ON RETARDED EVOLUTION OF PROTOSTARS*

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ABSTRACT

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Possible mechanisms for holding protostars in a state of suspended evolution for time scales of the order of 10^{10} years are discussed. Rotation seems to be the only viable conventional mechanism. Magneto-turbulence, a new concept, may provide long term distention forces.

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I. DORMANT STARS

The increasing number of difficulties found to be associated with star formation by fragmentation has led Layzer (1954), (1963), Ambartsumian (1958), and others to begin investigating alternative methods. In particular they favor the hypothesis that stars are formed in an expanding medium. Layzer postulates that this formation occurred at the time of the formation of the galaxy, while Ambartsumian thinks the process may be currently happening, in matter ejected from galactic nuclei. Since observations show a whole spectrum of stellar ages, Layzer supposes that stars formed ten billion years ago have in many cases had their early development arrested so they now appear young.

We wish, therefore, to discuss some of the facets of arrested development. We will consider stars as individuals, and stars in clusters. In the latter case there exists the possibility of many types of interactions between stars, and between stars and gas. The time scales we have in mind are of the order of billions of years.

II. ISOLATED STARS

The first question to investigate is whether a star can remain in a dormant condition, or a condition of greatly retarded evolution, by virtue of its intrinsic properties, i.e., without external disturbances.

If the dominant forces within a star are gas or radiation pressure and gravitational attraction then the conventional models of the contraction of a star toward the main sequence are relevant. However, it is well known that the time scales involved in this sort of process are at least as short as the Kelvin contraction time t_k:

$$t_k \approx 10^7 \times \frac{M^2}{LR} \text{ years} \tag{1}$$

where M and L are the mass and luminosity (assumed constant) of the star and R is its final radius, all given in solar units. If the recent models of Hyashi, et al (1961), (1962), are correct, then in the Hertzsprung-Russell diagram protostars will approach the main sequence vertically from above, with much higher luminosities than their final main sequence values. Accordingly the contraction times will be diminished considerably.

Rotation will, of course, cause a protostar to flatten, and prevent its collapse. A rapidly rotating star could remain in a primordial state for any desired length of time, neglecting considerations of dynamical stability. The usual problem associated with this condition lies in finding the mechanism for damping the rotation. Magnetic braking seems to be the only reasonable solution, but it does not represent a very well developed theory.

A large scale magnetic field bound in an ionized material may provide sufficient magnetic pressure to prevent collapse of a protostar. Chandrasekhar and Fermi (1953) have obtained a form of the virial theorem for a system containing thermal, gravitational and magnetic energy. For a bound, stable, monatomic gas cloud

$$2T + V + M = 0 \tag{2}$$

where M is the magnetic energy, $\int \frac{|H|^2}{8\pi} dV$, V the gravitational potential energy, and T the thermal and kinetic energy. Then, of course, the total energy is given by

$$E = T + V + M. \tag{3}$$

If the protostar is to remain relatively unchanged, then its size and consequently V must remain constant. If the magnetic field is bound in the star then M is constant. However, the protostar is bound to radiate, particularly if hot enough to be ionized in order to trap the magnetic field. The time scales for cooling of a transparent hydrogen cloud are much less than 10^6 years for protostellar conditions. Therefore if T is allowed to change, we conclude that it must be quite small, in order not to upset condition Eq. (2). Therefore the equilibrium of the star must be determined primarily by magnetic and gravitational forces. There are a few relevant comments to be made.

(a) If the magnetic field is oriented more or less in one direction, say along the z-axis, then collapse along that axis will be relatively unimpeded, especially in the absence of pressure forces resulting from the vanishing of T. Of course after collapse has proceeded to a large extent in the z-direction pressure forces must come into play. T can no longer be neglected in comparison with V and M, energy will be radiated away, and evolution will proceed.

(b) If $T \sim 0$ than Eq. (2) implies that V and M must be nearly of the same magnitude for a stable situation. If they are not precisely the same, which of course is likely to be the case, then we inquire, will the situation adjust to equalize them? Both V and M vary inversely as the first power of the stellar radius. Therefore their ratio remains constant during a scale change. Therefore a cloud with a slight initial unbalance of gravitational and magnetic energy will be unstable to collapse or to dissipation, depending on which form of energy dominates. In the case of predominance of gravitational energy, and collapse, eventually the thermal energy term will stabilize the situation, but then radiation and evolution will occur.

(c) It is necessary that the protostar remain ionized if the magnetic field is to be bound and inhibit collapse. However, under conditions where pressure forces supply a stabilizing influence there will be a high collision rate and consequently a method for recombination or ionization, depending on the temperature.

If the magnetic field pattern is more or less random, then at first glance (a) is not likely to occur, although comments (b) and (c) still hold. However the situation may be somewhat more complicated, since (a) may hold locally over each region of reasonably correlated field direction. Then the protostar would still collapse until thermal (i.e., pressure) effects prohibited it.

We conclude, therefore, that magnetic effects cannot maintain a protostar in a steady state, even if there is no field slippage. It is, however, quite possible that they would slow down the evolution somewhat, by diminishing the collapse forces, and lowering the temperature, thus lowering the rate of radiation loss.

III. PROTOCLUSTERS

1. Giant Star. The simplest notion of a protocluster would be that of a static giant star, of hundreds or thousands of solar masses. But if gas and radiation pressure are responsible for its maintenance, then we have the conventional massive star model. And it has been established that for stars heavier than about sixty solar masses (Schwarzschild and Härm (1959)) instability sets in. Stars of about ten to sixty solar masses will have a very short evolutionary time compared to the age of the galaxy. So a pressure distended giant star cannot provide a long period of dormancy. Even if it collapses and produces many stars in a subsequent explosion, their birth would have been delayed only a short while.

If magnetic pressure or rotation are present in a single giant star then the preceding discussion of the ordinary single star is applicable. It would again appear that rotational forces are the only ones capable of preventing collapse. However the resulting cluster would have more of a disk shape than a spherical shape. Also it should retain some of the rotational motion. The observational data on this point are not conclusive: some clusters may have rotation.

2. Star-star Interactions. A protocluster is more likely to consist of well defined protostars and perhaps a large amount of gas. If a star can be surrounded by a sufficiently opaque wall so that its radiation cannot

escape, then it will have its contraction considerably slowed.

An idealized model would consist of a star in a container with perfectly reflecting walls. Then the system would come into thermodynamic equilibrium with the star becoming an isothermal gas sphere. The radius of an isolated, isothermal, gas sphere is infinite (Emden (1907)) so in the present case there would be a non-zero pressure and density at the wall of the container. These conditions would be met in a cluster of many stars by having the reflecting wall at the point where a neighboring star begins.

Let us suppose that the stars of a cluster are packed sufficiently tightly so that a star located deep within the cluster sees only a sky full of stellar surface, at the same temperature as its own surface. This situation it cannot distinguish from that of a reflecting wall.

The mean free path of a photon travelling through a cluster consisting of stars and no gas will be

$$\lambda_p \quad \frac{mR_c^3}{r_s^2 M} \tag{4}$$

where m and r_s are the mass and radius of a protostar and M and R_c are the mass and radius of the cluster. It is clear that λ_c, the mean free path of a star before collision, differs from λ_p by a factor of order unity. In fact λ_c can never exceed λ_p. If now, a star is to survive without collision in a cluster it must have a mean free path at least as large as the cluster radius. Therefore since $\lambda_p \gtrsim \lambda_c$ a photon near the center of the cluster will have a high probability of escaping. Consequently a central star will not see a bright sky, but rather a fairly dark one.

We note that in general a collision between two protostars need not necessarily be disruptive. The gravitational binding energy of a star is approximately

$$E_G \approx \frac{Gm^2}{r_s} \quad . \tag{5}$$

Its kinetic energy will be

$$E_{KE} = \tfrac{1}{2}m\bar{v}^2 = \tfrac{1}{2}m\,\frac{GM}{R_c} \,, \tag{6}$$

by the virial theorem applied to the cluster as a whole. The ratio $E_{KE}:E_G$ is then

$$\frac{E_{KE}}{E_G} \approx \tfrac{1}{2}\frac{M}{m}\,\frac{r_s}{R_c} \tag{7}$$

If the cluster is so dense that the sky would appear white, then by Eq. (4) with $\lambda_p \approx R_c$,

$$\frac{M}{m}\,\frac{{r_s}^2}{{R_c}^2} \approx 1 \,,$$

so that Eq. (7) becomes

$$\frac{E_{KE}}{E_G} \approx \tfrac{1}{2}\,\frac{R_c}{r_s} \,. \tag{8}$$

Clearly $R_c >> r_s$ and in this case a collision is likely to be disruptive.

However, for a looser cluster, with smaller stars it would not be unusual to find $E_{KE} << E_G$.

The occurrence of close stellar encounters presents a mechanism for exchange of energy and spin between stars. This energy transfer has been evaluated by Burke (1965). The amount transferred in a single encounter is so small in comparison with, say, the solar energy radiated in a millenium, that a very large number of near collisions would be necessary to provide a protostar with a continuing energy supply, apart from contraction. And for such a large number of near (2 or 3 stellar radii) collisions there are bound to be head on, and sometimes disruptive, collisions. This point, of just what happens after a stellar collision, could bear further investigation. It may be that stars are not destroyed in the process.

We may likewise conclude that the spin exchanged between protostars will be small.

Of course, it may be desirable to let a majority of the stars be annihilated in collisions, while the remaining ones acquire some spin. Since the angular momentum vector added at each encounter will be randomly oriented, the net total angular momentum after N encounters will accumulate in a manner analogous to a random walk, and will therefore be $S\sqrt{N}$ where S is the spin added per encounter. Actually once the star begins rotating the results of Burke will no longer be strictly applicable, but they should still provide a reasonable approximation.

3. Star-gas Interactions. We will discuss briefly three phenomena associated with interactions between stars and gas: accretion, braking, and shock waves. Some aspects of the first two have been treated by Burke (1965). It is clear that accretion will change the mass and perhaps the chemical composition of a star. It may also provide braking of a star's motion through the cluster. Even without accretion braking may occur, in the same way as dynamical friction. Accretion may significantly affect the course of evolution of a star, although it is not likely to retard it noticeably, except in one way. It could add angular momentum, perhaps to the extent that its rotation could attain nearly the critical value. That would prevent stellar collapse, with the usual qualifying remark about stability appended.

Shock waves will be created in the interstellar gas by the passage of supersonic stars. If the gas is considerably rarer than the protostars the shock waves will not have a great perturbing influence on the bulk of stellar material upon impact. They may affect the stellar atmospheres. Again this will not be likely to directly affect the evolutionary time scale. On the other hand the shock waves may have quite a significant effect on the interstellar gas. However, we should add that if the protostars are in a state not greatly different from that of the gas, that the shocks might be disruptive, or at least provide considerable thermal energy. These points are now being investigated.

4. Cluster Atmosphere. We might inquire whether there could be a large, perhaps quite massive, atmosphere or cloud surrounding a protocluster. Could such an atmosphere be supported against gravitational collapse, and could it affect the evolution of stars within?

Clearly it cannot be in hydrostatic equilibrium, supported by gas pressure, for then the situation is that of the giant star, with short evolutionary time scale, and radial instability. In fact, the presence of a cluster of stars within should add to the gravitational collapse forces, without offering significant retarding influence.

If the cloud is opaque enough to retard the escape of radiation from within, then of course it will feel the pressure of the trapped radiation. However, the situation then becomes analogous to that of an early type, very massive star. The evolution and collapse of the cloud proceed very rapidly. For sufficiently massive stars the radiation pressure is an unstabilizing influence.

The cloud might remain in place, supported by macroscopic motions, i.e., turbulence. There are then two possible solutions: the turbulent energy is remnant in the primeval cloud, and is slowly dissipating, or the internal cluster of stars feeds turbulent energy into the surrounding cloud. If the large majority of the cluster mass is in the form of gas then the first alternative must be realized.

In a turbulent fluid the kinetic energy stored will be

$$E = \frac{1}{2} M\bar{v}^2 \tag{9}$$

The rate of dissipation of turbulent energy per gram of fluid is approximately (Batchelor (1960), p. 103)

$$\epsilon \approx \frac{\bar{v}^3}{\lambda} \tag{10}$$

where λ is the characteristic size of the largest eddies. Therefore the time scale for dissipation of E is

$$\tau \approx \frac{E}{\epsilon M} = \frac{\lambda}{2\bar{v}} \tag{11}$$

Clearly τ is the time which it takes for an eddy of size λ to move its own length. For a cloud to be in primarily dynamic equilibrium the time scale for internal motions must be considerably shorter than the free fall time of the cloud. This means that the cloud energy will be converted to thermal energy in a time short compared to the evolutionary time of a quasi-static giant star. And once in the form of thermal energy we are back to the situation of a giant star in hydrostatic equilibrium.

The alternative of the internal cluster feeding energy to the cloud is therefore likely to be a bit more promising. The energy drawn from the internal cluster will of course come from its gravitational supply, thereby causing it to contract. Let us begin by computing the approximate amount of contraction.

For a body held in equilibrium by pressure forces

$$E_G = -3 \int P dV \approx -3\bar{P}V \tag{12}$$

(Landau and Lifshitz (1958)). The pressure from turbulence is (Batchelor (1960), p. 182)

$$\bar{P} = \frac{1}{5} \rho \overline{v^2}. \tag{13}$$

Combining Eqs. (12) and (13) to eliminate $\bar{P}$, we obtain

$$E_G \approx -\frac{3}{5} M \overline{v^2} \tag{14}$$

where $M = \rho V$. We see this result could have been obtained from the virial theorem by neglecting the central body of stars, except in its contribution to E_G.

We take a gas cloud of radius R_2 and mass M_2 to have imbedded within it a stellar system of mass M_1 and radius R_1 (see Fig. 1). The gravitational energy of the cloud will then be

$$E_{G2} = -\frac{3}{5} G \frac{M_2}{R_2} \left[M_2 + \frac{5}{2} M_1 \right], \tag{15}$$

where the density has been taken to be uniform, and $R_1{}^2 << R_2{}^2$. The gravitational energy of the stellar system, with the same approximation, is

$$E_{G1} = -\frac{3}{5} G \frac{M_1{}^2}{R_1}. \tag{16}$$

If we then use Eq. (14) for E_{G2} and Eq. (10) for the rate of energy dissipation, we obtain

$$\frac{d}{dt}\left(\frac{1}{R_1}\right) = \frac{5M_2}{3GM_1{}^2\lambda_2}\left[\frac{G}{R_2}\left(M_2 + \frac{5}{2}M_1\right)\right]^{3/2}. \qquad (17)$$

Eq. (17) integrates to give

$$\frac{1}{R_{1f}} - \frac{1}{R_{1i}} = \frac{5M_2\,\Delta t}{3GM_1{}^2\lambda_2}\left[\frac{G}{R_2}\left(M_2 + \frac{5}{2}M_1\right)\right]^{3/2}, \qquad (18)$$

where the subscript "f" refers to final and the subscript "i" refers to initial. In Table 1 some specific evaluations of Eq. (18) have been tabulated. These values indicate that a contracting core of protostars can indeed provide sufficient turbulent energy (and therefore pressure to keep a surrounding cloud of gas in equilibrium. However, there are several qualifying remarks to be made.

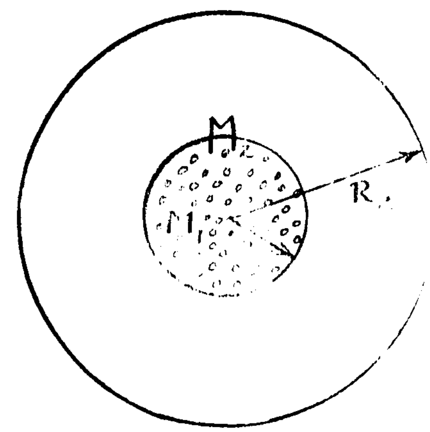

Fig. 1. Gas cloud with internal cluster.

How is the turbulent energy transferred to the cloud? We have obtimistically taken λ_2 larger than R_{1f} in most cases. This was necessary to prevent R_{1f} from being extremely small. But it will be difficult for a turbulent core of size R_1 to produce eddies much larger than R_1. And even if these eddies are produced, a reasonable fraction of them must propagate further than their characteristic sizes, λ_2, in order to provide turbulent energy to the outer parts of the cloud.

Table 1. Evaluations of Eq. (18).

M_1	M_2	Δt	R_2	λ_2	R_{1i}	R_{1f}
10^{37}g.	10^{36}g.	10^{17}sec.	10^{19}cm.	10^{18} cm.	10^{15}cm.	4×10^{13}cm.
10^{37}	5×10^{36}	10^{17}	10^{20}	10^{19}	10^{18}	3×10^{16}
10^{37}	10^{37}	10^{17}	10^{21}	10^{20}	10^{20}	9×10^{18}
10^{37}	10^{37}	10^{17}	10^{21}	10^{19}	9×10^{17}	5×10^{17}
10^{37}	10^{36}	10^{17}	10^{19}	10^{18}	10^{16}	6×10^{14}
10^{37}	10^{36}	10^{17}	10^{20}	10^{19}	10^{18}	2×10^{17}
10^{37}	10^{36}	10^{17}	10^{21}	10^{20}	6×10^{19}	3×10^{19}
10^{37}	10^{36}	10^{17}	10^{21}	10^{20}	10^{21}	6×10^{19}
10^{37}	10^{36}	10^{17}	10^{20}	10^{18}	2×10^{16}	10^{16}

To allow the cloud to contract does not help much since the dissipation will rise rapidly and require the internal stellar system to contract faster and farther. It appears that R_2 must always be considerably larger than R_1 in order to give R_{1f} a reasonably large value, but this is incompatible with transfer of energy to large eddies, comparable in size to R_2, which will have lower dissipation rates.

Perhaps very small values of R_{1f} should be considered. If the turbulent energy from a small core quickly becomes converted to thermal energy and this is really responsible for supporting the outer layers, then we have a problem concerning a very massive star with a somewhat peculiar core, and should treat at least the outer regions in terms of static gas pressure.

Without considering the stability problem, rotational forces are as usual capable of supporting an extended and massive atmosphere. Rotation alone however, will permit contraction along the rotation axis, which will make the cloud optically thin in that direction, even if opaque in the other directions. The addition of a magnetic field perpendicular to the rotation axis will tend to change the collapse to a uniform contraction, as Mestel (1964) has shown. A magnetic field without rotation will be subject to the same conditions mentioned in the case of a single star. The presence of the rotation and magnetic field greatly reduces the amount of gas pressure support needed, so that there is a better chance that radiation or turbulence generated by the central cluster could hold the atmosphere in distension.

Let us, at this point, consider crudely the sort of opacity a dense cluster atmosphere might have. Take

$$\tau_2 \sim K_2 \rho_2 R_2 \sim K_2 \frac{M_2}{R_2^2} \qquad (20)$$

where τ_2 is the optical path, and K_2 is the opacity. Gaustad (1963) has investigated the opacity of material expected to be found in the interstellar medium. He finds that known opacities will be very low. As an upper limit currently known values of opacity rarely exceed 1 or 10, even in stellar interiors. If we then evaluate Eq. (20) for a very favorable case, say $K_2 = 1$, $M_2 = 10^{37}$g, $R_2 = 10^{18}$cm. we obtain $\tau_2 = 10$. This value of τ_2 can be increased by diminishing R_2. However, then the rate of turbulent dissipation increases, and R_{1f} diminishes considerably.

An atmosphere with optical depth somewhat greater than unity will of course somewhat modify the temperature and pressure distribution inside the cluster. However, its effects on evolution should be second order, or else not a direct consequence of its opacity. It will no more prevent radiation from escaping from a star than does the very thin outer layer of the star itself.

We may conclude that conditions seem unfavorable for the maintenance of a massive, static, cluster atmosphere for time scales of the order of the age of the galaxy.

And if one does exist, its effects on the internal cluster will be not primarily through its opacity. This discussion does not preclude the existence of large amounts of interstellar gas in a cluster. The opacity of such gas would still be too low to retard evolution of the stars by a significant amount. But other effects, such as accretion, friction, and shock wave propagation would still be present.

The dissolution outwards of a cluster atmosphere would not affect the binding forces of the cluster. However, removal of gas from within the cluster will, of course, enhance its tendency to dissipate. Shock waves may help effect this removal. The situation is qualitatively analogous to that of shock waves in the solar atmosphere, and a "cluster wind" may result.

The applicability of much of the discussion in this paper depends on the time scale for some of the processes involved, and particularly for the magnetic processes, this was sometimes not discussed. Certainly the time scales for magnetic field slippage, collapse along the direction of the field (both with and without rotation) and maintenance of ionization, should be more thoroughly investigated.

IV. MAGNETOTURBULENCE

Finally, a recent suggestion advanced by Layzer (1965) may significantly modify the conventional processes so far considered in this paper. In considering the "Nature and Origin of Non-Thermal Radio Sources" he envisions the existence of "magnetoturbulence:" turbulence in the presence of a strong magnetic field. The property of magnetoturbulence which interests us in the present context is the very low rate of dissipation associated with it. Although Layzer's theory has not yet been fully developed, it is clear that non-decaying turbulence can offer a long term distending force. This mechanism may in fact prove to be more viable and effective than rotation. Magnetoturbulence in individual protostars, in protoclusters, or in giant stars may prove able to provide forces capable of retarding their evolution for time scales of the order of the age of the galaxy.

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