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GPO PRICE $ ______

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Hard copy (HC) 3.00

Microfiche (MF) .50

ff 653 July 65

FACILITY FORM 602

N66-14956 (ACCESSION NUMBER)

(THRU) 1

60 (PAGES)

(CODE) 07

CR 69207 (NASA CR OR TMX OR AD NUMBER)

(CATEGORY)

Determination of RMS Height of a Rough Surface Using Radar Waves

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Grant Number NsG-213-61

1388-19 31 August 1965

Prepared for:
National Aeronautics and Space Administration
Office of Grants and Research Contracts
Washington, D. C. 20546

## Department of ELECTRICAL ENGINEERING

THE OHIO STATE UNIVERSITY
RESEARCH FOUNDATION
Columbus, Ohio

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REPORT 1388-19

REPORT
by
THE OHIO STATE UNIVERSITY RESEARCH FOUNDATION
COLUMBUS, OHIO 43212

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|---|---|
| Sponsor | National Aeronautics & Space Administration<br>Office of Grants and Research Contracts<br>Washington, D. C. 20546 |
| Grant Number | NsG-213-61 |
| Investigation of | Theoretical and Experimental Analysis of the Electromagnetic Scattering and Radiative Properties of Terrain, with Emphasis on Lunar-Like Surfaces |
| Subject of Report | Determination of RMS Height of a Rough Surface Using Radar Waves |
| Submitted by | Donald E. Barrick<br>Antenna Laboratory<br>Department of Electrical Engineering |
| Date | 31 August 1965 |

The material contained in this report is also used as a dissertation submitted to the Department of Electrical Engineering, The Ohio State University as partial fulfillment for the degree Doctor of Philosophy.

# ABSTRACT

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An experiment is described which can be used to determine the mean square height of a rough surface in terms of the correlation between two backscattered waves at different frequencies as a function of frequency separation. This scheme is analyzed both for rough planar surfaces and rough spherical surfaces (e. g., planetary surfaces). Thus far, no experiment involving radar has been suggested which can measure the rms surface roughness height; only the rms surface slope can be estimated from radar measurements. The scheme is simple: a continuous radar carrier is amplitude modulated by a low frequency signal and the correlation between the two sidebands is measured as a function of the modulating frequency. In the analysis, the backscattering cross section of a large rough sphere is derived also.

The material for this report was used as Chapter IV of a dissertation by the author entitled "A More Exact Theory for Scattering of Electromagnetic Waves from Statistically Rough Surfaces". This accounts for the discontinuities in page numbering. Any equations referred to here having numbers before 4.1 may be found either in the dissertation or in Report 1388-18. Author

# CHAPTER IV
# AN EXPERIMENT YIELDING THE MEAN SQUARE HEIGHT, $\sigma^2$, OF A ROUGH SURFACE

## A. A Planar Rough Surface

### 1. Complex correlation coefficient of scattered fields at two frequencies

It has been shown in the preceding part that the predicted backscattered power from a planar rough surface (oriented as in Fig. 5) is a function only of the mean square surface slope, $S^2$, as a roughness parameter of the surface when the Gaussian and Bessel JPDF models are used in conjunction with the Gaussian class of correlation coefficients. Furthermore, as discussed previously, this mean square slope which is visible on the surface at a certain wavelength is a function of wavelength and increases as the wavelength decreases. The fact that backscattered power measurements can yield no information about the mean square height of the surface roughness, $\sigma^2$, is somewhat discouraging. Therefore, it would be most desirable to be able to find this mean square height parameter, $\sigma^2$, for an unknown rough surface by radar measurements.

There are two main reasons why the mean square height, $\sigma^2$, is sought.

(i) Mean square surface height $\sigma^2$, along with mean square surface slope $S^2$ both yield a quite complete picture of the type of rough surface under study. Either parameter alone is relatively meaningless in obtaining an idea of the roughness.

(ii) Mean square surface height $\sigma^2$, unlike mean square surface slope, is for all practical purposes independent of the examining wavelength; the effect of the addition of the height variation of the smaller scale surface structures, visible at decreased wavelengths, on the overall surface mean square height is very small. The mean square height of the surface remains essentially the mean square height of the largest scale structure.

It was mentioned in connection with equation (2.38) that such a general method of determination of mean square height from statistical properties of the scattered waves alone requires that wavelength of the measuring waves be both higher and lower (generally in the same order of magnitude) than the expected rms height, $\sigma$, of the surface. This requirement poses two difficulties.

(i) When wavelength is of the same order of magnitude as the rms surface height, the applicability of the physical optics approximation is questionable. Without the use of the physical optics approximation, the integral equation for the scattered field appears insoluble when one wishes to find a closed, general form of the solution in terms of the various statistical parameters.

(ii) Measurement of planetary surfaces with wavelengths of the order of magnitude of expected surface heights is nearly impossible as far as the instrumentation is concerned due to the low antenna gains at larger wavelength.

Due to the above difficulties of obtaining measurements from the surfaces at such low frequencies, the following experiment has been suggested. Illuminate the surface with two waves at two different frequencies, $f_1$ and $f_2$; keep the two frequencies both high enough (in the radar range) so that measurement is convenient and so that the physical optics approximation applies for at least the largest scale surface roughness. Then vary the frequency separation, $\Delta f = f_1 - f_2$, and measure the correlation between the signals at the two frequencies, $f_1$ and $f_2$, for an ensemble of such rough surfaces. (In measurement of correlation both amplitude and phase of the two signals will be preserved.) At very small frequency separation, it might be felt that the correlation should be nearly perfect (i. e., unity). However, as frequency separation $\Delta f$ increases to the point where "separation wavelength," i. e., $\lambda_s = \frac{C}{\Delta f}$ becomes of the same order of magnitude as surface heights, or $\sigma$, then the correlation should intuitively begin to decrease. Thus, the separation wavelength, $\lambda_s$, at which correlation begins to decrease should provide some measure of rms surface roughness height, $\sigma$.

Such an experiment has the advantages of using convenient radar frequencies at which physical optics may be applied. The frequency separation, $\Delta f$, can be produced by simply amplitude modulating the radar carrier; the correlation between the two returning may then be measured.

The analysis and theoretical prediction for the correlation coefficient will now be undertaken employing the Gaussian and Bessel JPDF statistical models. The analysis will be made for backscattering from a perfectly conducting surface at first, but the results will apply equally well to a non-perfectly conducting surface, since the integral to be evaluated in both cases has been shown to be the same. The wave numbers of the two frequencies are $k_1 = \frac{2\pi}{\lambda_1} = \frac{2\pi f_1}{C}$ and $k_2 = \frac{2\pi}{\lambda_2} = \frac{2\pi f_2}{C}$ ; therefore $\Delta f = k_1 - k_2 = \frac{2\pi\Delta f}{C} = \frac{2\pi}{\lambda_s}$ . The covariance between two backscattered complex waves from a surface at these two frequencies has been derived in equation (2. 2b). It is repeated here, neglecting the second term of the integrand which is zero for very rough surfaces.

$$(4.1) \quad \mathrm{Cov}[H_1^s H_2^{s*}] = \frac{k_1 k_2 e^{j\Delta k R_o} L_x L_y H_{01}^i H_{02}^i}{2\pi R_o^2} \left( \frac{\sin(\Delta k \sin\theta L_y)}{(\Delta k \sin\theta L_y)} \right) \cdot \cos^2\theta$$

$$\times \int_0^\infty \rho J_o(2k_1 \sin\theta\rho) M_{\zeta\zeta'}(j2k_1 \sec\theta, -j2k_2 \sec\theta;\rho)\, d\rho \quad .$$

a. Gaussian JPDF

When the Gaussian JPDF statistical model is employed, the joint characteristic function is $M_{\zeta\zeta'}(j2k_1 \sec\theta, -j2k_2 \sec\theta;\rho) = e^{-\frac{1}{2}[4k_1^2 \sec^2\theta\sigma^2 - 8k_1 k_2 \sec^2\theta\sigma^2 R(\rho) + 4k_2^2 \sec^2\theta\sigma^2]}$ .

Upon re-arranging, this becomes

$$(4.2) \quad M_{\zeta\zeta'}(j2k_1 \sec\theta, -j2k_2 \sec\theta;\rho) = e^{-2\sigma^2 \Delta k^2 \sec^2\theta} \cdot e^{-4\sigma^2 k_1 k_2 \sec^2\theta(1-R(\rho))} \quad .$$

The first factor is independent of $\rho$ and can be removed from the integrand. The second factor is almost identical to the previous joint characteristic function at a single frequency except that $k^2$ is replaced by $k_1 k_2$. By the same argument as was used before, only the first two terms of the series for $R(\rho)$ are significant because $\sigma^2 k_1 k_2$ is very large. Thus the integral becomes

$$(4.3) \quad \mathrm{Cov}[\, H_1^s H_2^{s*} \,] = \frac{k_1 k_2 e^{j\Delta k R_o} L_x L_y H_{01}^i H_{02}^i}{2\pi R_o^2} \left( \frac{\sin(\Delta k \sin\theta L_y)}{\Delta k \sin\theta L_y} \right) \cos^2\theta$$

$$\times e^{-2\sigma^2 \Delta k^2 \sec^2\theta} \times \begin{cases} \displaystyle\int_0^\infty \rho J_o(2k_1 \sin\theta\rho)\, e^{-4\sigma^2 k_1 k_2 \sec^2\theta \frac{\rho^2}{a^2}}\, d\rho & \text{for the Gaussian class correlation coefficient} \\ \displaystyle\int_0^\infty \rho J_o(2k_1 \sin\theta\rho)\, e^{-4\sigma^2 k_1 k_2 \cos^2\theta \frac{\rho}{a}} & \text{for the exponential class correlation coefficient .} \end{cases}$$

The factor which contains the information sought here is $e^{-2\sigma^2\Delta k^2 \sec^2\theta} = e^{-8\pi^2 \sec^2\theta \frac{\sigma^2}{\lambda_s^2}} = e^{-8\pi^2 \sec^2\theta\sigma^2 \frac{\Delta f^2}{C^2}}$ ; it is independent of the form of the surface correlation coefficient chosen. This factor shows that as $\Delta k$ increases, the covariance decreases accordingly. Theoretically, at $\Delta k = \frac{1}{\sqrt{2}\ \sigma \sec\theta}$ , the covariance has fallen to $\frac{1}{e}$ of its initial value at $\Delta k = 0$.

If the Gaussian class correlation coefficient is chosen to model the surface, the integral may be evaluated and the resulting covariance is:

$$(4.4) \quad \mathrm{Cov}[\, H_1^s H_2^{s*} \,] = \frac{H_{01}^i H_{02}^i}{4\pi R_o^2} \cdot \frac{L_x L_y}{S^2} \cdot \frac{k_1}{k_2} \cdot$$

$$e^{j\Delta k R_o} \left( \frac{\sin(\Delta k \sin\theta L_y)}{(\Delta k \sin\theta L_y)} \right) \cos^4\theta\, e^{-\frac{1}{S^2}\frac{k_2}{k_2}\sin^2\theta\cos^2\theta} \times e^{-2\sigma^2\Delta k^2 \sec^2\theta} .$$

The correlation between the scattered fields at the two frequencies is defined as follows:

$$\mathrm{Cor}[H_1^s H_2^{s*}] = \frac{\mathrm{Cov}[H_1^s H_2^{s*}]}{\sqrt{\mathrm{Var}[H_1^s]}\sqrt{\mathrm{Var}[H_2^s]}} ,$$

where the variance of the scattered field at a single frequency, denoted $\mathrm{Var}[H_1^s]$ is obtained from the covariance by setting $k_2 = k_1$ and $\Delta k = 0$. Thus the correlation, determined in this manner, becomes

$$\mathrm{Cor}[H_1^s H_2^{s*}] = \frac{k_1}{k_2}\, e^{j\Delta k R_o}\left(\frac{\sin(\Delta k \sin\theta\, L_y)}{(\Delta k \sin\theta\, L_y)}\right) e^{-\frac{1}{S^2}\frac{\Delta k}{k_2}\sin^2\theta\cos^2\theta}\left[e^{-2\sigma^2\Delta k^2\sec^2\theta}\right] . \tag{4.5}$$

If measurements are made at radar frequencies and frequency separation or modulation frequency is well below these radar frequencies, then $\Delta k << k_1 k_2$ , $\frac{k_1}{k_2} \simeq 1$ , and $\frac{\Delta k}{k_2} << 1$, so that the exponential factor containing $\frac{\Delta k}{k_2}$ is essentially always unity. At normal incidence, $\theta = 0$, and the diffraction pattern factor disappears; if such is the case, the correlation becomes

$$\mathrm{Cor}[H_1^s H_2^{s*}] = e^{j\Delta k R_o}\,[e^{-2\sigma^2\Delta k^2}] . \tag{4.6}$$

Thus the exponential in brackets, upon which the magnitude of the correlation depends, can be used to determine the mean square height, $\sigma^2$, of the surface when the correlation has been determined for various frequency separations, $\Delta k$.

b. Bessel JPDF

When the Bessel JPDF statistical model is employed, the joint characteristic function given in equation (3. 5d) is used, where $u = -2k_1 \sec\theta$ and $v = +2k_2 \sec\theta$; upon substitution of this joint characteristic function into (4. 1), the integration is performed and the result is simplified (the details are given in Appendix D). The covariance is

$$(4.7)\quad \mathrm{Cov}[H_1^s H_2^{s*}] = \frac{H_{01}H_{02}}{4\pi R_o^2}\cdot\frac{3L_xL_y}{S^2}\cdot\frac{k_1}{k_2}\, e^{j\Delta k R_o}\left(\frac{\sin(\Delta k \sin\theta L_y)}{(\Delta k \sin\theta L_y)}\right)\cos^4\theta\cdot e^{-\frac{\sqrt{6}}{S}\sin\theta\cos\theta}\times\left[\frac{1}{\left(1+\frac{4}{3}\sigma^2\Delta k^2\sec^2\theta\right)^{3/2}}\right].$$

The correlation is found in the same manner as before, and is

$$(4.8)\quad \mathrm{Cov}[H_1^s H_2^{s*}] = \frac{k_1}{k_2}\, e^{j\Delta k R_o}\left(\frac{\sin(\Delta k \sin\theta L_y)}{(\Delta k \sin\theta L_y)}\right)\cdot\left[\frac{1}{\left(1+\frac{4}{3}\sigma^2\Delta k^2\sec^2\theta\right)^{3/2}}\right].$$

At normal incidence, the above equation reduces to

$$(4.9)\quad \mathrm{Cov}[H_1^s H_2^{s*}] = e^{j\Delta k R_o}\left[\frac{1}{\left(1+\frac{4}{3}\sigma^2\Delta k^2\right)^{3/2}}\right].$$

As can be seen by comparison of (4.9) with (4.6), the expressions in brackets are quite similar in form as functions of $\sigma^2 \Delta k^2$, especially where the magnitude of the correlation is between 0.5 and unity. This is shown graphically in Fig. 21. This indicates that the statistical model has very little to do with the shape of the correlation curve in this region as a function of $\Delta k$.

To predict the mean squre height of the surface roughness, one would measure the correlation between the two reflected signals; when it fell to about 0.5, he would use the curves to find that $\sigma_{Gaussian} = \frac{\sqrt{0.585}}{\Delta k}$ or $\sigma_{Bessel} = \frac{\sqrt{0.665}}{\Delta k}$, depending upon which statistical model he wished to use. The percentage deviation in resulting rms height due to the choice of a particular statistical model in this case (where correlation = 0.5) is about 6.2 %, which is surprisingly low. If, for example, one could determine the rms height of the lunar surface roughness to within 6.2%, one would value his information quite highly. This serves to illustrate that such predictions for rms surface height should be quite insensitive to the form of the statistical model chosen.

Another use of such a correlation between the two signals should be pointed out. It should be noted that the difference between the two correlation curves for larger values of frequency separation, $\Delta f$, becomes more pronounced, especially if the curves had been plotted in decibels. The shape of each curve, as shown previously,

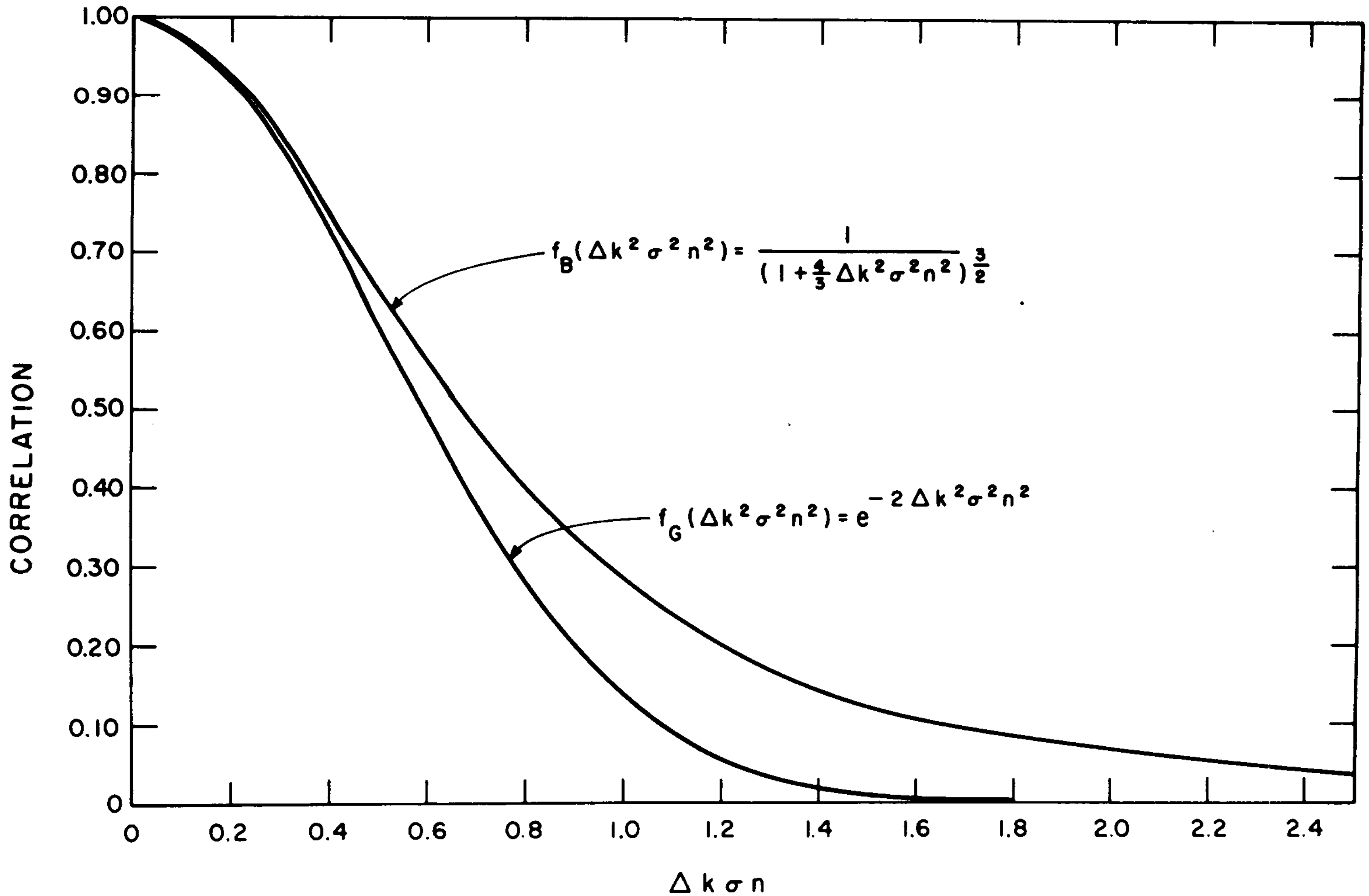


Fig. 21--Correlation coefficients of backscattered fields at two frequencies for two statistical models.

depends solely upon the form of the JPDF chosen to represent the surface and is independent of the surface correlation coefficient. Hence the curve for the Gaussian JPDF model is valid equally for the Gaussian class correlation coefficient as well as the exponential class correlation coefficient. This fact suggests the use of this experiment to obtain information about the JPDF of the rough surface. If, for example, the measured curve for correlation is closer to the predicted curve for the Gaussian JPDF model, one might assume that the true surface JPDF is close to Gaussian. With more investigation, it should be possible to determine almost uniquely the true surface JPDF from such correlation measurements, provided accurate enough measurements can be made at higher values of $\Delta f$.

2. A planar rough surface-real measurable correlation coefficient of scattered fields at two frequencies

The complex correlation coefficient discussed in the preceding subsection suffers from its lack of physical measurability. In a real world, how does one measure a so-called complex field? All fields are real quantities and antennas relate real time varying voltages at the antenna terminals to the real time varying field vectors producing these voltages. Hence, which real voltage quantities does one measure in order to obtain a functional dependence upon frequency separation, $\Delta f$, such as that in brackets in (4.6) or (4.9)? Various persons have attempted to measure a correlation between the demodulated amplitudes of two signals at separate frequencies reflected from the lunar surface ( Pettengill[ 20] ); such a correlation is proportional to the covariance $< |H_1^S| \, |H_2^S| >$, and not to $< H_1^S H_2^{S*} >$ . Thus such correlation curves do not contain the same information in the desired form, and should not be compared with the curves of Fig. 21.

A typical system is shown in Fig. 22 which can yield a pertinent correlation function. The arbitrary angle $\alpha$ (considered constant with respect to time) represents all the accumulative phase shift difference between the two channels introduced by the entire system. The magnitude voltages $v_1$ and $v_2$ are proportional to $|H_1^S|$ and $|H_2^S|$ respectively.

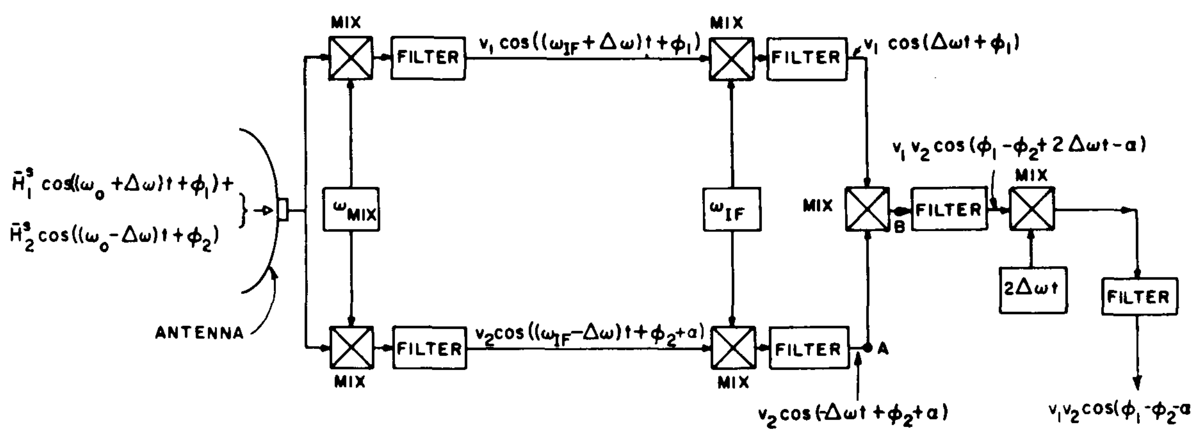


Fig. 22--System providing signals yielding correlation between scattered fields at two frequencies.

The following relationships then hold true; from consideration of (4. 6) (considering first the case of the perfectly conducting surface)

$$(4.10a) \qquad v_1 \cos((\omega_o + \Delta\omega)t + \phi_1) = \mathrm{Re}[\overline{H}_1^s(P')]$$

$$= \frac{k_1}{2\pi R_o} \left|\overline{H}_{01}^i\right| \iint_{D_1} \cos((\omega_o + \Delta\omega)t + k_1 R_o - 2k_1\zeta_1)\, dx_1\, dy_1$$

$$(4.10b) \qquad v_2 \cos((\omega_o - \Delta\omega)t + \phi_2) = \mathrm{Re}[\overline{H}_2^s(P')]$$

$$= \frac{k_2}{2\pi R_o} \left|\overline{H}_{02}^i\right| \iint_{D_1} \cos((\omega_o - \Delta\omega)t + k_2 R_o - 2k_2\zeta_1)\, dx_1\, dy_1 ,$$

where $\qquad \omega_1 = \omega_o + \Delta\omega \;\; ; \; \omega_2 = \omega_o - \Delta\omega \;\; ; \;\; \therefore \; k_1 = \dfrac{\omega_1}{C} \;\; , \; k_2 = \dfrac{\omega_2}{C}$

$$\Delta k = k_1 - k_2 = \frac{\omega_1 - \omega_2}{C} \qquad .$$

These equations are valid to within a constant phase angle which is removed from each for convenience. Hence, the signal at the output is given by

$$(4.11a) \qquad v_1 v_2 \cos(\phi_1 - \phi_2 - \alpha) = \frac{k_1 k_2}{4\pi^2 R_o^2} \left| H_{01}^i \right| \left| H_{02}^i \right|$$

$$\iiiint_{D_1} \cos[\Delta k R_o - 2k_1\zeta_1 + 2k_2\zeta_1' - \alpha]\, dx_1\, dy_1\, dx_1'\, dy_1'$$

This equation can be rewritten in exponential form.

$$(4.11b) \qquad v_1 v_2 \cos(\phi_1 - \phi_2 - \alpha) = \frac{k_1 k_2 |H_{01}^i| \, |H_{02}^i| \, e^{j(\Delta k R_o - \alpha)}}{8\pi^2 R_o^2}$$

$$\iiiint_{D_1} e^{-j2(k_1 \zeta_1 - k_2 \zeta_1')} \, dx_1 \, dy_1 \, dx_1' \, dy_1'$$

$$+ \frac{k_1 k_2 |H_{01}^i| \, |H_{02}^i| \, e^{-j(\Delta k R_o - \alpha)}}{8\pi^2 R_o^2}$$

$$\iiiint_{D_1} e^{j2(k_1 \zeta_1 - k_1 \zeta_1')} \, dx_1 \, dy_1 \, dx_1' \, dy_1' \qquad .$$

Upon averaging (2.40b), one arrives at the result,

$$(4.11c) \qquad \langle v_1 v_2 \cos(\phi_1 - \phi_2 - \alpha) \rangle = \frac{k_1 k_2 |H_{01}^i| \, |H_{02}^i| \, L_x L_y}{4\pi R_o^2 \sec^2\theta}$$

$$\left( \frac{\sin(\Delta k \sin\theta L_y)}{(\Delta k \sin\theta L_y)} \right) \left\{ e^{j(\Delta k R_o - \alpha)} \int_0^\infty \rho J_o(2k\phi \sin\theta\rho) \right.$$

$$M_{\zeta\zeta'}(j2k_1 \sec\theta, -j2k_2 \sec\theta; \rho) \, d\rho \; +$$

$$\left. e^{-j(\Delta k R_o - \alpha)} \int_0^\infty \rho J_o(2k_1 \sin\theta\rho) \, M_{\zeta\zeta'}(-j2k_1 \sec\theta, j2k_2 \sec\theta; \rho) \, d\rho \right\} .$$

But it has been shown previously that the joint characteristic function is a real function which is symmetric, such that $M_{\zeta\zeta'}(ju, -jv; \rho) = M_{\zeta\zeta'}(-ju, jv; \rho) = M^*_{\zeta\zeta'}(jv, -ju; \rho)$. Therefore, the integrals in (4.11c) are identical and are pure real numbers; (4.11c) can be expressed as

$$(4.11d) \qquad \langle v_1 v_2 \cos(\phi_1 - \phi_2 - \alpha) \rangle = \frac{k_1 k_2 |H_{01}^i| \, |H_{02}^i| L_x L_y}{2\pi R_o^2}$$

$$\left(\frac{\sin(\Delta k \sin\theta L_y)}{(\Delta k \sin\theta L_y)}\right) \cos^2\theta * \cos(\Delta k R_o - \alpha) \times$$

$$\int_0^\infty \rho J_o(2k_1 \sin\theta\rho) M_{\zeta\zeta'}(j2k_1 \sec\theta, -j2k_2 \sec\theta; \rho)\, d\rho \quad .$$

This equation is identical in form to (4.1), except that $e^{j\Delta k R_o}$ is replaced by $\cos(\Delta k R_o - \alpha)$; hence the analysis of the preceding subsection applies directly in this section when this substitution is made. Thus equations (4.6) and (4.9) for the two statistical models become

$$(4.12a) \qquad \mathrm{Cov}[v_1 v_2 \cos(\phi_1 - \phi_2 - \alpha)] = \cos(\Delta k R_o - \alpha)\left[e^{-2\sigma^2 \Delta k^2}\right]$$

for the Gaussian JPDF,

$$(4.12b) \qquad \mathrm{Cov}[v_1 v_2 \cos(\phi_1 - \phi_2 - \alpha)] = \cos(\Delta k R_o - \alpha)\left[\frac{1}{\left(1 + \frac{4}{3}\sigma^2 \Delta k^2\right)^{3/2}}\right]$$

for the Bessel JPDF .

The appearance of the factors $\cos(\Delta k R_o - \alpha)$ in the correlation function presents a complication; since $\Delta k R_o \gg \Delta k \sigma$ (i.e., $R_o$ for lunar measurements is the distance to the moon), (4.12a) and (4.12b) therefore appear as cosine waves modulated by the factors in brackets as functions of $\Delta k$. Hence for a discrete practical number of measurements (e. g., at ten different values of $\Delta k$), (4. 12a) and (4. 12b) would yield little information, since the correlation of each measurement might be positive or negative with limits equal to the factor in brackets. Only the factor in brackets is of interest.

The annoying cosine factor can be eliminated by introducting a third channel at A in Fig. 22 caused by shifting the phase at A by 90°. This scheme is shown in Fig. 23. Upon making a second average measurement of the output, one has for the correlation function

$$\text{(4. 13a)} \qquad \mathrm{Cov}[v_1 v_2 \sin(\phi_1 - \phi_2 - \alpha)] = \sin(\Delta k R_o - \alpha)\ [e^{-2\sigma^2 \Delta k^2}]$$

for the Gaussian JPDF

$$\text{(4. 13b)} \qquad \mathrm{Cov}[v_1 v_2 \sin(\phi_1 - \phi_2 - \alpha)] = \sin(\Delta k R_o - \alpha) \left[ \frac{1}{\left(1 + \frac{4}{3}\sigma^2 \Delta k^2\right)^{3/2}} \right]$$

for the Bessel JPDF.

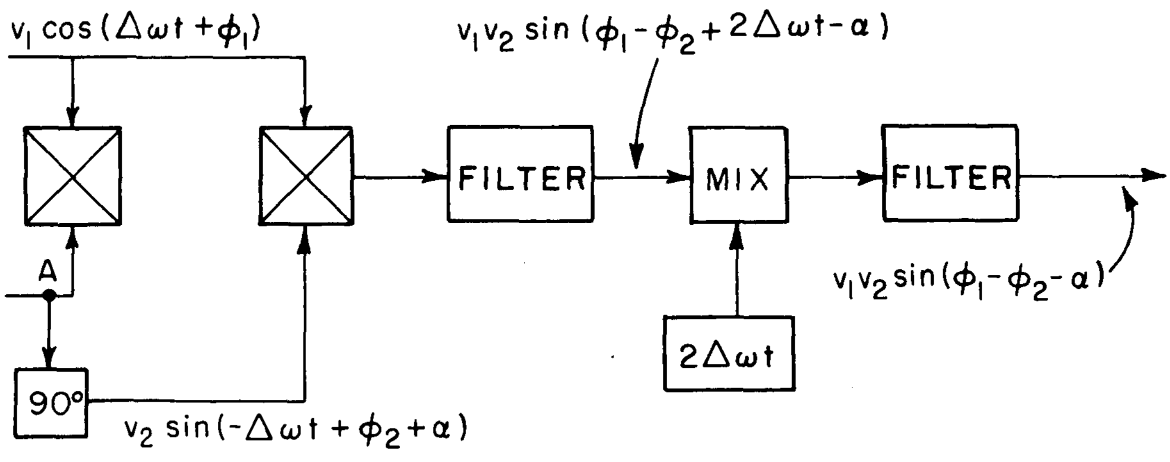


Fig. 23--Modification of system in Fig. 22 providing quadrature component of correlation coefficient.

Thus if one obtains the averages in equations (4. 12) and (4. 13), squares them, adds, and takes the square root, the sinusoidal factors disappear and only the quantities in brackets remain.

The purpose of this subsection is to show how measurements can be made yielding a physically significant correlation function which contains easily accessible information about rms surface height, $\sigma$, as a function of frequency separation, $\Delta f$. It is seen that the proper phase differences ($\cos(\phi_1 - \phi_2 - \alpha)$ or $\sin(\phi_1 - \phi_2 - \alpha)$) must be included in the averaging along with the signal magnitudes $v_1$ and $v_2$.*

## B. A Spherical Rough Surface

### 1. Complex correlation coefficient of scattered fields at two frequencies

The results of the preceding two subsections cannot be applied directly to the lunar surface (or other planetary surfaces) because when the incident radar wave is CW, the effective area illuminated is hemispheric instead of planar. One might intuitively feel that since the return from the forward cap of the moon's surface (which can be considered planar) is by far the strongest, then the correlation between the signals at two frequencies scattered from its entire

* It should be noted that at point B of Fig. 22, one can also obtain $v_1 v_2 \cos(\phi_1 + \phi_2 + \alpha)$; the average of this quantity does not result in the desired correlation, since this sum of the phase angles results in joint characteristic functions of the form $M_{\zeta\zeta'}(j2k_1 \sec\theta, j2k_2 \sec\theta; \rho)$, which are negligibly small for $2k_1\sigma \sec\theta >> 1$, as compared with the characteristic function $M_{\zeta\zeta'}(j2k_1 \sec\theta, -j2k_2 \sec\theta; \rho)$ obtained from the phase difference $\cos(\phi_1 - \phi_2 - \alpha)$.

surface should exhibit a functional dependence upon frequency separation, $\Delta f$, and upon rms surface height, $\sigma$, similar to that derived for the planar surfaces in (4.6) and (4.9). It will be shown in this section that such is indeed the case; the correlation for the Gaussian JPDF model has identically the functional relationship of (4.6).

The results of this subsection are in contradiction with a predicted correlation function of scattered waves from an hemispheric surface at two different frequencies (but at the same instant of time) done by Hagfors[2]. Hagfors' result does not depend upon rms surface height, and therefore cannot be used to predict this parameter of a rough spherical surface. Hagfors' loss of all dependency upon rms surface height is believed to be due to his approximations and mathematical reduction of the original integral.

For simplicity, only the Gaussian JPDF model will be analyzed here when used with the Gaussian class correlation coefficients; that the results may be extended by induction to the other JPDF model will be assumed. The surface is considered perfectly conducting, although this restriction may be easily removed later in the cases of circular polarization by multiplying by the appropriate factors involving the reflection coefficients. The mean surface is considered perfectly spherical with a mean radius A and with an actual radial distance to any point on the surface from the center of the sphere (chosen as the coordinate origin also) given by $r = A + h$, where h is the random variable representing surface roughness; h has a Gaussian JPDF, has zero mean, and it is assumed that $|h| << A$, i. e., that surface roughness is much smaller in height than the radius of the moon (see Fig. 24). The scattered field is given in this case by equation (2. 24) of the first part; $\zeta$ in this case, being the height of the surface in the z direction, is given by $\zeta_1 = r \cos \theta = (A + h) \cos \theta$. Therefore, the conjugate product of the complex scattered fields at two frequencies is given by

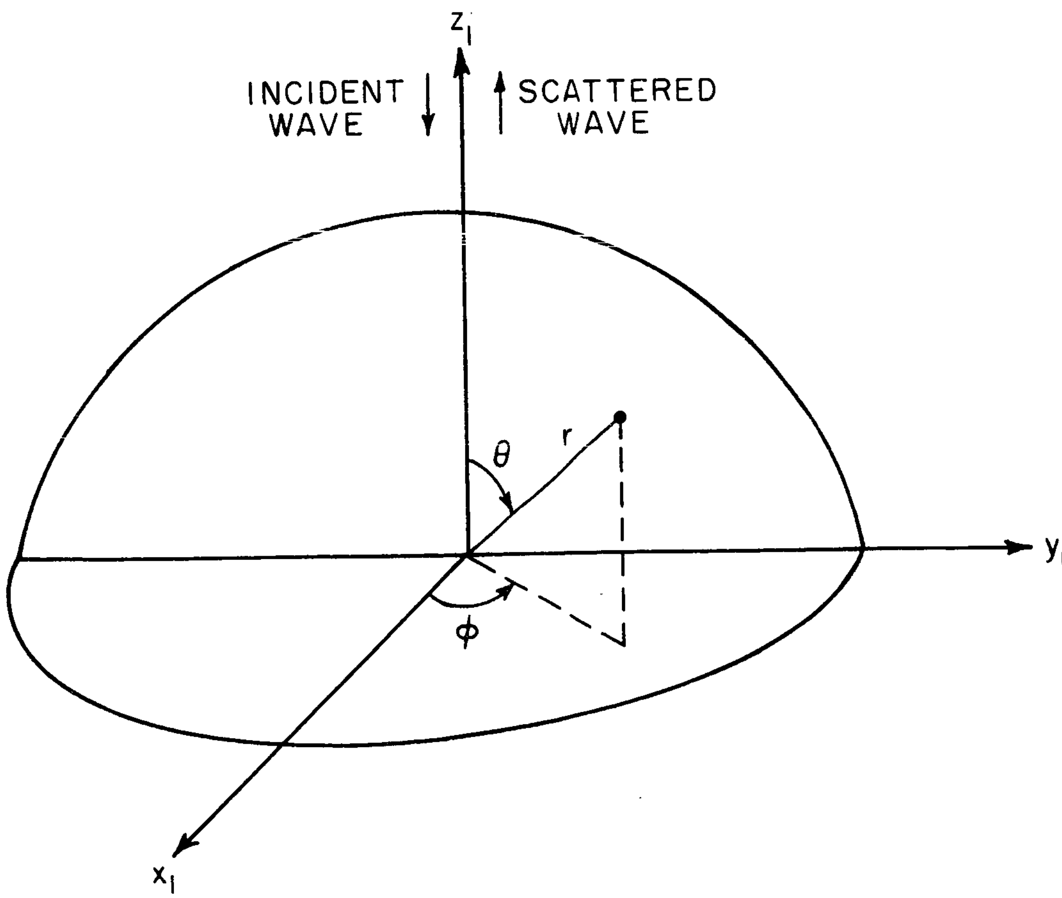


Fig. 24--Rough spherical scattering surface.

$$(4.14) \qquad H_1^s H_2^{s*} = \frac{k_1 k_2 e^{j\Delta k R_o}}{4\pi^2 R_o^2} H_{01}^i H_{02}^i$$

$$\iiiint\limits_{D_1} e^{-j2[k_1(A+h')\cos\theta - k_2(A+h')\cos\theta']} dx_1\, dy_1\, dx_1'\, dy_1' \ .$$

The domain $D_1$ of integration in this case is the circular area in the $z_1 = 0$ plane defined by $x_1^2 + y_1^2 \leq A^2$. At this point, however, it is convenient to change the variables of integration to the spherical angles $\theta$, $\phi$, $\theta'$, and $\phi'$ defined by the following equations:

$$(4.15) \qquad \begin{aligned} x_1 &= r \sin\theta\cos\phi\ , \\ y_1 &= r \sin\theta\sin\phi\ , \qquad \text{where } r = A + h(\theta,\phi)\ . \\ x_1' &= r' \sin\theta' \cos\phi', \\ y_1' &= r' \sin\theta' \sin\phi', \qquad \text{where } r' = A + h'(\theta',\phi')\ . \end{aligned}$$

In this case, h is considered a function of $\theta$ and $\phi$; therefore, the entire exponential in the integrand is already a function of $\theta$, $\phi$, $\theta'$, and $\phi'$. One must find the Jacobian in order to complete the integrand in terms of the new variables. In the determination of the Jacobian, r and $r'$ are considered essentially constant at A, and $|h| << A$ since the Jacobian occurs as a magnitude factor in the integrand. The Jacobian is then

$$J = \begin{vmatrix} \frac{\partial x_1}{\partial \theta} & \frac{\partial y_1}{\partial \theta} & \frac{\partial x_1'}{\partial \theta} & \frac{\partial y_1'}{\partial \theta} \\ \frac{\partial x_1}{\partial \phi} & \cdot & \cdot & \cdot \\ \frac{\partial x_1}{\partial \theta'} & \cdot & \cdot & \cdot \\ \frac{\partial x_1}{\partial \phi'} & \cdot & \cdot & \cdot \end{vmatrix} = A^4 \sin\theta \cos\theta \sin\theta' \cos\theta' \quad .$$

The domain of integration, $D_2$, is how $0 \leq \phi, \phi' < 2\pi$, $0 \leq \theta, \theta' \leq \frac{\pi}{2}$. Therefore the integral (2.45) becomes

$$(4.16) \qquad H_1^s H_2^{s*} = \frac{k_1 k_2 A^4 e^{j\Delta k R_o}}{4\pi^2 R_o^2} H_{01}^i H_{02}^i \int\int\int\int_{\substack{\theta,\theta'=0 \\ \phi,\phi'=0}}^{\substack{\phi,\phi'=2\pi \\ \theta,\theta'=\pi/2}} e^{-j2[k_1(A+h)\cos\theta - k_2(A+h')\cos\theta']} \sin\theta\cos\theta\sin\theta'\cos\theta' \, d\theta \, d\phi \, d\theta' \, d\phi' \quad .$$

Upon averaging the above expression, one obtains the covariance of the scattered fields in terms of the joint characteristic function of the random variables h and h' .

$$(2.47) \qquad \langle H_1^s H_2^{s*} \rangle = \frac{k_1 k_2 A^4 \, e^{j\Delta k R_o}}{4\pi^2 R_o^2} \, H_{01}^i H_{02}^i$$

$$\iiiint\limits_{D_2} M_{hh'}(-j2k_1 \cos\theta, +j2k_2 \cos\theta'; \rho)$$

$$e^{-j2A[k_1 \cos\theta - k_2 \cos\theta']} \sin\theta \cos\theta \sin\theta' \cos\theta' \, d\theta d\phi \, d\theta' \, d\phi' .$$

In this case the distance, $\rho$, between two points on the surface of a sphere at $(\theta, \phi)$ and $(\theta', \phi')$, must be determined in terms of the angular coordinates of the two points. Since it is the length of an arc on the spherical surface, it is given by $\rho = A\nu$, where $\nu$ is the angular separation (in radians) between the two points. The cosine of this angle, $\nu$, is determined by the dot product of the two unit vectors pointing from the origin towards the points $(\theta, \phi)$ and $(\theta', \phi')$; this is

$$\cos\nu = (\sin\theta\cos\phi\, \hat{x}_1 + \sin\theta\sin\phi\, \hat{y}_1 + \cos\theta\, \hat{z}_1) \cdot$$

$$(4.17) \qquad (\sin\theta'\cos\phi'\, \hat{x}_1 + \sin\theta'\sin\phi'\, \hat{y}_1 + \cos\theta'\, \hat{z}_1)$$

$$\therefore \cos\nu = \sin\theta\sin\theta' \cos(\phi - \phi') + \cos\theta\cos\theta' .$$

It is assumed here that since surface roughness is much smaller in scale than the radius of the sphere, then the correlation length of the surface roughness height, h, is much smaller than the radius also. Thus the joint characteristic function becomes vanishingly small when the angle $\nu$ is still very small. In this region where the joint characteristic function is non-zero and the angular separation between the two points is very small, the cosine function can be approximated by the first two terms of its series, i. e., $\cos \nu = 1 - \frac{\nu^2}{2}$; thus $\nu^2$ can be written

$$\nu^2 = 2[1-\cos \nu] = 2[1 - \sin\theta \sin\theta' \cos(\phi-\phi') - \cos\theta\cos\theta'] . \tag{4.18}$$

The Gaussian joint characteristic function in this case is

$$M_{hh'}(-j2k_1 \cos\theta, j2k_2 \cos\theta'; \rho) =$$

$$= e^{-2\sigma^2[k_1^2 \cos^2\theta - 2k_1 k_2 \cos\theta \cos\theta' e^{-\frac{A^2\nu^2}{a^2}} + k_2^2 \cos^2\theta']}$$

$$= e^{-2\sigma^2(k_1 \cos\theta - k_2 \cos\theta')^2} \cdot e^{-4\sigma^2 k_1 k_2 \cos\theta \cos\theta' (1-e^{\frac{A^2\nu^2}{a^2}})} .$$

It should be noted that the first exponential is always less than unity because the exponent is always negative; the second exponential becomes vanishingly small for $A^2\nu^2/a^2$ still very close to zero because the quantity $4\sigma^2 k_1 k_2 \cos\theta\cos\theta'$ is very large everywhere (except at or very near grazing incidence where $\theta$ or $\theta' \approx 90°$). Hence, when ignoring the effect of that portion of the surface very near grazing incidence, the correlation coefficient can be represented by the first two terms of its series expansion, so that $1 - e^{-A^2\nu^2/a^2} \simeq A^2\nu^2/a^2 = \frac{2A^2}{a^2}[1 - \sin\theta\sin\theta'\cos(\phi-\phi') - \cos\theta\cos\theta']$. Make the change of variables $\gamma = \phi-\phi'$. Then the covariance becomes

$$<H_1^s H_2^{s*}> = \frac{k_1 k_2 A^4}{4\pi^2 R_o^2} e^{j\Delta k R_o} H_{01}^i H_{02}^i \int_{\theta=0}^{\theta=\pi/2} \int_{\theta'=0}^{\theta'=\pi} \Bigg\{$$

$$\int_{\phi'=0}^{\phi'=2\pi} e^{-2\sigma^2(k_1 \cos\theta - k_2 \cos\theta')^2 - 8k_1 k_2 \frac{\sigma^2 A^2}{a^2} \cos\theta \cos\theta'(1-\cos\theta\cos\theta')} *$$

$$\cdot\, e^{-j2A(k_1 \cos\theta - k_2 \cos\theta')}$$

$$\times \left[ \int_{\gamma=\phi'}^{\gamma=2\pi+\phi'} e^{-8k_1 k_2 \frac{\sigma^2 A^2}{a^2} \sin\theta \cos\theta \sin\theta' \cos\theta' \cos\gamma'} d\gamma \right]$$

$$\times d\phi \Bigg\} \sin\theta \cos\theta \sin\theta' \cos\theta' \, d\theta \, d\theta' .$$

When one studies the integral included in the square brackets, one notices that the integrand is periodic, and since it is to be integrated over one period (from $\phi'$ to $\phi' + 2\pi$), the value of the integral is constant regardless of the initial value of $\phi'$; hence the limits of the integral may be replaced by 0 and $2\pi$ and the value of the integral in square brackets will remain unchanged. Then the integral over $\phi'$ may be made since the integrand is independent of $\phi'$. The integral in square brackets may be evaluated from the tables;

$$\int_0^{2\pi} e^{-8k_1 k_2 \frac{\sigma^2 A^2}{a^2} \sin\theta \cos\theta \sin\theta' \cos\theta' \cos\gamma} d\gamma =$$

$$2\pi I_o\left(8k_1 k_2 \frac{\sigma^2 A^2}{a^2} \sin\theta \cos\theta \sin\theta' \cos\theta'\right) ,$$

where $I_o$ is the modified Bessel function of the first kind. The integral may now be written

$$< H_1^s H_2^{s*} > = \frac{k_1 k_2 A^4}{R_o^2} e^{j\Delta k R_o} H_{01}^i H_{02}^i$$

$$\iint_{\theta,\theta'=0}^{\theta,\theta'=\pi/2} e^{-2\sigma^2(k_1\cos\theta - k_2\cos\theta')^2 - 8k_1 k_2 \frac{\sigma^2 A^2}{a^2}\cos\theta\cos\theta'(1-\cos\theta\cos\theta')}$$

$$e^{-j2A(k_1\cos\theta - k_2\cos\theta')} * I_o\left(8k_1 k_2 \frac{\sigma^2 A^2}{a^2} \sin\theta\cos\theta\sin\theta' \cos\theta'\right)$$

$$\sin\theta\cos\theta\sin\theta'\cos\theta'\, d\theta\, d\theta' \quad .$$

In the evaluation of the remaining double integral, it appears that asymptotic methods and approximations offer the simplest course. The constant $K \equiv 8k_1 k_2 \frac{\sigma^2 A^2}{a^2}$ is very large at radar frequencies; for instance, at wavelengths $\lambda_1 \simeq \lambda_2 \simeq 10$ cm, when applied to the lunar surface, this constant is of the order of magnitude $10^{15}$. Using this fact, the modified Bessel function will be written in terms of its large argument expansion where only the first term is retained; this is quite valid where the argument is greater than 10 in value. As can be seen, the argument $K \sin\theta\cos\theta\sin\theta'\cos\theta'$ is greater than 10 over the entire range of $\theta$ and $\theta'$ except for the two tiny ranges $0 \leq \theta,\theta' \leq \left(\frac{180}{\pi} \times 10^{-15}\right)^\circ$ and $\left(90 - \frac{180}{\pi} \times 10^{-15}\right)^\circ \leq \theta,\theta' \leq 90^\circ$. The error introduced from these two infinitesimal ranges of $\theta$ and $\theta'$ when using the large argument expansion for $I_o$ is hence negligibly small. Using this approximation, $I_o$ can be written

$$I_o(K\sin\theta\cos\theta\sin\theta'\cos\theta') \simeq \frac{1}{\sqrt{2\pi K\sin\theta\cos\theta\sin\theta'\cos\theta'}}$$

$$e^{K\sin\theta\cos\theta\sin\theta'\cos\theta'} .$$

Now this portion of the integrand can be combined with another exponential portion also containing the factor K; sines will be written in terms of cosines.

$$\therefore e^{-8k_1k_2\frac{\sigma^2A^2}{a^2}\cos\theta\cos\theta'(1-\cos\theta\cos\theta')} \cdot e^{K\sin\theta\cos\theta\sin\theta'\cos\theta'} =$$

$$e^{-K\cos\theta\cos\theta'(1-\cos\theta\cos\theta'-\sin\theta\sin\theta')}$$

$$= e^{-K\cos\theta\cos\theta'[1-\cos\theta\cos\theta'-\sqrt{1-\cos^2\theta}\sqrt{1-\cos^2\theta'}]} .$$

Let $f \equiv 1-\cos\theta\cos\theta'-\sqrt{1-\cos^2\theta}\sqrt{1-\cos^2\theta'}$ . Then several facts should be noted about f.

(i) $f = 0$ where $\cos\theta$ or $\cos\theta' = 0$.

(ii) f is always either zero or positive for any $\theta, \theta'$ between $0°$ and $90°$. This means that the exponent is always negative and in general, when $f \neq 0$, the exponential is very small due to the large constant K.

(iii) $f = 0$ where $\cos\theta = \cos\theta'$. Hence the exponential has its largest value when $\cos\theta$ is very close to $\cos\theta'$ (i. e., when the two points on the surface are very close to each other). This suggests making a change of variables such that $\cos\theta' = \cos\theta - \tau$.

Thus f becomes

$$f = 1-\cos\theta(\cos\theta-\tau) - \sqrt{(1-\cos^2\theta)}\ \sqrt{(1-\cos^2\theta) + 2\cos\theta\tau - \tau^2}\ .$$

Again, it can be seen that when the latitude separation, $\tau$, is zero, f becomes zero, and this exponential factor in the integrand has its largest value. However, when $\tau$ becomes different from zero but is still very small, the integrand becomes vanishingly small due to the very large factor K and the positive value of f. If one excludes again the regions of $\theta$ discussed previously very close to 0° and very close to 90°, then $(1-\cos^2\theta) >> \left| 2\cos\theta\tau - \tau^2 \right|$ in the region where $\tau$ is small enough so that the integrand does not vanish. In this region, the second square root in f may be written

$$\sqrt{(1-\cos^2\theta) + (2\cos\theta\tau-\tau^2)} \simeq \sqrt{(1-\cos^2\theta)} + \frac{\cos\theta\tau - \frac{1}{2}\tau^2}{\sqrt{(1-\cos^2\theta)}}\ .$$

Hence f can be written

$$f \simeq 1-\cos\theta(\cos\theta-\tau) - (1-\cos^2\theta) - \cos\theta\tau + \tfrac{1}{2}\tau^2 = \tfrac{1}{2}\tau^2\ .$$

The factor $\cos\theta\cos\theta'$ multiplying f can be written $\cos\theta(\cos\theta-\tau) \simeq \cos^2\theta$ since $|\tau| << \cos\theta$ in the range of interest. Also, the function $k_1\cos\theta - k_2\cos\theta'$ may be written $k_1\cos\theta - (k_1 - \Delta k)(\cos\theta-\tau) = \Delta k\cos\theta + \tau k_2$. Upon changing one variable of integration from $\theta'$ to $\tau$, one finds that the upper and lower limits of the integral on $\tau$ become $\cos\theta$ and $\cos\theta - 1$ respectively. However, since the integrand becomes vanishingly small for $\tau$ slightly greater than zero, these limits may be replaced by $\pm\infty$. Thus the integral becomes

$$<H_1^s H_2^{s*}> = \frac{k_1 k_2 A^4}{R_o^2} e^{j\Delta k R_o} H_{01}^i H_{02}^i$$

$$\int_{\theta=0}^{\theta=\pi/2} \int_{\tau=-\infty}^{\tau=\infty} e^{-2\sigma^2[\Delta k \cos\theta + \tau k_2]^2} \cdot e^{-j2\Delta k \cos\theta}$$

$$\cdot \frac{\sin\theta\cos\theta(\cos\theta-\tau)}{\sqrt{2\pi K \sin\theta\cos\theta(\cos\theta-\tau)}\,\sqrt{1-(\cos\theta-\tau)^2}}$$

$$\times\, e^{-K\cos^2\theta\frac{\tau^2}{2} - j2\Delta k_2\tau} \, d\tau\, d\theta \quad .$$

In this integral, the factor $\cos\theta-\tau$, where it occurs, may be replaced by $\cos\theta$, under the assumption that $|\tau| << \cos\theta$ in the non-vanishing range of interest. Also, in the first experimental factor of the integrand, it should be noted that since $2\sigma^2 k_2^2 << K$, then the effect of the factor $e^{-K\cos^2\theta\frac{\tau^2}{2}}$ in the integrand causes the integrand to vanish much faster for increasing $\tau$ than any affect caused by the first exponential factor, $e^{-2\sigma^2[\Delta k\cos\theta+\tau k_2]^2}$ for $\tau$ in this range near zero; hence, the first factor is considered constant in this range of $\tau$ at its value for $\tau = 0$, i. e. , $e^{-2\sigma^2[\Delta k\cos\theta+k_2\tau]^2} \simeq e^{-2\sigma^2\Delta k^2\cos^2\theta}$. Then the integral becomes

$$< H_1^s H_2^{s*} > = \frac{k_1 k_2 A^4 \; e^{j\Delta k R_o}}{\sqrt{2\pi}\;\sqrt{K}\;\; R_o^2} \; H_{01}^i H_{02}^i$$

$$\int_{\theta=0}^{\theta=\pi/2} e^{-2\sigma^2 \Delta k^2 \cos\,\theta} \, e^{-j2A\Delta k \cos\,\theta} \cos\,\theta \times$$

$$\left\{ \int_{-\infty}^{\infty} e^{-K\,\cos^2\theta \frac{\tau^2}{2} - j2Ak_2\tau} \; d\tau \right\} d\theta \quad .$$

The second integral in braces can be evaluated immediately from the tables:

$$\int_{-\infty}^{\infty} e^{-K\,\cos^2\theta \frac{\tau^2}{2} - j2Ak_2\tau} \; d\tau = \sqrt{\frac{2\pi}{K\,\cos^2\theta}} \; e^{-\frac{k_2}{k_1}\frac{a^2}{4\sigma^2}\sec^2\theta} \quad .$$

Employing the definition of mean square surface slope, $S^2 = \frac{4\sigma^2}{a^2}$ , the remaining integral becomes

$$(4.18 \qquad < H_1^s H_2^{s*} > = \frac{k_1 k_2 A^4}{K\;R_o^2} \; e^{j\Delta k R_o} \; H_{01}^i H_{02}^i$$

$$\int_0^{\pi/2} \left[ e^{-2\sigma^2 \Delta k^2 \cos^2\theta - \frac{k_2}{k_1} \cdot \frac{1}{S^2} \sec^2\theta} \right] e^{-j2A\Delta k\,\cos\,\theta} \, d\theta \; .$$

In the range of interest of $\Delta k$, i. e. , where $0.1 < \Delta k\sigma < 10$, the quantity $A\Delta k$ is very large. Therefore the method of stationary phase can be used to evaluate the remaining integral. Since the integrand is an even function of $\theta$, the integral limits may be changed, viz. , $\int_0^{\pi/2} = \frac{1}{2}\int_{-\pi/2}^{\pi/2}$ . If $e^{-j2A\Delta k \cos\,\theta} = e^{-\kappa f(\theta)}$ where $\kappa$ is very large, then $f(\theta) = \cos\theta$ has a saddle point at $\theta = 0$; expanding about $\theta = 0$ gives for the first two terms $f(\theta) = \cos \simeq 1 - \frac{\theta^2}{2}$. The expression in square brackets in the integrand is considered constant at $\theta = 0$ according to this method, and the remaining integral becomes

$$(4.19) \qquad < H_1^s H_2^{s*} > = \frac{k_1 k_2 A^4}{2 K R_o^2} e^{j\Delta k R_o - j2\Delta k A} H_{01}^i H_{02}^i$$

$$e^{-2\sigma^2 \Delta k^2 - \frac{k_2}{k_1} \cdot \frac{1}{S^2}} \int_{-\infty}^{\infty} e^{jA\Delta k\theta^2} d\theta \quad .$$

Evaluating the remaining Fresnel integral and employing the fact that $\frac{k_2}{k_1} \simeq 1$ for $\frac{\Delta k}{k_1} << 1$, the covariance becomes

$$(4.20 \qquad < H_1^s H_2^{s*} > = \frac{\sqrt{\pi} A^2 H_{01} H_{02}}{4 R_o^2} e^{j\Delta k R_o - j2\Delta k A + j\frac{\pi}{4}} \frac{1}{S^2}$$

$$e^{-1/S^2} \frac{1}{\sqrt{A\Delta k}} [ e^{-2\sigma^2 \Delta k^2} ] \quad .$$

It should be noted that the above covariance, being proportional to the signal strength of the two reflected waves from the surface, is a function of $\frac{1}{S^2} e^{-1/S^2}$; this factor is similar in form to that involved in the reflection from a rough planar surface when the Gaussian JPDF model is used.

The above form of the covariance at two different frequencies should be compared to Hagfors[2] result, which is $\mathrm{Cov}(H_1^S, H_2^{S*}) \sim \frac{1}{1/S^2 + jA\Delta k}$, where $S^2 = 4h_m^2/L^2 = 4\sigma^2/a^2$. His result is not at all a function of rms surface height alone, but depends only upon the parmeters rms surface slope, S, and the radius of the moon, A, as a function of frequency separation, $\Delta k$.

As shown in the preceding subsection, measurements can be made of the covariances $< v_1 v_2 \cos(\phi_1 - \phi_2 - \alpha) >$ and $< v_1 v_2 \sin(\phi_1 - \phi_2 - \alpha) >$ when the systems of Figs. 22 and 23 are employed. By squaring and adding these covariances and then taking the square root, the sinusoidal fluctuations of the covariances with the diameter of the moon (depending upon $2\Delta kA$) and the distance to the moon (depending upon $\Delta kR_o$) can be removed. Thus the magnitude or envelope of the covariance of equation (4. 20) obtained in this manner becomes

$$(4.21) \qquad \left| \mathrm{Cov}(H_1^s H_2^{s*}) \right| = \frac{\sqrt{\pi}\, A^2}{4\, R_o^2}\, H_{01}^i H_{02}^i\, \frac{1}{S^2}\, e^{-1/S^2}\, \frac{1}{\sqrt{A\Delta k}} \left[ e^{-2\sigma^2 \Delta k^2} \right] .$$

One might note with distress that the covariance of the scattered fields, as expressed in the above equation, goes to infinity as $\Delta k$, the frequency separation, goes toward zero. Since the covariance of the scattered fields at $\Delta k = 0$,( i. e. , at the same frequency) should give the scattered power, this would seemingly indicate that the scattered power is infinite. However, this apparent discrepancy is easily explained: equation (2. 51) was obtained from an asymptotic expansion where the parameter $2A\Delta k$ was assumed to be very large. Therefore it is valid only in this region and cannot explain the behavior of the covariance function when $\Delta k \to 0$ ( such that $2A\Delta k$ is small). For the lunar surface, frequency separation, $\Delta f$ need be only 1370 cycles per second in order that $2A\Delta k = 100$.

The correlation coefficient of the scattered fields at both frequencies is found by dividing the covariance in (4.21) by the square root of the variance of each of the two scattered components. In order to find the variance, one must go back to equation (4.18) and set $\Delta k = 0$. Then the variance becomes

$$(4.22) \qquad <|H^s|^2> = \frac{A^2}{2R_o^2 S^2} |H^i|^2 \int_0^{\pi/2} e^{-\frac{1}{S^2}\sec^2\theta} d\theta =$$

$$\frac{\pi}{4} \frac{A^2}{R_o^2} |H^i|^2 \frac{1}{S^2} \left[1 - \Phi\left(\frac{1}{S}\right)\right] .$$

The function $1-\Phi(x)$ is the complement of the error function, $\Phi(x)$, and is given by

$$1-\phi(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-t^2} dt .$$

Hence, the correlation coefficient of the backscattered fields is

$$(4.23) \qquad \mathrm{Cor}(H_1^s H_2^{s*}) = \frac{1}{\sqrt{\pi}} \frac{e^{-1/S^2}}{\left[1-\phi\left(\frac{1}{S}\right)\right]} \cdot \frac{1}{\sqrt{A\Delta k}} [ e^{-2\sigma^2 \Delta k^2} ] .$$

Since this is a true correlation coefficient only where $2A\Delta k$ is very large, then there must exist another correlation coefficient for $2A\Delta k$ small such that $\lim_{\Delta k \to 0} [\mathrm{Cor}(H_1^s H_2^{s*})] = 1$ (in order that the definition of a correlation coefficient not be violated). However, if one is interested in obtaining information about $\sigma$, the rms surface height, from the correlation coefficient of the scattered fields, he can employ (4.23) by restricting his range of $\Delta k$ which he uses for measurement such that $2A\Delta k$ is very large ($\Delta k > 1000$ cps in the case of the moon).

The functional dependency of the correlation upon the parameters $\sigma$ and $\Delta k$ can be enhanced for such large $A\Delta k$ by multiplying the correlation given in (2.54) by the factor $\sqrt{A\Delta k}$; thus one defines a new function

$$(4.24) \qquad P_a(\Delta k\sigma) = \sqrt{A\Delta k}\ \mathrm{Cor}(H_1^s H_2^{s*}) = K_{SG} \cdot e^{-2\sigma^2 \Delta k^2} \quad .$$

The constant $K_S$ is not a function of $\sigma$ or $\Delta k$, but is only a function of S, the rms surface slope. Thus when (4.24) is plotted after the correlation has been determined and multiplied by $\sqrt{A\Delta k}$ as a function of $\sigma^2\Delta k^2$, the curve is identical with that shown in Fig. 21. By noting where the experimentally obtained function $P(\Delta k\sigma)$ falls off to $\frac{1}{2}$ its initial value (the initial value still being where $2A\Delta k$ is large, e.g., at $\Delta f = 2000$ cps), one notes from the curve that $2\sigma^2\Delta k^2 = 0.693$, and thus one can find $\sigma$.

There are aspects of the preceding development which may be expanded and extended here.

(a) One might suspect that since the analysis of this subsection was done only for the model with the Gaussian JPDF and Gaussian correlation coefficient, then possibly other models will give different results. In particular, it was shown for this statistical model that one should multiply the correlation coefficient of the backscattered fields by $\sqrt{A\Delta k}$ in order to arrive at a function, $P(\sigma\Delta k)$, which varies in the manner predicted for the correlation coefficient of the fields from a rough planar surface, i. e., as $e^{-2\sigma^2 \Delta k^2}$. However, if the surface JPDF is not Gaussian, does one still multiply by $\sqrt{A\Delta k}$ to arrive at a result functionally similar to the correlation coefficient for the fields from a planar surface, or is the proper function of $A\Delta k$ different from the square root? The answer is that multiplication by the factor $\sqrt{A\Delta k}$ is correct for any statistical model regardless of the form of the JPDF so long as $2A\Delta k$ is large. This factor $\sqrt{A\Delta k}$ results from application of the asymptotic method of stationary phase; application of this method is possible because of the factor in the integrand $e^{-2A[k_1 \cos\theta - k_2 \cos\theta]} = e^{-j2k_2\tau}[e^{-j2A\Delta k \cos\theta}]$ (see equation 2. 47). This factor is independent of the form chosen for the joint characteristic function of the surface height, and it is this exponential in brackets which permits application of the stationary phase method; consequently, the factor $1/\sqrt{A\Delta k}$ always appears in the correlation coefficient. Hence, one would expect that the function $P(\sigma\Delta k)$ in the case where the surface has the Bessel JPDF model would have the form

$$(4.25) \qquad P_S(\sigma\Delta k) = K_{SB} \cdot \left[ \frac{1}{\left(1 + \frac{4}{3}\sigma^2\Delta k^2\right)^{3/2}} \right] .$$

This function is plotted as a function of $\sigma^2\Delta k^2$ and shown in the dotted curve of Fig. 21.

(b) If one wishes to add a second order correction to the covariance function for the scattered field, one may approximate the function $e^{-\frac{1}{S^2}\sec^2\theta}$ in the integrand of (2.50) by $e^{-\frac{1}{S^2}(1+\theta^2)}$ instead of by $e^{-1/S^2}$; this results in an integrand in (2.50a) of $e^{-\left(\frac{1}{S^2} - jA\Delta k\right)\theta^2}$ instead of $e^{j\Delta k\theta^2}$. The integrand is then evaluated in the same manner and the factor $1/\sqrt{A\Delta k}$ of equation (2.51) is replaced by $1/\sqrt{\frac{1}{S^2} - jA\Delta k}$; the resulting magnitude of the covariance function of (2.52) then has the factor $1/\sqrt{A\Delta k}$ replaced by $1/\left[\left(\frac{1}{S^2}\right)^2 + (A\Delta k)^2\right]^{1/4}$. This correction is necessary where $A\Delta k$ is not large with respect to $1/S^2$, but as can be seen, where $A\Delta k >> 1/S^2$, the results are the same in both cases.

(c) The results of this subsection have been derived for a perfectly conducting surface; however, they would differ from the results for a non-perfectly conducting surface when circular polarization is employed (and one considered only the polarized backscattered component) only by the appearance of the factor

$\frac{1}{4} \left| \rho_{\parallel}(\theta) - \rho_{\perp}(\theta) \right|^2$ in the integrand of (4.18). When the stationary phase method is applied, this factor is removed from the integrand and treated as a constant with its value at the saddle point $\theta = 0$; then this factor becomes $\left| \sqrt{\epsilon_r} - 1 \middle/ \sqrt{\epsilon_r} + 1 \right|^2$ for dielectric surfaces.

A similar result is obtained for the power from (4. 22) when one includes this factor in the integrand. The justification for removing it from the integrand as a constant in that case (when the stationary phase method is not used) is that the function $\frac{1}{4} \left| \rho_{\parallel}(\theta) - \rho_{\perp}(\theta) \right|^2$ is nearly constant as a function of $\theta$ all the way out to about 70°, as seen from Fig. 7. Since the integrand $e^{-\frac{1}{S^2} \sec^2\theta}$ becomes vanishingly small at $\theta$ large, this reflection coefficient factor is significant only in a neighborhood of $\theta = 0$ and is therefore a constant in this neighborhood. Hence this same factor appears in both the variance and covariance of the backscattered fields, and consequently cancels out of the correlation coefficient (equation (4. 23)) of the backscattered fields.

It should be noted that the results obtained for the variance and covariance of the backscattered fields for the cases of both the rough planar and rough hemispheric surfaces (when one employs a statistical model for the surface JPDF along with the Gaussian correlation coefficient) are not valid in the limit as slope, S, approaches either limit. In the case where slope approaches zero (i. e., the rough surface degenerates to a smooth surface), all of the variances and covariances approach zero indicating zero power from a smooth surface, which, of course, is not true. However, since $S^2 = \frac{4\sigma^2}{a^2}$, this behavior is a result of assuming either that correlation length, a, approaches infinity (which contradicts a previous assumption that surface height correlation length be much smaller than any overall dimension of the scattering surface) or that $\sigma$ approaches zero (which contradicts another stipulation which restricts the class of surfaces under investigation here to "very rough" surfaces where $k\sigma >> 1$). As rms slope, S, approaches infinity, the predicted power in all cases again approaches zero. That S would go to infinity means that either "$\sigma$" goes to infinity or "a" approaches zero; such a situation means that the surface is made up of very jagged and high spikes, for which the whole physical optics theory as employed here is inapplicable.

2. Backscattering cross section for a spherical rough surface

The predicted back scattering cross section for a spherical rough scatterer may be obtained from (4.22) in the usual manner and may be normalized to a dimensionless ratio by dividing by the geometrical cross section of the sphere ($\pi A^2$). The result is

(4.26a) $$\gamma_{sphere} = \frac{\pi}{S^2}\left[1 - \Phi\left(\frac{1}{S}\right)\right]$$ for a perfectly conducting surface, and

(4. 26b) $$\gamma_{sphere} = \pi \left| \frac{\sqrt{\epsilon_r} - 1}{\sqrt{\epsilon_r} + 1} \right|^2 \cdot \frac{1}{S^2} \left[ 1 - \Phi\left(\frac{1}{S}\right) \right]$$ for circular polarization and a non-perfectly conducting surface.

As mentioned previously, (4. 26a) unfortunately does not reduce to unity for a smooth sphere ($S \rightarrow 0$) as it should but instead approaches zero.

Theory of scattering from rough surfaces is little by little improving and coming of age, but there are many limitations and discrepancies as yet unconquered. However, its usefulness in its range of applicability is growing continuously and promises to reveal many statistical facts about surfaces from the properties of their scattered electromagnetic waves.

SUMMARY

An experiment is described and analyzed which offers a method of determining the root mean square height of a rough surface by measuring the correlation between two scattered waves at different frequencies from a stationary surface as a function of frequency separation. No satisfactory method to date exists for determining the rms height of a rough surface using radar waves. The problems of both rough planar surfaces and rough spherical surfaces (such as planetary surfaces) are analyzed, and the results are shown to be quite similar. Also, the backscattering cross section of rough spherical surfaces is found as a function of rms surface slope and dielectric constant.

## APPENDIX D

From equations ( 3. 5) , the statistical model for the JPDF to be used is

$$
\text{(D-1)} \qquad W(\zeta, \zeta ; \rho) = \frac{3}{\pi^2 \sigma^2 (1-R^2)^{1/2}} \left[\frac{\sqrt{3}\,|\zeta|}{\sigma}\right] \left[\frac{\sqrt{3}\,|\zeta' - R\zeta|}{\sigma(1-R^2)^{1/2}}\right] K_1\left[\frac{\sqrt{3}\,|\zeta|}{\sigma}\right] K_1\left[\frac{\sqrt{3}\,|\zeta' - R\zeta|}{\sigma(1-R^2)^{1/2}}\right] ,
$$

where $\zeta$ and $\zeta'$ are the two random variables representing the surface height at the points $(x, y)$, and $(x', y')$; R is the correlation coefficient between $\zeta$ and $\zeta'$, while the means of $\zeta$ and $\zeta'$ are both zero and the variances of both are $\sigma^2$. $K_1$ is a first order modified Bessel function of the second kind. R is a function of $\rho = \sqrt{(x-x')^2 + (y-y')^2}$, i. e. , separation between the two points.

### A. Determination of the Joint Characteristic Function

The joint characteristic function is defined as

$$
M_{\zeta\zeta'}(ju, jv; \rho) = \iint_{-\infty}^{\infty} e^{ju\zeta + jv\zeta'}\, W(\zeta, \zeta'; \rho)\, d\zeta d\zeta'
$$

$$
= \frac{3}{\pi^2 \sigma^2 (1-R^2)^{1/2}} \iint_{-\infty}^{\infty} e^{ju\zeta + jv\zeta'} \left[\frac{\sqrt{3}\,|\zeta|}{\sigma}\right] \left[\frac{\sqrt{3}\,|\zeta' - R\zeta|}{\sigma(1-R^2)^{1/2}}\right] K_1\left[\frac{\sqrt{3}\,|\zeta|}{\sigma}\right] K_1\left[\frac{\sqrt{3}\,|\zeta' - R\zeta|}{\sigma(1-R^2)^{1/2}}\right] d\zeta\, d\zeta' .
$$

Make the following change of variables

$$x \equiv \frac{\sqrt{3}\ \zeta}{\sigma} \quad ; \quad y \equiv \frac{\sqrt{3}}{\sigma(1-R^2)^{1/2}}\ [\zeta' - R\zeta] \quad .$$

Then the joint characteristic function becomes

$$M_{\zeta\zeta'}(ju, jv; \rho) = \frac{1}{\pi^2} \iint_{-\infty}^{\infty} e^{j\frac{u\sigma(1-R^2)^{1/2}}{\sqrt{3}} y + j\frac{u\sigma R}{\sqrt{3}} x + j\frac{u\sigma}{\sqrt{3}} x}$$

$$|x|\ |y|\ K_1(|x|)\ K_1(|y|)\ dx\, dy$$

$$= \left\{\frac{1}{\pi}\int_{-\infty}^{\infty} e^{jay}|y|\ K_1(|y|)\ dy\right\}$$

$$= \left\{\frac{1}{\pi}\int_{-\infty}^{\infty} e^{jbx}|x|\ K_1(|x|)\ dx\right\} \quad ,$$

where $a = \frac{u\sigma(1-R^2)^{1/2}}{\sqrt{3}}$ and $b = \frac{\sigma}{\sqrt{3}}(uR + v)$. The integrals in braces are identical in form and each is integrated as follows

$$\frac{1}{\pi}\int_{-\infty}^{\infty} e^{jay}|y|\ K_1(y)\ dy = \frac{1}{\pi}\left[-\int_{-\infty}^{0} e^{jay}\ yK_1(-y)\ dy\right.$$

$$\left. + \int_{0}^{\infty} e^{jay} yK_1(y)\ dy\right] \quad .$$

But $-\int_{-\infty}^{0} e^{jay} yK_1(-y)\, dy = +\int_{0}^{-\infty} e^{jay} yK_1(-y)\, dy = \int_{0}^{\infty} e^{-jay} yK_1(y)\ dy$ ,

where the last integral was obtained by replacing y by -y. Thus the original integral becomes

$$\frac{1}{\pi}\int_{-\infty}^{\infty} e^{jay}|y|K_1(|y|)\,dy = \frac{1}{\pi}\int_0^{\infty}(e^{jay}+e^{-jay})\,yK_1(y)\,dy$$

$$= \frac{2}{\pi}\int y\cos ay\,K_1(y)\,dy\,.$$

The last integral is evaluated in the tables[ Ref. 28, p. 763, #12] :

$$\frac{2}{\pi}\int_0^{\infty} y\cos ay\,K_1(y)\,dy = \frac{1}{(1+a^2)^{3/2}}\qquad .$$

Hence the joint characteristic function becomes

$$M_{\zeta\zeta'}(ju, jv;\rho) = \frac{1}{(1+a^2)^{3/2}}\cdot\frac{1}{(1+b^2)^{3/2}}$$

$$\text{(D-2)}\qquad \therefore\; M_{\zeta\zeta'}(ju, jv;\rho) = \frac{1}{\left(1+\frac{u^2\sigma^2}{3}(1-R^2)\right)^{3/2}}\cdot\frac{1}{\left(1+\frac{\sigma^2}{3}(uR+v)^2\right)^{3/2}}\,.$$

Several facts may be readily verified from this characteristic function:

$$\text{(i)}\qquad 1 = \iint_{-\infty}^{\infty} W(\zeta,\zeta')\,d\zeta d\zeta' = M_{\zeta\zeta'}(j0, j0;\rho)$$

$$\text{(ii)}\qquad 0 = \langle\zeta\rangle = \langle\zeta'\rangle = -j\left.\frac{\partial M_{\zeta\zeta'}}{\partial u}\right|_{u,v=0} = -j\left.\frac{\partial M_{\zeta\zeta'}}{\partial v}\right|_{u,v=0}$$

$$\text{(iii)}\qquad \sigma^2 = \langle\zeta^2\rangle = \langle\zeta'^2\rangle = -\left.\frac{\partial^2 M_{\zeta\zeta'}}{\partial u^2}\right|_{u,v=0} = -\left.\frac{\partial^2 M_{\zeta\zeta'}}{\partial v^2}\right|_{u,v=0}$$

$$\text{(iv)}\qquad \sigma^2 R = \langle\zeta\zeta'\rangle = -\left.\frac{\partial^2 M_{\zeta\zeta'}}{\partial u\,\partial v}\right|_{u,v=0}\qquad .$$

Thus the statistical model of (D-1) and (D-2) possesses all the necessary requirements of a true probability density function and characteristic function of a random surface height, except that $W(\zeta,\zeta';\rho)$ is not symmetric in $\zeta$ and $\zeta'$ ( and consequently $M_{\zeta\zeta'}(ju, jv;\rho)$ is not symmetric in u and v).

B. Evaluation of Integral (2. 26) and (2. 27)

Employing the statistical model of (D-2) for the joint characteristic function, the expressions for the covariance between the scattered fields at two frequencies (given in (2. 26) and (2. 27); (2. 26) is the same as (4. 1) when the average backscattered field is zero) can be evaluated; by making the frequencies equal to each other, the result is the average backscattered power. Since both integrals are similar in form, only the derivation for (2. 26) will be made here. The integral of this equation has the form

$$I \equiv \int_0^\infty \rho J_o(2k_1 \sin\theta\rho)\, M_{\zeta\zeta'}(-j2k_1 \sec\theta, +j2k_2 \sec\theta;\rho)\, d\rho,$$

where according to (D-2), the joint characteristic function has the form (defining $m \equiv \dfrac{2k_1 \sec\theta\sigma}{\sqrt{3}}$ and $n \equiv \dfrac{2k_2 \sec\theta\sigma}{\sqrt{3}}$

$$M_{\zeta\zeta'}(-j2k_1 \sec\theta, j2k_2 \sec\theta;\rho) = \frac{1}{\{[1+m^2(1-R^2)][1+(-mR+n)^2]\}^{3/2}}$$

$$\equiv \frac{1}{D^{3/2}}$$

Upon expanding D, one has

$$D= 1+m^2+n^2+m^2n^2(1-R^2)+m^4(R^2-R^4)-2mn(R+m^2(R-R^3)) \quad .$$

But as correlation function R approaches zero, (i. e., for separation, $\rho$, very large), this function becomes $D=1+m^2+n^2+m^2n^2$, which is a very large quantity because of the fact that $k_1\sigma$ and $k_2\sigma$ are very large. Hence $1/D^{3/2}$ is very small and the integrand is negligible. But for $R=1$ (i. e., $\rho=0$), then $D=1+m^2+n^2-2mn=1+(m-n)^2$, which may be small for m and n approximately equal. However, since D becomes very large for R varying slightly from unity, the first two terms in the series expansion of $R=e^{-\rho^2/a^2}$ may be used (only the Gaussian class correlation coefficient model is considered here); therefore, in this range, $R=1-\frac{\rho^2}{a^2}$ . The equation for D becomes,

$$D = 1+(m-n)^2+\frac{2}{a^2}[m^2(m-n)^2+mn]\,\rho^2 , \quad \text{or}$$

$$D \equiv \alpha^2+\beta^2\rho^2 ,$$

where $\alpha^2 = 1+(m-n)^2$ , $\beta^2 = \frac{2}{a^2}[m^2(m-n)^2+mn]$ .

Let $\gamma \equiv 2k_1 \sin\theta$. Therefore the integral I becomes

$$I = \frac{1}{\beta^3}\int_0^\infty \rho J_0(\gamma\rho)\,\frac{1}{(\rho^2+\alpha^2/\beta^2)^{3/2}}\,d\rho = \frac{1}{\alpha\beta^2}\; e^{-\gamma\frac{\alpha}{\beta}} \quad .$$

The value of this last integral was found in Ref. 28, p. 696, #6. 554-4. Substituting the actual constants into $\alpha$, and $\beta$ and noting that $k_2 = k_1 + \Delta k$, one obtains

$$\alpha = \sqrt{1 + \frac{4}{3}\sigma^2 \Delta k^2 \sec^2 \theta}$$

$$\beta = \frac{\sqrt{2}}{a}\sqrt{\frac{16}{9}\sigma^4 \sec^4\theta k_1^2 \Delta k^2 + \frac{4}{3}\sigma^2 \sec^2\theta\, k_1 (k_1 + \Delta k)} \; ;$$

however, since $\Delta k << k_1$, the expression for $\beta$ becomes

$$\beta = \frac{\sqrt{2}}{a}\frac{2}{\sqrt{3}}\sigma \sec\theta k_1 \sqrt{1 + \frac{4}{3}\sigma^2 \Delta k^2 \sec^2\theta} = 2\sqrt{\frac{2}{3}}\frac{\sigma}{a} k_1 \sec\theta\,\alpha \; .$$

Therefore, the integral I becomes

$$I = \frac{3}{8}\frac{a^2}{\sigma^2 k_1^2}\cos^2\theta \frac{1}{\left(1 + \frac{4}{3}\sigma^2 \Delta k^2 \sec^2\theta\right)^{3/2}} e^{-\sqrt{\frac{3}{2}}\frac{a}{\sigma}\sin\theta\cos\theta} \; ;$$

substituting this value for I into the covariance of equation (2.26) and employing the fact that $S^2 = \frac{4\sigma^2}{a^2}$, and $\frac{k_1}{k_2} = 1$ for $\Delta k << k_1$, one obtains

$$\text{(D-3)} \qquad \text{Cov}[H_1^s H_2^{s*}] = \frac{H_{01}^i H_{02}^i}{4\pi R_o^2}\frac{3}{S^2} L_x L_y e^{j\Delta k R_o}$$

$$\left(\frac{\sin(\Delta k \sin\theta L_y)}{\Delta k \sin\theta L_y}\right)\cos^4\theta\, e^{-\frac{\sqrt{6}}{S}\sin\theta\cos\theta}$$

$$\cdot \frac{1}{\left[1 + \frac{4}{3}\sigma^2 \Delta k^2 \sec^2\theta\right]^{3/2}}$$

The averaged backscattered power is obtained by setting $k_1 = k_2 = k$, $\Delta k = 0$ in the above equation; this gives

$$\text{(D-4)} \qquad \mathrm{Var}[H^s] = \langle |H^s|^2 \rangle = \frac{|H^i|^2 L_x L_y}{4\pi R_o^2} \gamma_{PC} =$$

$$\frac{|H^i|^2}{4\pi R_o^2} L_x L_y \cdot \frac{3}{S^2} \cos^4\theta \, e^{-\frac{\sqrt{6}}{S}} \sin\theta \cos\theta \quad .$$

Equation (D-3) is equation (4. 7) of the text while (D-4) is used to find (3. 10c).

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