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ATMOSPHERIC DENSITIES AND TEMPERATURES FROM THE DRAG  
ANALYSIS OF THE SAN MARCO SATELLITE

by

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ATMOSPHERIC DENSITIES AND TEMPERATURES FROM THE DRAG  
ANALYSIS OF THE SAN MARCO SATELLITE<sup>1</sup>

by

Luigi G. Jacchia<sup>2</sup> and Franco Verniani<sup>3</sup>

13840  
Abstract.--Atmospheric densities have been obtained from the drag analysis of the San Marco Satellite in the interval from December 19, 1964, to February 11, 1965. Temperatures have been computed from the densities using Jacchia's 1964 atmospheric model. The analysis is based on field-reduced Baker-Nunn observations and on Mini-track observations.

*Author*

Introduction

The San Marco Satellite (1964-84a) was launched on December 15, 1964, in an orbit with an eccentricity of 0.045, an inclination of 37.8, and a mean perigee height close to 200 km. It is a nearly spherical object with a diameter of 66 cm and a mass of 115 kg. According to information received from the National Aeronautics and Space Administration, the area-to-mass ratio is 0.032 cm<sup>2</sup>/g.

The San Marco Satellite was launched with the primary purpose of measuring atmospheric density through a dynamometer. Therefore, the densities obtained from the analysis of the atmospheric drag on this satellite are presented here for comparison with those obtained from the measurements. The results are based on the field-reduced Baker-Nunn observations and on Minitrack observations. The analysis covers the period from December 19, 1964, to February 11, 1965, a date close to the end of the active life of the satellite.

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## Densities and temperatures

The methods used to determine orbital accelerations and then to arrive at values of the atmospheric density are essentially those described by Jacchia and Slowey (1962). Densities were computed by numerical integration of Sterne's integral in a form that includes the effect of atmospheric rotation (1959); the value of the drag coefficient  $C_D$  was taken, as before, to be 2.2. Jacchia's last atmospheric model (1964) was used for the integration. The results of the analysis are given in Table 1, which contains the following quantities:

- MJD : time in Modified Julian Days.  $MJD = JD - 2400000.5$ .  
The tabulation is ordinarily at 0.5-day intervals, except during magnetic storms (0.2-day intervals). In the 5-day interval from MJD 38793 to MJD 38798, which was poorly covered by observations, the tabulation is at 1-day intervals.
- $\dot{P}$  : the observed rate of change of the anomalistic period  $P$ .  
Since this satellite has very low perigee heights, the effect of radiation pressure on  $P$  is completely negligible compared to atmospheric drag.
- $\log \rho_{\pi}$  : the decimal logarithm of the atmospheric density in  $g/cm^3$  at perigee height.
- $\log \rho_S$  : the decimal logarithm of the atmospheric density in  $g/cm^3$  at a standard height of 200 km.
- $T_{\pi}$  : the exospheric temperature in  $^{\circ}K$  at the perigee, computed from Jacchia's 1964 model.
- $z$  : the height of the perigee in km.
- $\alpha_{\pi} - \alpha_{\odot}$  : the difference in degrees between the right ascension  $\alpha_{\pi}$  of the perigee and the right ascension  $\alpha_{\odot}$  of the sun.
- $\delta_{\pi} - \delta_{\odot}$  : the difference in degrees between the declination  $\delta_{\pi}$  of the perigee and the declination  $\delta_{\odot}$  of the sun.
- $T_N$  : the nighttime exospheric temperature in  $^{\circ}K$  computed from  $T_{\pi}$  and Jacchia's 1964 model.

The logarithm of the observed atmospheric density at the standard height of 200 km, the logarithm of the density computed from Jacchia's 1964 model, and the residuals  $\log \rho_{\text{obs}} - \log \rho_{\text{comp}}$  are plotted as functions of time in Figure 1, together with the smoothed geomagnetic indices  $\bar{K}_p$  and  $\bar{a}_p$ , the solar decimetric flux  $F_{10.7}$ , and the theoretical curve of the diurnal variation.

The standard deviation of 1 observation is 0.022 in  $\log \rho$ . If we had assumed that all the scatter in the residuals is due to random errors in the determination of the atmospheric density, this figure would correspond to a probable error of 3.5 percent in our determinations. This is obviously not the case: the density variations of the atmosphere could not possibly have been taken into account without error, so the actual relative precision of our density determinations must be greater.

According to our information, experiments involving the extrusion of an antenna should have been performed on the satellite on January 29 (MJD 38789). These experiments may have had the effect of altering the presentation area of the satellite. If so, the effect must have been a minor one because our data do not show any discontinuity on that day.

#### Density variations with geomagnetic activity

During the period covered by this analysis there were several enhancements of geomagnetic activity as shown by the values of  $\bar{K}_p$  and  $\bar{a}_p$  (see Fig. 1). As usual, similar variations can be recognized in the density curve. In order to determine the time lag between the increase in the magnetic activity and the subsequent increase of the atmospheric density, we have plotted the 3-hour values of  $K_p$  and  $a_p$  and then drawn a smoothed curve directly comparable with the curve of the density, where the actual resolution in time is generally smaller than 3 hours. To increase the accuracy of the detection of the time of occurrence of the density peaks, we took these times directly from the mean anomaly curve and compared them with the times of occurrence of peaks in the smoothed curve for  $K_p$ . The result is an average time lag

(at the height of 200 km and at low latitudes) of 9 hours with a probable error of 1 or 2 hours. This lag is a little larger than the 5-hour lag determined by Jacchia and Slowey (1964) from a precision analysis of the Explorer IX Satellite and confirmed by Roemer (1965) on the basis of an extension of the analysis for the same satellite. But since the lag was derived from very limited data, the difference should not be taken too seriously. A more extensive survey of the time lag is in progress, involving satellites of different perigee heights and inclinations.

The mean of the residuals in  $\log \rho$  is  $-0.035$ ; in other words, the "observed" densities are systematically smaller than those of Jacchia's 1964 model by 8 percent. Since this discrepancy is within the margin of error caused by the uncertainties in the assumed drag coefficient and the area-to-mass ratio of the satellite, it should not necessarily be taken to indicate a real departure from the model atmosphere.

#### Orbital elements

As a preliminary step in the determination of the atmospheric drag, orbital elements were computed for the satellite from 4-day arcs at 1-day intervals. The time lapse covered by the observations was then divided into 2 intervals of the length of 33 and 32 days, respectively, with a 12-day overlap, and analytical functions were fitted by least squares to the elements within the 2 intervals. Table 2 gives the results of these least-squares fittings. It should be remembered that, while the expressions for  $\omega$ ,  $\Omega$ ,  $i$ , and  $e$  can be considered as representing the elements without systematic residuals, the same is not true of  $M$ , in which systematic residuals up to  $10^{-3}$  revolutions are tolerated.

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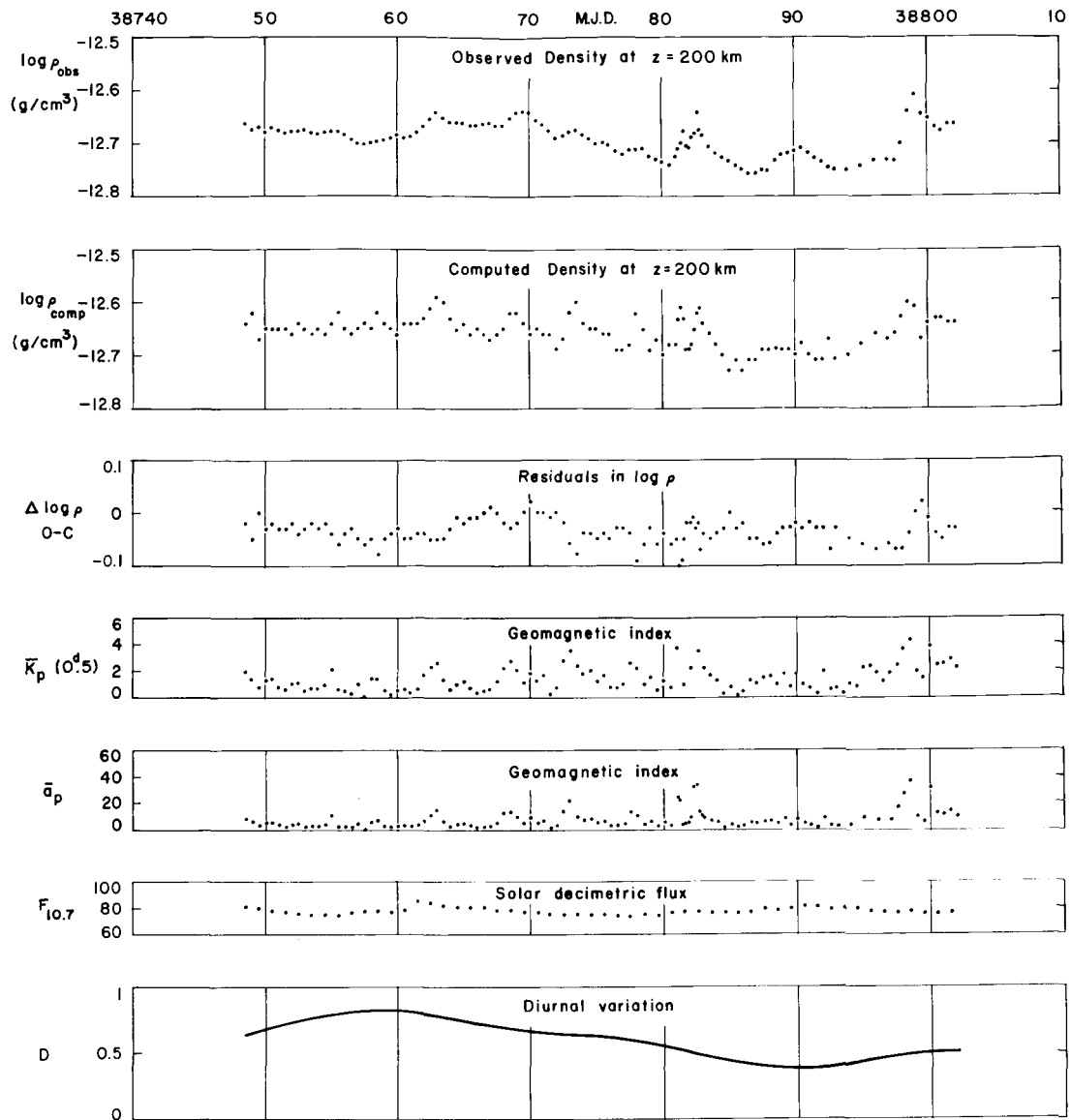


Figure 1.--Atmospheric densities determined from the orbital drag of Satellite 1964-84A (San Marco). Observed densities (top section) are compared with densities computed from Jacchia's 1964 model, taking into account the observed values of the geomagnetic index and the 10.7-cm solar flux, the diurnal factor, and the semiannual effect; residuals are plotted in the third section. MJD is the Modified Julian Day (JD minus 2,400,000.5). The half-day mean of the 3-hourly  $K_p$  index is  $\bar{K}_p(0.5)$ . The mean of the 3-hourly  $a_p$  index is  $\bar{a}_p$  taken over an interval equal to the resolution in the drag. The 10.7-cm solar flux  $F_{10.7}$  is in units of  $10^{-22}$  watts/m<sup>2</sup>/cycle/sec bandwidth.



Table 1.--Accelerations, atmospheric densities, atmospheric temperatures, and geometric parameters (see text for full explanation of the symbols)

MJD	$10^{-6} \ddot{P}$	$\log \rho_{\pi}$	$\log \rho_s$	$T_{\pi}$	$z$	$\alpha_{\pi} - \alpha_{\odot}$	$\delta_{\pi} - \delta_{\odot}$	$T_N$
38748.5	11.01	-12.700	-12.664	761	202.1	85.2	45.9	645
49.0	10.76	.704	.675	749	201.7	85.3	43.8	633
49.5	11.02	.693	.670	754	201.3	85.3	41.6	635
50.0	10.74	.699	.681	742	201.0	85.2	39.3	623
50.5	11.11	.686	.672	752	200.8	85.0	37.0	630
51.0	10.98	.688	.677	746	200.6	84.8	34.6	623
51.5	10.83	.691	.682	741	200.5	84.5	32.2	617
52.0	10.97	.686	.678	746	200.5	84.2	29.8	619
52.5	10.93	.687	.678	746	200.6	83.9	27.3	617
53.0	10.92	.688	.676	748	200.7	83.5	24.8	617
53.5	10.67	.697	.680	743	201.0	83.1	22.3	611
54.0	10.46	.705	.683	740	201.3	82.8	19.8	607
54.5	10.37	.710	.681	742	201.7	82.4	17.3	607
55.0	10.27	.716	.679	744	202.1	82.1	14.8	608
55.5	10.13	.724	.678	746	202.7	81.8	12.3	608
56.0	9.69	.743	.685	738	203.3	81.6	9.9	601
56.5	9.20	.765	.694	728	204.0	81.5	7.5	592
57.0	8.74	.787	.702	720	204.7	81.5	5.2	585
57.5	8.52	.800	.702	720	205.4	81.5	2.9	584
58.0	8.31	.814	.702	720	206.1	81.7	0.7	585
58.5	8.18	.824	.698	724	206.9	81.9	-1.4	588
59.0	8.08	.833	.694	728	207.7	82.3	-3.4	591
59.5	7.97	.842	.690	732	208.4	82.9	-5.3	595
60.0	7.87	.851	.687	736	209.1	83.6	-7.1	598
60.5	7.63	.866	.689	734	209.8	84.4	-8.7	597
61.0	7.50	.876	.687	735	210.4	85.4	-10.2	599
61.5	7.55	.877	.680	742	210.9	86.5	-11.5	606
62.0	7.74	.871	.670	754	211.4	87.8	-12.6	617
62.5	8.09	.859	.655	770	211.8	89.2	-13.5	631
63.0	8.45	.846	.642	785	212.0	90.7	-14.2	645
63.5	8.07	.863	.653	774	212.2	92.3	-14.7	636
64.0	7.76	.877	.662	763	212.3	93.9	-15.0	629
64.5	7.80	.876	.661	764	212.3	95.5	-15.0	631
65.0	7.82	.874	.662	763	212.2	97.1	-14.9	632
65.5	7.76	.876	.666	758	212.0	98.7	-14.5	629
66.0	7.89	.868	.665	759	211.7	100.2	-13.8	632
66.5	8.07	.859	.663	761	211.3	101.6	-13.0	635
67.0	8.26	.848	.662	763	210.8	102.8	-11.9	637
67.5	8.36	.841	.664	760	210.2	104.0	-10.7	636
68.0	8.53	.831	.665	759	209.6	105.0	-9.3	637
68.5	9.24	.800	.650	776	208.9	105.8	-7.8	652
69.0	9.83	.775	.641	788	208.2	106.5	-6.1	663
69.5	10.17	.760	.638	791	207.4	107.0	-4.3	666
70.0	10.32	.750	.641	787	206.6	107.4	-2.4	664
70.5	10.09	.753	.655	771	205.8	107.7	-0.3	651
71.0	10.10	.748	.662	763	205.1	107.8	1.8	645
71.5	9.95	.748	.674	749	204.3	107.9	3.9	634
72.0	9.78	.750	.688	735	203.6	107.8	6.2	623

Table 1.--Continued.

MJD	$10^{-6} \dot{p}$	$\log \rho_{\pi}$	$\log \rho_s$	$T_{\pi}$	$z$	$\alpha_{\pi} - \alpha_{\odot}$	$\delta_{\pi} - \delta_{\odot}$	$T_N$
38772.5	10.08	-12.736	-12.686	737	202.9	107.7	8.5	625
73.0	10.57	.717	.678	745	202.2	107.5	10.8	633
73.5	10.83	.706	.676	748	201.8	107.2	13.2	636
74.0	10.79	.704	.683	740	201.2	106.9	15.6	629
74.5	10.72	.704	.691	731	200.8	106.6	18.0	623
75.0	10.57	.708	.701	721	200.4	106.2	20.4	615
75.5	10.77	.700	.699	723	200.1	105.9	22.8	618
76.0	10.60	.706	.708	714	199.9	105.5	25.2	611
76.5	10.43	.711	.716	706	199.7	105.2	27.6	605
77.0	10.36	.714	.720	702	199.7	104.9	30.0	602
77.5	10.58	.708	.714	708	199.7	104.6	32.3	608
78.0	10.64	.708	.712	710	199.8	104.4	34.6	611
78.5	10.68	.709	.711	711	199.9	104.3	36.8	614
79.0	10.21	.727	.725	698	200.1	104.2	39.0	603
79.5	10.00	.737	.730	693	200.4	104.3	41.1	600
80.0	9.76	.749	.736	687	200.7	104.4	43.1	597
80.5	9.57	.759	.740	683	201.0	104.7	45.1	595
81.0	9.88	.751	.726	696	201.3	105.1	46.9	608
38781.2	10.32	-12.737	-12.711	711	201.5	105.3	47.6	622
81.4	10.62	.729	.700	722	201.6	105.5	48.3	631
81.6	11.35	.708	.678	745	201.8	105.7	48.9	653
81.8	10.36	.740	.706	716	201.9	106.0	49.6	628
82.0	10.23	.745	.709	713	202.0	106.3	50.2	626
82.2	10.85	.727	.689	733	202.2	106.6	50.8	645
82.4	11.05	.722	.682	741	202.3	106.9	51.4	652
82.6	12.42	.684	.644	783	202.4	107.2	51.9	690
82.8	11.22	.719	.675	748	202.6	107.6	52.4	660
38783.0	10.80	-12.733	-12.686	736	202.7	108.0	52.9	650
83.5	10.06	.759	.706	716	203.0	109.1	54.0	634
84.0	9.60	.777	.719	703	203.2	110.3	55.0	625
84.5	9.30	.790	.727	696	203.4	111.6	55.7	619
85.0	9.09	.798	.733	690	203.6	113.0	56.2	616
85.5	8.80	.810	.742	681	203.7	114.5	56.5	610
86.0	8.60	.819	.749	675	203.7	116.1	56.6	605
86.5	8.43	.826	.755	669	203.7	117.6	56.5	601
87.0	8.42	.825	.756	668	203.7	119.2	56.1	601
87.5	8.60	.817	.751	673	203.5	120.7	55.5	606
88.0	8.66	.814	.751	673	203.3	122.1	54.7	607
88.5	9.23	.790	.733	690	203.1	123.4	53.7	623
89.0	9.62	.774	.723	700	202.8	124.6	52.5	632
89.5	9.84	.764	.719	704	202.5	125.7	51.1	636
90.0	10.05	.754	.715	707	202.2	126.6	49.6	639
90.5	10.34	.742	.710	712	201.8	127.4	47.9	644
91.0	10.17	.744	.718	704	201.4	128.1	46.0	635
91.5	9.98	.748	.728	694	201.1	128.6	44.0	626
92.0	9.89	.748	.735	688	200.7	129.0	41.9	620
92.5	9.70	.751	.745	679	200.3	129.3	39.7	611

Table 1.--Continued.

MJD	$10^{-6} \dot{P}$	$\log \rho_{\pi}$	$\log \rho_S$	$T_{\pi}$	$z$	$\alpha_{\pi} - \alpha_{\odot}$	$\delta_{\pi} - \delta_{\odot}$	$T_N$
38793.0	9.67	-12.750	-12.749	675	200.0	129.4	37.4	607
94.0	9.78	.741	.750	674	199.5	129.4	32.6	604
95.0	10.06	.728	.743	681	199.2	129.1	27.5	607
96.0	10.34	.717	.733	690	199.2	128.6	22.3	613
97.0	10.22	.722	.733	690	199.4	128.0	16.9	611
38797.5	10.12	-12.727	-12.733	690	199.7	127.7	14.2	610
98.0	10.97	.702	.702	720	200.0	127.4	11.6	635
98.5	13.07	.647	.641	788	200.4	127.1	8.9	694
99.0	14.19	.624	.611	825	200.9	126.9	6.2	726
99.5	12.52	.668	.646	782	201.4	126.7	3.6	687
38800.0	12.02	-12.685	-12.653	773	202.0	126.6	1.0	678
00.5	11.35	.709	.666	759	202.6	126.5	-1.6	665
01.0	10.74	.732	.677	746	203.2	126.6	-4.1	654
01.5	10.95	.730	.665	759	203.9	126.7	-6.5	665
02.0	10.69	.743	.667	758	204.5	127.0	-8.8	664

Table 2.--Least-squares fitting of orbital elements for the San Marco Satellite;  $t$  = time in MJD;  $\omega$  = argument of perigee;  $\Omega$  = right ascension of the node;  $i$  = inclination;  $e$  = eccentricity;  $M$  = mean anomaly in node revolutions;  $\tau = t - t_0$ .

Section 1: MJD 38747 to 38780 (December 18, 1964, to January 20, 1965)

$$T_0 = 38747$$

$$\omega = 129.834 + 8.10353 \tau + 0.001555 \tau^2 + 0.738 \sin (226.97 + 8.19 \tau)$$

$$\Omega = 214.0175 - 6.053164 \tau - 8.402 \times 10^{-4} \tau^2 - 2.01 \times 10^{-6} \tau^3$$

$$i = 37.7970$$

$$e = 0.044696 - 1.1747 \times 10^{-4} \tau + 7.14 \times 10^{-7} \tau^2 + 4.16 \times 10^{-4} \sin (144.39476 + 8.194 \tau)$$

$$M = 0.5972 + 15.187150 \tau + 1.7866 \times 10^{-3} \tau^2 - 4.7484 \times 10^{-5} \tau^3 + 1.1727 \times 10^{-6} \tau^4 - 9.30 \times 10^{-9} \tau^5$$

Section 2: MJD 38768 to 38802 (January 8, 1965, to February 11, 1965)

$$T_0 = 38768$$

$$\omega = 301.015 + 8.1103 \tau + 3.071 \times 10^{-3} \tau^2 + 0.765 \sin (15.94 + 8.19 \tau)$$

$$\Omega = 86.5132 - 6.09070 \tau - 1.0134 \times 10^{-3} \tau^2 - 4.41 \times 10^{-7} \tau^3$$

$$i = 37.7963$$

$$e = 0.042866 - 1.132 \times 10^{-4} \tau + 5.12 \times 10^{-7} \tau^2 + 6.55 \times 10^{-4} \sin (297.50361 + 8.194 \tau)$$

$$M = 0.06515 + 15.232844 \tau + 1.22202 \times 10^{-3} \tau^2 + 6.0678 \times 10^{-6} \tau^3 - 4.4200 \times 10^{-7} \tau^4 + 7.0482 \times 10^{-9} \tau^5.$$

Errata to Special Report No. 193

Table 2. The fourth line of the heading should read  
"e = eccentricity; M = mean anomaly in revolutions;"  
In the table under Section 1 and Section 2  
"T<sub>0</sub>" should read "t<sub>0</sub>."