UNPUBLISHED PRELIMINARY DATA

CALIFORNIA INSTITUTE OF TECHNOLOGY.

NGR-05-002-028 6
ONR (Nonr-220(47))

THE GENERAL RELATIVISTIC INSTABILITY OF MASSIVE STARS,

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Presented in part at the Joint Discussion on Radio Galaxies on August 27, 1964 during the XIIth General Assembly of the INTERNATIONAL ASTROPHYSICAL UNION, Hamburg, Germany and in part at the XIIIth SOLVAY CONGRESS on September 24, 1964 at Brussels, Belgium.

ONE of THE "ORANGE AID" PREPRINT SERIES IN NUCLEAR ASTROPHYSICS.

November 6, 1964.

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XIIth GENERAL ASSEMBLY, IAU

Hamburg, Germany

I. INTRODUCTION

The pioneer theoretical investigations of Burbidge (1959) [1] and of Shklovsky (1960) [2] have shown that the observations on the extended radio sources imply the generation, storage and emission of prodigious amounts of energy, in round numbers of the order of $10^7 M_{\odot}c^2 \sim 10^{61}$ ergs or even more. On the very general grounds that the ultimate source of energy is the conversion of mass, it is thus clear that very large condensations of matter in some form or other are, or have been, associated with the radio sources. Burbidge (1962) [3] suggested supernovae explosions in large aggregates of stars as a possible mechanism for the original generation of the energy involved.

In the summer of 1962, after conversations with Geoffrey and Margaret Burbidge, Hoyle and I (1963a,b) [4,5] investigated what is perhaps the simplest of many possible models, namely that a mass of the order of $10^8 M_{\odot}c^2$ or greater has condensed into a single star in which the energy generation takes place. On this point of view, using the standard theory of stellar structure, one immediately obtains optical luminosities of the order of 10^{46} ergs/sec and lifetimes for nuclear energy generation of the order of 10^5 to 10^6 years so that the overall energy release is 10^{59} ergs. These figures roughly match the observational data for the so-called quasi-stellar objects subsequently discovered by Schmidt (1963) [6]. Hoyle and I were seeking an explanation of the energy requirements for extended radio sources and found that our model had a large optical luminosity. Problems in the stability of massive stars arise, as will be discussed in detail below. Questions of stability aside, it is apparent that nuclear energy generation by hydrogen burning in massive stars with $M \sim 10^8 M_\odot$ is adequate to match the energy requirements in the quasi-stellar objects.

However, the energy requirements for the extended radio sources involve nuclear burning in stars with $M \sim 10^{10} M_\odot$ or even more. This assumes that hydrogen burning with 0.7% conversion efficiency goes to completion in about 15% of the stellar mass, giving an overall efficiency of $\sim 0.1\%$ and an energy output $\sim 10^7 M_\odot c^2$. The efficiency of conversion of thermal energy into that of the high energy electron and magnetic fields necessary to give the synchrotron radio emission may only be of the order of 1% of even less. In this case nuclear burning in stellar masses approaching total galactic masses, $\sim 10^{12} M_\odot$, is required. Since there is no observational evidence for such wholesale nuclear conversion in the galaxies associated with the extended radio sources, Hoyle and I suggested gravitational collapse to the general relativistic limit as another possible source of energy. In principle all of the rest mass can be converted to energy in gravitational collapse although this requires that $2GM/Rc^2$ approach unity. It was realized that this ultimate 100% efficiency was probably not attainable during the collapse of an actual star because of the large red shifts in all forms of energy emission when $2GM/Rc^2 \sim 1$. Even so, the conversion of gravitational energy seemed more attractive to us than matter-antimatter annihilation which is also 100% efficient in the limit. We were unable to suggest a satisfactory model for the assembly of the matter and antimatter under realistic conditions. It did not seem unrealistic to suggest that a massive star of one type of matter could condense from the gas and smaller stars of a large galaxy, most probably in the galactic nucleus.

Feynman (1963) [7] first pointed out to us that general relativistic instabilities set in at a very early stage in the condensation of massive stars. Following Feynman's suggestion, Iben (1963) [8] carried out exact numerical integrations of the relativistic equations for a number of polytropes and confirmed Feynman's ideas. In my own work (Fowler, 1964a,b) [9,10] I have found that the first order post-Newtonian approximation is sufficient to illustrate the general physical principles involved and is particularly useful in investigations of the conditions under which nuclear reactions occur in massive stars.

Let me hasten to say that Chandrasekhar (1964a,b,c) [11,12,13] has now given in very elegant form the exact treatment of the dynamical instability of massive stars. After some initial disagreements concerning numerical values, when we both performed our sums correctly [10,12], agreement was reached on such matters as the radius for the onset of instability and so forth. Since the Dallas conference in 1963 this field of study has become a very active one, and in particular, McVittie (1964) [14] Gratton (1964) [15] and Zel'dovich (1964) [16] have independently made significant contributions in the approach to the solution of the problem. At the California Institute of Technology, James Bardeen is carrying out numerical calculations on the dynamical collapse using the IBM 7094.

2. BINDING ENERGY OF A MASSIVE POLYTROPE IN HYDROSTATIC EQUILIBRIUM

The binding energy E_B of a star is equal but opposite in sign to the total energy E exclusive of the rest mass energy and, when the star has radius R, is given by

$$- E_B = E = (M - M_o)c^2 \tag{1}$$

where $M = M(R)$ is the mass of the star and M_o is the total rest mass of its constituent particles. M is to be determined in principle by measuring the force exerted on a unit mass at a large distance (>> R) from the star and then using Newton's inverse square law of gravitational attraction. On the other hand, M_o can be measured by identifying and counting the constituent particles and multiplying by the appropriate rest mass.

One now employs the general relativistic equations for M and M_o and the general relativistic equation for hydrostatic equilibrium throughout the star. Each expression in these equations is appropriately expanded in terms proportional to integral powers of the gravitational constant G and only the Newtonian term and the next higher order term are retained. In this way [9,10] the post-Newtonian approximation for the total energy of a spherically symmetric, non-rotating star under hydrostatic equilibrium is found to be

$$E_{eq} = - 6\pi \int \beta p r^2 dr + \frac{8\pi G}{c^2} \int p r M_r dr + \frac{6\pi G^2}{c^2} \int \rho M_r^{\,2} dr \qquad (2)$$

where r is the radial coordinate, p is the pressure, ρ is the mass-energy density expressed in mass per unit volume, M_r is the mass interior to r and β is the ratio of gas pressure to total pressure (gas plus radiation). In order to appreciate the order of the terms in equation (2) it should be noted that p is linear in G in the approximation under discussion so that the first term on the right hand side of equation (2) is the classical Newtonian term and the last two are the post-Newtonian terms. In deriving equation (2) it was assumed that the stellar material is completely ionized into electrons and nuclei but that the temperature is below $T = 10^9$ degrees so that special relativistic effects for electrons and electron-positron pair formation can be neglected. Under these conditions the internal energy of particles and radiation per cm^3 is given by $3p(1 - \beta/2)$.

It is illuminating to express the classical term, which will be designated as $E_{eq}^{(1)}$, in terms of the appropriate average for β throughout the star. Thus

$$\begin{aligned} E_{eq}^{(1)} &= - 6\pi\langle\beta\rangle \int p r^2 dr = - \tfrac{1}{2} \langle\beta\rangle \int 3p dV \\ &= - \tfrac{1}{2} \langle\beta\rangle \Omega \qquad (3) \\ &= - \frac{3\langle\beta\rangle_n}{2(5-n)} \frac{GM^2}{R} \end{aligned}$$

Here the gravitational binding energy Ω, taken as a positive quantity, has been introduced. It is well known in classical hydrostatic equilibrium that $\Omega = \int 3p dV$ and that for a polytrope of index n, $\Omega = 3GM^2/(5-n)R$. In the approximation under consideration it is not necessary to distinguish between M and M_o and so the superfluous subscript has not been retained. In the last form of equation (3) the dependence of $\langle\beta\rangle$ on the polytropic index is made explicit by appending the subscript n.

The classical binding energy per unit mass is obtained by dividing equation (3) by Mc^2 to obtain

$$\frac{E_{eq}^{(1)}}{Mc^2} = - \frac{3\langle \beta \rangle_n}{4(5-n)} \left(\frac{R_g}{R} \right) \tag{4}$$

where $R_g = 2GM/c^2 = 3 \times 10^5 \ (M/M_\odot)$ cm is the limiting gravitational radius or <u>Schwarzschild limit</u> and the right hand side of equation (4) is the first and linear term in a power series in the dimensionless parameter $R_g/R = 2GM/Rc^2$. The post-Newtonian terms are, of course, quadratic in this parameter. For polytropes of index n, the post-Newtonian expression for the binding energy per unit mass can be reduced to

$$\frac{E_{eq}}{Mc^2} = - \frac{3\langle \beta \rangle_n}{4(5-n)} \left(\frac{R_g}{R} \right) + \zeta_n \left(\frac{R_g}{R} \right)^2 + \cdots \tag{5}$$

where

$$\zeta_n = \frac{3}{8(n+1)} \ \frac{R_n^{\ 2}}{M_n^{\ 3}} \left[\int_0^{R_n} \theta_n^{\ 2n+1} \ \xi^4 d\xi + \frac{10}{n+2} \int_0^{R_n} \theta_n^{\ n+2} \ \xi^2 d\xi \right] \tag{6}$$

In equation (6), ξ is the dimensionless radial variable used by Chandrasekhar (1938) [17] in treating polytropes, R_n is the value of ξ at the surface of the polytrope, $\theta_n = \theta_n(\xi)$ is the Lane-Emden function for the polytrope and $M_n = - \xi^2 d\theta_n/d\xi$ at the polytropic surface. M_n is a dimensionless measure of the mass of the polytrope. It will be recalled that the run of the variables throughout the polytrope are given by $\rho = \rho_c \theta_n^{\ n}$ and $p = p_c \theta_n^{\ n+1}$ where the subscript c designates central values. For a nondegenerate gas $(T/\mu\beta) = (T/\mu\beta)_c \theta_n$.

Equation (6) can be evaluated analytically for n = 0, 1, and 3 and the results are

$$\zeta_o = \frac{57}{280} = 0.2036, \quad \zeta_1 = \frac{1}{\pi} = 0.3183 \quad \text{and} \quad \zeta_3 = \frac{3}{16} \left(\frac{3}{\pi} \right)^{\frac{1}{2}} R_3 = 1.264 \tag{7}$$

where $R_3 = 6.897$ has been used.

3. THE CRITICAL RADIUS, TEMPERATURE AND DENSITY FOR THE ONSET OF DYNAMICAL INSTABILITY

The coefficient ζ_n is positive and thus the internal energy required for hydrostatic equilibrium eventually becomes positive, the binding energy is negative and the system is unbound rather than bound. The energy goes through a minimum or the binding energy through a maximum at a critical radius given by

$$\frac{R_c}{R_g} = \frac{8(5-n)}{3}\,\frac{\zeta_n}{\langle\beta\rangle_n} \approx \frac{4(5-n)}{9}\,\frac{\zeta_n}{\langle\Gamma_1 - 4/3\rangle_n} \tag{8}$$

This ratio is $19/7\ \langle\beta\rangle_0 = 2.714/\langle\beta\rangle_0$ for $n = 0$, $32/3\pi\ \langle\beta\rangle_1 = 3.395/\langle\beta\rangle_1$ for $n = 1$ and $(3/\pi)^{\frac{1}{2}}\ R_3/\beta_3 = 6.740/\beta_3$ for $n = 3$. For $n = 3$, β is a constant throughout the polytrope and averaging is unnecessary. In the last form of equation (8) the relation $\Gamma_1 - 4/3 \approx \beta/6$ has been employed as a fair approximation in massive stars. Γ_1 is the first of the adiabatic coefficients defined in [17]. The results for R_c/R_g are identical to those obtained in [11,12,13].

The early onset of stability can now be traced to the fact that R_c is inversely proportional to $\langle\beta\rangle_n$ which is small for massive stars. Fowler and Hoyle (1964) [18] have shown in massive stars that

$$\beta \approx \frac{1}{\mu}\left[\frac{3}{4\pi}\,(n+1)^3\,\frac{\Re^4}{aG^3}\right]^{\frac{1}{4}}\left(\frac{M_n}{M}\right)^{\frac{1}{2}}\ \theta_n^{(n-3)/4} \tag{9}$$

where μ is the mean molecular weight and the other symbols have the customary meanings. For a polytrope of index $n = 3$, β is constant throughout the polytrope and is given by

$$\beta \approx \frac{4.3}{\mu}\left(\frac{M_\odot}{M}\right)^{\frac{1}{2}} \tag{10}$$

This expression holds roughly for the average value throughout any polytrope and for hydrogen with $\mu = \frac{1}{2}$ yields $\beta \sim 10^{-3}$ in a polytrope with mass $M = 10^8\ M_\odot$. The upshot is that R_c is several thousand times R_g for such a mass, the actual factor being sensitive to the polytropic index. It is interesting to note that (5), (8) and (10) yield $E_{eq} \approx 2\ M_\odot c^2 \sim 4 \times 10^{54}$ ergs at the minimum for all large masses.

The onset of instability below the critical radius

$$R_c \sim 2.3 \times 10^5 \ (M/M_\odot)^{3/2} \text{ cm} \qquad (n=3,\ \mu=\tfrac{1}{2}) \qquad (11)$$

can be understood in the following way. Consider an adiabatic compression at a point below the critical radius. Hydrostatic equilibrium after the perturbation requires more internal energy and pressure than before and since this is not made available in the adiabatic compression, further collapse ensues. Consider an adiabatic expansion. Now hydrostatic equilibrium requires less internal energy and pressure than given adiabatically so expansion continues. Clearly the radius at which E reaches a minimum is critical in this regard. At larger radii the decrease in the equilibrium energy as R decreases gives the well known classical stability. When an actual star reaches the critical radius it will lose energy by radiation and the general relativistic instability will lead to collapse rather than expansion unless some internal energy resource can be called upon.

Can nuclear energy supply the energy necessary to halt the general relativistic collapse and perhaps even reverse the motion by supplying more than that required by equation (5) for hydrostatic equilibrium? This is a problem still under attack but this much can be made clear. The central temperature and the central density at criticality can be shown [9,10] to be relatively insensitive to the polytropic structure in contrast to the outer radius and are given by

$$T_c = 2.5 \times 10^{13} \ (M_\odot/M) \text{ degrees} \qquad (12)$$

$$\rho_c = 2.0 \times 10^{18} \ (M_\odot/M)^{7/2} \text{ gm cm}^{-3} \qquad (13)$$

It will be noted that the critical values are only $T_c = 2.5 \times 10^5$ degrees and $\rho_c = 2.0 \times 10^{-10}$ gm cm^{-3} for $M = 10^8\ M_\odot$. The density is very small indeed but it will be recalled that the central density at the Schwarzschild radius for a polytrope of index 3 is only ~ 100 gm cm^{-3}. The main point is that general relativistic considerations come into play in massive stars long before central temperatures and densities necessary for nuclear reactions to take place are reached. For hydrogen burning, $T \sim 8 \times 10^7$ degrees at $\rho \sim 10^{-2}$ gm cm^{-3} are required.

4. GENERAL RELATIVISTIC GRAVITATIONAL COLLAPSE

Nuclear energy or any form of energy must thus be generated during the collapse stage and the time scale for collapse becomes highly relevant in connection with generation rates per unit time. The hydrodynamic equation for the acceleration in the post-Newtonian approximation can be written

$$\frac{dv}{dt} = \frac{d^2r}{dt^2} \approx - \frac{1}{\rho}\frac{dp}{dr} - \frac{GM_r}{r^2}\left(1 + \frac{4GM_r}{rc^2}\right) + \tag{14}$$

where M_r is the mass interior to r. The numerical coefficient of the post-Newtonian term is approximately correct only for the polytrope with index $n = 0$ (constant ρ) and then only in hydrostatic equilibrium. However, equation (14) is sufficiently accurate for our present purposes.

In classical free fall the pressure gradient in a star is set equal to zero and the acceleration is just that due to the gravitational forces. The increase in kinetic energy of fall can be readily computed from the change in the gravitational potential energy. **Starting from rest at a radius large compared to R, the velocity of free fall at R is**

$$v_{ff} \approx c\left(\frac{2GM}{Rc^2}\right)^{\frac{1}{2}} \tag{15}$$

and the characteristic e-folding time in R or $T_8 = T/10^8$ is

$$\tau_{ff} \approx \frac{R}{c}\left(\frac{Rc^2}{2GM}\right)^{\frac{1}{2}} = 10^3\left(\frac{R}{R_\odot}\right)^{3/2}\left(\frac{M_\odot}{M}\right)^{\frac{1}{2}} \text{ sec}$$

$$\approx \frac{160}{(T_8)_c^{3/2}}\left(\frac{M}{M_\odot}\right)^{\frac{1}{4}} \text{ sec}$$

$$\sim 2 \times 10^4 \text{ sec for } M = 10^8 M_\odot,\ (T_8)_c = 0.8 \text{ (H-burning)} \tag{16}$$

It can be argued that the gravitational collapse is not free fall but arises from the post-Newtonian terms in the general relativistic expressions for the pressure gradient. The pressure gradient would just be balanced by the classical terms if general relativity were not taken into account and hence to order of magnitude the acceleration is equal to the post-Newtonian term. The kinetic energy per unit mass becomes equal to $\frac{1}{2} c^2 (2GM/Rc^2)^2$ not just $\frac{1}{2} c^2 (2GM/Rc^2)$ and so

$$v_{gc} \approx c\left(\frac{2GM}{Rc^2}\right)$$

Note that $v_{gc} \approx v_{ff} \approx c$ in the limit $2GM/Rc^2 = 1$.

The e-folding time is

$$\tau_{gc} \approx \frac{R}{c}\left(\frac{Rc^2}{2GM}\right) = 5 \times 10^5 \left(\frac{R}{R_\odot}\right)^2 \left(\frac{M_\odot}{M}\right) \text{ sec}$$

$$\approx \frac{4 \times 10^4}{(T_8)_c^{\,2}} \text{ sec} \quad \underline{\text{independent of } M}$$

$$\sim 10^5 \text{ sec} \sim 1 \text{ day}, \ (T_8)_c = 0.8 \qquad (17)$$

We are reminded of the quotation from The Lucky Chance by Aphra Behn (1640 -1689): "Faith, sir, we are here to day, and gone to morrow." In spherically symmetric general relativistic collapse the time scale for the release of nuclear energy is very short and for $M \geq 10^7 M_\odot$ the collapse is probably not stopped. However, for $M < 10^7 M_\odot$ the nuclear resources would seem to be adequate to stop and reverse the collapse. Oscillations of the star then become possible if adequate modes of energy transmission to and emission from the surface are available. It can be shown that ordinary thermal mechanisms are grossly inadequate. Shock wave phenomena leading to the generation of high energy particles presumably come into play and may well lead to the excitation of the HII and radio-emitting regions surrounding the quasi-stellar objects. Detailed calculations are underway in Pasadena on these problems.

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XIIIth SOLVAY CONGRESS

Brussels, Belgium

I. FIRST DISCUSSION

The formation of massive stars in the range 10^4 to 10^{10} $M_\odot$ has come to be of considerable interest in connection with the possible release of gravitational energy in order to meet the energy demands in extended radio sources. Numerous investigators in many countries, including Feynman, Iben, Chandrasekhar, Gratton, Zel'dovich, Hoyle and myself, have shown that massive stars with spherical symmetry exhibit an instability which follows directly from general relativistic considerations. Chandrasekhar has given a very elegant and exact proof of this instability. My own calculations deal only with the post-Newtonian approximation, but give results identical to Chandrasekhar's in the appropriate limit, while at the same time making the physical considerations involved quite clear and transparent.

Consider a spherically symmetric, non-rotating polytrope of index n, gravitational mass M, and radius R, in hydrostatic equilibrium. It is well known that the internal energy E, exclusive of the rest mass energy $M_o c^2$ of the constituent particles, and the binding energy E_B, are given on the basis of Newtonian mechanics by

$$E = - E_B = (M - M_o)c^2 = \tfrac{1}{2}\beta\Omega = - \frac{3\beta}{2(5-n)}\left(\frac{GM^2}{R}\right) \qquad (1)$$

where Ω is the gravitational potential energy and $\beta = p_g/(p_g + p_r)$ is the ratio of gas pressure, p_g, to total pressure, that of gas and radiation, p_r. When Eddington's quartic equation is solved for massive polytropes it is found that

$$\beta \approx \frac{4.3}{\mu}\left(\frac{M_\odot}{M}\right)^{\frac{1}{2}}, \qquad (2)$$

where μ is the mean molecular weight equal to $\frac{1}{2}$ for pure hydrogen. This expression holds throughout a polytrope with index n = 3 but is approximately correct in the central regions of any polytrope. It will be noted that β is quite small for massive stars being $\sim 10^{-3}$ for $M = 10^8\ M_\odot$. Thus the Newtonian term in the binding energy is relatively small in massive stars.

In order to fully appreciate the general relativistic effect in the binding energy it is best to divide by Mc^2 to obtain

$$\frac{E}{Mc^2} = -\frac{E_B}{Mc^2} = -\frac{3\beta}{4(5-n)}\left(\frac{2GM}{Rc^2}\right) \tag{3}$$

where now the dimensionless parameter $2GM/Rc^2$ appears. It will come as no surprise that the post-Newtonian approximation introduces a term in the square of this parameter so that for a polytrope of index $n = 3$, for example, one finds

$$\frac{E}{Mc^2} = -\frac{E_B}{Mc^2} = -\frac{3}{8}\beta\left(\frac{2GM}{Rc^2}\right) + 1.264\left(\frac{2GM}{Rc^2}\right)^2 + \cdots\cdot \tag{4}$$

In this approximation it is not necessary to distinguish between rest mass, M_o, and gravitational mass M as long as $2GM/Rc^2$ is small. Thus the internal energy required for hydrostatic equilibrium eventually becomes very large and positive, the binding energy is negative, and the system is unbound rather than bound. The energy goes through a minimum or the binding energy through a maximum at a critical radius given for $n = 3$ by

$$R_c = \frac{6.740}{\beta} R_g \tag{5}$$

and for $n = 0$ by

$$R_c = \frac{19}{7\beta} R_g \tag{6}$$

where

$$R_g = \frac{2GM}{c^2} = 3 \times 10^5 \left(\frac{M}{M_\odot}\right) \text{cm} \tag{7}$$

is the limiting Schwarzschild radius. Because of the smallness of β this critical radius is considerably greater than R_g and can be expressed for $\mu = \frac{1}{2}$, $n = 3$ as

$$R_c = 2.3 \times 10^5 \left(\frac{M}{M_\odot}\right)^{\frac{3}{2}} \text{cm} \tag{8}$$

which becomes 2.3×10^{17} cm for $M = 10^8\ M_\odot$. Unfortunately the critical radius is quite sensitive to the polytropic index just as are all radius parameters in polytropic structures. As n approaches 5, R_c rapidly diverges. Thus comparisons with observations on the radii of massive "cores" in quasi-stellar objects are subject to considerable uncertainty since details of the internal structure of the core are involved.

On the other hand, it can be shown that the central temperature and the central density are not greatly sensitive to the polytropic structure and the critical values are given by

$$T_c = 2.5 \times 10^{13} M_\odot/M \text{ degrees} \tag{9}$$

and

$$\rho_c = 2.0 \times 10^{18} (M_\odot/M)^{7/2} \text{ gm cm}^{-3} \tag{10}$$

The numerical coefficients displayed hold for n = 3 but are fair approximations for n = 0 to 5. It will be noted that the critical values are only $T_c = 2.5 \times 10^5$ degrees and $\rho_c = 2.0 \times 10^{-10}$ gm cm^{-3} for $M = 10^8 M_\odot$. The density is very small indeed but it will be recalled that the density at the Schwarzschild radius for a polytrope of index 3 is only ~ 100 gm cm^{-3}. The main point is that general relativistic considerations come into play in massive stars long before central temperatures and densities necessary for nuclear reactions to take place are reached. For hydrogen burning $T \sim 8 \times 10^7$ degrees at $\rho \sim 10^2$ gm cm^{-3} are required.

The general relativistic instability follows directly from the fact that E increases below the critical radius or above the critical temperature and density, this behavior being just the opposite to that before the critical conditions are reached. Consider an adiabatic compression at a point below the critical radius. Hydrostatic equilibrium after the perturbation requires more internal energy and pressure than before and since this is not made available in the adiabatic compression, further collapse ensues. Consider an adiabatic expansion. Now hydrostatic equilibrium requires less internal energy and pressure than given adiabatically so expansion continues. Clearly the radius at which E reaches a minimum is critical in this regard. At larger radii the decrease in the equilibrium energy as R decreases gives the well known classical stability. When an actual star reaches the critical radius it will lose energy by radiation and the general relativistic instability will lead to collapse rather than expansion unless some internal energy resource can be called upon. As noted above this cannot be nuclear energy until the collapse is well advanced. Whether the onset of nuclear energy generation can halt and eventually reverse the collapse is now a matter of extensive study by several groups. Considerations are also being given to the effectiveness of other stabilizing agents such as rotation, fragmentation, internal turbulence and entrained magnetic fields.

It is also of interest to point out that the luminosity of a massive star is not greatly sensitive to the polytropic structure being given by $L \sim 2 \times 10^{38}\ M/M_\odot$ ergs sec^{-1} which yields 2×10^{46} ergs sec^{-1} for $M = 10^{8}\ M_\odot$. It is this result which is of particular interest in connection with the observed luminosities of the quasi-stellar objects.

2. SECOND DISCUSSION

It is natural that nuclear physicists should be preoccupied with the source of the energy in radio galaxies and quasi-stellar objects. Energy generation in stars by means of exothermic nuclear reactions has stimulated many interesting and fruitful researches in nuclear physics. In approaching these new problems we are accustomed to begin with Einstein's mass-energy equation, $E = Mc^2$. However, this can be quite misleading. It is better to write

$$E = -E_B = (M - M_o)c^2 = 2 \times 10^{54}\ (M - M_o)/M_\odot \text{ ergs}, \qquad (11)$$

where E is the energy (exclusive of rest mass energy) of a system composed of particles with total rest mass M_o when by some mechanism of interaction the mass, measured gravitationally by an external observer, has been reduced to M. E_B is the positive binding energy of the system which has been released by the interaction and is the energy store available for transformation at varying efficiencies into various observable forms -- optical emission, radio emission, neutrinos, high energy particles and so forth. In the numerical expression the masses are expressed in solar units.

In principle it is possible to reduce M to zero but not to negative values and so the maximum available energy is indeed $M_o c^2$. One mechanism by which this can be accomplished is annihilation of equal amounts of matter and antimatter. No detailed theory is available of the way in which advantage can be taken of annihilation in the radio objects but this mechanism may indeed be the ultimate solution to the energy problem.

G. R. Burbidge has reviewed the energy requirements for us. For the quasi-stellar objects shining at 10^{46} ergs in the optical range over an estimated interval of 10^6 years, the total requirement is 3×10^{59} ergs. Radio emission by these objects is lower by one or two orders of magnitude. It is important to emphasize that this requirement can be met by the nuclear resources of a supermassive star or a number of massive stars. Hydrogen burning yields 0.007 of the rest mass in energy and perhaps 25% of a massive star can be converted to helium while on the main sequence. The overall yield is thus 3×10^{51} ergs per solar mass involved and the stellar mass required is $10^8\ M_\odot$, close to the mass assigned to the core of a quasi-stellar object by Schmidt in one of his models.

The problem does not lie in the energy resources but in the instability of massive stars which I have previously discussed at this Congress, and in more detail in the Reviews of Modern Physics <u>36</u>, 545 (1964). The central temperature at the onset of general relativistic instability in a non-rotating, spherically symmetric massive star is insensitive to the polytropic structure and is given by

$$T_c \approx 2.5 \times 10^{13}\ M_\odot/M \text{ degrees} \qquad (12)$$

which becomes $T_c \approx 2.5 \times 10^5$ degrees for $M = 10^8 M_\odot$. This is to be compared with the temperature at which hydrogen burning begins in massive stars, namely $T_c \approx 8 \times 10^7$ degrees. Thus general relativistic instability sets in long before hydrogen burning begins, collapse is initiated and it is a question of whether the onset of the burning can reverse the collapse and restore some semblance of stable equilibrium. Rotation, fragmentation, internal turbulence and entrained magnetic fields probably serve as additional mechanisms in this regard. The simpler problem, ignoring these agents, is under detailed study at Caltech, notably by Mr. James Bardeen using the full panoply of the general relativistically correct dynamic equations. My own interest lies primarily in the nuclear reactions involved. If carbon, nitrogen or oxygen nuclei are present, the hydrogen burning occurs through the rapid CNO bi-cycle in which proton capture by radioactive nuclei such as N^{13}, O^{15} and F^{17} occurs at a rate comparable to that of their intrinsic beta decays. The nuclear reactions in massive stars composed initially of pure hydrogen are of considerable interest in that such stars may well have produced substantial amounts of helium and even some heavier elements early in the history of the Galaxy. The pp-interaction to form deuterons is much too slow to be effective on the short time scale available under collapse conditions and the first effective nuclear process is electron capture by protons to produce neutrons and neutrinos according to

$$p + e^- \rightarrow n + \nu. \tag{13}$$

The neutrons are in turn captured by protons to form deuterons by

$$n + p \rightarrow d + \gamma, \tag{14}$$

and the deuterons then interact in a variety of ways to form alpha-particles by reactions which can be symbolically represented by

$$2d \rightarrow \alpha. \tag{15}$$

The temperatures and densities at which these hydrogen to helium conversion processes occur are high enough that the alpha-particles produce C^{12} through the well known Salpeter-Hoyle process

$$3\alpha \leftrightarrow C^{12*} \rightarrow C^{12}. \qquad (16)$$

The resultant C^{12} initiates the CNO bi-cycle and leads to rapid catalytic processing of hydrogen into helium. Additional proton, neutron and alpha-particle reactions lead to the formation of still heavier elements. In this way it may prove possible to account for the substantial amounts of helium and the traces of heavier elements thought to exist in the oldest stars, Pop II, in the Galaxy. Only detailed computer calculations can yield the ultimate solution to these problems.

We have emphasized that general relativistic instability and not lack of energy resources is the main difficulty in associating large masses with the cores of the quasi-stellar objects. The situation is quite different for the extended radio sources associated with elliptical galaxies where a realistic estimate of the various efficiency factors involved in the ultimate release of energy in the radio range leads on the basis of current knowledge and conventional ideas to a difficult and perhaps paradoxical situation.

Burbidge has told us that the minimum energy stored in relativistic electrons and magnetic fields in the extended radio sources is of the order of 10^{59} ergs. The magnetic field need not be far different from the equipartition value in either direction for this stored energy to be ten times higher, namely, 10^{60} ergs. If relativistic protons are associated with the electrons to give a neutral plasma the energy becomes 2×10^{61} ergs. Biermann has pointed out that known events in which relativistic particles are produced, such as solar flares, have efficiencies for energy transfer into the relativistic domain at most equal to 1%. On the basis the raw energy supply must be 2×10^{63} ergs, the full energy equivalent of 10^9 solar masses. Sandage has associated the radio sources with large elliptical galaxies for which the mass is thought to be 10^{12} $M_\odot$. The radio ellipticals differ very little in appearance from the ordinary ellipticals and only the nucleus has probably been involved in the violent event which led to the development of the radio source. On this basis one is tempted to associate not more than 1% or 10^{10} $M_\odot$ with the energy production. To obtain the energy equivalent of 10^9 $M_\odot$ it is then necessary that the raw energy production mechanism be at least 10% efficient. Nuclear production fails in this regard and as a consequence Hoyle and Fowler suggested the release of gravitational binding energy in massive stars as another possibility. It is now realized that relative red shifts greater than 10% occur when the energy release exceeds 10%. Thus the rate of emission of energy in <u>any</u> form is greatly retarded for strongly bound, collapsing systems and it is difficult to find actual physical mechanisms which release the gravitational energy rapidly enough under this handicap. Several calculations have indicated that 10% is a <u>practical</u> upper limit for the energy released before the Schwarzschild radius is reached and the red shift becomes infinite. If this practical limit is reached in 10^{10} $M_\odot$ the raw energy release just matches that required. However, it is difficult to assess the accuracy of our estimates for the various efficiency factors involved. It may well prove the case that large scale magnetohydrodynamic activity can lead to high energy particle production with far greater than 1% efficiency and may involve far more than 1% of the mass of a galaxy. On the other hand, the limitations arising from the instability and short time scales pertaining to massive systems according to general relativity theory may be removed if unconventional modifications of this theory, such as those of Hoyle and Narlikar, prove in the long run to correspond more closely to reality.