Buckling of Imperfect Cylindrical Shells under Axial Compression and External Pressure*

by

John Hutchinson
Harvard University, Cambridge, Massachusetts

ABSTRACT

15684

The results of an investigation of the effect of axisymmetric initial imperfections on the buckling of cylindrical shells under combinations of axial compression and pressure are presented and discussed. Interaction curves are obtained which have a branch determined by the present theory and a branch determined by linear buckling theory. Agreement between theory and experiment is quite good.

Author

GPO PRICE $ _____

CFSTI PRICE(S) $ _____

Hard copy (HC) _____ *1.00*

Microfiche (MF) _____ *.50*

ff 653 July 65

N66-15684

(ACCESSION NUMBER)   (THRU)

(PAGES)   (CODE)

(NASA CR OR TMX OR AD NUMBER)   (CATEGORY)

FACILITY FORM 602

Buckling of Imperfect Cylindrical Shells under

Axial Compression and External Pressure[*]

by

John Hutchinson

Harvard University, Cambridge, Massachusetts

INTRODUCTION

This note presents the results of a brief investigation of the effect

of initial imperfections on the buckling of cylindrical shells under

combinations of axial compression and external pressure. Although

consideration is restricted to shells with axisymmetric imperfections, the

important features of previously reported experimental findings are

reproduced.

ANALYSIS OF CYLINDRICAL SHELL WITH AXISYMMETRIC IMPERFECTIONS

In this note we consider a cylindrical shell with an initial

imperfection in the form of the axisymmetric buckling mode of the perfect

shell under axial compression. That is

$$w_0 = \bar{\xi} h \sin q_0 \frac{x}{R} \tag{1}$$

where w_0 is the initial radial displacement, $\bar{\xi}$ is the magnitude of the

imperfection relative to the shell thickness h , x is the axial distance,

R is the shell radius and

$$q_0^4 = 12(1-\nu^2)\left(\frac{R}{h}\right)^2$$

The length of the shell L is assumed to be sufficiently long to insure

- -

that the end conditions (assumed sufficiently strong) do not have an appreciable effect on the buckling load of the cylinder when loaded under axial compression alone. Theoretical analyses[1] seem to indicate that this requirement is met for thin shells of length greater than or about equal to the radius.

Koiter[2] has obtained an upper-bound to the buckling load of the imperfect shell described above for the case of pure axial loading. His results are shown in Figure 1. In this plot the buckling load of the imperfect shell P has been normalized with respect to the axial buckling load of the perfect shell

$$P_0 = (3(1-\nu^2))^{-1/2} Eh^2$$

Koiter's analysis has been extended to obtain an upper-bound to the axial buckling load of pressurized cylinders.[3] The upper-bound expression in Reference (3) is also valid for buckling under combined axial compression and external pressure. Here, these results will be exploited without repeating the details of the analysis other than to give a very brief description of the method employed.

The large-deflection Donnell equations for an imperfect cylindrical shell yield a very simple prebuckling solution for the loading combinations considered here and for an initial imperfection of the form given by Equation (1). Bifurcation from the prebuckling solution occurs at a certain value of the axial load (for a given external pressure) in the form of a non-axisymmetric buckling mode. The load-deflection curve falls subsequent to bifurcation; and thus, the bifurcation value is the buckling load.

The eigenvalue equations for the bifurcation load are solved approximately in a manner which insures that the approximate eigenvalue

expression yields an upper-bound to the exact bifurcation load (again, for fixed values of external pressure and initial imperfection). To effect this approximate solution it is necessary to assume a form for the non-axisymmetric buckling deflection. The form assumed was

$$w = \sin \frac{1}{2} q_0 \frac{x}{R} \sin \frac{\gamma}{2} q_0 \frac{y}{R} \qquad (2)$$

where y is the circumferential distance and γ is a free parameter to be chosen to minimize the upper-bound value.

The eigenvalue equation, obtained in Reference (3), is

$$(1-P/P_0)^2 [\frac{(1+\gamma^2)^2}{4} + \frac{4}{(1+\gamma^2)^2} - 2\bar{p}\gamma^2 - 2P/P_0]$$

$$- c\gamma^2\bar{\xi}(1-P/P_0)[P/P_0 + \frac{8}{(1-\gamma^2)^2}] + 4(c\gamma^2\bar{\xi})^2[\frac{1}{(1+\gamma^2)^2} + \frac{1}{(9+\gamma^2)^2}] = 0 \qquad (3)^*$$

where $c^2 = 3(1-\nu^2)$ and the pressure parameter is

$$\bar{p} = \frac{cpR^2}{Eh^2}$$

Here P is the total axial load (the load resulting from external pressure on the capped ends plus the additional applied axial load).

To find the upper-bound extimate of the axial buckling load P/P_0 for a given value of $\bar{p}$ and initial imperfection $\bar{\xi}$, it is necessary to solve Equation (3) for P/P_0 in terms of γ and then find the value of γ such that P/P_0 is minimized. An equivalent, but considerably easier, procedure is to solve Equation (3) for $\bar{p}$ in terms of P/P_0, $\bar{\xi}$ and γ and then minimize $\bar{p}$ with respect to γ. This procedure leads to the upper-bound curves plotted as solid lines in Figure 2. These curves correspond to three values of $\bar{\xi}$ chosen such that shell buckles at $P/P_0 = .3$, .5 and .7 when subject to axial compression with no external pressure. The upper-bound

- - - - - - - - - - - - - - - - - - - -

* With $\bar{p} = 0$, this is the expression Koiter used to obtain the results shown in Figure 1.

character can be construed in either of two ways: (1) for a given value

of external pressure $\bar{p}$ the associated value of axial load is an upper-

bound to the actual axial buckling load, or (2) for a given value of total

axial load P/P_0 the associated value of $\bar{p}$ is an upper-bound to the

actual buckling pressure.

Included in Figure 2 are buckling load curves (dashed) for combined

axial compression and external pressure for an initially perfect shell.

These curves are straight line approximations to those obtained, for example,

in Reference 1 on the basis of linear buckling equations for a perfect

cylinder. The several curves shown correspond to different values of the

frequently defined length parameter

$$z = (1-\nu^2)^{1/2} \frac{R}{h}\left(\frac{L}{R}\right)^2$$

where L is the length of the shell. While the upper-bound results are

independent of shell length and end conditions, the dashed buckling curves

for the perfect cylinders are dependent on both. The curves in Figure 2

correspond to a shell which is simply supported with respect to the radial

displacement, free with respect to additional axial stresses and clamped

with respect to the circumferential displacement. Completely damped end

conditions can raise the buckling load under pure external pressure as much

as forty percent.[1] For other end conditions, completely clamped for example,

the curves for the perfect cylinder would also be essentially straight line

curves. However, they would intersect the $\bar{p}$ axis at different values than

shown. The effect of end conditions on the axial buckling load can be

considered negligible for the purposes of this note as long as they are

sufficiently strong.

Reduction of the load carrying capacity resulting from the presence of

the imperfection of the assumed axisymmetric form occurs only if the shell

buckles in a mode which is, more or less, of the form given by Equation (2).

Under pure external pressure, for example, the shell buckles in a mode with

only one half wave length over the entire length of the cylinder. The

assumed imperfection has essentially no effect on the buckling load and,

consequently, the predictions of the linear theory are valid. We expect the

upper-bound predictions to be appropriate for combinations of axial load and

external pressure such that the upper-bound predictions fall below those for

the linear buckling theory. On the other hand, the linear theory will be

valid, both with respect to buckling load and mode form, for combinations

such that results of the linear theory fall below the upper-bound predictions.

Thus, the curve of critical load combinations (sometimes referred to as the

interaction curve) consists of two branches -- (1) the upper-bound branch

on which the shell buckles in a mode of the form of Equation (2); and (2)

the linear buckling theory branch.

For a given value of external pressure the axial buckling load will be

either the upper-bound value (actually less than or equal to this value) or

the value predicted for the perfect shell, whichever is less.

DISCUSSION OF RESULTS AND COMPARISON WITH EXPERIMENT

Recently published experimental data by Weingarten and Seide[4] are

reproduced from their paper in Figure 3. A typical specimen of the nine

plotted was buckled repeatedly for loads ranging from axial compression with

no external pressure to pure external pressure. In effect, the buckling

load data for each specimen represents a complete interaction curve for a

cylindrical shell with a given initial imperfection. It is therefore

meaningful to make a direct comparison with the interaction curves of

Figure 2. Of course, it should be remembered that the theoretical inter-
action curves are strictly applicable only to shells with the assumed
axisymmetric imperfection.

A typical experimental point in Figure 3 represents a critical
combination of total axial load P and external pressure p . The total
axial load has been normalized with respect to $2\pi Eh^2$; and thus, the value
of the ordinate for p = 0 reflects the extent to which the axial buckling
load falls below that for a perfect shell, $P/(2\pi Eh^2) = .6$. In addition,
the abscissa value is p/p_0 where p_0 is the experimentally determined
pressure for buckling under external pressure alone. The dotted line
labeled "interaction curve" is the curve of critical load combinations as
predicted by the linear buckling theory for a perfect cylinder.

Theory and experiment are compared in Figure 4 for a specimen for which
the agreement is quite good. For this specimen z = 1526 , and the
theoretical interaction curve in Figure 4 corresponds to this value of z .
The external pressure has been normalized with respect to the buckling
pressure for buckling under external pressure alone in the same manner
in which the experimental results are presented in Figure 3. The value of
$\bar{\xi}$ associated with the upper-bound branch of the interaction curve was
chosen such that the theoretical buckling load under pure axial compression
agreed with the experimental value.

The two main features of the theoretical results are: (1) the two
branches of the interaction curve and (2) the lack of dependence of the
axial buckling load on pressure on the upper-bound branch. Both these
features are characteristic, to a certain extent, of almost all the
specimens. Apparently, however, some specimens exhibit a greater
dependence on pressure than is predicted for a shell with axisymmetric

imperfections.

At least two possible explanations can be suggested to account for the discrepancy between the theory, as presented here, and experiment. Firstly, the predictions on upper-bound branch are, as has been emphasized, of an upper-bound nature and, most likely, over-estimates. Probably more important is the failure to account for other than axisymmetric imperfections. In this connection, it is noted that a significant distinction was found between the roles of axisymmetric and asymmetric imperfections in reducing the axial buckling load of pressurized cylinders.[3] While asymmetric imperfections are "ironed out" by internal pressure, axisymmetric ones are not. An "ironing in" effect due to external pressure (that is, the effect of the external pressure in "pushing in" the initial asymmetric imperfection) is expected to give a greater pressure dependence than is predicted for a shell with only axisymmetric imperfections.

A multiplicity of buckling modes are associated with the critical load of a perfect cylindrical shell under axial compression. Imperfections in the form of any of the buckling modes are particularly degrading. On the basis of linear shell theory, it is a simple matter to show that the smaller the number of circumferential wavelengths associated with one of these imperfections the less internal or external pressure will be necessary to iron it out or in. Thus, a cylinder whose buckling behavior is determined by an asymmetric imperfection with relatively few circumferential wavelengths should be expected to show a definite pressure dependence over the entire interaction curve. This effect, however, will be less noticeable in the case of externally pressurized cylinders than for those internally pressurized. This follows because the external pressure on a thin shell with z greater than 50, say, can never exceed $\bar{p} = .2$, while the ironing

out effect only becomes significant at internal pressures of this order
or larger.

REFERENCES

1. Almroth, B. O., "Influence of Edge Conditions on the Stability of Axially Compressed Cylindrical Shells", NASA CR-161, (1965).

2. Koiter, W. T., "The Effect of Axisymmetric Imperfections on the Buckling of Cylindrical Shells under Axial Compression", Koninkl. Nederl. Akademie Van Weterschappen - Amsterdam, Series B, 66, No. 5, (1963).

3. Hutchinson, J. W., "Axial Buckling of Pressurized Imperfect Cylindrical Shells", to be published in AIAA Journal.

4. Weingarten, V. I. and Seide, P., "Elastic Stability of Thin-Walled Cylindrical and Conical Shells under Combined External Pressure and Axial Compression", AIAA Journal, Vol. 3, 5, May (1965).
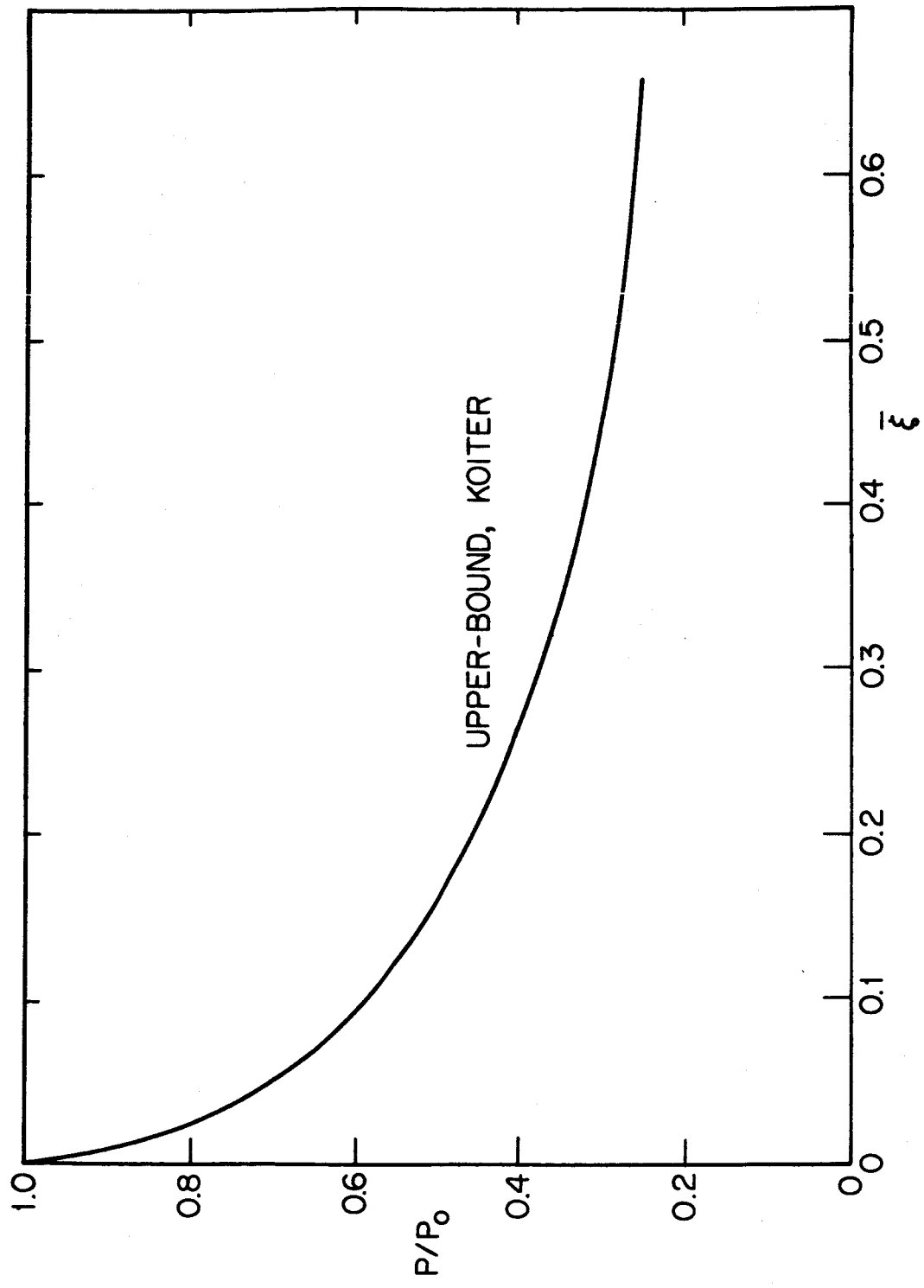
FIG. 1   EFFECT OF AXISYMMETRIC IMPERFECTION ON BUCKLING OF UNPRESSURIZED SHELL.
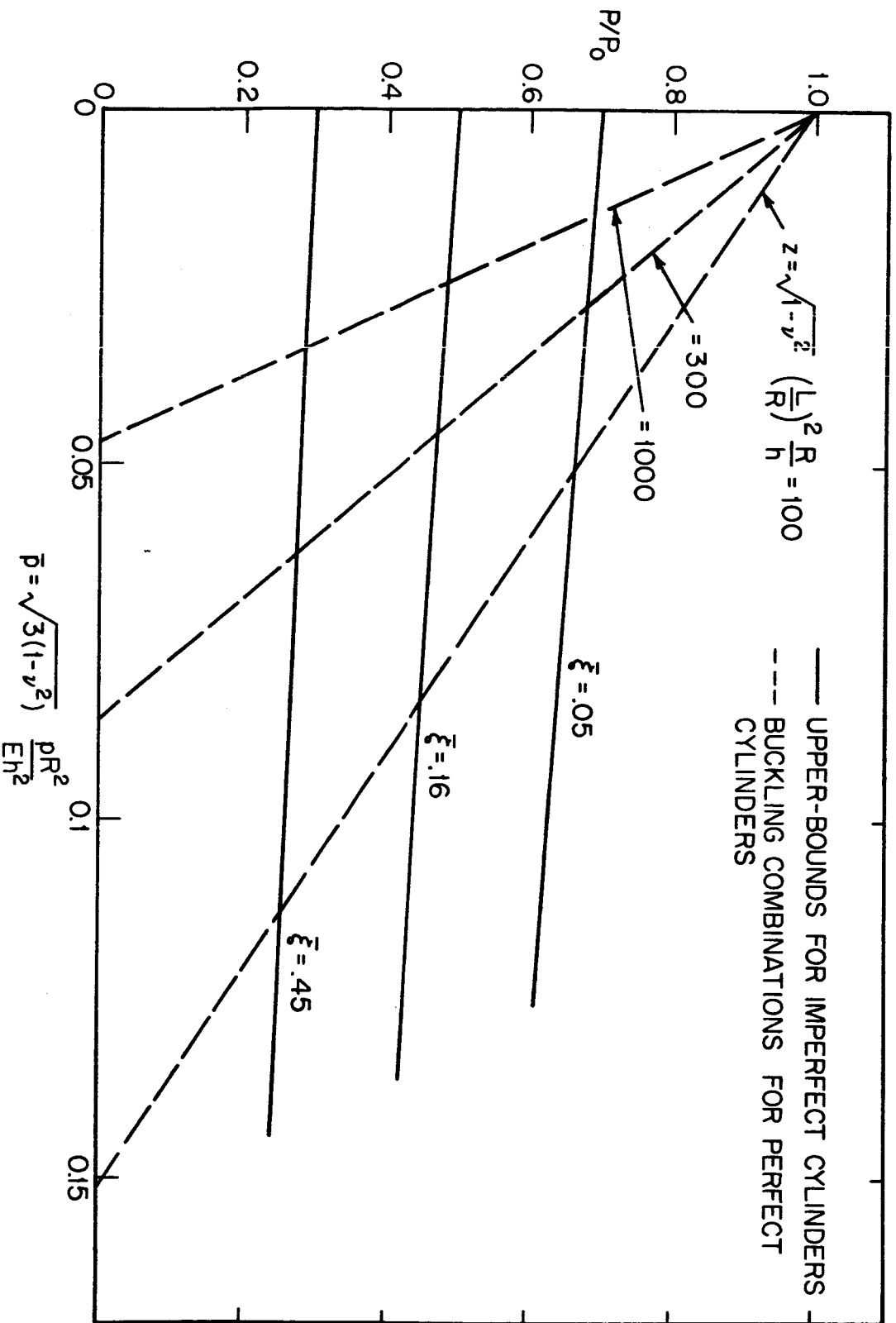
$z = \sqrt{1-\nu^2} \left(\frac{L}{R}\right)^2 \frac{R}{h} = 100$

=300

=1000

$\bar{\xi} = .05$

$\bar{\xi} = .16$

$\bar{\xi} = .45$

———— UPPER-BOUNDS FOR IMPERFECT CYLINDERS

– – – – BUCKLING COMBINATIONS FOR PERFECT CYLINDERS

P/P_0

$\bar{p} = \sqrt{3(1-\nu^2)} \dfrac{pR^2}{Eh^2}$

FIG. 2   EFFECT OF EXTERNAL PRESSURE ON BUCKLING LOAD ON INITIALLY IMPERFECT CYLINDRICAL SHELL.
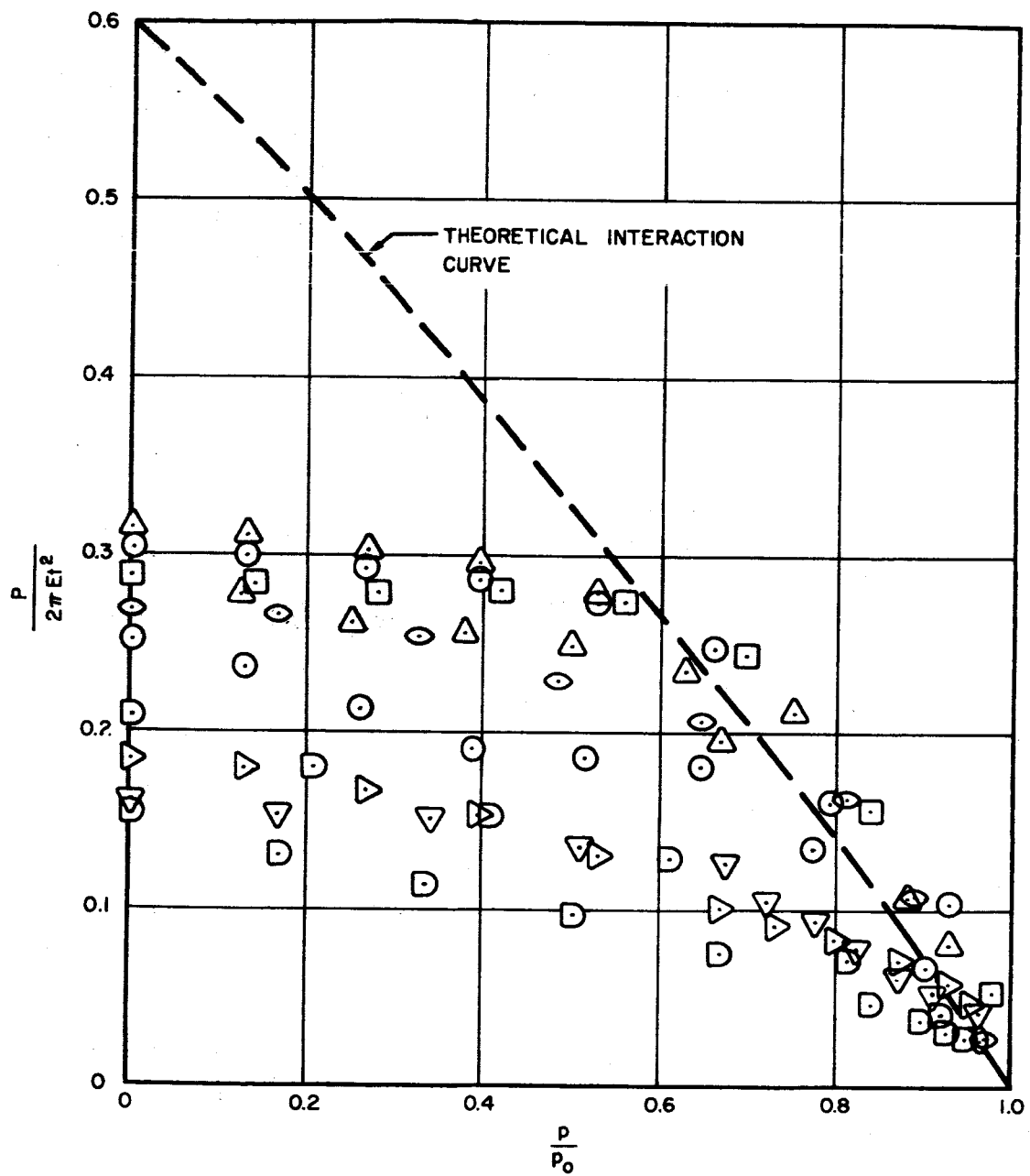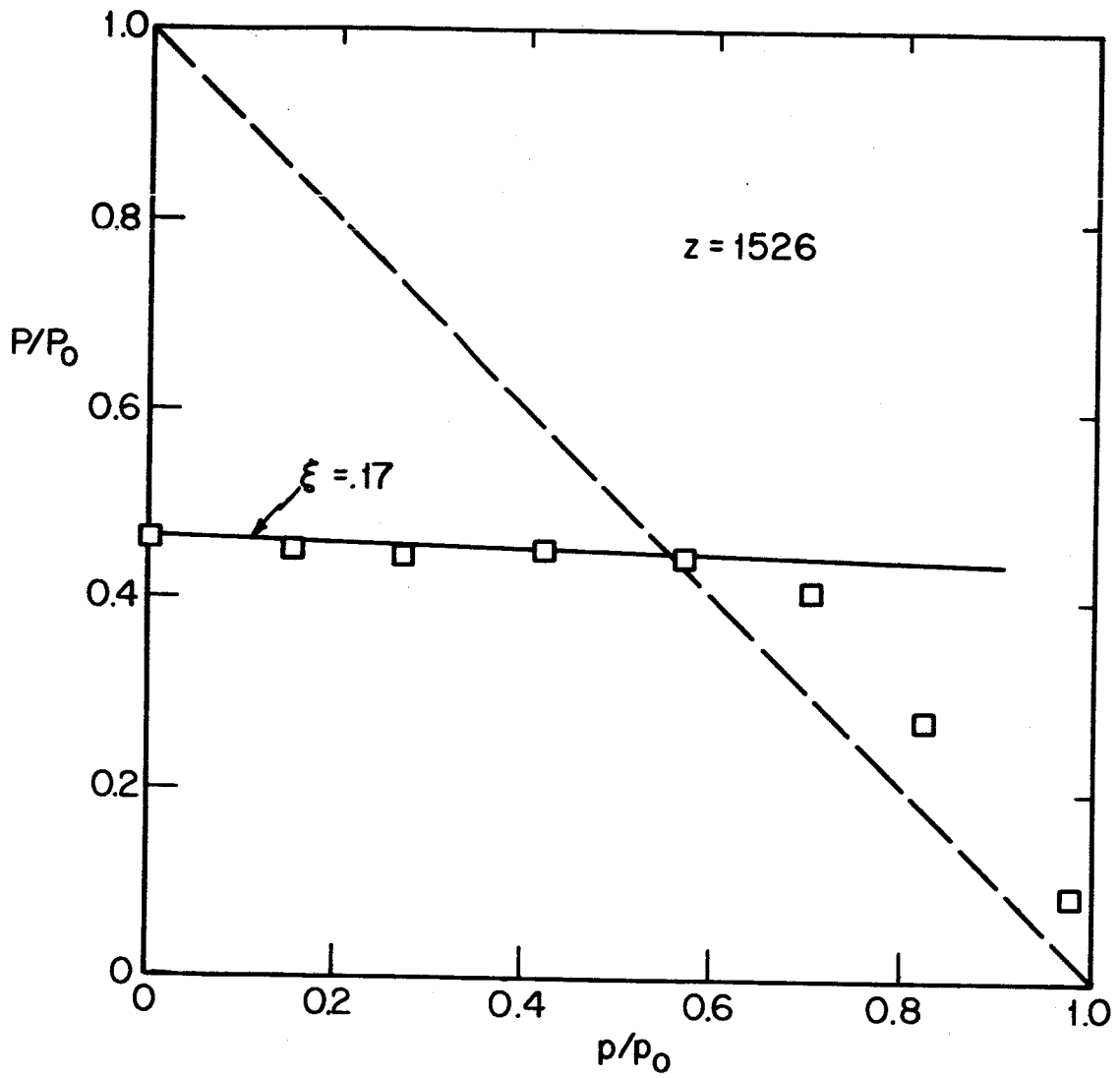
FIG. 3   EXPERIMENT DATA -WEINGARTEN AND SEIDE

FIG. 4 COMPARISON BETWEEN THEORY AND EXPERIMENT.