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# Virtual Mass Technique for Computing Space Trajectories

## Final Report



MARTIN

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Final Report

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## FOREWORD

The work described in this report was performed by the Martin Company for the NASA Manned Spacecraft Center under Contract No. NAS 9-4370.

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## SUMMARY

This study has demonstrated the feasibility of the Virtual Mass Technique for computing space trajectories and has developed a FORTRAN IV digital computer program for solving the restricted three-body problem by this procedure. The virtual mass at any instant of time replaces the combined gravitational effects of all the real celestial bodies upon a spacecraft. The magnitude and location of this fictitious body, along the line of the instantaneous resultant force vector, are uniquely computed by formulas derived from the generalization of the gravispheric force center concept. The computational procedure is based upon the assumption that, over a small time interval, the spacecraft motion can be represented as a two-body conic section arc relative to the moving and varying virtual mass. In this manner the complete trajectory is computed as a series of such arcs, pieced together in a stepwise manner--updating the position and magnitude of the apparent force center at each step. Thus, the virtual mass technique is like the patched conic approximation in that no differential equations are integrated numerically. It is similar to the Cowell method in that the equations for the virtual mass are much like the acceleration contribution terms in the differential equations of motion. As the spacecraft nears one of the real physical bodies, those terms dominate the contributions of the other bodies and the effective force center approximates that real body in size and location. Finally, this technique displays a kinship to the Encke method in the computation of a reference trajectory relative to the dominant body. This dominating body, however, is the continuously moving and varying virtual mass rather than one of the physical bodies. Since the perturbing effects of all bodies are included in the determination of this apparent force center, effectively a perfect rectification is made at each step and there is no need to numerically integrate these perturbations.

A single compact computer program embodying this procedure can be controlled very simply to compute an approximate solution rapidly as a series of relatively few patched conics or a highly accurate trajectory as a large number of such arcs at the expense of proportionately longer computation time. For example, a 70. 33-hr insertion-to-pericynthion circumlunar trajectory was computed (and a large amount of output data were printed) in 160 seconds on an IBM 7094 computer. This trajectory gave the spacecraft position at pericynthion accurate to within 0. 02 naut mi and exhibited a total variation of the Jacobi energy of less than 2 parts out of $7 \times 10^6$.

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## I. INTRODUCTION

It is well known that there is no closed-form general solution for the trajectory of an infinitesimal spacecraft freely falling in the combined gravity fields of two or more large celestial bodies. Therefore, each case must be solved individually by an approximate numerical procedure. Currently, two alternative procedures are used for finding such solutions; namely, the patched conic approximation technique and the accurate numerical integration of the differential equations of motion.

The patched conic technique makes the simplifying assumption that, while the spacecraft is within the sphere of influence of any gravitating body, the motion is dominated by that large body to the complete exclusion of all others. Since the general solution to the two-body problem is known to be a Keplerian conic section, a crude approximation to the n-body solution can be computed as a series of these preintegrated conic sections, patched together appropriately at the boundaries of the spheres of influence.

The precise numerical solution of the differential equations involves rather laborious step-by-step computation procedures, based upon one of two fundamental approaches.

The straightforward method of Cowell treats all terms in the differential equations as contributors of equal importance. Most of the time, however, the acceleration experienced by the infinitesimal body is dominated by a single one of the gravitating bodies, and all other contributions are small by comparison. This requires that great care be exercised when combining all the terms so as not to lose the significance of the small contributions. This computational difficulty tends to offset the advantage of the formulational simplicity of this method.

The other basic approach (due to Encke) consists of recognizing this domination of the motion by one body and computing the trajectory in two stages. First, the position and velocity at some time (epoch) are considered to define the elements of the osculating Keplerian conic section relative to the dominant body. Then, the perturbing relative accelerations of the less influential bodies are numerically integrated, carrying comparatively few significant digits, to obtain the path correction to be applied to the basic Keplerian motion. As the magnitude of the perturbed motion grows larger, accuracy would be lost without carrying more significant figures. Instead, when this happens, the reference conic section is "rectified" to obtain new osculating elements at a later epoch, thus reducing the magnitude of the perturbations.

This procedure works well for the case where the motion is always dominated by one particular body, as is the case for the planetary motions about the sun. Spacecraft on lunar or interplanetary trajectories, on the other hand, traverse from one sphere of influence to another--falling under the domination of successively different reference bodies. During the transitions from one reference body to another, the spacecraft is literally torn between the two major attractions. Sophisticated logic is required to enable the computer to select the dominant body and to switch from one reference body to another to ensure that this computational discontinuity does not disturb the continuity of the trajectory being integrated.

This sophistication is generally considered worthwhile, for the Encke integration step size can be much larger than that of the Cowell method. The final selection of one or the other probably is more a matter of personal preference, however, since the Cowell step can be be executed much faster.

Regardless of which integration procedure is used, the solution methods described offer the choice between a very crude, rapidly computed, patched conic trajectory and a high precision comparatively slow- running integrated solution. The former type is useful for parametric studies and early mission planning purposes to determine approximate injection conditions. The latter is needed for the refinement of rough initial conditions into an accurate determination of the requirements for a specific mission. Aside from the fact that two different programs are needed, this refinement process may involve iterative computation with the accurate but slow-running program. This is due to the wide gap between the crude approximation and the precision solution and to the very high sensitivity of space trajectories to errors in initial conditions.

This report describes a unique method of computing n-body trajectories which offers, in a single digital computer program, the capability of efficiently covering the complete spectrum from rapid crude solutions to more time-consuming accurate solutions. Chapter II describes the basic concept upon which the computation is based, and Chapter III discusses various considerations which must be made in mechanizing this concept for digital computation. Chapter IV presents the quantitative results of the study of these considerations, thereby showing how these items have been implemented. Chapter V gives a general description of the computer program and complete instructions in the use of it. For the reader who is interested or who desires to make changes for his own requirements, a detailed description is given in Chapter VI, including a complete FORTRAN listing of the program.

## II. BASIC PRINCIPLES OF THE VIRTUAL MASS

The concept of the virtual mass is based upon the idea of replacing the combined gravitational effects of many large celestial bodies upon an infinitesimal spacecraft by the attraction of a single equivalent body. This fundamental idea is not new. Its natural applicability to the restricted three-body problem (two large masses and one infinitesimal mass) is described in Refs. 1 and 2. A rather arbitrary attempt was made to make a similar reduction of the n-body problem in Ref. 3. The latter consisted of singling one point out of the infinite number of possibilities along the line of the instantaneous resultant gravitational force on the vehicle. Once the location (assumed inertially fixed) was chosen, of course, the mass magnitude was determined to give the correct force. The virtual mass location and magnitude, described in this report however, are derived as the n-body generalization of the gravispheric force center associated with the restricted three-body problem. Therefore, the presentation begins with a brief review of what is already known about the restricted three-body problem and proceeds from there with the generalization to the case of more than two gravitating bodies.

### A. REVIEW OF THE RESTRICTED THREE-BODY PROBLEM IN TERMS OF THE GRAVISPHERE

Consider the simple system comprised of only two large magnitude point masses $\mu_1$ and $\mu_2$ and (by comparison) an infinitesimal mass spacecraft S. The designation of the mass by the symbol $\mu$ is intended to suggest that the real quantity of interest is the mass times the Universal Gravitation Constant. The locus of all spacecraft positions S with constant ratio $\rho$ of distances $r_{1s}$, $r_{2s}$ to the two masses is a sphere with center G on the line through $\mu_1$ and $\mu_2$ as shown in Fig. 1. Since the gravitational attraction depends only upon displacement from the mass, the ratio of the gravitational attractions is also constant on such a spherical surface; hence, it is called a gravisphere.

The gravisphere exhibits an interesting intrinsic physical property; namely that, for all points on its surface, the resultant $\vec{F}_R$ of the attractions $\vec{F}_1$, $\vec{F}_2$ of the two bodies passes through a single focal point V on the line between $\mu_1$ and $\mu_2$ as shown in Fig. 2. The location of V relative to $\mu_1$ can be shown (e.g. from relations derived in Ref. 2) to be

$$\vec{r}_{v1} = \vec{r}_{21} \frac{\dfrac{\mu_2}{r_{2s}^3}}{\dfrac{\mu_1}{r_{1s}^3} + \dfrac{\mu_2}{r_{2s}^3}}$$

where

$$\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$$

$$r_{ij} = \left| \vec{r}_{ij} \right|$$

The location of this gravispheric force center can also be expressed relative to the same frame to which the masses are referred:

$$\vec{r}_v = \vec{r}_1 + \vec{r}_{v1} = \vec{r}_1 + (\vec{r}_2 - \vec{r}_1) \frac{\dfrac{\mu_2}{r_{2s}^3}}{\dfrac{\mu_1}{r_{1s}^3} + \dfrac{\mu_2}{r_{2s}^3}}$$

or

$$\vec{r}_v = \frac{\dfrac{\mu_1 \vec{r}_1}{r_{1s}^3} + \dfrac{\mu_2 \vec{r}_2}{r_{2s}^3}}{\dfrac{\mu_1}{r_{1s}^3} + \dfrac{\mu_2}{r_{2s}^3}} \qquad \text{(II-1)}$$

The magnitude of the effective mass (times Universal Gravitation Constant) $\mu_v$ which must be concentrated at V to replace the combined effects $\vec{F}_R$ of $\mu_1$ and $\mu_2$ also can be derived from expressions given in Ref. 2 as

$$\mu_v = r_{vs}^3 \left( \frac{\mu_1}{r_{1s}^3} + \frac{\mu_2}{r_{2s}^3} \right) \qquad \text{(II-2)}$$

Note that, unlike the fixed focal point location, the gravispheric mass magnitude varies according to the radial displacement $r_{vs}$ of the point on the surface from V.

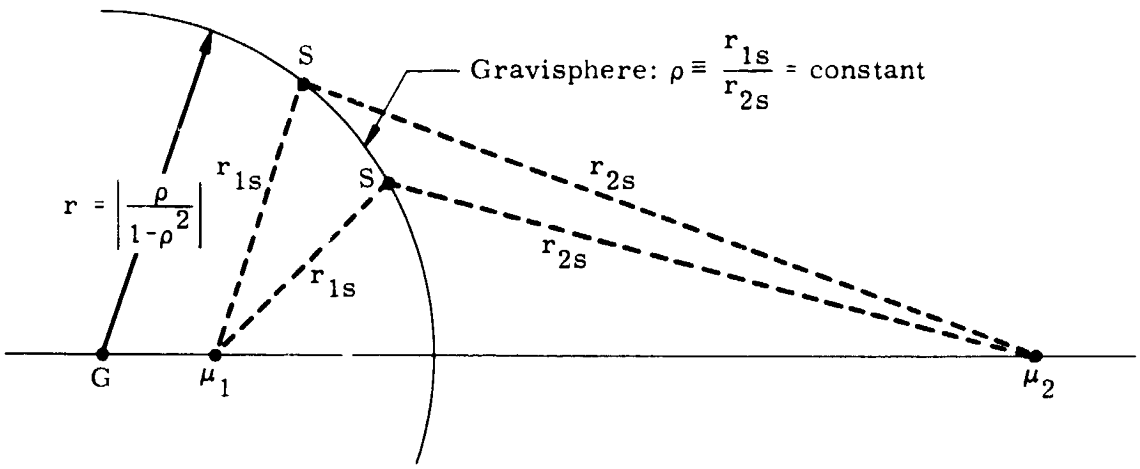


Fig. 1. Illustration of a Gravisphere

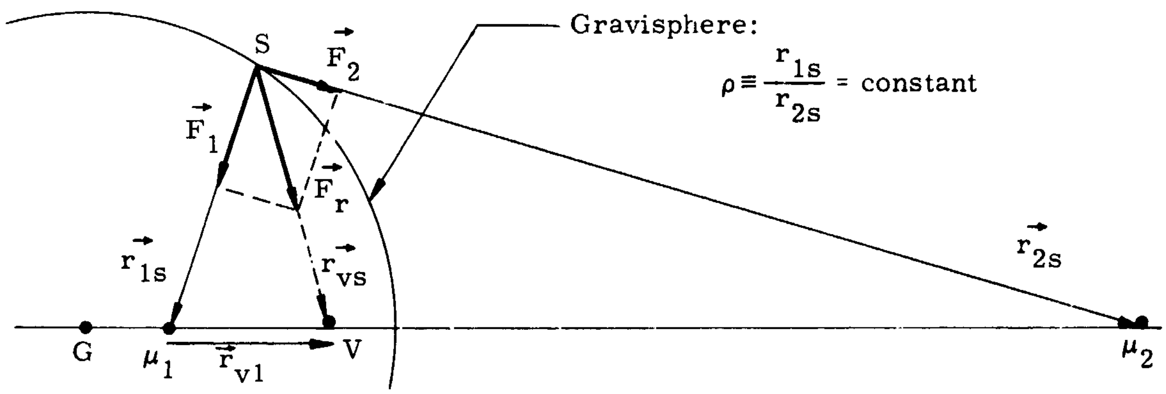


Fig. 2. The Gravispheric Force Center

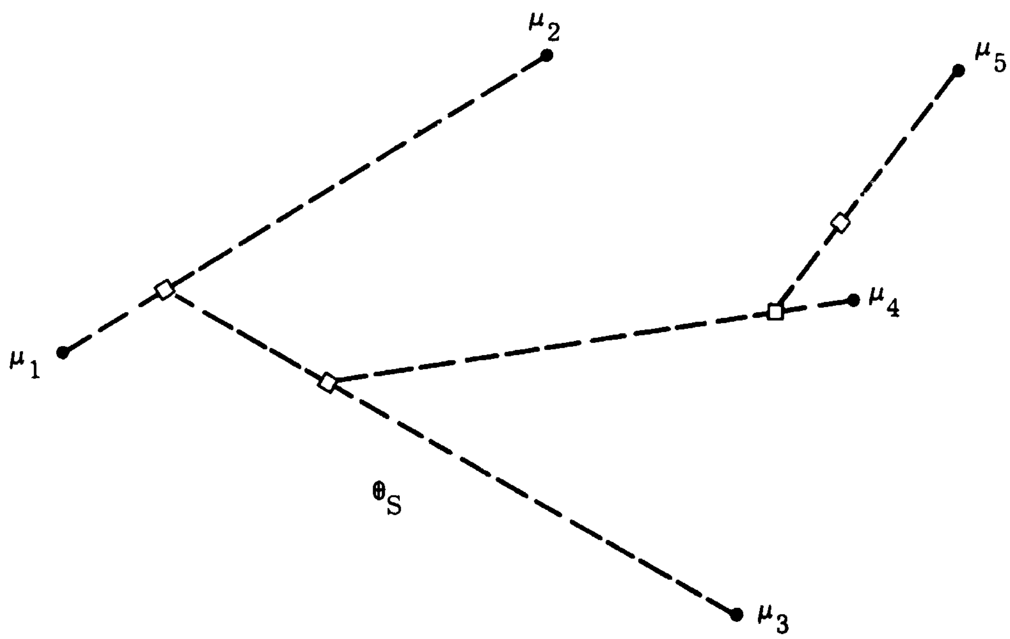


Fig. 3. Extension of Gravispheric Force Center Concept to More than Two Bodies

These considerations show how the attractions of two masses on an infinitesimal spacecraft can be reduced to the instantaneously equivalent attraction of a single mass. The magnitude and location of this equivalent mass on the line between the gravitating masses can be easily computed from equations (II-1, 2), knowing $\vec{r}_s$, $\vec{r}_1$, $\vec{r}_2$, $\mu_1$ and $\mu_2$. Observe that when the spacecraft is equidistant from both bodies $r_{1s} = r_{2s}$ and Eq (II-1) reduces to the usual expression for the center of mass. Thus, only in this case does the gravispheric force center coincide with the barycenter. (In this case the gravisphere is the plane dividing the space between $\mu_1$ and $\mu_2$.) Note also that the mass magnitude equals the total of the two real masses when the spacecraft is infinitely far displaced.

## B. GENERALIZATION OF THE GRAVISPHERIC FORCE CENTER CONCEPT TO THE CASE OF MORE THAN TWO GRAVITATING BODIES

Extension of the concept of the gravisphere itself to the case of three or more bodies is impossible. Except under very special circumstances, there simply are no surfaces of constant ratios of distances or gravitational attractions. However, now that the expressions (II-1) and (II-2) have been derived, it is no longer necessary to think in terms of these surfaces used in the derivation. The simpler condition expressed by these relations suggests the method by which the concept can be extended to n bodies. Consider the geometry sketched in Fig. 3. First select any two masses (say $\mu_1$ and $\mu_2$) and via Eqs (II-1 and 2) replace them by an equivalent mass appropriate to the spacecraft position relative to them. Now take this fictitious mass and another one of the real gravitating bodies ($\mu_3$, say) and replace these two by a new fictitious mass. Continue this process, stepping around the system, until all gravitating masses have been replaced by a single equivalent mass.

This geometric description can be expressed analytically by a straightforward application of the formulas (II-1, 2). The first step, of course, yields

$$\vec{r}_{v_{12}} = \frac{\dfrac{\mu_1 \vec{r}_1}{r_{1s}^3} + \dfrac{\mu_2 \vec{r}_2}{r_{2s}^3}}{\dfrac{\mu_1}{r_{1s}^3} + \dfrac{\mu_2}{r_{2s}^3}}$$

$$\mu_{v_{12}} = r_{v_{12}s}^{3}\left(\frac{\mu_1}{r_{1s}^{3}} + \frac{\mu_2}{r_{2s}^{3}}\right)$$

where the subscripts 12 indicate that these values obtain for masses $\mu_1$, $\mu_2$. Now again apply the basic formulas, treating $\mu_{v_{12}}$ as $\mu_1$, $\vec{r}_{v_{12}}$ as $\vec{r}_1$ and $\mu_3$ as $\mu_2$, $\vec{r}_3$ as $\vec{r}_2$:

$$\vec{r}_{v_{123}} = \frac{\dfrac{\mu_{v_{12}}\vec{r}_{v_{12}}}{r_{v_{12}s}^{3}} + \dfrac{\mu_3\vec{r}_3}{r_{3s}^{3}}}{\dfrac{\mu_{v_{12}}}{r_{v_{12}s}^{3}} + \dfrac{\mu_3}{r_{3s}^{3}}}$$

$$= \frac{\dfrac{\mu_1\vec{r}_1}{r_{1s}^{3}} + \dfrac{\mu_2\vec{r}_2}{r_{2s}^{3}} + \dfrac{\mu_3\vec{r}_3}{r_{3s}^{3}}}{\dfrac{\mu_1}{r_{1s}^{3}} + \dfrac{\mu_2}{r_{2s}^{3}} + \dfrac{\mu_3}{r_{3s}^{3}}}$$

$$\mu_{v_{123}} = r_{v_{123}s}^{3}\left(\frac{\mu_{v_{12}}}{r_{v_{12}s}^{3}} + \frac{\mu_3}{r_{3s}^{3}}\right)$$

$$= r_{v_{123}s}^{3}\left(\frac{\mu_1}{r_{1s}^{3}} + \frac{\mu_2}{r_{2s}^{3}} + \frac{\mu_3}{r_{3s}^{3}}\right)$$

With repeated application of the procedure, one gets for n gravitating bodies:

$$
\left.
\begin{aligned}
\vec{r}_v &= \frac{\vec{M}}{M_s} \\
\mu_v &= r_{vs}^{\,3}\, M_s \\
\text{where} \\
\vec{M} &= \sum_{i=1}^{n} \frac{\mu_i\, \vec{r}_i}{r_{is}^{\,3}} \\
M_s &= \sum_{i=1}^{n} \frac{\mu_i}{r_{is}^{\,3}}
\end{aligned}
\right\}
\qquad \text{(II-3)}
$$

and where

$\mu_i$ = mass of ith gravitating body (times Universal Gravitation Constant)

$\vec{r}_i$ = position of ith gravitating body

$\vec{r}_s$ = position of spacecraft

$r_{is} = \left| \vec{r}_i - \vec{r}_s \right|$

$r_{vs} = \left| \vec{r}_v - \vec{r}_s \right|$

Equations (II-3) are very simple in form and represent the generalization of the gravispheric force center for two gravitating bodies to the case of n attractive masses. Since the concept of the gravisphere itself is inappropriate for the larger number of bodies, this generalized effective force center is called the "virtual mass."

Interchanging the indices in Eqs (II-3) does not alter the numerical values of these expressions. This independence of the order in which the physical masses are taken demonstrates the uniqueness of the virtual mass.

It is a simple matter to show that these equations for the virtual mass define a fictitious body which has the same effect upon the spacecraft as the combined effects of all the real bodies. Consider the vector differential equation of motion of the spacecraft:

$$\ddot{\vec{r}}_s = \sum_{i=1}^{n} \frac{\mu_i (\vec{r}_i - \vec{r}_s)}{r_{is}^3}$$

This equation can be written as

$$\ddot{\vec{r}}_s = \sum_{i=1}^{n} \frac{\mu_i \vec{r}_i}{r_{is}^3} - \vec{r}_s \sum_{i=1}^{n} \frac{\mu_i}{r_{is}^3}$$

$$= \vec{M} - \vec{r}_s M_s$$

by Eq (II-3c, d). By Eq (II-3a, b) it becomes

$$\ddot{\vec{r}}_s = M_s (\vec{r}_v - \vec{r}_s) = \frac{\mu_v}{r_{vs}^3} \vec{r}_{vs}$$

Thus, the virtual mass acceleration of the spacecraft is identical with the acceleration by the real gravitating bodies.

Equations (II-3) can be differentiated to give the velocity and mass rate of the virtual mass as functions of the positions and velocities of the spacecraft and the gravitating bodies:

$$\dot{\vec{M}} = \sum_{i=1}^{n} \frac{\mu_i}{r_{is}^3} \left[ \dot{\vec{r}}_i - \vec{r}_i \left( \frac{V_{is}}{r_{is}} \right) \right] \qquad \text{(II-4)}$$

$$
\left.
\begin{aligned}
\dot{M}_s &= -\sum_{i=1}^{n} \frac{\mu_i}{r_{is}^3} \left( \frac{V_{is}}{r_{is}} \right) \\
\dot{\vec{r}}_v &= \frac{\dot{\vec{M}} - \vec{r}_v \dot{M}_s}{M_s} \\
\dot{\mu}_v &= \mu_v \left[ \frac{V_{vs}}{r_{vs}} + \frac{\dot{M}_s}{M_s} \right]
\end{aligned}
\right\} \qquad \text{(II-4)}
$$

where

$$
\frac{V_{is}}{r_{is}} = \frac{3\, \vec{r}_{is} \cdot \dot{\vec{r}}_{is}}{r_{is}^2}
$$

## C. THE SOLUTION TO THE N-BODY PROBLEM AS VIEWED IN THE LIGHT OF THE VIRTUAL MASS

It was shown in the preceding section that at any instant the virtual mass replaces the aggragate effect on the spacecraft of all the real gravitating bodies and thereby reduces the n-body problem to an unusual type of restricted two-body problem. This reduced problem is unusual in that the gravitating body does not remain in uniform motion but accelerates in inertial space and the mass magnitude varies. As Eqs (II-3) clearly show, whenever the spacecraft is very near to one of the real bodies (e.g., the jth one), that body's contribution to the virtual mass position and magnitude is highly favored (because of the division by the small $r_{js}^3$). In such a situation, the virtual mass is near to the dominant physical body ($\vec{r}_v \approx \vec{r}_j$) and essentially matches it in size ($\mu_v \approx \mu_j$). Slight differences occur due to the perturbing influences of the other bodies. As the trajectory carries the spacecraft far away from this real body and under the dominant influence of another one, the virtual mass continuously moves to the vicinity of the new body and grows or shrinks to nearly its mass magnitude. Thus, every spacecraft trajectory in an n-body gravity field has associated with it a separate phantom trajectory of the related virtual mass.

A simple example of this behavior is illustrated in Fig. 4. The trajectories shown are for the restricted three-body problem, where the two-dimensional circumlunar spacecraft trajectory is flown in the earth-moon orbital plane. Of course, for this case of only two gravitating bodies, the virtual mass motion is restricted along the earth-moon line. The two paths are depicted as the solid lines in an inertially oriented barycentric coordinate system. The moon trajectory is shown, however, the earth motion has been omitted to keep the curves uncluttered near the origin. Relative position lines between the virtual mass and the spacecraft are shown at several time points by the dashed lines. To the scale of the plot, the initial virtual mass displacement from the center of earth is indistinguishable. Note also that the virtual mass coincides with the barycenter at approximately 22 hr, where the spacecraft is equidistant from earth and moon. Figure 5 shows the corresponding variation of the virtual mass magnitude for this example. The abscissa is the virtual mass displacement along the earth-moon line. Time points corresponding to those appearing in Fig. 4 are spotted on the curve.

Of course, the idea is immediately suggested of using the virtual mass as a means of constructing the spacecraft n-body trajectory in a stepwise numerical procedure. Consider that the spacecraft position and velocity are given in some reference frame at some instant of time. Assume also that an ephemeris gives the positions and velocities of the gravitating bodies (of known masses) in this same reference frame. These data are sufficient to compute the virtual mass position, velocity, mass magnitude and magnitude rates from Eqs (II-3) and (II-4). Then by simple subtractions, the spacecraft position and velocity vectors can be computed relative to the virtual mass at this instant of time. If now the relative motion is computed over some increment of time, the spacecraft trajectory can be propagated and transformed back to the reference coordinate frame. The whole process can now be repeated with the new position and velocity of the vehicle at the new time.

If the virtual mass were fixed in magnitude and unaccelerated, one could compute the spacecraft relative motion over any finite arc with no error as the conic section solution to the two-body problem. The absolute motion would be exact as well for this case where the fixed magnitude virtual mass moves with constant velocity. The mass and velocity do change, however, and hence, the characterizations of the spacecraft relative motion as a conic section and of the virtual mass magnitude and velocity as constant are not exact. But this is no different from any other approximation scheme associated with the numerical integration of differential equations. The fundamental theorem of the calculus guarantees that theoretically, the errors of this approximation will vanish in the limit as the arc length (time increment) approaches zero. There is, of course, a practical limit to the accuracy which can be achieved due to the limitation of the number

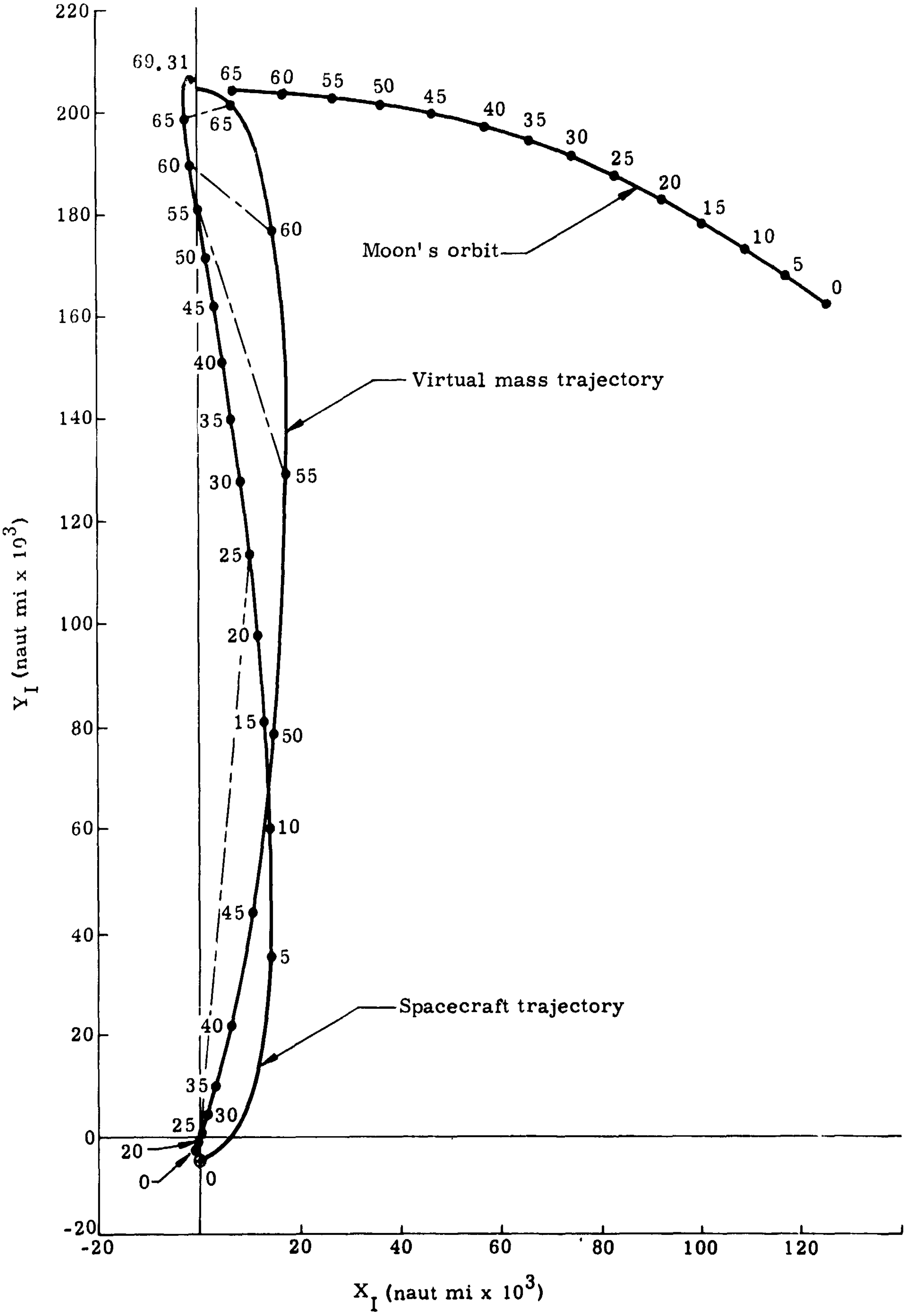


Fig. 4. Virtual Mass Trajectory for an In-Plane Spacecraft Trajectory

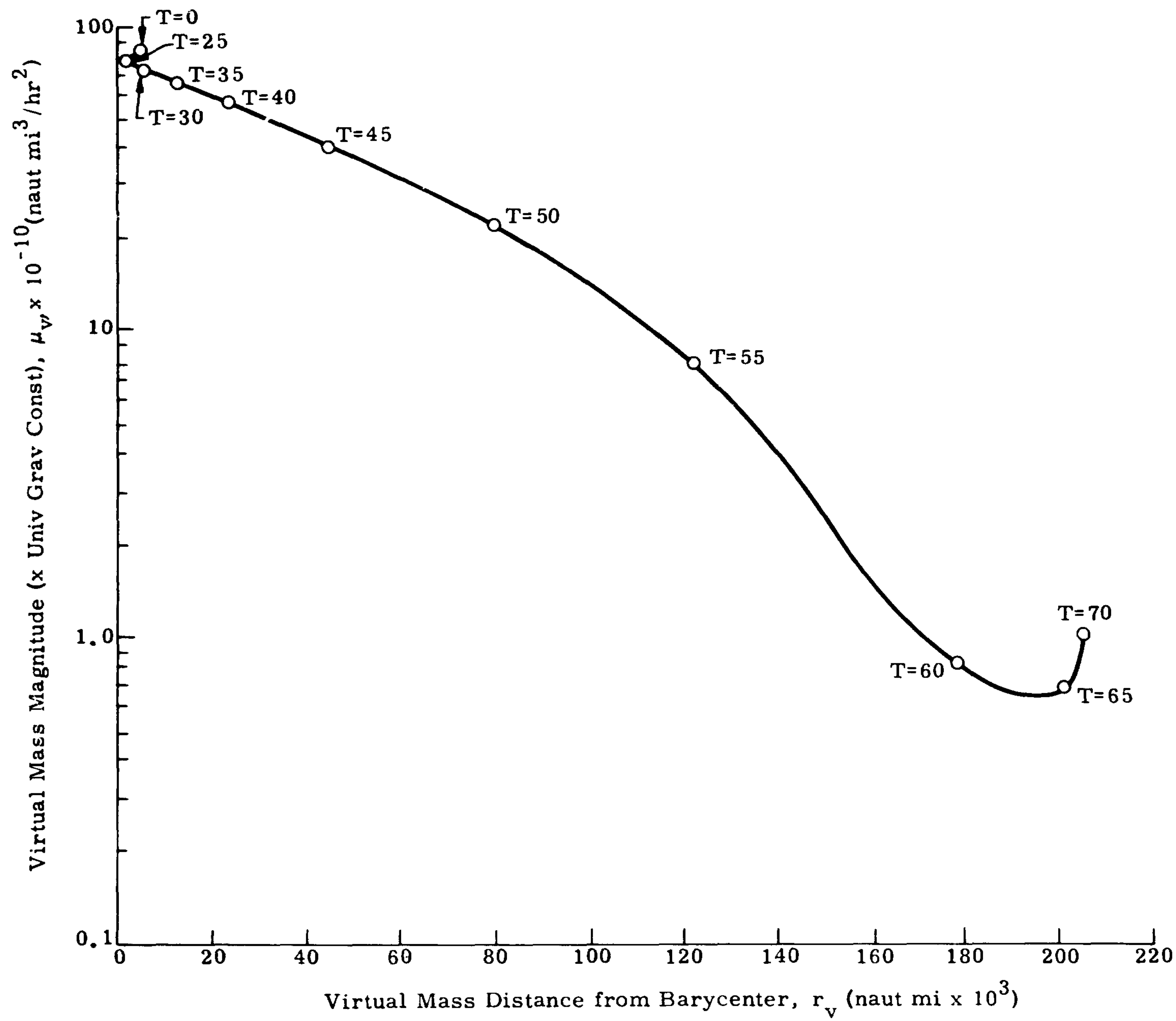


Fig. 5. Virtual Mass Magnitude Versus Distance from the Barycenter

of digits which can be carried in the computations and the length of time available to perform them. The next two chapters will treat these practical aspects of the numerical calculation.

This chapter will be concluded with some observations concerning this new procedure for solving the n-body problem. There is a similarity to the Cowell method in the procedure of adding up the attractions of all the real gravitating bodies at each computing step (see Eqs (II-3)). This summation, however, is not expressed in terms of the resultant force; but rather as the magnitude and location of a "virtual mass" which instantaneously produces identically the same resultant force on the spacecraft. It is like the Encke procedure in that a Keplerian conic section is computed relative to the virtual mass as the reference body. Of course, there are no discontinuous jumps from one reference body to another, since the virtual mass moves continuously from the vicinity of one real body to that of another as the spacecraft trajectory is dominated by successively different bodies. Since all the perturbing effects are included in the computation of the virtual mass, a perfect rectification is made at each computing interval. This then eliminates entirely the need for numerically integrating higher order acceleration perturbations. Thus, finally, the procedure is like the patched conic technique in that only preintegrated conic section solutions are pieced together.

## III. DIGITAL COMPUTATION

## FORMULATIONAL CONSIDERATIONS

It has been truly said that numerical computation is more of an art than a science. This Chapter in fact is an exposition of a primitive form of the art practiced here to implement the concepts discussed in the last Chapter in a digital computer program. Where alternative approaches and variable mechanizations are described here, they were tested and compared in the computer. The results are reported in the following Chapter.

### A. VECTOR ORBITAL ELEMENTS

A number of complications and inefficiencies would result if the computation scheme outlined in the preceding Chapter were implemented in terms of the conic section equations as generally written in polar coordinates in the plane of motion. The complications would arise in the special procedures required to handle cases of zero inclination, zero eccentricity and unity eccentricity. The principal inefficiency would manifest itself in the necessity for a large number of coordinate transformations. Each computation cycle would require a rotational transformation from the reference (ephemeris) frame to the instantaneous plane of motion, defined by the position and velocity relative to the virtual mass, and back again.

The transformations can be eliminated entirely and the other difficulties minimized by using the three-dimensional vector formulation of the two-body conic section solution. These relations will be developed here for the sake of including in this report a complete listing of the equations required for the computation.

If both sides of the vector equation of motion for the two-body problem: *

$$\ddot{\vec{r}} = \frac{-\mu \vec{r}}{r^3} \qquad \text{(III-1)}$$

are cross-multiplied by $\vec{r}$, the equation

$$\vec{r} \times \ddot{\vec{r}} = -\frac{\mu}{r^3} \vec{r} \times \vec{r} = 0$$

results.

---

*The quantities are not subscripted here for the sake of simplicity of notation. It is to be understood, nevertheless, that the spacecraft motion relative to the virtual mass is implied.

This can be integrated to obtain

$$\vec{k} = \vec{r} \times \dot{\vec{r}} \qquad \text{(III-2)}$$

The constant of integration $\vec{k}$ will be called the "kepler vector" since it obviously represents twice the areal rate. Now form the vector product of Eq (III-2) and Eq (III-1), divided by $-\mu$:

$$-\frac{1}{\mu}\,\vec{k} \times \ddot{\vec{r}} = \frac{(\vec{r} \times \dot{\vec{r}}) \times \vec{r}}{r^3}$$

It can easily be shown that the right side is $\frac{d}{dt}\left(\frac{\vec{r}}{r}\right)$ and hence this equation can be integrated to yield

$$\vec{e} = -\frac{\vec{r}}{r} - \frac{\vec{k} \times \dot{\vec{r}}}{\mu} \qquad \text{(III-3)}$$

This integration constant $\vec{e}$ will be called the "eccentricity vector." The magnitude of $\vec{e}$ is the eccentricity of the conic section and the vector points along the major axis toward periapsis.

The equation of the conic section is easily derived from Eq (III-3) by forming its inner product with $\vec{r}$:

$$\vec{e} \cdot \vec{r} = -\frac{\vec{r}}{r} \cdot \vec{r} - \frac{\vec{k} \times \dot{\vec{r}}}{\mu} \cdot \vec{r}$$

Interchanging the dot and cross in the last term on the right and substituting from Eq (III-2) gives finally

$$\vec{e} \cdot \vec{r} = -r + \frac{k^2}{\mu} \qquad \text{(III-4)}$$

Actually Eq (III-4) defines a three-dimensional surface rather than a path. The orbit is specified as the intersection of this surface with the plane normal to $\vec{k}$.

The velocity $\dot{\vec{r}}$ can easily be determined at any position $\vec{r}$ on a given orbit $\vec{k}$, $\vec{e}$. Observe first that since $\vec{k}$ is orthogonal to $\dot{\vec{r}}$:

$$\frac{\vec{k}}{k} \times \dot{\vec{r}}$$

is a vector in the plane of motion, perpendicular to the velocity vector and equal to it in magnitude. The cross product of this resulting vector by the same unit normal to the plane gives the original velocity identically:

$$\dot{\vec{r}} \equiv \left(\frac{\vec{k} \times \dot{\vec{r}}}{k}\right) \times \frac{\vec{k}}{k} = \frac{\vec{k}}{k^2} \times \left(-\vec{k} \times \dot{\vec{r}}\right)$$

Substitute for the expression in parentheses from Eq (III-3) to obtain

$$\dot{\vec{r}} = \frac{\vec{k}}{k^2} \times \mu \left(\vec{e} + \frac{\vec{r}}{r}\right) = \left(\frac{\mu}{k^2}\right) \vec{k} \times \left(\vec{e} + \frac{\vec{r}}{r}\right) \qquad \text{(III-5)}$$

$\vec{k}$ and $\vec{e}$ are completely determined in any three-dimensional coordinate system by Eqs (III-2) and (III-3), having given the position $\vec{r}$, velocity $\dot{\vec{r}}$ and central mass $\mu$. These vectors define the geometry of the orbit just as do the classical orbital elements a, e, i, $\Omega$, $\omega$. Of course, six elements are defined by the three components each of $\vec{k}$ and $\vec{e}$, but the identical satisfaction of the orthogonality condition

$$\vec{e} \cdot \vec{k} \equiv 0$$

implies that, in fact, there are only five independent elements.

The behavior of the $\vec{k}$ and $\vec{e}$ orbital elements of the spacecraft motion relative to the virtual mass is illustrated in Figs. 6 and 7 for the example circumlunar trajectory of Chapter II, Section C. Recall that, in this simple case, the motion is two-dimensional in the earth-moon orbital plane. Therefore, the $\vec{k}$ vector is everywhere orthogonal to this plane and hence its magnitude variation (shown in Fig. 7) is the only significant feature. The eccentricity vector, on the other hand, lies in the plane and varies in both magnitude and direction. Figure 6 depicts $\vec{e}$ as a series of arrows, emanating from the virtual mass focal points, pointing in the indicated directions and equal in lengths to the eccentricities appropriate to the positions.

This section is concluded with an explanation of the direct method for computing the conic section time of flight from given initial position $\vec{r}_1$ to final position $\vec{r}_2$ on a known orbit:

$$t_2 - t_1 = f\left(\vec{r}_1, \vec{r}_2, \vec{k}, \vec{e}\right)$$

No derivations are given. Known results are simply expressed in terms of the vector notation adopted here.

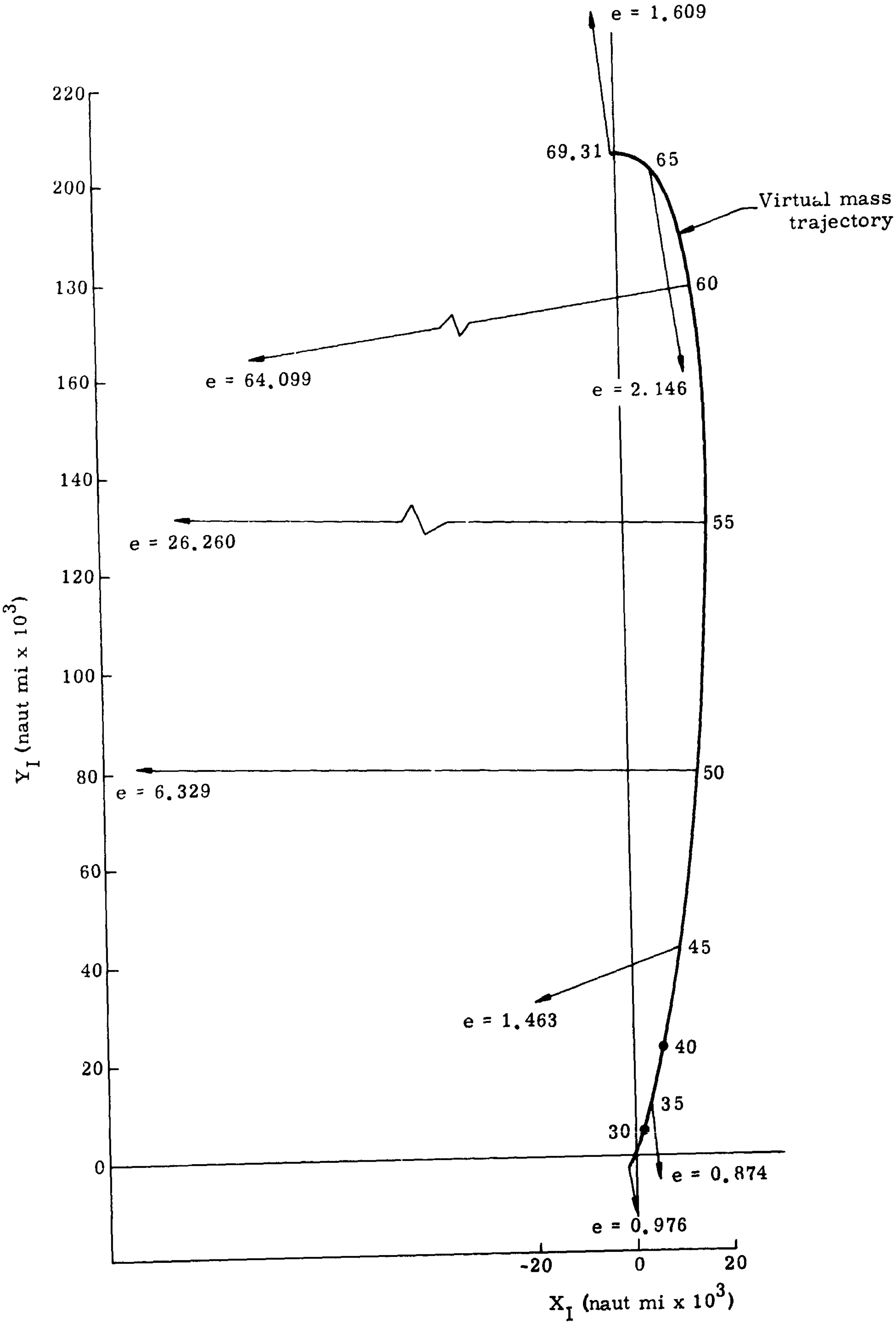


Fig. 6. Eccentricity Vector for Various Virtual Mass Positions for In-Plane Trajectory

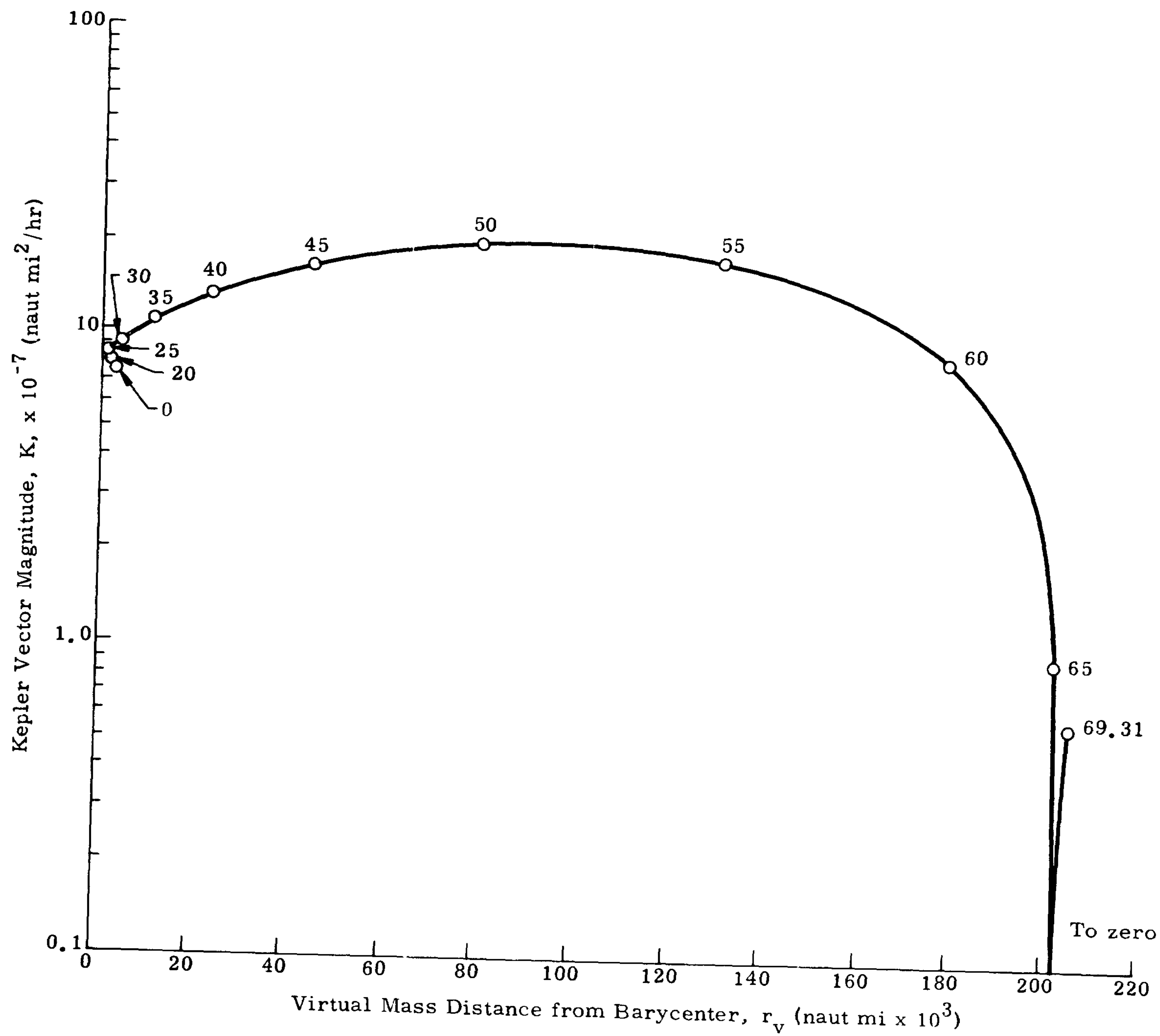


Fig. 7. Kepler Vector Magnitude Versus Virtual Mass Displacement from Barycenter for In-Plane Trajectory

Section C of this Chapter describes how to handle the inverse problem of finding the final position $\vec{r}_2$ on a given orbit, with a prescribed flight time from an initial position $\vec{r}_1$:

$$\vec{r}_2 = \vec{g}\,(\vec{r}_1,\ \vec{k}_1,\ \vec{e},\ t_2 - t_1)$$

The conic section time of flight can be computed from

$$t_2 - t_1 = \frac{M_2 - M_1}{\omega_M} \qquad \text{(III-6)}$$

In this expression, when the orbit is elliptic or hyperbolic ($e \neq 1$), M is interpreted as the mean anomaly and $\omega_M$ as the mean angular rate. For the parabolic case ($e = 1$), M is taken to be the area swept out by the radius vector as it rotates from periapsis and $\omega_M$ the (constant) areal rate. The value M can be represented in the algebraic form

$$M_i = E_i - \psi_i \qquad (i = 1,\ 2) \qquad \text{(III-7)}$$

in all cases. When $e \neq 1$, E represents the eccentric anomaly and $\psi = e \sin E$ or $e \sinh E$. In the hyperbolic case the sign of M should be reversed; but, as will be shown later, this can be accommodated in the sign of $\omega_M$. When $e = 1$, E represents the area obtained by projection of the parabolic arc normal to the major axis and $\psi$ defines the triangular area obtained by similar projection of the radius vector to the position defining the end of the arc. The parabolic triangular area is signed negatively when the true anomaly is less than 90° so that Eq (III-7) is always valid.

It remains now to show how the values of E, $\psi$, and $\omega_M$ are computed for the various cases.

First, some preliminary computations are defined. The in-plane unit normal to the major axis is

$$\left.\begin{aligned} \vec{n} &= \frac{\vec{k} \times \vec{e}}{ke} && (\vec{e} \neq 0) \\ \vec{n} &= \frac{\vec{k} \times \vec{r}_1}{k\, r_1} && (\vec{e} = 0) \end{aligned}\right\} \qquad \text{(III-8)}$$

Note that in the circular case the major axis is arbitrarily assumed along the initial position vector. The length of the semi-minor axis is

$$\left.\begin{aligned} b &= \frac{k^2}{\mu\left(\left|1 - e^2\right|\right)^{1/2}} && (e \neq 1) \\ b_i &= \frac{2}{r_i - k^2/\mu} && (e = 1) \end{aligned}\right\} \qquad \text{(III-9)}$$

The semi-minor axis is infinite in the parabolic case, hence Eq (III-9b) is written to give the reciprocal of one-half the base of the aforementioned triangular area (the denominator is $-\vec{e} \cdot \vec{r}$ by Eq (III-4)). The projection of the radius vector orthogonal to the major axis, divided by b, is simply

$$X_i = \frac{\vec{n} \cdot \vec{r}_i}{b_i} \qquad \text{(III-10)}$$

These auxiliary computations now make it easy to display the necessary values. First $\omega_M$ is given by

$$\left.\begin{aligned} \omega_M &= \frac{\mu\,(1 - e^2)}{kb} && (e \neq 1) \\ \omega_M &= \frac{k}{2} && (e = 1) \end{aligned}\right\} \qquad \text{(III-11)}$$

The first value represents the mean angular rate, the second is the areal rate. As noted earlier, $\omega_M < 0$ for hyperbolic orbits ($e > 1$). The value of $\psi$ is given by

$$\psi_i = e\, X_i \qquad \text{(III-12)}$$

in all cases. Note that when $e = 0$, $\psi_i = 0$ (or $M_i = E_i$) and that $\psi_i$ indeed is the triangular area for $e = 1$ (by Eqs (III-9b) and (III-10)). Finally, the eccentric anomaly (or parabolic arc area) E is

$$\left.\begin{aligned} E_i &= \sin^{-1} X_i && (e < 1) \\ E_i &= \frac{\left(k^2/\mu \cdot X_i\right)^3}{3} && (e = 1) \\ E_i &= \sinh^{-1} X_i && (e > 1) \end{aligned}\right\} \qquad \text{(III-13)}$$

There is no ambiguity in the hyperbolic case since the orbit is aperiodic. This is reflected in the fact that the inverse hyperbolic sine is a monotonically increasing function of the argument. The ambiguity which does exist in the periodic elliptic case can be easily resolved. When $e \neq 0$

$$\left.\begin{array}{ll} E_i = \text{principal value} = PV & \text{for } r_i \leq a \\ E_i = \pi - PV & \text{for } X_i > 0,\ r_i > a \\ E_i = -\pi - PV & \text{for } X_i < 0,\ r_i > a \end{array}\right\} \qquad \text{(III-14)}$$

When $e = 0$, the above test on $r - a$ must be replaced by a test on $\vec{r}_1 \cdot \vec{r}_2$.

Note that the time can be negative in the case where $e < 1$ and the cut $E = \pi$ (or $-\pi$) is crossed. If this should happen merely add $2\pi/\omega_M$ to the time given by Eq (III-6).

## B. NONITERATED VERSUS ITERATED COMPUTATION

The characterization of the virtual mass motion as a constant-velocity straight line and of the mass magnitude as held constant over each computing interval is dynamically consistent with the characterization of the spacecraft relative motion as a conic section. Therefore, an important problem concerning the computation is the determination of a method for establishing appropriate values of the virtual mass velocity and mass to hold constant over the interval.

The simplest approach, of course, is to merely take the values given by the virtual mass equations themselves at the beginning of the step. These values can be used, much as in the classical Euler integration scheme, to propagate the motion to the end of the interval, where new values are discontinuously assumed consistent with the new situation. This procedure is fast since just one computation (no iteration) is required per time interval. Unfortunately, accuracy suffers due to the fact that initial values, rather than mean values, are used over the step. Whereas the spacecraft trajectory itself would be continuous in this case, perhaps the most serious failing would result from the discontinuities in the virtual mass trajectory. The virtual mass position propagated to the end of an interval, for the purpose of locating the spacecraft, would not, in general, correspond to the position computed by Eq (II-3) for the start of the next interval.

If the correct position and magnitude of the virtual mass at the end of the interval were known a priori, there would be no problem whatever in establishing the required average velocity or in choosing some linearly interpolated value of the mass to hold constant:

$$\dot{\vec{r}}_{v_{av}} = \frac{\vec{r}_{v_e} - \vec{r}_{v_B}}{\Delta t}$$

$$\mu_{v_{av}} = C_1\, \mu_{v_e} + (1 - C_1)\, \mu_{v_B} \quad (0 \leq C_1 \leq 1 \text{ is a specified constant}) \qquad \text{(III-15)}$$

Since these final values are not known at the outset, but are in fact part of the answer sought, an iterative computation procedure, analogous to the modified Euler scheme, is suggested. The final values $\vec{r}_{v_e}$ and $\mu_{v_e}$ are estimated initially and then iteratively improved by computation based upon the resulting spacecraft final position. When the difference between successive values becomes acceptably small, the iteration can be discontinued and the computation can proceed to the next interval. The better the initial estimate, naturally, the faster the convergence. The method decided upon for study was to assume a second order variation with time in computing this first guess.

$$\left.\begin{aligned} \vec{r}_{v_e} &= \vec{r}_{v_B} + \dot{\vec{r}}_{v_B}(\Delta t) + \ddot{\vec{r}}_{v_{av}}(\Delta t)^2 \\ \mu_{v_e} &= \mu_{v_B} + \dot{\mu}_{v_B}(\Delta t) + \ddot{\mu}_{v_{av}}(\Delta t)^2 \end{aligned}\right\} \qquad \text{(III-16)}$$

The constant terms are given by Eqs (II-3), the linear term coefficients by Eqs (II-4). The (acceleration) coefficients of the squared terms are assumed to hold for this interval from the previous one. Thus, they would be computed as

$$\left.\begin{aligned} \ddot{\vec{r}}_{v_{av}} &= \frac{\vec{r}_{v_e} - \vec{r}_{v_B} - \dot{\vec{r}}_{v_B}(\Delta t)}{(\Delta t)^2} \\ \ddot{\mu}_{v_{av}} &= \frac{\mu_{v_e} - \mu_{v_B} - \dot{\mu}_{v_B}(\Delta t)}{(\Delta t)^2} \end{aligned}\right\} \qquad \text{(III-17)}$$

after convergence was achieved in the preceding in . rval. At the starting time step, they are set to 0. Although this iterative scheme is slightly more complicated and requires multiple looping each interval, there will be no discontinuity in the virtual mass trajectory, and accuracy should be better, for a given step size, than with the simple noniterative approach.

## C. THE COMPUTING INTERVAL

It is intuitively obvious that different computing interval sizes are required for different parts of a trajectory. Large step sizes can be used when the spacecraft is far away from a relatively constant magnitude and slowly moving virtual mass (such as would be the case during the heliocentric arc of an interplanetary mission). Small increments should be taken, however, whenever the vehicle is close to the virtual mass (as for a trajectory grazing by a planet) or whenever a sphere-of-influence crossing occurs and the virtual mass moves and changes magnitude rapidly from one dominant physical body to another.

If the step size is controlled to maintain equal increments of true anomaly in the motion relative to the virtual mass, the time increment variation will behave qualitatively as desired. The simple formula for converting the true anomaly increment into the corresponding time interval is

$$\Delta t = \frac{C_2 \, r_{vs}^2}{k} \qquad \text{(III-18)}$$

where k is the magnitude of $\vec{k}$ and where $C_2$ is an input constant defining the desired angular step size in radians. Multiplication of $C_2$ by $r_{vs}^2$, of course, converts the angle into twice the area increment. Dividing by double the instantaneous areal rate relative to the virtual mass gives (for small steps) the time to cover this angle.

A practical difficulty could arise in attempting to use Eq (III-18) as it is. Figure 7 shows that k vanishes at one point along the trajectory for the in-plane case. At this point, of course, the relative motion is directly toward or away from the virtual mass. Although k does not vanish for more general trajectories, it still can become small enough to cause Eq (III-18) to compute a very large time increment. This problem can be circumvented by replacing k by $r_{vs} V_{vs}$, the scalar product of the position and velocity magnitudes. This then represents a fictitious areal rate which assumes that the velocity is always normal to the position vector and thus, in general, is larger than the true areal rate. Substituting this into Eq (III-18) gives

$$\Delta t = \frac{C_2 r_{vs}}{V_{vs}} \qquad \text{(III-19)}$$

as the time increment. This form can give computational difficulty only when $V_{vs} \rightarrow 0$--a highly unlikely occurrence.

It is one thing to set the desired time increment, but the realization of it is another matter. Since the conic section time of flight is a transcendental form, it is not possible to invert it to determine in closed form the final position corresponding to a given flight time from a given initial position. Two alternatives are possible, depending upon whether the basic computation philosophy is noniterative or iterative (see Section B). In the noniterative approach, the final position is estimated so as to approximate the desired time. Once the estimate is made, the desired time is disregarded and the conic section time of flight equations are used to ascertain what time actually did elapse from $\vec{r}_1$ to the estimated $\vec{r}_2$. The trajectory time and ephemeris time are then updated by this true time increment in preparation for the next step.

The iterative procedure cannot be treated so simply, however, because the procedures for estimating and updating the final values of the virtual mass and for computing the average velocity all depend upon achieving a predetermined time increment with very high accuracy. A double iteration could be mechanized in which the spacecraft final position is iterated within the outer loop of the virtual mass final condition iteration. Such a procedure is cumbersome and time-consuming. It is also unnecessary if accurate initial estimates of both the spacecraft and the virtual mass final conditions can be made and then simultaneously updated within a single iteration loop. This latter course was decided upon for the mechanization of the iterated virtual mass procedure. The logical details of the technique are not of primary concern here and, hence, are deferred until Chapters V and VI. The establishment of the computation interval is the subject of interest here.

It has been shown that an accurate estimation procedure for the spacecraft final relative position is required for both the noniterative and the iterative approaches. Since this estimate, itself, will be repeatedly applied in the iterative scheme, the objective is to develop an estimation procedure which improves with each iteration.

The spacecraft and virtual mass data are known at the beginning of the interval. The virtual mass magnitude and velocity are given by Eqs (II-3) and (II-4) for the noniterated case or by Eqs (III-15) for the iterated case. The initial relative position and velocity and the mass, therefore, are trivially determined and from them, the vector orbital elements $\vec{k}$, $\vec{e}$ are obtained by Eqs (III-2) and (III-3). Equation (III-19) gives the desired step size, $\Delta t$. The final position $\vec{r}_{vs_2}$ must lie in the plane of relative motion defined by $\vec{r}_{vs_1}$, $\dot{\vec{r}}_{vs_1}$ and, hence, can be expressed as a linear combination of them:

$$\vec{r}_{vs_2} = B\left[\vec{r}_{vs_1} + (\Delta\tau)\,\dot{\vec{r}}_{vs_1}\right] \equiv B\,\vec{\sigma}_{vs_2} \qquad \text{(III-20)}$$

The geometry is illustrated in Fig. 8 and shows that $\Delta\tau$ determines the time (or true anomaly) increment and B ensures satisfaction of the orbital equation. Once $\Delta\tau$ is given, B is easily computed since Eq (III-20) must satisfy Eq (III-4):

$$B = \frac{k^2/\mu_v}{\vec{e}\cdot\vec{\sigma}_{vs_2} + \sigma_{vs_2}} \qquad \text{(III-21)}$$

The question therefore is reduced to that of relating $\Delta\tau$ to the desired $\Delta t$. As in the case of the virtual mass estimation procedure, a second order variation will be assumed. Here the constant term is 0 and the linear coefficient is 1, for it must be true that $\Delta\tau \rightarrow \Delta t$ as $\Delta t \rightarrow 0$

$$\Delta\tau = \Delta t + \kappa\,(\Delta t)^2 \qquad \text{(III-22)}$$

The procedure for evaluating $\kappa$ is similar to that used for the second order coefficients in Eq (III-16). After the computation of the conic section time of flight $\Delta t_k = t_2 - t_1$ (from Eq (III-6)) in the preceding iteration, that $\Delta t_k$ value and the $\Delta\tau$ value used to obtain it specify the exact $\kappa$ for that case as

$$\kappa = \frac{\Delta\tau - \Delta t_k}{(\Delta t_k)^2} \qquad \text{(III-23)}$$

This value will be assumed to hold for the present iteration from the last one. Clearly, this assumption gets better and better as $\Delta t_k \rightarrow \Delta t$, the desired time increment. In the noniterated case $\kappa$ is merely updated to provide the best first (and only) estimate of the next interval.

The study reported in Ref. 2 showed that, under some circumstances, the conic section time of flight may be different from the true time. In the event such a time bias might prove desirable in this case, the provision was made to cause $\Delta t_k \rightarrow C_3 \Delta t$ by rewriting Eq (III-22) as

$$\Delta \tau = C_3 \Delta t + \kappa \left[ C_3 \Delta t \right]^2 \qquad \text{(III-24)}$$

where $C_3$ is an input constant.

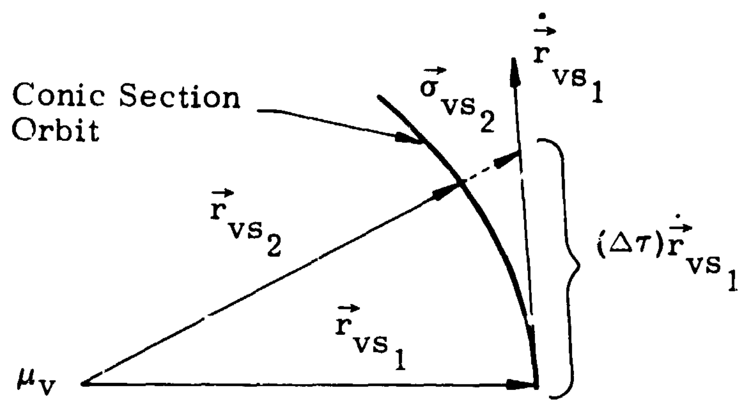


Fig. 8. Geometry of Spacecraft Final Relative Position Determination

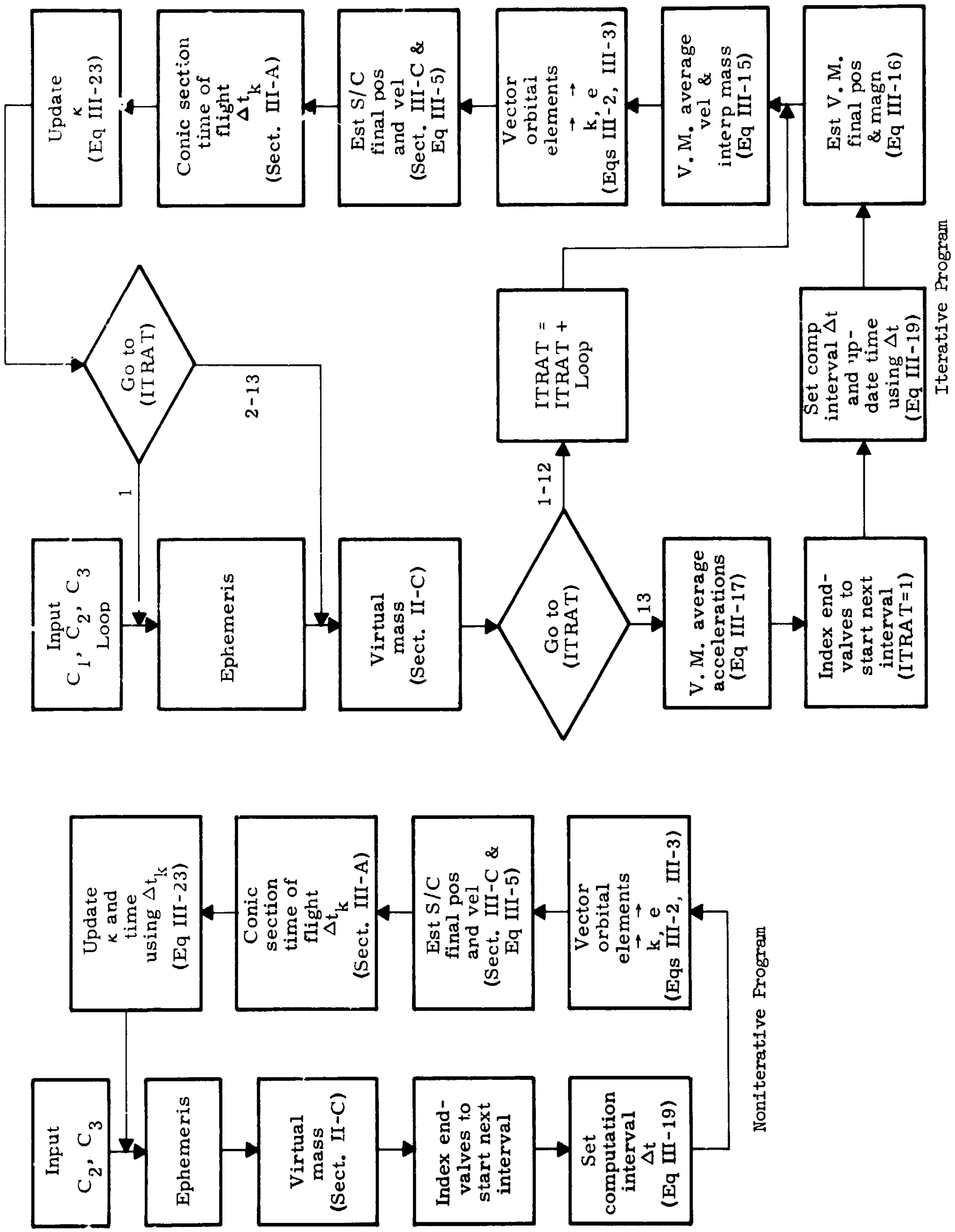


Fig. 9. Iterative and Noniterative Program Logic

# IV. DIGITAL COMPUTER STUDY

## A. APPROACH

The suitability of the virtual mass technique as a flexible integration method for the n-body problem could be assessed only by trying some numerical examples. Accordingly, the basic concept described in Chapter II was mechanized, in conformity with the considerations of Chapter III, as two separate computer programs: one a simple noniterative procedure, the other a somewhat more complicated iterative procedure. Salient features of the flow diagrams for the two programs are sketched in Fig. 9. Details such as special logic paths for starting the computation and tests for stopping conditions and printout have been omitted in order to emphasize the basic principles of operation. Reference is made on the flow diagrams to the sections of the two previous chapters where the appropriate equations may be found. Note that the iterated program loops through the ephemeris subroutine only once each computation interval. Since the desired time increment $\Delta t$ is fixed and the iteration procedure is intended to make repetitive improvement to achieve this objective, the final time and, hence, the gravitating body data are fixed. Improvement in the virtual mass data, therefore, is effected by improvement in the spacecraft final position and velocity.

The input constants provide the means by which the computations are controlled within the programs. $C_2$ sets the desired computation interval size (see Section III-C). $C_3$ biases the Keplerian flight time for values different from unity (see Section III-C). In the iterated program $C_1$ $(0 \leq C_1 \leq 1)$ linearly interpolates the virtual mass magnitude to some value between the initial and final values (see Section III-B). The constant LOOP controls the number of iterations per computing interval according to the following:

| Value of LOOP | No. of iterations (after first pass) |
|---|---|
| 12 | 1 |
| 6 | 2 |
| 4 | 3 |
| 3 | 4 |
| 2 | 6 |
| 1 | 12 |

In order to properly assess the effects of variations of the program controls and to compare the two programs with each other, an index to the accuracy of the solution is necessary. The constancy of the Jacobi integral is a necessary condition to any solution to the restricted three-body problem and, therefore, could be used for just such an accuracy index.

In addition, this case of just two gravitating bodies is the simplest n-body problem and would serve adequately as a test of the integration method. Therefore, the ephemeris subroutine was programmed for the restricted three-body problem by representing two bodies in circular orbits about their common center of mass. The expression for the Jacobi integral is classically derived in the rotating barycentric coordinate system. Since all computations for the virtual mass procedure are carried out in an inertially oriented barycentric frame, the Jacobi constant was transformed to that reference:

$$C_J = 2\left(\frac{\mu_1}{r_{1s}} + \frac{\mu_2}{r_{2s}}\right) - (\dot{x}_s)^2 - (\dot{y}_s)^2 - (\dot{z}_s)^2 - 2\,\omega\,(y_s\,\dot{x}_s - x_s\,\dot{y}_s)$$

It was recognized that some of the computations represented by the equations in Chapters II and III may involve differences between nearly equal numbers. Loss of significance in such cases can be alleviated by carrying out these computations in double precision. Rather than attempt an analysis in detail to isolate those computations where increased accuracy would be required, all computations were done in double precision on the IBM 7094 digital computer.

A circumlunar trajectory, inclined initially nearly 30° to the earth-moon plane, was chosen as the principal test trajectory. The pericynthion altitude of about 210 naut mi (lunar radius ≈ 938.5 naut mi) was reached in slightly more than 70.3 hr from insertion at earth. This trajectory is given in Chapter V as a sample problem solved by the final version of the program. All the details, including the initial conditions, the physical constants describing the earth-moon system and the trajectory time-history, appear there.

## B. RESULTS

Although a great number of exploratory studies and parametric runs had to be made, the pertinent results can be summarized quite concisely.

As expected, the iterated program was more accurate than the simple noniterated one. A direct comparison of the two is shown in Fig. 10 in terms of the Jacobi energy accuracy index. The gains controlling the computing interval size, $C_2 = 0.0005$, and the time bias, $C_3$, were set to the same values for the two programs. The curve shows the difference between the Jacobi energies at corresponding time points on the test trajectory as computed by the two programs. This method of presentation was chosen because, although the gains selected caused the iterated program to compute with high accuracy, there was a small variation of the Jacobi energy. The difference shown in the curve of Fig. 10 shows how much worse the variation was using the noniterative

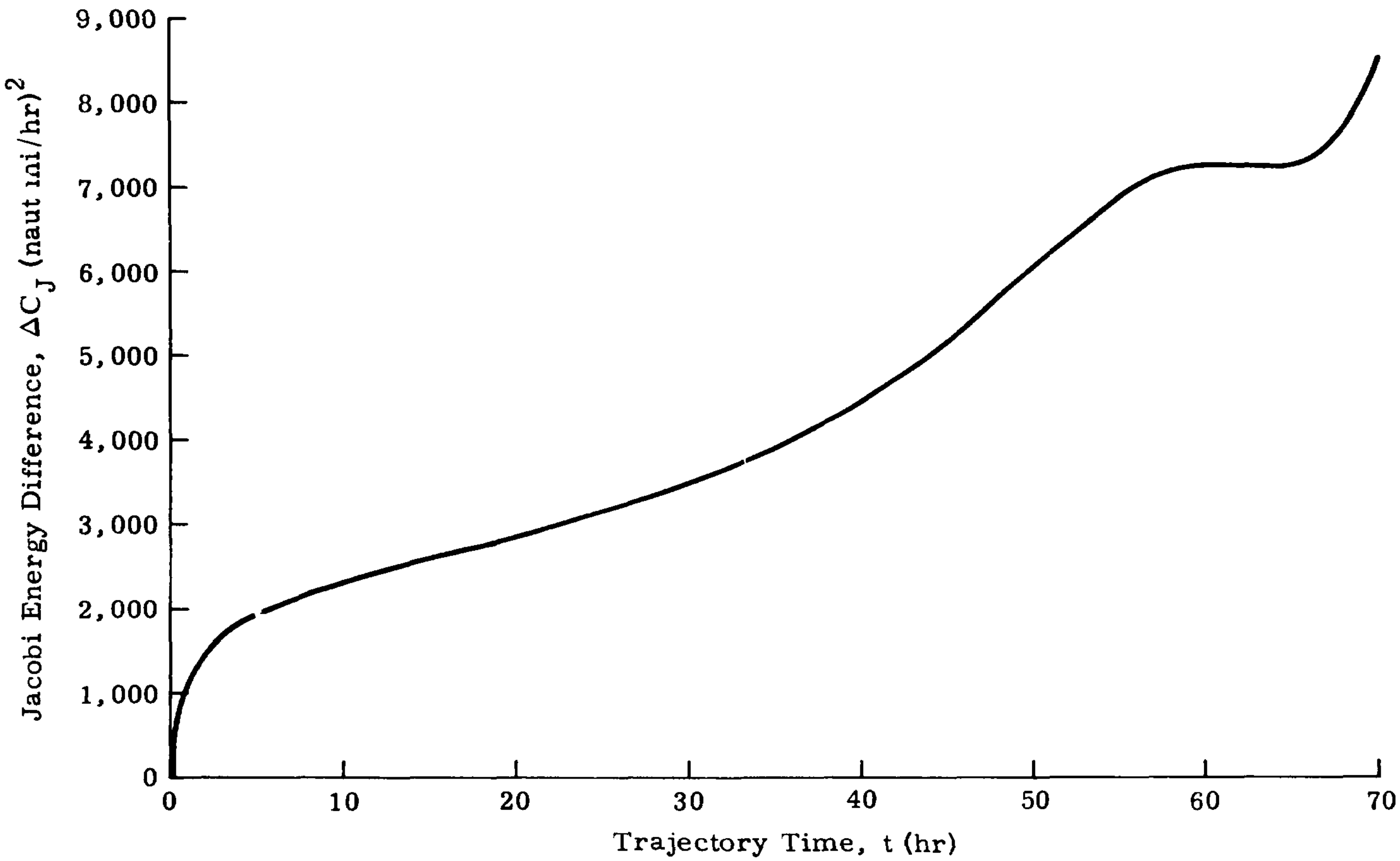


Fig. 10. Variation of Noniterated Trajectory Jacobi Energy from Iterated Solution for Same Gain $C_2$ on Computation Interval Size

program. The maximum difference was an appreciable 9000 (naut mi/hr)$^2$ out of 7033989.7388 (naut mi/hr)$^2$. As will become more apparent in the other studies of the iterative program, the superior performance of the more sophisticated method well justifies the slight additional complication. Without it, the calculation of precision trajectories would be impractical.

Restricting attention, therefore, to the iterative program, the first question to be resolved was that of the number of iterations required per computing interval. With any reasonable gain $C_2$ on the computing interval size, it was found that just one extra loop (after the first pass) was sufficient to produce answers which, to at least eight significant figures, were identical with those for more repetitions. Therefore, the final program has been fixed to loop the first time through the computations, including an access to the ephemeris subroutine, and then just one additional iteration (by-passing the ephemeris).

The studies of the interpolated virtual mass value and the conic section time bias are summarized in Figs. 11 and 12. For both of these comparisons, a base run was made with the mass interpolated at the midpoint ($C_1 = 0.5$) and no time bias ($C_3 = 1.0$). The curve of Fig. 11 was generated as the time-history of the differences in the Jacobi energy between the base run and two others in which $C_1 = 0.4$ and 0.6. The incremental error buildup portrayed by this graph clearly shows that the best mass value to use is the arithmetic mean between the initial and final values. Similarly, the incremental error curve of Fig. 12 was generated as deviations from the base run of two time biases of $C_3 =$ 0.99999 and 1.00001. This curve shows the extreme sensitivity of the solution accuracy to this parameter and indicates that no bias ($C_3 = 1.0$) is best. On the basis of these results, the final version of the program is coded to calculate the virtual mass magnitude as the simple average of the end-values and to achieve an unbiased match between the desired and the conic section time increments.

The constant $C_2$, since it controls the computation interval size, is the basic accuracy selector. Final comparative studies of this parameter were made with the other program controls set to the optimized values as described above. The variation of the Jacobi energy with time along the test trajectory is shown in Fig. 13 for various gains $C_2$. A value of $C_2 \leq 0.001$ maintains the maximum deviation to less than 2 (naut mi/hr)$^2$ out of the 7033989.7388 (naut mi/hr)$^2$ initial value. The discontinuity in the curves at approximately 38.9 hr is due to a computational inaccuracy which occurs as a result of the fact that the computing interval spans the apocenter relative to the virtual mass. As noted, the discussion of the time of flight in Chapter III Section A, such an occurrence

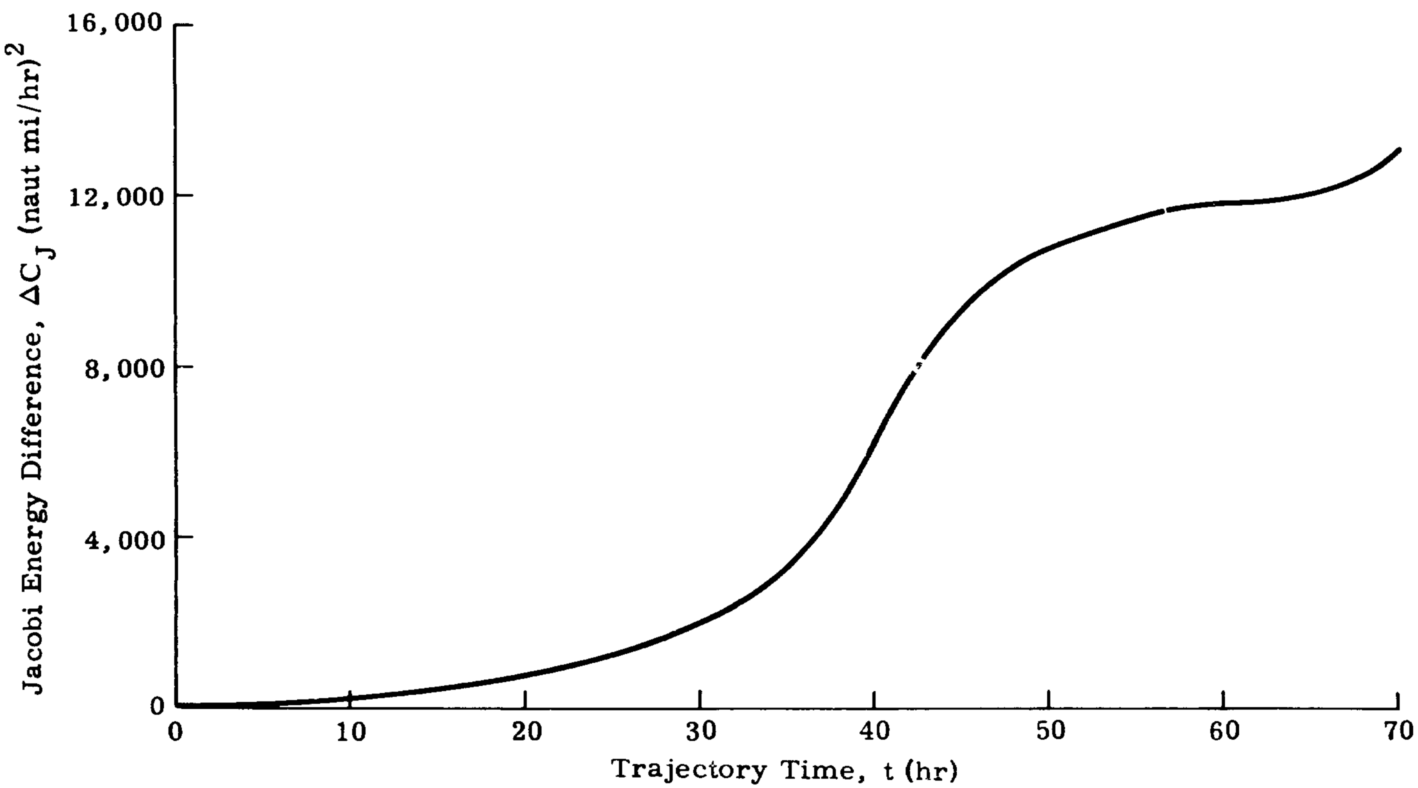


Fig. 11. Variation of Jacobi Energy Due to Change of Virtual Mass Interpolated Magnitude away from Midpoint by $\Delta C_1 = 0.1$ ($C_J = 7033989.7$)

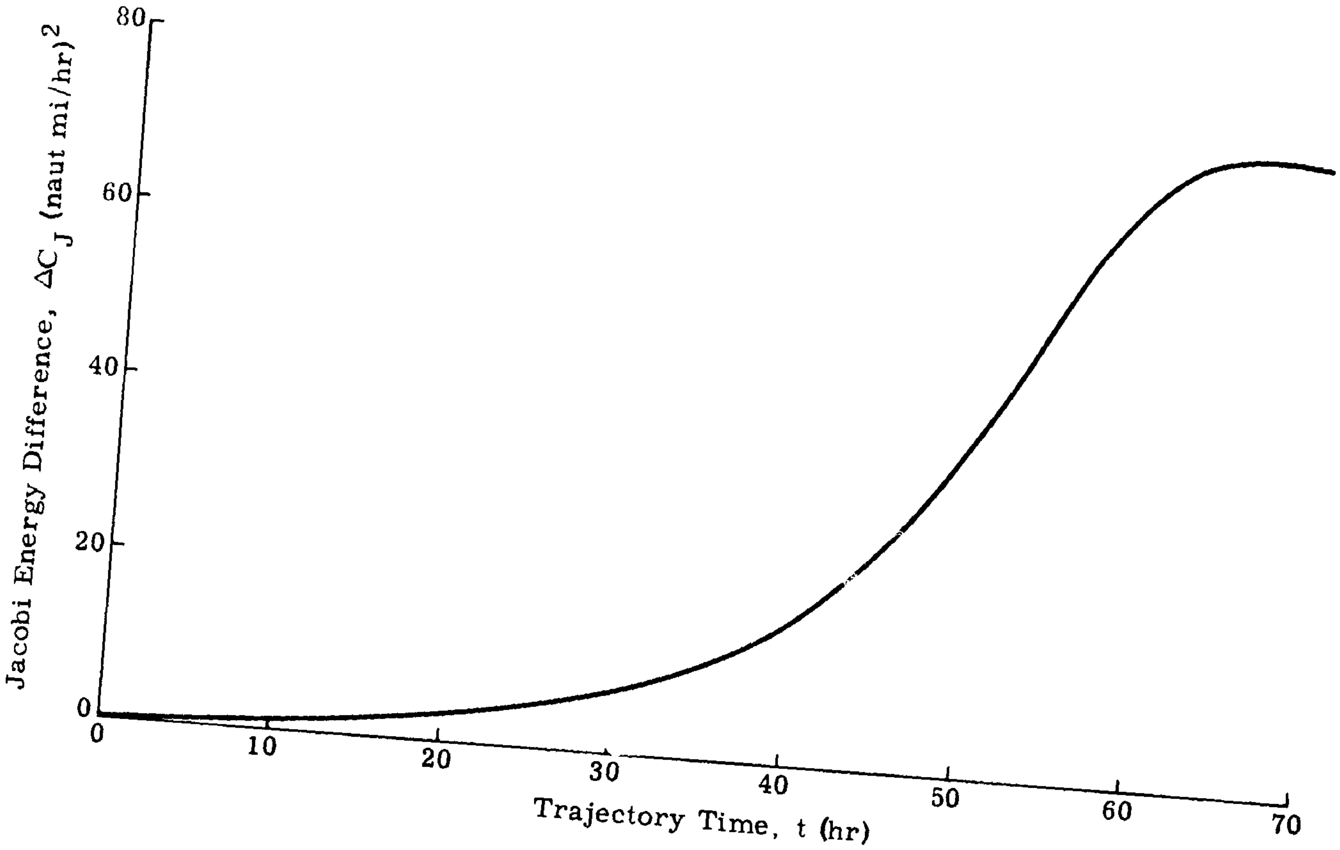


Fig. 12. Variation of Jacobi Energy Due to Time Bias of $|\Delta C_3| = 0.00001$ ($C_J = 7033989.7$)

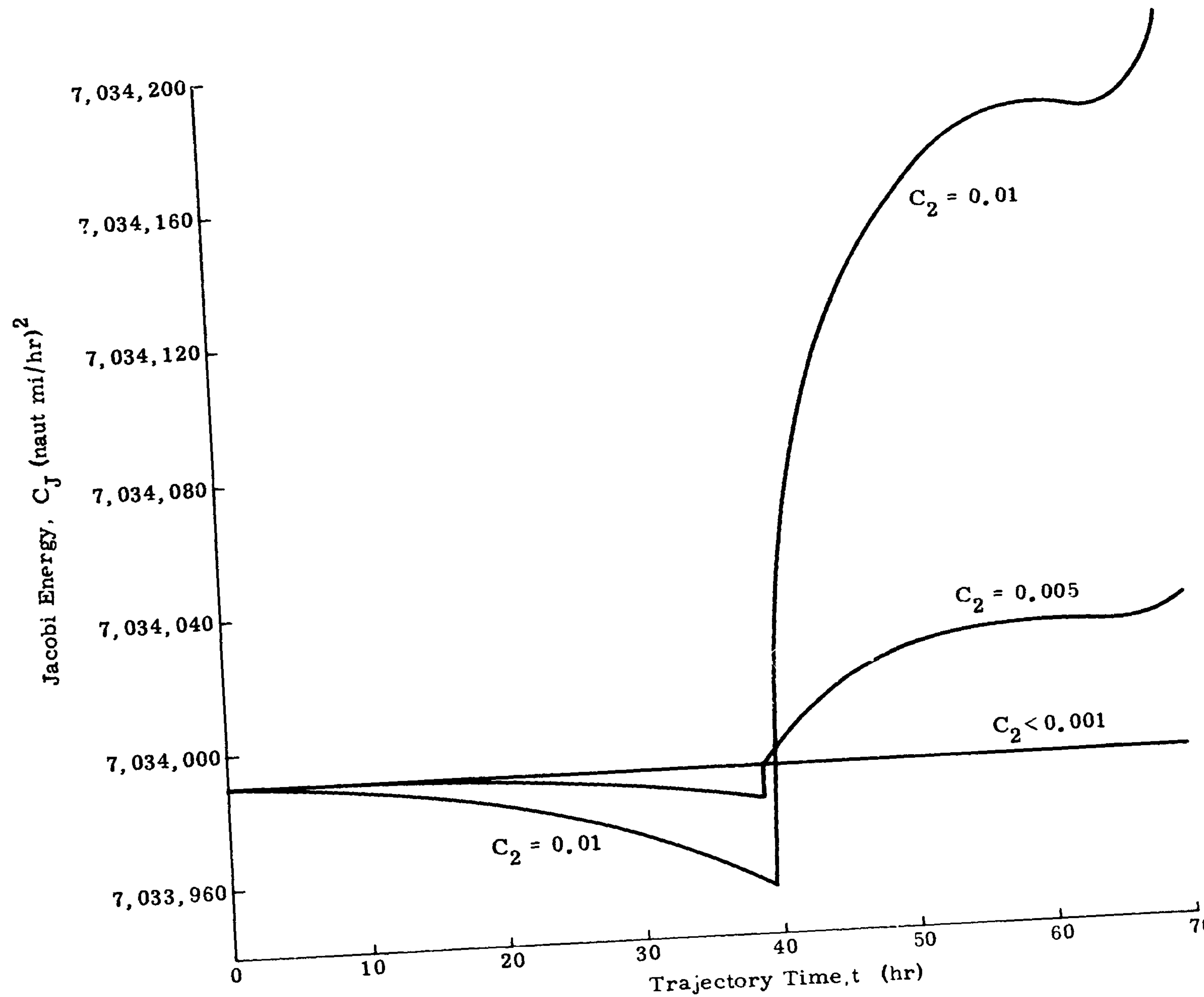


Fig. 13. Jacobi Energy Variation with Computation Interval Gain $C_2$

would cause the formulas to compute a large negative value for the time of flight. The program was provided with the capability to theoretically correct this mistake by adding a time equal to one period of the orbit. Unfortunately, some loss of significance occurs due to the subtraction of two nearly equal numbers. Apparently, the resulting inaccuracy is enough to cause the Jacobi energy variation indicated. This difficulty should be less serious for a machine such as the CDC 3600 than as shown here for the IBM 7094. The former carries 20 digits in double precision and also has automatic rounding, whereas the latter carries only 16 digits and simply truncates.

To gain some insight into what these variations in the Jacobi energy mean in terms of the spacecraft positional deviations, the differences $\Delta r$ were computed between the base run with $C_2 = 0.001$ and other trajectories run with gains $C_2 > 0.001$. These differences, divided by the magnitude of the spacecraft position vector from the barycenter, are shown plotted in Fig. 14. They show, as an example, that a gain of $C_2 = 0.005$ gives a positional displacement of $\Delta r = 0.307$ naut mi for a total position vector distance of $r = 197299.51$ naut mi at 65.0 hr (just prior to the pericynthion at 70.338787 hr). Figure 13 shows that this same trajectory showed a Jacobi energy variation of $\Delta C_J = 44.66$ $(\text{naut mi/hr})^2$ at this time.

An independent check of this Jacobi energy versus position deviation correspondence was made by comparing the same base run with identically the same trajectory numerically integrated by a standard procedure in an entirely different computer program. The integration tolerance happened to be set in that program so that the resulting solution displayed a Jacobi energy variation at 65 hr which was very nearly the same as that noted above (for the $C_2 = 0.005$ case). The positional difference between the numerically integrated trajectory and the base run was also approximately the same $\Delta r = 0.384$ naut mi. Thus, it is concluded that the Jacobi energy does provide a good index to accuracy and that the base run apparently is an accurate solution to the problem.

Of course, the price of accuracy is computation time. The run with gain $C_2 = 0.005$ calculated the trajectory in 2369 increments and took 57 sec on the computer. This time was measured for a complete problem cycle, from the time the instruction was given to read the input data until the program again sought data for the next problem.

Program accuracy control by means of the constant $C_2$ would require some study on the part of the user to determine what value to use to obtain a certain accuracy and to estimate the expected running time on the computer. The problem has been simplified somewhat by utilizing some

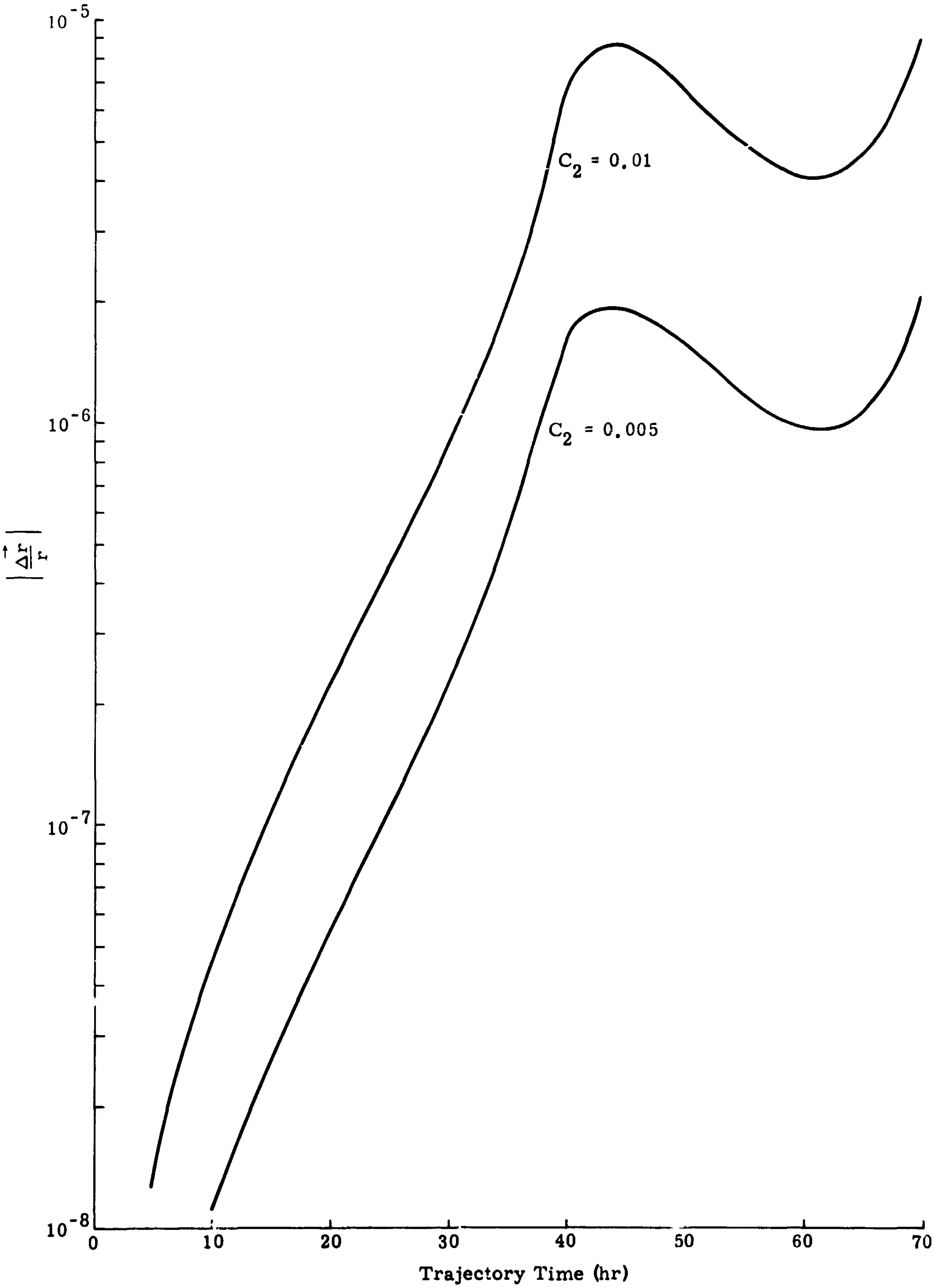


Fig. 14. Spacecraft Position Vector Error as a Function of Time for Various Computing Step Size Gains, $C_2$

information obtained from a cross-plot of Fig. 14. Since the maximum error occurs at the 70-hr point near pericynthion, a plot of $C_2$ versus $\Delta r/r$ at 70 hr was made as shown in Fig. 15. This curve was approximated by a second degree polynomial fit and built into the initialization section of the program. Thus, the user can input the more intuitively meaningful number $\Delta r/r$, or fractional accuracy desired at pericynthion, and the program will internally set its own gain appropriately.

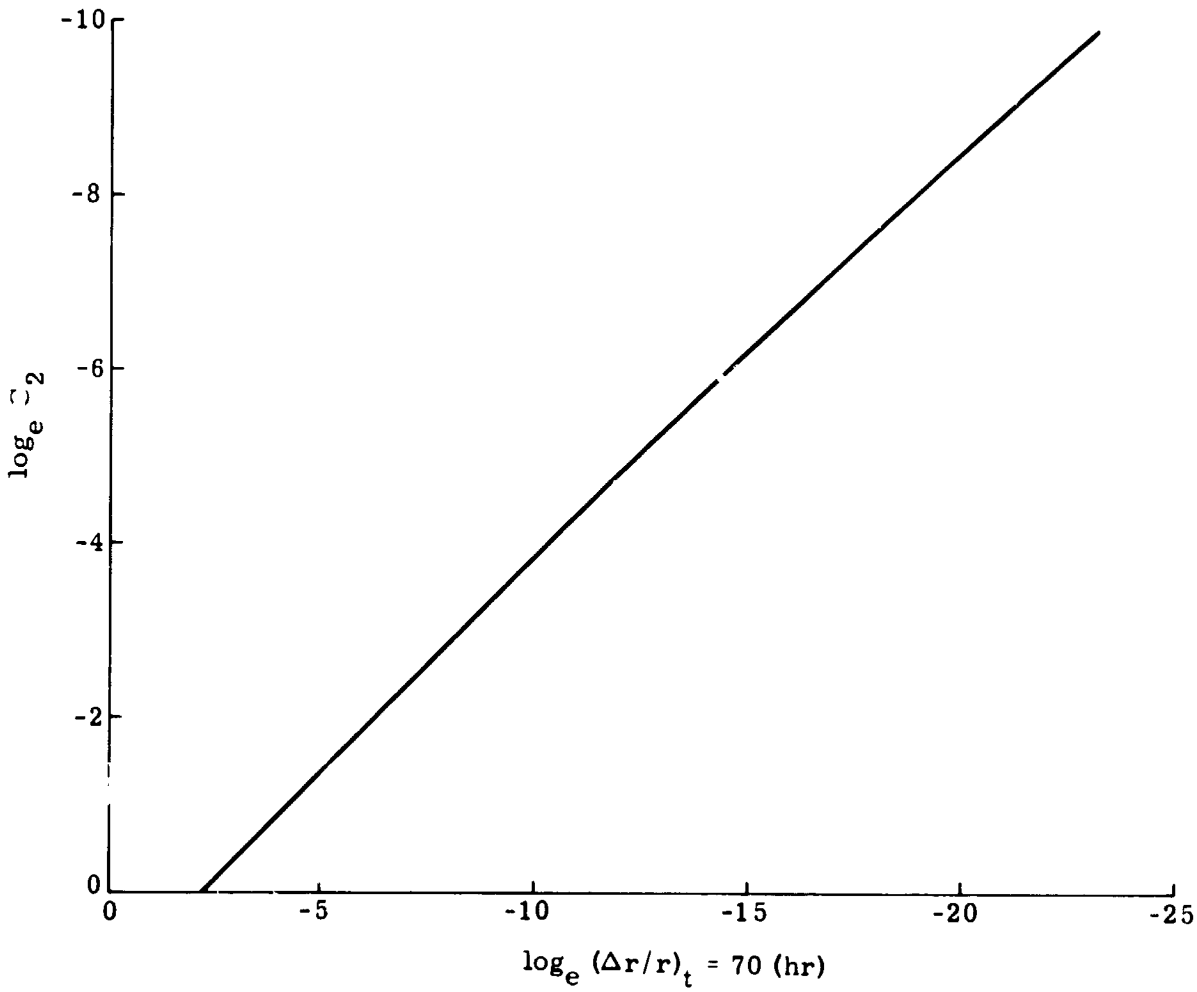


Fig. 15. Correlation of Computing Interval Gain $C_2$ with Position Error at t = 70 (hr)

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# V. GENERAL DESCRIPTION AND USE OF THE VIRTUAL MASS PROGRAM FOR COMPUTING SPACE TRAJECTORIES

This chapter is intended to serve two purposes.

(1) It can be used, independently of the rest of this report, by the mission analyst. Usually he is not so much concerned about the computation process or the implementation of the procedure. Instead he is more interested in what general problem is solved, what is the solution accuracy and how can he use this digital computer program to calculate trajectories.

(2) It also provides a broad overview of the digital program and its use, uncluttered by details. Thus it serves as an introduction to the trajectory analyst who may be interested in the details given in Chapter VI.

## A. GENERAL DESCRIPTION

The purpose of this contract was to investigate the feasibility of the technique of computing space trajectories as a series of conic section solutions relative to a moving and varying virtual mass. At every instant of time this virtual mass replaces the combined effects of all the gravitating bodies upon the spacecraft. As explained in the preceding chapter, this was done by testing the procedure in a digital computer program. The simplest n-body problem, the restricted three-body problem, was used for this purpose. The final version of this FORTRAN IV source program is delivered with this report in fulfillment of the obligations under the contract.

Although this program solves only the restricted three-body problem, the virtual mass subroutine was formulated in completely general form. Thus, the only changes required in the program to make it capable of computing trajectories for more than two gravitating bodies are to replace the ephemeris and the input and output subroutines.

The reference coordinate system is a set of inertially oriented axes, centered at the barycenter. The xy plane is the earth-moon orbital plane and the initial position of the moon (and hence the earth) is given in terms of the ephemeris time $t_{eph}$, or time since the moon crossed the positive x-axis. Specification of the earth-moon distance D, the angular rate $\omega$ and the ratio of moon mass to total system mass $\mu$ complete the description of the gravitational environment. The total mass of the system (times the Universal Gravitation Constant) is computed internally in the program as

$$\mu_e + \mu_m = \omega^2 D^3 \qquad \text{(V-1)}$$

to ensure dynamical consistency of the system. The spacecraft initial position and velocity components are specified in the reference barycentric coordinate system.

To permit the greatest freedom possible in the use of the program and to scale numbers for greatest computational accuracy, all calculations within the program are carried out using dimensionless quantities. Thus, no units are specified (except that $\omega$ is presumed given in degrees per unit time) and the user may input the necessary data in any system of units he chooses. The value of $\omega^{-1}$ is chosen as the unit of time and D as the unit of length. Thus, conversion factors

$\omega^{-1}$ = time

D = length

$\omega D$ = velocity

$\omega^2 D^3$ = mass times Universal Gravitation Constant

$\omega^3 D^3$ = mass rate

etc.

are used to nondimensionalize input data and to convert dimensionless computed values to output data in the same system of units as the input.

The program has been discussed here in terms of the earth-moon system but, of course, is applicable to any two bodies (e. g., sun and a planet). The input constants determine the details of the system. It is only necessary to remember that the ephemeris time gives the time elapsed since the body designated by mass ratio $\mu$ ($0 < \mu < 1$) crossed the x-axis.

As described in Chapter IV, Section B, the accuracy of the solution is controlled by an input number $\Delta r/r$. This quantity represents the allowable error (nondimensionalized by D) in spacecraft position at the pericynthion point for an earth-moon trajectory. The program automatically converts this number to an equivalent gain controlling the computing interval size to maintain roughly equal steps in true anomaly relative to the virtual mass (see Chapter III, Section C). The gain correspondence was established for the lunar trajectory and it is not known at present what performance could be expected for an interplanetary trajectory (i. e., one which does not pass close to the large body).

The computing interval is adjusted as a print time is approached to cause the printout to occur exactly at a specified time increment. A similar adjustment is made whenever a major axis (pericenter or apocenter) crossing is imminent. The simple logic for this latter adjustment is not adequate for all cases. The pericenter crossings are picked up rather consistently except when such a crossing occurs shortly after the initial point (this is the case for the test trajectory selected) and a very loose gain is used. In this situation the program steps over this region before it has a chance to anticipate it. The apocenter crossing (which occurs at about 38.9 hr for the test trajectory) is only rarely caught by the routine provided.

Three stopping conditions are specified as input data, and the problem will terminate on whichever condition is met first. The conditions are a maximum allowable trajectory time and an impact with either of the two gravitating bodies. The radii of the two bodies must be given.

## B. INPUT AND OUTPUT

A series of from 2 to 7 cards (1 to 6 data cards and 1 problem card) is used to input the data for a given problem. A number of problems may be run consecutively by inputting a sequence of such series of cards. Formats for the 7 cards are given below. The control word on each card begins in Column 1 and must appear exactly as specified. The variables begin in the columns indicated and must be punched according to the standard FORTRAN formats supplied (D18.0 indicates a double precision numeric field of 18 columns and I1 indicates an integer field of 1 column).

Data Card 1--

| Col. 1 | Col. 9 (D18.0) | Col. 27 (D18.0) | Col. 45 (D18.0) | Col. 63 (D18.0) |
|---|---|---|---|---|
| POSITION | $t$ | $x_s$ | $y_s$ | $z_s$ |

Data Card 2--

| Col. 1 | Col. 9 (D18.0) | Col. 27 (D18.0) | Col. 45 (D18.0) |
|---|---|---|---|
| VELOCITY | $\dot{x}_s$ | $\dot{y}_s$ | $\dot{z}_s$ |

Data Card 3--

| Col. 1 | Col. 9 (D18. 0) | Col. 27 (D18. 0) | Col. 45 (D18. 0) | Col. 63 (D18. 0) |
|---|---|---|---|---|
| EFEMERIS | $t_{eph}$ | $\omega$ | D | $\mu$ |

Data Card 4--

| Col. 1 | Col. 9 (D18. 0) |
|---|---|
| ACCURACY | $\Delta r/r$ |

Data Card 5--

| Col. 1 | Col. 9 (D18. 0) | Col. 27 (D18. 0) | Col. 45 (D18. 0) |
|---|---|---|---|
| STOP | $t_f$ | $r_{1s_F}$ | $r_{2s_F}$ |

Data Card 6--

| Col. 1 | Col. 9 (D18. 0) | Col. 42 (I 1) | Col. 43 (I 1) | Col. 44 (I 1) |
|---|---|---|---|---|
| PRINT | $\Delta t_p$ | IPRT1 | IPRT2 | IPRT3 |

IPRT1, IPRT2, and IPRT3 are used to indicate printout option requests. Ordinarily (and in case Columns 42 to 44 of this card are left blank), only the standard block of printout is given (see discussion below on output). However, any or all of the optional printout blocks may be obtained at each print interval by using the integers 1, 2, and 3 in Columns 42 to 44. A 1 appearing anywhere in these columns would request the first optional block in addition to the standard block of output. A 2 would request the second optional block, and a 3, the third. Thus, any combination of the integers 1, 2, and 3 may appear in any of the three columns to request any combination of the 3 optional blocks in addition to the standard block.

Problem Card--

| Col. 1 | Col. 9 (D18. 0) |
|---|---|
| PROBLEM | NPROB |

NPROB is the problem number and will be truncated to an integer before being stored by the program. This number is used to identify the output.

To make the program as convenient to use as possible, certain flexibilities of the input have been incorporated in the program:

(1) For any problem, the six data cards may appear in any order. The problem card, however, must always appear last.

(2) On the first problem of a job, the data cards PRINT and ACCURACY may be omitted. If the PRINT card is omitted, $\Delta t_p$ is assumed to be 5., and only the standard block of output is given. If the ACCURACY card is omitted, $\Delta r/r$ is assumed to be 1. D-7.

(3) On any problem after the first problem of a job, any of the 6 data cards may be omitted. For those cards which do not appear in a given problem, the variables used in the first problem are always assumed. The problem card can never be omitted and must always be the last input card for a problem.

(4) Any of the variable fields, if left blank, are assumed to be 0. D0.

For each problem, the input data are printed out as the first page of output. The sequence of fields corresponds to those on the input cards, but the cards are ordered in a standard sequence and any assumed cards (by omission assumed same as first problem) are also printed.

Subsequent pages of output for each problem give the standard block of output, followed by any optional blocks requested, at each printing interval. The optional blocks are always ordered 1, 2, and 3 if they appear. (See the PRINT data card for the method of requesting optional blocks of output.) All variables are dimensioned in the same units as the input.

Standard output block (option 0)

TRAJECTORY TIME = t

SPACECRAFT INERTIAL TRAJECTORY

| | | | | |
|---|---|---|---|---|
| POSITION. . . . . | $x_s$ | $y_s$ | $z_s$ | $r_s$ |
| VELOCITY . . . . | $\dot{x}_s$ | $\dot{y}_s$ | $\dot{z}_s$ | $\dot{r}_s$ |

Optional output block 1

EPHEMERIS TIME = $t_{eph}$

EPHEMERIS DATA

| | | | | |
|---|---|---|---|---|
| POSITION OF EARTH . . . . | $x_E$ | $y_E$ | $z_E$ | $r_E$ |
| VELICITY OF EARTH . . . . | $\dot{x}_E$ | $\dot{y}_E$ | $\dot{z}_E$ | $\dot{r}_E$ |
| POSITION OF MOON . . . . | $x_M$ | $y_M$ | $z_M$ | $r_M$ |
| VELOCITY OF MOON . . . . | $\dot{x}_M$ | $\dot{y}_M$ | $\dot{z}_M$ | $\dot{r}_M$ |

Optional output block 2

SPACECRAFT RELATIVE TRAJECTORIES

| | | | | |
|---|---|---|---|---|
| POSITION REL TO EARTH . . . . | $x_{Es}$ | $y_{Es}$ | $z_{Es}$ | $r_{Es}$ |
| VELOCITY REL TO EARTH . . . . | $\dot{x}_{Es}$ | $\dot{y}_{Es}$ | $\dot{z}_{Es}$ | $\dot{r}_{Es}$ |
| POSITION REL TO MOON . . . . | $x_{Ms}$ | $y_{Ms}$ | $z_{Ms}$ | $r_{Ms}$ |
| VELOCITY REL TO MOON . . . . | $\dot{x}_{Ms}$ | $\dot{y}_{Ms}$ | $\dot{z}_{Ms}$ | $\dot{r}_{Ms}$ |

Optional output block 3

VIRTUAL MASS DATA

| | | | | |
|---|---|---|---|---|
| VIRTUAL MASS POSITION . . . . . . . . | $x_v$ | $y_v$ | $z_v$ | $r_v$ |
| VIRTUAL MASS VELOCITY . . . . . . . | $\dot{x}_v$ | $\dot{y}_v$ | $\dot{z}_v$ | $\dot{r}_v$ |
| SPACECRAFT POS REL TO VM. . . . | $x_{vs}$ | $y_{vs}$ | $z_{vs}$ | $r_{vs}$ |
| SPACECRAFT VEL REL TO VM . . . . | $\dot{x}_{vs}$ | $\dot{y}_{vs}$ | $\dot{z}_{vs}$ | $\dot{r}_{vs}$ |
| KEPLER (ANG MOM.) VECTOR . . . . | $k_x$ | $k_y$ | $k_z$ | $k$ |
| ECCENTRICITY VECTOR. . . . . . . . . | $e_x$ | $e_y$ | $e_z$ | $e$ |

VM MAGN $= \mu_v$

VM MAGN RATE $= \dot{\mu}_v$

## C. SAMPLE PROBLEM

The foregoing descriptions of certain features of the program and of the input and output formats are best illustrated by an example. This section, therefore, lists the output of a sample problem. As noted in Chapter IV, Section A, this example is the problem used as a reference for the digital computer study reported there.

Observe that all output options were requested. Distances are expressed in terms of nautical miles and times are in hours. In addition to the printout at the requested 5-hr increments, there are also outputs at t = 0.002900911 · · ·, t = 70.338748577 and t = 70.4000645948 hr. The first occurred because of the fact that the trajectory insertion was made with a slightly negative flight path angle. Since the virtual mass almost exactly coincides with the location of the earth, periapsis relative to the fictitious body was achieved almost immediately. The second time corresponds to the periapsis passage at pericynthion. Both of these occurrences were preceded by the notation "MAJOR AXIS CROSSING" in the printout. The last time point was printed as one of the stopping conditions (maximum time t = 70.4 hr) was met.

For the ACCURACY used in this example, the Jacobi energy variation was less than 2 parts out of $C_J$ = 7033989.7 $(\text{naut mi/hr})^2$.

VIRTUAL MASS PROGRAM FOR COMPUTING SPACE TRAJECTORIES

PAGE 1

| | | 1 | 2 | 3 |
|---|---|---|---|---|
| PRINT | 5.00000000000D 00 | | | |
| POSITION | 0.00000000000D-39 | -1.12608800000D 03 | -5.43309510000D 03 | 1.95972700000D 02 |
| VELOCITY | 1.83648790000D 04 | 3.15253210000D 03 | 1.06248890000D 04 | |
| EPHEMERIS | 9.35911770000D 01 | 5.49014930000D-01 | 2.07747200000D 05 | 1.21432890000D-02 |
| ACCURACY | 1.00000000000D-07 | | | |
| STEP | 7.04000000000D 01 | 3.44400000000D 03 | 9.38000000000D 02 | |
| PROBLEM | 1 | | | |

TRAJECTORY TIME = 0.000000000000D-39

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| **SPACECRAFT INERTIAL TRAJECTORY** | | | | |
| POSITION . . . . . . . . . . . . . . . | -1.126088000000D 03 | -5.433095100000D 03 | 1.959727000000D 02 | 5.55202682347D 03 |
| VELOCITY . . . . . . . . . . . . . . . | 1.836487900000D 04 | 3.152532100000D 03 | 1.062488900000D 04 | 2.14498369595D 04 |

EPHEMERIS TIME = 5.35911770000D 01

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| **EPHEMERIS DATA** | | | | |
| POSITION OF EARTH . . . . . . . . . | -1.57446895747D 03 | -1.97110014777D 03 | 0.000000000000D-39 | 2.52273428854D 03 |
| VELOCITY OF EARTH . . . . . . . . . | 1.88873145430D 01 | -1.50867476072D 01 | 0.000000000000D-39 | 2.41731380671D 01 |
| POSITION OF MOON . . . . . . . . . | 1.28083069249D 05 | 1.60349021507D 05 | 0.000000000000D-39 | 2.05224465711D 05 |
| VELOCITY OF MOON . . . . . . . . . | -1.53648327270D 03 | 1.22730710526D 03 | 0.000000000000D-39 | 1.96648508205D 03 |
| **SPACECRAFT RELATIVE TRAJECTORIES** | | | | |
| POSITION REL. TO EARTH . . . . . . . | 0.448380957470D 03 | -0.346199495220D 04 | 0.195972700000D 03 | 0.349640670280D 04 |
| VELOCITY REL. TO EARTH . . . . . . . | 0.183459916850D 05 | 0.316761884760D 04 | 0.106248890000D 05 | 0.214358924780D 05 |
| POSITION REL. TO MOON . . . . . . . | -0.129209157250D 06 | -0.165782116610D 06 | 0.195972700000D 03 | 0.210187428050D 06 |
| VELOCITY REL. TO MOON . . . . . . . | 0.199013622730D 05 | 0.192522499470D 04 | 0.106248890000D 05 | 0.226419738060D 05 |
| **VIRTUAL MASS DATA** | | | | |
| VIRTUAL MASS POSITION . . . . . . . | -1.57446162103D 03 | -1.97109096317D 03 | 0.00000000000D-39 | 2.52272253353D 03 |
| VIRTUAL MASS VELOCITY . . . . . . . | 1.88874807386D 01 | -1.50863590675D 01 | -0.00000000000D-39 | 2.41732254326D 01 |
| SPACECRAFT POS. REL. TO V.M. . . . | 4.48373621026D 02 | -3.46200413683D 03 | 1.95972700000D 02 | 3.49641485620D 03 |
| SPACECRAFT VEL. REL. TO V.M. . . . | 1.83459915193D 04 | 3.16761845907D 03 | 1.06248890000D 04 | 2.14358922788D 04 |
| KEPLER (ANG. MOM.) VECTOR. . . . . | -3.74041764134D 07 | -1.16860646172D 06 | 6.49341750925D 07 | 7.49458814690D 07 |
| ECCENTRICITY VECTOR . . . . . . . | 1.39947861281D-01 | -9.63351041496D-01 | 6.32772532211D-02 | 9.75517628648D-01 |

V.M. MAGN. 6.13252860752D 11
V.M. MAGN. RATE = -3.17598630455D 07

MAJOR AXIS CROSSING

VIRTUAL MASS PROGRAM FOR COMPUTING SPACE TRAJECTORIES

PROBLEM 1 PAGE 3

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| TRAJECTORY TIME = 2.50091149148D-03 | | | | |
| **SPACECRAFT INERTIAL TRAJECTORY** | | | | |
| POSITION . . . . . . . . . . . . . . | -1.07265043077D 03 | -5.42367294454D 03 | 2.26778049489D 02 | 5.53341346185D 03 |
| VELOCITY . . . . . . . . . . . . . . | 1.83386593200D 04 | 3.34338477204D 03 | 1.06132203544D 04 | 2.14505266527D 04 |
| EPHEMERIS TIME = 9.35940779115D 01 | | | | |
| **EPHEMERIS DATA** | | | | |
| POSITION OF EARTH . . . . . . . . . | -1.57441415644D 03 | -1.97114391233D 03 | 0.00000000000D-39 | 2.52273428854D 03 |
| VELOCITY OF EARTH . . . . . . . . . | 1.88877335002D 01 | -1.50962225931D 01 | 0.00000000000D-39 | 2.41731380671D 01 |
| POSITION OF MOON . . . . . . . . . | 1.28078611998D 05 | 1.60352581754D 05 | 0.00000000000D-39 | 2.05224465711D 05 |
| VELOCITY OF MOON . . . . . . . . . | -1.53651738741D 03 | 1.22726439535D 03 | 0.00000000000D-39 | 1.96648598205D 03 |
| **SPACECRAFT RELATIVE TRAJECTORIES** | | | | |
| POSITION REL. TO EARTH . . . . . . | 0.50156373567D 03 | -0.34525290322D 04 | 0.22677804949D 03 | 0.34961337479D 04 |
| VELOCITY REL. TO EARTH . . . . . . | 0.18319771586D 05 | 0.33584709946D 04 | 0.10613220354D 05 | 0.21436739600D 05 |
| POSITION REL. TO MOON . . . . . . | -0.12915146243D 06 | -0.16577625470D 06 | 0.22677804949D 03 | 0.21014737281D 06 |
| VELOCITY REL. TO MOON . . . . . . | 0.19875176767D 05 | 0.21161203767D 04 | 0.10613220354D 05 | 0.22630533818D 05 |
| **VIRTUAL MASS DATA** | | | | |
| VIRTUAL MASS POSITION . . . . . . | -1.57440682777D 03 | -1.97113472442D 03 | 0.00000000000D-39 | 2.52272252956D 03 |
| VIRTUAL MASS VELOCITY . . . . . . . | 1.88890995124D 01 | -1.50843323219D 01 | 0.00000000000D-39 | 2.41730255034D 01 |
| SPACECRAFT POS. REL. TO V.M. . . . | 4.99513674925D 02 | -3.45283910072D 03 | 2.25826373689D 02 | 3.49614202928D 03 |
| SPACECRAFT VEL. REL. TO V.M. . . . | 1.83206247224D 04 | 3.35257731156D 03 | 1.06136065268D 04 | 2.14367373533D 04 |
| KEPLER (ANG. MOM.) VECTOR. . . . . | -3.74041759921D 07 | -1.16860679822D 06 | 6.49341686354D 07 | 7.49458756695D 07 |
| ECCENTRICITY VECTOR . . . . . . . . | 1.39947711249D-01 | -9.63350981262D-01 | 6.32771682740D-02 | 9.75517542132D-01 |
| V.M. MACH. = 6.13252768477D 11 | | | | |
| V.M. MACH. RATE = -3.18195525874D 07 | | | | |

VIRTUAL MASS PROGRAM FOR COMPUTING SPACE TRAJECTORIES

PROBLEM 1 PAGE 4

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| TRAJECTORY TIME = 5.0000000000D 06 | | | | |
| SPACECRAFT INERTIAL TRAJECTORY | | | | |
| POSITION . . . . . . . . . . . . . . . | 1.17906753598D 04 | 3.51562791216D 04 | 8.31258610287D 03 | 3.80010930742D 04 |
| VELOCITY . . . . . . . . . . . . . . . | 3.60489095030 02 | 5.85064127505D 03 | 3.04962149480D 02 | 5.87000826627D 03 |
| EPHEMERIS TIME = 5.855127700000 01 | | | | |
| EPHEMERIS DATA | | | | |
| POSITION OF EARTH . . . . . . . . . | -1.47826181744D 03 | -2.04424320708D 03 | 0.000000000000D-39 | 2.52273428854D 03 |
| VELOCITY OF EARTH . . . . . . . . . | 1.95881799807D 01 | -1.41648794225D 01 | 0.000000000000D-39 | 2.41731380671D 01 |
| POSITION OF MOON . . . . . . . . . | 1.20256617212D 05 | 1.66299210290D 05 | 0.000000000000D-39 | 2.05224465711D 05 |
| VELOCITY OF MOON . . . . . . . . . | -1.59349868476D 03 | 1.15231311699D 03 | 0.000000000000D-39 | 1.96648508205D 03 |
| SPACECRAFT RELATIVE TRAJECTORIES | | | | |
| POSITION REL. TO EARTH . . . . . . | 0.132689371770 05 | 0.372005223290 05 | 0.831258610290 04 | 0.403614004100 05 |
| VELOCITY REL. TO EARTH . . . . . . | 0.346460729520 03 | 0.586480615450 04 | 0.304962149480 03 | 0.588294043650 04 |
| POSITION REL. TO MOON . . . . . . | -0.108465941850 06 | -0.131142931170 06 | 0.831258610290 04 | 0.170389049020 06 |
| VELOCITY REL. TO MOON . . . . . . | 0.195954759430 04 | 0.469832815810 04 | 0.304962149480 03 | 0.509971726350 04 |
| VIRTUAL MASS DATA | | | | |
| VIRTUAL MASS POSITION . . . . . | -1.45837523774D 03 | -2.01674266219D 03 | 0.000000000000D-39 | 2.48879675738D 03 |
| VIRTUAL MASS VELOCITY . . . . . . | 2.92720372128D 01 | -2.13408135518D-01 | 0.000000000000D-39 | 2.92728520084D 01 |
| SPACECRAFT POS. REL. TO V.M. . . . | 1.32462819121D 04 | 3.71250031591D 04 | 8.10080597[illegible]D 03 | 4.02838340230D 04 |
| SPACECRAFT VEL. REL. TO V.M. . . . | 3.38135968792D 02 | 5.85465573314D 03 | 3.05807143[illegible]D 02 | 5.87238007081D 03 |
| KEPLER (ANG. MOM.) VECTOR. . . . . | -3.72[illegible]956983[illegible]D 07 | -1.24087048703D 06 | 6.49991214300D 07 | 7.49512071701D 07 |
| ECCENTRICITY VECTOR . . . . . . . | 1.40[illegible]12295450D-01 | -9.62727733781D-01 | 6.23109608158D-02 | 9.74935466003D-01 |
| V.M. MAGN. = 6.11453964736D 11 | | | | |
| V.M. MAGN. RATE = -[illegible].2[illegible]E171218[illegible]2D 08 | | | | |

VIRTUAL MASS PROGRAM FOR COMPUTING SPACE TRAJECTORIES

PROBLEM 1 PAGE 5

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| TRAJECTORY TIME = 1.00000000000D 01 | | | | |
| SPACECRAFT INERTIAL TRAJECTORY | | | | |
| POSITION . . . . . . . . . . . . . . . | 1.23535191087D 04 | 6.02643693423D 04 | 9.03071162528D 03 | 6.21768236488D 04 |
| VELOCITY . . . . . . . . . . . . . . . | -4.75403929667D 01 | 4.40544614403D 03 | 3.98602440315D 01 | 4.40598295986D 03 |
| EPHEMERIS TIME = 1.03551177000D 02 | | | | |
| EPHEMERIS DATA | | | | |
| POSITION OF EARTH . . . . . . . . . | -1.37866208772D 03 | -2.11269475752D 03 | 0.00000000000D-39 | 2.52273428854D 03 |
| VELOCITY OF EARTH . . . . . . . . . | 2.02440908261D 01 | -1.32105030425D 01 | 0.00000000000D-39 | 2.41731380671D 01 |
| POSITION OF MOON . . . . . . . . . | 1.12154177962D 05 | 1.71867744769D 05 | 0.00000000000D-39 | 2.05224465711D 05 |
| VELOCITY OF MOON . . . . . . . . . | -1.64685704018D 03 | 1.07467458662D 03 | 0.00000000000D-39 | 1.96648508205D 03 |
| SPACECRAFT RELATIVE TRAJECTORIES | | | | |
| POSITION REL. TO EARTH . . . . . . . | 0.13732181196D 05 | 0.62377064100D 05 | 0.90307116253D 04 | 0.64506004981D 05 |
| VELOCITY REL. TO EARTH . . . . . . . | -0.67784483793D 02 | 0.44186566471D 04 | 0.39860244031D 02 | 0.44193563038D 04 |
| POSITION REL. TO MOON . . . . . . . | -0.99800658854D 05 | -0.11160337543D 06 | 0.90307116253D 04 | 0.14999012856D 06 |
| VELOCITY REL. TO MOON . . . . . . . | 0.15993166472D 04 | 0.33307715574D 04 | 0.39860244031D 02 | 0.36950363926D 04 |
| VIRTUAL MASS DATA | | | | |
| VIRTUAL MASS POSITION . . . . . . . | -1.26775690715D 03 | -1.94274100623D 03 | 0.00000000000D-39 | 2.31979529117D 03 |
| VIRTUAL MASS VELOCITY . . . . . . . | 4.84323730232D 01 | 3.35441796301D 01 | 0.00000000000D-39 | 5.89144018361D 01 |
| SPACECRAFT POS. REL. TO V.M. . . . . | 1.36228074793D 04 | 6.21370519206D 04 | 9.03006959064D 03 | 6.42505740193D 04 |
| SPACECRAFT VEL. REL. TO V.M. . . . . | -9.52719779346D 01 | 4.37500068141D 03 | 4.02996592181D 01 | 4.37622346032D 03 |
| KEPLER (ANG. MOM.) VECTOR. . . . . | -3.70024585950D 07 | -1.40930708980D 06 | 6.55197118441D 07 | 7.52595597300D 07 |
| ECCENTRICITY VECTOR . . . . . . . . | 1.43265270023D-01 | -9.61217462908D-01 | 6.02340353117D-02 | 9.73700204171D-01 |
| V.M. MAGN. = 8.06949006637D 11 | | | | |
| V.M. M..N. RATE = -1.21325061061D 09 | | | | |

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| TRAJECTORY TIME = 1.50000000000D 01 | | | | |
| SPACECRAFT INERTIAL TRAJECTORY | | | | |
| POSITION . . . . . . . . . . . . . . | 1.17294410835D 04 | 8.03260654006D 04 | 8.96937698387D 03 | 8.16719431283D 04 |
| VELOCITY . . . . . . . . . . . . . . | -1.82826650792D 02 | 3.68115184175D 03 | -5.22325491505D 01 | 3.68605923792D 03 |
| EPHEMERIS TIME = 1.08551177000D 02 | | | | |
| EPHEMERIS DATA | | | | |
| POSITION OF EARTH . . . . . . . . . | -1.27589834827D 03 | -2.17629770378D 03 | 0.00000000000D-39 | 2.52273428854D 03 |
| VELOCITY OF EARTH . . . . . . . . . | 2.08535417731D 01 | -1.22258087474D 01 | 0.00000000000D-39 | 2.41731380671D 01 |
| POSITION OF MOON . . . . . . . . . | 1.03794346481D 05 | 1.77041845237D 05 | 0.00000000000D-39 | 2.05224457111D 05 |
| VELOCITY OF MOON . . . . . . . . . | -1.69643588230D 03 | 9.94569693477D 02 | 0.00000000000D-39 | 1.96648508205D 03 |
| SPACECRAFT RELATIVE TRAJECTORIES | | | | |
| POSITION REL. TO EARTH . . . . . . | 0.130053394320D 05 | 0.825023631040D 05 | 0.896937698390D 04 | 0.840013600780D 05 |
| VELOCITY REL. TO EARTH . . . . . . | -0.203680192560D 03 | 0.369337765050D 04 | -0.522325491510D 02 | 0.369935836720D 04 |
| POSITION REL. TO MOON . . . . . . | -0.920649053580D 05 | -0.967157798370D 05 | 0.896937698390D 04 | 0.133829513180D 06 |
| VELOCITY REL. TO MOON . . . . . . | 0.151360923150D 04 | 0.268658214830D 04 | -0.522325491510D 02 | 0.308406627430D 04 |
| VIRTUAL MASS DATA | | | | |
| VIRTUAL MASS POSITION . . . . . . | -9.57473308409D 02 | -1.63316063960D 03 | 0.00000000000D-39 | 1.89313729324D 03 |
| VIRTUAL MASS VELOCITY . . . . . . | 7.76069842279D 01 | 9.65067731063D 01 | 0.00000000000D-39 | 1.23840224710D 02 |
| SPACECRAFT POS. REL. TO V.M. . . . | 1.26877843857D 04 | 8.19472487168D 04 | 8.96955142830D 03 | 8.34073395913D 04 |
| SPACECRAFT VEL. REL. TO V.M. . . . | -2.60363498490D 02 | 3.58504982895D 03 | -5.21912987403D 01 | 3.59487070130D 03 |
| KEPLER (ANG. MOM.) VECTOR. . . . . | -3.64332221525D 07 | -1.67315184453D 06 | 6.68224116093D 07 | 7.61276152706D 07 |
| ECCENTRICITY VECTOR . . . . . . . | 1.47646810792D-01 | -9.58335913091D-01 | 5.65051075842D-02 | 9.71287871972D-01 |
| V.M. MAGN. = 7.98869802623D 11 | | | | |
| V.M. MAGN. RATE = -2.07788153337D 09 | | | | |

## V I R T U A L   M A S S   P R O G R A M   F O R   C O M P U T I N G   S P A C E   T R A J E C T O R I E S

PROBLEM 1 PAGE 7

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| TRAJECTORY TIME = 2.00000000000D 01 | | | | |
| SPACECRAFT INERTIAL TRAJECTORY | | | | |
| POSITION . . . . . . . . . . . . . . | 1.06405113563D 04 | 9.74879838799D 04 | 8.58164825248D 03 | 9.84417196600D 04 |
| VELOCITY . . . . . . . . . . . . . . | -2.45715971548D 02 | 3.21170766826D 03 | -9.83651581642D 01 | 3.22259494653D 03 |
| EPHEMERIS TIME = 1.13551177000D 02 | | | | |
| EPHEMERIS DATA | | | | |
| POSITION OF EARTH . . . . . . . . . | -1.17020644039D 03 | -2.23490077799D 03 | 0.00000000000D-39 | 2.52273428854D 03 |
| VELOCITY OF EARTH . . . . . . . . . | 2.14151341406D 01 | -1.12130563964D 01 | 0.00000000000D-39 | 2.41731380671D 01 |
| POSITION OF MOON . . . . . . . . . | 9.51963084627D 04 | 1.81809637208D 05 | 0.00000000000D-39 | 2.05224465711D 05 |
| VELOCITY OF MOON . . . . . . . . . | -1.74212142837D 03 | 9.12182277143D 02 | 0.00000000000D-39 | 1.96648508205D 03 |
| SPACECRAFT RELATIVE TRAJECTORIES | | | | |
| POSITION REL. TO EARTH . . . . . . | 0.11810717797D 05 | 0.99722889958D 05 | 0.85816482525D 04 | 0.10078587462D 06 |
| VELOCITY REL. TO EARTH . . . . . . | -0.26713110569D 03 | 0.32229207247D 04 | -0.98365158164D 02 | 0.32354679305D 04 |
| POSITION REL. TO MOON . . . . . . | -0.84555797106D 05 | -0.84321653328D 05 | 0.85816482525D 04 | 0.1197[illegible]246544D 06 |
| VELOCITY REL. TO MOON . . . . . . | 0.14964054568D 04 | 0.22995253911D 04 | -0.98365158164D 02 | 0.274[illegible]3090937D 04 |
| VIRTUAL MASS DATA | | | | |
| VIRTUAL MASS POSITION . . . . . . | -4.68642463312D 02 | -8.95031725608D 02 | 0.00000000000D-39 | 1.01030072170D 03 |
| VIRTUAL MASS VELOCITY . . . . . . | 1.20694286677D 02 | 2.09636733803D 02 | 0.00000000000D-39 | 2.41898059099D 02 |
| SPACECRAFT POS. REL. TO V.M. . . . | 1.11147726979D 04 | 9.83369476705D 04 | 8.58315647661D 03 | 9.93346063785D 04 |
| SPACECRAFT VEL. REL. TO V.M. . . . | -3.66194087799D 02 | 3.00350791854D 03 | -9.82597263791D 01 | 3.02734419921D 03 |
| KEPLER (ANG. MOM.) VECTOR. . . . . | -3.54421400146D 07 | -2.05096663233D 06 | 6.93937166600D 07 | 7.79476726069D 07 |
| ECCENTRICITY VECTOR . . . . . . . | 1.53218718321D-01 | -9.53168420025D-01 | 5.00835350865D-02 | 9.66702835965D-01 |
| V.M. MAGN. = 7.85392301980D 11 | | | | |
| V.M. MAGN. RATE = -3.41317727450D 05 | | | | |

VIRTUAL MASS PROGRAM FOR COMPUTING SPACE TRAJECTORIES

PROBLEM 1 PAGE 8

TRAJECTORY TIME = 2.500000000000D 01

EPHEMERIS TIME = 1.185511170000D 02

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| **SPACECRAFT INERTIAL TRAJECTORY** | | | | |
| POSITION . . . . . . . . . . . . . . | 9.320542801010D 03 | 1.126517076380D 05 | 8.016590201140D 03 | 1.133205430200D 05 |
| VELOCITY . . . . . . . . . . . . . . | -2.789134409520D 02 | 2.869478425120D 03 | -1.255196686550D 02 | 2.885732892530D 03 |
| **EPHEMERIS DATA** | | | | |
| POSITION OF EARTH . . . . . . . . . | -1.061828925510D 03 | -2.288385374790D 03 | 0.000000000000D-39 | 2.522734288540D 03 |
| VELOCITY OF EARTH . . . . . . . . . | 2.192757908220D 01 | -1.017457024170D 01 | 0.000000000000D-39 | 2.417313806710D 01 |
| POSITION OF MOON . . . . . . . . . | 8.637979628040D 04 | 1.861601786660D 05 | 0.000000000000D-39 | 2.052244657110D 05 |
| VELOCITY OF MOON . . . . . . . . . | -1.783808830740D 03 | 8.277014155550D 02 | 0.000000000000D-39 | 1.966485082050D 03 |
| **SPACECRAFT RELATIVE TRAJECTORIES** | | | | |
| POSITION REL. TO EARTH . . . . . . . | 0.103623717270D 05 | 0.114940093010D 06 | 0.801659020110D 04 | 0.115686145860D 06 |
| VELOCITY REL. TO EARTH . . . . . . . | -0.300841020030D 03 | 0.287965299540D 04 | -0.125519668660D 03 | 0.289804449250D 04 |
| POSITION REL. TO MOON . . . . . . . | -0.770592534790D 05 | -0.735084710290D 05 | 0.801659020110D 04 | 0.106798359440D 06 |
| VELOCITY REL. TO MOON . . . . . . . | 0.150489538980D 04 | 0.204177700960D 04 | -0.125519668660D 03 | 0.253955088120D 04 |
| **VIRTUAL MASS DATA** | | | | |
| VIRTUAL MASS POSITION . . . . . . . | 2.833434766730D 02 | 6.106436286340D 02 | 0.000000000000D-39 | 6.731784064900D 02 |
| VIRTUAL MASS VELOCITY . . . . . . . | 1.842263596020D 02 | 4.123580842540D 02 | 0.000000000000D-39 | 4.516398357340D 02 |
| SPACECRAFT POS. REL. TO V.M. . . . | 9.046571510270D 03 | 1.119912960770D 05 | 8.019130484730D 03 | 1.126418985400D 05 |
| SPACECRAFT VEL. REL. TO V.M. . . . | -4.628867322450D 02 | 2.458873674420D 03 | -1.254330118220D 02 | 2.505206042240D 03 |
| KEPLER (ANG. MOM.) VECTOR. . . . . | -3.376543440530D 07 | -2.577210394320D 06 | 7.408366161110D 07 | 8.145634101600D 07 |
| ECCENTRICITY VECTOR . . . . . . . . | 1.578549316140D-01 | -9.437617561980D-01 | 3.911480167320D-02 | 9.576713421660D-01 |

V.M. MAGN. = 7.634361827070D 11

V.M. MAGN. RATE = -5.536250446470D 05

## VIRTUAL MASS PROGRAM FOR COMPUTING SPACE TRAJECTORIES

PROBLEM 1 PAGE 9

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| TRAJECTORY TIME = 3.00000000000D 01 | | | | |
| SPACECRAFT INERTIAL TRAJECTORY | | | | |
| POSITION . . . . . . . . . . . . . . . | 7.87652929213D 03 | 1.26307020359D 05 | 7.34226801497D 03 | 1.26765184515D 05 |
| VELOCITY . . . . . . . . . . . . . . . | -2.96779738133D 02 | 2.60257457398D 03 | -1.43034735930D 02 | 2.62334358440D 03 |
| EPHEMERIS TIME = 1.23591177000D 02 | | | | |
| EPHEMERIS DATA | | | | |
| POSITION OF EARTH . . . . . . . . . | -9.51014528453D 02 | -2.33661285994D 03 | 0.00000000000D-39 | 2.52273428854D 03 |
| VELOCITY OF EARTH . . . . . . . . . | 2.23897005441D 01 | -9.11273359407D 00 | 0.00000000000D-39 | 2.41731380671D 01 |
| POSITION OF MOON . . . . . . . . . | 7.73650437036D 04 | 1.90083485183D 05 | 0.00000000000D-39 | 2.05224465711D 05 |
| VELOCITY OF MOON . . . . . . . . . | -1.82140241740D 03 | 7.41320991081D 02 | 0.00000000000D-39 | 1.96648508205D 03 |
| SPACECRAFT RELATIVE TRAJECTORIES | | | | |
| POSITION REL. TO EARTH . . . . . . . | 0.88275438206D 04 | 0.12864363322D 06 | 0.73422680150D 04 | 0.12915501847D 06 |
| VELOCITY REL. TO EARTH . . . . . . . | -0.31916943868D 03 | 0.26116873076D 04 | -0.14303473593D 03 | 0.26350025918D 04 |
| POSITION REL. TO MOON . . . . . . . | -0.69488514411D 05 | -0.63776464824D 05 | 0.73422680150D 04 | 0.94604439643D 05 |
| VELOCITY REL. TO MOON . . . . . . . | 0.15246226793D 04 | 0.18612535829D 04 | -0.14303473593D 03 | 0.24102278211D 04 |
| VIRTUAL MASS DATA | | | | |
| VIRTUAL MASS POSITION . . . . . . . | 1.42427561429D 03 | 3.49940050008D 03 | 0.00000000000D-39 | 3.77814304724D 03 |
| VIRTUAL MASS VELOCITY . . . . . . . | 2.78284995206D 02 | 7.79771338774D 02 | 0.00000000000D-39 | 8.27940746268D 02 |
| SPACECRAFT POS. REL. TO V.M. . . . | 6.48995967530D 03 | 1.22687615854D 05 | 7.35165390168D 03 | 1.23078907517D 05 |
| SPACECRAFT VEL. REL. TO V.M. . . . | -5.74153972134D 02 | 1.82915223576D 03 | -1.42846593802D 02 | 1.92246087988D 03 |
| KEPLER (ANG. MOM.) VECTOR. . . . . | -3.09728021973D 07 | -3.29391265588D 06 | 8.23127062245D 07 | 8.80087833261D 07 |
| ECCENTRICITY VECTOR . . . . . . . . | 1.53393733053D-01 | -9.25841573210D-01 | 2.06698644093D-02 | 9.38690310656D-01 |
| V.M. MAGN. = 7.27876689071D 11 | | | | |
| V.M. MAGN. RATE = -8.96390169470D 09 | | | | |

V I R T U A L   M A S S   P R O G R A M   F O R   C O M P U T I N G   S P A C E   T R A J E C T O R I E S

PROBLEM 1 PAGE 10

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| TRAJECTORY TIME = 3.56000000000D 01 | | | | |
| SPACECRAFT INERTIAL TRAJECTORY | | | | |
| POSITION . . . . . . . . . . . . . | 6.368141024280 03 | 1.387595539670 05 | 6.595334161740 03 | 1.390620921390 05 |
| VELOCITY . . . . . . . . . . . . . | -3.053082398550 02 | 2.385280366990 03 | -1.550352454040 02 | 2.409732656910 03 |
| EPHEMERIS TIME = 1.28591177000D 02 | | | | |
| EPHEMERIS DATA | | | | |
| POSITION OF EARTH . . . . . . . . | -8.380175666870 02 | -2.379477852070 03 | 0.000000000000D-39 | 2.522734288540 03 |
| VELOCITY OF EARTH . . . . . . . . | 2.280043796410 01 | -8.029983353460 00 | 0.000000000000D-39 | 2.417313806710 01 |
| POSITION OF MOON . . . . . . . . | 6.817273546030 04 | 1.935705528250 05 | 0.000000000000D-39 | 2.052244657110 05 |
| VELOCITY OF MOON . . . . . . . . | -1.854815911620 03 | 6.532392455560 02 | 0.000000000000D-39 | 1.966485082050 03 |
| SPACECRAFT RELATIVE TRAJECTORIES | | | | |
| POSITION REL. TO EARTH . . . . . . | 0.720615859100 04 | 0.141139031820 06 | 0.659533416170 04 | 0.141476688740 06 |
| VELOCITY REL. TO EARTH . . . . . . | -0.328106677820 03 | 0.239331035030 04 | -0.155035245400 03 | 0.242066636790 04 |
| POSITION REL. TO MOON . . . . . . | -0.618604598436D 05 | -0.548109988580 05 | 0.659533416170 04 | 0.828706969700 05 |
| VELOCITY REL. TO MOON . . . . . . | 0.154950767180 04 | 0.173204112140 04 | -0.155035245400 03 | 0.232915787320 04 |
| VIRTUAL MASS DATA | | | | |
| VIRTUAL MASS POSITION . . . . . . | 3.139649688270 03 | 8.914761686950 03 | 0.000000000000D-39 | 9.451474810850 03 |
| VIRTUAL MASS VELOCITY . . . . . . | 4.163885845560 02 | 1.454932770880 03 | 0.000000000000D-39 | 1.513343589910 03 |
| SPACECRAFT POS. REL. TO V.M. . . . | 3.289969847860 03 | 1.297645804700 05 | 6.608560162460 03 | 1.299743948380 05 |
| SPACECRAFT VEL. REL. TO V.M. . . . | -7.202092234110 02 | 9.414134610840 02 | -1.548631974270 02 | 1.195384139140 03 |
| KEPLER (ANG. MOM.) VECTOR. . . . . | -2.631714533960 07 | -4.250050732390 06 | 9.655486962780 07 | 1.001673495650 08 |
| ECCENTRICITY VECTOR . . . . . . . | 1.091795348230-01 | -8.886707848360-01 | -9.358432540930-03 | 8.954013150000-01 |
| V.M. MAGN. = 6.70353749408D 11 | | | | |
| V.M. MAGN. RATE = -1.44614515165D 10 | | | | |

VIRTUAL MASS PROGRAM FOR COMPUTING SPACE TRAJECTORIES

PROBLEM 1 PAGE 11

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| TRAJECTORY TIME = 4.00000000000D 01 | | | | |
| SPACECRAFT INERTIAL TRAJECTORY | | | | |
| POSITION . . . . . . . . . . . . . . | 4.83448639174D 03 | 1.50218402288D 05 | 5.79750266337D 03 | 1.50407950859D 05 |
| VELOCITY . . . . . . . . . . . . . . | -3.07157835474D 02 | 2.20330961002D 03 | -1.63657017363D 02 | 2.23062834036D 03 |
| EPHEMERIS TIME = 1.33551177000D 02 | | | | |
| EPHEMERIS DATA | | | | |
| POSITION OF EARTH . . . . . . . . . | -7.23097366616D 02 | -2.41688197663D 03 | 0.00000000000D-39 | 2.52273428854D 03 |
| VELOCITY OF EARTH . . . . . . . . . | 2.31588487057D 01 | -6.92880441613D 00 | 0.00000000000D-39 | 2.41731380671D 01 |
| POSITION OF MOON . . . . . . . . . | 5.88233797569D 04 | 1.96613378823D 05 | 0.00000000000D-39 | 2.05224465711D 05 |
| VELOCITY OF MOON . . . . . . . . . | -1.88397262990D 03 | 5.63658325325D 02 | 0.00000000000D-39 | 1.96648508205D 03 |
| SPACECRAFT RELATIVE TRAJECTORIES | | | | |
| POSITION REL. TO EARTH . . . . . . | 0.55575837384D 04 | 0.15263528426D 06 | 0.57975026634D 04 | 0.15284641892D 06 |
| VELOCITY REL. TO EARTH . . . . . . | -0.33031668418D 03 | 0.22102384144D 04 | -0.16365701736D 03 | 0.22407691938D 04 |
| POSITION REL. TO MOON . . . . . . | -0.53989493365D 05 | -0.46394976535D 05 | 0.57975026634D 04 | 0.71421077272D 05 |
| VELOCITY REL. TO MOON . . . . . . | 0.15768147944D 04 | 0.16396512847D 04 | -0.16365701736D 03 | 0.22806983252D 04 |
| VIRTUAL MASS DATA | | | | |
| VIRTUAL MASS POSITION . . . . . . | 5.67990735188D 03 | 1.89845328464D 04 | 0.00000000000D-39 | 1.98159994682D 04 |
| VIRTUAL MASS VELOCITY . . . . . . | 6.08570869614D 02 | 2.69653707423D 03 | 0.00000000000D-39 | 2.76435719400D 03 |
| SPACECRAFT POS. REL. TO V.M. . . . | -6.99592180509D 02 | 1.31308101791D 05 | 5.82364420045D 03 | 1.31439042362D 05 |
| SPACECRAFT VEL. REL. TO V.M. . . . | -9.12292455296D 02 | -4.61705529687D 02 | -1.63419454518D 02 | 1.03545820012D 03 |
| KEPLER (ANG. MOM.) VECTOR. . . . . | -1.87694896382D 07 | -5.42725187536D 06 | 1.20115709243D 08 | 1.21694422268D 08 |
| ECCENTRICITY VECTOR . . . . . . . | -9.17737988795D-02 | -8.04881478359D-01 | -5.07081206813D-02 | 8.11682165548D-01 |
| V.M. MAGN. = 5.78491854009D 11 | | | | |
| V.M. MAGN. RATE = -2.27768451418D 10 | | | | |

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| TRAJECTORY TIME = 4.50000000000D 01 | | | | |
| SPACECRAFT INERTIAL TRAJECTORY | | | | |
| POSITION . . . . . . . . . . . . . . . | 3.30616350511D 03 | 1.60837401770D 05 | 4.96226597639D 03 | 1.60947894080D 05 |
| VELOCITY . . . . . . . . . . . . . . . | -3.03199452227D 02 | 2.04829603976D 03 | -1.70166279431D 02 | 2.07759551814D 03 |
| EPHEMERIS TIME = 1.36551177400D 02 | | | | |
| EPHEMERIS DATA | | | | |
| POSITION OF EARTH . . . . . . . . . | -6.06517668451D 02 | -2.44873939169D 03 | 0.00000000000D-39 | 2.52273428854D 03 |
| VELOCITY OF EARTH . . . . . . . . . | 2.34641102215D 01 | -5.81172397197D 00 | 0.00000000000D-39 | 2.41731380671D 01 |
| POSITION OF MOON . . . . . . . . . | 4.93402198630D 04 | 1.99204979934D 05 | 0.00000000000D-39 | 2.05224465711D 05 |
| VELOCITY OF MOON . . . . . . . . . | -1.90880565800D 03 | 4.72783817316D 02 | 0.00000000000D-39 | 1.96648508205D 03 |
| SPACECRAFT RELATIVE TRAJECTORIES | | | | |
| POSITION REL. TO EARTH . . . . . . . | 0.39126811736D 04 | 0.16328614116D 06 | 0.49622659764D 04 | 0.16340837510D 06 |
| VELOCITY REL. TO EARTH . . . . . . . | -0.32666356245D 03 | 0.20541077637D 04 | -0.17016627943D 03 | 0.20868695098D 04 |
| POSITION REL. TO MOON . . . . . . . | -0.46034056358D 05 | -0.38367573164D 05 | 0.49622659764D 04 | 0.60131767665D 05 |
| VELOCITY REL. TO MOON . . . . . . . | 0.16056062058D 04 | 0.15755122224D 04 | -0.17016627943D 03 | 0.225591[illegible]1310D 04 |
| VIRTUAL MASS DATA | | | | |
| VIRTUAL MASS POSITION . . . . . . . | 9.27677558187D 03 | 3.74538236507D 04 | 0.00000000000D-39 | 3.85855863148D 04 |
| VIRTUAL MASS VELOCITY . . . . . . . | 8.27263845730D 02 | 4.87782702078D 03 | 0.00000000000D-39 | 4.94748036025D 03 |
| SPACECRAFT POS. REL. TO V.M. . . . . | -5.93207559588D 03 | 1.23479752299D 05 | 4.96806942942D 03 | 1.23721948184D 05 |
| SPACECRAFT VEL. REL. TO V.M. . . . . | -1.12984241884D 03 | -2.81916120508D 03 | -1.70126923105D 02 | 3.0419[illegible]022216D 03 |
| KEPLER (ANG. MOM.) VECTOR. . . . . | -7.00144172479D 06 | -6.62234134986D 06 | 1.56236139401D 08 | 1.56533085467D 08 |
| ECCENTRICITY VECTOR . . . . . . . . | -9.56553700123D-01 | -5.93783734885D-01 | -6.80347940575D-02 | 1.12791969494D 00 |
| V.M. MAGN. = 4.39040512181D 11 | | | | |
| V.M. MAGN. RATE = -3.30183437654D 10 | | | | |

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| TRAJECTORY TIME = 5.00000000000D 01 | | | | |
| **SPACECRAFT INERTIAL TRAJECTORY** | | | | |
| POSITION . . . . . . . . . . . . . . | 1.81316420197D 03 | 1.70738382388D 05 | 4.09786822194D 03 | 1.70797175928D 05 |
| VELOCITY . . . . . . . . . . . . . . | -2.92764407134D 02 | 1.91559320167D 03 | -1.75457232865D 02 | 1.94576297450D 03 |
| EPHEMERIS TIME = 1.43591177000D 02 | | | | |
| **EPHEMERIS DATA** | | | | |
| POSITION OF EARTH . . . . . . . . . | -4.88546020930D 02 | -2.47497698495D 03 | 0.00000000000D-39 | 2.52273428854D 03 |
| VELOCITY OF EARTH . . . . . . . . . | 2.37155219405D 01 | -4.68130570458D 00 | 0.00000000000D-39 | 2.41731380671D 01 |
| POSITION OF MOON . . . . . . . . . | 3.57432248716D 04 | 2.01339408471D 05 | 0.00000000000D-39 | 2.05224465711D 05 |
| VELOCITY OF MOON . . . . . . . . . | -1.92925800447D 03 | 3.80824277221D 02 | 0.00000000000D-39 | 1.96648508205D 03 |
| **SPACECRAFT RELATIVE TRAJECTORIES** | | | | |
| POSITION REL. TO EARTH . . . . . . . | 0.230170922229D 04 | 0.173213359370D 06 | 0.409786822190D 04 | 0.173277114050D 06 |
| VELOCITY REL. TO EARTH . . . . . . . | -0.316479929070D 03 | 0.192027450740D 04 | -0.175457232870D 03 | 0.195407240650D 04 |
| POSITION REL. TO MOON . . . . . . . | -0.379300616700D 05 | -0.306010260830D 05 | 0.409786822190D 04 | 0.489071047960D 05 |
| VELOCITY REL. TO MOON . . . . . . . | 0.163649359730D 04 | 0.153476892440D 04 | -0.175457232870D 03 | 0.225042489010D 04 |
| **VIRTUAL MASS DATA** | | | | |
| VIRTUAL MASS POSITION . . . . . . . | 1.37318870290D 04 | 6.95658195964D 04 | 0.00000000000D-39 | 7.09081658026D 04 |
| VIRTUAL MASS VELOCITY . . . . . . . | 9.03292297176D 02 | 8.08460279893D 03 | -0.00000000000D-39 | 8.13490868976D 03 |
| SPACECRAFT POS. REL. TO V.M. . . . . | -1.19068333798D 04 | 1.01233849557D 05 | 4.09961226688D 03 | 1.02014076470D 05 |
| SPACECRAFT VEL. REL. TO V.M. . . . . | -1.19620270976D 03 | -6.16537557767D 03 | -1.75447383784D 02 | 6.28279704599D 03 |
| KEPLER (ANG. MOM.) VECTOR. . . . . | 7.51443529307D 06 | -6.99299006837D 06 | 1.94506304887D 08 | 1.94776978333D 08 |
| ECCENTRICITY VECTOR . . . . . . . . | -4.56483950882D 00 | -9.01086160087D-02 | 1.73115532039D-01 | 4.56900955256D 00 |
| V.M. MAGN. = 2.56227401054D 11 | | | | |
| V.M. MAGN. RATE = -3.81673786623D 10 | | | | |

VIRTUAL MASS PROGRAM FOR COMPUTING SPACE TRAJECTORIES

PROBLEM 1 PAGE 14

TRAJECTORY TIME = 5.50000000000D 01

EPHEMERIS TIME = 1.48551177000D 02

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| **SPACECRAFT INERTIAL TRAJECTORY** | | | | |
| POSITION . . . . . . . . . . . . . . . | 3.94300961204D 02 | 1.80028087368D 05 | 3.20829994657D 03 | 1.80057104562D 05 |
| VELOCITY . . . . . . . . . . . . . . . | -2.72669013933D 02 | 1.80390397235D 03 | -1.80399550370D 02 | 1.83329264723D 03 |
| **EPHEMERIS DATA** | | | | |
| POSITION OF EARTH . . . . . . . . . | -3.69431672930D 02 | -2.49553454149D 03 | 0.00000000000D-39 | 2.52273428854D 03 |
| VELOCITY OF EARTH . . . . . . . . . | 2.39125068766D 01 | -3.54014390769D 00 | 0.00000000000D-39 | 2.41731380671D 01 |
| POSITION OF MOON . . . . . . . . . | 3.00550197488D 04 | 2.03011765951D 05 | 0.00000000000D-39 | 2.05224465711D 05 |
| VELOCITY OF MOON . . . . . . . . . | -1.94528273147D 03 | 2.87990750868D 02 | 0.00000000000D-39 | 1.96648508205D 03 |
| **SPACECRAFT RELATIVE TRAJECTORIES** | | | | |
| POSITION REL. TO EARTH . . . . . . . | 0.76375412850D 03 | 0.18252362191D 06 | 0.32082999466D 04 | 0.18255341428D 06 |
| VELOCITY REL. TO EARTH . . . . . . . | -0.29658152081D 03 | 0.18074441163D 04 | -0.18039955037D 03 | 0.18404778808D 04 |
| POSITION REL. TO MOON . . . . . . . | -0.29660718788D 05 | -0.22983678582D 05 | 0.32082999466D 04 | 0.37660336014D 05 |
| VELOCITY REL. TO MOON . . . . . . . | 0.16726137175D 04 | 0.15159132215D 04 | -0.18039955037D 03 | 0.22645470940D 04 |
| **VIRTUAL MASS DATA** | | | | |
| VIRTUAL MASS POSITION . . . . . . . | 1.73786949218D 04 | 1.17387364090D 05 | 0.00000000000D-39 | 1.18666812063D 05 |
| VIRTUAL MASS VELOCITY . . . . . . . | 4.24257698694D 02 | 1.06300319533D 04 | -0.00000000000D-39 | 1.06384949087D 04 |
| SPACECRAFT POS. REL. TO V.M. . . . . | -1.69815133568D 04 | 6.26771827172D 04 | 3.20904517692D 03 | 6.50161441505D 04 |
| SPACECRAFT VEL. REL. TO V.M. . . . . | -6.97317106346D 02 | -8.82567097610D 03 | -1.80395[illegible]451D 02 | 8.8[illegible]5013370340D 03 |
| KEPLER (ANG. MOM.) VECTOR. . . . . | 1.70153077967D 07 | -5.30110726705D 06 | 1.93579121250D 08 | 1.[illegible]3977845080D 08 |
| ECCENTRICITY VECTOR . . . . . . . | -1.91445993638D 01 | 5.33525022810D-01 | 1.69739134093D 00 | 1.92271025148D 01 |

V.M. MAGN. = 8.80350936020D 10

V.M. MAGN. RATE = -2.57255743304D 10

VIRTUAL MASS PROGRAM FOR COMPUTING SPACE TRAJECTORIES

PROBLEM 1 PAGE 15

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| TRAJECTORY TIME = 6.000000000000 01 | | | | |
| SPACECRAFT INERTIAL TRAJECTORY | | | | |
| POSITION . . . . . . . . . . . . . . . . . | -8.813893114090 02 | 1.888190703790 05 | 2.292127455510 03 | 1.888350392120 05 |
| VELOCITY . . . . . . . . . . . . . . . . . | -2.325142-28610 02 | 1.717706656030 03 | -1.864696386340 02 | 1.743373154390 03 |
| EPHEMERIS TIME = 1.535911770000 02 | | | | |
| EPHEMERIS DATA | | | | |
| POSITION OF EARTH . . . . . . . . . . | -2.495124239310 02 | -2.510364882020 03 | 0.000000000000-39 | 2.522734288540 03 |
| VELOCITY OF EARTH . . . . . . . . . . | 2.405461295210 01 | -2.390857531260 00 | 0.000000000000-39 | 2.417313806710 01 |
| POSITION OF MOON . . . . . . . . . . | 2.029783878630 04 | 2.042182143370 05 | 0.000000000000-39 | 2.052244657110 05 |
| VELOCITY OF MOON . . . . . . . . . . | -1.956843062470 03 | 1.944962898690 02 | 0.000000000000-39 | 1.966485082050 03 |
| SPACECRAFT RELATIVE TRAJECTORIES | | | | |
| POSITION REL. TO EARTH . . . . . . . | -0.631876887480 03 | 0.191329435260 06 | 0.229212745550 04 | 0.191344207940 06 |
| VELOCITY REL. TO EARTH . . . . . . . | -0.256568855810 03 | 0.172009751360 04 | -0.186469638630 03 | 0.174909518320 04 |
| POSITION REL. TO MOON . . . . . . . | -0.211792280980 05 | -0.153991439580 05 | 0.229212745550 04 | 0.262858742620 05 |
| VELOCITY REL. TO MOON . . . . . . . | 0.172432881960 04 | 0.152321036620 04 | -0.186469638630 03 | 0.230830037560 04 |
| VIRTUAL MASS DATA | | | | |
| VIRTUAL MASS POSITION . . . . . . . | 1.671913218540 04 | 1.682125548500 05 | 0.000000000000-39 | 1.690413943090 05 |
| VIRTUAL MASS VELOCITY . . . . . . . | -7.571029004060 02 | 8.759709821290 03 | -0.000000000000-39 | 8.792367198600 03 |
| SPACECRAFT POS. REL. TO V.M. . . . | -1.760081359800 04 | 2.061043735210 04 | 2.292231300000 03 | 2.719987300510 04 |
| SPACECRAFT VEL. REL. TO V.M. . . . | 5.245105953280 02 | -7.042248910640 03 | -1.864687878760 02 | 7.064216282990 03 |
| KEPLER (ANG. MOM.) VECTOR. . . . . | 1.229926010470 07 | -2.079702773460 06 | 1.131389176210 08 | 1.138244773470 08 |
| ECCENTRICITY VECTOR . . . . . . . . | -5.879982016060 01 | -5.354258021080 00 | 6.293670047230 00 | 5.937758173370 01 |
| V.M. MAGN. = 1.340772613350 10 | | | | |
| V.M. MAGN. RATE = -5.575568953950 05 | | | | |

V I R T U A L   M A S S   P R O G R A M   F O R   C O M P U T I N G   S P A C E   T R A J E C T O R I E S

PROBLEM 1   PAGE 16

TRAJECTORY TIME = 6.500000000000 01

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| SPACECRAFT INERTIAL TRAJECTORY | | | | |
| POSITION . . . . . . . . . . . . . . | -1.82849144238D 03 | 1.97286528550D 05 | 1.33385464294D 03 | 1.97299510634D 05 |
| VELOCITY . . . . . . . . . . . . . . | -1.24571292659D 02 | 1.68359102394D 03 | -1.99077278613D 02 | 1.69989073346D 03 |

EPHEMERIS TIME = 1.58591177000D 02

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| EPHEMERIS DATA | | | | |
| POSITION OF EARTH . . . . . . . . . | -1.28999053132D 02 | -2.51943397113D 03 | 0.00000000000D-39 | 2.52273428854D 03 |
| VELOCITY OF EARTH . . . . . . . . . | 2.41415140356D 01 | -1.23608417107D 00 | 0.00000000000D-39 | 2.41731380671D 01 |
| POSITION OF MOON . . . . . . . . . | 1.04940745748D 04 | 2.04955984849D 05 | 0.00000000000D-39 | 2.05224465711D 05 |
| VELOCITY OF MOON . . . . . . . . . | -1.96391246670D 03 | 1.00555462672D 02 | 0.00000000000D-39 | 1.96648[illegible]8205D 03 |
| SPACECRAFT RELATIVE TRAJECTORIES | | | | |
| POSITION REL. TO EARTH . . . . . . . | -0.16994923893D 04 | 0.19980596252D 06 | 0.13338546429D 04 | 0.19981764212D 06 |
| VELOCITY REL. TO EARTH . . . . . . . | -0.14871280669D 03 | 0.16848271081D 04 | -0.19907727861D 03 | 0.17030530367D 04 |
| POSITION REL. TO MOON . . . . . . . | -0.12322566017D 05 | -0.76694562992D 04 | 0.13338546429D 04 | 0.14575505527D 05 |
| VELOCITY REL. TO MOON . . . . . . . | 0.18393411740D 04 | 0.15830355613D 04 | -0.19907727861D 03 | 0.24349146403D 04 |
| VIRTUAL MASS DATA | | | | |
| VIRTUAL MASS POSITION . . . . . . . | 1.01689297120D 04 | 1.98605697828D 05 | 0.00000000000D-39 | 1.98865860171D 05 |
| VIRTUAL MASS VELOCITY . . . . . . . | -1.73898827975D 03 | 3.30192925380D 03 | -0.00000000000D-39 | 3.73186508790D 03 |
| SPACECRAFT POS. REL. TO V.M. . . . | -1.20071759976D 04 | -1.30936900086D 03 | 1.33505780051D 03 | 1.21519176244D 04 |
| SPACECRAFT VEL. REL. TO V.M. . . . | 1.61362992377D 03 | -1.62148091976D 03 | -1.99050456061D 02 | 2.29635983596D 03 |
| KEPLER (ANG. MOM.) VECTOR. . . . . | 2.42540124707D 06 | -2.35477623240D 05 | 2.1582505[illegible]7D 07 | 2.17196356222D 07 |
| ECCENTRICITY VECTOR . . . . . . . . | -4.89034967591D 00 | -5.81611210785D 00 | 4.86111117557D-01 | 7.61439321432D 00 |

V.M. MAGN. = 5.96340902734D 05

V.M. MAGN. RATE = 7.[illegible]D 08

TRAJECTORY TIME = 7.00000000000D 01

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| SPACECRAFT INERTIAL TRAJECTORY | | | | |
| POSITION . . . . . . . . . . . . . . | -7.78639828824D 02 | 2.06033723406D 05 | 1.56422005888D 02 | 2.06035254091D 05 |
| VELOCITY . . . . . . . . . . . . . . | 1.74610803693D 03 | 1.62831086309D 03 | -4.01883513362D 02 | 2.42111542514D 03 |

EPHEMERIS TIME = 1.63591177000D 02

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| EPHEMERIS DATA | | | | |
| POSITION OF EARTH . . . . . . . . . | -8.18963135505D 00 | -2.52272099538D 03 | 0.00000000000D-39 | 2.52273428854D 03 |
| VELOCITY OF EARTH . . . . . . . . . | 2.41730106904D 01 | -7.84740154219D-02 | 0.00000000000D-39 | 2.41731380671D 01 |
| POSITION OF MOON . . . . . . . . . | 6.66226612469D 02 | 2.05223384312D 05 | 0.00000000000D-39 | 2.05224465711D 05 |
| VELOCITY OF MOON . . . . . . . . . | -1.96647471996D 03 | 6.38386212941D 00 | 0.00000000000D-39 | 1.96648508205D 03 |

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| SPACECRAFT RELATIVE TRAJECTORIES | | | | |
| POSITION REL. TO EARTH . . . . . . | -0.77045019747D 03 | 0.20855644440D 06 | 0.15642200589D 03 | 0.20855792616D 06 |
| VELOCITY REL. TO EARTH . . . . . . | 0.17219350262D 04 | 0.16283893371D 04 | -0.40188351336D 03 | 0.24037933410D 04 |
| POSITION REL. TO MOON . . . . . . | -0.14448664413D 04 | 0.81033909324D 03 | 0.15642200589D 03 | 0.16639580293D 04 |
| VELOCITY REL. TO MOON . . . . . . | 0.37125827569D 04 | 0.16219270010D 04 | -0.40188351336D 03 | 0.40712931952D 04 |

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| VIRTUAL MASS DATA | | | | |
| VIRTUAL MASS POSITION . . . . . . | 6.66198750314D 02 | 2.05214801699D 05 | 0.00000000000D-39 | 2.05215883052D 05 |
| VIRTUAL MASS VELOCITY . . . . . . | -1.96626767535D 03 | 4.48283241705D 01 | -0.00000000000D-39 | 1.96677862246D 03 |
| SPACECRAFT POS. REL. TO V.M. . . . | -1.44630511882D 03 | 8.18295937761D 02 | 1.56580766261D 02 | 1.66910882655D 03 |
| SPACECRAFT VEL. REL. TO V.M. . . . | 3.71113714904D 03 | 1.58414901476D 03 | -4.01749451259D 02 | 4.05505482843D 03 |
| KEPLER (ANG. MOM.) VECTOR. . . . . | -5.76797210567D 05 | 4.04106574225D 01 | -5.32797128257D 06 | 5.35910188479D 06 |
| ECCENTRICITY VECTOR . . . . . . . | 2.85758979880D-02 | 1.49576049431D 00 | -3.08223387723D-03 | 1.49603661000D 00 |

V.M. MAGN. = 1.00726969020D 10
V.M. MAGN. RATE = 3.64496538046D 07

MAJOR AXIS CROSSING

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| TRAJECTORY TIME = 7.03387485769D 01 | | | | |
| SPACECRAFT INERTIAL TRAJECTORY | | | | |
| POSITION . . . . . . . . . . . . . . | 8.25522846803D-02 | 2.06373023657D 05 | 1.33621192150D-02 | 2.06373023657D 05 |
| VELOCITY . . . . . . . . . . . . . . | 2.69325849722D 03 | 4.66115622573D-02 | -5.04436194953D 02 | 2.74009073022D 03 |
| EPHEMERIS TIME = 1.63529925577D 02 | | | | |
| EPHEMERIS DATA | | | | |
| POSITION OF EARTH . . . . . . . . . | -1.02962098692D-03 | -2.52273428854D 03 | 0.00000000000D-39 | 2.52273428854D 03 |
| VELOCITY OF EARTH . . . . . . . . . | 2.41731380671D 01 | -9.86594996807D-06 | 0.00000000000D-39 | 2.41731380671D 01 |
| POSITION OF MOON . . . . . . . . . | 8.37596800760D-02 | 2.05224465711D 05 | 0.00000000000D-39 | 2.05224465711D 05 |
| VELOCITY OF MOON . . . . . . . . . | -1.96648508205D 03 | 8.02595152462D-04 | 0.00000000000D-39 | 1.96648508205D 03 |
| SPACECRAFT RELATIVE TRAJECTORIES | | | | |
| POSITION REL. TO EARTH . . . . . . . | 0.835819056670-01 | 0.208895757950 06 | 0.133621192150-01 | 0.208895757950 06 |
| VELOCITY REL. TO EARTH . . . . . . . | 0.266908535920 04 | 0.466214282070-01 | -0.504436194950 03 | 0.271633439240 04 |
| POSITION REL. TO MOON . . . . . . . | -0.120739539570-02 | 0.114855794570 04 | 0.133621192150-01 | 0.114855794580 04 |
| VELOCITY REL. TO MOON . . . . . . . | 0.465974357930 04 | 0.458039671050-01 | -0.504436194950 03 | 0.468696768730 04 |
| VIRTUAL MASS DATA | | | | |
| VIRTUAL MASS POSITION . . . . . . . | 8.37585336061D-02 | 2.05221656679D 05 | 0.00000000000D-39 | 2.05221656679D 05 |
| VIRTUAL MASS VELOCITY . . . . . . . | -1.96645816557D 03 | 5.47403109109D-04 | 0.00000000000D-39 | 1.96645816557D 03 |
| SPACECRAFT POS. REL. TO V.M. . . . | -8.81242798553D-01 | 1.15136683611D 03 | 1.08630203821D-01 | 1.15136717848D 03 |
| SPACECRAFT VEL. REL. TO V.M. . . . | 4.65971511284D 03 | 1.47097402218D 00 | -5.04436118946D 02 | 4.68694060280D 03 |
| KEPLER (ANG. MOM.) VECTOR. . . . . | -5.80791178084D 05 | 6.16552139361D 01 | -5.36504389432D 06 | 5.39638901343D 06 |
| ECCENTRICITY VECTOR . . . . . . . | -1.51785442138D-05 | 1.51153576275D 00 | 1.90137541074D-05 | 1.51153576265D 00 |
| V.M. MAGN. = 1.00705539998D 10 | | | | |
| V.M. MAGN. RATE = 5.27152254732D 03 | | | | |

STOPPING CONDITION--EXCEEDED MAXIMUM TRAJECTORY TIME

VIRTUAL MASS PROGRAM FOR COMPUTING SPACE TRAJECTORIES

PROBLEM 1 PAGE 19

| | X - COMP. | Y - COMP. | Z - COMP. | RESULTANT |
|---|---|---|---|---|
| TRAJECTORY TIME = 7.44000045547D 01 | | | | |
| **SPACECRAFT INERTIAL TRAJECTORY** | | | | |
| POSITION . . . . . . . . . . . . . . | 1.64062254532D 02 | 2.06358906721D 05 | -3.07912235034D 01 | 2.06358974236D 05 |
| VELOCITY . . . . . . . . . . . . . . | 2.63716455146D 03 | -4.55376737925D 02 | -4.98368994467D 02 | 2.72220067217D 03 |
| EPHEMERIS TIME = 1.63551241555D 02 | | | | |
| **EPHEMERIS DATA** | | | | |
| POSITION OF EARTH . . . . . . . . . | 1.48117085760D 00 | -2.52273385372D 03 | 0.00000000000D-39 | 2.52273428854D 03 |
| VELOCITY OF EARTH . . . . . . . . . | 2.41731339006D 01 | 1.41927541891D-02 | 0.00000000000D-39 | 2.41731380671D 01 |
| POSITION OF MOON . . . . . . . . . | -1.20493267666D 02 | 2.05224430339D 05 | 0.00000000000D-39 | 2.05224465711D 05 |
| VELOCITY OF MOON . . . . . . . . . | -1.96648474311D 03 | -1.15458072960D 00 | 0.00000000000D-39 | 1.96648508205D 03 |
| **SPACECRAFT RELATIVE TRAJECTORIES** | | | | |
| POSITION REL. TO EARTH . . . . . . . | 0.16258108367D 03 | 0.20888164058D 06 | -0.30791223503D 02 | 0.20888170612D 06 |
| VELOCITY REL. TO EARTH . . . . . . . | 0.26129914176D 04 | -0.45539093068D 03 | -0.49836899447D 03 | 0.26987917116D 04 |
| POSITION REL. TO MOON . . . . . . . | 0.28455552220D 03 | 0.11344763825D 04 | -0.30791223503D 02 | 0.11700241908D 04 |
| VELOCITY REL. TO MOON . . . . . . . | 0.46036492946D 04 | -0.45422215720D 03 | -0.49836899447D 03 | 0.46527708143D 04 |
| **VIRTUAL MASS DATA** | | | | |
| VIRTUAL MASS POSITION . . . . . . . | -1.20491523838D 02 | 2.05221460247D 05 | 0.00000000000D-39 | 2.05221495619D 05 |
| VIRTUAL MASS VELOCITY . . . . . . . | -1.96645317647D 03 | -6.44618502212D 00 | 0.00000000000D-39 | 1.96646374199D 03 |
| SPACECRAFT POS. REL. TO V.M. . . . | 2.83125202533D 02 | 1.13758543925D 03 | -3.06365716788D 01 | 1.17268883828D 03 |
| SPACECRAFT VEL. REL. TO V.M. . . . | 4.60416761938D 03 | -4.46743603741D 02 | -4.98428494741D 02 | 4.65256598870D 03 |
| KEPLER (ANG. MOM.) VECTOR. . . . . | -5.80691690561D 05 | 6.17572292164D 01 | -5.36411841694D 06 | 5.39545820429D 06 |
| ECCENTRICITY VECTOR . . . . . . . . | -3.48094948298D-03 | 1.51098547114D 00 | 3.94225584102D-04 | 1.51098953220D 00 |
| V.M. MAGN. = 1.00710286334D 10 | | | | |
| V.M. MAGN. RATE = 1.47648570654D 07 | | | | |

# VI. DETAILS OF THE COMPUTER PROGRAM

The detailed derivations of all the equations have been given in Chapters II and III. It is the purpose of this chapter, therefore, merely to facilitate the thorough understanding of how these equations have been implemented in the digital computer program, the FORTRAN listing of which appears in Section C of this chapter.

## A. FLOW DIAGRAMS

There are only two areas concerned with the basic computation procedure where the logic becomes at all involved. These are the MAIN program structure itself and the time of flight calculation within the subroutine VECTOR. The other computational subroutines are straightforward procedures for evaluating the equations as derived. The logic is somewhat complicated within the INPUT and PRINT subroutines to provide the very flexible operational features described in Chapter V, Section B. These subroutines, however, are not essential to the basic computational procedure of the program and hence will not be flow diagrammed here. Flow diagrams for the two sections mentioned above (MAIN and VECTOR) are shown in Figs. 16 and 17, respectively, with the equations written in the algebraic notation introduced earlier. The numbers appearing in the left-hand margins of the blocks are external formula numbers and can be correlated directly with the FORTRAN listing of the program in Section C. The titles appearing above some of the blocks correspond with the comments in the listings.

In the MAIN program sketched in Fig. 16, the indicated subroutines are as follows:

| Subroutine | Description |
|---|---|
| INPUT | In conjunction with other subroutines (DINPT, SPACE (LINES), NEWPGE) and with BLOCK DATA, reads in data, performs conversions and initialization calculations, and prints out the input data. Sets ITRAT = 3, KOUNT = 1. |
| EPHEM | Computes the position and velocity components of two gravitating bodies (in circular orbits) from the known ephemeris time, $t_{eph}$. |
| VMASS | Computes the position, magnitude, velocity and magnitude rate of the virtual mass for known positions and velocities of the spacecraft and gravitating bodies of known masses. |

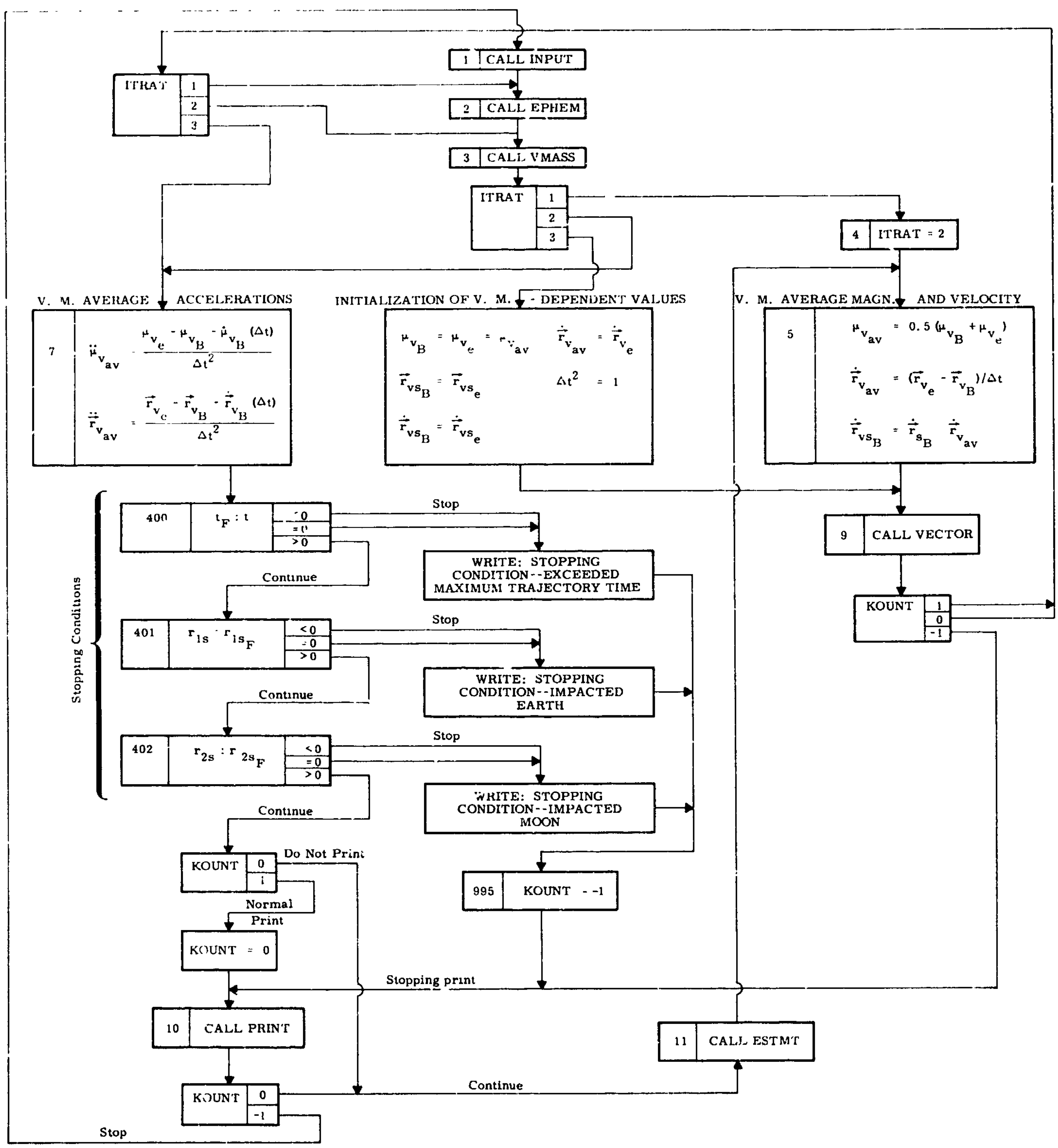


Fig. 16. Flow Diagram of MAIN Program

| Subroutine | Description |
|---|---|
| VECTOR | Calculates the vector orbital elements $\vec{k}$, $\vec{e}$, computes the spacecraft final position on the orbit to accurately approximate the desired time interval and then computes the conic section time of flight. |
| ESTMT | Updates final values of preceding computing interval to serve as initial values for new step (sets ITRAT = 1), determines desired size of time increment on basis of modified true anomaly, major axis crossing or requested print time (sets KOUNT = 1 or 0 depending upon whether regular print is indicated or not), and estimates the final position and magnitude of the virtual mass. |
| PRINT | In conjunction with subroutines SPACE (LINES) and NEWPGE, performs output conversions and prints out the requested data. |

The fixed-point variables ITRAT and KOUNT provide the program logic controls according to:

| Variable | Value | Action |
|---|---|---|
| ITRAT | 1 | First pass through computation cycle (including ephemeris) |
| ITRAT | 2 | Second and last pass through cycle (excluding ephemeris) |
| ITRAT | 3 | Initialization flag |
| KOUNT | -1 | Stopping flag |
| KOUNT | 0 | Continue normal computation |
| KOUNT | 1 | Print flag |

The subroutine VECTOR, shown in Fig. 17, contains two blocks with stars beneath the external formula numbers. This is intended to indicate that the details of the internal logic are not shown for the sake of brevity. In block 510, there are a number of tests to ensure that the argument of the arc sine does not exceed 1. by more than a specified tolerance for the elliptic case. If it does, a stopping condition (KOUNT = -1) is flagged, a return is made to the MAIN program and the logic paths will then terminate the problem. In addition, tests in the listing are not shown for proper quadrant determinations. These tests are straightforward implementations of the procedures described in Chapter III, Section A. Block 520 merely includes some logic to handle the special circumstance where the apocenter for the elliptic case is crossed and the uncorrected equations give a large negative flight time. This, too, is discussed in Chapter III, Section A.

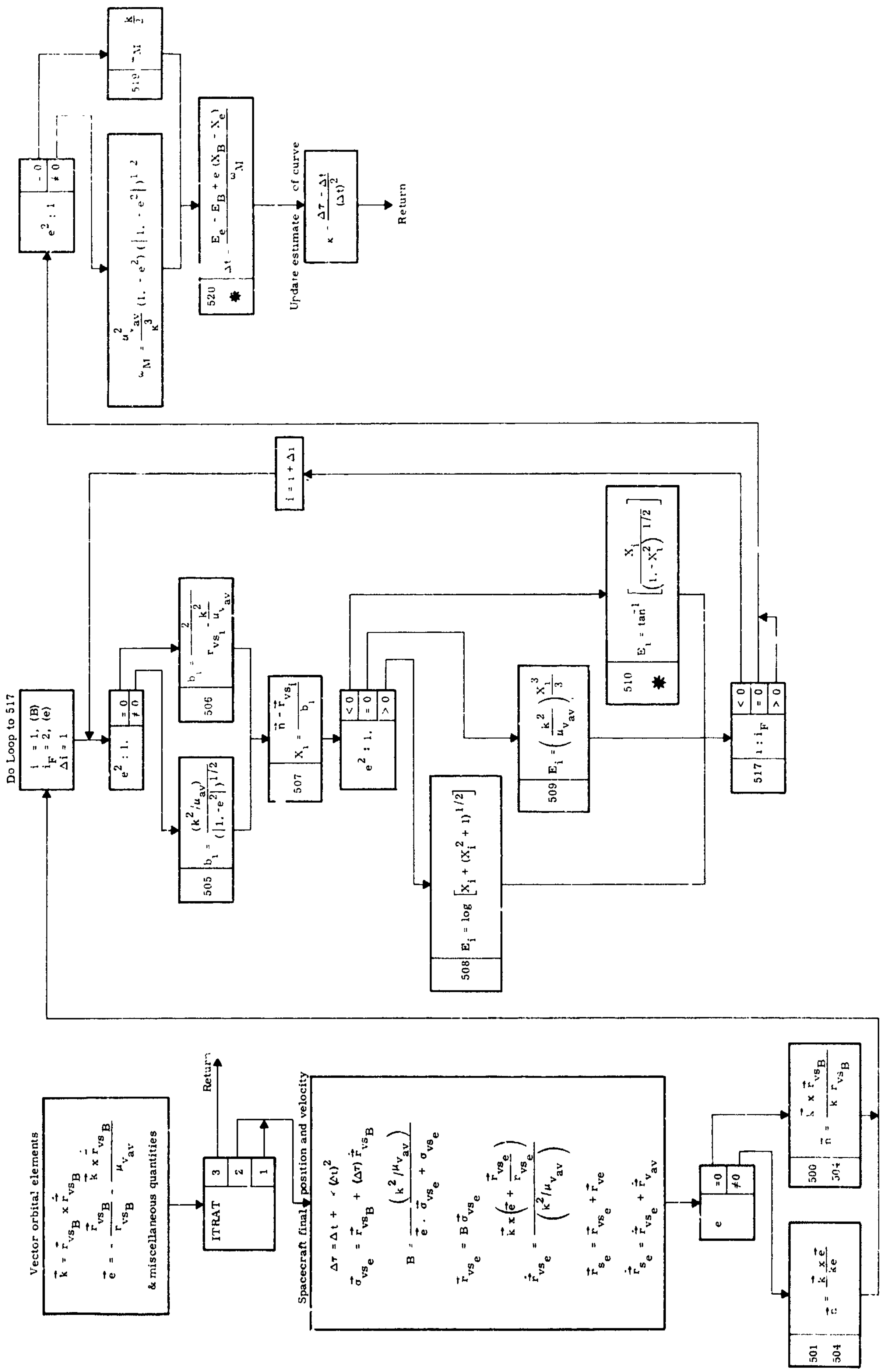


Fig. 17. Flow Diagram of Subroutine Vector

## B. ARRAY NOTATION

A glance at the FORTRAN listing in the following section reveals that nearly all the floating point variables have been written in array notation. This makes the job of following the listing all the more difficult. This difficulty is lessened considerably through the aid of Tables I and II in this section. These tables relate the locations in the F (I, J) and V (I, J) arrays to the corresponding algebraic variables--a set of equivalence statements as it were. This slight increase in complexity is deemed well justified on the basis of the conciseness of formulation it affords. With the establishment of appropriate DO loops, all three components of many of the vector expressions can be evaluated by single FORTRAN statements which are essentially identical with the standard vector forms. A testament to the compactness of the basic program is given in the fact that, without the elaborate input and output provisions, the program easily fits in an IBM 1620 computer.

Note that many of the locations in the V (I, J) array find multiple use throughout the program. No attempt has been made to optimize the arrangements and it is quite obvious that more efficient schemes are possible. Such an optimization of the program, though not of immediate interest here, would be required for most efficient machine coding for a computer onboard a spacecraft.

Note further that, although the F (I, J) array appears as an 8 x 4 matrix in Table II for the simple restricted three-body problem, it is dimensioned in the program as F (80, 4). Thus, it can accommodate as many many as twenty gravitating bodies with no change in that part of the program. As mentioned earlier, the subroutine VMASS is also completely general. The DO loops have been established to increment by 4 to the final value

NBODY = 4* NBODY -3

where NBODY on the right is stipulated as the number of gravitating bodies. In this program, NBODY = 2 is coded in the INPUT section, but that is the only statement which would have to be changed to consider more than two attractive masses.

# TABLE I

## V (I, J) Array

| I \ J | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|
| 1 | $(t_e)_{dim}, t_B$ | $(x_{s_e})_{dim}, x_{s_B}$ | $(y_{s_e})_{dim}, y_{s_B}$ | $(z_{s_e})_{dim}, z_{s_B}$ | $\omega$ (deg/t), $\omega$ (rad/t) | D | $\mu$, $-\mu$ |
| 2 | $t_e$ | $x_{s_e}$ | $y_{s_e}$ | $z_{s_e}$ | $(t_F)_{dim}, t_F$ | $(r_{1s_F})_{dim}, r_{1s_F}$ | $(r_{2s_F})_{dim}, r_{2s_F}$ |
| 3 | $(t_{eph_e})_{dim}, t_{eph_B}$ | $(\dot{x}_{s_e})_{dim}, \dot{x}_{s_B}$ | $(\dot{y}_{s_e})_{dim}, \dot{y}_{s_B}$ | $(\dot{z}_{s_e})_{dim}, \dot{z}_{s_B}$ | $(\Delta t_P)_{dim}, \Delta t_P$ | $C_2$ | |
| 4 | $t_{eph_e}$ | $\dot{x}_{s_e}$ | $\dot{y}_{s_e}$ | $\dot{z}_{s_e}$ | $\frac{\Delta r}{r}$ | | $\omega D$ (velocity) |
| 5 | $(\mu_{v_e})_{dim}, \mu_{v_B}$ | $(x_{v_e})_{dim}, x_{v_B}$ | $(y_{v_e})_{dim}, y_{v_B}$ | $(z_{v_e})_{dim}, z_{v_B}$ | $\omega D^2$ (area rate) | $\omega^2 D^2$ (velocity)$^2$ | $1 - \mu$ |
| 6 | $\mu_{v_e}$ | $M_x, x_{v_e}$ | $M_y, y_{v_e}$ | $M_z, z_{v_e}$ | $\omega^2 D^3$ (mass) | $\omega^3 D^3$ (mass rate) | $\kappa$, $\Delta\tau$ |
| 7 | $(\dot{\mu}_{v_e})_{dim}, \dot{\mu}_{v_B}$ | $(\dot{x}_{v_e})_{dim}, \dot{x}_{v_B}$ | $(\dot{y}_{v_e})_{dim}, \dot{y}_{v_B}$ | $(\dot{z}_{v_e})_{dim}, \dot{z}_{v_B}$ | $\Delta t_k$ | $\Delta t$ | $\mu_{v_{average}}$ |
| 8 | $\dot{\mu}_{v_e}$ | $\dot{M}_x, \dot{x}_{v_e}$ | $\dot{M}_y, \dot{y}_{v_e}$ | $\dot{M}_z, \dot{z}_{v_e}$ | $(\Delta t)^2$ | $\dot{\mu}_{v_{average}}$ | B, $\omega_M$ |
| 9 | $r_{vs_B}$ | $x_{vs_B}$ | $y_{vs_B}$ | $z_{vs_B}$ | $x_{vs_B}, (\sigma_{vs_e})_x$ | $y_{vs_B}, (\sigma_{vs_e})_y$ | $z_{vs_B}, (\sigma_{vs_e})_z$ |
| 10 | $r_{vs_e}$ | $x_{vs_e}$ | $y_{vs_e}$ | $z_{vs_e}$ | $\dot{x}_{v_{avg}}, \ddot{x}_{v_{avg}}$ | $\dot{y}_{v_{avg}}, \ddot{y}_{v_{avg}}$ | $\dot{z}_{v_{avg}}, \ddot{z}_{v_{avg}}$ |
| 11 | $V_{vs_B}$ | $\dot{x}_{vs_B}$ | $\dot{y}_{vs_B}$ | $\dot{z}_{vs_B}$ | $\dot{x}_{vs_B}, n_x$ | $\dot{y}_{vs_B}, n_y$ | $\dot{z}_{vs_B}, n_z$ |
| 12 | | $\dot{x}_{vs_e}, e_x + \frac{x_{vs_e}}{r_{vs_e}}$ | $\dot{y}_{vs_e}, e_y + \frac{y_{vs_e}}{r_{vs_e}}$ | $\dot{z}_{vs_e}, e_z + \frac{z_{vs_e}}{r_{vs_e}}$ | $M_s, e_x + \frac{x_{vs_e}}{r_{vs_e}}$ | $\dot{M}_s, e_y + \frac{y_{vs_e}}{r_{vs_e}}$ | $e_z + \frac{z_{vs_e}}{r_{vs_e}}$ |
| 13 | | | $t_p$ | $\Delta t_{MA}$ | $1. - e_e^2$ | $(\lvert 1. - e_e^2 \rvert)^{1/2}$ | $\frac{k^2}{\mu_{v_{avg}}}$ |
| 14 | $e_e$ | $e_{x_e}$ | $e_{y_e}$ | $e_{z_e}$ | $e_e^2, e_{x_e}$ | cos $(t_e)$, $e_{y_e}$ | sin $(t_e)$, $e_{z_e}$ |
| 15 | $(k)_{dim}$ | $(k_x)_{dim}$ | $(k_y)_{dim}$ | $(k_z)_{dim}$ | $b_B, X_B$ | $E_B$ | $a - r_{vs_B}$; or $\vec{r}_{vs_B} \cdot \vec{r}_{vs_B}$ |
| 16 | k | $k_x$ | $k_y$ | $k_z$ | $k_x, k_e^2, b_e, X_e$ | $k_y, E_e$ | $k_z, a - r_{vs_e}$; or $\vec{r}_{vs_e} \cdot \vec{r}_{vs_B}$ |

## TABLE II

F (I, J) Array

| I \ J | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | $x_1$ | $y_1$ | $z_1$ | $\mu_1$ |
| 2 | $\dot{x}_1$ | $\dot{y}_1$ | $\dot{z}_1$ | $\frac{3V_{1s}}{r_{1s}^2}$ |
| 3 | $x_{1s}$ | $y_{1s}$ | $z_{1s}$ | $r_{1s}$ |
| 4 | $\dot{x}_{1s}$ | $\dot{y}_{1s}$ | $\dot{z}_{1s}$ | $\frac{\mu_1}{r_{is}^3}$ |
| 5 | $x_2$ | $y_2$ | $z_2$ | $\mu_2$ |
| 6 | $\dot{x}_2$ | $\dot{y}_2$ | $\dot{z}_2$ | $\frac{3V_{2s}}{r_{2s}^2}$ |
| 7 | $x_{2s}$ | $y_{2s}$ | $z_{2s}$ | $V_{2s}$ |
| 8 | $\dot{x}_{2s}$ | $\dot{y}_{2s}$ | $\dot{z}_{2s}$ | $\frac{\mu_2}{r_{2s}^3}$ |

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## C. FORTRAN IV LISTING OF PROGRAM

```
$IBFTC MAIN     M94/2,XR7
      COMMON /COM/ V,            F,           PI,          RAD,          VI
      DOUBLE PRECISION  V(16,7),F(80,4),PI,RAD,VI(16)
      COMMON /COM/ ITRAT,KOUNT
      COMMON /COM/ NBODYI,    NBODY
      COMMON /COM/ IPRT(4),   IPRTI(4)
      COMMON /COM/ KL,        IPG,         LINCT,      LINPGE
      COMMON /COM/ NBLOCK
    1 CALL INPUT
    2 CALL EPHEM
    3 CALL VMASS
      IF(ITRAT .EQ. 1) GO TO 4
      IF(ITRAT .EQ. 2) GO TO 7
C INITIALIZATION OF VIRTUAL MASS-DEPENDENT VALUES
      V(7,7) = V(6,1)
      DO 600 J=2,4
      V(10,J+3)=V(8,J)
      V(9,J)=V(10,J)
  600 V(11,J)=V(12,J)
      V(9,1)=V(10,1)
      V(8,5)=1.
      V(5,1)=V(6,1)
      GO TO 9
    4 ITRAT = 2
C VIRTUAL MASS AVERAGE MAGNITUDE AND VELOCITY
    5 V(7,7)=.5*V(5,1)+.5*V(6,1)
      DO 390 J=2,4
      V(10,J+3)=(V(6,J)-V(5,J))/V(7,6)
  390 V(11,J)=V(3,J)-V(10,J+3)
    9 CALL VECTOR
      IF(KOUNT .LT. 0) GO TO 10
      IF(ITRAT .EQ. 1) GO TO 2
      IF(ITRAT .EQ. 2) GO TO 3
C VIRTUAL MASS AVERAGE ACCELERATIONS
    7 V(8,6)=(V(6,1)-V(5,1)-V(7,1)*V(7,6))/V(8,5)
      DO 340 J=2,4
  340 V(10,J+3)=(V(6,J)-V(5,J)-V(7,J)*V(7,6))/V(8,5)
C TEST FOR STOPPING CONDITIONS
  400 IF (V(2,5) .GT. V(2,1)) GO TO 401
      CALL SPACE (3)
      WRITE (6,4000)
 4000 FORMAT (//53H STOPPING CONDITION--EXCEEDED MAXIMUM TRAJECTORY TIME
     $)
      GO TO 995
  401 IF (F(3,4) .GT. V(2,6)) GO TO 402
      CALL SPACE (3)
```

```
      WRITE (6,4010)
 4010 FORMAT (//35H STOPPING CONDITION--IMPACTED EARTH)
      GO TO 995
  402 IF (F(7,4) .GT. V(2,7)) GO TO 403
      CALL SPACE (3)
      WRITE (6,4020)
 4020 FORMAT (//35H STOPPING CONDITION--IMPACTED MOON )
      GO TO 995
  403 CONTINUE
      IF(KOUNT .EQ. 0) GO TO 11
      KOUNT = 0
   10 CALL PRINT
      IF(KOUNT .LT. 0) GO TO 1
   11 CALL ESTMT
      GO TO 5
  995 KOUNT=-1
      GO TO 10
      END
```

```
$IBFTC INP      M94/2,XR7
      SUBROUTINE INPUT
      COMMON /COM/ V,          F,          PI,          RAD,          VI
      DOUBLE PRECISION  V(16,7),F(80,4),PI,RAD,VI(16)
      COMMON /COM/ ITRAT,KOUNT
      COMMON /COM/ NBODYI,    NBODY
      COMMON /COM/ IPRT(4),   IPRTI(4)
      COMMON /COM/ KL,        IPG,        LINCT,      LINPGE
      COMMON /COM/ NBLOCK
      COMMON /GCDIN/  ICARD(14)
      DIMENSION  INCHK(6),FMT(2),CRDTYP(2,6),APRT(4),AAPRT(2,4),IX(3)
      DIMENSION   NBLK(4)
      DOUBLE PRECISION WD(16)
      EQUIVALENCE (WPD,WD(1))
      DATA  INCHK /0,1,1,1,0,1 /
      DATA NBLK /10,11,9,12 /
      DATA  FMT(1) /12H(A24,4D24.0)/
      DATA CRDTYP (1,1) /12HPRINT       /
      DATA CRDTYP (1,2) /12HPOSITION    /
      DATA CRDTYP (1,3) /12HVELOCITY    /
      DATA CRDTYP (1,4) /12HEFEMERIS    /
      DATA CRDTYP (1,5) /12HACCURACY    /
      DATA CRDTYP (1,6) /12HSTOP        /
      DATA AAPRT /6H      ,6H      ,
     $            6H      ,6H     1 ,
     $            6H      ,6H     2 ,
     $            6H      ,6H     3  /
      DATA INERR /0/
      DATA PRINT    / 6HPRINT  /
      DATA POSITI   / 6HPOSITI /
      DATA VELOCI   / 6HVELOCI /
      DATA EFEMER   / 6HEFEMER /
      DATA ACCURA   / 6HACCURA /
      DATA STOP     / 6HSTOP   /
      DATA PROBLE   / 6HPROBLE /
      DATA IENTRY /0/
      IPG=0
      KL=-77777
    1 DO 10 I=1,16
      DO 10 J=1,7
   10 V(I,J)=0.
      DO 20 I=1,80
      DO 20 J=1,4
   20 F(I,J)=0.
      V( 3,5)=VI(1)
      V( 1,1)=VI(2)
      V( 1,2)=VI(3)
      V( 1,3)=VI(4)
      V( 1,4)=VI(5)
      V( 3,2)=VI(6)
```

```
      V( 3,3)=VI(7)
      V( 3,4)=VI(8)
      V( 3,1)=VI(9)
      V( 1,5)=VI(10)
      V( 1,6)=VI(11)
      V( 1,7)=VI(12)
      V( 4,5)=VI(13)
      V( 2,5)=VI(14)
      V( 2,6)=VI(15)
      V( 2,7)=VI(16)
      DO 30 I=1,4
   30 IPRT(I)=IPRTI(I)


  100 CALL DINPT (5,NOUT,FMT,WD)
      IF(WRD.EQ.PRINT ) GO TO 110
      IF(WRD.EQ.POSITI) GO TO 120
      IF(WRD.EQ.VELOCI) GO TO 130
      IF(WRD.EQ.EFEMER) GO TO 140
      IF(WRD.EQ.ACCURA) GO TO 150
      IF(WRD.EQ.STOP  ) GO TO 160
      IF(WRD.EQ.PROBLE) GO TO 170
      GO TO 500
  110 CONTINUE
      V(3,5)=WD(2)
      BACKSPACE 5
      READ (5,111) IX
  111 FORMAT (41X3I1)
      DO 113 I=2,4
  113 IPRT(I)=0
      DO 112 I=1,3
      ISUB=IX(I)
      IF (ISUB.EQ.0) GO TO 112
      IPRT(ISUB+1)=1
  112 CONTINUE
      INCHK(1)=0
      GO TO 100
  120 CONTINUE
      V(1,1)=WD(2)
      V(1,2)=WD(3)
      V(1,3)=WD(4)
      V(1,4)=WD(5)
      INCHK(2)=0
      GO TO 100
  130 CONTINUE
      V(3,2)=WD(2)
      V(3,3)=WD(3)
      V(3,4)=WD(4)
```

```
      INCHK(3)=0
      GO TO 100
  140 CONTINUE
      V(3,1)=WD(2)
      V(1,5)=WD(3)
      V(1,6)=WD(4)
      V(1,7)=WD(5)
      INCHK(4)=0
      GO TO 100
  150 CONTINUE
      V(4,5)=WD(2)
      INCHK(5)=C
      GO TO 100
  160 CONTINUE
      V(2,5)=WD(2)
      V(2,6)=WD(3)
      V(2,7)=WD(4)
      INCHK(6)=0
      GO TO 100
  170 CONTINUE
      KLT=WD(2)
      GO TO 600
  500 IF (INERR.NE.1) CALL NEWPGE
      CALL SPACE (5)
      WRITE (6,501) ICARD
  501 FORMAT (/56H AN ERROR HAS BEEN DETECTED READING THE FOLLOWING CARD
     $--/10X,13A6,A2/18H EXECUTION DELETED/1H )
      INERR=1


      GO TO 100
  600 IF (IENTRY.NE.0) GO TO 650
      DO 601 I=1,6
  601 IF (INCHK(I).NE.0) GO TO 602
      GO TO 649
  602 CALL NEWPGE
      WRITE (6,604)
  604 FORMAT (/91H THE FOLLOWING REQUIRED CARD TYPES WERE NOT INPUT ON T
     $HE FIRST PROBLEM -- EXECUTION DELETED)
      DO 605 I=1,6
      IF (INCHK(I).EQ.0) GO TO 605
      WRITE (6,606) (CRDTYP(J,I),J=1,2)
  606 FORMAT (10X,2A6)
  605 CONTINUE
      INERR=1
  649 IENTRY=1
      DO 610 I=1,4
  610 IPRTI(I)=IPRT(I)
```

```
      VI( 1)=V(3,5)
      VI( 2)=V(1,1)
      VI( 3)=V(1,2)
      VI( 4)=V(1,3)
      VI( 5)=V(1,4)
      VI( 6)=V(3,2)
      VI( 7)=V(3,3)
      VI( 8)=V(3,4)
      VI( 9)=V(3,1)
      VI(10)=V(1,5)
      VI(11)=V(1,6)
      VI(12)=V(1,7)
      VI(13)=V(4,5)
      VI(14)=V(2,5)
      VI(15)=V(2,6)
      VI(16)=V(2,7)
  650 IF (INERR.NE.0) GO TO 100
      NBODY=NBODYI
      NBLOCK=0
      DO 660 I=1,4
  660 IF (IPRT(I).NE.0) NBLOCK=NBLOCK+NBLK(I)
      CALL NEWPGE
      KL=KLT
      DO 651 I=1,4
      J=IPRT(I)+1
  651 APRT(I)=AAPRT(J,I)
      WRITE (6,652) V(3,5),
     $                  APRT,V(1,1),(V(1,J),J=2,4),(V(3,J),J=2,4),
     $                  V(3,1),(V(1,J),J=5,7),V(4,5),(V(2,J),J=5,7),KL
  652 FORMAT (12H PRINT        1PD20.11,6X4A6/
     $          12H POSITION    4D20.11/
     $          12H VELOCITY    3D20.11/
     $          12H EFEMERIS    4D20.11/
     $          12H ACCURACY     D20.11/
     $          12H STOP        3D20.11/
     $          12H PROBLEM     I5)
      CALL NEWPGE
C DIMENSIONAL CONVERSION FACTORS
      V(1,5)=V(1,5)/RAD
      V(4,7)=V(1,5)*V(1,6)
      V(5,5)=V(4,7)*V(1,6)
      V(5,6)=V(5,5)*V(1,5)
      V(6,5)=V(5,6)*V(1,6)
      V(6,6)=V(6,5)*V(1,5)
```

```
C EPHEMERIS DATA
      V(1,7)=-V(1,7)
      V(5,7)=1.+V(1,7)
      F(1,4)=V(5,7)
      F(5,4)=-V(1,7)
      NBODY = 2
      NBODY=4*NBODY-3
C NONDIMENSIONALIZATION
      DO 50 J=2,3
      V(J,5)=V(J,5)*V(1,5)
   50 V(2,J+4)=V(2,J+4)/V(1,6)
      V(2,1)=V(1,1)*V(1,5)
      V(4,1)=V(3,1)*V(1,5)
      DO 51 J=2,4
      V(2,J)=V(1,J)/V(1,6)
   51 V(4,J)=V(3,J)/V(4,7)
C INITIALIZATION OF MISCELANEOUS VALUES
      V(3,6)=DEXP(1.13756474179255 + .509713741462307*DLCG(V(4,5))
     $           +.145601812792780-2 * DLOG(V(4,5))**2 )
      ITRAT = 3
      KOUNT = 1
      V(13,3)=V(2,1)+V(3,5)
      RETURN
      END
```

```
$IBFTC EPH      XR7,M94/2
      SUBROUTINE EPHEM
      COMMON /COM/ V,          F,          PI,          RAD,          VI
      DOUBLE PRECISION  V(16,7),F(80,4),PI,RAD,VI(16)
      COMMON /COM/ ITRAT,KOUNT
      COMMON /COM/ NBODYI,   NBODY
      COMMON /COM/ IPRT(4),  IPRTI(4)
      COMMON /COM/ KL,       IPG,        LINCT,     LINPGE
      COMMON /COM/ NBLOCK
      V(14,7)=DSIN(V(4,1))
      V(14,6)=DCOS(V(4,1))
      DO 101 I=1,5,4
      DO 100 J=1,2
  100 F(I,J)=V(I,7)*V(14,J+5)
      F(I+1,1)=-F(I,2)
  101 F(I+1,2)=F(I,1)
      RETURN
      END
```

```
$IBFTC VMS      M94/2,XR7
      SUBROUTINE VMASS
      COMMON /COM/ V,          F,          PI,          RAD,          VI
      DOUBLE PRECISION  V(16,7),F(80,4),PI,RAD,VI(16)
      COMMON /COM/ ITRA ,KOUNT
      COMMON /COM/ NBODYI,   NBODY
      COMMON /COM/ IPRT(4),  IPRTI(4)
      COMMON /COM/ KL,          IPG,          LINCT,     LINPGE
      COMMON /COM/ NBLOCK
C VIRTUAL MASS POSITION AND MAGNITUDE
      V(12,5)=0.
      DO 201  I=1,NBODY,4
      DO 200  J=1,3
  200 F(I+2,J)=V(2,J+1)-F(I,J)
      F(I+2,4)=DSQRT(F(I+2,1)**2+F(I+2,2)**2+F(I+2,3)**2)
      F(I+3,4)=F(I,4)/F(I+2,4)**3
  201 V(12,5)=V(12,5)+F(I+3,4)
      DO 203  J=1,3
      V(6,J+1)=0.
      DO 202  I=1,NBODY,4
  202 V(6,J+1)=V(6,J+1)+F(I+3,4)*F(I,J)
      V(6,J+1)=V(6,J+1)/V(12,5)
  203 V(10,J+1)=V(2,J+1)-V(6,J+1)
      V(10,1)=DSQRT(V(10,2)**2+V(10,3)**2+V(10,4)**2)
      V(6,1)=V(10,1)**3*V(12,5)
C VIRTUAL MASS VELOCITY AND MAGNITUDE RATE
      V(12,6)=0.
      DO 301  I=1,NBODY,4
      DO 300  J=1,3
  300 F(I+3,J)=V(4,J+1)-F(I+1,J)
      F(I+1,4)=3.*(F(I+2,1)*F(I+3,1)+F(I+2,2)*F(I+3,2)+F(I+2,3)*F(I+3,3)
     1)/F(I+2,4)**2
  301 V(12,6)=V(12,6)-F(I+1,4)*F(I+3,4)
      DO 303  J=1,3
      V(8,J+1)=0.
      DO 302  I=1,NBODY,4
  302 V(8,J+1)=V(8,J+1)+F(I+3,4)*(F(I+1,J)-F(I,J)*F(I+1,4))
      V(8,J+1)=(V(8,J+1)-V(6,J+1)*V(12,6))/V(12,5)
  303 V(12,J+1)=V(4,J+1)-V(8,J+1)
      V(8,1)=V(6,1)*(3.*(V(10,2)*V(12,2)+V(10,3)*V(12,3)+V(10,4)*V(12,4)
     1)/V(10,1)**2+V(12,6)/V(12,5))
      RETURN
      END
```

```
$IBFTC VTR      XR7,M94/2
      SUBROUTINE VECTOR
      COMMON /COM/ V,          F,          PI,         RAD,        VI
      DOUBLE PRECISION  V(16,7),F(80,4),PI,RAD,VI(16)
      COMMON /COM/ ITRAT,KOUNT
      COMMON /COM/ NBODYI,   NBODY
      COMMON /COM/ IPRT(4),  IPRTI(4)
      COMMON /COM/ KL,       IPG,        LINCT,      LINPGE
      COMMON /COM/ NBLOCK
C VECTOR ORBITAL ELEMENTS
      DO 401  J=2,4
      DO 400  I=9,11,2
  400 V(I,J+3)=V(I,J)
  401 V(16,J)=V(9,J+1)*V(11,J+2)-V(9,J+2)*V(11,J+1)
      DO 403  J=2,4
      DO 402  I=11,16,5
  402 V(I,J+3)=V(I,J)
  403 V(14,J)=-V(9,J)/V(9,1)-(V(16,J+1)*V(11,J+2)-V(16,J+2)*V(11,J+1))/V
     1(7,7)
      DO 404  I=14,16,2
      V(I,5)=V(I,2)**2+V(I,3)**2+V(I,4)**2
  404 V(I,1)=DSQRT(V(I,5))
      V(13,5)=1.-V(14,5)
      V(13,6)=DSQRT(DABS(V(13,5)))
      V(13,7)=V(16,5)/V(7,7)
      IF  (ITRAT.EQ.3)  RETURN
C SPACECRAFT FINAL POSITION AND VELOCITY
      V(6,7)=V(7,6)+V(7,6)*V(7,6)*V(6,7)
      DO 410  J=2,4
  410 V(9,J+3)=V(9,J)+V(6,7)*V(11,J)
      V(8,7)=V(13,7)/(V(14,2)*V(9,5)+V(14,3)*V(9,6)+V(14,4)*V(9,7)+DSQRT
     1(V(9,5)**2+V(9,6)**2+V(9,7)**2))
      DO 411  J=2,4
  411 V(10,J)=V(8,7)*V(9,J+3)
      V(10,1)=DSQRT(V(10,2)**2+V(10,3)**2+V(10,4)**2)
      DO 414  J=2,4
  414 V(12,J)=V(14,J)+V(10,J)/V(10,1)
      DO 413  J=2,4
      DO 412  I=12,16,4
  412 V(I,J+3)=V(I,J)
      V(12,J)=(V(16,J+1)*V(12,J+2)-V(16,J+2)*V(12,J+1))/V(13,7)
      V(2,J)=V(10,J)+V(6,J)
  413 V(4,J)=V(12,J)+V(10,J+3)
C KEPLERIAN TIME OF FLIGHT
      IF(V(14,1) .NE. 0.) GO TO 501
  500 M=9
      GO TO 502
  501 M=14
  502 NN=16-M
      DO 504  J=2,4
      DO 503  I=M,16,NN
```

```
503 V(I,J+3)=V(I,J)
504 V(11,J+3)=(V(16,J+1)*V(M,J+2)-V(M,J+1)*V(16,J+2))/(V(16,1)*V(M,1))
    DO 517 I=9,10
    IF(V(13,5) .EQ. 0.) GO TO 506
505 V(I+6,5)=V(13,7)/V(13,6)
    GO TO 507
506 V(I+6,5)=2./(V(I,1)-V(13,7))
507 V(I+6,5)=(V(11,5)*V(I,2)+V(11,6)*V(I,3)+V(11,7)*V(I,4))/V(I+6,5)
    IF(V(13,5)) 508,509,510
508 V(I+6,6)=DLOG(V(I+6,5)+DSQRT(V(I+6,5)**2+1.))
    GO TO 517
509 V(I+6,6)=V(14,1)**2*(V(13,7)*V(I+6,5))**3/3.
    GO TO 517
510 IF(DABS(V(I+6,5)) .LT. 1.) GO TO 524
    IF (DABS(V(I+6,5)) .LE. 1.0001) GO TO 597
    KOUNT=-1
    CALL SPACE (2)
    WRITE (6,596)
596 FORMAT (/27H UNACCEPTABLE ERROR IN ATAN)
    RETURN
597 CALL SPACE (2)
    WRITE (6,598)
598 FORMAT (/27H   ACCEPTABLE ERROR IN ATAN)
    V(I+6,6)=DSIGN(PI/2.,V(I+6,5))
    GO TO 523
524 V(I+6,6)=DATAN(V(I+6,5)/DSQRT(1.-V(I+6,5)**2))
523 IF(V(14,1) .LT. 0.) GO TO 512
511 V(I+6,7)=V(I,2)*V(9,2)+V(I,3)*V(9,3)+V(I,4)*V(9,4)
    GO TO 513
512 V(I+6,7)=V(13,7)/V(13,5)-V(I,1)
513 IF(V(I+6,7) .GE. 0.) GO TO 517
    IF(V(I+6,5) .GE. 0.) GO TO 516
    V(I+6,6)=-PI-V(I+6,6)
    GO TO 517
516 V(I+6,6)=PI-V(I+6,6)
517 CONTINUE
    IF(V(13,5) .EQ. 0.) GO TO 519
    V(8,7)=V(7,7)**2/V(16,1)**3*V(13,5)*V(13,6)
    GO TO 520
519 V(8,7)=V(16,1)*V(14,1)**2/2.
520 V(7,5)=V(16,6)-V(15,6)+V(14,1)*(V(15,5)-V(16,5))
    IF(V(7,5) .GE. 0.) GO TO 522
    IF (V(8,7).LT.0.) GO TO 522
    V(7,5)=V(7,5)+2.*PI
522 V(7,5)=V(7,5)/V(8,7)
C UPDATE ESTIMATE OF CURVE
    IF (V(7,5).EQ.0.) GO TO 525
    V(6,7)=(V(6,7)-V(7,5))/V(7,5)**2
525 RETURN
    END
```

```
$IBFTC PRT     XR7,M94/2
      SUBROUTINE PRINT
      COMMON /COM/ V,          F,          PI,          RAD,        VI
      DOUBLE PRECISION  V(16,7),F(80,4),PI,RAD,VI(16)
      COMMON /COM/ ITRAT,KOUNT
      COMMON /COM/ NBODYI,   NBODY
      COMMON /COM/ IPRT(4),  IPRTI(4)
      COMMON /COM/ KL,        IPG,       LINCT,    LINPGE
      COMMON /COM/ NBLOCK
      DOUBLE PRECISION TEMP(16),RV,VV,RS,VS
      DO 351 J=2,4
      DO 350 I=1,5,4
      V(I,J)=V(1,6)*V(I+1,J)
      V(I+2,J)=V(4,7)*V(I+3,J)
      F(I,J-1)=V(1,6)*F(I,J-1)
  350 F(I+1,J-1)=V(4,7)*F(I+1,J-1)
  351 CONTINUE
      DO 352 J=1,4
  352 V(15,J)=V(5,5)*V(16,J)
      V(1,1)=V(2,1)/V(1,5)
      V(3,1)=V(4,1)/V(1,5)
      V(5,1)=V(6,1)*V(6,5)
      V(7,1)=V(8,1)*V(6,6)
      RV=DSQRT(V(5,2)**2+V(5,3)**2+V(5,4)**2)
      VV=DSQRT(V(7,2)**2+V(7,3)**2+V(7,4)**2)
      RS=DSQRT(V(1,2)**2+V(1,3)**2+V(1,4)**2)
      VS=DSQRT(V(3,2)**2+V(3,3)**2+V(3,4)**2)
  410 CALL SPACE (NBLOCK)
      WRITE (6,411) (V(1,I),I=1,4),RS,(V(3,I),I=2,4),VS
  411 FORMAT (/////20H TRAJECTORY TIME =  1PD20.11//
     $               40H SPACECRAFT INERTIAL TRAJECTORY         /
     $               40H     POSITION . . . . . . . . . . . . . . 4D20.11/
     $               40H     VELOCITY . . . . . . . . . . . . . . 4D20.11)
  420 IF (IPRT(2).EQ.0) GO TO 430
      TEMP( 1) = F(1,1)
      TEMP( 2) = F(1,2)
      TEMP(3 ) = F(1,3)
      TEMP(4) =DSQRT(TEMP(1)**2+TEMP(2)**2+TEMP(3)**2)
      TEMP( 5) = F(2,1)
      TEMP(6 ) = F(2,2)
      TEMP( 7) = F(2,3)
      TEMP(8) =DSQRT(TEMP(5)**2+TEMP(6)**2+TEMP(7)**2)
      TEMP( 9) = F(5,1)
      TEMP(10) = F(5,2)
      TEMP(11) = F(5,3)
      TEMP(12)=DSQRT(TEMP(9)**2+TEMP(10)**2+TEMP(11)**2)
      TEMP(13) = F(6,1)
      TEMP(14) = F(6,2)
      TEMP(15) = F(6,3)
      TEMP(16)=DSQRT(TEMP(13)**2+TEMP(14)**2+TEMP(15)**2)
      WRITE (6,421) V(3,1),(TEMP(I),I=1,16)
  421 FORMAT (///  20H EPHEMERIS TIME   =  1PD20.11//
     $               40H EPHEMERIS DATA                          /
     $               40H     POSITION OF EARTH . . . . . . . . . 4D20.11/
```

```
$                     40H      VELOCITY OF EARTH  . . . . . . . . . . 4D20.11//
$                     40H      POSITION OF MOON   . . . . . . . . . . 4D20.11/
$                     40H      VELOCITY OF MOON   . . . . . . . . . . 4D20.11)
430 IF (IPRT(3).EQ.0) GO TO 440
    TEMP ( 1) = V(1,2)-F(1,1)
    TEMP ( 2) = V(1,3)-F(1,2)
    TEMP ( 3) = V(1,4)-F(1,3)
    TEMP( 4)=DSQRT(TEMP(1)**2+TEMP(2)**2+TEMP(3)**2)



    TEMP ( 5) = V(3,2)-F(2,1)
    TEMP ( 6) = V(3,3)-F(2,2)
    TEMP ( 7) = V(3,4)-F(2,3)
    TEMP( 8)=DSQRT(TEMP(5)**2+TEMP(6)**2+TEMP(7)**2)
    TEMP (9 ) = V(1,2)-F(5,1)
    TEMP (10) = V(1,3)-F(5,2)
    TEMP (11) = V(1,4)-F(5,3)
    TEMP(12)=DSQRT(TEMP(9)**2+TEMP(10)**2+TEMP(11)**2)
    TEMP (13) = V(3,2)-F(6,1)
    TEMP (14) = V(3,3)-F(6,2)
    TEMP (15) = V(3,4)-F(6,3)
    TEMP(16)=DSQRT(TEMP(13)**2+TEMP(14)**2+TEMP(15)**2)
    WRITE (6,431) (TEMP(I),I=1,16)
431 FORMAT(///    40H SPACECRAFT RELATIVE TRAJECTORIES        /
$                     40H      POSITION REL. TO EARTH . . . . . . . 4D20.11/
$                     40H      VELOCITY REL. TO EARTH . . . . . . . 4D20.11//
$                     40H      POSITION REL. TO MOON  . . . . . . . 4D20.11/
$                     40H      VELOCITY REL. TO MOON  . . . . . . . 4D20.11)
440 IF (IPRT(4).EQ.0) RETURN
    TEMP(1)=V( 9,2)*V(1,6)
    TEMP(2)=V( 9,3)*V(1,6)
    TEMP(3)=V( 9,4)*V(1,6)
    TEMP(4)=V( 9,1)*V(1,6)
    TEMP(5)=V(11,2)*V(4,7)
    TEMP(6)=V(11,3)*V(4,7)
    TEMP(7)=V(11,4)*V(4,7)
    TEMP(8)=DSQRT(TEMP(5)**2+TEMP(6)**2+TEMP(7)**2)
    WRITE (6,441) (V(5,I),I=2,4),RV,(V(7,I),I=2,4),VV,
$       (TEMP(I),I=1,8),
$                     (V(15,I),I=2,4),V(15,1),(V(14,I),I=2,4),V(14,1),
$                     V(5,1),V(7,1)
441 FORMAT(///    20H VIRTUAL MASS DATA   /
$                     40H      VIRTUAL MASS POSITION  . . . . . . . 4(1PD20.11
$)/                   40H      VIRTUAL MASS VELOCITY  . . . . . . . 4D20.11/
$                     40H      SPACECRAFT POS. REL. TO V.M. . . . . 4D20.11/
$                     40H      SPACECRAFT VEL. REL. TO V.M. . . . . 4D20.11/
$                     40H      KEPLER (ANG. MOM.) VECTOR. . . . . . 4D20.11/
$                     40H      ECCENTRICITY VECTOR  . . . . . . . . 4D20.11/
$                     24H      V.M. MAGN.      =  D20.11/
$                     24H      V.M. MAGN. RATE =  D20.11)
    RETURN
    END
```

```
$IBFTC EST     XR7,M94/2
      SUBROUTINE ESTMT
      COMMON /COM/ V,          F,          PI,         RAD,        VI
      DOUBLE PRECISION  V(16,7),F(80,4),PI,RAD,VI(16)
      COMMON /COM/ ITRAT,KOUNT
      COMMON /COM/ NBODYI,   NBODY
      COMMON /COM/ IPRT(4),  IPRTI(4)
      COMMON /COM/ KL,       IPG,       LINCT,      LINPGE
      COMMON /COM/ NBLOCK
C INDEX VALUES IN V ARRAY
      DO 361 I=1,9,2
      DO 360 J=1,4
  360 V(I,J)=V(I+1,J)
  361 CONTINUE
      ITRAT=1
C ESTABLISH COMPUTING TIME INCREMENT
  395 V(13,4)=V(7,6)*V(16,5)/(V(15,5)-V(16,5))
      IF (V(7,6).EQ.0.) V(13,4)=-1.
      V(11,1)=DSQRT(V(11,2)**2+V(11,3)**2+V(11,4)**2)
      V(7,6)=V(3,6)*V(9,1)/V(11,1)
      IF(V(13,4) .LT. 0.) GO TO 394
      IF(V(13,4) .GT. 1.1*V(7,6)) GO TO 394
      V(7,6)=V(13,4)
      CALL SPACE (5)
      WRITE (6,6000)
 6000 FORMAT (//2X,19HMAJOR AXIS CROSSING//1H )
      GO TO 400
  394 IF(V(1,1)+1.1*V(7,6) .LT. V(13,3)) GO TO 378
  390 V(7,6)=V(13,3)-V(1,1)
      V(13,3)=V(13,3)+V(3,5)
  400 KOUNT = 1
C INCREMENT TIMES
  378 DO 379 I=1,3,2
  379 V(I+1,1)=V(I,1)+V(7,6)
      V(8,5)=V(7,6)**2
      IF (V(2,1).GE.V(13,3))  V(13,3)=V(13,3)+V(3,5)
C ESTIMATE VIRTUAL MASS FINAL POSITION AND MAGNITUDE
      V(6,1)=V(5,1)+V(7,1)*V(7,6)+V(8,6)*V(8,5)
      DO 380 J=2,4
  380 V(6,J)=V(5,J)+V(7,J)*V(7,6)+V(10,J+3)*V(8,5)
      RETURN
      END
```

```
$IBFTC BLKDAT  XR7,M94/2
      BLOCK DATA
      COMMON /COM/ V,        F,        PI,        RAD,        VI
      DOUBLE PRECISION  V(16,7),F(80,4),PI,RAD,VI(16)
      COMMON /COM/ ITRAT,KOUNT
      COMMON /COM/ NBODYI,   NBODY
      COMMON /COM/ IPRT(4),   IPRTI(4)
      COMMON /COM/ KL,        IPG,        LINCT,     LINPGE
      COMMON /COM/ NBLOCK
      DATA  PI  /3.141592653589793 /
      DATA  RAD /57.29577951308232 /
      DATA NBODYI  /     2        /
      DATA IPRTI   / 1,0,0,0      /
      DATA LINPGE/60/
      DATA VI / 5.D0, 11*0.D0, 1.D-7, 3*0.D0 /
      END
```

```
$IBFTC DIN      XR7,M94/2
      SUBROUTINE DINPT (X,XX,XXX,WD)
      DOUBLE PRECISION WD(5)
      COMMON/GCDIN/ ICARD(14)
      READ (5,1) ICARD
    1 FORMAT (13A6,A2)
      BACKSPACE 5
      READ (5,2) WD
    2 FORMAT (A6,2X,4D18.0)
      RETURN
      END
```

```
$IBFTC SPC      XR7,M94/2
      SUBROUTINE SPACE (LINES)
      COMMON /COM/ V,         F,          PI,         RAD,        VI
      DOUBLE PRECISION  V(16,7),F(80,4),PI,RAD,VI(16)
      COMMON /COM/ ITRAT,KOUNT
      COMMON /COM/ NBODYI,    NBODY
      COMMON /COM/ IPRT(4),   IPRTI(4)
      COMMON /COM/ KL,        IPG,        LINCT,      LINPGE
      COMMON /COM/ NBLOCK
      IF (LINPGE.LT.(LINCT+LINES)) CALL NEWPGE
      LINCT=LINCT+LINES
      RETURN
      END
```

```
$IBFTC NPG      XR7,M94/2
      SUBROUTINE NEWPGE
      COMMON /COM/ V,        F,        PI,        RAD,        VI
      DOUBLE PRECISION  V(16,7),F(80,4),PI,RAD,VI(16)
      COMMON /COM/ ITRAT,KOUNT
      COMMON /COM/ NBODYI,   NBODY
      COMMON /COM/ IPRT(4),   IPRTI(4)
      COMMON /COM/ KL,        IPG,       LINCT,     LINPGE
      COMMON /COM/ NBLOCK
      IPG=IPG+1
      WRITE (6,1)
    1 FORMAT                                                   (120H
     $1   V I R T U A L    M A S S    P R O G R A M    F O R    C O M P
     $U T I N G    S P A C E    T R A J E C T O R I E S       )
C
C        WHEN  KL = -77777, ONLY TITLE AND PAGE NUMBER ARE GIVEN, AS
C     THIS SIGNALS INPUT DIAGNOSTICS ARE TO BE GIVEN, OR INPUT DATA
C     IS TO BE LISTED.
C
      IF (KL.EQ.-77777) G. TO 10
      WRITE (6,2) KL,IPG
    2 FORMAT (90X8HPROBLEM I5,6X5HPAGE I4///
     $                                  40X80H          X - COMP.
     $    Y - COMP.              Z - COMP.          RESULTANT    )
      LINCT=6
      RETURN
   10 WRITE (6,3) IPG
    3 FORMAT (109X5HPAGE I4)
      LINCT=2
      RETURN
      END
```

# VII. CONCLUSIONS AND RECOMMENDATIONS

## A. CONCLUSIONS

The results of this study have shown that the virtual mass technique provides a practicable and very flexible method for solving the n-body problem. Identically the same computer program can be controlled by a single input to obtain approximate solutions very quickly, or highly accurate trajectories in a proportionately longer time. The sample circumlunar trajectory included in this report gives the spacecraft position accurate to within 0.02 naut mi at t = 70 hr (approximately 0.33 hr before pericynthion) in 160 sec on an IBM 7094 digital computer.

The use of rectangular coordinates and the formulation of the conic section relationships in terms of the vector orbital elements $\vec{k}$, $\vec{e}$ have resulted in a computationally compact program. Without the elaborate input-output provisions which have been incorporated to provide operational flexibility, the basic computational program easily fits in an IBM 1620 computer.

## B. RECOMMENDED FURTHER STUDIES

A number of further studies are suggested and will be listed here without elaboration.

(1) Derive analytical expressions for trajectory sensitivities from the simple conic section forms relative to the virtual mass. Use these to propagate the state transition matrix analytically.

(2) Study techniques for representing aspherical gravitational potentials by appropriate planet-fixed distributions of discrete point masses similar to the method of Ref. 4. Investigate the integration of such trajectories by the virtual mass technique.

(3) Study the problem of computing dynamically consistent trajectories (see Refs. 2 and 5) by investigating an extension of the virtual mass technique to compute the simultaneous trajectories followed by n gravitating bodies. The procedure would be to reduce the problem to a series of n two-body systems at every instant. Each two-body system would consist of a different one of the n real bodies and a corresponding fictitious body lumping the effects of all others on the one of immediate interest. The numerical computation accuracy would be controlled so as to conserve known integrals: energy, momentum and uniformity of motion of the center of mass.

(4) Develop an Encke-like procedure for computing low thrust trajectories. Here the thrust "perturbation" would be integrated separately and added as a correction to the reference gravitational trajectory relative to the virtual mass.

(5) Perform a general study of trajectories of the virtual mass to ascertain, if possible, the fundamental characteristics of its motion. Also try to find an analytical solution to the variable mass two-body problem.

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