RE-16

OPTIMUM TRANSFER TO MARS VIA VENUS

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ABSTRACT

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This report describes an investigation into the advantages of a thrust maneuver near Venus during a round trip mission to Mars. Optimum thrusted flybys of Venus have been computed for practical dates at Earth and Mars between 1970 and 1990. It was found that the use of thrust during the flyby of Venus does offer savings over the pure flyby without thrust, but from a practical point of view they do not appear significant. The conclusions apply only to the specific type of Earth-Mars transfer for a round-trip stopover mission which was studied. The method of analysis is applicable to any interplanetary flyby trajectory.

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OPTIMUM TRANSFER TO MARS VIA VENUS

Introduction

Several investigators have recognized the fact that the energy requirements for fast, round-trip missions to Mars can be substantially reduced if Venus is encountered enroute. This first became apparent in the consideration of round-trip missions which would fly by both Venus and Mars.[1][2][3][4] It has also been recognized that a flyby of Venus enroute to or upon return from a stopover mission to Mars[5][6][7] will produce savings over a stopover mission which proceeds direct to Mars and direct to Earth upon return. In addition, several investigators have pointed out that a thrust impulse applied during the hyperbolic encounter with Venus can provide further savings in comparison with those trajectories which make a pure flyby of Venus without thrust.[8][9][10] The purpose of this study is to investigate the magnitude of any saving associated with the additional thrust impulse at Venus. It is also an objective to develop a clearer picture of the saving which the Venus flyby flights make in comparison to the direct flights and the dates when the flyby flights are superior. Consequently, there are three types of Earth-Mars, free-fall transfers which are to be compared. These are:

1) Direct, free-fall, transfer from Earth to Mars.

2) Venus flyby enroute to Mars (no thrust at Venus).

3) A free-fall transfer which includes a thrust maneuver during a flyby of Venus enroute to Mars.

When Venus Is Available

The major advantage of making a flyby of Venus is that the hyperbolic encounter with the planet changes the velocity of the vehicle relative to the Sun. The magnitude of the velocity change can be large enough to make a significant change in the solar orbit. The limiting magnitude of the velocity change is the planet's circular satellite velocity at the point of closest approach to the planet. A Venus encounter has the potential of providing a velocity change of above 20,000 feet per second. A fast, round-trip to Mars by direct transfer will already take the vehicle to the vicinity of the Venusian orbit. The only major question is whether Venus will be in the right part of its orbit to be available to the spacecraft.

Two physical principles help in the analysis. The first principle is that bodies in near circular orbits of the Sun tend to get ahead of Earth when at distances less than one astronomical unit from the Sun. Likewise they tend to get behind Earth when at distances greater than one a.u. from the Sun. In order to return to Earth without circling the Sun, the fast, round-trip to Mars has to have the vehicle spend time inside of Earth's orbit catching up for the time it spends outside of Earth's orbit getting behind. The further the vehicle proceeds inside Earth's orbit, the greater the angle by which it gets ahead. The second principle is that low-energy transfers between planets are characterized by the fact that the planetary alignment of the two planets with the Sun occurs about half way between the launch and arrival dates. The combination of these two principles leads to the conclusion that the location of the Sun-Venus-Mars alignment, relative to the Earth, is indicative of the availability of Venus for a low energy transfer to Mars. The situation is shown pictorially in Fig. 1. The motion of Venus and Mars is relative to a frame in which the Earth-Sun line is non-rotating. Venus at less than one a. u. from the Sun is continually getting ahead relative to Earth. Mars at greater than one a. u. from the Sun is continually falling behind. For a Venus encounter en-route to Mars the encounter must take place ahead of Earth. If the transfer is to be of low energy requirement, Venus and Mars must be aligned with the Sun when the vehicle is about half way to Mars. Consequently, the location of the Sun-Venus-Mars alignment in the reference frame of Fig. 1 is sufficient to predict whether or not Venus is available. The same argument applies to trips which flyby Venus upon return from Mars. The first low energy transfer to Mars will cause the vehicle to get behind relative to Earth. Consequently, the location of the Sun-Venus-Mars alignment must be behind relative to Earth.

There are other methods for investigating the availability of Venus. One method is to look at the optimum direct trip and see if Venus is at all close to the vehicle when it crosses the Venusian orbit. [6] Another method is to look at the dates of planetary alignment. [8] Still another method is to search for a flyby opportunity during dates for which low energy transfers between Earth and Venus are known to exist. The advantage of Fig. 1 over all these other techniques is that it gives the information at a glance and provides more physical insight into what is going on. For example, it can be seen that Venus is available on one leg during every Mars opposition period. During every third Mars opposition period it is possible to utilize Venus both going and returning. Trips which utilize Venus can be found when the Sun-Venus-Mars alignment is outside the "available" region shown in Fig. 1, but there will probably be direct trips which arrive or depart Mars on the same date, have shorter flight times, and require smaller launch velocities. An example of this is discussed later in connection with Fig. 6.

Optimization of Launch, Flyby, and Arrival Dates

A planetary flyby is a maneuver which, although it occurs in the immediate vicinity of one planet, actually involves three planets: the launch planet, the flyby planet, and the destination planet. The flyby maneuver itself can be thought of as a connecting link between two trajectories: (1) the trajectory originating at the launch planet on a given date and terminating at the flyby planet on a specified date; and, (2) the trajectory originating at the flyby planet on the same flyby date and terminating at the destination planet on a specified arrival date.

The techniques for a mission involving a single planetary flyby are also directly applicable to a transfer involving more than one planetary flyby, such as a non-stop, round-trip mission flying by both Venus and Mars. In the discussion of a planetary flyby in this more general context, the "launch" planet is the planet previously encountered by the vehicle. Thus the "launch" planet may be the planet at which the mission originated or it may be a planet at which a previous flyby took place. Analogously, in the more general context, the "destination" planet is the planet subsequently encountered by the vehicle and may be either the termination planet or a subsequent flyby planet.

For a specified launch date at the launch planet and arrival date at the destination planet (sufficient information to uniquely determine a direct transfer), there exist many possible flyby dates. For most of the dates a thrust maneuver during the flyby is required to connect the inbound and outbound trajectories. Specifically, a thrust maneuver is required for those dates on which either

(1) The inbound and outbound hyperbolic velocities (relative to the planet) differ in magnitude, since no energy change relative to the planet can occur in a pure (unthrusted) flyby, or

(2) some portion of the pure flyby trajectory passes beneath the surface of the flyby planet.

In mission planning a meaningful criterion defining the optimum transfer is the minimization of the over-all velocity requirement for the mission, subject to a constraint on maximum admissable mission duration time. One might think that a thrusted flyby would result in a larger velocity requirement than a pure flyby, since a pure flyby requires no velocity impulse at the flyby planet. However, this is not necessarily true. Allowing a thrusted flyby permits the choice of date at the flyby planet for given launch and destination dates. Among this much larger set of possible flyby dates, there may exist a date for which the initial velocity impulse at the launch planet is small compared with launch velocities for other flyby dates. If the velocity saving at the launch planet is great enough to compensate for the necessary velocity impulse required during the flyby, then this mission is optimum even though a flyby thrust maneuver is necessary.

To find the transfer which results in the smallest over-all mission velocity requirement, it was decided to investigate (for a given launch and destination date) each possible date at the flyby planet. From this the best flyby date was selected for the given launch and destination dates. This procedure was then extended to include all launch and destination dates of interest. The direct computation of trajectories for each of the dates at the flyby planet was chosen out of preference to many currently popular optimization techniques[11] because of the presence of more than one local minimum. It was considered more desirable to gain an understanding of the complete picture of velocity requirements for different combinations of dates, rather than converge on a single trip for which the velocity requirement possesses a local minimum.

General Description of Thrust Impulse Optimization

For a specified flyby date, the hyperbolic excess velocity vector inbound to the flyby planet is uniquely determined by the launch date at the previous planet. In like manner, the hyperbolic excess velocity vector outbound from the flyby planet is specified by the arrival date at the next planet to be encountered. Thus for a given set of launch, flyby, and destination dates the magnitudes of the inbound and outbound hyperbolic excess velocities and the angle through which the vehicle must turn during the flyby maneuver are specified.

With this information the computation of the point of application of the flyby velocity impulse requires a two-dimensional optimization. The optimization problem can be stated as follows: Given the inbound and outbound hyperbolic excess velocity vectors, determine the point of application of the smallest vector velocity impulse which will connect the inbound and outbound trajectories. This optimization must in addition include the constraint that the vehicle must not pass beneath the surface of the flyby planet during the maneuver.

To solve this optimization problem, one must consider the mechanics of the flyby maneuver in more detail. Consider a vehicle entering the sphere of influence of the flyby planet along the inbound asymptote with a specified hyperbolic excess velocity vector. As the vehicle approaches the planet, the fact that the planet is not a mass point but a mass of finite size becomes important. During the flyby maneuver the distance of closest approach to the planet must be greater than the planet radius to avoid collision with the planet.

As the vehicle enters the sphere of influence of the flyby planet, the energy of the trajectory is specified by the magnitude of the hyperbolic excess velocity. The one additional constant of motion to completely describe the inbound hyperbolic trajectory (the angular momentum) is arbitrary at the sphere of influence.

In discussing flyby trajectories, the hyperbolic excess velocity is a convenient parameter describing the orbit of the vehicle, but the angular momentum is less convenient. The specification of the distance of closest approach to the planet (the peripoint radius) is equivalent to specifying the angular momentum and results in a more easily visualized orbital parameter. The peripoint radius is related to the angular momentum by the relation,

$$r_\pi = \frac{-1 + \sqrt{1 + V_H{}^2 h^2}}{V_H{}^2} \tag{1}$$

where

r_π is the normalized peripoint radius

V_H is the normalized hyperbolic excess velocity magnitude

h is the normalized angular momentum per unit mass

NOTE: Unless otherwise noted, in all equations lengths are normalized in multiples of flyby planet radius.
Velocities are normalized in multiples of circular satellite velocity at the planet's surface.

The inbound hyperbolic trajectory of the vehicle as it enters the sphere of influence is characterized by a specified value of the hyperbolic excess velocity and a peripoint radius which is arbitrary in magnitude. The magnitude of the peripoint radius will be determined by the flyby velocity impulse optimization. In the optimization it is assumed that the angular orientation of the peripoint is chosen so that the point lies in the plane defined by the radius vector of the approaching vehicle and the predetermined direction of the outbound asymptote. This assumption allows the flyby maneuver to be analyzed as a planar problem.

The actual process of guiding the vehicle toward this determined peripoint is accomplished by applying small velocity corrections to the vehicle as it approaches the planet from the sphere of influence. Assuming perfect guidance of this type, the entire flyby trajectory can be considered to be a planar trajectory. Thus the optimization to determine the point of application of the optimum thrust impulse is two-dimensional instead of three-dimensional.

Details of Thrust Impulse Optimization

In the preceding section the characteristics of a planetary flyby were described geometrically and the assumption of coplanar inbound and outbound trajectories was introduced. To perform the two-dimensional optimization which has been described, the flyby maneuver must be described analytically.

For a given position of the vehicle, the vector velocity impulse which is necessary to transfer the vehicle from the inbound trajectory to the desired outbound trajectory is the vector difference between the velocities of the inbound and outbound hyperbolas at that point. Since the inbound and outbound hyperbolic trajectories are described only in terms of their hyperbolic excess velocity vectors, it would be convenient to express the velocity at any point along the hyperbola in terms of the hyperbolic excess velocity vector.

An equation which expresses the velocity vector along a hyperbola in terms of the hyperbolic excess velocity vector and the radius vector to that point is the hyperbolic injection velocity equation given by Battin.[4] This equation describes the velocity necessary at a given point to achieve specified hyperbolic conditions (magnitude and direction of hyperbolic excess velocity). Since the hyperbolic excess velocity vectors are known for both the inbound and outbound hyperbola and since the vehicle position is a point which is necessarily common to both hyperbolas, this equation can be adapted to describe the flyby maneuver. The hyperbolic injection velocity equation can be written as:

$$\bar{V} = \frac{1}{2}\left(\sqrt{1 + \frac{4}{V_{H_o}^2\, r\, (1 + \cos\theta)}} + 1\right) \bar{V}_{H_o} + \frac{V_{H_o}}{2\,r}\left(\sqrt{1 + \frac{4}{V_{H_o}^2\, r\, (1 + \cos\theta)}} - 1\right) \bar{r} \qquad (2)$$

where the variables are defined in Fig. 2.

This equation is adapted for the analysis of a flyby as follows. At any point the velocity which will result in the desired outbound hyperbolic excess velocity vector is given directly by Eq (2). Thus the outbound velocity can be written as:

$$\bar{V}_o = \frac{1}{2}\left(\sqrt{1 + \frac{4}{V_{H_o}^2 \, r \, (1 + \cos\theta_o)}} + 1\right)\bar{V}_{H_o}$$
$$+ \frac{V_{H_o}}{2r}\left(\sqrt{1 + \frac{4}{V_{H_o}^2 \, r \, (1 + \cos\theta_o)}} - 1\right)\bar{r} \tag{3}$$

where a subscripted variable such as $\bar{V}_o$ denotes that the variable refers to the outbound hyperbola.

The velocity, $\bar{V}_I$, of the inbound hyperbola at the same point is obtained by noting that for the same radius vector $\bar{r}$, $-\bar{V}_I$ is the necessary velocity to achieve the hyperbolic excess velocity $-V_{H_I}$. This is shown in Fig. 3.

Thus the original injection equation can be adapted to describe the inbound hyperbola by changing the algebraic signs of the appropriate terms to yield:

$$\bar{V}_I = \frac{1}{2}\left(\sqrt{1 + \frac{4}{V_{H_I}^2 \, r \, (1 + \cos\theta_I)}} + 1\right)\bar{V}_{H_I}$$
$$- \frac{V_{H_I}}{2r}\left(\sqrt{1 + \frac{4}{V_{H_I}^2 \, r \, (1 + \cos\theta_I)}} - 1\right)\bar{r} \tag{4}$$

The necessary velocity impulse applied at $\bar{r}$ which will transfer the vehicle from the inbound to the outbound hyperbola is then $\Delta\overline{V} = \overline{V}_o - \overline{V}_I$

Noting that $\theta_o = \pi + A - \theta_I$, the general expression for the velocity impulse $\Delta\overline{V}$, can be written in terms of the co-ordinates of the point of application (r, θ_I) and the known dynamical properties of the hyperbolas (V_{H_O}, V_{H_I}, A). Thus for an arbitrary point described by (r, θ_I), the velocity impulse $\Delta\overline{V}$ can be computed.

For computational purposes the vector quantities are co-ordinatized by defining a two-dimensional Cartesian reference frame having its origin at the center of the flyby planet, x-axis parallel to the inbound asymptote, and y-axis perpendicular to the inbound asymptote. The expressions for the components of $\Delta\overline{V}$ are then:

$$\Delta V_x = \frac{V_{H_O}}{2}\left[\sqrt{1 + \frac{4}{V_{H_O}^2 r\left[1 - \cos(\theta_I - A)\right]}}(\cos A - \cos\theta_I) + \cos A + \cos\theta_I\right] \tag{5}$$

$$- \frac{V_{H_I}}{2}\left[\sqrt{1 + \frac{4}{V_{H_I}^2 r(1 + \cos\theta_I)}}(1 + \cos\theta_I) + 1 - \cos\theta_I\right]$$

$$\Delta V_y = \frac{V_{H_O}}{2}\left[\sqrt{1 + \frac{4}{V_{H_O}^2 r\left[1 - \cos(\theta_I - A)\right]}}(\sin\theta_I - \sin A) - \sin A - \sin\theta_I\right] \tag{6}$$

$$+ \frac{V_{H_I}}{2}\sin\theta_I\left(\sqrt{1 + \frac{4}{V_{H_I}^2 r(1 + \cos\theta_I)}} - 1\right)$$

The magnitude of the impulse is given by:

$$\Delta V = \sqrt{\Delta V_x^2 + \Delta V_y^2} \tag{7}$$

The Computation of the Flyby Impulse

To compute the magnitude of the flyby thrust impulse as a function of position a MAD computer program for use on the IBM 7094 was written. From the results of this program a map of the required velocity impulse magnitude as a function of vehicle position in the plane of motion was made for values of inbound and outbound hyperbolic excess velocity and turn angle which are typical for a flyby of Venus.

This map shows in pictoral form the sensitivity of the magnitude of the impulse to position variation in the radial and circumferential directions. It also suggests a simplified method for dealing with the constraint of finite planet size in determining the optimum thrusting point.

The program to compute the flyby impulse magnitude by solving Eqs 5, 6, and 7 was written to compute the ΔV, not at arbitrary points in the plane of motion, but at points lying along hyperbolas, representing various inbound trajectories having different peripoint radii. In this way the required impulse magnitudes for adjacent points along a specific inbound trajectory are easily compared. By doing this computation along many different hyperbolas, the impulse magnitudes can be computed in the region of the plane of motion bounded by the inbound and outbound asymptotes and extending radially from the center of the planet out to several planet radii. A map of the impulse magnitudes can them be made by plotting the loci of constant ΔV, as shown in Fig. 4.

In Fig. 4 two points are labelled: the optimum thrusting point (characterized by the minimum value of ΔV) and the common peripoint (the point which is the peripoint of both the inbound and outbound hyperbolas). The common peripoint is of interest because the impulse magnitude is easily computed at this point. The impulse magnitude at the common peripoint is simply the scalar difference between the peripoint velocity magnitudes of the two hyperbolas, since the velocities are parallel at this point (both velocities are perpendicular to the radius vector).

Thus

$$\Delta V = V_{\pi_O} - V_{\pi_I} \tag{8}$$

where

$$V_{\pi_O} = \sqrt{\frac{2}{r_\pi} + V_{H_O}^2}$$

$$V_{\pi_I} = \sqrt{\frac{2}{r_\pi} + V_{H_I}^2} \tag{9}$$

V_{π_O} and V_{π_I} are the normalized magnitudes of the peripoint velocities of the outbound and inbound hyperbolas, respectively.

r_π is the normalized common peripoint radius, the computation of which requires a short iteration. As shown in Fig. 4 for typical values of turn angle and hyperbolic excess velocities, the common peripoint lies very close to the optimum thrusting point. As a consequence, the magnitude of ΔV at the common peripoint is only slightly larger than the minimum ΔV for the maneuver. The magnitude of ΔV at the common peripoint is less than 3% higher than the minimum value of ΔV for the typical case shown in Fig. 4. This phenomenon has been noted by Gobetz [9] and others.

Because the flyby impulse magnitude is easily computed at the common peripoint and is only slightly larger than the minimum value of ΔV to accomplish the specified maneuver, the following approximation can be made:

> If the common peripoint radius is greater than the radius of the flyby planet, the minimum value of the flyby impulse magnitude can be approximated by the impulse magnitude computed at the common peripoint.

If the common peripoint radius is less than the planet radius (or any arbitrarily defined sphere which the vehicle must not penetrate), this approximation can not be used, since it would imply that the vehicle must pass beneath the surface of the planet. Thus the computed value of the common peripoint radius when compared with the planet radius determines whether the constraint of finite planet size must be considered. If the common peripoint lies beneath the surface of the planet, a different method for computing the minimum impulse magnitude must be used.

To determine the constrained optimum thrusting point in the case where the common peripoint and (presumably) the true optimum thrusting point lie beneath the surface, one must be able to identify those possible thrusting points which are not permissible when the constraint of finite planet size is included. Clearly, only requiring the thrusting point to lie at a radius greater than the planet radius is not a stringent enough constraint, since a velocity impulse applied at a distance of several planet radii could place the vehicle on a trajectory which would eventually impact the planet.

The regions of the plane of motion in which velocity corrections are not permissible are bounded by portions of the inbound and outbound trajectories having peripoint radii equal to the planet radius. As shown in Fig. 4, the bounding curve in the region which the vehicle flies through when approaching the planet is the extension of the outbound hyperbola which skims the surface of the planet. A velocity impulse applied in this region is not permissible since it would put the vehicle on an outbound hyperbola whose peripoint radius was less than the planet radius, resulting in collision with the planet. In the region of the plane which the vehicle flies through after passing the planet, the bounding curve is the continuation of the inbound hyperbola which skims the surface of the planet. Applying a velocity impulse in this region is not permissible since it would require that the vehicle had previously flown beneath the planet's surface.

Rather than searching for an optimum thrusting point in the permissible correction region a more efficient procedure was devised. The scheme which was used to compute the constrained optimum thrusting point is an approximation based on observations of the general form of the loci of constant ΔV and on physical arguments based on characteristics of hyperbolic motion.

As shown in Fig. 4, each locus of constant ΔV is a closed contour which necessarily contains all contours for smaller values of ΔV, since the impulse magnitude is a continuous, single-valued function of position. Two contours cannot intersect, since at the point of intersection the magnitude of the impulse would simultaneously have two values. Thus, is some sense, the closer a point is to the true optimum thrusting point, the smaller the value of ΔV required. If the true optimum thrusting point lies beneath the surface of the planet, this implies that the constrained optimum thrusting point should lie on a trajectory which brings the vehicle as close as possible to the planet's surface. One then expects the constrained optimum thrusting point to lie along the boundary of the permissible correction region.

More specifically, the constrained optimum thrusting point should lie along the bounding hyperbola having the smaller hyperbolic excess velocity. The argument for this is the fact that at a given point near the flyby planet, a smaller velocity along a hyperbola passing through that point results in a larger turn angle around the planet.

Because of the constraint of finite planet size, the constrained optimum thrusting point must lie on a hyperbola having a larger peripoint radius than the hyperbola passing through the true optimum thrusting point. This larger peripoint radius results in a smaller turn angle. The additional ΔV required in the constrained case compared to the minimum ΔV is due in part to the fact that the inbound hyperbola has not been able to supply enough turn angle for the maneuver. To make up for this deficiency in turn angle, it is more efficient for the vehicle to pass closest to the planet with a small velocity. Thus an approximate procedure for computing the magnitude of the impulse in the case of the finite planet constraint can be stated:

> If the common peripoint radius is less than the radius of the flyby planet, the constrained optimum thrust magnitude can be approximated by computing the magnitude of the required impulse along the less energetic boundary of the permissible correction region. The smallest value of the impulse magnitude computed along this boundary is then the constrained optimum flyby impulse magnitude.

Results

Optimum transfers to Mars via Venus have been computed for most of the attractive launch and arrival dates between 1970 and 1990 using the procedures previously described. Fig. 5 is a summary plot of the launch velocity requirements as a function of date at Earth and date at Mars.

The contours represent the dates for which the launch velocity requirement from an initial parking orbit is .2 EMOS. Contours are given for both the direct flights and the optimum via Venus trips. The minimum velocity requirement is given in EMOS units next to the location of each local minimum. It can be seen that Venus is available both going and returning from Mars during the opposition period of 1971. During the 1973 opposition period Venus is available enroute to Mars only. During the 1975 opposition period Venus is available only upon return. During the 1978 opposition period the cycle repeats with Venus again being available both going and returning. The pattern shown in Fig. 5 is exactly that predicted by Fig. 1.

Several comparisons between the flyby trips and the direct trips are informative. Fig. 6 shows dates for which the pure flyby of Venus is not possible without striking the surface of Venus. In addition the flyby with thrust is better than a direct flight which utilizes the same dates at Earth and at Mars. When this situation occurs, however, there are shorter direct trips which leave Earth at a later date, arrive at Mars on the same date, and require less velocity. The date at Mars in Fig. 6 is during the 1976 opposition period. This opposition period would not be considered a favorable one for going by Venus enroute to Mars because direct trips arriving at Mars on the same dates are shorter and more economical. This result is in agreement with the information derived from Fig. 1. Although the information presented in Fig. 6 is for a single date at Mars, the situation which it represents was observed to be true in general; that is, whenever the pure flyby of Venus requires the vehicle to go below the planet's surface, neighboring direct flights are more attractive than the resulting thrusted flyby.

Figure 7 shows a situation when the flyby of Venus offers significant savings over direct flight. The optimum via-Venus trips tend to be of slightly longer duration than the best direct flights but this is offset by the significant saving in velocity. For all of the attractive via-Venus trips represented in Fig. 7 the pure flyby is possible. This sitution was also found to be true in general; that is, whenever the via-Venus trips offer velocity savings over the best direct flights, the pure flyby is possible without striking the planet's surface.

Figure 8 shows the effect of varying the date at Venus while the dates at Earth and Mars are held fixed. The dates chosen at Earth and Mars are those of an attractive via-Venus trip shown in Fig. 7. It is seen that the date of the pure flyby at Venus occurs very close to the date for the over-all optimum This is due to the fact that the ΔV required at Venus changes more rapidly with date at Venus than the ΔV required at Earth. The ΔV required at Mars is not shown in the figure, but it too changes comparatively slowly with the date at Venus. These characteristics, although shown for only one pair of Earth and Mars dates, are representative of most atttractive via-Venus trajectories which permit a pure flyby at Venus. Consequently, the use of thrust during the flyby of Venus when a pure flyby is available does offer savings, but the savings are very small, typically a few hundred feet per second.

Figure 9 shows the 1984 opposition in slightly greater detail. The shaded area inside the .2 EMOS launch contours represents trips which in addition arrive with less than .2 EMOS hyperbolic approach velocity. A realistic cost function will depend on both the launch and arrival velocities. It is fortunate that those trips with low launch velocity tend to have low arrival velocity.

The dotted lines are the probable contours which would result if the optimum plane change were made for each interplanetary transfer that approached 180 degrees [15]. The 1984 period is shown primarily because the availability of Venus both going and returning offers a great deal of flexibility for mission planning. It is possible for instance to utilize Venus during both halves of the journey and realize a stay time on Mars of over a hundred days at the expense of a longer expedition. It is also possible to launch one vehicle to Mars via Venus and have it arrive at Mars just prior to launching a second vehicle which proceeds direct to Mars. The vehicles could rendezvous at Mars and still have time for an economical return via Venus. Other combinations are feasible which utilize this unique flexibility.

Conclusions

As a result of this study the following conclusions are presented.

1) The method described here can be used to compute optimum transfers to Mars via Venus for comparison with pure flyby and direct trips. These computations have been performed for practical dates between 1970 and 1990 and the results plotted in a form useful for mission planning.

2) For practical flybys of Venus enroute to Mars the common-peripoint solution of the thrusted flyby maneuver is an excellent approximation to the optimum maneuver. The approximate optimum velocity increment is only a few percent larger than the true optimum.

3) For dates when a pure flyby of Venus would take the vehicle beneath the surface of the planet, there are neighboring direct trips to Mars which are more economical than the resulting flyby with thrust.

4) For dates at Earth and Mars which produce attractive pure flybys of Venus, the date at Venus for the pure flyby is very close to the date for the optimum flyby with thrust. Consequently, the saving associated with the thrusted flyby in comparison with the pure flyby is small, of the order of a few hundred feet per second.

5) The plots of the computed trajectories verify that Venus is available when the Mars-Venus alignment occurs in the range shown in Fig. 1.

6) The conclusions of this study are applicable only to the specific type of trips investigated. Other investigators[12][13][14] have shown that the addition of a velocity increment during a

flyby of Mars will produce significant savings over a pure flyby of Mars for certain missions.

Since the addition of a thrust impulse has the potential of providing savings for flyby missions, the magnitude of these savings should be investigated in each case. The method described here can be applied to the optimization of other flyby missions.

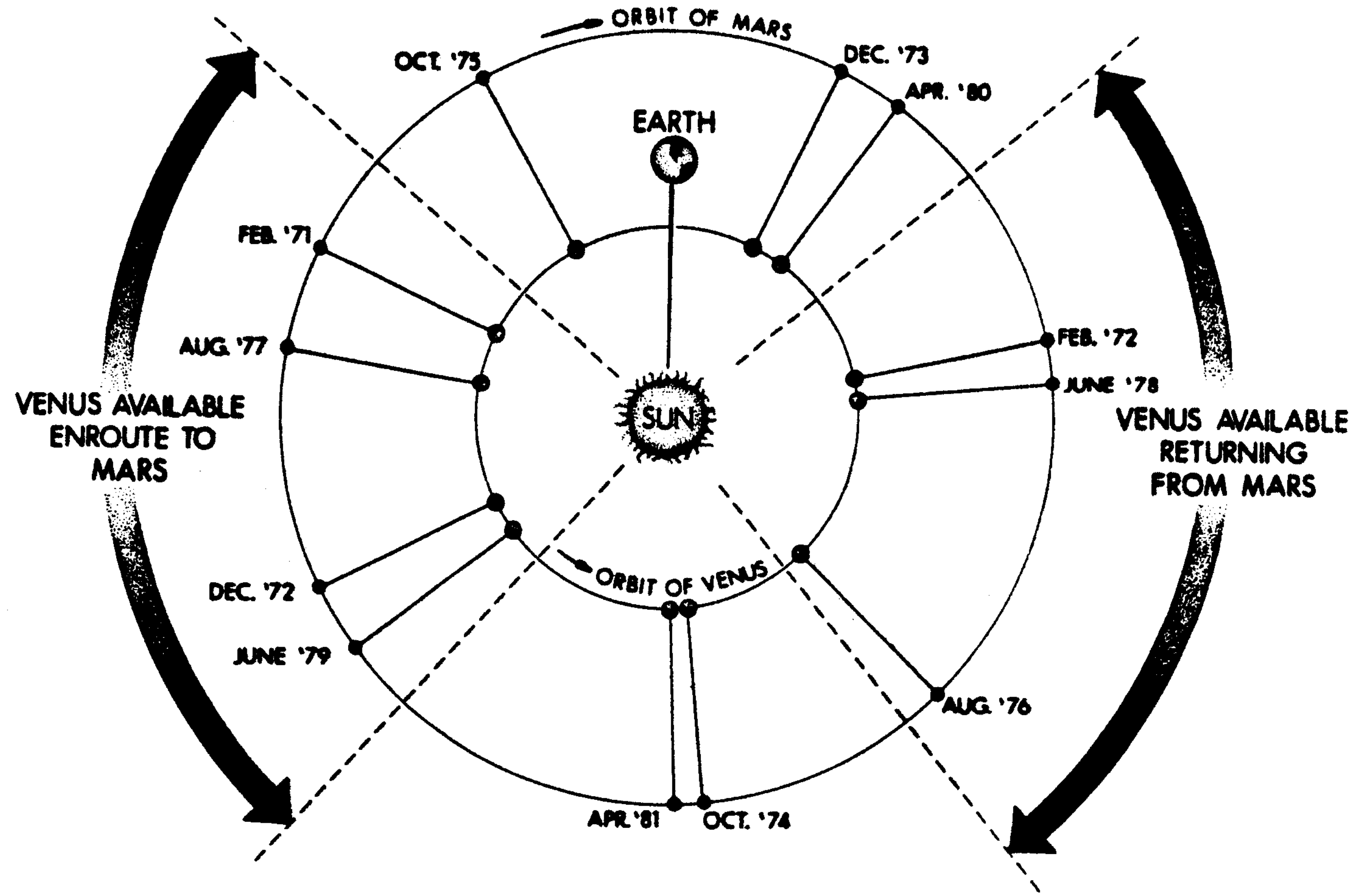


Fig. 1 Angular orientation of Venus - Mars alignments relative to Earth

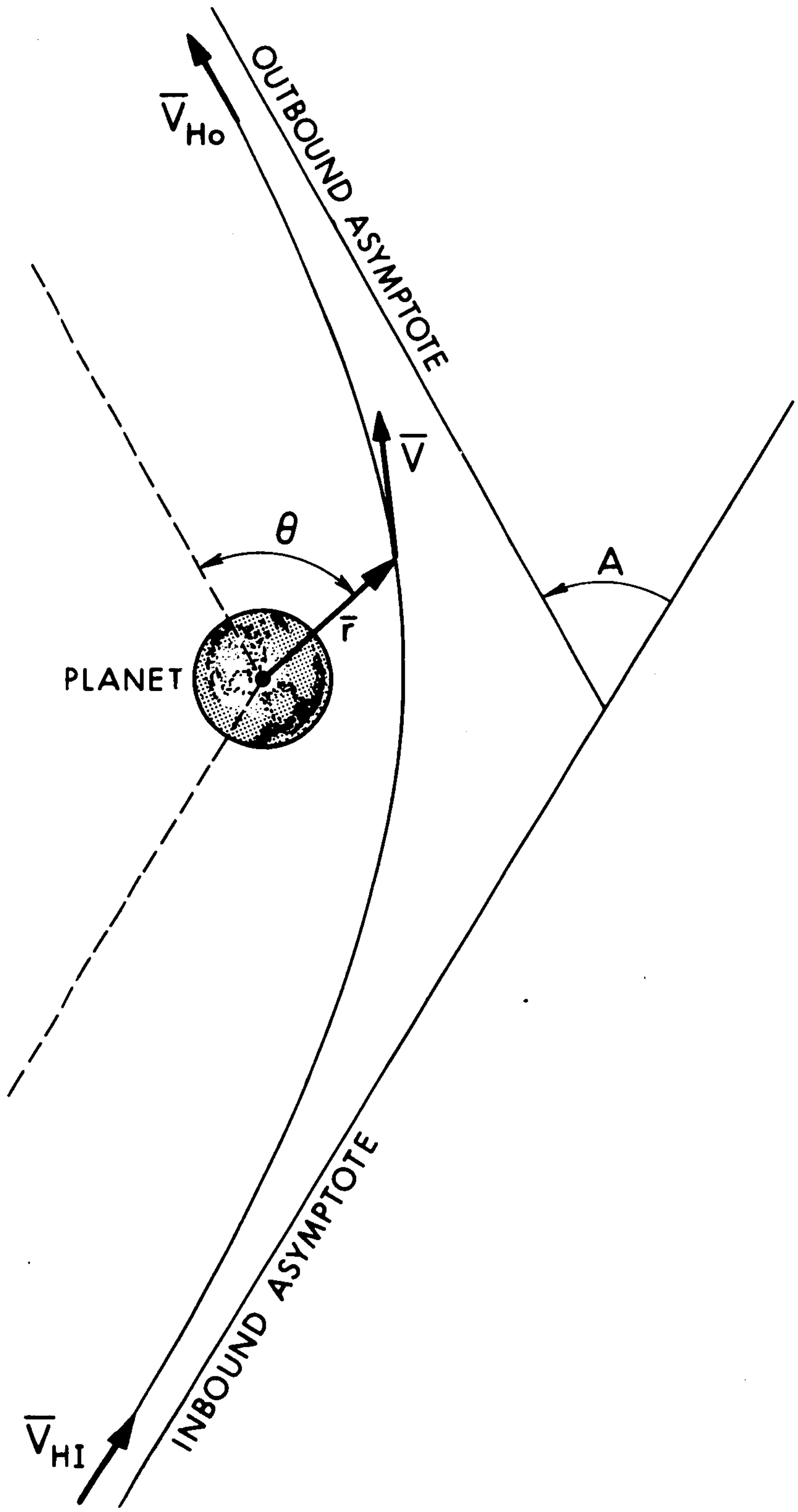


Fig. 2 The planetary encounter.

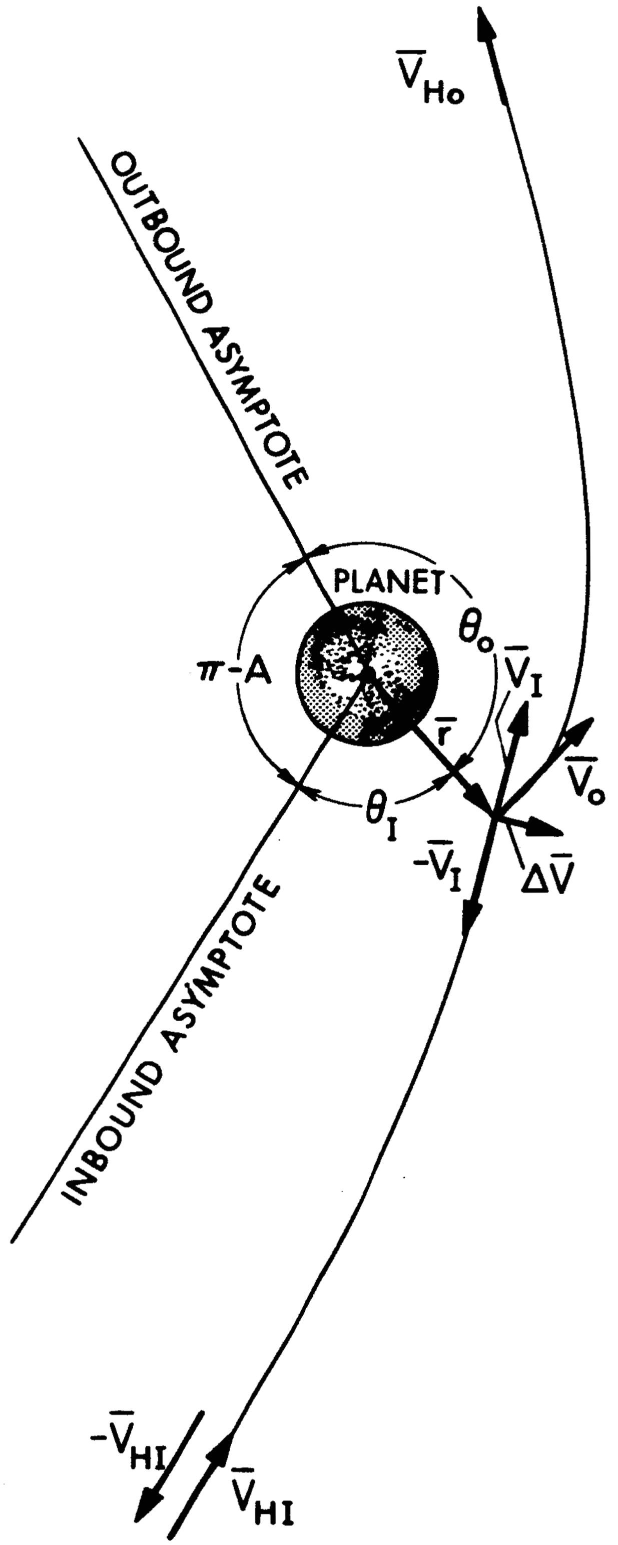


Fig. 3 The flyby parameters.

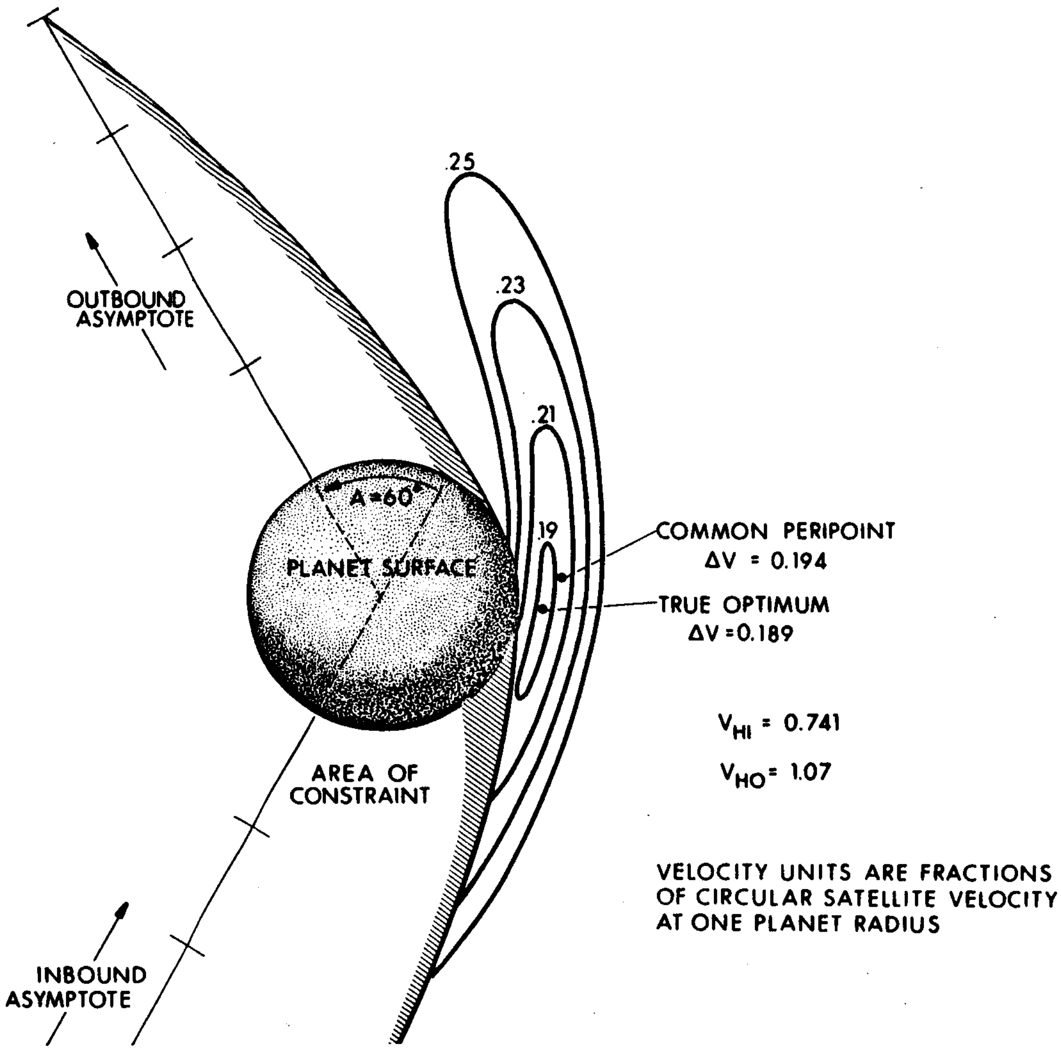
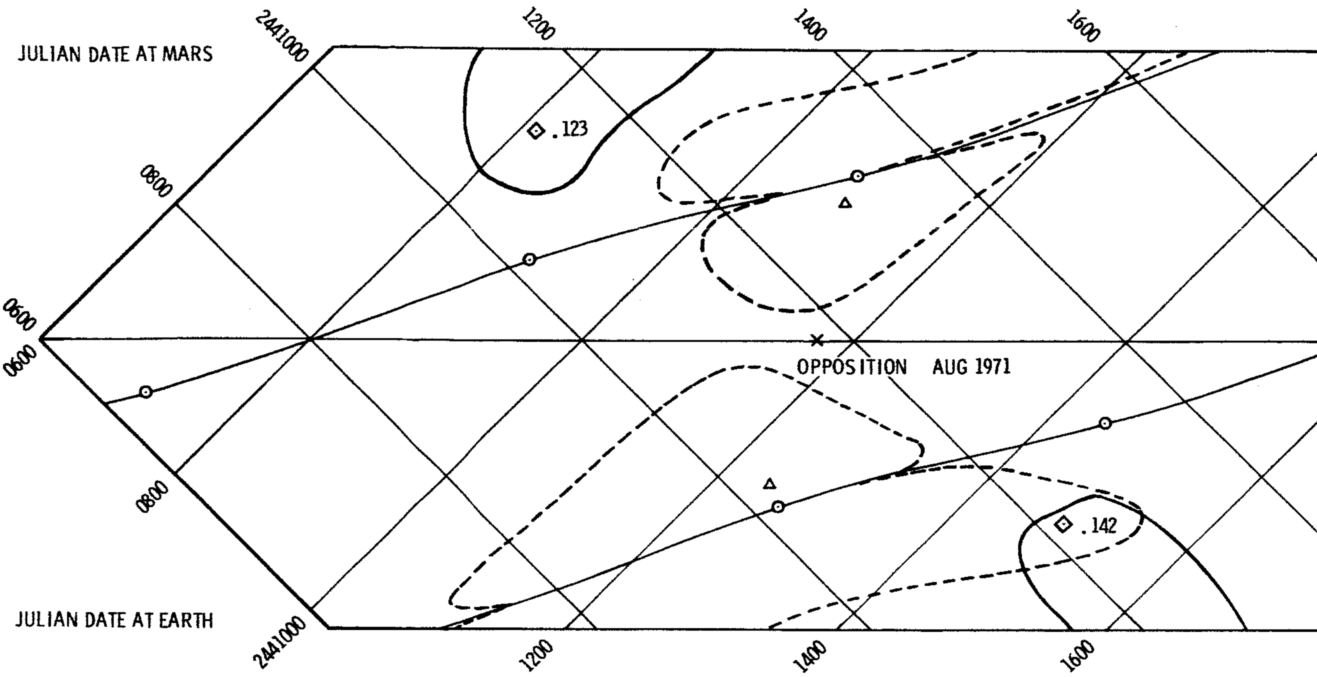


Fig. 4 Loci of constant ΔV.



LEGEND

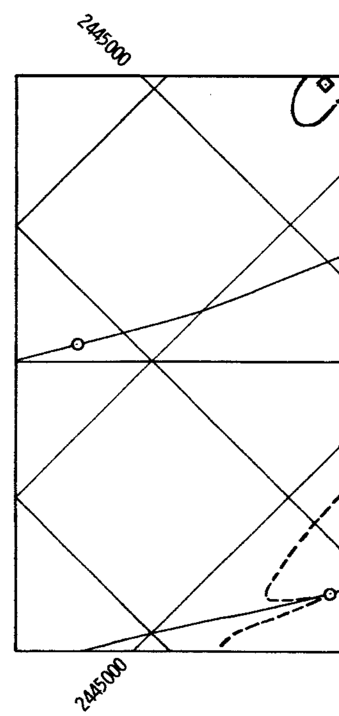
AREA REPRESENTS DIRECT TRANSFERS REQUIRING LESS THAN .2 EMOS LAUNCH VELOCITY

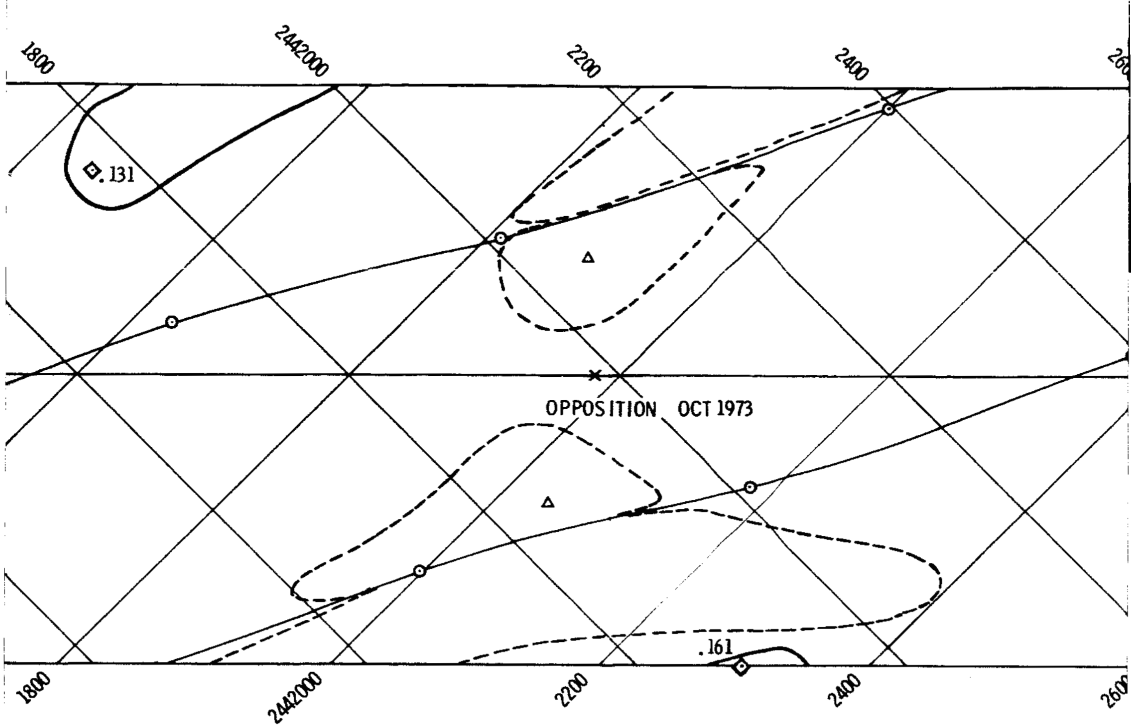
△ INDICATES DIRECT TRANSFER REQUIRING MINIMUM LAUNCH VELOCITY INSIDE THE CONTOUR

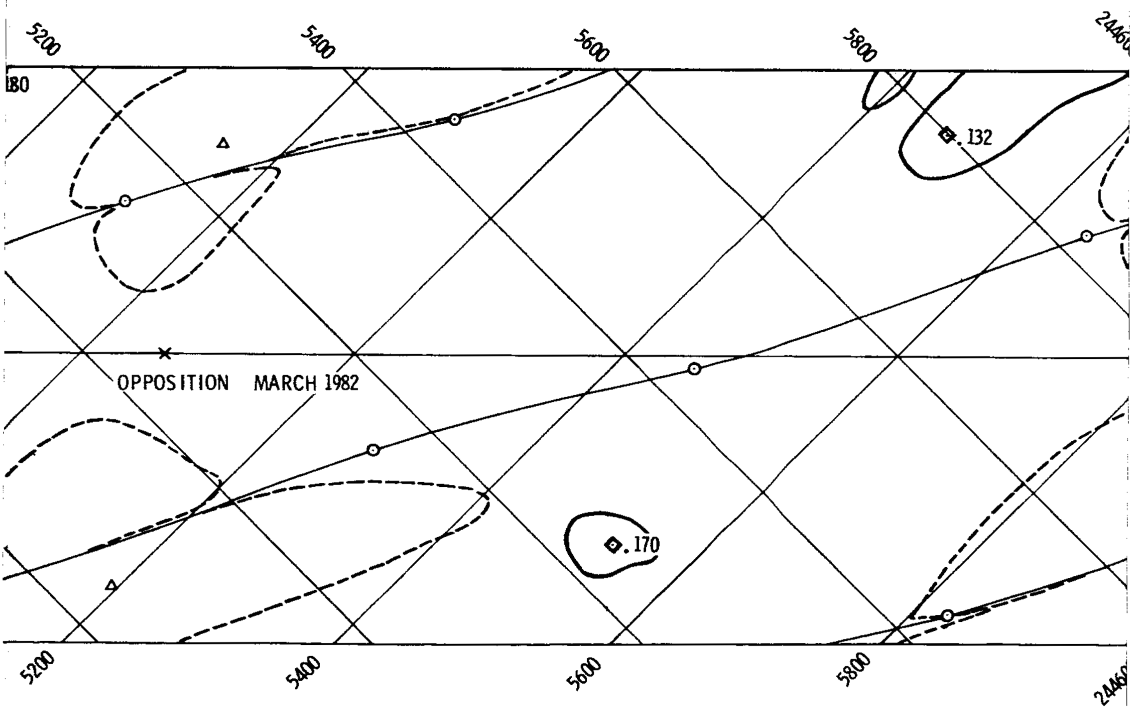
AREA REPRESENTS VIA VENUS TRANSFERS REQUIRING LESS THAN .2 EMOS LAUNCH VELOCITY

◻ INDICATES VIA VENUS TRANSFER REQUIRING MINIMUM LAUNCH VELOCITY INSIDE THE CONTOUR NUMBER DENOTES THE MINIMUM LAUNCH VELOCITY IN EMOS

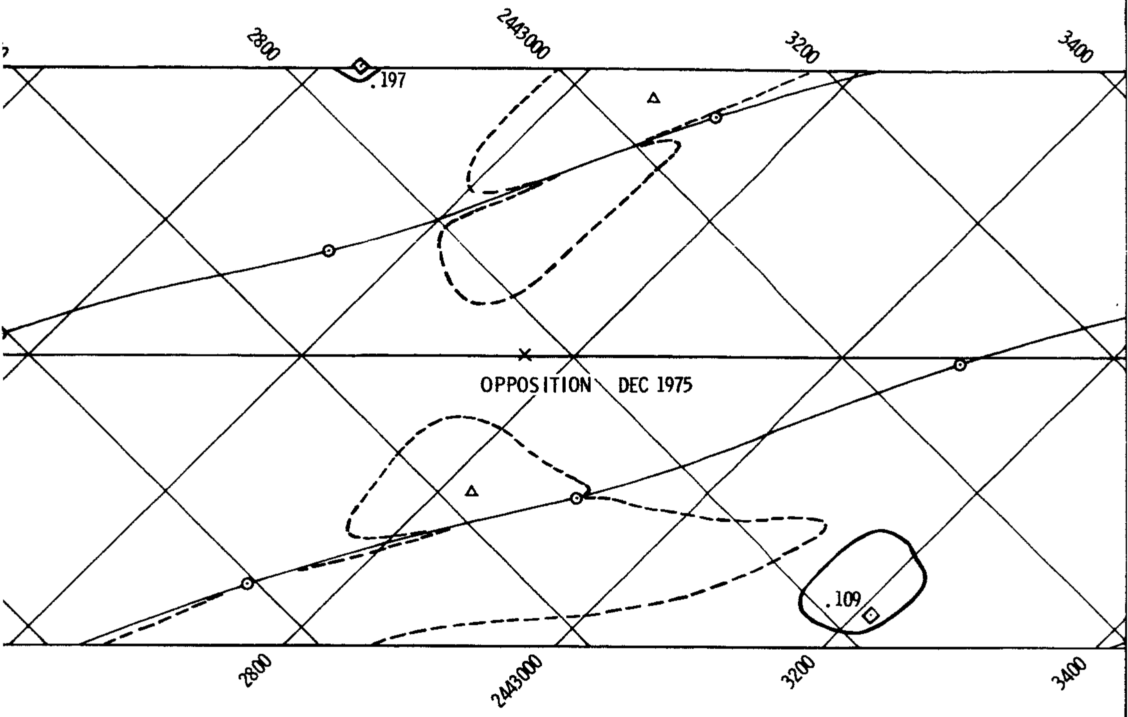
⊙ REPRESENTS 180^0 NODAL TRANSFER

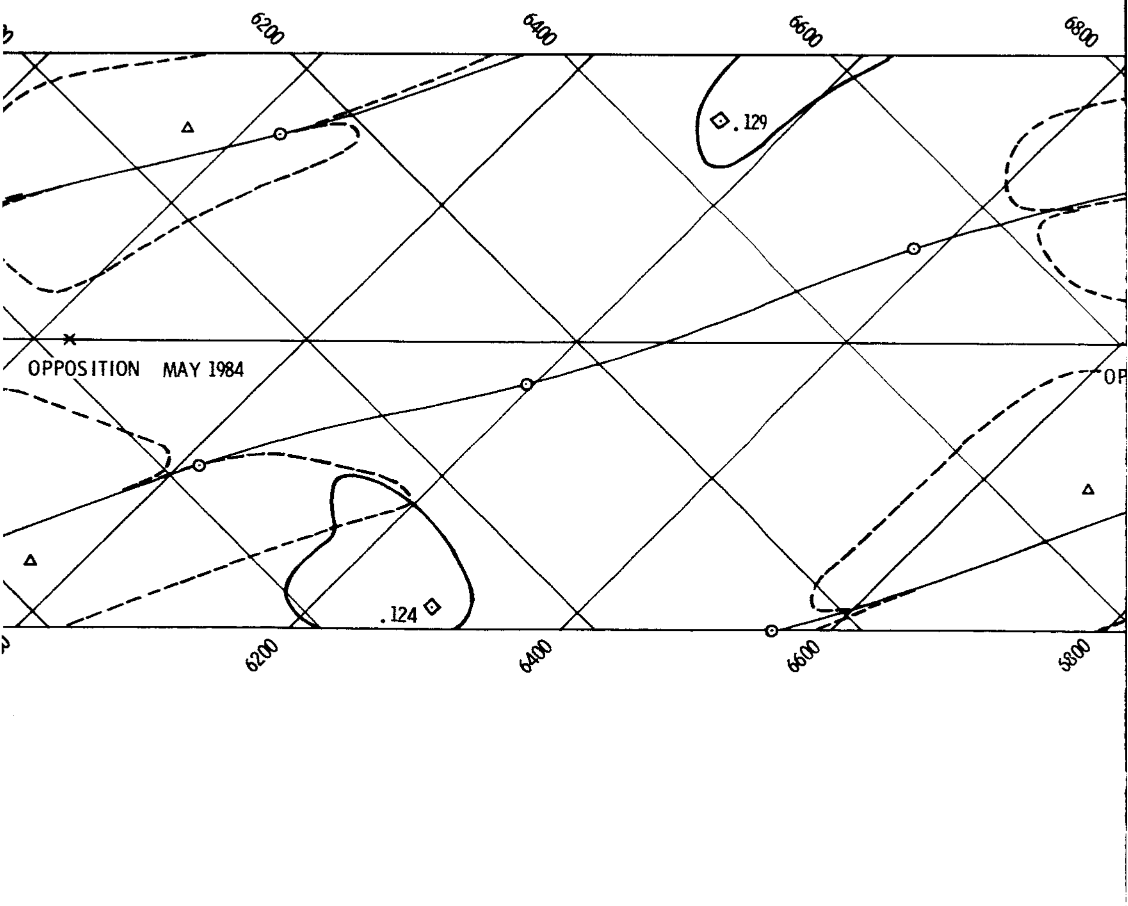


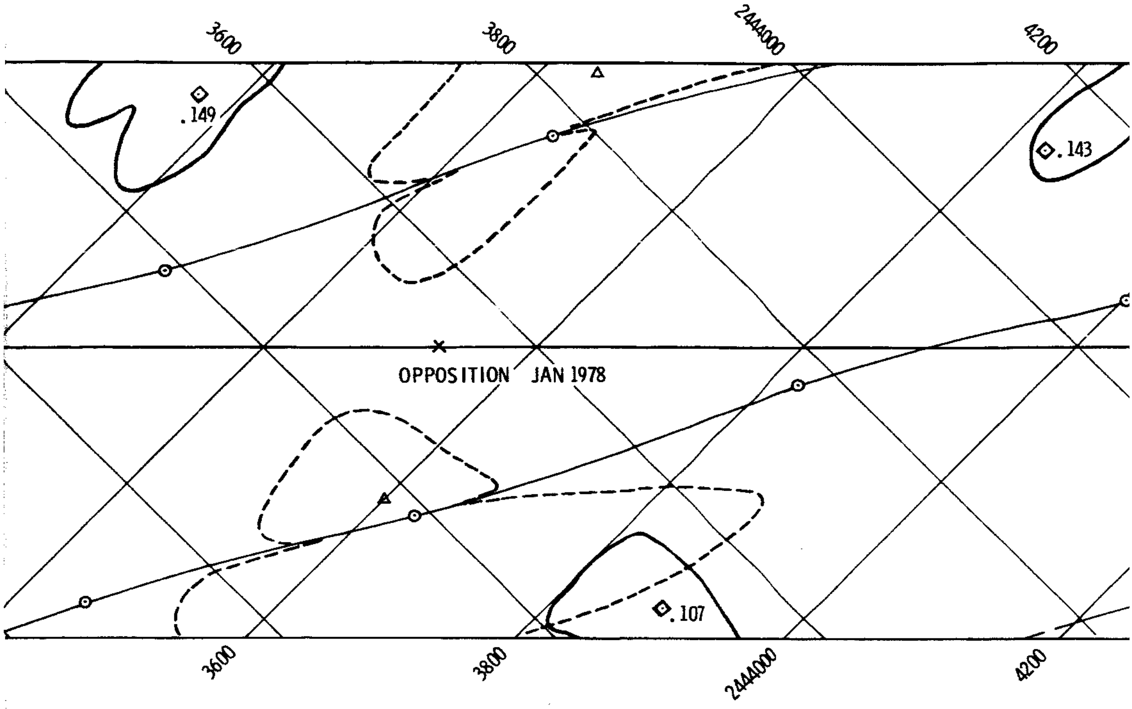

1800
2442000
2200
2400
.131
OPPOSITION OCT 1973
.161
1800
2442000
2200
2400
2600

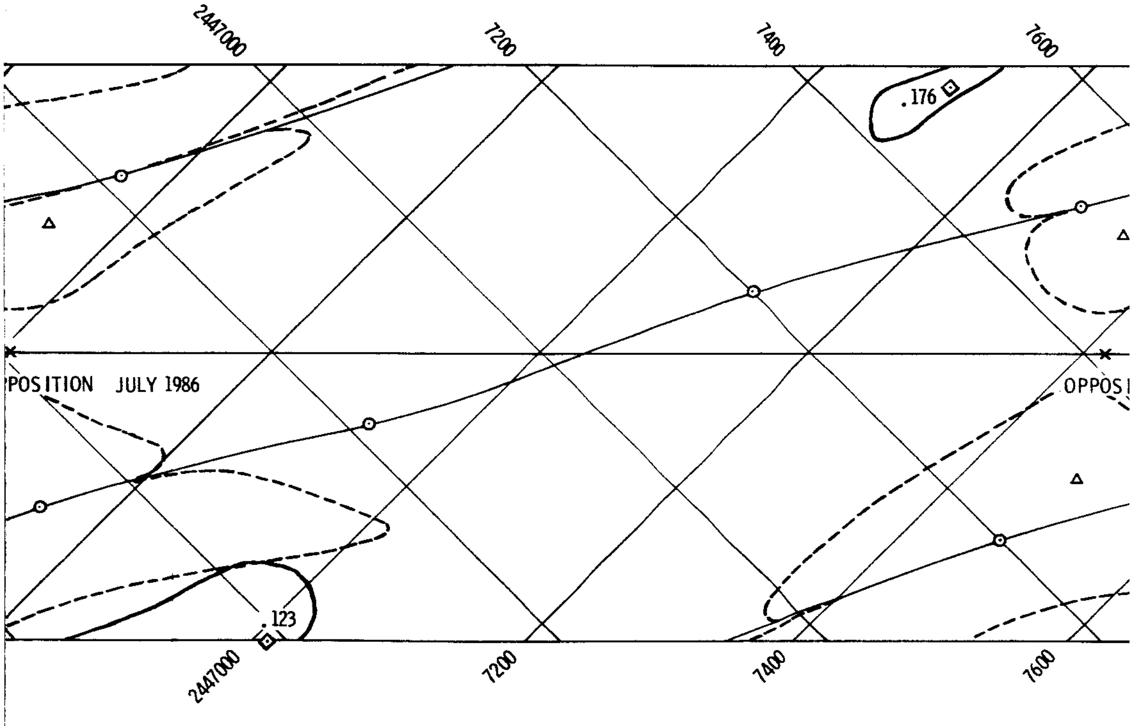

5200
5400
5600
5800
180
.132
OPPOSITION MARCH 1982
.170
5200
5400
5600
5800

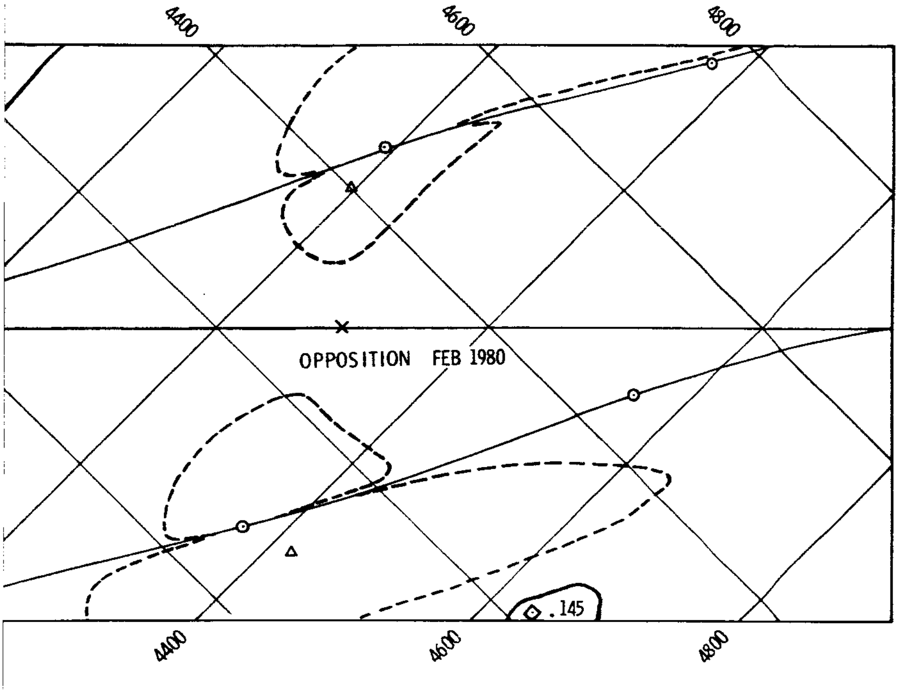
2


2800
2443000
3200
3400
.197
OPPOSITION DEC 1975
.109
2800
2443000
3200
3400


6200
6400
6600
6800
.129
OPPOSITION MAY 1984
.124
6200
6400
6600
5800


3600
3800
2444000
4200
.149
.143
OPPOSITION JAN 1978
.107
3600
3800
2444000
4200


2447000
7200
7400
7600
.176
POSITION JULY 1986
OPPOS
.123
2447000
7200
7400
7600



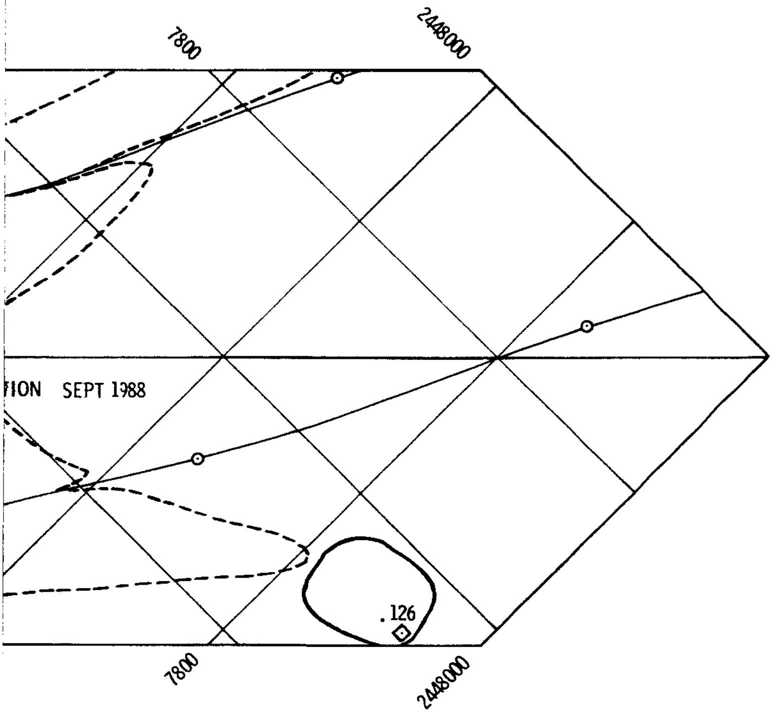


Fig. 5 Contours of 0.2 EMOS launch velocity

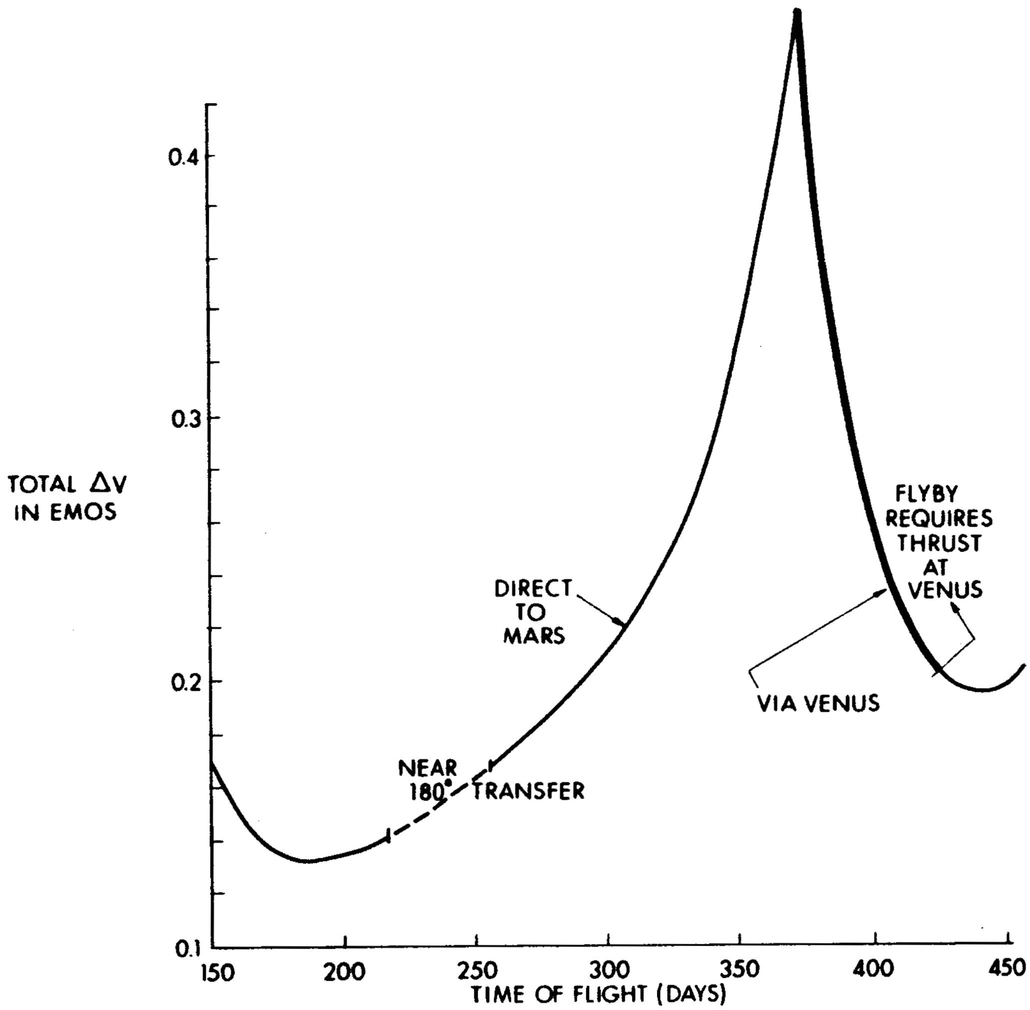


Fig. 6 Total ΔV vs trip time for arrival at Mars on Julian date 244 2860 (1976).

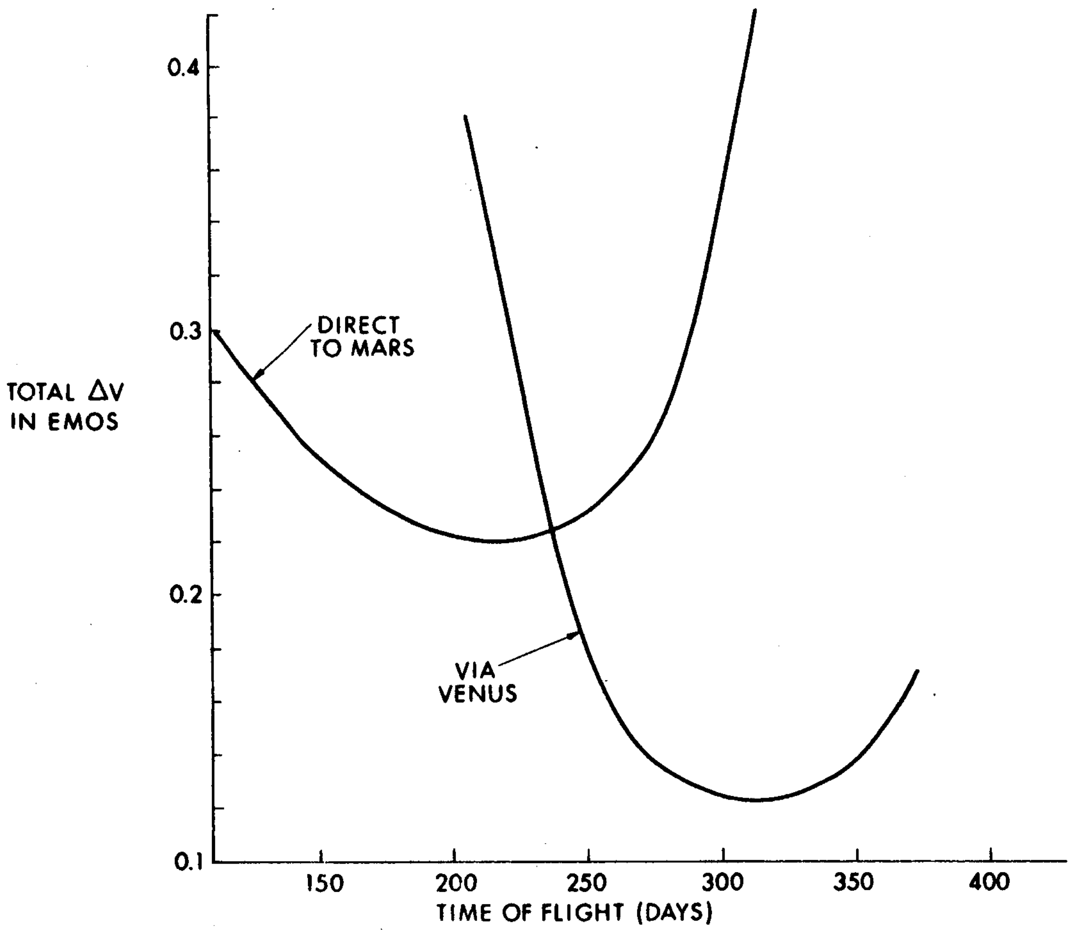


Fig. 7 Total ΔV vs trip time for arrival at Mars on Julian date 244 1120(1971).

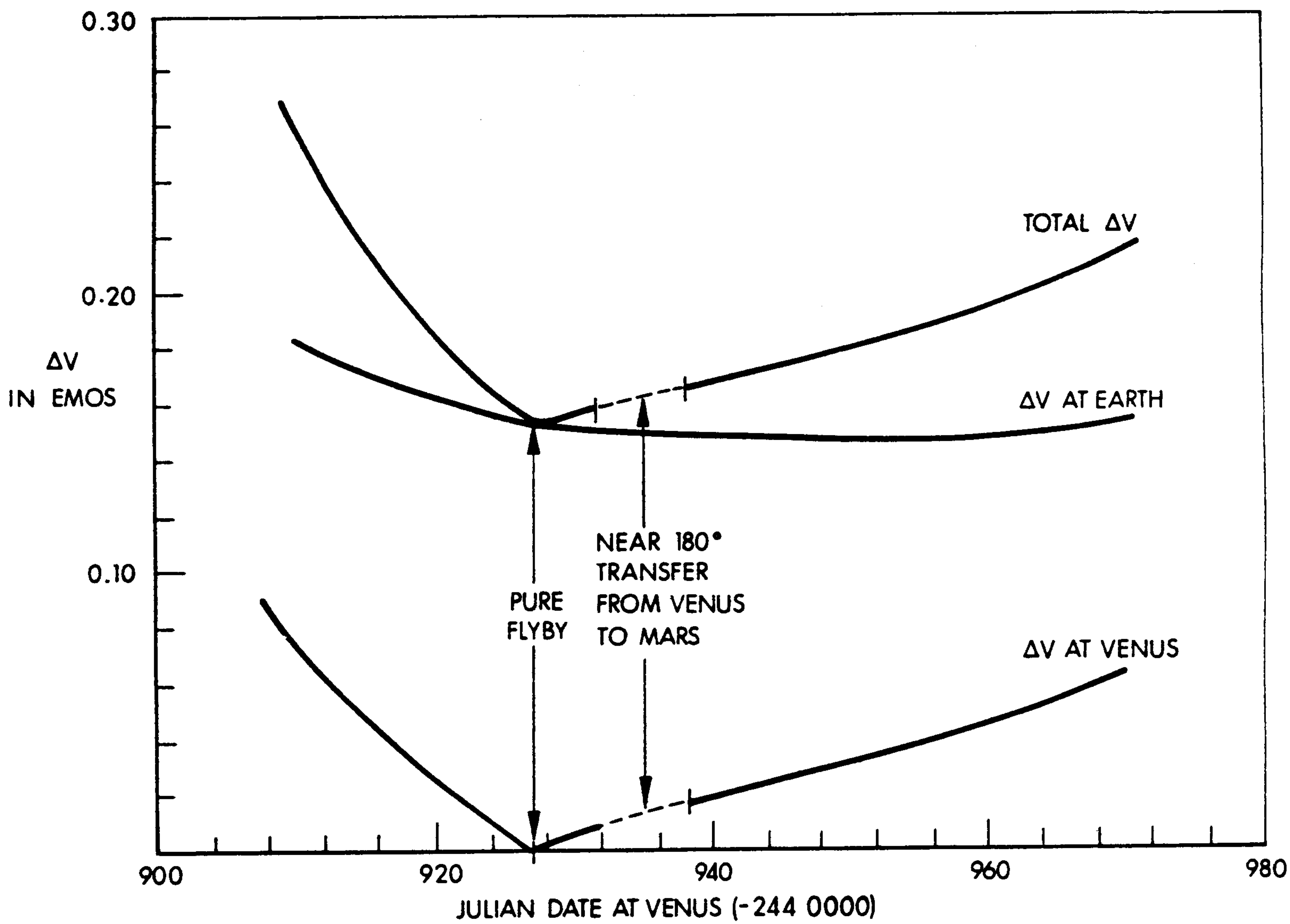


Fig. 8 ΔV vs date at Venus for Earth launch on J. D. 244 0860 and Mars arrival J. D. 244 1120

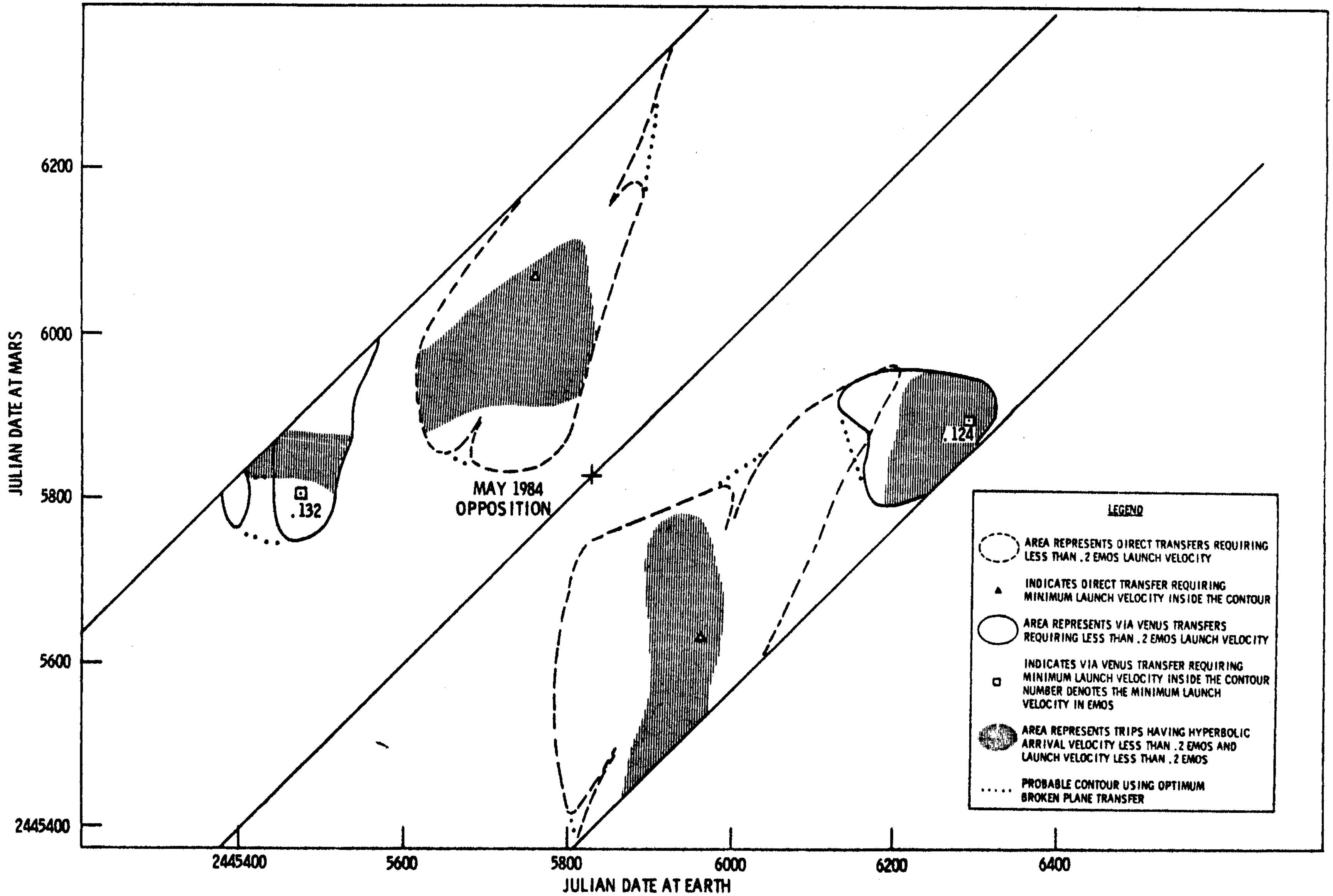


Fig. 9 Contours for 1984 opposition

REFERENCES

1. Crocco, G. A., "One Year Exploration Trip", Proceedings of the VIIth International Astronomical Congress, Rome, 1956.

2. Lockheed Missiles and Space Company, Final Report: A Study of Interplanetary Transportation Systems, No. 3-17-62-1, Sunnyvale, California, 2 Jun 1962.

3. Aeronautic Division of Ford Motor Company, EMPIRE - A Study of Early Manned Interplanetary Missions, Publication No. U-1951, Newport Beach, California, 21 Dec 1962.

4. Battin, R. H., Astronautical Guidance, McGraw Hill, New York, 1964.

5. Hollister, W. M., The Mission for a Manned Expedition to Mars, Sc. D. Thesis, M. I. T., Cambridge, Massachusetts, 1963.

6. Sohn, R. L., "Venus Swingby Mode for Manned Mars Missions", Journal of Spacecraft and Rockets, Vol. 1, No. 5, Sept-Oct 1964.

7. Deerwester, J. M., "Initial Mass Savings Associated with the Venus Swingby Mode of Mars Round Trips", AIAA 2nd Aerospace Sciences Meeting, New York, Jan 1965.

8. Hollister, W. M., "Mars Transfer Via Venus", AIAA/ION Astrodynamics Guidance and Control Conference, Los Angeles, Aug 1964.

9. Gobetz, F. W., "Optimum Transfers between Hyperbolic Asymptotes", AIAA Journal, Vol. 1, No. 9, Sept 1963.

10. Ross, S., "A Systematic Approach to the Study of Nonstop Interplanetary Round Trips," Interplanetary Missions Conference, 9th Annual AAS Meeting, Los Angeles, Jan 1963.

11. Leitmann, G., Optimization Techniques, Academic Press, New York, 1962.

12. Lee, V. A., "Determination and Analysis of Optimum Constrained Ballistic Interplanetary Missions," AIAA/ION Astrodynamics Guidance and Control Conference, Los Angeles, Aug 1964.

13. Luidens, R. W. and Kappraff, J. M., Mars Nonstop Round-Trip Trajectories, NASA TN D-2605, Jan 1965.

14. Titus, R. R., "Powered Flybys of Mars," AIAA 2nd Annual Meeting, San Francisco, July 1965.

15. Fimple, W. R., "Optimum Midcourse Plane Changes for Ballistic Interplanetary Trajectories," ARS 17th Annual Meeting and Space Flight Exposition, Los Angeles, Nov 1962.