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ELECTRODYNAMICS OF MOVING MEDIA

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I
INTRODUCTION

Several problems are currently being investigated under this grant. The details are described in Sections II, III and IV. As a result of this grant, the Radiation Laboratory of The University of Michigan is able to support, in a part-time basis, two graduate students working in this area. The work by V. Pyati on the radiation due to an oscillating dipole over a lossless semi-infinite moving dielectric medium has been supported in part by Grant NGR-23-005-107. This study provides the thesis material for Mr. Pytai's Ph. D dissertation and will soon be published as a technical report. The study on the field of a moving charge in a bounded region has yielded some interesting results which may have some close bearing on the understanding of the excitation of Cerenkov radiation. The foundation of electrodynamics of media is further consolidated by a thorough investigation of the EHPMv formulation based upon Born's equations. It is hoped that this will end the lack of comprehension that has resulted from considering the EHPMv formulation as a new and independent formulation of the electrodynamics of moving media.

II

DEFINITE AND INDEFINITE FORMS OF MAXWELL-MINKOWSKI EQUATIONS

Much work has been published recently about the different formulations of electrodynamics of moving media. The problem was practically resolved in a previous study (Tai, 1964) however much confusion remains as can be seen in the articles appearing in recent issues of the Proceedings of the Institute of Electrical and Electronics Engineers (Szablya, 1965; Unz, 1965; Penfield and Szablya, 1965 and Tai and Szablya, 1965). In summarizing these articles, it appears that some of the authors still do not appreciate Minkowski's great work, while others still think the EHPMv formulation is an independent formulation. To clarify the matter further we have re-examined the definite and indefinite forms of Maxwell-Minkowski equations based upon Born's equations. The treatment is contained in a paper by Tai (1965b). The contents of the paper are outlined below.

The relativistic transformation between the polarization and magnetization vectors in two inertial systems is given by

$$\bar{P}' = \bar{\bar{\gamma}} \cdot (\bar{P} - \frac{1}{c^2}\bar{v} \times \bar{M}) \tag{2.1}$$

$$\bar{M}' = \bar{\bar{\gamma}} \cdot (\bar{M} + \bar{v} \times \bar{P}) \tag{2.2}$$

where

$$\bar{\bar{\gamma}} = \begin{bmatrix} (1-\beta^2)^{-1/2} & 0 & 0 \\ 0 & (1-\beta^2)^{-1/2} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\beta = v/c \quad .$$

The velocity of separation between the two inertial systems is assumed to be in the z-direction. Born's equations (Born, 1910) are obtained by solving (2.1) and (2.2) for $\bar{P}$ and $\bar{M}$ and substituting them into the Maxwell equations defined in the unprimed system.

This yields

$$\nabla \times \bar{E} = -\frac{\partial}{\partial t}\mu_o\left[\bar{H} + \bar{\bar{\gamma}}\cdot(\bar{M}' - \bar{v}\times\bar{P}')\right]$$

$$\nabla \times \bar{H} = \bar{J} + \frac{\partial}{\partial t}\left[\epsilon_o\bar{E} + \bar{\bar{\gamma}}\cdot(\bar{P}' + \frac{1}{c^2}\bar{v}\times\bar{M}')\right] .$$

If we define

$$\bar{P}_c = \bar{\bar{\gamma}}\cdot\bar{P}' \tag{2.3}$$

$$\bar{M}_c = \bar{\bar{\gamma}}\cdot\bar{M}' \tag{2.4}$$

$$\epsilon_o\bar{E}_c = \epsilon_o\bar{E} + \frac{1}{c^2}\bar{\bar{\gamma}}\cdot(\bar{v}\times\bar{M}') = \epsilon_o\bar{E} + \frac{1}{c^2}\bar{v}\times\bar{M}_c \tag{2.5}$$

$$\bar{H}_c = \bar{H} - \bar{\bar{\gamma}}\cdot(\bar{v}\times\bar{P}') = \bar{H} - \bar{v}\times\bar{P}_c \quad . \tag{2.6}$$

Then (2. 1) and (2. 2) can be written in the form

$$\nabla\times(\bar{E}_c - \mu_o\,\bar{v}\times\bar{M}_c) = \bar{J} + \frac{\partial}{\partial t}\mu_o(\bar{H}_c + \bar{M}_c) \tag{2.7}$$

$$\nabla\times(\bar{H}_c + \bar{v}\times\bar{P}_c) = \bar{J} + \frac{\partial}{\partial t}(\epsilon_o\bar{E}_c + \bar{P}_c) \quad . \tag{2.8}$$

Equations (2. 7) and (2. 8) are identical to the two equations derived by Chu (Fano, Chu and Adler, 1960), using a kinematic method. The undesirable feature of that derivation is that it postulates the so-called magnetic charge model. As we see here, such a fictitious model is not necessary. We can trace the origin of the field quantities in a precise manner from the relativistic formulation. Other indefinite forms of Maxwell's equations can be derived accordingly in a similar manner. This work shows the precise relations between the field vectors in various formulations using only Minkowski's covariant formulation of electrodynamics of moving media within the framework of the special theory of relativity.

III

RADIATION DUE TO AN OSCILLATING DIPOLE OVER A LOSSLESS SEMI-INFINITE MOVING DIELECTRIC MEDIUM

The solution to two boundary value problems in the electrodynamics of moving media is nearing completion. In both problems, as shown in Fig. 3-1, the upper half space ($z > 0$) is free space and the lower half space ($z < 0$) is filled by a lossless dielectric which is moving with a uniform velocity v in the y-direction.

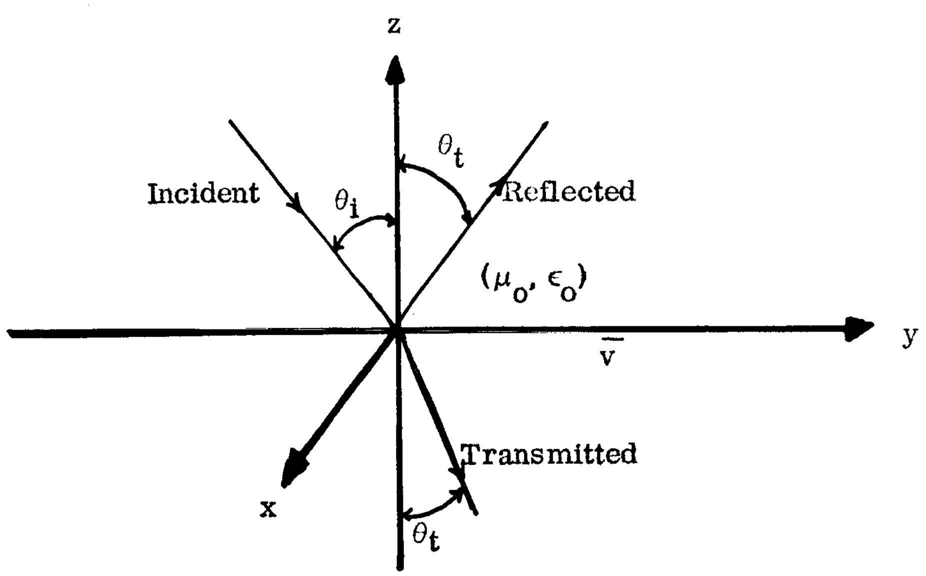


FIG 3-1: GEOMETRY OF THE PROBLEM

The primary problem involves the determination of the radiation field due to an oscillating (Hertzian) dipole located in a vacuum. A secondary problem, the reflection and refraction of a plane electromagnetic wave incident on the moving dielectric, is considered as a preliminary to the more difficult problem above. The latter is a generalization of the work reported by Tai (1964) in the sense that the plane of incidence is not restricted to the yz-plane

Reflection-Refraction Problem

The Maxwell-Minkowski equations in the moving medium, assuming $e^{-i\omega t}$ variation, are given by

$$\nabla \times \underline{E} = i\omega \underline{B} \tag{3.1}$$

$$\nabla \times \underline{H} = -i\omega \underline{D} \tag{3.2}$$

$$\underline{B} = \mu \underline{\underline{\alpha}} \cdot \underline{H} - \underline{\Omega} \times \underline{E} \tag{3.3}$$

$$D = \epsilon \underline{\underline{\alpha}} \cdot \underline{E} + \underline{\Omega} \times \underline{H} \tag{3.4}$$

where

$$\left.\begin{aligned} \underline{\Omega} &= \left(\frac{n^2-1}{1-n^2\beta^2}\right) \frac{\beta}{c}\hat{y} \\ a &= \frac{1-\beta^2}{1-n^2\beta^2} \\ \underline{\underline{\alpha}} &= \begin{bmatrix} a & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & a \end{bmatrix} \end{aligned}\right\} \tag{3.5}$$

$$\beta = \frac{v}{c}, \quad n = \left(\frac{\mu\,\epsilon}{\mu_o\,\epsilon_o}\right)^{1/2}$$

$$k^2 = \omega^2 \mu\, \epsilon \quad .$$

Plane wave solutions of the above equations which represent the transmitted wave can be constructed thus:

$$\underline{E} = (E_{ox}, E_{oy}, E_{oz})e^{iK\delta} \tag{3.6}$$

where

E_{ox}, E_{oy}, E_{oz} are independent of the coordinates

$\delta = x \sin\theta_t \cos\phi + y \sin\theta_t \sin\phi - z\cos\theta_t$,

and K, the propagation constant satisfies the dispersion relation

$$K^2\sin^2\theta_t\cos^2\phi+\frac{1}{a}(K\sin\theta_t\sin\phi+\omega\Omega)^2+K^2\cos^2\theta_t-ak^2=0\ . \tag{3.7}$$

The amplitudes must satisfy the relation

$$K\sin\theta_t\cos\phi_t E_{ox}+\frac{1}{a}(K\sin\theta_t\sin\phi_t+\omega\Omega)E_{oy}-K\cos\theta_t E_{oz}=0. \tag{3.8}$$

The magnetic field is given by

$$\underline{H}=-\frac{i}{\mu\omega}\,\underline{\underline{\alpha}}^{-1}\cdot\left[(\nabla+i\omega\underline{\Omega})\times\underline{E}\right] \tag{3.9}$$

Alternatively, one may start with the magnetic field

$$\underline{H}=(H_{ox},H_{oy},H_{oz})e^{iK\delta} \tag{3.10}$$

and proceed in a similar manner. The rest of the analysis is quite straightforward and the salient results are:

1. Snell's law is modified, thus

$$\sin\theta_t=\sin\theta_i\left[1+\frac{n^2-1}{1-\beta^2}(1-\beta\sin\theta_i\sin\phi)^2\right]^{-1/2} \tag{3.11}$$

2. The reflected and the transmitted waves possess components not present in the incident wave, except when $\phi=\pi/2$, and the phenomenon of Brewster's angle is absent.

3. When $\phi=\pi/2$, the results are quite similar to the non-moving case.

Dipole Problem

Two cases corresponding to a vertical dipole or a horizontal dipole (in the direction of velocity) are treated in detail. In each case, the problem is formulated in terms of the vector and scalar potentials appropriate to each region shown in Fig. 3-1. Formal solution is obtained by applying the boundary conditions at the interface z = 0. Asymptotic expansion of the Fourier integrals is obtained by using the saddle point method. The results will appear in a report now under preparation.

Some allied problems for future research are; 1) magnetic dipole over moving medium, 2) complementary problem in which the sources are located in the moving medium, and 3) extensions to lossy (conducting) media.

IV

FIELD OF A MOVING CHARGE IN A BOUNDED REGION: CERENKOV PHENOMENA IN A WAVEGUIDE

4.1 Introduction

The original analytical work on Cerenkov radiation (Frank and Tamm 1937, Tamm, 1939) treated the problem of a point charge moving in an infinite medium. Their method assumed a point charge moving with velocity $v\hat{z}$ with respect to the stationary medium, giving a current density of

$$\bar{J} = q\,\bar{v}\,\delta(x)\,\delta(y)\,\delta(z-vt) \quad .$$

Nag and Sayied (1956) approached the problem from a moving medium point of view, using the Minkowski theory. That is, they considered the problem of a stationary charge in a moving medium, and transformed the resulting fields.

The problem discussed here is the case of a charge moving along the axis of a cylindrical medium bounded by a conducting shell. This differs from the case above in that there are reflections, and the radiation is enclosed. All three cases discussed above assume a point charge and constant velocity, which result in infinite energy density. The assumption of constant velocity is clearly unreal since the radiation must result in a loss of kinetic energy of the charge, and there are no sources of energy available to maintain the velocity in the problem. A third difficulty is the assumption of constant permeability and permittivity: all real materials exhibit dispersion, a variation of these parameters with frequency. This assumption results in infinite energies, but the character of the fields remains intact.

The problem will be presented in the following manner. First, Tamm's method is used to derive an expression for the vector potential; then the moving medium approach is used to yield the same expression. Finally, we solve the the integrals and interpret the results.

4.2 Solution of the Vector Potential - Tamm's Method

Here the problem is characterized as a point charge q moving along the z-axis with velocity $\bar{v} = -v\hat{z}$. The vector potential in the MKS system satisfies the differential equation

$$\nabla^2\bar{A} - \frac{n^2}{c^2}\frac{\partial^2\bar{A}}{\partial t^2} = \hat{z}\mu q v\ \delta(x)\delta(y)\delta(z+vt)$$

where

$$n^2 = \frac{\mu\,\epsilon}{\mu_o\epsilon_o}\ .$$

Casting this in cylindrical coordinates, applying the boundary condition that $n \times \bar{E} = 0$ at r=b, and applying the Fourier transform ethod, it can be shown that

$$\bar{A}(r,z,t) = \frac{-i\mu q v\hat{z}}{16\pi b\sqrt{n^2\beta^2-1}}\int_0^\infty \left\{ e^{iu\tau}\left[H_0^{(2)}(\alpha u) - \frac{J_o(\alpha u)H_0^{(2)}(u)}{J_o(u)}\right] - e^{-iu\tau}\left[H_0^{(1)}(\alpha u) - \frac{J_o(\alpha u)H_0^{(1)}(u)}{J_o(u)}\right]\right\} du$$

where

$$\alpha = r/b$$

$$u = \frac{\omega b}{v}\sqrt{n^2\beta^2-1}$$

$$\tau = \frac{z+vt}{b\sqrt{n^2\beta^2-1}}\ .$$

4.3 Solution for the Vector Potential - Moving Medium Method

Here the charge is assumed stationary in a medium moving with velocity $\bar{v} = v\hat{z}$. Due to the constitutive relation

$$\bar{D} = \epsilon'\,\bar{\bar{\alpha}}\cdot\bar{E} + \bar{\Omega}\times\bar{H}$$

where the quantities are defined in Section III, the scalar potential satisfies the

$$\frac{a}{r}\frac{\partial}{\partial r} r \frac{\partial \phi}{\partial r} + \frac{\partial^2 \phi}{\partial z^2} = - \frac{q}{\epsilon'} \frac{\delta(r)}{2\pi r} \delta(z) \quad .$$

By Fourier analysis, this has the solution

$$\phi = \frac{+iq}{16\pi\epsilon'} \frac{(n^2\beta^2-1)}{(1-\beta^2)} \int_0^\infty \left\{ e^{i\lambda z}\left[H_0^{(2)}\left(\frac{\lambda r}{a'}\right) - \frac{J_o\left(\frac{\lambda r}{a'}\right) H_0^{(2)}\left(\frac{\lambda b}{a'}\right)}{J_o\left(\frac{\lambda b}{a'}\right)}\right]\right.$$

$$\left. -e^{-i\lambda z}\left[H_0^{(1)}\left(\frac{\lambda r}{a'}\right) - \frac{J_o\left(\frac{\lambda r}{a}\right) H_0^{(1)}\left(\frac{\lambda b}{a'}\right)}{J_o\left(\frac{\lambda b}{a'}\right)}\right]\right\} d\lambda \quad .$$

The relation between the vector potential in the primed system and the scalar potential in the unprimed (stationary) system is

$$\bar{A}(x', y', z') = \hat{z} \; \frac{\mu \epsilon' \, v \sqrt{1-\beta^2}}{(n^2\beta^2-1)} \; \phi(x, y, z) \quad .$$

Using the coordinate transformation

$$x = x', \quad y = y', \quad z = \frac{z' + vt'}{\sqrt{1-\beta^2}}$$

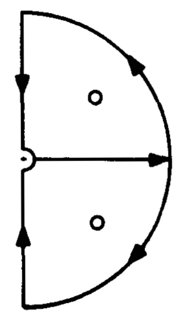
It can be shown that the expression for the vector potential is the same as for Section 4.2, where,

$$\alpha = r'/b, \quad \tau = \frac{z' + vt'}{b\sqrt{n^2\beta^2-1}} \quad , \quad a' = |a|^{1/2}$$

The primed system here corresponds to the unprimed system in Section 4.2.

4.4 Evaluation of the Integral and Interpretation

The integral in the expression for the vector potential can be put in a more tractable form by using the method of contour integration. If the proper contour is chosen, the integral along the infinite quarter-circle will vanish, and



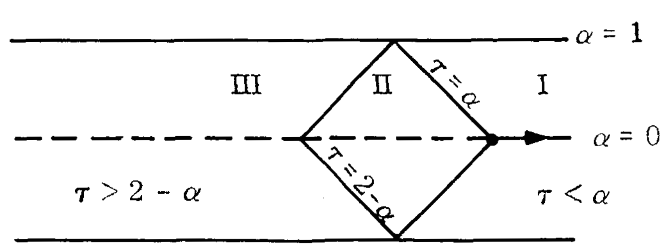
$$\int_0^{\infty} f(u)du = \int_0^{\pm i\infty} f(u)du \pm \text{(residues)}$$

where the plus sign applies when the contour is closed in the upper quarter-circle, and the minus sign below. The Hankel functions have a branch point at u=0, but there is no difficulty here. The Hankel functions have a logarithmic behavior near the origin, and the integral exists as the lower limit approaches zero.

The presence of $J_0(u)$ in the denominator poses a problem, since the Bessel function has its zeros on the real axis. In order to obtain physically realistic results corresponding to the retarded potential, a small imaginary part is introduced into n, negative for the first bracketed term in the integral, positive for the second. This has the effect of lowering or raising the contour below or above the real axis. Thus some contours will enclose these zeros - some will not.

The integrals along the imaginary axis become Laplace transforms of the modified Bessel functions, which are known .

The resulting expression has three distinct regions, shown in the following Fig. 4-1. In regions I and II the solution is the same as in the unbounded case.



$$\bar{A}(r,z,t) = \frac{-\mu q v \hat{z}}{4\pi b\sqrt{n^2\beta^2 - 1}} \left[\begin{array}{lll} 0 & , & \tau < \alpha \\ \dfrac{1}{\sqrt{\tau^2 - \alpha^2}} & , & \alpha < \tau < 2 - \alpha \\ \pi \displaystyle\sum_{m=1}^{\infty} \dfrac{\sin \tau u_m \, J_o(\alpha u_m) N_o(u_m)}{J_1(u_m)} & , & \tau > 2 - \alpha \end{array} \right] ,$$

where u_m denotes the mth zero of $J_o(u)$. Since $J_1(u_m) \approx N_o(u_m)$ for all m, the last sum is approximately

$$\sum_{m=1}^{\infty} \sin \tau \, u_m \, J_o(\alpha u_m)$$

and since

$$u_m \approx \pi\left(m - \frac{1}{4} \right) ,$$

this is approximately

$$\sum_{m=1}^{\infty} \sin \pi \tau \left(m - \frac{1}{4}\right) J_o\left(\alpha\pi \left(m - \frac{1}{4}\right)\right) .$$

This can be seen to be periodic in τ with a period of 8 .

V

PUBLICATIONS AND PAPERS PRESENTED AT CONFERENCES

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VI

PLANS FOR FUTURE WORK

The constitutive relationship of a moving conductive medium will be investigated. We hope to resolve the question "Whether or not a moving conductive medium is anisotropic". There is evidence that the previous discussions on this subject by Sommerfeld and Cullwick are not completely satisfactory.

VII

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