## Technical Report

## JUNCIURE STRESS FIEL.DS IN MULIICELLULAR SHELL STRUCTURES

Final Report<br>Nine Volumes

## Vol. V Influence Coefficients of Segmental Shells

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## FOREWORD

This report is the result of a study on the numerical analysis of stiffness influence coefficients for cylindrical, conical and spherical shell segments. Work on this study was performed by staff members of Lockheed Missiles \& Space Company in cooperation with the George C. Marshall Space Flight Center of the National Aeronautics and Space Administration under Contract NAS 8-11480. Contract technical representative was $H$. Coldwater.

This volume is the fifth of a nine-volume final report of studies conducted by the department of Solid Mechanics, Aerospace Sciences Laboratory, Lockheed Missiles \& Space Company. Project Manager was K. J. Forsberg; E. Y. W. Tsui was Technical Director for the work.

The nine volumes of the final report have the following titles:
Vol. I Numerical Methods of Solving Large Matrices
Vol. II Stresses and Deformations of Fixed-Edge Segmental Cylindrical Shells

Vol. III Stresses and Deformations of Fixed-Edge Segmental Conical Shells

Vol. IV Stresses and Deformations of Fixed-Edge Segmental Spherical Shells

Vol. V Influence Coefficients of Segmental Shells
Vol. VI Analysis of Multicellular Propellant Pressure Vessels by the Stiffness Method

Vol. VII Buckling Analysis of Segmental Orthotropic Cylinders under Uniform Stress Distribution

Vol. VIII Buckling Analysis of Segmental Orthotropic Cylinders under Non-uniform Stress Distribution

Vol. IX Summary of Results and Recommendations

This volume presents a technique using the modified digital programs developed for intermediate loads or thermal gradients to determine stiffness influence coefficients of cylindrical, conical and spherical shell segments. The problem is solved numerically by the finite difference method, using a direct method for solving a large system of simultaneous equations. For completeness as a self-contained report, a portion of the information presented in Vol. II, III and IV is repeated here.

Author

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## NOTATION


dimensional quantities
dimensionalizing matrix for displacements diagonal matrix $\delta=\mu \delta$
$\lambda$
dimensionalizing matrix for forces diagonal matrix $\hat{F}=\lambda F$ Additional notations and symbols are defined in the text.

Section 1
INIRODUCTION

As a results of an investigation of juncture stress fields peculiar to the multicellular pressure vessels ( $F \div g$. l), a theory for the prediction of the membrane and bending stresses and the corresponding deformations for such shell structures was formulated.*


Fig. 1 Multicellular Shell Structure

[^0]Since analytic solutions are still lacking, it was decided to solve the problem numerically by means of a finite-difference technique. To ensure the feasibility of such a numerical solution, a direct method of solving large matrices with a high-speed digital computer was also developed.

According to the previous work, if the stiffness or displacement method is used, the total forces and hence the corresponding stresses along the juncture of the shell segments (Fig. 2) may be expressed concisely in the following matrix form

$$
\begin{equation*}
F=k \delta+F^{f} \tag{1.1}
\end{equation*}
$$

where $k$ is the stiffness matrix, $\delta$ are the deformations, and $F^{f}$ are the fixed-end forces due to applied loads or thermal gradients. In view of this situation, it is logical to solve the problem systematically by the established general procedure of analysis already described. This procedure may be stated briefly as follows:

1. Determination of the fixed-end forces, $\mathrm{F}^{\mathrm{f}}$, along the boundary as well as stresses and deformations in the interior of shell segments due to loads
2. Determination of the influence coefficients, $k_{i j}$, along the boundaries of shell segments, i.e., the induced forces at points i due to unit deformations $(\delta=1)$ at points $j$
3. Determination of the actual deformations, $\delta$, along the shell boundaries; this requires the satisfaction of both compatibility and equilibrium conditions at the junctures of the structure

Once all the work involved in these three steps is completed, the total stresses and deformations in the specific discrete interior locations may be obtained.

This volume presents results of the work involved in Step 2 to determine the local stiffness influence coefficients $k_{i j}$ of cylindrical, conical, and spherical shell segments. The problem is formulated as a homogeneous


Fig. a Basic Shell Elements of Multicellular Structure
differential equation with inhomogeneous boundary conditions. The numerical technique of finite differences is used to obtain a solution to this problem.

The operation of three digital programs is given and three numerical examples are presented to demonstrate the program input and output.

It should be noted that influence coefficients for rectangular plates can be determined by the program for cylinders if the radius ( $\hat{R}$ ) is increased appropriately.

Section 2
FORMULATION OF THE PROBLEM

### 2.1 General Considerations

The basic equations governing segmental cylindrical, conical, and spherical shells subjected to intermediate loads such as pressure, temperature, etc., are formulated in Vols. II, III, and IV. In order to mathematically join two or more of these continuous elastic shell segments together to form a multicellular structural shape, it is necessary to establish the constitutive boundary equations for each of the segments. These equations relate the boundary loads to boundary displacements as a function of the boundary coordinate $\eta$. If boundary displacements are known then the boundary loads can be uniquely determined by these linear relations.

One technique to accomplish this is known as the stiffness method which describes the boundary force as a function of the boundary coordinate due to unit deformations introduced at a point $s$ on the boundary. For an element the constitutive equations can be written as

$$
\begin{equation*}
F=k \delta \tag{2.1}
\end{equation*}
$$

where $F$ and $\delta$ are column vectors defined by

$$
\mathrm{F}=\left(\begin{array}{c}
\bar{N}_{\eta} \\
\bar{N}_{\zeta} \\
\bar{Q} \\
\bar{M}
\end{array}\right) ; \delta=\left(\begin{array}{c}
u_{\eta} \\
u_{\zeta} \\
w \\
\omega_{\eta}
\end{array}\right)
$$

and $k$ is a stiffness matrix which is a function of the boundary coordinate and the point at which the displacements are applied. A general load can be obtained by superposition of the effects of a given boundary displacement. To
establish the stiffness matrix, unit values of the displacement components are applied at a point on the shell boundary. At all other points the displacement vector is set to zero. This boundary-value problem involves solving a set of homogeneous differential equations subjected to homogeneous boundary conditions except at the point at which the unit displacement is applied.

### 2.2 Governing Differential Equations

The governing differential equations required to find the stiffness matrix of a segmental cylindrical, conical, and spherical shell are given in Vols. II, III, and IV. All that must be done is to set the right hand terms to zero.

The governing equations for a cylindrical shell become [see Eqs. (2.8a-c, Vol. II)]:

$$
\begin{aligned}
& a_{1} u_{,_{x x}}+a_{2}{ }^{u,}{ }_{\theta \theta}+a_{3}{ }^{v,}{ }_{x \theta}+a_{4} w,{ }_{x x x}+a_{5} w,{ }_{x}=0
\end{aligned}
$$

$$
\begin{aligned}
& +c 8^{w,}{ }_{\theta \theta \theta \theta}+c_{9}{ }^{w}{ }_{\theta \theta}+c_{10}=0 \quad(2 . c c)
\end{aligned}
$$

For a conical shell, the homogeneous equations are given by:

$$
\begin{align*}
& a_{1}{ }^{u},_{x x}+a_{2}{ }^{u,}{ }_{\theta \theta}+a_{3} u_{x}+a_{4} u+a_{5} v,{ }_{x \theta}+a_{6}{ }^{v,} \theta_{\theta}+a_{7}{ }^{w},_{x}+a_{8} w=0 \tag{2.3a}
\end{align*}
$$

$$
\begin{align*}
& +b_{9}{ }^{w}{ }_{x \theta}+b_{10}{ }^{w}{ }_{\theta}=0 \tag{2.3b}
\end{align*}
$$

$$
\begin{aligned}
& c_{1} u_{x}+c_{2} u+c_{3} v_{x x \theta}+c_{4}{ }^{v,}{ }_{\theta \theta \theta}+c_{5}{ }^{v}{ }_{x \theta}+c_{6}^{v v_{\theta}}+c_{7}{ }^{w,}{ }_{x x x x}
\end{aligned}
$$

$$
\begin{align*}
& +c_{13^{W}{ }_{\theta \theta}}+c_{14^{W}{ }_{x}}+c_{15^{W}}=0 \tag{2.3c}
\end{align*}
$$

The corresponding equations for spherical shells are:

$$
\begin{align*}
& a_{1} u_{\varphi \varphi}+a_{2} u_{,} \theta_{\theta}+a_{3} u_{\varphi}+a_{4} u+a_{5} v,_{\varphi \theta}+{ }^{a_{6}} \sigma^{v} \theta+a_{7}{ }^{w,} \varphi_{\varphi \varphi} \\
& +a_{8}{ }^{w,} \varphi_{\varphi \theta}+a_{9}{ }^{w,}{ }_{\varphi \varphi}+a_{10^{w,} \theta_{\theta}}+a_{11}{ }^{w, \varphi}=0  \tag{2.4a}\\
& b_{1} u_{\varphi} \varphi_{\varphi}+b_{2} u_{\theta}+b_{3} v{ }_{\varphi \varphi}+b_{4}^{v,} \theta \theta+b_{5} v \varphi_{\varphi}+b_{6} v+b_{7}^{w,}{ }_{\varphi \varphi \rho} \\
& +b_{8}{ }^{w}{ }_{\theta \theta \theta}+b_{9} w_{\rho}+b_{10^{w} \theta}=0  \tag{2.4b}\\
& c_{1}^{u_{,}} \varphi_{\varphi \varphi \varphi}+c_{2} u_{\varphi} \varphi_{\varphi \theta}+c_{3} u_{\varphi \varphi}+c_{4} u_{,} \theta_{\theta}+c_{5} u_{\varphi}+c_{6}^{u}+c_{7} v_{\varphi \varphi}{ }_{\varphi \varphi} \\
& +c_{8}{ }^{v}{ }_{\theta \theta \theta \theta}+c_{9} v_{\varphi \theta}+c_{10^{v} v_{\theta}}+c_{11}{ }^{W},_{\varphi \varphi \varphi \varphi \varphi}
\end{align*}
$$

$$
\begin{align*}
& +\mathrm{c}_{16^{W}{ }_{\varphi \varphi}}+\mathrm{c}_{17^{\mathrm{W},}{ }_{\theta \theta}}+\mathrm{c}_{18^{\mathrm{W}}, \varphi}+\mathrm{c}_{19^{\mathrm{W}}}=0 \tag{2.4c}
\end{align*}
$$

All of the $a_{i}, b_{i}, c_{i}$ coefficients of the preceding equations are given in Vols. II, III, and IV and are not repeated in this volume.

### 2.3 Boundary Conditions

It has been established previously (see Sect. 2.2, Vol. II) that four boundary conditions are required at each point on the shell boundary. Consider a general shell boundary given in Fig. 3. The boundary coordinates are given as


Fig. 3 Boundary Coordinates, Deformations, and Forces
$5, \eta, z$; positive deformations $u_{\zeta}, u_{\eta}, w, \omega_{\eta}$ and their corresponding forces $\bar{N}_{\zeta}, \bar{N}_{\eta}, \bar{Q}, \bar{M}$ are defined as shown. The boundary constitutive equations [Eqs. (1)] can be written as

$$
\left[\begin{array}{l}
\bar{N}_{\eta}  \tag{2.5}\\
\bar{N}_{\zeta} \\
\bar{Q} \\
\bar{M}
\end{array}\right]=\left[\begin{array}{llll}
k_{11} & \cdots & \\
k_{21} & \cdots & \\
k_{31} & \cdots & \cdots \\
k_{41} & \cdots & & \\
k_{44}
\end{array}\right]\left(\begin{array}{l}
u_{\eta} \\
u_{5} \\
w \\
\omega_{\eta}
\end{array}\right)
$$

where $k_{i j}$ are functions of the coordinate $\eta$ and the point $s$ at which the displacement is specified.

In order to find $k_{11}, k_{21}, k_{31}$, and $k_{41}$, the required boundary conditions are

$$
\begin{array}{ll}
u_{\zeta}=w=\omega_{\eta}=0 & \text { for all } \eta \\
u_{\eta}=1.0 & \text { for } \eta=s, 0 \leq s \leq \ell  \tag{2.6}\\
u_{\eta}=0 & \text { for } \eta \neq s
\end{array}
$$

where $\ell$ is the total length of the boundary curve. With the above boundary conditions the governing equations must be solved for the boundary forces which are the desired stiffness functions. The other stiffness functions are fourd in a similar manner.

For the cylindrical, conical, and spherical elements of the multicellular shell structure, the boundary curves are shown in Fig. 4. The nondimensional displacement vector and the corresponding force vector for these shapes are shown respectively in Tables 1 and 2.

The pertinent differential equations have been nondimensionalized so as to cover a wide range of shell parameters. Once the stiffness matrix is found in nondimensional form, it can be transformed to dimensional quantities if it is so desired. The nondimensional boundary loads and displacements can be converted to dimensional values by the following multiplications

$$
\begin{align*}
& \hat{F}=\lambda F=\left[\begin{array}{cccc}
\lambda_{11} & & & 0 \\
& \lambda_{c \cdot 2} & & \\
& & \lambda_{33} & \\
& 0 & & \lambda_{44}
\end{array}\right]\left[\begin{array}{c}
\bar{N}_{\eta} \\
\overline{N_{5}} \\
\bar{Q} \\
\bar{M}
\end{array}\right)  \tag{2.7}\\
& \hat{\delta}=\mu \delta=\left[\begin{array}{cccc}
\mu_{11} & & & 0 \\
& \mu_{22} & & \\
& & \mu_{33} & \\
& 0 & & \mu_{44}
\end{array}\right]\left(\begin{array}{l}
u_{\eta} \\
u_{5} \\
w \\
\omega_{5}
\end{array}\right) \tag{2.8}
\end{align*}
$$

where $\lambda$ and $\mu$ are diagonal matrices. The coefficients of these matrices are given in Tables 3 and 4 for the cylinder, cone, and sphere.

$$
\begin{aligned}
& \overline{\mathrm{ab}}-\text { upper boundary } \\
& \overline{\mathrm{bc}}-\text { right boundary } \\
& \overline{\mathrm{cd}}-\text { lower boundary } \\
& -\quad \text { symmetry line }
\end{aligned}
$$

a) Cylinder
b) Cone

c) Sphere - Orientation 1

d) Sphere - Orientation 2

Fig. 4 Boundary Curves for Cylindrical, Conical and Spherical Segments of Multicellular Structure

|  | $\delta$ | Cylinder | Cone | Sphere |
| :---: | :---: | :---: | :---: | :---: |
| 10 | $\begin{aligned} & u_{\eta} \\ & u_{\zeta} \\ & \omega_{n} \end{aligned}$ | $\begin{gathered} -v \\ -u \\ w \\ +\left(\rho / \theta_{c}\right) w, x \end{gathered}$ | $-\mathrm{V}$ <br> $-\mathbf{u}$ $+\mathrm{W}, \mathrm{x}$ | $-V$ $-u$ <br> W $-u+w_{1}$ |
| 10 | $\begin{aligned} & u_{\eta} \\ & u_{\delta} \\ & w \\ & \omega_{n} \end{aligned}$ | $\begin{gathered} -u \\ v \\ v=\left(1 / \theta_{c}\right) w, \theta \end{gathered}$ | $\begin{gathered} -u \\ v \\ v / x \tan \varphi_{3}-w, \theta_{\theta} / x \theta_{c} \sin \varphi_{3} \end{gathered}$ | $\begin{gathered} -u \\ v \\ w \\ v-(1 / \sin \varphi)^{w} \theta \end{gathered}$ |
| 10 | $\begin{aligned} & u_{\eta} \\ & u_{\zeta} \\ & w \\ & w_{\eta} \end{aligned}$ | V <br> $u$ <br> W $-\left(p / \theta_{c}\right) w_{x}$ | V <br> $u$ <br> w $\text { -W, } x$ | $\begin{gathered} v \\ u \\ w \\ u-w_{1} \varphi \end{gathered}$ |
|  |  | 1 Nondimen | 1 Displacement Components Cor ne Boundary Displacement $\delta$ | sponding |


|  | F | Cylinder | Cone | Sphere |
| :---: | :---: | :---: | :---: | :---: |
| $1 \times$ | $\begin{aligned} & \bar{N}_{\eta} \\ & \bar{N}_{5} \\ & \bar{Q} \\ & \bar{M}_{\zeta} \end{aligned}$ | $\begin{aligned} & N_{x \theta}+M_{x \theta} \\ & N_{x} \\ & -\left[Q_{x}+\frac{1}{\theta_{c}} M_{x \theta, \theta}\right] \\ & M_{x} \end{aligned}$ | $\begin{aligned} & N_{x \theta}+\frac{h^{2}}{6 x(1-v) \tan \varphi_{3}} M_{x \theta} \\ & N_{x} \\ & -\left[Q_{x}+\frac{1}{x \theta_{c} \sin \varphi_{3}} M_{x \theta, \theta}\right] \\ & M_{x} \end{aligned}$ | $\begin{aligned} & N_{\varphi \theta}+2 x M_{\varphi \theta} \\ & N_{\varphi} \\ & -\left[Q_{\varphi}+\frac{(1-v)}{\sin \varphi} M_{\varphi \theta, \theta}\right. \\ & M_{\varphi} \end{aligned}$ |
| 18 | $\begin{aligned} & \bar{N}_{\eta} \\ & \bar{N}_{5} \\ & \overline{\mathrm{Q}} \\ & \overline{\mathrm{M}}_{5} \end{aligned}$ | $\begin{aligned} & -N_{\theta x} \\ & N_{\theta} \\ & {\left[Q_{\theta}+\frac{\rho}{\theta_{c}} M_{\theta x, x}\right]} \\ & M_{\theta} \end{aligned}$ | $\begin{aligned} & -N_{\theta x} \\ & N_{\theta} \\ & {\left[Q_{\theta}+M_{\theta x, x}\right]} \\ & M_{\theta} \end{aligned}$ | $\begin{aligned} & -N_{\theta \varphi}-\Sigma x M_{\varphi \theta} \\ & N_{\theta} \\ & Q_{\theta}+(1-v) M_{\varphi \theta, \varphi} \\ & M_{\theta} \end{aligned}$ |
| 10 | $\begin{aligned} & \overline{\mathrm{N}}_{\mathrm{T}} \\ & \overline{\mathrm{~N}}_{6} \\ & \overline{\mathrm{Q}} \\ & \overline{\mathrm{M}}_{\zeta} \end{aligned}$ | $\begin{aligned} & N_{x \theta}+M_{x \theta} \\ & N_{x} \\ & Q_{x}+\frac{1}{\theta_{c}} M_{x \theta, \theta} \\ & M_{x} \end{aligned}$ | $\begin{aligned} & N_{x \theta}+\frac{h^{2}}{6 x(1-v) \tan \varphi_{3}} M_{x \theta} \\ & N_{x} \\ & Q_{x}+\frac{1}{x \theta} \sin \varphi_{3} M_{x \theta, \theta} \\ & M_{x} \end{aligned}$ | $\begin{aligned} & N_{\varphi \theta}+2 \kappa M_{\varphi \theta} \\ & N_{\varphi} \\ & Q_{\varphi}+\frac{(1-v)}{\sin \varphi} M_{\varphi \theta, \theta} \\ & M_{\varphi} \end{aligned}$ |

ensions 1 Boundary Forces Corresponding to
the Boundary Force F

|  | Cylinder | Cone | Sphere |
| :--- | :--- | :--- | :--- |
| $\lambda_{11}$ | Eh | $\mathrm{Eh} / 2(1+v)$ | $\mathrm{Eh} / 2(1+v)$ |
| $\lambda_{22}$ | $E h$ | $\mathrm{Eh} /\left(1-v^{2}\right)$ | $\mathrm{Eh} /\left(1-v^{2}\right)$ |
| $\lambda_{33}$ | Eh | $\mathrm{D} / \mathrm{X}_{\mathrm{L}}^{2}$ | $\mathrm{D} / \mathrm{R}^{2}$ |
| $\lambda_{44}$ | EhR | $\mathrm{D} / \mathrm{X}_{\mathrm{L}}$ | $\mathrm{D} / \mathrm{R}$ |

Table 3 Components oi $\lambda$ which Dimensionalizes the Boundary Forces of Cylindrical, Conical, and Spherical Shell Segments

|  | Cylinder | Cone | Sphere |
| :--- | :--- | :--- | :--- |
| $\mu_{11}$ | R | $\mathrm{X}_{\mathrm{L}}$ | R |
| $\mu_{22}$ | R | $\mathrm{X}_{\mathrm{L}}$ | R |
| $\mu_{33}$ | R | $\mathrm{X}_{\mathrm{L}}$ | R |
| $\mu_{44}$ | 1.0 | 1.0 | 1.0 |

Table 4 Components of $\mu$ which Dimensionalize the Boundary Displacements of Cylindrical, Conical and Spherical Shell Segments

To dimensionalize the stiffness matrix requires

$$
\hat{F}=\lambda F=\lambda k \delta=\left(\lambda k \mu^{-1}\right) \delta
$$

Thus the dimensional stiffness matrix is

$$
\begin{equation*}
\hat{k}=\lambda k \mu^{-1} \tag{2.9}
\end{equation*}
$$

and each component can be found by

$$
\begin{equation*}
\hat{k}_{i j}=\lambda_{i i} k_{i j} \frac{l}{\mu_{j j}} \tag{2.10}
\end{equation*}
$$

## Section 3

NUMERICAL ANALYSIS

### 3.1 General

The boundary value problem formulated in Section 2 for the determination of the stiffness matrix will be solved by the finite-difference method. This method is described in Vols. II, III, and IV for the analysis of cylindrical, conical and spherical shell segments with fixed edges and subjected to surface and thermal loads. This numerical method of solution replaces the continuous coordinate system defining the shell segments by a finite number of coordiaate points. To accomplish this discretization, the continuous two-dimensional domain of the shell reference surface is covered by a rectangular net. Lattice points of this net which fall on the boundary curve are called boundary pcints, and all other lattice points interior to the boundary are called mesh points. At these lattice points the dependent variables ( $u, v, w$ ) of the governing differential equations are replaced by discrete variables.

The difference equations which are a set of algebraic relations representing the governing equations and boundary conditions are formed by first approximating the derivatives at a given point by a function of the variable at neighboring points. These functions replace the derivatives of the governing equations. Thus, at each mesh point three algebraic equations can be written in terms of neighboring points. When the boundary conditions are accounted for in these equations the resulting set of simultaneous algebraic equations

$$
\underset{\sim}{A X}=\underset{\sim}{B}
$$

replaces the continuous problem. The solution of this set of algebraic equations can be accomplished by methods described in Vol. I.

Difference equations for interior points of the shell segments undur consideration are available in Vols. II, III, and IV and are not repeated in this volume. As noted in Section 2 the only difference is that the right hand side is zero. The main concern in this section is the presentation of the bourdary conditions necessary to generate the stiffness matrix and a description of the procedure to find this matrix.

### 3.2 Stiffness Matrix

The stiffness matrix given in Section 2 which relates the boundary displacements to the boundary forces as a function of the boundary coordinate $\eta$ and the point $s$ at which unit displacements are applied. The discretizatior of this problem by finite differences leads to a stiffness matrix defined by the boundary points.

Figure 5 represents a shell segment covered by a rectangular net required for a finite difference solution of the problem. Instead of the continuum boundary coordinate $\eta$, the boundary curve is defined by a sequence of points denoted by $i=1,2,3 \ldots n$ as shown.


Fig. 5 Boundary of a Shell Segment for a
Finite Difference Solution

At each point i four boundary forces and four displacement quantities are given as

$$
F_{i}=\Delta \eta_{i}\left(\begin{array}{c}
\bar{N}_{\eta}  \tag{3.1}\\
\bar{N}_{\zeta} \\
\bar{Q} \\
\bar{M}
\end{array}\right)_{i} \quad, \quad \delta_{i}=\left(\begin{array}{c}
u_{\eta} \\
u_{\zeta} \\
w \\
\omega_{\eta}
\end{array}\right)_{i}
$$

The boundary point is considered to be the mid-point of a small line segment $\Delta \eta_{i}$ over which the load and displacement act.

In general the constitutive boundary equation in matrix form is

$$
F=k \delta
$$

For the shell segment shown in Fig. 5 this equation in an expanded form becomes

$$
\left.\left(\begin{array}{c}
F_{1}  \tag{3.2}\\
F_{2} \\
\cdot \\
F_{n}
\end{array}\right]=\left[\begin{array}{llllll}
K_{11} & K_{12} & \cdots & \cdots & \cdots & K_{1 n} \\
K_{21} & K_{22} & \cdots & \cdots & \cdots & \\
\cdots & & \cdots & \cdots & \cdots & \cdots
\end{array}\right] K_{n n}\right]\left[\begin{array}{c}
\delta_{1} \\
\delta_{2} \\
\cdot \\
\delta_{n}
\end{array}\right]
$$

where $K_{i, j}$ are $4 \times 4$ submatrices.
The coefficients of the submatrices can be defined by four indices as $k_{i, j}^{\ell, m}$. The subscript $i$ denotes the point at which the force is acting, $j$ shows the point the displacement is applied. The superscript $k$ and $m$ clarify the boundary forces and displacements, $\ell=1$ designates the load $\bar{N}_{\eta}$, $\boldsymbol{\ell}=2,3,4$ designates $\bar{N}_{S}, \bar{Q}$ and $\bar{M}$ respectively. The superscript $m=1,2$, 3, or 4 shows that the load is due to a displacement $u_{\eta}, u_{5}$, w, or $\omega_{\eta}$
respectively. Thus $k_{i . j}^{2,3}$ is the boundary force $\bar{N}_{5}$ at point $i$ due to a unit displacement of $w$ at point $j$. From the above matrix it is evident that the boundary forces can be obtained once the displacements are known.

### 3.3 Boundary Conditions

To find the influence coefficients of a fixed edge segmental shell, a unit value of $u_{\eta}, u_{\zeta}, w$, or $\omega_{\eta}$ is applied at a boundary point $j$ while the other boundary displacements are set to zero. The influence coefficients are then equal to the respective forces due to the unit displacement.

In specific to find the influence coefficients due to a unit displacement, the boundary conditions $u_{\eta}$ are

$$
\begin{align*}
& u_{\eta}=0 \text { at points } i=1,2 \ldots j-1, j+1 \ldots n \\
&=1  \tag{3.3}\\
& u_{5}=w=a_{\eta}=0 \text { at all points }
\end{align*}
$$

With the above boundary conditions, the boundary forces obtained by solving the governing equations yield the influence coefficients

$$
k_{i, j}^{\ell, 1} \quad \text { for } \ell=1,2,3,4 \text { and } i=1,2,3 \ldots \eta
$$

This is equivalent to a complete colum of the influence matrix $k$. The other columns are obtained by giving $u_{5}, w, \omega_{\eta}$ unit values at all boundary points in a manner similar to that for $u_{\eta}$.

# Section 4 <br> DIGITAL PROGRAMS 

### 4.1 General Description

The digital programs for fixed-edge cylindrical, conical, and spherical shell segments described in Vols. II, III and IV have been modified so as to allow unit nondimensional dispiacements ( $u, v, w$ ) and rotation ( $\omega_{\eta}$ ) to be prescribed on the segment boundary at specified locations. Output of the programs consist of the boundary forces ( $\bar{N}_{\eta}, \bar{N}_{\zeta}, \bar{Q}, \bar{M}_{\zeta}$ ) resulting from the boundary disturbance. As shown in Section 3, these boundary forces are the local stiffness coefficients $k_{i j}$. This information along with identification can be outputed on a reserve tape (B6) for use in the overall juncture problem. This detail will be explained in Vol. VI.

The program modification consisted of changes in the difference equations at specific points and their neighboring points where unit displacements are introduced. To write an efficient program for the solution of the simultaneous equation system

$$
\underset{\sim}{A} X=\underset{\sim}{B}
$$

the right hand term $\underset{\sim}{B}$ was expanded to 61 columns. Since $\underset{\sim}{A}$ is factored this permits 61 right hand vectors to be solved in a minimum of time. These 61 columns correspond to 15 points at which the boundary can be disturbed and the case of intermediate loads. Displacements can only be applied at the intersection of the mesh lines and the boundary. Likewise the boundary forces are computed at all boundary points. When a point is chosen to yield the boundary displacement, all four quantities $u, v, w$, and $\omega_{\eta}$ are given a unit value consecutively. In the program these can be applied along the upper, right, and lower boundaries. Figure 4 defines these boundary lines for the cylinder, cone, and sphere.

The finite-difference mesh network is specified completely by prescribing the number of rows and columns exclusive of the boundaries, together with the grading options which have been chosen. Rows in the finite-difference mesh are parallel to the $\theta$-axis, and columns are parallel to the $x$ - or $\varphi$-axis. The number of rows may vary from 4 to 24 and the number of colums from 4 to 80. Thus, a maximum of 5760 unknowns can be solved. Greater accuracy near the boundaries can often be obtained by selecting grading. By this means, it is possible to use a mesh spacing at the boundary as little as $1 / 32$ of that at the middle portion of the panel.

Tlere are certain restrictions on the use of the grading option. When such an option is used, a separate input card is required to specify a mesh spacings exponent $M M(J)$ for each row $J$. The finite-difference equations written along row $J$, then use the mesh spacing $X H / 2^{*} * M M(J)$. This distance must be the least of the two distances from row $J$ to the row above and the row below. XH is the basic input mesh spacing along the $x$-direction. For any row $J$, $\operatorname{MM}(J)$ and $M(J+1)$ must not differ by more than 1 . Also, three consecutive rows cannot have three distinct exponents. $M M(J)$ may vary from 0 to 5 .

The description of symbols, and input data for the cylindrical, conical and spherical shell segments are given in Tables 5 and 6 .

Table 5
DESCRIPTION OF SYMBOLS

Symbol
Description

## Cylinder

IOPII 0

IOPT2

IOPT3

RHO
THC
R1H
DIH
D1B
T1H
ClH
0
1

号

IOPTI

THC
HBO
H工
XOXI
PH3

0
1

Cone
Uniform mesh spacing
Graded mesh spacing in x-direction
Half aygle of segment (see Fig. 3)
Nondimensional reference thickness ( $\hat{h}_{0} / \hat{X}_{L}$ )
Rate of change of thickness in $x$
Nondimensional distance (see Fig. 3) $\left(=\hat{x}_{\theta} / \hat{X}_{L}\right)$
Half cone angle

## Table 5

## DESCRIPTION OF SYMBOLS (cont'd)

## Symbol

Description

## Sphere



## Common to All Programs

Hollerith information describing problem
Number of rows in the finite-difference mesh Number of columns in the finite-difference mesh Basic distance between rows in the mesh Basic distance between columns in the mesh Poisson's retio

Grading mesh constants; mesh spacing used for difference equations on row $J$ is equal to $\mathrm{XH} / 2 . * * \operatorname{MM}(J)$

Number of points at which unit displacements are specified on upper boundary
NJB

NKB
$I B$

Number of points at which unit displacements are specified on right boundary

Number of points at which unit displacements are specified on lower boundary
Column number at which unit displacements are specified on upper boundary

| $\begin{aligned} & \text { Table } 5 \\ & \text { ION OF SYMBOLS (conc'd) } \end{aligned}$ |  |  |
| :---: | :---: | :---: |
| Symbol | Description |  |
| JB |  | Row number at which unit displacements are specified on right boundary |
| KB |  | Column number at which unit displacements are specified on lower boundary |
| ICAS |  | Case Identification |
| IREC |  | Number of records previously written on data tape |
| ITP | -1 | Transfer old records from reserve tape A6 to reserve tape B6 |
|  | +1 | Skip old records on reserve tape B6 |
| IEND | 0 | The program expects more records to be read on data tape |
|  | 1 | Data tape B6 is unloaded from the machine |

# Table 6 <br> INPUT DATA SEQUENCE AND FORMAT 

## Cylinder

| Card | Fortran Symbol | Format |
| :---: | :---: | :---: |
| 1 | RECORD | 72 H |
| 2 | IOPTI, IOPT2, IOPT3 | 5 II |
| 3 | ROW, COL, XH, XK | $6 \mathrm{El2} .8$ |
| 4 | ZNU, RHO, THC, RIH | 6 E 12.8 |
| $5^{(a)}$ | DIB, DlH, TlH, ClH | $6 \mathrm{El2.8}$ |
| $6^{(b)}$ | MM(J), J=1, ROW | 3512 |
| 7 | NIB, NJB | 212 |
| $8^{(c)}$ | $\operatorname{IB}(\mathrm{I}), \mathrm{I}=1, \mathrm{NIB}$ | 36 I 2 |
| $9^{(d)}$ | JB(I), $\mathrm{I}=1, \mathrm{NVB}$ | 36 I 2 |
| 10 | ICAS, IREC, ITP, IEND | 4 I 4 |

(a) Omitted unless IOPMI $=1$
(b) Omitted unless IOPT3 $=1$
(c) Omitted if NIB $=0$
(d) Omitted if NJB $=0$

# Table 6 <br> INPUT DATA SEQUENCE AND FORMAT (cont'd) 

## Cone

| Card | Fortran Symbol | Format |
| :---: | :---: | :---: |
| 1 | RECORD | 72 H |
| 2 | IOPTI | 3 Il |
| 3 | ROW, COL, XH, XK | $3 \mathrm{El2.8}$ |
| 4 | ZNU, THC, HBO, H1, XOXL, PH3 | $6 \mathrm{El2} .8$ |
| $5^{\text {(a) }}$ | MM(J), J=1, ROW | 35 I 2 |
| 6 | NIB, NJB, NKB | 3 I 2 |
| $7^{\text {(b) }}$ | $\mathrm{IB}(\mathrm{I}), \mathrm{I}=1, \mathrm{NIB}$ | 36 I 2 |
| $8^{(c)}$ | $J B(I), I=1, N J B$ | 3612 |
| $9^{\text {(d) }}$ | $\mathrm{KB}(\mathrm{I}), \mathrm{I}=1, \mathrm{NKB}$ | 3612 |
| 10 | ICAS, IREC, ITP, IEND | 4 I 4 |

(a) Omitted unless IOPTl $=1$
(b) Omitted if NIB $=0$
(c) Omitted if NJB $=0$
(d) Omitted if $\mathrm{NKB}=0$

## Table 6 <br> INPUT DATA SEQUENCE AND FORMAT (conc'd)

## Sphere

| Card | Fortran Symbol | Format |
| :---: | :---: | :---: |
| 1 | RECORD | 72 H |
| 2 | IOPT1, IOPT2, IOPT3, IOPT4, IOPT5 | 10 Il |
| 3 | ROW, COL, XH, XK | 3E12.8 |
| 4 | ZNU, THC, PHI, FF, RH | $6 \mathrm{El2.8}$ |
| $5^{(a)}$ | $\operatorname{MM}(J), J=1, \mathrm{ROW}$ | 3512 |
| 6 | NIB, NJB, NKB | 3 I 2 |
| $7^{(b)}$ | $\mathrm{IB}(\mathrm{I}), \mathrm{I}=1, \mathrm{NIB}$ | 3612 |
| $8^{(c)}$ | $J B(I), ~ I=1, ~ N J B$ | 3612 |
| $9^{(d)}$ | $\mathrm{KB}(\mathrm{I}), \mathrm{I}=1, \mathrm{NKB}$ | 3612 |
| 10 | ICAS, IREC, ITP, IEND | 4 I 4 |

(a) Omitted unless IOPTI $=1$
(b) Omitted if NIB $=0$
(c) Omitted if NJB $=0$
(d) Omitted if $N K B=0$

### 4.2 Numerical Examples

Output of the influence coefficient programs is demonstrated for cylindrical, conical, and spherical shell elements shown in Fig. 6. Sample output for the three elements given in Table 7 consists of the boundary stress resultants noted as NTAN, NNORM, $Q$, and $M\left(\bar{N}_{T_{i}}, \bar{N}_{\xi}, \bar{Q}, \bar{M}_{\xi}\right)$ is printed out at each boundary mesh point denoted by a row and column number. These stress resultants are due to a unit value of $u_{\xi}$, $u_{\eta}$, w or $\omega_{\eta}\left(u, v, w, w^{*}\right)$ introduced at a boundary point which is also given in the print out. As shown in Section 3 the boundary stress resultants correspond to sjiffness influence coefficients. The program always computes the boundary resultants at each boundary point specified in the order $u, v, w, w^{*}$. The cylindrical element has two lines of symmetry and in the finite difference mesh ten columns and fifteen rows with grading are used. Results shown in Table 7 for this element are for deformations introduced on the upper row corresponding to column 3, row 15 . The conical element has seventeen rows and ten columns and the deformations are introduced on the lower boundary corresponding to column 3, row 0 . For the sphere thirteen rows and ten columns are used and the deformations are introduced on the lower boundary corresponding to column 3, row 0 . Due to grading and the column spacing of the examples, accuracy of the boundary resultants vary along different boundary lines.


side View


Fig. 6 Configurations of shells in the numerical examples

Table 7
Sample Output of Influence Coefficients - Cylinder

COL 3 ROW $15 \quad U=-1.0$
BOUNDARY STRESS RESULTANTS.

| ROW | COL | NTAN | NNORM | 0 | M |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 15 | 1 | 0. | -9.0222E-01 | -1.5998E-01 | -9.2569E-04 |
| 15. | 2 | -1.3355E 00 | -3.1935E 00 | $-2.3816 E-01$ | -8.9420E-05 |
| 15. | 3 | -6.0558E-02 | 8.6910E 00 | 1.9975E-01 | -6.3680E-03 |
| 15. | 4 | $1.2271 E 00$ | -3.0156E 00 | -8,0657E-02 | $1.8081 E=03$ |
| 15. | 5 | -6.6651E-02 | -4.0185E-01 | 5.8793E-03 | 6.3627E-04 |
| 15. | 6 | -6.9635E-02 | -1.1362E-01 | $1.0013 E-01$ | 1.4712E-03 |
| 15. | 7 | -5.5749E-02 | -4.3395E-03 | $1.1364 E-01$ | $1.5277 E-03$ |
| 15. | 8 | -3.7963E-02 | 3.3326E-02 | 9.6847E-02 | 1.2600E-03 |
| 15. | 9 | -2.2407E-02 | 3.6860E-02 | 6.5395E-02 | 8.3474E-04 |
| 15. | 10 | -1.1557E-02 | 2.5179E-02 | 2.8230E-02 | 3.5110E-04 |
| 15. | 11 | 0 . | 0 . | -7.0119E-07 | -0. |
| 15. | 11 |  |  |  |  |
| 15. | 11 | -0. | 0. | 0. | -0. |
| 14. | 11 | 6.9335E-04 | 2.0095E-03 | 5.2537E-07 | -1.3148E-06 |
| 13. | 11 | 1.2679E-03 | 3.5461E-03 | 1.8371E-05 | -8.5504E-07 |
| 12. | 11 | 2.1916E-03 | $5.9968 E=03$ | 7.3118E-05 | 1.9774E-06 |
| 11. | 11 | 2.9859E-03 | 7.6058E-03 | 1.5408E-04 | 7.4666E-06 |
| 10. | 11 | 3.6715E-03 | 8.6119E-03 | 2.5252E-04 | $1.4743 E-05$ |
| 9. | 11 | 4.8553E-03 | $9.8314 E-03$ | 4.6869E-04 | 3.1247E-05 |
| 8. | 11 | 5.8192E-03 | $1.0146 E-02$ | 7.0549E-04 | 4.9932E-05 |
| 7. | 11 | 7.4301E-03 | 9.8311E-03 | 1.1982E-03 | 8.9270E-05 |
| 6. | 11. | 8.5517E-03 | 8.8183E-03 | 1.7010E-03 | 1.2933E-04 |
| 5. | 11 | $1.0111 \mathrm{E}-02$ | $6.8135 E-03$ | 2.7074E-03 | 2.0803E-04 |
| 4. | 11 | 1.0494E-02 | 6.0139E-03 | 3.7405E-03 | 2.7935E-04 |
| 3. | 11 | 9.0161E-03 | $7.1269 E=03$ | 5.7412E-03 | 4.0590E-04 |
| 2. | 11 | 4.9961E-03 | 1.0160E-02 | 7.0846E-03 | 4,8230E-04 |
| 1. | 11 | -0. | 1.1648E-02 | 7.5441E-03 | 5.0738E-04 |

Table 7 (cont'd)
Sample Output of Irfluence Coefficients - Cylinder

COL 3 ROW $15 \quad V=-1.0$
BOUNDARY STRESS RESUL"ANTS.


Table 7 (cont'd)

## Sample Output of Influence Coefficients - Cylinder

COL 3 ROW $15 \quad W=1.0$
BOUNDARY STRESS RESULTANTS.

| ROW | COL | NTAN | NNORM | 0 | M |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 15 | 1 | 0. | 9.8734E-04 | 5.2553E-02 | 6.9594E-04 |
| 15 | 2 | $1.3910 E-01$ | -2.0642E-02 | -1.2157E-01 | -1.5999E-03 |
| 15 | 3 | -1.8303E-02 | 1.4618E-01 | 7.8872E-01 | $1.0676 E-02$ |
| 15. | 4 | -1.8295E-01 | $-2.2180 E-02$ | -1.1550E-01 | -1.5238E-03 |
| 15. | 5 | -8.2147E-02 | 6.4510E-03 | 3.8968E-02 | 5.1218E-0.4 |
| 15. | 6 | -3.4978E-02 | 1.1512E-02 | 2.9290E-02 | 3.8626E-04 |
| 15. | 7 | -1.7541E-02 | 8.7393E-03 | 2.1495E-02 | 2.8050E-04 |
| 15. | 8 | -8.8091E-03 | 6.6128E-03 | 1.4529E-02 | 1.8825E-04 |
| 15, | 9 | -4.1358E-03 | 4.5697E-03 | 8.5025E-03 | 1.0926E-04 |
| 15. | 10 | -1.7051E-03 | 2.6539E-03 | 3.3954E-03 | 4.2847E-05 |
| 15. | 11 | 0 . | 0 . | -1.0320E-07 | -0. |
| 15. | 11 |  |  |  |  |
| 15. | 11 | -0. | 0. | 0. | -0. |
| 14. | 11 | B.0805E-05 | 2.9704E-04 | -2.1336E-07 | -2.1979E-07 |
| 13. | 11 | 1.4499E-04 | 5.3817E-04 | 1.4215E-06 | -2.2668E-07 |
| 12. | 11. | $2.4175 E-04$ | 9.4629E-04 | $6.7143 E-06$ | -1.0794E-08 |
| 11. | 11 | 3.1748E-04 | 1.2484E-03 | $1.4804 E-05$ | 5.2951E-07 |
| 10. | 11 | 3.7534E-04 | $1.4696 E-03$ | 2.4760E-05 | 1.2926E-06 |
| 9. | 11 | 4.6061E-04 | $1.7940 \mathrm{E}-03$ | 4.6754E-05 | 3.0751E-06 |
| 8. | 11 | 5.0724E=04 | $1.9435 E-03$ | 7.0966E-05 | 5.1663E-06 |
| 7. | 11 | 5.5312E-04 | $1.9892 E-03$ | 1.2143E-04 | 9.6847E-06 |
| 6. | 11 | 5.3914E-04 | $1.7366 E-03$ | 1.6931E-04 | 1.4064E-05 |
| 5. | 11 | 5.0419E-04 | 9.5598E-04 | 2.5775E-04 | 2.2164E-05 |
| 4. | 11 | 4.7398E-04 | 5.5718E-04 | 3.3671E-04 | 2.7353E-05 |
| 3. | 11 | 3.4109E-04 | 6.2437E-04 | 4.8266E-04 | 3.3927E-05 |
| 2. | 11 | 1.4700E-04 | 1.1230E-03 | 5.5081E-04 | 3.6089E-05 |
| 1. | 11 | -0. | $1.3200 E-03$ | $5.6602 E-04$ | 3.6468E-05 |

Table 7 (cont'd)
Sample Output of Influence Coefficients - Cylinder

## COL 3 ROW 15 W*a 1.0

BOUNDARY STRESS RESULTANTS.

| ROW | COL | NTAN | NNORM | 0 | M |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 15. | 1 | 0. | 1.5359E-03 | 1.0410E-03 | 1.2722E-05 |
| 15. | 2 | 4.2671E-03 | 3.5108E-03 | -2.0270E-03 | -3.5636E-05 |
| 15. | 3 | -3.6495E-04 | 7.2915E-03 | 1.2007E-01 | 1.6081E-03 |
| 15. | 4 | -5.1618E-03 | 3.2174E-03 | -2.0864E-03 | -3.6226E-05 |
| 15. | 5 | -1.6026E-03 | 8.4927E-04 | 7.1016E-04 | 8.8313E-06 |
| 15. | 6 | -7.2826E-04 | 4.5910E-04 | 5.7449E-04 | 7.3054E-06 |
| 15. | 7 | -3.5301E-04 | 2.3933E-04 | 3.7818E-04 | 4.8494E-06 |
| 15. | 8 | -1.6652E-04 | 1. $2544 \mathrm{E}-04$ | 2.2632E-04 | 2.9157E-06 |
| 15. | 9 | -7.2070E-05 | 6. $3752 \mathrm{E}-05$ | 1.1798E-04 | 1.5212E-06 |
| 15. | 10 | -2.6279E-05 | 3.0237E-05 | 4.3311E-05 | 5.5323E-07 |
| 15. | 11 | 0 . | 0 . | -1.5695E-09 | -0. |
| 15. | 11 |  |  |  |  |
| 15. | 11 | -0. | 0. | 0 | -0. |
| 14. | 11 | 9.9735E-07 | 4.4834E-06 | -4.3720E-09 | -3.5769E-09 |
| 13. | 11 | 1.7571E-06 | 8.2647E-06 | $1.1247 E-08$ | -4.4429E-09 |
| 12. | 11 | 2.8247E-06 | 1.4886E-05 | 6.4157E-08 | -3.2453E-09 |
| 11. | 11 | 3.5583E-06 | 2.0090E-05 | $1.4698 E-07$ | 2.1573E-09 |
| 10. | 11 | 4.0062E-06 | 2.4153E-05 | 2.4951E-07 | 1.0500E-08 |
| 9, | 11. | -4.4175E-06 | 3.0496E-05 | 4.7619E-07 | 3.0724E-08 |
| 8. | 11 | 4.1818E-06 | 3.3803E-05 | 7.2378E-07 | 5.5416E-08 |
| 7. | 11 | 2.9796E-06 | 3.5497E-05 | 1.2345 E-06 | $1.1023 E-07$ |
| 6. | 11 | 9.6550E-07 | 3.0690E-05 | 1.6524E-06 | 1.6017E-07 |
| 5. | 11 | -2.2608E-06 | 1.3919E-05 | 2.2月85E-06 | 2.4522E-07 |
| 4. | 11 | -3.6468E-06 | 4.5749E-06 | 2.6605E-06 | 2.6620E-07 |
| 3. | 11 | -4.8563E-06 | 3.8886E-06 | 3.2643E-06 | $2.3389 E-07$ |
| 2. | 11 | -3.7482E-06 | 1.2205E-05 | $2.9458 E-06$ | $1.6620 \mathrm{E}=07$ |
| 1. | 11 | -0. | 1.5192E-05 | $2.6238 E=06$ | $1.3484 E-07$ |

Table 7 (cont'd)
Sample Output of Influence Coefficients - Cylinder

COL 3 ROW 15 W- 1.0
BOUNDARY STRESS RESULTANTS,

|  | COL | NTAN | M | 0 | M |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 15. | 1 | 0. | 1.5359E-03 | 1.0410E-03 | 1.2722E-05 |
| 15. | 2 | 4.2671E-03 | 3.5108E-03 | -2.0270E-03 | -3.5636E-05 |
| 15. | 3 | -3.6495E-04 | 7.2915E-03 | 1.2007E-01 | $1.6081 \mathrm{E}-03$ |
| 15. | 4 | -5.1618E-03 | 3.2174E-03 | -2.0864E-03 | $-3.6226 E-05$ |
| 15. | 5 | -1.6026E-03 | 8.4927E-04 | 7.1016E-04 | 8.8313E-06 |
| 15. | 6 | -7.2826E-04 | 4.5910E-04 | 5.7449E-04 | 7.3054E-06 |
| 15. | 7 | -3.5301E-04 | 2.3933E-04 | 3.7818E-04 | 4.8494E-06 |
| 15. | 8 | -1.6652E-04 | 1.2544E-04 | 2.2632E-04 | 2.9157E-06 |
| 15. | 9 | -7.2070E-05 | 6.3752E-05 | 1.1798E-04 | $1.5212 \mathrm{E}-06$ |
| 15. | 10 | -2.6279E-05 | 3,0237E-05 | 4.3311E-05 | 5.5323E-07 |
| 15. | 11 | 0. | 0 | -1.5695E-09 | -0. |
| 15. | 11 |  |  |  |  |
| 15. | 11 | -0. | 0. |  | -0. |
| 14. | 11. | 9.9735E-07 | 4.4834E-06 | -4.3720E-09 | -3.5769E-09 |
| 13. | 11 | 1.7571E-06 | 8.2647E-06 | $1.1247 E-08$ | -4.4429E-09 |
| 12. | 11 | 2,8247E-06 | 1.4886E-05 | 6.4157E-08 | -3.2453E-09 |
| 11. | 11 | 3.5583E-06 | 2.0090E-05 | $1.4698 \mathrm{E}-07$ | 2.1573E-09 |
| 10. | 11 | 4.0062E-06 | 2.4153E-05 | 2.4951E-07 | 1.0500E-08 |
| 9. | 11 | 4.4175E-06 | 3.0496E-05 | 4.7619E-07 | 3.0724E-08 |
| 8. | 11 | 4.1818E-06 | 3.3803E-05 | 7.2378E-07 | 5.5416E-08 |
| 7. | 11 | 2.9796E-06 | 3.5497E-05 | 1.2345E-06 | 1.1023E-07 |
| 6. | 11 | $9.6550 \mathrm{E}-07$ | 3.0690E-05 | 1.6524E-06 | 1.6017E-07 |
| 5. | 11 | -2.2608E-06 | 1.3919E-05 | 2.2885E-06 | 2.4522E-07 |
| 4. | 11 | -3.6468E-06 | 4.5749E-06 | 2.6605E-06 | 2.6620E-07 |
| 3. | 11 | -4.8563E-06 | 3.8886E-06 | 3.2643E-06 | $2.3389 \mathrm{E}-07$ |
| 2. | 11 | -3.7482E-06 | 1.2205E-05 | 2.9458E-06 | 1.6620E-07 |
| 1. | 11 | -0. | 1.5192E-05 | $2.6238 \mathrm{E}-06$ | $1.3484 E-07$ |

Table 7 (cont'd)
Sample Output of Influence Coefficients - Cone

COL 3 NCIN $C \quad U=-3,0$
boUndary sipress kfsuliants.

| ROW | COL | ntan | NNORM | 0 | M |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 14. | 1 | 0. | 9.8012t-02 | 1,2647E 01 | -3,1620E OO |
| 14. | 2 | -9.462SE-U4 | 9.7266F-02 | 1.2511E 01 | -2.7781F 00 |
| 14, | 3 | -3.12R1E-03 | 9.2780E-02 | 1.1337 E 01 | -1.7030F 00 |
| 14. | 4 | -5.f.108F-03 | 6.3909t-02 | 8.3033 E 00 | -1.6490E-0\% |
| 14. | 5 | -A.0966F-0S | 7.2536E-02 | 4.2417F 00 | 1.5009 F 00 |
| 14. | 6 | -7.1992E-03 | 6.0227t-02 | 6.6385E-01 | 2.9025 F 00 |
| 14. | 7 | -6.8120F-03 | 4.CC18E-02 | -1.6471E 00 | 3.6727F 00 |
| 14. | 8 | -6.5531E-03 | 3.6196F-02 | -2.74S4F 00 | 3,5653E 00 |
| 14. | 9 | -6.0299E-03 | 2.4544F-02 | -3.187SF 00 | 2.5735 F 00 |
| 14 | 10 | -4.2914E-03 | 1.20391-02 | -3.624SE 00 | 1.0731F 00 |
| 14. | 11 | -0 |  | -2.4277E 00 | 0 . |
| 14, 11 |  |  |  |  |  |
| 14. | 11 | -0. | 0. | O. | -0, |
| 13. | 11 | 1.0954F-01 | 6.1583F-02 | $4.3781 E 03$ | 8.4094 F 01 |
| 12. | 11 | 3.7332F-01 | t.7033F-02 | 7.0938 E 03 | 1.7209F 02 |
| 11. | 11 | 3. $2227 F-01$ | 7.0278E-02 | 7.6181E 03 | 2,26n1F 02 |
| 10. | 11 | 2.4830F-01 | 4.0139F-02 | $6.2024 E 03$ | 2,0940F 02 |
| 9. | 11 | 1.4270F-01 | 2.2022E-03 | 3.5865F 03 | 1,4569F 02 |
| 8. | 11 | 9.63C2F-02 | -i.0773E-02 | 2.283bF 03 | 1.0400 F 02 |
| 7. | 11 | 5.729.3F-U2 | -1.3918E-02 | 1.0797 F 03 | 5.7082 F 01 |
| 6. | 11 | 4.0562E-02 | -1.4257t-02 | 6.3103 E 02 | 3.6722 F 01 |
| 5. | 11 | 2.5965F-02 | -1.5511E-02 | 3.1116F 02 | 1.9002 F 01 |
| 4. | 11 | 1.0236t-02 | -1.2293t-02 | 1.60R1F 02 | 1.2246 F 01 |
| 3. | 11 | 1.2845E-02 | -9.9897E-03 | 8.0043 EL | $6.3086 F 00$ |
| 2. | 11 | 0, 5298E-03 | -6.0856t-03 | 2.8085 E 02 | 2.1880 E 00 |
| 1. | 11 | 3.3306E-0S | -3.4645t-03 | 7.8911F 00 | 6,3808F-0: |
| 0. | 11 | -0. |  |  | -0. |
| 0.11 |  |  |  |  |  |
| 0. | 11 | -0. | 0. | 9.5225E-01 |  |
| 0. | 10 | -1.0043E-02 | -2.8511t-02 | 2.9410 ES | 3,9854F 01 |
| 0. | 9 | 2.C599E-02 | -6.8008F-02 | $9.8975 E 03$ | 1.2595E 02 |
| 0. | 8 | 1.2673E-01 | -i.5099F-01 | 1.7031504 | 2,1551F 02 |
| 0 , | 7 | 3.0062E-U1 | -3,3148E-01 | 2.2429 F 04 | 2.8409 F 02 |
| 0. | 6 | 5.0.733E-01 | -7.4013t-01 | 2.4322504 | 3.0775 F 02 |
| 0. | 5 | 1.0.776E U0 | -1.6760t 00 | 1.9117F 04 | 2,3054F 02 |
| 0. | 4 | -n.2818E 00 | -9.9173F 00 | -2.6229E 04 | -1,9874F 02 |
| 0. | 3 | $3.0 .0105-01$ | 2.8084 El | 5.8898F 04 | 4.1602f 02 |
| 0, | 2 | 7.0597E 0U | -1.0503E 01 | $-2.7352 \mathrm{E} 04$ | -2.1306F 02 |
| 0. | 1 | 0. | -3.6404F 00 | 1.1375 O | 1,2271E |

Table 7 (cont'd)
Sample Output of Influence Coefficients - Cone

## COL 3 HOW O $V=1 . C$

## goundary spress resultants.



Table 7 (cont'd)
Sample Output of Influence Coefficients - Cone

COL 3 ROW $0 \quad W=1.0$
BOUNDARY STRESS RESULTANTS

| ROW | COL | n man | NNOHM | 0 | M |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 14. | 1 | 0. | 1.7458E-03 | 1.4760 E 00 | -7.7389E-02 |
| 14. | 2 | -4.6212E-C4 | 1.6438E-03 | 1.3614E 00 | -6,9391E-02 |
| 14. | 3 | -8, $6457 E-04$ | 1,430CE-03 | 1.0270E OO | -4,6980E-02 |
| 14. | 4 | -1.1292E-03 | 1.2057E-03 | 5.2360E-01 | -1,4693E-02 |
| 14. | 5 | -1.1970E-03 | 1.0359E-03 | -3.6429E-02 | 2,0653F-02 |
| 14. | 6 | -1.C736E-03 | 8.5088E-04 | -5.1752E-01 | 5,1066F-02 |
| 14. | 7 | -6.1508E-C4 | 6.1841E-04 | -8.0605E-C1 | 6.8909E-02 |
| 14. | 8 | -5.1302E-04 | 3.7639E-04 | -8.3961E-01 | 6,8894F-02 |
| 14. | 9 | -2.5504E-04 | 1.8140E-04 | -6.3487E-01 | 5,0560E-02 |
| 14 | 10 | $-8.9100 E-05$ | $5.998 \mathrm{CF}-05$ | -3,0303E-01 | 2,1302F-02 |
| 14. | 11 | -0. | 0. | -5,2064E-02 | 0. |
| 14, | 11 |  |  |  |  |
| 14. | 11 | 0 | 0. |  | -0. |
| 13. | 11 | 4.3444E-C3 | 1.5406E-03 | 8.8283E 01 | 1,6698E 00 |
| 12. | 11 | 9.1098E-C3 | 2,8961E-03 | 2.3153E 02 | 5.5704E 00 |
| 11. | 11 | 1.C948E-C2 | 4,0330E-03 | 3.9160F 02 | 1.1546F 01 |
| 10. | 11 | 8,7577E-03 | 2.9588E-03 | 4,0032E 02 | 1,3546E 01 |
| 9. | 11 | 5.c244F-03 | 6,8278t-04 | 3.3248F 02 | 1,3883E 01 |
| 8. | 11 | 3.2705E-03 | 1,2ち47E-03 | 2,7011E 02 | 1,2323E 01 |
| 7. | 11 | 1.C014E-c3 | 3.4193E-03 | 1,6664E 02 | B,1378F 00 |
| 6, | 11 | -1.7980E-04 | 4.0950 E-03 | 1.1572E 02 | 5,8216F 00 |
| 5. | 11 | -1.c374E-03 | 3.9406E-03 | 6.3582 E 01 | 3,2829F 00 |
| 4. | 11 | -1.2777E-03 | 3.5692t-03 | 4.0533E 01 | 2,12世SE 00 |
| 3. | 11 | -1, $2343 \mathrm{E}-03$ | 2.8227E-03 | 2,1259E 01 | 1.1384 E 00 |
| 2. | 11 | -7.9916E-04 | $1.6098 E-03$ | 7.3224E 00 | 3,9913F-01 |
| 1. | 11 | -4.0484E-04 | 8.7835E-04 | 2.1311F 00 | 1,1862F-01 |
| 0 , | 11 | -0. | 0. | 0. | -0, |
| 0. | 11 |  |  |  |  |
|  | 11 | -0 | 6 | -3.878CE-01 | 0. |
| 0. | 10 | 8.7908E-03 | 3.7831E-03 | 5.5569 E 02 | 7.2359F 00 |
| 0. | 9 | 2.6227E-02 | t.6236E-03 | 1.5935E 03 | 2.0185 F 01 |
| 0. | 8 | 6.0302E-02 | 1.0153E-02 | 2.9053E 03 | 3.6713 E 01 |
| 0. | 7 | 1.2236E-01 | 1.3985E-02 | 4.6189E 03 | 5.8162 E 01 |
| 0 , | 6 | 2.3865E-01 | $1.0837 \mathrm{E}-02$ | 7.1278E 03 | 8,6407E 01 |
| 0. | 5 | 5.1355E-C1 | 1.3244E-02 | 1.3895 E 04 | 1,0686F 02 |
| 0. | 4 | $1.1206 E 00$ | -9,3345E-02 | -7.3086E 04 | -8,9062F 02 |
| 0. | 3 | 1.2076E-C1 | 1.3268E-01 | 2.3941E 05 | 3,0127E 03 |
| 0. | 2 | -8.3023E-01 | -9.0315t-02 | -6,9772E 04 | -8,5250F 02 |
| 0. | 1 | C. | 1.2792E-02 | 2.4167E 04 | 1,6797F |

Table 7 (cont'd)
Sample Output of Influence Coefficients - Cone

COL 3 REW $\quad k=1.0$ BOUNDARY STRESS RESULTANTS,

| ROW | COL | ATAN | NNORM | 0 | M |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 14. | 1 | 0. | 1.8073E-05 | 2.4752E-02 | -1.0371E-03 |
| 14. | 2 | -7.6726E-C6 | 1.6665E-05 | 2.2747E-02 | -9,3916E-04 |
| 14. | 3 | -1.4284E-05 | 1. $5625 E=05$ | $1.6950 \mathrm{E}=02$ | -6,6448E-04 |
| 14. | 4 | -1.8564E-05 | 1.0531E-05 | 8,3000E-03 | -2,6789E-04 |
| $14 \%$ | 5 | -1.9569E-05 | 8.2599E-06 | -1.3001E-03 | 1,6816F-04 |
| 14 | 6 | -1.7352E-05 | $6.0460 E-06$ | -9.5187E-03 | 5,4696E-04 |
| 14. | 7 | -1,2883E-05 | 3.4724E-06 | -1.4359E-02 | $7.7600 \mathrm{~F}=04$ |
| 14. | 8 | -7.7346E-C6 | 1.U36CE-06 | -1.4735E-02 | 7.9104E-04 |
| 14: | 9 | -3.4379E-06 | -4.9656E-07 | -1.0923E-02 | 5.85R1F-04 |
| 14 | 10 | -9.C692E-07 | -7.7710E-07 | -4.8674E=03 | 2.4782F-04 |
| 14 | 11 | -0. | 0. | -5.8786E-04 | 0. |
| 14 | 11 |  |  |  |  |
| 1 | 11 |  | . | 0 | , |
| 13 | 11 | 5.1292E-05 | $1.6032 E-05$ | $1.0333 E 00$ | 1.9427F-01 |
| 12, | 11 | 1.182JE-04 | 4.0323E-05 | $3.2838 E 00$ | 7,9088F-02 |
| 11 | 11 | 1.4585E-04 | 6.0903E-05 | $6.0724 E 00$ | 1.7948F-0I |
| 10 | 11 | 1.1535E-04 | 4.5943E-05 | 6.4906 E 00 | 2,1923F-01 |
| 9 | 11 | 5.8888E-05 | 1.7879E-05 | 5.9122E 00 | 2,3875E-01 |
| 8 | 11 | $3.0801 E-05$ | 2.8239E-05 | $5.0020 E 00$ | 2.1600E-01 |
| 7 | 11 | -4,2257E-06 | 6.5688E-05 | $3.1738 E \quad 00$ | 1.4405E-01 |
| 6 | 11 | -2.1258E-05 | 7.6967E-05 | $2.2343 E 00$ | 1.0360F-01 |
| 5. | 11 | -3.1064E-05 | 7.3514E-05 | 1.2382E OO | 5,8697F-02 |
| 4 | 11 | -3.2380E-05 | $6.6610 t-05$ | 7.9476E-01 | 3.8196F-02 |
| 3 | 11 | -2.8495E-05 | 5.2915E-05 | $4.1973 E=01$ | 2.0500F-02 |
| 2. | 11 | -1.7542E-05 | 3.0465E-05 | 1.4494E=01 | 7.1962F-03 |
| 1. | 11 | -9.9597E-06 | $1.6734 \mathrm{E}-05$ | 4.2355E-02 | 2,1429E-03 |
| 0. | $\begin{gathered} 11 \\ 11 \end{gathered}$ | $-0$ | 0 . | 0. | -0. |
| 0. | 11 | -0 |  | -7.4227E-03 |  |
| 0. | 10 | 1.7068E-04 | 8,2829E-05 | 1.0112 E 01 | 1.3062E-01 |
| 0. | 9 | 4.7635E-04 | 1.4404E-04 | $2.7644 E 01$ | 3,4832F-01 |
| 0. | 8 | 1.0375E-03 | $2.1639 \mathrm{E}-04$ | $4.7863 E 01$ | 6,0015E-01 |
| 0. | 7 | $1.9837 E-03$ | 2.8922E-04 | $6.9293 E 01$ | 8.5807E-01 |
| 0. | 6 | 3.5659E-03 | 3.3241E-04 | 8.3865E 01 | 9,6963F-01 |
| 0, | 5 | 6.5d8JE-03 | 2.3936E-04 | 3.4573E 01 | -7.3646F-02 |
| 0 \% | 4 | 3.4172E-03 | -4,6623E-04 | -1.2313E 02 | $-1.2827 \mathrm{~F} 01$ |
| 0. | 3 | $1.9376 E-03$ | 6,1453E-04 | 8.4356E 03 | 1.3030 E 02 |
| 0 . | 2 | $1.0030 E-03$ | -4,2410E-04 | -1.0298E 02 | -1,2658F 01 |
| 0. | 1 | 0 . | 1,8995E-04 | $7.3602 E 00$ | -9,2483F-01 |

Table 7 (cont'd)
Sample Output of Influence Coefficients - Sphere

COL 3 ROW $0 \quad U=1.0$
BOUNDARY STRESS RESUGTANTS.

| R | COL | NTAN | NNORM | 0 | M |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0. | 1.3763E-01 | $4.7462 E 02$ | 3.6565E 01 |
| 14 | 2 | -9.9868E-03 | 1.4741E-01 | 4.7490E 02 | 3.6323E 01 |
| 14. | 3 | -2.8676E-02 | 1.6966E-01 | $4.7125 E 02$ | 3.4621 E |
| 14. | 4 | -4.4474E-02 | 1.2452E-01 | $4.6159 E 02$ | $2.9554 E 01$ |
| 14, | 5 | -4.6420E-02 | 8.8871E-02 | 4.1047E 02 | $2.3130 E \quad 01$ |
| 14. | 6 | F-4.3637E-02 | 6.4787E-02 | 3.3317E 02 | 1.7078 El |
| 14, | 7 | -3.7394E-02 | 4.7068E-02 | 2.5275E 02 | 1.2119 E |
| 14. | 8 | -2.8501E-02 | 3.2800E-02 | $1.8319 E 02$ | B,2257E 00 |
| 14, | 9 | +1.8557E-02 | 2.1288E-02 | $1.2606 E 02$ | 5.0135E 00 |
| 1 | 10 | -9.3547E-03 | $1.1681 \mathrm{E}-02$ | 7.2055E 01 | 2.1227E 00 |
| 14 | 11 | 0. | 0. | 1.8206E-0 | -0. |
| 14. | 11 |  |  |  |  |
| 14 | 11 | 0. | 0 | 1.5717E-01 | 0. |
| 13 | 11 | 2.8057E-02 | 8.5952E-03 | 7.6629E 01 | $5.7043 E 00$ |
| 12. | 11 | 5.0054E-02 | 8.6125E-03 | 7.1942 E 01 | $5.3948 E \quad 00$ |
| 11. | 11 | $5.6083 \mathrm{E}=02$ | 7.5590E-03 | $3.0547 E 01$ | 2.0839 E (0) |
| 10. | 11 | 4.1540E-02 | 3.1938E-03 | -1.5886E 01 | -1.2877E 00 |
| 9 | 11 | 3.115BE-02 | -1.4525E-03 | -3.0562E 01 | -2.2938E 00 |
| 8. | 11 | 2.0455E-02 | -6.9996E-03 | -2.7669E 01 | -2.0075E 00 |
| 7. | 11 | 1.5527E-02 | -8.8109E-03 | -2.2805E 01 | -1.6333E 00 |
| 6. | 11 | 1.0849E-02 | -8.9641E-03 | -1.4205E 01 | -1,0083E 00 |
| 5, | 11 | 8.4962E-03 | -8.2758E-03 | -9.8344E 00 | -6.9149E-01 |
| 4. | 11 | 5.9930E-03 | -6.6481E-03 | -5.5745E 00 | -3.8396E-01 |
| 3. | 11 | 4.6729E-03 | -5.5316E-03 | -3.6014E 00 | -2.4081E-01 |
| 2. | 11 | 3.2562E-03 | -4.0806E-03 | -1.9475E 00 | -1.2015E-01 |
| 1. | 11 | 1.7107E-03 | -2.2542E-03 | -7.9784E-01 | -3.5122E-02 |
| 0. | 11 | 0. | 0. | -1.5101E-01 | 0.0 |
| 0. | 11 |  |  |  |  |
| 0. | 11 | 0 | -0. | 6.3037E-02 | -0 |
| 0. | 10 | -1.9314E-02 | -1.4983E-02 | -1.0995E 02 | -3.8645E 00 |
| 0 | 9 | -3.4542E-02 | -3.1835E-02 | -3.0090E 02 | $-8.8485 E 00$ |
| 0 | 8 | -5.6964E-02 | -6.1673E-02 | -5.6817E 02 | -1.5298E 01 |
| 0. | 7 | -9.2699E-02 | -1.2005E-01 | -9.6466E 02 | -2.3082E 01 |
| 0 \% | 6 | -I.509IE-01 | -2.4894E-01 | -1.5262E 03 | -2.9026E 01 |
| 0. | 5 | -2.6217E-01 | -5.7823E-01 | -2.0923E 03 | -2.2194E 01 |
| 0. | 4 | -4.8324E 00 | -4.1010E 00 | -1.0046E 04 | -3.4467E 00 |
| 0. | 3 | -8,0612E-02 | 1.0972E O1 | 4.0048 E 04 | 6.2078E 02 |
| 0. | 2 | 4.6512E 00 | -4.2748E 00 | -3.8526E 03 | -1.3218E 01 |
| 0. |  | -0. | -1.0933E 00 | -\$.0257E 03 | -2.7190E 01 |

# Table 7 (cont'd) <br> Sample Output of Influence Coefficients - Sphere 

COL 3 ROH $0 \quad V=1.0$
boundary stress resultants.

| ROW | COL | NTAN | NNORM | 0 | M |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 14. | 1 | 0. | 2.6398E-02 | 8.3172E 01 | 1.1485 E | 01 |
| 14 | 2 | 2.9931E-02 | 2.3504E-02 | 1.1746E 02 | 1.0013 E | 01 |
| 14 | 3 | 5.2320E-02 | 7.3411E-03 | 1.2238E 02 | $6.1353 E$ | 011 |
| 14 | 4 | 6.5180E-02 | -7.8173E-03 | $9.2618 E 01$ | 1.6691E | 00 |
| 14 | 5 | 6.7209E-02 | -8.2424E-03 | 4.3777E 01 | -1.7018E | 00 |
| 14 | 6 | 6.0483E-02 | -5.7284E-03 | -6.6452E-01 | -3.6315E | 00 |
| 14 | 7 | 4.9350E-02 | -4.3269E-03 | -3.1183E 01 | -4.4068E | 00 |
| 14. | 8 | 3.6032E-02 | -4.0579E-03 | -4.8645E 01 | -4.3301E | 00 |
| 14. | 9 | 2.2232E-02 | -4.4477E-03 | -5.5212E 01 | -3.4656E | 00 |
| 14, | 10 | 1.0067E-02 | -4,6089E-03 | -4.5687E 01 | -1.7843E | 00 |
| 14. | 11 | 0. | 0. | 1.3028E-01 | -0. |  |
| 14. | 11 |  |  |  |  |  |
| 14. | 11 | - | 0. | -6.5705E-02 | . |  |
| 13. | 11 | -1.2180E-02 | -1.2090E-02 | -8.0858E 01 | -4.8137E | 00 |
| 12. | 11 | -1.8586E-02 | -2.0945E-02 | -1.4318E 02 | -8.7958E | 00 |
| 11. | 11 | -2.0979E-02 | -2.9255E-02 | -1.8105E 02 | -1.1455E | 01 |
| 10. | 11 | -1.5642E-02 | -3.5156E-02 | -1.7309E 02 | -1.1357E | 1 |
| 9. | 11 | -1.0097E-02 | -3.4427E-02 | -1.4622E 02 | -9.6868E | 00 |
| 8 | 11 | -2.9320E-03 | -2.9025E-02 | -9.4369E 01 | -6.2357E | 00 |
| 7 | 11 | 3.5086E-04 | -2.4191E-02 | -6.7532E 01 | -4.4479E | 00 |
| 6 | 11 | 2.3770E-03 | -1.7329E-02 | -4.0024E 01 | -2.6272E | 00 |
| 5 | 11 | 2.8536E-03 | -1.3484E-02 | -2.6817E 01 | -1.7547E | 00 |
| 4 | 11 | 2.6481E-03 | -9.2746E-03 | -1.4712E 01 | -9.5960 | 01 |
| 3. | 11 | 2.3452E-03 | -7.0774E-03 | -9.1521E 00 | -5.9360 | 01 |
| 2. | 11 | 1.8212E-03 | -4.7969E-03 | -4.5546E 00 | -2.904 | 1 |
| 1. | 11 | 1.0500E-03 | $-2.4379 E-03$ | -1.4142E 00 | -8.18 |  |
| 0 | 11 | 0 | 0. | -6.7217E-02 | 0. |  |
| 0. | 11 |  |  |  |  |  |
| 0. | 11 | -0. | -0, | 1.7247E-01 | -0. |  |
| 0. | 10 | -2.6136E-02 | -3.5458E-03 | -2.2226E 02 | -9.3098E | 00 |
| 0 | 9 | -7.1519E=02 | 2.5244E-03 | -5.5635E 02 | -1.9562E | 01 |
| 0. | 8 | -1.5675E-01 | 8.8278E-03 | -9.1747E 02 | -3.1974E | 01 |
| 0. | 7 | -3.2268E-01 | 1.4791E-02 | -1.4883E 03 | -5.1890E | 01 |
| 0. | 6 | -6.8954E-01 | 1.3327E-02 | -2.5395E 03 | -8.5319E | 01 |
| 0. | 5 | -1.6749E 00 | -4.6969E-02 | -4.6458E 03 | -1.3622E | 02 |
| 0. | 4 | -1.1931E 01 | $1.3024 E 00$ | -1.6704E 04 | -2.2662E | 02 |
| 0. | 3 | $3.3447 E 01$ | -1.8773E-02 | -5.2308E 02 | 5.5850 E | 01 |
| 0. | 2 | -1.1369E 01 | -1.3338E 00 | 1.7572E 04 | 3.5339 E | 02 |
| 0. |  |  | 7.8168E-02 | $1.2174 \mathrm{O4}$ | 3.1104E | 02 |

Table 7 (cont'd)
Sample Output of Influence Coefficients - Sphere

COL 3 ROW $0 \quad W=1.0$
BOUNDARY STRESS RESULTANTS.

| RO | COL | NTAN | NNORM | 0 | M |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 14. | 1 | 0. | 1.4487E-02 | 5.2860E 01 | 4.0013E 00 |
| 14. | 2 | -1.4747E-03 | 1,5445E-02 | 5.3209 E 01 | 4.0256E 00 |
| 14. | 3 | -5.4012E-03 | 1,8814E-02 | 5,5300E 01 | 3.9225E 00 |
| 14 | 4 | -9.0683E-03 | $1.2868 \mathrm{E}-02$ | $5.3836 E 01$ | 3.3333E |
| 14. | 5 | -9.5494E-03 | 9.3669E-03 | 4.5940 E 01 | 2.5847E 00 |
| 14. | 6 | -8.5011E-03 | 7.0535E-03 | 3.6007E 01 | 1.9189E |
| 14. | 7 | -6.8375E-03 | 5.1350E-03 | 2.7158E 01 | 1,4021E |
| 14. | 8 | -4.9551E-03 | 3.5535E-03 | $2.0265 E 01$ | 1.0017E 00 |
| 14. | 9 | -3.0882E-03 | 2.3425E-03 | $1.4825 E 01$ | 6.4989E-01 |
| 14. | 10 | -1.4670E-03 | 1,3759E-03 | 9.1736 E 00 |  |
| . | 11 | 0 . | , | 2.4459E-02 |  |
| 4. 11 |  |  |  |  |  |
| 14, | 11 | 0. | . | 1.8358E-02 | 0. |
| 13. | 11 | 3.2580E-03 | 1.4864E-03 | 1.1438E 01 | 7.8639E-01 |
| 12. | 11 | 5.5464E-03 | 1.9590E-03 | 1.4042E 01 | 9,4605E-01 |
| 11. | 11 | 6.0923E-03 | 2.2951E-03 | $1.1895 E 01$ | 7.7823E-01 |
| 0. | 11 | 4.4420E-03 | 2.2491E-03 | 6.9819 E 00 | 4.9451E-01 |
| 9. | 11 | 3.2330E-03 | 1.8254E-03 | 4.3028 E 00 | 3,2281E-01 |
| 8. | 11 | 1.9244E-03 | 1.0670E-03 | 2.0759E 00 | 1.6193E-01 |
| 7. | 11 | 1.3218E-03 | 6.5548E-04 | 1.2239E 00 | 9.8143E-02 |
| 6. | 11 | 8.0631E-04 | 2.6458E-04 | 6.6763E-01 | 5.4126E-02 |
| 5. | 11 | 5.7566E-04 | $1.0456 \mathrm{E}-04$ | 4.2086E-01 | 3.4544E-02 |
| 4. | 11 | 3.6919E-04 | -4,0693E-06 | 2,1601E-01 | $1.8363 \mathrm{E}-02$ |
| 3. | 11 | 2.7128E-04 | -3,9976E-05 | 1.2347E-01 | 1.1058E-02 |
| . | 11 | 1.7775E-04 | -5.4077E-05 | 4.8999E-02 | 5.2005E-03 |
| 1. | 11 | 8.7659E-05 | -4,2351E-05 | 3.7478E-05 | 1.3596E-03 |
|  | 11 | 0. | 0. | 9.1847E-03 | 0. |
| 0.11 |  |  |  |  |  |
|  | 11 | -0. | -0. | -3.2619E-03 | 0. |
| 0 . | 10 | -1.7190E-04 | -1.0886E-03 | 1.3810E 00 | 1.6310E-01 |
| O | 9 | 1.2787E-03 | -2.8096E-03 | 3.1835E 00 | 3.8804E-01 |
| 0. | 8 | 5.0179E-03 | -5.5001E-03 | 6.6353 E 00 | 9.5049E-01 |
| 0. | 7 | 1.4063E-02 | -1.0024E-02 | 3.3231E 01 | 2.7340E 00 |
| 0. | 6 | 3.6913E-02 | -1.8004E-02 | $1.4736 \mathrm{E} ~ 02$ | 7.5655 E 00 |
| 0. | 5 | 9.7414E-02 | -3.3513E-02 | 2.5756E 03 | 2.1409 El |
| 0. | 4 | 3.6916E-01 | -1.1819E-01 | -1.4697E 04 | -2.1394E 02 |
| 0 . | 3 | 1.4112E-02 | 6.4658E-01 | $3.7237 E 04$ | 8.8205E 02 |
| 0. | 2 | -3.2737E-01 | -1.2934E-01 | -6.5630E 03 | -2.0723E 02 |
| 0. | 1 | -0. | -6.1073E-0 | 3.5344 E | 4.2344E |

Table 7 (conc'd)
Sample Output of Influence Coefficients - Sphere

COL 3 ROW 0 W* $W=-1.0$
BOUNDARY STRESS RESULTANTS.



[^0]:    *"Investigation of Juncture Stress Fields in Multicellular Shell Structures," by E. Y W. Tsui, F. A. Brogan, J. M. Massard, P. Stern, and C. E. Stuhlman, Technical Report M-03-63-1, Lockheed Missiles \& Space Company, Sunnyvale, Calif., Feb 1964 - NASA CR-61050.

