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AN INVESTIGATION OF CONTROLLED TETHERING IN SPACE

Contract NAS 1-3912

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AN INVESTIGATION OF CONTROLLED TETHERING IN SPACE

Contract NAS 1-3912

Project 349

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PREFACE AND ACKNOWLEDGEMENTS

This technical investigation is based upon, and is an extension of a previous NASA investigation into the retrieval of an incapacitated extra-vehicular astronaut by controlled tethering conducted by The Marquardt Corporation. The present investigation examines the dynamics of the tethering/retrieval process and develops mathematical expressions and design criteria required for further development of this technology.

This study contract was performed within the Advanced Products Department of the Power Systems Division. Personnel contributing to this contract include: R. W. Adlhoch, J. H. Clements, W. E. Warner, U. E. Sbaraglia, D. P. Muhonen, W. H. Straly, and T. A. Sedgwick.

ABSTRACT

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This report summarizes the work performed under Task I of NASA Contract NAS 1-3912 dated 6 May 1964. This study was sponsored and funded by the Langley Research Center, National Aeronautics and Space Administration. The purpose of the study was to provide the necessary analyses and preliminary designs, as well as development program plans, for controlled tethering/retrieval operations. The results of this study may be applied to further development and flight testing of a controlled tethering system. The Task I investigations reported in this document include:

1. Development of the equations of motion of connected bodies in orbit
2. Development of various operational philosophies for tethered retrieval
3. Investigation of tether line dynamics during retrieval
4. Analysis of gravity gradient effects
5. Development of a digital computer program for rapid investigation of overall orbiting system
6. Development of preliminary design concepts for operational hardware.

author

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ILLUSTRATIONS

I. SUMMARY

The study reported in this document had, as primary objective, the extension of controlled tethering technology by analytical, computational, and design procedures. The work performed in this study, NASA Contract NAS 1-3912, was a follow-on to a previous study, NASA Contract NAS 1-2577. One of the important results of this total effort was the development of a computational technique to analyze the motion of multiple bodies in orbit while they are connected by a flexible tether. This result provides a versatile aid for conducting investigations into the nature of tethered body motions. Although developed primarily to investigate the dynamics of tethered retrieval of extra-vehicular astronauts, the technique can be used for a multiplicity of missions involving the use of tethers, both controlled and uncontrolled, connecting bodies in space.

The investigations leading up to the development of a digital computer program to examine the retrieval dynamics of a tethered astronaut were of the following nature. Analytical expressions were derived to represent, in mathematical form, the physical nature of the problem. These expressions were examined to ascertain the significance of particular system parameters and simplified to the extent that hand computations could be made. These simplified results then formed the basis for establishing the boundary conditions for more precise machine computations. At the same time, design investigations established the practical range of operations for the mechanical system, thus furnishing additional guidelines for the computations.

The major ground rules established at the beginning of the present contract that affected the selection of the operational mode of retrieval were as follows:

1. The tethered body was an incapacitated astronaut

2. The operational envelope of the astronaut should extend to a few thousand feet from his space vehicle at a velocity of a few tens of feet per second.

Although operational models involving other retrieval techniques were considered, these ground rules excluded them from the present study because they either required astronaut inputs or exceeded physiological and/or mechanical system limits. The two competitive techniques which did fit the ground rules were the use of propulsion and angular momentum redistribution. The decision was made to pursue the retrieval technique which used angular momentum redistribution on the basis of its flexibility within the astronaut extra-vehicular operational region. Within the family of angular momentum redistribution techniques, the 3-body concept was selected. Using this concept, as the astronaut is reeled in, a third body is extended on a separate tether to absorb system momentum. It was selected for its simpler control problems, its suitability to an inactive tethered body,

and its lower design complexity. Further, the 3-body retrieval system is competitive in overall system weight and developmental costs. Upon development, the basic configuration can be used for other applications without major modification.

The significant conclusion drawn from this study was that during the tethered retrieval of an incapacitated astronaut from an extra-vehicular assignment angular momentum must, in most cases, be removed from the system. This is best done by its redistribution to a third body, termed a momentum module or "anchor mass". During the deployment of the anchor mass on the secondary tether, and the simultaneous or subsequent retrieval of the astronaut, control must be effected on both the reeling rates of the tethers and the tension in the primary tether. A simpler control logic increases difficulty for the general case. Results showed that control over these two basic parameters would be sufficient to maintain a proper phase angle between the primary and secondary tether lines. If the phase angle relationship between tethers is not controlled, undesirable angular momentum transfer to the astronaut occurs during some portion of the retrieval process. This is especially true when the primary tether is short and the secondary tether is long.

Although additional analyses are indicated in order to permit detailed design of some segments of the controlled tethering system for a general application, the investigations reported here permit the initiation of the development of hardware leading to an operational system. The development and the simulation study plans developed as a part of this investigation, reported in Reference 1, together with the results of this study, will permit the development and qualification of man-rated controlled tethering equipment to proceed with confidence.

II. INTRODUCTION

With the advent of lengthier and more complex orbital operations by manned vehicles, an increasing number of missions may be expected to involve some extra-vehicular activity of man or machinery. Since many unusual and unexpected situations may arise in the course of a complex maneuver, human intelligence and dexterity may often be required to be applied at some location remote to the central vehicle. Examples would include the servicing of distant equipment, trouble shooting or even rescue operations. To prevent his drifting away and to enable recovery in the event of an emergency, as well as to provide him with a sense of well being, a physical connection or tether between the astronaut and the vehicle will probably be required.

For most extra-vehicular operations it may be assumed that the astronaut's departure from and return to the vehicle are accomplished by the same mode of transportation, his personal propulsion unit. When all systems are operating, the return trip is expected to involve no greater difficulty than the departure. During the period in which the astronaut is separated from the vehicle, the total system angular momentum may undergo dramatic changes. It is not unlikely that its peak value may be greater than its initial value by several orders of magnitude. This change is not usually accompanied by large energy or velocity changes in the system components and is therefore easily overlooked. Upon successful completion of the maneuver, the system angular momentum is returned to essentially its initial state through mass expulsion so that its existence during the operation causes no real problem.

If, however, a failure occurs in some portion of the system when its angular momentum is at a high level, the entire complexion of the operation is changed. If the angular momentum is allowed to remain in the astronaut while retrieval is effected, the retrieval state will differ appreciably from the initial state in that a large increase in the astronaut angular velocity will be experienced. If during retrieval angular momentum is transferred from the astronaut, his final angular velocity will be diminished proportionally.

The objectives of this study are the investigation of means by which astronaut retrieval through controlled tethering may be affected and the selection and preliminary development of that method which provides the greatest potential with regard to simplicity, reliability, weight, and cost. This work is an extension of and an elaboration upon the results of a previous Marquardt study on the retrieval of tethered astronauts, performed under NASA Contract NAS 1-2577, Reference 2.

Following a preliminary selection of the retrieval method (with NASA's assistance), the scope of the work included the following:

1. Retrieval Philosophy

   The three-body retrieval system was adopted. In this scheme, a third body (anchor mass) in addition to the astronaut and vehicle is employed to act as a sink for undesirable system angular momentum during retrieval. After the initiation of retrieval, this anchor mass is deployed on its own tether from the astronaut location and always remains distant from the system mass center. As the retrieval operation progresses, the proper relative position between the astronaut and anchor mass is maintained to provide a net transfer of angular momentum from the astronaut to the anchor mass. Upon completion of the retrieval, the bulk of the system angular momentum has been transferred to the anchor, so that the angular velocity of the astronaut remains within tolerable limits.

2. Operational Limits

   Although the results of the study are qualitatively applicable to a wide range of environmental conditions, the analysis was performed under the assumption that the system was in a circular earth orbit of no more than several hundred miles altitude. Separation distances between the three bodies were taken as several thousand feet and initial tangential velocity of the astronaut was limited to a nominal value of 25 fps. No active control was exercised on the tether at the astronaut location, and retrieval times of more than fifteen minutes were not considered.

III. DISCUSSION OF PROBLEM

The method of retrieval under consideration is the three-body momentum redistribution scheme. Using this method, the astronaut is drawn in toward the vehicle by his tether while a third body is extended outward from him on its own tether. As this occurs, the tendency of the astronaut to increase his angular velocity is offset by the tendency of the third body (anchor mass) to decrease its angular velocity as it moves outward from the system mass center. Each of these bodies attempts to restrain the motion of the other through the mechanism of tension in the cable connecting them. This tension and the relative misalignment of the two tethers under consideration effects a net transfer of system angular momentum to the anchor mass. Throughout a successful retrieval operation, sufficient angular momentum is transferred from the astronaut to the anchor mass so that radial accelerations and therefore line tensions are maintained within tolerable limits.

The retrieval philosophy has been simply stated, but its analysis offers considerable difficulties. Even when the rotational motions of the astronaut and anchor mass are ignored, these bodies still possess six degrees of freedom and one constraint between them. Since the vehicle may be expected to possess relatively large polar moments of inertia, it exhibits three rotational as well as three translational degrees of freedom. Therefore, as the dynamics of the vehicle are introduced, six additional degrees of freedom and another constraint are added to the problem. Moreover, the internal angular momentum transfer is not limited to the astronaut and anchor mass alone; the vehicle angular momentum may be expected to undergo continuous change also.

In general, two approaches to the problem may be followed, analytical and computational. The analytical approach, of necessity, requires that certain simplifying assumptions be made which to some extent compromise the validity and applicability of the results. However, the results appear in the form of equations whose properties may be extensively examined by a suitable variation of their parameters. In contrast, the digital computation approach may be used to solve particular cases and configurations of the entire complex of equations simultaneously - the overall problem - but each solution stands alone and only indirectly gives any measure of important system trends. For these reasons, analytical and computational work has been done in a parallel effort to provide a common solution.

In the interest of accuracy and completeness, certain special considerations were introduced into the problem during the analysis. Among these were gravity gradient effects and both orbital and hardware characteristics. The orbital effects were the apparent accelerations of Euler and Coriolis. Hardware characteristics are concerned with the vehicle geometry, with the mechanical properties of the tether such as elasticity and hysteresis, and with the flexible nature of the tether which makes it incapable of sustaining a compressive load. These considerations are not of primary importance insofar as order-of-magnitude studies are concerned, but show significant contributions in more detailed work. All the above mentioned effects are discussed in detail elsewhere.

Upon the establishment of suitable design criteria from the results of the analysis, it remains to incorporate the appropriate properties into the design of actual hardware. Preliminary designs are advanced in the succeeding discussion that indicate that the controlled tethering system analyses can be implemented.

IV. ANALYSIS

A. Mathematical Model

The nature of the analysis depends heavily upon the mathematical model describing the system under consideration. As was mentioned earlier, the application of strictly analytic methods requires an overall simplification of the problem statement to permit any solutions at all. This simplification is that the mathematical model reduces to two point masses, the astronaut and the anchor, moving about a stationary vehicle in a Newtonian reference frame. For many applications it has been shown that this is a reasonable approximation of the problem. A complete discussion of this approach appears in Appendix A.

For use in digital computer solutions the mathematical model of the problem as derived in Appendix B is far more extensive. Again, the astronaut and anchor are treated as point masses, and in addition the vehicle is represented as a freely moving rigid body having a finite mass and non-zero moments of inertia. Also, in this model the astronaut is connected to the vehicle by an elastic tether which in general attaches to the vehicle at some point other than the mass center. Consequently, during a solution of the tethered maneuver, the tension in the primary tether exerts a torque about the vehicle mass center as well as transmitting a linear force to both the bodies it connects. The mathematical model in use here describes the dynamically coupled angular motion of the vehicle in terms of its principal moments of inertia, its existing angular velocity components, and the components of applied torque due to primary tether tension. As well as describing the angular motion of the vehicle, the model also includes the linear accelerations of all three bodies involved.

Since the entire configuration is assumed to move in a circular earth orbit, the equations of motion are established relative to an earth oriented coordinate system and are not Newtonian. To render the classical equations of motion applicable to this situation, the artificial accelerations of Coriolis and Euler are introduced along with the real gravity gradient forces. These additional terms are added as their first order terms in their power series expansions since the higher order terms converge very rapidly in the area of interest in this problem.

The tethers connecting the three masses are assumed to be massless but are assigned the mechanical properties of elasticity, internal damping and hysteresis. A separate analysis of a tether having a distributed mass and moving in a rotating reference frame is presented in Appendix C. This consideration was not involved in the analysis reported herein; however, the inclusion of this effect is considered beneficial to overall system operation and would be an asset.

It has been shown in earlier studies (References 2, 3, and 4) that all environmental effects other than those already discussed may be dismissed as being insignificant for times involved in manned tethering operations. These neglected effects include aerodynamic drag, solar radiation pressure, magnetic effects, and others.

B. Operational Envelopes

For those cases to which the analytic methods of Appendix A are applied, it is assumed that all motion occurs in a single plane. Since precessional forces and other out-of-plane effects are not considered, this assumption is merely a consequence of all initial motion being limited to the plane in question.

It is further assumed that the retrieval operation is sufficiently slow that no appreciable transients or oscillations occur. The transfer of angular momentum is taken to occur slowly and smoothly so that both the astronaut and anchor mass traverse the vehicle with equal angular rates at all points in time during the retrieval.

The operational philosophies and envelopes in the case of the digital computer analysis developed in Appendix B, however, are practically unlimited. The retrieval may be effected at conditions which induce severe oscillations and transients under any combination of retrieval conditions and an accurate time history of actual behavior may be obtained. In addition to the equations of motion in which gravity gradient effects and orbital characteristics are embodied in the basic program structure, a variety of system characteristics is also available to the operator. Among these are the following:

1. Selection of component masses.

2. Selection of vehicle moments of inertia.

3. Selection of tether attachment location on vehicle.

4. Direction of local vertical to vehicle surface at tether attachment point.

5. Gain constants and deadband range of second order attitude control system for vehicle to track astronaut during tethered operations.

6. Linear thrust capabilities at both vehicle and astronaut locations.

7. Selection of circular orbital altitude.

8. Tether elasticity.

9. Tether hysteresis.

A complete listing of the system inputs and an explanation of each appears in Appendix D.

C. Utilization of Mathematical Models

The mathematical model developed for analysis is contained in Appendix A. As previously discussed, this model has incorporated a number of simplifications to make it amenable for hand computation. In no way do these simplifications affect the validity of the results when first order effects are sought. This model then serves as a basis for rapid hand or analog computation to study the system dynamics for qualitative review. A wide range of conditions can thus be surveyed and boundaries established for more detailed, exhaustive digital computer studies.

In order to investigate the effects of more subtle ingredients of a system of interconnected orbiting bodies, mathematical expressions were derived to study the overall system simultaneously. Such a mathematical model, described in detail in Appendix B, necessitated the aid of a digital computer facility. A series of digital computer runs were made during the study. A portion of them are plotted in Figures 1 through 13.

The astronaut's true velocity and the tension in the primary tether are plotted as a function of time during the retrieval process in Figures 1, 2, and 3. The initial conditions were held constant with the exception of the secondary tether length. It can be observed that a secondary tether length of 3,000 feet resulted in a better system behavior for the retrieval mode employed which maintained a constant reel-in speed. It should be noted that a modification of this constant reel-in mode will be required for the general case. This modification will entail control of both primary tether tension and reel-in rate, particularly in the terminal phase. This terminal increase in tension is emphasized in Figures 4, 5, and 6 where the same parameters are plotted as a function of distance.

The digital computer solution also frees the restraint concerning the angular relationship of the astronaut and the anchor mass during the retrieval process. The angles $\phi_{astronaut}$ and $\phi_{anchor\ mass}$, as shown in Figure 7, represent the angular position of the tetherlines of the astronaut and anchor mass as projected onto the horizontal plane of the orbiting reference frame. The angles $\Theta_{astronaut}$ and $\Theta_{anchor\ mass}$ are the elevation angles of the tetherlines to the astronaut and anchor mass as measured from this horizontal plane. It can be seen that the angles Θ and ϕ are the Euler angles as discussed in Appendix D. The phase angle relationship between the primary and the secondary tethers is such that angular momentum is transferred from the astronaut to the anchor mass during the majority of the operation. Angular momentum transfer in the proper sense takes place only when the astronaut leads the anchor mass in angular position about the vehicle. In this case the line tension produces a torque on the anchor mass which is of the same sense as its angular momentum vector and the system operation is as desired. When the angular lead of the astronaut exceeds a straight angle, an effective lag angle results and angular momentum is transferred back to the astronaut. Near the termination of the retrieval process it is observed that control over the system configuration is reduced to the extent that angular momentum may be returned to the astronaut. This behavior further emphasizes the need for closer operational control in the terminal phase.

Figure 8 is a plot of the primary tether angle, η , relative to the local normal vector at its point of attachment at the vehicle surface. In the tracking philosophy employed, no attitude control torque is applied to the vehicle until the angle in question (the error signal in this case) exceeds a certain deadband. For the system under consideration the deadband is arbitrarily taken as 30°. It may be seen from the figure that the relative angle between the tether and the vehicle remained well within its deadband during most of the retrieval operation. Since during this time the astronaut and anchor mass executed several complete revolutions about the system mass center, it is evident that the vehicle followed the astronaut angular motions without employing outside torque from the on-board altitude control system. This is indicative of very smooth performance wherein angular momentum is actually transferred to the vehicle from the astronaut through the mechanism of tether tension. When the space vehicle increases in mass and moments of inertia, active attitude control will be required. The Gemini vehicle was used for these representative computer runs.

The astronaut's radial velocity is plotted as a function of time for a constant rate retrieval process in Figure 9. Due to tether line elasticity the commanded reel-in velocity varies slightly from that experienced by the astronaut. It can be seen that near the retrieval process termination, when the momentum module is out of phase, the astronaut's radial velocity fluctuates slightly.

Again, a simple control fix will permit smooth retrieval over the entire operational regime. As illustrated in Figure 10, it is observed that the distance between the astronaut and the anchor mass remains nearly constant, with a slight increase observed near the retrieval termination due to line stretch. The length of the primary tether is shown as constantly decreasing with time.

The motion of the space vehicle, astronaut, and anchor mass is shown in Figure 11 as projected on the plane of original astronaut motion. It can be observed that for the majority of the retrieval, the phase angle relationship between the tethers is such as to transfer angular momentum from the astronaut to the momentum module. The same motion is shown in Figure 12 as projected on the plane of orbital motion. This figure illustrates the precession of the plane of motion during the retrieval operation. A combination of Figures 11 and 12 would show the system motion as the retrieved bodies passed in and out of the initial plane of motion as well as the angular orientation of the bodies relative to each other.

A representative constant tension retrieval philosophy is presented in Figure 13. Here the tension was deliberately set below the required retrieval level to observe the oscillatory motion of the astronaut about the space vehicle. A judicious control exercised over both line tension and reel-in rate suggests itself as a method of combining the desirable and eliminating the undesirable features of both modes of retrieval discussed above.

V. HARDWARE CONSIDERATIONS

A. General Considerations

The controlled tethering electromechanical system was analyzed to determine functional and operational requirements necessary to satisfy the retrieval of an incapacitated astronaut from an extra-vehicular assignment. Each subsystem was studied in detail, with trade-off designs based on overall system requirements. The subsystems that were chosen in this study to make up the controlled tethering system include those listed below. They are described in following paragraphs.

1. Tethering system controller module
   a. Drum assembly
   b. Capstan drive assembly
   c. Electronic controller
   d. Drive mechanism

2. Angle sensor and activation assembly

3. Cable and attachment equipment assembly

4. Momentum module assembly

5. Display module.

The preliminary design studies were based upon the general design specifications contained in Table I. Other specifications resulting from different initial conditions will result in different system weights and volumes, but unless the retrieval requirements vary considerably, the design solutions remain essentially unchanged.

The controlled tethering system is illustrated in Figure 14 in block diagram form. This system has been developed with consideration of either automatic or manned control over its operation.

B. Tethering System Controller Module

The main function of this unit is to provide control and power for the primary tether during system operation. That portion of this assembly which was designed by The Marquardt Corporation is shown in Marquardt Drawing No. 228201, Figure 15. The basic elements of this assembly are listed as follows:

1. Prime mover and power transmission

2. Drive capstan

3. Cable storage reel

4. Level wind mechanism

5. Load sensors

6. Cable idlers

7. Static brake.

The prime mover is a 28 V d-c electric motor having a rated torque of 1.0 in.-lb at 13,500 rpm. The power transmission consists of appropriate gearing and electromagnetic clutches to deliver driving torque to both the cable capstan and the take-up reel.

A dual channel rotary servo actuator such as shown on Drawing No. 56-65-42R1 (Figure 16) was selected to provide the drag tension on the winch cable during an excursion of an astronaut. During retrieval it will provide tension in the cable plus power for cable and astronaut retrieval. Each servo actuator consists of a case enclosing a direct-current space type motor, two spur gear trains, and two pairs of counter-rotating clutches. The motor used in this design is very similar to the one used on the Apollo Service Module and others; therefore, considerable design information and test data is available for use in designing this motor.

The magnetic particle clutch, which forms an important part of this unit, has demonstrated its high level of performance capabilities in almost every field of servo or automatic control applications. The proportional torque producing capabilities of this device adapt themselves quite well to the smooth and sensitive operation demanded in an application of this nature. Since the clutch transmits torque by using dry magnetic powder, it has excellent starting characteristics and no leakage problems even at operating conditions. The gear trains consist of envolute spur gears. Gear systems similar to this have been produced in production quantities for other space applications.

The primary power output of the system is transmitted to the cable through the drive capstan rather than the take-up reel. This design eliminates the difficulties usually associated with control of line tension and proper cable lay when the system power is delivered directly to the reel. A sufficient torque is delivered to the reel to permit winding and to insure proper capstan operation.

Several cable idlers and guides are included in the design to provide proper line routing and compact packaging. In addition to these functions, two idlers serve in a dual capacity. One idler pulley is mounted on a load measuring device to provide a tension sensing capability and a second idler is spring loaded to compensate for transient differences in capstan and reel speeds. In addition to the system elements whose functions are associated with the dynamics of its operation, a static brake is provided to lock the reel in a fixed position during periods of inactivity.

Functional descriptions of the controlled tethering system controller module components are given in Table II. Preliminary specifications, as shown in Table III, were the basis for these designs.

C. Angle Sensor and Activation Assembly

The general function of this assembly is to provide a universal guide for the tether at the vehicle skin, provide a capability for measurement of the tether angle relative to the vehicle, and provide an attachment point for the fully retrieved astronaut. (See Marquardt Drawing No. 228202, Figure 17). The cable is provided a suitable guide within an angle of 45 degrees in any direction from the normal to the vehicle skin at the attachment point by means of suitable contoured surfaces which constrain the motion of the tether in that region. At all possible points of contact, these surfaces are limited to a curvature of 1.5 inch radius so that the cable will not be unduly loaded in bending while passing through the assembly. Within the working range of 45 degrees from the normal, the relative cable angle in two planes is sensed by a pair of moveable guides, or bails, which are freely pivoted to follow the motion of the cable. When the cable angle exceeds the working range, the cable contacts and depresses the limit sensing ring which actuates a switch to stop cable reel-in until the entire system is again oriented so that the cable angle is within the working range.

The astronaut attachment hook is at the end of the cable. The first three feet of tether outward from this point is covered with a protective shroud to prevent it from abraiding the astronaut's space suit. This terminal shroud also serves several other important functions. By reason of its increased diameter, it is easily seen by the astronaut, thereby providing him with an immediate reference to his orientation relative to the space vehicle. Since this section is also stiffer in bending than the basic cable, it tends to maintain itself as an approximately straight shaft, holding the thin unprotected portion of the cable away from the immediate vicinity of the astronaut. The shroud is also compressible so that it may function as a shock absorber upon completion of the retrieval operation. When this unit is fully compressed, it can easily be stored.

Fixed to the end of the shroud section on the far side of the astronaut attach hook is the shutoff ring which engages the limit sensing ring at the completion of a retrieval operation. This action activates the limit switch which shuts off the retrieval drive mechanism.

Concentric with the entire assembly is an area which provides space for mounting a lanyard ring. This ring is attached to the free end of a short connecting cable, or lanyard, which extends from the vehicle interior and is trailed by the astronaut as he approaches the location where his propulsion unit is stored. The ring is transported to the assembly location at the time when extra-vehicular activity is to be initiated and is held in its place by small permanent magnets mounted around the assembly. The purpose of this ring is twofold: first to provide a redundancy in the system so that at no time during the maneuver the astronaut is without a positive link to his vehicle - and second, it provides some means for the onboard astronaut to return an incapacitated extra-vehicular astronaut to within reach of the entry hatch. The function of this and other components is more clearly seen in the proposed operational procedure which is shown in Table IV. The procedure listed is primarily an automatic one, with the on-board astronaut serving as a monitor of the extra-vehicular activity. The participation of the on-board astronaut will depend to a great extent on the mission, the space vehicle, and the retrieval requirements. As an operational system is developed, the machine-operator interface should be determined to optimize the overall mission.

Functional descriptions of the angle sensor mechanism as well as the astronaut attachment assembly are given in Table II. It should be noted that the angle sensor assembly serves to obtain the relation of the tether to the space vehicle surface, activates and shuts off the controller module, and guides the tether from the controller module through the space vehicle surface.

The attachment mechanism discussed in this section is typical of that required for an extra-vehicular excursion of an astronaut. Detailed design of such features will depend upon interfaces between the astronaut's suit, his personal propulsion unit, any equipment carried by the astronaut, and the momentum mass philosophy adopted.

D. Cable and Attachment Equipment Assembly

The connector serving as the tether to secure and retrieve the astronaut is one of the most vital of the controlled tethering system components. Significant lengths demand the utmost of cable technology. The specific tensile strength should be high, the cable should possess high flexibility, there should be a built-in redundancy in case of partial malfunction, and the surface should be smooth. In addition, the tether should lose none of its desirable characteristics in the space environment. To meet these stringent requirements, a multiple stranded cable encased in teflon with an overall diameter of 0.070 inch was tentatively selected. Extensive testing of this component is required before final selection.

One proposed cable configuration is illustrated in Figure 18. This cable, denoted as Type A, would provide direct communication with the tethered astronaut. Variations on this approach will provide solutions to the flexible connector problems previously noted as well as a communications link through the tether. Type B cable, that requiring no telemetry circuit, is not as complex as the Type A tethering cable. A reduction of the cable diameter for an equal load carrying capacity is a significant factor in the Type B cable. A dip process for coating of both cable types may allow an additional reduction in the overall diameter as opposed to a wrapping process.

E. Momentum Module Assembly

The anchor mass assembly consists of a release mechanism, a cable storage reel, a reel-out speed limiter, and an inert mass to act as an angular momentum sink. See Marquardt Drawing No. 228205, Figure 19.

The release mechanism is a mechanical latch which may be released either directly by the astronaut or on signal from the space vehicle. If the astronaut were incapacitated, the latch would be actuated by the latter method. An acceleration sensitive type release would be a highly filtered mechanical accelerometer which releases the latch when an acceleration in excess of a preset value is imposed on the system for a given time interval, probably three or four seconds. This is accomplished by the compression of a spring by a mass moving through a resisting medium. The mass moves under the influence of the centripetal acceleration imposed on the astronaut when the retrieval of an incapacitated astronaut is initiated.

The cable storage device is a reel of standard design which rotates to pay out cable during deployment of the unit. The reel-out speed is limited by a friction brake acting directly on the end faces of the reel. The magnitude of the braking force is determined as a scheduled function of the amount of remaining cable on the reel as determined by a roller follower. As the roller follower moves inward on the reel, brake discs linked to the follower move radially across the end faces engaging areas of differing coefficients of friction. The braking is selected to halt the outward motion of the anchor mass smoothly near the end of its tether so that undesirable impacting is avoided. In the event that a portion of the astronaut's equipment is used for the momentum mass, similar provision for deployment would be made.

The controlled tethering system specifications that guided the design and selecting of subsystem components are tabulated in Table III. The design specifications for the remainder of the system components were generated on the basis of the previously stated ground rules and established engineering practices. As development activities continue, the design specifications become increasingly more detailed.

F. Sensors for Flight System

The parameters to be measured would normally lend themselves to rather conventional forms of instrumentation; however, the hard vacuum environment creates a special problem because of the materials and rubbing or contacting surfaces usually found in conventional transducers. Thus it becomes evident that in nearly every case, special attention will be required in adapting components to the space environment.

In addition to modifying available components, it appears advisable to consider seriously the use of electro-optical, magnetically-coupled, or capacitive-coupled devices with the objective of eliminating the problem areas associated with materials.

Some brief comments are offered in the following, summarizing some of the considerations involved in selecting sensor components.

1. Tether Tension

The mechanical spring element for the basic force sensor was designed for long term accuracy and stability. Spring deflection is sensed by a differential transformer or variable reluctance displacement transducer. Strain gages are also a possible choice, however, gage cement poses a problem in the vacuum environment. An unbonded gage design could be worked out, but would be more fragile, requiring extra protection. The tether tension sensor should have the following characteristics:

a. Low signal threshold

b. High response

c. Stretch compensation

d. A range from 0 to 100 lbs force.

2. Tether Length (Range)

The length of tether deployed is measured by counting the number of revolutions of the drive capstan. A magnetic pickup or a hermetically-sealed reed switch actuated by a permanent magnet on the rotating member, generates one or more pulses for each revolution of capstan. A calibrated counter integrates the count to display the tether length. Pickup and counter configuration must be capable of bi-directional operation. A photoelectric pickup would also be practical. This sensor should have 1% accuracy, stretch compensation, offer digital measurement, and be capable of measuring the total tether length.

3. Tether Velocity (Range Rate)

The rotational speed of the capstan drive is the basic parameter sensed. The pickup can be on a-c tachometer generator with voltage output accurately proportioned to velocity. Photoelectric or magnetic pickups are also satisfactory, generating pulses at a frequency proportional to velocity. A frequency discriminator circuit converts pulse frequency to analog d-c voltage for display and control signal. The range rate sensor should have an accuracy of 1% over its range of 0 to $\pm$ 25 fps. A positive range rate servo is preferred.

4. Tether Angle

Angle sensing guide bails are the basic elements. The angular displacement of the bails is sensed by synchros, differential transformers, or a-c induction potentiometers. Synchros would require special Teflon sleeve bearings and replacement of rotor brushes with pigtail leads. Any of the above transducers would require special insulation and coil impregnation. The wire bail low inertia angle sensor selected for this function should have an accuracy of 5% of the linear band. The envelope of operation should include a cone of about 30 degrees half angle for normal sensitivity with less accuracy required to the full range of twice this angle. Components that meet these requirements are readily available.

VI. OBSERVATIONS AND CONCLUDING REMARKS

A. General Comments

During the course of the investigations of the controlled tethering/retrieval system it was observed that the angular momentum of the orbiting system was the primary deterrent to a successful operation. Further, it was concluded that to retrieve an inactive tethered object it was necessary to either furnish propulsive capability to one or more of the connected bodies or effect a redistribution of the system angular momentum. In most cases requiring retrieval or control via the flexible connector, angular momentum redistribution techniques were concluded to offer the most straight forward solution. Comparative studies between the two concepts have shown that the momentum redistribution techniques for retrieval allow for a greater range of extra-vehicular activity with any basic system. Also, this technique permits the basic system to be used for a wider variety of mission applications. The specific angular momentum redistribution technique that is the most versatile is based on a 3-body concept. Through a variety of analyses, it has been concluded that this approach lends itself to easier mechanization, simpler control requirements, and is competitive in weight, cost, and efficiency with more sophisticated techniques. Specific observations and conclusions are listed below.

B. Specific Comments

1. Analysis of the overall retrieval problem entails the use of a digital computer facility. Although segments of the problem may be solved by analog computer techniques, the significance of comparatively long-term, second-order effects may be masked. Precession of the planes of motion, influence of orbital motion on the dynamic system, etc. require computational accuracies beyond that of the analog computer. Analysis by hand techniques is prohibitively lengthy if these effects are sought. Although hand analysis is slow and, as well as analog techniques, lacks the accuracy of digital computer solutions, they have been used and should be continued for rapid verification and survey purposes.

2. The effect of a uniformly distributed mass in the form of a flexible connection between the orbiting bodies has been concluded to be of significance in the study of system dynamics. Although the computations did not include this parameter, it was concluded that its presence will assist in reducing any undesirable operational characteristics. Future digital computer program modifications can readily accommodate this feature.

3. It was observed that both the astronaut and the momentum module deviated from the original plane of motion. These deviations do not adversely affect the retrieval operation involving a relatively low mass vehicle such as considered here. This motion must be predicted, however, for retrieval from large space complexes whose other operational characteristics might impose boundaries on the retrieval plane.

4. It was observed that retrieval attempts with constant tension created sustained oscillations or reciprocations in tether length as a function of time. Unless the tension selected is so high as to demand unreasonable rates on the reel-in mechanism, the retrieval stalls. At this point in the retrieval attempt, the astronaut begins to separate from the space vehicle. The momentum module contributes to this reciprocation by alternately assuming and then releasing system angular momentum to the astronaut. However, judicious manipulations of both tension and reel-in speed can circularize the astronaut's trajectory at any tether length prior to reaching maximum tension. After the oscillations have been damped from the system, retrieval may again be initiated. It can therefore be concluded that constant tension retrieval attempts must have other controlling inputs to prevent system limits from being exceeded.

5. A retrieval by controlling the power level of the reel-in mechanism was one method used to alleviate this terminal condition; however, there still existed some oscillations in tension and corresponding reel-in rate. It was observed that for similar maximum tether loadings, the constant power method could effect a successful retrieval but took about twice as long as the constant reel-in retrieval when used over the entire operation. A modified philosophy would further alleviate this terminal condition.

6. This modified philosophy appears to be one that returns the astronaut to his parent vehicle using an essentially constant reel-in rate initially. Upon approaching a predetermined primary tetherline tension or angular velocity value, the reel-in rate should then be reduced so that the angular velocity of the anchor mass will approach that of the astronaut. In this manner, the proper phase angle relationship between the primary and secondary tetherlines can be maintained. By a proper maintenance of the tetherline phase angles, the astronaut can be retrieved to the immediate proximity of the parent vehicle. At this point, the secondary tetherline with its anchor mass, can be released and the retrieval operation successfully culminated.

C. Concluding Remarks

The significant conclusion drawn from this study was that during the tethered retrieval of an incapacitated astronaut from an extra-vehicular assignment angular momentum must, in most cases, be removed from the system. This is best done by its redistribution to a third body, termed a momentum module or "anchor mass". During the deployment of the anchor mass on the secondary tether and the simultaneous or subsequent retrieval of the astronaut, control must be effected on both the reeling rates of the tethers and the tension in the primary tether. A simpler control logic increases difficulty for the general case. It was concluded that control over these two basic parameters would be sufficient to maintain a proper phase angle between the primary and secondary tether lines. If the phase angle relationship between tethers is not controlled, undesirable angular momentum transfer to the astronaut occurs during some portion of the retrieval process. This is especially true when the primary tether is short and the secondary tether is long.

It is expected that an optimum, or near optimum, retrieval philosophy would incorporate the desirable features of several of the methods which were investigated singularly. One promising method would include the combination of the constant tension and constant reel-in rate methods described above. In this case, the astronaut would be retrieved at a constant rate until a pre-determined level of tension was reached, at which time the reel-in rate would be controlled to maintain tension at this constant level.

Although additional analyses are indicated in order to permit detailed design of some segments of the controlled tethering system for a general application, the investigations reported here permit the initiation of the development of hardware leading to an operational system. The development and the simulation study plans developed as part of this investigation, reported in Marquardt Report No. 6079, Reference 1, together with the results of this study, will permit the development and qualification of man-rated controlled tethering equipment to proceed with confidence.

D. Direction of Future Activity

Further effort should be expended on advancing controlled tethering technology in three basic areas: analyses, simulation, and hardware development. In a companion report produced as Phase II of the contract reported herein, The Marquardt Corporation Report No. 6079, "Program Plan for Controlled Tethering Experiment", Reference 1, the requirements for a flight test program are set forth. In addition to the recommendations made in that report for further activity on tethering technology, the following is recommended.

1. Analysis

a. Tethers with Distributed Mass

The investigations of long tether lines have dealt primarily with massless cables, insofar as the system dynamics are concerned. Although this situation is recognized as a simplification of the problem, the solution of the overall problem is considered as conservative when tether mass is excluded. A great deal of background work has been performed on the general subject of tethers of distributed mass in space, however, a detailed treatment of the problem is considered beyond the scope of this investigation. Therefore, it is recommended that further analyses of this problem be made. The objective of these analyses would be to define suitable expressions that predict the overall motion of connected bodies in orbit where line mass is a significant portion of any both within the system.

b. Tethering System Interface

The problems associated with the integration of controlled tethering with other elements of the total system should be thoroughly considered. It is recommended that analyses and preliminary designs of solutions to these interface problems be continuously conducted during controlled tethering system development. By this approach, many design conflicts can be avoided by judicious compromise; thus, a more nearly optimum overall system can be developed.

2. Simulation

In addition to the research simulations recommended in TMC Report No. 6079, procedural simulations should be conducted on the deployment and retrieval of tethered bodies. Investigations of the degree to which the operator participates in normal and emergency situations should be carried out and valid simulations conducted prior to system design finalization. The on-board operator controls and displays should be examined for optimum mission performance. These findings should then be validated by meaningful simulations.

3. Development

Hardware development of the controlled tethering system is a requirement prior to experimental flight tests. In this development sequence, however, certain items have longer development lead times than others. One such item involves the communications link between the tethered astronaut and the space vehicle. Cable technology must be advanced to provide this capability without exceeding weight limitations.

Therefore, it is recommended that additional effort be expended developing suitable tethers that offer desirable characteristics regarding strength, flexibility, hysteresis, resistance to degradation in the space environment, etc., as well as serve as an efficient communications carrier.

VII. REFERENCES

1. The Marquardt Corporation Report 6079, "Program Plan for Controlled Tethering Experiment," 4 December 1964. UNCLASSIFIED.

2. The Marquardt Corporation Report S-356, "Dynamics of Retrieval of Tethered Astronaut," Contract NAS 1-2577, 1 November 1963. (Reprinted as NASA Contractor Report CR-185). UNCLASSIFIED.

3. The Marquardt Corporation Report 20,155, "Moment Requirements for Satellite Attitude Control," March 1963. UNCLASSIFIED.

4. Jensen, J., G. Townsend, J. Kork, and D. Kraft, "Design Guide to Orbital Flight," McGraw-Hill, New York, New York, 1962.

5. Goldstein, H., "Classical Mechanics," Addison-Wesley Publishing Co., Inc., Reading, Mass., 1956.

6. AMRL-TDR-62-123, "An Analysis of the Behavior of Long Tetherlines in Space," Mueller, D. D., Capt., USAF, November 1962. UNCLASSIFIED.

TABLE I

PRELIMINARY SPECIFICATIONS FOR
CONTROLLED TETHERING SYSTEM

(Established for designs used in this study)

System/Parameters	Design Specification
Overall System Operating Envelope	
1. Maximum ΔV of astronaut	25 ft/sec
2. Maximum excursion distances	2000 ft
3. Maximum weight to be retrieved	425 lbs
4. Maximum acceleration (by operational astronaut propulsion system)	1/40"g"
5. Maximum acceleration (by tether)	1/4 "g"
6. Normal working load on tether	1/8 "g"
7. Maximum tension in tether	100 lbs
8. Working tension in tether	50 lbs
9. Assumed maximum retrieval time	15 min

TABLE II

FUNCTIONAL DESCRIPTION OF COMPONENTS

A. Tethering System Controller Module

Design Feature or Functional Component	Function Performed	Description
1. Dual Output Electro-Mechanical Actuator	Provides torque to capstan and reel for driving power.	A rotary servo-actuator consisting of D.C. motor; four dry magnetic particle clutches, two stages of spur gearing, two output shafts.
2. Gear Train and Housing	Gear train encased in a housing provides transmission of power from actuator to winding mechanism.	Gear train composed of a set of double reduction gears, consisting of 2 pinions and 2 gears, all enclosed in a gear housing.
3. Reel, support and cover	Reel provides storage for cable and provides ground for electromechanical brake. Reel support and cover provides support for one end of the reel and for an electromechanical brake. This reel support and cover completely encases the reel.	The reel is a spool shaped item allowing storage of the cable. The reel support and cover is a cylinder, with one end open and one end closed.
4. Level Winding Mechanism	Allows cable to be reeled on and off smoothly and evenly.	Mechanism consist of a lead screw with a double helix thread and a level winder follower which tracks the helix screw. The follower also has a spiral-shaped covered groove to allow the cable to pass from the reel to the capstan.
5. Pulleys and Cable Guards	The pulleys guide the cable to its destination - providing tension and change of direction. Cable guards prevent cable from detaching itself from pulleys.	The pulleys are constructed with deep grooves for retention of cable and integral low friction bearings. Cable guards are constructed of thin sheet metal which covers a minimum of cable-pulley contact area.

TABLE II (Continued)

Design Feature or Functional Component	Function Performed	Description
6. Idler Arm Assembly	Supports the cable tensioning pulley. Idler is preloaded by means of a "Bendix flexural pivot" which maintains tension within a prescribed limit.	The idler arm is a clevis fitting with one-half of the flexural pivot fixed to it; and the clevis end accepts the pulley.
7. Capstan and Support Structure	Transmits power from actuator to cable. Structure ties capstan to mounting plane.	Capstan is drum shaped pulley which accepts multiple wraps of cable. Capstan driven by spline shaft from activator.
8. Cable Velocity and Length Sensor	Senses cable velocity and cable length through rotation of capstan.	An electro-mechanical device that translates rotation into electrical impulses which read out length and velocity.
9. Tension Sensor	Senses cable tension as cable leaves unit.	A strain-gage device that measures bending moment in pulley pin and provides an electrical signal which after conversion is read out as cable tension.
B. Angle Sensor and Activation Assembly		
1. Cable Guide	Directs cable from deployment unit to angle sensing bails.	Funnel type structure formed internally to a radius equal to 45 times the cable diameter and flared at small end to receive cable from deployment unit.
2. Bails and Synchros	Senses azimuth and co-elevation of cable.	Two double pail handle type "frictionless" guides 90° apart - synchros are directly attached to pivoting end of bails - cable passes thru slot in each of bails - as cable changes direction (due to astronaut maneuvers), "frictionless" bails follow cable thereby rotating synchros which register azimuth and co-elevation on screen in spacecraft.

TABLE II (Continued)

Design Feature or Functional Component	Function Performed	Description
3. Pivots	Supports and allows bails to pivot with minimum friction - stops bails when maximum angle of 45° from vertical is reached.	Four "frictionless" TFE pillow blocks - max. bail angle stop integral part of pillow block.
4. Magnetic Bolts	Retains capsule lanyard ring while astronaut is performing space maneuvers.	Dual purpose magnetic bolts (8 required) - attaches limit switch actuator retaining ring to main structure and secures capsule lanyard ring in stowage position.
5. Capsule Lanyard Ring	Twofold function: provides for positive link of astronaut to space vehicle while performing maneuvers and provide means of returning incapacitated extra-vehicular astronaut to within reach of entry hatch.	Ring made of steel tubing - has cable connecting it to vehicle and quick-disconnect snap hook for attachment to astronaut.
6. Limit Switch Actuator Retaining Ring	Retain limit switch actuator against pressure of compression spring.	Ring (7.5 in. O.D.) is bolted to structure by eight magnetic bolts.
7. Limit Switch Actuator	Actuates limit switch when depressed by shut-off ring or tether cable.	Curved shell ring - curve form equal to 45 times the cable diameter.
8. Limit Switch	When actuated, stops cable reel-in until system is reoriented.	Miniature arm-type limit switch.
9. Shut-Off Ring	Depresses limit switch actuator when retrieved astronaut arrives at spacecraft.	Steering wheel shaped ring/internal guide for cable - bayonet disconnect for cable cover attachment.

TABLE II (Continued)

Design Feature or Functional Component	Function Performed	Description
10. Bayonet Disconnects	Two-fold function: provides attachment of anti-abrasion cover to shut off ring and also for compression of anti-abrasion cover for stowage.	Disconnects are of opposite latching for safety in operation.
11. Anti-Abrasion Cable Cover	Primary Function: To provide protection for astronaut's spacesuit from abrasion caused by the cable - other functions: acts as shock absorber upon completion of retrieval operation; due to stiffness in bending, it tends to hold the thin unprotected portion of the cable away from astronaut; also its increased diameter makes it easily seen by the astronaut, thereby providing him with an immediate reference to his orientation relative to the space vehicle.	Fabric covered 7/8 in. diameter x 30 in. (extended) compression spring made of small diameter and low resistance wire - can be compressed to 5 in. length.
12. Spherical Bearing	Prevents any torsion in cable due to maneuvers of astronaut.	TFE lined spherical bearing located between anti-abrasion cable cover and quick disconnect snap hook.
13. Quick-Disconnect Snap Hook	Attaches astronaut to vehicle by cable.	Standard Air Force parachute harness snap hook - modified to take spherical bearing.

TABLE III

COMPONENT SPECIFICATIONS FOR
CONTROLLED TETHERING SYSTEM

System/Component	Design Specifications
Capstan Drive:	Assume f = 0.1, Ti/To = 12.3 (See Appendix A) Low tension paying out Tension difference between capstan and reel
Electric Motor:	
1. Power	1/5 HP
2. Power source	28 V d.c.
3. Running time	Up to 1/2 hr
4. Speed ratio	Variable
5. Environment	Cannistered in space
6. Cooling	Radiation and conduction
Power Supply:	
1. Required total energy at 100% efficiency	35 w-hr
2. Average HP at 100% efficiency	100 ft-lb-sec^{-1}
Momentum Module:	
1. Module + reel at module	15 lb
2. Tether line length (15 lb total weight)	3000 ft
3. Tether line tension	50 lbs

TABLE III (Continued)

System/Component	Design Specifications
4. Detachment mechanism	2 lbs Tension sensitive
5. Secondary tethering cable	10 lbs
Control and Display:	
1. On-board display and controls	Desired for command astronaut for either manual or automatic control modes
a. Range	
b. Tension	
c. Range rate	
d. Display of position	
e. Power switch (manual)	
f. Activator switch (manual)	
g. Overload switch (automatic)	
h. Override range rate controller (manual)	

TABLE IV

EXTRA-VEHICULAR OPERATIONAL PROCEDURE
(Based on Gemini Mission Profile)

Normal Procedure for Extra-Vehicular Astronaut

Step

1. Perform pre-depressurization check list.
2. Go on PLSS and perform check.
3. Depressurize the spacecraft.
4. Perform pre-open hatch check list.
5. Open hatch, attach capsule lanyard ring hook, and egress.
6. Move to adapter section of spacecraft and obtain maneuvering pack.
7. Don maneuvering pack and move to controlled tethering system access hatch (do not use maneuvering pack to reach access hatch).
8. Open tethering system access hatch and attach tethering line hook, passing it through capsule lanyard ring.
9. Detach capsule lanyard ring hook from harness and secure ring to magnetic latches on spacecraft.
10. Release tethering system safety locks and activate tethering system (either astronaut).
11. Perform tethered maneuvering tests using maneuvering pack.
12. Return to spacecraft tethering access hatch with automatic take-up of controlled tether.
13. Tether reel drive mechanism automatically shuts off and engages brake.
14. Remove capsule lanyard ring from stowage and attach hook to harness.
15. Remove maneuvering pack and tethering line hook, secure and leave outside spacecraft (normal procedure in Gemini mission).

TABLE IV (Continued)

16. Switch to spacecraft ECS and ingress spacecraft.

17. Perform strap-in operations and remove lanyard ring hook.

18. Close spacecraft hatch.

19. Re-pressurize spacecraft.

20. Perform post-EVA check list.

Emergency Procedure

Retrieval of Active Astronaut

1. Steps 1 through 11 of Normal Procedure.

2. Initiate retrieval mode by on-board astronaut.

3. Deploy momentum module.

4. Reel tethered astronaut to spacecraft.

5. Repeat Step 13 of Normal Procedure.

6. Jettison momentum module.

7. Steps 14 through 20 of Normal Procedure.

Retrieval of Incapacitated Astronaut

1. Steps 1 through 6 of Emergency Procedure, Retrieval of Active Astronaut.

2. On-board astronaut pulls on capsule lanyard ring line, detaching ring from stowage, causing ring to engage extra-vehicular astronaut.

3. On-board astronaut releases tethering system brake.

4. On-board astronaut pulls incapacitated astronaut to spacecraft hatch.

5. On-board astronaut attaches spacecraft ECS and lanyard ring hook to extra-vehicular astronaut.

6. On-board astronaut removed maneuvering pack and tethering line hook and leaves outside spacecraft.

TABLE IV (Continued)

7. On-board astronaut assists in ingress maneuvers.

8. Administer to extra-vehicular astronaut.

9. Steps 17 through 20 of Normal Procedure by command astronaut.

TENSIONS AND VELOCITIES AT ASTRONAUT DURING RETRIEVAL
AS A FUNCTION OF TIME

WITH 2000 - ft SECONDARY TETHER

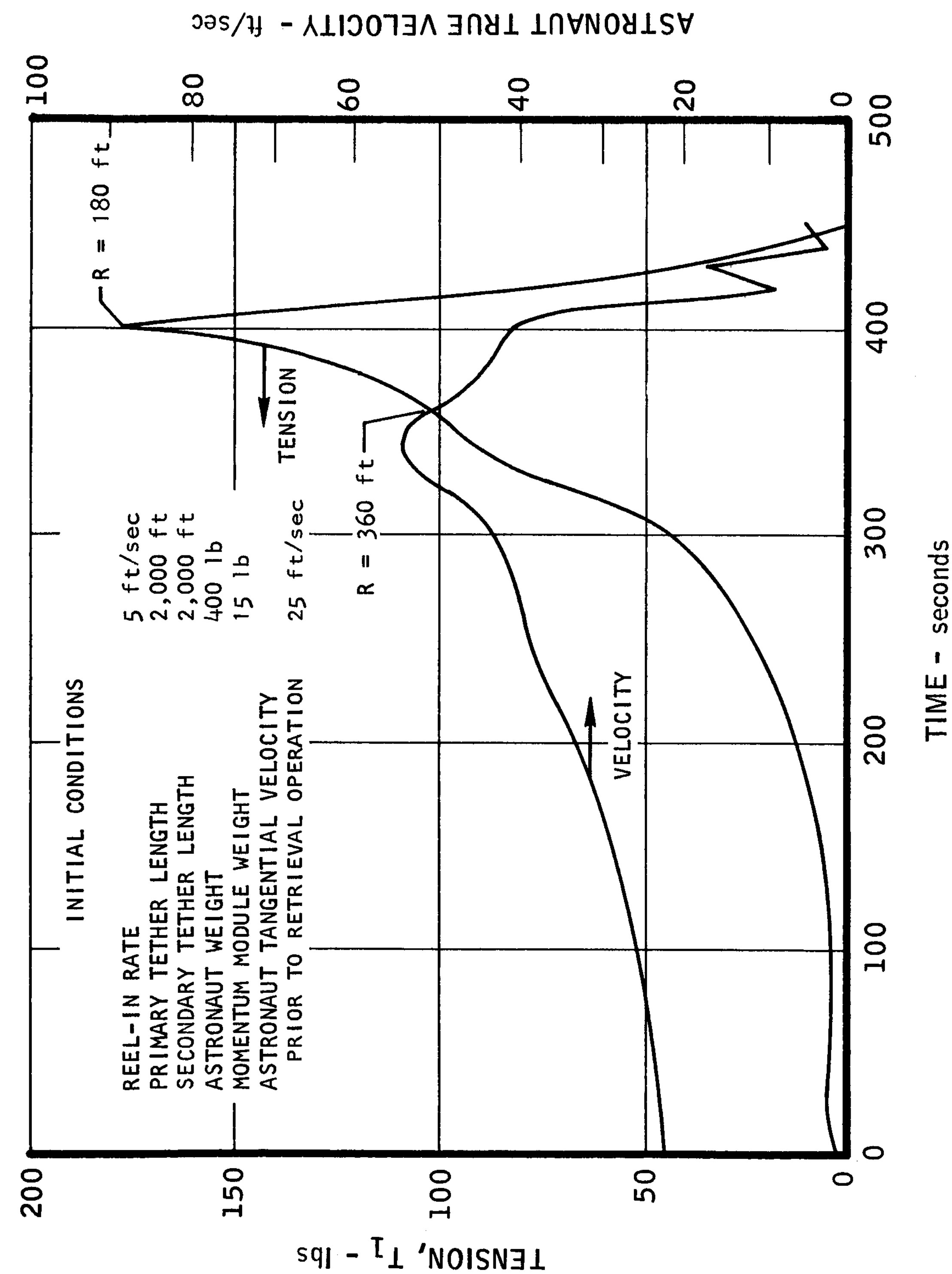


FIGURE 1

TENSIONS AND VELOCITIES AT ASTRONAUT DURING RETRIEVAL AS A FUNCTION OF TIME

WITH 3000 - ft SECONDARY TETHER

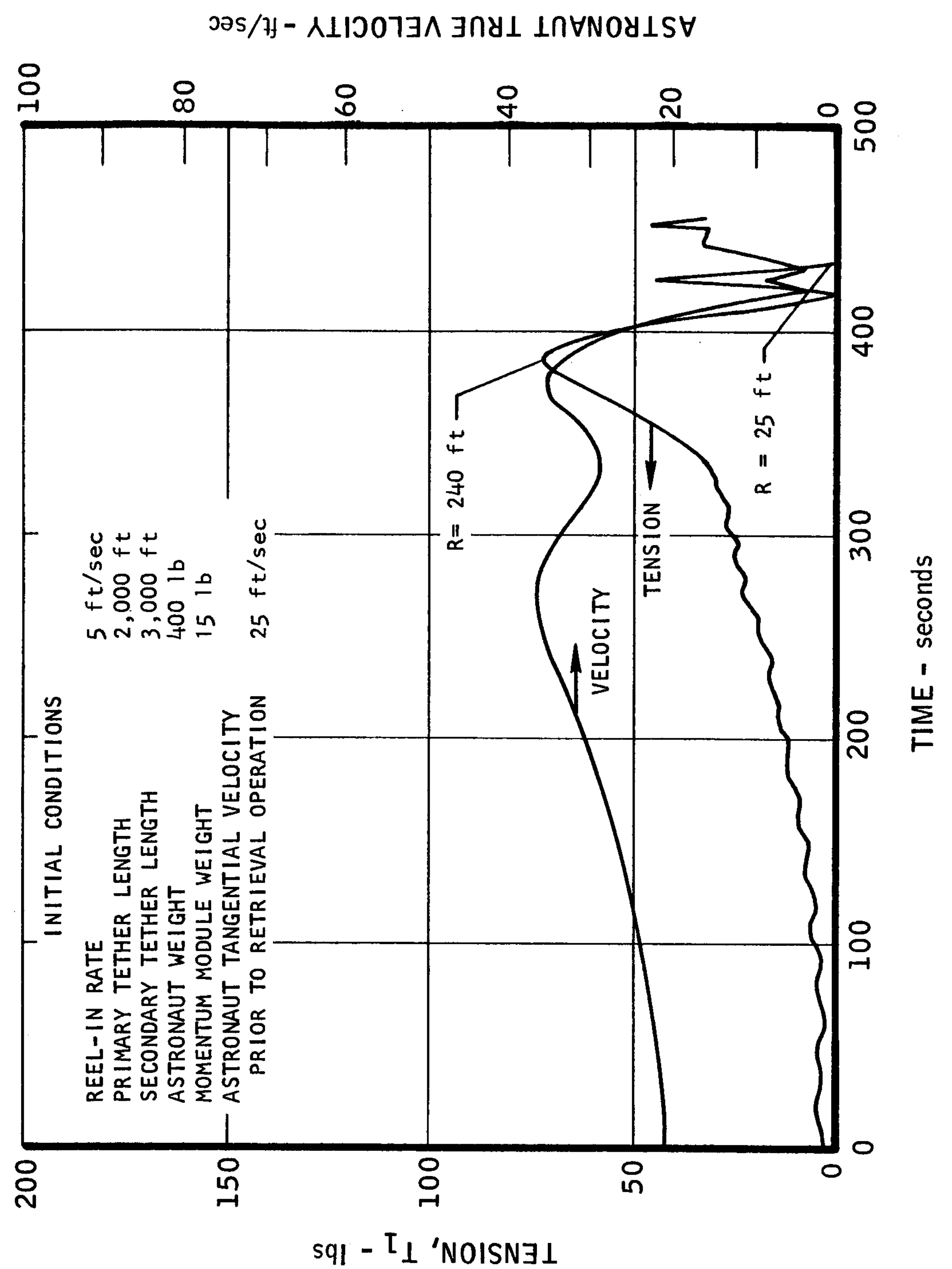
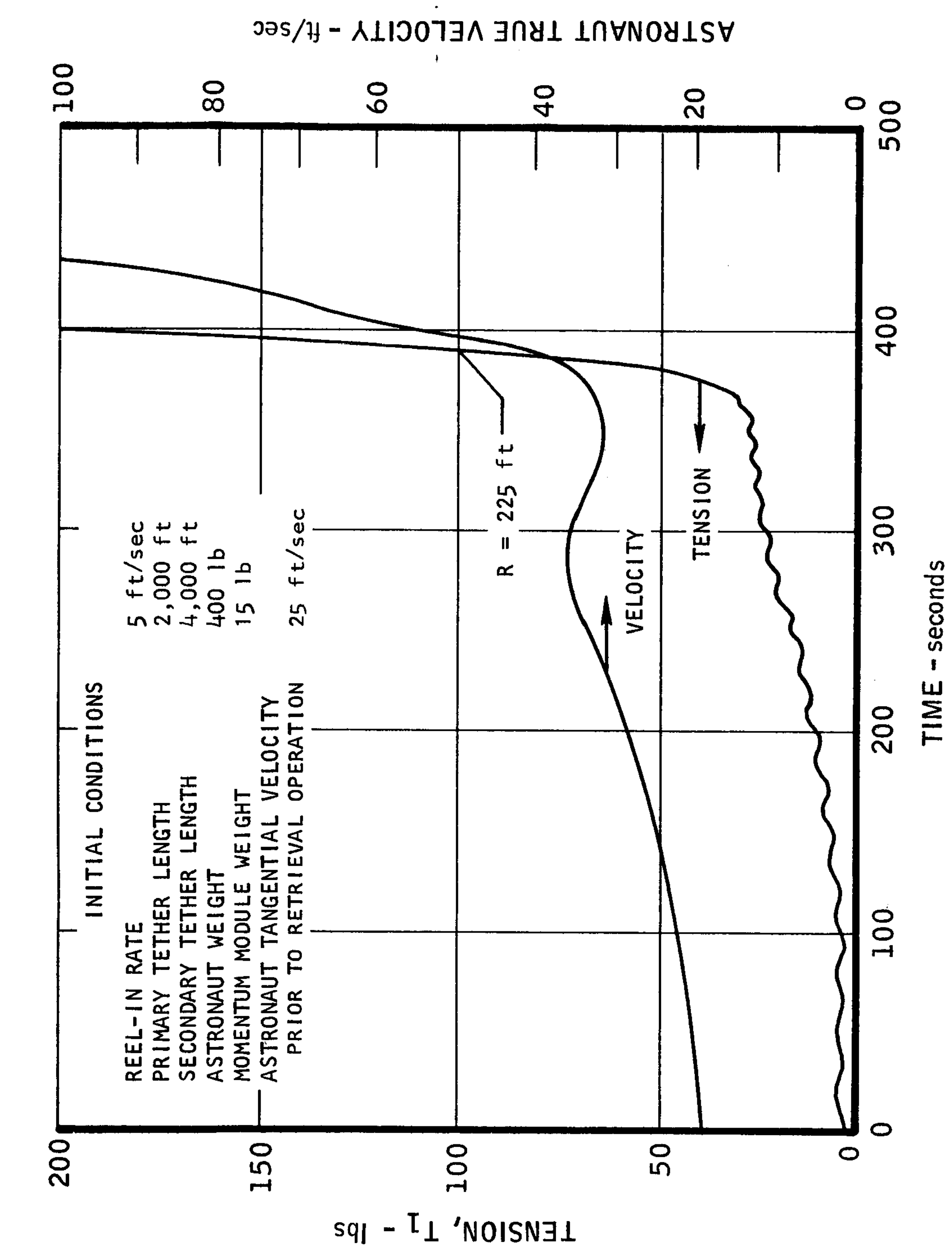


FIGURE 2

TENSIONS AND VELOCITIES AT ASTRONAUT DURING RETRIEVAL AS A FUNCTION OF TIME
WITH 4000-ft SECONDARY TETHER

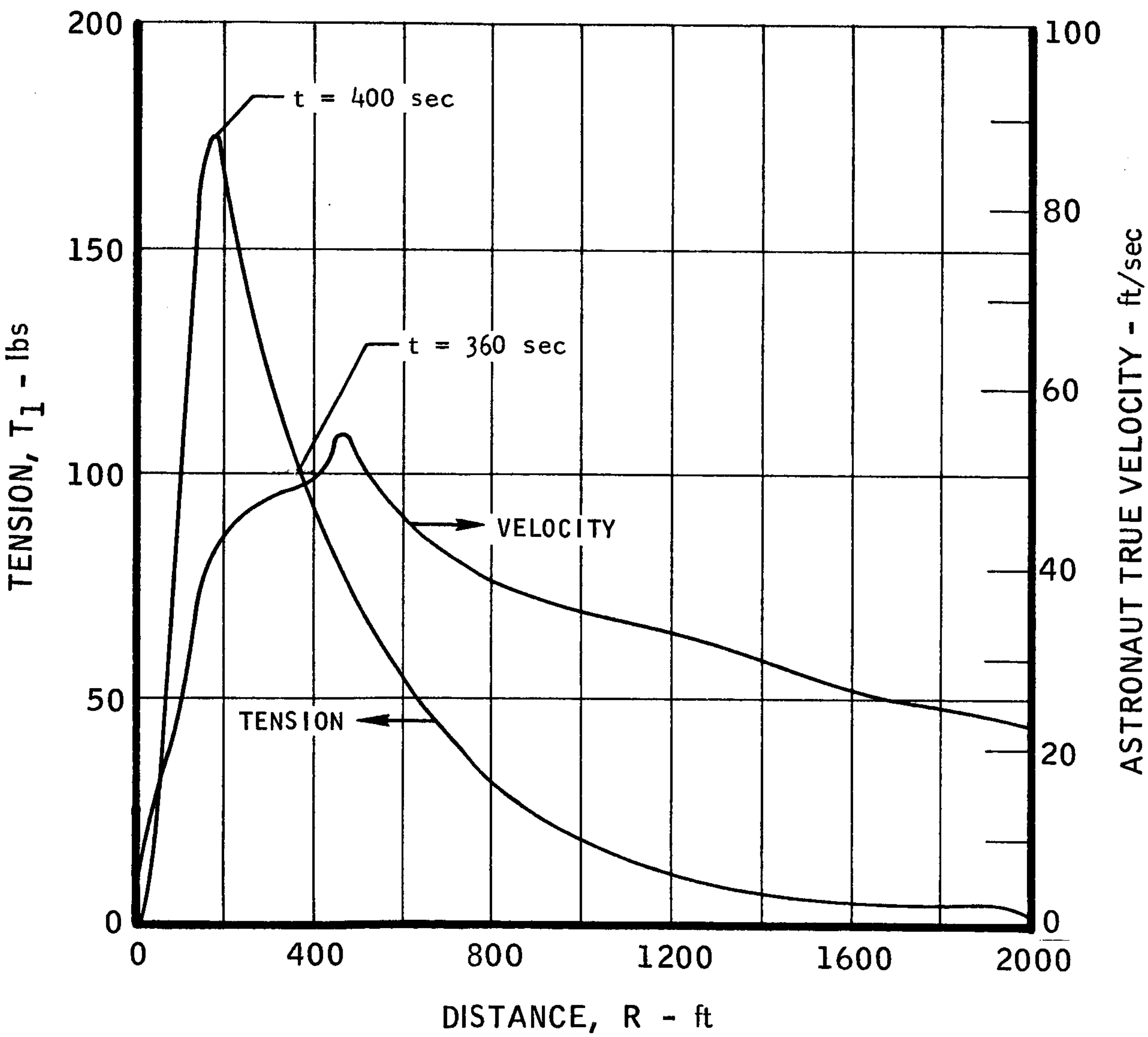


TENSIONS AND VELOCITIES AT ASTRONAUT DURING RETRIEVAL AS A FUNCTION OF DISTANCE

WITH 2000 -ft SECONDARY TETHER

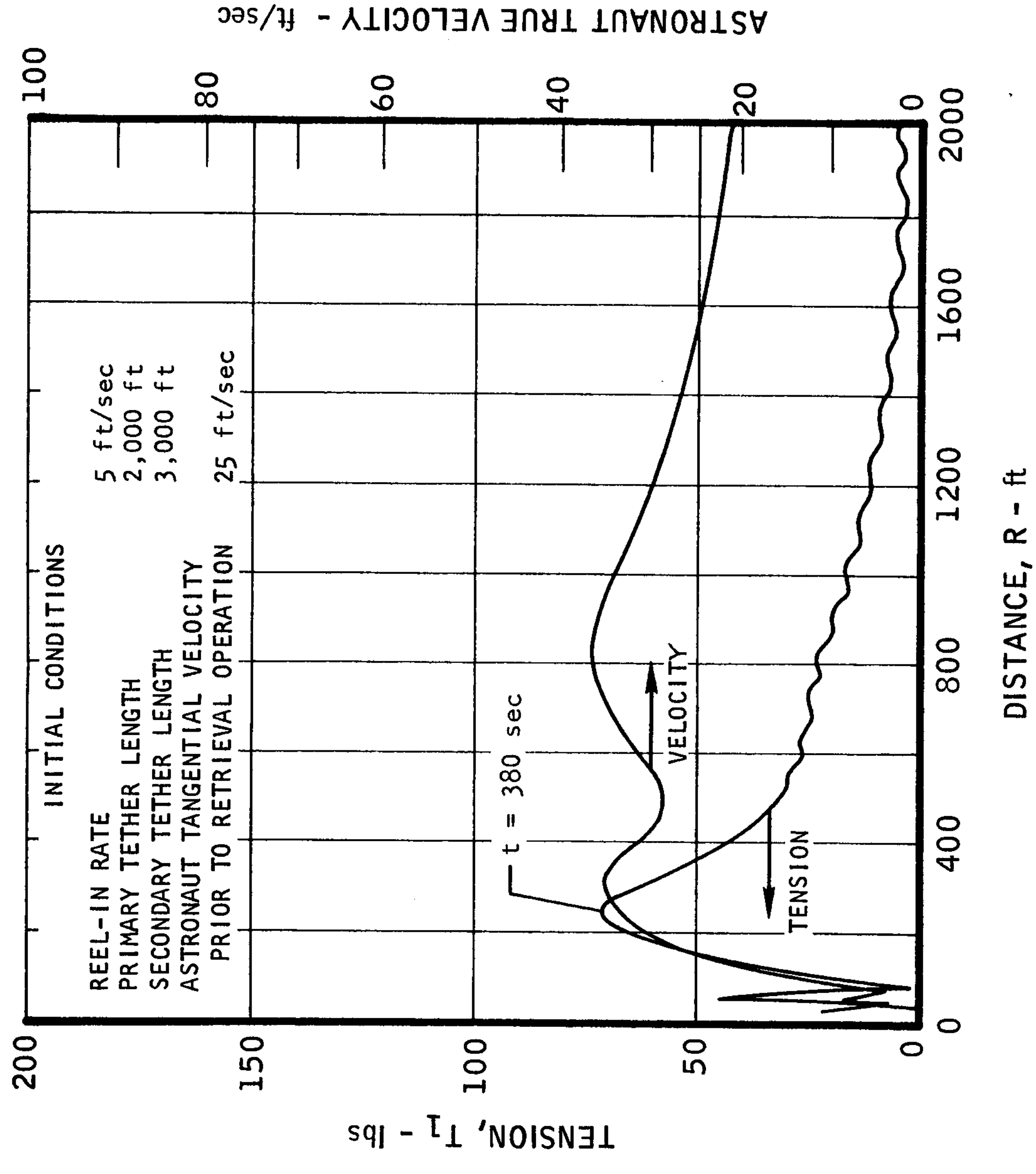
INITIAL CONDITIONS

REEL-IN RATE	5 ft/sec
PRIMARY TETHER LENGTH	2,000 ft
SECONDARY TETHER LENGTH	2,000 ft
ASTRONAUT WEIGHT	400 lb
MOMENTUM MODULE WEIGHT	15 lb
ASTRONAUT TANGENTIAL VELOCITY PRIOR TO RETRIEVAL OPERATION	25 ft/sec

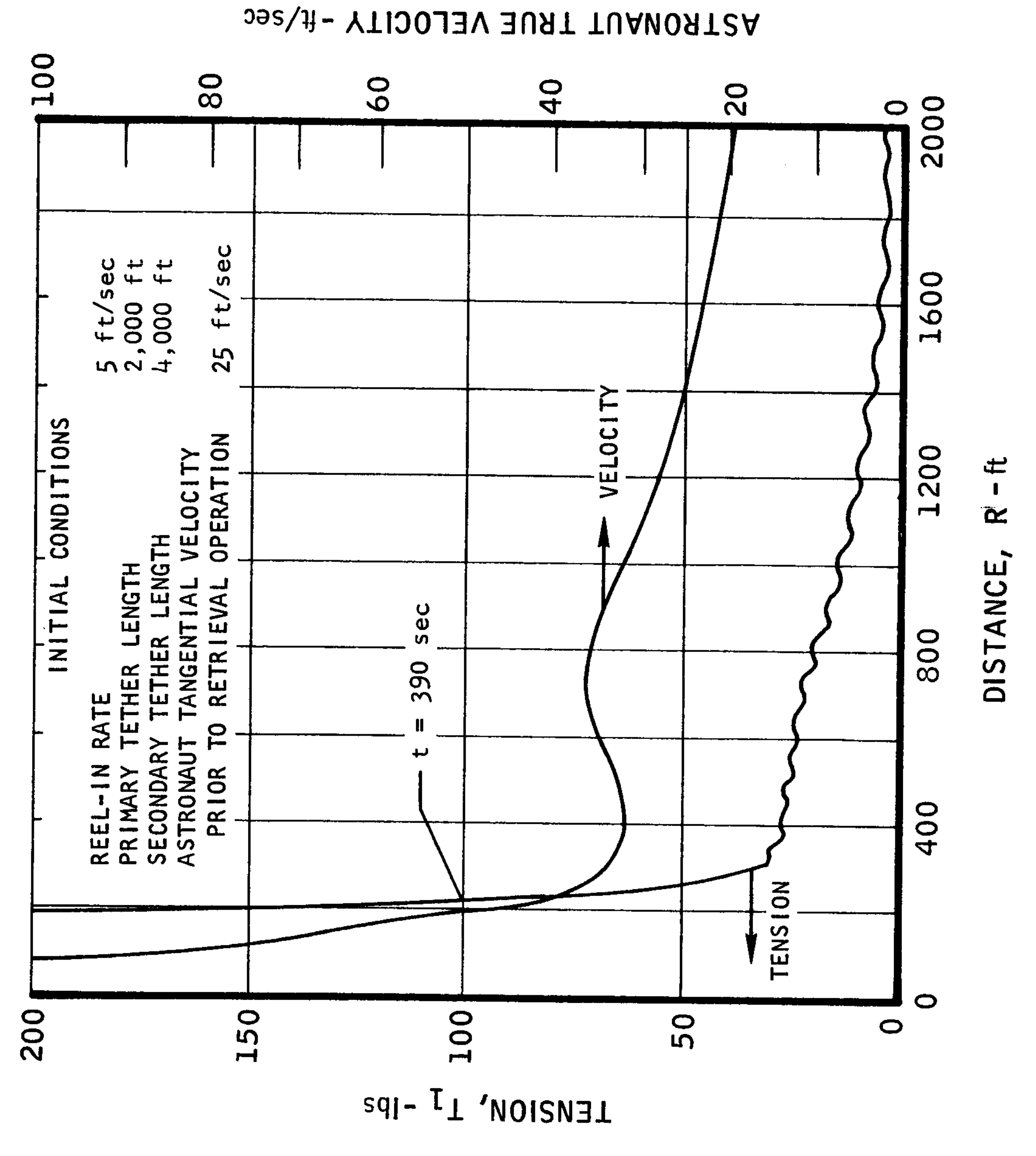


TENSIONS AND VELOCITIES AT ASTRONAUT DURING RETRIEVAL AS A FUNCTION OF DISTANCE

WITH 3000- ft SECONDARY TETHER



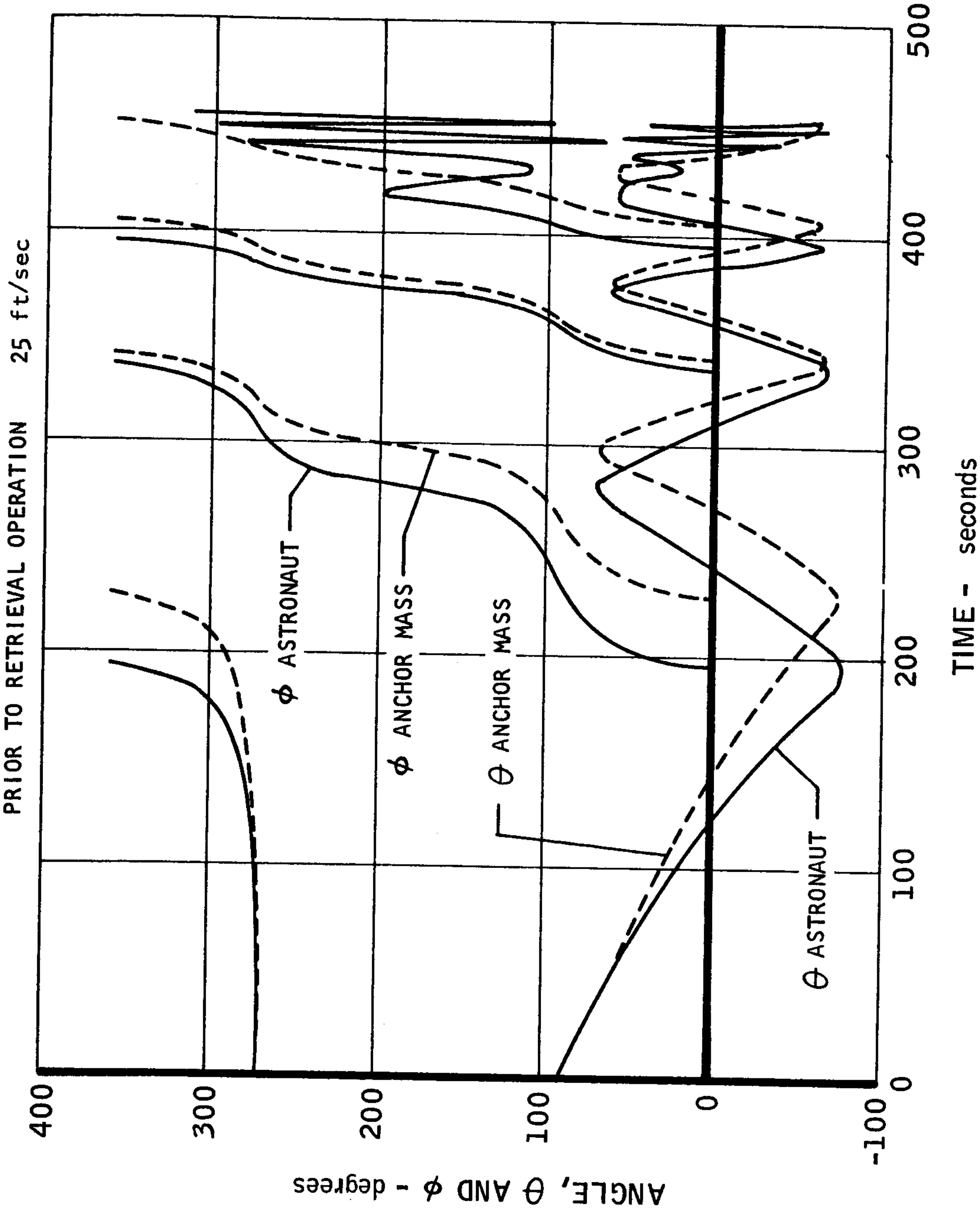
TENSIONS AND VELOCITIES AT ASTRONAUT DURING RETRIEVAL
AS A FUNCTION OF DISTANCE
WITH 4000-ft SECONDARY TETHER



ANGULAR ORIENTATION OF TETHERS DURING RETRIEVAL PROCESS

INITIAL CONDITIONS

REEL-IN RATE	5 ft/sec
PRIMARY TETHER LENGTH	2,000 ft
SECONDARY TETHER LENGTH	3,000 ft
ASTRONAUT TANGENTIAL VELOCITY PRIOR TO RETRIEVAL OPERATION	25 ft/sec



PRIMARY TETHER ANGULAR DEPARTURE FROM NULL POSITION

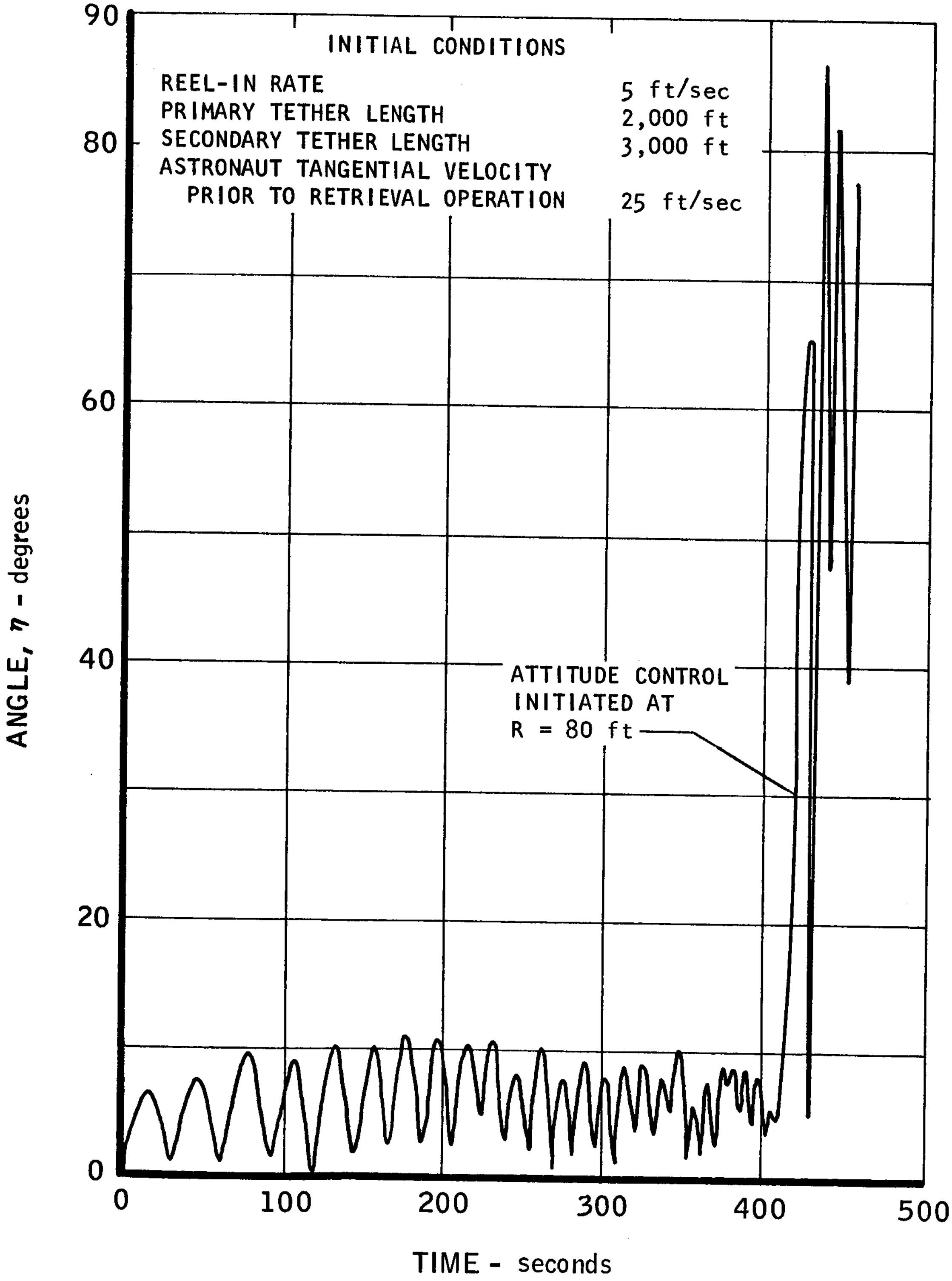


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FIGURE 8

ASTRONAUT RADIAL VELOCITY DURING RETRIEVAL PROCESS

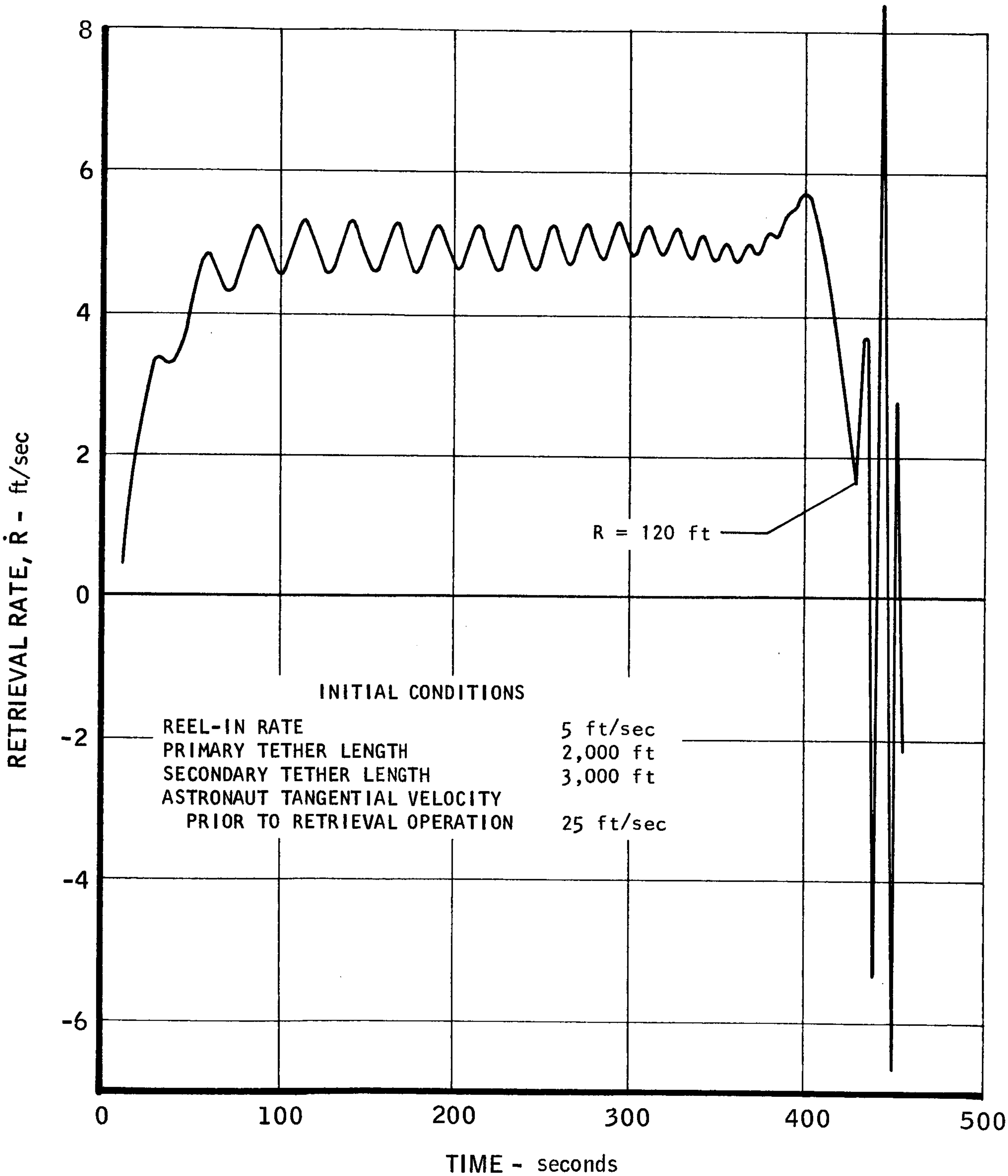

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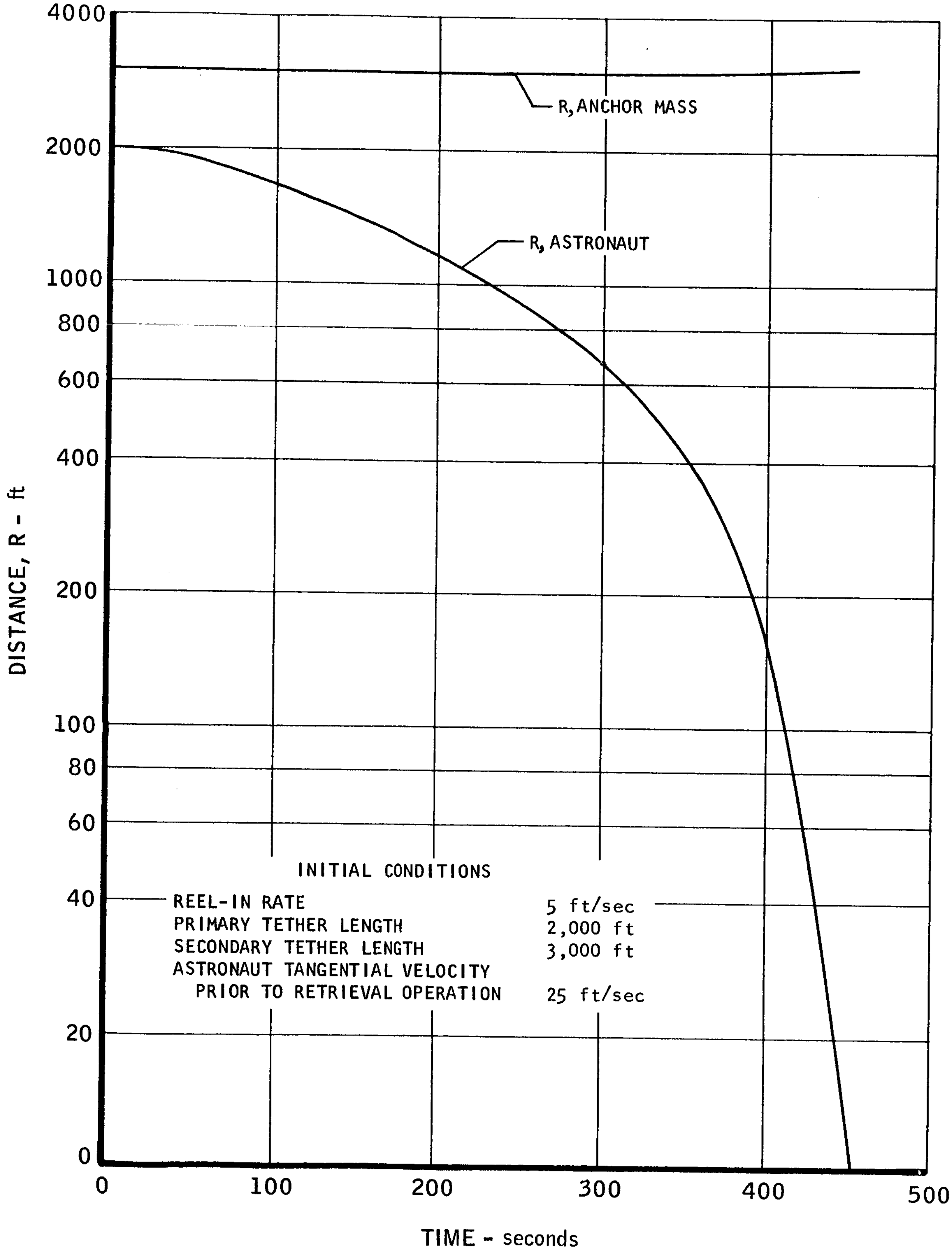
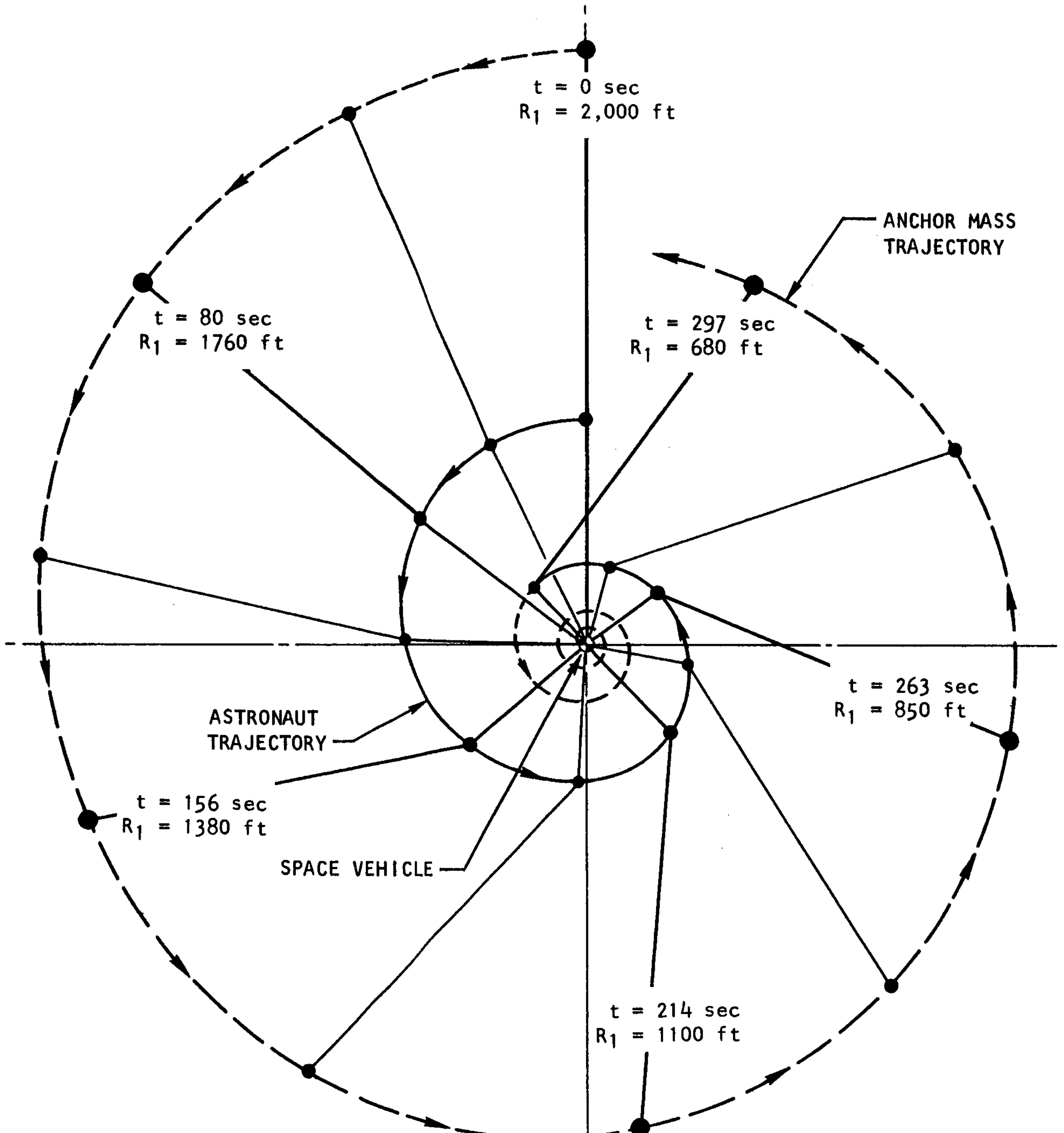
FIGURE 9

TETHER LENGTHS DURING RETRIEVAL PROCESS AS A FUNCTION OF TIME



PROJECTION OF SYSTEM MOTION ONTO PLANE OF INITIAL MOTION



INITIAL CONDITIONS

REEL-IN RATE	5 ft/sec
PRIMARY TETHER LENGTH	2,000 ft
SECONDARY TETHER LENGTH	3,000 ft
ASTRONAUT WEIGHT	400 lb
MOMENTUM MODULE WEIGHT	15 lb
ASTRONAUT TANGENTIAL VELOCITY PRIOR TO RETRIEVAL OPERATION	25 ft/sec

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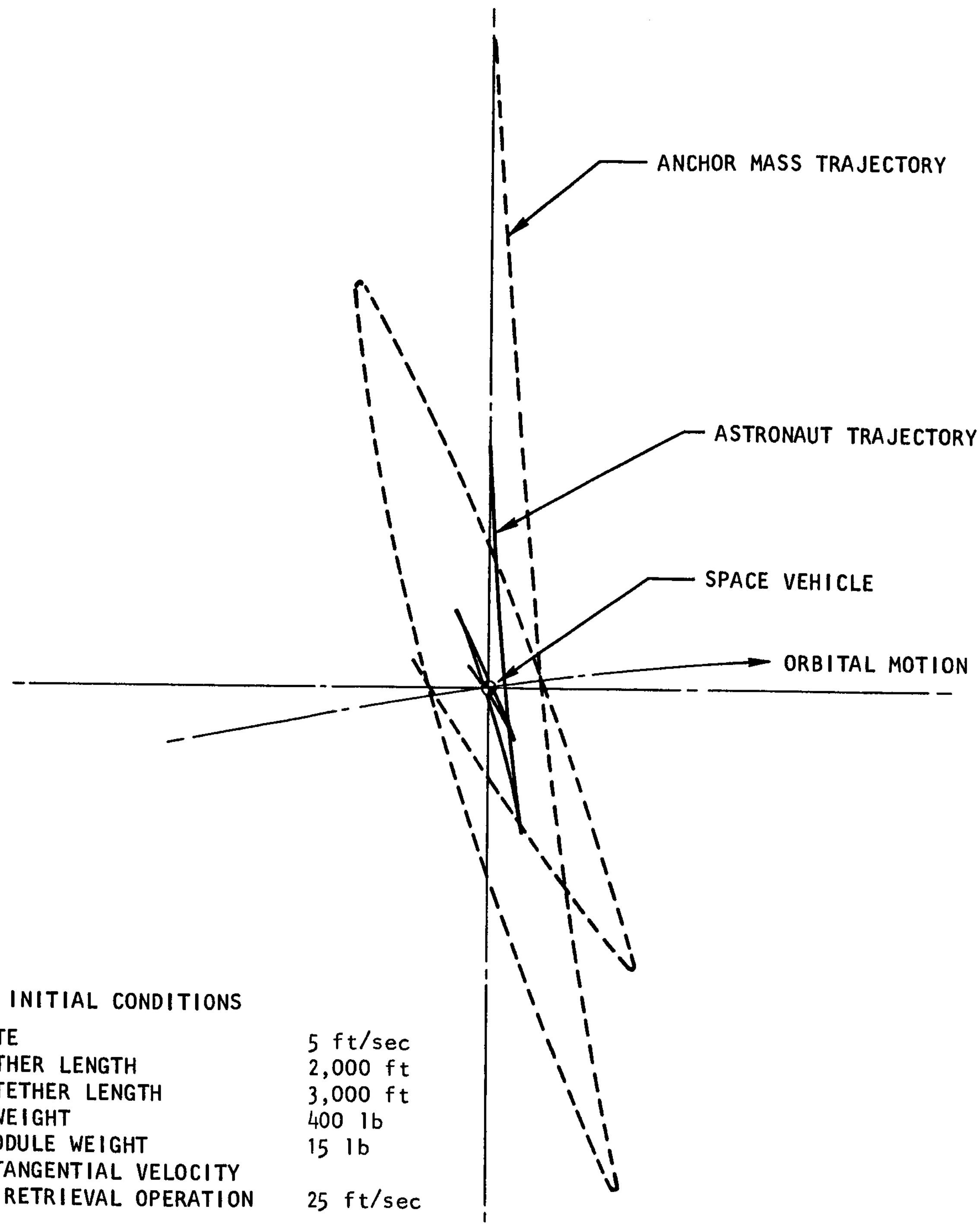
PROJECTION OF SYSTEM MOTION ONTO PLANE OF ORBITAL MOTION



INITIAL CONDITIONS

REEL-IN RATE	5 ft/sec
PRIMARY TETHER LENGTH	2,000 ft
SECONDARY TETHER LENGTH	3,000 ft
ASTRONAUT WEIGHT	400 lb
MOMENTUM MODULE WEIGHT	15 lb
ASTRONAUT TANGENTIAL VELOCITY PRIOR TO RETRIEVAL OPERATION	25 ft/sec

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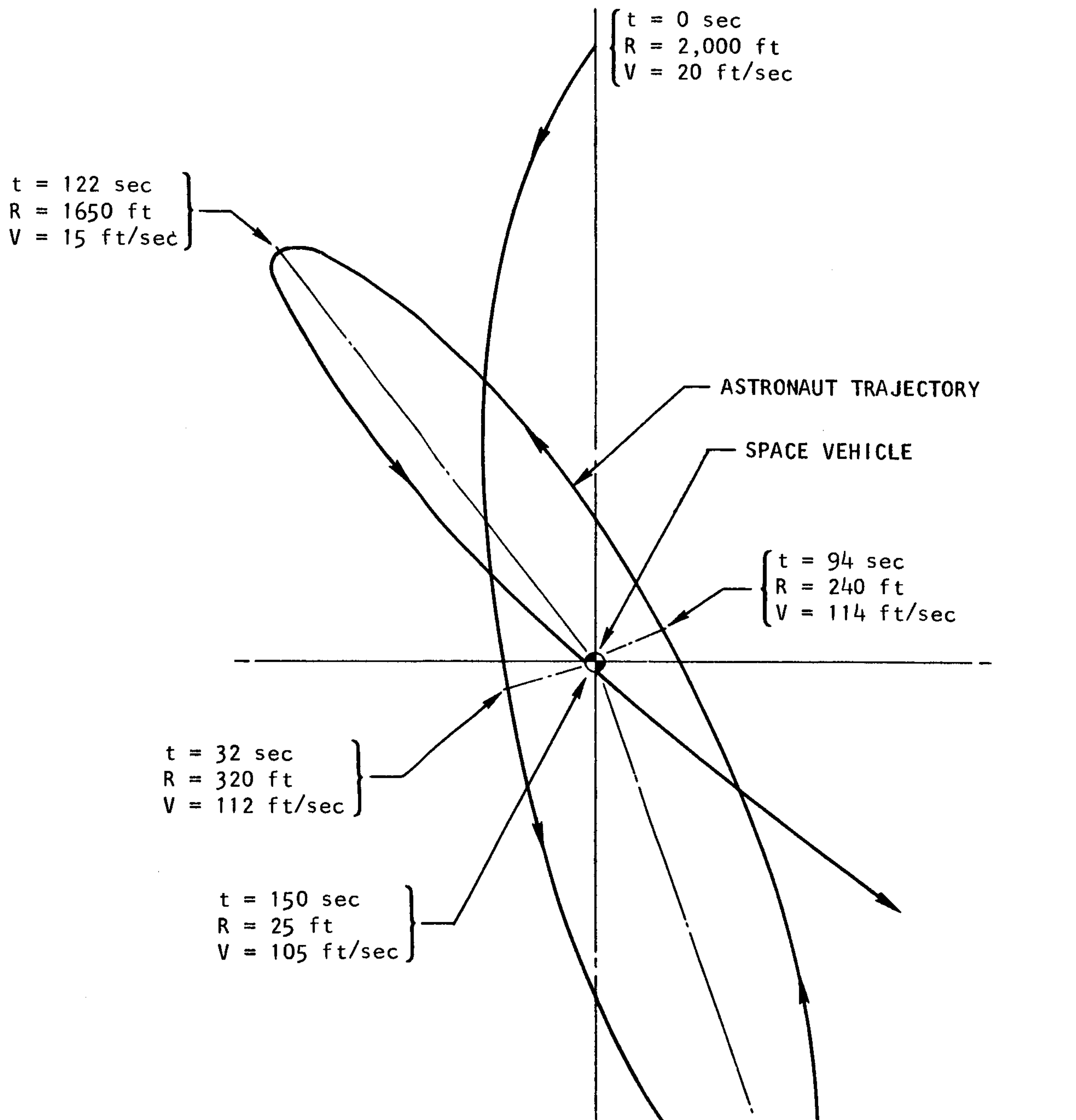
FIGURE 12

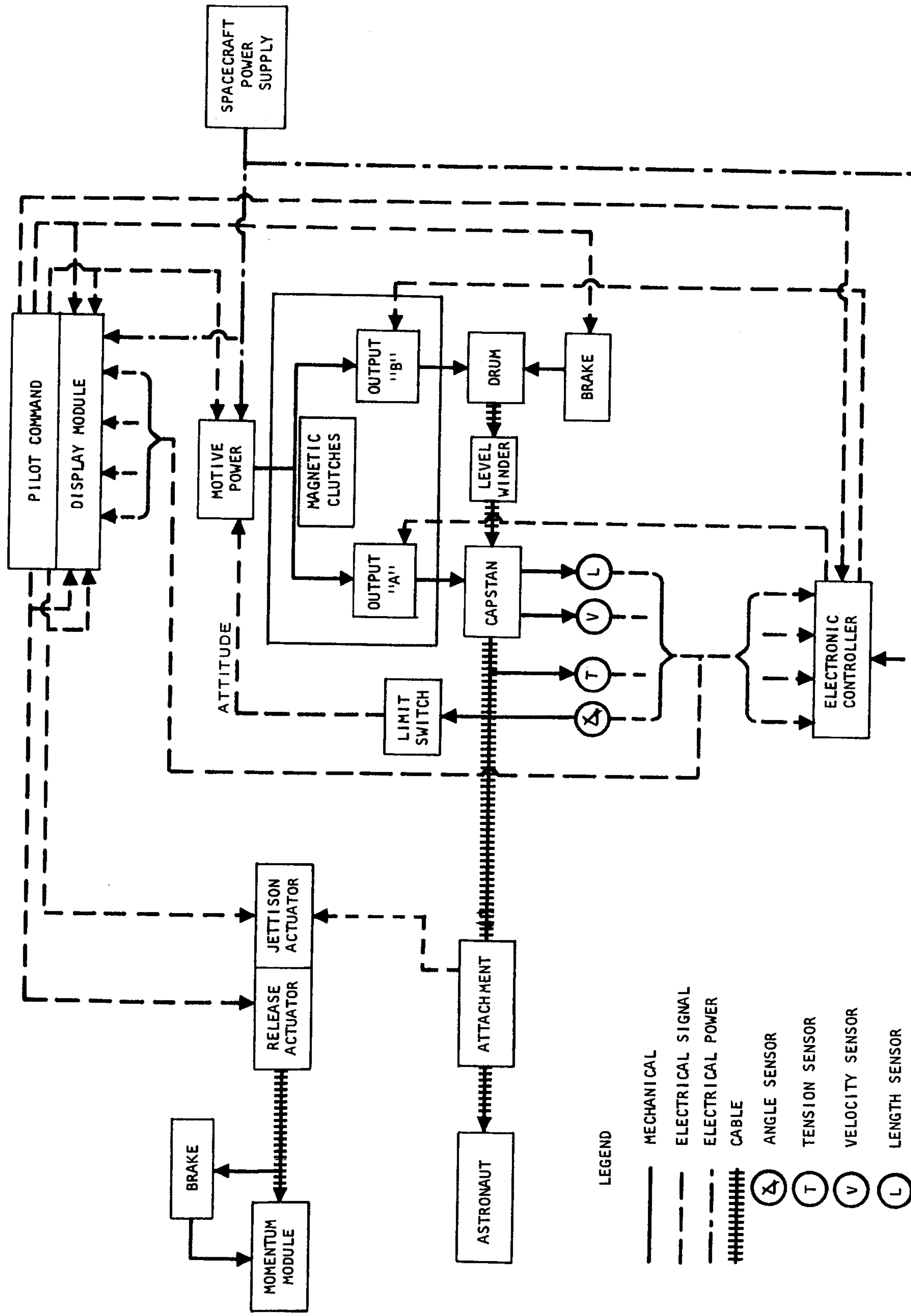
PROJECTION OF ASTRONAUT TRAJECTORY ONTO PLANE OF INITIAL MOTION FOR CONSTANT TENSION RETRIEVAL



INITIAL CONDITIONS

REEL-IN RATE, FUNCTION OF:	50 lb tension
PRIMARY TETHER LENGTH	2,000 ft
SECONDARY TETHER LENGTH	4,000 ft
ASTRONAUT WEIGHT	400 lb
MOMENTUM MODULE WEIGHT	15 lb
ASTRONAUT TANGENTIAL VELOCITY PRIOR TO RETRIEVAL OPERATION	25 ft/sec

CONTROLLED TETHERING RETRIEVAL SYSTEM BLOCK DIAGRAM



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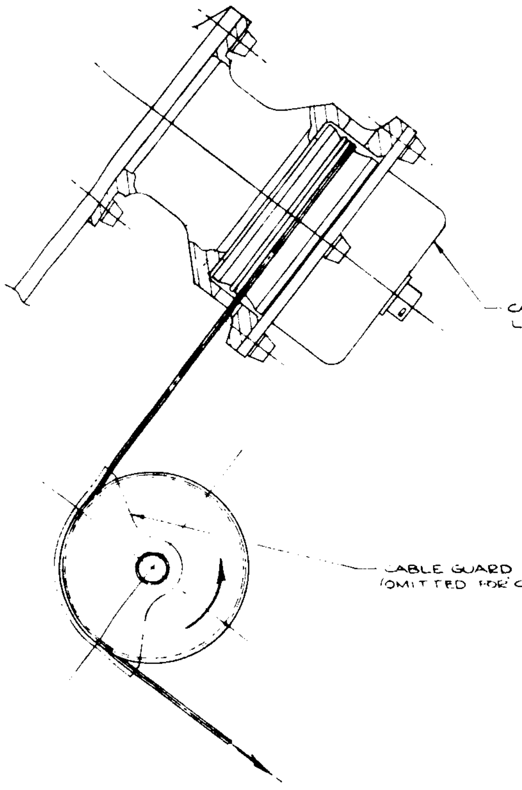

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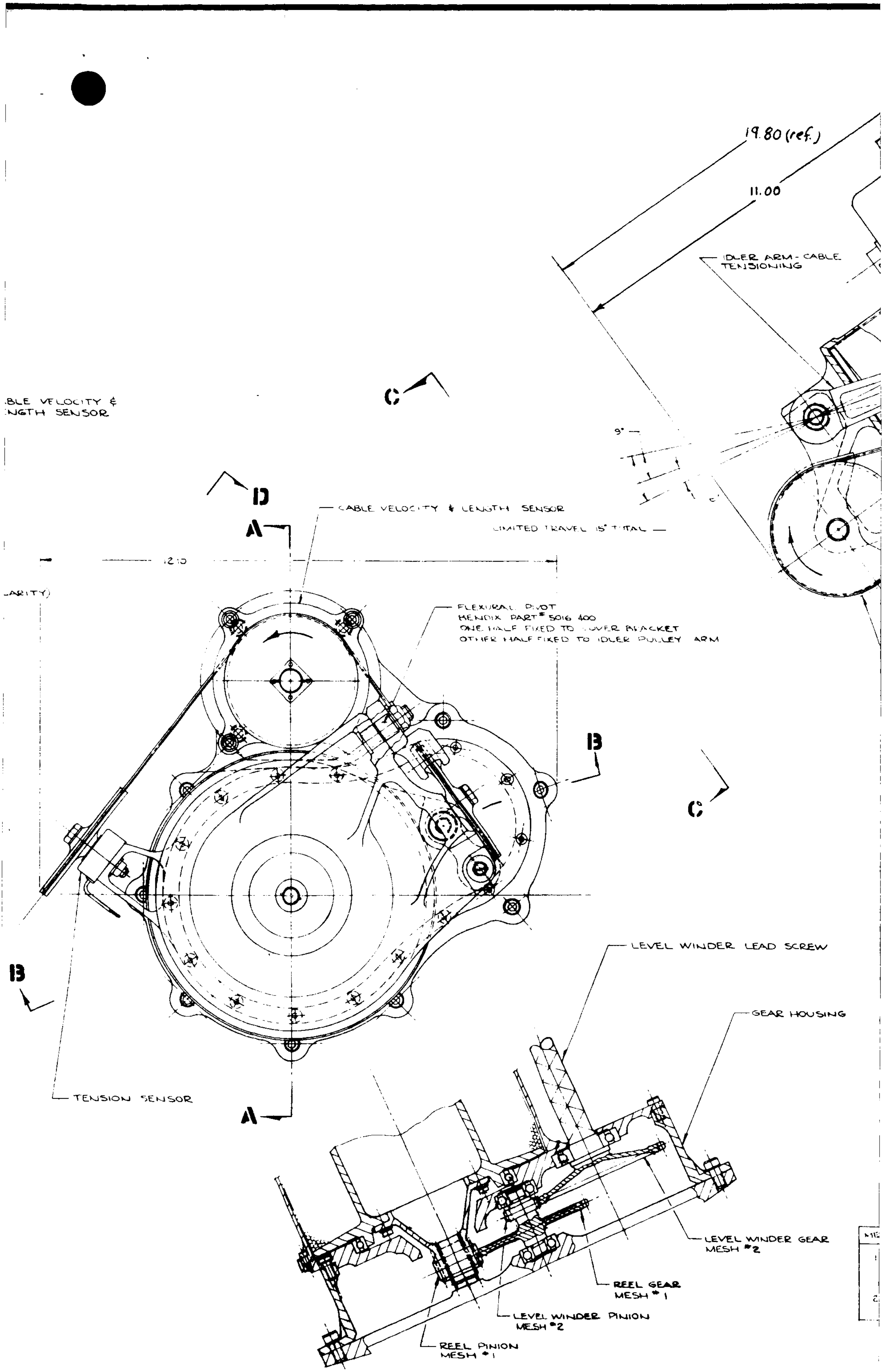
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FLEXURAL PIVOT
BENDIX PART # 5016 400
ONE HALF FIXED TO COVER BRACKET
OTHER HALF FIXED TO IDLER PULLEY ARM
B
LEVEL WINDER LEAD SCREW
GEAR HOUSING
TENSION SENSOR
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REEL PINION MESH #1
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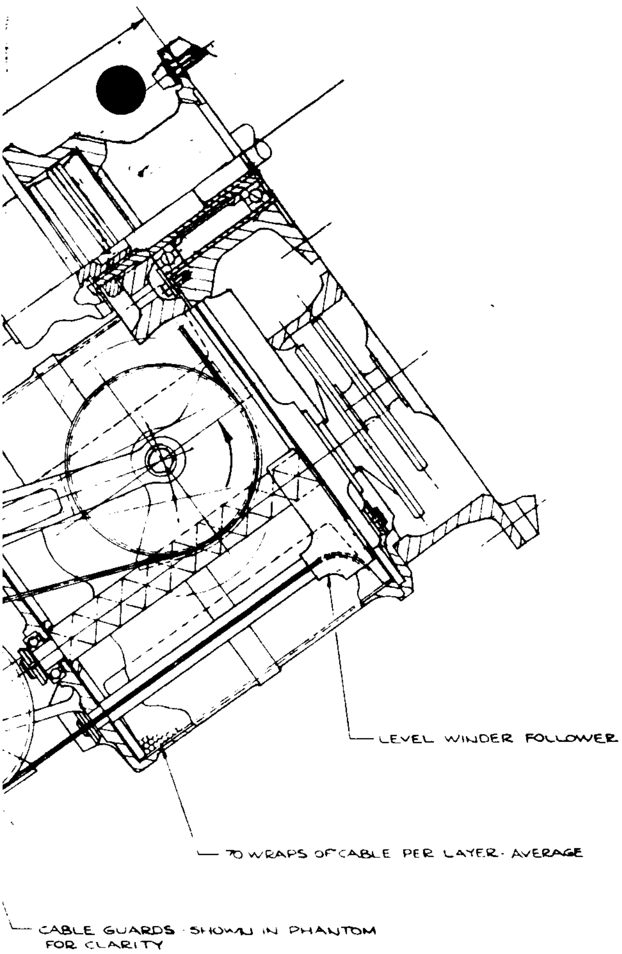
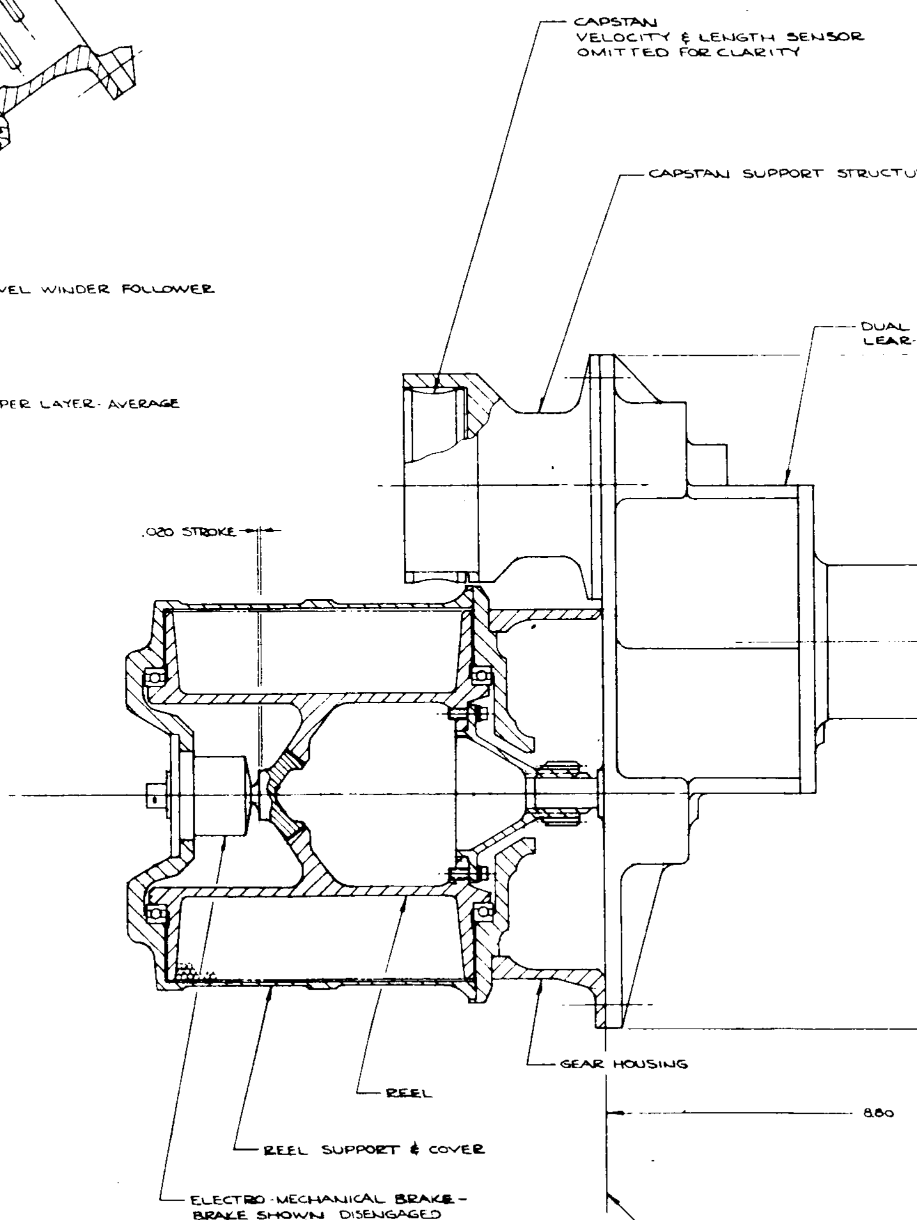


VIEW **C C**



SECTION **A A**

GEAR DATA

H NO.	DP	P.D.	TEETH	TOOTH TYPE	RATIO	OVERALL RATIO	REEL FOLLOWER
PINION	32	1.000	32	20° STUB	2.875	13.852	~ 70
GEAR	32	2.875	92				
PINION	32	0.688	22		4.818		
GEAR	32	3.312	106				

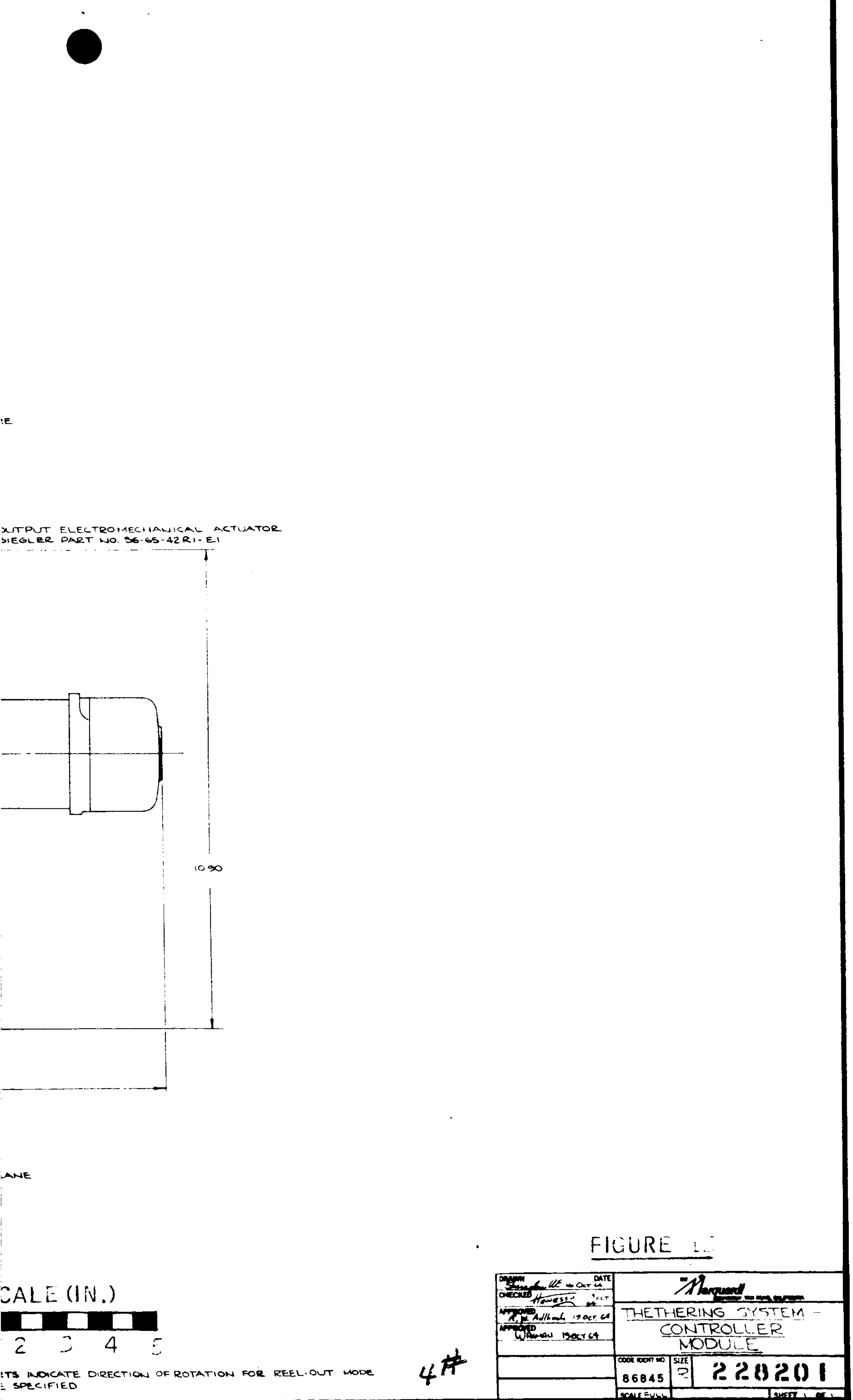
NOTE - FULL FILLET TOOTH FORM

S

0 1

1. ARROWS ON ROTATING PA
NOTES - UNLESS OTHERWIS

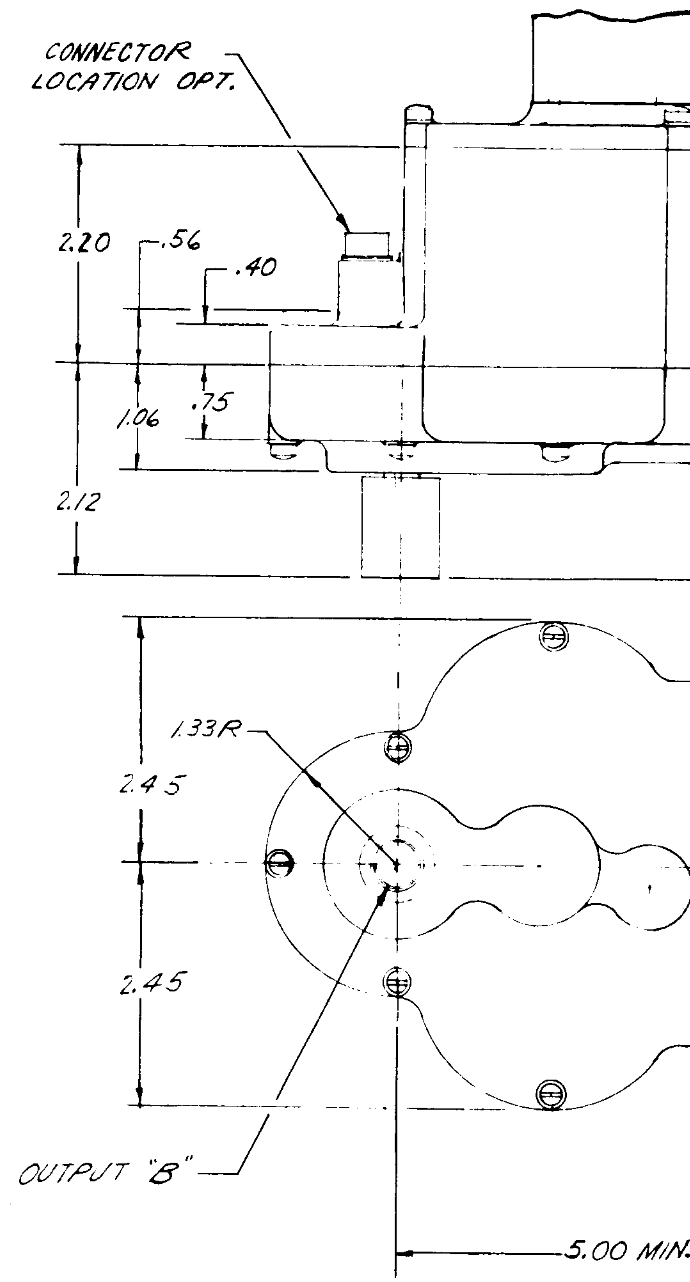
3#



OUTPUT ELECTROMECHANICAL ACTUATOR
SIEGLER PART NO. 56-65-42 R1-E1
10.90
ANE
CALE (IN.)
2 3 4 5
TS INDICATE DIRECTION OF ROTATION FOR REEL-OUT MODE
SPECIFIED
4#
DRAWN
DATE
CHECKED
APPROVED
APPROVED
Marquardt
THETHERING SYSTEM - CONTROLLER MODULE
CODE IDENT NO.
86845
SIZE
228201
SCALE FULL
SHEET

FIGURE

D

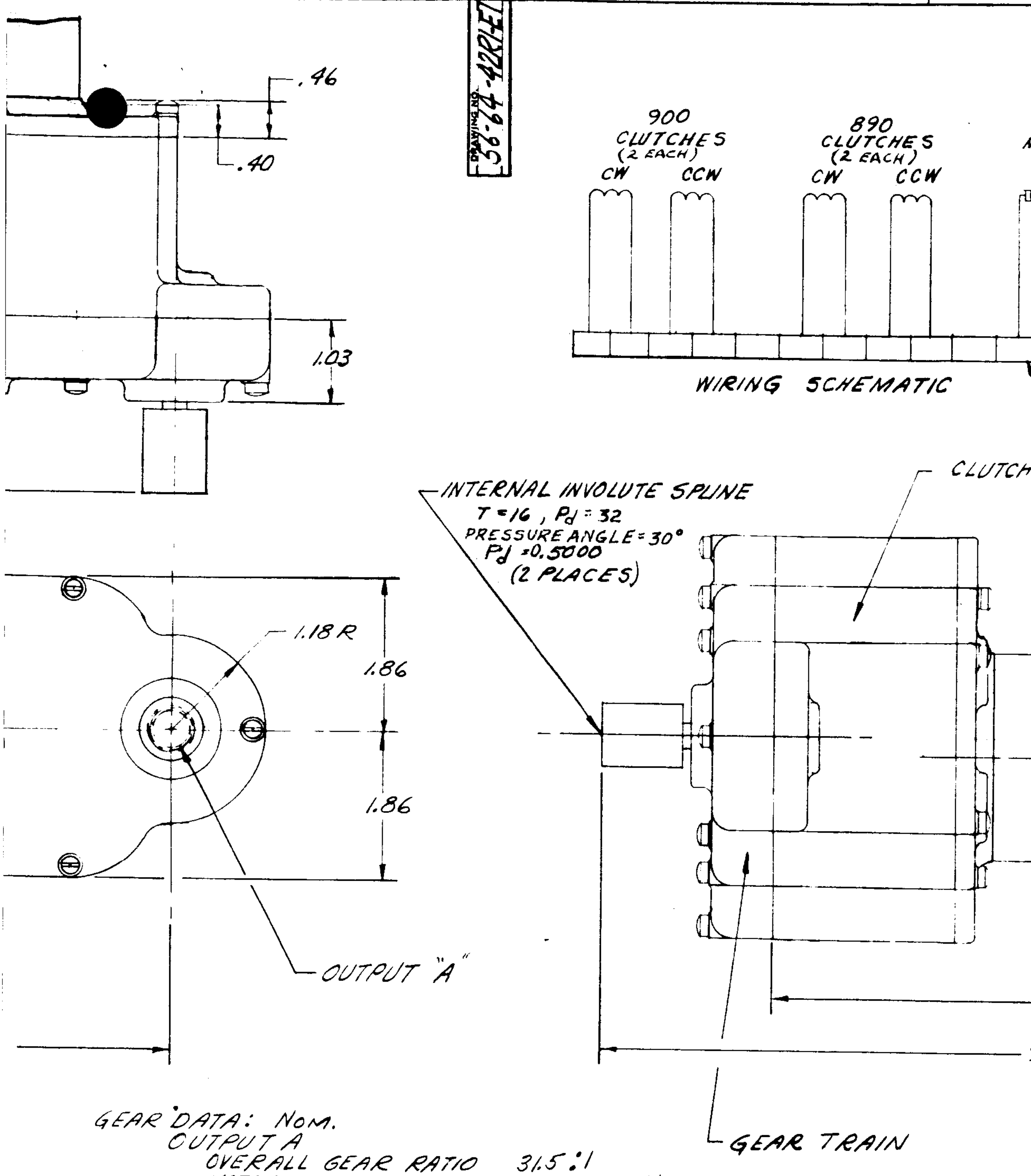

CONNECTOR LOCATION OPT.
2.20
.56
.40
1.06
.75
2.12
1.33R
2.45
2.45
OUTPUT "B"
5.00 MIN.

C

B

A

1



GEAR DATA: NOM.

OUTPUT A

OVERALL GEAR RATIO 31.5 : 1
MOTOR TO CLUTCH GEAR RATIO 13.53 : 1
CLUTCH TO OUTPUT GEAR RATIO 2.33 : 1

OUTPUT B

OVERALL GEAR RATIO 117.00 : 1
MOTOR TO CLUTCH GEAR RATIO 13.50 : 1
CLUTCH TO OUTPUT GEAR RATIO 8.70 : 1

CLUTCH DATA:
MODEL 900, LeSi

MODEL 890, LeSi

LOAD DATA: (POWER PHASE) (NOM)
OUTPUT A : 1.5 IN-LB @ 432 RPM
TO 4 IN-LB @ 430 RPM
OUTPUT B : 103 IN-LB @ 115 RPM
TO 5 IN-LB @ 120 RPM

MOTOR DATA:
28 VOLT D.C.
SPEED 13,500 RPM ± 10 % AT 1.0 IN LB

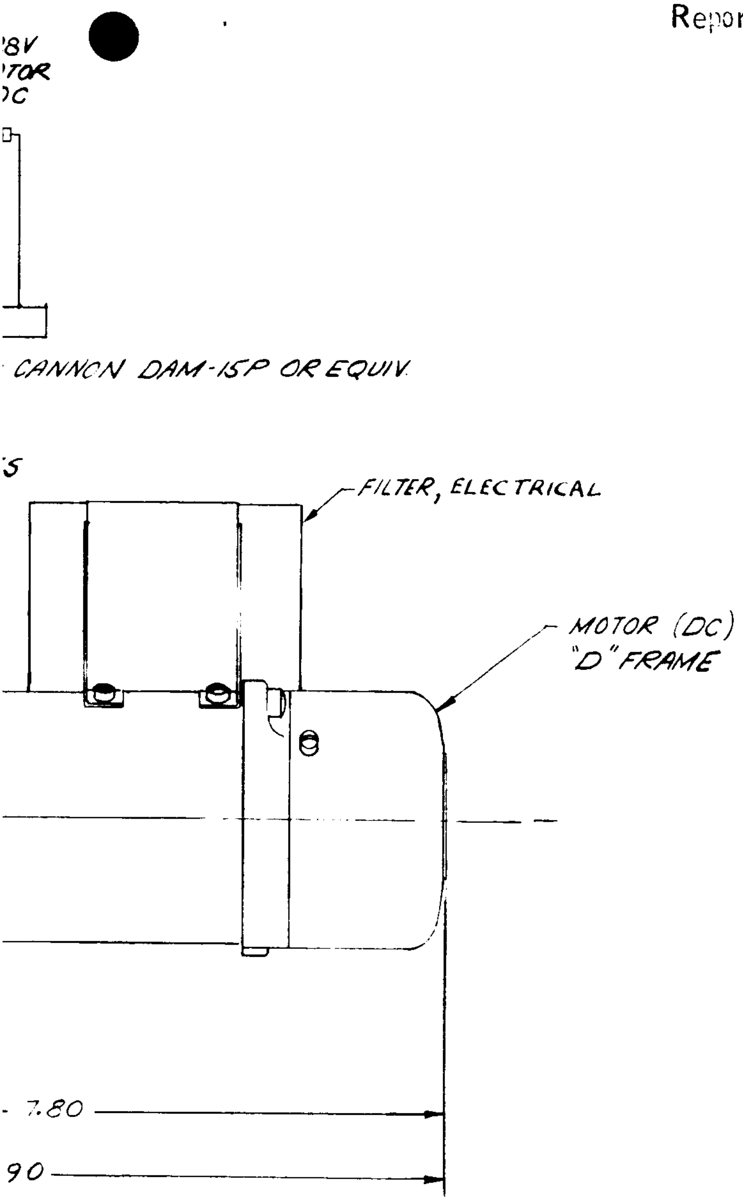
#1

UNLESS OTHERWISE S
DIMENSIONS ARE IN
TOLERANCES C
DECIMALS
.XX ± .01 .XXX ± .005

MATERIAL

REVISIONS			
SYM	DESCRIPTION	DATE	APPROVED

Report [illegible]092



DRAWING NUMBER				
56-65-42R1-E1				
PE	E	FILES	CLASS	REV SYM

FIGURE 1

THIS DOCUMENT CONTAINS INFORMATION PROPRIETARY TO LEAR SIEGLER, INC., AND IS FURNISHED UPON THE EXPRESS CONDITION THAT THE INFORMATION CONTAINED HEREIN WILL NOT BE USED DIRECTLY OR INDIRECTLY IN ANY WAY DETRIMENTAL TO THE INTERESTS OF LEAR SIEGLER, INC., POWER EQUIPMENT DIVISION.

NEXT ASSY NO.	MODEL NO.
FIRST APPLICATION	

ECIFIED
NCHES

ANGLES
± 0° 30'

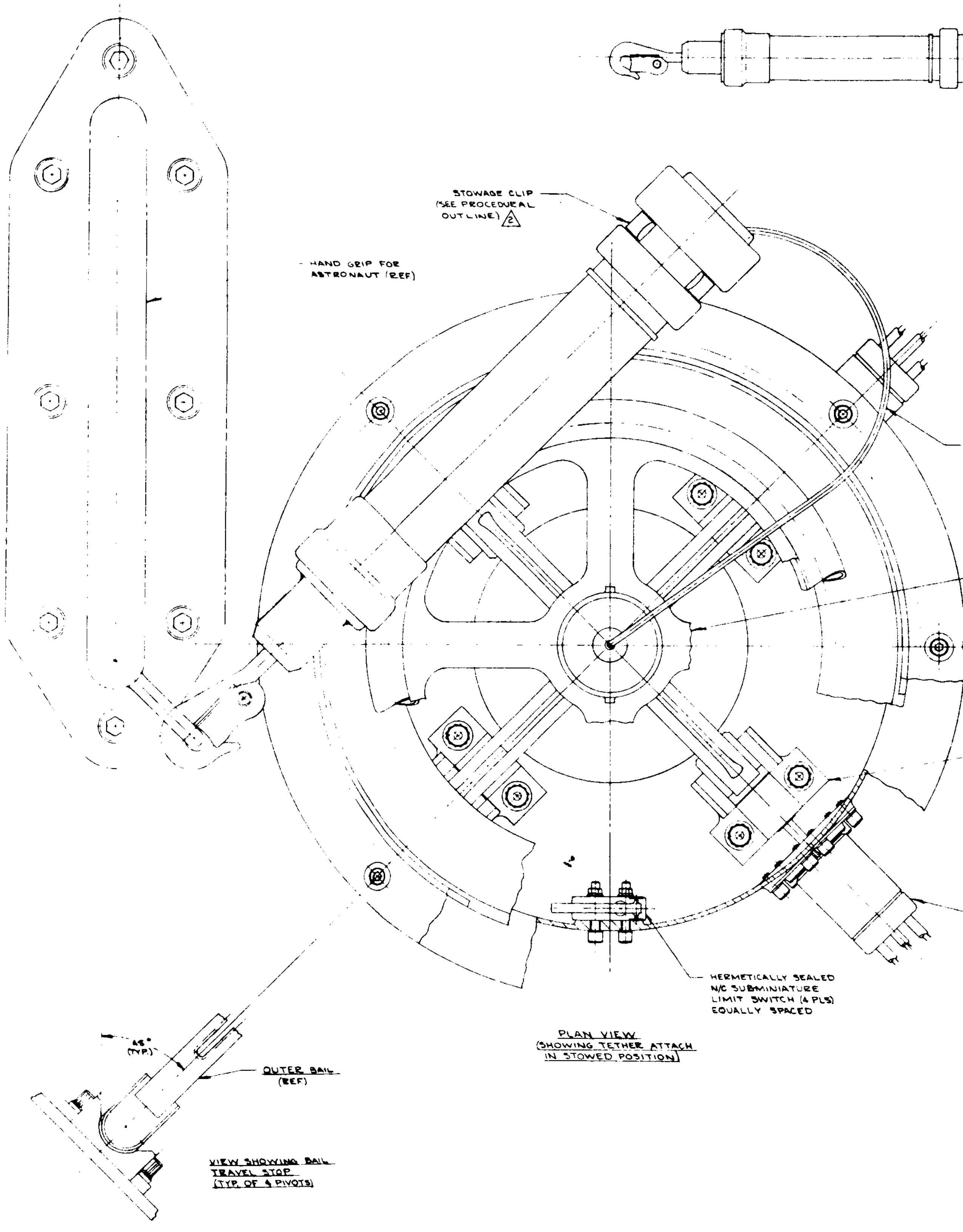
DRAWN BY	Bednarz	DATE 8/27/64
CHECKED BY		DATE
MAT'L APPROVED		DATE
PROJECT ENGR.	Bednarz	DATE 8/27/64

LEAR SIEGLER, INC — LSi POWER EQUIPMENT DIVISION, CLEVELAND 1, OHIO

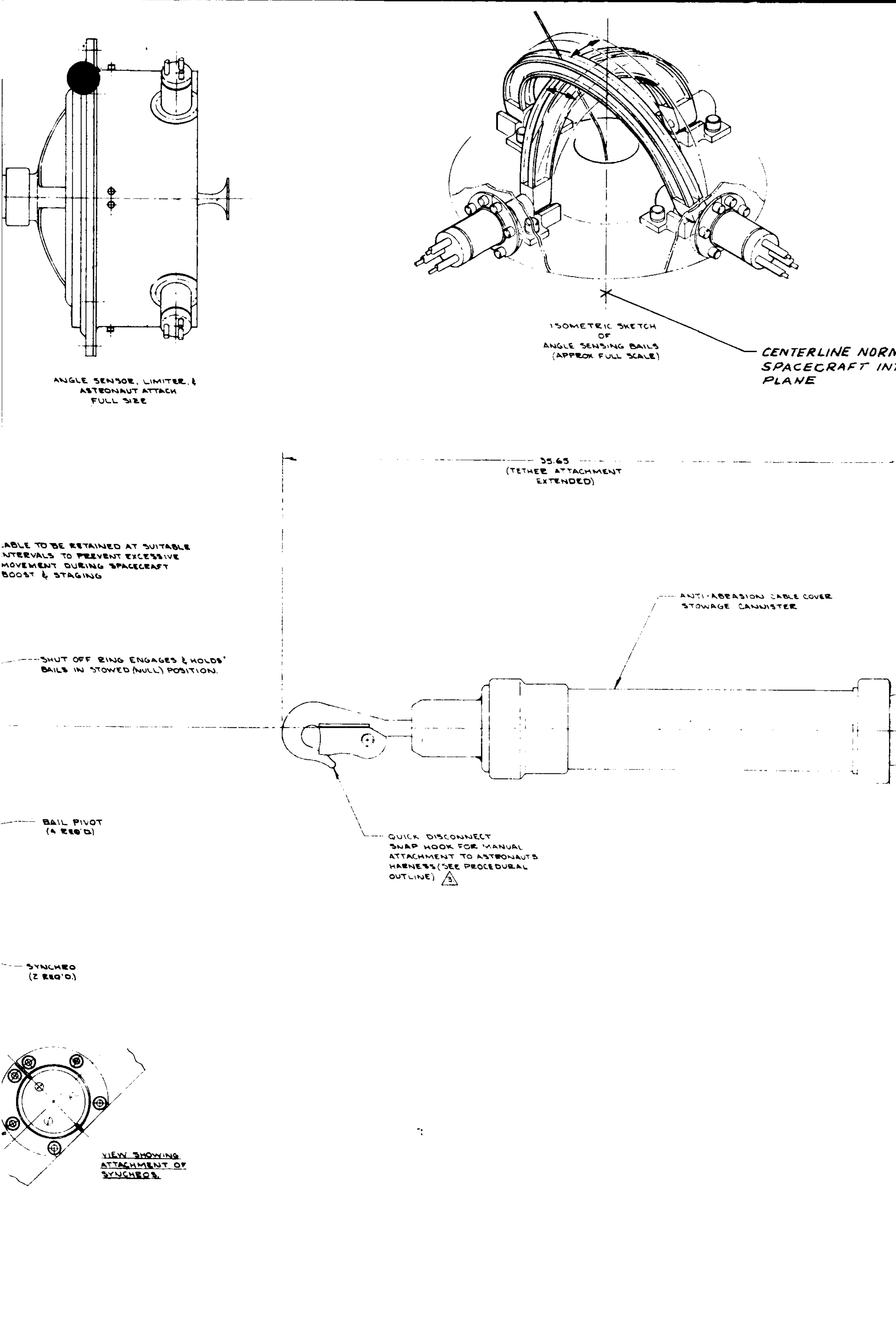
ACTUATOR, ELECTRO-MECHANICAL SERVO-ROTARY - INSTALLATION

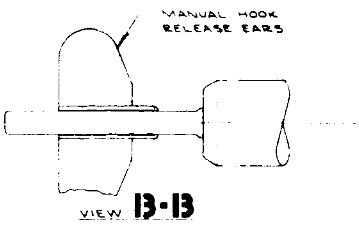
CODE IDENT. NO.	SIZE	DWG. NO.
31435	D	56-65-42R1-E1
SCALE	WEIGHT 10.5	SHEET 1 OF 1

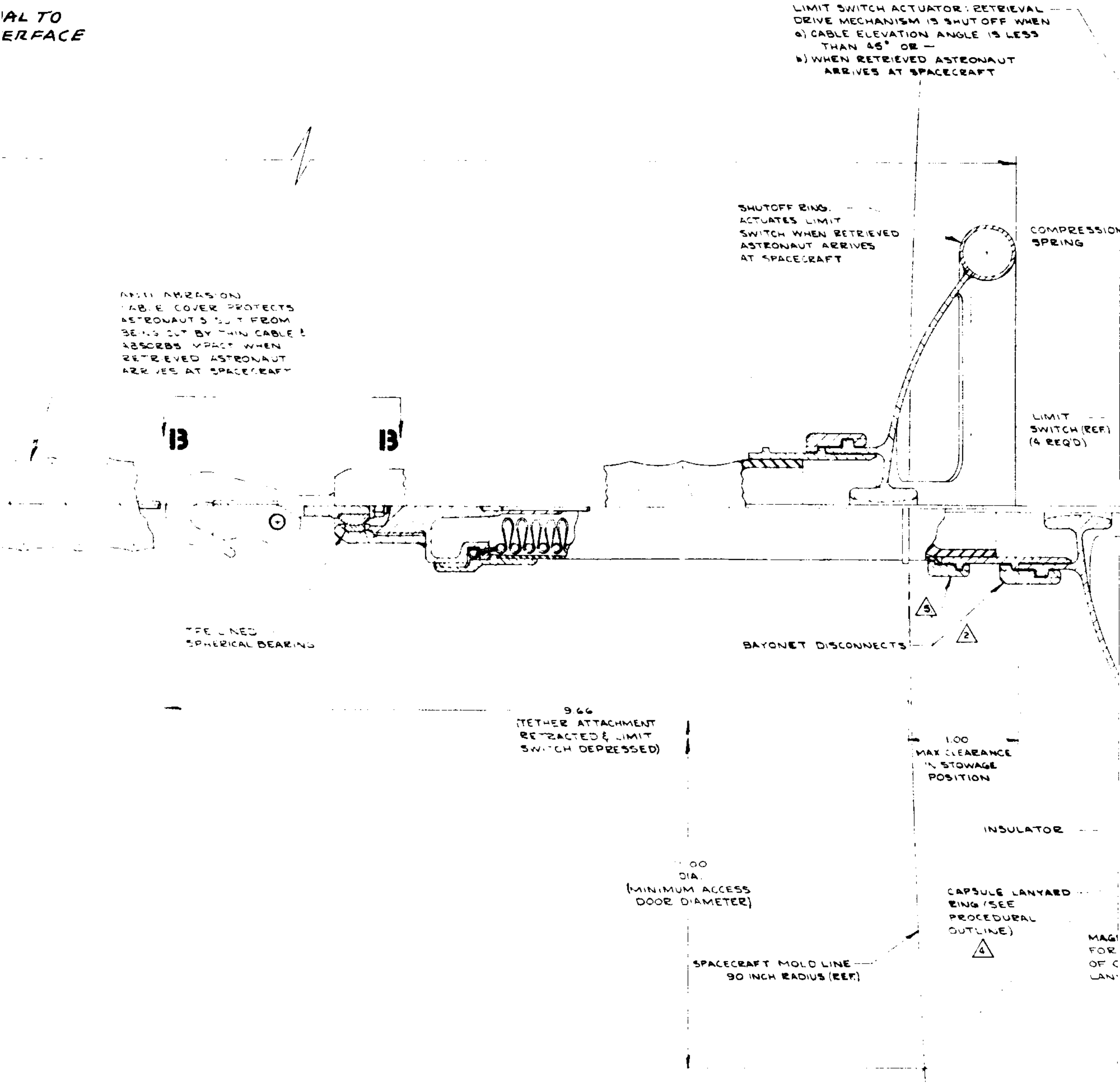
3 #



STOWAGE CLIP (SEE PROCEDURAL OUTLINE) 2
HAND GRIP FOR ASTRONAUT (REF)
HERMETICALLY SEALED N/C SUBMINIATURE LIMIT SWITCH (4 PLS) EQUALLY SPACED
PLAN VIEW (SHOWING TETHER ATTACH IN STOWED POSITION)
45° (TYP.)
OUTER BAIL (REF)
VIEW SHOWING BAIL TRAVEL STOP (TYP. OF 4 PIVOTS)


ISOMETRIC SKETCH OF ANGLE SENSING BAILS (APPROX FULL SCALE)
CENTERLINE NORM SPACECRAFT IN PLANE
ANGLE SENSOR, LIMITER & ASTRONAUT ATTACH FULL SIZE
35.65 (TETHER ATTACHMENT EXTENDED)
ABLE TO BE RETAINED AT SUITABLE NTERVALS TO PREVENT EXCESSIVE MOVEMENT DURING SPACECRAFT BOOST & STAGING
ANTI-ABRASION CABLE COVER STOWAGE CANNISTER
SHUT OFF RING ENGAGES & HOLDS BAILS IN STOWED (NULL) POSITION.
BAIL PIVOT (4 REQ'D)
QUICK DISCONNECT SNAP HOOK FOR MANUAL ATTACHMENT TO ASTRONAUTS HARNESS (SEE PROCEDURAL OUTLINE) 3
SYNCHRO (2 REQ'D.)
VIEW SHOWING ATTACHMENT OF SYNCHROS.





TETHERING/RETRIEVAL SYSTEM ACTIVATION: PROCEDURAL OUTLINE

ASTRONAUT PERFORMS NORMAL SPACECRAFT EGRESS PROCEDURE AND PROCEEDS TO TETHERING/RETRIEVAL SYSTEM ACCESS HATCH, SECURED TO SPACECRAFT BY CAPSULE LANYARD.

THEN,

1. OPEN AND SECURE ACCESS DOOR
2. REMOVE TETHER ATTACHMENT FROM STOWAGE POSITION AND ATTACH, WITH BAYONET DISCONNECT, TO SHUT-OFF RING
3. ATTACH SNAP HOOK TO HARNESS, PASSING IT THROUGH CAPSULE LANYARD RING.
4. DETACH CAPSULE LANYARD HOOK FROM HARNESS, SECURE CAPSULE LANYARD RING AT INDICATED LOCATION AND STOW CAPSULE LANYARD HOOK
5. DETACH BAYONET DISCONNECT FOR EXTENSION OF ANTI-ABRASION CABLE COVER
6. AUTOMATIC SYSTEM TAKES OVER AND TETHERING MISSION IS INITIATED.

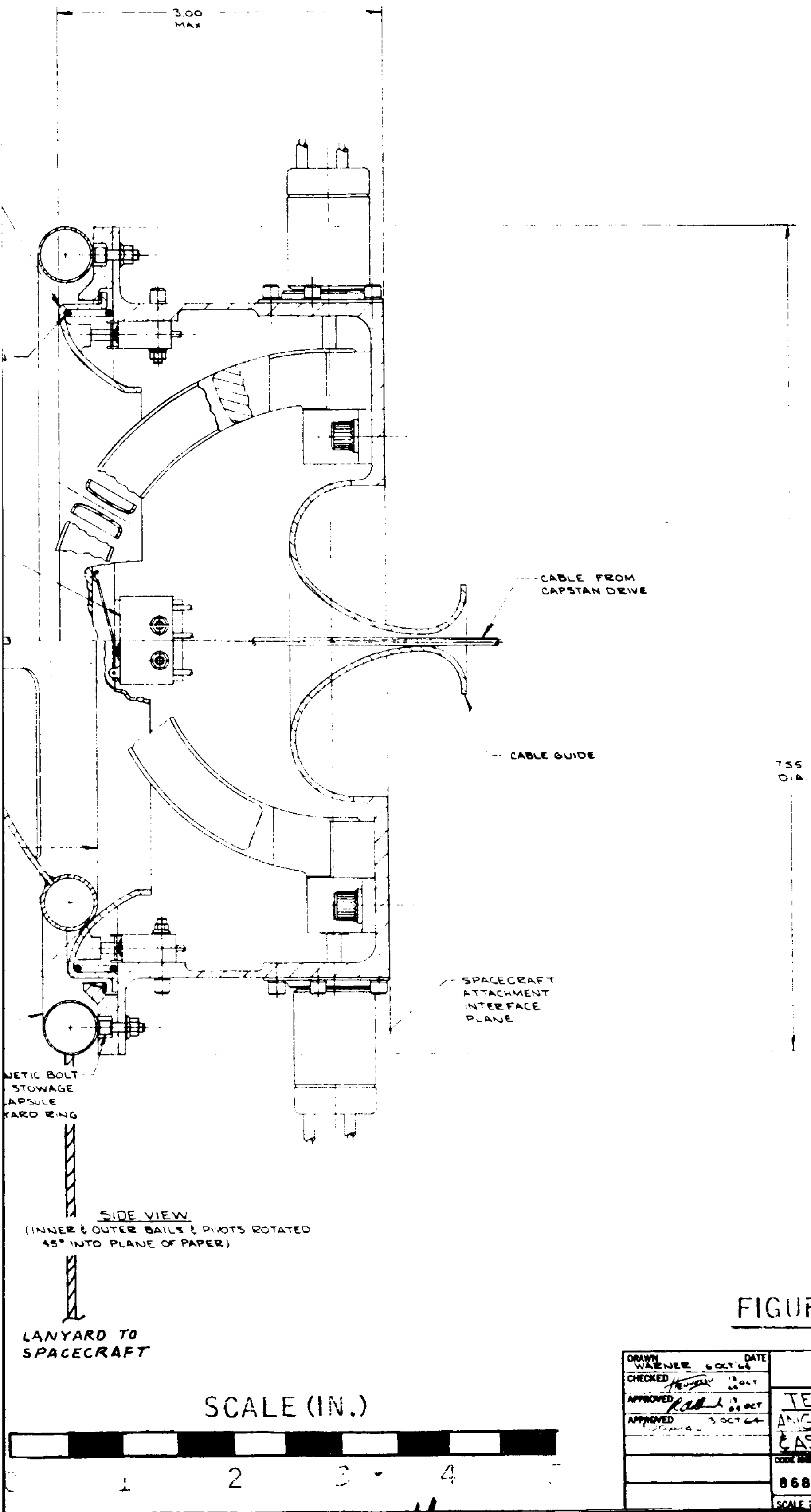
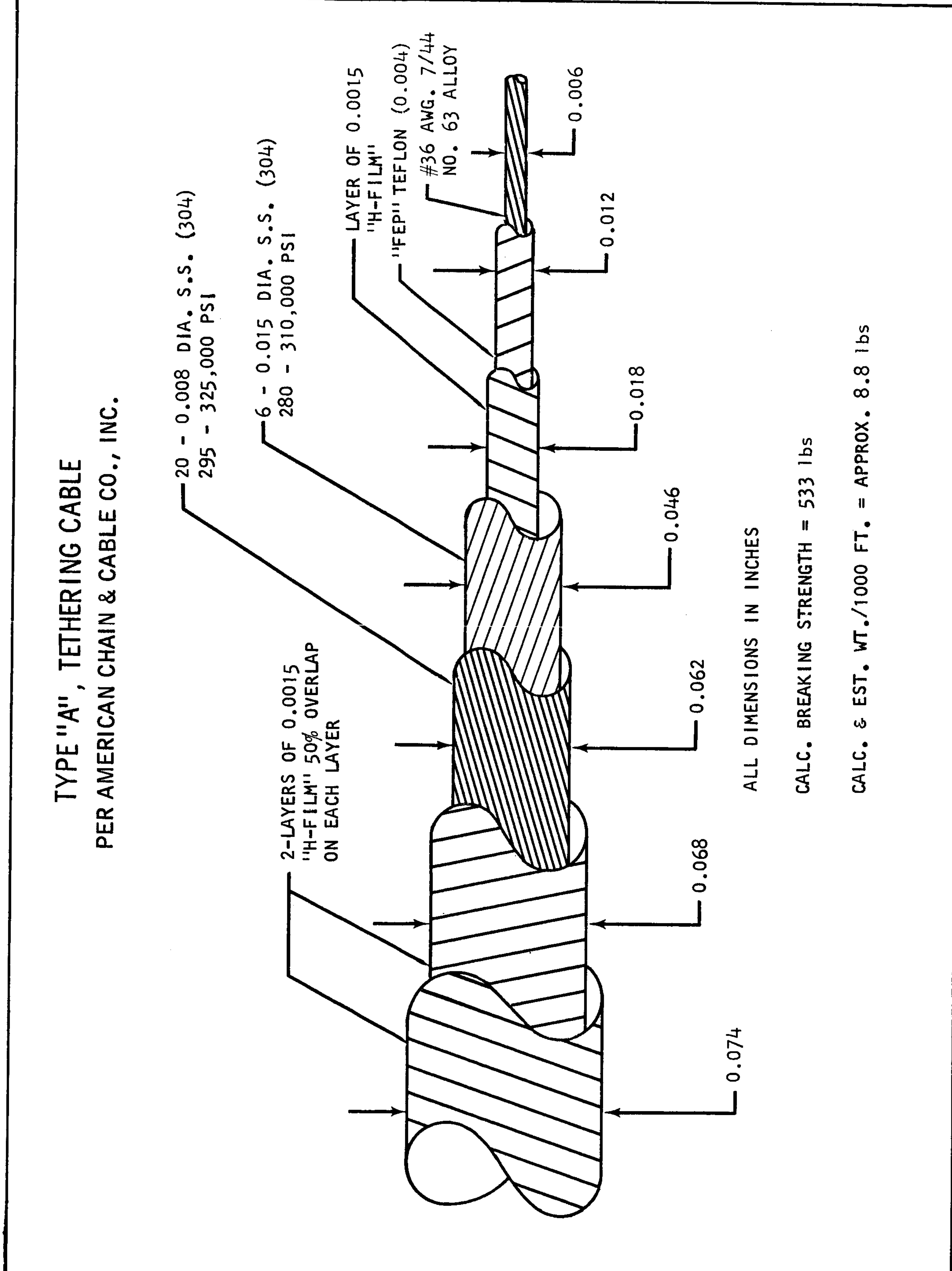


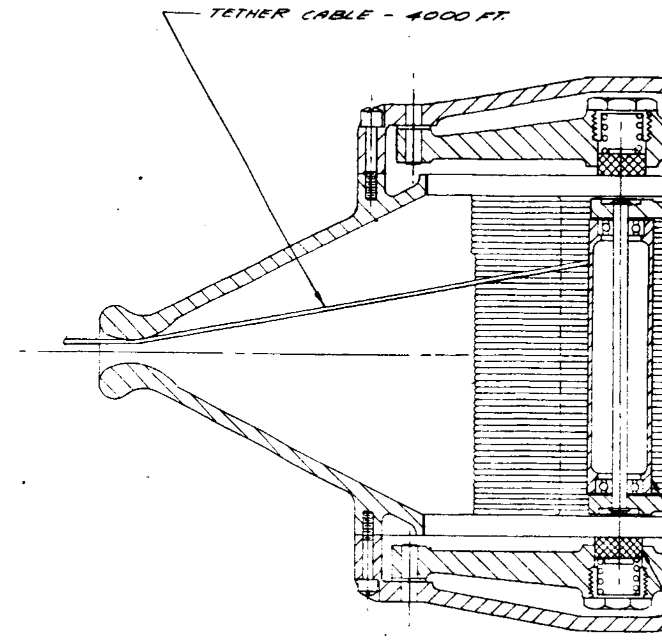
FIGURE 1.

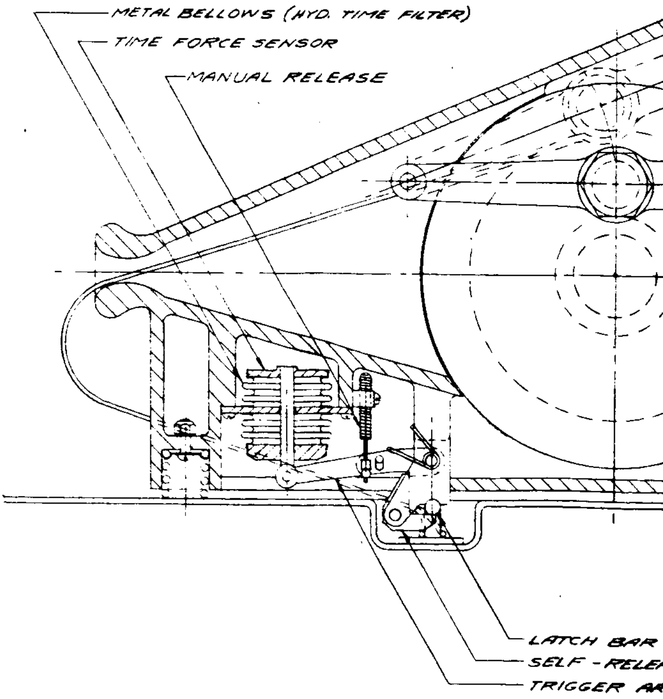
DRAWN WARNER 6 OCT 64	Marquardt
CHECKED	TETHERING SYSTEM ~ ANGLE SENSOR LIMITER, & ASTRONAUT ATTACH.
APPROVED	
APPROVED 13 OCT 64	
CODE IDENT NO. 86845	SIZE R — 228202
SCALE 2/1 NOTED	SHEET 1 OF 1

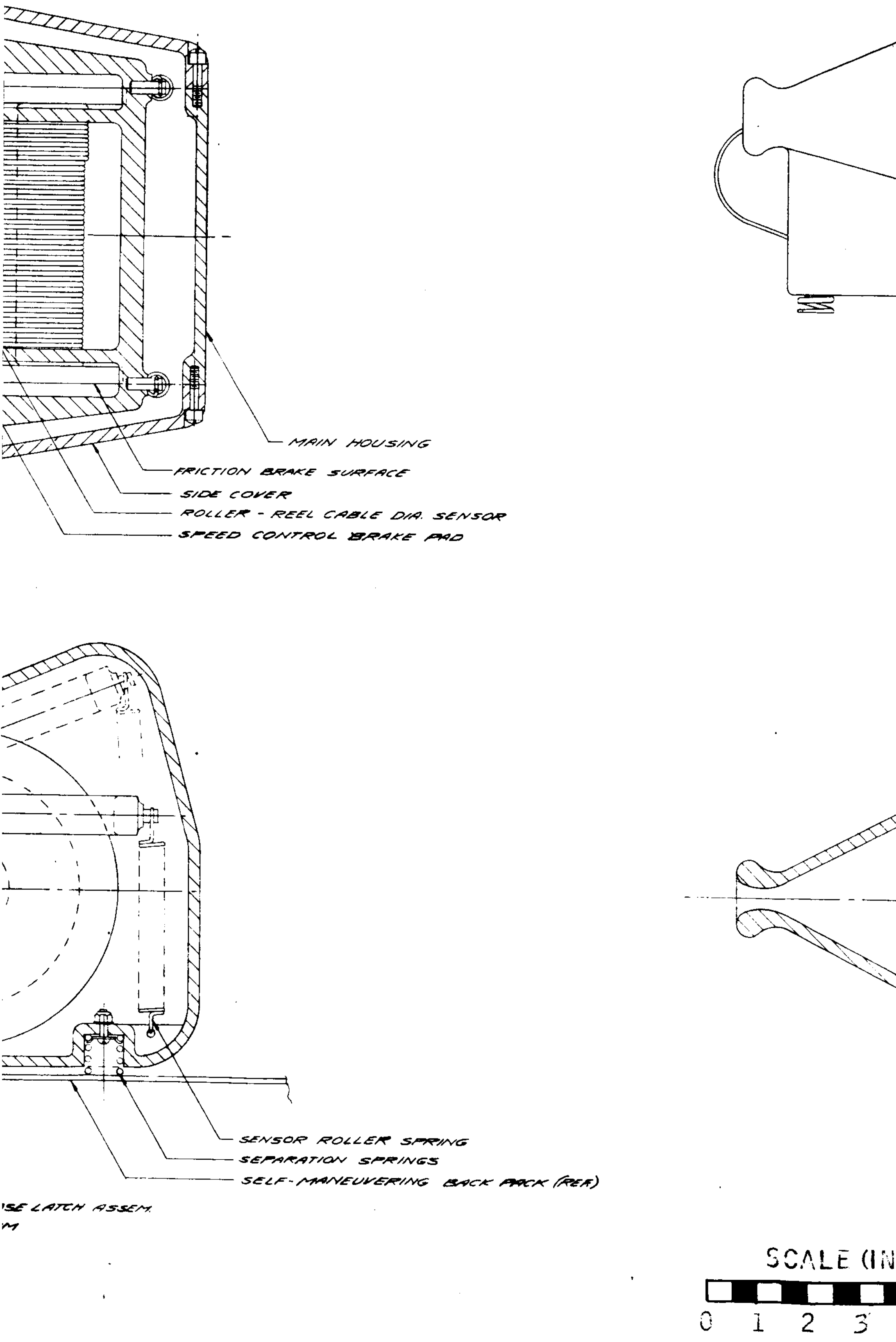


TMC A673

FIGURE 18


TETHER CABLE - 4000 FT.


METAL BELLOWS (HYD. TIME FILTER)
TIME FORCE SENSOR
MANUAL RELEASE
LATCH BAR


MAIN HOUSING
FRICTION BRAKE SURFACE
SIDE COVER
ROLLER - REEL CABLE DIA. SENSOR
SPEED CONTROL BRAKE PAD
SENSOR ROLLER SPRING
SEPARATION SPRINGS
SELF-MANEUVERING BACK PACK (REF)
SE LATCH ASSEM.
M
SCALE (IN
0 1 2 3

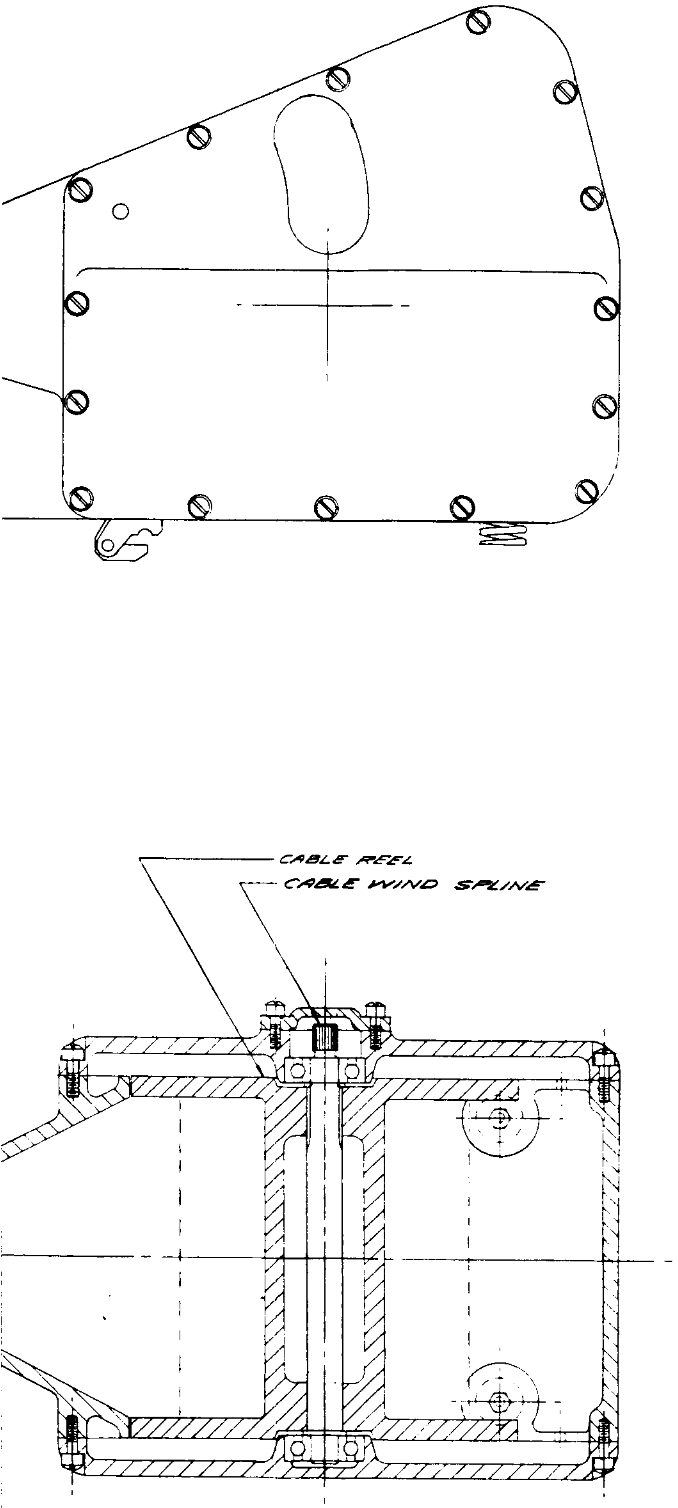


FIGURE 19

QTY REQD	CODE IDENT	PART OR IDENTIFYING NO.	NOMENCLATURE OR DESCRIPTION	SPECIFICATION	MATERIAL OR NOTE	ZONE	ITEM NO.

LIST OF MATERIALS OR PARTS LIST

UNLESS OTHERWISE SPECIFIED
DIMENSIONS ARE IN INCHES
TOLERANCES ON
DECIMALS .XX ± .XXX ± ANGLES ±
SURFACE FINISH
DO NOT SCALE THIS DRAWING
FIRST APPL.

DRAWN R.E. TIMBS DATE 9-2-?
CHECKED
APPROVED R.W. [illegible] 10/12/64
APPROVED

Marquardt

TETHERING SYSTEM -
ANCHOR MASS ASSEMBLY

CODE IDENT NO. 86845 SIZE 228205

SCALE SHEET OF

APPENDIX A

ANALYSIS OF IDEALIZED SYSTEM BEHAVIOR

APPENDIX A

ANALYSIS OF IDEALIZED SYSTEM BEHAVIOR

To obtain a comprehensive evaluation of the multiple body retrieval method for system sizing, certain simplications are introduced so that analytic solutions to the problem may be obtained.

Specifically, it is assumed that the space vehicle, astronaut and inert anchor mass remain coplanar during retrieval and that their angular velocities about the combined mass center are equal. It is also assumed that the angular misalignment between the tethers does not experience large excursions. When the system is performing well, the angular misalignment between the bodies in question is not shown to be large so that the assumed configuration does not appreciably deviate from the actual one. The assumed configuration is admittedly an idealized situation which may not be reached but is expected to be closely approached.

It has been shown previously that when the astronaut is reeled in at a rate which is independent of other system variables and parameters, oscillations in line tension and relative angular positions occur. The presence of these oscillations does not negate the validity of the previous assumptions; however, reduction of these effects is definitely advantageous insofar as system operation is concerned. In addition, the mathematical model under consideration now applies more suitably to a non-oscillatory system. It may safely be assumed that these oscillations can be held to a suitably low level or eliminated entirely by means of appropriate system control design.

In keeping with the assumptions stated above, the approximate dynamics of the retrieval system will be examined. It is assumed that initially the anchor mass is situated at the astronaut location. For any given tangential velocity, the presence of the anchor mass at this location increases the total angular momentum of the system above that which would exist in its absence. The alternative initial configuration has the anchor mass located at the mother ship with effectively zero angular momentum. When the anchor mass is transported to the astronaut location, the astronaut reduces his tangential velocity at the expense of imparting angular momentum to the anchor mass. Each of the two alternate initial configurations do not differ substantially from each other and the results of one are expected to apply approximately to the other. For this reason, the choice of initial configurations was entirely arbitrary. The initial configurations define the angular momentum which is conserved in the system throughout the retrieval operations.

$$H = r_o \, v_o \, (m_1 + m_2) = \text{constant} \tag{1}$$

When retrieval is in progress, the anchor mass is extended to the end of its tether. Since the angular velocities of the astronaut and the anchor are also assumed equal,

$$\frac{v_1}{r} = \frac{v_2}{r + L} \tag{2}$$

APPENDIX A (Continued)

and since the angular momentum is constant,

$$H = r\ v_1\ m_1 + (r + L)\ v_2\ m_2 = r_o\ v_o\ (m_1 + m_2) \tag{3}$$

or

$$r\ v_1\ m_1 \left[1 + \left(\frac{r + L}{r}\right)^2 \frac{m_2}{m_1}\right] = r_o\ v_o\ (m_1 + m_2) \tag{4}$$

Line tension (T) may generally be expressed as,

$$T = m\ r\ \omega^2 \tag{5}$$

where:

$$\omega = v/r \tag{6}$$

or

$$T = \frac{m\ v^2}{r} \tag{7}$$

The total line tensions between the astronaut and the space vehicle includes the accelerations of both the astronaut and the anchor mass.

$$T = \frac{m_1\ v_1^2}{r} + \frac{m_2\ v_2^2}{r + L} \tag{8}$$

or

$$T = \frac{m_1\ v_1^2}{r} \left[1 + \frac{m_2}{m_1} \left(\frac{r + L}{r}\right)\right] \tag{9}$$

Combining Equations (4) and (9):

$$T = \frac{r_o^2\ v_o^2\ (m_1 + m_2)^2}{m_1\ r^3} \frac{\left[1 + \frac{m_2}{m_1}\left(\frac{r + L}{r}\right)\right]}{\left[1 + \frac{m_2}{m_1}\left(\frac{r + L}{r}\right)^2\right]^2} \tag{10}$$

MAC A673

APPENDIX A (Continued)

The initial line tension is designated as T_o where

$$T_o = (m_1 + m_2) \frac{v_o^2}{r_o} \tag{11}$$

Combining Equations (10) and (11):

$$\frac{T}{T_o} = \left(\frac{r_o}{r}\right)^3 \left(1 + \frac{m_2}{m_1}\right) \frac{\left[1 + \frac{m_2}{m_1} \left(1 + \frac{L}{r}\right)\right]}{\left[1 + \frac{m_2}{m_1} \left(1 + \frac{L}{r}\right)^2\right]^2} \tag{12}$$

Applying the following relations to Equation (12)

$$a = \frac{m_2}{m_1}$$

$$b = \frac{L}{r_o}$$

$$X = \frac{r}{r_o}$$

gives

$$\frac{T}{T_o} = \frac{1 + a}{X^3} \frac{\left[1 + a \left(1 + \frac{b}{X}\right)\right]}{\left[1 + a \left(1 + \frac{b}{X}\right)^2\right]^2} \tag{13}$$

The value of X for which T/T_o assumes a maximum is now determined. Equation (13) is first rewritten as shown below.

$$\frac{T}{T_o} = (1 + a) \frac{\left[X + a (X + b)\right]}{\left[X^2 + a (X + b)^2\right]^2} \tag{14}$$

When the derivative of Equation (14) is taken and equated to zero, the following expression results:

$$(1 + a)\left[X^2 + a (X + b)^2\right]^2 = 2 \left[X^2 + a (X + b)^2\right]\left[2 X + 2 aX + 2 ab\right]\left[X + a (X + b)\right] \tag{15}$$

It may be shown from Equation (15) that at the completion of the retrieval (X = 0) the derivative of T/T_o is as follows.

APPENDIX A (Continued)

$$\frac{d}{dX}\left(\frac{T}{T_o}\right)\bigg|_{X=0} = \frac{(1+a)\ (1-3\,a)}{a^2\ \ b^4} \tag{16}$$

Equation (16) indicates that when $a < 1/3$, the line tension at the fully retrieved configuration is not at its maximum. Since both the line strength and the astronaut acceleration tolerance concern themselves with the maximum line load, it is necessary to determine where this occurs. Equation (15) is simplified to the following:

$$\left\{X^2 + a\ (X + b)^2\right\}\ \left\{\left[(1+a)\ X + a\ b\right]^2 - \frac{a\ b^2}{3}\right\} = 0 \tag{17}$$

The maximum value of T/T_o corresponds to a solution of Equation (17) when $a < 1/3$. Since only those values of a and b which are greater than zero are of interest in this investigation and the first factor of Equation (17) never reduces to zero, the equation is therefore equivalent to

$$\left[(1+a)\ \ X + a\ b\right]^2 - \frac{a\ b^2}{3} = 0 \tag{18}$$

or

$$(1+a)\ \ X + ab = b\sqrt{\frac{a}{3}} \tag{19}$$

which reduces to

$$X = \frac{b}{1+a}\left(\sqrt{\frac{a}{3}} - a\right) \tag{20}$$

The value of X in Equation (20) is that for which T/T_o is maximum. This variable is defined as $\overline{X}$, so that the previous equation is now written as

$$\overline{X} = \frac{b}{1+a}\left(\sqrt{\frac{a}{3}} - a\right) \tag{21}$$

It is seen from Equation (21) that the point of maximum tension occurs at the fully extended condition ($\overline{X} = 1$) when

$$b = \frac{1+a}{\sqrt{\frac{a}{3}} - a} \tag{22}$$

MAC A673

APPENDIX A (Continued)

It may be shown that the right side of Equation (22) assumes a minimum value when $a = 7 - 4\sqrt{3}$.

Since $0 \leq \overline{X} \leq 1$, the following restriction is imposed:

$$b \leq \frac{1 + a}{\sqrt{\frac{a}{3}} - a} \tag{23}$$

When the appropriate value of a is substituted into the above expression it is found that

$$\left[\frac{1 + a}{\sqrt{\frac{a}{3}} - a}\right]_{min} = 13.16 \tag{24}$$

Therefore the following expression covers all cases:

$$b \leq 13.16 \tag{25}$$

Now, when Equations (13) and (21) are combined, the following result is obtained:

$$\left(\frac{T}{T_o}\right)_{max} = \frac{9}{16} \frac{(1 + a)^3}{b^3 \; a^2} \sqrt{\frac{a}{3}} \tag{26}$$

where

$$a \leq 1/3; \quad b \leq \frac{1 + a}{\sqrt{\frac{a}{3}} - a}$$

When $a \geq 1/3$, $\overline{X} = 0$ and

$$\left(\frac{T}{T_o}\right)_{max} = \frac{1 + a}{b^3 \; a} \tag{27}$$

APPENDIX A (Continued)

which results from setting X = 0 in Equation (14). It is to be noted that no restriction is placed upon b in the case stated above.

When $a < 1/3$ and $b \geq \dfrac{1 + a}{\sqrt{\frac{a}{3}} - a}$, $\bar{X} = 1$ and

$$\left(\frac{T}{T_o}\right)_{max} = (1 + a) \frac{\left[1 + a\,(1 + b)\right]}{\left[1 + a\,(1 + b)^2\right]^2} \qquad (28)$$

which results from setting X = 1 in Equation (14). Equations (27) and (28) apply to a range of parameters which is not usually considered practical, and are introduced essentially for the sake of completeness.

Equations (26) and (27) cover practically the entire working range of the variables a and b. It is noted that each of these equations may be expressed as

$$\left(\frac{T}{T_o}\right)_{max} = f_1\,(a)\,f_2\,(b) \qquad (29)$$

where

$$f_1\,(a) = \frac{9}{16}\,\frac{(1 + a)^3}{a^2}\sqrt{\frac{a}{3}} \qquad \text{for } a \leq \frac{1}{3}$$

$$= \frac{1 + a}{a} \qquad \text{for } a \geq \frac{1}{3}$$

and

$$f_2\,(b) = \frac{1}{b^3}$$

Since the tension ratio is expressible as the product of functions of each of the pertinent system variables, the contribution of each of these variables is independent of the other. Figure A-1 shows a plot of the relative contributions of each of these functions.

APPENDIX A (Continued)

Figure A-2 shows a plot of T/T_o as a function of b with a as a parameter. The values of X for which the maximum tension occurs are also shown in Figure A-2.

The net energy requirement to effect a retrieval is now considered. This energy is represented by the increase in kinetic energy of the system components following retrieval.

Define:

E_o = Initial kinetic energy of system

E_f = Final kinetic energy of system

$$E_o = \frac{(m_1 + m_2)}{2} v_o^2 = \frac{r_o T_o}{2} \tag{30}$$

$$E_f = \frac{m_2 v_f^2}{2} = \frac{L T_f}{2} \tag{31}$$

where

T_f = T at X = 0

$$\Delta E = 1/2 \, (L \, T_f - r_o \, T_o) \tag{32}$$

From Equation (14) it is seen that

$$T_f = \frac{1 + a}{a \, b^3} T_o \tag{33}$$

therefore

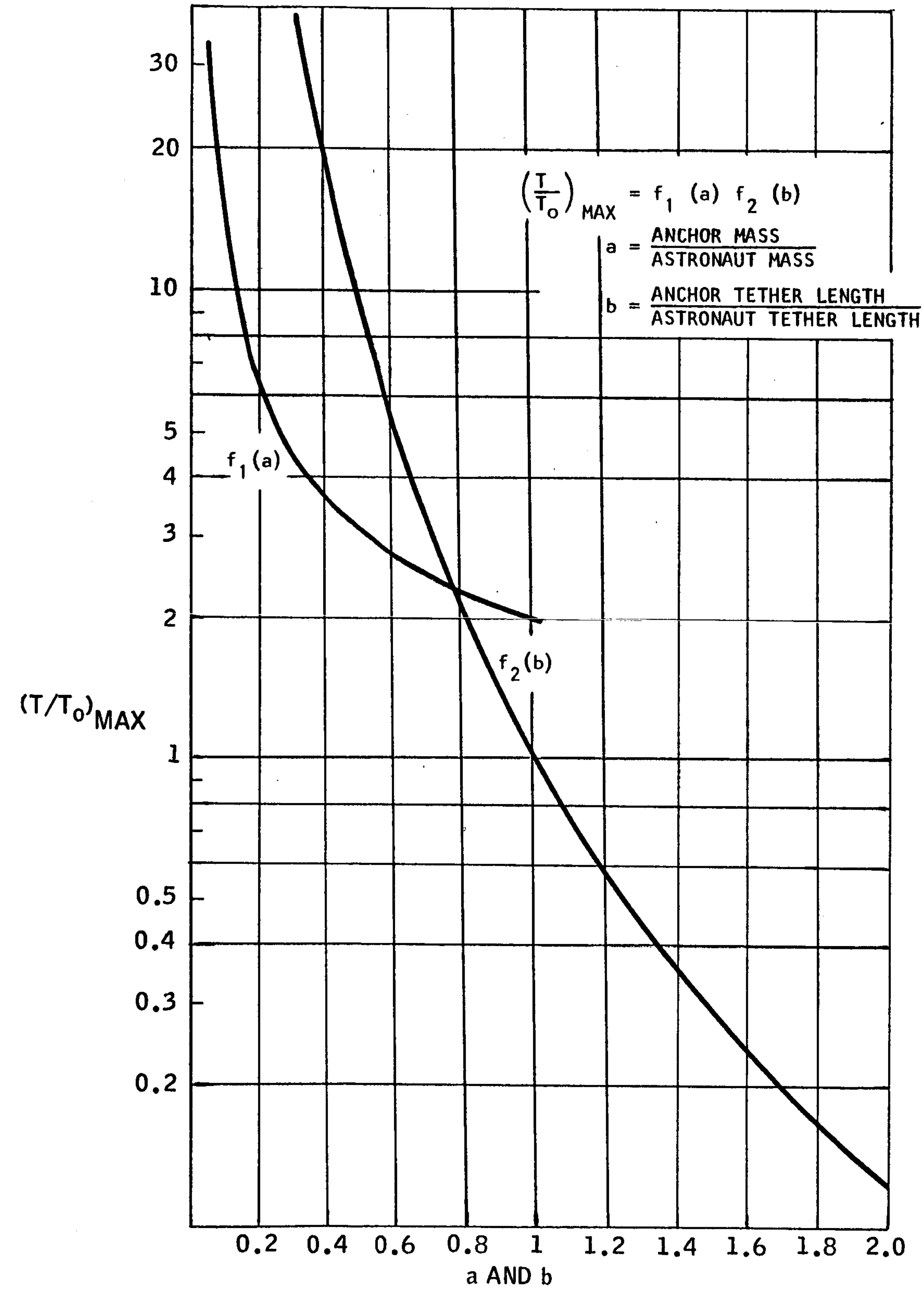
$$\Delta E = \frac{r_o T_o}{2} \left(\frac{1 + a}{a \, b^3} - 1 \right) = \frac{r_o T_o}{2a \, b^2} \left[1 + a \, (1 - b^2) \right] \tag{34}$$

APPENDIX A (Continued)

NOMENCLATURE

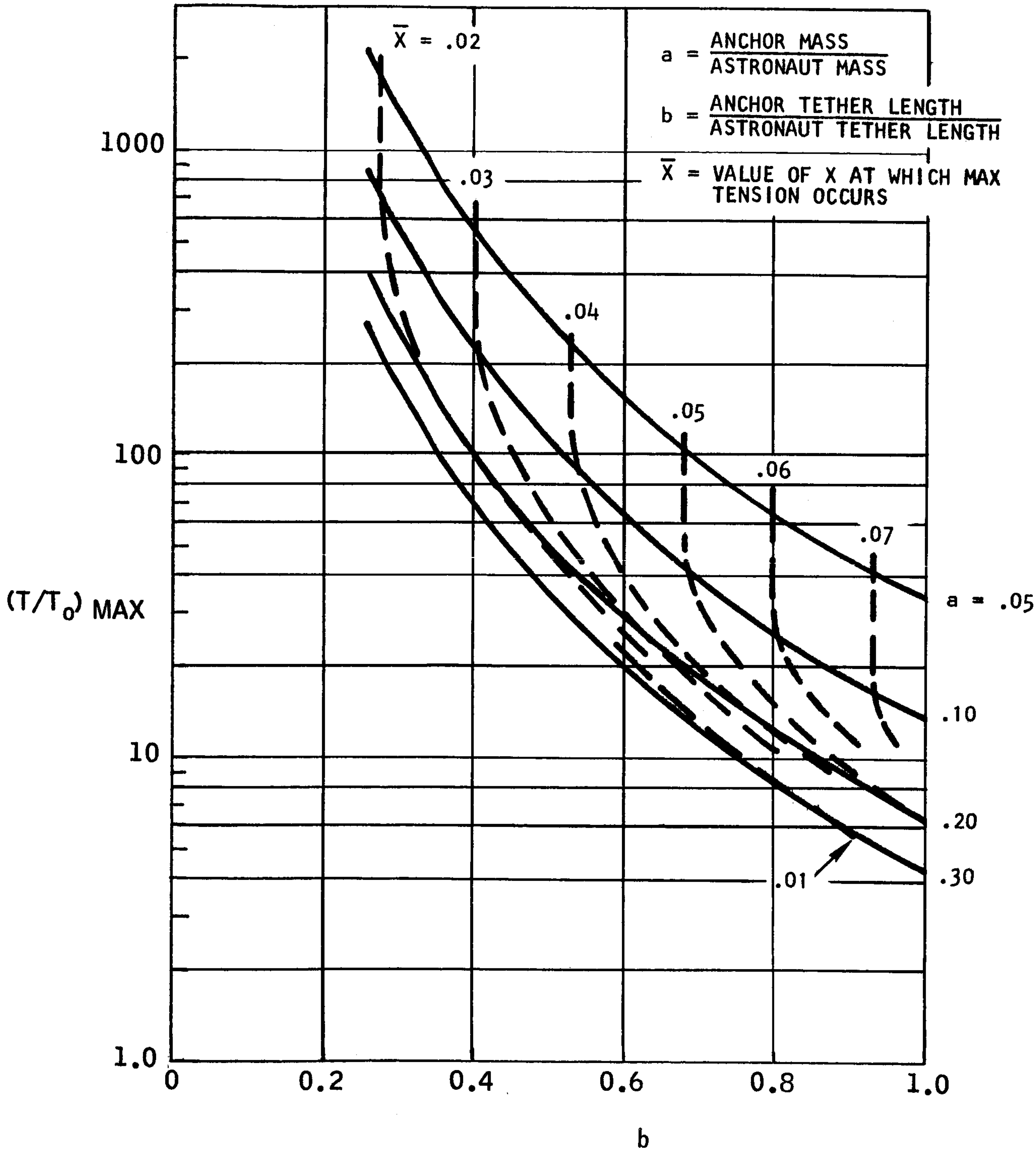
Symbol	Description
m_1	Mass of astronaut
m_2	Anchor mass
r	Length of astronaut tether
r_o	Initial astronaut tether length
L	Length of anchor mass tether
H	System angular momentum
v_1	Astronaut tangential velocity
v_2	Anchor mass tangential velocity
v_o	Initial tangential velocity
T	Line tension
a	$\frac{m_2}{m_1}$
b	$\frac{L}{r_o}$
X	$\frac{r}{r_o}$
T_o	Initial line tension
$\bar{X}$	Point of maximum tension
E	Energy expended during retrieval

TENSION RATIO FUNCTION vs. MASS AND TETHERLINE LENGTH RATIOS



TMC A673

DETERMINATION OF PRIMARY TETHERLINE LENGTHS AT MAXIMUM TENSIONS



APPENDIX B

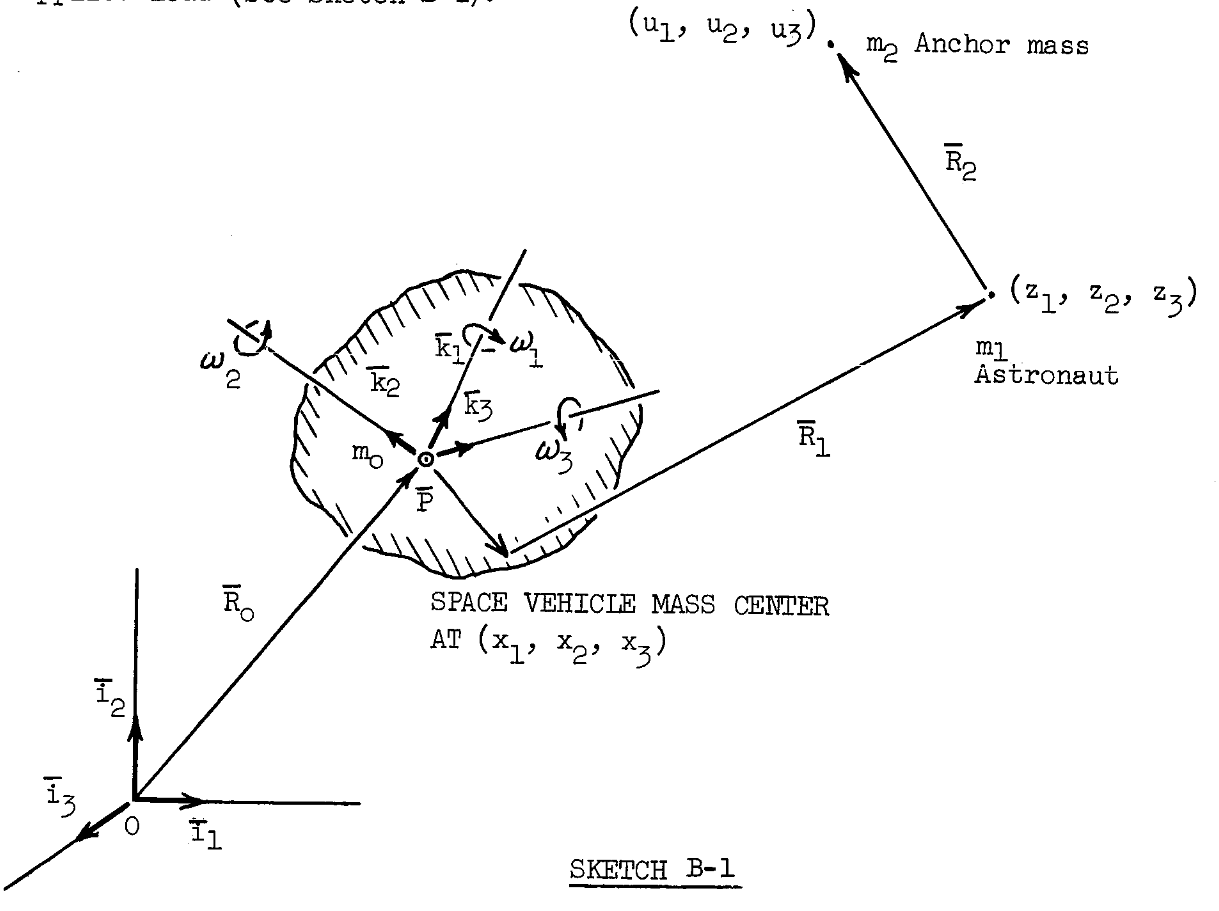
EQUATIONS OF MOTION OF AN ORBITING SYSTEM OF CONNECTED MASSES

APPENDIX B

EQUATIONS OF MOTION OF AN ORBITING SYSTEM OF CONNECTED MASSES

The behavior of an orbiting system of connected masses which include a space vehicle, astronaut, and anchor mass, is described here in terms of their differential equations of motion. In addition to the translatory motion of all bodies, the cross-coupled rotational motion of the vehicle is also included. Provision is made for the simulation of applied force and/or torque on the system so that the results of external thrust may be evaluated.

Consider a space vehicle which is free to translate and rotate under an applied load (See Sketch B-1).



SKETCH B-1

Except for the angular orientation of the space vehicle, all coordinates are given relative to the point O and the triad, $(\bar{i}_1, \bar{i}_2, \bar{i}_3)$. The triad $(\bar{k}_1, \bar{k}_2, \bar{k}_3)$ is aligned with the principal axes of inertia of the vehicle and is located at its mass center. The vectors $\bar{R}_1$ and $\bar{R}_2$ represent cable connections between the masses. The vector $\bar{R}_o$ represents vehicle position only. In all cases, connecting cables are assumed to have no mass. The astronaut and anchor mass are considered as point masses. The vector $\bar{P}$ is fixed relative to the $(\bar{k}_1, \bar{k}_2, \bar{k}_3)$ triad and defines the point of attachment of the cable to the vehicle.

APPENDIX B (Continued)

Now

$$\overline{R}_o = X_1 \overline{i}_1 + X_2 \overline{i}_2 + X_3 \overline{i}_3 \tag{1}$$

and

$$\overline{P} = S_1 \overline{k}_1 + S_2 \overline{k}_2 + S_3 \overline{k}_3 \tag{2}$$

Both ($\overline{i}_1$, $\overline{i}_2$, $\overline{i}_3$) and ($\overline{k}_1$, $\overline{k}_2$, $\overline{k}_3$) are right handed unit orthogonal triads and therefore transform as follows (see Reference 5).

$$\begin{bmatrix} \overline{i}_1 \\ \overline{i}_2 \\ \overline{i}_3 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \overline{k}_1 \\ \overline{k}_2 \\ \overline{k}_3 \end{bmatrix} \tag{3}$$

or more conveniently

$$\left[\overline{i}\right] = \left[A\right] \left[\overline{k}\right] \tag{4}$$

It is easily shown that the general element a_{ij} of $\left[A\right]$ is the direction cosine between $\overline{i}_i$ and $\overline{k}_j$ or

$$a_{ij} = \overline{i}_i \cdot \overline{k}_j \tag{5}$$

Similarly,

$$\left[\overline{k}\right] = \left[B\right] \left[\overline{i}\right] \tag{6}$$

Where the elements of $\left[B\right]$ are also stated as

$$b_{ij} = \overline{k}_i \cdot \overline{i}_j \tag{7}$$

APPENDIX B (Continued)

It is thus evident that

$$b_{ij} = a_{ji} \tag{8}$$

The vector $\overline{P}$ may be expressed in terms of either $(\overline{k}_1, \overline{k}_2, \overline{k}_3)$ or $(\overline{i}_1, \overline{i}_2, \overline{i}_3)$. Thus

$$\overline{P} = S_1 \overline{k}_1 + S_2 \overline{k}_2 + S_3 \overline{k}_3 = y_1 \overline{i}_1 + y_2 \overline{i}_2 + y_3 \overline{i}_3 \tag{9}$$

and it may be shown that

$$[y] = [A] [S] \tag{10}$$

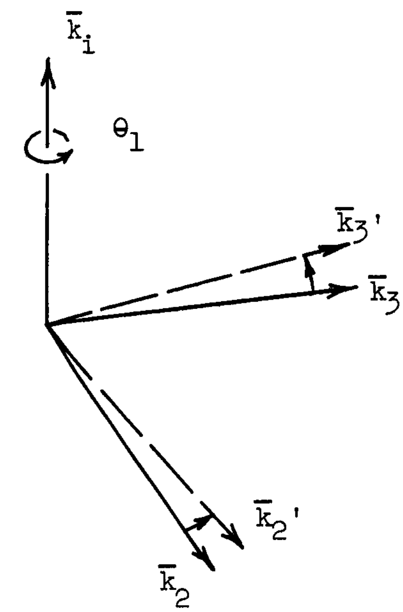
where

$$[y] = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \quad ; \quad [S] = \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} \tag{11}$$

and $[A]$ is as previously defined.

It is easily seen that the matrix $[A]$ is entirely determined by the orientation of the triad $(\overline{k}_1, \overline{k}_2, \overline{k}_3)$. When this triad is rotating with an angular velocity ω, having components ω_1, ω_2, ω_3 along the local coordinate axes, the elements of $[A]$ change with time. Consider the triad of Sketch B-2 executing a differential rotation $\delta\theta_1$ about $\overline{k}_1$

APPENDIX B (Continued)



SKETCH B-2

It may be shown that

$$\begin{bmatrix} \bar{k}'_1 \\ \bar{k}'_2 \\ \bar{k}'_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \delta\theta_1 \\ 0 & -\delta\theta_1 & 1 \end{bmatrix} \begin{bmatrix} \bar{k}_1 \\ \bar{k}_2 \\ \bar{k}_3 \end{bmatrix} \tag{12}$$

or

$$\left[\bar{k}'\right] = \left[\Delta\theta_1\right]\left[\bar{k}\right] \tag{13}$$

Similar expressions exist for rotations about the other axes. Thus,

$$\left[\Delta\theta_2\right] = \begin{bmatrix} 1 & 0 & -\delta\theta_2 \\ 0 & 1 & 0 \\ \delta\theta_2 & 0 & 1 \end{bmatrix} ; \left[\Delta\theta_3\right] = \begin{bmatrix} 1 & \delta\theta_3 & 0 \\ -\delta\theta_3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{14}$$

In the general case, a differential rotation will have components along all three axes. Since the sum of differential rotations is independent of the order of summation, the result of rotation about all three axes simultaneously is given as

APPENDIX B (Continued)

$$\left[\bar{k}''\right] = \left[\Delta\theta_3\right]\left[\Delta\theta_2\right]\left[\Delta\theta_1\right]\left[\bar{k}\right] \tag{15}$$

When the three rotation matrices are expanded and only first order differentials are retained

$$\left[\Delta\theta_3\right]\left[\Delta\theta_2\right]\left[\Delta\theta_1\right] = \begin{bmatrix} 1 & \delta\theta_3 & -\delta\theta_2 \\ -\delta\theta_3 & 1 & \delta\theta_1 \\ \delta\theta_2 & -\delta\theta_1 & 1 \end{bmatrix} = \left[\Delta\theta\right] \tag{16}$$

As the triad is rotated, the transformation matrix $\left[B\right]$ becomes $\left[B'\right]$ where

$$\left[B'\right] = \left[\Delta\theta\right]\left[B\right] \tag{17}$$

The expansion of Equation (17) is as follows:

$$\left[B'\right] = \begin{bmatrix} (a_{11} + a_{12}\,\delta\theta_3 - a_{13}\,\delta\theta_2)(a_{21} + a_{22}\,\delta\theta_3 - a_{23}\,\delta\theta_2) \\ \qquad (a_{31} + a_{32}\,\delta\theta_3 - a_{33}\,\delta\theta_2) \\ (-a_{11}\,\delta\theta_3 + a_{12} + a_{13}\,\delta\theta_1)(-a_{21}\,\delta\theta_3 + a_{22} + a_{23}\,\delta\theta_1) \\ \qquad (-a_{31}\,\delta\theta_3 + a_{32} + a_{33}\,\delta\theta_1) \\ (a_{11}\,\delta\theta_2 - a_{12}\,\delta\theta_1 + a_{13})(a_{21}\,\delta\theta_2 - a_{22}\,\delta\theta_1 + a_{23}) \\ \qquad (a_{31}\,\delta\theta_2 - a_{32}\,\delta\theta_1 + a_{33}) \end{bmatrix} \tag{18}$$

Define

$$\delta\theta_1 = \omega_1\,dt, \quad \delta\theta_2 = \omega_2\,dt, \quad \delta\theta_3 = \omega_3\,dt \tag{19}$$

Now

$$\frac{d}{dt}\left[B\right] = \lim_{dt \to 0}\left\{\frac{\left[B'\right] - \left[B\right]}{dt}\right\} \tag{20}$$

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APPENDIX B (Continued)

Therefore

$$\frac{d}{dt}\left[B\right] = \begin{bmatrix} (a_{12}\,\omega_3 - a_{13}\,\omega_2) & (a_{22}\,\omega_3 - a_{23}\,\omega_2) & (a_{32}\,\omega_3 - a_{33}\,\omega_2) \\ (a_{13}\,\omega_1 - a_{11}\,\omega_3) & (a_{23}\,\omega_1 - a_{21}\,\omega_3) & (a_{33}\,\omega_1 - a_{31}\,\omega_3) \\ (a_{11}\,\omega_2 - a_{12}\,\omega_1) & (a_{21}\,\omega_2 - a_{22}\,\omega_1) & (a_{31}\,\omega_2 - a_{32}\,\omega_1) \end{bmatrix} \qquad (21)$$

The derivative of $\left[B\right]$ is by definition the derivatives of the elements of $\left[B\right]$. These are equated to the elements of Equation (21) to obtain the following:

$$\left.\begin{aligned} \frac{d}{dt}\,a_{11} &= a_{12}\,\omega_3 - a_{13}\,\omega_2 \\ \frac{d}{dt}\,a_{12} &= a_{13}\,\omega_1 - a_{11}\,\omega_3 \\ \frac{d}{dt}\,a_{13} &= a_{11}\,\omega_2 - a_{12}\,\omega_1 \\ \frac{d}{dt}\,a_{21} &= a_{22}\,\omega_3 - a_{23}\,\omega_2 \\ \frac{d}{dt}\,a_{22} &= a_{23}\,\omega_1 - a_{21}\,\omega_3 \\ \frac{d}{dt}\,a_{23} &= a_{21}\,\omega_2 - a_{22}\,\omega_1 \\ \frac{d}{dt}\,a_{31} &= a_{32}\,\omega_3 - a_{33}\,\omega_2 \\ \frac{d}{dt}\,a_{32} &= a_{33}\,\omega_1 - a_{31}\,\omega_3 \\ \frac{d}{dt}\,a_{33} &= a_{31}\,\omega_2 - a_{32}\,\omega_1 \end{aligned}\right\} \qquad (22)$$

or, written in matrix form

$$\frac{d}{dt}\left[B\right] = \left[C\right]\left[B\right] \qquad (23)$$

APPENDIX B (Continued)

where

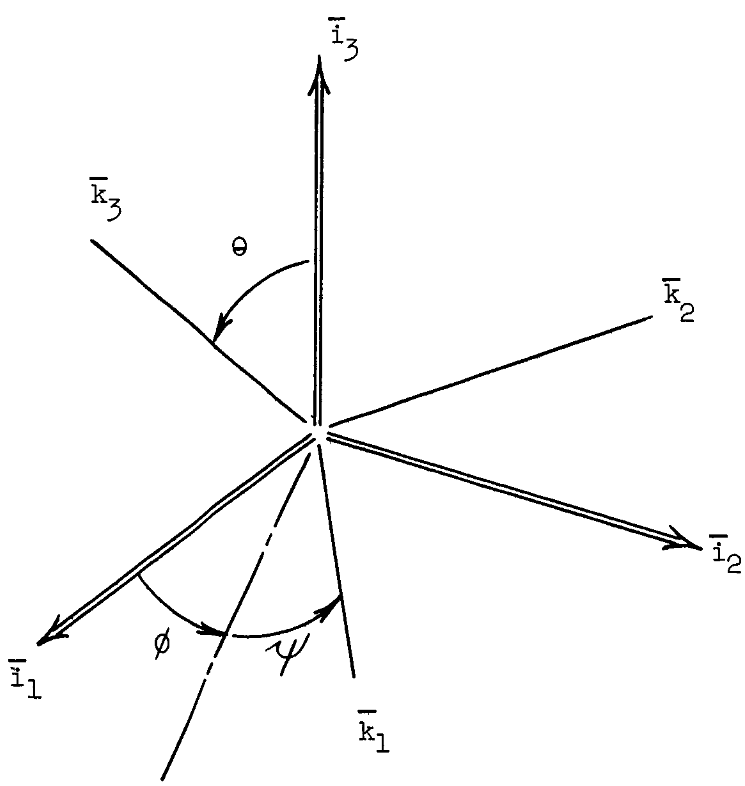
$$[C] = \begin{bmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{bmatrix} \tag{24}$$

and since $[B]$ is the transpose as well as the inverse of $[A]$, the following is also true.

$$\frac{d}{dt}[A] = [A][C]^T \tag{25}$$

Equation (25) is not essential to the development and is included merely for the sake of completeness.

Of the nine elements of $[A]$, only three are independent. Therefore, these nine elements may be expressed in terms of the three Eulerian angles of the vehicle relative to the $(\bar{i}_1, \bar{i}_2, \bar{i}_3)$ triad. These angles are specified as ϕ, θ, ψ and are defined as shown in Sketch B-3.



SKETCH B-3

APPENDIX B (Continued)

These angles are defined by the rotations required to revolve the $(\bar{i}_1, \bar{i}_2, \bar{i}_3)$ triad into the $(\bar{k}_1, \bar{k}_2, \bar{k}_3)$ triad. These are as follows:

1. Rotate $\bar{i}_1$, and $\bar{i}_2$ about $\bar{i}_3$ through an angle, ϕ
2. Rotate $\bar{i}_2$ and $\bar{i}_3$ about $\bar{i}_1$ through an angle, θ
3. Rotate $\bar{i}_1$ and $\bar{i}_2$ about $\bar{i}_3$ through an angle, ψ

It is easily verified that the matrix elements a_{ij} may be expressed in terms of the Eulerian angles as follows:

$$\left.\begin{aligned}
a_{11} &= \cos\psi\cos\phi - \cos\theta\sin\phi\sin\psi \\
a_{12} &= -\sin\psi\cos\phi - \cos\theta\sin\phi\cos\psi \\
a_{13} &= \sin\theta\sin\phi \\
a_{21} &= \cos\psi\sin\phi + \cos\theta\cos\phi\sin\psi \\
a_{22} &= -\sin\psi\sin\phi + \cos\theta\cos\phi\cos\psi \\
a_{23} &= -\sin\theta\cos\phi \\
a_{31} &= \sin\psi\sin\theta \\
a_{32} &= \cos\psi\sin\theta \\
a_{33} &= \cos\theta
\end{aligned}\right\} \tag{26}$$

Now consider tension in the connecting cables. To each cable denoted by $\bar{R}_1$ and $\bar{R}_2$, assign a spring constant μ_1 and μ_2 such that

$$\left.\begin{aligned}
R_1 &= (1 + \mu_1 T_1)\, R_1' \\
R_2 &= (1 + \mu_2 T_2)\, R_2'
\end{aligned}\right\} \tag{27}$$

where

$$\mu = \frac{1}{Ea}$$

and

E = Young's modules

a = line cross section area

APPENDIX B (Continued)

Where

$$\left.\begin{aligned} R_1 &= \left|\bar{R}_1\right| = \text{Actual cable length} \\ R_2 &= \left|\bar{R}_2\right| = \text{Actual cable length} \end{aligned}\right\} \qquad (28)$$

R_1', R_2' are unstressed cable lengths

T_1 , T_2 are cable tensions

Now

$$R_1^{\,2} = (x_1 + y_1 - z_1)^2 + (x_2 + y_2 - z_2)^2 + (x_3 + y_3 - z_3)^2 \qquad (29)$$

and

$$R_2^{\,2} = (u_1 - z_1)^2 + (u_2 - z_2)^2 + (u_3 - z_3)^2 \qquad (30)$$

From Equation (27)

$$T_1 = \frac{R_1 - R_1'}{\mu_1 \; R_1'} \qquad (31)$$

$$T_2 = \frac{R_2 - R_2'}{\mu_2 \; R_2'} \qquad (32)$$

The quantities, R_1' and R_2', will generally not be expressed as explicit functions of time, but will be specified indirectly by the time derivative

$$\dot{R}' = f\,(q_1, q_2, \ldots q_n) \qquad (33)$$

Where the q's are as yet undetermined variables such as tension, cable length, or time. It is assumed here that Equation (33) will yield to integration without difficulty.

APPENDIX B (Continued)

Now consider the equations of motion of the various system bodies relative to the point O and the ($\bar{i}_1$, $\bar{i}_2$, $\bar{i}_3$) triad. It is assumed here that the above-mentioned coordinate frame is moving in a circular orbit about a central attracting body (planet) with the vector $\bar{i}_1$ pointing in the direction of travel and the vector $\bar{i}_2$ pointing radially outward from the center of attraction. The vector $\bar{i}_3$ is oriented to complete the right hand triad. Since this frame is not Newtonian, the usual equations of motion must be modified to account for this. A point mass m having position coordinates x_1, x_2, x_3 in a reference frame as described above has the following equations of motion:(see Reference 6).

$$F_1 = m\left\{\ddot{x}_1 + 2\Omega\dot{x}_2\right\} \qquad \text{Force in } x_1 \text{ direction}$$

$$F_2 = m\left\{\ddot{x}_2 - 3\Omega^2 x_2 - 2\Omega\dot{x}_1\right\} \qquad \text{Force in } x_2 \text{ direction} \qquad (34)$$

$$F_3 = m\left\{\ddot{x}_3 + \Omega^2 x_3\right\} \qquad \text{Force in } x_3 \text{ direction}$$

Where Ω is the orbital angular velocity

$$\Omega = \sqrt{\frac{k}{L^3}} \qquad (35)$$

k = Gravitational constant of planet

L = Orbit radius

Now consider translational motion of the system bodies. The motion of the vehicle mass center is the same as that of a particle of equal mass experiencing a force equal to that of the summation of forces acting on the vehicle. The total outside force on the vehicle is due to cable tension and reaction jets.

Define

H_i = Jet thrust in the i direction

Now, the direction cosine of the vector $\bar{R}_1$, in the i direction is given as

$$-\left(\frac{x_i + y_i - z_i}{R_1}\right)$$

APPENDIX B (Continued)

The equations of motion of vehicle translation are now obtained from Equations (31), (34) and the above definitions. Thus,

$$\left.\begin{aligned} H_1 - \left(\frac{R_1 - R_1'}{\mu_1 R_1' R_1}\right)(x_1 + y_1 - z_1) &= m_o(\ddot{x}_1 + 2\Omega \dot{x}_2) \\ H_2 - \left(\frac{R_1 - R_1'}{\mu_1 R_1' R_1}\right)(x_2 + y_2 - z_2) &= m_o(\ddot{x}_2 - 3\Omega^2 x_2 - 2\Omega \dot{x}_1) \\ H_3 - \left(\frac{R_1 - R_1'}{\mu_1 R_1' R_1}\right)(x_3 + y_3 - z_3) &= m_o(\ddot{x}_3 + \Omega^2 x_3) \end{aligned}\right\} \quad (36)$$

The thrust components H_i are as yet undefined, but will probably be functions of y_i and z_i (and possibly $\dot{y}_i$ and $\dot{z}_i$).

The motion of the astronaut is determined by the tensions in two cables and by jet thrust having components F_i in the i directions. The direction cosines of the second cable are defined as

$$\frac{u_i - z_i}{R_2}$$

The nature of F_i is similar to that of H_i.

The equations of motion for the astronaut are now as follows:

$$\left.\begin{aligned} F_1 + \left(\frac{R_1 - R_1'}{\mu_1 R_1' R_1}\right)(x_1 + y_1 - z_1) + \left(\frac{R_2 - R_2'}{\mu_2 R_2' R_2}\right)(u_1 - z_1) &= \\ m_1(\ddot{z}_1 + 2\Omega \dot{z}_2) & \\ F_2 + \left(\frac{R_1 - R_1'}{\mu_1 R_1' R_1}\right)(x_2 + y_2 - z_2) + \left(\frac{R_2 - R_2'}{\mu_2 R_2' R_2}\right)(u_2 - z_2) &= \\ m_1(\ddot{z}_2 - 3\Omega^2 z_2 - 2\Omega \dot{z}_1) & \\ F_3 + \left(\frac{R_1 - R_1'}{\mu_1 R_1' R_1}\right)(x_3 + y_3 + z_3) + \left(\frac{R_2 - R_2'}{\mu_2 R_2' R_2}\right)(u_3 - z_3) &= \\ m_1(\ddot{z}_3 + \Omega^2 z_3) & \end{aligned}\right\} \quad (37)$$

APPENDIX B (Continued)

Similarly, the equations of motion for the anchor mass are as follows:

$$\left.\begin{aligned}
\left(\frac{R_2 - R_2'}{\mu_2 R_2' R_2}\right)(z_1 - u_1) &= m_2 (\ddot{u}_1 + 2\Omega \dot{u}_2) \\
\left(\frac{R_2 - R_2'}{\mu_2 R_2' R_2}\right)(z_2 - u_2) &= m_2 (\ddot{u}_2 - 3\Omega^2 u_2 - 2\Omega \dot{u}_1) \\
\left(\frac{R_2 - R_2'}{\mu_2 R_2' R_2}\right)(z_3 - u_3) &= m_2 (\ddot{u}_3 + \Omega^2 u_3)
\end{aligned}\right\} \tag{38}$$

It is noteworthy that no force other than cable tension is applied to the anchor mass.

Now consider the space vehicle rotational dynamics. The torque produced by the cable tension is given by

$$\text{Torque} = Q_1 \bar{k}_1 + Q_2 \bar{k}_2 + Q_3 \bar{k}_3 \tag{39}$$

Where

$$Q_i = \frac{T_1}{R_1} (\bar{k}_i \cdot \bar{P} \times \bar{R}_1) \tag{40}$$

The vector $\bar{P}$ is originally specified in terms of the unit vectors $(\bar{k}_1, \bar{k}_2, \bar{k}_3)$

$$\bar{P} = S_1 \bar{k}_1 + S_2 \bar{k}_2 + S_3 \bar{k}_3 \tag{41}$$

But $\bar{R}_1$ is expressed in the $(\bar{i}_1, \bar{i}_2, \bar{i}_3)$ system

$$\bar{R}_1 = r_1 \bar{i}_1 + r_2 \bar{i}_2 + r_3 \bar{i}_3 \tag{42}$$

Where

$$r_i = z_i - x_i - y_i \tag{43}$$

APPENDIX B (Continued)

Now, $\overline{R}_1$ may be written in the form

$$\overline{R}_1 = r_1' \, \overline{k}_1 + r_2' \, \overline{k}_2 + r_3' \, \overline{k}_3 \tag{44}$$

Where

$$\left[r'\right] = \left[B\right] \left[r\right] \tag{45}$$

As was shown earlier, $\left[B\right]$ is simply the transpose of $\left[A\right]$.

Now,

$$\begin{aligned} \overline{P} \times \overline{R}_1 = \overline{k} \, (S_2 \, r_3' - S_3 \, r_2') + \overline{k}_2 \, (S_3 \, r_1' - S_1 \, r_3') \\ + \overline{k}_3 \, (S_1 \, r_2' - S_2 \, r_1') \end{aligned} \tag{46}$$

So that

$$\left.\begin{aligned} Q_1 &= \frac{T_1}{R_1} (S_2 \, r_3' - S_3 \, r_2') \\ Q_2 &= \frac{T_1}{R_1} (S_3 \, r_1' - S_1 \, r_3') \\ Q_3 &= \frac{T_1}{R_1} (S_1 \, r_2' - S_2 \, r_1') \end{aligned}\right\} \tag{47}$$

The rotational dynamics are now written as

$$\left.\begin{aligned} I_1 \, \dot{\omega}_1 &= (I_2 - I_3) \omega_2 \, \omega_3 + Q_1 + G_1 \\ I_2 \, \dot{\omega}_2 &= (I_3 - I_1) \omega_1 \, \omega_3 + Q_2 + G_2 \\ I_3 \, \dot{\omega}_3 &= (I_1 - I_2) \omega_1 \, \omega_2 + Q_3 + G_3 \end{aligned}\right\} \tag{48}$$

APPENDIX B (Continued)

Where G_i is the applied torque due to jet reaction about $\bar{k}_i$. In general, G_i and H_i will not be independent.

Equations (10), (22), (26), (31), (32), (33), (36), (37), (38), (43), (45), (47), and (48) express the complete equations of motion of the system.

NOMENCLATURE

Symbol	Description
$\bar{i}_1$, $\bar{i}_2$, $\bar{i}_3$	Unit vectors of the coordinate system
$\bar{k}_1$, $\bar{k}_2$, $\bar{k}_3$	Unit vectors fixed in the vehicle
x_1, x_2, x_3	Coordinates of vehicle mass center
y_1, y_2, y_3	Components of vehicle lever arm in ($\bar{i}_1$, $\bar{i}_2$, $\bar{i}_3$) system
S_1, S_2, S_3	Components of vehicle lever arm in ($\bar{k}_1$, $\bar{k}_2$, $\bar{k}_3$) system
z_1, z_2, z_3	Coordinates of astronaut
u_1, u_2, u_3	Coordinates of anchor mass
m_o	Vehicle mass
m_1	Astronaut mass
m_2	Anchor mass
$\bar{R}_o$	Vehicle position vector
$\bar{R}_1$	Relative position vector between vehicle and astronaut
r_1, r_2, r_3	Components of $\bar{R}_1$ in ($\bar{i}_1$, $\bar{i}_2$, $\bar{i}_3$) system
r_1', r_2', r_3'	Components of $\bar{R}_1$ in ($\bar{k}_1$, $\bar{k}_2$, $\bar{k}_3$) system
$\bar{R}_2$	Relative position vector between astronaut and anchor mass
R_1, R_2	Magnitudes of $\bar{R}_1$ and $\bar{R}_2$
R_1', R_2'	Unstressed cable lengths
μ_1, μ_2	Cable spring constants

APPENDIX B (Continued)

Symbol	Description
T_1, T_2	Cable tensions
H_1, H_2, H_3	Thrust components applied to vehicle
F_1, F_2, F_3	Thrust components applied to astronaut
G_1, G_2, G_3	Outside torque components applied to vehicle
$\overline{P}$	Vehicle lever arm lever
Q_1, Q_2, Q_3	Vehicle torque components in ($\overline{k}_1$, $\overline{k}_2$, $\overline{k}_3$) system due to cable tension
ω_1, ω_2, ω_3	Vehicle angular velocity about principal axes
I_1, I_2, I_3	Principal moments of inertia
A	Transformation matrix from ($\overline{k}_1$, $\overline{k}_2$, $\overline{k}_3$) to ($\overline{i}_1$, $\overline{i}_2$, $\overline{i}_3$)
B	Transformation matrix from ($\overline{i}_1$, $\overline{i}_2$, $\overline{i}_3$) to ($\overline{k}_1$, $\overline{k}_2$, $\overline{k}_3$)
ϕ, θ, ψ	Eulerian angles of vehicle
Ω	Angular velocity of orbit

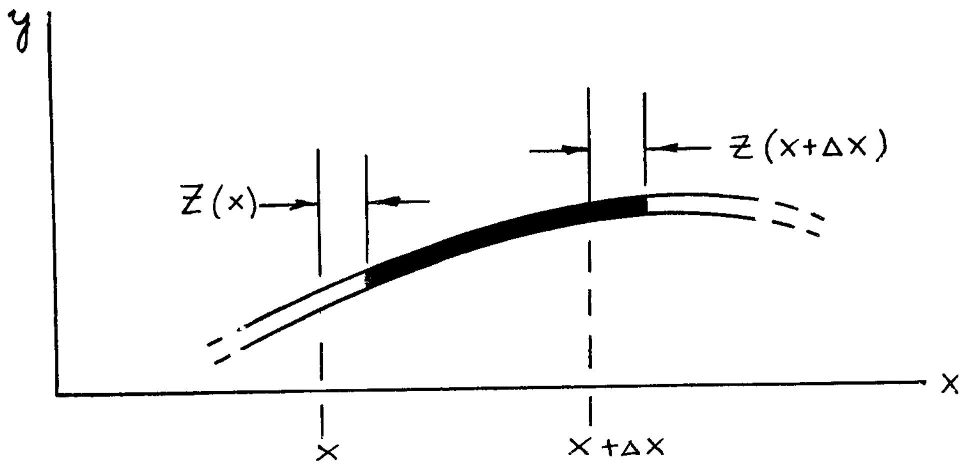
APPENDIX C

TETHER LINE VIBRATIONS

APPENDIX C

TETHER LINE VIBRATIONS

Consider a flexible line whose equilibrium position lies along the X-axis. Small displacements from the equilibrium as shown in Sketch C-1 are now of interest.



SKETCH C-1

When the slope of the line is small, the forces acting on the element of Sketch 1 are given as,

F_y = Net force in positive Y direction

F_x = Net force in positive X direction

Where

$$F_y = T(X + \Delta X) \left.\frac{\partial y}{\partial x}\right|_{X + \Delta X} - T(X) \left.\frac{\partial y}{\partial X}\right|_{X} \tag{1}$$

And

$$F_x = E \left\{ a(X + \Delta X) \left.\frac{\partial Z}{\partial X}\right|_{X + \Delta X} - a(X) \left.\frac{\partial Z}{\partial X}\right|_{X} \right\} \tag{2}$$

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APPENDIX C (Continued)

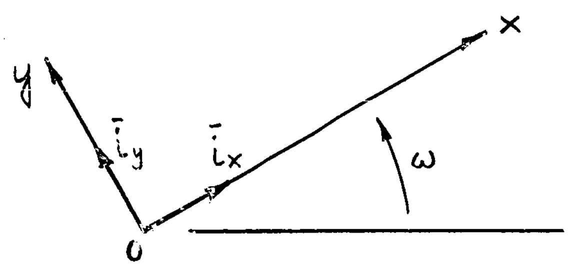
And in passing to the limit,

$$\frac{F_y}{dX} = T(X)\,\frac{\partial^2 y}{\partial X^2} + \frac{\partial T}{\partial X}\,\frac{\partial y}{\partial X} \qquad (3)$$

And

$$\frac{F_X}{dX} = E\left\{a(X)\,\frac{\partial^2 Z}{\partial X^2} + \frac{\partial a}{\partial X}\,\frac{\partial Z}{\partial X}\right\} \qquad (4)$$

Now consider a rotating coordinate system which pivots about its origin. See Sketch 2.



SKETCH C-2

Now it may be shown that the acceleration of a point on the X-axis is

$$a = \left\{\ddot{X} - X\left(\frac{\dot{y}}{X} + \omega\right)^2\right\}\bar{i}_X + \left\{\ddot{y} + 2\,\dot{X}\left(\frac{\dot{y}}{X} + \omega\right) + X\,\dot{\omega}\right\}\bar{i}_y \qquad (5)$$

The mass of the line element is given as

$$m = \rho\; a(X)\; dX$$

APPENDIX C (Continued)

So that when the proper components of Equation (5) are combined with Equations (3) and (4), the following equations result:

$$T(x) \frac{\partial^2 y}{\partial x^2} + \frac{\partial T}{\partial x} \frac{\partial y}{\partial x} = \rho\, a(x) \left\{ \frac{\partial^2 y}{\partial t^2} + Z \frac{\partial Z}{\partial t} \left(\frac{1}{x} \frac{\partial y}{\partial t} + \omega \right) + X \dot{\omega} \right\} \qquad (7)$$

$$E \left\{ a(x) \frac{\partial^2 Z}{\partial x^2} + \frac{\partial a}{\partial x} \frac{\partial Z}{\partial x} \right\} = \rho\, a\,(x) \left\{ \frac{\partial^2 Z}{\partial t^2} - X \left(\frac{1}{X} \frac{\partial y}{\partial t} + \omega \right)^2 \right\} \qquad (8)$$

In the above equations, the following substitutions have been made:

$$\dot{x} = \frac{\partial Z}{\partial t} \qquad (9)$$

$$\ddot{x} = \frac{\partial^2 Z}{\partial t^2}$$

It may be shown that the ratio of longitudinal to transverse wave velocity is approximately equal to the ratio of the modulus of elasticity of the line material to the nominal line stress. For all line materials under consideration, these wave velocities differ by several orders of magnitude. Therefore, the two vibrational modes are essentially dynamically uncoupled.

Equations (7) and (8) are now written as

$$T(x) \frac{\partial^2 y}{\partial x^2} + \frac{\partial T}{\partial x} \frac{\partial y}{\partial x} = \rho\, a\,(x) \left\{ \frac{\partial^2 y}{\partial t^2} + x\, \dot{\omega} \right\} \qquad (10)$$

$$E \left\{ a(x) \frac{\partial^2 Z}{\partial x^2} + \frac{\partial a}{\partial x} \frac{\partial Z}{\partial x} \right\} = \rho\, a\,(x) \left\{ \frac{\partial^2 Z}{\partial t^2} - \omega^2 \right\} \qquad (11)$$

APPENDIX C (Continued)

If in addition it is assumed

$$a(x) = \text{constant}$$

$$\omega = \text{constant}$$

The equations are further reduced to

$$T(x)\frac{\partial^2 y}{\partial x^2} + \frac{\partial T}{\partial x}\frac{\partial y}{\partial x} = \rho a \frac{\partial^2 y}{\partial t^2} \tag{12}$$

$$\frac{\partial^2 Z}{\partial x^2} = \frac{\rho}{E}\left\{\frac{\partial^2 Z}{\partial t^2} - \omega^2\right\} \tag{13}$$

It may also be assumed that

$$\frac{\partial y}{\partial x} \ll 1$$

$$\frac{\partial^2 Z}{\partial t^2} \gg \omega^2$$

So that Equations (12) and (13) reduce to

$$T(x)\frac{\partial^2 y}{\partial x^2} = \rho a \frac{\partial^2 y}{\partial t^2} \tag{14}$$

$$\frac{\partial^2 Z}{\partial x^2} = \frac{\rho}{E}\frac{\partial^2 Z}{\partial t^2} \tag{15}$$

APPENDIX C (Continued)

Equation (15) is recognized as the classical linear wave equation which offers no difficulty in its solution. Only Equation (14), which represents the transverse motion of the line is non-linear by reason of the non-constant tension coefficient T(x).

Now,

$$T(x) \approx T_L + \rho a \omega^2 \int_x^1 x dx = T_L + \frac{\rho a \omega^2}{2} (L^2 - x^2) \qquad (16)$$

where

T_L = Tension at X = L

or,

$$T(x) = To - kx^2 \qquad (17)$$

where

$$To = T_L + \frac{\rho a \omega^2}{2} L^2 = \text{Tension at } x = 0$$

$$k = \frac{\omega^2 \rho a}{2}$$

The approximate equation for transverse line motion is therefore given as

$$(To - kx^2) \frac{\partial^2 y}{\partial x^2} = \rho a \frac{\partial^2 y}{\partial t^2} \qquad (18)$$

Certain analytic solutions of Equations (18) may be obtained after the following normalizations are applied:

$$x = (To/k)^{1/2} \xi \qquad (19)$$

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APPENDIX C (Continued)

$$t = (\rho a/k)^{1/2}\,\theta \tag{20}$$

to yield

$$(1-\xi^2)\left(\frac{\partial^2 y}{\partial \xi^2}\right) = \left(\frac{\partial^2 y}{\partial \theta^2}\right) \tag{21}$$

A product solution of the form

$$y(\xi,\theta) = \left\{ f(\xi) \right\} \left\{ g(\theta) \right\} \tag{22}$$

is assumed to exist. Whence

$$\frac{(1-\xi^2)}{f}\left(\frac{d^2 f}{d\xi^2}\right) = \frac{1}{g}\left(\frac{d^2 g}{d\theta^2}\right) \tag{23}$$

Since, from Equation (22), f is a function of ξ only and g is a function of θ only, Equation (23) can only be satisfied if the quantities on either side of the equals sign are equal to a constant, namely, $-\beta^2$, the eigenvalue. (A negative sign is chosen to yield periodic solutions.) The following two ordinary differential equations are thus obtained:

$$\left(\frac{d^2 g}{d\theta^2}\right) + \beta^2 g = 0 \tag{24}$$

$$(1-\xi^2)\left(\frac{d^2 f}{d\xi^2}\right) + \beta^2 f = 0 \tag{25}$$

APPENDIX C (Continued)

The solutions of Equation (24) are given by

$$g = C_1 \sin \beta\theta + C_2 \cos \beta\theta \tag{26}$$

where C_1 and C_2 are arbitrary constants. The solutions of Equation (8) can be obtained by expressing f as a Maclaurin series in ξ and determining the constants of the series by substituting the series in Equation (25). However, except for the trivial case of β equal to zero, the solutions of Equation (25) have zeros at ξ equal to $\pm$ 1. To emphasize this point the solutions are expressed herein as

$$f = F\,(1-\xi^2) \tag{27}$$

and F is expressed as a Maclaurin series in ξ. Substituting Equation (27) in Equation (25) yields the following equation for F:

$$(1-\xi^2)\left(\frac{d^2F}{d\xi^2}\right) - 4\xi F + (\beta^2 - 2) F = 0 \tag{28}$$

Expressing F as a Maclaurin series in ξ

$$F = a_o + a_1 \xi + a_2\xi^2 + a_3\xi^3 + \ldots + a_n\xi^n + \ldots \tag{29}$$

Substituting Equation (29) in Equation (28) yields the following recurrence equation for the coefficients of the series:

$$a_n = \left\{1 - \frac{\beta^2}{n(n-1)}\right\} a_{n-2} \tag{30}$$

APPENDIX C (Continued)

From this result the solutions of Equation (29) are easily separated into an even function:

$$F_e = a_o + a_2\xi^2 + a_4\xi^4 + \ldots + a_n\xi^n + \ldots \quad \text{(n even)} \qquad (31)$$

and an odd function:

$$F_o = a_1\xi + a_3\xi^3 + a_5\xi^5 + \ldots + a_n\xi^n + \ldots \quad \text{(n odd)} \quad (32)$$

The convergence criterion for Equations (31) and (32) may be deduced as follows. For sufficiently large n, namely n much greater than β, the recurrence equation reduces to

$$a_n = a_{n-2} \qquad n \gg \beta \qquad (33)$$

Beyond such a sufficiently large value of n the terms of Equations (31) and (32) form two infinite geometric progressions each with a ratio of terms equal to ξ^2. Clearly then for convergence

$$\xi^2 < 1 \qquad (34)$$

These considerations also lead to an expression for the upper bounds on the errors associated with truncating the series expressions of Equations (31) and (32). Equation (30) indicates that for a sufficiently large value of n, namely $n(n-1)$ greater than β^2, and beyond, the terms of either series do not change sign. Also from Equation (30), under the same condition

$$/a_n/ < /a_{n-2}/ \qquad n(n-1) > \beta^2 \qquad (35)$$

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APPENDIX C (Continued)

Consequently the absolute magnitude of every term in the following series is greater than that of the corresponding term in Equation (31) or Equation (32) depending upon whether n is even or odd, respectively.

$$S_1 = a_n \xi^{n+2} + a_n \xi^{n+4} + a_n \xi^{n+6} + \ldots \tag{36}$$

Hence, the absolute magnitude of the error associated with truncating either Equation (31) or Equation (32) at the nth term is less than the absolute magnitude of S_1. Since Equation (36) is an infinite geometric progression and it is assumed that the converge criterion, Equation (34) is met, S_1 may be expressed in closed form as follows,

$$S_1 = \frac{a_n \xi^{n+2}}{1 - \xi^2} \tag{37}$$

A roughly similar approach yields lower bounds on the errors associated with truncating the series expressions of Equations (31) and (32). As before it is assumed that n(n-1) is greater than β^2. Then from Equation (30)

$$\frac{a_n}{a_{n-2}} < \frac{a_{n+2}}{a_n} < \frac{a_{n+4}}{a_{n+2}} < \ldots \tag{38}$$

Consequently the absolute magnitude of every term in the following series is less than that of the corresponding term in Equation (31) or Equation (32) depending upon whether n is even or odd, respectively.

$$S_2 = a_n \left(\frac{a_n}{a_{n-2}}\right) \xi^{n+2} + a_n \left(\frac{a_n}{a_{n-2}}\right)^2 \xi^{n+4} + a_n \left(\frac{a_n}{a_{n-2}}\right)^3 \xi^{n+6} + \ldots \tag{39}$$

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APPENDIX C (Continued)

Hence, the absolute magnitude of the error associated with truncating either Equation (31) or (32) at the nth term is greater than the absolute magnitude of S_2. Since Equation (39) is an infinite geometric progression, S_2 may be expressed in closed form as follows:

$$S_2 = \frac{a_n \xi^{n+2}}{\frac{a_{n-2}}{a_n} - \xi^2} \tag{40}$$

In computing values of F_e and F_o, S_2 should be added to the truncated series. The absolute magnitude of the error will then lie between zero and $(S_1 - S_2)$.

One special situation where the computation of solutions is particularly simple is the case

$$\beta^2 = \alpha(\alpha - 1) \tag{41}$$

where α is an integer. From the recurrence Equation (30), it is apparent that the series for F terminates at the $(\alpha-2)$ term. That is, for α an even integer F_e is a polynomial of order $(\alpha-2)$, and f_e is therefore a polynomial of order α. Similarly for α an odd integer, F_o and f_o are polynomials of order $(\alpha-2)$ and α, respectively. A few of these polynomials and their non-zero positive zeros are listed in Table C-1.

APPENDIX C (Continued)

Another special situation where the computation of solutions is particularly simple occurs when β^2 is much greater than n(n-1). In this case the recurrence Equation (13) reduces to:

$$a_n = \frac{-\beta^2}{n\,(n-1)}\, a_{n-2} \qquad \beta^2 \gg n(n-1) \tag{42}$$

the recurrence equation for the sine ($\beta\xi$) and cosine ($\beta\xi$) series. For any finite value of β the condition cannot, of course, be satisfied throughout the infinite series since n takes on all integer values from zero to infinity. However, this may be ignored if the truncation error becomes negligible while β^2 is still much greater than n(n-1). For the sine or cosine series the absolute magnitude of the truncation error is less than the absolute magnitude of the last term in the truncated series. As a result the truncation error can be shown to be negligible if the absolute magnitude of the last term is much less than unity. Thus, if for any value of n

$$\frac{(\beta x)^n}{n!} \ll 1 \tag{43}$$

and

$$\beta^2 \gg n\,(n-1) \tag{44}$$

then

$$F_e = C_3 \cos\,(\beta x) \tag{45}$$

and

$$F_o = C_4 \sin\,(\beta x) \tag{46}$$

where C_3 and C_4 are arbitrary constants.

APPENDIX C (Continued)

It is interesting to compare these results with the polynomial solutions. However, be careful in interpreting the comparison results as being representative of the solutions in general. In Table C-1 under each of the $(\beta\xi)$'s, corresponding to the zeros of the polynomials, are listed the $(\beta\xi)$'s corresponding to zeros of the cosine and sine (depending upon whether the polynomial is even or odd, respectively). It can be seen that in several cases the zeros of the polynomial solutions are predicted within several percent by the zeros of the appropriate approximate solution (either Equation (45) or Equation (46). Interestingly, conditions of Equations (43) and (44) can not be simultaneously satisfied by any value of n for these cases. However, for these cases the following less restrictive modification of conditions of Equations (43) and (44) can be satisfied.

$$\frac{(\beta x)^n}{n!} < 1 \qquad (47)$$

and

$$\beta^2 > n(n-1) \qquad (48)$$

Further, conditions of Equations (47) and (48) cannot be satisfied for those cases for which the zeros of the polynomial solutions are not accurately predicted by the approximate solutions. Thus, it appears that for the polynomial solutions at least, conditions of Equations (47) and (48) should be employed to determine the applicability of the approximate solutions given by Equations (45) and (46).

APPENDIX C (Continued)

NOMENCLATURE

Symbol	Description
y (x)	Lateral displacement of a line element from its equilibrium
Z (x)	Longitudinal displacement of a line element from its equilibrium position
T (x)	Line tension at x
a (x)	Line cross-section area
ρ	Line density
E	Modulus of elasticity
ξ	Normalized x
Θ	Normalized t
β	Eigenvalue

APPENDIX C (Continued)

Table C-1

Polynomial solutions of Equation (11) and Their Non-Zero Positive Zeros

$\beta^2 = 1.2$

$F = 1$

$\beta^2 = 2.3$

$F = \xi$

$\beta^2 = 3.4$

$F = (1-5\xi^2)$

$\xi = 0.44721$

$\beta\xi = 1.5491$

1.5708

$\beta^2 = 4.5$

$F = \xi\ (3-7\xi^2)$

$\xi = 0.65465$

$\beta\xi = 2.9277$

3.1416

$\beta^2 = 5.6$

$F = (1-14\xi^2 + 21\xi^4)$

$\xi = 0.28523,\ 0.76505$

$\beta\xi = 1.5623,\ 4.1904$

$1.5708,\ 4.7124$

$\beta^2 = 6.7$

$F = (5-30\xi^2 + 33\xi^4)$

$\xi = 0.46984,\ 0.83023$

$\beta\xi = 3.0449,\ 5.3805$

$3.1416,\ 6.2832$

APPENDIX D

DIGITAL COMPUTER PROGRAM

APPENDIX D

DIGITAL COMPUTER PROGRAM

I. GENERAL DISCUSSION

The Astronaut Retrieval Program is designed to simulate the motion of an orbiting system of three bodies joined by two elastic tether lines. One body, the space vehicle, is assumed to have a distributed mass and therefore has non-zero dimensions and moments of inertia. The other two bodies, the astronaut and the anchor, are simulated as point masses. The system therefore contains twelve degrees of freedom consistent with the constraints imposed by the connecting tethers. Provision is made for the reel-in and reel-out of the tether lines, so the program is particularly applicable to astronaut retrieval problems where more than two bodies are used. In the present version of the program variations in tether lengths are handled by describing the second order time derivatives of the unstressed cable lengths as functions of time, tension, unstressed cable length and its first derivative. Constants in these functions are input, allowing flexibility in the description of these functions; provision is also made for maintaining a constant tension on the astronaut tether line. If the desired reel-in reel-out philosophy cannot be described by these functions, another philosophy can be easily inserted with minor reprogramming.

The computer deck is written in FORTRAN IV and MAP, and it is set up for use with IBSYS on the 7040 after the leading and trailing control cards are added. Subroutine MCKLØK examines the computer interval timer for the printout of computer execution time. If the computer is not provided with an interval timer similar to that in the 7040, MCKLØK is replaced with a dummy subroutine having the same name. If IBSYS is not used, the $IBFTC cards are removed from the beginning of each subroutine deck; and MCKLØK, the only MAP subroutine, is replaced with a FORTRAN dummy.

Input and output for this program are in polar coordinates, but because the analytical description is simpler, the computations are handled in cartesian coordinates. The logic of the program largely consists of the solution of a set of simultaneous differential equations. The set includes eleven second-order and twelve first-order differential equations, which are converted into a system of 34 first order equations. Adams-Moulton fourpoint method is used in the numerical integration procedure. This method provides for an estimation of local truncation error and allows the user to vary the integration step size as needed. A maximum local error is input in the program, and step size is changed by a doubling-halving procedure to maximize computer efficiency. In high frequency oscillations, very small step sizes are required to retain sufficient accuracy. Since the Adams-Moulton method requires past information at constant integration step size, the Runge-Kutta fourth-order formula is used to "restart" whenever step size is changed.

The program deck consists of several subroutines which are described below:

Main Program

Contains basic program logic. All subroutines are called by the main program.

APPENDIX D (Continued)

Subroutine Input

Handles all input logic and conversion of input polar coordinates to cartesian coordinates.

Subroutine DIFEQ

Contains evaluation of all derivatives and related functions.

Subroutine AMRK

Contains numerical integration logic.

Subroutine Output

Handles conversion of cartesian coordinates to output polar form, together with all output logic.

Subroutine MCKLØK

Examines computer interval timer for printout of computer execution time.

Multiple cases can be stacked in the data deck, and they will be run successively. The first card of each run is the description header card of alphanumeric information, the first 72 columns of which will be printed at the top of each page of output. Nine or more data cards must follow the header card; data must be punched as indicated in the input form provided. All data is floating point and can be punched in either E or F format in fields of 12. The last parameter on card 9, CNUMBR, is the number of C(I) constants to be read in. Up to 100 C(I) constants can be used; they are to be used as additional input constants in case changes are made in the program.

II. TETHER LINE CONTROL

Simulation of Line Properties

The tether lines may be considered as the most critical elements of the entire system from a standpoint of their influence upon the dynamics of the remainder of the system as well as their provisions of the means by which retrieval is possible at all. For this reason capabilities have been designed into the computer program for the simulation of pertinent lumped mechanical properties of the lines - the elasticity, internal damping, and hysteresis.

The elasticity is so programmed that its contribution to line tension is a linear function of line elongation. For most cases under consideration this source of line load predominates all others, but it provides a purely

APPENDIX D (Continued)

conservative means of energy transfer within the tether. The contribution of damping and hysteresis to the line tension is usually considerably lower than that due to elastic stretch, but it provides the only means by which energy dissipation takes place internal to the tether. Specifically, the tension in each tether is expressed by an equation of the form

$$T = \frac{R - R'}{\mu R'} (1 + \epsilon)$$

where μ is as defined in Appendix B, and

where

$$\epsilon = k_1 \frac{(\dot{R} - \dot{R}')}{R} \quad \text{for} \quad \left| \frac{\dot{R} - \dot{R}'}{R} \right| \le \frac{k_2}{k_1}$$

$$\epsilon = \frac{\dot{R} - \dot{R}'}{\left| \dot{R} - \dot{R}' \right|} k_2 \quad \text{for} \quad \left| \frac{\dot{R} - \dot{R}'}{R} \right| \ge \frac{k_2}{k_1}$$

and

R = Actual (stressed) line length

R' = Unstressed line length

μ = Elastic factor

k_1 = Damping coefficient

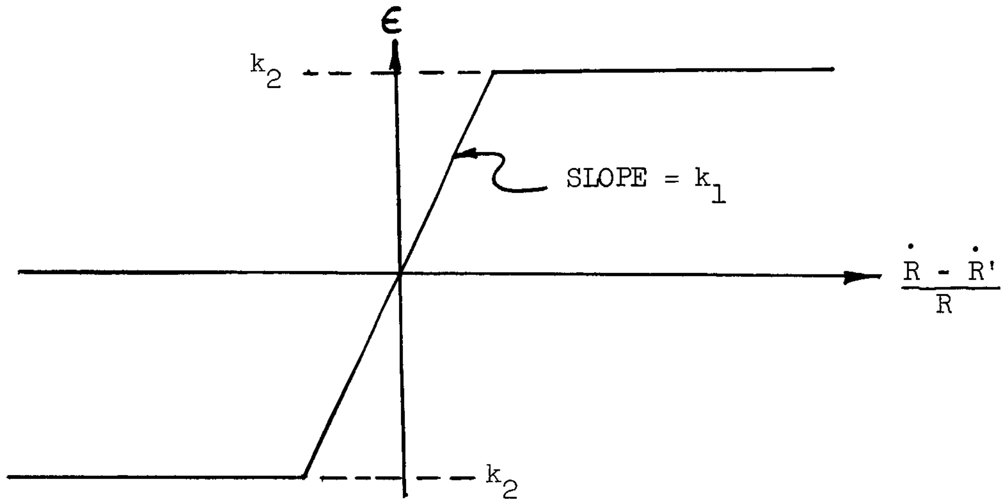
k_2 = Maximum hysteresis level.

The relation between ϵ and the line lengths is best shown graphically in Sketch D-1.

From the preceding equations it is seen that negative values of tension could occur when R' is greater than R. Since negative tension corresponds to compression which a flexible line cannot sustain, a provision is made in the program to set line tension equal to zero whenever a negative value is commanded.

However, the option of simulating an elastic connection capable of sustaining compression between the bodies is retained in the program also. This option is exercised by setting the input constant, C25, equal to zero. When this constant is non-zero, a purely flexible tether is simulated.

APPENDIX D (Continued)



SKETCH D-1

It is also seen that within a specific range, the only dissipative component of line tension is provided by linear damping of line stretch velocity. It is only when this linear range is exceeded that this restraining force assumes the character of true hysteresis. Since these values are selected as input constants, a wide range of control over the nature of the disspative forces is available. If velocity damping forces are to predominate, the saturation level of the damping term may be selected to be very large so that the appropriate parameters always remain within the linear region. If hysteresis forces are to predominate, the linear gain may be set arbitrarily large so that this portion of the line tension is forced to saturation by very small values of line stretch velocity. If desired, any degree of mixing of the two effects may be obtained by an appropriate selection of the two above mentioned input constants.

Regardless of the values of the input constants which determine damping and hysteresis, the line tension will always be uniquely determined as a function of its stretch and stretch velocity. This is due to the fact that no discontinuities exist in the expression defining tension and therefore there is no indeterminate 'set' or residual stretch in the line following a flexure.

A. TETHER LENGTH CONTROL

Given a particular system configuration, all aspects of a retrieval operation are completely determined when tether lengths are specified as functions of the system parameters. Because of its dominant role the generation of these quantities, the line lengths, is performed within a particular computer subroutine. This isolation permits removal and complete replacement with relative ease of any retrieval scheme which is expressible in terms of the system variables.

APPENDIX D (Continued)

Expressions for tether lengths could either be stated explicitly or in terms of higher order differential equations in which tether length (R) is the dependent variable. From the nature of the situation it has been established that R is most suitably expressed as the solution of a linear second-order differential equation whose coefficients are functions of the system parameters. Although this representation requires two additional integration steps, making it considerably more complex than an explicit expression, it offers definite advantages over the simpler method. Primarily, it avoids severe transients and possible discontinuities in certain of the important system parameters.

The analytic expressions for the differential equations defining the unstressed tether lengths of the astronaut and the anchor mass are

$$\ddot{R}_1' = C_4 + C_5 t + C_6 t^2 + \frac{C_7}{T_1 + C_{10}} + C_8 R_1' + C_9 \dot{R}_1'$$

and

$$\ddot{R}_2' = C_{14} + C_{15} t + C_{16} t^2 + \frac{C_7}{T_2 + C_{20}} + C_{18} R_2' + C_{19} \dot{R}_2'$$

respectively. The above equations are defined for the linear ranges of operation, however, certain parameter limitations are also imposed. In the first equation, t is not allowed to exceed the value of the input constant C_2, R_1' is not allowed to be less than C_3, and $\ddot{R}_1'$ is not allowed to exceed C_{11}. Stated mathematically,

$$t = \min\left[\text{time},\ C2\right] \qquad \text{or} \qquad t \le C_2$$

$$R_1' = \max\left[R_1',\ C3\right] \qquad\qquad R_1' \ge C_3$$

$$\left|\ddot{R}_1'\right| = \min\left[\left|\ddot{R}_1'\right|,\ C11\right]. \qquad\qquad \left|\ddot{R}_1'\right| \le C_{11}$$

Similar expressions exist for the anchor tether;

$$t = \min\left[\text{time},\ C_{12}\right]$$

$$R_2' = \min\left[R_2',\ C_{13}\right]$$

$$\left|\ddot{R}_2'\right| = \min\left[\left|\ddot{R}_2'\right|,\ C_{21}\right].$$

APPENDIX D (Continued)

When appropriate values are assigned to the input constants defining the astronaut and anchor tether line accelerations, numerous modes of operation may be commanded. Among these are the following:

a. Constant reel-in (or reel-out) rate. This is accomplished by setting all constants but C_4, C_5, C_8 and C_9 (or the corresponding ones in the case of the anchor mass) to zero. An alternate method is to set all constants but C_4 and C_9 to zero. With the first method mentioned above, the equation assumes the form:

$$\ddot{R}_1' = K\left(vt + D - R_1' - B\dot{R}_1'\right)$$

where

$$K = -C_8, \quad D = -\frac{C_4}{C_8}, \quad v = -\frac{C_5}{C_8} \quad \text{and } B = \frac{C_9}{C_8}$$

It is evident that the above equation defines a second order damped system which has a steady state velocity, v. The transient response of the system is determined by the selection of the input constants.

b. Constant line acceleration. Set all constants but C_6, C_8 and C_9 to zero so that the equation assumes the form:

$$\ddot{R}_1' = K\left(\frac{at^2}{z} - R_1' - B\dot{R}_1'\right)$$

where the command acceleration, a, is equal to $-\frac{C_6}{2C_8}$. The desired effect could be obtained directly by setting all constants but C_4 to zero, but the former method provides greater flexibility in the selection of transient conditions.

c. Constant power consumption. Set all constants but C_7 and C_9 to zero so that the equation is of the form:

$$\ddot{R}_1' = -K\left(\frac{P}{T_1} - \dot{R}_1'\right)$$

where P is the command power consumption rate. To avoid difficulties which would be encountered when T_1 approaches zero, the constant C_{10} may be given a small (non-zero) value.

d. Radial velocity proportional to separation distance. Set all constants but C_8 and C_9 equal to zero. The equation is now of the form:

$$\ddot{R}_1' = -K\left(kR_1' - \dot{R}_1'\right)$$

APPENDIX D (Continued)

e. Combinations of the above. Many of the effects of the above methods of line length manipulation may be mixed by appropriate selections of the input constants.

The astronaut tether length may also be controlled indirectly by means of a separate subroutine which assigns a constant value of tension to this line throughout the retrieval operation. When this subroutine is activated, the general equation describing the tether length by its second derivative is bypassed. The constant tension retrieval simulation is selected by assigning a non-zero value to the input constant, C_{26}. This constant represents the tension level imposed on the primary tether.

For most cases it is suggested that at least an approximate solution of the line length equation be obtained in the time domain to assist in the selection of values for input constants. An example is given below for the case of a constant reel-in velocity command.

$$\ddot{R}'_1 = C_4 + C_5 t + C_8 R'_1 + C_9 \dot{R}'_1$$

Since this equation is linear the methods of the Laplace transform are applicable, and the expression transforms to the following:

$$R'_1 = \frac{S^3 R'_{10} + S^2 (\dot{R}'_{10} - C_9 R'_{10}) + S C_4 + C_5}{S^2 (S^2 - C_9 S - C_8)}$$

Since the constants C_8 and C_9 must always be negative in this application, the expression is always stable. The pertinent properties of the expression are now listed as,

$$\text{Undamped natural frequency} = \sqrt{- C_8}$$

$$\text{Damping coefficient} = - \frac{C_9}{2\sqrt{- C_8}}$$

To avoid undesirable oscillations, the system should be at least critically damped, therefore

$$2\sqrt{- C_8} \geq - C_9.$$

APPENDIX D (Continued)

III. VEHICLE TRACKING PHILOSOPHY

In any real situation the orientation of the astronaut tether line relative to the vehicle must be controlled to avoid interference between them. If this were not the case, the tether would very likely wrap around the vehicle in the course of an attempted retrieval. To simulate real conditions, an astronaut tracking control system is contained in the computer program. The function of this system is to sense the angle of departure of the tether from the normal to the vehicle skin at the attachment point, and to provide a restoring torque in the proper plane when the departure angle exceeds a predetermined limit. This system is discussed in detail elsewhere in this report.

Entered as an input to the program are the components of a unit vector which is fixed in the vehicle and defines the direction of the local normal at the point of the tether attachment. As the retrieval operation progresses, it is necessary to monitor the angle between this normal vector and the astronaut tether. At no time must this angle be allowed to exceed a certain limiting value determined by the general vehicle geometry to avoid interference between the vehicle and the tether in the form of line abrasion, snagging or windup. Control over this relative angle is exercised by means of the vehicle attitude control system, using input signals obtained from the tether angle sensors. The basic control philosophy may be stated as follows:

a. No restoring torque is applied to the vehicle when the tether angle lies within a specified dead zone.

b. When the tether angle exceeds the dead zone, the restoring torque is proportional to the amount of the excess and to the rate of change of the angle in question.

c. The contribution to the restoring torque due to the angular rate is limited to a specified value (saturation of the damping term).

d. The first and second order gain constants are chosen to provide an undercritically damped system. This is done so that when a corrective torque is initiated, the tether angle will be caused to return to within the dead zone at a reduced rate rather than asymptotically approach the dead zone limit from the "outside". Since a dead zone exists, the general motion of the tether relative to the vehicle is expected to resemble a wandering of the tether angle within the conical angle defining the dead zone limits with short excursions across its boundaries.

e. When the magnitude and direction of the restoring torque vector has been determined, the vehicle is torqued about each of its three control axes in proportion to the projection of the basic torque vector on these axes. Precession of the vehicle will undoubtedly exist in the general case, but it is not considered an important factor. The vehicle angular velocity will always be small when the system is performing well and these precessional effects will be correspondingly small. In addition, no great accuracy is required of the vehicle in tracking the astronaut so slight irregularities may be disregarded.

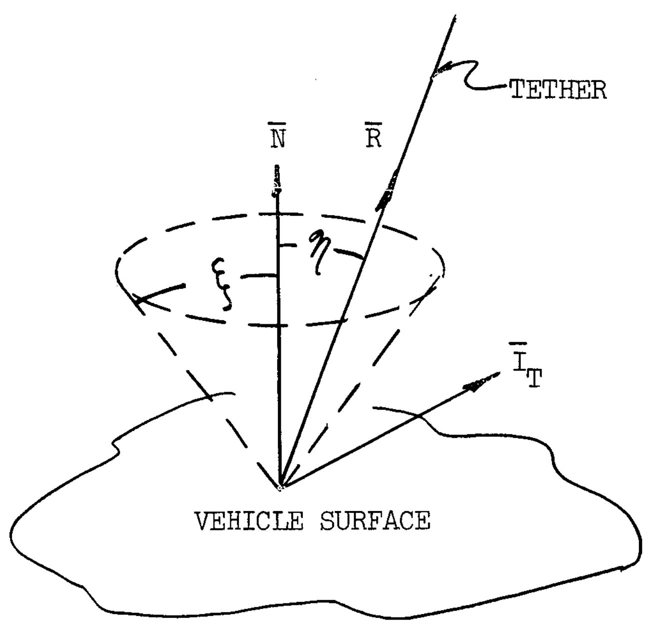
APPENDIX D (Continued)

The presence of an attitude control system is necessary to a successful retrieval operation, but otherwise exerts little influence on the nature of overall system behavior. For this reason, it is not expected that major modifications will be made to this portion of the program. The first and second order gain constants as well as the dead zone and saturation values, however, are specified as input parameters.

Mathematically stated, the command torque vector applied to the vehicle to provide the required attitude control is

$$\overline{G} = \overline{I}_T \left\{ K_1 (\eta - \xi) + K_2 \dot{\eta} \right\} \quad \text{when } \eta \geq \xi$$

$$\overline{G} = 0 \quad \text{otherwise.}$$



SKETCH D-2

$\overline{I}_T$ is the unit vector normal to the plane of the unit vectors $\overline{N}$ and $\overline{R}$ and has the sense of $\overline{N} \times \overline{R}$. Specifically,

$$I_T = \beta_1 \overline{k}_1 + \beta_2 \overline{k}_2 + \beta_3 \overline{k}_3$$

APPENDIX D (Continued)

where the $\bar{k}_i$ are unit vectors along the vehicle axes and β_i are the direction cosines of $\bar{I}_T$ with these axes. The values of the β_i are calculated by applying the following relation to each axis:

$$\beta_i = \frac{\bar{k}_i \cdot \bar{N} \times \bar{R}}{\left| \bar{N} \times R \right|}$$

The three applied torque components are therefore given as

$$G_1 = \beta_1 \left[K_1 (\eta - \xi) + K_2 \dot{\eta} \right]$$

$$G_2 = \beta_2 \left[K_1 (\eta - \xi) + K_2 \dot{\eta} \right]$$

$$G_3 = \beta_3 \left[K_1 (\eta - \xi) + K_2 \dot{\eta} \right]$$

The value of $\dot{\eta}$ is determined within the program by means of an approximate differentiation technique. This calculated value is used whenever it does not exceed a given input constant, C_1, otherwise this constant is substituted for the value. This condition is no more than a saturation of the system damping.

The approximate dynamics of the tracking system may be derived from the following linear equation.

$$G_i = I_i \ddot{\psi} = -\beta_i (K_1 \psi + K_2 \dot{\psi})$$

where

$$\psi = \eta - \xi \text{ (error signal).}$$

Taking Laplace transforms yields

$$\left(\frac{I}{\beta} S^2 + K_2 S + K_1\right) \mathcal{L}\{\psi\} = \left(\frac{I}{\beta} - K_2\right) \psi_o + \frac{I}{\beta} S \dot{\psi}_o$$

where the subscripts, i, have been removed for convenience.

APPENDIX D (Continued)

The solution of the above equation involves either sinusoids or exponentials, depending upon the nature of the coefficients of the characteristic equation,

$$\frac{I}{\beta} S^2 + K_2 S + K_1$$

The undamped natural frequency is given as

$$\omega_o = \sqrt{\frac{K_1 \beta}{I}}$$

and the damping ratio is

$$\zeta = \frac{K_2}{2} \sqrt{\frac{\beta}{K_1 I}}$$

It must be remembered that the linear equations defining the system are valid only for that time during which the angular deadband, ξ, is exceeded. Within the deadband, the vehicle may be expected to drift with a nearly constant angular velocity.

With regard to the nature of the vehicle motion induced by the tracking system, it is desirable to select the system constants so that this motion is oscillatory. If this condition were not provided, the system would be overcritically damped and the vehicle position would asymptotically approach the deadband from the higher values of error angle. Since a relative angular drift is almost certainly imposed on the system during retrieval, there will exist a constant tendency of the vehicle to exceed its deadband. A non-oscillatory system would probably require continuous thrusting to compensate for this tendency while an underdamped system would at least assure intermittent quiescent periods.

The time domain solution of the underdamped system is given as

$$\psi = \psi_o e^{-at} \sin \omega t$$

where

$$a = \frac{K_2 \beta}{2I}$$

$$\omega = \frac{\beta}{2I} \sqrt{\frac{4 K_1 I}{\beta} - K_2^2}$$

APPENDIX D (Continued)

It is generally desirable to damp the system motion quickly but still retain a sufficiently high angular velocity on the first return oscillation to overcome any existing drift velocity and drive the vehicle angular position well into its deadband.

In a truly linear system, the maximum velocity experienced in any particular half cycle is reduced from that of the previous half cycle by a factor, ρ. Therefore, the vehicle angular velocity upon returning to the deadband is ρ times the angular velocity with which it entered. It may be shown that

$$\ln \rho = - \frac{K_2 \pi}{\sqrt{\dfrac{4 K_1 I}{\beta} - K_2^2}}$$

Generally, β is assumed equal to unity.

When K_1 and ρ have been selected the value of K_2 is then established using the following relationship:

$$K_2 = 2 \sqrt{\frac{I (\ln \rho)^2}{\pi^2 + (\ln \rho)^2}}$$

A value of $\rho = 0.4$ has been shown to give suitable results under most conditions.

IV. DATA HANDLING

Input and Output

Due to the large number of data associated with a computer program of this type, a special effort was made to maintain both the input and output formats as easily readable as possible. Many large computer output displays often induce fatigue and weariness in the reader when large numbers of runs are analyzed, or even scanned. To prevent this occupational hazard, extra space was allotted in the output format to clearly identify and assemble all related parameters in proper groupings.

Parameter Identifications

The input format provides for the following parameters to be introduced into the program:

1. Vehicle attitude
2. Vehicle position

APPENDIX D (Continued)

3. Astronaut position
4. Anchor mass position
5. Vehicle linear velocity
6. Astronaut velocity
7. Anchor mass velocity
8. Vehicle rotational velocity
9. Tether attach point on vehicle relative to vehicle mass center
10. Polar moments of inertia of the vehicle
11. Local vertical to vehicle skin at point of tether attachment
12. Vehicle mass
13. Astronaut mass
14. Anchor mass
15. Tether line elastic modulus
16. Orbit altitude
17. Vehicle attitude control system gain constants
18. Tether line elasticity
19. Tether hysteresis and damping
20. Line accelerations during anchor deployment and astronaut retrieval
21. All necessary initial conditions.

The output format provides for the following parameters to be printed:

1. Vehicle position
2. Astronaut position
3. Anchor mass position relative to astronaut
4. Anchor mass position relative to vehicle
5. Vehicle linear velocity
6. Astronaut velocity
7. Anchor mass velocity
8. Vehicle angular velocity
9. Vehicle attitude
10. Angle between tether line and vehicle
11. Tether line tensions
12. Normalized angular momentum of astronaut
13. Normalized angular momentum of anchor mass
14. Tether reel-in (or reel-out) rates
15. Torque required for attitude control

All positions, linear velocities and normalized angular momentums in both the input and output are given in terms of their spherical coordinates (magnitude, azimuth angle, elevation angle). This is done purely as a convenience in reading and interpreting the computer results. These coordinates were selected because they are basically those sensed directly by observers at the vehicle and astronaut locations, and because they yield the magnitudes of the quantities in question without additional manipulation. Although the equations of motion are integrated most easily in terms of cartesian coordinates within the program itself, they are rather awkward to deal with directly in the output and are converted into spherical coordinates for the above stated reasons.

APPENDIX D (Continued)

In both the input and output, the vehicle position is given in terms of its coordinates relative to the origin of the orbiting reference frame. The astronaut position, however, is given in both cases in terms of its coordinates relative to its attachment point on the vehicle skin. Similarly, the anchor mass position is given in relation to that of the astronaut. The output also contains the coordinates of the anchor mass relative to the vehicle attach point. Although this is a redundant output, it provides a convenient monitor of the anchor mass as observed from the vehicle.

All linear velocities in both the input and output are given in terms of their coordinates relative to the orbiting reference frame. The vehicle angular velocity is given in terms of its angular velocity components along the vehicle principal axes.

Vehicle attitude is given in terms of a modified Eulerian angle set. This is elaborated upon elsewhere.

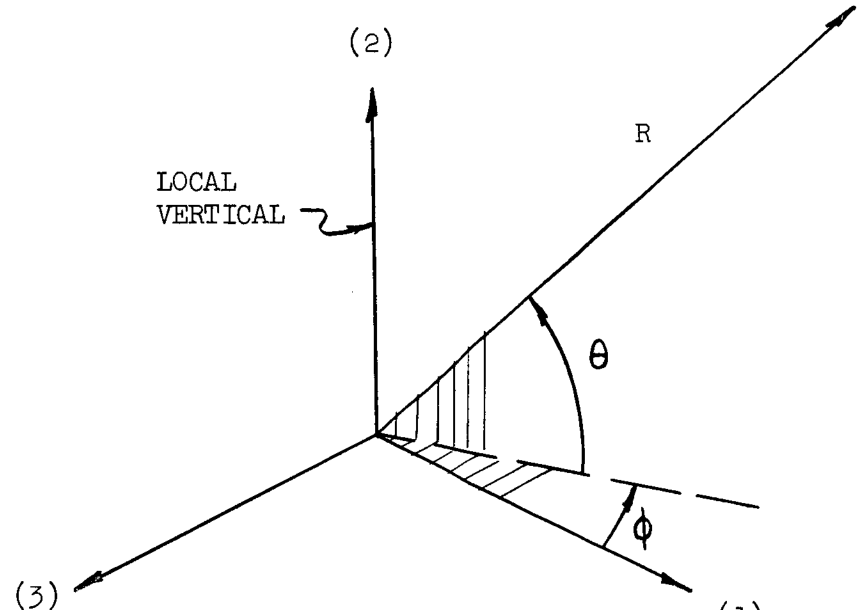
The normalized angular momentums of the astronaut and anchor mass are defined as the angular momentums per unit mass and are given relative to the system mass center. These outputs serve several functions. In addition to providing a measure of the magnitude of the component angular momentum in question, they define the planes of motion of the bodies under consideration. Small perturbations in the body motions are made readily apparent through this output.

It is to be noted that the velocities and momentums provided in the printout are not taken relative to inertial space and are therefore not "absolutes" in the usual sense. These measurements correspond with those made by an observer stationed in the orbital reference frame, and are for most applications more useful than those made relative to inertial coordinates.

Coordinate System Conventions

As was previously discussed, all pertinent parameters are given in terms of their spherical coordinates. The relation between these coordinates and the cartesian coordinates in which the program operates is best illustrated by Sketch D-3. The Cartesian axes are identified by subscripts 1, 2, and 3, where the 1-axis points in the direction of orbital motion, the 2-axis points upward in the direction of the local vertical, and the 3-axis is mutually perpendicular to the other two to complete a right handed triad.

APPENDIX D (Continued)



SKETCH D-3

The spherical coordinates bear the following relation to the cartesian coordinates: the angle Θ is the elevation of the radius vector R from the 1 - 3 plane and the angle Φ, is the counter-clockwise rotation from the 1-axis of the projection of R on the 1 - 3 plane (as viewed from above). The angular ranges employed in the input and output formats are as follows:

$$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$0 \leq \Phi < 2\pi$$

The vehicle attitude is defined in terms of modified Eulerian angles, $(\Phi\, \theta\, \Psi)$ as shown in Sketch D-4. The unit vector triad (i_1, i_2, i_3) is oriented relative to the orbiting coordinate system while the (k_1, k_2, k_3) triad is fixed in the vehicle.

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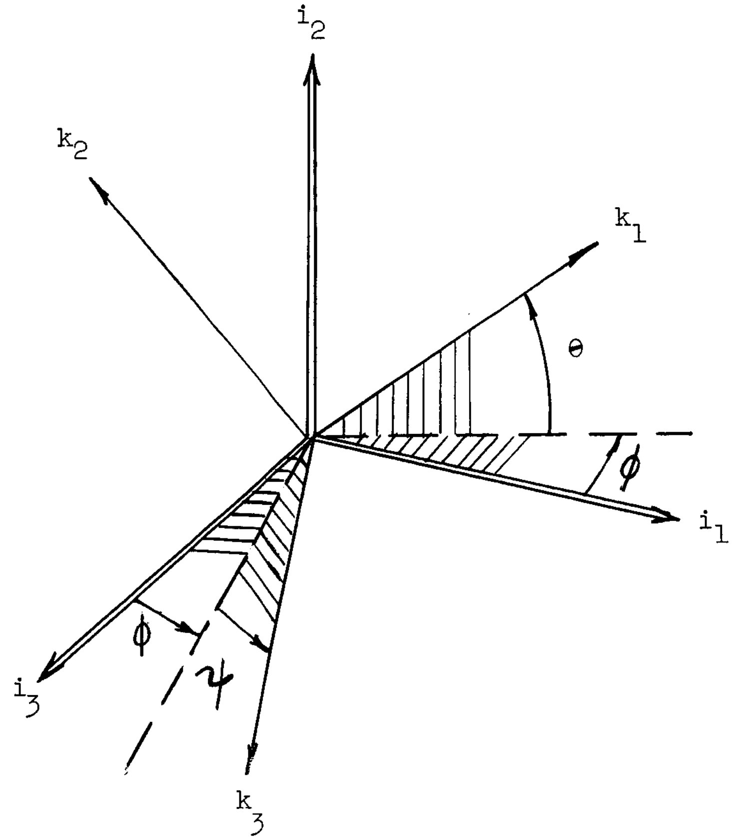
APPENDIX D (Continued)



SKETCH D-4

The relation between the (i_1, i_2, i_3) and (k_1, k_2, k_3) triads is best given by the rotations required to achieve the configurations shown in Sketch 4 assuming the triads are initially coincident:

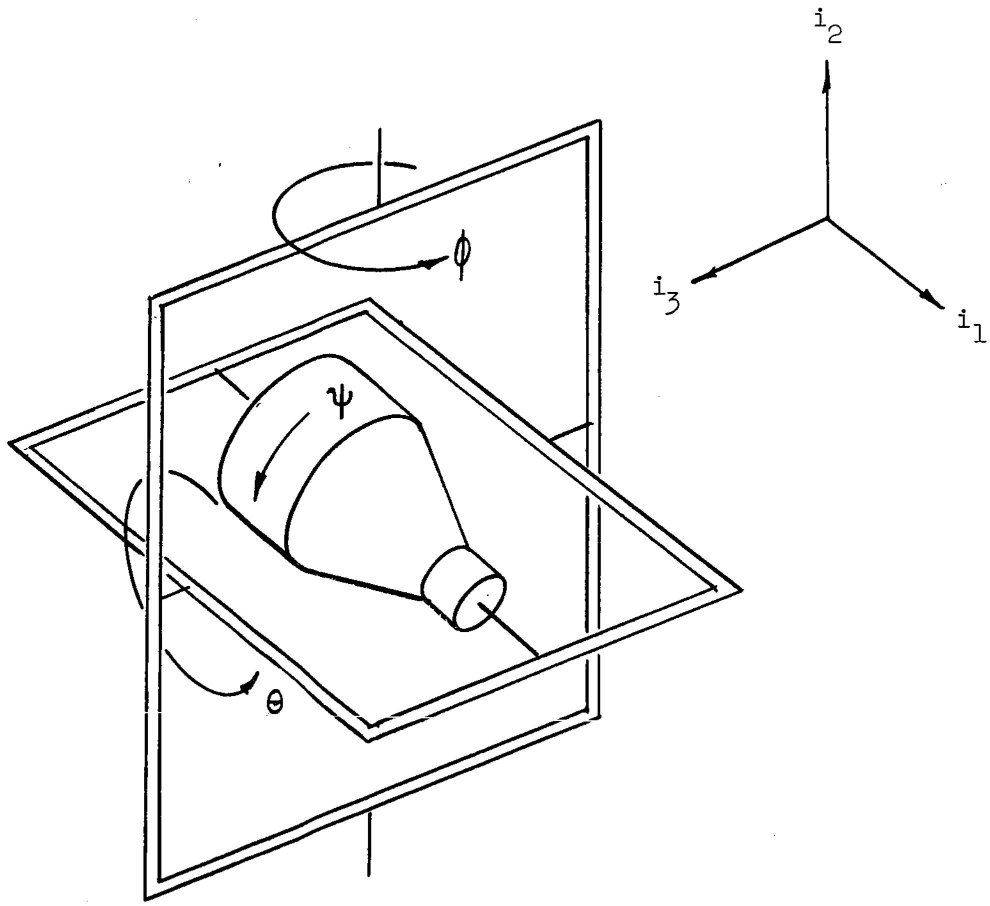
1. Rotate the k-triad about k_2 through an angle Φ.

2. Rotate the k-triad about the new k_3 direction through an angle θ.

3. Rotate the k-triad about the new k_1 direction through an angle ψ.

The angular motion described above is equivalent to that obtained if the vehicle were suspended in a gimballed cage whose elements are rotated through the Euler angles in question. This model is shown in Sketch D-5.

APPENDIX D (Continued)



SKETCH D-5

The above defined angular coordinates (Φ, θ, Ψ) are seen to be different from Eulerian angles as established in most of the classical works. The modification is incorporated here because of its similarity to the spherical coordinate convention in use. It may easily be shown that the angles θ and Φ have identical meanings when used as either Euler angles or spherical coordinates. The angular ranges employed in the input and output formats are as follows:

$$-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

$$0 \leq \Phi < 2\pi$$

$$0 \leq \Psi < 2\pi$$

APPENDIX D (Continued)

Dimensional Units

The selection of units was made entirely on the basis of convenience to the program operator. Within the program itself, there exists only one set of dimensions, the pound, foot, second, radian system. Wherever a different set of units is deemed desirable for either the input or output displays, the appropriate conversion is made within the computer program. The dimensions of the various input/output parameters are listed below:

1.	Length	Always given in feet, with the exception of orbit altitude which is in statute miles.
2.	Mass	Not introduced as such - pound weight used instead.
3.	Time	Independent variable of program - printed out in seconds.
4.	Force	Used for line tension - pound force
5.	Torque	Pound feet
6.	Linear Velocity	Feet/second
7.	Moment of inertia	Slug feet^2
8.	Normalized	Feet^2/second
9.	Angles	Degrees
10.	Angular rates	Radians/second

APPENDIX D (Continued)

GLOSSARY OF TERMS USED IN COMPUTER PROGRAM

Input	Description
TIN	Initial time
DTIN	Minimum integration interval
DTPRIN	Printout interval
TMAX	Maximum run time - run is terminated when reached
RMIN	Minimum primary tether length - run is terminated when reached
H	Orbit altitude
X1	ξ tracking system angular deadband
AK1	K_1 tracking system first order gain
AK2	K_2 tracking system second order gain
THEA	θ_A Modified vehicle Euler angles
PHIA	Φ_A
PSIA	Ψ_A
THEO	θ_o Vehicle spherical coordinates
PHIO	Φ_o
RO	R_o
THE1	θ_1 Astronaut spherical coordinates relative to tether attach point on vehicle
PHI1	Φ_1
R	R_1
THE2	θ_2 Anchor mass spherical coordinates relative to astronaut
PHI2	Φ_2
RR	R_2
THEVO	θ_{vo} Vehicle velocity spherical coordinates
PHIVO	Φ_{vo}
RVO	R_{vo}
THEV1	θ_{v1} Astronaut velocity spherical coordinates
PHIV1	Φ_{v1}
RV1	R_{v1}
THEV2	θ_{v2} Anchor velocity spherical coordinates
PHIV2	Φ_{v2}
RV2	R_{v2}

APPENDIX D (Continued)

GLOSSARY (Continued)

Input		Description
OM1	ω_1	Vehicle angular rates about its principal axes
OM2	ω_2	
OM3	ω_3	
S1	S_1	Tether attach point coordinates relative to vehicle principal axes
S2	S_2	
S3	S_3	
AI1	I_1	Vehicle principal moments of inertia
AI2	I_2	
AI3	I_3	
AN1	N_1	Unit normal vector to vehicle at tether attach point (components taken along vehicle principal axis)
AN2	N_2	
AN3	N_3	
WO	W_o	Vehicle weight
W1	W_1	Astronaut weight
W2	W_2	Anchor weight
AMV1	μ_1	Elastic factor of astronaut tether
AMV2	μ_2	Elastic factor of anchor tether
CNUMBER		Integer indicating number of input constants to follow
C1		Upper bound for $\dot{\eta}$
C2 to C11		Parameters defining astronaut retrieval method defined elsewhere
C12 to C21		Parameters defining anchor mass deployment method
C22		k_1 tether line damping rate
C23		k_2 maximum hysteresis level
C24		Maximum allowable integration error
C25		Line slack command
C26		Command for constant tension in astronaut tether

Output	Description
TIME	Simulation time
THEA	Modified vehicle Euler angles
PHIA	
PSIA	

APPENDIX D (Continued)

GLOSSARY (Continued)

Output		Description
T1		Astronaut line tension
T2		Anchor line tension
OM1		Same as in input
OM2		Same as in input
OM3		Same as in input
ETA	η	Angle between astronaut tether and normal to vehicle surface
RDOT	$\dot{R}_1$	Rate of change of separation between vehicle and astronaut
RRDOT	$\dot{R}_2$	Rate of change of separation between astronaut and anchor mass
G1	G_1	Attitude control torque applied to vehicle
G2	G_2	Attitude control torque applied to vehicle
G3	G_3	Attitude control torque applied to vehicle
VEHICLE		Position vector of vehicle relative to origin of coordinates
ASTRONAUT		Position vector of astronaut relative to tether attach point on vehicle
ANK1		Position vector of anchor relative to astronaut
ANK2		Position vector of anchor relative to vehicle
VEH VEL		Velocity vector of vehicle
ASTR VEL		Velocity vector of astronaut
ANK1 VEL		Velocity vector of anchor mass
ANG-MOM1		Specific angular momentum vector of astronaut
ANG-MOM2		Specific angular momentum vector of anchor

APPENDIX E

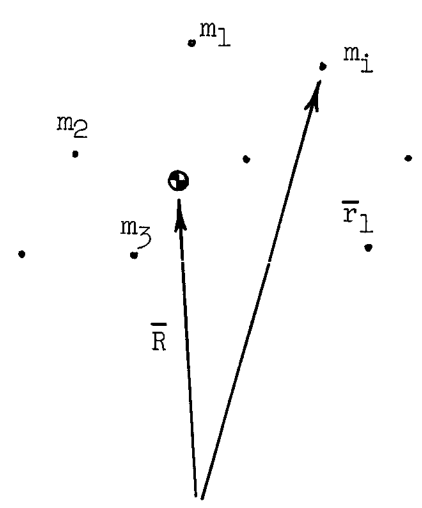
GRAVITY GRADIENT EFFECTS ON AN ORBITING SYSTEM OF CONNECTED MASSES

APPENDIX E

GRAVITY GRADIENT EFFECTS ON AN ORBITING SYSTEM OF CONNECTED MASSES

Excluding the motion of the system under consideration, the only true forces external to an orbital system are environmental. Accelerations such as those of Coriolis and Euler are in reality not forces and cannot change the angular momentum of the system.

When the system has sufficient altitude, aerodynamic forces vanish and the only environmental forces of any consequence are those due to the gravitational force gradient. The first order effects of this phenomenon are developed below. Consider a system of N particles having a mass center designated by the vector $\overline{R}$ originating from the center of attraction of the central body.



SKETCH E-1

Define

$\overline{r}_i$ = Position vector of i^{th} mass particle

k = Gravitational constant of central body

$\overline{T}_i$ = Force moment of m_i about the mass center

Now

$$\overline{T}_i = (\overline{r}_i - \overline{R}) \times \left(\frac{k\, \overline{r}_i\, m_i}{r_i^3} \right) \tag{1}$$

APPENDIX E (Continued)

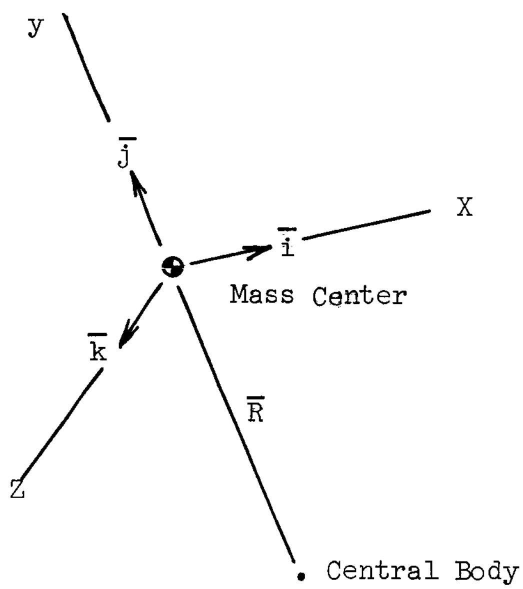
Where

$$r_i = \left| \bar{r}_i \right|$$

Since $\bar{r}_i \times \bar{r}_i = 0$, Equation (1) reduces to

$$\bar{T}_i = \frac{k}{r_i^3} \bar{R} \times \bar{r}_i m_i \tag{2}$$

Now define a coordinate system centered about the mass center as shown in Sketch E-2.



SKETCH E-2

The unit vector $\bar{j}$ is colinear with $\bar{R}$ while $\bar{i}$ and $\bar{k}$ are normal to $\bar{R}$. Now the local position vectors may be written as

$$\bar{r}_i = \bar{R} + X_i \bar{i} + y_i \bar{j} + Z_i \bar{k} \tag{3}$$

and

$$\bar{R} \times \bar{r}_i = R (\bar{i} Z_i - \bar{k} X_i) \tag{4}$$

Where $R = \left| \bar{R} \right|$

APPENDIX E (Continued)

The total torque about the mass center is given as

$$\sum_i \overline{T}_i = \tilde{T} = k\,R \sum_i \left(\frac{\bar{i}\,Z_i - \bar{k}\,X_i}{r_i^{\,3}}\right) m_i \tag{5}$$

Since X, y, and Z are very small compared with r_i

$$r_i \approx R + y_i \tag{6}$$

and

$$\frac{1}{r_i^{\,3}} = \frac{1}{(R + y_i)^3} \approx \frac{1}{R^3}\left(1 - 3\,\frac{y_i}{R}\right) \tag{7}$$

so that

$$\tilde{T} = \frac{k}{R^2} \sum_i \left(1 - 3\,\frac{y_i}{R}\right)(\bar{i}\,Z_i - \bar{k}\,X_i)\,m_i \tag{8}$$

Equation (8) may be expanded to

$$\tilde{T} = \frac{k}{R^2} \sum_i (\bar{i}\,Z_i - \bar{k}\,X_i)\,m_i - \frac{3k}{R^3} \sum_i y_i\,(\bar{i}\,Z_i - \bar{k}\,X_i)\,m_i \tag{9}$$

By definition of the mass center

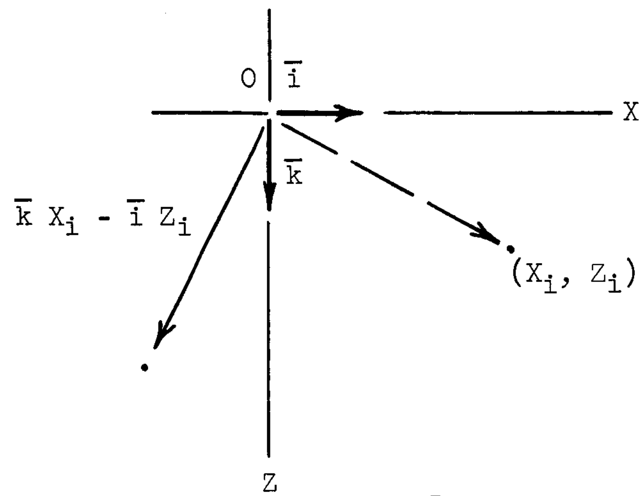
$$\sum_i X_i\,m_i = \sum_i Z_i\,m_i = 0$$

Therefore Equation (9) becomes

$$\tilde{T} = \frac{3k}{R^3} \sum_i y_i\,(\bar{k}\,X_i - \bar{i}\,Z_i)\,m_i \tag{10}$$

APPENDIX E (Continued)

It is now of interest to consider the plane of $\tilde{T}$. From Equation (10), it is seen that $\tilde{T}$ lies in the plane normal to $\bar{j}$. This is depicted in Sketch E-3.



SKETCH E-3

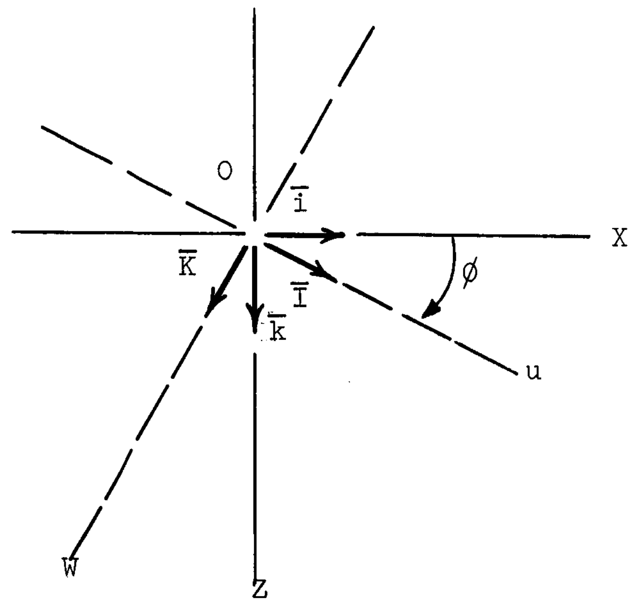
It is seen that the vector in the parentheses under the summation sign in Equation (10) is equivalent to a clockwise rotation through a right angle of the projection of the position vector on the X-Z plane. Therefore $\tilde{T}$ may be calculated by the following expression.

$$\tilde{T}' = \frac{3k}{R^3} \sum_i y_i \left(\bar{i}\, X_i + \bar{k}\, Z_i\right) m_i \tag{11}$$

and then rotated to establish the proper direction.

It is convenient to consider a rotation of the original coordinate system about the y axis as shown in Sketch E-4.

APPENDIX E (Continued)



SKETCH E-4

Since y_i does not change in this transformation, and the expression in the parentheses of Equation (11) merely represents a radius vector in the plane of Sketch 4, the equation may now be written as

$$\tilde{T}' = \frac{3k}{R^3} \sum_i y_i \left(\bar{I}\, u_i + \bar{K}\, W_i\right) m_i \qquad (12)$$

The new coordinate system is rotated through an angle ϕ, until the u-axis is colinear with $\tilde{T}'$. Now, Equation (12) is expanded to yield

$$\tilde{T}' = \frac{3k}{R^3}\, \bar{I} \sum_i y_i\, u_i\, m_i + \frac{3k}{R^3}\, \bar{K} \sum_i y_i\, W_i\, m_i \qquad (13)$$

Since $\tilde{T}'$ has no component in the $\bar{K}$ direction, the second term on the right of Equation (13) vanishes and

$$\tilde{T}' = \frac{3k}{R^3}\, \bar{I} \sum_i y_i\, u_i\, m_i \qquad (14)$$

To restore the torque vector to its proper orientation, it is rotated clockwise through a right angle. Therefore

APPENDIX E (Continued)

$$\tilde{T}' = \frac{3k}{R^3} \bar{K} \sum_i y_i u_i m_i \tag{15}$$

It is a simple matter to show that the magnitude of $\sum_i y_i u_i m_i$ assumes its maximum value as a function of ϕ when the u-axis is colinear with $\tilde{T}'$. Therefore, the gravity gradient torque vector is normal to the vertical plane on which the product of inertia $\sum_i y_i u_i m_i$ of the constituent particles assumes its maximum. When the system mass is distributed on a line, the maximum torque is experienced when the line is inclined at 45° to the vertical. In this case, $u_i = y_i$ and $\ell_i^2 = 2 y_i^2$, where ℓ_i is the distance of m_i from the origin. Equation (15) now becomes

$$\tilde{T} = \frac{3k}{R^3} \sum_i y_i^2 m_i = \frac{3}{2} \frac{k}{R^3} \sum_i \ell_i^2 m_i \tag{16}$$

The unit vector, $\bar{K}$, is omitted from the above expressions since only the magnitude of the torque vector is of concern at this time.

When the system is composed of only two masses, then

$$\ell_1 m_1 = - \ell_2 m_2$$

and

$$\tilde{T} = \frac{3}{2} \frac{k}{R^3} \left\{ \ell_1 (\ell_1 m_1) - \ell_2 (-\ell_1 m_1) \right\} = \frac{3}{2} \frac{k}{R^3} (\ell_1 m_1)(\ell_1 + \ell_2) \tag{17}$$

It may be shown that

$$m_1 \ell_1 = \frac{m_1 m_2}{m_1 + m_2} L \tag{18}$$

Where

$$L = \ell_1 + \ell_2$$

APPENDIX E (Continued)

Now define

$$m_2 = c\, m_1$$

so that

$$\tilde{T} = \frac{3}{2} \frac{k}{R^3} \left(\frac{c}{c+1}\right) m_1 L^2 \tag{19}$$

and by a previous definition

$$\frac{k\, m_1}{R^2} = W_1 = \text{Weight of } m_1 \text{ at altitude } R \tag{20}$$

Therefore

$$\tilde{T} = W_1 L \left(\frac{3}{2} \cdot \frac{L}{R} \cdot \frac{c}{c+1}\right) \tag{21}$$

When

$$m_2 >> m_1$$

Then

$$\frac{c}{c+1} \approx 1$$

and

$$\tilde{T} = W_1 L \left(\frac{3}{2} \frac{L}{R}\right) \tag{22}$$

APPENDIX E (Continued)

From the form of Equation (22), it is seen that the moment applied to the system is equivalent to a force W_1 (1.5 L/R) acting through a lever L. By definition, the factor (1.5 L/R) is the local "G" exerted on m_1 by the above mentioned force. Therefore, the effective "G" acting on the smaller body by reason of gravity gradient forces is:

$$"G" = \frac{3}{2}\frac{L}{R} \tag{23}$$

NOMENCLATURE

Symbol	Description
X, y, Z	Coordinates of mass particle
$\bar{i}, \bar{j}, \bar{k}$	Unit vectors in X, y, Z directions
u, w	Transformed coordinates
$\bar{I}, \bar{K}$	Unit vectors in u, w directions
$\bar{r}_i$	Position vector of i^{th} particle
m_i	Mass of i^{th} particle
$\bar{R}$	Position vector of mass center
R	$\lvert \bar{R} \rvert$
T	Torque acting on system
k	Gravitational constant
W_1	Weight of m_1
L	Distance separating two masses
c	m_2/m_1

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