MASSACHUSETTS INSTITUTE OF TECHNOLOGY

RE-14

TWO-BODY LINEAR GUIDANCE MATRICES

by

Linda P. Abrahamson
Robert G. Stern

March 1965
(Revised June 1965)

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ABSTRACT

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The analytical expressions for the two-body linear guidance matrices are presented in handbook form in a velocity-dependent coordinate system. These matrices are the solution to the two-body variant equations of motion.

author

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TWO-BODY LINEAR GUIDANCE MATRICES

1. SYMBOL TABLE

General Notation

An underlined lower-case letter represents a column vector.

A capital letter with an asterisk over it represents a matrix.

A superscript T following a vector or a matrix indicates the transpose of the vector or matrix.

A superscript -1 following a square matrix indicates the inverse of the matrix.

Symbols

a	semi-major axis
e	eccentricity of the reference trajectory
$\delta \underline{e}$	variation in grouping of six orbital elements
E	eccentric anomaly on elliptical reference trajectory
f	true anomaly on reference trajectory
F	eccentric anomaly on hyperbolic reference trajectory
$\overset{*}{F}_m$	3-by-6 matrix relating components of $\delta \underline{r}_m$ to components of $\delta \underline{e}$
$\overset{*}{G}$	3-by-3 matrix relating components of $\delta \underline{\ddot{r}}$ to components of $\delta \underline{r}$
h	orbital angular momentum per unit mass of space vehicle
$\overset{*}{H}_{ij}$	6-by-3 matrix relating components of $\delta \underline{e}$ to components of $\delta \underline{r}_i$ when $\delta \underline{r}_j$ is constant
i	$\sqrt{-1}$
δi	angle between the actual trajectory plane and the reference trajectory plane
$\overset{*}{I}_N$	N-by-N identity matrix
$\overset{*}{J}_{ij}$	3-by-3 matrix relating components of $\delta \underline{v}_i$ to components of $\delta \underline{r}_i$ when $\delta \underline{r}_j$ is constant

$\overset{*}{K}_{ij}$	3-by-3 matrix relating components of $\delta \underline{v}_i$ to components of $\delta \underline{r}_j$ when $\delta \underline{r}_i$ is constant
$\overset{*}{L}_m$	3-by-6 matrix relating components of $\delta \underline{v}_m$ to components of $\delta \underline{e}$
$\overset{*}{M}_{mk}$	3-by-3 matrix relating components of $\delta \underline{r}_m$ to components of $\delta \underline{r}_k$ when $\delta \underline{v}_k$ is constant
n	mean angular motion
$\overset{*}{N}_{mk}$	3-by-3 matrix relating components of $\delta \underline{r}_m$ to components of $\delta \underline{v}_k$ when $\delta \underline{r}_k$ is constant
$\overset{*}{0}_N$	N-by-N zero matrix
p	distance along first axis of reference trajectory flight path coordinate system
q	distance along second axis of reference trajectory flight path coordinate system
r	magnitude of position vector on reference trajectory
$\underline{r}$	position vector on reference trajectory
$\delta \underline{r}$	variation in position vector
$\delta \ddot{\underline{r}}$	variation in inertial acceleration vector
$\overset{*}{R}_k$	6-by-3 matrix relating components of $\delta \underline{e}$ components of $\delta \underline{r}_k$ when $\delta \underline{v}_k$ is constant
$\overset{*}{S}_{mk}$	3-by-3 matrix relating components of $\delta \underline{v}_m$ to components of $\delta \underline{r}_k$ when $\delta \underline{v}_k$ is constant
t	time
t_o	time of perihelion passage for reference trajectory
$\overset{*}{T}_{mk}$	3-by-3 matrix relating components of $\delta \underline{v}_m$ to components of $\delta \underline{v}_k$ when $\delta \underline{r}_k$ is constant
$\delta \underline{v}$	variation in velocity vector
$\overset{*}{V}_k$	6-by-3 matrix relating components of $\delta \underline{e}$ to components of $\delta \underline{v}_k$ when $\delta \underline{r}_k$ is constant
$\delta \underline{x}$	six component vector consisting of $\delta \underline{r}$ and $\delta \underline{v}$
X	simplifying factor (see section 12)
z	distance along axis normal to reference trajectory plane
δ	operator indicating the first variation
μ	gravitational constant in sun's gravitational field
$\overset{*}{\Phi}_{ji}$	state transition matrix; 6-by-6 matrix relating components of $\delta \underline{x}_j$ to components of $\delta \underline{x}_i$

$\delta \phi$	longitude of perihelion of actual trajectory relative to perihelion of reference trajectory
ω	latitude of perihelion of reference trajectory
$\delta \Omega$	angle, in reference trajectory plane, between positive half of semi-major axis and positive half of line of nodes

2. INTRODUCTION

The purpose of this report is to present in handbook form the analytic expressions for the two-body linear guidance matrices. These matrices are the solution to the two-body variant equations of motion. Each matrix is written in terms of each of three independent variables, namely the elliptical eccentric anomaly, the hyperbolic eccentric anomaly, and the true anomaly. For the eccentric anomaly, it is necessary to have two sets of matrices, one of which applies to ellipses and the other to hyperbolas. However, since true anomaly is defined in the same way for both ellipses and hyperbolas, the set of matrices expressed in terms of that variable is applicable to both types of orbits without modification. The material presented here is an extension of the work done by Stern in Reference 6. The analytic forms of the matrices were verified numerically on a digital computer.

3. PATH DEVIATION VECTORS

The variant equations of two-body motion can be written as follows:

$$\delta \ddot{\underline{r}} = \overset{*}{G} \delta \underline{r} \tag{3-1}$$

where

$$\overset{*}{G} = \left[\frac{\mu}{r^5} \left(3 \underline{r}\underline{r}^T - \underline{r}^T \underline{r} \overset{*}{I}_3 \right) \right]$$

The solution of these equations contains six integration constants which may be considered as a six-component path deviation vector. This vector defines the difference between the vehicle's actual trajectory and the reference trajectory. Three possible path deviation vectors are

$$(1) \quad \begin{Bmatrix} \delta \underline{r}_i \\ \\ \delta \underline{v}_i \end{Bmatrix}$$

which represents the three components of position variation and the three components of velocity variation at time t_i,

$$(2) \quad \begin{Bmatrix} \delta \underline{r}_i \\ \\ \delta \underline{r}_j \end{Bmatrix}$$

which represents the three components of position variation at times t_i and t_j,

$$(3) \quad \begin{Bmatrix} \delta \underline{e} \end{Bmatrix}$$

which represents the variations in a set of six orbital elements. The first path deviation vector is called the state vector and may be written as $\delta \underline{x}_i$.

4. SOLUTIONS OF THE VARIANT EQUATIONS OF MOTION

The various path deviation vectors may be related to each other as follows:

$$\delta \underline{e} = \begin{Bmatrix} \overset{*}{R}_k & \overset{*}{V}_k \end{Bmatrix} \begin{Bmatrix} \delta \underline{r}_k \\ \delta \underline{v}_k \end{Bmatrix} \tag{4-1}$$

$$= \begin{Bmatrix} \overset{*}{R}_k & \overset{*}{V}_k \end{Bmatrix} \delta \underline{x}_k \tag{4-2}$$

$$= \overset{*}{H}_{ij} \, \delta \underline{r}_i + \overset{*}{H}_{ji} \, \delta \underline{r}_j \tag{4-3}$$

where

$$\overset{*}{R}_k = \left\{ \frac{\partial \underline{e}}{\partial \underline{r}_k} \middle| \; \delta \underline{v}_k = \text{constant} \right\} \tag{4-4}$$

$$\overset{*}{V}_k = \left\{ \frac{\partial \underline{e}}{\partial \underline{v}_k} \middle| \; \delta \underline{r}_k = \text{constant} \right\} \tag{4-5}$$

$$\overset{*}{H}_{ij} = \left\{ \frac{\partial \underline{e}}{\partial \underline{r}_i} \middle| \; \delta \underline{r}_j = \text{constant} \right\} \tag{4-6}$$

$\overset{*}{R}$, $\overset{*}{V}$, and $\overset{*}{H}$ are 6-by-3 matrices whose subscripts indicate the relevant times.

The variation in position may be calculated from

$$\delta \underline{r}_m = \overset{*}{F}_m \, \delta \underline{e} \tag{4-7}$$

$$= \overset{*}{F}_m \left\{ \overset{*}{H}_{ij} \, \delta \underline{r}_i + \overset{*}{H}_{ji} \, \delta \underline{r}_j \right\} \tag{4-8}$$

$$= \overset{*}{F}_m \left\{ \overset{*}{R}_k \quad \overset{*}{V}_k \right\} \delta \underline{x}_k \tag{4-9}$$

$$= \left\{ \overset{*}{M}_{mk} \quad \overset{*}{N}_{mk} \right\} \delta \underline{x}_k \tag{4-10}$$

where

$$\overset{*}{F}_m = \left\{ \frac{\partial \underline{r}_m}{\partial \underline{e}} \right\} \tag{4-11}$$

$\overset{*}{M}$ and $\overset{*}{N}$ are 3-by-3 matrices, and $\overset{*}{F}$ is a 3-by-6 matrix.

The variation in velocity may be obtained from

$$\delta \underline{v}_m = \overset{*}{L}_m \, \delta \underline{e} \tag{4-12}$$

$$= \overset{*}{L}_m \left\{ \overset{*}{H}_{ij} \, \delta \underline{r}_i + \overset{*}{H}_{ji} \, \delta \underline{r}_j \right\} \tag{4-13}$$

$$= \overset{*}{L}_m \left\{ \overset{*}{R}_k \quad \overset{*}{V}_k \right\} \delta \underline{x}_k \tag{4-14}$$

$$= \left\{ \overset{*}{S}_{mk} \quad \overset{*}{T}_{mk} \right\} \delta \underline{x}_k \tag{4-15}$$

where

$$\overset{*}{L}_m = \left\{ \frac{\partial \underline{v}_m}{\partial \underline{e}} \right\} \qquad (4\text{-}16)$$

$\overset{*}{S}$ and $\overset{*}{T}$ are 3-by-3 matrices, and $\overset{*}{L}$ is a 3-by-6 matrix.

Since

$$\delta \underline{r}_m = \left\{ \overset{*}{M}_{mk} \quad \overset{*}{N}_{mk} \right\} \delta \underline{x}_k \qquad (4\text{-}17)$$

and

$$\delta \underline{v}_m = \left\{ \overset{*}{S}_{mk} \quad \overset{*}{T}_{mk} \right\} \delta \underline{x}_k \qquad (4\text{-}18)$$

then

$$\delta \underline{x}_m = \overset{*}{\Phi}_{mk} \; \delta \underline{x}_k \qquad (4\text{-}19)$$

where

$$\overset{*}{\Phi}_{mk} = \left\{ \begin{matrix} \overset{*}{M}_{mk} & \overset{*}{N}_{mk} \\ \overset{*}{S}_{mk} & \overset{*}{T}_{mk} \end{matrix} \right\} \qquad (4\text{-}20)$$

$\overset{*}{\Phi}_{mk}$ is known as the state transition matrix. It relates the state at time t_m to the state at time t_k.

5. USEFUL GUIDANCE EQUATIONS

The following equations are helpful in two-body guidance problems:

$$\delta \underline{v}_i = \overset{*}{L}_i \left\{ \overset{*}{H}_{ij} \, \delta \underline{r}_i + \overset{*}{H}_{ji} \, \delta \underline{r}_j \right\} \qquad (5\text{-}1)$$

$$= \overset{*}{J}_{ij} \, \delta \underline{r}_i + \overset{*}{K}_{ij} \, \delta \underline{r}_j \qquad (5\text{-}2)$$

$$\delta \underline{r}_j = - \overset{*}{K}_{ij}{}^{-1} \, \overset{*}{J}_{ij} \, \delta \underline{r}_i + \overset{*}{K}_{ij}{}^{-1} \, \delta \underline{v}_i \qquad (5\text{-}3)$$

$$= \overset{*}{K}_{ij}{}^{-1} \left\{ - \overset{*}{J}_{ij} \quad \overset{*}{I}_3 \right\} \delta \underline{x}_i \qquad (5\text{-}4)$$

The 3-by-3 matrices $\overset{*}{J}$ and $\overset{*}{K}$ may also be used to compute velocity corrections as explained in Volume II, Appendices L and M of

Reference 6. They are related to $\overset{*}{M}$ and $\overset{*}{N}$ by the equations

$$\overset{*}{M}_{ji} = - \overset{*}{K}_{ij}{}^{-1} \overset{*}{J}_{ij} \tag{5-5}$$

and

$$\overset{*}{N}_{ji} = \overset{*}{K}_{ij}{}^{-1} \tag{5-6}$$

It is shown in Reference 6 that $\overset{*}{J}$ is a symmetric matrix.

6. PROPERTIES OF THE STATE TRANSITION MATRIX

Since

$$\delta \underline{x}_j = \overset{*}{\Phi}_{ji} \, \delta \underline{x}_i \tag{6-1}$$

$$= \overset{*}{\Phi}_{ji} \overset{*}{\Phi}_{ij} \, \delta \underline{x}_j \tag{6-2}$$

then

$$\overset{*}{\Phi}_{ji} \overset{*}{\Phi}_{ij} = \overset{*}{I}_6 \tag{6-3}$$

and

$$\overset{*}{\Phi}_{ji}{}^{-1} = \begin{pmatrix} \overset{*}{M}_{ji} & \overset{*}{N}_{ji} \\ \overset{*}{S}_{ji} & \overset{*}{T}_{ji} \end{pmatrix}^{-1} \tag{6-4}$$

$$= \begin{pmatrix} \overset{*}{M}_{ij} & \overset{*}{N}_{ij} \\ \overset{*}{S}_{ij} & \overset{*}{T}_{ij} \end{pmatrix} \tag{6-5}$$

It is shown in Reference 1 that

$$\overset{*}{\Phi}_{ji}{}^{-1} = \begin{pmatrix} \overset{*}{T}_{ji}{}^{T} & -\overset{*}{N}_{ji}{}^{T} \\ -\overset{*}{S}_{ji}{}^{T} & \overset{*}{M}_{ji}{}^{T} \end{pmatrix} \tag{6-6}$$

Therefore, the state transition matrix can be inverted by inspection. As a consequence of the fact that $\overset{*}{\Phi}_{ji}$ is symplectic, the determinant of $\overset{*}{\Phi}_{ji}$ is equal to +1.

7. MATRIX CHECKS

Listed below are certain matrix equations which are useful in the detection of errors.

$$\overset{*}{M}_{jj} = \overset{*}{I}_3 \quad (7\text{-}1)$$

$$\overset{*}{S}_{jj} = \overset{*}{0}_3 \quad (7\text{-}2)$$

$$\overset{*}{N}_{jj} = \overset{*}{0}_3 \quad (7\text{-}3)$$

$$\overset{*}{T}_{jj} = \overset{*}{I}_3 \quad (7\text{-}4)$$

$$\overset{*}{M}_{mk} = \overset{*}{F}_m \overset{*}{R}_k \quad (7\text{-}5)$$

$$\overset{*}{N}_{mk} = \overset{*}{F}_m \overset{*}{V}_k \quad (7\text{-}6)$$

$$\overset{*}{S}_{mk} = \overset{*}{L}_m \overset{*}{R}_k \quad (7\text{-}7)$$

$$\overset{*}{T}_{mk} = \overset{*}{L}_m \overset{*}{V}_k \quad (7\text{-}8)$$

$$\overset{*}{J}_{mk} = \overset{*}{L}_m \overset{*}{H}_{mk} \quad (7\text{-}9)$$

$$\overset{*}{K}_{mk} = \overset{*}{L}_m \overset{*}{H}_{km} \quad (7\text{-}10)$$

$$\overset{*}{M}_{mk} \overset{*}{T}_{mk}{}^T - \overset{*}{N}_{mk} \overset{*}{S}_{mk}{}^T = \overset{*}{I}_3 \quad (7\text{-}11)$$

$$\overset{*}{M}_{mk}{}^T \overset{*}{T}_{mk} - \overset{*}{S}_{mk}{}^T \overset{*}{N}_{mk} = \overset{*}{I}_3 \quad (7\text{-}12)$$

As a consequence of Eqs (4-20) and (6-6), the following matrix products are symmetric: $\overset{*}{M}_{mk} \overset{*}{S}_{mk}{}^{-1}$, $\overset{*}{N}_{mk} \overset{*}{T}_{mk}{}^{-1}$, $\overset{*}{N}_{mk}{}^{-1} \overset{*}{M}_{mk}$, $\overset{*}{T}_{mk}{}^{-1} \overset{*}{S}_{mk}$, $\overset{*}{M}_{mk} \overset{*}{N}_{mk}{}^T$, $\overset{*}{S}_{mk} \overset{*}{T}_{mk}{}^T$, $\overset{*}{N}_{mk}{}^T \overset{*}{T}_{mk}$, and $\overset{*}{M}_{mk}{}^T \overset{*}{S}_{mk}$.

8. THE COORDINATE SYSTEM

The coordinate system used in this presentation is the reference trajectory flight path coordinate system whose axes are labeled p, q, and z. The origin of the system is the center of the central body being considered. The positive z-axis is in the direction of the angular momentum vector of the vehicle's motion about the central body, and the p-q plane is the reference trajectory plane. The positive q-axis is parallel to the relative velocity vector of the vehicle's nominal motion with respect to the central body. The positive p-axis is 90° behind the positive q-axis. See Fig. 1.

The pqz system was chosen because the matrix elements are simpler than those expressed in any of the other systems considered.

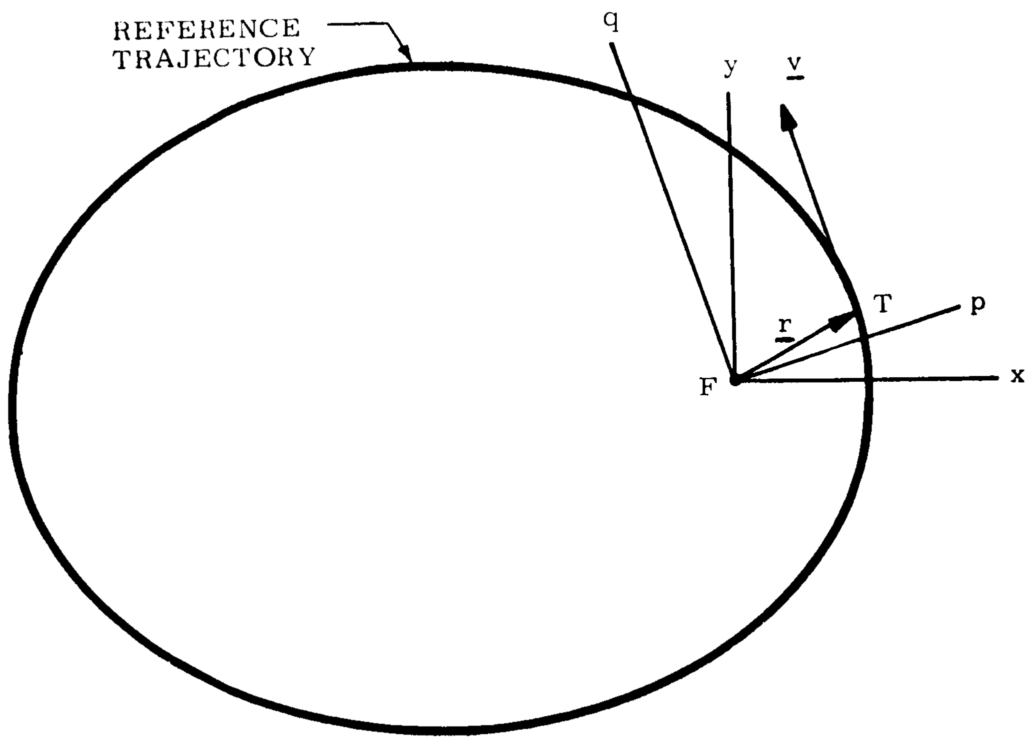
9. THE INDEPENDENT VARIABLE

Three independent variables have been considered: 1. the elliptical eccentric anomaly, 2. the hyperbolic eccentric anomaly, and 3. the true anomaly. The guidance matrices written in terms of these independent variables can be found in Appendices A, B, and C, respectively. When true anomaly is used, all secular terms contain the time t in addition to the true anomaly.

10. ORBITAL ELEMENTS

The grouping of orbital elements used in the path deviation vector $\delta \underline{e}$ is as follows:

$$\delta \underline{e} = \begin{Bmatrix} \delta a/a \\ \delta e \\ \delta \phi \\ \delta t_o \\ \delta i \cos \delta \Omega \\ \delta i \sin \delta \Omega \end{Bmatrix} \qquad (10\text{-}1)$$



F - attractive focus

T - vehicle position on reference trajectory

r - position vector

v - velocity vector

p, q - flight path coordinate axes

x, y - stationary system coordinate axes

Fig. 1 Flight path coordinate system.

The angles $\delta\Omega$, δi, and $\delta\phi$ relate the axes of the actual trajectory to the axes of the reference trajectory. Let p' q' z' be the axes of the actual trajectory. Then $\delta\Omega$ is the angle between the p-axis and the line of nodes and δi is the angle between the z and z' axes. $\delta\phi = \delta(\omega + \Omega)$, see Fig. 2. Since $\delta\Omega$ is not necessarily small, $\cos\delta\Omega$ and $\sin\delta\Omega$ are used instead of $\delta\Omega$ in the path deviation vector.

11. RELATIONSHIPS USED

The guidance matrices written in terms of the elliptical eccentric anomaly are derived in Reference 5. The matrices expressed in terms of the true anomaly may be obtained from those of Reference 5 by the substitutions

$$\sin E = \frac{(1 - e^2)^{1/2} \sin f}{1 + e\cos f} \tag{11-1}$$

$$\cos E = \frac{\cos f + e}{1 + e\cos f} \tag{11-2}$$

The secular term E is derived from Kepler's equation and Eq (11-1)

The matrices written in terms of the hyperbolic eccentric anomaly are easily obtained from those written in terms of the elliptical eccentric anomaly by the following substitutions:

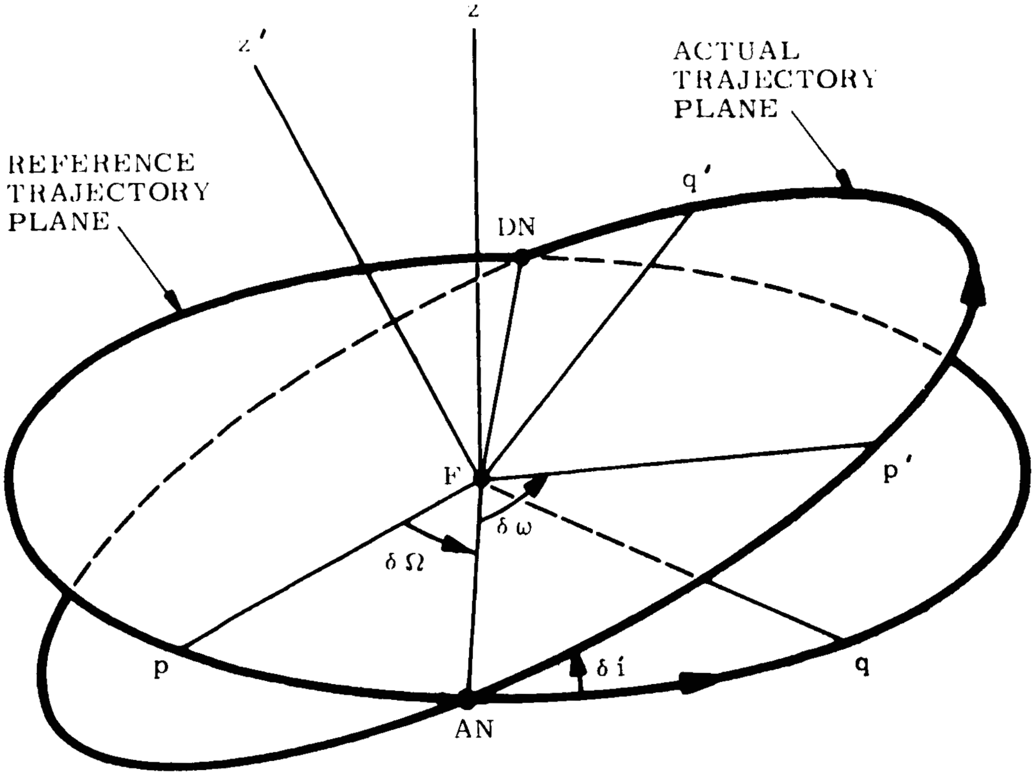
$$E \rightarrow iF \tag{11-3}$$

$$\sin E \rightarrow i(\sinh F) \tag{11-4}$$

$$\cos E \rightarrow \cosh F \tag{11-5}$$

$$(1 - e^2)^{1/2} = i(e^2 - 1)^{1/2}$$

$$n = \frac{h}{ia^2 (e^2 - 1)^{1/2}} \tag{11-6}$$



F - origin at center of central body
AN - ascending node
DN - descending node
pqz - reference trajectory pqz system
p'q'z' - actual (variant) trajectory pqz system
δΩ - longitude of ascending node
δ i - inclination of actual trajectory plane
δ ω - latitude of perihelion of actual trajectory

Fig. 2 Orientation of actual trajectory to reference trajectory.

where $i = \sqrt{-1}$

12. SIMPLIFICATION SUBSTITUTIONS

The following equations are used to simplify the matrix expressions:

$$E_P = \frac{1}{2}(E_j + E_i) \qquad E_M = \frac{1}{2}(E_j - E_i) \tag{12-1}$$

$$F_P = \frac{1}{2}(F_j + F_i) \qquad F_M = \frac{1}{2}(F_j - F_i) \tag{12-2}$$

$$f_P = \frac{1}{2}(f_j + f_i) \qquad f_M = \frac{1}{2}(f_j - f_i) \tag{12-3}$$

where the subscripts M and P refer to "minus" and "plus", respectively. X is defined by

$$X_E = (3E_M - e \sin E_M \cos E_P)(\cos E_M + e \cos E_P) - 4 \sin E_M \tag{12-4}$$

$$X_F = (3F_M - e \sinh F_M \cosh F_P)(\cosh F_M + e \cosh F_P) - 4 \sinh F_M \tag{12-5}$$

$$X_f = \left[\frac{3ht_M}{a^2} + \frac{2e(1 - e^2) \sin f_M (\cos f_P + e \cos f_M)}{(1 + e \cos f_i)(1 + e \cos f_j)}\right]\left[(1 + e^2) \cos f_M + 2e \cos f_P\right] - 4(1 - e^2) \sin f_M \tag{12-6}$$

where the subscripts on X refer to the independent variable used and $t_M = \frac{1}{2}(t_j - t_i)$.

13. APPLICATIONS

The guidance matrices in this handbook have a two-fold purpose. First, they are useful in making analytic studies of the two-body variational problem. Secondly, they can be readily programmed on a digital computer for numerical investigations.

APPENDIX A

GUIDANCE MATRICES FOR ELLIPTICAL ORBITS

$$
\overset{*}{V} = \frac{1}{na} \left\{ \frac{(1 - e\cos E)}{(1 - e^2\cos^2 E)^{1/2}} \begin{pmatrix} 0 \\ (1-e^2)^{1/2}\sin E \\ -\frac{1}{e}(\cos E + e) \\ \frac{(1-e^2)^{1/2}}{en}\left[e - \cos E\right] \\ 0 \\ 0 \end{pmatrix} \;\Bigg|\; \frac{1}{(1 - e^2\cos^2 E)^{1/2}} \begin{pmatrix} 2(1 + e\cos E) \\ 2(1-e^2)\cos E \\ \frac{2(1-e^2)^{1/2}\sin E}{e} \\ \frac{1}{en}\left[2\sin E(1 - e^3\cos E) - 3e(1 + e\cos E)(E - e\sin E)\right] \\ 0 \\ 0 \end{pmatrix} \;\Bigg|\; \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{(\cos E - e)}{(1-e^2)^{1/2}} \\ \sin E \end{pmatrix} \right\}
$$

$$
\overset{*}{R} = \frac{1}{a}
\left\{
\begin{array}{c}
\dfrac{1}{(1 - e\cos E)^2 (1 - e^2\cos^2 E)^{1/2}}
\begin{pmatrix}
2(1 - e^2)^{1/2} & 2e\sin E & 0 \\
(1 - e^2)^{1/2}\left[(1 - e\cos E)\, e\cos^2 E + (\cos E - e)\right] & (1 - e^2)\sin E & 0 \\
\dfrac{\sin E}{e}\left[(1 - e\cos E)\, e\cos E + (1 - e^2)\right] & \dfrac{-(1 - e^2)^{1/2}(\cos E - e)}{e} & 0 \\
\dfrac{(1 - e^2)^{1/2}}{en}\left\{\left[(1 - e\cos E)\, e\cos E + (1 - e^2)\right]\sin E - 3e(E - e\sin E)\right\} & \dfrac{1}{en}\left[-(1 - e^2)(\cos E - e) - 3e^2\sin E\,(E - e\sin E) + e(1 + e\cos E)(1 - e\cos E)^2\right] & 0
\end{pmatrix} \\
\hline
\dfrac{1}{(1 - e\cos E)}
\begin{pmatrix}
0 & 0 & \dfrac{\sin E}{(1 - e^2)^{1/2}} \\
0 & 0 & -\cos E
\end{pmatrix}
\end{array}
\right\}
$$

$$
\overset{*}{H}_{ij} = \frac{1}{2a} \left\{ \frac{1}{(1 - e^2 \cos^2 E_i)^{1/2}} \times \begin{pmatrix} -4(1-e^2)^{1/2} \sin E_M \\ (1-e^2)^{1/2} \left[-(1 + e\cos E_i) \sin E_j \cdot \left(\frac{3 E_M}{\sin E_M} - e \cos E_P \right) + 4 (\sin E_P + e \sin E_M) \right] \\ \frac{1}{e} \left[(1 + e \cos E_i)(\cos E_j + e) \cdot \left(\frac{3 E_M}{\sin E_M} - e \cos E_P \right) \cdot 4 (\cos E_P + e \cos E_M) \right] \\ \frac{(1-e^2)^{1/2}}{en} \left[(1 + e \cos E_i) \left(\frac{3 E_M}{\sin E_M} - e \cos E_P \right) (\cos E_j - e) - 4 (\cos E_P - e \cos E_M) + 2e (3 E_j - e \sin E_j) \sin E_M \right] \\ 0 \\ 0 \end{pmatrix} \;\middle|\; \frac{(1 - e \cos E_i)}{(1 - e^2 \cos^2 E_i)^{1/2}} \times \begin{pmatrix} 2(\cos E_M + e \cos E_P) \\ 2(1-e^2) \cos E_P \\ \frac{2(1-e^2)^{1/2} \sin E_P}{e} \\ \frac{1}{en} \left[2 \sin E_P (1 + e^2) + 4 e \sin E_M - e (3 E_j - e \sin E_j)(\cos E_M + e \cos E_P) \right] \\ 0 \\ 0 \end{pmatrix} \frac{1}{\sin E_M (\cos E_M - e \cos E_P)} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{-(\cos E_j - e)}{(1-e^2)^{1/2}} \\ -\sin E_j \end{pmatrix} \right\}
$$

$$
\overset{*}{F} = a \left\{ \begin{array}{c|cccccc}
\dfrac{(1 - e\cos E)}{(1 - e^2\cos^2 E)^{1/2}} & (1-e^2)^{1/2} & -\dfrac{(\cos E + e)}{(1-e^2)^{1/2}} & -e\sin E & 0 & 0 & 0 \\
\hline
\dfrac{1}{(1 - e^2\cos^2 E)^{1/2}} & \begin{array}{l} -3/2\,(1 + e\cos E)(E - e\sin E) \\ + e\sin E\,(1 - e\cos E) \end{array} & 2\sin E & (1-e^2)^{1/2}(1 - e\cos E) & -n\,(1 + e\cos E) & 0 & 0 \\
\hline
 & 0 & 0 & 0 & 0 & (1-e^2)^{1/2}\sin E & -(\cos E - e)
\end{array} \right\}
$$

$$\overset{*}{L} = n a \left\{ \begin{array}{c} \dfrac{1}{(1 - e\cos E)^2 (1 - e^2\cos^2 E)^{1/2}} \begin{pmatrix} \frac{3}{2}(1 - e^2)^{1/2}(E - e\sin E) & \dfrac{-\sin E}{(1 - e^2)^{1/2}}\left[(1 - e\cos E)\, e\cos E + (1 - e^2)\right] & -(1 - e\cos E)^2 (1 + e\cos E) & n(1 - e^2)^{1/2} & 0 & 0 \\ \frac{1}{2}\left[3e\sin E(E - e\sin E) - (1 + e\cos E)(1 - e\cos E)^2\right] & (\cos E - e) & 0 & e n \sin E & 0 & 0 \end{pmatrix} \\ \hline \dfrac{1}{(1 - e\cos E)} \begin{pmatrix} 0 & 0 & 0 & 0 & (1 - e^2)^{1/2}\cos E & \sin E \end{pmatrix} \end{array} \right\}$$

$$
\overset{*}{M}_{ji} = \left\{ \begin{array}{c|c}
\dfrac{1}{(1-e^2\cos^2 E_i)^{1/2}(1-e^2\cos^2 E_j)^{1/2}} \begin{pmatrix}
\begin{array}{l} (1+e\cos E_i)(1-e\cos E_j) \\ +\dfrac{2\sin E_M(1-e\cos E_j)}{(1-e\cos E_i)^2}\cdot\Big[(1-e^2)\sin E_M \\ -(1-e\cos E_i)e\sin E_i\cos E_M\Big] \end{array} &
\begin{array}{l} \dfrac{2(1-e^2)^{1/2}(1-e\cos E_j)}{(1-e\cos E_i)^2} \\ \cdot\sin E_M(\cos E_M - e\cos E_P) \end{array} \\[2ex]
\begin{array}{l} \dfrac{2(1-e^2)^{1/2}}{(1-e\cos E_i)^2} \\ \cdot\Big\{-(1+e\cos E_j) \\ \cdot(3E_M - e\sin E_M\cos E_P) \\ +2\sin E_M\Big[e\cos E_P \\ +(1+e\cos E_i - e^2\cos^2 E_i)\cos E_M\Big]\Big\} \end{array} &
\begin{array}{l} (1-e\cos E_i)(1+e\cos E_j) \\ +\dfrac{2}{(1-e\cos E_i)^2}\Big\{-(1+e\cos E_j) \\ \cdot e\sin E_i(3E_M - e\sin E_M\cos E_P) \\ +2\sin E_M\Big[2e\sin E_P \\ -(1+e^2)\sin E_M\Big]\Big\} \end{array}
\end{pmatrix} & \begin{matrix} 0 \\[2ex] 0 \end{matrix} \\ \hline
\begin{pmatrix} 0 & \qquad 0 \end{pmatrix} & 1-\dfrac{2\sin^2 E_M}{1-e\cos E_i}
\end{array} \right\}
\tag{K-39}
$$

$$
\hat{\hat{N}}_{ji} = \frac{2}{n} \left\{ \begin{array}{c|c} \dfrac{1}{(1-e^2\cos^2 E_i)^{1/2}(1-e^2\cos E_j)^{1/2}} \begin{pmatrix} (1-e\cos E_i)(1-e\cos E_j) \cdot \sin E_M (\cos E_M + e\cos E_p) & 2(1-e^2)^{1/2}(1-e\cos E_j)\sin^2 E_M & 0 \\ -2(1-e^2)^{1/2}(1-e\cos E_i)\sin^2 E_M & -(1+e\cos E_i)(1+e\cos E_j) \cdot (3E_M - e\sin E_M \cos E_P) + 4\sin E_M(\cos E_M + e\cos E_P) & 0 \end{pmatrix} \\ \hline \begin{pmatrix} 0 & 0 & \sin E_M(\cos E_M - e\cos E_P) \end{pmatrix} \end{array} \right\}
$$

(K-40)

$$
\ddot{S}_{ji} = 2n \left\{ \begin{array}{c}
\dfrac{1}{(1 - e\cos E_i)^2 (1 - e\cos E_j)^2 (1 - e^2\cos^2 E_i)^{1/2} (1 - e^2\cos^2 E_j)^{1/2}}
\left(\begin{array}{lll}
\begin{array}{l}
(1-e^2)(3E_M - 2e \sin E_M \cos E_P) \\
- \Big\{ (1 - e\cos E_i)(1 - e\cos E_j) e^2 \sin E_i \sin E_j \\
+ (1-e^2) \left[1 + (1 - e\cos E_i)\, e \cos E_i \right. \\
\left. + (1 - e\cos E_j) e \cos E_j \right] \Big\} \sin E_M \cos E_M
\end{array}
&
\begin{array}{l}
(1-e)^{1/2} \Big\{ 3e \sin F_i (E_M - e \sin E_M \cos F_P) \\
+ \left[(1-e^2) + (1 - e\cos E_j) e \cos E_j \right] \sin^2 E_M \\
\quad - e^2 \left[2(1 - e\cos E_i) \cos E_M \cos E_P \right. \\
\left. + (1 - e\cos E_j) \cos E_j \right] \sin E_M \sin E_P \Big\}
\end{array}
& 0 \\
& & \\
\begin{array}{l}
(1-e^2)^{1/2} \Big\{ 3e \sin E_j (E_M - e \sin E_M \cos E_P) \\
- \left[(1-e^2) + (1 - e\cos E_i) e \cos E_i \right] \sin^2 E_M \\
- e^2 \left[2(1 - e\cos E_j) \cos E_M \cos E_P \right. \\
\left. + (1 - e\cos E_i) \cos E_i \right] \sin E_M \sin E_P \Big\}
\end{array}
&
\begin{array}{l}
e^2 \sin E_i \sin E_j \left[3E_M - 4e \sin E_M \cos E_P \right. \\
\left. + e^2 \sin E_M \cos E_M (\cos^2 E_P - \sin^2 E_P) \right] \\
- (1-e^2) \sin E_M \left[(1+e^2) \cos E_M - 2e \cos E_P \right]
\end{array}
& 0
\end{array} \right) \\
\hline
\left(\begin{array}{ccc}
0 & 0 & - \dfrac{\sin E_M \cos E_M}{(1 - e\cos E_i)(1 - e\cos E_j)}
\end{array} \right)
\end{array} \right\}
$$

(K-41)

$$
{}^{*}T_{ji} = \left\{ \begin{array}{c|c}
\dfrac{1}{(1-e^2\cos^2 E_i)^{1/2}\,(1-e^2\cos^2 E_j)^{1/2}} \begin{pmatrix}
\begin{array}{l} (1+e\cos E_j)(1-e\cos E_i) \\ + \dfrac{2\sin E_M (1-e\cos E_i)}{(1-e\cos E_j)^2}\Big[(1-e^2)\sin E_M \\ +(1-e\cos E_j)\, e\sin E_j \cos E_M\Big] \end{array} &
\begin{array}{l} \dfrac{2(1-e^2)^{1/2}}{(1-e\cos E_j)^2} \\ \cdot\Big\{(1+e\cos E_i) \\ \cdot (3E_M - e\sin E_M \cos E_P) \\ -2\sin E_M \Big[e\cos E_P \\ +(1+e\cos E_j - e^2\cos^2 E_j)\cos E_M\Big]\Big\} \end{array} \\[2ex]
\begin{array}{l} -\dfrac{2(1-e^2)^{1/2}(1-e\cos E_i)}{(1-e\cos E_j)^2} \\ \cdot \sin E_M(\cos E_M - e\cos E_P) \end{array} &
\begin{array}{l} (1-e\cos E_j)(1+e\cos E_i) \\ + \dfrac{2}{(1-e\cos E_j)^2}\Big\{(1+e\cos E_i) \\ \cdot e\sin E_j\,(3E_M - e\sin E_M\cos E_P) \\ -2\sin E_M\Big[2e\sin E_P \\ +(1+e^2)\sin E_M\Big]\Big\} \end{array}
\end{pmatrix} & \begin{pmatrix} 0 \\ 0 \end{pmatrix} \\ \hline
\begin{pmatrix} 0 & 0 \end{pmatrix} & \left(1 - \dfrac{2\sin^2 E_M}{1-e\cos E_j}\right)
\end{array} \right\}
\qquad \text{(K-42)}
$$

$$
J_{ij}^{*} = n \left\{ \begin{array}{c} \dfrac{1}{1 + e\cos E_i} \begin{pmatrix} \begin{array}{l} \dfrac{e \sin E_i}{(1 - e\cos E_i)^2} \\ + \dfrac{1}{2(1 - e\cos E_i) X} \Big\{ (1 + e\cos E_i) \\ \cdot \left[2\sin^2 E_M - (1 + e\cos E_i) \right] \\ \cdot \left[\dfrac{3 E_M}{\sin E_M} - e\cos E_P \right] \\ + 4 \Big[(1 + e\cos E_i)\cos E_M \\ + e\sin E_i \sin E_M \Big] \Big\} \end{array} & -(1 - e^2)^{1/2} \left[\dfrac{1}{(1 - e\cos E_i)^2} + \dfrac{\sin E_M}{X} \right] & 0 \\ -(1 - e^2)^{1/2} \left[\dfrac{1}{(1 - e\cos E_i)^2} + \dfrac{\sin E_M}{X} \right] & \begin{array}{l} -\dfrac{e\sin E_i}{(1 - e\cos E_i)^2} \\ + \dfrac{1}{2X}(1 - e\cos E_i)(\cos E_M + e\cos E_P) \end{array} & 0 \end{pmatrix} \\ \hline \begin{pmatrix} 0 & 0 & \dfrac{2\sin^2 E_M - (1 - e\cos E_i)}{2\sin E_M (\cos E_M - e\cos E_P)(1 - e\cos E_i)} \end{pmatrix} \end{array} \right\}
$$

(K-47)

$$\overset{*}{K}_{ij} = \frac{n}{2} \left\{ \begin{array}{c} \dfrac{1}{(1 - e^2 \cos^2 E_i)^{1/2} (1 - e^2 \cos^2 E_j)^{1/2}} \times \begin{pmatrix} \begin{array}{l} (1 + e \cos E_i)(1 + e \cos E_j) \\ \cdot \left(\dfrac{3 E_M}{\sin E_M} - e \cos E_P \right) \\ - 4 (\cos E_M + e \cos E_P) \end{array} & 2 (1 - e^2)^{1/2} (1 - e \cos E_j) \sin E_M & 0 \\ - 2 (1 - e^2)^{1/2} (1 - e \cos E_i) \sin E_M & \begin{array}{l} - (1 - e \cos E_i)(1 - e \cos E_j) \\ \cdot (\cos E_M + e \cos E_P) \end{array} & 0 \end{pmatrix} \\ \hdashline \begin{pmatrix} 0 & 0 & \dfrac{1}{\sin E_M (\cos E_M - e \cos E_P)} \end{pmatrix} \end{array} \right\} \tag{K-48}$$

APPENDIX B

GUIDANCE MATRICES FOR HYPERBOLIC ORBITS

$$\overset{*}{V} = \frac{a(1-e^2)^{1/2}}{h} \left\{ \frac{(e \cosh F - 1)}{(e^2 \cosh^2 F - 1)^{1/2}} \begin{pmatrix} 0 \\ (e^2-1)^{1/2} \sinh F \\ \frac{1}{e}(\cosh F + e) \\ \frac{a^2(e^2-1)(e - \cosh F)}{eh} \\ 0 \\ 0 \end{pmatrix} \Bigg| \frac{1}{(e^2 \cosh^2 F - 1)^{1/2}} \begin{pmatrix} 2(e \cosh F + 1) \\ -2(e^2-1) \cosh F \\ \frac{-2(e^2-1)^{1/2} \sinh F}{e} \\ \frac{a^2(e^2-1)^{1/2}}{eh} \Big[2 \sinh F \, (e^3 \cosh F - 1) + 3e(1 + e \cosh F)(F - e \sinh F) \Big] \\ 0 \\ 0 \end{pmatrix} \Bigg| \frac{1}{a} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{(\cosh F - e)}{(e^2-1)^{1/2}} \\ -\sinh F \end{pmatrix} \right\}$$

$$\overset{*}{R} = \frac{1}{a} \left\{ \begin{array}{c|c} \dfrac{1}{(e \cosh F - 1)^2 (e^2 \cosh^2 F - 1)^{1/2}} \begin{pmatrix} 2(e^2-1)^{1/2} & 2\, e \sinh F & 0 \\ (e^2-1)^{1/2} \left[-(e \cosh F - 1)(e \cosh^2 F) + (\cosh F - e) \right] & -(e^2-1) \sinh F & 0 \\ -\frac{1}{e} \left[(e \cosh F - 1)\, e \cosh F + (e^2-1) \right] \sinh F & -\dfrac{(e^2-1)^{1/2} (\cosh F - e)}{e} & 0 \\ \dfrac{a^2(e^2-1)}{eh} \left\{ \left[(e \cosh F - 1)\, e \cosh F + (e^2-1) \right] \sinh F + 3e(F - e \sinh F) \right\} & -\dfrac{a^2 (e^2-1)^{1/2}}{eh} \left[(e^2-1)(e - \cosh F) - 3e^2 \sinh F\, (F - e \sinh F) - e(e \cosh F + 1)(e \cosh F - 1)^2 \right] & 0 \end{pmatrix} \\ \hline \dfrac{1}{e \cosh F - 1} \begin{pmatrix} 0 & 0 & \dfrac{-\sinh F}{(e^2-1)^{1/2}} \\ 0 & 0 & \cosh F \end{pmatrix} \end{array} \right\}$$

$$
\overset{*}{H}_{ij} = \frac{1}{2a} \left\{ \frac{1}{(e^2\cosh^2 F_i - 1)^{1/2}} \times \begin{pmatrix} -4(e^2-1)^{1/2} \sinh F_M \\[2ex] (e^2-1)^{1/2}\Big[-(e\cosh F_i + 1)\sinh F_j \cdot \left(\frac{3F_M}{\sinh F_M} - e\cosh F_P\right) + 4(\sinh F_P + e\sinh F_M)\Big] \\[2ex] -\frac{1}{e}\Big[(e\cosh F_i + 1)(\cosh F_j + e) \cdot \left(\frac{3F_M}{\sinh F_M} - e\cosh F_P\right) - 4(\cosh F_P + e\cosh F_M)\Big] \\[2ex] \frac{a^2(e^2-1)}{eh}\Big[(e\cosh F_i + 1)(\cosh F_j - e) \cdot \left(\frac{3F_M}{\sinh F_M} - e\cosh F_P\right) - 4(\cosh F_P - e\cosh F_M) - 2e(3F_j - e\sinh F_j)\sinh F_M\Big] \\[2ex] 0 \\[2ex] 0 \end{pmatrix} \;\Bigg|\; \frac{2(e\cosh F_i - 1)}{(e^2\cosh^2 F_i - 1)} \times \begin{pmatrix} (\cosh F_M + e\cosh F_P) \\[2ex] -(e^2-1)\cosh F_P \\[2ex] -\frac{(e^2-1)^{1/2}\sinh F_P}{e} \\[2ex] \frac{-a^2(e^2-1)^{1/2}}{eh}\Big[(e^2+1)\sinh F_P - \frac{e}{2}(3F_j - e\sinh F_j)(\cosh F_M + e\cosh F_P) + 2e\sinh F_M\Big] \\[2ex] 0 \\[2ex] 0 \end{pmatrix} \;\Bigg|\; \frac{1}{\sinh F_M(\cosh F_M - e\cosh F_P)} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{(\cosh F_j - e)}{(e^2-1)^{1/2}} \\ -\sinh F_j \end{pmatrix} \right\}
$$

$$F^* = a \left\{ \begin{array}{c} \dfrac{(e\cosh F-1)}{(e^2\cosh^2 F-1)^{1/2}} \begin{pmatrix} -(e^2-1)^{1/2} & \dfrac{-(\cosh F+e)}{(e^2-1)^{1/2}} & e\sinh F & 0 & 0 & 0 \end{pmatrix} \\ \hdashline \dfrac{1}{(e^2\cosh^2 F-1)^{1/2}} \begin{pmatrix} \begin{array}{c} -{}^{1}/_{2}\,[3(e\cosh F+1) \\ \cdot(F-e\sinh F)+ \\ 2e\sinh F(e\cosh F-1)] \end{array} & 2\sinh F & -(e^2-1)^{1/2}(e\cosh F-1) & \dfrac{h(e\cosh F+1)}{a^2(e^2-1)^{1/2}} & 0 & 0 \end{pmatrix} \\ \hdashline \begin{pmatrix} 0 & 0 & 0 & 0 & -(e^2-1)^{1/2}\sinh F & -(\cosh F-e) \end{pmatrix} \end{array} \right\}$$

$$
L^{*} = \frac{h}{a(e^2-1)^{1/2}}
\left\{
\begin{array}{l}
\dfrac{1}{(e\cosh F - 1)^2 (e^2\cosh^2 F - 1)^{1/2}}
\begin{pmatrix}
\frac{3}{2}(e^2-1)^{1/2}(F - e\sinh F) & -\dfrac{\sinh F}{(e^2-1)^{1/2}}\Big[(e\cosh F - 1) \cdot e\cosh F + (e^2-1)\Big] & (e\cosh F - 1)^2 \cdot (e\cosh F + 1) & \dfrac{-h}{a^2} & 0 & 0 \\
{}^{1}/_{2}\Big[3e\sinh F(F - e\sinh F) + (e\cosh F + 1)(e\cosh F - 1)^2\Big] & -(\cosh F - e) & 0 & \dfrac{-eh\sinh F}{a^2(e^2-1)^{1/2}} & 0 & 0
\end{pmatrix} \\
\hline
\dfrac{-1}{(e\cosh F - 1)}
\begin{pmatrix}
0 & 0 & 0 & 0 & (e^2-1)^{1/2}\cosh F & \sinh F
\end{pmatrix}
\end{array}
\right\}
$$

$$
\overset{*}{M}_{ji} = \left\{ \begin{array}{c|c}
\dfrac{1}{(e^2\cosh^2 F_i - 1)^{1/2}(e^2\cosh^2 F_j - 1)^{1/2}} \begin{pmatrix}
\begin{array}{l}
(e\cosh F_i + 1)(e\cosh F_j - 1) \\
+ \dfrac{2\sinh F_M (e\cosh F_j - 1)}{(e\cosh F_i - 1)^2} \\
\cdot \Big[(e^2 - 1)\sinh F_M - (e\cosh F_i - 1) \\
\quad \cdot\, e\sinh F_i \cosh F_M \Big]
\end{array}
&
\begin{array}{l}
\dfrac{-2(e^2-1)^{1/2}(e\cosh F_j - 1)}{(e\cosh F_i - 1)^2} \\
\cdot \sinh F_M(\cosh F_M - e\cosh F_P)
\end{array}
\\[2ex]
\begin{array}{l}
\dfrac{2(e^2-1)^{1/2}}{(e\cosh F_i - 1)^2} \Big\{ -(e\cosh F_j + 1) \\
\cdot (3F_M - e\sinh F_M \cosh F_P) \\
\quad + 2\sinh F_M \Big[e\cosh F_P - (e^2\cosh^2 F_i \\
\quad\quad - 1 - e\cosh F_i)\cosh F_M \Big] \Big\}
\end{array}
&
\begin{array}{l}
(e\cosh F_i - 1)(e\cosh F_j + 1) \\
+ \dfrac{2}{(e\cosh F_i - 1)^2} \Big\{ -(e\cosh F_j + 1) \\
\cdot\, e\sinh F_i (3F_M - e\sinh F_M \cosh F_P) \\
\quad + 2\sinh F_M \Big[2e\sinh F_P \\
\quad\quad - (1 + e^2)\sinh F_M \Big] \Big\}
\end{array}
\end{pmatrix}
& \begin{matrix} 0 \\[6ex] 0 \end{matrix} \\
\hline
\begin{matrix} 0 & \qquad\qquad & 0 \end{matrix} & 1 - \dfrac{2\sinh^2 F_M}{(e\cosh F_i - 1)}
\end{array} \right\}
$$

$$
\mathring{N}_{j1} = 2a^2 \frac{(e^2-1)^{1/2}}{h}
\left\{
\begin{array}{c}
\dfrac{1}{(e^2\cosh^2 F_j - 1)^{1/2}\,(e^2\cosh^2 F_i - 1)^{1/2}}
\begin{pmatrix}
\begin{array}{l}(e\cosh F_i - 1)(e\cosh F_j - 1)\\ \cdot \sinh F_M(\cosh F_M + e\cosh F_P)\end{array} & 2(e^2-1)^{1/2}(e\cosh F_j - 1)\sinh^2 F_M & 0 \\[2ex]
-2(e^2-1)^{1/2}(e\cosh F_i - 1)\sinh^2 F_M & \begin{array}{l} -(1 \bullet e\cosh F_i)(1 \bullet e\cosh F_j) \\ \cdot(3F_M - e\sinh F_M\cosh F_P) \\ +4\sinh F_M(\cosh F_M + e\cosh F_P)\end{array} & 0
\end{pmatrix} \\
\hdashline
\begin{pmatrix}
0 & 0 & \sinh F_M(e\cosh F_P - \cosh F_M)
\end{pmatrix}
\end{array}
\right\}
$$

$$
S_{ji} = \frac{2h}{a^2 (e^2-1)^{1/2}} \left\{ \frac{1}{(e\cosh F_i - 1)^2 (e\cosh F_j - 1)^2 (e^2\cosh^2 F_i - 1)^{1/2} (e^2\cosh^2 F_j - 1)^{1/2}} \right.
$$

$$
\left(\begin{array}{lll}
\begin{array}{l}
(e^2-1)(3F_M - 2e\sinh F_M \cosh F_P) \\
-\Big\{(e\cosh F_i - 1)(e\cosh F_j - 1) \\
\cdot\, e^2 \sinh F_i \sinh F_j + (e^2-1) \\
\cdot\Big[1 - \big\{(e\cosh F_i - 1) e\cosh F_i \\
+ (e\cosh F_j - 1)\, e\cosh F_j\big\}\Big]\Big\} \sinh F_M \cosh F_M
\end{array}
&
\begin{array}{l}
(e^2-1)^{1/2} \Big\{3e\sinh F_i (F_M - e\sinh F_M \cosh F_P) \\
-\big[(e^2-1) - (e\cosh F_j - 1) e\cosh F_j\big] \sinh^2 F_M \\
+ e^2 \big[2(e\cosh F_i - 1)\cosh F_M \cosh F_P \\
+ (e\cosh F_j - 1)\cosh F_j\big] \sinh F_M \sinh F_P\Big\}
\end{array}
& 0 \\
\\
\begin{array}{l}
(e^2-1)^{1/2} \Big\{3e\sinh F_j (F_M - e\sinh F_M \cosh F_P) \\
+\big[(e^2-1) + (e\cosh F_i - 1)(e\cosh F_i)\big] \sinh^2 F_M \\
+ e^2 \big[2(e\cosh F_j - 1)\cosh F_M \cosh F_P \\
+ (e\cosh F_i - 1)\cosh F_i\big] \sinh F_M \sinh F_P\Big\}
\end{array}
&
\begin{array}{l}
e^2 \sinh F_i \sinh F_j \big[3F_M - \\
4e\sinh F_M \cosh F_P + \\
e^2 \sinh F_M \cosh F_M (\cosh^2 F_P \\
+ \sinh^2 F_P)\big] - (e^2-1)\sinh F_M \\
\cdot \big[(1+e^2)\cosh F_M - 2e\cosh F_P\big]
\end{array}
& 0
\end{array}\right)
$$

$$
\left.
\left(\begin{array}{ccc}
0 & 0 & \dfrac{-\sinh F_M \cosh F_M}{(e\cosh F_i - 1)(e\cosh F_j - 1)}
\end{array}\right)
\right\}
$$

$$
\dot{T}_{ji} = \left\{ \begin{array}{c}
\dfrac{1}{(e^2\cosh^2 F_i - 1)^{1/2}(e^2\cosh^2 F_j - 1)^{1/2}}
\begin{pmatrix}
\begin{array}{l}
(e\cosh F_j + 1)(e\cosh F_i - 1) \\
+ \dfrac{2\sinh F_M (e\cosh F_i - 1)}{(e\cosh F_j - 1)^2} \\
\cdot \Big[(e^2 - 1)\sinh F_M \\
+ (e\cosh F_j - 1)\, e\sinh F_j \cosh F_M \Big]
\end{array}
&
\begin{array}{l}
\dfrac{2(e^2-1)^{1/2}}{(e\cosh F_j - 1)^2} \Big\{ (e\cosh F_i + 1) \\
\cdot (3F_M - e\sinh F_M \cosh F_P) \\
-2\sinh F_M \Big[e\cosh F_P \\
-(e^2\cosh^2 F_j - 1 - e\cosh F_j)\cosh F_M \Big] \Big\}
\end{array}
& 0 \\[2ex]
\begin{array}{l}
\dfrac{2(e^2-1)^{1/2}(e\cosh F_i - 1)}{(e\cosh F_j - 1)^2} \\
\cdot \sinh F_M (\cosh F_M - e\cosh F_P)
\end{array}
&
\begin{array}{l}
(e\cosh F_j - 1)(e\cosh F_i + 1) \\
+ \dfrac{2}{(e\cosh F_j - 1)^2} \Big\{ (e\cosh F_i + 1) \\
\cdot e\sinh F_j (3F_M - e\sinh F_M \cosh F_P) \\
-2\sinh F_M \Big[2e\sinh F_P + \\
(1 + e^2)\sinh F_M \Big] \Big\}
\end{array}
& 0
\end{pmatrix} \\
\hline
\begin{pmatrix}
0 & 0 & 1 - \dfrac{2\sinh^2 F_M}{(e\cosh F_j - 1)}
\end{pmatrix}
\end{array} \right\}
$$

$$
J^{*}_{ij} = \frac{h}{a^2 (e^2-1)^{1/2}} \left\{ \begin{array}{c}
\dfrac{1}{(e \cosh F_i + 1)} \left(\begin{array}{ccc}
\begin{array}{l} \dfrac{e \sinh F_i}{(e \cosh F_i - 1)^2} \\ + \dfrac{1}{2(e \cosh F_i - 1) X} \Big\{ (e \cosh F_i + 1) \\ \cdot \left[2 \sinh^2 F_M + (e \cosh F_i + 1) \right] \\ \cdot \left[\dfrac{3 F_M}{\sinh F_M} - e \cosh F_P \right] \\ + 4 \left[(e \cosh F_i + 1) \cosh F_M \right. \\ \left. - e \sinh F_i \sinh F_M \right] \Big\} \end{array} & -(e^2-1)^{1/2} \left[\dfrac{1}{(e \cosh F_i - 1)^2} + \dfrac{\sinh F_M}{X} \right] & 0 \\
-(e^2-1)^{1/2} \left[\dfrac{1}{(e \cosh F_i - 1)^2} + \dfrac{\sinh F_M}{X} \right] & \begin{array}{l} - \dfrac{e \sinh F_i}{(e \cosh F_i - 1)^2} \\ + \dfrac{1}{2X} \left[(e \cosh F_i - 1)(\cosh F_M + e \cosh F_P) \right] \end{array} & 0
\end{array} \right) \\
\hline
\dfrac{1}{(e \cosh F_i - 1)} \left(\begin{array}{ccc} 0 & 0 & \dfrac{-2 \sinh^2 F_M + (e \cosh F_i - 1)}{2 \sinh F_M (\cosh F_M - e \cosh F_P)} \end{array} \right)
\end{array} \right\}
$$

$$
\overset{*}{K}_{ij} = \frac{h}{2a^2(e^2-1)^{1/2}} \left\{ \begin{array}{c} \dfrac{1}{(e^2\cosh^2 F_i - 1)^{1/2}(e^2\cosh^2 F_j - 1)^{1/2}} \times \begin{pmatrix} \begin{array}{l}(e\cosh F_j + 1)(e\cosh F_i + 1) \\ \cdot \left(\dfrac{3F_M}{\sinh F_M} - e\cosh F_P\right) \\ - 4(\cosh F_M + e\cosh F_P)\end{array} & 2(e^2-1)^{1/2}(e\cosh F_j - 1)\sinh F_M & 0 \\ -2(e^2-1)^{1/2}(e\cosh F_i - 1)\sinh F_M & \begin{array}{l} -(e\cosh F_i - 1)(e\cosh F_j - 1) \\ \cdot (\cosh F_M + e\cosh F_P)\end{array} & 0 \end{pmatrix} \\ \hline \begin{pmatrix} 0 & 0 & \dfrac{1}{\sinh F_M(e\cosh F_P - \cosh F_M)} \end{pmatrix} \end{array} \right\}
$$

APPENDIX C

GUIDANCE MATRICES FOR ELLIPTICAL AND HYPERBOLIC ORBITS

$$
\overset{*}{V} = \frac{a}{h} \left\{ \frac{(1 - e^2)}{(1 + e^2 + 2e\cos f)^{1/2}\,(1 + e\cos f)} \begin{pmatrix} 0 \\ (1 - e^2)\sin f \\ -\frac{1}{e}\left[2e + (1 + e^2)\cos f\right] \\ -\frac{a^2 (1 - e^2)^2 \cos f}{eh} \\ 0 \\ 0 \end{pmatrix} \;\vdots\; \frac{1}{(1 + e^2 + 2e\cos f)^{1/2}} \begin{pmatrix} 2(1 + e^2 + 2e\cos f) \\ 2(1 - e^2)(\cos f + e) \\ \frac{2(1 - e^2)\sin f}{e} \\ \frac{a^2}{eh}\left\{ 2(1 - e^2)^2 \sin f - \left[\frac{3eh(1 + e^2 + 2e\cos f)(t - t_o)}{a^2} - \frac{2e^2 (1 - e^2)^2 \sin f}{(1 + e\cos f)} \right] \right\} \\ 0 \\ 0 \end{pmatrix} \;\vdots\; \frac{(1 - e^2)}{(1 + e\cos f)} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \cos f \\ \sin f \end{pmatrix} \right\}
$$

$$
\overset{*}{R} = \left\{ \begin{array}{c}
\dfrac{1}{a\,(1 - e^2)\,(1 + e^2 + 2\,e\cos f)^{1/2}} \begin{pmatrix} \dfrac{2\,(1 + e\cos f)^3}{(1 - e^2)} & \dfrac{2\,e\sin f\,(1 + e\cos f)^2}{(1 - e^2)} & 0 \\ \left[e\,(\cos f + e)^2 + \cos f\,(1 + e\cos f)^2 \right] & (1 + e\cos f)^2 \sin f & 0 \\ \dfrac{\sin f}{e} \left[e\,(\cos f + e) + (1 + e\cos f)^2 \right] & -\dfrac{1}{e}\,(1 + e\cos f)^2 \cos f & 0 \end{pmatrix} \\
\hline
\dfrac{a}{e\,h\,(1 + e^2 + 2\,e\cos f)^{1/2}} \begin{pmatrix} \begin{array}{l} \sin f\,\left[e\,(\cos f + e) \right. \\ \left. + (1 + e\cos f)^2 \right] \\ - \dfrac{3\,e\,h\,(t - t_o)\,(1 + e\cos f)^3}{a^2\,(1 - e^2)^2} \end{array} & \begin{array}{l} -(1 + e\cos f)^2 \cos f \\ \dfrac{-3\,e^2\,h\,(t - t_o)\,\sin f\,(1 + e\cos f)^2}{a^2\,(1 - e^2)^2} \\ + e\,(1 + e^2 + 2\,e\cos f) \end{array} & 0 \end{pmatrix} \\
\hline
\dfrac{1}{a\,(1 - e^2)} \begin{pmatrix} 0 & 0 & \sin f \\ 0 & 0 & -(\cos f + e) \end{pmatrix}
\end{array} \right\}
$$

$$
\tilde{H}_{ij} = \frac{1}{a} \left\{ \frac{(1-e^2)}{(1+e^2+2e\cos f_i)^{1/2}(1+e\cos f_j)} \times
\begin{pmatrix}
-2\sin^2 f_M \\[2ex]
-\left[(1+e^2+2e\cos f_i)\sin f_j\right] \cdot \left[\frac{3ht_M}{2a^2(1-e^2)} + \frac{e\sin f_M(\cos f_P + e\cos f_M)}{(1+e\cos f_i)(1+e\cos f_j)}\right] + 2\sin f_M(\sin f_P + e\sin f_M) \\[2ex]
\frac{1}{e(1-e^2)}\left\{\left[1+e^2+2e\cos f_i\right]\left[2e+(1+e^2)\cos f_j\right] \cdot \left[\frac{3ht_M}{2a^2(1-e^2)} + \frac{e\sin f_M(\cos f_P + e\cos f_M)}{(1+e\cos f_i)(1+e\cos f_j)}\right] - 2\sin f_M\left[(1+e^2)\cos f_P + 2e\cos f_M\right]\right\} \\[2ex]
\frac{a^2(1-e^2)}{eh}\left\{(1+e^2+2e\cos f_i)\left[\frac{3ht_M}{2a^2(1-e^2)} + \frac{e\sin f_M(\cos f_P + e\cos f_M)}{(1+e\cos f_i)(1+e\cos f_j)}\right]\cos f_j + \sin f_M\left[-2\cos f_P + e\sin f_M\left(\frac{3h(t_j - t_o)}{a^2(1-e^2)} + \frac{2e\sin f_j}{1+e\cos f_j}\right)\right]\right\} \\[2ex]
0 \\[2ex]
0
\end{pmatrix}
\frac{(1-e^2)}{(1+e\cos f_i)(1+e\cos f_j)(1+e^2+2e\cos f_i)^{1/2}} \times
\begin{pmatrix}
\sin f_M\left[(1+e^2)\cos f_M + 2e\cos f_P\right] \\[2ex]
(1-e^2)\sin f_M\left[\cos f_P + e\cos f_M\right] \\[2ex]
\frac{(1-e^2)\sin f_M \sin f_P}{e} \\[2ex]
\frac{a^2(1-e^2)\sin f_M}{eh}\left\{\left[(1+e^2)\sin f_P + 2e\sin f_M\right] - \left[\frac{3eh(t_j - t_o)}{2a^2(1-e^2)} + \frac{e^2\sin f_j}{(1+e\cos f_j)}\right]\left[(1+e^2)\cos f_M + 2e\cos f_P\right]\right\} \\[2ex]
0 \\[2ex]
0
\end{pmatrix}
\right.
$$

$$
\overset{*}{F} = a \left\{ \begin{array}{l}
\dfrac{1}{(1+e^2+2e\cos f)^{1/2}} \begin{pmatrix} (1-e^2) & \dfrac{\left[-2e+(1+e^2)\cos f\right]}{(1+e\cos f)} & \dfrac{-e(1-e^2)\sin f}{(1+e\cos f)} & 0 & 0 & 0 \\ -3(1+e^2+2e\cos f)\left[\dfrac{h(t-t_o)}{2a^2(1-e^2)}\right] + \dfrac{e\sin f(1-e^2)}{(1+e\cos f)} & 2\sin f & (1-e^2) & \dfrac{-h(1+e^2+2e\cos f)}{a^2(1-e^2)} & 0 & 0 \end{pmatrix} \\
\hline
\dfrac{(1-e^2)}{(1+e\cos f)} \begin{pmatrix} 0 & 0 & 0 & 0 & \sin f & -\cos f \end{pmatrix}
\end{array} \right\}
$$

$$\overset{*}{L} = \frac{h}{a(1-e^2)} \left\{ \begin{array}{c} \dfrac{1}{(1+e^2+2e\cos f)^{1/2}} \begin{pmatrix} \dfrac{3h(t-t_o)(1+e\cos f)^3}{2a^2(1-e^2)^2} & \dfrac{-\sin f}{(1-e^2)}\left[e(\cos f+e) + (1+e\cos f)^2\right] & -\left[2e\cos f+(1+e^2)\right] & \dfrac{h(1+e\cos f)^3}{a^2(1-e^2)^2} & 0 & 0 \\ \dfrac{1}{2}\left[\dfrac{3eh(t-t_o)\sin f(1+e\cos f)^2}{a^2(1-e^2)^2} - (1+e^2+2e\cos f)\right] & \dfrac{(1+e\cos f)^2\cos f}{(1-e^2)} & 0 & \dfrac{h(1+e\cos f)^2 e\sin f}{a^2(1-e^2)^2} & 0 & 0 \end{pmatrix} \\ \hdashline \begin{pmatrix} 0 & 0 & 0 & 0 & (\cos f+e) & \sin f \end{pmatrix} \end{array} \right\}$$

$$
\tilde{M}_{ji} = \left\{ \begin{array}{c}
\dfrac{1}{(1+e^2+2e\cos f_i)^{1/2}\,(1+e^2+2e\cos f_j)^{1/2}}
\begin{pmatrix}
(1+e^2+2e\cos f_i)+\dfrac{2\sin f_M}{(1+e\cos f_j)}\Big[\sin f_M\cdot(1+e\cos f_i)^2-e\sin f_i(\cos f_M+e\cos f_P)\Big] & \dfrac{2(1+e\cos f_i)^2\sin f_M\cos f_M}{(1+e\cos f_j)} & 0 \\
\begin{array}{l}\dfrac{2(1+e\cos f_i)^2}{(1-e^2)^2}\Bigg\{-(1+e^2+2e\cos f_j)\cdot\left[\dfrac{3ht_M(1+e\cos f_i)}{a^2(1-e^2)}\right] \\ \quad +2\sin f_M\left[\dfrac{-e^2(\cos f_P+e\cos f_M)(e+\cos f_j)}{(1+e\cos f_j)} +(\cos f_M+e\cos f_P)\left(1+e\dfrac{(e+\cos f_i)(1-e^2)}{(1+e\cos f_i)^2}\right)\right]\Bigg\}\end{array} & \begin{array}{l}(1+e^2+2e\cos f_j)+\dfrac{2(1+e\cos f_i)^2}{(1-e^2)^2} \\ \cdot\Bigg\{-\dfrac{(1+e^2+2e\cos f_j)e\sin f_i}{(1+e\cos f_i)(1+e\cos f_j)} \\ \cdot\left[\dfrac{3ht_M(1+e\cos f_i)(1+e\cos f_j)}{a^2(1-e^2)} +2e\sin f_M(\cos f_P+e\cos f_M)\right] \\ +2\sin f_M\Big[2e\sin f_P-(1+e^2)\sin f_M\Big]\Bigg\}\end{array} & 0
\end{pmatrix} \\
\hline
\begin{pmatrix} 0 & 0 & 1-\dfrac{2\sin^2 f_M}{(1+e\cos f_j)} \end{pmatrix}
\end{array} \right\}
$$

$$
\overset{*}{N}_{ji} = \frac{2a^2}{h}
\left\{
\begin{array}{c}
\dfrac{1}{(1+e^2+2e\cos f_i)^{1/2}\,(1+e^2+2e\cos f_j)^{1/2}}
\begin{pmatrix}
\dfrac{(1-e^2)^2 \sin f_M}{(1+e\cos f_i)(1+e\cos f_j)} \cdot \left[(1+e^2)\cos f_M + 2e\cos f_P\right] & \dfrac{2(1-e^2)^2 \sin^2 f_M}{(1+e\cos f_j)} & 0 \\[2ex]
\dfrac{-2(1-e^2)^2 \sin^2 f_M}{(1+e\cos f_i)} & \begin{array}{l} \dfrac{-(1+e^2+2e\cos f_i)(1+e^2+2e\cos f_j)}{(1+e\cos f_i)(1+e\cos f_j)} \\ \cdot \left[\dfrac{3ht_M(1+e\cos f_i)(1+e\cos f_j)}{a^2(1-e^2)} \right. \\ \left. +2e\sin f_M(\cos f_P + e\cos f_M)\right] + \\ 4\sin f_M \left[(1+e^2)\cos f_M + 2e\cos f_P\right] \end{array} & 0
\end{pmatrix} \\
\hdashline
\begin{pmatrix}
0 & 0 & \dfrac{(1-e^2)^2 \sin f_M \cos f_M}{(1+e\cos f_i)(1+e\cos f_j)}
\end{pmatrix}
\end{array}
\right\}
$$

$$
\hat{S}_{ji} = \frac{2h}{a^2(1-e^2)^2}
\left\{
\begin{array}{c}
\dfrac{(1+e\cos f_i)^2(1+e\cos f_j)^2}{(1-e^2)^2(1+e^2+2e\cos f_i)^{1/2}(1+e^2+2e\cos f_j)^{1/2}}
\left(
\begin{array}{ccc}
\begin{array}{l}
\dfrac{3ht_M(1+e\cos f_i)(1+e\cos f_j)}{a^2(1-e^2)} \\
+e\sin f_M(\cos f_P+e\cos f_M) \\
-\sin f_M(\cos f_M+e\cos f_P) \\
\cdot\left\{1+e(1-e^2)\left[e(1-e^2)\right.\right. \\
\cdot\dfrac{\sin f_i\sin f_j}{(1+e\cos f_i)^2(1+e\cos f_j)^2} \\
\left.\left.+\dfrac{(e+\cos f_i)}{(1+e\cos f_i)^2}+\dfrac{(e+\cos f_j)}{(1+e\cos f_j)^2}\right]\right\}
\end{array}
&
\begin{array}{l}
-\dfrac{3eh\sin f_i t_M(1+e\cos f_j)}{a^2(1-e^2)} \\
+(1-e^2)\sin f_M\left\{\sin f_M\left[1+\dfrac{e(e+\cos f_j)}{(1+e\cos f_j)^2}\right]\right. \\
-e^2\sin f_P\left[\dfrac{2e(\cos^2 f_P+\cos^2 f_M)+2(1+e^2)\cos f_M\cos f_P}{(1+e\cos f_i)^2(1+e\cos f_j)}\right. \\
\left.\left.+\dfrac{(e+\cos f_j)}{(1+e\cos f_j)^2}\right]\right\}
\end{array}
& 0 \\
\\
\begin{array}{l}
\dfrac{3eh\sin f_j t_M(1+e\cos f_i)}{a^2(1-e^2)} \\
-(1-e^2)\sin f_M\left\{\sin f_M\left[1+e\dfrac{(e+\cos f_i)}{(1+e\cos f_i)^2}\right]\right. \\
+\dfrac{e^2\sin f_P\left[2e(\cos^2 f_P+\cos^2 f_M)+2(1+e^2)\cos f_P\cos f_M\right.}{(1+e\cos f_j)^2(1+e\cos f_i)} \\
\left.\left.+\dfrac{(e+\cos f_i)}{(1+e\cos f_i)^2}\right]\right\}
\end{array}
&
\begin{array}{l}
e^2\sin f_i\sin f_j\left\{\dfrac{3ht_M}{a^2(1-e^2)}\right. \\
+\dfrac{e\sin f_M}{(1+e\cos f_i)(1+e\cos f_j)}\left[e(\cos f_M+e\cos f_P)\right. \\
\cdot\left\{\dfrac{(e+\cos f_i)(e+\cos f_j)-(1-e^2)\sin f_i\sin f_j}{(1+e\cos f_i)(1+e\cos f_j)}\right\} \\
\left.\left.-(\cos f_P+e\cos f_M)\right]\right\} \\
-\sin f_M(1-e^2)(\cos f_M-e\cos f_P)
\end{array}
& 0
\end{array}
\right) \\
\hdashline
\left(
\begin{array}{ccc}
0 & 0 & -\sin f_M(\cos f_M+e\cos f_P)
\end{array}
\right)
\end{array}
\right\}
$$

$$
\overset{*}{T}_{ji} = \left\{ \begin{array}{c}
\dfrac{1}{(1+e^2+2e\cos f_i)^{1/2}(1+e^2+2e\cos f_j)^{1/2}}
\begin{pmatrix}
\begin{array}{l} (1+e^2+2e\cos f_j) + \dfrac{2\sin f_M}{(1+e\cos f_i)}\Big[\sin f_M(1+e\cos f_j)^2 \\ \quad + e\sin f_j(\cos f_M + e\cos f_P)\Big] \end{array}
&
\begin{array}{l} \dfrac{2(1+e\cos f_j)^2}{(1-e^2)^2}\left\{\dfrac{3ht_M(1+e\cos f_j)(1+e^2+2e\cos f_i)}{a^2(1-e^2)}\right. \\ \quad + \dfrac{2e^2\sin f_M(\cos f_P + e\cos f_M)(e+\cos f_i)}{(1+e\cos f_i)} \\ \quad - 2\sin f_M(\cos f_M + e\cos f_P) \\ \quad \cdot\left.\left[\dfrac{e(1-e^2)(\cos f_j + e) + (1+e\cos f_j)^2}{(1+e\cos f_j)^2}\right]\right\} \end{array}
& 0 \\
-\dfrac{2(1+e\cos f_j)^2\sin f_M\cos f_M}{(1+e\cos f_i)}
&
\begin{array}{l} (1+e^2+2e\cos f_i) + \dfrac{2(1+e\cos f_j)^2}{(1-e^2)^2} \\ \quad \cdot\left\{\dfrac{(1+e^2+2e\cos f_i)e\sin f_j}{(1+e\cos f_i)(1+e\cos f_j)}\left[\dfrac{3ht_M(1+e\cos f_i)(1+e\cos f_j)}{a^2(1-e^2)}\right.\right. \\ \quad + \left. 2e\sin f_M(\cos f_P + e\cos f_M)\right] \\ \quad - \left. 2\sin f_M\left[2e\sin f_P + (1+e^2)\sin f_M\right]\right\} \end{array}
& 0
\end{pmatrix}
\\ \hline
\begin{pmatrix} 0 & 0 & 1 - \dfrac{2\sin^2 f_M}{(1+e\cos f_i)} \end{pmatrix}
\end{array} \right\}
$$

$$
J^{*}_{ij} = \frac{h}{a^2(1-e^2)}
\begin{Bmatrix}
\begin{array}{l}
\dfrac{e\sin f_i\,(1+e\cos f_i)^2}{(1+e^2+2e\cos f_i)(1-e^2)} + \dfrac{(1+e\cos f_i)}{2X} \\[2ex]
\cdot\left\{\left[2(1-e^2)\sin^2 f_M - (1+e^2+2e\cos f_i)(1+e\cos f_j)\right]\right. \\[2ex]
\cdot\left[\dfrac{3ht_M}{a^2(1-e^2)\sin f_M} + \dfrac{2e(\cos f_P + e\cos f_M)}{(1+e\cos f_i)(1+e\cos f_j)}\right] \\[2ex]
\left. \quad + 4(e\cos f_P + \cos f_M) + \dfrac{(1-e^2)e\sin f_i\,\sin f_M}{(1+e^2+2e\cos f_i)}\right\}
\end{array}
&
\begin{array}{l}
-\dfrac{(1+e\cos f_i)}{(1+e^2+2e\cos f_i)}\left[\dfrac{(1+e\cos f_i)^2}{(1-e^2)}\right. \\[2ex]
\left. \quad + \dfrac{(1-e^2)^2\sin f_M}{X}\right]
\end{array}
& 0 \\[8ex]
-\dfrac{(1+e\cos f_i)}{(1+e^2+2e\cos f_i)}\left[\dfrac{(1+e\cos f_i)^2}{(1-e^2)} + \dfrac{(1-e^2)^2\sin f_M}{X}\right]
&
\begin{array}{l}
\dfrac{1}{(1+e^2+2e\cos f_i)}\left\{-\dfrac{e\sin f_i(1+e\cos f_i)^2}{(1-e^2)}\right. \\[2ex]
\left. \quad + \dfrac{(1-e^2)^2}{2X}\left[(1+e^2)\cos f_M + 2e\cos f_P\right]\right\}
\end{array}
& 0 \\[8ex]
0 & 0 & \dfrac{\left[2\sin^2 f_M - (1+e\cos f_j)\right](1+e\cos f_i)}{2(1-e^2)\sin f_M\cos f_M}
\end{Bmatrix}
$$

$$\overset{*}{K}_{ij} = \frac{h(1+e\cos f_i)(1+e\cos f_j)}{2a^2(1-e^2)} \left\{ \begin{array}{c} \dfrac{1}{(1+e^2+2e\cos f_i)^{1/2}(1+e^2+2e\cos f_j)^{1/2}} \times \begin{pmatrix} \begin{array}{l} \left[(1+e^2+2e\cos f_i)(1+e^2+2e\cos f_j)\right] \\ \cdot \left[\dfrac{3ht_M}{a^2(1-e^2)\sin f_M} + \dfrac{2e(\cos f_P + e\cos f_M)}{(1+e\cos f_i)(1+e\cos f_j)}\right] \\ -4\left[(1+e^2)\cos f_M + 2e\cos f_P\right] \end{array} & \dfrac{2(1-e^2)^2 \sin f_M}{(1+e\cos f_j)} & 0 \\ -\dfrac{2(1-e^2)^2 \sin f_M}{(1+e\cos f_i)} & -(1-e^2)^2\left[(1+e^2)\cos f_M + 2e\cos f_P\right] & 0 \end{pmatrix} \\ \hdashline \begin{pmatrix} 0 & 0 & \dfrac{1}{\sin f_M \cos f_M} \end{pmatrix} \end{array} \right\}$$

LIST OF REFERENCES

1. Battin, R. H., Astronautical Guidance, McGraw-Hill Book Company, New York, 1964.

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5. Stern, R. G., Analytic Solution of the Equations of Motion of an Interplanetary Space Vehicle in the Midcourse Phase of its Flight, Experimental Astronomy Laboratory Report RE-4, M. I. T., November 1963.

6. Stern, R. G., Interplanetary Midcourse Guidance Analysis, Sc. D. thesis in Department of Aeronautics and Astronautics, M. I. T., May 1963.

ERRATA

p. 5, Appendix A begins on p. 21, not p. 19; Appendix B begins on p. 33, not p. 31; Appendix C begins on p. 45, not p. 43; and References begin on p. 57, not p. 55.

p. 19, Eq. (12-2) for "E_p" read "F_p".

p. 24, The coefficient of the first column should read

$$\frac{1}{(1-e^2\cos^2 E_i)^{1/2}\,X} .$$

p. 26, Insert "0" for element L_{25}.

p. 34, The factor of element V_{31} should read $\frac{1}{e}(\cosh F+e)$. Insert a multiplication sign at the beginning of the second line in the factor of element V_{42}.

p. 35, In the factor of element R_{31}, the first term is "$-\frac{1}{e}$".

p. 36, In the factor of element F_{22}, for "(e^2-1)" read "$-(e^2-1)$". On the second line of the factor of the element H_{42}, for "$-2e$" read "$-\frac{e}{2}$". In the denominator of the coefficient of the third column, for "$(\cosh F_M-\cosh F_p)$" read "$(\cosh F_M - e\cosh F_p)$".

p. 38, In the second line of the factor of element L_{12}, for "cosh F" read "$\cdot e\cosh F$".

p. 39, In the element M_{33} for "$\frac{1-2\sinh^2 F_M}{(e\cosh F_i - 1)}$" read

"$1-\frac{2\sinh^2 F_M}{(e\cosh F_i - 1)}$".

p. 40, In the denominator of the coefficient of the 2x3 block, for "$(e^2\cosh F_j - 1)^{1/2}$" read "$(e^2\cosh^2 F_j - 1)^{1/2}$".

p. 41, The third term in the denominator of the coefficient of the 2x3 block is "$(e^2\cosh^2 F_i - 1)^{1/2}$".

p. 43, Insert "0" for element J_{13}. In the denominator of the factor of element J_{33}, for "$2\sinh F_M(\cosh F_M - e\cosh F_p)$

(e cosh F_i - 1)" read "2 sinh F_M(cosh F_M - e cosh F_p)". In the unbound version p. 43 is incorrectly numbered as p. 52.

p. 47, In the factor of element R_{21} for "$(1+e \cos f)^2]$" read "$\cos f(1 + e \cos f)^2]$".

p. 48, In the denominator of the second line of the factor of element H_{42}, for "$2a(1 - e^2)$" read "$2a^2(1 - e^2)$".

p. 49, In the numerator of the factor of element F_{12} for "$[-2e + (1 + e^2) \cos f]$" read "$-[2e + (1+e^2) \cos f]$".

p. 51, In the second line of the factor of element M_{11}, for "$(e \cos f_M$" read "$(\cos f_M$". In the element M_{33} for "$\frac{1-2 \sin^2 f_M}{1+e \cos f_j)}$" read "$1 - \frac{2 \sin^2 f_M}{(1+e \cos f_j)}$".

p. 52, The coefficient of the 2x3 block is

$$\text{"}\frac{1}{(1+e^2+ 2e \cos f_i)^{1/2}(1+e^2+ 2e \cos f_j)^{1/2}}\text{"}$$

In the numerator of the factor of element N_{21}, for "$-2(1 - e^2)$" read "$-2(1 - e^2)^2$". In the unbound version p. 52 is incorrectly numbered as p. 43.

p. 53, In the numerator of the last line of the factor of element S_{12}, for "$(e + \cos f_i)$" read "$(e + \cos f_j)$". At the end of the third line of the factor of element S_{22}, insert a brace "}".

p. 54, In the numerator of the second line of the factor of element T_{12}, for "$2e^2$" read "2e". The factor of element T_{12} is

$$\text{"}\frac{2(1+e \cos f_j)^2}{(1 - e^2)^2} \left\{ \frac{3ht_M(1 + e \cos f_j)(1 + e^2 + 2e \cos f_i)}{a^2(1 - e^2)} \right.$$

$$+ \frac{2e\ \sin f_M(\cos f_P + e\cos f_M)(e + \cos f_i)}{(1 + e\cos f_i)}$$

$$- 2\sin f_M(\cos f_M + e\cos f_P)$$

$$\cdot \left[\frac{e(1 - e^2)(\cos f_j + e) + (1 + e\cos f_j)^2}{(1 + e\cos f_j)^2}\right]\Big\}"$$

The factor of element T_{22} is

$$"(1+e^2+2e\cos f_i) + \frac{2(1+e\cos f_i)^2}{(1 - e^2)^2}\ \Big\{\frac{(1+e^2+2e\cos f_i)e\sin f_j}{(1+e\cos f_i)(1+e\cos f_j)}$$

$$\cdot \left[\frac{3ht_M(1+e\cos f_i)(1+e\cos f_j)}{a^2(1-e^2)} + 2e\sin f_M(\cos f_P + e\cos f_M)\right]$$

$$- 2\sin f_M[2e\sin f_P + (1+e^2)\sin f_M]\Big\}".$$

p. 55, The second term in the factor of element J_{22} is

$$" - \frac{e\sin f_i(1 + e\cos f_i)^2}{(1 - e^2)}".$$

p. 56, The first term in the second line of the factor of element K_{11} is $"\frac{3ht_M}{a^2(1-e^2)\sin f_M}"$.