EFINS-65-103

THE KINETIC PROPERTIES OF THE GALACTIC COSMIC RAY GAS*

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ABSTRACT

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The elementary formulae for the pressure and the speed of sound in a statistically isotropic homogeneous cosmic ray gas are worked out. Cosmic ray observations near Earth give a cosmic ray energy density of 1.29×10^{-12} ergs/cm^3, a pressure of 0.45×10^{-12} dynes/cm^2, and a compressibility $\delta P / \delta N = 0.66 \times 10^{-12}$ ergs/cm^3. There is reason to believe that the interstellar values are not significantly higher than near Earth.

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It has been shown elsewhere (Parker 1958a, 1965 a,b) that the cosmic ray gas pressure in the galaxy has novel and important effects on the dynamical state of the interstellar gas and fields. For this reason there is a need to work out some of the elementary kinetic properties of the cosmic ray gas. It will be assumed that the cosmic ray gas is statistically isotropic and homogeneous, the isotropy being maintained during a slow, and possibly anisotropic, compression by the scattering from weak irregular magnetic fields within the gas. We fix our attention first on a collisionless gas of N particles per cm^3, each with a rest energy $\mathcal{E}_0 = Mc^2$ and each with the same total energy $\mathcal{E}$.

The pressure is defined as twice the rate of transport of momentum in any given direction. Thus, in say the x-direction, the pressure contributed by N particles per cm^3 with random velocity v, and with velocity v_x and momentum p_x in the x-direction, is

$$P_{xx} = \frac{1}{2} N \langle 2 p_x v_x \rangle,$$

where the angular brackets denote the statistical mean. Note, then, that $p_x v_x = M v_x^2 / (1 - v^2/c^2)^{1/2}$ and for statistical isotropy, $v_x^2 = v^2/3$. Then since

$$\frac{v^2}{c^2} = 1 - \frac{\mathcal{E}_0^2}{\mathcal{E}^2}, \quad \mathcal{E}^2 = \mathcal{E}_0^2 + p^2 c^2,$$

it follows that the scalar pressure P is

$$P = \tfrac{1}{3} N\left(\mathcal{E} - \mathcal{E}_0^2/\mathcal{E}\right). \qquad (1)$$

Consider how $\mathcal{E}$ and P vary during a slow adiabatic compression of a volume V. The total particle energy in V is $NV\mathcal{E}$, so with $\delta(NV) = 0$ the first law of thermodynamics gives

$$P\delta V + NV\delta\mathcal{E} = 0 .$$

Integration of this equation gives

$$\mathcal{E}^2 - \mathcal{E}_0^2 = (\mathcal{E}_1^2 - \mathcal{E}_0^2)(N/N_1)^{2/3} \qquad (2)$$

for a compression from the density N_1, at which all particles have the energy $\mathcal{E}_1$, to the density N, with particle energy $\mathcal{E}$. The pressure varies as

$$P = P_1\left(\frac{N}{N_1}\right)^{5/3} \frac{\mathcal{E}_1}{\left[\mathcal{E}_0^2 + (\mathcal{E}_1^2 - \mathcal{E}_0^2)(N/N_1)^{2/3}\right]^{1/2}} \qquad (3)$$

and

$$\frac{\delta P}{\delta N} = \frac{(\mathcal{E}^2 - \mathcal{E}_0^2)(4\mathcal{E}^2 + \mathcal{E}_0^2)}{9\mathcal{E}^3} .$$

The speed of propagation of a sound wave in the cosmic ray gas is readily computed. Assuming again that there are weak magnetic fields present which maintain isotropy, the equation of motion for a small disturbance velocity $\underset{\sim}{u}$ associated with a density perturbation δN and an adiabatic pressure perturbation δP is described by the equation

$$\rho \frac{\partial \underset{\sim}{u}}{\partial t} = - \nabla \delta P$$

$$= - \nabla \left(\frac{\delta P}{\delta N} \, \delta N \right) \tag{4}$$

to first order, where $\rho = N \mathcal{E} / c^2$. The equation for conservation of particles is

$$\frac{\partial \delta N}{\partial t} + N \, \nabla \cdot \underset{\sim}{u} = 0 \tag{5}$$

to first order. It is a simple matter to use (4) to eliminate $\underset{\sim}{u}$ from (5), yielding

$$\frac{\partial^2 \delta N}{\partial t^2} - \frac{N}{\rho} \frac{\delta P}{\delta N} \nabla^2 \delta N = 0 \tag{6}$$

It is evident from (6) that the speed of sound can be written

$$b = \left(\frac{N}{\rho} \frac{\delta P}{\delta N} \right)^{1/2} = c \, \frac{\left(\mathcal{E}^2 - \mathcal{E}_0^{\,2} \right)^{1/2} \left(4\mathcal{E}^2 + \mathcal{E}_0^{\,2} \right)^{1/2}}{3 \, \mathcal{E}^2} . \tag{7}$$

The speed of sound reduces to $5^{1/2} v/3$ in the nonrelativistic limit $(\mathcal{E} - \mathcal{E}_0 \ll \mathcal{E}_0)$ and to $2c/3$ in the extreme relativistic limit $(\mathcal{E} \gg \mathcal{E}_0)$.

So far the calculations refer to a monoenergetic gas. The actual cosmic ray gas consists of an energy distribution $F(\mathcal{E})$, so that $N = \int d\mathcal{E}\, F(\mathcal{E})$ and the total pressure P_t is

$$P_t = \frac{1}{3} \int d\mathcal{E}\; F(\mathcal{E}) (\mathcal{E} - \mathcal{E}_0^2/\mathcal{E})$$

where the limits of integration are $\mathcal{E}_0$ to ∞ . For the simple power law $F(\mathcal{E}) = C\,\mathcal{E}^{-\alpha}$, where C and α are constants, it is readily shown that

$$P_t = \frac{2N\mathcal{E}_0(\alpha-1)}{3\alpha(\alpha-2)} = \frac{2N}{3\alpha}\langle\mathcal{E}\rangle$$

$$= \frac{2N(\alpha-1)}{3\alpha}\left[\langle\mathcal{E}\rangle - \mathcal{E}_0\right]. \tag{8}$$

The mean energy is $\langle\mathcal{E}\rangle = \mathcal{E}_0(\alpha-1)/(\alpha-2)$ and the number density is $N_t = C/\mathcal{E}_0^{\alpha-1}(\alpha-1)$. The speed of sound is

$$b^2 = \frac{N_t}{\rho_t}\frac{dP_t}{dN} = \frac{c^2}{\langle\mathcal{E}\rangle}\int d\mathcal{E}\; F(\mathcal{E})\,\frac{\delta P}{\delta N}$$

$$= c^2\,\frac{2(5\alpha+6)}{9\alpha(\alpha+2)} \,. \tag{9}$$

It is this speed of sound which appears in the earlier calculation (Parker, 1965b) of the suprathermal hydromagnetic waves, as is readily shown by rewriting those equations in terms of δN in place of $\delta\rho$.

Tidman (1965) has pointed out that the sound waves may be subject to severe Landau damping if small scale weak magnetic fields are not present throughout the cosmic ray gas. In the presence of a large-scale magnetic field, sound waves propagating parallel to the field may be severely damped.

Cosmic ray observations in the vicinity of Earth give the energy spectrum $F(\mathcal{E})$ of the cosmic ray gas in the inner solar system. Now the outward convection of cosmic ray particles in the fields carried in the solar wind (Parker, 1958b, 1961, 1963) reduces the cosmic ray density in the inner solar system below the level in interstellar space. The amount of the reduction has not yet been determined by direct observation, but at solar minimum the reduction is presumed to be significant only at low energies ($\sim 10^9$ ev and below). Most of the cosmic ray energy density and pressure is contributed by particles above 10^9 ev, so it appears that the cosmic ray energy density and pressure measured in space near Earth is perhaps not greatly different from what it is outside the solar system in interstellar space.

The observational data (McDonald 1956, Vogt, 1962, Waddington and Freier, 1965; Ormes and Webber, 1965; Fan, Gloeckler and Simpson, 1965; Hagge, Ludwig, and McDonald, 1965) has been combined* to give the cosmic

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* The author is indebted to Mr. S. D. Verma and Mr. J. J. L'Heureux for compiling this data.

ray proton spectrum above 100 Mev shown in Fig. 1 for solar minimum. The number of particles per cm^3 Bev is plotted as a function of the particle kinetic energy $T = \mathcal{E} - \mathcal{E}_0$ in Bev up to 10 Bev. Numerical values are given in Table I so that the reader can easily make appropriate revisions if future observations should show the present values near Earth to be a poor representation of the cosmic rays in interstellar space. Above 10 Bev the distribution $F(T)$ over the proton energy is approximated by $C/T^{2.5}$ with $C = 1.84 \times 10^{-10}$ so that $F = 5.8 \times 10^{-14}$ protons/cm^2 ster Bev. at $T = 10$ Bev. The exponent 2.5 is chosen to fit to the small air shower data at 10^{11} ev. At higher energies the exponent should be larger, but there is so little contribution to the total energy there that the error introduced by the exponent 2.5 is negligible. The uncertainty in the present cosmic ray density at the orbit of Earth is perhaps $\pm$ 20 percent for $10^8 - 10^{10}$ ev.

The proton kinetic energy distribution and the proton pressure distribution are both plotted in Fig. 1 in units of Bev/cm^3 ster Bev. Note that both distributions necessarily vanish at zero energy, so that they are not sensitive to the rather uncertain interstellar particle density distribution below 10^9 ev.

Numerical integration gives a total proton energy density of 7.5×10^{-13} ergs/cm^3, of which half lies below 10^{10} ev and half above. Helium nuclei are about one seventh as abundant as protons at the same energy per nucleon so that helium contributes about 4/7 as much as the protons. All nuclei heavier than helium contribute perhaps one quarter as many nucleons as helium, so that the total energy of all cosmic ray particles is about 12/7 that of the protons alone, or 1.3×10^{-12} ergs/cm^3 total, which is in rough agreement with the number 1 ev/cm^3 quoted by Ginzburg and Syrovatsky (1964).

The pressure exerted by the protons is readily shown to be 0.26×10^{-12} $dynes/cm^2$, giving a total of 0.45×10^{-12} $dynes/cm^2$ for all the cosmic ray gas. The pressure is thus 0.35 of the energy density, compared to 2/3 for a nonrelativistic gas and 1/3 for an extreme relativistic gas such as photons.

The integral $\int dT\, F(T)\, \delta P/\delta N$ involved in the speed of sound is readily computed and is 0.38×10^{-12} $ergs/cm^3$ for the protons alone and 0.66×10^{-12} $ergs/cm^3$ total. The integrand $F(T)\, \delta P/\delta N$ is shown in Fig. 1 for protons.

These numbers, it will be remembered, apply to cosmic rays as they are observed in the inner solar system at sunspot minimum. The numbers represent a lower limit on the cosmic ray gas in interstellar space outside the solar system, but, on the other hand, there is no compelling reason to believe that the energy density and the pressure are much higher in interstellar space. The energy density in interstellar space will be known only when space vehicles venture beyond the region swept out by the solar wind and measure the interstellar cosmic ray energy density directly. For the present we shall approximate the interstellar cosmic ray gas energy density with the value measured locally in space.

TABLE I

proton energy Bev.	protons per cm^2 ster. Bev	v/c	protons per cm^3 Bev.
0.1	2000	0.427	1.96×10^{-10}
0.2	2000	0.570	1.47
0.4	1600	0.715	0.937
1.0	900	0.875	0.431
2.0	410	0.949	0.196
4.0	146	0.982	0.062
10.0	22	0.996	0.0110

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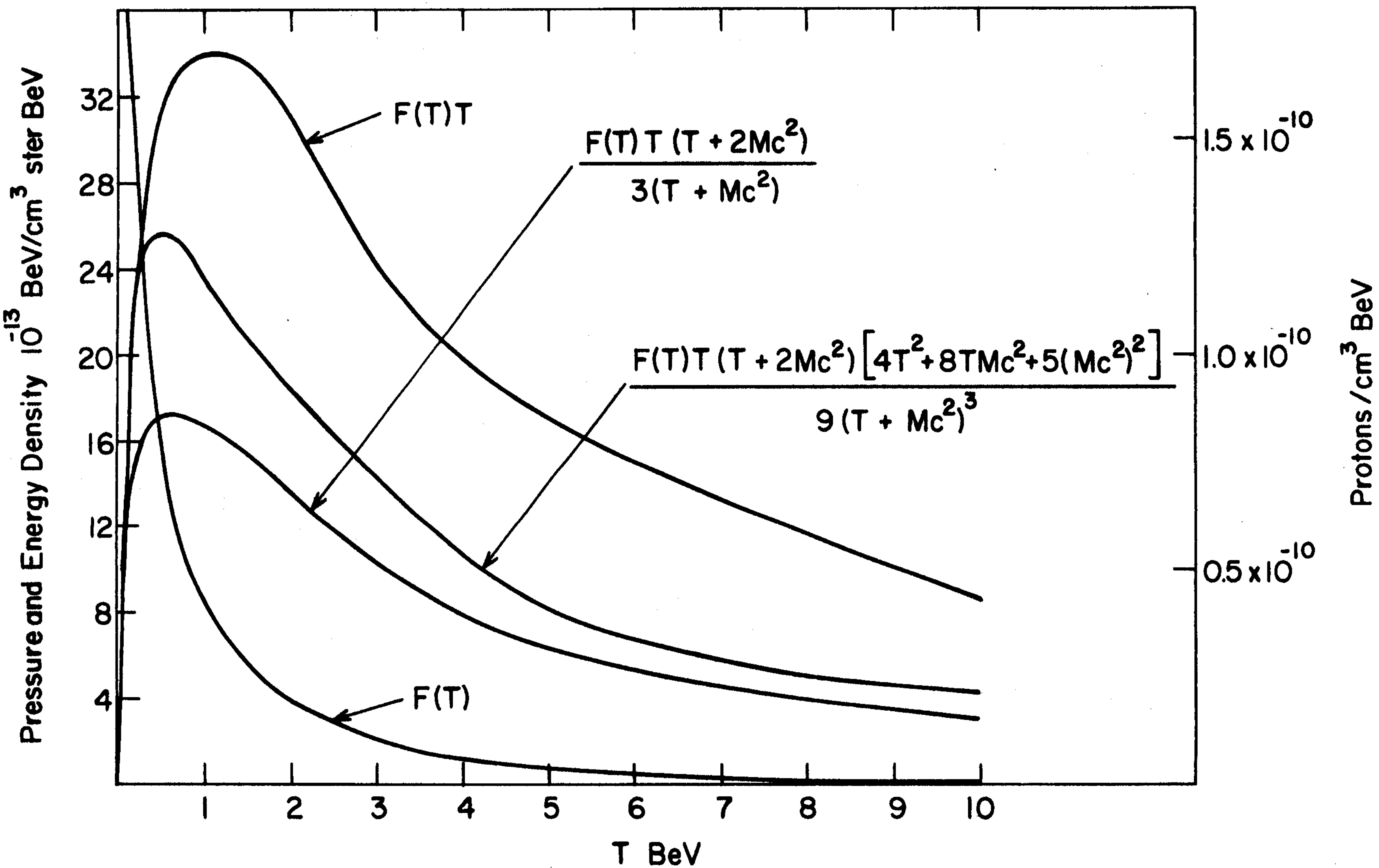
FIGURE CAPTIONS

Fig. 1. A plot of the cosmic ray proton density distribution $F(T)$ protons/ cm^3 Bev of proton kinetic energy T, together with a plot of the energy distribution $F(T)\,T$, the pressure distribution $F(T)\,T(T+2Mc^2)/3(T+Mc^2)$, and the distribution $F(T)\,\delta P/\delta N$, all in units of Bev/cm^3 ster Bev.



Pressure and Energy Density 10^{-13} BeV/cm^3 ster BeV
32
28
24
20
16
12
8
4
F(T)T
$\frac{F(T)T(T+2Mc^2)}{3(T+Mc^2)}$
$\frac{F(T)T(T+2Mc^2)\left[4T^2+8TMc^2+5(Mc^2)^2\right]}{9(T+Mc^2)^3}$
F(T)
1.5×10^{-10}
1.0×10^{-10}
0.5×10^{-10}
Protons/cm^3 BeV
1
2
3
4
5
6
7
8
9
10
T BeV

Fig. 1