## Ionospheric Research

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on
Variations in the Total Electron Content of the Ionosphere at Mid-Latitudes During Quiet Sun Conditions
by
Steven L. Solomon
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Submitted by:


Approved by:

A. H. Waynick, Professor of Electrical Engineering, Director, Ionosphere Research Laboratory

The Pennsylvania State University
College of Engineering
Department of Electrical Engineering

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#### Abstract

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The Faraday rotation effect on satellite signals has been used to determine the electron content of the ionosphere at midlatitudes. The ambiguity in the total number of rotations along the ray path has been resolved by using the method of closelyspace frequencies.

Observations of the total electron content of the ionosphere made over a five month period are presented and compared with values obtained in similar experiments in previous years. Horizontal gradients are discussed with sunrise effects being considered. Diurnal and day-to-day variations in content are shown, and conclusions are drawn associating variations in the magnitude and gradient of the total electron content with magnetic activity.




## I. INTRODUCTION

1. The Ionosphere ${ }^{(20)}$

The ionosphere is a region of the atmosphere, extending from about 60 km upwards in altitude, in which the constituent gases are partially ionized.

The production of free electrons and positive ions in the atmosphere is due to photoionization by ultraviolet and X-radiation from the sun. As the radiation penetrates deeper into the atmosphere, the production rate per unit volume increases because the gaseous density in increasing; however, the radiation is absorbed in the production process and a level is reached below which the production rate decreases. The principal loss process for free electrons in dissociative recombination which is the recombination of a free electron and an ionized molecule with subsequent dissociation of the molecule.

## 2. Methods of Studying the Ionosphere

The most widely used method of studying the height distribution of free electrons in the ionosphere has been the pulse sounding method. This consists of sending radio frequency pulses vertically from the ground into the ionosphere and measuring the delay time for an echo to return. Delay time is measured as a function of frequency by an instrument known as an ionosonde, and the resulting records are known as ionograms. The recording of ionograms has taken place for many years at stations throughout the world; methods have been developed to deduce electron density as a function of
height from these records. They have been used to obtain the electron density height profile up to the electron density maximum, $\mathrm{N}_{\text {max }}$. The frequency at this point is critical, and above it the radio waves penetrate the ionosphere and do not return to the earth. Measurements of this nature have led to a general description of the bottomside ionosphere.

Techniques have been developed to study the total ionosphere. Muastrements above $N_{\text {max }}$ have been made using rockets, artificial sathlites, incoherent scatering of radio waves by free electrons, radio star emisolors and radio echoes from the moon. Since 1962, the "Aloncte" topside sounder satellite has been used for radio Wave probing abow $N_{m a x}$ and has yielded a large number of electron densily height prolites.
3. Satelite Beacon Experiments

The placing of the firstartificial earth satellite in orbit by the USSR marked the beginning of new scientific experiments making use of sateities as a new experimental tool. This satellite allowed the meandrement of propagation effects over the transmission path from the satelite to earth.

Satcitite expesiments based on both pols rization rotation and pher pilh thepersion meagurements have made avaidable data of



station is different each day. Over a number of weeks the satellite pass time precesses through twenty-four hours, making observations of diurnal variations in electron content possible. Seasonal variations in electron content can also be observed, provided the active life of the satellite is sufficiently long.

A disadvantage in satellite total electron content measurements is that there is no way of determining the distribution of electrons between the satellite and the observer.

## 4. Statement of the Problem

The S-66 Ionosphere Beacon Satellite was specifically designed for ionosphere research by means of radio transmissions. It was launched on October 10, 1964 by the National Aeronautics and Space Administration. Signals are recorded whenever the satellite is in the field of view of the receiving station at University Park, Pennsylvania ( $40.8^{\circ} \mathrm{N}, 77.9^{\circ} \mathrm{W}$ ).

It is proposed that the method of Faraday rotation be used to calculate the total columnar electron content between the station and the path of the satellite using the 40 MHz and 41 MHz signals from the S-66 satellite. Although total electron content is not an ionospheric parameter that has been widely used in the past, it is very useful in interpreting other ionospheric data, it serves as a check on the determinations of the electron density profile, and it is readily correlated with other geophysical phenomena.

It is further proposed to compare the total electron content with results obtained from similar experiments in previous years.

The horizontal gradient of the content will be examined and variations in both the magnitude and the gradient of the electron content will be analyzed. An attempt will be made to associate these variations with magnetic activity.

## ii. THEORY

## 1. First-order Theory of Faraday Rotation

A linearly polarized radio wave traveling through a homogeneous ionized medium, such as the ionosphere, is equivalent to two components, the ordinary and extraordinary waves of the magnetoionic theory, which are generally elliptically polarized with opposite senses of rotation and which travel with different velocities. For a sufficiently high frequency (wave frequency >> collision, plasma, and gyromagnetic frequencies) and a direction of propagation which is not too nearly normal to the magnetic field, these waves will be nearly circularly polarized. They also have complex, anisotropic, refractive indices, i.e. they are absorbed as they propagate, and for neither characteristic wave do the wave-normal and ray directions coincide. Since the refractive indices are different, i.e., the two components have different phase velocities, the plane of polarization of the resultant wave field gradually rotates as the wave progresses through the ionosphere. This phenomenon is known as the ionospheric Faraday effect.

An expression for the total number of rotations of the plane of polarization of the resultant wave will be derived. Since the instantaneous electric vector for each component rotates through $2 \pi$ radians for each wavelength of phase path, their incremental rotations are given by

$$
d \phi_{o}=2 \pi \frac{d s}{\lambda_{0}} \text { and } d \phi_{e}=2 \pi \frac{d s}{\lambda_{e}}
$$

where

$$
\begin{aligned}
d \phi_{o}= & \text { rotation of the ordinary wave } \\
d \phi_{e}= & \text { rotation of the extraordinary wave } \\
d s= & \text { an increment of path length } \\
\lambda_{o}= & \text { wavelength corresponding to ordinary phase } \\
& \text { velocity } \\
\lambda_{e}= & \text { wavelength corresponding to extraordinary } \\
& \text { phase velocity }
\end{aligned}
$$

Figure 1 shows the relation between $d \phi_{o}, d \phi_{e}$, and $d \Omega$, the number of rotations (one rotation $=\pi$ radians) through which the plane of polarization rotates as the wave travels the distance ds. From this figure
or

$$
\begin{gathered}
d \phi_{e}+(d \Omega) \pi=d \phi_{o}-(d \Omega) \pi \\
d \Omega=\frac{d \phi_{o}-d \phi_{e}}{2 \pi}
\end{gathered}
$$

Substituting for $d \phi_{o}$ and d $\phi_{e}$ yields

$$
\mathrm{d} \Omega=\frac{2 \pi \mathrm{ds} / \lambda_{o}-2 \pi \mathrm{ds} / \lambda_{e}}{2 \pi}=\left(\frac{1}{\lambda_{o}}-\frac{1}{\lambda_{e}}\right) \mathrm{ds} .
$$

Using the relations $\lambda=v / f$ and $v=c / n, \lambda$ can be expressed in terms of index of refraction $n$, frequency $f$, and free space velocity c; this gives the expression

$$
\begin{equation*}
d \Omega=\frac{f}{c}\left(n_{o}-n_{e}\right) d s \tag{1}
\end{equation*}
$$

Appleton and Hartree ${ }^{(19)}$ have shown that the expression for the index of refraction of an ionized medium with a magnetic


RELATIONSHIP BETWEEN $d_{\phi_{0}}, d_{\phi_{e}}$ AND $d \Omega$

FIGURE I
field is

$$
n_{o, e}^{2}=1-\frac{X}{1-i Z-\frac{Y_{T}^{2}}{2(1-X-i Z)} \pm\left[\frac{Y_{T}^{4}}{4(1-X-i Z)^{2}}+Y_{L}^{2}\right]^{\frac{1}{2}}}
$$

where

$$
\begin{aligned}
& \mathrm{X}=\mathrm{f}_{\mathrm{N}}^{2} / \mathrm{f}^{2} \\
& \mathrm{Y}_{\mathrm{L}}=\mathrm{f}_{\mathrm{L}} / \mathrm{f} \\
& \mathrm{Y}_{\mathrm{T}}=\mathrm{f}_{\mathrm{T}} / \mathrm{f} \\
& \mathrm{Z}=v / 2 \pi \mathrm{f} \\
& \mathrm{f}_{\mathrm{N}}^{2}=\mathrm{Ne}^{2} / 4 \pi^{2} \epsilon_{\mathrm{o}} \mathrm{~m} \\
& \mathrm{f}_{\mathrm{L}}=(\text { Be/2mm) cos } \theta \\
& \mathrm{f}_{\mathrm{T}}=(\text { Be/2 } \pi m) \text { sin } \theta \\
& \mathrm{f}=\text { wave frequency } \\
& \nu=\text { frequency of collision of electrons with heavy } \\
& \\
& \mathrm{N}=\text { particles } \\
& \mathrm{e}=\text { number density of electrons } \\
& \mathrm{m}=\text { the charge of an electron } \\
& \epsilon=\text { electric permittivity of free space } \\
& \mathrm{B}=\text { induction of the imposed magnetic field } \\
& \theta=\text { the angle between the magnetic field and } \\
& \text { the wave normal }
\end{aligned}
$$

All quantities are in rationalized MKS units.

For a wave frequency of 40 MHz which is much greater than the collision frequency, the difference between the absorption indices of the two modes is much less than one so the previous equation can be reduced to

$$
\mathrm{n}_{\mathrm{O}, \mathrm{e}}^{2}=1-\frac{\mathrm{X}}{1-\frac{\mathrm{Y}_{\mathrm{T}}^{2}}{2(1-\mathrm{X})} \pm\left[\frac{\mathrm{Y}_{\mathrm{T}}^{4}}{4(1-\mathrm{X})^{2}}+\mathrm{Y}_{\mathrm{L}}^{2}\right]^{\frac{1}{2}}}
$$

Use will now be made of the QL (quasi-longitudinal) approximation. The condition for this is that

$$
\frac{\mathrm{Y}_{\mathrm{T}}^{4}}{4 \mathrm{Y}_{\mathrm{L}}^{2}} \ll|1-\mathrm{X}|^{2}
$$

Rewriting in terms of frequency

$$
\left(\frac{\mathrm{Be}}{2 \mathrm{~m}} \sin \theta \tan \theta\right)^{2} \ll\left(2 \pi \frac{\left(\mathrm{f}^{2}-\mathrm{f}_{\mathrm{N}}^{2}\right)}{\mathrm{f}}\right)^{2}
$$

The QL inequality is at least $1: 10$ for a wave frequency of 40 MHz provided $\theta$ is not between $87^{\circ}$ and $93^{\circ}$. In other words, except for rays that are within three degrees of being perpendicular to the geomagnetic field, the propagation is quasi-longitudinal. This means that the index of refraction assumes the form

$$
n_{o, e}=\left(1-\frac{X}{1 \pm\left|Y_{L}\right|}\right)^{\frac{1}{2}}
$$

At 40 MHz both $\mathrm{X} \ll 1$ and $\left|Y_{\perp}\right| \ll 1$ so that $n_{o, e}$ may be
written approximately

$$
n_{o, e}=1-\frac{X}{2}\left(1 \mp\left|Y_{L}\right|\right)
$$

Thus

$$
n_{o}-n_{e}=1-\frac{X}{2}\left(1-\left|Y_{L}\right|\right)-\left(1-\frac{X}{2}\left(1+\left|Y_{L}\right|\right)=X\left|Y_{L}\right|\right.
$$

Substituting this expression into equation (1)

$$
\begin{aligned}
d \Omega & =\frac{f}{c}\left(X Y_{L}\right) d s=\frac{f}{c}\left(\frac{f_{N}^{2}}{f^{2}} \frac{f}{f}\right) d s=\frac{f}{c}\left(\frac{N e^{2} B e \cos \theta}{4 \pi^{2} \epsilon_{o} m f^{2} 2 \pi m f}\right) d s \\
& =\frac{N e^{2} B \cos \theta}{8 \pi^{3} \mathrm{~cm}^{2} \epsilon_{o} f^{2}} d s=\frac{e^{3}}{8 \pi^{3} c \epsilon_{o} m^{2}} \frac{B \cos \theta N d s}{f^{2}}
\end{aligned}
$$

If we wish to find the total number of rotations of the plane of polarization, assuming that the two modes follow a common path $P$, we integrate to find

$$
\Omega=\frac{e^{3}}{8 \pi^{3} \mathrm{c} \mathrm{\epsilon}_{\mathrm{o}} \mathrm{~m}^{2}} \frac{\int_{\mathrm{P}} \mathrm{~B} \cos \theta \mathrm{Nds}}{\mathrm{f}^{2}}=7.53 \times 10^{3} \frac{\int_{P} B \cos \theta N d s}{f^{2}}
$$

If it is assumed that the ionosphere does not vary horizontally and that the ray is straight, it can be written that

$$
\mathrm{ds}=\mathrm{dh} \sec \chi,
$$

where $d h$ is ar element of height and $X$ is the angle between the ray and the vertical. Then $\Omega$ becomes

$$
\Omega=\frac{7.53 \times 10^{3}}{f^{2}} \int_{0}^{h} \mathrm{~s} N \cos \theta \sec x d h
$$

where $h_{s}$ is the upper extreme of one end of the path. Enough is known about the electron distribution so that the mean ionosphere height is determined reasonably accurately. Since $B \cos \theta \sec \chi$ is slowly varying, its value calculated at this height can replace the expression in the integral. The equation then becomes

$$
\begin{equation*}
\Omega=\frac{7.53 \times 10^{3}}{f^{2}} \overline{B \cos \theta \sec \chi} \int_{0}^{h} \mathrm{~s} N \mathrm{dh} . \tag{2}
\end{equation*}
$$

Thus the plane of polarization of the wave undergoes a rotation which is directly proportional to the integrated electron content.

Since the initial polarization is not known and the plane of polarization is unaltered by the addition to $\Omega$ of $n \pi(n=0,1,2, \ldots)$, the total angle of rotation cannot be found unambiguously using a single frequency at mid-latitudes. In order that this total angle be unambiguously determined, use may be made of the relation $\Omega \alpha l / f^{2}$. This technique involves the observation of slow fading on two closely-spaced frequencies.

If there is a difference of $\Delta \Omega$ in the amounts of rotations of the two frequencies which are separated by a small amount $\Delta f$, then $\Omega$ can be found in terms of $\Delta \Omega$.

$$
\Omega^{\prime}-\Omega=\Delta \Omega
$$

where the primes will denote the second frequency so that

$$
\begin{equation*}
\frac{\Delta \Omega}{\Omega}=\frac{\Omega^{\prime}}{\Omega}-1=\frac{\mathrm{f}^{2}}{\mathrm{f}^{\prime 2}}-1=\frac{\mathrm{f}^{2}-\mathrm{f}^{\prime 2}}{\mathrm{f}^{\prime 2}} . \tag{3}
\end{equation*}
$$

2. Application to Satellite Observations $(2,9,13)$

For the determination of integrated electron content, transmissions from the S-66 satellite are being used. The satellite is in a near-polar orbit well above the $F$ region of the ionosphere. Maximum Faraday rotation will be obtained for propagation directions that are not too nearly perpendicular to the earth's magnetic field and for frequencies that are low (because of $1 / f^{2}$ dependence) but sufficiently above the plasma frequency of the Flayer. For this latter reason, frequencies of 40 MHz and 41 MHz are transmitted from the beacon satellite.

When the satellite is to the south, the rotation angle is relatively large since both $\cos \theta$ and $\sec \chi$ are large. When the satellite is at the point of closest approach, the rotation angle is found to be smaller, owing largely to the reduction in sec $X$. As the satellite continues its travel to the north, the rotation angle still continues to decrease because the reduction of $\cos \theta$ more than offsets the increase in sec $X$. If the ray path could become transverse to the magnetic field, the total rotation angle would be reduced to a small fraction of the overhead value which to a first approximation may be equated to zero. The locus of points where this
theoretically takes place is below the radio horizon, and from one of these points we have effectively counted the number of Faraday rotations of the received signal as the satellite moves through this point to any point on the trajectory. From this number of rotations the electron content of that part of the ionosphere between observer and satellite is calculated for various points along the trajectory and hence the variation of the electron content along a slice of the ionosphere can be determined.

Going back to equation (2), if 41 MHz is inserted for the wave frequency the expression becomes

$$
\Omega=4.48 \times 10^{-12} \overline{B \cos \theta \sec \chi} \int_{0}^{h} \mathrm{~s} N \mathrm{dh}
$$

In terms of electron content, this can be written as

$$
\begin{equation*}
\int_{0}^{h} \mathrm{~s} N \mathrm{dh}=2.23 \times 10^{11} \frac{\Omega}{\overline{\mathrm{~B} \cos \theta \sec \chi}} \tag{4}
\end{equation*}
$$

Now use must be made of equation (3) to get $\Omega$ in terms of $\Delta \Omega$. Letting the prime terms refer to 40 MHz and the unprimed terms to the 41 MHz signal, $\Omega=19.75 \Delta \Omega$.
3. Second-Order Theory ${ }^{\text {(2 }}$ (22)

The Faraday effect equations have been extended to include second-order effects. First-order theory is based on straight line propagation and second-order theory concerns the following departures from this:
(1) The non-uniform distribution of ionization causes the various rays to be refracted and follow different paths between source and receiver.
(2) Since the medium is anisotropic, the wave normal and ray for a particular mode of propagation are not coincident in direction.
(3) The refractive index is non-linear in electric density and magnetic field intensity.

The second-order polarization rotation angle was derived as

$$
\Omega_{2}=\Omega\left[1+\frac{\beta \bar{X}}{2}+\frac{(\beta-1)}{2} G \bar{X}\right]
$$

where

$$
\begin{aligned}
& \Omega_{2}=\text { second-order polarization rotation angle } \\
& \Omega=\text { first-order polarization rotation angle as } \\
& \text { previously defined } \\
& \bar{X}=\frac{e^{2}}{\epsilon_{0} m 4 \pi^{2} f^{2}} \frac{\int_{0}^{h} s}{h^{\prime}}=\frac{4.91 \times 10^{-14} \int_{0}^{h} s}{h^{\prime}} N d h \\
& h^{\prime}=h_{s}+R_{o}(1-\cos C 2) \\
& R_{o}=\text { radius of the earth } \\
& \text { C2 = central angle between station and satellite } \\
& \beta=\frac{X^{2}}{\overline{X^{2}}}=\text { a measure of the non-uniformity of the } \\
& \text { ionization distribution over the height of } \\
& \text { integration to the satellite }
\end{aligned}
$$

$G=a \operatorname{geometrical}$ parameter involving the direction of straight line propagation, the magnetic field, and the vertical at the ionosphere point near which the bulk of ionization lies

When applying the theory to close-spaced frequency polarization rotation dispersion, the second-order correction is found to be twice as great as that for the polarization rotation itself, for

$$
\mathrm{d} \Omega_{2}=-2 \Omega \frac{\mathrm{df}}{\mathrm{f}}[1+\beta \overline{\mathrm{X}}+(\beta-1) \mathrm{G} \overline{\mathrm{X}}]
$$

instead of $d \Omega=-2 \Omega \mathrm{df} / \mathrm{f}$ as in first-order theory. The measured value of $\Omega$ must then be adjusted to a corrected value which is done by multiplying it by ( $1-\beta \bar{X}-(\beta-1) G \bar{X})$. This new value of $\Omega$ is then used in the first-order equations previously derived.

## III. INSTRUMENTATION

1. Satellite

The S-66 Ionospheric Beacon Satellite provides a means for obtaining Faraday rotation and differential Doppler ionospheric measurements on all parts of the globe over an extended period of time.

The orbit is almost polar and circular to obtain worldwide coverage and simplify interpretation of the measurements, respectively. The satellite beacon is nominally at 1000 Km altitude so that it is above the major portion of the ionosphere's electron content.

Initial orbit properties were:

| Anomalistic period | 104.76 minutes |
| :--- | :---: |
| Inclination | 79.61 degrees |
| Perigee | 880 Km |
| Apogee | 1080 Km |

There are four transmitters operating continuously at $20 \mathrm{MHz}, 40 \mathrm{MHz}, 41 \mathrm{MHz}$ and 360 MHz ; the first three have a nominal power of 250 mw while the 360 MHz signal has a radiated power of 100 mw .

The satellite radiates linear polarization from dipoles normal to its axis. The axis is magnetically oriented along the local magnetic field at the orbit height; therefore, at mid-latitudes it is seen from one end only. The spin about the axis is damped mechanically to a rate designed to be less than 0.02 rotations per minute.

The satellite period and the precession of the orbit plane are such that almost fourteen complete orbits occur in one day. If the first orbit is directly over the receiving station, the fifteenth will pass approximately one day and twenty-seven minutes later and 8. 94 degrees further west in longitude. Consecutive orbits cross the station latitude separated by 104.76 minutes in time and $26.35^{\circ}$ in longitude. Only satellite passes which cross the receiving station latitude within 25 degrees east or west of the receiving station longitude are recorded. Therefore, the precession of satellite pass time is such that observations are made twenty-seven minutes later each day for two days; on the third day another satellite pass is observed seventy-eight minutes earlier. Thus, the overall trend of time of satellite passes is backward through time of day, an average of eight minutes per day. During a six month period, the time of a satellite pass precesses backward through twenty-four hours.

Usually two consecutive north-going passes were observed, and eleven hours later two consecutive south-going passes each day.

## 2. Receiving and Analysis Equipment

The block diagram of the receiving and analysis equipment is shown in Figure 2. Used for Faraday rotation are the 20 MHz , 40 MHz , and 41 MHz signals received on fixed linearly polarized dipole antennas and recorded on magnetic tape, together with the standard time. When the tapes are played back, the signals are

BLOCK DIAGRAM OF FARADAY RECEIVING AND ANALYSIS EQUIPMENT
figure 2
filtered and envelope detected and recorded on a multichannel Sanborn chart recorder as shown in Figure 3.
$+$

SAMPLE RECORDING OF SATELLITE SIGNALS
FIGURE 3

## IV. METHOD OF ANALYSIS

## 1. Counting of Rotations

The sample recording of part of a pass in Figure 3 contains the WWV trace showing the Eastern Standard Time every second. The reading accuracy is limited by the sharpness of the nulls of the 40 MHz and 41 MHz signals whose times must be read.

Each successive null to the south represents one more rotation of the plane of polarization of the propagated signal since the satellite passed through the theoretical transverse point which is below the radio horizon to the north; it is not known how many rotations have been made before the northernmost null recorded. Use must be made of the previously stated relationship

$$
\Omega=19.75 \Delta \Omega
$$

to solve this problem which is now reduced to finding the fractional difference of rotation between the 40 MHz and 41 MHz nulls. This can be done by linear interpolation to the 41 MHz null, assuming a constant rate of rotation between the adjacent 40 MHz nulls. The procedure has been simplified by using an expanding scale and reading the fraction directly.

In many cases the assumption of a linear rate of rotation between adjacent pairs of nulls proved to be erroneous and yielded misleading results; this necessitated altering the method of counting the rotations. By observing the 20 MHz nulls, it is possible to determine where, in fact, the 40 MHz rotation is essentially linear.

For each pass then, a pair of 40 MHz nulls is carefully chosen with regard to the aforementioned linearity. The fractional and then the total rotation is calculated for the bracketed 41 MHz null. For each 41 MHz null to the south of this null, one rotation is added successively; for each to the north, one is subtracted successively. The time is read at each 41 MHz null, thus giving a time and a value of $\Omega$ for each null.
2. Resolution of Ambiguities

Unlike fractional differences, an integral difference of rotation cannot be measured directly. In other words, there is ambiguity involved as to whether to any measured fractional difference on integer should be added.

Several methods are used to resolve this ambiguity. First of all, the chosen null is picked as far to the north as possible, still observing the linearity previously discussed; the fractional rotation is generally less than unity a reasonable distance north of the station. Also by looking at the entire trace, it can sometimes be determined whether the fractional rotation is greater than one, since $\Delta \Omega$ must always be greater than zero. Lastly, the results can be compared for the different possibilities, and generally only one will give reasonable results for electron content, especially with regard to magnitude, gradient, and time of day. All of these checks will combine to resolve the ambiguity in virtually all instances.

Another problem which arises is the detection of a reversal of the sense of rotation. This takes place rarely and briefly and
only when there is a comparatively large variation in ionization over a small interval. Thus for a southgoing pass, a sudden decrease in electron content may cause the plane of polarization to stop and reverse its direction for a short while. From the 41 MHz signal alone there is no sure way to determine when this happens. Usually a reversal can be recognized when there is a change in the pattern of the two sets of nulls at 40 MHz and 41 MHz . The 20 MHz nulls are then used to determine when the rotation reversed and then where it returned to its original sense. This is possible to do since there is a constant ratio of nulls from the 20 MHz signal to the others of approximately four-to-one. Subsequent nulls must then be adjusted to their proper numerical value.

## 3. Calculation of Results

It has been shown that the observed data yield the number of polarization rotations at different times. The orbital data supplied by NASA give the latitude, longitude, and height of the satellite every minute, and since the satellite travels at approximately constant speed, linear interpolation is used to find the location at the time of each null.

The next step is to find $\overline{B \cos \theta \sec \chi}$ at each of these locations. This is also done with sufficient accuracy by linear interpolation of values which have been pre-computed at integral latitudes in satellite position from $22 \mathrm{~N}-60 \mathrm{~N}$, integral longitudes from $50 \mathrm{~W}-103 \mathrm{~W}$, and 100 Km intervals of satellite height. The values of $G$ are found in a similar manner by interpolation of a
pre-computed grid.
At this point, the first-order value of $\int N d h$ is found using equation (4). An ellipsoidal earth is assumed in converting satellite position to ionosphere position at the mean ionosphere height that has been used.

It is then a simple matter to calculate $\bar{X}$ and then with values of $G$ and $\beta$ (which is assumed to be 2.5 based on electron density profiles) determine the second-order electron content for a meter square column extending vertically from the earth through the ionosphere up to the satellite height.

Most of the calculations were done on the IBM 7074 digital computer at The Pennsylvania State University.
4. Errors

The second-order theory has eliminated most of the systematic errors encountered when making simplifying approximations. In the extreme northern part of the pass, the second order content will be as much as $5 \%$ higher than first order. This difference gradually diminishes and becomes zero slightly north of the station at about $43^{\circ} \mathrm{N}$. Further south, the second order content gradually becomes lower than the first order value, with as much as a $5 \%$ difference at the extreme southern end.

As stated before, much care was taken in determining where to calculate the fractional difference of rotation. For the most part, any error incurred here will virtually disappear except in the northern end of the pass where the number of rotations are very
small. Even then, this error will be substantial only when there is non-linearity between the 20 MHz nulls which enclose the chosen 41 MHz null, or if this null cannot be read accurately. By choosing a point in the north instead of the south to determine the rotation, the percent error would remain the same, but the actual number of rotations in error would be significantly less. This actual rotation is the important thing since the rotations at all other nulls would contain the same error. Nevertheless, a one-tenth second difference in the time of the null can result in as much as a $1 \%$ error in $\Delta \Omega$ during the day. For this reason, a null which cannot be read this closely should not be considered then. The allowable tolerance will increase with the time between nulls.

An error could be caused by rotation of the plane of polarization due to the spinning of the satellite about its magnetic axis; however, in the original specifications published, the satellite is said to spin less than 0.02 revolutions every minute, which is negligible compared with the measured polarization rotation rates.

Another source of error is in the scaling process; $i . e$. the reading of the times of the observed nulls. Most nulls are of sufficient sharpness to be read to the nearest tenth of a second; however, there are a few others in which the uncertainty is as much as a full second, most of these being at night when the rotation rate is slow. A time displacement of one second will result in a distance displacement of less than $0.06^{\circ}$ latitude for the satellite with the longitude and height displacements being negligible. This will result in a change of less than $0.25 \%$ in $\overline{\mathrm{B} \cos \theta \sec \chi}$ which is very small
considering the accuracy of this term.
In the calculations the mean ionosphere height was assumed to be 250 km based on a study of electron density profiles. A height of 300 km would have been closer to the true value at night but the daytime results will be of more use for this report, so the lower value was chosen. A comparison in the results using the two heights is shown in Figure 4. The difference in the content calculations is greater as the satellite is further to the north of the station. Also, being further to the west will increase the difference to as much as $15 \%$. It should be noted that the content calculation for a particular null will be for different meter square columns in the ionosphere for each of the heights. The effect of both of these differences is best shown in the curves.
(In this and all other plots of electron content versus latitude the six digit number at the end of each curve will signify the date. The first two digits represent the year; the second two, the month; and the last two, the day. The four digit number is the approximate Eastern Standard Time the satellite was travelling across the station latitude.)


ELECTRON CONTENT vs. LATITUDE FOR DIFFERENT MEAN IONOSPHERE HEIGHTS

FIGURE 4

## V. RESULTS

Total electron content has been calculated for 285 passes of the S-66 satellite observed at University Park, Pennsylvania $\left(40.8^{\circ} \mathrm{N}, 77.9^{\circ} \mathrm{W}\right)$ between October 24, 1964 and March 16, 1965. During this five month span both the northgoing and southgoing passes precessed 20 hours thus yielding results during two different months for most periods of the day or night.

An analysis of these results with regard to magnitudes, gradients, and variations of the electron content is found in subsequent sections. An overhead value of the electron content of the ionosphere has been determined from each pass and will be found in the Appendix.

The observed daily values of the solar flux at 2800 MHz , $S$, varied from 69.9 to $85.6 \times 10^{-22}$ watts $/ \mathrm{m}^{2} / \mathrm{Hz}$. The magnitude of $S$ is an indication of the intensity of the solar radiation which is a source of electron production. These values of $S$ are relatively small so that the total electron content is expected to be small compared to what it is during a period of high solar activity. Furthermore, the changes in $S$ were always less than $5 \%$ from one day to the next.

## 1. Diurnal Variation

The total electron content for southgoing and northgoing passes has been plotted in Figure 5A and 5B respectively with local time as the abscissae. The content at $41^{\circ} \mathrm{N}$ was chosen, regardless of any horizontal gradient. The midday period for the


# DIURNAL VARIATION OF ELECTRON CONTENT FROM SOUTHGONG SATELLITE PASSES 


southgoing passes was in November, 1964, while that for the northgoing was in February, 1965.

As expected there seems to be a steady average increase in total electron content from about sunrise through the day to midafternoon. Also the ratio of daytime maximum to nighttime minimum is between 5 and 10 to 1 which confirms previous results. (14, 16) The maximum values of the electron content seem to occur a little later in the day in November than in February.

In Table 1 these results are compared to those of other years extending from the last sunspot maximum. The solar index $S$ is tabulated as a measure of solar activity. It is apparent from the table that during the years of low solar activity, a slight increase in $S$ corresponds to a small increase in the electron content. During the periods of high solar activity when $S$ has increased by a factor of about three, the content increases by about a factor of about six. The exact relationship between total content and the solar flux index is not clear from this table because the data is grouped to either end of the range. It does seem as though $S$ is a more accurate indicator of electron content during periods of low solar activity.

## 2. Seasonal Variation

Since the data being used cover a span from October to March, it is unlikely that any strong seasonal effect will be noticed. Nevertheless, it is seen from the two diurnal curves that the midday values during February are approximately $5 \%$ to $10 \%$ higher than those in November. It is also noticed that during

| YEAR | LA TITUDE | AUTHOR | ELECTRON CONTENT | S |
| :---: | :---: | :---: | :---: | :---: |
| 1958 | $37.4{ }^{\circ} \mathrm{N}$ | Garriott ${ }^{(13)}$ | $7 \times 10^{17} / \mathrm{m}^{2}$ | 250 |
| 1959 | $40.0^{\circ} \mathrm{N}$ | Yeh ${ }^{(24)}$ | $7.5 \times 10^{17} / \mathrm{m}^{2}$ | 225 |
| 1961 | $40.8{ }^{\circ} \mathrm{N}$ | Hibberd ${ }^{(14)}$ | $1.5 \times 10^{17} / \mathrm{m}^{2}$ | 90 |
| 1963 | $43.0{ }^{\circ} \mathrm{N}$ | Lyon ${ }^{(16)}$ | $1.3 \times 10^{17} / \mathrm{m}^{2}$ | 80 |
| 1964 | $40.8{ }^{\circ} \mathrm{N}$ | Solomon | $0.95 \times 10^{17} / \mathrm{m}^{2}$ | 73 |

## ELECTRON CONTENT IN NOVEMBER

TABLE lA

| YEAR | LATITUDE | AUTHOR | ELECTRON CONTENT | S |
| :---: | :---: | :---: | :---: | :---: |
| 1959 | $37.4{ }^{\circ} \mathrm{N}$ | Garriott ${ }^{(13)}$ | $7 \times 10^{17} / \mathrm{m}^{2}$ | 200 |
| 1959 | $40.8^{\circ} \mathrm{N}$ | Ross ${ }^{(21)}$ | $6 \times 10^{17} / \mathrm{m}^{2}$ | 200 |
| 1962 | $40.8{ }^{\circ} \mathrm{N}$ | Hibberd ${ }^{(14)}$ | $1.5 \times 10^{17} / \mathrm{m}^{2}$ | 100 |
| 1963 | $43.0{ }^{\circ} \mathrm{N}$ | Lyon ${ }^{(16)}$ | $1.3 \times 10^{17} / \mathrm{m}^{2}$ | 80 |
| 1965 | $40.8{ }^{\circ} \mathrm{N}$ | Solomon | $1.05 \times 10^{17} / \mathrm{m}^{2}$ | 75 |

## ELECTRON CONTENT IN FEBRUARY

TABLE 1B

November the values lie much closer to the curve with not nearly as much scattering as there is in February.

## 3. Gradients

Gradients will be measured by normalizing the average slope of an entire pass. In other words, the change in electron content per degree will be divided by the mean content for each pass so that the units of the gradient will be inverse degrees. This method will give the best basis for comparison among gradients from different passes.

The path of the satellite makes a $10.4^{\circ}$ eastward angle with the meridian. This results in a 3 degree longitudinal difference in the ionosphere from one end of the path to the other ( $10^{\circ}$ of latitude) which is equivalent to a thirteen minute difference in local time. Also, the satellite takes nine minutes to travel this path so that the electron content of the ionosphere at the ends of the pass is measured with a twenty-two minute difference in local time. This is much more important near sunrise when there is a large diurnal variation than at midday when a state of equilibrium between electron production and electron loss is nearly reached. The effect of this west-east gradient after sunrise will be discussed in the next section.

A study of all passes between 1800 in the evening and 0700 the following morning reveals that at these times there is generally a north-south gradient (content increases to the south) averaging 0.1; a few representative nighttime gradients are shown in Figure 6A.


NIGHTTIME GRADIENTS

FIGURE 6A

In fact only a small number of passes between these times have virtually no gradient, and only three of them have $\mathrm{s}-\mathrm{n}$ gradients. These three gradients are all rather small and exist only from the station northward. There is no obvious pattern to the magnitude of these gradients; they seem to be entirely unrelated to the values of the previous day.

The average value for daytime gradients between 1000 and 1800 is 0.01 which is ten times smaller than the nighttime value. The first group from the middle of October to the middle of December contains small gradients in either direction. Late afternoon passes are level for the most part with a few having a s-n gradient from the station northward. Two steep gradients from November passes are shown in Figure 6B. Their values of 0.022 and 0.045 are not nearly as high as nighttime values only because the mean value of electron content is so much higher. Around noon the gradient is $\mathrm{n}-\mathrm{s}$ for the most part but it is not very large.

The northgoing passes are for times between 1000 and 1800 from the middle of January to the middle of March. These follow almost the same pattern as the southgoing passes with small gradients in the late afternoon, mostly $\mathrm{n}-\mathrm{s}$, and two spectacular deviations in February similar to those in November. The gradients gradually become predominantly $n-s$ with an unexplained exception once in a while. As before, these exceptions are generally from the station northward.

All satellite passes between 0700 and 1000 in the morning


DAYTIME GRADIENTS

FIGURE 6B
have apparent $n-s$ gradients, the average of the measured values being 0.058. This is about half the nighttime value, but only because of the higher magnitudes of electron content. The reasons for these gradients being much higher than the midday values will be seen in the following section.

The observations of nighttime gradients are in strong agreement with Butler ${ }^{(7)}$ who has indicated a sharp increase in ionization toward the south during all seasons. For midday gradients he gets an average of zero for the fall and a slight $s-n$ gradient for the winter. These results yield slight $\mathrm{n}-\mathrm{s}$ gradients in November and February. One probable reason for the difference is that thesc data cover a larger geographical area. In many cases there is a s-n gradient from the station northward which is overshadowed by n-s gradients from the station southward. Thus, while the overall gradient is $\mathrm{n}-\mathrm{s}$, a ridge of electron content may very well exist to the north of the station. Curves of this nature are shown in Figure 7A.

Over the entire five months of passes, it is observed that many of the electron content vs. latitude curves seem to reach a local minimum just to the south of the station as in Figure 7A. The plot of these minimum points in Figure 7B shows that they are all located within two degrees latitude of the line drawn. This gives rise to the possibility that there is a trough to the south of the station.


LOCAL MINIMUMS SOUTH OF The station

FIGURE 7A

LOCATIONS OF LOCAL MINIMUMS IN ELECTRON CONTENT
FIGURE 7B

## 4. Sunrise Effects

In the morning from 0700 to 1000 , the slope of the diurnal curve is $0.00285 \times 10^{17}$ electrons per minute which is a difference of $0.063 \times 10^{17}$ electrons for twenty-two minutes or ten degrees of ionosphere latitude. In other words, for the period immediately following sunrise, the longitudinal component of the satellite orbit and the travel time of the satellite are the source of a positive slope in the direction of travel of $0.0063 \times 10^{17}$ electrons per degree.

For midday southgoing passes the average slope is $0.0112 \times 10^{17}$ electrons per degree while the slope of electron content from southgoing passes from 0700 to 1000 is $0.0264 \times 10^{17}$ electrons per degree. When the east-west effect is removed, the gradient becomes 0.47 instead of 0.58 and the slope reduces to $0.0201 \times 10^{17}$ electrons per degree. This is almost a factor of two higher than the midday slope.

During the five months observations, sunrise was earlier to the south along a given meridian in the northern hemisphere. As a result, from sunrise on, the ionosphere to the south has been exposed for a longer period of time to the sun's ultraviolet rays which are the source of electron production. In the course of the day, this becomes negligible, but immediately after sunrise a pronounced effect is expected; i. e., there should be a large n-s gradient which gradually levels off to the midday value.

In Figure 8 there are curves from four passes when sunrise was at 0722 EST for $40^{\circ} \mathrm{N}$ and $75^{\circ} \mathrm{W}$. The gradient is largest for the pass right around sunrise when the ionosphere below $40^{\circ} \mathrm{N}$ has


SATELLITE PASSES AFTER SUNRISE

FIGURE 8
already been exposed to the sun and that above $40^{\circ} \mathrm{N}$ has not. On the whole, the gradient decreases as expected.

During the end of December the difference between sunrise times at each end of a ten-degree satellite pass is thirty minutes. In the morning hours, this is equivalent to a difference of $0.0086 \times 10^{17}$ electrons per degree of latitude. Since the average morning slope was shown to be $0.0201 \times 10^{17}$ electrons per degree in the north-south direction, that part of the slope not contributed by the sunrise difference is $0.0115 \times 10^{17}$ electrons per degree. This is essentially equal to the average midday slope of $0.0112 \times 10^{17}$ electrons per degree.
5. Day-to-day Variation

Day-to-day variations are observed in total electron content as well as in the gradient. The variations are believed to be real and not due to random experimental error. The usual day-to-day variation is about $10 \%$ from the mean but occasionally there is an extremely large variation; this is generally in the form of an increase in electron content. The cause of these variations may be associated either with a variable electron production process or with the mechanisms of ionization removal or transfer.

The daily electron content at the station latitude is plotted in Figures 9A and 9B for two large intervals of time. In both graphs the value nearest to noon was used when there were two daytime values; the approximate times are shown. Below each curve is a plot of the planetary magnetic disturbance index for each

figure 9a


FIGURE 98
day. The upper curve clearly shows the day-to-day variations; care must be taken to distinguish diurnal from daily variations. The beginning of Figure 9 B is in the late afternoon when the diurnal variation is rapid. Since the times of observation vary each day, daily variations are inaccurate at those times when the diurnal variation is large. During the middle of the day, however, the observation time will not be as important.

An attempt is made to correlate variations in electron content with magnetic activity as measured by the planetary magnetic index $K_{p}$. This index is calculated for eight 3-hour intervals every day. Figures 9A and 9B show no readily apparent correlation between the two quantities in question. This is in line with the results of Ross ${ }^{(21)}$ and Garriott ${ }^{(13)}$ who found no systematic dependence during the winter months although both Ross and Lyon ${ }^{(16)}$ have shown an inverse relationship during the summer months.

Individual passes where the electron content has undergone a large variation from the me an or expected value are now examined. The behavior of $K_{p}$ for the period preceding the pass will then be discussed. (Unless stated otherwise, $\Sigma K_{p}$ will refer to the 24 hours before the pass, and all times will be in Eastern Standard Time.)

The first group of passes are shown in Figure 10. All but one of them have steep n-s gradients and the magnitudes of the electron content are considerably higher than those of other days. Previous to the two November 15 passes, $\Sigma K_{p}=4-1 / 3$ for the


STEEP N-S GRADIENTS AND LARGE ELECTRON CONTENT

FIGURE 10
period from 1300 on November 13 to 0700 on November 15. Then the value of $\mathrm{K}_{\mathrm{p}}$ immediately rose to between 3 and 4 through the times of the passes at 1245 and 1429. The run of November 26 at 1231 was also preceded by a quiet period, but earlier that day $K_{p}$ went up to over 3. On February 6 there was a sudden commencement between 0700 and 1000 and $K_{p}$ went up to $4-2 / 3$ for the next three hours; the pass was two hours later at 1513. Another pass which was 105 minutes later yielded normal results. The final pass of the set on February 23 at 1415 was characterized by $K_{p}=4-1 / 3$ from 1000-1300 and $K_{p}=4-2 / 3$ from 1300-1600. Also there had been a sudden commencement at 0600. The previous pass at 1230 showed an electron content which was slightly higher than normal and a small $\mathrm{s}-\mathrm{n}$ gradient. Further sets of results will be examined before attempting to draw any conclusions.

In Figure 11 there are three early afternoon passes whose magnitudes are somewhat low for that particular time of day. Preceding the November 25 pass, $\Sigma K_{p}=1-1 / 3$ which is extremely low. On the other hand, for the November 30 pass $\Sigma K_{p}=18$, and $\Sigma K_{p}=11$ before the February 22 pass. There certainly cannot be any correlation inferred from this set of results.

The electron content curves in Figure 12 are slightly higher than usual in magnitude with a substantial s-n gradient. For the November 22 pass, $\Sigma K_{p}=11-1 / 3$ which is not an unusual value. Similarly, $\Sigma K_{p}=12-1 / 3$ before the March 2 results and $\Sigma K_{p}=6-2 / 3$,


EARLY AFTERNOON PASSES DISPLAYING SMALL ELECTRON CONTENT
figure II


S-N GRADIENTS

FIGURE 12
a low value, before the March 9 pass. Once again there is no clear relationship to be found.

The three passes shown in Figure 13 have relatively steep n-s gradients although the magnitudes of their electron contents are as expected. The December 16 run at 0925 is characterized by $K_{p}$ being equal to $5-1 / 3$ during the period from 0700-1000. For the March 4 passes, $\Sigma K_{p}=29-1 / 3$ which is a relatively large index.

Figure 14 shows passes displaying unusual behavior at night. The early morning pass on December 5 had a higher electron content than would be expected for that time and was associated with a relatively low value of $\Sigma \mathrm{K}_{\mathrm{p}}=7-2 / 3$. Prior to the January 22 pass at $0521_{n} K_{p}=5-2 / 3$ from 0100 to 0400, and before the March 3 pass 2t $2336 K_{p}=6$ from 2200 to 0100 the next day. Both of these curves have steep n -s gradients.

It seems as though there might be an association between the electron content and the magnetic activity. For both sets of afternoon passes having steep $n-s$ gradients, $K_{p}$ had risen up to between three and six before each of these passes. Other daytime passes which follow times when $K_{p}$ rises to over three are examined. Four of the six passes in this category showed either a higher electron content than usual or a reasonably steep gradient; the other two had average results.

Re-examining the nighttime results, it is noticed that the passes with steep $\mathrm{n}-\mathrm{s}$ gradients were preceded by high values of $K_{p}$. Checking back through the entire five months, it is seen that


STEEP N-S GRADIENTS

FIGURE I3


UNUSUAL BEHAVIOR AT NIGHT
FIGURE 14
every time the value of $K_{p}$ rises above five, there is a steep n-s gradient in the plot of the following nighttime pass.

For both the day and night there is an association of magnetic activity with horizontal north-south gradients in the electron content. This gradient is a result of an increase in electron content to the south and/or a decrease to the north. If the agency responsible for magnetic disturbances increases the temperature of the upper atmosphere to the north there would be a corresponding increase in the vertical transport of oxygen molecules. This in turn would increase the recombination rate, thereby causing a decrease in content to the north.

## VI. SUMMARY

## 1. Conclusions

Studies of the variation of the total electron content of the ionosphere at mid-latitudes have been made based on data taken at University Park, Pennsylvania ( $40.8^{\circ} \mathrm{N}, 77.9^{\circ} \mathrm{W}$ ) from October 24, 1964 to March 16, 1965. The S-66 Beacon Satellite has been used for all observations, and the equipment used for analysis has been described.

Values for $\overline{B \cos \theta \sec \chi}$ were precomputed and the method of unambiguously determining the number of rotations of the plane of polarization of the 41 MHz signal has been discussed. The total electron content was then calculated using this data, with precautionary measures being taken to keep the error to a minimum.

The horizontal gradients are generally positive from north to south averaging 0.1 per degree at night, 0.047 per degree in the morning, and 0.01 per degree during the day. Morning results have been corrected for westueast slopes resulting from the eastward direction of the satellite path and the travel time of the satellite from one end of the observed path to the other. The contribution of an eariier sunrise in the south has also been determined.

There was a large diurnal variation as expected with the ratio of daytime maximum to nighttime minimum being between 5 and 10 to l. In February, the maximum values of electron content occur slightly earlier in the day and are $5 \%$ to $10 \%$ higher than those in November. A comparison between electron content
results during periods of high and low solar activity showed that when the value of the solar flux index was a factor of about three higher the electron content was larger by a factor of about six.

Day-to-day variations averaging about $10 \%$ were observed in the total electron content. There was no systematic dependence between the planetary magnetic disturbance index and the magnitude of the electron content. It was noted that following increases in magnetic activity, there were increases in the horizontal northsouth gradients for both the day and the night. The decrease of content to the north is believed to be the result of an increase in the recombination rate at higher latitudes.

The locations of local minimums in electron content led to the observation of an apparent trough just to the south of the station.

## 2. Suggestions for Further Work

The data presented here were observed over a five-month interval. Seasonal effects on the total electron content and its horizontal gradient cannot be deduced until results are calculated for the remaining seven months of the year. Calculations at times when there is more magnetic activity, will aid in determining a correlation with electron content. Observations must be made continuously over a period of years to ascertain the solar cycle variation.

Dispersive doppler results should be computed and compared with Faraday rotation results; this would be of use in the
investigation of irregularities. The possible trough to the south of the station should be verified by further studies.

An improvement in the determination of the mean ionosphere height at which to evaluate the geomagnetic field would increase the accuracy of the Faraday rotation method.

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## APPENDIX

## Experimental Data

The following tabulations of data list the more important quantities derived from the satellite passes which have been analyzed. For each pass, a null was picked at the time closest to when the satellite crossed the station latitude. The height of the satellite is given at the time of this null, along with the latitude and longitude of the ionosphere point, which determines the location of the vertical meter square column for which the electron content is calculated. The results are given in chronological order with the following notation.

DATE - date of record in Eastern Standard Time
TIME - time of the null in Eastern Standard Time
HEIGHT - height of the satellite in Km at TIME
LONG - longitude of the meter square column in degrees west

LAT - latitude of the meter square column in de grees north
$\int N d h$ - electron content of the meter square column up to HEIGHT in units of $10^{17}$.

| DA TE | TIME | HEIGHT | LONG | LAT | $\int \frac{N d h}{}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 641024 | 1640:49 | 906 | 76.07 | 40.87 | 0.851 |
| 641026 | 1550:47 | 910 | 73.10 | 40.98 | 0.982 |
| 641026 | 1736:10 | 912 | 81.66 | 40.29 | 0.531 |
| 641027 | 1617:53 | 912 | 76.30 | 41.10 | 0.920 |
| 641029 | 1712:51 | 919 | 81.90 | 40.98 | 0.647 |
| 641031 | 1622:45 | 924 | 79.10 | 40.88 | 0.753 |
| 641101 | 1505:19 | 927 | 73.59 | 41.03 | 1.079 |
| 641102 | 1717:45 | 931 | 85.93 | 41.05 | 0.711 |
| 641104 | 1627:27 | 937 | 82.18 | 41.02 | 0.848 |
| 641105 | 1509:45 | 939 | 76. 79 | 41.12 | 1.022 |
| 641105 | 1655:14 | 942 | 86.17 | 40.83 | 0.843 |
| 641106 | 0441:26 | 909 | 78. 34 | 41.30 | 0.198 |
| 641106 | 1352:54 | 946 | 70.01 | 40.93 | 0.878 |
| 641106 | 1537:24 | 945 | 79.36 | 40.86 | 0.985 |
| 641107 | 0323:18 | 908 | 72.68 | 40.78 | 0.154 |
| 641107 | 0508:02 | 909 | 81.49 | 40.60 | 0.230 |
| 641108 | 0341:08 | 906 | 75.81 | 41.10 | 0.163 |
| 641109 | 1330:03 | 956 | 70.46 | 41.11 | 0.972 |
| 641109 | 1514:28 | 954 | 79.54 | 41.11 | 0.924 |
| 641110 | 1357:14 | 959 | 74.25 | 41.03 | 0.913 |
| 641110 | 1542:14 | 960 | 82.43 | 40.89 | 0.819 |
| 641111 | 0328:08 | 901 | 76.01 | 40.79 | 0.240 |
| 641111 | 1424:31 | 963 | 77.06 | 40.98 | 0.862 |
| 641112 | 0355:24 | 900 | 78.81 | 40.67 | 0.159 |
| 641112 | 1451:54 | 967 | 79.65 | 41.02 | 0.994 |
| 641113 | 0238:04 | 898 | 73.00 | 40.97 | 0.218 |
| 641113 | 1334:33 | 972 | 74.46 | 41.02 | 1.020 |
| 641113 | 1519:30 | 973 | 82.56 | 40.96 | 0.893 |
| 641114 | 1547:08 | 978 | 86.58 | 41.00 | 0.884 |
| 641115 | 0332:34 | 896 | 79.00 | 40.55 | 0.172 |
| 641115 | 1244:34 | 980 | 71.19 | 41.21 | 1.454 |
| 641115 | 1429:24 | 980 | 79.74 | 40.85 | 1.416 |
| 641116 | 0215:10 | 895 | 73. 20 | 40.75 | 0.206 |
| 641117 | 1339:05 | 987 | 77. 38 | 41.06 | 0.940 |
| 641118 | 1222:03 | 993 | 71.39 | 41.03 | 0.895 |
| 641118 | 1406:27 | 991 | 79.92 | 41.13 | 0.950 |
| 641119 | 0152:47 | 894 | 73.34 | 41.12 | 0.165 |
| 641119 | 0337:54 | 894 | 81.92 | 41.45 | 0.193 |
| 641120 | 0219:45 | 893 | 76.51 | 40.54 | 0.177 |
| 641120 | 1316:28 | 1000 | 77.51 | 41.02 | 0.940 |
| 641121 | 0102:10 | 893 | 69.47 | 40.76 | 0.248 |
| 641121 | 0246:52 | 893 | 79.39 | 40.27 | 0.143 |
| 641121 | 1149:32 | 1007 | 71.61 | 40.88 | 0.878 |
| 641121 | 1343:51 | 1004 | 80.03 | 41.07 | 0.856 |
| 641122 | 1226:41 | 1009 | 75.00 | 40.88 | 1.050 |
| 641123 | 0157:26 | 894 | 76.57 | 40.97 | 0.134 |
| 641123 | 1253:56 | 1013 | 77.62 | 40.91 | 0.976 |
| 641125 | 1203:47 | 1020 | 75. 26 | 41.09 | 0.678 |


| DATE | TIME | HEIGHT | LONG | LAT | $\int \underline{N d h}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 641126 | 1231:24 | 1026 | 77. 72 | 40.80 | 1. 343 |
| 641127 | 1114:02 | 1029 | 72.26 | 41.05 | 0.913 |
| 641127 | 1258:31 | 1028 | 80.28 | 41.12 | 0.820 |
| 641128 | 1141:12 | 1032 | 75.42 | 41.03 | 0.868 |
| 641128 | 1326:07 | 1032 | 83.25 | 41.16 | 0.932 |
| 641129 | 0112:18 | 902 | 76.80 | 41.29 | 0.053 |
| 641129 | 1208:24 | 1035 | 77.97 | 41.13 | 0.873 |
| 641130 | 1051:11 | 1040 | 72.62 | 41.23 | 0.967 |
| 641130 | 1234:12 | 1047 | 78.73 | 40.31 | 0.653 |
| 641201 | 0021:28 | 903 | 74. 17 | 40.58 | 0.206 |
| 641201 | 1303:41 | 1045 | 83.37 | 40.95 | 0.875 |
| 641202 | 1146:12 | 1048 | 78.01 | 40.74 | 0.841 |
| 641203 | 1028:43 | 1050 | 72. 77 | 41.05 | 0.758 |
| 641203 | 1213:37 | 1051 | 80.48 | 40.78 | 0.871 |
| 641204 | 1056:03 | 1053 | 75. 71 | 40.90 | 0.782 |
| 641204 | 1240:45 | 1053 | 83.54 | 41.28 | 0.836 |
| 641205 | 0026:36 | 914 | 77.17 | 40.99 | 0.388 |
| 641205 | 1123:35 | 1057 | 78.13 | 40.73 | 0.895 |
| 641205 | 2312:06 | 938 | 67.13 | 36.75 | 0.067 |
| 641206 | 1006:06 | 1060 | 73.00 | 41.03 | 0.696 |
| 641206 | 1150:45 | 1059 | 80.64 | 41.02 | 0.742 |
| 641206 | 2337:07 | 922 | 74.37 | 41.76 | 0.145 |
| 641207 | 0120:35 | 916 | 83.48 | 40.40 | 0.112 |
| 641207 | 1033:31 | 1063 | 75.84 | 40.80 | 0.831 |
| 641207 | 1218:18 | 1063 | 83.68 | 41.11 | 0.894 |
| 641207 | 0003:51 | 921 | 77. 32 | 40.95 | 0.212 |
| 641208 | 1100:49 | 1064 | 78. 28 | 40.85 | 0.853 |
| 541209 | 0943:33 | 1068 | 73.17 | 40.93 | 0.636 |
| 641209 | 1128:04 | 1067 | 80. 78 | 41.08 | 0.750 |
| 641210 | 0058:38 | 928 | 83.18 | 41.26 | 0.197 |
| 641210 | 1155:31 | 1070 | 83.86 | 41.30 | 0.902 |
| 641211 | 1037:59 | 1071 | 78.45 | 41.04 | 0.676 |
| 641212 | 1105:31 | 1073 | 80.91 | 41.03 | 0.636 |
| 641213 | 0947:52 | 1075 | 76.25 | 41.14 | 0.606 |
| 641213 | 1132:49 | 1075 | 84.04 | 41.41 | 0.884 |
| 641214 | 1015:30 | 1077 | 78.55 | 40.91 | 0.816 |
| 641214 | 2200:43 | 943 | 71.72 | 40.95 | 0.187 |
| 641215 | 0858:12 | 1078 | 73.61 | 41.00 | 0.674 |
| 641215 | 1042:58 | 1079 | 81.03 | 40.97 | 0.768 |
| 641215 | 2228:38 | 950 | 75.02 | 41.30 | 0.112 |
| 641216 | 0925:23 | 1079 | 76.36 | 41.03 | 0.741 |
| 641216 | 1110:44 | 1080 | 84.19 | 40.86 | 0.936 |
| 641217 | 0952:57 | 1081 | 78.67 | 40.86 | 0.709 |
| 641218 | 0835:39 | 1082 | 73.76 | 40.92 | 0.537 |
| 641218 | 1020:08 | 1081 | 81.21 | 41.19 | 0.733 |
| 641219 | 0902:58 | 1082 | 76.44 | 40.84 | 0.712 |
| 641219 | 1048:10 | 1083 | 84.39 | 40.83 | 0.779 |


| DATE | TIME | HEIGHT | LONG | LAT | $\int N d h$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 641219 | 2233:00 | 965 | 77.95 | 40.89 | 0.112 |
| 641220 | 0930:06 | 1083 | 78.84 | 41.06 | 0.646 |
| 641220 | 2115:31 | 968 | 72.27 | 41.12 | 0.107 |
| 641220 | 2259:43 | 965 | 80.83 | 40.30 | 0.140 |
| 641221 | 0812:43 | 1083 | 74.07 | 41.21 | 0.428 |
| 641221 | 0957:30 | 1083 | 81.37 | 41.22 | 0.579 |
| 641222 | 0840:05 | 1084 | 76.66 | 41.08 | 0.452 |
| 641222 | 1025:35 | 1083 | 84.62 | 40.83 | 0.728 |
| 641222 | 2211:25 | 985 | 77.76 | 41.93 | 0.113 |
| 641223 | 0907:42 | 1083 | 78.94 | 40.88 | 0.515 |
| 641229 | 2007:16 | 1004 | 73.04 | 40.82 | 0.185 |
| 641229 | 2152:38 | 1008 | 80.83 | 41.24 | 0.163 |
| 641230 | 0704.54 | 1079 | 74.52 | 41.14 | 0.216 |
| 641230 | 0850:14 | 1077 | 81.81 | 40.71 | 0.384 |
| 641230 | 2034:36 | 1008 | 76.01 | 41.61 | 0.180 |
| 641231 | 0732:33 | 1076 | 76.97 | 40.80 | 0.252 |
| 641231 | $0917: 24$ | 1077 | 85.35 | 41.24 | 0.565 |
| 641231 | 2102:18 | 1014 | 78.46 | 40.87 | 0.176 |
| 650101 | 0800:10 | 1075 | 79.30 | 40.62 | 0.304 |
| 650101 | 1944:47 | 1018 | 73.27 | 41.00 | 0.191 |
| 650102 | 0827:04 | 1074 | 82.04 | 41.25 | 0.436 |
| 650102 | 2012:09 | 1021 | 76. 14 | 40.80 | 0.305 |
| 650102 | 2157:15 | 1024 | 84.25 | 41.44 | 0.209 |
| 650103 | 0855:12 | 1069 | 85.64 | 40.81 | 0.420 |
| 650103 | 2040:05 | 1029 | 78.49 | 41.27 | 0.175 |
| 650104 | 0737:16 | 1069 | 79.50 | 40.88 | 0.286 |
| 650104 | 1922:04 | 1029 | 73.51 | 40.92 | 0.240 |
| 650104 | 2106:21 | 1026 | 81.49 | 40.34 | 0.165 |
| 650105 | 0805:05 | 1065 | 82.19 | 40.63 | 0.327 |
| 650105 | 1950:34 | 1039 | 76.11 | 41.78 | 0.198 |
| 650106 | 0647:27 | 1063 | 77.20 | 40.67 | 0.203 |
| 650106 | 0832:07 | 1064 | 85.94 | 41.34 | 0.397 |
| 650106 | 20177.54 | 1042 | 78.50 | 41.67 | 0.145 |
| 650107 | 0714:47 | 1061 | 79.62 | 40.75 | 0.194 |
| 650107 | 2044:51 | 1044 | 81.14 | 41.42 | 0.257 |
| 650108 | 1927:31 | 1048 | 76.32 | 41.38 | 0.193 |
| 650109 | 1809:27 | 1047 | 70.33 | 41.28 | 0.376 |
| 650110 | 0507:12 | 1053 | 71.88 | 41.19 | 0.293 |
| 650110 | 0534:25 | 1050 | 75.06 | 41.09 | 0.199 |
| 650111 | 1904:16 | 1054 | 76.59 | 40.82 | 0.274 |
| 650112 | 0602:24 | 1044 | 77.41 | 40.47 | 0.088 |
| 650112 | 1746:37 | 1055 | 70.63 | 41.02 | 0.493 |
| 650112 | 1932:18 | 1059 | 78.83 | 41.38 | 0.268 |
| 650113 | 0444:29 | 1044 | 73.08 | 41.25 | 0.163 |
| 650113 | 0629:32 | 1042 | 79.93 | 40.74 | 0.149 |
| 650113 | 1814:05 | 1039 | 74.12 | 40.82 | 0.334 |
| 650113 | 1959:02 | 1060 | 81.62 | 40.93 | 0.203 |


| DATE | TIME | HEIGHT | LONG | LAT |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 650114 | 0511:58 | 1039 | 75.11 | 40.91 | 0.149 |
| 650114 | 1842:08 | 1065 | 76.65 | 41.26 | 0.165 |
| 650115 | 0539:09 | 1037 | 77.67 | 41.00 | 0.198 |
| 650115 | 1724:06 | 1064 | 70.91 | 41.12 | 0.289 |
| 650115 | 1909:15 | 1066 | 79.06 | 41.02 | 0.188 |
| 650116 | 0606.05 | 1036 | 80.22 | 41.51 | 0.165 |
| 650116 | 1751:27 | 1066 | 74.31 | 40.80 | 0.363 |
| 650116 | 1937:04 | 1070 | 81.49 | 41.55 | 0.224 |
| 650117 | 0449:27 | 1027 | 75. 20 | 40.79 | 0.221 |
| 650117 | 0634:19 | 1027 | 83.09 | 40.90 | 0.165 |
| 650117 | 1819:04 | 1070 | 76.88 | 40.87 | 0.371 |
| 650117 | 2003:04 | 1067 | 85.82 | 40.51 | 0.326 |
| 650118 | 1846:48 | 1073 | 79.15 | 41.17 | 0.191 |
| 650119 | 0359:27 | 1019 | 72.20 | 40.95 | 0.227 |
| 650119 | 0544:59 | 1014 | 80. 18 | 40.04 | 0.203 |
| 650119 | 1914:52 | 1078 | 81.47 | 41.94 | 0.188 |
| 650120 | 0426:34 | 1017 | 75.41 | 40.99 | 0.200 |
| 650120 | 0611:56 | 1013 | 83.34 | 40.62 | 0.159 |
| 650120 | 1756.34 | 1077 | 76.99 | 40.97 | 0.356 |
| 650121 | 0453:16 | 1016 | 78. 10 | 41.55 | 0.182 |
| 650121 | 1638:38 | 1077 | 71.44 | 40.86 | 0.673 |
| 650121 | 1823:52 | 1078 | 79.36 | 40.90 | 0.404 |
| 650122 | 0336:52 | 1007 | 72.29 | 40.85 | 0.157 |
| 650122 | 0521:02 | 1011 | 80.52 | 41.28 | 0.221 |
| 650122 | 1706:22 | 1079 | 74.64 | 40.93 | 0.574 |
| 650122 | 1851:26 | 1080 | 81.94 | 41.23 | 0.282 |
| 650123 | 0403:37 | 1006 | 75.63 | 41.25 | 0.225 |
| 650123 | 0549:28 | 1000 | 83.61 | 40.43 | 0.200 |
| 650123 | 1733:49 | 1080 | 77. 15 | 40.86 | 0.462 |
| 650123 | 1918:17 | 1080 | 85.87 | 41.05 | 0.267 |
| 650124 | 0432:05 | 996 | 77.94 | 40.17 | 0.094 |
| 650124 | 1616:03 | 1081 | 71.67 | 40.87 | 0.643 |
| 650125 | 0458:33 | 997 | 80.68 | 41.12 | 0.116 |
| 650125 | $1644{ }^{\circ} 00$ | 1083 | 74. 78 | 41.15 | 0.601 |
| 650125 | 1828.30 | 1082 | 82.24 | 40.95 | 0.343 |
| 650126 | 0340:41 | 996 | 75.83 | 41.50 | 0.141 |
| 650126 | 0526:27 | 989 | 83.89 | 40.82 | 0.216 |
| 650126 | 1711:22 | 1083 | 77.24 | 41.01 | 0.549 |
| 650128 | 0436:19 | 982 | 80.81 | 40.69 | 0.171 |
| 650128 | 1621:20 | 1083 | 74.93 | 41.10 | 0.895 |
| 650128 | 1805:55 | 1084 | 82.39 | 41.01 | 0.640 |
| 650129 | 0318:13 | 982 | 75.88 | 41.29 | 0.160 |
| 650129 | 1648:58 | 1084 | 77.33 | 41.19 | 0.662 |
| 650130 | 1531:10 | 1084 | 73.08 | 41.20 | 0.776 |
| 650131 | 0229:04 | 968 | 72.55 | 40.56 | 0.138 |
| 650131 | 0413:20 | 971 | 81.05 | 41.01 | 0.159 |
| 650131 | 1558:44 | 1084 | 75.08 | 41.11 | 0.870 |


| DATE | TIME | HEIGHT | LONG | $\underline{\text { LA T }}$ | $\int N d h$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 650131 | 1743:26 | 1084 | 82.51 | 41. 14 | 0.657 |
| 650201 | 0255:26 | 970 | 76.04 | 41.38 | 0.230 |
| 650201 | 1625:50 | 1084 | 77.56 | 40.75 | 0.606 |
| 650201 | 1811.06 | 1084 | 86.08 | 41.83 | 0.525 |
| 650202 | 0138:48 | 961 | 68.57 | 41.11 | 0.331 |
| 650202 | 0323:26 | 962 | 78.46 | 40.74 | 0.189 |
| 650202 | 1653:15 | 1083 | 79.95 | 40.82 | 0.627 |
| 650203 | 0350:18 | 961 | 81.29 | 41.37 | 0.174 |
| 650203 | 1536:11 | 1082 | 75.21 | 41.16 | 0.757 |
| 650203 | 1720:28 | 1082 | 82.86 | 40.85 | 0.641 |
| 650204 | 0417:53 | 956 | 84.80 | 41.47 | 0.157 |
| $650<04$ | 1603:0, | 1081 | 77.70 | 40.69 | 0.911 |
| 650204 | 1749:02 | 1080 | 85. 94 | 42.45 | 0.721 |
| 650205 | 0301:11 | 948 | 78.51 | 40.30 | 0.306 |
| 650205 | 1445:56 | 1080 | 72.46 | 41.21 | 0.925 |
| 650205 | 1631:02 | 1079 | 79.98 | 41.17 | 0.684 |
| 650206 | 0143:32 | 945 | 72.92 | 40.69 | 0.196 |
| 650206 | 0327:50 | 948 | 81.46 | 41.14 | 0.106 |
| 650206 | 1513:26 | 1079 | 75.37 | 41.03 | 1.113 |
| 650206 | 1658:09 | 1078 | 82.91 | 41.14 | 0.768 |
| 550207 | 1540:50 | 1076 | 77. 76 | 40.96 | 0.886 |
| 650208 | 1423:22 | 1075 | 72.61 | 41.22 | 0.785 |
| 650208 | 1608:13 | 1075 | 80.18 | 41.03 | 1.025 |
| 650209 | 1450:53 | 1073 | 75.49 | 41.10 | 1.027 |
| 650209 | 1635:14 | 1074 | 83. 26 | 40.90 | 0.834 |
| 650210 | 0148:05 | 932 | 76. 17 | 40.63 | 0.199 |
| 650210 | 1518:24 | 1070 | 77.81 | 41.13 | 0.997 |
| 650211 | $1400: 24$ | 1069 | 72.80 | 40.88 | 0.981 |
| c50211 | 1545:34 | 1068 | 80.34 | 41.01 | 0.847 |
| 650212 | 0057:40 | 927 | 73.42 | 41.13 | 0.197 |
| 650212 | 0242:41 | 926 | 81.84 | 40.88 | 0.125 |
| 6.50212 | 1428:15 | 1056 | 75.62 | 41.10 | 0.801 |
| 650212 | 1612:42 | 1067 | 83.43 | 41.01 | 0.792 |
| 650113 | 1455:40 | 1063 | 78.02 | 41.05 | 0.975 |
| 650214 | 0151:59 | 923 | 79. 18 | 41.37 | 0.102 |
| 650214 | 1337:48 | 1062 | 72.95 | 40.89 | 1. 088 |
| 650214 | 1522:55 | 1061 | 80.51 | 41.02 | 0.867 |
| 650214 | 0035:02 | 917 | 73.52 | 41.02 | 0.205 |
| 650215 | 0220:11 | 916 | 82.03 | 40.63 | 0.154 |
| 650215 | 1405:25 | 1059 | 75.78 | 40.91 | 0.935 |
| 650215 | 1550:13 | 1058 | 83.60 | 41.17 | 0.891 |
| 650216 | 1432:47 | 1056 | 78.22 | 40.83 | 0.847 |
| 650217 | $0129: 51$ | 912 | 79.23 | 40.78 | 0.142 |
| 650217 | 1315:27 | 1053 | 73.07 | 41.17 | 0.853 |
| 650217 | 1500:06 | 1063 | 80.74 | 40.86 | 0.870 |
| 650218 | 0012:22 | 910 | 73.64 | 40.95 | 0.154 |
| 650218 | 0157:11 | 909 | 82.27 | 40.95 | 0.159 |


| DATE | TIME | HEIGHT | $\underline{\text { LONG }}$ | LAT | $\int N d h$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 650218 | 1342:47 | 1050 | 75. 91 | 40.92 | 1. 100 |
| 650219 | 0039:21 | 909 | 76.76 | 41.19 | 0.151 |
| 650219 | 1410:11 | 1046 | 78. 34 | 40.08 | 1.052 |
| 650220 | 0106:52 | 906 | 79.43 | 41.04 | 0.163 |
| 650220 | 1252:46 | 1043 | 73.21 | 41.12 | 1.083 |
| 650220 | 1437:47 | 1042 | 80.80 | 41.17 | 0.944 |
| 650220 | 2349:06 | 905 | 74. 04 | 41.52 | 0.124 |
| 650221 | 0134:24 | 903 | 82.49 | 41.03 | 0.142 |
| 650221 | 1320:02 | 1040 | 76.05 | 40.82 | 0.994 |
| 650221 | 1604:54 | 1039 | 84.11 | 41.20 | 0.712 |
| 650222 | 0017:16 | 900 | 76. 70 | 40.51 | 0.098 |
| 650222 | 1347:41 | 1034 | 78.43 | 41.02 | 0.764 |
| 650223 | 1229:58 | 1032 | 73.36 | 40.97 | 1. 225 |
| 650223 | 1414:57 | 1031 | 81.04 | 41.03 | 1.328 |
| 650224 | 0109:58 | 903 | 83.08 | 43.10 | 0.131 |
| 650224 | 0257:34 | 1027 | 76. 13 | 41.00 | 0.950 |
| 650224 | 1442:05 | 1028 | 84.49 | 41.07 | 0.991 |
| 650225 | 1324:49 | 1024 | 78.62 | 40.85 | 0.875 |
| 650227 | 1234:47 | 1015 | 76. 28 | 40.88 | 0.969 |
| 650227 | 2331:31 | 894 | 77.07 | 40.78 | 0.158 |
| 650228 | 1116:57 | 1014 | 69.88 | 40.92 | 0.788 |
| 650228 | 1302:09 | 1012 | 78. 76 | 40.86 | 0.999 |
| 650311 | 1144:46 | 1008 | 73.60 | 41.08 | 1.095 |
| 650301 | 1329:26 | 1008 | 81.50 | 40.88 | 1. 205 |
| 650301 | 2241:41 | 893 | 74.15 | 40.58 | 0.218 |
| 650302 | 1212:09 | 1003 | 76.40 | 40.92 | 1.088 |
| 650302 | 1356:44 | 1004 | 85.10 | 41.10 | 1.231 |
| 650303 | 2335.50 | 893 | 80.07 | 41.14 | 0.347 |
| ó50304 | 1122:06 | 995 | 73.72 | 41.09 | 0.960 |
| 650304 | 1306:52 | 996 | 81.65 | 41.02 | 0.883 |
| 650304 | 2218:44 | 893 | 74.41 | 40.80 | 0.237 |
| 650305 | 1149:36 | 991 | 76.49 | 40.99 | 1.077 |
| 650306 | 1217:01 | 986 | 79.00 | 41.09 | 1.005 |
| 650307 | 1059:15 | 984 | 73.86 | 40.91 | 1.139 |
| 650307 | 1244:02 | 984 | 81.92 | 40.89 | 1.113 |
| 650307 | 2155:54 | 894 | 74.61 | 40.88 | 0.210 |
| 650307 | 2340.34 | 895 | 83.52 | 41.22 | 0.141 |
| 650308 | 1126:59 | 977 | 76.60 | 41.13 | 0.964 |
| 650308 | 2222:37 | 896 | 77.67 | 41.47 | 0.201 |
| 650309 | 1009:08 | 975 | 70.31 | 41.15 | 1.035 |
| 650309 | 1154:11 | 974 | 78.72 | 40.95 | 1.256 |
| 650309 | 2106:35 | 898 | 70.63 | 40.16 | 0.312 |
| 650310 | 1036:38 | 971 | 73.97 | 40.98 | 0.907 |
| 650310 | 1221:31 | 970 | 82.06 | 41.09 | 0.983 |
| 650310 | 2133:06 | 898 | 74.81 | 70.94 | 0.264 |
| 650311 | 1104:04 | 966 | 76.77 | 40.92 | 0.742 |
| 650311 | 1248:29 | 968 | 86.24 | 40.96 | 0.941 |


| DATE | TIME | HEIG HT | LONG | LAT | $\int \underline{N d h}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 650311 | 2200:38 | 900 | 77.61 | 40.66 | 0.231 |
| 650312 | 1132:32 | 961 | 79.33 | 41.01 | 0.826 |
| 650312 | 2043:03 | 901 | 71.44 | 41.11 | 0.360 |
| 650312 | 2227:28 | 900 | 80.54 | 41.29 | 0.241 |
| 650313 | 1013:53 | 959 | 74. 10 | 40.94 | 0.901 |
| 650313 | 1158:42 | 959 | 83.34 | 40.98 | 1. 230 |
| 650313 | 2110:34 | 903 | 74.90 | 40.69 | 0.357 |
| 650314 | 1041:14 | 955 | 76.92 | 40.80 | 0.965 |
| 650314 | 2137:23 | 905 | 77.94 | 41.19 | 0.401 |
| 650315 | 0923:31 | 952 | 70.62 | 40.86 | 0.775 |
| 650315 | 1108:46 | 950 | 79.50 | 40.97 | 0.785 |
| 650316 | 1135:49 | 948 | 82.65 | 40.82 | 0.873 |

## Acknowledgments

I wish to express my thanks to Dr. William J. Ross for his suggestion of this problem and his guidance throughout the course of the work. The assistance of the technicians and computresses of the Ionosphere Research Laboratory is acknowledged.

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