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Wave Length Dependence of Polarization by Small Graphite Flakes*

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Abstract

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The extinction by oblate spheroidal particles whose complex dielectric constants are anisotropic in the sense of graphite is calculated in Rayleigh approximation. It is shown that for sufficiently small graphite flakes (maximum radius ∼ .05 micron) as represented by the oblate spheroid model, and not too thick dielectric mantles, the wave length dependence of polarization may be expected to follow the curve for the wave length dependence for extinction. This is in contradiction to the observed tendency for the polarization to remain fairly constant through the visual part of the spectrum.

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Introduction

In a series of papers starting with that of Hoyle and Wickramasinghe (1962) there has been a resurgence of interest in the possibility that graphite may be of significant and perhaps primary importance in explaining the interstellar extinction and polarization. With the exception of the work of Wickramasinghe (1962) the main attention has been directed to the problems of extinction particularly in the infrared and the ultraviolet (see for example Wichramasinghe and Guillaume 1965, Greenberg 1965). This is a little surprising when one considers that the original motivation for invoking graphite (Cayrel and Schatzman 1954) was the deep concern that dielectric needles "apparently" were incapable of producing the maximum observed ratio of polarization to extinction. Cayrel and Schatzman demonstrated experimentally that even a small contribution by graphite flakes would be sufficient to account for the polarization (a figure of 5 percent contribution to the total extinction was given by Greenberg 1962). The more theoretical and quantitative treatment of Wickramasinghe (1962) based on the Rayleigh approximation to absorption by oblate spheroids with anisotropic conductivity derives a figure for the required degree of orientation on the assumption that all of the extinction is produced by graphite. However, neither in this latter paper nor in any of the others has the problem of the wave length dependence of polarization by graphite been seriously considered.

The astronomically observed character of the wave length dependence of polarization is on at least as good and dependable a foundation as is the extinction curve.

The purpose of this paper is to apply the most recent optical data on graphite to a calculation which gives at least a semi-quantitative estimate of the wave length dependence of polarization to be expected from small graphite flakes.

It should be noted that the original qualms about the polarizing abilities of dielectric particles have already been discussed and resolved semi-quantitatively on the basis of incomplete data on prolate spheroids (Greenberg et al 1963). Recent exact calculations (Greenberg and Shah 1965) on polarization by cylinders with arbitrary degrees of orientation confirms this work.

2. Extinction Cross-sections by Oblate Spheroids with Anisotropic Refractive Indices

Following Wickramasinghe (1962) we consider graphite to be represented by an oblate spheroid whose thickness along the symmetry axis is 2c and whose width is 2a.

We shall be concerned with extinction, absorption, and scattering cross-sections in which the electric vector of the incident radiation is either along c or a. Within the framework of the Rayleigh approximation cross-sections for arbitrary orientation are easily obtained by appropriate linear combinations of these basic cross-sections.

It is assumed that the tensor representing the complex index of refraction has the same principal axes as the spheroid (a very reasonable assumption). Following the generalizations by Clark Jones (1945) of the electrostatic polarization of an ellipsoid of anisotropic material we may describe the polarizability along the a and c axes in the notation of van de Hulst (1957) by

$$\alpha_a = \frac{a^2 c}{3} \; \frac{m_a^2 - 1}{L_a(m_a^2 - 1) + 1}$$

$$\alpha_c = \frac{a^2 c}{3} \; \frac{m_c^2 - 1}{L_c(m_c^2 - 1) + 1} \qquad (1)$$

where m_a^2 and m_c^2 are the complex dielectric constants along the a and c directions respectively, and the L_a and L_c are numerical factors depending on the ratio a/c.

It is now easily shown that the various absorption and scattering cross-sections are given by

$$C_{abs}^{(a)} = 4\pi k \, \mathrm{Re} \left\{ i \, \alpha_a \right\}$$

$$C_{abs}^{(c)} = 4\pi k \, \mathrm{Re} \left\{ i \, \alpha_c \right\}$$

$$C_{sc}^{(a)} = \frac{8}{3}\pi k^4 |\alpha_a|^2 \qquad (2)$$

$$C_{sc}^{(c)} = \frac{8}{3}\pi k^4 |\alpha_c|^2$$

where $k = 2\pi/\lambda$, and the superscripts a and c correspond to the electric vector, of the incident radiation along the a and c directions, respectively.

The total or extinction cross-sections are given by $C_{ext} = C_{abs} + C_{sc}$.

3. Polarization by "Model" Graphite Particles

It can be shown (Davis and Greenstein 1951) for particles whose scattering is given adequately by the Rayleigh approximation that the ratio of polarization to extinction is

$$\left(\frac{\Delta m_p}{\Delta m}\right)_{calc} = \text{constant} \; \frac{C^{(a)}_{ext}/C^{(c)}_{ext} - 1}{2\; C^{(a)}_{ext}/C^{(c)}_{ext} + 1} \qquad (3)$$

where the subscript calc is used to denote the theoretically calculated value and where the "constant" is dependent on, among other factors, the degree and kind of orientation of the particles.

We do not use the quantity $(\Delta m_p)_{calc}$ directly to estimate the wave length dependence of polarization. A somewhat better estimate of this quantity is obtained by multiplying Eq. (3) by the observed wave length dependence of extinction to give

$$\Delta m_p(\lambda) = \left(\frac{\Delta m_p}{\Delta m}\right)_{calc} (\Delta m)_{obs} \qquad (4)$$

We present the following table as a reasonable representation (See Wickramasinghe and Guillaume 1965) of the complex dielectric constants of graphite

$\lambda^{-1}(\mu^{-1})$	m_a^2	m_c^2
.9	4-10i	4-10βi
1.2→2.8	4- 7i	4- 7βi

where β is the ratio of conductivities along the c direction and the a direction and is to be expected to be less than unity. In order to span a range of possibilities we will consider in turn $\beta = 10^{-2}, 10^{-1}, 1$.

The size of the graphite flake is taken to be such that a = 0.05 micron which corresponds to the radius for spherical graphite particles which roughly give the extinction curve. Graphite flakes are quite flat and we shall consider two values of the degree of oblateness: $\frac{a}{c} = 5$ and $\frac{a}{c} = \infty$ (disk).

A summary of the numerical results is presented in the following tables.

Table 1

Cross-sections for Oblate Spheroids

$a/c = 5,\ L_c = .75,\ L_a = .125$

$\lambda^{-1} \backslash \beta$	$C^{(a)}_{abs}/C^{(c)}_{abs}$ 10^{-2}	10^{-1}	1
0.9	305	32	19
1.2-2.8	396	41	14

$\lambda^{-1} \backslash \beta$	$C^{(a)}_{abs}/C^{(c)}_{sc}$ 10^{-2}	10^{-1}	1
0.9	11.0	99	91
2.8	.3	2.8	4.5

λ^{-1}	$C^{(a)}_{abs}/C^{(a)}_{sc}$
.9	40
2.8	4.5

Table 2

Cross-sections for Flat Disks

$a/c \rightarrow \infty,\ L_c = 1.0,\ L_a = 0$

$\lambda^{-1} \backslash \beta$	$C^{(a)}_{abs}/C^{(c)}_{abs}$ 10^{-2}	10^{-1}	1
0.9	1600	170	116
1.2-2.8	1600	165	65

$\lambda^{-1} \backslash \beta$	$C^{(c)}_{abs}/C^{(c)}_{sc}$ 10^{-2}	10^{-1}	1
0.9	1.25 a/c	12.5 a/c	17.2 a/c
2.8	.03 a/c	.32 a/c	8 a/c

λ^{-1}	$C^{(a)}_{abs}/C^{(a)}_{sc}$
.9	18 a/c
2.8	.9 a/c

We see from the tables that in Eq. (3) we may effectively replace $C^{(a)}_{ext}/C^{(c)}_{ext}$ by $C^{(a)}_{abs}/C^{(c)}_{abs}$. Furthermore, it can be seen that the value of

$$\left(\frac{\Delta m_p}{\Delta m}\right)_{calc}$$

is a constant within 5 percent over the entire visible spectrum. Consequently, to this approximation, the wave length dependence of the polarization is just that of the extinction; i.e., it rises monotonically from the far infrared to the ultraviolet. This dependence of polarization in wave length has also been predicted (Greenberg 1962) for the Platt (1956) type particles. However it is generally observed that the polarization is rather flat at least through the range $1.3 \sim \lambda^{-1} \sim 3$ with, perhaps on the average a maximum in the green.

In figure 1 are shown the observations of Behr (1963) a solid theoretical curve (Greenberg 1962, and Greenberg et al 1963) which fits the observation of Gehrels (1961) and a dotted theoretical curve based on the result that the polarization by small graphite flakes follows the extinction curve. The solid theoretical curve is normalized to unity at $\lambda^{-1} = 2$, and is exceedingly close to 1 at $\lambda^{-1} = 1.5$ which we have for convenience chosen as the point of normalization for the dotted curve. It appears that even where the Rayleigh approximation should be valid, the graphite model does not give a good representation of the polarization. The values of ka at $\lambda^{-1} = 0.9$ and 2.8 are $ka_{.9} = .28$ and $ka_{2.8} = .88$ and one should expect the approximation to be valid up to $ka \sim 0.5$.

All of the above calculations have assumed uncoated graphite flakes. If graphite flakes do indeed exist in interstellar space, it is to be expected that they would accrete dielectric (dirty ice) mantles. If the mantles are not too thick (relative to the core thickness) the grain would probably still preserve a reasonable degree of oblateness. Since the ratio $C^{(a)}_{ext}/C^{(c)}_{ext}$ would, at least for the longer wave lengths, still be determined primarily by core absorption (a Rayleigh approximation result) we should find that the wave length dependence of polarization would still have the tendency to increase, although perhaps not so steeply. This latter is admittedly a rather qualitative statement, but when taken with the quantitative results for pure graphite flakes, it leads one to conclude that, at the very least, the graphite flakes do not readily lend themselves to a theoretical explanation of the wave length dependence of polarization.

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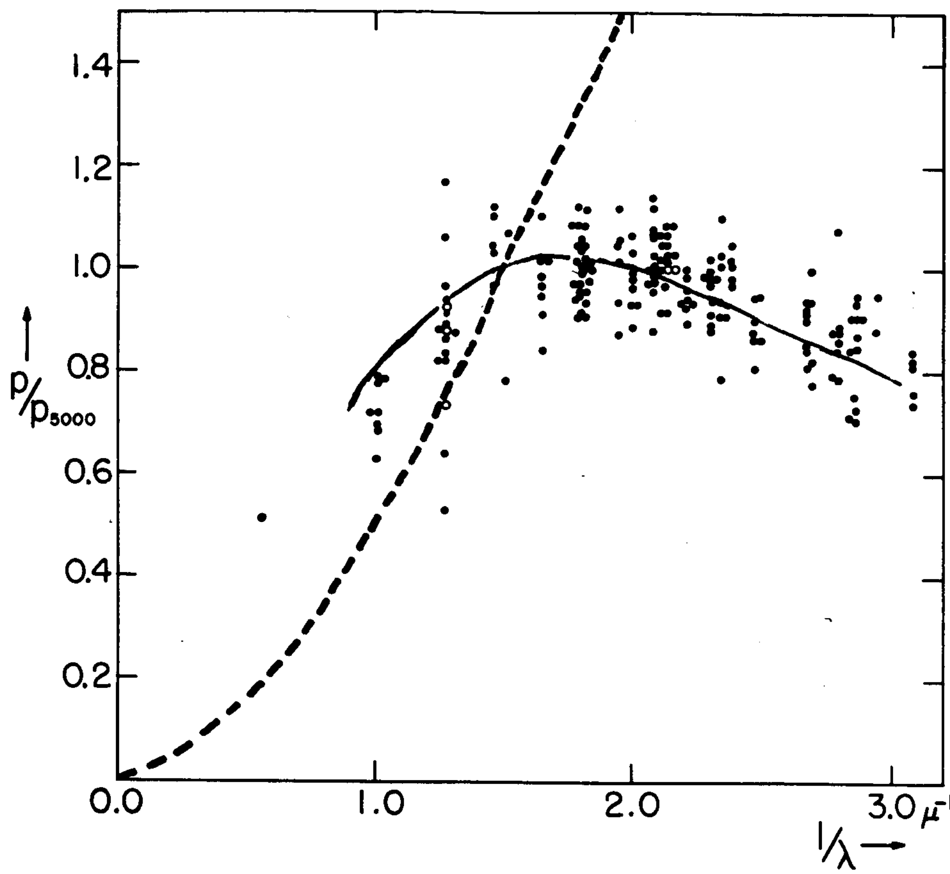

Figure 1. Wave length dependence of polarization. Experimental points by Behr (1964). Solid curve is best theoretical fit (Greenberg et al. 1963) to observations of Gehrels (1960). Dashed curve is based on theoretical results of this paper and is normalized to unity at $\lambda^{-1} = 1.5$.