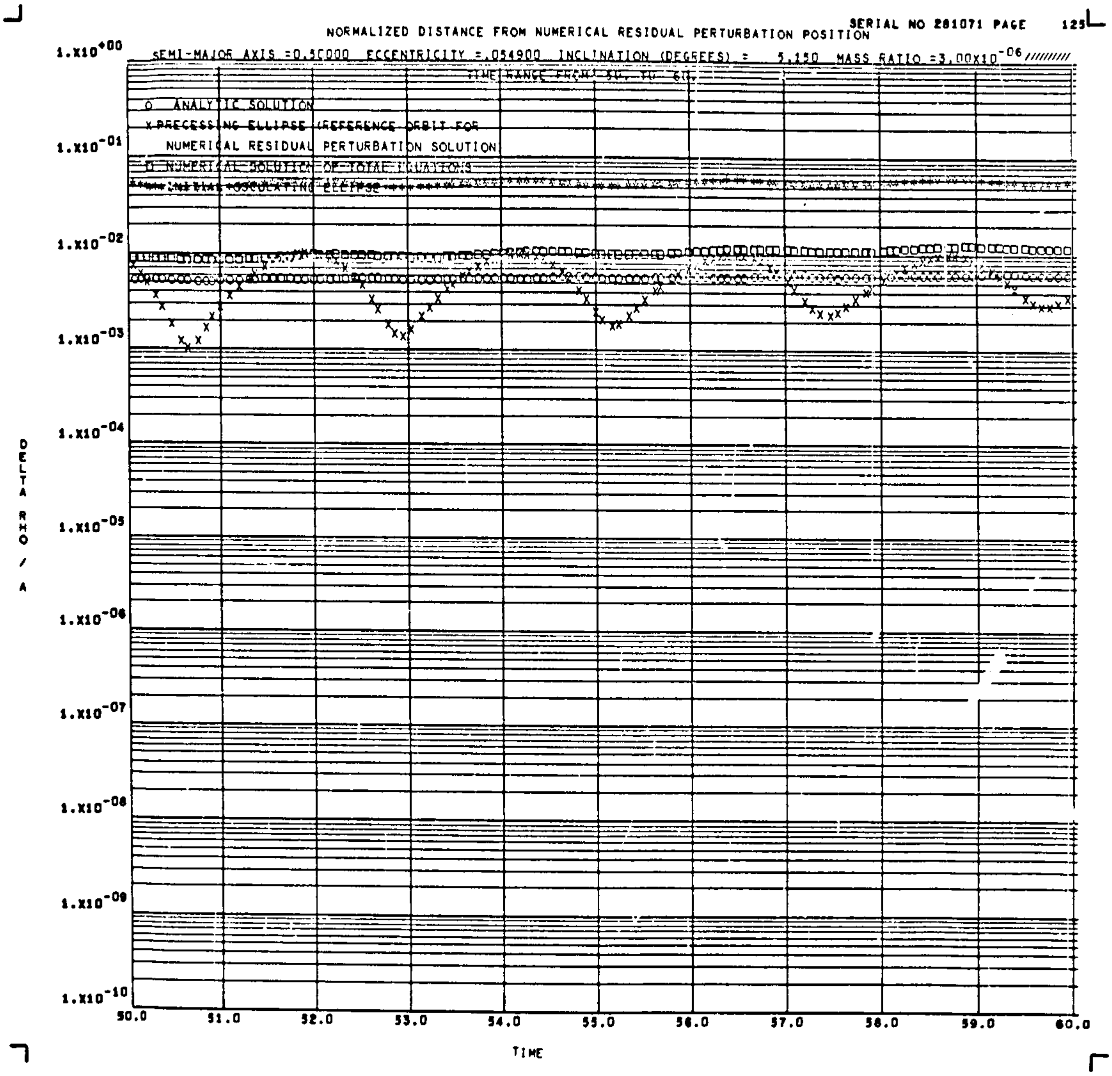

SERIAL NO 281071 PAGE 109
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =0.50000 ECCENTRICITY =.054900 INCLINATION (DEGREES) =
O ANALYTIC SOLUTION
NUMERICAL RESIDUAL PERTURBATION SOLUTION
INITIAL OSCULATING ELLIPSE
NUMERICAL RESIDUAL PERTURBATION SOLUTION
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
ABSOLUTE VALUE OF DELTA C / C
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
TIME

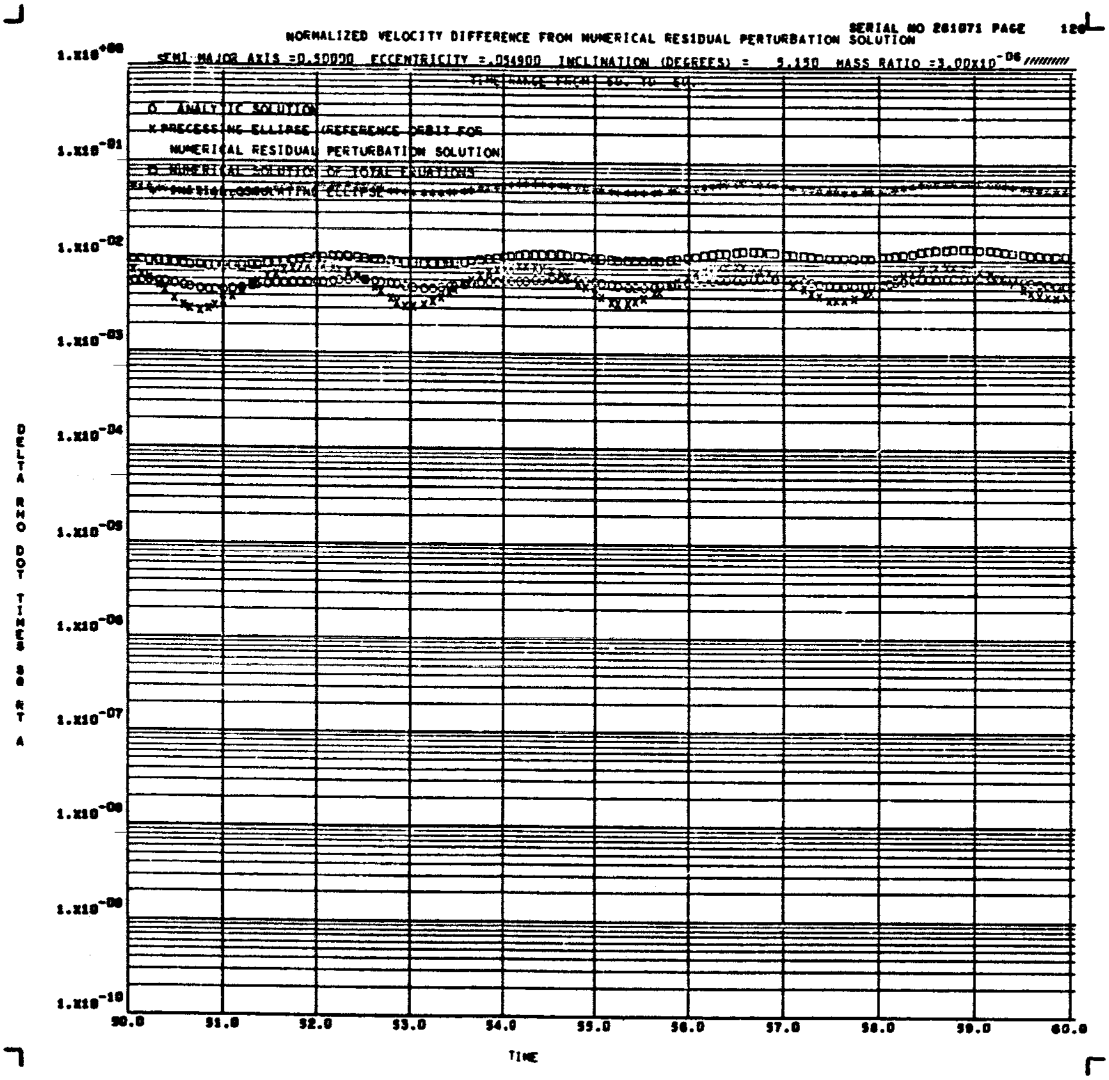

SERIAL NO 281071 PAGE 110
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =0.50000 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00x10-06
TIME RANGE FROM 0. TO 10.
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
1.x10+00
1.x10-01
1.x10-02
1.x10-03
1.x10-04
1.x10-05
1.x10-06
1.x10-07
1.x10-08
1.x10-09
1.x10-10
DELTA RHO / A
0
1.0
2.0
3.0
4.0
5.0
6.0
7.0
8.0
9.0
10.0
TIME

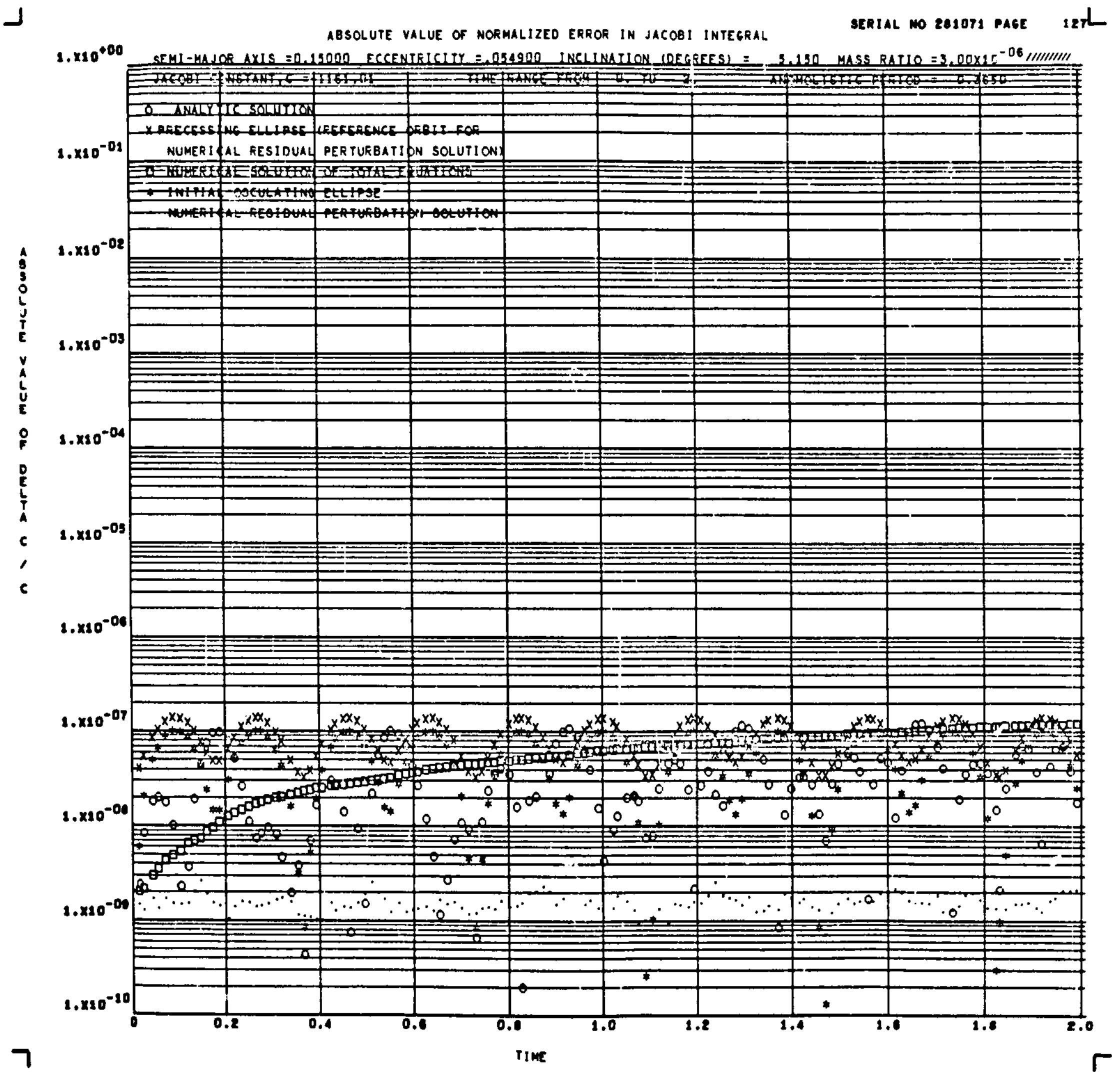

SERIAL NO 281071 PAGE 111
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =0.50000 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00x10-06
TIME RANGE FROM 0. TO 10.
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
DELTA RHO DOT TIMES SQ RT A
TIME



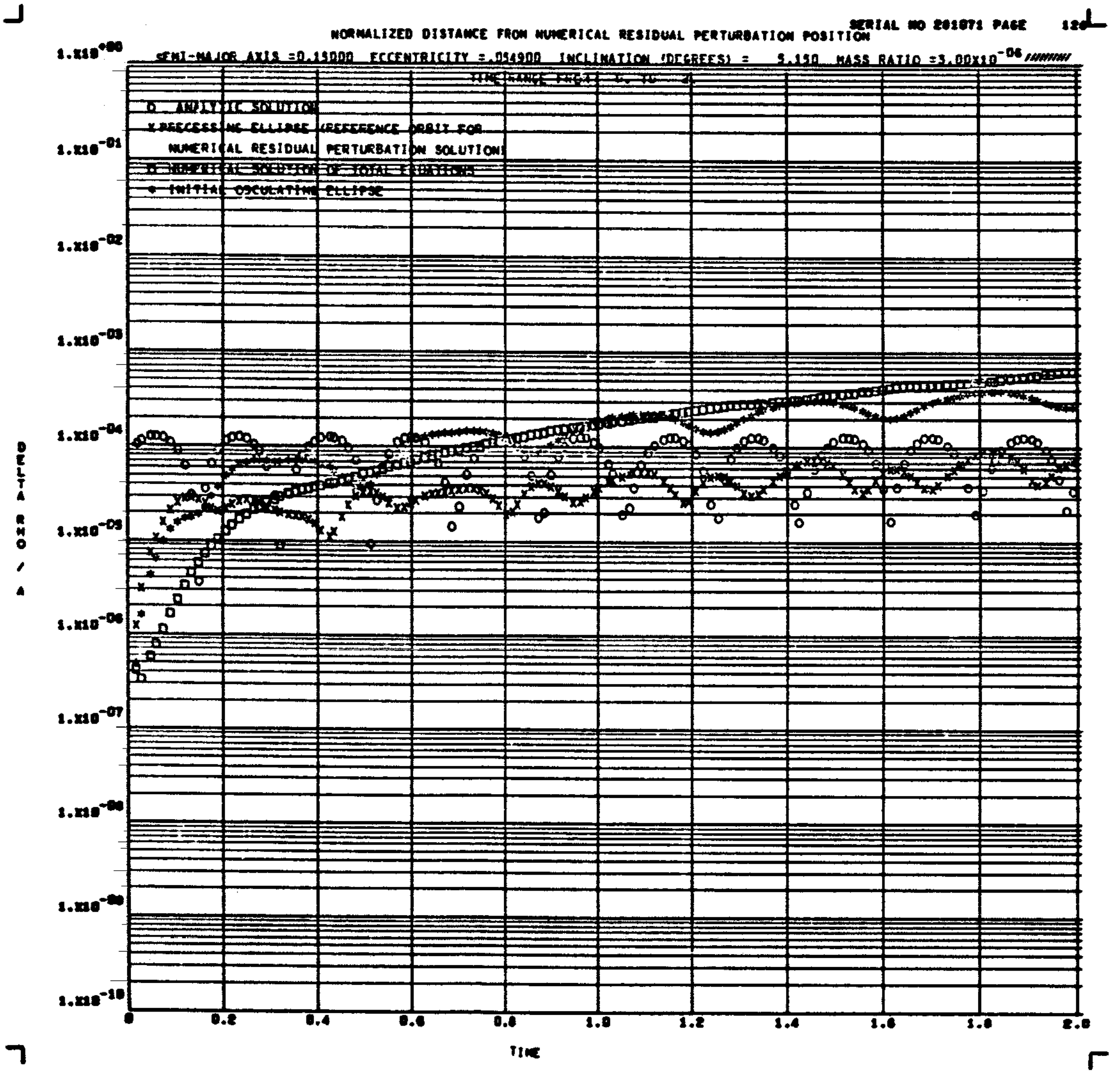
SERIAL NO 281071 PAGE 124
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =0.50000 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00X10-06
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
NUMERICAL RESIDUAL PERTURBATION SOLUTION
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
50.0
51.0
52.0
53.0
54.0
55.0
56.0
57.0
58.0
59.0
60.0
TIME
ABSOLUTE VALUE OF DELTA C / C

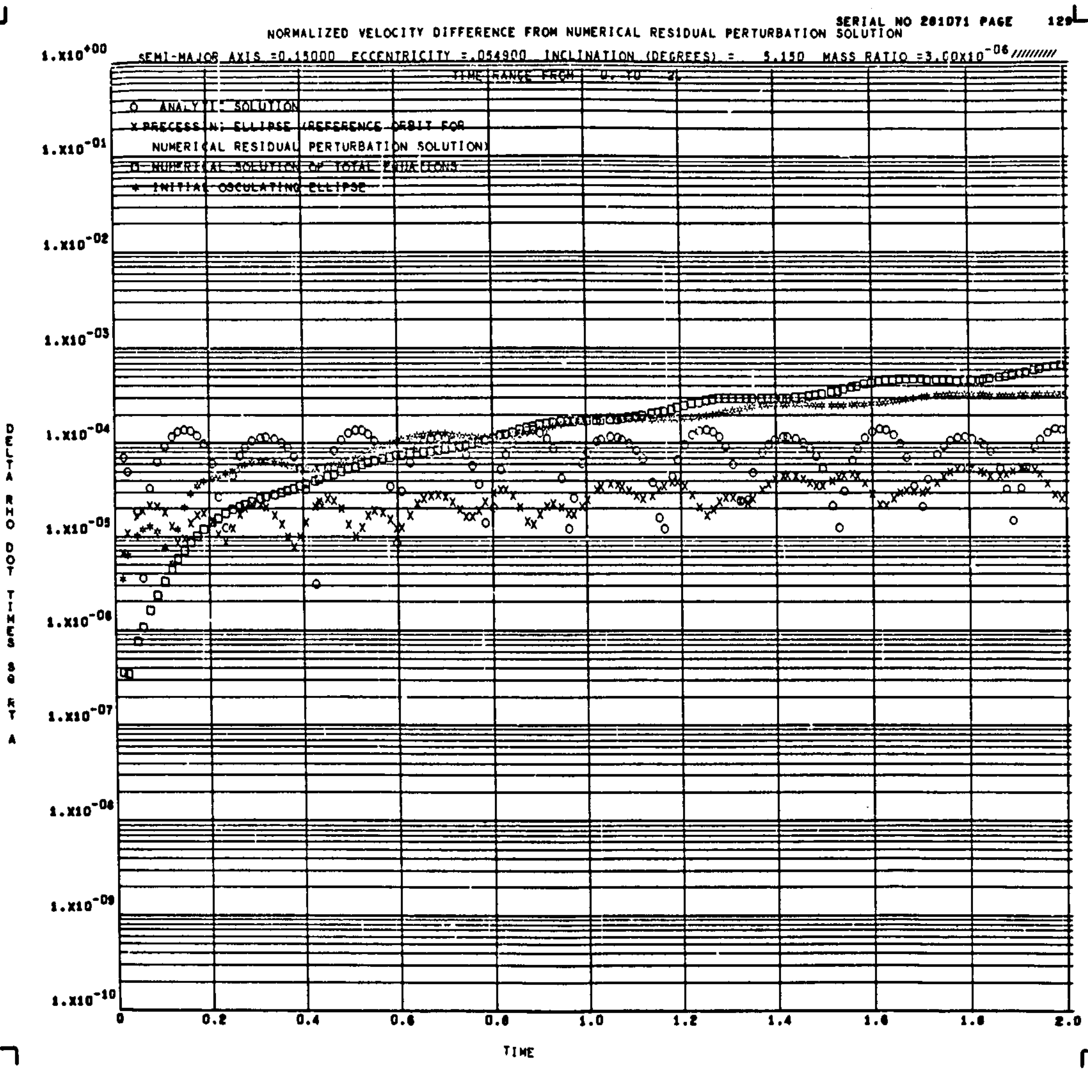

SERIAL NO 281071 PAGE 125
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =0.50000 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00X10-06
TIME RANGE FROM 50. TO 60.
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO / A
50.0
51.0
52.0
53.0
54.0
55.0
56.0
57.0
58.0
59.0
60.0
TIME

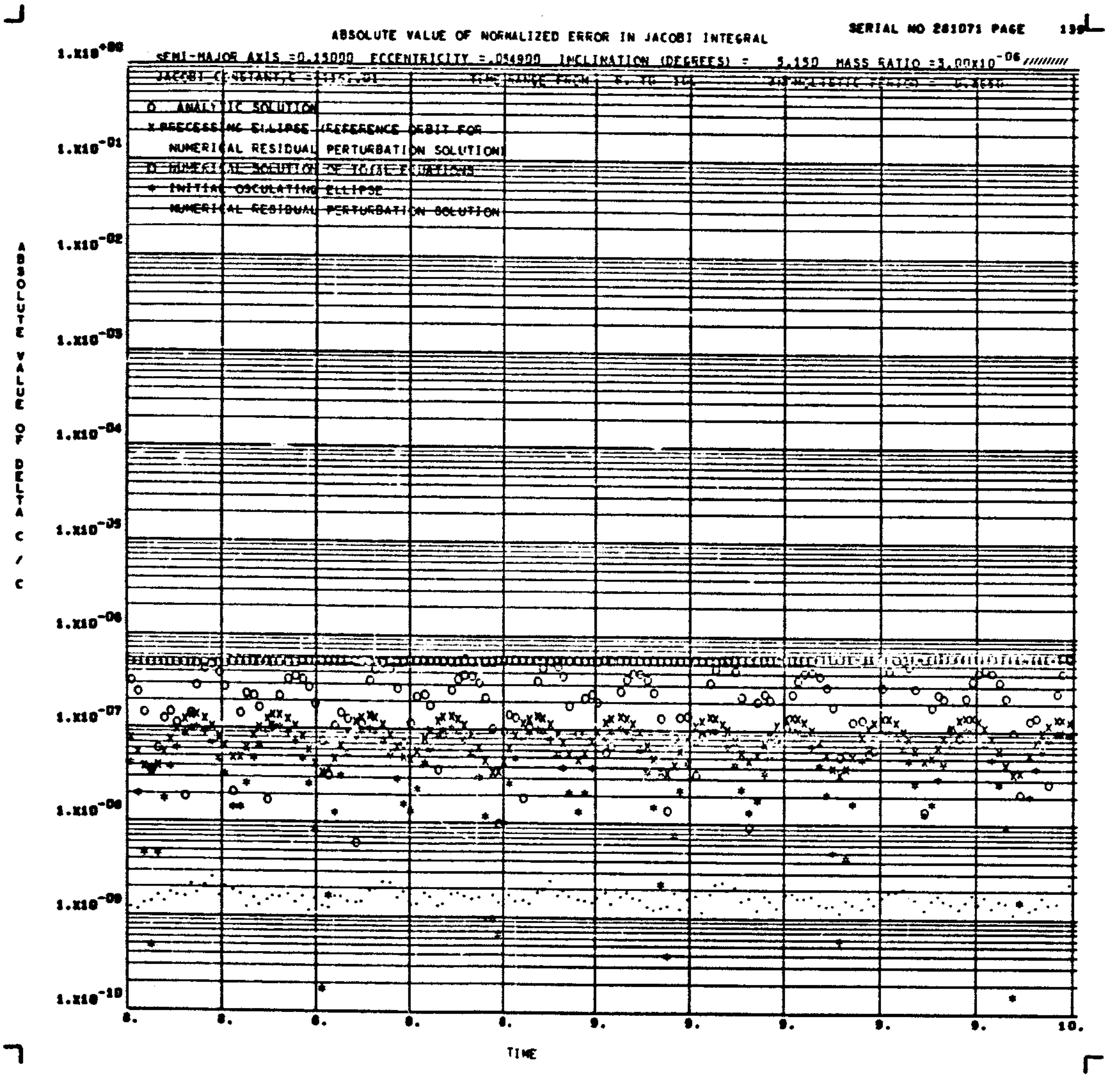

SERIAL NO 261071 PAGE 120
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =0.50000 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00x10-06
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
NUMERICAL SOLUTION OF TOTAL EQUATIONS
1.x10+00
1.x10-01
1.x10-02
1.x10-03
1.x10-04
1.x10-05
1.x10-06
1.x10-07
1.x10-08
1.x10-09
1.x10-10
DELTA RHO DOT TIMES SQ RT A
50.0
51.0
52.0
53.0
54.0
55.0
56.0
57.0
58.0
59.0
60.0
TIME

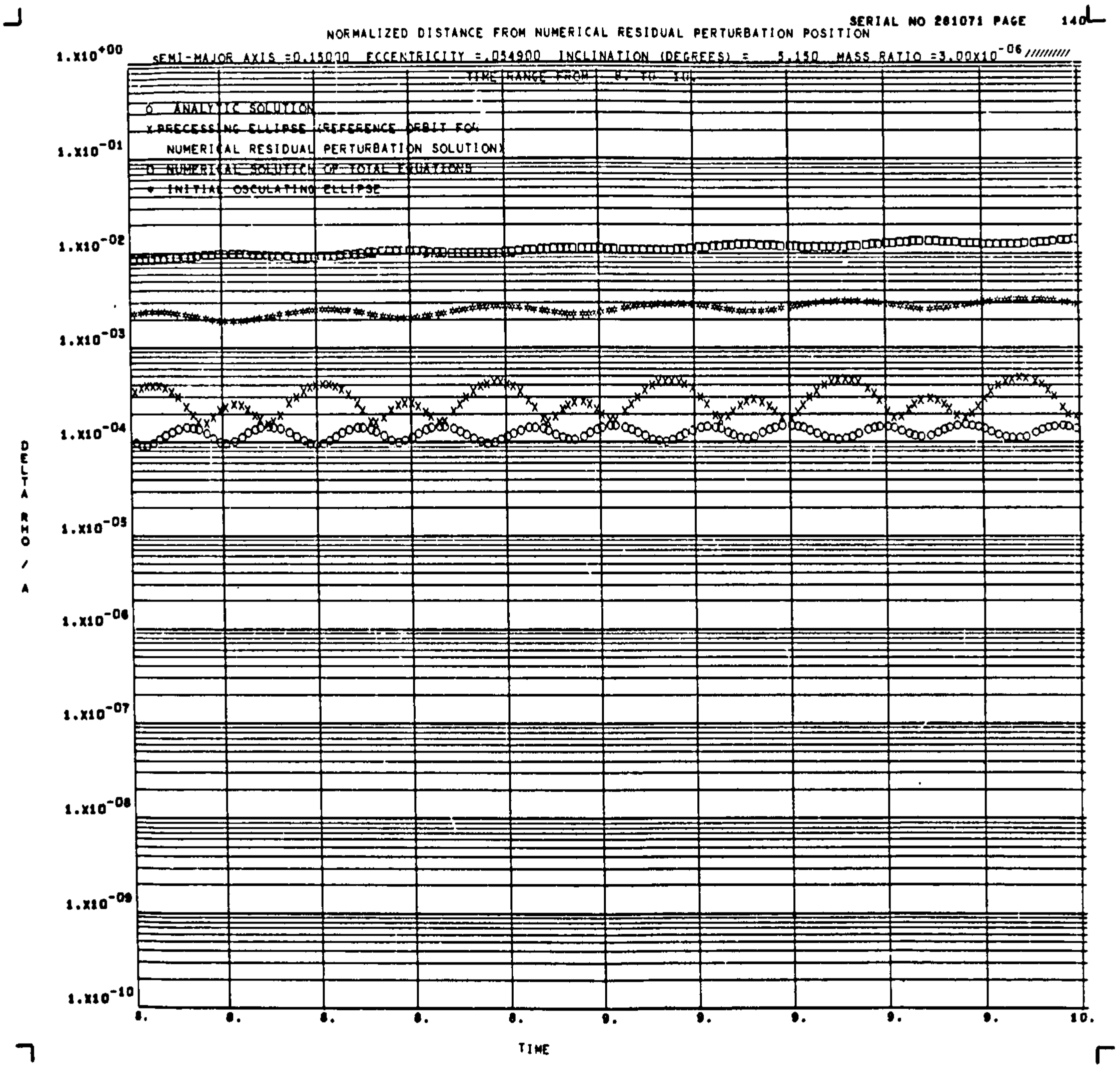


SERIAL NO 281071 PAGE 127
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00X10^-06
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
· NUMERICAL RESIDUAL PERTURBATION SOLUTION
1.X10^+00
1.X10^-01
1.X10^-02
1.X10^-03
1.X10^-04
1.X10^-05
1.X10^-06
1.X10^-07
1.X10^-08
1.X10^-09
1.X10^-10
ABSOLUTE VALUE OF DELTA C / C
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
TIME

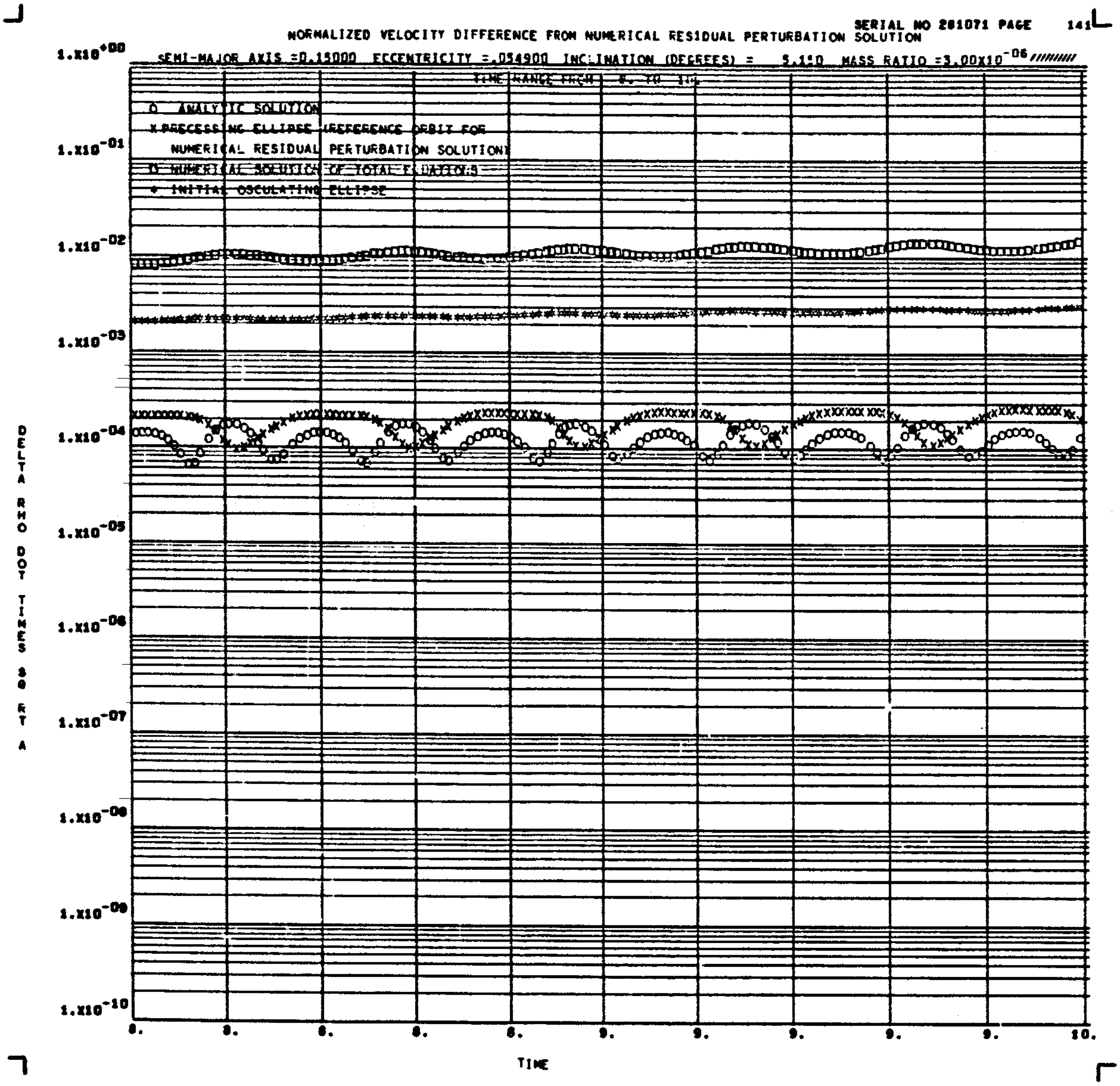

SERIAL NO 281871 PAGE
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00x10-06
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
DELTA RHO / A
TIME


SERIAL NO 281071 PAGE 129
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00X10-06
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO DOT TIMES SQ RT A
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
TIME

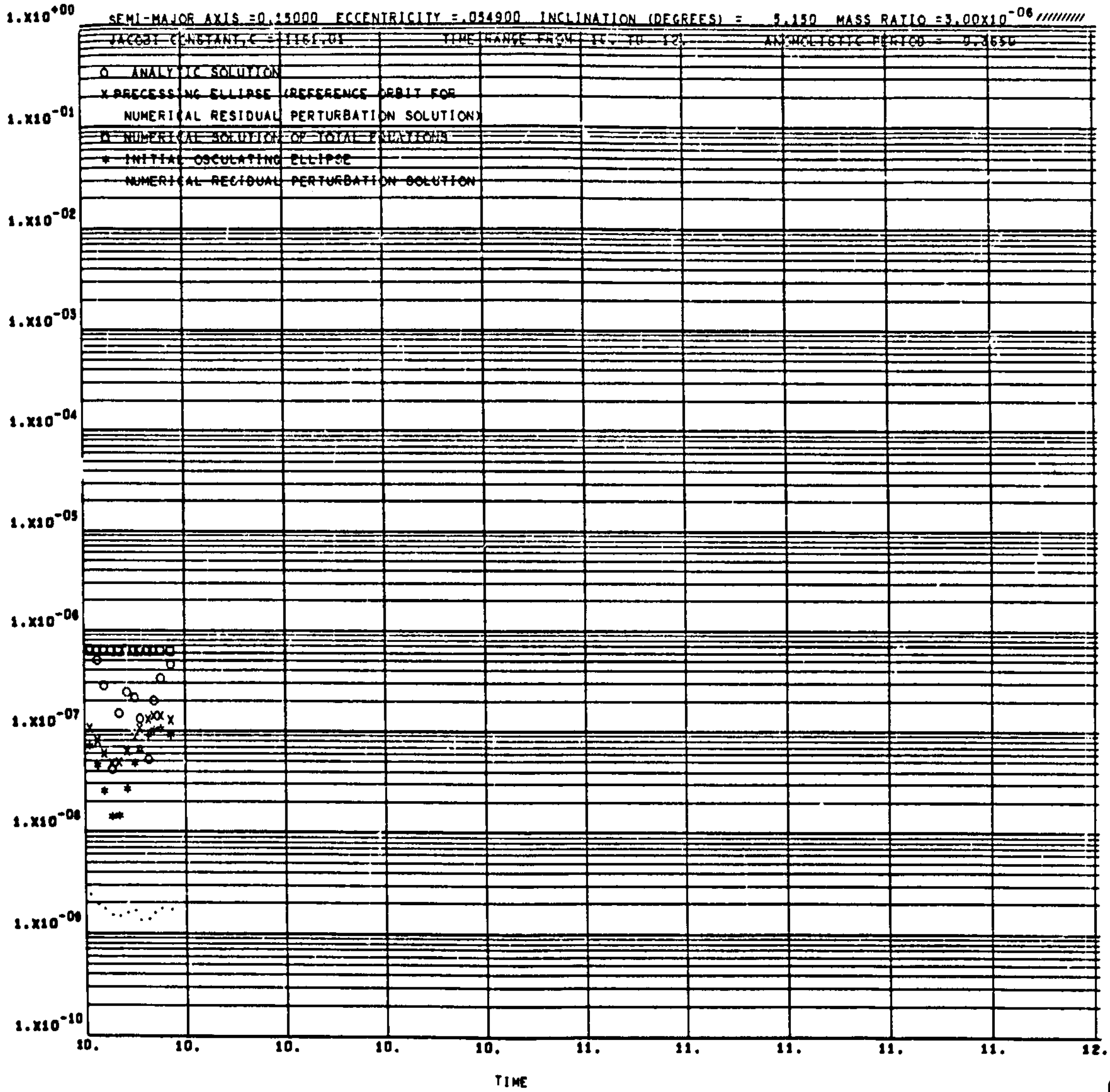

ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SERIAL NO 281071 PAGE
SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00x10-06
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
NUMERICAL RESIDUAL PERTURBATION SOLUTION
ABSOLUTE VALUE OF DELTA C / C
TIME

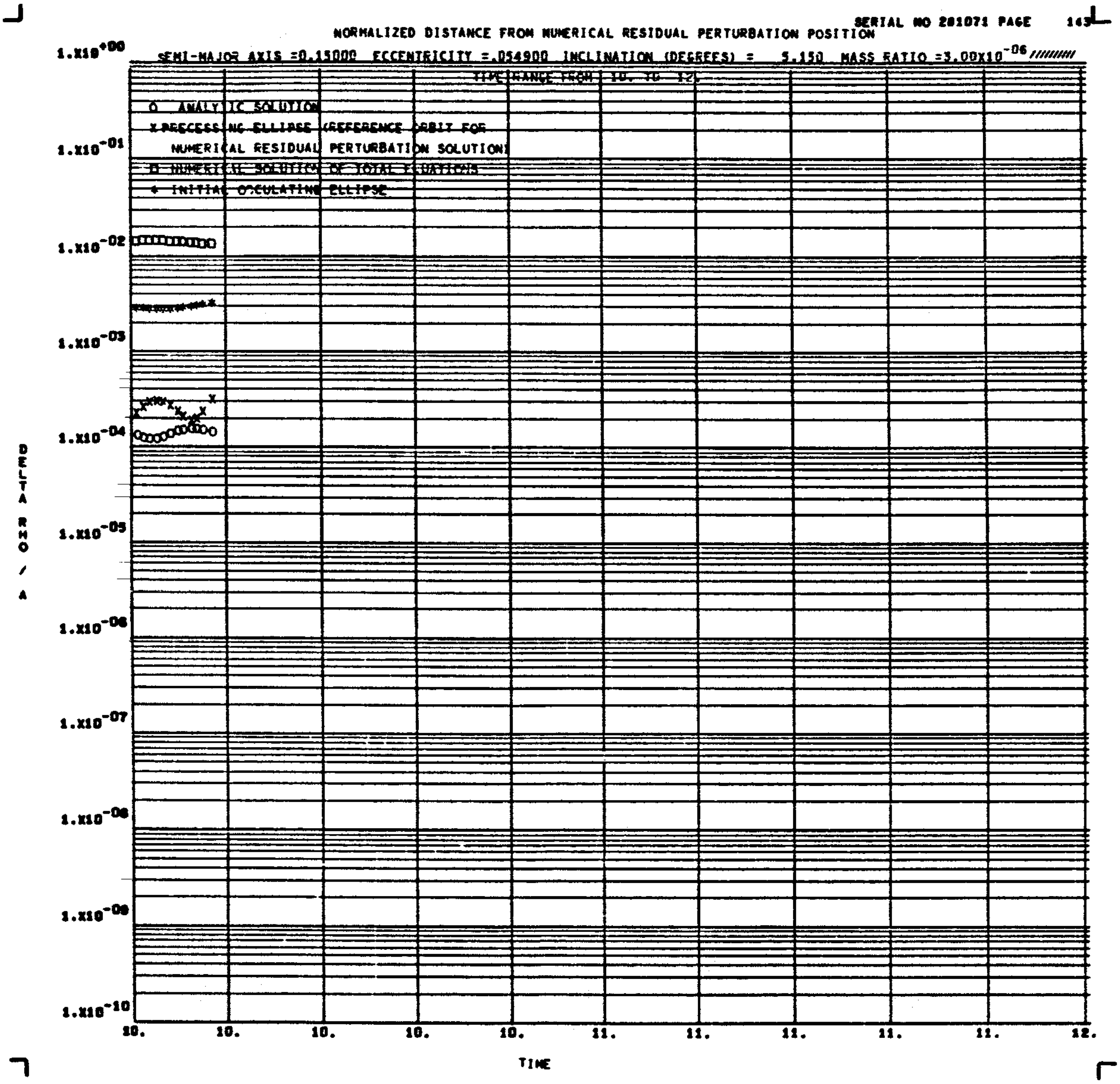


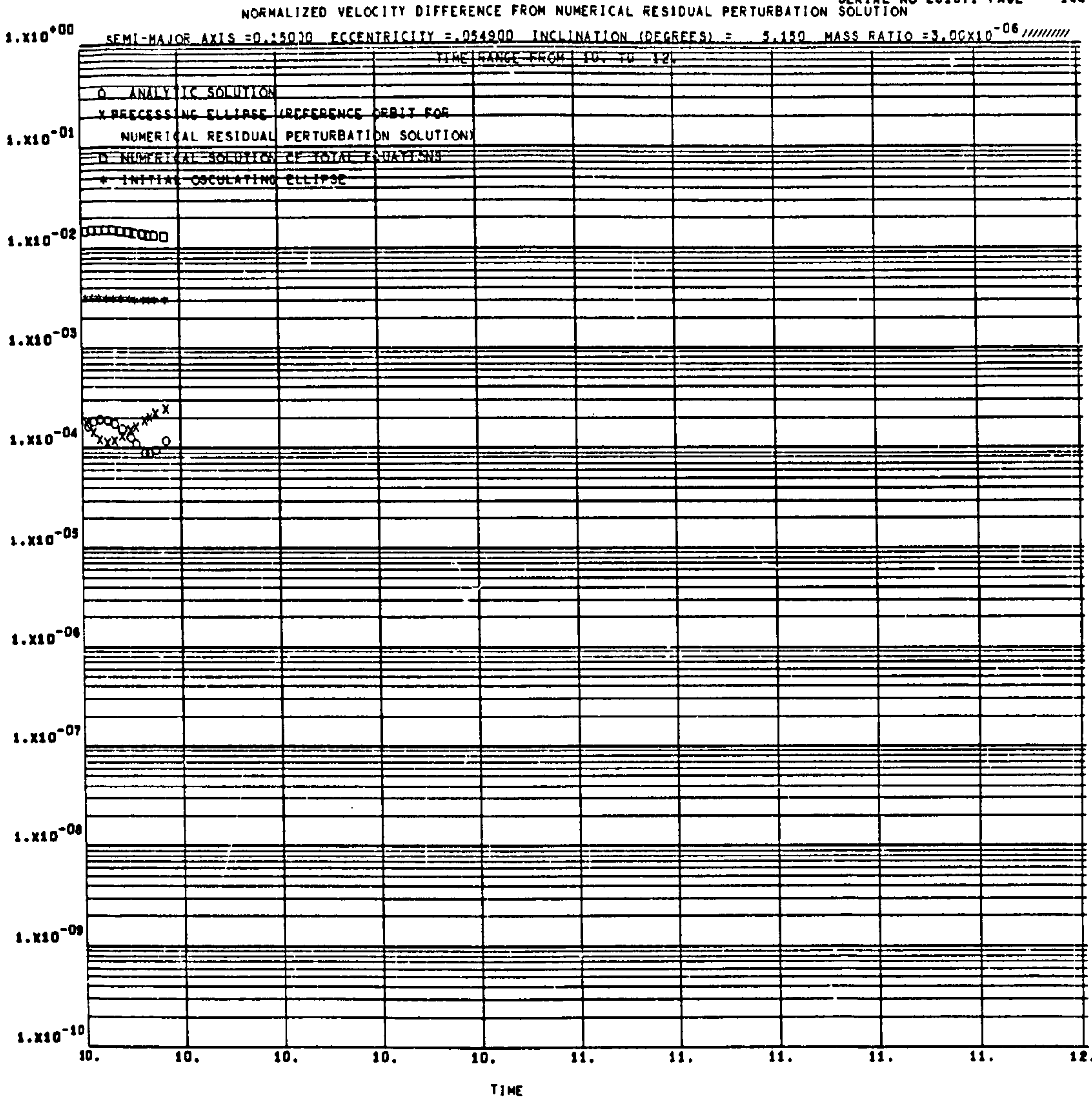
SERIAL NO 281071 PAGE 140
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00X10-06
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO / A
8.
8.
8.
8.
8.
9.
9.
9.
9.
9.
10.
TIME

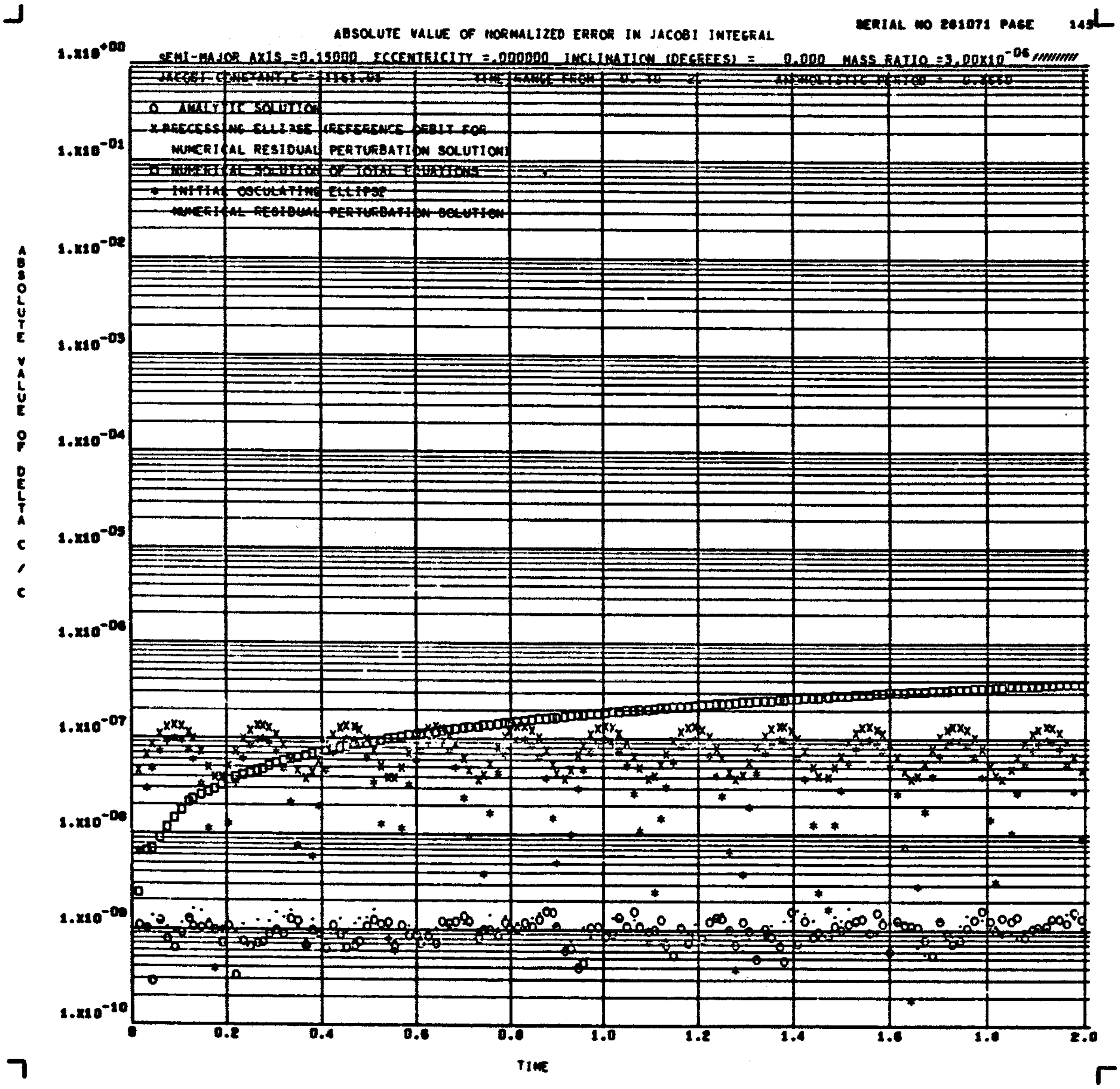

NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00X10^-06
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO DOT TIMES SQ RT A
8.
8.
8.
8.
8.
9.
9.
9.
9.
9.
10.
TIME

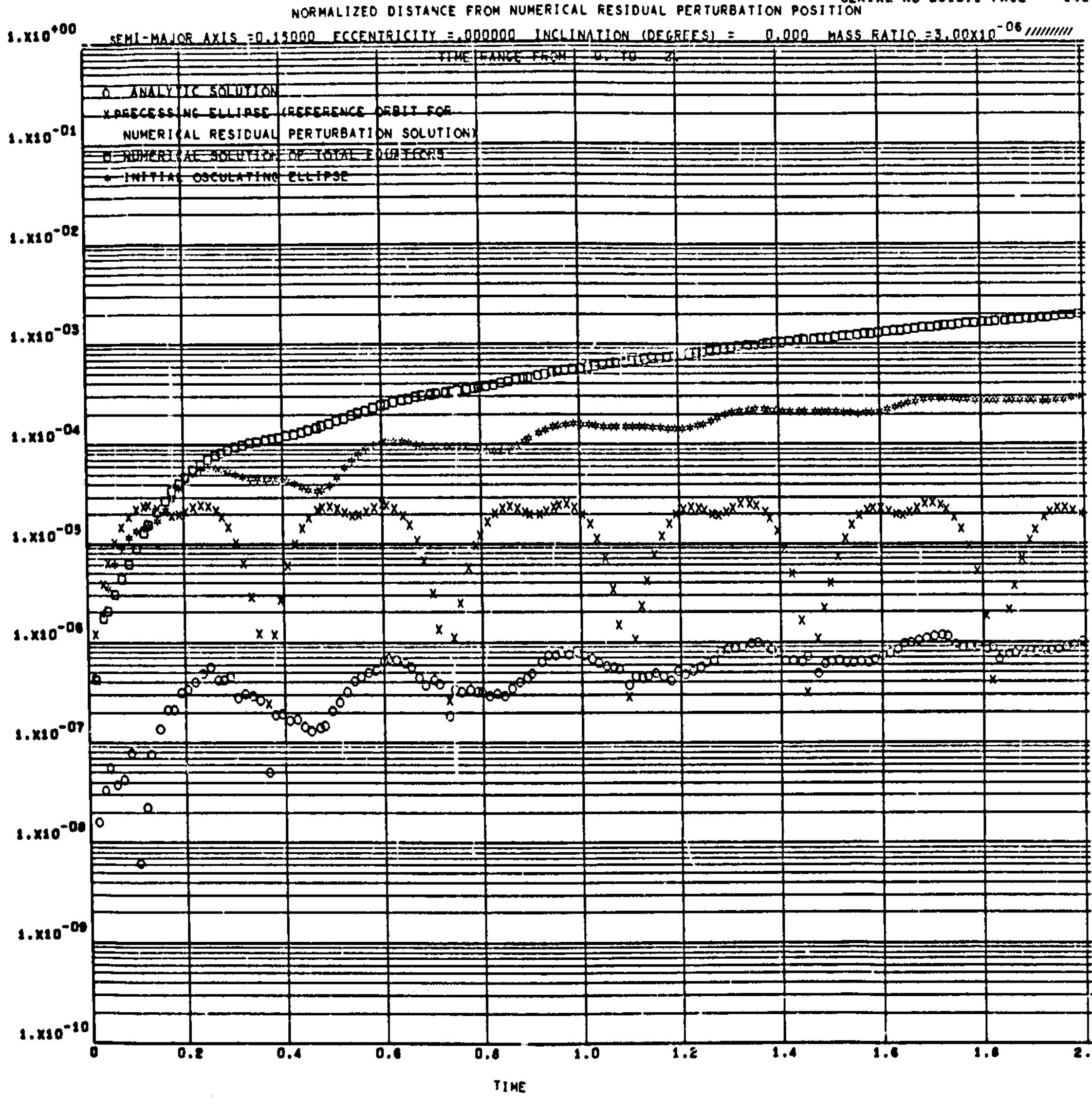
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL

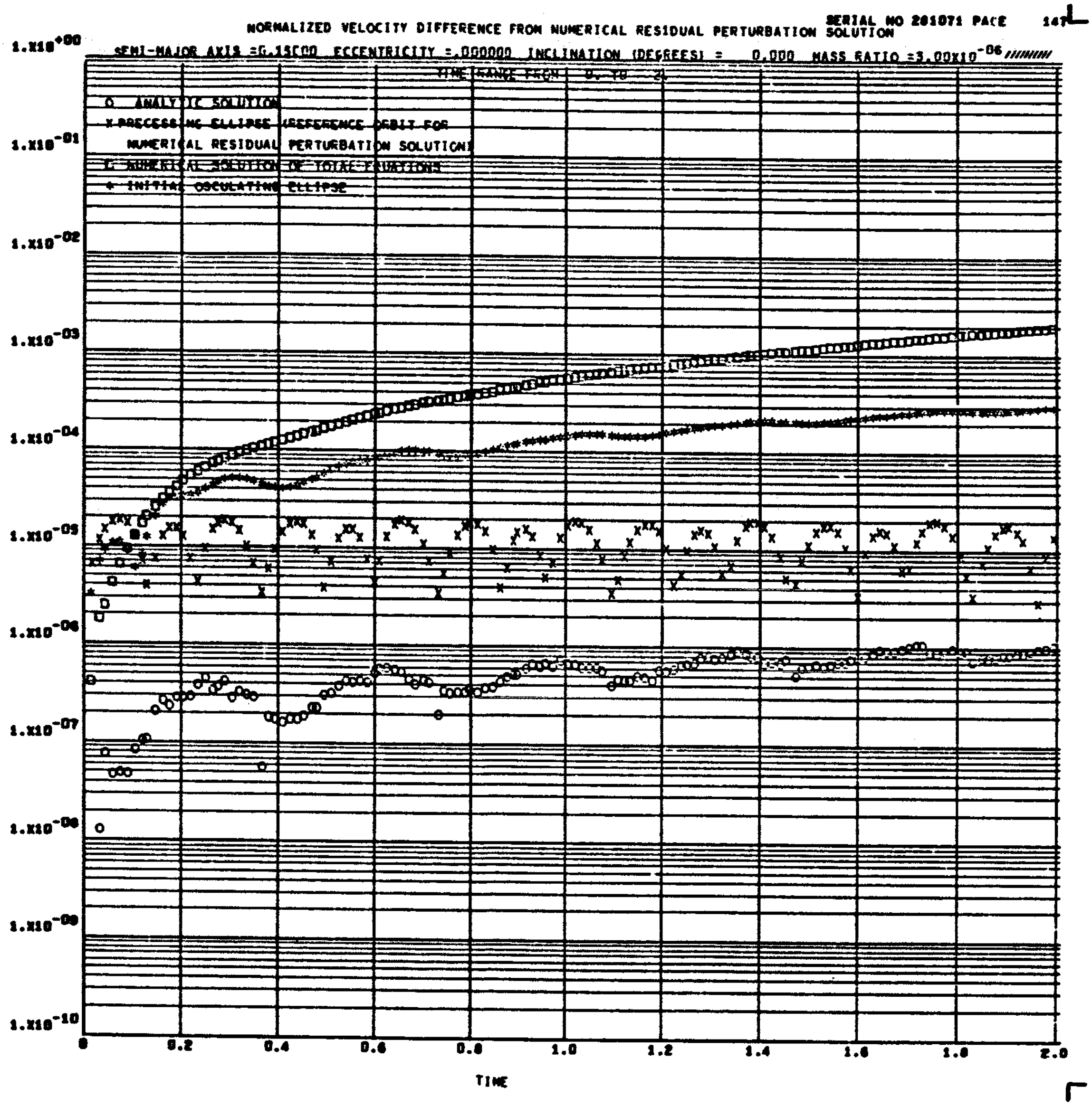



SERIAL NO 281071 PAGE 143
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00X10-06
TIME RANGE FROM 10. TO 12.
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO / A
10.
10.
10.
10.
10.
11.
11.
11.
11.
11.
12.
TIME

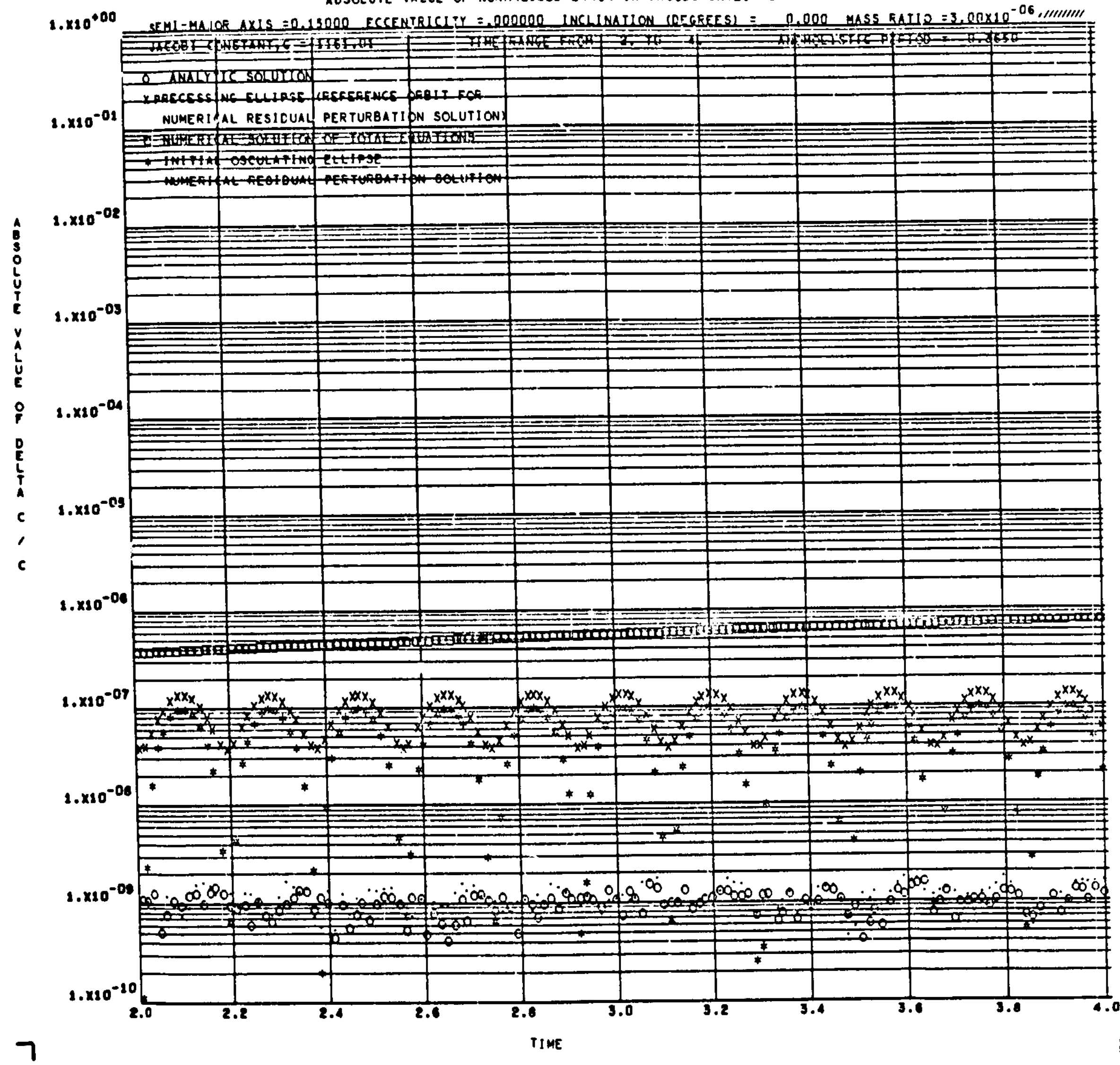

NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =0.15030 ECCENTRICITY =.054900 INCLINATION (DEGREES) = 5.150 MASS RATIO =3.00X10-06
TIME RANGE FROM 10. TO 12.
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO DOT TIMES SQ RT A
10.
10.
10.
10.
10.
11.
11.
11.
11.
11.
12.
TIME

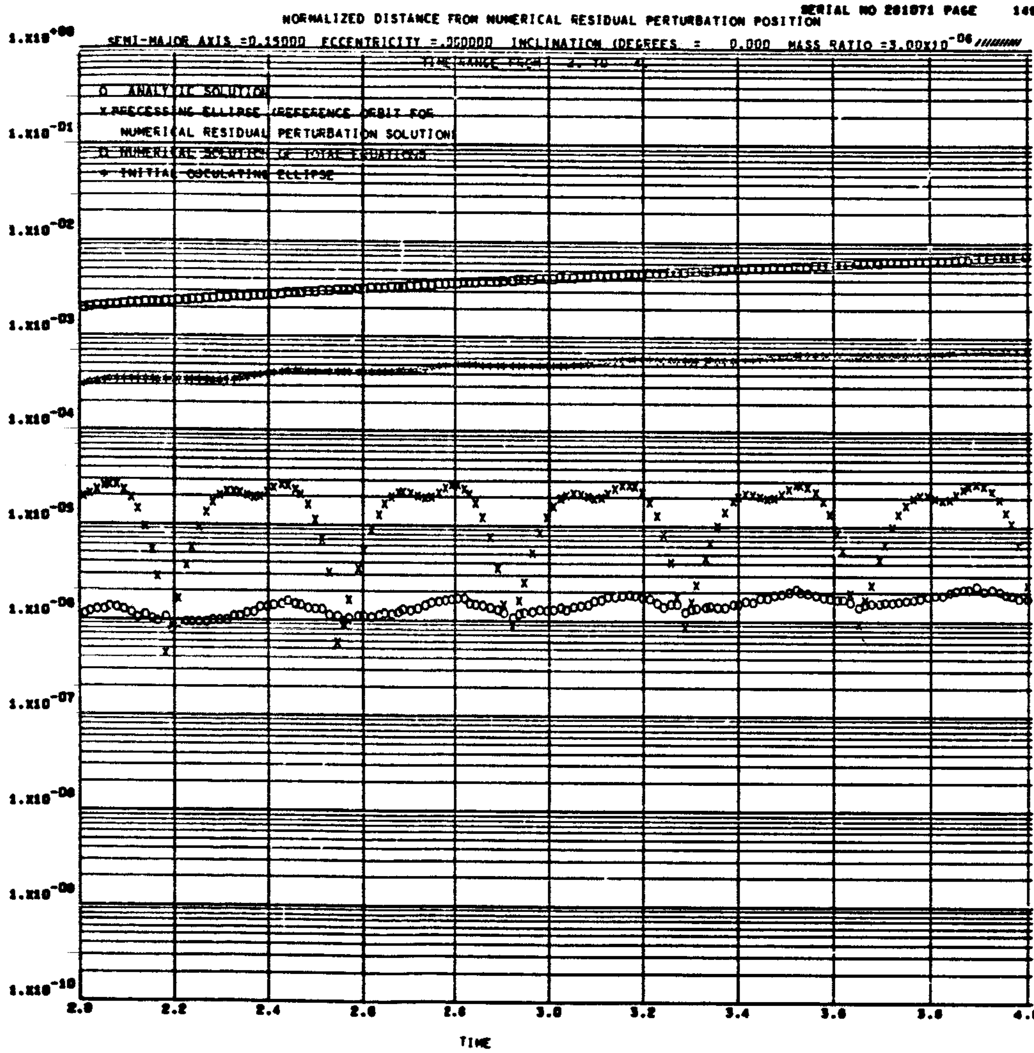

SERIAL NO 201071 PAGE 145
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.00X10^-06
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
· NUMERICAL RESIDUAL PERTURBATION SOLUTION
1.X10^+00
1.X10^-01
1.X10^-02
1.X10^-03
1.X10^-04
1.X10^-05
1.X10^-06
1.X10^-07
1.X10^-08
1.X10^-09
1.X10^-10
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
TIME
ABSOLUTE VALUE OF DELTA C / C


NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.00X10-06
TIME RANGE FROM 0. TO 2.
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO / A
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
TIME

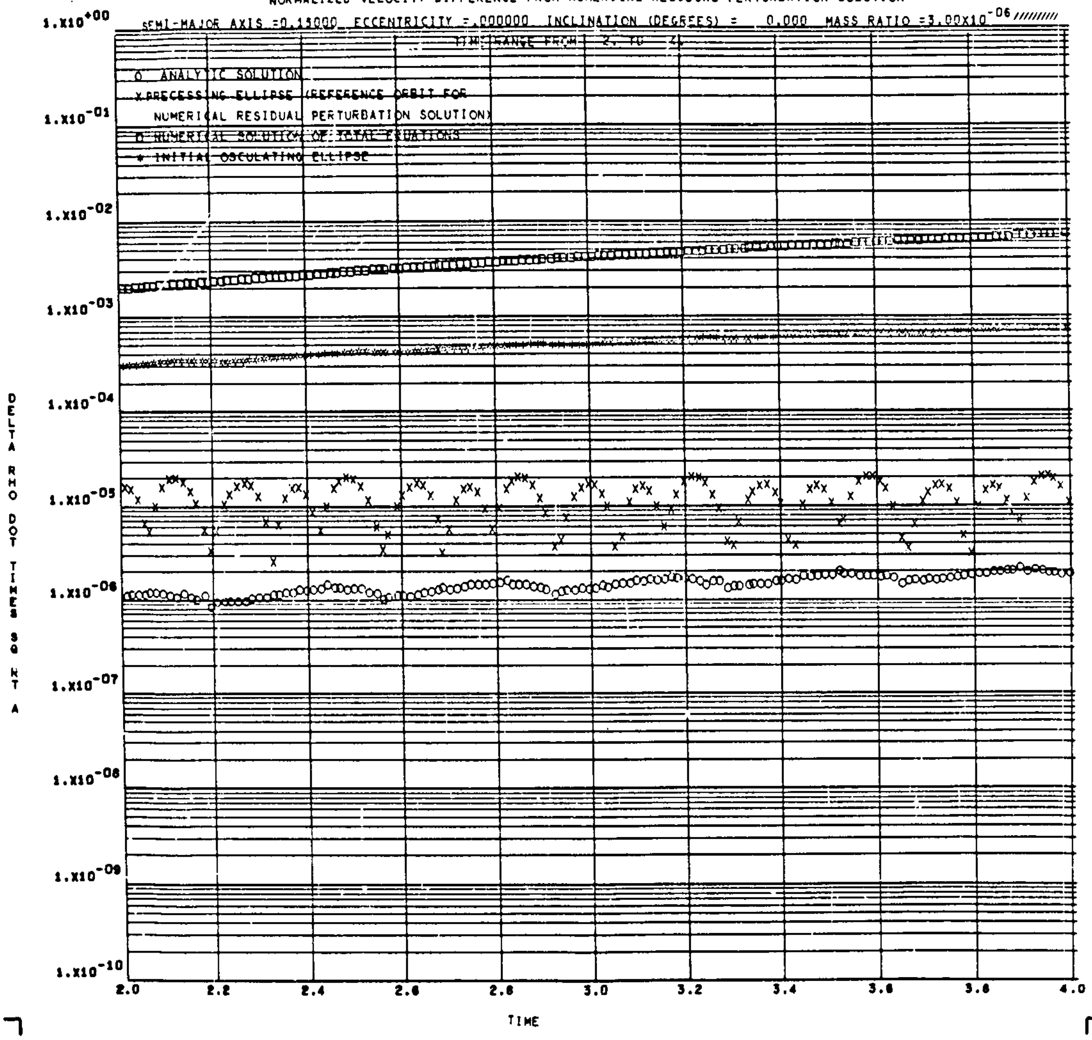

SERIAL NO 201071 PAGE 147
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.00X10-06
TIME RANGE FROM 0. TO 2.
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO DOT TIMES SQ RT A
0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
1.6
1.8
2.0
TIME

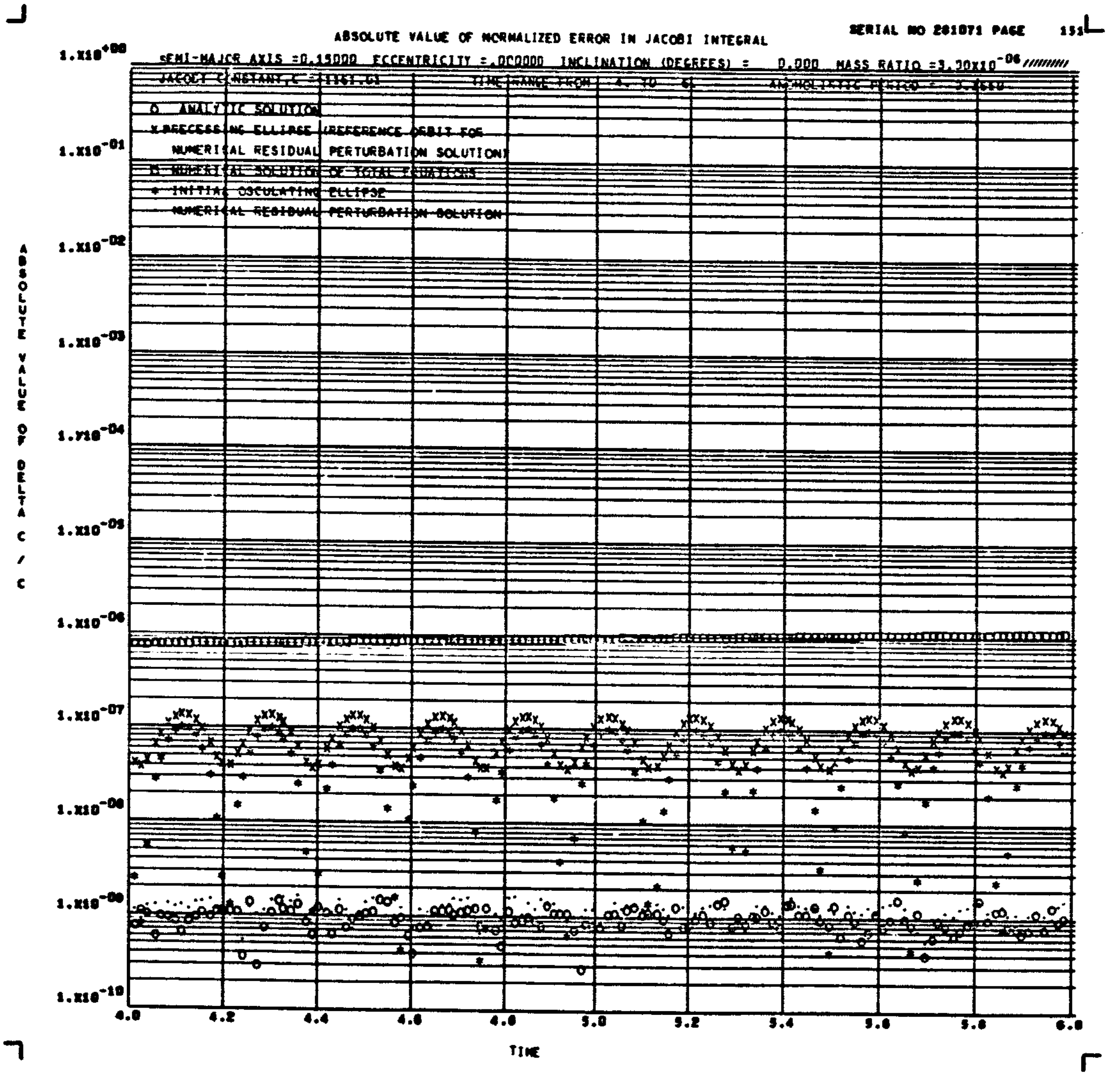
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL

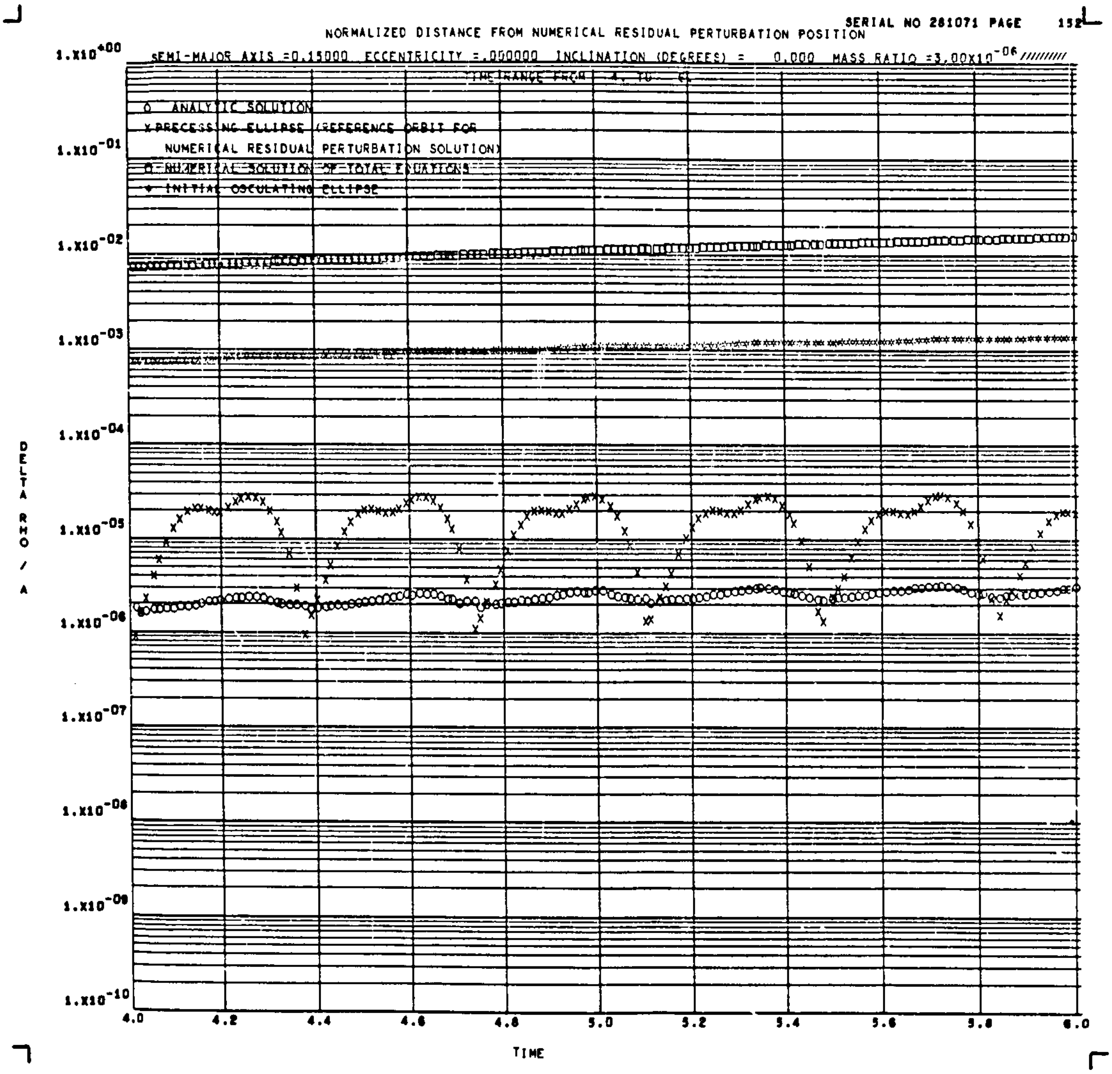



NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.000000 INCLINATION (DEGREES = 0.000 MASS RATIO =3.00x10^-06
TIME RANGE FROM 2. TO 4.
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
1.x10^+00
1.x10^-01
1.x10^-02
1.x10^-03
1.x10^-04
1.x10^-05
1.x10^-06
1.x10^-07
1.x10^-08
1.x10^-09
1.x10^-10
DELTA RHO / A
2.0
2.2
2.4
2.6
2.8
3.0
3.2
3.4
3.6
3.8
4.0
TIME

NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION



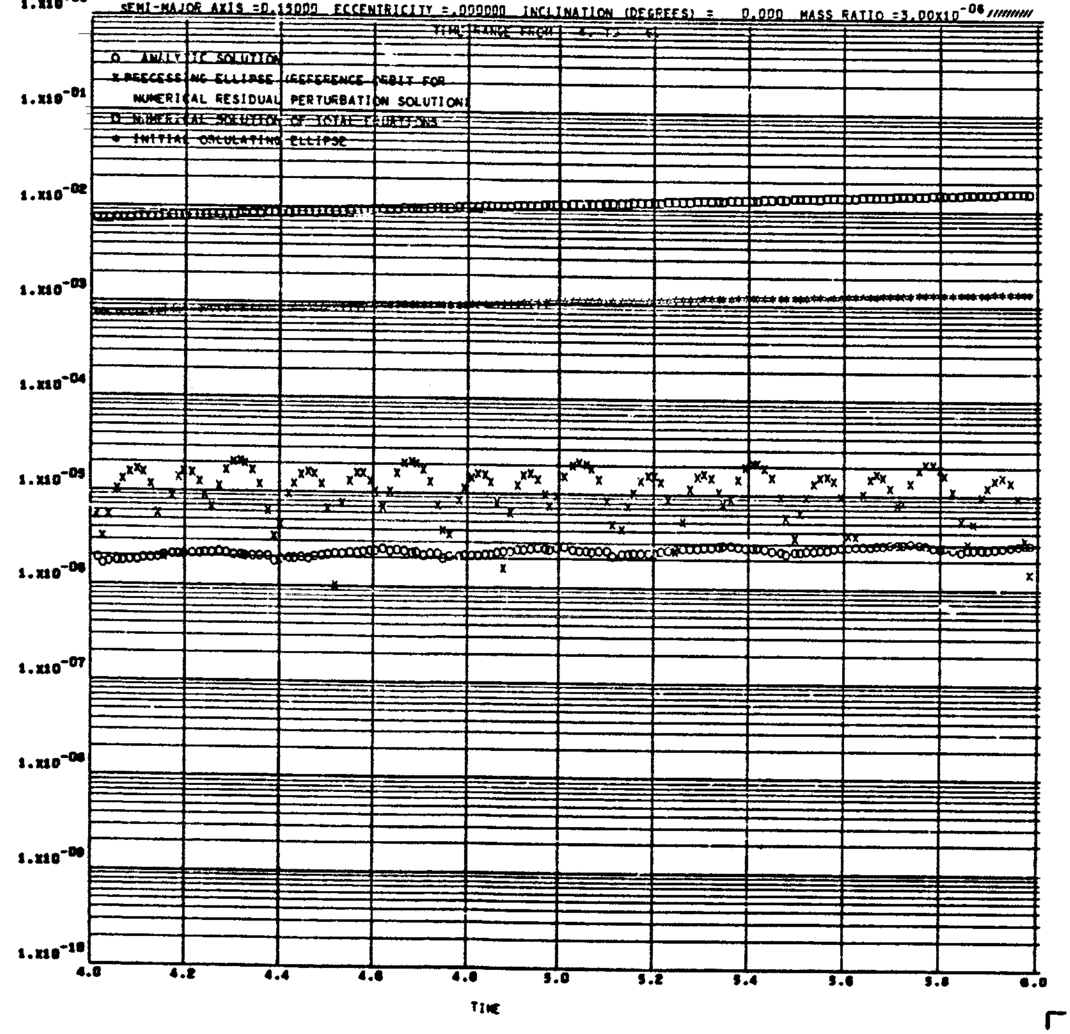

SERIAL NO 281071 PAGE 151
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.30x10^-06
JACOBI CONSTANT,C = 1161.61 TIME RANGE FROM 4. TO 6.
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
· NUMERICAL RESIDUAL PERTURBATION SOLUTION
ABSOLUTE VALUE OF DELTA C / C
TIME

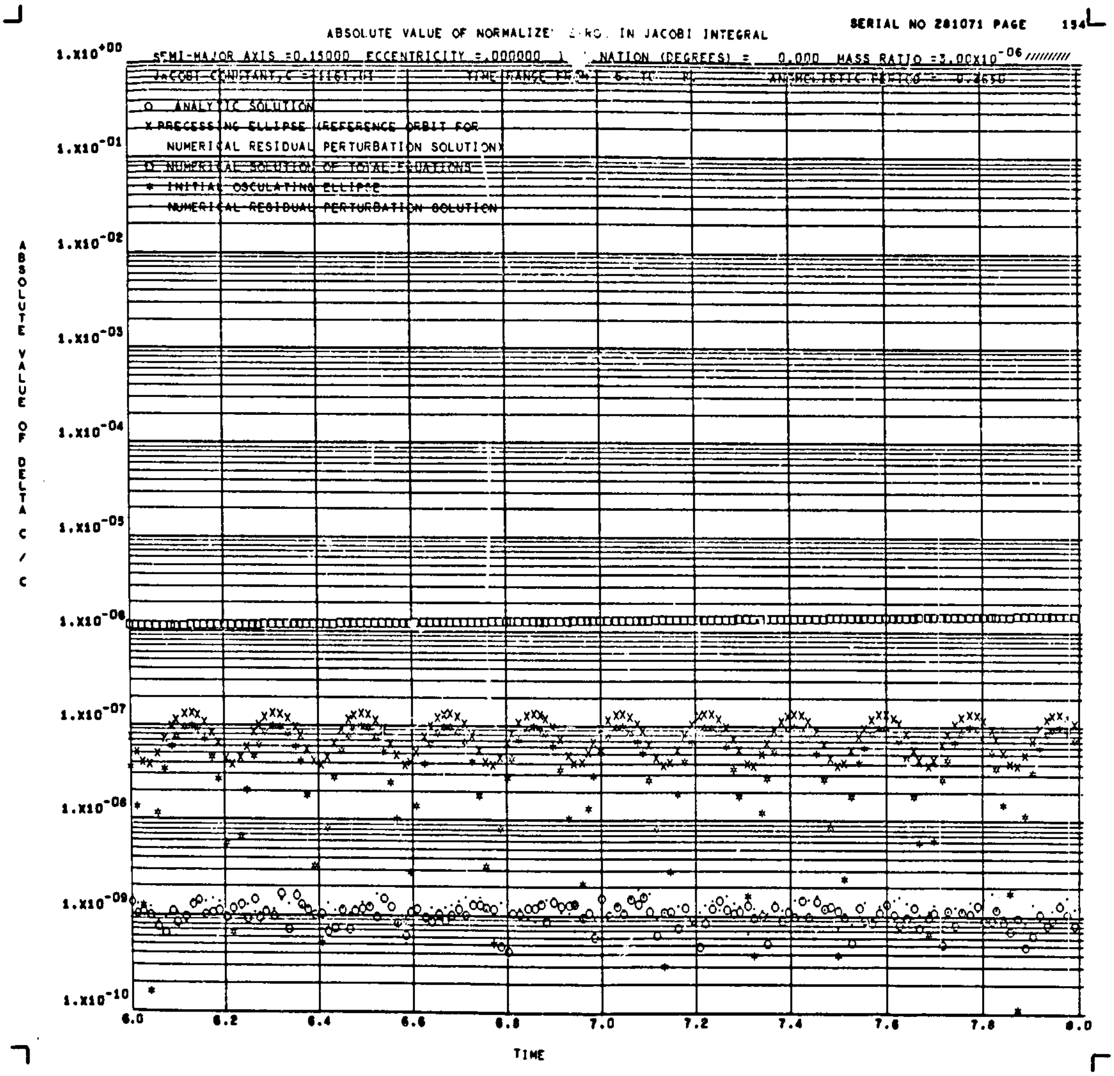

SERIAL NO 281071 PAGE 152
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.00X10-06
TIME RANGE FROM 4. TO 6.
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
O NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO / A
4.0
4.2
4.4
4.6
4.8
5.0
5.2
5.4
5.6
5.8
6.0
TIME

NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION

SERIAL NO 281071 PAGE 153

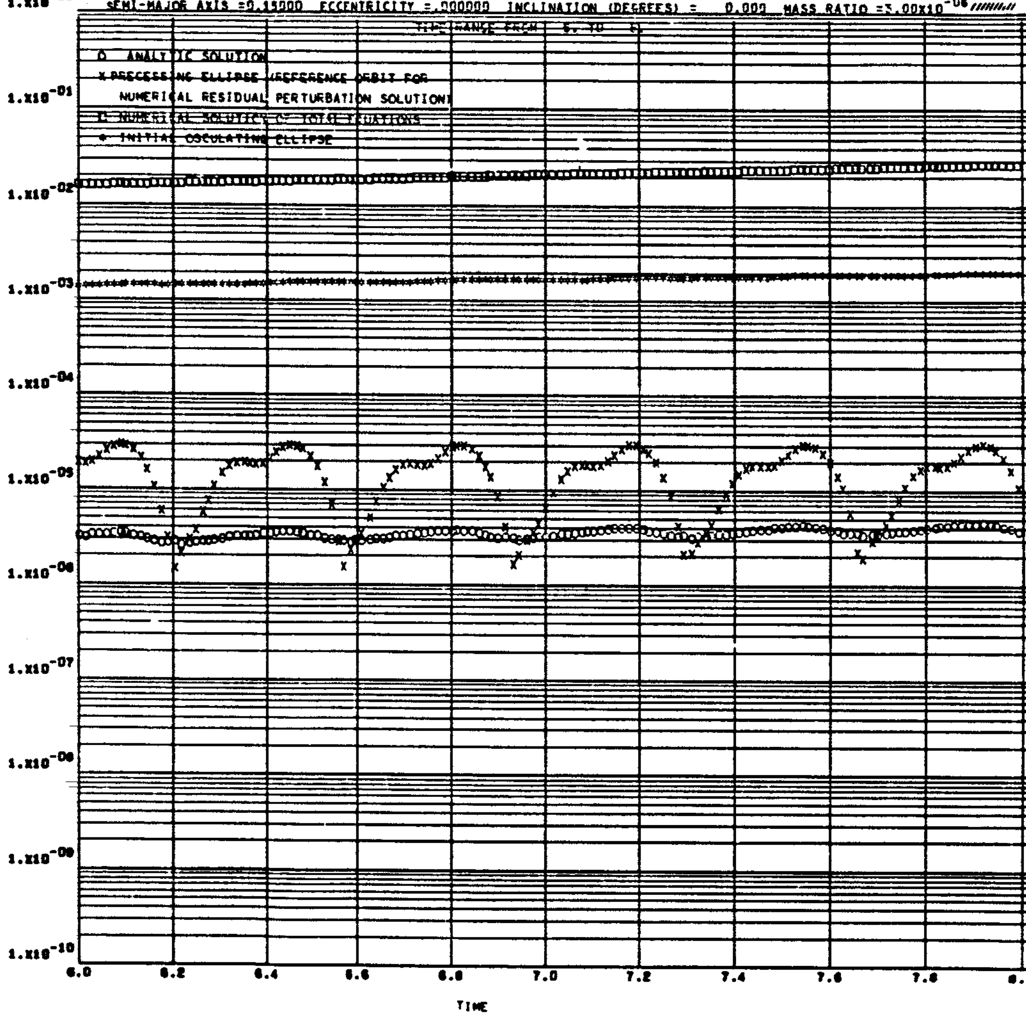
SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.00x10^{-06}



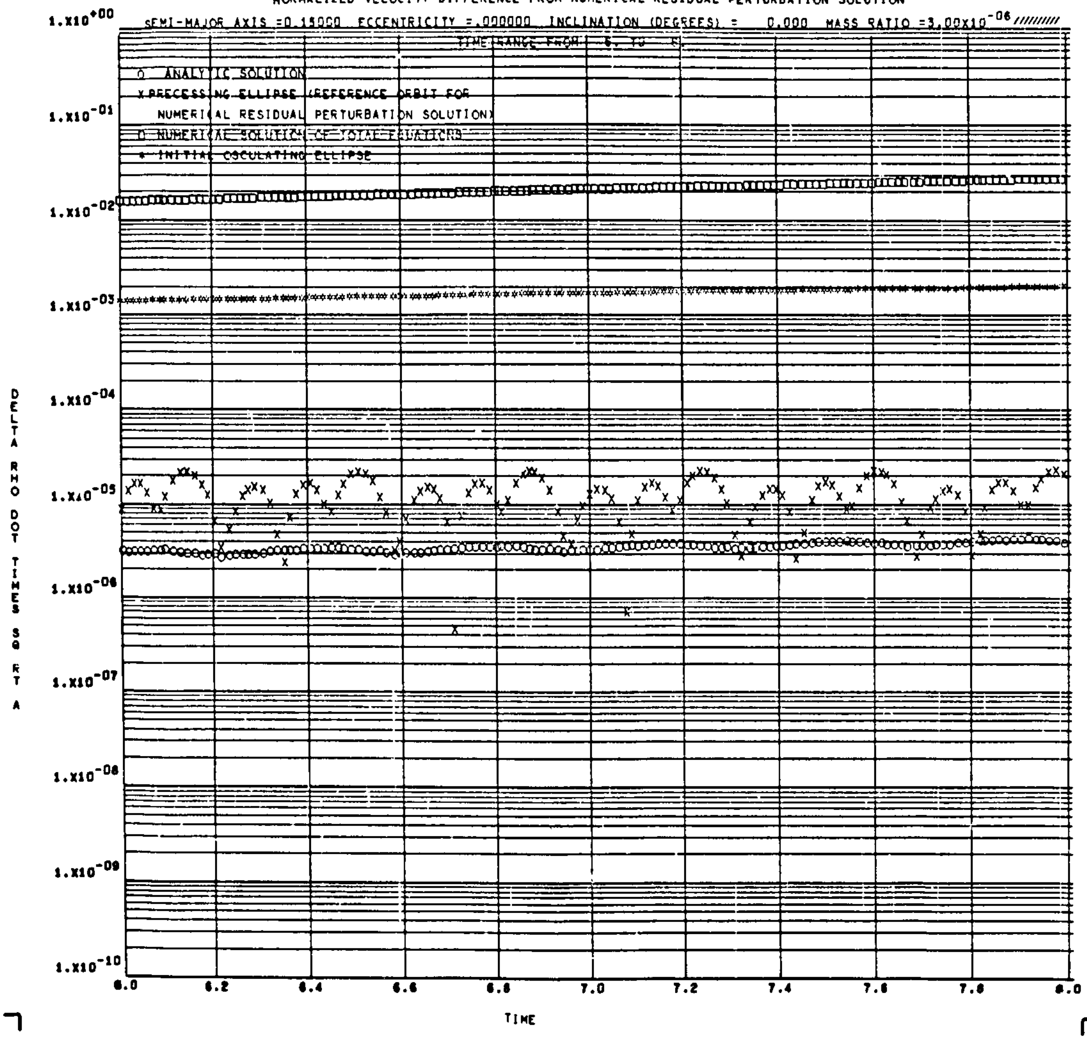

SERIAL NO 281071 PAGE 154
ABSOLUTE VALUE OF NORMALIZE IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.000000 NATION (DEGREES) = 0.000 MASS RATIO =3.00X10-06
JACOBI CONSTANT,C = TIME RANGE 6. TO 8.
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
· NUMERICAL RESIDUAL PERTURBATION SOLUTION
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
ABSOLUTE VALUE OF DELTA C / C
6.0
6.2
6.4
6.6
6.8
7.0
7.2
7.4
7.6
7.8
8.0
TIME

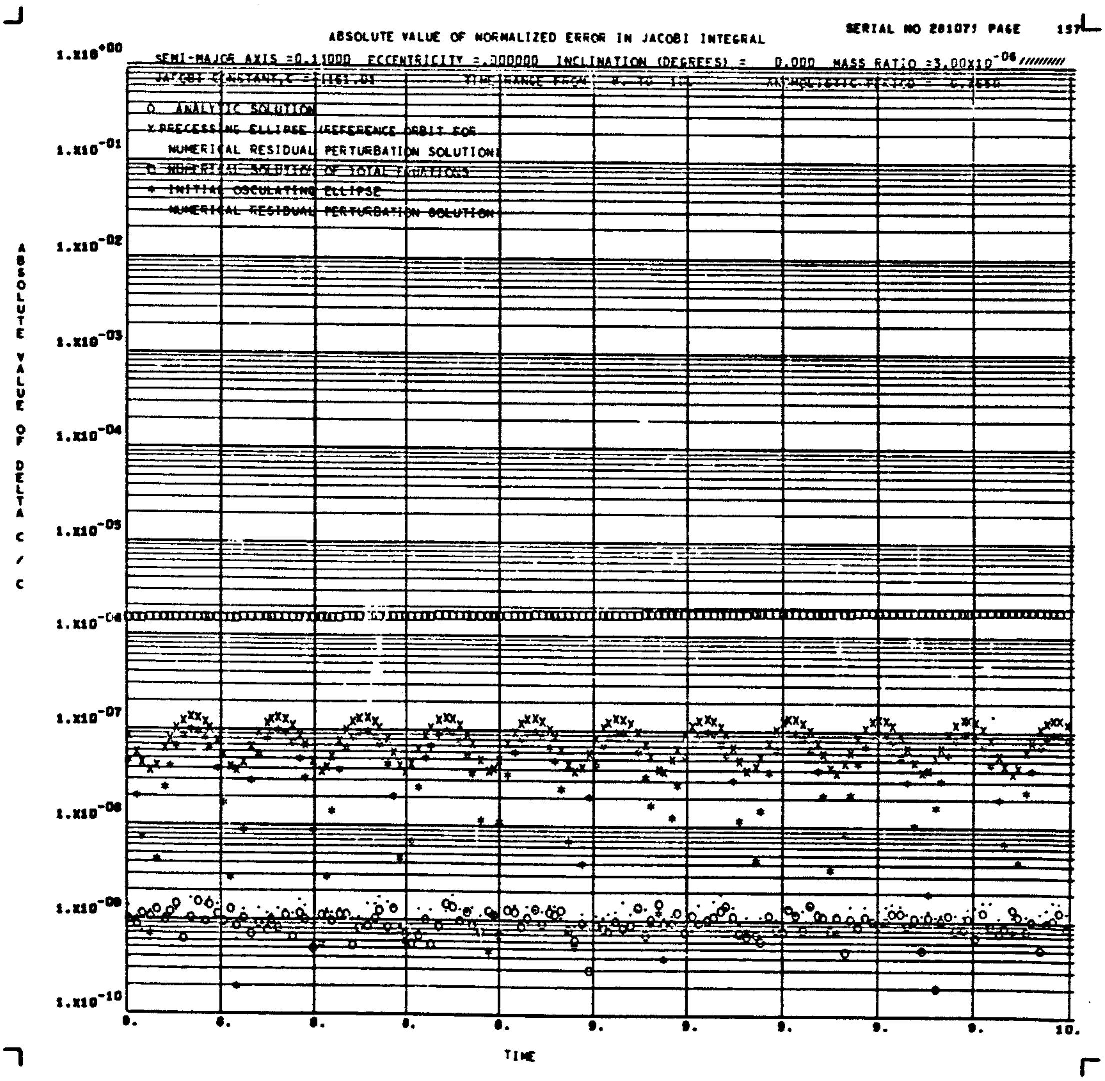
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION

SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.00x10^{-06}

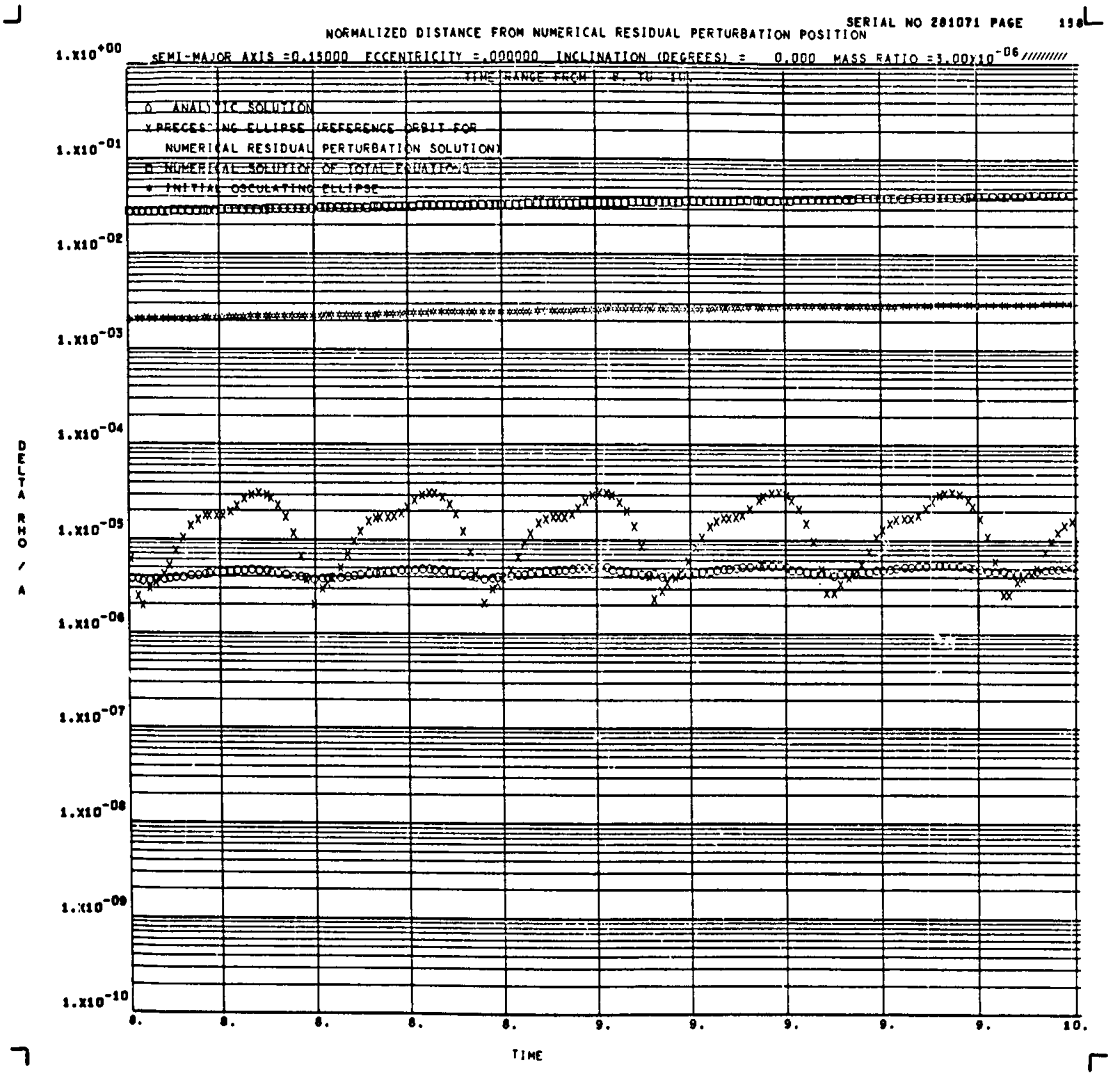


NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION

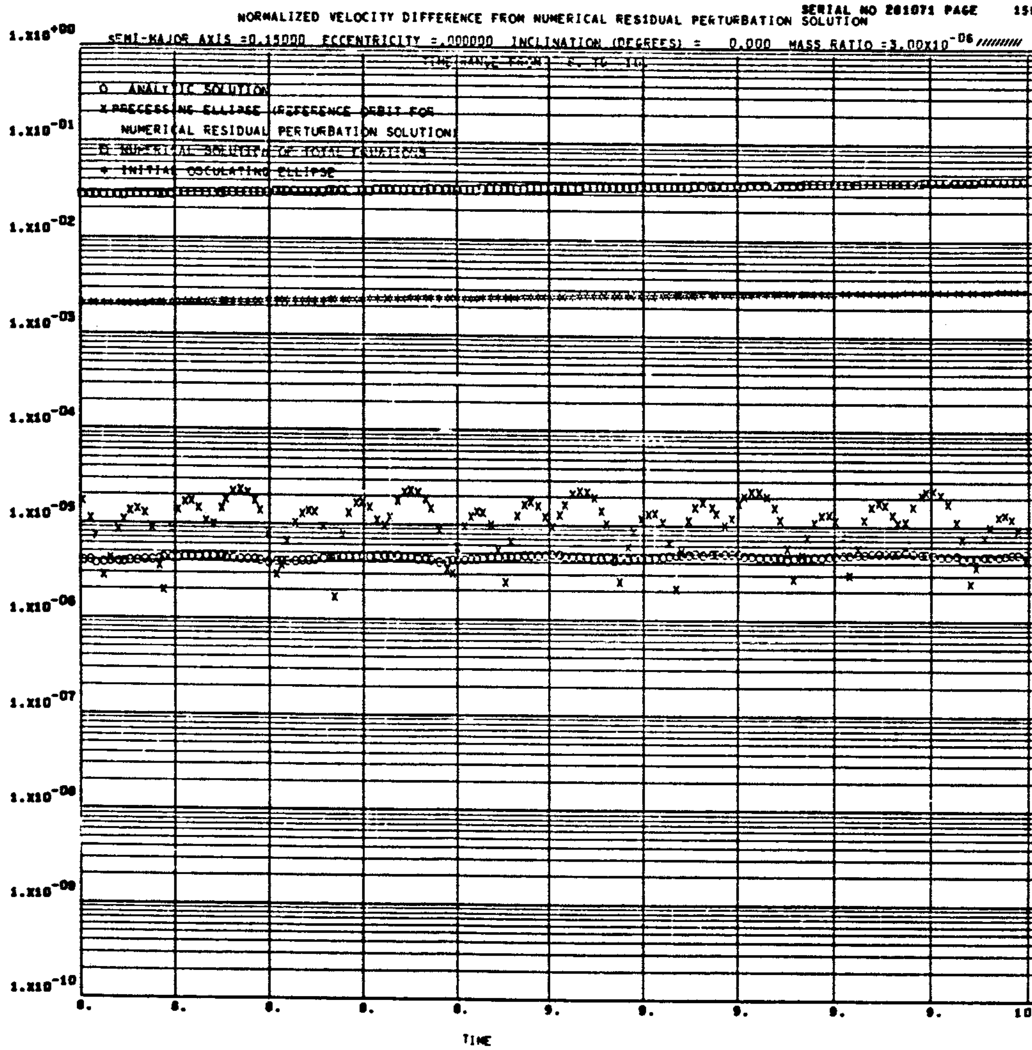


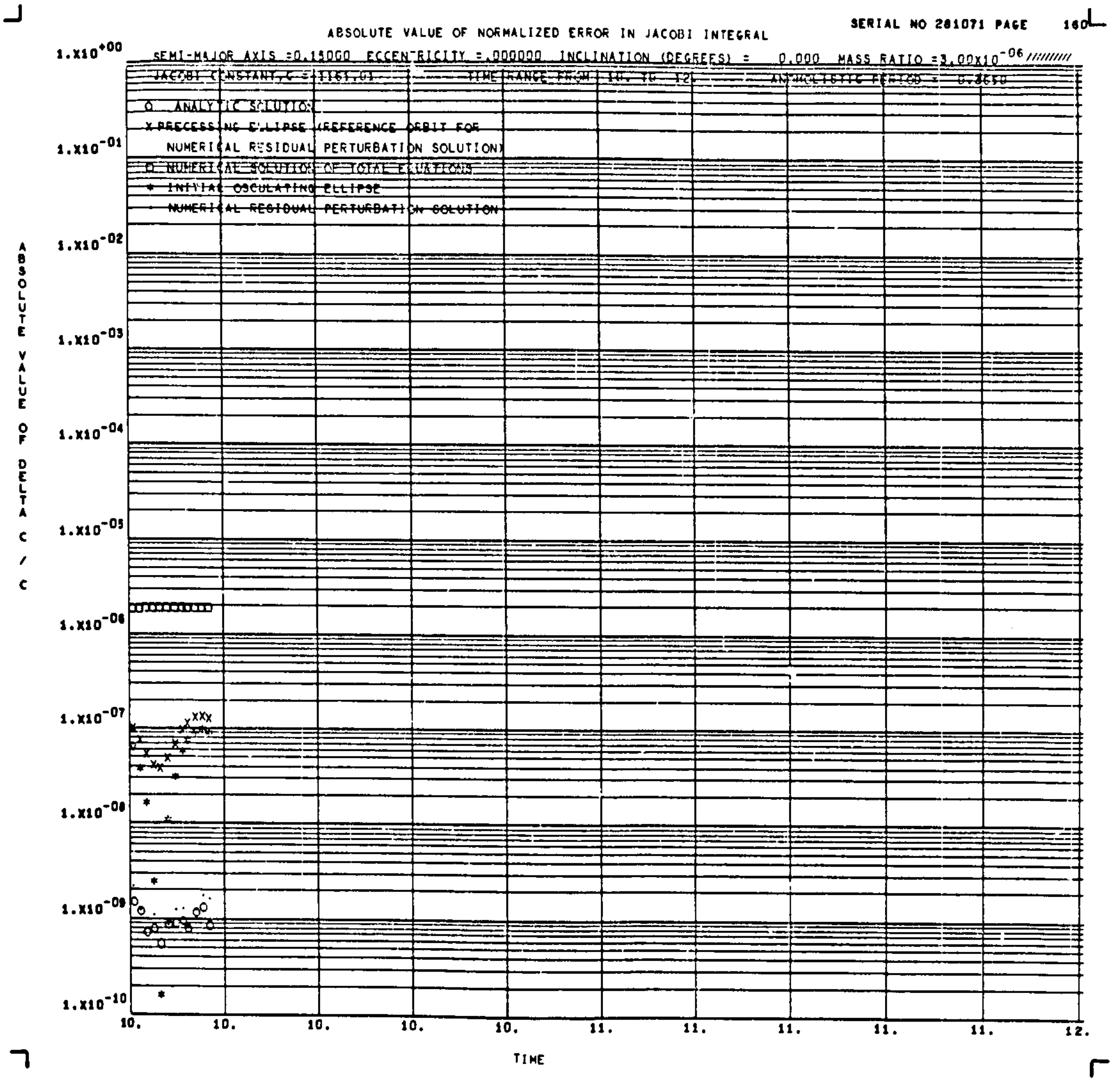


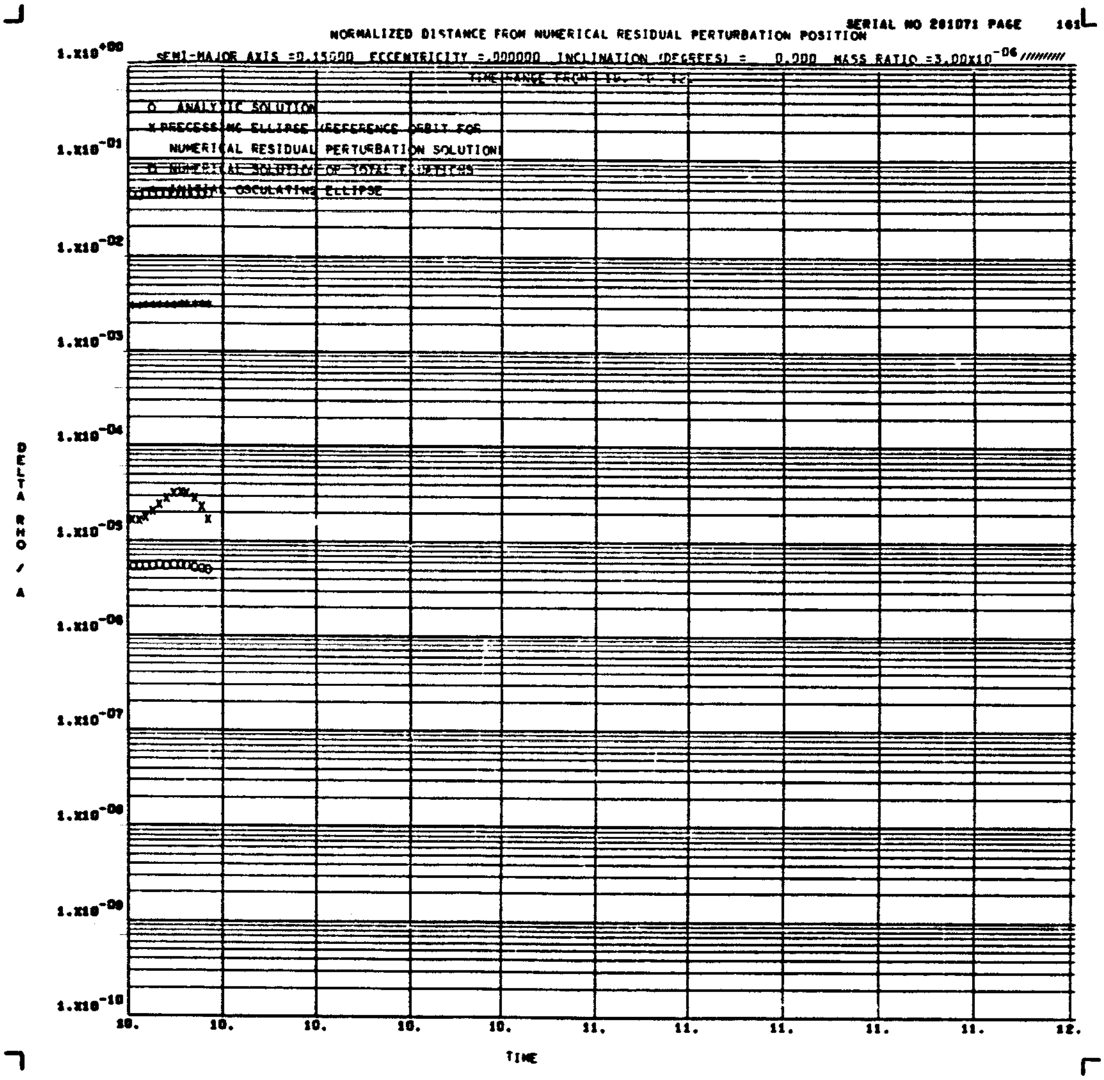
SERIAL NO 281071 PAGE 157
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =0.11000 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.00x10-06
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
+ INITIAL OSCULATING ELLIPSE
NUMERICAL RESIDUAL PERTURBATION SOLUTION
1.x10+00
1.x10-01
1.x10-02
1.x10-03
1.x10-04
1.x10-05
1.x10-06
1.x10-07
1.x10-08
1.x10-09
1.x10-10
ABSOLUTE VALUE OF DELTA C / C
8.
9.
10.
TIME


SERIAL NO 281071 PAGE 198
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.00X10-06
TIME RANGE FROM 8. TO 10.
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO / A
8.
8.
8.
8.
8.
9.
9.
9.
9.
9.
10.
TIME

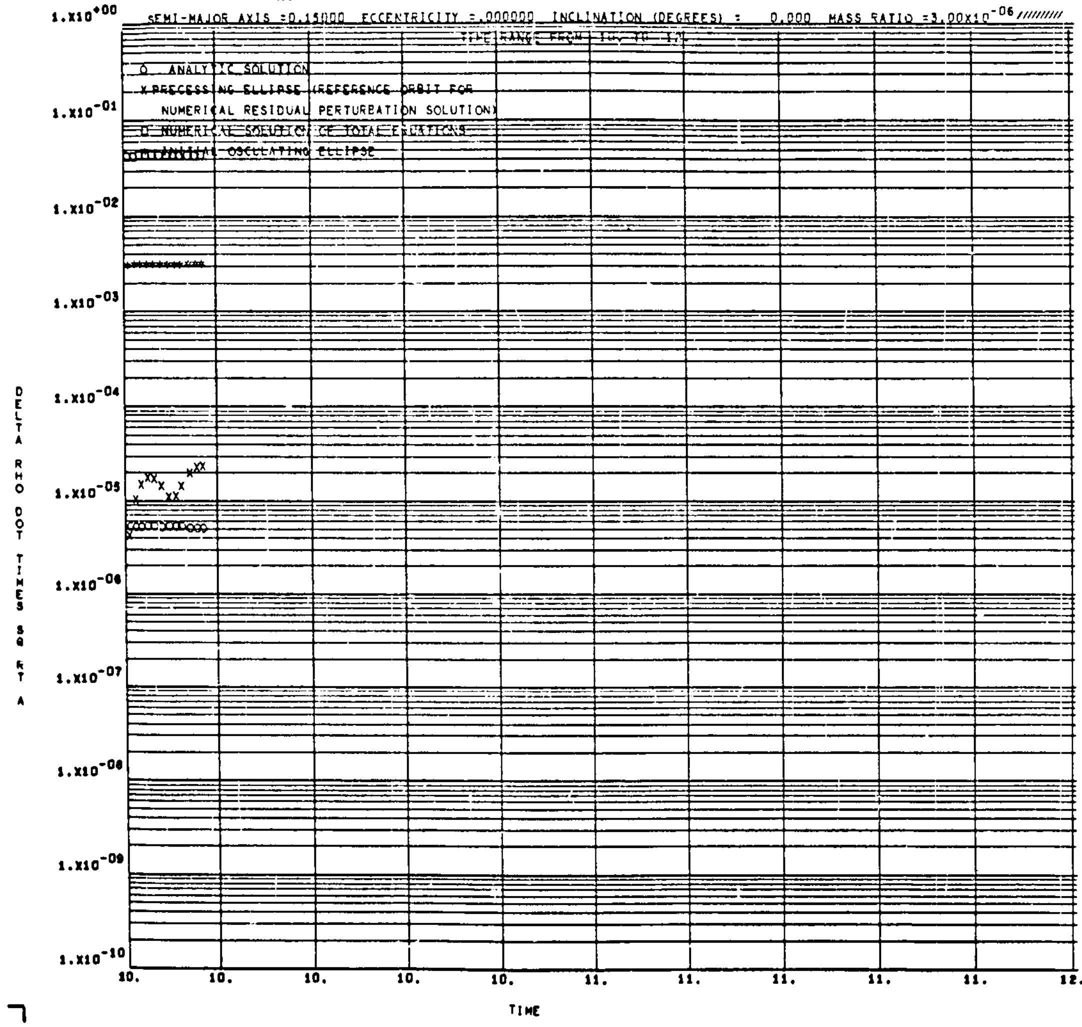
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION

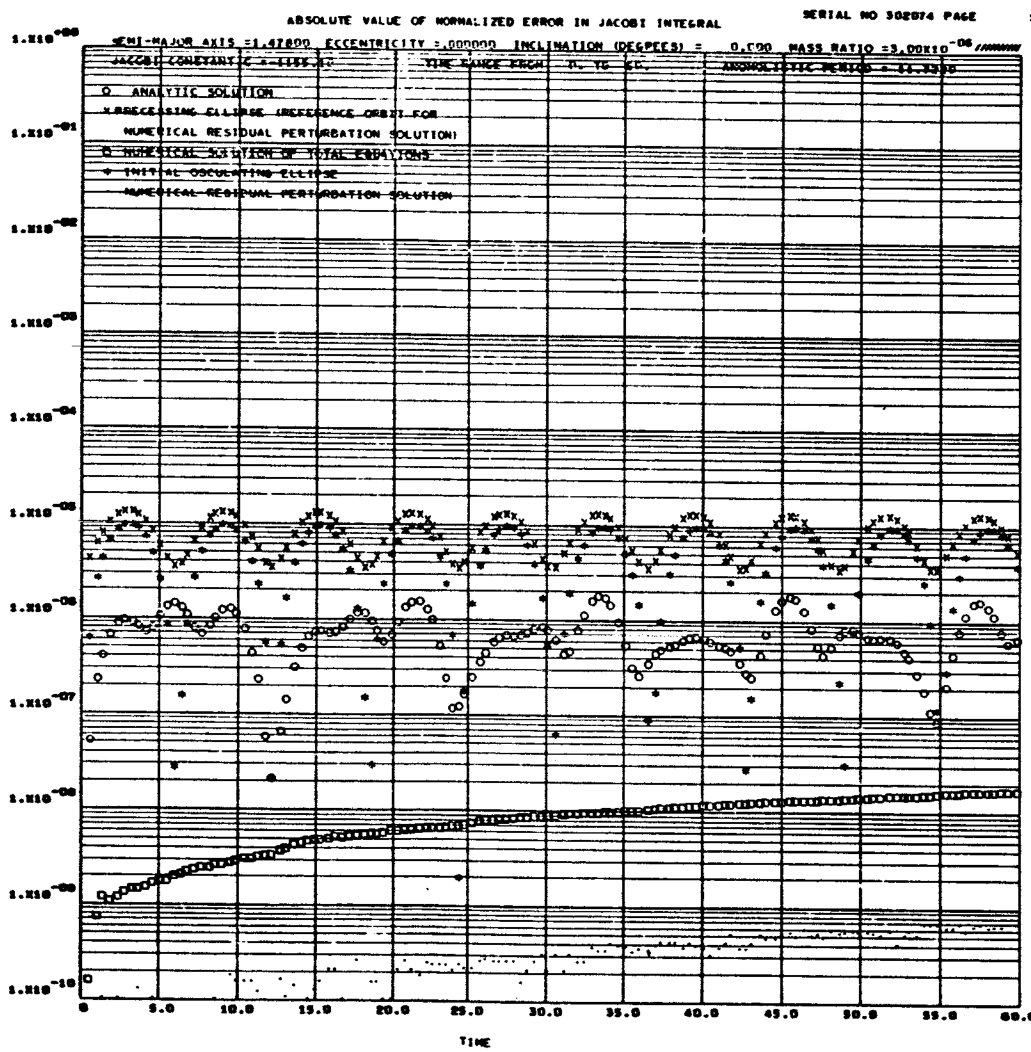


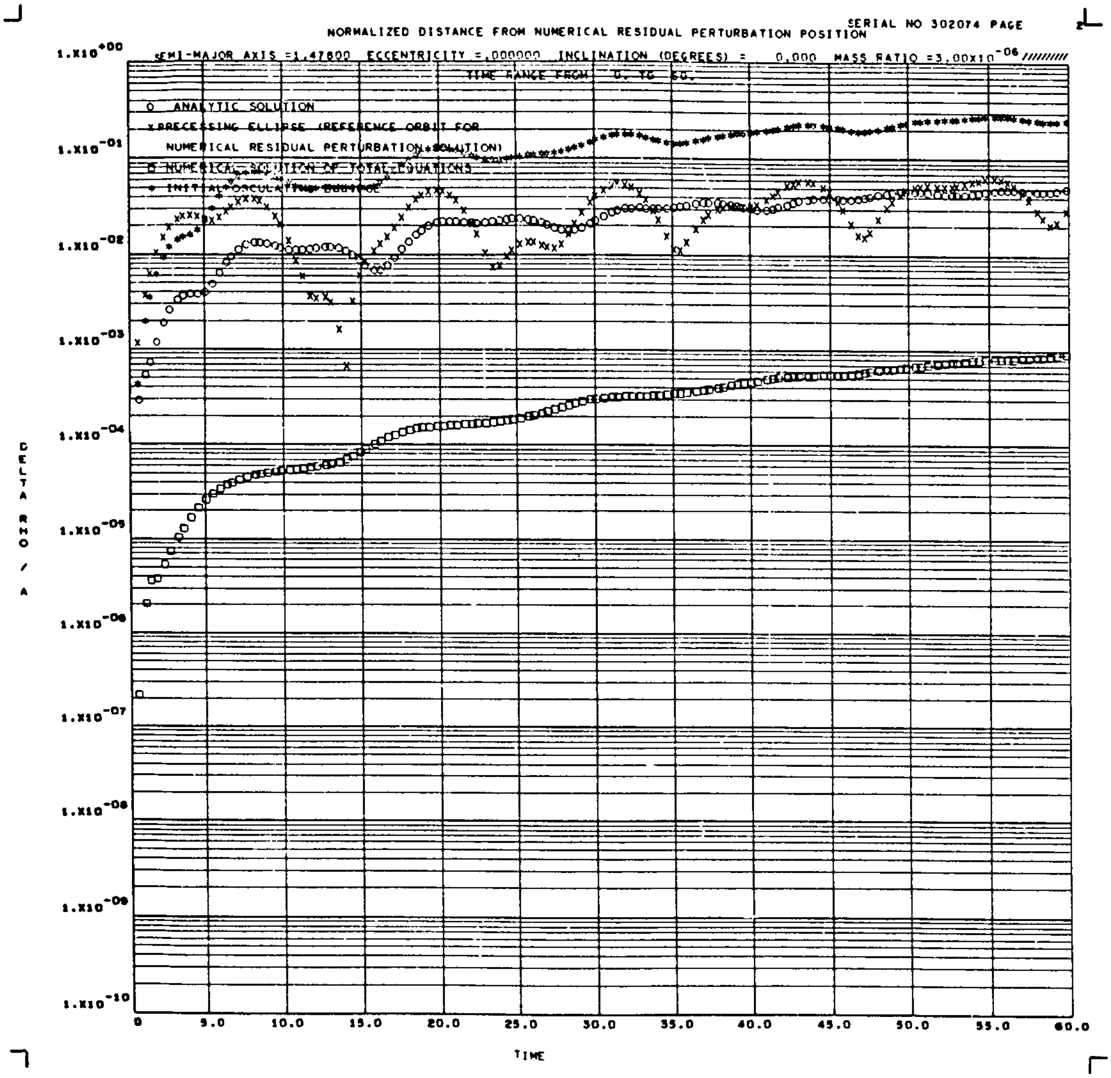

SERIAL NO 281071 PAGE 160
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.00x10^-06
JACOBI CONSTANT,C = 1161.01 TIME RANGE FROM 10. TO 12.
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
· NUMERICAL RESIDUAL PERTURBATION SOLUTION
ABSOLUTE VALUE OF DELTA C / C
1.x10^+00
1.x10^-01
1.x10^-02
1.x10^-03
1.x10^-04
1.x10^-05
1.x10^-06
1.x10^-07
1.x10^-08
1.x10^-09
1.x10^-10
10. 10. 10. 10. 10. 11. 11. 11. 11. 11. 12.
TIME

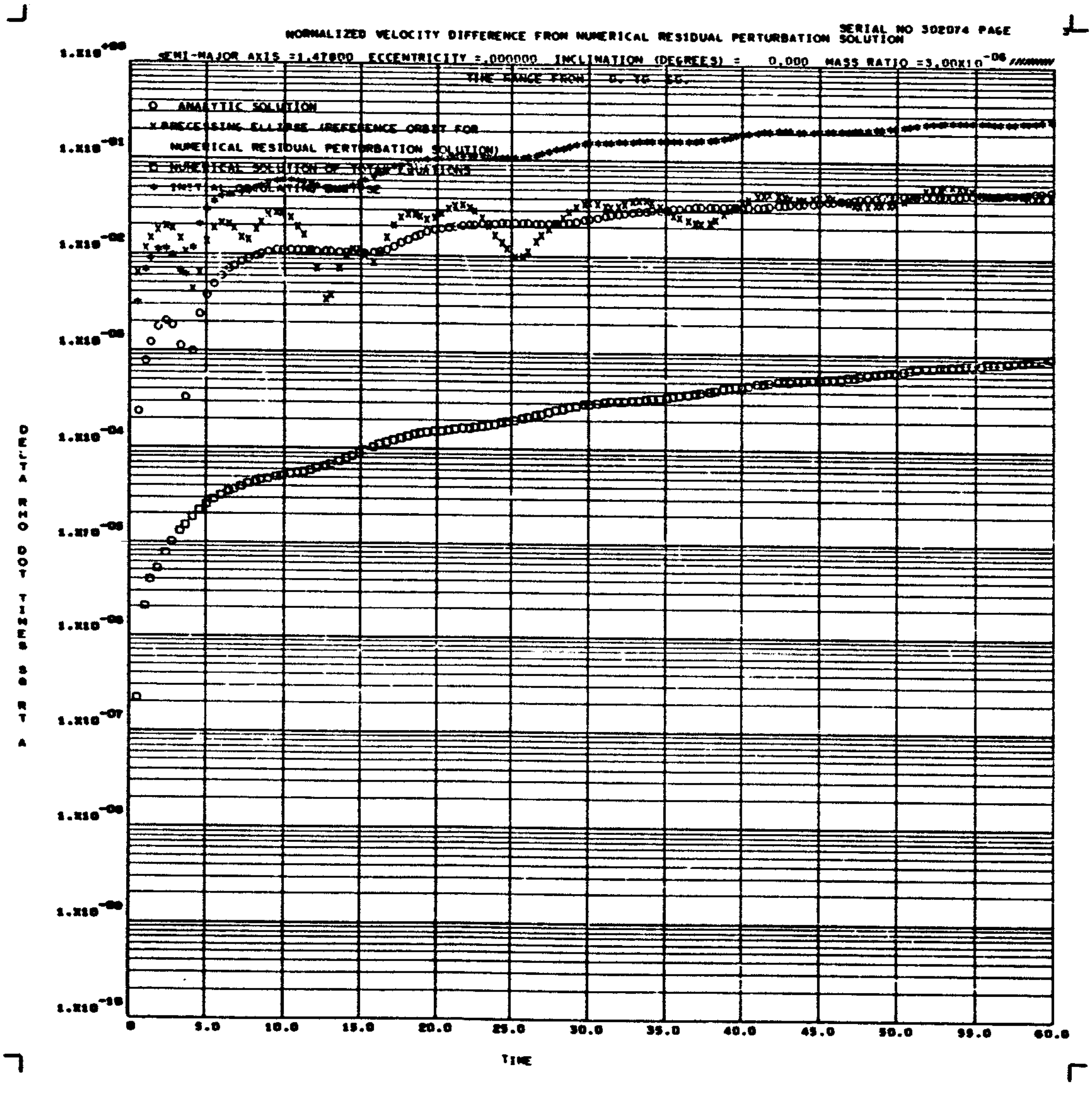

NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =0.15000 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.00x10-06
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
NUMERICAL OSCULATING ELLIPSE
DELTA RHO / A
TIME
1.x10+00
1.x10-01
1.x10-02
1.x10-03
1.x10-04
1.x10-05
1.x10-06
1.x10-07
1.x10-08
1.x10-09
1.x10-10
10.
11.
12.

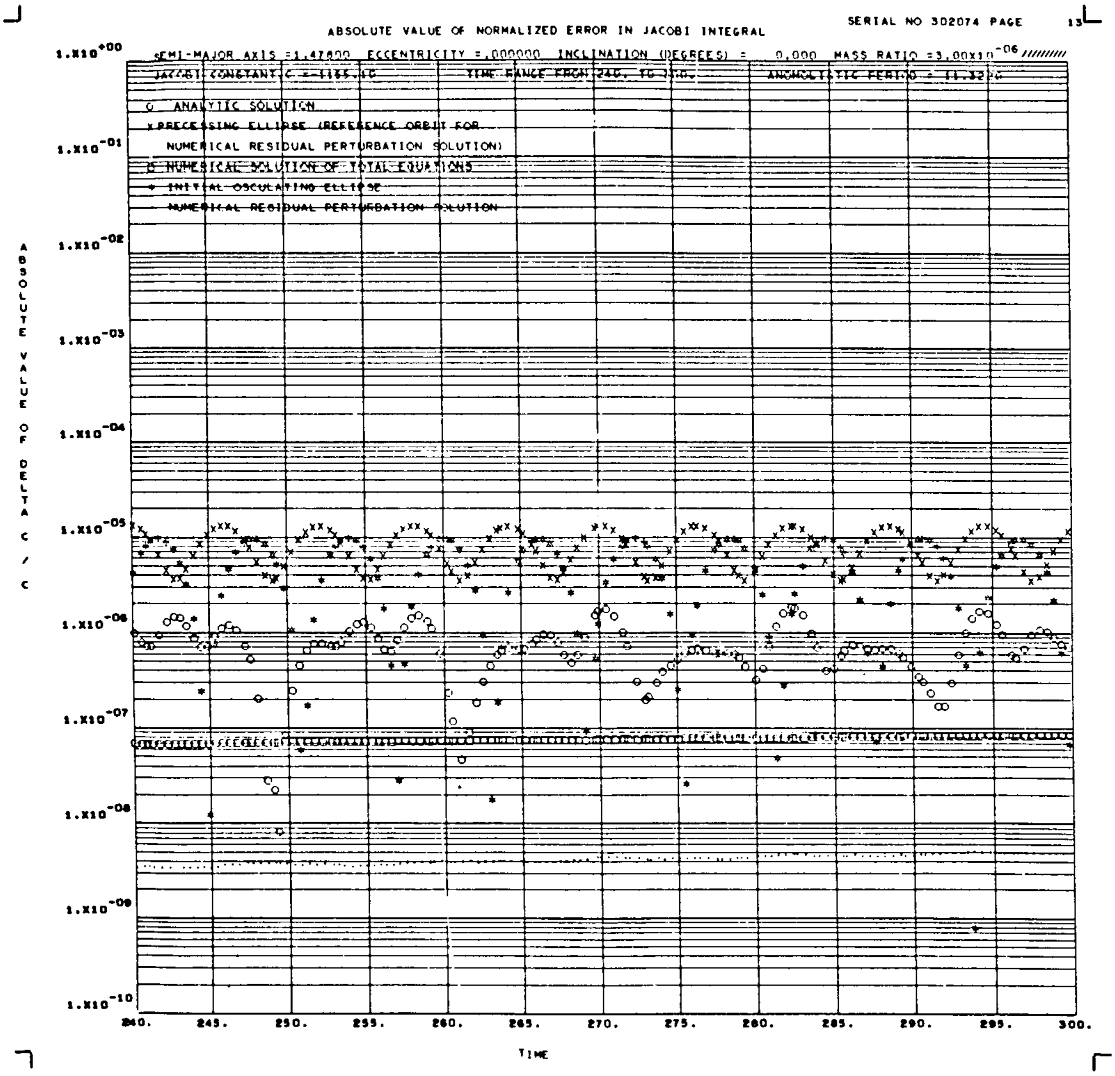
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION



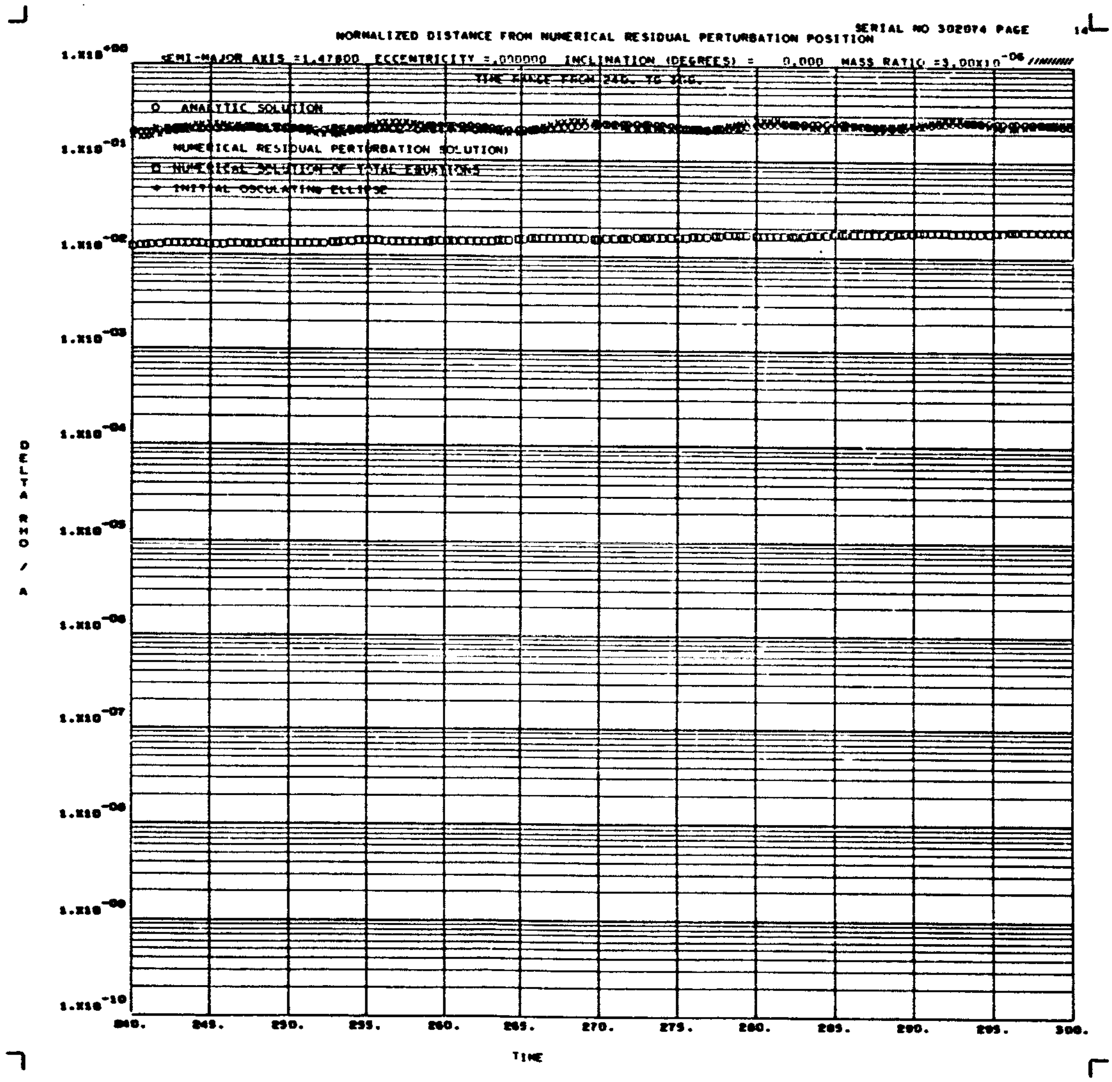

ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SERIAL NO 302074 PAGE
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.00X10-06
O ANALYTIC SOLUTION
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
ABSOLUTE VALUE OF DELTA C / C
TIME
0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
55.0
60.0


SERIAL NO 302074 PAGE
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.00x10-06
TIME RANGE FROM 0. TO 60.
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
DELTA RHO / A
TIME

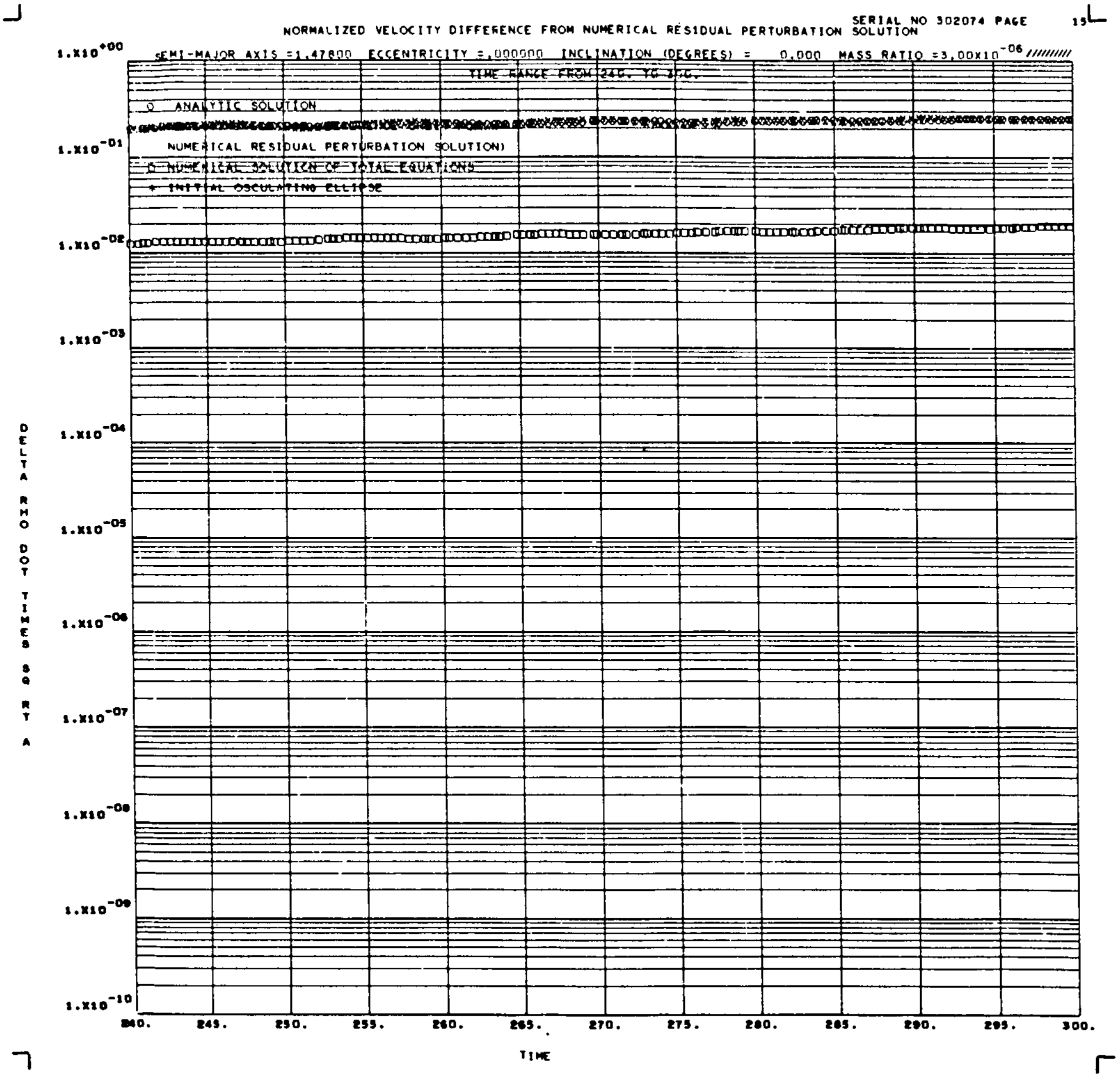

NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SERIAL NO 302074 PAGE
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.00X10-06
O ANALYTIC SOLUTION
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
0
5.0
10.0
15.0
20.0
25.0
30.0
35.0
40.0
45.0
50.0
55.0
60.0
TIME
DELTA RHO DOT TIMES SQ RT A

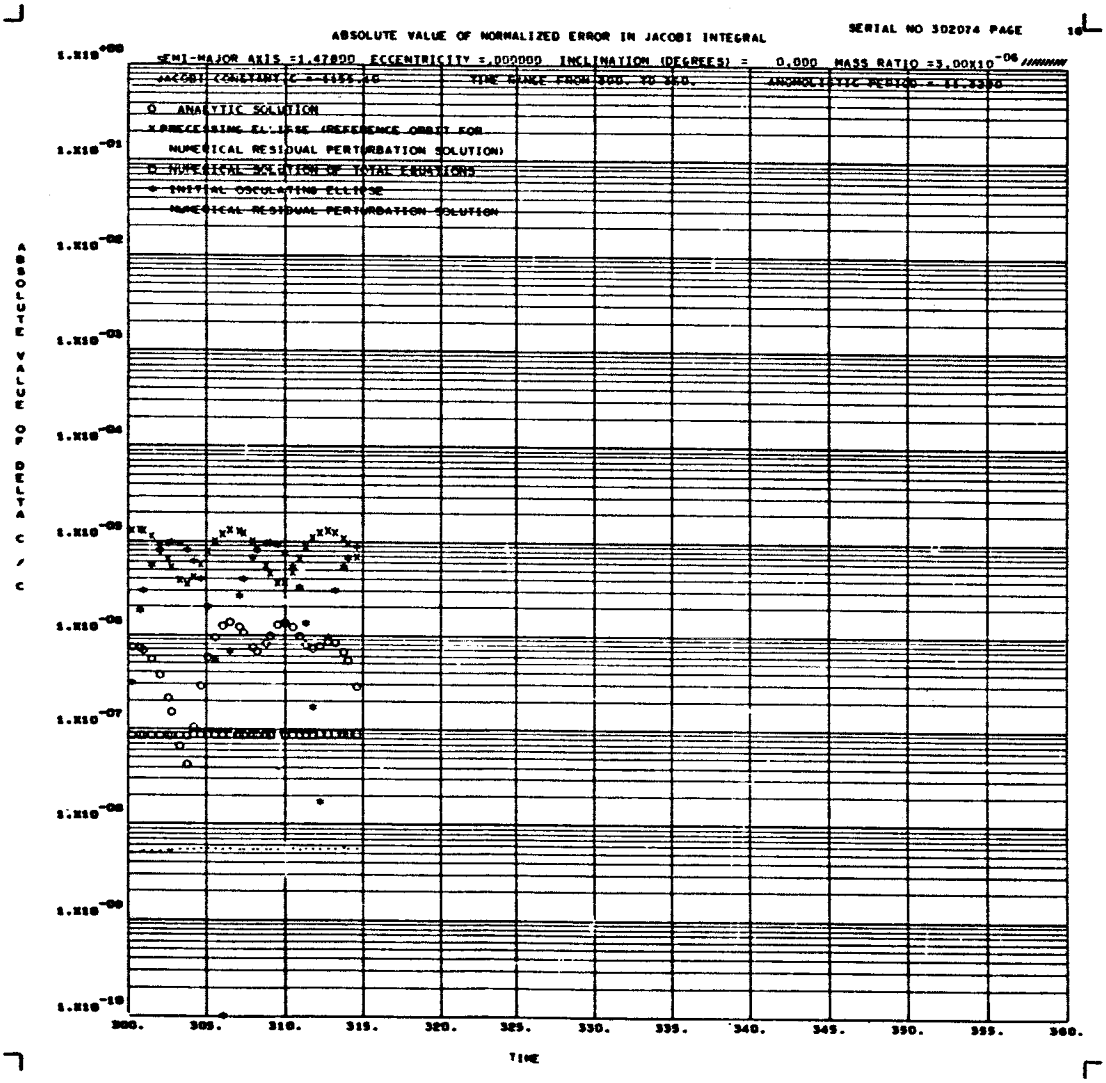


SERIAL NO 302074 PAGE 13
ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.00x10-06
O ANALYTIC SOLUTION
x PRECESSING ELLIPSE (REFERENCE ORBIT FOR NUMERICAL RESIDUAL PERTURBATION SOLUTION)
NUMERICAL SOLUTION OF TOTAL EQUATIONS
* INITIAL OSCULATING ELLIPSE
NUMERICAL RESIDUAL PERTURBATION SOLUTION
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
ABSOLUTE VALUE OF DELTA C / C
240. 245. 250. 255. 260. 265. 270. 275. 280. 285. 290. 295. 300.
TIME

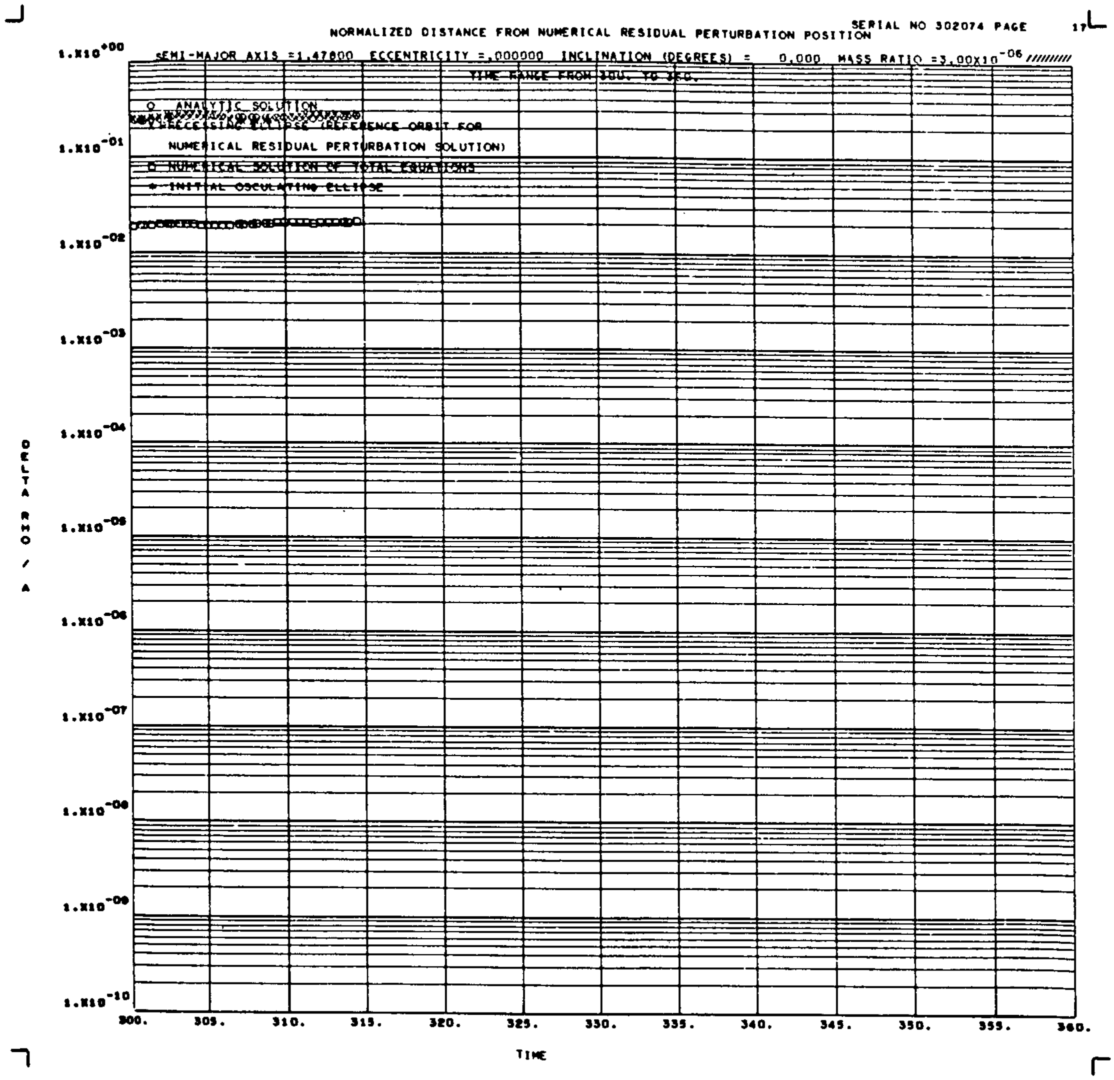


SERIAL NO 302074 PAGE 14
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.00x10^-06
O ANALYTIC SOLUTION
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
DELTA RHO / A
TIME


SERIAL NO 302074 PAGE 15
NORMALIZED VELOCITY DIFFERENCE FROM NUMERICAL RESIDUAL PERTURBATION SOLUTION
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.00x10^-06
O ANALYTIC SOLUTION
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
DELTA RHO DOT TIMES SQ RT A
TIME



ABSOLUTE VALUE OF NORMALIZED ERROR IN JACOBI INTEGRAL
SERIAL NO 302074 PAGE
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.00X10-06
O ANALYTIC SOLUTION
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
ABSOLUTE VALUE OF DELTA C / C
TIME
300. 305. 310. 315. 320. 325. 330. 335. 340. 345. 350. 355. 360.


SERIAL NO 302074 PAGE 17
NORMALIZED DISTANCE FROM NUMERICAL RESIDUAL PERTURBATION POSITION
SEMI-MAJOR AXIS =1.47800 ECCENTRICITY =.000000 INCLINATION (DEGREES) = 0.000 MASS RATIO =3.00X10-06
TIME RANGE FROM 300. TO 360.
O ANALYTIC SOLUTION
X PRECESSING ELLIPSE (REFERENCE ORBIT FOR
NUMERICAL RESIDUAL PERTURBATION SOLUTION)
□ NUMERICAL SOLUTION OF TOTAL EQUATIONS
+ INITIAL OSCULATING ELLIPSE
1.X10+00
1.X10-01
1.X10-02
1.X10-03
1.X10-04
1.X10-05
1.X10-06
1.X10-07
1.X10-08
1.X10-09
1.X10-10
DELTA RHO / A
300.
305.
310.
315.
320.
325.
330.
335.
340.
345.
350.
355.
360.
TIME

SERIAL NO 302074 PAGE