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AEROTHERMOCHEMICAL ANALYSIS OF NON-OSCILLATORY AND OSCILLATORY CHARACTERISTICS OF LIQUID BIPROPELLANT ROCKET MOTORS: WAVE MODEL, DROPLET HEATING, AND EVAPORATION PROGRAM COMPUTATION

Prepared For
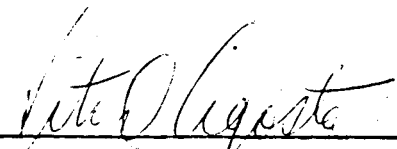
Jet Propulsion Laboratory
Pasadena, California

under NAS 7-100

Report No. 65-1

15 January 1965

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DROPLET HEATING, AND EVAPORATION PROGRAM COMPUTATION

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Report No. 65-1
Third Quarterly Report
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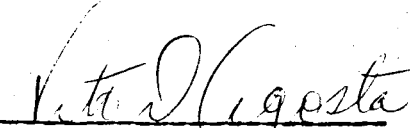
by

V. D. Agosta and S. Z. Burstein

ABSTRACT

This report is divided into three parts: (1) a description of the wave propagation model, (2) fuel droplet heating analysis, and (3) discussion of further work done on the non-oscillatory aerothermochemical computer program.

It is decided to use a wave propagation model in which the wave lengths are coupled to the cavity dimensions.

From the droplet heating analysis, it is found that very large temperature gradients exist near the surface, the interior of the droplet remaining relatively unheated. In addition, heat-up time to saturation conditions for a $(0.375 \times 10^{-3} ft)$ drop is about 40 microseconds.

The computer program presented herein is the modification of an earlier code that was designed to test some basic ideas on the theory of evaporation rate controlled combustion. It is designed to solve automatically and simultaneously the interaction between the nonlinear gas dynamic equations and the two components of the spray, i.e., fuel and oxidizer. In addition, the effect of equilibrium dissociating combustion products on this solution is included.

NOMENCLATURE

a	droplet radius
C_p	specific heat
h	thermal film coefficient
k	thermal conductivity
r	radius
T	temperature
t	time
ρ	density

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I. INTRODUCTION

This report is divided into three parts. These are (1) a description of the wave propagation model, (2) fuel droplet heating analysis, and (3) discussion of further work done on the non-oscillatory aerothermochemical computer program.

It has become apparent as work progressed on the instability program that the wave equations, the wave energy coupling mechanisms, the method of solution for these wave equations, and the injector geometry must be considered in formulating and solving the non-oscillatory aerothermochemical program. Review of the oscillatory data and injector configuration supplied by the Jet Propulsion Laboratory suggested the approach assumed and given below. The most significant finding in the analysis of the oscillatory data is that the tangential mode of oscillation exhibits the wave characteristics of the rocket chamber cavity. In other words, the wave period and estimated Mach number agree with those expected for the classical wave behavior in the tangential mode. Thus the conclusion made is to analyze this wave behavior in terms of the field equations (e. g. , Moore and Maslen) in contradistinction to analyzing wave propagation along a "one-dimensional" circumferential strip (Priem). The analysis remains nonlinear because of

the large wave amplitudes observed. In this case, the wave slope is not

predominately indicative of the gas dynamic behavior of the waves, since

the rocket chamber wall continuously modifies the wave shape due to reflec-

tions as the wave sweeps along the wall. The question arises - how does

this model manifest itself to the previous approach assumed? The answer

is "very well;" the development of the wave model is given below.

II. WAVE MODEL

Essentially, the wave model consists of a resonating cavity in which

the oscillations are three-dimensional in a two-phase reacting fluid flowing

in one dimension, i. e., axially. The effect of these waves is to impose a

circulatory motion to the fluid, and to cause the liquid phase of the fluid

flowing to evaporate and react at rates different than those in the non-

oscillatory case. These excesses serve as energy sources to drive the

wave if the transport relaxation times are in the proper phase. The problem

reduces to: (1) generating the wave equations in three dimensions,

(2) determining the axial droplet distribution from the one-dimensional non-

oscillatory aerothermochemical analysis, (3) determining the radial and

tangential droplet distribution from the injector geometry, (4) determining

the relaxation time from the non-steady droplet evaporation analysis and

a diffusion analysis of the evaporated fuel vapor through the film surrounding

the fuel droplet, (5) from (2), (3) and (4) calculating the excess $\nabla \cdot \varphi$ term,

and (6) applying (5) to the wave dynamic equations of (1) and solving.

The gas dynamic equations given in the previous Quarterly Report,

Report No. 64-3, are general and serve as the basis for the wave equations.

These wave equations will be derived shortly and given in the Fourth Quarterly
Report. It is assumed that viscous and thermal effects are important only
in regions near the chamber walls, and thus will not be included at this time.
Droplet heating and evaporation is discussed in the following section below.

III. DROPLET HEATING AND EVAPORATION

The droplet heat transfer model is given in Figure 1. The basic
equation for heat transfer is

$$T_t = \varkappa \nabla^2 T \tag{1}$$

where the subscript t means $(\frac{\partial}{\partial t})$.

Spherical symmetry is assumed; therefore, Eq. (1) is written as

$$T_t = \varkappa [T_{rr} + \frac{2}{r} T_r] \tag{2}$$

with the boundary and initial conditions

$$h(T_f - T) = k T_r \quad ; \quad r = a \quad t \geq 0 \tag{3a}$$

$$T(o\,t) \text{ is finite} \tag{3b}$$

$$T(r,o) = T_i \tag{3c}$$

The solution to this equation is (H. S. Carslaw and J. C. Jaeger,
" Conduction of Heat in Solids," Oxford at the Clarendon Press, 1950):

$$\frac{T_f - T}{T_f - T_i} = \frac{2h}{kr} \sum_{n=1}^{\infty} \frac{(a^2 \alpha_n^2 + \mu^2) \sin \alpha_n a \sin \alpha_n r}{\alpha_n^2 [a^2 \alpha_n^2 + \mu(\mu+1)]} \rho^{-\alpha_n^2 \varkappa t} \tag{4}$$

Flame T_f

Q

T_a

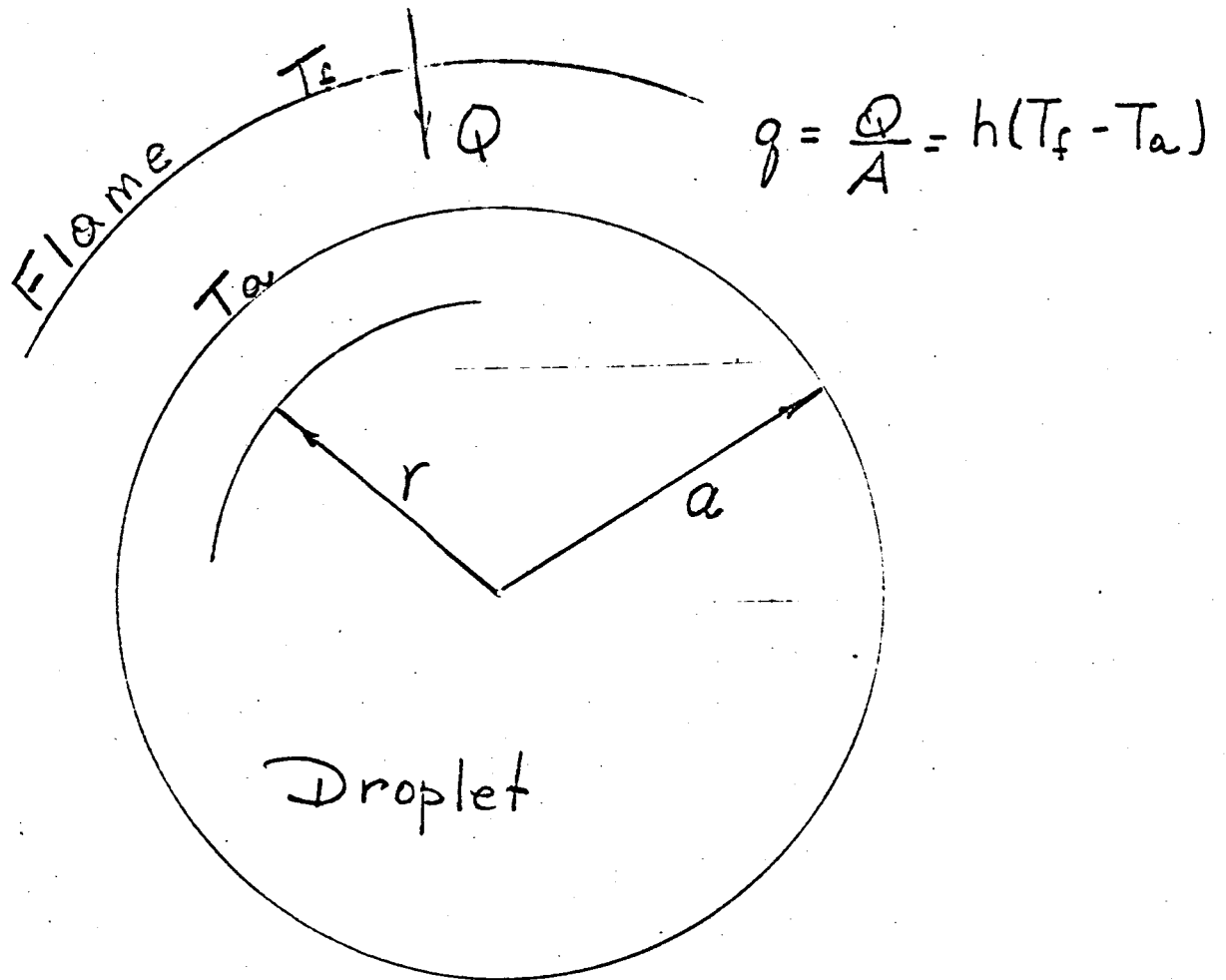$$q = \frac{Q}{A} = h(T_f - T_a)$$

r

a

Droplet

FIGURE 1.   HEAT TRANSFER
TO BURNING DROPLET

where α_n is given by the solution of

$$\mu + \alpha_n a \cot \alpha_n a = 0 \tag{5}$$

and

$$\mu \equiv (\frac{ah}{k} - 1). \tag{6}$$

For 50/50 hydrazine-UDMH blend at about $650^\circ R$:

$\rho \simeq 53$ lbm/ft^3

$C_p \simeq 0.694$ Btu/lbm$^\circ R$

$k \simeq 0.42 \times 10^{-4}$ Btu/sec ft $^\circ R$

$\varkappa \simeq 1.15 \times 10^{-6}$ ft^2/sec.

For O/F $\simeq 2.20$, the flame temperature near the injector end of the chamber is $T_f \simeq 5800^\circ R$ and $h = 1.5$ B/ft^2 sec$^\circ R$ to a droplet $a \simeq 0.375 \times 10^{-3}$ ft.

Reference is now made to Eq. (2). For short time intervals, i.e., $10^{-6} \leq t \leq 10^{-3}$, $e^{-\alpha_n^2 \varkappa t} \simeq e^0 = 1$. In addition, $\sin \alpha_n a \leq 1$ and $\sin \alpha_n r \leq 1$. Therefore, the rate of convergence of this solution for short time intervals is dependent on the factor

$$\frac{2h}{kr} \sum_{n=1}^{\infty} \frac{(a^2 \alpha_n^2 + \mu^2)}{\alpha_n^2 (a^2 \alpha_n^2 + \mu(\mu+1))},$$

and this converges very slowly, making the solution of no practical use in the time range of interest. Thus, a short term solution is sought.

The method of solution again is found in H. S. Carslaw and J. C. Jaeger, "Operational Methods in Applied Mathematics," Dover Publications, Inc., 1963, and was used by Richard F. Parisse in "Ablation of a

Solid Sphere of a Low Conductivity Material," PIBAL Report No. 683, Polytechnic Institute of Brooklyn, 1964. The heating up solution is reproduced below, and modified to the extent that the terms apply to a liquid fuel droplet.

A short-time solution for the temperature distribution in a liquid sphere subjected to a point symmetric heat input is obtained by utilizing Laplace transformations. The appropriate form of the heat conduction equation is

$$\frac{\partial T}{\partial t} = \varkappa \left(\frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} \right) \quad ; \quad 0 \leq r < a \quad . \tag{7}$$

with the conditions

$$h(T_f - T_a) = k(\partial T / \partial r)_{r=a} \tag{8a}$$

$$T(r, o) = T_i \tag{8b}$$

$$T(o, t) \text{ is finite} \tag{8c}$$

where the symbols are defined as

a   initial radius of sphere

h   heat transfer coefficient

k   thermal conductivity

r   radial coordinate

t   time

T   temperature

T_a heated surface temperature

T_i initial temperature of material

T_f stagnation temperature of flow field

$\varkappa$ thermal diffusivity

Eq. (7) can be transformed into a more convenient form by the substitution

$$u = r(T_f - T)$$

The result is

$$\frac{\partial u}{\partial t} = \varkappa \frac{\partial^2 u}{\partial r^2} \quad ; \quad 0 \leq r < a \qquad (9)$$

and Eqs.(8) become

$$\frac{\partial u}{\partial r} + \frac{\mu}{a} u = 0 \quad ; \quad at \ r = a \qquad (10a)$$

$$u(r,o) = r(T_f - T_i) \qquad (10b)$$

$$u(o,t) = 0 \qquad (10c)$$

where $\mu = (ah-k)/k$.

The Laplace transformation is now applied to Eq. (9)

$$\int_o^\infty e^{-pt} \frac{\partial^2 u}{\partial r^2} \, dt - \frac{1}{\varkappa} \int_o^\infty e^{-pt} \frac{\partial u}{\partial t} \, dt = 0$$

and by using Eq. (10b), the resulting subsidiary equation is

$$\frac{d^2 \bar{u}}{dr^2} - q^2 \bar{u} = - \frac{(T_f - T_i)r}{\varkappa} \ , \qquad (11)$$

where

$$q^2 = p/\varkappa .$$

Eqs. (10a) and (10c) become

$$\frac{d\overline{u}}{dr} + \frac{\mu}{a}\,\overline{u} = 0 \quad , \quad \text{at } r = a \qquad\qquad (12a)$$

$$\overline{u} = 0 \qquad\qquad , \quad \text{at } r = 0 . \qquad\qquad (12b)$$

A general solution to Eq. (11) is

$$\overline{u} = C_1 e^{qr} + C_2 e^{-qr} + \frac{(T_f - T_i)\,r}{q^2 \varkappa}$$

where C_1 and C_2 are constants. These constants are evaluated by applying

Eqs. (12a) and (12b). The result in series form is as follows:

$$\overline{u} = \frac{(T_f - T_i)r}{p} - \frac{(1+\mu)(T_f - T_i)}{p} \sum_{n=0}^{\infty} (-1)^n \frac{(q-\mu/a)^n}{(q+\mu/a)^{n+1}} \left\{ e^{-q[(2n+1)a - r]} - e^{-q[(2n+1)a+r]} \right\}$$

$$(13)$$

For a short-time solution, Eq. (13) can be truncated after the n=0 term.

For the range of parameters considered in this report, the higher order

terms of Eq. (13) have been shown to be negligible.

The inverse transformation of Eq. (13) for n=0 is

$$\overline{T} = \frac{1}{r'} \left[\frac{1+\mu}{\mu}\right] \left\{ \text{erfc}\,\frac{1-r'}{2\sqrt{t'}} - \text{erfc}\,\frac{1+r'}{2\sqrt{t'}} \right.$$

$$- \exp[\mu(1-r') + \mu^2 t']\,\text{erfc}[\frac{1-r'}{2\sqrt{t'}} + \mu\sqrt{t'}] \qquad (14)$$

$$\left. + \exp[\mu(1+r') + \mu^2 t']\,\text{erfc}[\frac{1+r'}{2\sqrt{t'}} + \mu\sqrt{t'}] \right\}$$

where

$$t' = \frac{\varkappa t}{a^2}$$

$$r' = r/a$$

$$\overline{T} = \frac{T - T_i}{T_f - T_i} \; .$$

A plot was made of the heating up of a fuel droplet, (50/50 N_2H_4/UDMH, for a = 0.375 x 10^{-3} ft) and is shown in Figure 2. It is seen that in a very short time the surface of the drop heats up. In the absence of saturation vapor pressure-temperature data, a saturation temperature of 480°F was assumed. Note the saturation temperature at p = 14.7 psia is 146°F for UDMH and 235°F for N_2H_4; and at the critical pressure of 1696 psia, T_i = 634°F. For this temperature of 480°F, the heat-up time of the droplet before evaporation occurs is about 40 microseconds.

The temperature distribution in the droplet was determined for the case where the surface temperature was equal to the evaporation temperature. It is seen in Figure 3 that the heating effect is concentrated in the outer one percent of the droplet radius. This agrees with results from the previous aerothermochemical analysis, namely, that the drop, considered at constant temperature, does not heat up significantly.

The next step in this analysis is to determine the evaporation rate for the droplet. An equation has been derived and, unfortunately, it is highly nonlinear. Several techniques for the solution of this equation are currently being investigated.
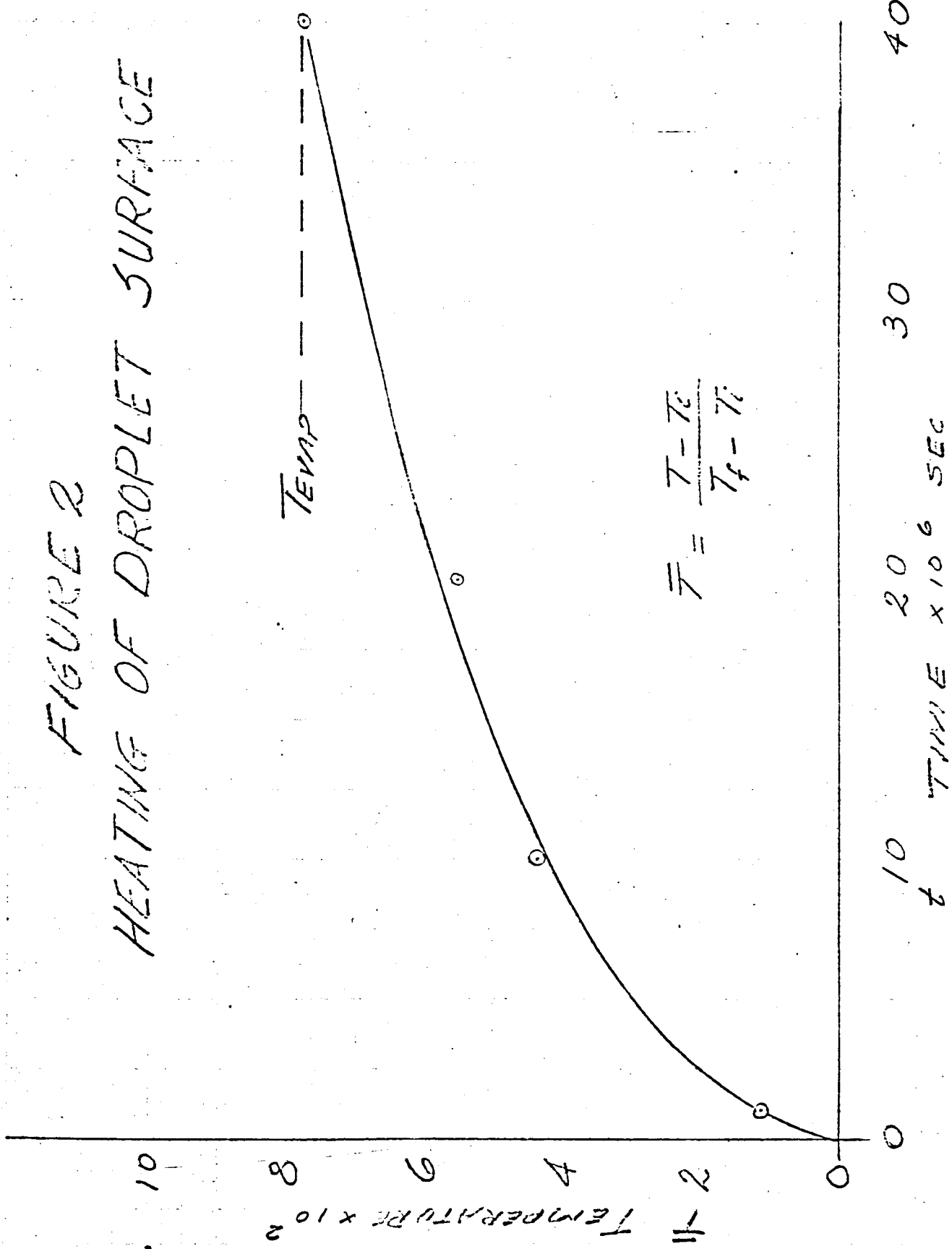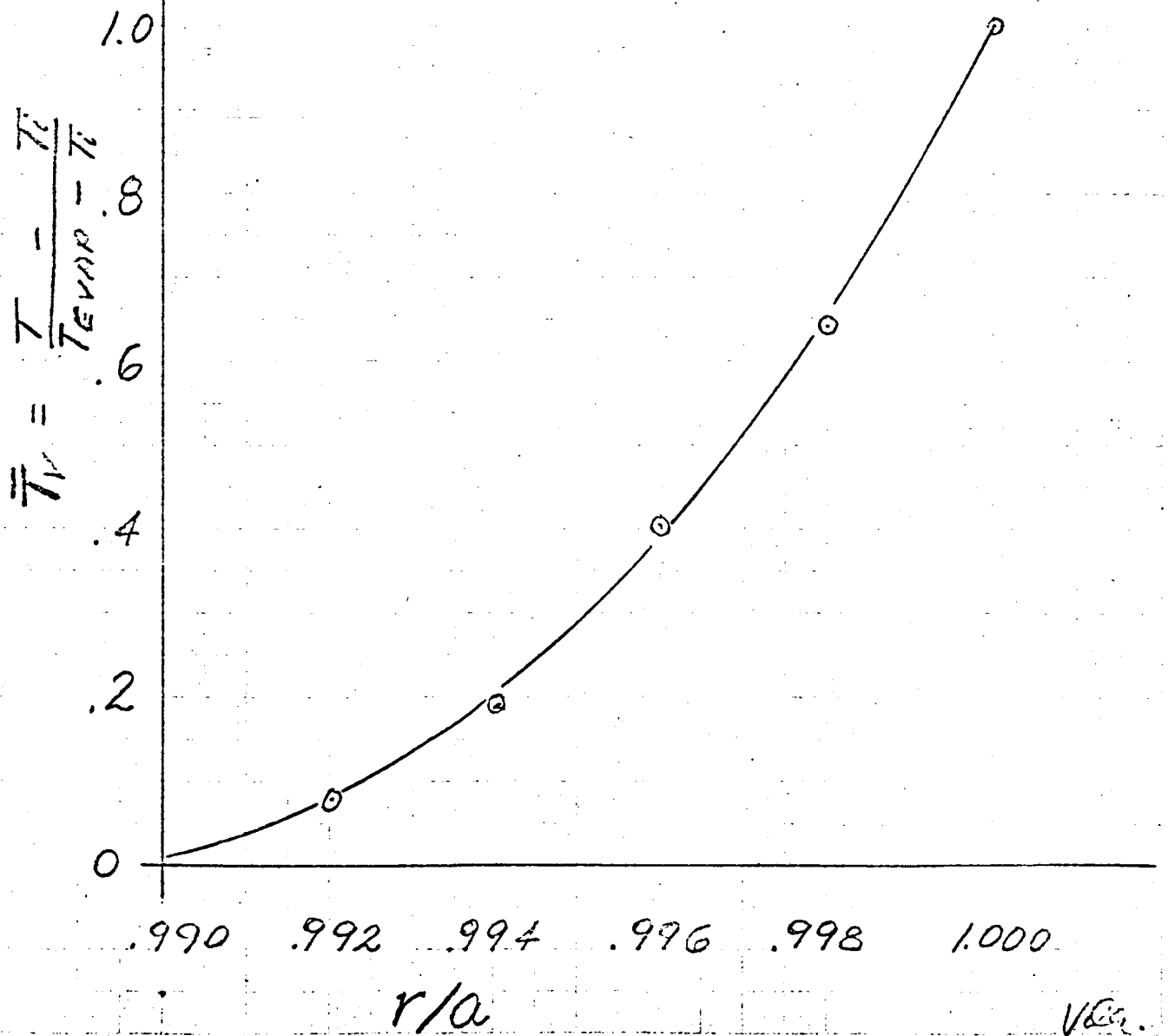
FIGURE 2

HEATING OF DROPLET SURFACE

$$\bar{T} = \frac{T - T_i}{T_f - T_i}$$

Temp

$\bar{T}$ TEMPERATURE $\times 10^2$

t TIME $\times 10^6$ SEC

0   10   20   30   40

0   2   4   6   8   10

FIGURE 3
TEMPERATURE DISTRIBUTION
IN DROPLET

$$\overline{T}_V = \frac{T - T_i}{T_{evap} - T_i}$$

r/a

IV. DESCRIPTION OF THE NON-OSCILLATORY STEADY STATE
    COMPUTER PROGRAM

The computer program, which is presented in this quarterly report, is a modification of an earlier code that was designed to test some basic ideas on the theory of evaporation rate controlled combustion. The earlier version, although successful, only considered evaporation of either fuel or oxidizer. The final solution obtained was dependent on the accuracy of the guess of the rate of the evaporation of the most volatile constituent of the fuel-oxidizer combination. For components which evaporate at approximately the same rate, this program could be used but only in the most awkward manner, i. e. , iterations between fuel trajectories and gas dynamics and oxidizer trajectories and gas dynamics could be handled, but one must also carry out some manipulations of intermediate results manually.

These deficiencies appear to be resolved by the present code. It is designed to solve automatically and simultaneously the interaction between the nonlinear gas dynamic equations and the two components of the spray, fuel and oxidizer. In addition, the effect of the equilibrium dissociating combustion products on the solution is included. An additional optional feature comprising the Weber number effect on fuel and oxidizer (droplet breakup) is included. It must be stated though, that the present version, although essentially complete, is not totally debugged. The final code will be presented in the fourth quarterly report. Its utility will be demonstrated with some solutions pertaining to steady flows in a J. P. L. combustion chamber (Resonant Chamber 1; referenced to drawings D9111263, D9111264, D9111248) using N_2H_4-C_2H_8 as fuel and N_2O_4 as oxidizer.

A glossary of terms will be given when the final version is presented. Although the present version considers the oxidizer ballistics to behave similarly to the fuel ballistics, an additional program will be presented in the fourth quarterly report. It will contain the formulation of oxidizer evaporation as contained in the first and second quarterly reports. This version has the property that it can predict the transient behavior during startup conditions in the combustion chamber. Comparison between these two approaches to oxidizer evaporation will be presented.