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4 December 1964

INTERIM STUDY REPORT

ON

NAVIGATION AND GUIDANCE ANALYSIS

FOR

A MARS MISSION

NASA CR70635

Contract No: NAS8-11198

Prepared by

PHILCO CORPORATION
Western Development Laboratories
Palo Alto, California

for

Marshall Space Flight Center
Huntsville, Alabama

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FOREWARD

Although a formal report is not required at this time under Contract NAS-8-11198, sufficient progress in certain areas and the difficulties encountered in other areas is believed to be of sufficient general interest to warrant documentation. In addition, there is a certain degree of dependence of the scope and detail of this study on the IBM 7094 analysis programs under development for NASA, GSFC (Contract NAS 5-9700). Furthermore, the manned-mars mission study for MSFC requires the definition of mathematical models of practical onboard measurements and guidance laws which have a strong influence on the capabilities being designed into the IBM 7094 programs for GSFC. Thus, the two contracts compliment each other strongly in certain areas and this fact is being used to enhance the quality of the products to be delivered in both cases.

In order to show the complimenting features, when they can enter the study, and the possible study scope changes, a program plan for the remainder of the MSFC contract is also included. This plan is not intended to be final but rather to provide a working document from which each contract can benefit from the other.

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SECTION 1

1.0 INTRODUCTION

1.1 General Objectives and Scope

The primary objective of Contract NAS 8 - 11198 is to establish the basic sensor requirements for an Advanced Spaceborne Detection, Tracking, and Navigation System capable of performing interplanetary missions. The scope of the study is to consider missions up to and including manned interplanetary flight.

In order to make the problem amenable to study, certain restrictions on the scope of the study had to be made. These restrictions were either suggested by or approved by MSFC personnel. They are as follows:

1. Primary emphasis and calculations are to be for the 1975 opportunity for a round trip to Mars. The trajectory is to include a stop over of about forty days in orbit about Mars at an altitude higher than the sensible Martian atmosphere (500 km.). The outbound flight time is 235 days and return flight time is 297 days. This restriction was made at the onset of the study since obtaining data for all possible missions would be out of the question. Also, in order to accomplish the objective, it is necessary to develop techniques for calculations, graphical interpretation and data organization which when developed can be followed for any other mission. Other missions may be studied if sufficient time and funds exist and if their study appears noteworthy.

2. The Saturn guidance injection covariance matrix shall be applicable. This matrix, which is a function of parking time in orbit about the earth, is intended to be representative of the capabilities of future launch vehicle guidance systems. The primary influence of this quantity is on the magnitude of the midcourse outbound velocity requirements at the first guidance correction. Parametric variations of its size are of interest, however, any other adjustments need definitive reasons before they are to be studied.

3. The study is to emphasize but not necessarily be restricted to the following phases:
   - (a) midcourse from earth to Mars
   - (b) orbital navigation at Mars
   - (c) midcourse from Mars to earth

   These phases are probably the most demanding on the sensor requirements if one neglects the inertial components required during the accelerating (or decelerating) portions of the total mission. It is hoped that some information pertinent to the alignment and inertial component requirements during entry into and from the Martian orbit can be obtained late in the study. As a minimum it is planned to outline a future study effort which includes these and other phases of a manned interplanetary mission late in the current contract.

1.2 Plan of Approach

In order to initiate the study effort, a plan of approach was suggested in which one would gain data on the trade off between onboard system complexity and attainable accuracies. The simplest onboard system utilizes both earth based tracking and computational facilities with reliance on telemetry to the spacecraft for command signals.

The following systems in the order of increasing onboard system complexity represent the plan of approach.

System I: Onboard equipment consists of

(a) an attitude control system with a reference alignment procedure.

(b) telemetry for receipt of command signals to carry out thrusting maneuvers.

All computations and tracking are to be earth based for this system.

System II: Same as I with the addition of onboard angular measurements of the target as the target is approached. The measurements are to be restricted in a manner which allows earth based computation to do the necessary orbit determiation.

System III: Same as II with the addition of an onboard digital computer for usage during the terminal part of the outbound midcourse phase, orbital navigation phase and the initial part of the midcourse return phase. This system would allow rapid onboard calculations when they may be required during the rapidly changing portions of the flight which occur at great distances (and consequent time delays) from the earth.

System IV. Complete onboard system for all phases of the mission. In this instance, the system is to take on a configuration and mode of operation which is deemed desirable for a manned vehicle. Angular measurements using a sextant type instrument would be emphasized. for those purposes where they are useful. The reason for this is that the sextant is probably the best single instrument considering its simplicity and

<u>System IV Continued</u>: reliability if a man is available to operate it.

1.3 Current Status

In carrying out this plan of approach, three areas of analytical effort are required:

1. Definition of the mathematical model of the system configuration and its various error sources and the consequent analysis program development.
2. Calculation of data on the system configuration with parametric variations on those important error sources (e.g. the sensor accuracy). This data reveals the tradeoffs between measurement and execution errors and the guidance accuracy attainable.
3. Analysis and graphical representation of the data in a form suitable for later system studies.

In view of the fact that Item 1 was essentially completed for the Type I system under GSFC Contract NAS 5-3342, the study plan was to initiate the second and third analytical areas on this system as the first effort. Since the error propagation program developed under Contract NAS 5-3342 does not have the capability of treating resolution type errors in midcourse maneuvers, investigation of the manner of inclusion of this capability was also initiated. The progress report for August on this contract and Section 5 of reference 1* contains the mathematical derivation of the method of treating resolution errors and other improvements in the manner of calculating guidance data. These changes have been incorporated in a modified error propagation program for use in this study (see section II of this report). Note that the material in reference 1 is more complete than that of the other report and should be used as the reference for those interested.

*Reference 1 - First Quarterly Report for "Certain Computer Programs Contract NAS5-9700, WDL TR2332, Nov. 1, 1964.

Certain unforeseen difficulties were encountered in the use of the error propagation program for the initial study effort. The difficulties were (a) extremely long run times for evaluation of the earth based tracking accuracy and (b) numerical accuracy problems in the evaluation of guidance accuracy data. Section II of this report describes these problems and the steps which have been made to reduce or eliminate the difficulties.

In order to study the influence of feasible onboard measurements as implied by Systems II, III, and IV, an analytical effort was initiated to define reasonable mathematical models, a manner of organizing data calculations for good efficiency, and techniques for generating sensible onboard schedules for the observations. The results of this analysis are given in Section III of this report.

During the study to date a number of reports by other investigators for simular missions have been reviewed. The influence of these reports on the recommended direction the current study should take is discussed in Section IV. In addition, the potential added capability being designed in the IBM 7094 programs, for delivery to GSFC, approximate dates they will be available and their utilization is included in Section IV. Section IV also takes both of these factors in mind and suggests a program study plan for data generation during the remainder of this contract.

Section V discusses some additional analytical problems whose solution is important in setting up overall requirements and techniques for the onboard computer needed for a manned interplanetary mission. The extent to which these problems can be resolved under the current contract is not known at this time.

SECTION 2

2.0 NUMERICAL PROBLEMS AND THEIR SOLUTION

2.1 Description of the Problems

In the process of generating guidance and navigation data using the Error Propagation Program to simulate earth based tracking, a number of problems were encountered which required a solution before further progress could be made. The problems which were encountered were in two areas: (1) extremely long run times were required when using earth based tracking for navigation and (2) a number of numerical problems associated with the use of single precision in the Error Propagation Program.

(1) RUN TIME

If all the DSIF data which are available for the mission being studied are processed, the computer run time amounts to approximately one minute per day of actual flight. This would amount to approximately 4 hours for each run from earth to mars. Consequently, a parametric study of the earth-based navigation system, if it is required, will require an increase in the 7094 time allowed at Ames.

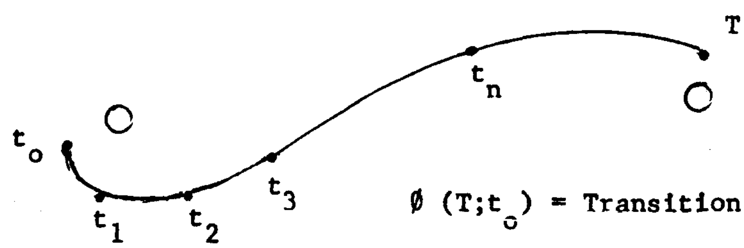
(2) NUMERICAL PROBLEMS

There are two difficulties which have been encountered as a result of the use of single precision in the major part of the Error Propagation Program (eight significant figures). The first problem is that the nominal trajectory is sensitive to the eighth figure of the starting value of the injection positions and velocities. The sensitivity of the end point position to initial conditions is on the order of 10^5 km/km for position and 10^8 km/km/sec for velocity. Therefore, for injection from an earth park orbit with a radius of 6500 km and a velocity of 16 km/sec, the significance in terms of position variations at the target of the eighth figures of position and velocity is on the order of 10 km and 100 km respectively.

The problem arose first in the process of iteration to find the nominal trajectory. The trajectory obtained during the iteration process had a radius of closest approach which oscillllated about the 500 km desired approach with a magnitude of approximately 100 km. Once a nominal trajectory is selected the program will repeat the trajectory, but the difficulty is that on a long run, when it is desirable to stop the program at some point and restart with new parameters, the new trajectory is different from the original one.

The second problem, which is much more significant in terms of the data which are to be generated, is the numerical accuracy of the transition matrix from the running time (t) to the end point (T). The need for the matrix and the method of computation of this matrix in the Error Propagation Program are described in the following paragraphs.

Prior to a data generation run, variational equations are integrated from the starting point to the end point to obtain a matrix of sensitivity partials, the transition matrix from injection to the end point (Figure 1).



$\phi\ (T;t_o)$ = Transition Matrix

Figure 1

When a guidance correction is to be considered or it is desired to know what the knowledge of the target miss is for the observations which have been taken, the transition matrix from the specified time (t) to the end point is required. The need for the matrix is illustrated by the following equations.

The knowledge of the target miss is determined by the following equations.

$$\underset{6 \times 6}{\phi(T;t)} = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} = \text{transition matrix from time (t) to the end point (T)}$$

$$\underset{3 \times 3}{KM} = \underset{3 \times 6}{\begin{pmatrix} \phi_{11} & \phi_{12} \end{pmatrix}} \underset{6 \times 6}{P(t)} \underset{6 \times 3}{\begin{pmatrix} \phi_{11} & \phi_{12} \end{pmatrix}^T}$$

where:

T = Transpose

KM is a 3 x 3 covariance matrix of the knowledge of miss for a fixed time of arrival guidance law.

P(t) is the knowledge of state covariance matrix at time (t).

The expected guidance correction at the time (t) is obtained as follows.

$$\underset{3 \times 3}{E\left\{ \dot{x}_g \dot{x}_g^T \right\}} = \underset{3 \times 6}{\begin{bmatrix} \phi_{12}^{-1} \phi_{11} & I \end{bmatrix}} \left\{ PAR(t) - P(t) \right\} \begin{bmatrix} \phi_{12}^{-1} \phi_{11} & I \end{bmatrix}^T$$

where

$E\left\{ \dot{x}_g \dot{x}_g^T \right\}$ = covariance matrix of the guidance correction at time (t).

PAR(t) = covariance matrix of the deviation of state from the nominal at time (t).

I = 3 x 3 unit diagonal matrix

As can be seen from the above equations the transition matrix, $\phi(T;t)$, from time (t) to the end point (T) is essential in generating output data.

The computation of the transition matrix, $\phi(T;t)$ is performed in the following manner:

$$\phi(T;t_1) = \phi(T;t_o)\ \phi^{-1}(t_1;t_o)$$

$$\phi(T;t_2) = \phi(T;t_1)\ \phi^{-1}(t_2;t_1)$$

$$\vdots$$

$$\phi(T;t_n) = \phi(T;t_n)\ \phi^{-1}(t_n;t_{n-1})$$

The transition matrix is updated at each time point at which output data are desired. The above method of calculation although analytically correct encounters numerical difficulties. The inverse matrix after a period of 3 or 4 days becomes numerically ill-conditioned and the above operation yields a transition matrix to the end point which is also ill-conditioned. The matrix inversion was compared for two methods of inversion but both yielded numerical problems. In one method, the matrix was inverted directly by using a single precision matrix inverse routine. In the second method the matrix was inverted by use of the following identity which is valid for the transition matrix along coast flight trajectories.

$$\phi = \begin{pmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{pmatrix} \qquad \phi^{-1} = \begin{pmatrix} \phi_{22}^T & -\phi_{12}^T \\ -\phi_{21}^T & \phi_{11}^T \end{pmatrix}$$

This numerical accuracy problem was found in the process of checking the magnitudes of the second velocity correction which were presented in the August progress report, reference (1).* The velocity requirements shown in reference (1) for the second velocity correction and the miss after the correction have been found to be incorrect and the data should be disregarded. The reference also contained data for the magnitudes of the first velocity correction. This data is valid. The reason that no numerical difficulties arise for the first correction is due to the fact that for short time intervals (12 hours in this case) the inverse transistion matrix used in the manner described yields a sufficiently accurate transition matrix.

2.2 Description of Solution

In view of the previous numerical problems, particularly the one associated with the accuracy of the transition matrix, it was decided that a modified Error Propagation Program need be constructed.

The modified program was designed with the precision trajectory replaced by a patched conic trajectory. It also incorporates closed form analytic expressions for the transition matrices. A description of the method used for calculating the transition matrices is contained in reference 2. The modified program has been checked out and is operating. The transition matrix generated by integrating the variational equations along the precision trajectory in the Error Propagation Program checked quite well with the one obtained from the closed form solution along the patched conic. The modified program requires about half the time of the Error Propagation Program to calculate the same amount of data. The most important feature of the program is that at each data output time, the transition matrix to the end point is obtained by an evaluation of the closed form solution from the running time (t) to the end point time T. Using this procedure there is no inversion of the matrix from injection time, t_o, to the running time, t, required as was shown previously.

* Reference List is presented at the end of the Section.

The result is that the knowledge of the target miss and guidance velocity requirements may be computed at each time point without the numerical inversion problem.

In addition to modifications previously described, the guidance subroutines have been modified. The modifications consist of implementing the results of the analysis which are part of the August progress report reference (1) entitled "Variable Time of Arrival Guidance" (a corrected and extended version of this analysis is in Section 5 of reference 3). The three changes which were made and are described by the aforementioned analysis are the following: (1) The program now has the capability of properly handling a resolution type error in making the midcourse correction. This is an error which is in the direction of the velocity correction but independent of the magnitude of the correction. (2) After a guidance correction is made the state deviation covariance matrix, PAR(t), is maintained with respect to the new nominal which is based on the knowledge of state at the time of the correction. Previously, it has been maintained with respect to the injection nominal. (3) The arrival time deviation from the nominal is computed and presented as part of the output.

In addition to the above modifications to the program, an output was added which is the magnitude of the velocity correction required when the guidance law being used is for variable time of arrival with minimum fuel as the third constraint on the correction.

The analytic derivation of the guidance law is contained in reference (4) and repeated here for convenience. Let the miss parameters be Δ B·T, Δ B·R, and ΔA. For small variations, we assume linearization is valid and obtain:

$$\begin{pmatrix} \Delta B\cdot T \\ \Delta B\cdot R \\ \Delta A \end{pmatrix} = \underset{3x6}{\begin{bmatrix} J \end{bmatrix}} \; \underset{6x1}{\bar{x}(t)} = \underset{3x6}{\begin{bmatrix} J \end{bmatrix}} \; \underset{6x6}{\emptyset(T;t)} \underset{6x1}{\bar{x}(t)} = \underset{3x3}{A} \; \underset{3x1}{x(t)} + \underset{3x3}{B} \; \underset{3x1}{\dot{x}(t)}$$

where J denotes the matrix of partials relating the miss parameters and the state at time (T)

$\emptyset$ is the transition matrix from time (t) to time (T)

The quantity, ΔA, is primarily associated with a miss in the time of arrival since $\Delta B \cdot T$ and $\Delta B \cdot R$ define the target miss in a plane normal to the trajectory. Since two of the components of a guidance velocity correction are sufficient to correct $\Delta B \cdot T$ and $\Delta B \cdot R$, we may choose the third degree of freedom so that $|\dot{x}_g|$ is minimized. The guidance correction may be obtained as follows.

$$\begin{pmatrix} \Delta B \cdot T \\ \Delta B \cdot R \\ \Delta A \end{pmatrix} + B \, \dot{x}_g(t) = 0$$

$$\dot{x}_g(t) = -B^{-1} \begin{pmatrix} \Delta B \cdot T \\ \Delta B \cdot R \\ \Delta A \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} AB.T \\ \Delta B \cdot R \\ \Delta A \end{pmatrix}$$

For this case we select the value of ΔA which minimizes

$$\dot{x}_{1g}^2 + x_{2g}^2 + x_{3g}^2 = (\Delta V)^2$$

$$\dot{x}_{1g} = b_{11} \, \Delta B \cdot T + b_{12} \, \Delta B \cdot R + b_{13} \, \Delta A$$

$$\dot{x}_{2g} = b_{21} \, \Delta B \cdot T + b_{22} \, \Delta B \cdot R + b_{23} \, \Delta A$$

$$\dot{x}_{3g} = b_{31} \, \Delta B \cdot T + b_{32} \, \Delta B \cdot R + b_{33} \, \Delta A$$

Now we determine the value of ΔA which minimizes $(\Delta V)^2$:

$$\frac{\partial \Delta V}{\partial \Delta A} = 0 = \frac{1}{V}\left(\dot{x}_{1g} b_{13} + \dot{x}_{2g} b_{23} + \dot{x}_{3g} b_{33}\right)$$

$$-\Delta A = \frac{\left(b_{11}\, b_{13} + b_{21}\, b_{23} + b_{31}\, b_{33}\right) \Delta B\cdot T + \left(b_{12}\, b_{13} + b_{22}\, b_{23} + b_{32}\, b_{33}\right) \Delta B\cdot R}{b_{13}^2 + b_{23}^2 + b_{33}^2}$$

$$\Delta A = C(\Delta B\cdot T) + D\,(\Delta B\cdot R)$$

Now in order to compute the covariance matrix of the expected minimum velocity correction, the above value of ΔA is used in the computations.

$$E^*(\dot{x}_g \dot{x}_g^T) = E\left\{ B^{-1} \begin{pmatrix} \Delta B\cdot T \\ \Delta B\cdot R \\ \Delta A \end{pmatrix} \begin{pmatrix} \Delta B\cdot T \\ \Delta B\cdot R \\ \Delta A \end{pmatrix}^T \left(B^{-1}\right)^T \right\}$$

$$E(\dot{x}_g \dot{x}_g^T) = B^{-1} \begin{bmatrix} E(\Delta B\cdot T)^2 & E(\Delta B\cdot T\ \Delta B\cdot R) & E(\Delta B\cdot T\ \Delta A) \\ E(\Delta B\cdot R\ \Delta B\cdot T) & E(\Delta B\cdot R)^2 & E(\Delta B\cdot R\ \Delta A) \\ E(\Delta A\ \Delta B\cdot T) & E(\Delta A\ \Delta B\cdot R) & E(\Delta A)^2 \end{bmatrix} (B^{-1})^T$$

where

$$E(\Delta A\ \Delta B\cdot T) = C\ E(\Delta B\cdot T)^2 + D\ E(\Delta B\cdot R\ \Delta B\cdot T)$$

$$E(\Delta A\ \Delta B\cdot R) = C\ E(\Delta B\cdot T\ \Delta B\cdot R) + D\ E(\Delta B\cdot R)^2$$

$$E(\Delta A)^2 = C^2 E(\Delta B\cdot T)^2 + D^2\ E(\Delta B\cdot R)^2 + 2CD\ E(\Delta B\cdot T\ \Delta B\cdot R)$$

---

*E is the expected value of the quantity in brackets.

The right hand sides of the above equations are all known quantites and are used to fill in the guidance correction covariance matrix. The square root of the trace of the matrix is then the RMS minimum fuel required. These computations have been added to the modified program and checked.

As part of the check of the operation of the new conic program, a simulation of an onboard navigation and guidance system for a Mars mission was made. The run was a duplication of a Mars mission studied at Ames. The data which were generated were in close agreement with the unpublished Ames data.

2.3 References:

1. "Monthly Progress Report on Contract NAS 8-11198", dated 30 September, 1964.

2. Woolston, D. S., Mohan, J., "Program Manual for Minimum Variance Precision Tracking and Orbit Predictioh Program", Report X-640-63-144, Goddard Space Flight Center (July 1, 1963).

3. Philco WDL, "First Quarterly Report for Certain Computer Programs", WDL-TR-2332 for Goddard Space Flight Center.

4. Schmidt, S. F., "The Application of State Space Methods to Navigation Problems", Guidance and Control System Engineering Department Technical Report No. 4 (July 1964).

SECTION 3

3.0 THE SCHEDULING OF MEASUREMENTS AND ORGANIZATION OF CALCULATIONS FOR ANALYSIS OF AN ONBOARD NAVIGATION SYSTEM

3.1 Introduction

One of the problems associated with studies of navigation instrumentation requirements of space missions is the selection of a measurement schedule for the onboard instrumentation. This problem usually arises for manned missions where it is desirable to provide the capability of navigation but it can arise in unmanned probes when a high accuracy orbit relative to the target body is needed to fulfill the mission objectives.

The basic difficulty is that it is completely unfeasible to calculate data for all of the large number of possible schedules and observations. One usually is interested in establishing which observations are the most important on a given schedule and the effect of scheduling. To plan data runs to establish the interrelationships between these factors some guidelines are needed. In addition, since each data calculation on a digital computer requires a reasonable amount of time, it is desirable to organize the calculations such that every run provides as much information as possible.

This section discusses some of the basic problems and suggests a certain organization of the calculations and auxiliary data which should be calculated prior to the basic study. It is believed that this auxiliary data should be quite useful in forming the guidelines needed to reduce the number of observation schedules to be calculated.

3.2 Possible Celestial Body Measurements From Onboard A Space Vehicle

Considering the measuring devices which can be used onboard a spacecraft which will give information useful for the determination of position and velocity, two categories generally exist. These are

1. Radar type devices which in their simplest form provide measurements of altitude and altitude rate. In more complex forms where several devices are coupled with an inertial reference, it is conceivable that one could obtain additional quantities such as

   a. direction of the apparent local vertical in inertial coordinates

   b. velocity relative to the surface and inertial direction of this velocity vector

2. Optical type devices such as

   a. Scanners (visible or infrared region). These devices are generally automatic and can be arranged to track such that their optical center remains near the center (or rim) of a celestial body of the solar system. If the device is combined with an inertial reference platform, the inertial direction (two angles) of the body center can be obtained. With other modifications the subtended angle can be obtained.

      Another scanner possibility would combine the instrument with a star tracker. The star tracker will establish an inertial direction and an appropriately designed and mounted planet tracker could provide the angle from the star to the planet rim or center.

   b. Theodolite. This device may be used on manned space vehicles where the man directs the theodolite to the apparent center of the body in the solar system . If the theodolite is mounted on an inertial platform, then the measurements possible are the direction (two angles) of the celestial body (apparent center or landmark) in a known inertial reference system. Modifications could also allow the measurement of the subtended angle.

c. Sextant. This device is also used on manned vehicles. It has the capability of measuring a single angle. One can measure an inertial angle (eg. from some star to the apparent center, rim, or a landmark, of a body of the solar system) without the need of an inertial reference platform. An appropriately designed sextant can be used for a variety of purposes in addition to the above such as

   1. measurement of the subtended angle of the body, or perhaps the angle between two landmarks
   2. the angle between two given bodies of the solar system.

   In addition, the instrument may be calibrated onboard by star to star measurements.

d. Photograph of the celestial body in the solar system in the star background. The reduction of the photograph by the onboard personnel can provide the inertial direction (two angles) and perhaps the subtended angle of the body.

The radar type devices for onboard usage are invariably restricted (as a result of power considerations) to usefulness in the immediate vicinity of a celestial body. The optical devices, however, can provide useful information throughout an interplanetary flight.

In conducting a study of manned or unmanned interplanetary flight it is generally desirable to determine what onboard instrumentation is required. Invariably, all that can be determined is the added benefits gained by the use of onboard measurements.

For navigation purposes, the added benefits probably are judged on the basis of how much more accurately one can obtain the mission objectives.

The problems associated with such studies are that many interrelated variables exist such as

a. accuracy of the measurements
b. frequency and sequence of the measurements
c. type of measurements
d. measurement implications on vehicle design (gas consumption for attitude orientation, view window, size, etc.).

It is desirable, therefore, to uncouple as many of these variables as possible.

3.3 Methods of Decoupling Interrelated Variables

3.3.1 Organization of Calculations

The simulation of a navigation system on a digital computer may calculate (under the assumption of linearity in the vicinity of a nominal trajectory) statistical data relating such things as rms miss due to rms errors in the observations, etc. However, if we simulate only one system in each run, then the run provides only one datum point.

Two ways of decoupling the interrelated variables and thus gaining more information per data run are, therefore, considered. One factor which may be used in this regard is that (1) if the sequence and type of measurement remains constant and measurement errors are independent random variables and (2) the final orbit established is determined only from these measurements, then the covariance matrix of orbit determination errors is proportional to the covariance matrix of the measurement errors. This is proved as follows:

Assume a set of measurements, y_i, which are related to the initial state vector, x_o, by

$$y_i(t_i) = H_i(t_i)\, x_o + \epsilon_i \tag{1}$$

where ϵ_i is a gaussian random variable with zero mean. In vector form

$$y = H\, x_o + \epsilon \tag{2}$$

and

$$p(y-Hx_o) = p(\epsilon) = \frac{1}{(2\pi)^{n/2}\ |Q|^{\frac{1}{2}}}\ e^{-\frac{1}{2}(y-Hx_o)^T Q^{-1}(y-Hx_o)^T Q^{-1}(y-Hx_o)} \tag{3}$$

where

$$Q = E(\epsilon\epsilon^T)$$

Assume we wish to determine x_o such that we have the highest probability that the estimate of x_o, $\hat{x}_o$, is correct. That is,

$$H\,\hat{x}_o = \bar{y} = \text{mean value of } y.$$

This occurs when the exponent in equation (3) is a minimum. In order to find the relationship between the estimate of x_o, $\hat{x}_o$, and the measurements, y, one takes the gradient of the exponent of equation (3) with respect to x_o. The resultant expression is then set equal to zero and $\hat{x}_o$ is determined to be the x_o which satisfies the equation.

Thus,

$$H^TQ^{-1}(y-Hx_o) = 0$$

or, if $H^TQ^{-1}H$ is invertible, then

$$\hat{x}_o = (H^TQ^{-1}H)^{-1}\,H^TQ^{-1}y \tag{4}$$

Equation (4) is the so-called "maximum likelihood" estimate.

The covariance matrix of the error in this estimate, P_o, is found as follows

$$P_o = E(x-\hat{x}_o)(x-\hat{x}_o) = E\left(\left[x-(H^TQ^{-1}H)^{-1}H^TQ^{-1}y\right]\left[(H^TQ^{-1}H)^{-1}H^TQ^{-1}y\right]^T\right) \tag{5}$$

If one substitutes $y = Hx+\epsilon$ and recognizes that Q^{-1} and $(H^TQ^{-1}H)^{-1}$ are symmetric in equation (5), then a number of quantities cancel and

$$P_o = (H^TQ^{-1}H)^{-1} \tag{6}$$

As can be seen in equation (6), a direct scaling of the measurement errors by scaling the variances in Q results in a direct scaling of P_o, Q.E.D.

If one assumes equation of motion error sources do not exist (or if they do, that they are additional components of x_o), then the target miss covariance matrix, TM, is found from

$$TM = B(t;t_o)\ P(t_o)\ B^T(t;t_o) \tag{7}$$

where $B(t;t_o)$ of equation (7) represents the sensitivity matrix relating variations in the initial state x_o to the miss vector, b. Since B is not a random variable, then measurement error variances also directly scale into the knowledge of the target miss.

A second factor which can be used in reducing the number of calculations required can be obtained if one assumes the measurement errors, ϵ, of equation (2) are independent. In this case, Q and Q^{-1} are diagonal matrices so one may write the inverse of equation (6) as

$$P_o^{-1} = H^T Q^{-1} H = \sum_{i=1}^{n} H_i^T Q_i^{-1} H_i \tag{8}$$

One run on the computer may calculate as many different measurement combinations as desired. The data for each configuration is contained in the quantity $H_i^T Q_i^{-1} H_i$. Thus, one may change the accuracies and combinations of the $\Sigma H_i^T Q_i^{-1} H_i$ at will, following the data run, and calculate P_o and TM. In this manner the computer time required per datum point obtained is considerably reduced.

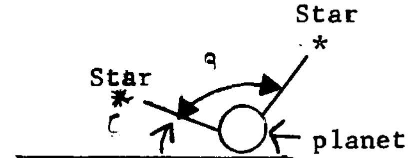
3.3.2 Organization of Data Runs

Another method of reducing the number of data runs to be made is to calculate certain quantities on the nominal trajectory which determine which measurements are the most promising as a function of time.

The technique suggested here also allows a study of the influences of the measurement sequence on the vehicle design following the definition of the sequence. The method suggested is primarily for consideration of a sextant type device where the following questions are important.

1. Which star should be used?
2. Which celestial body should be used?
3. Is the star-celestial body angle within limits?
4. Is the sun in a bad position?

As can be noted in considering a sextant, two different simultaneous star-celestial body measurements are equivalent to one theodolite (or planet tracker) measurement. These two angles would be from two stars to some point, such as the apparent center of the target.



The two stars would be located such that the angle θ of the sketch is 90°. The angle ζ of the sketch is arbitrary in this case and, as a consequence, one questions that if ζ is properly selected, one of the measurements is more important than the other.

To establish this importance we recognize that the principal cause of uncertainties in the knowledge of the miss will be attributed to the knowledge of current velocity. Since we are dealing with angular measurements they must be smoothed over a given time period in order to estimate the velocity. Thus we may choose the reference direction of the sketch such that for the two measurements under consideration:

1. $\zeta = 0$ is in the direction of maximum rotation rate of the line of sight due to the vehicle's inertial velocity

2. $\zeta = 90^{\circ}$ is in the direction of zero rotation rate of the line of sight due to vehicle's inertial velocity

The first measurement then gives the greatest information on the vehicle velocity if it is taken periodically over a given time period. Thus, we might expect that if for some reason the number of total measurements is restricted, then more measurements of this type should be assigned than those of the second type.

Note that in the consideration the words, "due to the vehicle motion" are added. This is simply a result of the fact that the celestial body ephemeris is known and knowledge of the body's velocity gives no information on the vehicles position or velocity.

The direction of the maximum line of sight rate due to vehicles motion is given by

$$u = \frac{r \times v}{|r \times v|} \times \frac{r}{|r|} \tag{9}$$

where

r = instantaneous inertial vector between the vehicle and the body

v = the inertial velocity vector of the vehicle relative to the body of greatest attractive force, (the central body)

The velocity vector v is defined in this manner since the central body's inertial velocity is known and does not contribute any knowledge about the vehicle motion.

If we define a plane by r and v then one may calculate the following quantities. Let the inertial unit vectors be defined as i, j, k in a right handed orthogonal system

$$\ell = k \times \left(\frac{r \times v}{|r \times v|} \right) = \begin{pmatrix} \ell_1 \\ \ell_2 \\ 0 \end{pmatrix} \tag{10}$$

Right ascension of ascending node, $\omega_o = \tan^{-1} \frac{\ell_2}{\ell_1}$ (11)

Inclination of plane $i_o = \cos^{-1}\left(k \cdot \left(\frac{r \times v}{|r \times v|}\right)\right)$ (12)

The right ascension anddeclination of the body are

$$\left.\begin{aligned} RA_b &= \tan^{-1} \frac{r_2}{r_1} \qquad \text{where} \quad r = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} \\ DEC_b &= \tan^{-1} \frac{r_3}{\sqrt{r_1^2 + r_2^2}} \end{aligned}\right\} \quad (13)$$

The direction of greatest line of sight rate due to the vehicles motion is an equation relating δ = declination, and ω = right ascension as follows

$$\tan \delta = \tan i_o \sin(\omega - \omega_o) \quad (14)$$

Now assume we have a star catalog which defines the right ascension, declination, and brightness of a large number of stars. (It is believed at least one is available for 7094 usage which has 250,000 star locations and brightness stored on tape.) Let us say we also desire to have those stars which are within some view window described by a cone of some half-angle about the body.

Then

$$t = \frac{r \times v}{|r \times v|} \quad (15)$$

is normal to r and u.

Thus $t\cos\theta + u\sin\theta$ describe a unit vector normal to r. Let γ = the half-cone angle, then

$$s = \frac{r}{|r|} + \tan\gamma(t\cos\theta + u\sin\theta) \tag{16}$$

$$DEC(\theta) = \tan^{-1} \frac{s_3}{\sqrt{s_1^2 + s_2^2}}$$

$$RA(\theta) = \tan^{-1} \frac{s_2}{s_1} \tag{17}$$

Equation (17) gives the right ascension and declination boundaries of the stars of interest as θ is varied from $\underline{0 \rightarrow 2\pi}$. This boundary could also be used in viewing star charts to determine what magnitude stars are available without using the tapes.

Of interest also is to have the right ascension and declination of points for $\theta = 0, \pm\frac{\pi}{2}$, and π since one then could mark on the chart that direction which occurs for the maximum line of sight rate due to vehicle motion, ($\theta = \pm\frac{\pi}{2}$), and the direction for which it is zero, ($\theta = 0, \pi$).

If the 7094 tape were available it could rapidly be scanned to find those stars above a given brightness level for which the unit vector toward the stars, s_d, obeys.

$$\frac{1}{|r|} \begin{pmatrix} \cos DEC_s \cos RA_s \\ \cos DEC_s \sin RA_s \\ \sin DEC_s \end{pmatrix} \begin{pmatrix} r_1 \\ r_2 \\ r_3 \end{pmatrix} = s_d \cdot \frac{r}{|r|} \leq \cos\gamma \tag{18}$$

From this set, two stars may be selected such that

1. $\left(\bar{s}_d \times \frac{\bar{r}}{|r|} \right) \cdot u$ were a minimum

   This would be the star that lay closest to the direction of maximum rotation rate of the line of sight

2. $\left(s_d \times \frac{r}{|r|}\right) \cdot t$ were a minimum

This would be that star which lay closest to the direction of zero line of sight due to vehicle's motion.

With sextant measurements defined in this way we may obtain a criteria on how to distribute the number of measurements between the two categories in order to reduce the knowledge of the miss to a minimum.

With the previous considerations a direction of measurements is defined. We now consider calculations which can be used to indicate which body is important. Two quantities to each celestial body which should be useful in this regard are

$$\dot{\psi} = \frac{u \cdot v}{|r|} \tag{19}$$

$$\epsilon_p = \sigma_\epsilon |r| \tag{20}$$

σ_ϵ = standard deviation of angular measurement error.

The quantity, $\dot{\psi}$, given by equation (19) is the value of the maximum line of sight rate caused by the vehicle's motion. That body, of those possible having the highest average value of this quantity over a given time period is probably the most useful in determining the velocity of the vehicle.

The quantity, ϵ_p, given by equation (20) is the standard deviation of the error in position knowledge (normal to the line of sight) as a result of one angular measurement. That body, of those possible, at any particular time having the lowest value of this quantity is probably the most valuable in establishing the position of the vehicle.

The standard deviation of angular measurement error, σ_ϵ, should probably be taken in the form

$$\sigma_\epsilon = \sqrt{k_1^2 + 4k_2^2 \left(\sin^{-1} \frac{rad}{|r|}\right)^2} \tag{21}$$

where

k_1^2 = the variance of the angular error when the instrument is used for star-star measurements

k_2^2 = the variance of the angular error when the instrument is used to measure the subtended angle of the body at some low altitude

rad = radius of the body

r = range to the body center.

The error model given by equation (21) is based on the assumption that as the body is approached, its size in the field increases which causes a greater error in detecting the apparent center (or the rim). It is likely that the magnitude of the total line of sight rate should also enter in this error model due to the difficulties of tracking a moving object. This line of sight rate effect will be neglected until further knowledge indicates how the error should occur in the error model.

In addition to the time histories along a nominal trajectory of the quantities given by equations (19) and (20) the direction of the bodies of interest as a function of time should be calculated. This direction will be calculated as the right ascension and declination in ecliptic coordinates. The graphical plots of the data indicated by equations (19) and (20) and the trajectory and body direction information can be readily calculated in one computer run along the nominal trajectory. These quantities should aid in reducing the data runs of a simulated navigation system to those runs where the sequence of measurements is the most promising.

Some credibility of these statements can be made by showing they verify some information gained from other investigations. For example:

1. In-plane angle measurements need be made at a greater rate than out of plane measurements.

This is quite obviously verified by the previous material since if one is near the central body and views the central body, the direction of the maximum line of slight rate is also in the orbital plane of the vehicle about the central body. Also, one should use the nearest body since for that body

$\dot{\psi}$ is the greatest value and

$|r|\ \epsilon_p$ is its smallest value

2. If one has two trajectories passing a body then one can better determine the orbit for the tracjectory which bends more.

This is probably true for the reason that the trajectory which bends the most

a. has a higher value of the $\int \dot{\psi}\ dt$ over a given time period i.e.,

$\sum_{i=0}^{n} \dot{\psi}_i\ T$ is greater

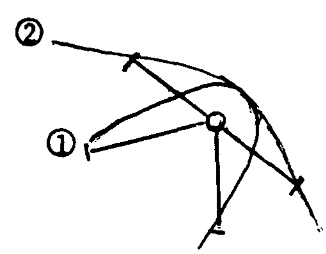
n = number of measurements

T = measurement interval

b. $\sum_{i=0}^{n} \epsilon_{pi}$ is smaller (closer to body for a greater time period)

c. the direction of the position determination by angle measurements spans a greater arc segment as shown below

As illustrated by the planes normal to the radius vectors, trajectory (1) should be determined better by angular measurements.



3. Observations of the moon in addition to the earth can be very helpful in determining the orbit in the early phases of an interplanetary mission.

This is also borne out by considering the fact that $\dot{\psi}$ for the moon could be large and ϵ_p small if it is favorably located. Also note that a high energy trajectory leaving the earth will be moving in a nearly straight line away from the earth after a short time; consequently, the value of the quantity $\dot{\psi}$ of the earth could be smaller than that of the moon even when the moon is at a greater distance.

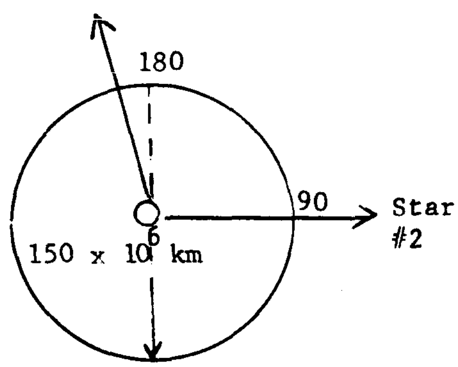
The agreement between this interpretation of relatively simple quantities and results of a more complete analysis seem to verify the need for the calculations mentioned. Further studies using these quantities as guides are certainly warranted to verify their usefulness.

3.3.3 Illustration Example

As a simple demonstration of how in-plane and out-of-plane measurements might compare let us hypothesize the following problem.

A vehicle is in a circular orbit about the sun at a radius of 150,000,000 km and has a period of 360 days. We are allowed to make measurements up to 90 days and are to predict the position at 180 days. We observe only the sun and the error in the angle measurements is a random independent variable having zero mean and standard deviation of .1 mil.

Let us hypothesize for simplicity that one star lies normal to the orbit plane and one star lies in the orbit plane.



Time of Flight Days

The out of plane measurement, star #1-vehicle-sun angle is always a constant, 90°. The variance of the error in angle knowledge as a function of the number of measurements, n, is therefore

$$\frac{\sigma_1^2}{n} = \frac{(.1 \times 10^{-3})^2}{n}$$

After 180 days, the out-of-plane error in distance is simply $\frac{15000}{\sqrt{n}}$ km where n is the number of measurements in the first 90 days.

As a result of the circular orbit assumptions, the star #2-vehicle-sun angle for the in-plane measurements obeys the equation

$$\dot{\gamma} = \text{constant}$$

or

$$\gamma(t) = \alpha_o + \dot{\gamma} t$$

If we can determine γ_o and $\dot{\gamma}$ to an accuracy as given by the covariance matrix

$$Q(0) = E\left(\begin{pmatrix} \gamma_o \\ \dot{\gamma} \end{pmatrix} \begin{pmatrix} \gamma_o & \dot{\gamma} \end{pmatrix} \right)$$

then

$$Q(180) = \begin{pmatrix} 1 & 180 \\ 0 & 1 \end{pmatrix} Q(0) \begin{pmatrix} 1 & 0 \\ 180 & 1 \end{pmatrix}$$

and $\sigma_{\gamma}^2(180) = \sigma_{\gamma_o}^2(0) + 360\, \rho \sigma_{\gamma_o} \sigma_{\dot{\gamma}}(0) + 32{,}400\, \sigma_{\dot{\gamma}}^2(0)$

If one uses the maximum likelihood estimate then the covariance matrix of the errors in the estimate is

$$(Q(0))^{-1} = A^T W^{-1} A$$

where

$$A^T = \begin{pmatrix} 1 & 1 & 1 & \\ t & t_1 & t_2 & \cdots\cdots \end{pmatrix}$$

$$W^{-1} = \frac{1}{(.1 \times 10^{-3})^2} I$$

Therefore

$$Q(0)^{-1} = \frac{1}{(.1 \times 10^{-3})^2} \begin{pmatrix} n & \Sigma t_i \\ \Sigma t_i & \Sigma t_i^2 \end{pmatrix}$$

If we let n = 10 and the time of the measurements be every ten days we find

$$\Sigma t_i = 450$$

$$\Sigma t_i^2 = 28,500$$

Thus for ten measurements

$$Q(0) = \begin{pmatrix} 10 & 450 \\ 450 & 28500 \end{pmatrix}^{-1} \times (.1 \times 10^{-3})^2$$

$$= \begin{pmatrix} \frac{28,500}{72500} & \frac{-450}{72500} \\ \frac{-450}{72500} & \frac{10}{72500} \end{pmatrix} \times (.1 \times 10^{-3})^2$$

After 10 measurements of this type the angular error at time zero has a standard deviation of .0630 mils. This error is about the same as the out-of-plane error for only 2-3 measurements.

The in-plane angular variance at 180 days is given by $\sigma_{\gamma}^2(180)$. The standard deviation of this quantity is .162 mils. This value is poorer than that given by a single out-of-plane measurement.

This simple example demonstrates the fact that measurements in the direction of maximum line of sight rate need be made at a higher frequency than those in the direction of zero line of sight rate. This guide can also help in the planning of simulation runs.

4.0 PROGRAM PLAN

4.1 Introduction

In order to make the scope of this study as large as possible, it is desirable to use the results and ideas from previous studies wherever they are applicable. Consequently, a number of studies* have been reviewed to reveal the type and scope of available data. This literature survey along with the knowledge of the program capabilities and scheduled dates of completion of the 7094 programs under development (GSFC Contract NAS5-9700) has helped to define the data which should be generated during the remainder of this study.

In summary, the previous studies which have applicable data and have been reviewed contain the following assumptions.

1) The trajectory can be defined by a conic section.

2) The error sources are random uncorrelated variables

3) The midcourse execution errors omit resolution type errors (see section 5 of reference 12).

In addition, the previous results primarily emphasized

1) Onboard navigation measurements

2) The navigation aspects of the mission. That is, with the exception of reference (2) not much data on guidance law comparisons is available.

Under the above assumptions the overall results indicate that the Interplanetary Navigation and Guidance can be performed successfully with onboard optical measurements. Measurement accuracies required are on the order of 10 seconds of arc.

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* See list of references at the end of this section.

The data presented in these studies is generally given in the form

1) Accuracy of knowledge of the miss at the target.
2) Accuracy of guidance at the target.
3) Number and magnitude of midcourse corrections.

On the basis of the material which has been reviewed to date additional study and/or data for the following areas is needed.

1) The effects of guidance execution errors on the terminal accuracy. Harry and Friedlander (reference 7) have published some data on this subject for the planet approach phase of a mission.

2) The effects of uncertainties in knowledge of physical constants and measurement errors of a bias type on the navigation and guidance accuracies. The effects of such uncertainties as the knowledge of (a) the AU, (b) gravitational constants, (c) velocity of light, (d) time of the measurements, (e) biases in the measurements have not been studied. The ability to include these factors in the study is being developed under Contract NAS5-9700. (See reference 12). The theory for treating these type uncertainties is contained in reference 1, 3, 10, and 11. Reference 11 contains some data of these effects for a lunar mission.

3) Techniques for selection of onboard measurement schedules. Battin (reference 1) suggests one technique for optimizing the schedule. Denham and Speyer (reference 13) later showed that Battin's technique did not yield an optimum schedule. The undesirable features of both the above approaches are (a) the number of stars considered for sextant type measurements must be limited; (b) the calculation time to obtain data for an interplanetary mission would be intolerable. Section 3 of this report discusses simple techniques which require additional study.

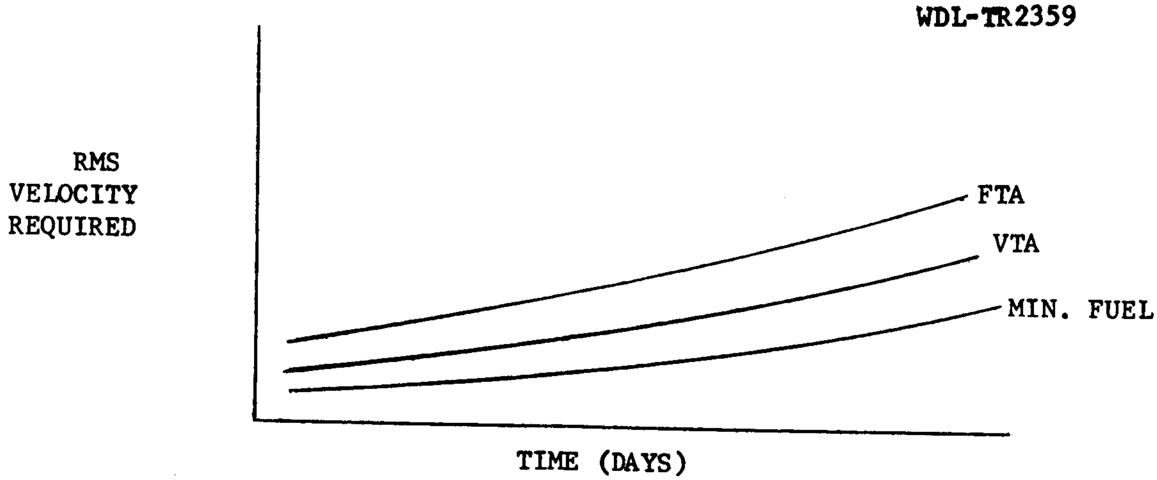


Figure (2a)

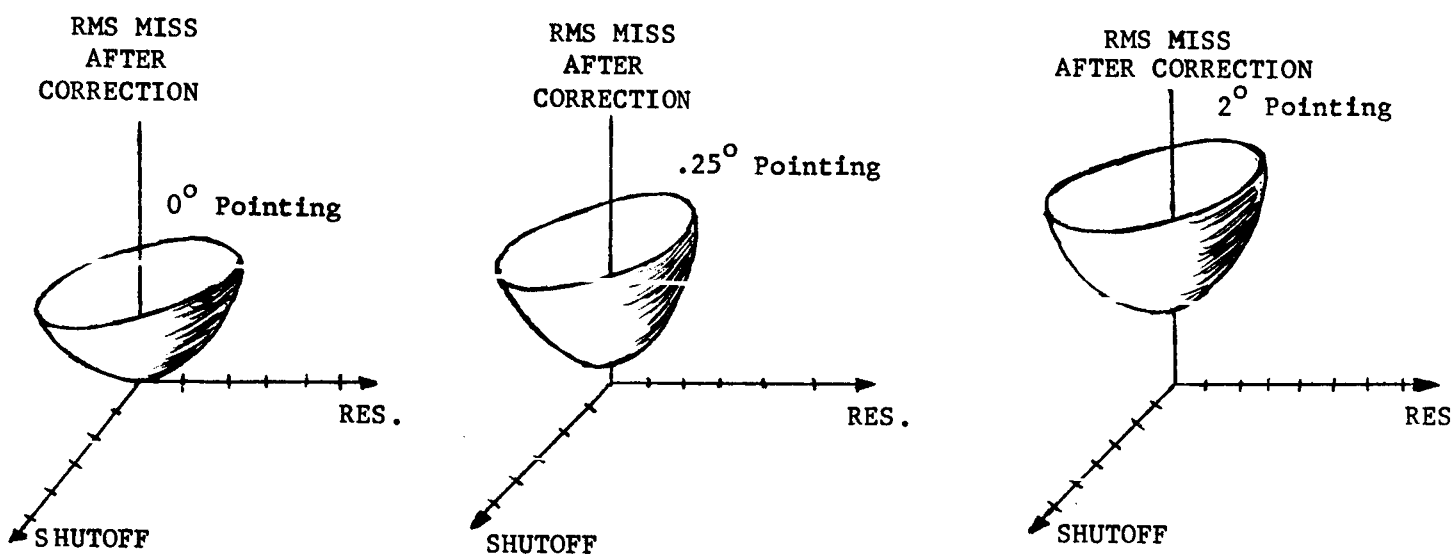


Figure (2b)

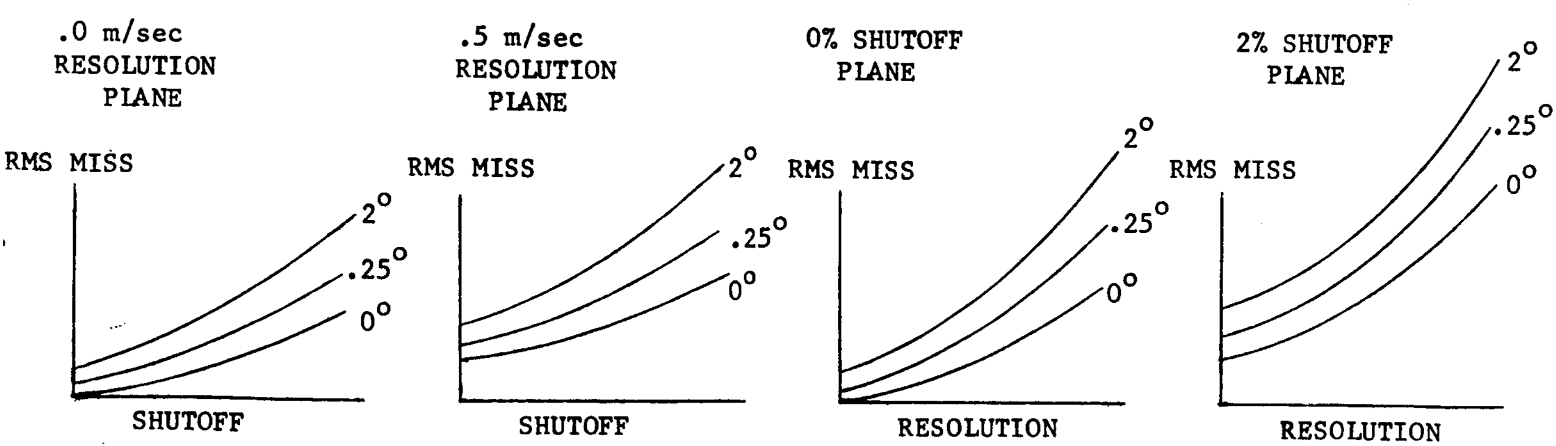


Figure (2c)

FIRST CORRECTION DATA
FIGURE 4-2

4) Data using earth-based tracking and combinations of earth-based and onboard tracking is not available. Search is continuing for information on these topics.

5) There is a lack of data on the interrelationships between the various phases of the mission. In particular, no data on the orbital phase at Mars has been found.

6) No information relating approximations in the calculations for the onboard computer to navigation and guidance errors has been found. This subject is discussed in section 5 of this report.

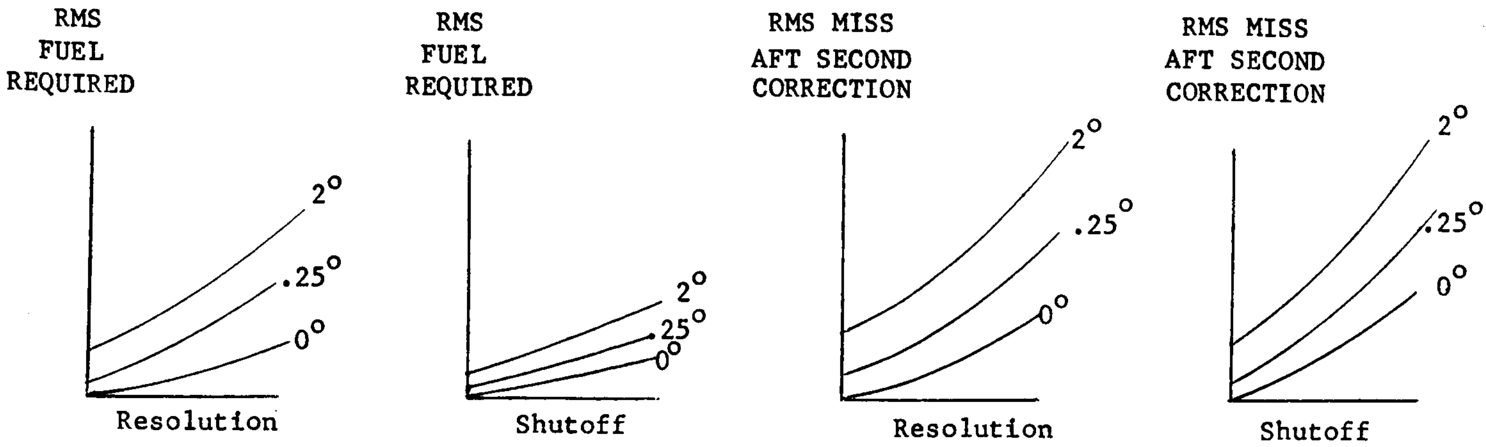
The next section outlines a schedule for those areas which can be considered during this contract. This schedule is based on the above considerations and on experience and knowledge obtained during the first four months of this contract. The practical problems associated with data generation on the IBM 7094 programs are dominating factors on the ability to maintain this schedule. The schedule will be used as a guide to insure that important areas are not neglected.

In each phase of the mission, routine data will be generated for comparison with the data in the references. The depth and direction of study in each area will be determined by the significance of the results which are first obtained.

4.2 Study Schedule

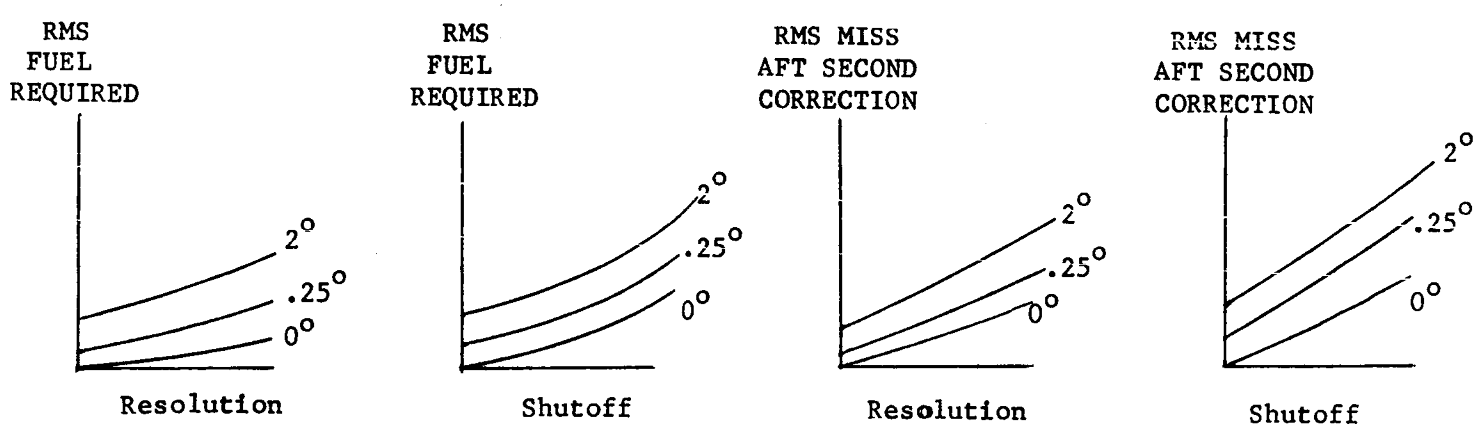
The five areas in which new or additional data will be generated are as follows:

1) Guidance Systems Analysis
2) Selection of Onboard Observation Schedule
3) Effect of Physical Model Unknowns



SECOND CORRECTION TIME #1

Figure (3a)



SECOND CORRECTION TIME #2

Figure (3b)

SECOND CORRECTION DATA

FIGURE 4-3

4) Effects of Correlated Measurement Errors

5) Comparison of Conic Section Data Versus Precision Trajectory Data

The ability to study the last three areas will be dependent on completion of certain phases of Goddard Contract NAS5-9700. The schedule reflects the expected time of completion of the programs under development.

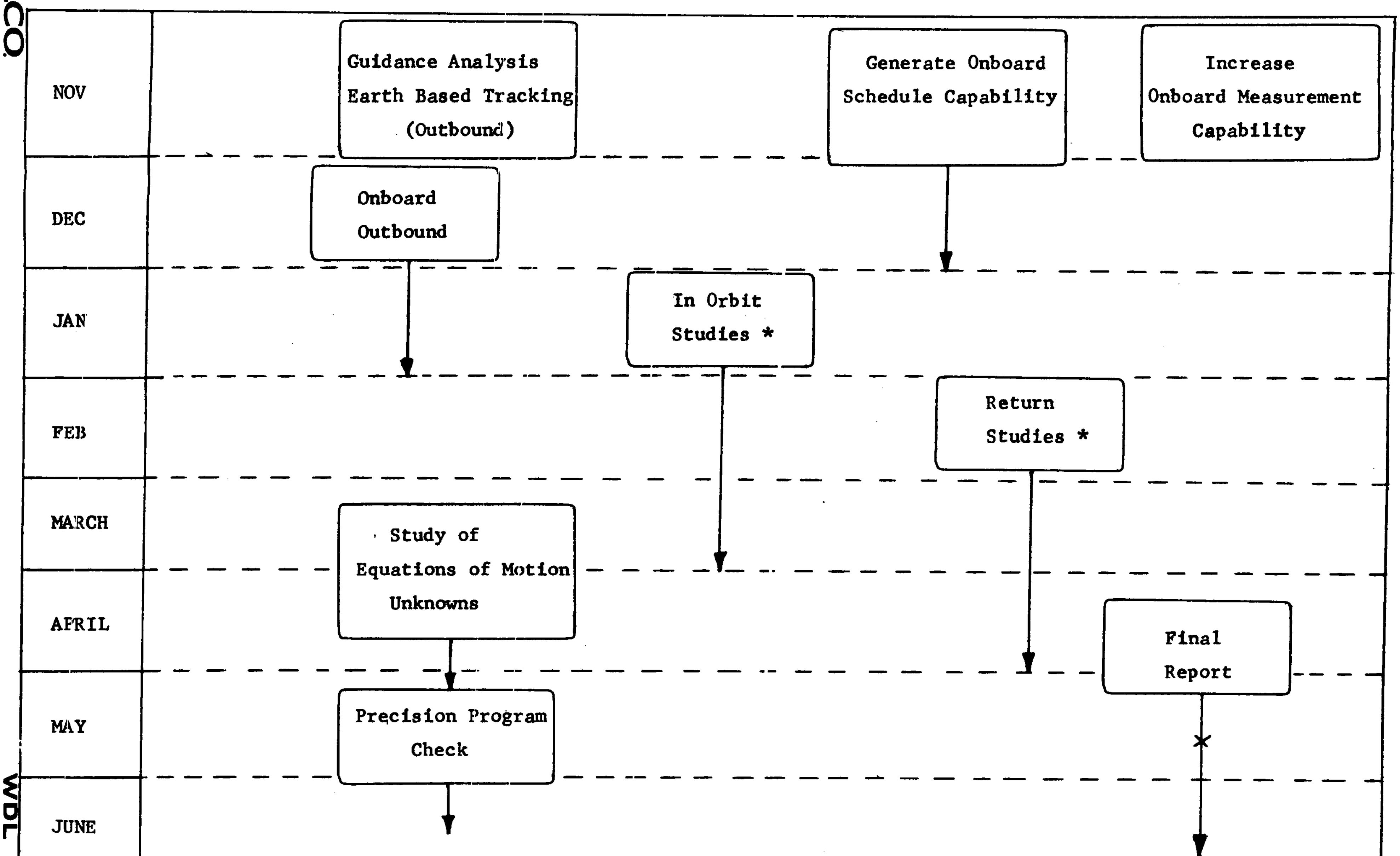
Table 1 presents a time schedule of the overall areas of effort for the remainder of the contract. Sections 4.2.1 and 4.2.2 present a detailed outline of the effort to be performed in November and December. This effort is in areas 1 and 2 of the above list.

Sections 4.2.3, 4.2.4, and 4.2.5 discuss items 3, 4, and 5 of the above list. A detailed study schedule for these efforts will be presented in a later monthly report.

4.2.1 Guidance Analysis Earth-Based Tracking

Due to the long run time required when processing all the available data, the navigation system using the DSIF tracking network will not be parametrically studied at this time. It will be assumed that there is continuous tracking early in the flight and thereafter tracking for a day every ten days. Using this type of navigation system, the onboard guidance system will be studied in terms of its error sources; pointing, shutoff, and resolution. The value of this portion of the proposed study is that it will provide parametric data concerning these three fundamental guidance system errors. The resolution type of guidance error has been included in the guidance system simulation. As mentioned previously data presented in the references were obtained either omitting or incorrectly treating the resolution type error. The first data which have been generated indicate that the resolution error is a significant factor in the error associated with making a guidance correction.

TABLE 1



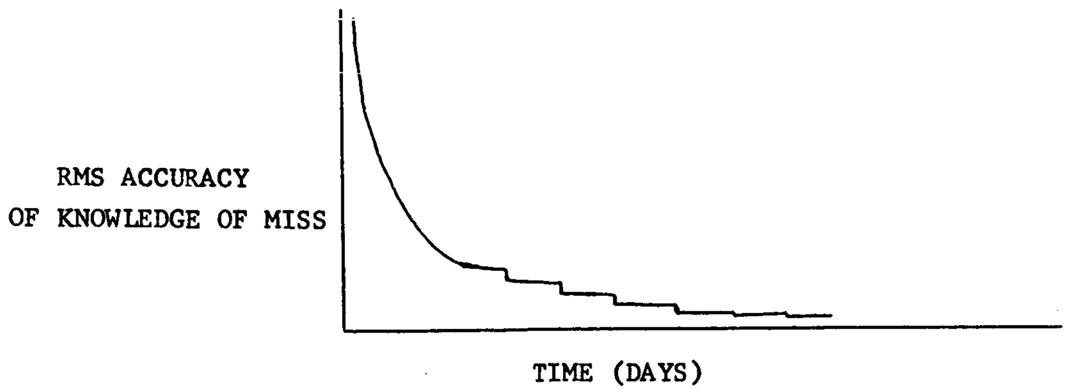
* Earth Based & Onboard Navigation

X First Submission of Final Report

The tradeoffs between guidance system errors, navigation accuracy, and fuel requirements will be studied in detail. Examples of the type of data which will be generated are illustrated in Figures 1, 2, and 3.

Figure 1 presents the knowledge of target miss as a function of time for earth-based tracking with tracking every tenth day.

Figure 2 presents data dealing with the first correction. Figure 2a illustrates the fuel required for the first correction for various guidance laws as a function of time. Figure 2 b is an illustration of a family of three dimensional surfaces which are generated by selecting various pointing errors and allowing the shutoff error and resolution error to vary. The dependent variable in the figure is shown as the RMS position miss following a correction. The series of graphs in Figure 2c represents planes which are passed through the three dimensional surfaces. The data shown in Figure 2 would be representative of a single set of conditions for the following factors; knowledge of the target miss, time of the correction, guidance law, and the injection covariance matrix. These parameters will be examined to establish their importance in the generation of the illustrated guidance system miss surface.



NAVIGATION DATA

Figure 4-1

Figure 3 presents, the same type of surface as shown in Figure 2 with the exception that the dependent variable is the RMS fuel required for the second correction and RMS target miss after the second correction. As shown by Figures 3a and 3b these surfaces will be generated for various times of the second midcourse correction. This type of data will be extended to more than 2 corrections if it appears noteworthy. These data again imply particular knowledge of miss, time of first correction, etc. These parameters will be evaluated as to their importance in any conclusions which are drawn from the data. The parameters which will be studied initially are:

A) Guidance Laws

1. Fixed time of arrival
2. Variable time of arrival (B·T, B·R, VINF)
3. Minimum Velocity (B·T, B·R,MIN FUEL)

B) Guidance System

1. Pointing Error (0^{o}-2^{o})
2. Shutoff Error (0%-2%)
3. Resolution Error (0^{m}/sec-.5^{m}/sec)

C) Times for Corrections

1. First correction
2. Second correction

The proposed presentation of data shown for earth-based tracking, at least initially, is representative of the type of presentation which will be used for the other navigation systems to be studied in succeeding months.

4.2.2 Selection of Onboard Observation Schedule

Initially, the major effort in the evaluation of the onboard navigation system will be in three areas, which are primarily concerned with increasing the capability and the flexibility of the simulation program.

The three areas of effort are outlined below.

A) Provide additional program measurement capability discussed in Section 3.

1. Range through subtended angle measurements

2. Include range and target size dependent errors in the angle measurements

3. Measurement of onboard angles in specified coordinate systems

4. Evaluate possibility of including treatment of uncertainties in the speed of light, the AU and the onboard clocktime as they affect the measurements.

B) Provide the capability in the Quick-Look Program to scan all the bodies of interest along a trajectory and establish desirable portions of the celestial sphere in which a star would be of significant value in providing navigation data. This is an implementation of the analysis presented in section 3. The data which will be generated will be used to evaluate the proposed technique for scheduling.

C) Add a program modification which will permit storage of measurement information referred to some selected time. As discussed in section 3 this will allow the change of variances of measurements, kinds of measurements, and the number of measurements along a trajectory without the need of making a separate run for each case of interest.

The first data to be generated in this area would be information which would permit selections of reasonable schedules. These data would be obtained following completion of program modifications (b) described above. These data would generate information on desirable star locations to be used in conjunction with planet measurements. It would also establish which planets are most desirable to observe as a function of time.

4.2.3 Effect of Physical Model Unknowns

The knowledge of the physical model is a factor both in navigation and guidance. In the case of navigation, the data which have been obtained from observations are fit to the equations of motion. The manner in which these data are included and weighted is dependent on the accuracy of knowledge of the physical model. The past data must in general be weighted less heavily than current data. The actual distribution of the weighting is dependent on the accuracy of knowledge of the physical constants.

The physical model is also an important factor in predicting future states. This prediction is necessary in order to determine the control action (guidance) necessary to achieve an objective at the end point.

These uncertainties in the physical model also affect certain types of measurements. For example, the uncertainties in the knowledge of the speed of light and the AU are factors which affect the treatment of onboard optical measurements. Data demonstrating the affects of some of these uncertainties will be accomplished with the aid of the computer program being developed for GSFC. The following list is the tentative plan.

1) Establish the importance of uncertainties in the following on a typical navigation system

    a. Gravitational constants
    b. AU
    c. Solar Pressure

2) Establish the influence of the uncertainty in the AU on the guidance accuracy at Mars.

3) Establish the effects of model unknowns on the transistion matrices used to generate information on the future state for guidance information.

4.2.4 Effects of Correlated Errors in the Measurements

The influence of bias type of errors in the measurements will be studied. Data which have been generated at Philco indicate that, dependent on the degree of the curvature of the trajectory, the effects of bias errors may be eliminated by the fitting of data to the equations of motion. The results of data fitting for two simple cases are shown in Figure 4. The data were treated so that the bias type of error was correlated from measurement to measurement. There were no random errors in the measurement. One case is the solution of an equation of motion of the form:

$$\dot{x} = 0$$

The second case had an equation of motion of the form:

$$\ddot{x} = kx$$

The second type of equation of motion which has an oscillatory type solution allows the bias error effect to be removed as can be seen in figure 4. The influence of possible bias type of errors in the onboard measurements or earth-based tracking may be evaluated in a similar manner along the trajectory. The influence of the bias error will be a function of the position along the trajectory. A description of the data to be generated which will determine the effects will be submitted at a later date.

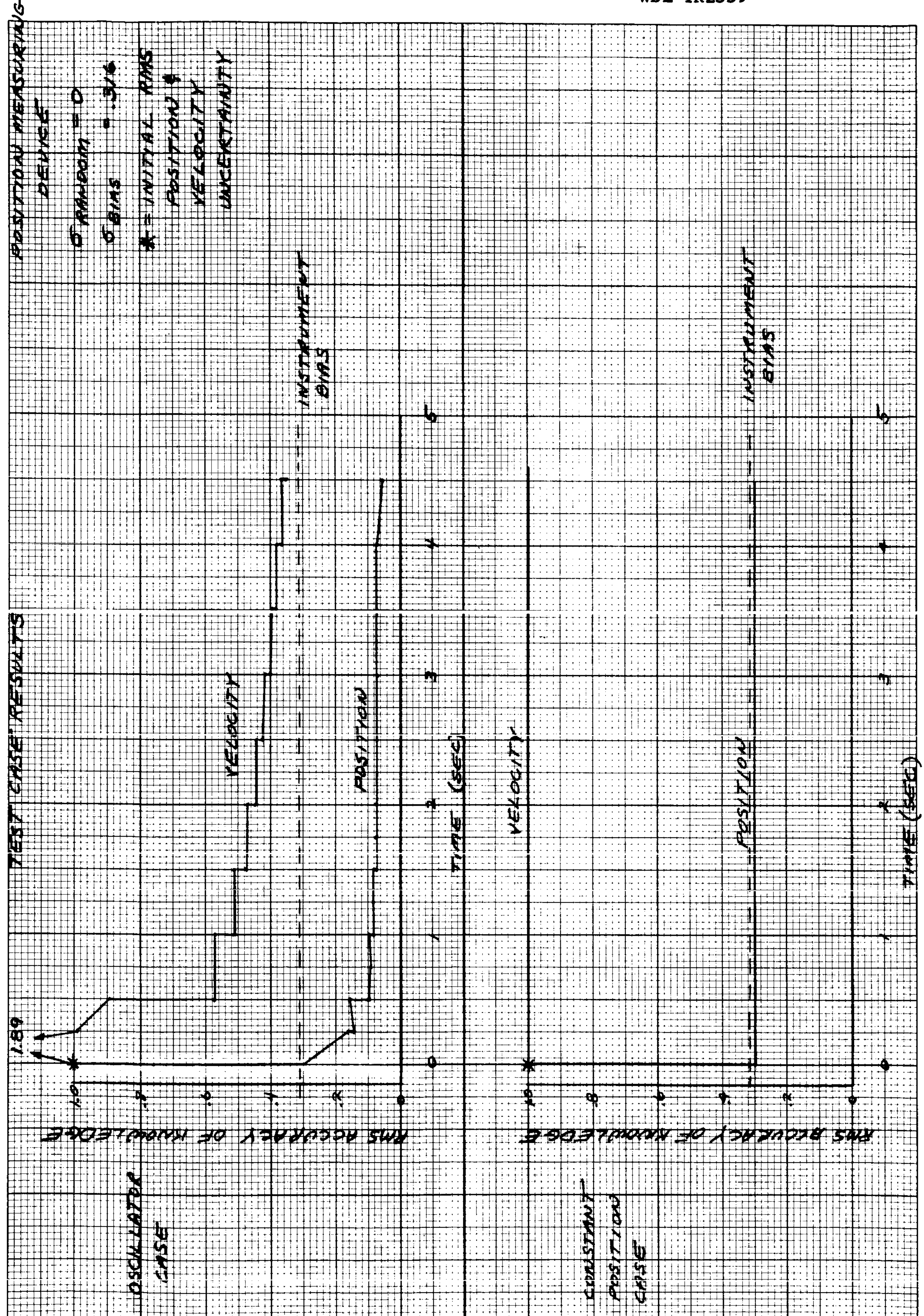

TEST CASE RESULTS
POSITION MEASURING DEVICE
σ RANDOM = 0
σ BIAS = .316
* = INITIAL RMS POSITION & VELOCITY UNCERTAINTY
OSCILLATOR CASE
RMS ACCURACY OF KNOWLEDGE
1.89
1.0
.8
.6
.4
.2
0
VELOCITY
POSITION
INSTRUMENT BIAS
TIME (SEC)
0
1
2
3
4
5
CONSTANT POSITION CASE
RMS ACCURACY OF KNOWLEDGE
VELOCITY
POSITION
INSTRUMENT BIAS
TIME (SEC)

FIGURE 9-4

4.2.5 Comparison of Conic Section Versus Precision Trajectory Data

The data generated will be primarily along conic trajectories. Therefore to indicate the degree of dependence of the data on the conic particular data of significant importance will be regenerated using the precision error propagation program.

4.3 References

1) Battin, R. H., "A Statistical Optimizing Navigation Procedure for Space Flight," ARS Journal, 1681-1696 (November 1962).

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SECTION 5

5.0 ANALYTICAL PROBLEMS RELATIVE TO ONBOARD COMPUTATION REQUIREMENTS

5.1 Introduction

The navigation (or navigation and guidance) of a vehicle may be subdivided into the following five tasks.

1) Navigation Instrumentation: The measurement of quantities which are related to the state (position or velocity, or both and other pertinent quantities of the vehicle).

2) The Determination of the State of the Vehicle: This involves the calculation of the state of the vehicle from the observations.

3) The Prediction of the Future State of the Vehicle: In general this involves the calculation of the future path of the vehicle if no changes are made in the future control action.

4) The Application of the Guidance Law: This involves the calculation of the control action required to modify the future path of the vehicle in some desired manner.

5) The Application of Control Action: This involves the execution of the control action in accordance with the guidance law.

The accuracy attainable by any navigation (or navigation and guidance system) depends on all five of these tasks. The accuracy by which task numbers 1 and 5 are performed depends to a large degree on the basic sensor accuracy. The accuracy by which tasks number 2-4 are performed depends on various analytical approximations which are made to reduce the complexity of these calculations.

When considering a manned space vehicle for interplanetary flight it would be highly desirable to make the onboard system self-sufficient, i.e., that all five of the tasks can be done onboard the vehicle. Whether this is an emergency mode or the primary mode of operation will depend on many factors which need not concern us at this time.

In order to make the onboard system self-sufficient a digital computer is required to carry out the calculations indicated by tasks 2-4. The reasons for this requirement is a result of the complexity of the calculations for the known methods of solution. Admitting the need, however, does not automatically resolve the problem since digital computers can be made in varying degrees of complexity. The complexity required in turn governs factors of importance such as reliability, weight and power.

In view of these considerations, simple calculation procedures for carrying out tasks 2-4 are important. Certain techniques for this purpose which are believed to be worthy of investigation are the subjects of this section.

5.2 Some Techniques for Simplifying the Calculations for the Determination of State Problems

Prior to describing some potential techniques for approximating the solution of the determination of state problem it is necessary to review the solutions which are in common usage. First, one must recognize that the cause of the basic computational difficulty of determination of the state lies in the fact that there are no practical onboard sensors which allow one to simultaneously measure the necessary three components of position and velocity of a spacecraft. Furthermore, those ground based radar stations which can measure six directly related quantities cannot perform to the required accuracies at very large ranges from the ground station. Thus, all methods in use today work on the principle of solving for a set of initial conditions on the states at some reference time from measurements taken over a time interval.

In the use of the onboard sextant instrument discussed in section 3, at least six measurements need be made over some time interval before a computation of both the position and velocity vectors is possible. In general one needs many more measurements than six before the accuracy of the "fix" is adequate for tasks 3 and 4. The techniques in common use today for these "smoothing of the measurements" calculations differ only in computational procedures. They all also rely on the fortunately good approximation that in the vicinity of the true solution, causes and effects obey linear relationships.

One method, commonly referred to as the "maximum likelihood" or "weighted least squares" method was discussed in section 3. The numerical implementation of this method is depicted in Figure (5-1). The actual measurements at times t are shown by the crosses. From some guess of the states of the system at time, t_o, the equations of motion are solved to yield the resultant calculated measurements as shown by curve (1). The differences between true and calculated measurements are used to find an incremental change in the states at time t_o. The process is again repeated as shown by the curve (2). After n iterations (if the process is convergent) the solution might be as shown by curve (n).

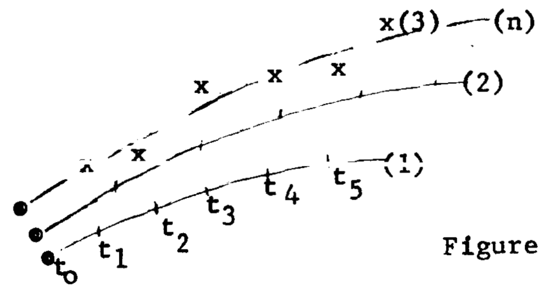

Figure 5-1

A second method is described in the reference (4) on page 2-9 . This method is sometimes referred to as the "minimum variance method." The principal difference between the two techniques is that for the minimum variance method one starts the solution by a guess of the starting state and covariance matrix of errors of this guess. Following this the technique can be implemented to work in a manner identical to that depicted in Figure (5-1) or alternatively as depicted in Figure (5-2).

As illustrated in Figure (5-2) each observation is used as it occurs to improve the estimate of the state.

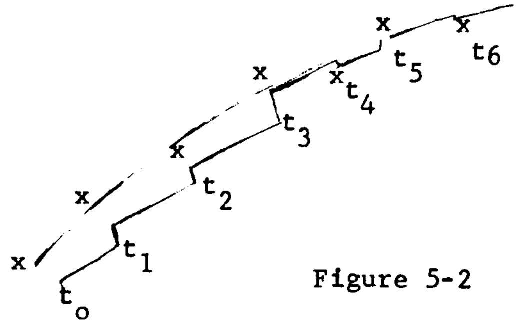

Figure 5-2

The "maximum likelihood" or "weighted least squares" methods can also be made to work in exactly the same manner as Figure (5-2). There is a problem of a 6 x 6 matrix inversion if this is done using these methods which does not occur with the minimum variance method. There is an advantage to the implementation indicated by Figure (5-2) if the time interval over which the measurements are to be smoothed is large. This is a result of the fact that if convergence occurs (i.e., the calculated solution moves toward the true solution) then as more measurements are included the estimate approaches the true solution. This tends to make the linear assumptions more valid and as a consequence iteration as shown in Figure (5-1) is not required in many instances. In order to simplify the onboard computer one area of study then is to make a detailed comparison of storage, speed, and accuracy (or significant figures) required for the two techniques and arrive at a modification of the procedures which is optimum.

A second area of study is in the details considered in the solution of the equations of motion. As an example the usage of conic segments (for the solution of the two body problem) could represent a substantial reduction in computer speed and storage requirements. Pre-calculated information stored in manuals for onboard personnel to use and perhaps input to the computer represents another technique of simplification.

5.3 Some Techniques for Simplifying Calculations for the Prediction of Future State Problem

The basic problem here is: given the solution (from the measurements) for the state of the vehicle, find the future path. The most obvious solution is to simply solve the equations of motion over the future time period of interest. Again, the detailed complexity of the equations of motion utilized dictates the computation requirements.

Known techniques which can be considered for the purpose of reducing the complexity of the calculations are similar to those given in the previous section.

1) Conic sections pieced together over the prediction time interval.

2) Pre-calculated sensitivity coefficient data relating future deviations of a family of "nominal" (pre-calculated) trajectories. The navigator could select that trajectory from the family which was closest (in some sense) to his estimated state. Stored data for this trajectory could be input to the onboard digital computer.

5.4 Some Techniques for Simplifying the Calculations of the Guidance Law.

The function of the guidance law is to calculate that control motion which will cause the future path to follow some desired plan. The normal approach is to define some desired constraints at some intermediate (or terminal) point of the trajectory which is a function of the state at that point. The calculation of deviations of the unperturbed trajectory from these desired constraints is most likely a part of the prediction problem discussed in section 5.3. Sensitivity coefficients relating control changes to constraint changes need be calculated or are part of the stored data. Given the deviations and the sensitivity coefficients, the required control action can be calculated. If the deviations are large and the sensitivity coefficients nonlinear an iteration is required in this calculation.

When considering a manned mission two modes of operation of the Guidance law are deemed desirable. These are:

1) What control action need be taken if the planned mission is to be carried through?

2) What control action need be taken for maximum survival probability in case of an emergency?

Both problems could be handled by pre-calculated data and a study could be made to find how much and what data are required.

Both problems might also be solved by the piece wise conic section approach. This method could be far superior in terms of overall reliability. This is simply because not all possible situations can be pre-calculated. Thus in the emergency mode a situation could arise wherein no course of action is available to the astronaut.

5.5 Recommended Study Action:

What is actually needed in these three areas is a detailed plan of approach. The plan would detail techniques and methods of generation of data to gain a basic understanding of the tradeoffs involved. Following the development of this plan, programs can be developed to implement the data generation.