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REPORT WR 65-1

THE EFFECTS UPON SHOCK MEASUREMENTS OF LIMITED FREQUENCY RESPONSE INSTRUMENTATION

By

M. J. Crocker and L.C. Sutherland

This Report Reflects Work Under Contract No. NAS 8-11217

January, 1965

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Director of Research

Date January, 1965

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LIST OF SYMBOLS

C	capacitance
d	span of blast gauge
f	frequency
$H_1(\omega)$	frequency response function of high frequency pass system
$H_2(\omega)$	frequency response function of low frequency pass system
p_A	static pressure after a shock front
p_B	static pressure before a shock front
$P(t)$	non-dimensionalized pressure - time history of shock pulse
$P'(t)$	measured non-dimensionalized pressure - time history of shock pulse
$P(\omega)$	Fourier transform of non-dimensionalized pressure - time history $P(t)$
$P'(\omega)$	Fourier transform of measured non-dimensionalized pressure time history $P'(t)$
r	resistance
s	Laplace operator
t	time
t_o	duration of positive phase of pressure pulse
T_1	time constant for high frequency pass filter
T_2	time constant for low frequency pass filter
V	velocity of shock pulse front
x	space variable
X	position of shock front in x - direction

GREEK ALPHABET

α	T_1/t_o
β	T_2/t_o
γ	ratio of specific heats for a gas
ξ	"dummy" space variable in x-direction
θ	function of α and τ
θ'	first differential of θ with respect to T_1
θ''	second differential of θ with respect to T_2
τ	t/t_o non-dimensionalized time
ϕ	function of α
ϕ'	first differential of ϕ with respect to T_1
ϕ''	second differential of ϕ with respect to T_1
ω	angular frequency

SUBSCRIPTS

1	refers to high frequency pass system
2	refers to low frequency pass system
lower	refers to low frequency cut-off
upper	refers to high frequency cut-off

SUPERSCRIPT

$\wedge$	refers to maximum value of function

SUMMARY

In this report, the modifications to an ideal shock pulse provided by various measuring systems with different frequency response limitations are determined analytically. The measuring systems considered are one with low frequency attenuation, one with high frequency attenuation and a system with both high and low frequency attenuation.

The approach which is adopted is to obtain the Fourier transform of an ideal shock pulse and by the use of the System Frequency Response Function to obtain the measured time history of the shock pulse. The measured time histories for each system are computed for various system cut-off frequencies. The computations indicate the effects upon the measured shock peak and length of its positive phase due to limitations in frequency response of the measuring systems.

1.0 INTRODUCTION

A system designed to measure a shock pulse should ideally have a uniform frequency response from 0 to ∞. In practice a system is always limited at high frequencies and usually limited at low frequencies as well. A piezo-electric blast gauge, for example may have a low frequency cut off at about 1 to 5 c/s and a high frequency cut off beginning at about 10^4 to 10^5 c/s.

The purpose of this report is to analyze the effects upon the measured time history of a shock pulse when such a measuring system is used. In particular it is necessary to determine the errors in the observed shock peak and in the duration of the positive phase of the pulse, caused by a finite frequency bandwidth of the measuring instrumentation.

2.0 IDEAL SHOCK PULSE

Consider the variation of pressure with time at a point on the ground, some distance from the center of a surface blast. A suitable approximation for the pressure time history of an actual blast wave is provided by the non-dimensionalized pressure.

$$P(t) = (1 - t/t_o)\, e^{-\frac{t}{t_o}} \tag{1}$$

which is illustrated in Figure 1.

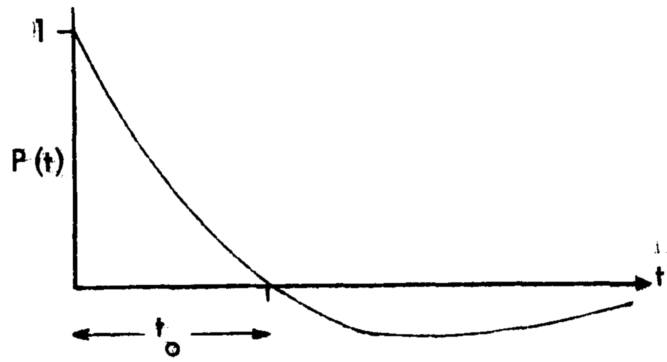


Figure 1: Ideal Blast Wave

The distortion of P(t) due to frequency limitations of the measuring system is best determined by considering the frequency spectrum of P(t).

The Fourier spectrum of such a wave is defined by:

$$P(\omega) = \int_{-\infty}^{\infty} P(t) \cdot e^{-j\omega t} \cdot dt$$

$$= \int_{0}^{\infty} (1 - t/t_o) \cdot e^{-\frac{t}{t_o} - j\omega t} \cdot dt$$

$$P(\omega) = t_o \int_{0}^{\infty} (1 - \tau) \cdot e^{-\tau(1 + j\omega t_o)} \cdot d\tau$$

where $\tau = t/t_o$

Integrating by parts:

$$P(\omega) = t_o \int_0^{\infty} e^{-\tau(1 + j\omega t_o)} . d\tau - \int_0^{\infty} \tau \, e^{-\tau(1 + j\omega t_o)} . d\tau$$

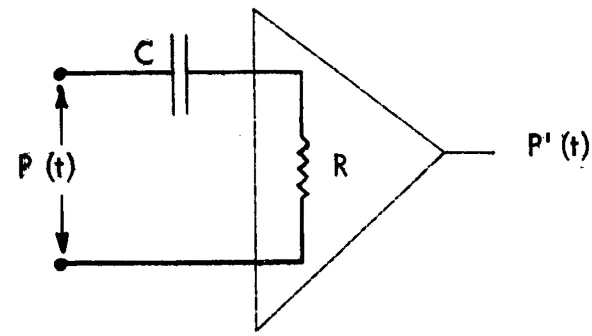
$$= \frac{-t_o}{1 + j\omega t_o} \left[e^{-\tau(1 + j\omega t_o)} \right]_0^{\infty} + t_o \left[\frac{\tau . e^{-\tau(1 + j\omega t_o)}}{1 + j\omega t_o} \right]_0^{\infty} + t_o \left[\frac{e^{-\tau(1 + j\omega t_o)}}{(1 + j\omega t_o)^2} \right]_0^{\infty}$$

$$= \frac{t_o}{1 + j\omega t_o} - \frac{t_o}{(1 + j\omega t_o)^2}$$

$$P(\omega) = \frac{j\omega t_o^{\ 2}}{(1 + j\omega t_o)^2} \tag{2}$$

3.0 MEASURED SHOCK PULSE - HIGH FREQUENCY PASS FILTER

Figure 2: Simple High Frequency Pass Filter.



Consider now the observed pressure-time history $P'(t)$ from a transducer having a simple R C high frequency pass filter. For such a system, the frequency response function, $H_1(\omega)$ is:

$$H_1(\omega) = \frac{P'(\omega)}{P(\omega)} = \frac{R}{R + \frac{1}{j\omega C}} = \frac{j\omega RC}{1 + j\omega RC}$$

$$H_1(\omega) = \frac{j\omega T_1}{1 + j\omega T_1} \tag{3}$$

where $T_1 = RC$ the time constant for the filter.

The Fourier Transform of the observed pressure-time history, $P'(\omega)$, may be determined from equations 2 and 3.

$$P'(\omega) = H_1(\omega) \cdot P(\omega)$$

$$= \frac{-\omega^2 t_o^2 T_1}{(1 + j\omega t_o)^2 (1 + j\omega T_1)}$$

$$P'(\omega) = \frac{-\omega^2}{(j\omega + \frac{1}{t_o})^2 \; (j\omega + \frac{1}{T_1})}$$

It can be shown that the Laplace Transform of a function approaches the Fourier transform of that function, if the function is zero for all negative time(See Ref. 1). Substituting $s = j\omega$, the Laplace Transform is:

$$P'(s) = \frac{s^2}{(s + \frac{1}{t_o})^2 \; (s + \frac{1}{T_1})}$$

The inverse transform $P'(t)$ may now be obtained from Reference 2.

$$P'(t) = \left[\frac{\frac{1}{t_o}\left(\frac{1}{t_o} - \frac{2}{T_1}\right)}{\left(\frac{1}{t_o} - \frac{1}{T_1}\right)^2} - \frac{\left(\frac{1}{t_o}\right)^2 t}{\left(\frac{1}{t_o} - \frac{1}{T_1}\right)}\right] e^{-\frac{t}{t_o}} + \frac{\left(\frac{1}{T_1}\right)^2}{\left(\frac{1}{t_o} - \frac{1}{T_1}\right)^2} e^{-\frac{t}{T_1}}$$

$$P'(t) = \left[\frac{\alpha\;(\alpha - 2)}{(\alpha - 1)^2} - \frac{\alpha \; t/t_o}{(\alpha - 1)}\right] e^{-\frac{t}{t_o}} + \frac{e^{-\frac{t}{t_o \alpha}}}{(\alpha - 1)^2} \tag{4}$$

where $\alpha = T_1/t_o$

The observed or measured pressure $P'(t)$ is plotted in Figure 3, for several values of $\alpha = T_1/t_o$.

It is noticed that when $\alpha = T_1/t_o \longrightarrow \infty$ (i.e. for a perfect measuring system with a low frequency cut-off at zero), equation 4 reduces to

$$P'(t) = \left(1 - t/t_o\right) e^{-\frac{t}{t_o}} \qquad \alpha = \frac{T_1}{t_o} \longrightarrow \infty$$

the ideal pressure defined by equation 1.

At first sight it appears that the measured pressure $P'(t)$, as defined in equation 4 may become infinite when $\alpha \longrightarrow 1$. Thus some further examination of this case is necessary.

Re-arranging equation 4

$$P'(t) = \frac{\left[\alpha\ (\alpha - 2) - \alpha\ (\alpha - 1)\ \tau\right] e^{-\tau} + e^{-\frac{\tau}{\alpha}}}{(\alpha - 1)^2} = \frac{\theta(\alpha)}{\phi(\alpha)}$$

where $\tau = t/t_o$

It is seen that when $\alpha \longrightarrow 1$, $P'(t) \rightarrow \frac{0}{0}$, that is the ratio of two small quantities which is indeterminate. The limiting value of $P'(t)$ when $\alpha = 1$, may thus be found by the use of L'Hospital's rule, which may be formulated by expansion of the numerator and denominator:

$$\lim_{\alpha \to \alpha_o} \frac{\theta(\alpha)}{\phi(\alpha)} = \lim_{\alpha \to \alpha_o} \frac{\theta(\alpha_o) + \theta'(\alpha_o) \cdot \delta\alpha/1! + \theta''(\alpha_o) \cdot \delta\alpha^2/2! + \dots}{\phi(\alpha_o) + \phi'(\alpha_o) \cdot \delta\alpha/1! + \phi''(\alpha_o) \cdot \delta\alpha^2/2! + \dots}$$

$$= \frac{\theta'(\alpha_o)}{\phi'(\alpha_o)} \quad \text{if determinate,} \qquad \text{where } \alpha = \alpha_o + \delta\alpha,\ \theta(\alpha_o) = \phi(\alpha_o) = 0, \text{ and } \delta\alpha \longrightarrow 0$$

$$= \frac{\theta''(\alpha_o)}{\phi''(\alpha_o)} \quad \text{if } \frac{\theta'(\alpha_o)}{\phi'(\alpha_o)} \text{ is indeterminate}$$

etc.

Now

$$\lim_{\alpha \to 1} \frac{\theta'(\alpha)}{\phi'(\alpha)} = \left\{ \frac{\left[2\alpha - 2 - (2\alpha - 1)e^{-\tau}\right]e + (\tau/\alpha^2)\, e^{-\frac{\tau}{\alpha}}}{2(\alpha - 1)} \right\}_{\alpha \to 1} = \frac{0}{0}, \text{ indeterminate.}$$

$$\lim_{\alpha \to 1} \frac{\theta''(\alpha)}{\phi''(\alpha)} = \left\{ \frac{2(1 - \tau)e^{-\tau} - (2\tau/\alpha^3)\, e^{-\frac{\tau}{\alpha}} + (\tau^2/\alpha^4)\, e^{-\frac{\tau}{\alpha}}}{2} \right\}_{\alpha \to 1}$$

$$= (1 - 2\tau + \tau^2/2)\, e^{-\tau} \tag{5}$$

Thus

$$\lim_{\alpha \to 1} P'(t) = (1 - 2\tau + \tau^2/2)\, e^{-\tau}$$

$P'(t)$, for $\alpha = 1$, given by equation 5 is also plotted in Figure 3 and it is seen that the measured pressure $P'(t)$ is finite when $\alpha \to 1$, that is $T_1 \to t_o$.

A study of Figure 3 shows that the major effect of low frequency attenuation is to reduce the apparent duration of the positive phase significantly for ratios of the RC time constant to pulse duration of less than about twenty to one. This error in measured positive phase duration is plotted in Figure 4 as a function of T_1/t_o.

To measure pulses with a nominal duration of about 12 milliseconds, the RC time constant of the transducer system should be $>$ 240 milliseconds, corresponding to a low frequency cut-off:

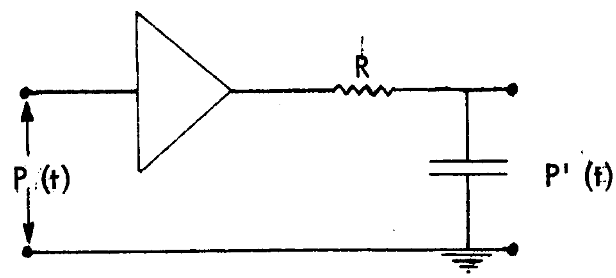
$$f_{lower} = \frac{1}{2\pi T} = \frac{10^3}{2\pi\, 240}$$

$$= \underline{0.66 \text{ c/sec.}}$$

f_{lower} is defined by $\omega T_1 = 1$, from equation 3.

4.0 MEASURED SHOCK PULSE - LOW FREQUENCY PASS FILTER

Figure 5: Simple Low Frequency Pass Filter.



$RC = T_2$

Consider now the measured pressure-time history recorded by a simple system with high frequency attenuation. The Frequency Response Function for such a system, illustrated in Figure 5 is given by:

$$H_2(\omega) = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j\omega T_2} \qquad (6)$$

If a measurement is made of P (t), the pressure defined by equation 1, the Fourier spectrum of the measured pulse may be determined from equations 2 and 6. Thus the Fourier spectrum:

$$P'(\omega) = H_2(\omega) \quad P(\omega) = \frac{j\omega t_o^{\,2}}{(1 + j\omega t_o)^2} \cdot \frac{1}{1 + j\omega T_2}$$

Substituting $s = j\omega$, the Laplace Transform is:

$$P'(s) = \frac{1}{T_2} \cdot \frac{s}{\left(s + \frac{1}{T_2}\right)\left(s + \frac{1}{t_o}\right)^2}$$

The inverse transform $P'(t)$ may now be determined from Reference 2. Thus the measured pressure-time history $P'(t)$ is:

$$P'(t) = \left[\frac{1}{(1 - T_2/t_o)^2} - \frac{t/t_o}{(1 - T_2/t_o)}\right] . e^{-\frac{t}{t_o}} - \frac{e^{-\frac{t/t_o}{T_2/t_o}}}{(1 - T_2/t_o)^2}$$

$$P'(t) = \left[\frac{1}{(1 - \beta)^2} - \frac{\tau}{(1 - \beta)}\right] . e^{-\tau} - \frac{e^{-\frac{\tau}{\beta}}}{(1 - \beta)^2} \qquad (7)$$

where $\beta = T_2/t_o$ and $\tau = t/t_o$. It is seen that when $\beta \to 1$, $P'(t)$ becomes indeterminate.*

It is observed again that when $\beta \longrightarrow 0$ (corresponding to a perfect measuring system with a high frequency cut off at ∞), then:

$$P'(t) \longrightarrow (1 - \tau) . e^{-\tau} = (1 - t/t_o) . e^{-\frac{t}{t_o}}$$

which is the "true" input pressure experienced by the transducer.

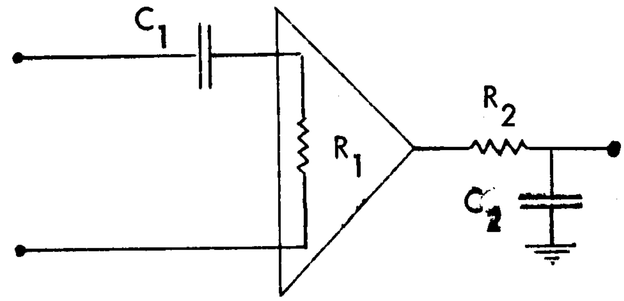
The measured pressure $P'(t)$, as defined by equation 7 is plotted in Figure 6, for several values of β. High frequency attenuation is seen to cause a reduction in the measured shock peak and an increase in the duration of the shock pulse measured.

The error in the recorded shock peak as a function of β is given in Figure 7. The graph shows that the measured value of the shock peak is very sensitive to the value of the high frequency time constant; a value of $\beta = T_2/t_o = 1/500$ still causes an error of about 2 percent in the measured shock peak.

* Use of L'Hospital's rule may again be used to show that $P'(t)$ has a finite value when $\beta = 1$, (see Page 6).

5.0 MEASURED SHOCK PULSE - BAND PASS FILTER

Figure 8: Simple Band Pass Filter.



$$T_1 = R_1 C_1 \; ; \quad T_2 = R_2 C_2$$

Most measuring systems (e.g. those employing Piezo-electric gauges) are limited at both the high and low frequency ends of the spectrum.

In order to simplify the analysis the system illustrated in Figure 8, consisting of simple single-order RC networks, is assumed to simulate the frequency pass band characteristics of a practical measuring system.

The Frequency Response Function is given by equations 3 and 6 and may be written:

$$H(\omega) = H_1(\omega) \cdot H_2(\omega)$$

$$H(\omega) = \frac{j\omega T_1}{1 + j\omega T_1} \cdot \frac{1}{1 + j\omega T_2} = \frac{j\omega}{\left(j\omega + \frac{1}{T_1}\right)} \cdot \frac{\frac{1}{T_1}}{\left(j\omega + \frac{1}{T_2}\right)} \qquad (8)$$

The Fourier spectrum of the measured pulse, when the system experiences a shock P (t) given by equation 1, is derived from equations 2 and 8:

$$P'(\omega) = H(\omega)\, P(\omega) = \frac{j\omega}{\left(j\omega + \frac{1}{t_o}\right)^2} \cdot \frac{j\omega}{\left(j\omega + \frac{1}{T_1}\right)} \cdot \frac{\frac{1}{T_2}}{\left(j\omega + \frac{1}{T_2}\right)}$$

Substituting $s = j\omega$, the Laplace Transform is:

$$P'(s) = \frac{1}{T_2} \cdot \frac{s^2}{\left(s+\frac{1}{t_o}\right)^2 \left(s+\frac{1}{T_1}\right) \left(s+\frac{1}{T_2}\right)} \tag{9}$$

The inverse transform $P'(t)$ is now determined from any Table of Laplace Transforms (such as Reference 2), using the method of partial fractions to simplify the inverse transform required in equation 9, if necessary.

The measured pressure - time history $P'(t)$ is:

$$P'(t) = \frac{e^{-\frac{t/t_o}{T_1/t_o}}}{\left(1-\frac{T_2}{T_1}\right)\left(\frac{T_1}{t_o}-1\right)^2} - \frac{e^{-\frac{t/t_o}{T_2/t_o}}}{\left(1-\frac{T_2}{T_1}\right)\left(\frac{T_2}{t_o}-1\right)^2} +$$

$$+\left\{\frac{\left(T_1/t_o\right)\left[\left(\frac{T_2}{T_1}+1\right)\left(\frac{T_1}{t_o}\right)-2\right]}{\left(\frac{T_1}{t_o}-1\right)^2\left(\frac{T_2}{t_o}-1\right)^2} + \frac{\left(T_1/t_o\right)\left(t/t_o\right)}{\left(\frac{T_1}{t_o}-1\right)\left(\frac{T_2}{t_o}-1\right)}\right\} e^{-\frac{t}{t_o}}$$

Thus

$$P'(\tau) = \frac{e^{-\tau/\alpha}}{\left(1-\frac{\beta}{\alpha}\right)(\alpha-1)^2} - \frac{e^{-\tau/\beta}}{\left(1-\frac{\beta}{\alpha}\right)(\beta-1)^2} + \left\{\frac{\alpha\left(1+\frac{\beta}{\alpha}\right)-2}{(\alpha-1)(\beta-1)} + \tau\right\} \frac{\alpha\, e^{-\tau}}{(\alpha-1)(\beta-1)} \tag{10}$$

where, as before, $\tau = t/t_o$, $\alpha = T_1/t_o$ and $\beta = T_2/t_o$. It is seen again that if if α or $\beta \rightarrow 1$, the $P'(t)$ becomes indeterminate.*

From Sections 3 and 4 it is seen that for good transducer systems, $\alpha > 1$ and $\beta < 1$.

Thus $\beta/\alpha \longrightarrow 0$, and to a good approximation equation 10 can be written:

$$P'(\tau) = \frac{e^{-\tau/\alpha}}{(\alpha-1)^2} - \frac{e^{-\tau/\beta}}{(\beta-1)^2} + \left\{ \frac{\alpha-2}{(\alpha-1)(\beta-1)} + \tau \right\} \frac{\alpha\, e^{-\tau}}{(\alpha-1)(\beta-1)} \tag{11}$$

Again it is seen that for a perfect transducer system where $\alpha \longrightarrow \infty$ and $\beta \longrightarrow 0$, then

$P'(\tau) \longrightarrow (1-\tau).e^{-\tau}$ the pressure pulse as defined by equation 1.

$P'(\tau)$, as defined by equation 10 was computed, for 9 cases using different values of α and β. A specimen curve plotted by the computer is presented directly in Figure 9. The curve is compiled from 100 points. The curves have been grouped and plotted in Figures 10 through 12 for comparison with the "true" pressure pulse defined by equation 1.

Figures 9 through 12 demonstrate that high and low frequency attenuation present in a transducer system results in errors both in the measured peak and in the length of positive phase duration of an explosion pressure pulse. For a constant low frequency cut-off, increasing high frequency attenuation results in increasing reduction of the peak pressure measured and increasing length of the measured duration of the positive phase. For a constant high frequency cut-off, increasing low frequency attenuation results in decreasing values of the measured peak pressure and the measured positive phase duration.

It is interesting to notice that for particular combinations of high and low cut-off frequencies there is zero error in the value of the measured positive phase duration. The locus of α and β for zero error in the measured positive phase duration is plotted in Figure 13.

It is clear that for good reproduction of the shock pulse, the time constants for the upper and lower frequency cut-offs should be: -

* Use of L'Hospital's rule may again be used to show that $P'(t)$ has a finite value when $\beta = 1$, (see page 6).

Lower Frequency: $T = \dfrac{1}{2\pi f_{lower}} \gg t_o$

Upper Frequency: $T = \dfrac{1}{2\pi f_{upper}} \ll t_o$

6.0 EFFECTS OF FINITE RISE TIME

6.1 Effects of Finite Shock Thickness

The idealized pressure pulse defined by equation 1, assumes an instantaneous change in pressure or a zero rise time. In practice, an actual shock pulse requires a finite time to reach a maximum. For a blast wave in air, the rise time is dependant upon the Mach number of the wave front. In the lower pressure, $0 \longrightarrow 10$ p.s.i. range, the rise time is of the order of microseconds.

TABLE 1

Rise Time For An Air-Blast

Δp Pressure Rise Lb/in^2	Mach Number M	Shock Thickness Inch	Rise Time μ Second
0.0034	1.0001	6×10^{-2}	4.5
0.034	1.001	6×10^{-3}	0.45
0.34	1.01	7×10^{-4}	0.052
3.60	1.1	9×10^{-5}	0.007

Table 1 gives the rise time to be expected for a low pressure blast. The shock thickness (column 3) is given, for the Mach numbers shown, in Reference 3. The pressure rise through the shock, Δp, is obtained from the well known shock relationship (Reference 4):

$$\frac{\Delta p}{p_1} = \frac{p_A - p_B}{p_1} = \frac{2\gamma (M^2 - 1)}{\gamma + 1} \qquad (12)$$

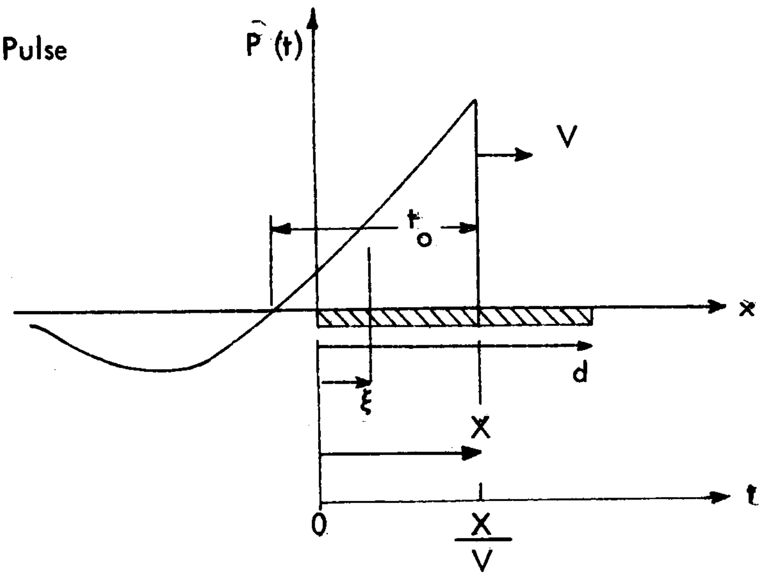
where p_B and p_A are the static pressures before and after the shock front respectively and $\gamma = 1.4$ for air.

The rise time is seen to be so extraordinarily small for blast pressures in the normal range of interest that it can be neglected.

6.2 Effects of Shock Passage

Another cause of finite build up time in the measured shock pulse is the time required for an air shock to pass over the sensitive diaphragm of the transducer, provided that it is mounted "side-on" to the blast. Assume the diaphragm is a rigid rectangular piston of length d and unit width, whose output is proportional to the average pressure over its face.

Figure 14: Passage of Shock Pulse Over Diaphragm.



The pressure sensed by the diaphragm can be divided into two time regimes, thus using the symbols defined by Figure 14, and the pressure pulse defined by equation 1: Firstly, when the shock front is passing over the diaphragm

For $X \leq d, \quad t \leq d/V$:

During this phase the pressure at point $\xi \leq X$ on the surface will be:

$$P(\xi, t) = \left[1 - \frac{t - \xi/V}{t_o}\right] \cdot e^{-\frac{t - \xi/V}{t_o}}$$

where ξ is a dummy space variable, in the x - direction.

By integrating $P(\xi, t)$, for the range of ξ from 0 to $X = Vt$, the mean pressure on the surface at time t is defined:

$$\overline{P}(t) = \frac{1}{d}\int_0^{Vt}\left[1 - \frac{t - \xi/V}{t_o}\right] \cdot e^{-\frac{t - \xi/V}{t_o}} \cdot d\xi$$

which gives on evaluation:

$$\overline{P}(t) = \frac{V t_o}{d} \cdot \frac{t}{t_o} \cdot e^{-\frac{t}{t_o}} \tag{13}$$

Secondly, when the shock front has passed over the diaphragm:
For $X \geq d, \quad t \geq d/V$:

$$\overline{P}(t) = \frac{1}{d}\int_0^{d}\left[1 - \frac{t - \xi/V}{t_o}\right] \cdot e^{-\frac{t - \xi/V}{t_o}} \cdot d\xi$$

$$\overline{P}(t) = \left[e^{d/Vt_o} - \frac{V t_o}{d} \cdot \frac{t}{t_o}\left(e^{d/Vt_o} - 1\right)\right] \cdot e^{-\frac{t}{t_o}} \tag{14}$$

The peak pressure detected by the transducer reaches a maximum when $t = d/V$ and both equations 13 and 14 on substition show this peak pressure to be:

$$\hat{P} = e^{-d/Vt_o} \tag{15}$$

The duration of the positive pulse detected by the transducer can be seen by inspection to be:

$$t = t_o \left(1 + \frac{d}{V t_o}\right) . \qquad (16)$$

This result can also be obtained from equation 14, if $p(t)$ is set equal to zero, t set equal to $\frac{d}{V}$ and d/Vt_o assumed small.

Consider the following values of the variables which are typical of those likely to be found in blast measurements:

Blast gauge span	d	$\approx$	$0.02 \longrightarrow 0.04$ ft.
Blast velocity	V	$\approx$	1100 ft/sec.
Positive duration	t_o	$\approx$	$.002 \longrightarrow 5.0$ seconds.

For the worst case of maximum span d, and minimum duration t_o, the maximum value of $d/Vt_o \approx 0.02$. This would predict an observed peak equal to 98 percent of the true value (see equation 15) and a positive phase duration 2 percent greater than the true value (see equation 16).

Since the worst case of d/Vt_o likely to be encountered was considered, it is seen that for most blast measurements, the errors due to finite rise time may be assumed to be negligible. The more important source of error in the measured shock peak and positive phase duration are due to the limitation in the frequency response of the transducer and measuring system.

7.0 CONCLUSIONS

In conclusion, a practical shock pulse measurement system should have:-

a) A low frequency response such that the low frequency time - constant is at least 20 times the positive phase duration, for less than 4 percent error in the measured positive phase duration .

Thus the low frequency cut-off should be:

$$f_{lower} \leq \frac{1}{40\pi t_o}$$

b) A high frequency time - constant (for the entire system) equal to less than 1/250 th of the positive phase duration t_o, for less than 4 percent error in the measured shock peak.

Thus the high frequency cut-off should be:

$$f_{upper} \geq \frac{125}{\pi t_o}$$

Errors due to the finite size of the transducer and the finite time for a shock to pass over it can normally be assumed to be negligible.

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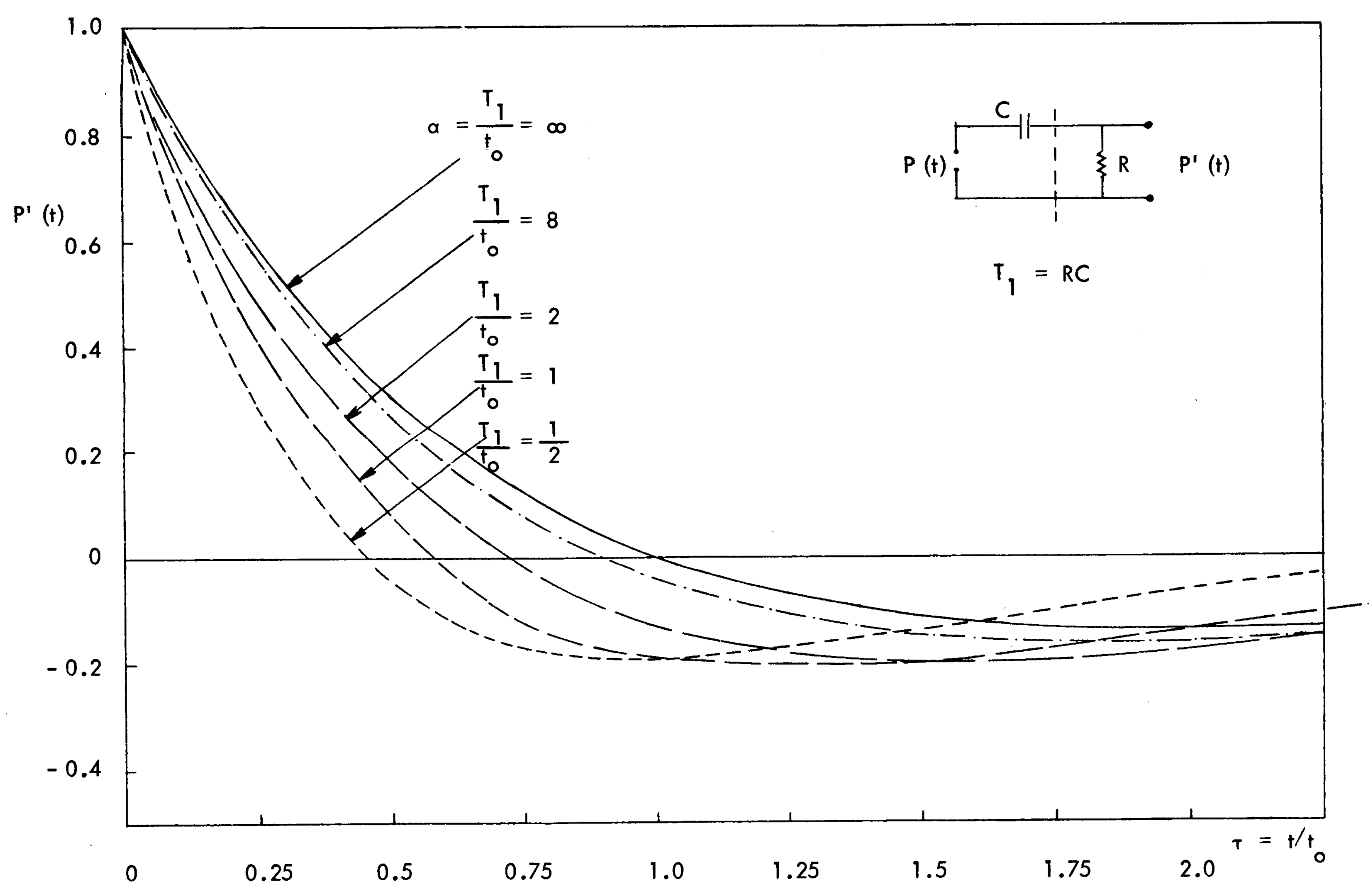

Figure 3: Effects of System Response Limited at Low Frequencies on Ideal Explosion Pressure-Time History

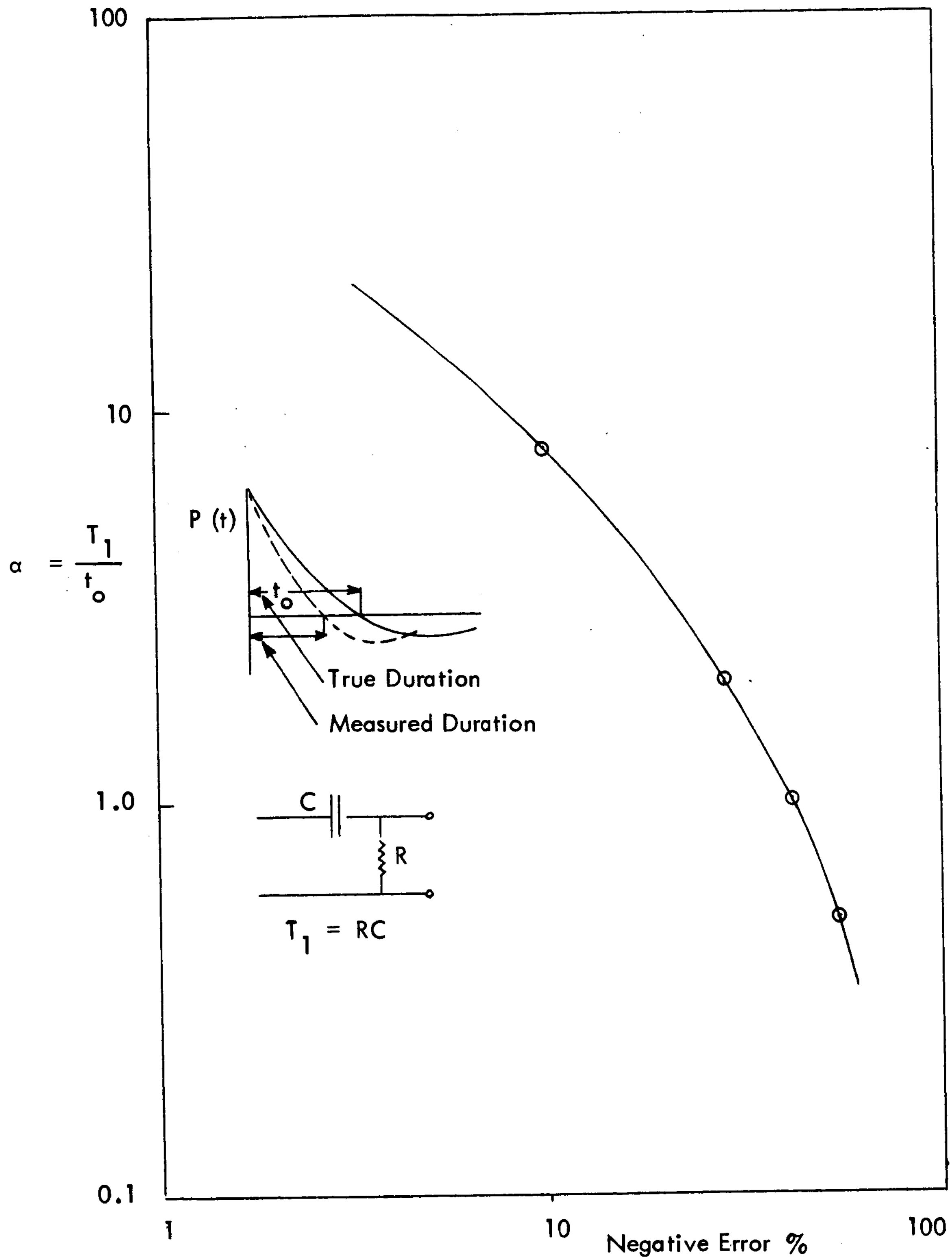

Figure 4: Error in Measured Duration of Positive Phase of Shock Pulse Due to Limited Low Frequency Response

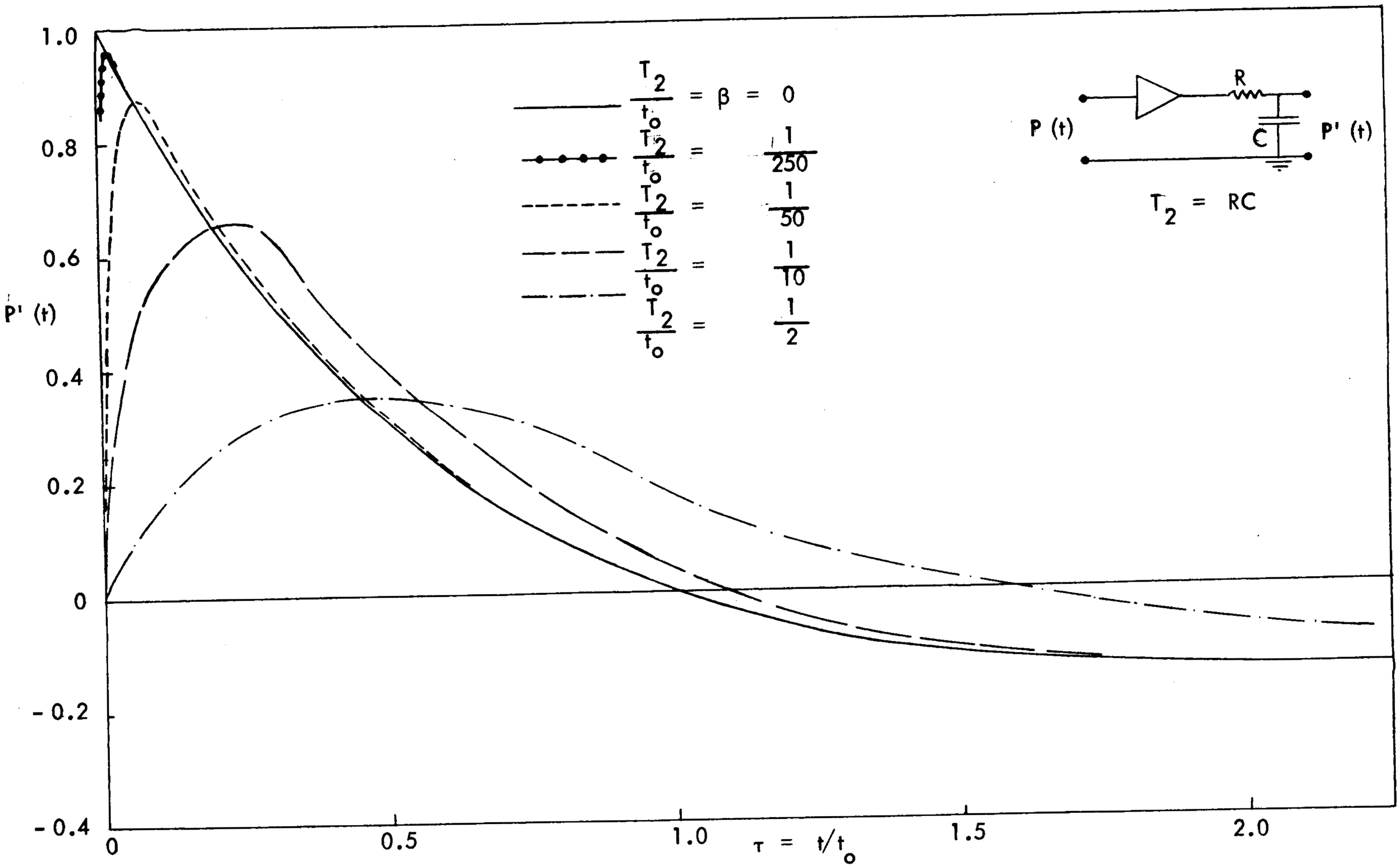

Figure 6: Effects of System Response, Limited at High Frequencies, on Ideal Explosion Pressure-Time History

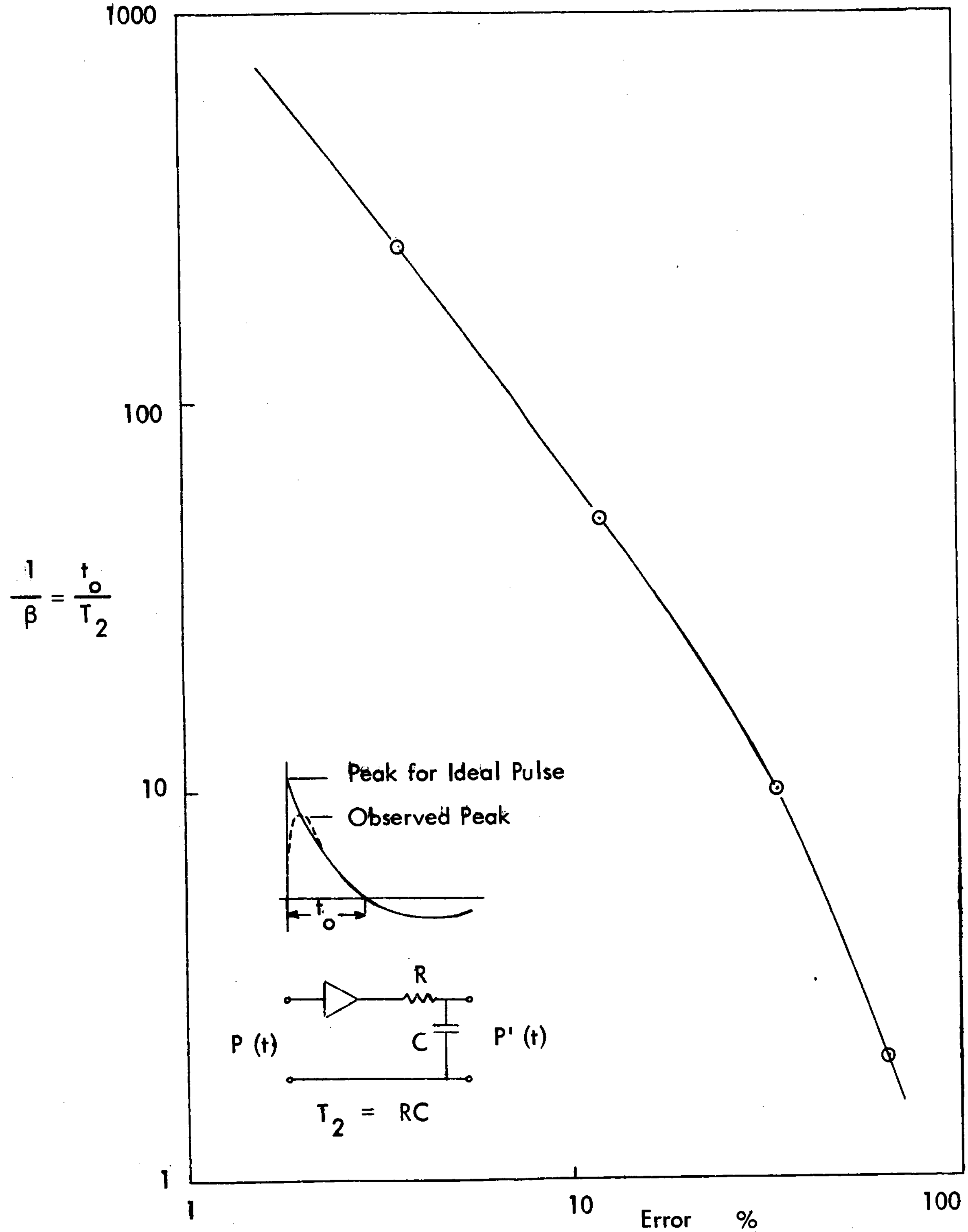

Figure 7: Error in Observed Peak Pressure of Shock Pulse Due to Limited High Frequency Response

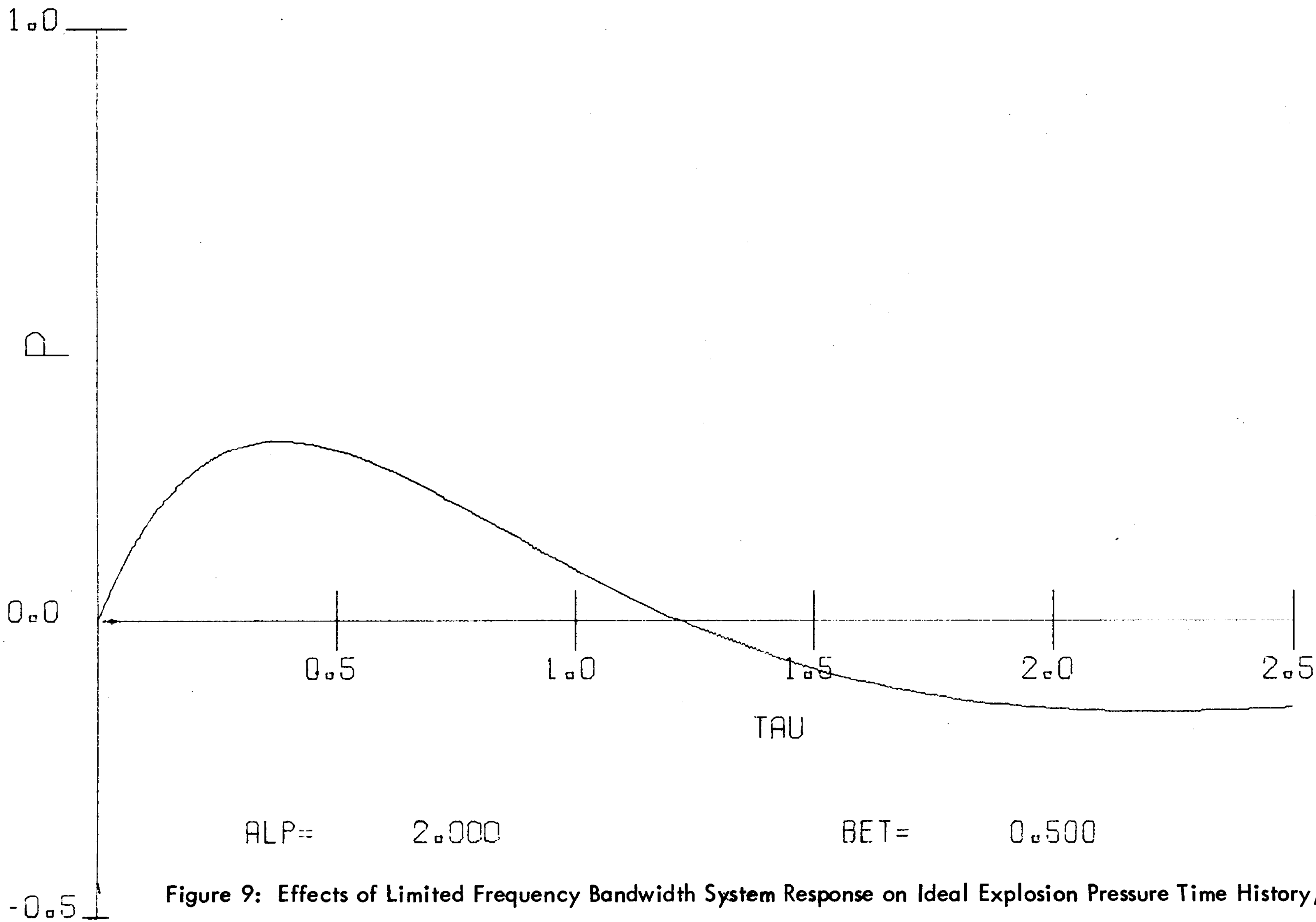

Figure 9: Effects of Limited Frequency Bandwidth System Response on Ideal Explosion Pressure Time History, (α =2, β = .5)

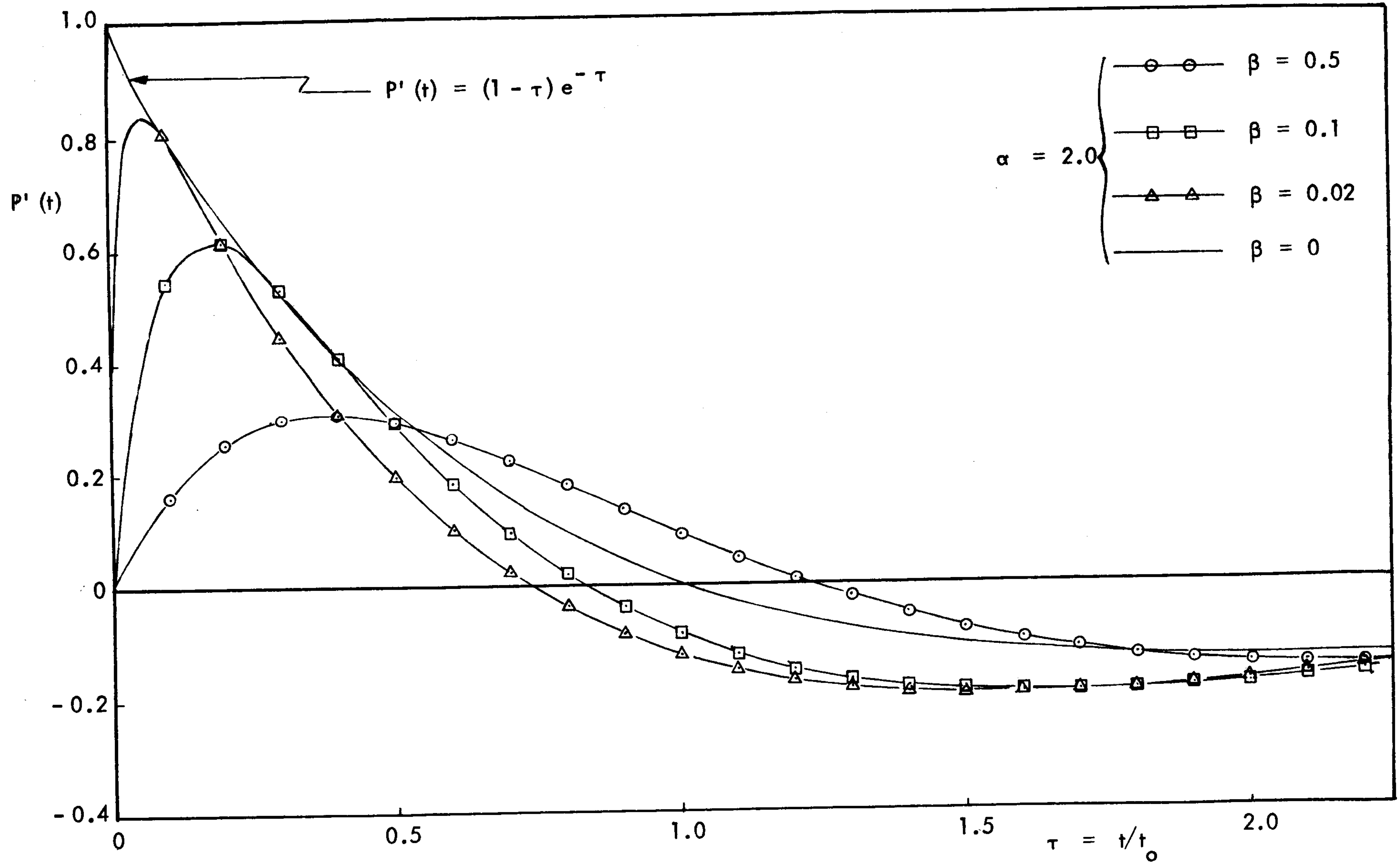

Figure 10: Comparison of Limited Frequency Bandwidth System Response ($\alpha = 2.0$) with Ideal Explosion Pressure-Time History

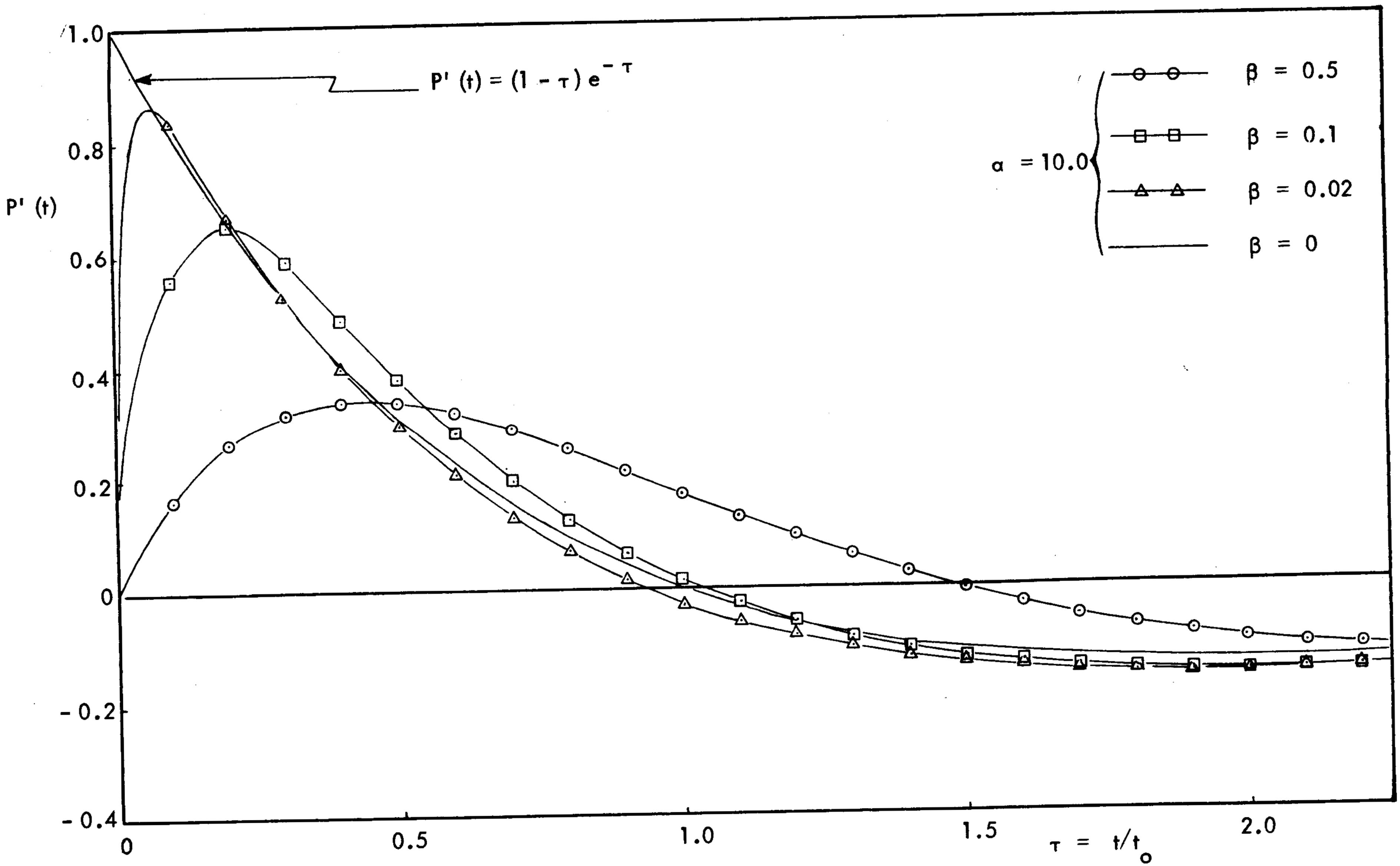

Figure 11: Comparison of Limited Frequency Bandwidth System Response (α = 10.0) with Ideal Explosion Pressure-Time History

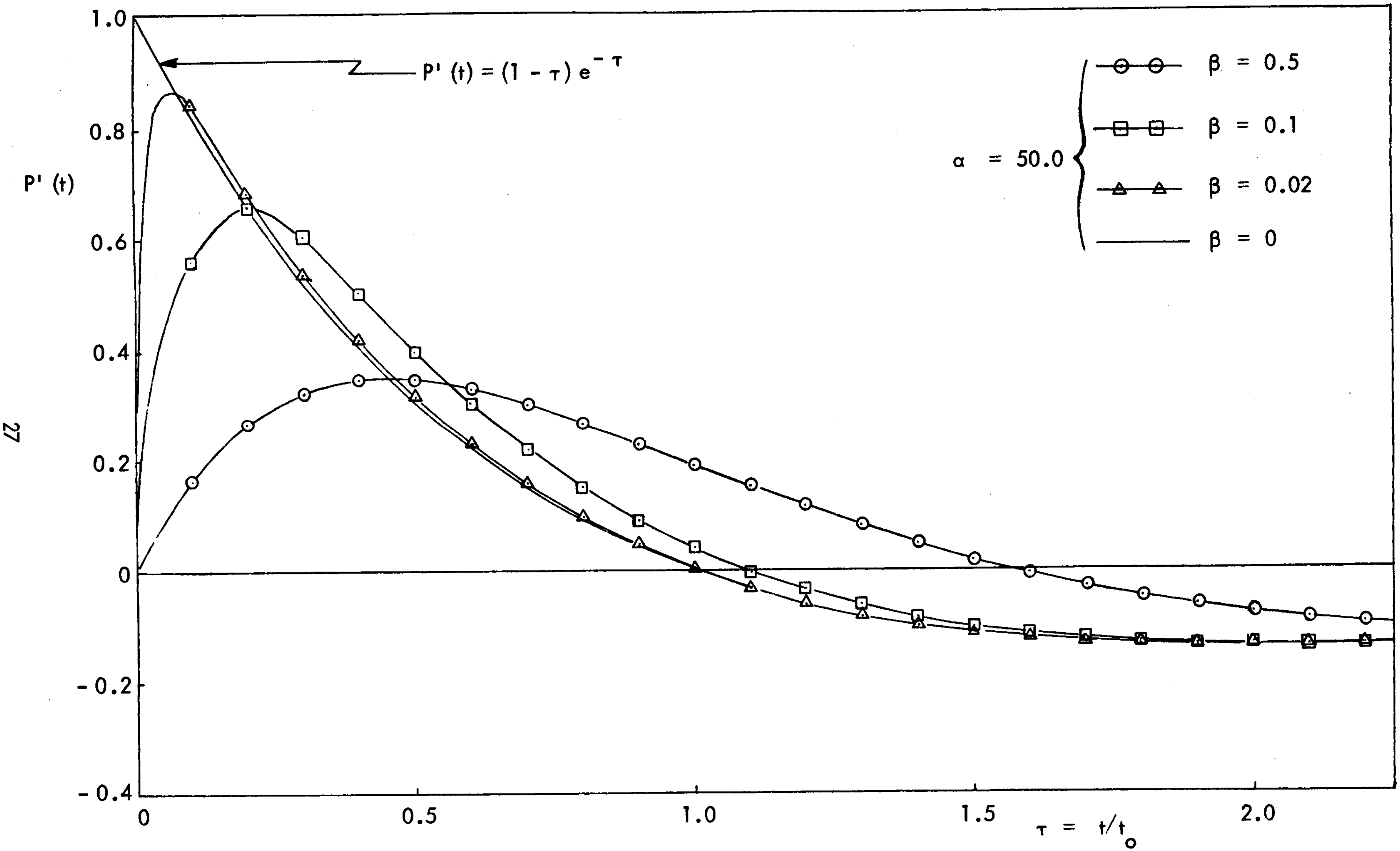

Figure 12: Comparison of Limited Frequency Bandwidth System Response (α = 50.0) with Ideal Explosion Pressure-Time History

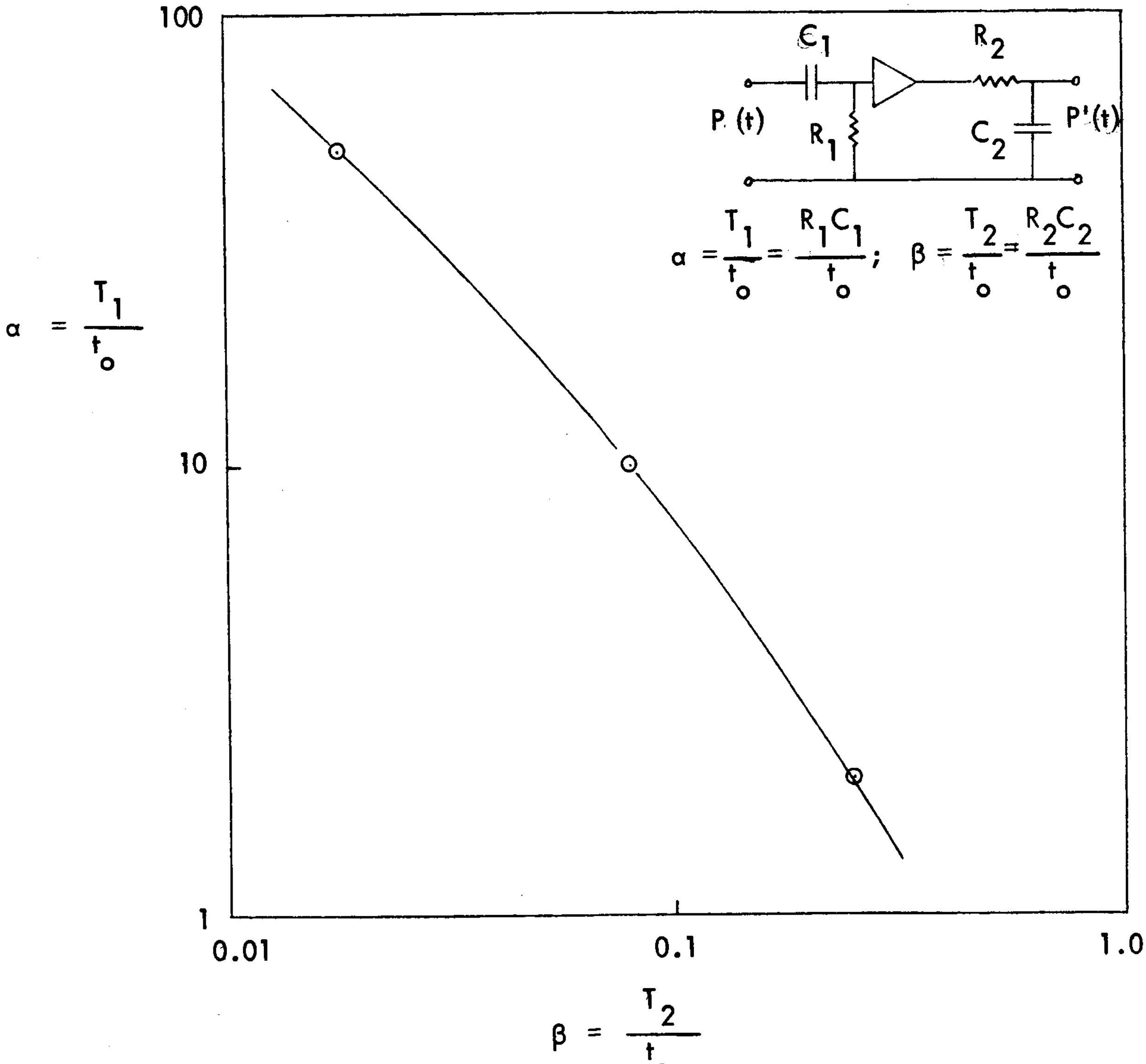

Figure 13: Locus of α and β for Zero Error in Measured Duration of Positive Phase